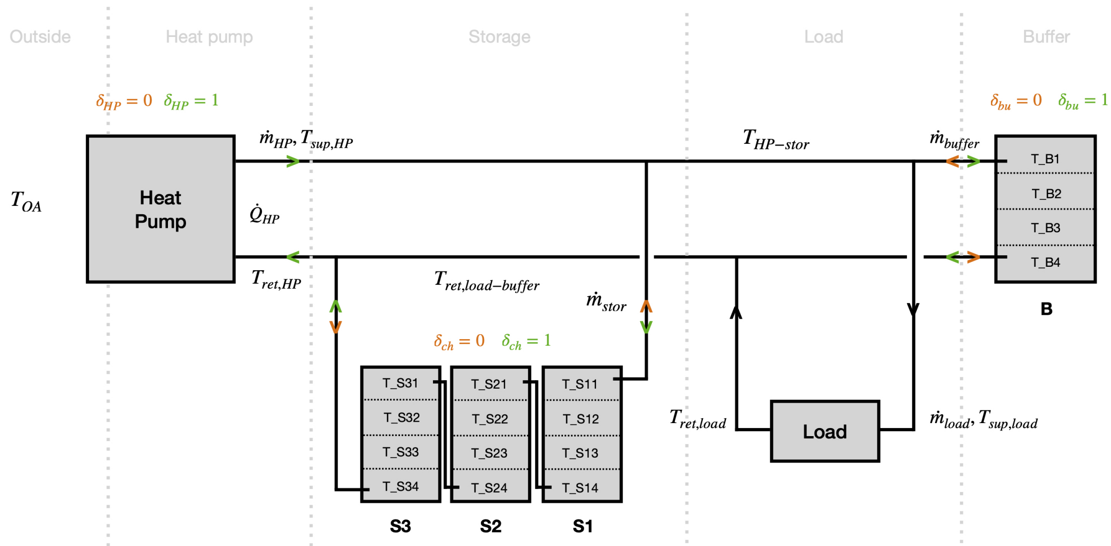
1 Gridworks' system



1.1 Variables

Horizon: $t = 0, 1, \dots, N$

Time step: $\Delta t_s = 300\ [s],\ \Delta t_m = 5\ [min],\ \Delta t_h = 1/12\ [h]$

State variables [K]:

$$x = (x(0), ..., x(N))$$
$$x(t) = (x_0(t), ..., x_{15}(t)) = (T_{B1}(t), \dots, T_{B4}(t), T_{S11}(t), T_{S12}(t), \dots, T_{S34}(t))$$

Input variables:

$$u = (u(0), ..., u(N-1))$$
$$u(t) = (u_0(t), \dots, u_5(t)) = (T_{sup,HP}(t), \dot{m}_{stor}(t), \delta_{ch}(t), \delta_{bu}(t), \delta_{HP}(t), \delta_R(t))$$

$T_{sup,HP}$ in [K], m_{stor} in [kg/s]

$\delta_{ch} = 1$ if charging the storage tanks, 0 otherwise

$\delta_{bu} = 1$ if charging the buffer tank, 0 otherwise

$\delta_{HP} = 1$ if the heat pump is on, 0 otherwise

$\delta_R = 1$ if using the resistive heating, 0 otherwise

(1)

1.2 Parameters

For $t = 0, \dots, N-1$, define $p(t) = (c_{el}(t), Q_{load}(t), T_{OA}(t), \dot{Q}_{HP,max}(t), COP1(t))$.

Electricity prices [$/Wh]:	$c_{el}(t)$	
Required load [Wh]:	$Q_{load}(t)$	
Outside air temperature [K]:	$T_{OA}(t)$	(2)
⇒ Maximum heat delivered [W]:	$\dot{Q}_{HP,max}(t)$	
⇒ 1/COP term [-]:	$COP1(t)$	

1.3 Objective

$$\min_{x,u} \sum_{t=0}^{N-1} c_{el}(t) \cdot (\dot{W}_{HP}(t) + \dot{W}_R(t)) \cdot \Delta t_h \tag{3}$$

$$\begin{aligned}
\dot{W}_{HP}(t) &= \dot{Q}_{HP}(t) \cdot \tfrac{1}{COP}(t) \\
&= \dot{m}_{HP} \cdot c_p \cdot (T_{sup,HP}(t) - T_{ret,HP}(t)) \cdot \delta_{HP} \cdot \tfrac{1}{COP}(t) \\
\dot{W}_R(t) &= \dot{Q}_{R,tot} \cdot \delta_R(t)
\end{aligned} \tag{4}$$

