5 Model description

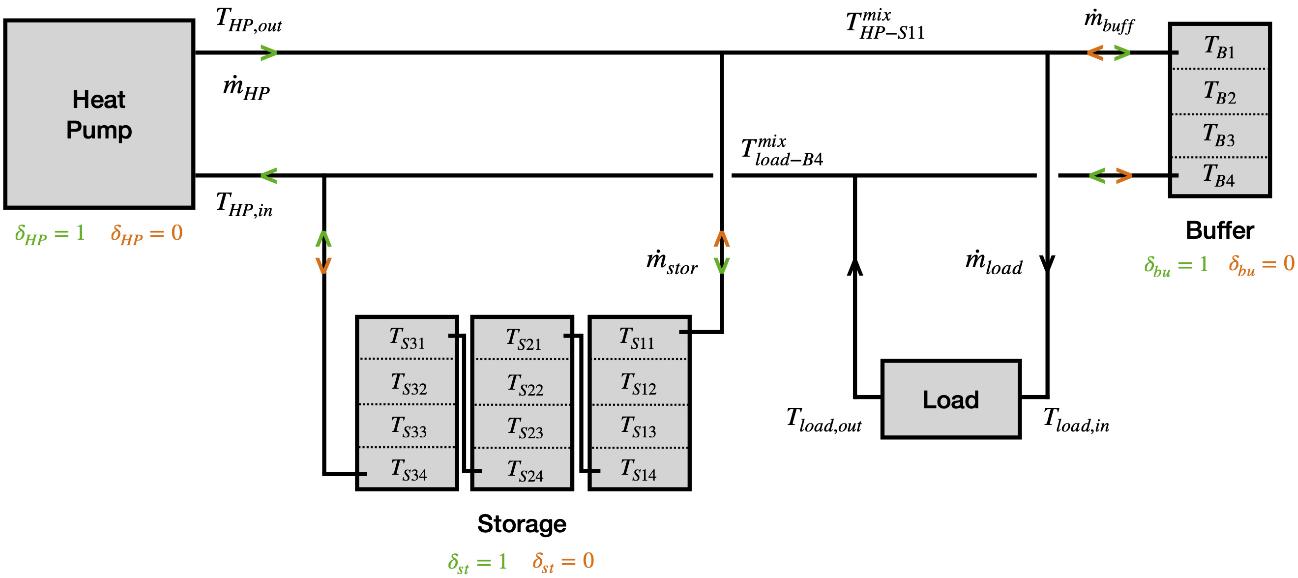


Fig. 1 – Overview of the system

5.1 Variables

5.1.1 Input variables

- Heat pump supply temperature: $T_{HP,sup}$
- Storage mass flow rate: $\dot{m}_{stor}$
- Operating mode: δ_{HP}, δ_{st}, δ_{bu}

5.1.2 State variables

- Buffer: T_{Bj} $\forall j = 1, ..., 4$
- Storage: T_{Sij} $\forall j = 1, ..., 4$ $\forall i = 1, ..., 3$

5.2 Intermediate variables

- Mass flow rate of buffer: $\dot{m}_{buff}$
- Temperature of mixed water from HP and storage: T^{mix}_{HP-S11}
- Temperature of water reaching the load: $T_{load,in}$
- Temperature of water exiting the load: $T_{load,out} = T_{load,in} - \overline{\Delta T_{load}}$
- Temperature of mixed water from load and buffer: $T^{mix}_{load-B4}$
- Temperature of water reaching the HP: $T_{HP,ret}$

