Planning, Learning and Intelligent Decision Making

2022/2023

Homework 1 - Group 25

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Introduction

This project was developed for the course Planning, Learning and Intelligent Decision Making taught at Instituto Superior Técnico under the professor [Francisco Saraiva de Melo](#).

Exercise 1

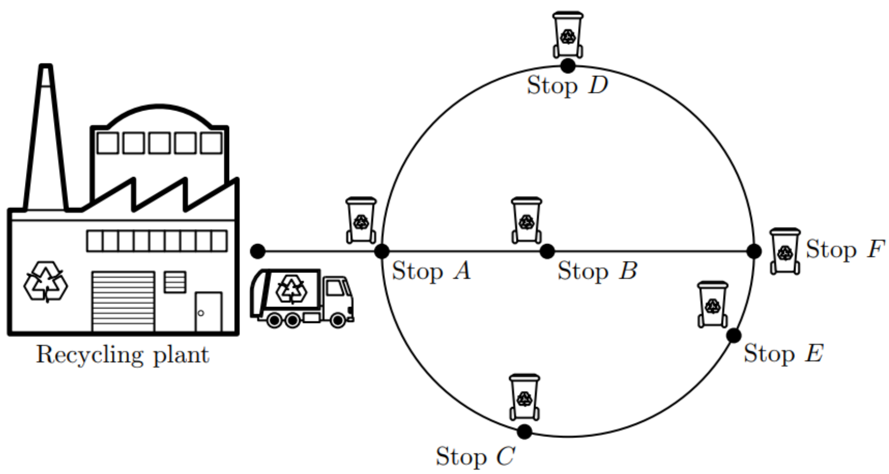
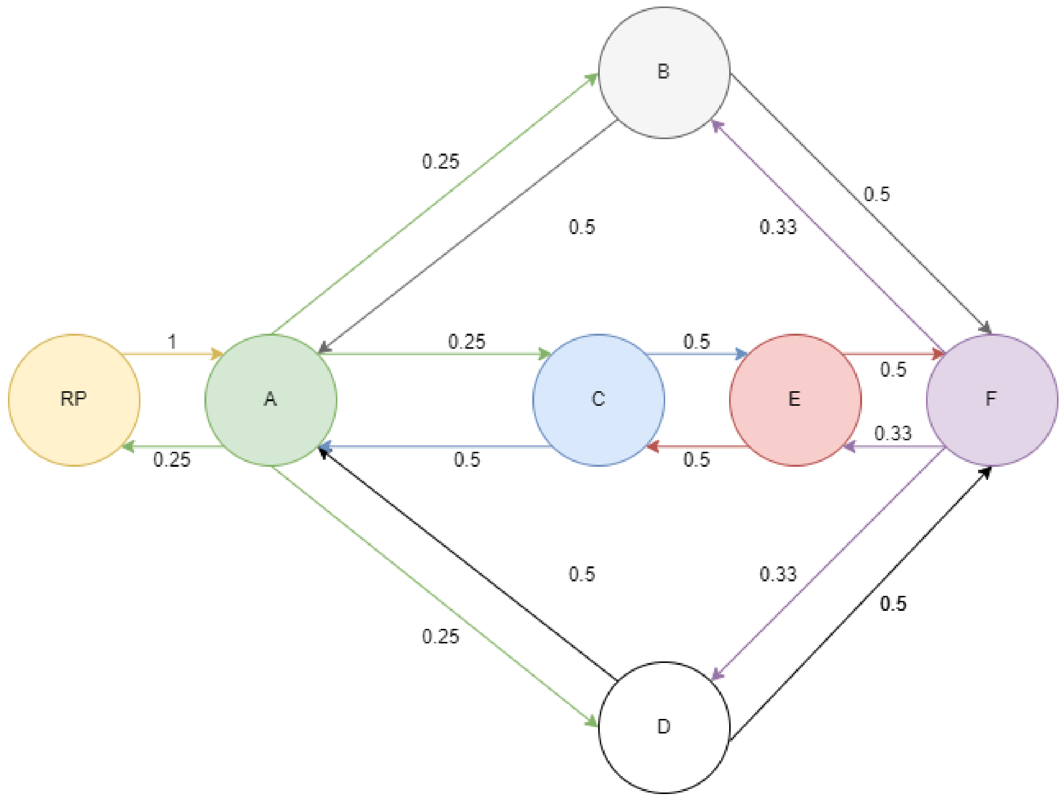


Figure 1: The garbage truck visits the different sites to empty the garbage containers before returning to the recycling plant.

