Glossary of Linear Algebra Terms

Prepared by Vince Zaccone

For Campus Learning Assistance Services at UCSB

Basis (for a subspace)
A linearly independent set of vectors that spans the space

Basic Variable
A variable in a linear system that corresponds to a pivot column in the coefficient matrix.

Column Space
The span of the columns of a matrix.
For an mxn matrix A, $Col\ (A) = \{\vec{y}: \vec{y} = A\vec{x}\ for\ some\ \vec{x} \in \mathbb{R}^n\}$

Consistent linear system
A linear system with at least one solution.

Determinant (of a square matrix A)
The number det(A) defined inductively by a cofactor expansion along any row or column of A. Also, $(-1)^r$ times the product of the diagonal entries in any echelon form U obtained from A by row replacements and r row interchanges (but no scaling).
If det(A)=0, A is a singular matrix (not invertible).

Diagonal Matrix
A square matrix whose entries NOT on the diagonal are all zero.

Diagonalizable Matrix
A matrix that is similar to a diagonal matrix.
That is, if A is similar to diagonal matrix D, we can write $A = PDP^{-1}$ where D is diagonal and P is an invertible matrix whose columns are eigenvectors of A.

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Dimension (of a vector space V)
The number of vectors in a basis for V. The dimension of the zero space is 0.

Domain (of a transformation T)
The set of all vectors for which T($\vec{x}$) is defined, i.e. the inputs to the transformation.

Dot Product (also Inner Product or Scalar Product)
The scalar $\vec{u}^T\vec{v}$, usually written $\vec{u}\cdot\vec{v}$, defined as the sum of the products of corresponding components of the vectors.

Echelon Matrix
A rectangular matrix with three properties:
(1) All nonzero rows are above each row of zeros.
(2) The leading entry in each row is in a column to the right of any leading entry in a row above it.
(3) All entries in a column below a leading entry are 0.

Eigenspace (of A corresponding to λ)
The set of all solutions to $A\vec{x} = \lambda\vec{x}$, where λ is an eigenvalue of A.

Eigenvalue (of A)
A scalar λ such that the equation $A\vec{x} = \lambda\vec{x}$ has a solution for some nonzero vector $\vec{x}$.
Found by solving the characteristic equation $\det(A - \lambda I) = 0$.

Eigenvector (of A)
A nonzero vector $\vec{x}$, such that $A\vec{x} = \lambda\vec{x}$ for some scalar λ.
Eigenvectors are in the null space of $(A - \lambda I)$.

Elementary Matrix
An invertible matrix that results by performing one elementary row operation on an identity matrix.

Elementary Row Operation
(1) Replace one row by the sum itself and another row.
(2) Switch two rows.
(3) Multiply all entries in a row by a constant.

Free Variable
A variable in a linear system that does not correspond to a pivot column.

Gaussian Elimination (row reduction)
A systematic method using elementary row operations that reduces a matrix to echelon form or reduced echelon form.

General Solution (of a linear system)
A parametric description of a solution set that expresses the basic variables in terms of the free variables (the parameters).

Gram-Schmidt Process
An algorithm for producing an orthogonal basis for a subspace that is spanned by a given set of vectors.

Homogeneous Equation
An equation of the form $A\vec{x} = \vec{0}$, possibly written as a system of linear equations.

Identity Matrix (denoted by I or I_n)
A square matrix with ones of the diagonal and zeros elsewhere.

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Image (of a vector $\vec{x}$ under a transformation T)
The vector T($\vec{x}$) assigned to $\vec{x}$ by T. The set of all the images is called the Range of T.

Inconsistent Linear System
A linear system with no solution.

Inner Product (also Dot Product)
A function on a vector space that assigns to each pair of vectors $\vec{u}$ and $\vec{v}$ a number $\langle\vec{u},\vec{v}\rangle$.

Inverse (of an nxn matrix A)
An nxn matrix A^{-1} such that $AA^{-1}=A^{-1}A=I_n$.

Isomorphism
A one-to-one linear mapping from one vector space onto another.

Kernel (of a linear transformation T:V→W)
The set of $\vec{x}$ in V such that T($\vec{x}$)= $\vec{0}$. Also see Null Space.

Length (or Norm or Magnitude) of a vector $\vec{v}$.
The scalar $\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}}$

Linear Combination.
A sum of scalar multiples of vectors. The scalars are called the weights.

Linear Equation
An equation that can be written in the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$.

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Linear Dependence (of vectors)

For a set of vectors $\{\vec{v}_1, \cdots, \vec{v}_n\}$, if the equation $c_1\vec{v}_1 + \cdots c_n\vec{v}_n = \vec{0}$ has non-trivial solutions then the set of vectors is dependent.

If the equation $c_1\vec{v}_1 + \cdots c_n\vec{v}_n = \vec{0}$ has only the trivial solution then the set of vectors is independent.

The trivial solution is $c_1 = c_2 = \cdots 0$.