1.4 Constraints

Bounds

$$\begin{aligned}
T_{sup,HP} &\in [T_{sup,HP,min}, T_{sup,HP,max}] \\
\dot{m}_{stor} &\in [0, \dot{m}_{stor,max}] \\
\delta_{ch}, \delta_{bu}, \delta_{HP}, \delta_R &\in \{0, 1\} \\
T_{Bj}, T_{Sij} &\in [T_{w,min}, T_{w,max}]
\end{aligned} \tag{5}$$

Heat pump operation

$$\dot{Q}_{HP} \quad \in [\dot{Q}_{HP,min} \cdot \delta_{HP}(t), \dot{Q}_{HP,max}(t)] \tag{6}$$

Load supply temperature

The water going to the load must be hot enough.

$$T_{sup,load} \geq T_{sup,load,min} \tag{7}$$

Mass flow rates

$$\dot{m}_{load} = \frac{\dot{Q}_{load}(t)}{c_p \cdot \Delta T_{load}} = \frac{Q_{load}(t)}{\Delta t_h \cdot c_p \cdot \Delta T_{load}} \tag{8}$$

$$\dot{m}_{buffer} = [\dot{m}_{HP} \cdot \delta_{HP} - \dot{m}_{stor} \cdot (2\delta_{ch} - 1) - \dot{m}_{load}] \cdot \frac{1}{2\delta_{bu} - 1} \tag{9}$$

$$\dot{m}_{buffer} \geq 0 \tag{10}$$

Operation constraints

The storage can only be charging if the heat pump is on.

$$\delta_{ch} \leq \delta_{HP} \tag{11}$$

System dynamics

The system dynamics are further detailed in the next two sections, which address the buffer and storage tanks separately.

$$\begin{aligned}
x(0) &= x_{initial} \\
x(t+1) &= f(x(t), u(t), p(t))
\end{aligned} \tag{12}$$

1.5 Buffer tank

For every water layer $j = 1, .., 4$ in the buffer tank:
(change in energy stored in layer j) = (rate of energy transfer in) - (rate of energy transfer out)

$$\begin{aligned} m_j \cdot c_p \cdot \tfrac{dT_{Bj}}{dt} &= \dot{Q}_{in,j} - \dot{Q}_{out,j} \\ \rho A z \cdot c_p \cdot \tfrac{T_{Bj}(t+1) - T_{Bj}(t)}{\Delta t_s} &\approx \dot{Q}_{in,j} - \dot{Q}_{out,j} \end{aligned} \tag{13}$$

Re-arranging for $T_{B_j}(t+1)$ we get:

$$\Rightarrow T_{Bj}(t+1) = T_{Bj}(t) + \frac{\Delta t_s}{\rho A z \cdot c_p} \cdot (\dot{Q}_{in,j} - \dot{Q}_{out,j}) \tag{14}$$

$$\Rightarrow \begin{cases} T_{B1}(t+1) = T_{B1}(t) + \frac{\Delta t_s}{\rho A z \cdot c_p} \cdot (\dot{Q}_{conv,1} - \dot{Q}_{losses,1} + \dot{Q}_{Top,B}) \\ T_{B2}(t+1) = T_{B2}(t) + \frac{\Delta t_s}{\rho A z \cdot c_p} \cdot (\dot{Q}_{conv,2} - \dot{Q}_{losses,2}) \\ T_{B3}(t+1) = T_{B3}(t) + \frac{\Delta t_s}{\rho A z \cdot c_p} \cdot (\dot{Q}_{conv,3} - \dot{Q}_{losses,3}) \\ T_{B4}(t+1) = T_{B4}(t) + \frac{\Delta t_s}{\rho A z \cdot c_p} \cdot (\dot{Q}_{conv,4} - \dot{Q}_{losses,4} + \dot{Q}_{Bottom,B}) \end{cases} \tag{15}$$

1.5.1 Heat and mass transfer between layers $\dot{Q}_{conv,j}$

Layer j exchanges heat with the layer above it (j-1) and below it (j+1). The heat exchange is due to both natural convection and water flowing through the layers at mass flow rate $\dot{m}_{buffer}$.

$$\begin{aligned} \dot{Q}_{conv,1} &= (1-\delta_{bu}) \cdot (\dot{m}_{buffer} \cdot c_p \cdot (T_{B2} - T_{B1})) && + \delta_{bu} \cdot 0 \\ \dot{Q}_{conv,2} &= (1-\delta_{bu}) \cdot (\dot{m}_{buffer} \cdot c_p \cdot (T_{B3} - T_{B2})) && + \delta_{bu} \cdot (\dot{m}_{buffer} \cdot c_p \cdot (T_{B1} - T_{B2})) \\ \dot{Q}_{conv,3} &= (1-\delta_{bu}) \cdot (\dot{m}_{buffer} \cdot c_p \cdot (T_{B4} - T_{B3})) && + \delta_{bu} \cdot (\dot{m}_{buffer} \cdot c_p \cdot (T_{B2} - T_{B3})) \\ \dot{Q}_{conv,4} &= (1-\delta_{bu}) \cdot 0 && + \delta_{bu} \cdot (\dot{m}_{buffer} \cdot c_p \cdot (T_{B3} - T_{B4})) \end{aligned} \tag{16}$$

Also need to add free convection.

