

## Homework 1. Markov chains

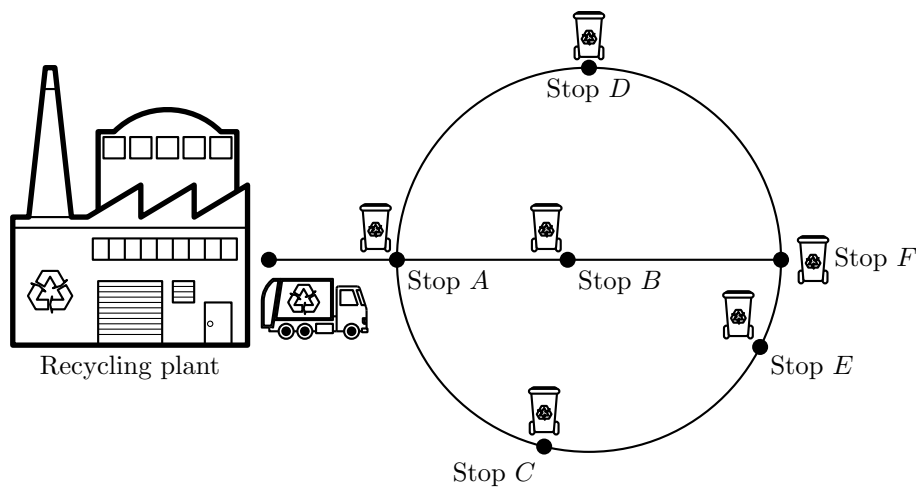


Figure 1: The garbage truck visits the different sites to empty the garbage containers before returning to the recycling plant.

Consider the diagram in Fig. 1. The building on the left corresponds to a recycling plant. A truck leaves the recycling plant and traverses the road network depicted in the diagram. At each intersection, the truck randomly chooses one of the roads in the intersection (including the one by which it arrived) with equal probability.

### Exercise 1.

- Write down the Markov chain model (state space and transition probabilities) describing the motion of the truck. Consider that a new time step  $t$  takes place whenever the truck arrives at one of the seven dotted locations (Recycling plant and stops A to F).
- Suppose that the truck is in the recycling plant at time step  $t = 0$ . Compute the probability of the truck being in each stop at time step  $t = 2$ .

(c) Suppose that the truck takes:

- 20 minutes to travel between stops  $E$  and  $F$ ;
- 30 minutes to travel between the recycling plant and stop  $A$ ;
- 40 minutes to travel between stops  $A$  and  $B$ ;
- 55 minutes to travel between stops  $A$  and  $C$  and between stops  $C$  and  $E$ ;
- 1h10 to travel between stops  $A$  and  $D$  and between stops  $D$  and  $F$ ;
- 1h20 to travel between stops  $B$  and  $F$ .

If the truck leaves the recycling plant at 10am on Monday, when is it expected to return to the plant? Assume that, in each segment, the truck takes the same amount time in both directions.