(a) The motion of the truck can be represented by the following Markov chain model, where RP is the recycling plant and the state space is X = {RP, A, B, C, D, E, F}:



The transition probability matrix, *P*, is as follows, with the order of the columns/rows from up to bottom, left to right, being RC, A, B, C, D, E and F:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1/3 & 0 \end{bmatrix}$$

(b) Considering the truck is in the recycling plant at time step t = 0, the probability of the truck being in each stop at time step t = 2 is the first row of the transition probability matrix *P*, raised to the power of 2:

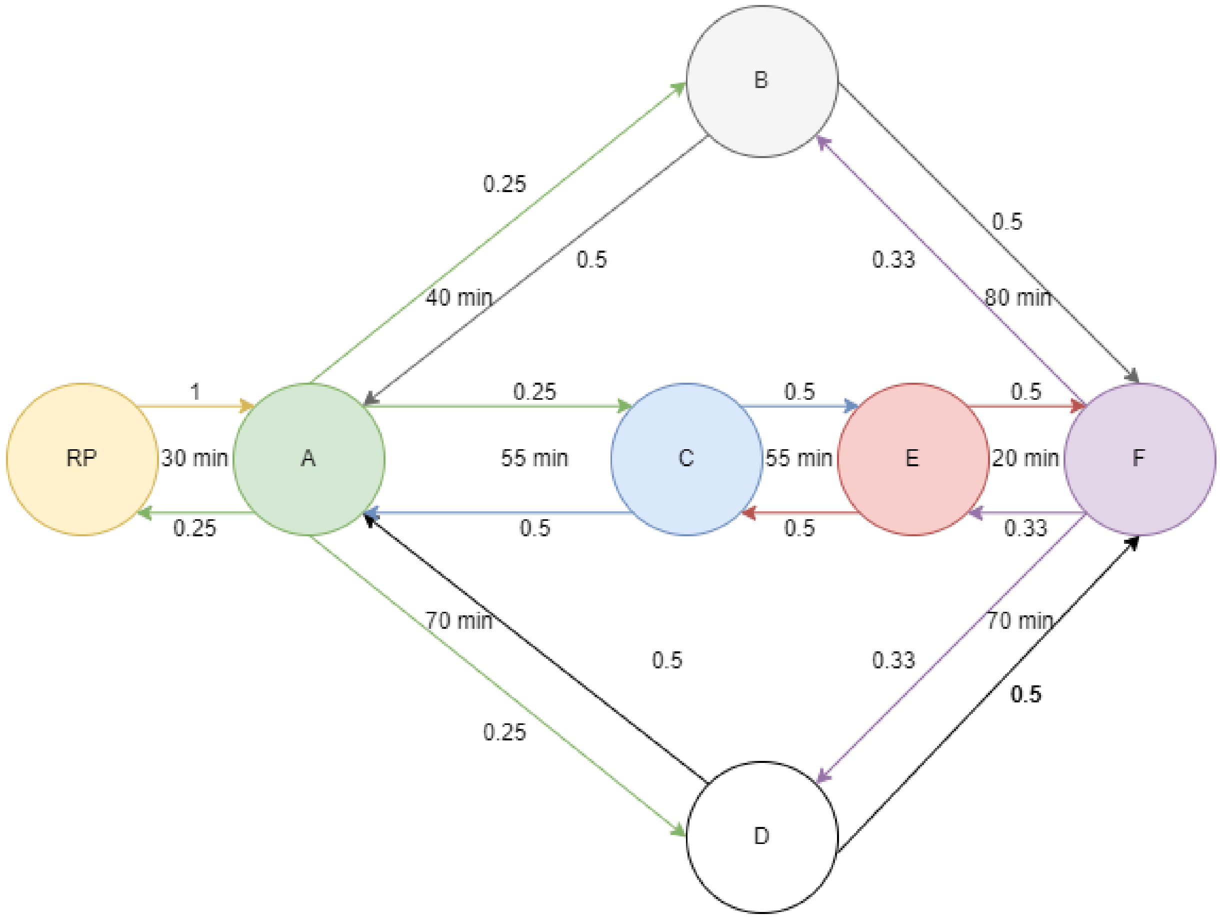
$$P^2 = \begin{bmatrix} 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0.625 & 0 & 0 & 0 & 0.125 & 0.25 \\ 0.125 & 0 & 0.292 & 0.125 & 0.292 & 0.167 & 0 \\ 0.125 & 0 & 0.125 & 0.375 & 0.125 & 0 & 0.25 \\ 0.125 & 0 & 0.292 & 0.125 & 0.292 & 0.167 & 0 \\ 0 & 0.25 & 0.167 & 0 & 0.167 & 0.417 & 0 \\ 0 & 0.33 & 0 & 0.167 & 0 & 0 & 0.5 \end{bmatrix}$$