1.5.2 Water entering at top of buffer $\dot{Q}_{Top,B}$

$\delta_{bu} = 1$ when buffer is being charged (water entering top $\Rightarrow$ heat transfer to top) .
$\delta_{bu} = 0$ when buffer is being discharged (water exiting top $\Rightarrow$ no heat transfer to top).

$$\dot{Q}_{Top,B} = \delta_{bu} \cdot \dot{m}_{buffer} \cdot c_p \cdot (T_{HP-stor} - T_{B1}) \tag{17}$$

1.5.3 Water entering at bottom of buffer $\dot{Q}_{Bottom,B}$

$\delta_{bu} = 1$ when buffer is being charged (water exiting bottom $\Rightarrow$ no heat transfer to bottom).
$\delta_{bu} = 0$ when buffer is being discharged (water entering bottom $\Rightarrow$ heat transfer to bottom).

$$\dot{Q}_{Bottom,B} = (1-\delta_{bu}) \cdot \dot{m}_{buffer} \cdot c_p \cdot (T_{ret,load} - T_{B4}) \tag{18}$$

1.5.4 Losses $\dot{Q}_{losses,j}$

$$\dot{Q}_{losses} = 0 \text{ for now} \tag{19}$$

1.6 Storage tanks

For every water layer $j = 1, .., 4$ in every storage tank $i = 1, ..., 3$:
(change in energy stored in layer ij) = (rate of energy transfer in) - (rate of energy transfer out)

$$\begin{aligned} m_{ij} \cdot c_p \cdot \tfrac{dT_{Sij}}{dt} &= \dot{Q}_{in,ij} - \dot{Q}_{out,ij} \\ \rho Az \cdot c_p \cdot \tfrac{T_{Sij}(t+1) - T_{Sij}(t)}{\Delta t_s} &\approx \dot{Q}_{in,ij} - \dot{Q}_{out,ij} \end{aligned} \tag{20}$$

Re-arranging for $T_{Sij}(t+1)$ we get:

$$\Rightarrow T_{Sij}(t+1) = T_{Sij}(t) + \frac{\Delta t_s}{\rho Az \cdot c_p} \cdot (\dot{Q}_{in,ij} - \dot{Q}_{out,ij}) \tag{21}$$

$$\Rightarrow \begin{cases} T_{Si1}(t+1) = T_{Si1}(t) + \frac{\Delta t_s}{\rho Az \cdot c_p} \cdot (\dot{Q}_R + \dot{Q}_{conv,i1} - \dot{Q}_{losses,i1} + \dot{Q}_{Top,Si}) \\ T_{Bi2}(t+1) = T_{Si2}(t) + \frac{\Delta t_s}{\rho Az \cdot c_p} \cdot (\dot{Q}_R + \dot{Q}_{conv,i2} - \dot{Q}_{losses,i2}) \\ T_{Si3}(t+1) = T_{Si3}(t) + \frac{\Delta t_s}{\rho Az \cdot c_p} \cdot (\dot{Q}_R + \dot{Q}_{conv,i3} - \dot{Q}_{losses,i3}) \\ T_{Si4}(t+1) = T_{Si4}(t) + \frac{\Delta t_s}{\rho Az \cdot c_p} \cdot (\dot{Q}_R + \dot{Q}_{conv,i4} - \dot{Q}_{losses,i4} + \dot{Q}_{Bottom,Si}) \end{cases} \tag{22}$$

1.6.1 Heat and mass transfer between layers $\dot{Q}_{conv,ij}$

$$\begin{aligned} \dot{Q}_{conv,i1} &= (1-\delta_{ch}) \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si2} - T_{Si1})) & &+ \delta_{ch} \cdot 0 \\ \dot{Q}_{conv,i2} &= (1-\delta_{ch}) \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si3} - T_{Si2})) & &+ \delta_{ch} \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si1} - T_{Si2})) \\ \dot{Q}_{conv,i3} &= (1-\delta_{ch}) \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si4} - T_{Si3})) & &+ \delta_{ch} \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si2} - T_{Si3})) \\ \dot{Q}_{conv,i4} &= (1-\delta_{ch}) \cdot 0 & &+ \delta_{ch} \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si3} - T_{Si4})) \end{aligned} \tag{23}$$

Also need to add free convection.