Linear Transformation T (from vector space V to vector space W)

A rule T that to each vector $\vec{x}$ in V assigns a unique vector T($\vec{x}$) in W, such that:

(1) T($\vec{u} + \vec{v}$)=T($\vec{u}$)+T($\vec{v}$) for all $\vec{u}, \vec{v}$ in V.

(2) T($\vec{u}$)=cT($\vec{u}$) for all $\vec{u}$ in V and all scalars c.

Magnitude (or Length or Norm) of a vector $\vec{v}$

The scalar $\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$

Nonhomogeneous equation

An equation of the form $A\vec{x} = \vec{b}$ with $\vec{b} \neq \vec{0}$.

Nonsingular (matrix)

An invertible matrix.

Nontrivial solution

A nonzero solution of a homogeneous equation.

Norm (or Length or Magnitude) of a vector $\vec{v}$

The scalar $\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$

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Normalizing (of a vector $\vec{v}$)

The process of creating a unit vector $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$

Null Space (of an mxn matrix A)

The set Nul(A) of all solutions to the homogeneous equation $A\vec{x} = \vec{0}$.

One-To-One (mapping)

A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ such that each $\vec{b}$ in $\mathbb{R}^m$ is the image of *at most* one $\vec{x}$ in $\mathbb{R}^n$.

Onto (mapping)

A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ such that each $\vec{b}$ in $\mathbb{R}^m$ is the image of *at least* one $\vec{x}$ in $\mathbb{R}^n$.

Origin

The zero vector.

Orthogonal (set of vectors)

A set S of vectors such that $\vec{u} \cdot \vec{v} = 0$ for each distinct pair of vectors in S.

Orthonormal (set of vectors)

A orthogonal set of unit vectors.

Overdetermined System

A system of equations with more equations than unknowns.

Parametric Equation of a Line

An equation of the form $\vec{x} = \vec{p} + t\vec{v}$, where t is a parameter.
You can think of this as a line that goes through “point” $\vec{p}$ with “slope” $\vec{v}$.

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Pivot Column
A column that contains a pivot position.

Pivot Position
A position that will contain a leading entry when the matrix is reduced to echelon form.

Range (of a linear transformation T)
The set of all vectors of the form T($\vec{x}$) for some $\vec{x}$ in the domain of T.

Rank (of a matrix A)
The dimension of the column space of A, denoted by Rank(A).

Reduced Echelon Matrix
A rectangular matrix in echelon form that also has the following properties:
The leading entry in each nonzero row is 1, and each leading 1 is the only nonzero entry in its column

Row Equivalent (matrices)
Two matrices for which there exists a (finite) sequence of row operations that transforms one matrix into the other.

Scalar
A (real) number used to multiply a vector or matrix.

Similar (matrices)
Matrices A and B such that $P^{-1}AP=B$ (or $A=PBP^{-1}$) for some invertible matrix P.

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Singular (matrix)
A square matrix that has no inverse.

Span $\{\overrightarrow{v_1}, \cdots, \overrightarrow{v_n}\}$
The set of all linear combinations of $\overrightarrow{v_1}, \cdots, \overrightarrow{v_n}$.

Standard Basis
For $\mathbb{R}^n$: the basis $\mathcal{E} = \{\vec{e}_1, \cdots, \vec{e}_n\}$, consisting of the columns of the nxn identity matrix.
For $\mathbb{P}_n$: the basis $\{1, t, \cdots, t^n\}$.

Standard Matrix (for a linear transformation T)
The matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x}$ in the domain of T.

Subspace
A subset H of a vector space V such that H is itself a vector space under the operations of vector addition and scalar multiplication defined on V.

Symmetric Matrix
A matrix A such that $A^T = A$.

Trace (of a square matrix A)
The sum of the diagonal entries in A, denoted by tr(A).

Transformation (or Function or Mapping) T from $\mathbb{R}^n$ to $\mathbb{R}^m$.
A rule that assigns to each vector $\vec{x}$ in $\mathbb{R}^n$ a unique vector $T(\vec{x})$ in $\mathbb{R}^m$.

Transpose (of matrix A)
An nxm matrix A^T whose columns are the corresponding rows of the mxn matrix A.

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Trivial Solution
The solution $\vec{x} = \vec{0}$ of a homogeneous equation $A\vec{x} = \vec{0}$.

Underdetermined System
A system of equations with fewer equations than unknowns.

Unit Vector
A vector $\vec{v}$ such that $\|\vec{v}\| = 1$.

Vector
A list of numbers; a matrix with only one column; any element of a vector space.

Vector Space
A set of objects, called vectors, on which two operations are defined, called addition and multiplication by scalars (real numbers). Ten axioms must be satisfied.

Weights
The scalars used in a linear combination.

Zero Subspace
The subspace $\{\vec{0}\}$ consisting of only the zero vector.

Zero Vector
The unique vector, denoted by $\vec{0}$, such that $\vec{u} + \vec{0} = \vec{u}$ for all $\vec{u}$.
In $\mathbb{R}^n$, $\vec{0}$ is the vector whose entries are all zero.

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