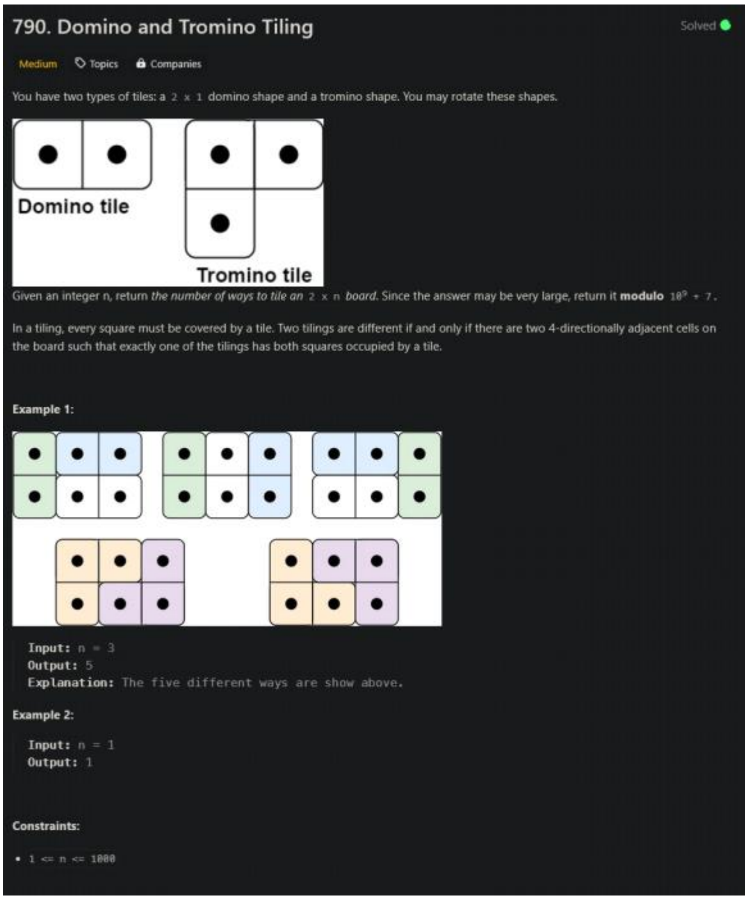
Domino and Tromino Tiling



790. Domino and Tromino Tiling

Solved

Medium Topics Companies

You have two types of tiles: a 2 x 1 domino shape and a tromino shape. You may rotate these shapes.

Domino tile

Tromino tile

Given an integer n, return *the number of ways to tile an* 2 x n *board*. Since the answer may be very large, return it **modulo** $10^9 + 7$.

In a tiling, every square must be covered by a tile. Two tilings are different if and only if there are two 4-directionally adjacent cells on the board such that exactly one of the tilings has both squares occupied by a tile.

Example 1:

```
Input: n = 3
Output: 5
Explanation: The five different ways are show above.
```

Example 2:

```
Input: n = 1
Output: 1
```

Constraints:

- 1 <= n <= 1000

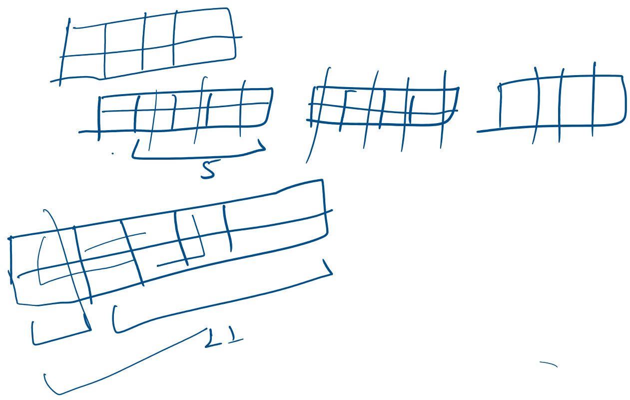
nums(3)

dp[1] = 1

dp(2) = 2

dp[3] = 5

dp(4)