5.3 Parameters

- Mass flow rate of load: $\dot{m}_{load}$

5.4 Heat pump

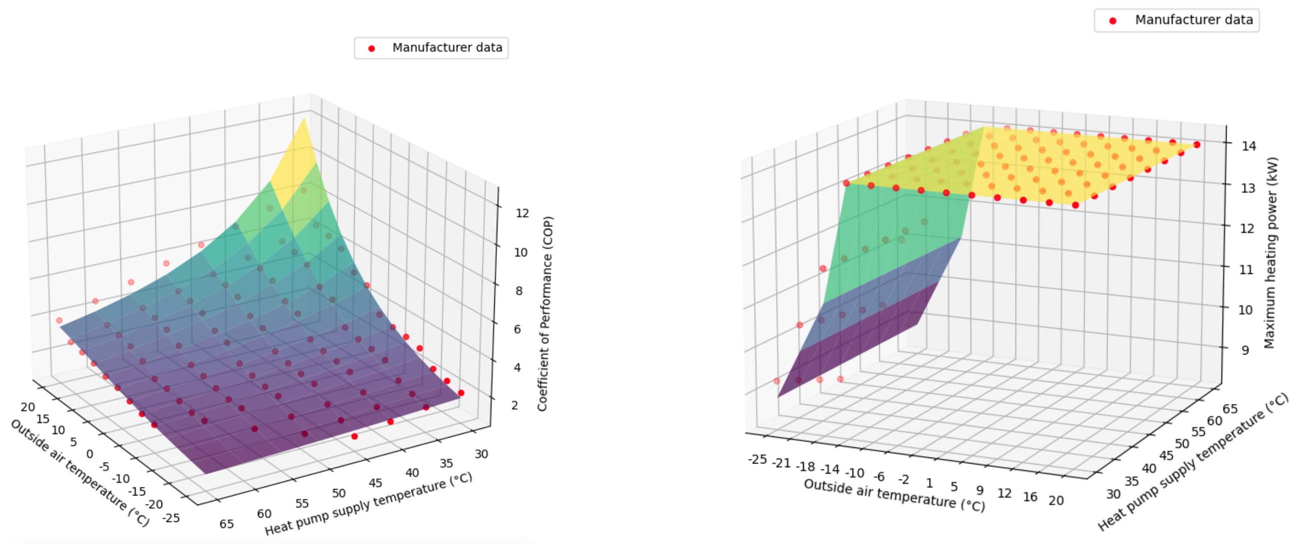
$$\dot{W}_{HP} = \frac{\dot{Q}_{HP}}{COP} \tag{1}$$

$$\dot{Q}_{HP} = \dot{m}_{HP} \cdot c_p \cdot (T_{HP,sup} - T_{HP,ret}) \cdot \delta_{HP} \tag{2}$$

$$COP \approx \beta_{0,C} + \beta_{1,C} \cdot T_{OA} + \beta_2 \cdot T_{OA}^2 + \beta_3 \cdot T_{HP,sup} \tag{3}$$

$$\dot{Q}_{HP,max} \approx \begin{cases} \beta_{0,Q} + \beta_{1,Q} \cdot T_{OA} & \text{if } T_{OA} < -7°\text{C} \\ 14\,kW & \text{otherwise} \end{cases} \tag{4}$$

$$T_{HP,ret} = \frac{\dot{m}_{load} \cdot T_{load,out} + \dot{m}_{stor} \cdot T_{S34} \cdot \delta_{st} + \dot{m}_{buff} \cdot T_{B4} \cdot \delta_{bu}}{\dot{m}_{load} + \dot{m}_{stor} \cdot \delta_{st} + \dot{m}_{buff} \cdot \delta_{bu}} \tag{5}$$



(a) Coefficient of Performance (COP)

(b) Maximum heating power $\dot{Q}_{HP,max}$

Fig. 2 – Estimating heat pump parameters based on manufacturer data

5.5 Load

$$\dot{m}_{load} = \frac{\dot{Q}_{load}}{c_p \cdot \overline{\Delta T_{load}}} = \frac{Q_{load}/\Delta t}{c_p \cdot \overline{\Delta T_{load}}} \tag{6}$$

$$T_{load,in} = \frac{\dot{m}_{HP} \cdot T_{HP,sup} \cdot \delta_{HP} + \dot{m}_{stor} \cdot T_{S11} \cdot (1 - \delta_{st}) + \dot{m}_{buff} \cdot T_{B1} \cdot (1 - \delta_{bu})}{\dot{m}_{HP} \cdot \delta_{HP} + \dot{m}_{stor} \cdot (1 - \delta_{st}) + \dot{m}_{buff} \cdot (1 - \delta_{bu})} \tag{7}$$

$$T_{load,out} = T_{load,in} - \overline{\Delta T_{load}} \tag{8}$$

5.6 Buffer tank

$$\dot{m}_{buff} = [\dot{m}_{HP} \cdot \delta_{HP} - \dot{m}_{stor} \cdot (2\delta_{st} - 1) - \dot{m}_{load}] \cdot \frac{1}{2\delta_{bu} - 1} \tag{9}$$