1.6.2 Water entering top of storage tank i $\dot{Q}_{Top,Si}$

$\delta_{ch} = 1$ when storage is being charged (water entering at top $\Rightarrow$ heat transfer to top).
$\delta_{ch} = 0$ when storage is being discharged (water exiting at top $\Rightarrow$ no heat transfer to top).

$$\begin{aligned} \dot{Q}_{Top,S1} &= \delta_{ch} \cdot \dot{m}_{stor} \cdot c_p \cdot (T_{sup,HP} - T_{S11}) \\ \dot{Q}_{Top,S2} &= \delta_{ch} \cdot \dot{m}_{stor} \cdot c_p \cdot (T_{S14} - T_{S21}) \\ \dot{Q}_{Top,S3} &= \delta_{ch} \cdot \dot{m}_{stor} \cdot c_p \cdot (T_{S24} - T_{S31}) \end{aligned} \tag{24}$$

1.6.3 Water entering bottom of storage tank i $\dot{Q}_{Bottom,Si}$

$\delta_{ch} = 1$ when storage is being charged (water exiting at bottom $\Rightarrow$ no heat transfer to bottom).
$\delta_{ch} = 0$ when storage is being discharged (water entering at bottom $\Rightarrow$ heat transfer to bottom).

$$\begin{aligned} \dot{Q}_{Bottom,S1} &= (1-\delta_{ch}) \cdot \dot{m}_{stor} \cdot c_p \cdot (T_{S21} - T_{S14}) \\ \dot{Q}_{Bottom,S2} &= (1-\delta_{ch}) \cdot \dot{m}_{stor} \cdot c_p \cdot (T_{S31} - T_{S24}) \\ \dot{Q}_{Bottom,S3} &= (1-\delta_{ch}) \cdot \dot{m}_{stor} \cdot c_p \cdot (T_{ret,load-buffer} - T_{S34}) \end{aligned} \tag{25}$$

1.6.4 Electrical resistors $\dot{Q}_R$

$$\dot{Q}_R = \delta_R \cdot \dot{Q}_{res} \tag{26}$$

1.6.5 Losses $\dot{Q}_{losses,ij}$

$$\dot{Q}_{losses} = 0 \text{ for now} \tag{27}$$

1.7 Water in the circuit

1.7.1 Temperatures entering and exiting the load

Entering the load: mix of water from HP (if on), storage (if discharging) and buffer (is discharging).

$$T_{sup,load} = \frac{\dot{m}_{HP} \cdot T_{sup,HP} \cdot \delta_{HP} + \dot{m}_{stor} \cdot T_{S11} \cdot (1 - \delta_{ch}) + \dot{m}_{buffer} \cdot T_{B1} \cdot (1 - \delta_{bu})}{\dot{m}_{HP} \cdot \delta_{HP} + \dot{m}_{stor} \cdot (1 - \delta_{ch}) + \dot{m}_{buffer} \cdot (1 - \delta_{bu})} \tag{28}$$

Exiting the load: assumed constant change of temperature across the load ΔT_{load}.

$$T_{ret,load} = T_{sup,load} - \Delta T_{load} \tag{29}$$

1.7.2 Temperatures entering and exiting the heat pump

Leaving the HP: one of the input variables

$$T_{sup,HP} \tag{30}$$

Returning to the HP: mix water returning from load, storage (if charging), buffer (if charging).

$$T_{ret,HP} = \frac{\dot{m}_{load} \cdot T_{ret,load} + \dot{m}_{stor} \cdot T_{S34} \cdot \delta_{ch} + \dot{m}_{buffer} \cdot T_{B4} \cdot \delta_{bu}}{\dot{m}_{load} + \dot{m}_{stor} \cdot \delta_{ch} + \dot{m}_{buffer} \cdot \delta_{bu}} \tag{31}$$

1.7.3 Intermediate temperatures

Mixing water from HP (if on) and storage (if discharging)

$$T_{HP-stor} = \frac{\dot{m}_{HP} \cdot T_{sup,HP} \cdot \delta_{HP} + \dot{m}_{stor} \cdot T_{S11} \cdot (1 - \delta_{ch})}{\dot{m}_{HP} \cdot \delta_{HP} + \dot{m}_{stor} \cdot (1 - \delta_{ch})} \tag{32}$$

Mixing return from the load and buffer (if charging)

$$T_{ret,load-buffer} = \frac{\dot{m}_{load} \cdot T_{ret,load} + \dot{m}_{buffer} \cdot T_{B4} \cdot \delta_{bu}}{\dot{m}_{load} + \dot{m}_{buffer} \cdot \delta_{bu}} \tag{33}$$

1.7.4 Confronting errors

In certain combinations of the fixed binary variables $(\delta_{ch}, \delta_{HP}, \delta_{bu})$, some of the derivatives of the system dynamics equations with respect to input variables can produces NaN values, making the solver crash. This issue was resolved by adding small $\epsilon = 10^{-7}$ values to the denominators of such equations to avoid possible divisions by 0.