Therefore, the probability of the truck being:

- in the recycling plant (RP) = 0.25
- in the stop B = 0.25
- in the stop C = 0.25
- in the stop D = 0.25
- in any other stop = 0

These results can also be easily verified in the diagram presented in a) .

(c) For the sake of convenience, the updated diagram presented in a) is as follows:



In order to calculate the total expected return time to RP, we should consider the transition probabilities and the time taken traveling between each pair of states by using the expected time, E_x (where X is a generic state):

$E_{RP} = 30 + E_A$

$E_A = 0{,}25(E_B + 40) + 0{,}25\ (E_C + 55) + 0{,}25(E_D + 70) + 0{,}25(30)$

$E_B = 0{,}5(E_F + 80) + 0{,}5(E_A + 40)$

$E_F = 0.33(E_B + 80) + 0{,}33(E_E + 20) + 0.33(E_D + 70)$

$E_D = 0{,}5(E_F + 70) + 0{,}5(E_A + 70)$

$E_E = 0{,}5(E_F + 20) + 0{,}5(E_C + 55)$

$E_C = 0{,}5(E_E + 55) + 0{,}5(E_A + 55)$

Let us now solve this system of linear equations (we used *QuickMath.com* to make the calculations quicker):

$$\begin{cases} a = 0.25\ (b+40) + 0.25\ (c+55) + 0.25\ (d+70) + 0.25 \cdot 30 \\ b = 0.5\ (f+80) + 0.5\ (a+40) \\ c = 0.5\ (e+55) + 0.5\ (a+55) \\ d = 0.5\ (f+70) + 0.5\ (a+70) \\ e = 0.5\ (f+20) + 0.5\ (c+55) \\ f = \dfrac{1}{3}\ (b+80) + \dfrac{1}{3}\ (e+20) + \dfrac{1}{3}\ (d+70) \end{cases}$$

$$\begin{cases} a = 810 \\ b = 1015 \\ c = 1005 \\ d = 1025 \\ e = 1090 \\ f = 1100 \end{cases}$$

From these results, we can reach E_{RP} = 30 + E_A = 30 + a = 840 minutes, which in turn corresponds to 14 hours. That being said, considering the truck departs from the Recycling Plant at 10 am on a Monday, we can infer it is expected to return at midnight of the same day (aka, 12pm).

References

[1] - De Melo, F. et al. (2023) Theoretical Lectures, Planning, Learning and Intelligent Decision Making. Instituto Superior Técnico (IST). Available at: https://fenix.tecnico.ulisboa.pt/disciplinas/ADI/2022-2023/2-semestre/material-de-apoio (Accessed: March 4, 2023).