$$m_j c_p \frac{dT_{Bj}}{dt} = \dot{Q}_{in,j} - \dot{Q}_{out,j} = \begin{cases} \dot{Q}_{mix,1} - \dot{Q}_{loss,1} + \dot{Q}_{Top,B} & j = 1 \\ \dot{Q}_{mix,2} - \dot{Q}_{loss,2} & j = 2 \\ \dot{Q}_{mix,3} - \dot{Q}_{loss,3} & j = 3 \\ \dot{Q}_{mix,4} - \dot{Q}_{loss,4} + \dot{Q}_{Bottom,B} & j = 4 \end{cases} \tag{10}$$

5.6.1 Mixing between layers

$$\begin{aligned}
\dot{Q}_{mix,1} &= (1-\delta_{bu}) \cdot (\dot{m}_{buff} \cdot c_p \cdot (T_{B2} - T_{B1})) && +\delta_{bu} \cdot 0 \\
\dot{Q}_{mix,2} &= (1-\delta_{bu}) \cdot (\dot{m}_{buff} \cdot c_p \cdot (T_{B3} - T_{B2})) && +\delta_{bu} \cdot (\dot{m}_{buff} \cdot c_p \cdot (T_{B1} - T_{B2})) \\
\dot{Q}_{mix,3} &= (1-\delta_{bu}) \cdot (\dot{m}_{buff} \cdot c_p \cdot (T_{B4} - T_{B3})) && +\delta_{bu} \cdot (\dot{m}_{buff} \cdot c_p \cdot (T_{B2} - T_{B3})) \\
\dot{Q}_{mix,4} &= (1-\delta_{bu}) \cdot 0 && +\delta_{bu} \cdot (\dot{m}_{buff} \cdot c_p \cdot (T_{B3} - T_{B4}))
\end{aligned} \tag{11}$$

5.6.2 Top and bottom layers

$$\dot{Q}_{Top,B} = \dot{m}_{buff} \cdot c_p \cdot (T^{mix}_{HP-S11} - T_{B1}) \cdot \delta_{bu} \tag{12}$$

$$\dot{Q}_{Bottom,B} = \dot{m}_{buff} \cdot c_p \cdot (T_{load,out} - T_{B4}) \cdot (1-\delta_{bu}) \tag{13}$$

$$T^{mix}_{HP-S11} = \frac{\dot{m}_{HP} \cdot T_{HP,sup} \cdot \delta_{HP} + \dot{m}_{stor} \cdot T_{S11} \cdot (1-\delta_{st})}{\dot{m}_{HP} \cdot \delta_{HP} + \dot{m}_{stor} \cdot (1-\delta_{st})} \tag{14}$$

5.7 Storage tanks

$$m_j c_p \frac{dT_{Sij}}{dt} = \dot{Q}_{in,ij} - \dot{Q}_{out,ij} = \begin{cases} \dot{Q}_{mix,i1} - \dot{Q}_{loss,i1} + \dot{Q}_{Top,Si} & j=1 \\ \dot{Q}_{mix,i2} - \dot{Q}_{loss,i2} & j=2 \\ \dot{Q}_{mix,i3} - \dot{Q}_{loss,i3} & j=3 \\ \dot{Q}_{mix,i4} - \dot{Q}_{loss,i4} + \dot{Q}_{Bottom,Si} & j=4 \end{cases} \tag{15}$$

5.7.1 Mixing between layers

$$\begin{aligned}
\dot{Q}_{mix,i1} &= (1-\delta_{st}) \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si2} - T_{Si1})) && +\delta_{st} \cdot 0 \\
\dot{Q}_{mix,i2} &= (1-\delta_{st}) \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si3} - T_{Si2})) && +\delta_{st} \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si1} - T_{Si2})) \\
\dot{Q}_{mix,i3} &= (1-\delta_{st}) \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si4} - T_{Si3})) && +\delta_{st} \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si2} - T_{Si3})) \\
\dot{Q}_{mix,i4} &= (1-\delta_{st}) \cdot 0 && +\delta_{st} \cdot (\dot{m}_{stor} \cdot c_p \cdot (T_{Si3} - T_{Si4}))
\end{aligned} \tag{16}$$