Approach # 1 : Recursion + Memoization

```
class Solution{
    int mod = 1e9+7;
  public:
    int numTilings(int n){
    vector<vector<int>> dp(n+1, vector<int>(n+1,-1));
    return help(n,n,dp); }

  // maintaining |r1-r2| <1
  int help (int r1, int r2, vector<vector<int>>&dp){
  if(r1==0 && r2==0) return 1;
  if(r1<=0 || r2<=0) return 0;
  if(dp[r1][r2]!=-1)return dp[r1][r2];
  if(r1==r2){
  dp[r1][r2]= help(r1-1,r2-1,dp)%mod;
  dp[r1][r2]= (dp[r1][r2]+ help(r1-2,r2-2,dp))%mod;
                  help(r1-1,r2-2,dp))%mod;
```

```
dp[r1][r2] = (dp[r1][r2] + help(r1-2, r2-2, dp))%mod;
dp[r1][r2] = (dp[r1][r2] + help(r1-1, r2-2, dp))%mod;
dp[r1][r2] = (dp[r1][r2] + help(r1-2, r2-1, dp))%mod;

}
else if (r1>r2){
    dp[r1][r2] = help(r1-2, r2, dp)%mod;
    dp[r1][r2] = (dp[r1][r2] + help(r1-2, r2-1, dp))%mod;

}
else {
    dp[r1][r2] = help(r1, r2-2, dp)%mod;
    dp[r1][r2] = (dp[r1][r2] + help(r1-1, r2-2, dp))%mod;

    }
return dp[r1][r2];

}
};
```

```
C++   Auto

class Solution {
    int mod=1e9+7;
public:
    int numTilings(int n) {
        vector<vector<int>> dp(n+1, vector<int>(n+1, -1));
        return help(n, n, dp);
    }

// maintaining |r1-r2|<1

    int help(int r1, int r2, vector<vector<int>> &dp){
        if(r1==0 && r2==0)return 1;
        if(r1<=0 || r2<=0)return 0;
        if(dp[r1][r2]!=-1)return dp[r1][r2];
        if(r1==r2){
            dp[r1][r2]=help(r1-1, r2-1, dp)%mod;
            dp[r1][r2]=(dp[r1][r2]+help(r1-2, r2-2, dp))%mod;
            dp[r1][r2]=(dp[r1][r2]+help(r1-1, r2-2, dp))%mod;
            dp[r1][r2]=(dp[r1][r2]+help(r1-2, r2-1, dp))%mod;
        }
        else if(r1>r2){
            dp[r1][r2]=help(r1-2, r2, dp)%mod;
            dp[r1][r2]=(dp[r1][r2]+help(r1-2, r2-1, dp))%mod;
        }
        else{
            dp[r1][r2]=help(r1, r2-2, dp)%mod;
            dp[r1][r2]=(dp[r1][r2]+help(r1-1, r2-2, dp))%mod;
        }
    return dp[r1][r2];
    }
};
```

S.C. $O(n^2)$

T.C. $O(n)$

why $O(n)$ T.C.

Max. difference b/w r1 & r2 = 1

$|r1 - r2| \leq 1$ (always)

because of how we handle r1 & r2 3-cases:-

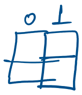
because of how we handle r1 & r2 states.

For any tile $\Rightarrow$ $width_{max} = 2$ (horizontal)

we are putting horizontal in only r1 where $r1 > r2$.

Tabulation:- Let $dp[0] = 1$ (for identity)

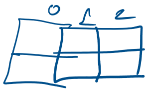
$N = 1$



$dp[0] |$

$dp[1] = 1$

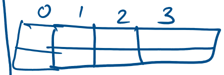
$N = 2$



$dp[1] | + dp[0] =$

$dp[2] = 2$

$N = 3$

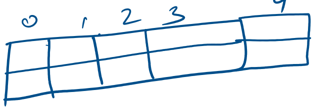


$dp[2] | + dp[1] = + dp[0]$

$\underset{2}{\uparrow} \quad \underset{1}{\uparrow} \quad \underset{2}{\uparrow}$

$dp[3] = 5$

$N = 4$

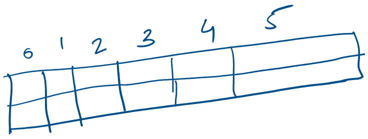


$dp[4] = dp[3] | + dp[2] = + dp[1] + dp[0]$

$\underset{5}{\uparrow} \quad \underset{2}{\uparrow} \quad \underset{2}{\uparrow} \quad \underset{2}{\uparrow}$

$= 11$

$N = 5$



$dp[4] | + dp[3] = + dp[2] + dp[1]$ $\Rightarrow$ These will continue for future N

$\therefore dp[N] = dp[N-1] + dp[N-2] + dp[N-3] \times 2 + dp[N-4] \times 2 \ldots$

$dp[N] = dp[N-1] + dp[N-2] + 2 \times (dp[N-3] + dp[N-4] \ldots)$

$= dp[N-1] + (dp[N-2] + dp[N-3]) + dp[N-3] + 2 \times (dp[N-4] + dp[N-5] \ldots)$ — 1

$\therefore dp[N-1] = dp[N-2] + dp[N-3] + 2 \times (dp[N-4] + dp[N-5] + \ldots)$

$\Rightarrow 2 \times (dp[N-4] + dp[N-5] + \ldots) = dp[N-1] - dp[N-2] - dp[N-3]$

Now, put this in 1

$dp[N] = dp[N-1] + (\cancel{dp[N-2]} + \cancel{dp[N-3]}) + dp[N-3] + dp[N-1] - \cancel{dp[N-2]} - \cancel{dp[N-3]}$

$= 2 \times dp[N-1] + dp[N-3]$

$dp[N] = dp[N-1] + $ [illegible]

$\Rightarrow \quad dp[N] = 2 \times dp[N-1] + dp[N-3]$

C++ Code :-

```
class Solution{
public :
int mod = 1e9+7;
 int numTilings (int n){
   vector<int> dp(1001);
   dp[0]= 1;
   dp[1]=1;
   dp[2] = 2;
   for(int i=3; i<=n; i++){
   dp[i]= (2*dp[i-1])%mod;
   dp[i]= (dp[i] + dp[i-3]) %mod;
   }
   return dp[n]; }};
```

```
C++ ∨   Auto
class Solution {
public:
int mod=1e9+7;
    int numTilings(int n) {
        vector<int> dp(1001);
        dp[0]=1;
        dp[1]=1;
        dp[2]=2;
        for(int i=3;i<=n;i++){
            dp[i]=(2*dp[i-1])%mod;
            dp[i]=(dp[i]+dp[i-3])%mod;
        }
        return dp[n];
    }
};
```

T.C. $O(n)$

S.C. $O(1)$