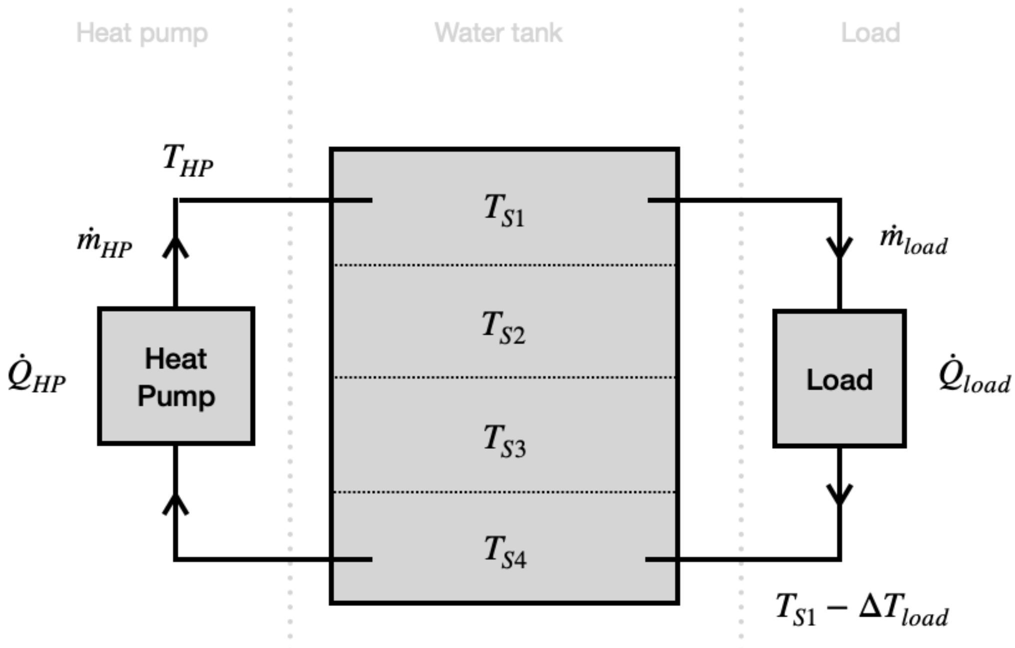
1 Setup



2 Variables

$$
\begin{aligned}
&\text{Time step:} && \Delta t_m = 5 \text{ [min]} \\
& && \Delta t_s = \Delta t_m \cdot 60 \text{ [s]} \\
& && \Delta t_h = \Delta t_m / 60 \text{ [hours]} \\
&\text{Horizon:} && t = 0, 1, \ldots, N \\
&\text{State variables [K]:} && x = (x(0), ..., x(N)) \\
& && x(t) = (x_1(t), ..., x_4(t)) = (T_{S1}(t), \ldots, T_{S4}(t)) \\
&\text{Input variable [K]:} && u = u(t) \\
& && u(t) = T_{HP}(t)
\end{aligned} \tag{1}
$$

3 Parameters

$$
\begin{aligned}
&\text{Electricity prices [\$/Wh]:} && c_{el} = (c_{el}(0), ..., c_{el}(N-1)) \\
&\text{Required load [W]:} && \dot{Q}_{load} = (\dot{Q}_{load}(0), ..., \dot{Q}_{load}(N-1))
\end{aligned} \tag{2}
$$

4 Objective

$$\min_{x,u,s} \sum_{t=0}^{N-1} c_{el}(t) \cdot \Delta t_h \cdot \dot{W}_{HP}(t) + c \cdot \dot{Q}_{load} \cdot s(t) \tag{3}$$

$$\Rightarrow \min_{x,u,s} \sum_{t=0}^{N-1} c_{el}(t) \cdot \Delta t_h \cdot \frac{\dot{m}_{HP} \cdot c_p \cdot (T_{HP}(t) - T_{S4}(t))}{COP} + c \cdot \dot{Q}_{load} \cdot s(t) \tag{4}$$

5 Constraints

$$\begin{array}{ll} \text{Initial state:} & x(0) = x_{initial} \\ \text{System dynamics:} & x(t+1) = f(x(t), u(t), p(t)) \\ & \\ \text{Storage water:} & T_{S1}(t), ..., T_{S4}(t) \in [T_{S,min}, T_{S,max}] \\ \text{Heat pump leaving water:} & T_{HP}(t) \in [T_{HP,min}, T_{HP,max}] \\ \text{Heat pump operation:} & \dot{Q}_{HP}(t) \in [\dot{Q}_{HP,min}, \dot{Q}_{HP,max}] \\ & \\ \text{Water supplied to the load :} & T_{S1}(t) \geq T_{sup,min} - s(t) \\ & s(t) \geq 0 \end{array} \tag{5}$$

6 System dynamics

For every water layer $i = 1, .., 4$ in the tank:

$$\begin{array}{ll} \text{(change in energy stored in layer i)} & = \text{(rate of energy transfer in) - (rate of energy transfer out)} \\ m_{layer} \cdot c_p \cdot \frac{dT_{Si}}{dt} & = \dot{Q}_{in,i} - \dot{Q}_{out,i} \\ m_{layer} \cdot c_p \cdot \frac{T_{Si}(t+1) - T_{Si}(t)}{\Delta t_s} & \approx \dot{Q}_{in,i} - \dot{Q}_{out,i} \end{array}$$

$$\Rightarrow \begin{cases} T_{S1}(t+1) = T_{S1}(t) + \frac{\Delta t_s}{m_{layer} \cdot c_p} \cdot (\dot{m}_{HP} \cdot c_p \cdot (T_{HP} - T_{S1}) + \dot{m}_{load} \cdot c_p \cdot (T_{S2} - T_{S1})) \\ T_{S2}(t+1) = T_{S2}(t) + \frac{\Delta t_s}{m_{layer} \cdot c_p} \cdot (\dot{m}_{HP} \cdot c_p \cdot (T_{S1} - T_{S2}) + \dot{m}_{load} \cdot c_p \cdot (T_{S3} - T_{S2})) \\ T_{S3}(t+1) = T_{S3}(t) + \frac{\Delta t_s}{m_{layer} \cdot c_p} \cdot (\dot{m}_{HP} \cdot c_p \cdot (T_{S2} - T_{S3}) + \dot{m}_{load} \cdot c_p \cdot (T_{S4} - T_{S3})) \\ T_{S4}(t+1) = T_{S3}(t) + \frac{\Delta t_s}{m_{layer} \cdot c_p} \cdot (\dot{m}_{HP} \cdot c_p \cdot (T_{S3} - T_{S4}) + \dot{m}_{load} \cdot c_p \cdot (T_{ret,load} - T_{S4})) \end{cases} \tag{6}$$

Where a constant temperature drop (ΔT_{load}) is assumed at the load:

$$\begin{array}{rl} T_{ret,load} & = T_{S1} - \Delta T_{load} \\ \dot{m}_{load} & = \frac{\dot{Q}_{load}}{c_p \cdot \Delta T_{load}} \end{array} \tag{7}$$