5.7.2 Top and bottom layers

$$\begin{aligned}
\dot{Q}_{Top,S1} &= \dot{m}_{stor} \cdot c_p \cdot (T_{HP,sup} - T_{S11}) \cdot \delta_{st} \\
\dot{Q}_{Top,S2} &= \dot{m}_{stor} \cdot c_p \cdot (T_{S14} - T_{S21}) \cdot \delta_{st} \\
\dot{Q}_{Top,S3} &= \dot{m}_{stor} \cdot c_p \cdot (T_{S24} - T_{S31}) \cdot \delta_{st}
\end{aligned} \tag{17}$$

$$\begin{aligned}
\dot{Q}_{Bottom,S1} &= \dot{m}_{stor} \cdot c_p \cdot (T_{S21} - T_{S14}) \cdot (1-\delta_{st}) \\
\dot{Q}_{Bottom,S2} &= \dot{m}_{stor} \cdot c_p \cdot (T_{S31} - T_{S24}) \cdot (1-\delta_{st}) \\
\dot{Q}_{Bottom,S3} &= \dot{m}_{stor} \cdot c_p \cdot (T^{mix}_{load-B4} - T_{S34}) \cdot (1-\delta_{st})
\end{aligned} \tag{18}$$

$$T^{mix}_{load-B4} = \frac{\dot{m}_{load} \cdot T_{load,out} + \dot{m}_{buff} \cdot T_{B4} \cdot \delta_{bu}}{\dot{m}_{load} + \dot{m}_{buff} \cdot \delta_{bu}} \tag{19}$$

6 Optimal Control Problem Formulation

6.1 MINLP formulation

$$\min_{x,u,p} \int_0^{N-1} c_{el}(t) \cdot \dot{W}_{HP}(t)\, dt \tag{20}$$

$$\begin{aligned}
\text{s.t. } x(0) &= x_0 \\
\dot{x}(t) &= f(x(t), u(t), p(t))
\end{aligned}$$

$$\begin{aligned}
\dot{Q}_{HP}(t) &\in [\dot{Q}_{HP,min}(t) \cdot \delta_{HP}(t), \dot{Q}_{HP,max}(t)] \\
T_{load,in}(t) &\geq T_{load,in}^{min} \\
\dot{m}_{HP} \cdot \delta_{HP}(t) &\geq \dot{m}_{stor}(t) \cdot (2\delta_{st}(t) - 1) + \dot{m}_{load}(t)
\end{aligned} \tag{21}$$

$$\begin{aligned}
T_{HP,sup} &\in [T_{HP,sup}^{min}, T_{HP,sup}^{max}]^{N-1} \\
\dot{m}_{stor} &\in [0, \dot{m}_{stor}^{max}]^{N-1} \\
\delta_{ch}, \delta_{bu}, \delta_{HP} &\in \{0,1\}^{N-1} \\
T_{Bj}, T_{Sij} &\in [T_{tanks}^{min}, T_{tanks}^{max}]^N \ \forall i, j
\end{aligned}$$

6.2 Scheduling MILP

$$\min_{x',u',p'} \sum_{t=0}^{N-1} c_{el}(t) \cdot \frac{Q_{HP}^{on-off}(t)}{COP(t)} \tag{22}$$

$$\begin{aligned}
\text{s.t. } Q_{stor}(0) &= Q_{stor,0} \\
Q_{stor}(t+1) &= Q_{stor}(t) + Q_{HP}(t) - Q_{load}(t)
\end{aligned}$$

$$\begin{aligned}
Q_{HP,min} \cdot \delta_{HP} &\leq Q_{HP}^{on-off} \leq Q_{HP,max} \cdot \delta_{HP} \\
Q_{HP} + Q_{HP,max} \cdot (\delta_{HP} - 1) &\leq Q_{HP}^{on-off} \leq Q_{HP} + Q_{HP,min} \cdot (\delta_{HP} - 1)
\end{aligned} \tag{23}$$

$$\begin{aligned}
Q_{stor}(t) &\in [Q_{stor}^{min}, Q_{stor}^{max}]^N \\
Q_{HP}(t) &\in [Q_{HP}^{min}(t) \cdot \delta_{HP}(t), Q_{HP}^{max}(t)]^{N-1}
\end{aligned}$$

Explain how storage is calculated (see notebook).