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Unbraiding the Bounce

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Abstract. We study a recently proposed by Ijjas and Steinhardt particular realization [1] of the cosmological bounce scenario. First, we reveal the exact construction of the Lagrangian used in [1]. This explicit construction allowed us to study other cosmological solutions in this theory. In particular we found solutions with superluminal speed of sound and discuss the consequences of this feature for a possible UV-completion. Further, following the originally constructed background history, we evaluated the tensor and scalar spectra during the bouncing phase characterized by the violation of null (and strong) energy condition. We found that the change of the speed of sound is the cause of the dominance of the tensor power spectrum over the scalar part through most of the bounce. Moreover, we observe that none of the spectra evaluated across the bouncing phase is scale invariant. In addition to this, we present our results for particle production by showing the evolution of the occupation number of scalar fluctuations through the bounce.

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1 Introduction

Since 2011 it is well known that scalar-tensor theories with Kinetic Gravity Braiding (which are also called first two terms of Horndeski or simple generalized Galileons) allow for [2, 3] a classical bouncing evolution manifestly free from ghost and gradient instabilities around the bounce. The possibility of the bounce in such systems was briefly mentioned in Moreover, in [3] it was demonstrated that one can easily construct spatially flat bouncing universes, without ghosts and gradient instabilities and with bouncing solutions of a non-vanishing measure. Moreover, in [3] it was demonstrated that there is a continuum of such minimally coupled theories of the type of Kinetic Gravity Braiding. This Ref. [3] provided inequalities on two free functions of kinetic term present in Lagrangian of the theory, which are sufficient to guaranty a healthy bounce. It was also showed that this setup also works in a presence (in fact unavoidable) of the normal matter - like radiation etc. In one of examples ("Hot G-Bounce") it was even showed that such a bouncing universes can smoothly transit to the radiation dominated stage. In this work it was also discussed that there is always a singularity (either in the gravitational or acoustic metric).

In 2016, Ijjas and Steinhardt proposed a particular realization [1] of the cosmological bounce scenario in a particular subclass of these theories. For convenience we denote this realization as *IS-bounce*. The IS-bounce was claimed to be free from ghost and gradient instabilities and free from superluminal propagation of perturbations. The explicit case of the IS-bounce uses a class of Kinetic Gravity Braiding theories with explicitly strongly *broken* shift-symmetry $\phi \to \phi + c$

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\left(k(\phi)(\partial\phi)^2 + \frac{1}{2}q(\phi)(\partial\phi)^4 + (\partial\phi)^2\Box\phi\right), \tag{1.1}$$

where

$$(\partial\phi)^2 \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \equiv 2X\,, \qquad \Box\phi \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu\phi\,, \tag{1.2}$$

where ∇_μ is the usual Levi-Civita connection [1]. The scalar field is supposed to be minimally coupled to gravity. Hence the theory is defined by two free functions $k(\phi)$ and $q(\phi)$. In

[1]Further we use: the standard notation $\sqrt{-g} \equiv \sqrt{-\mathrm{det} g_{\mu\nu}}$ where $g_{\mu\nu}$ is the metric, the signature convention $(+,-,-,-)$ (contrary to [1]), and the units $c = \hbar = 1$, $M_{\mathrm{Pl}} = (8\pi G_{\mathrm{N}})^{-1/2} = 1$.

notation of [4] we have[2]

$$K(X,\phi) = k(\phi)\,X + q(\phi)\,X^2\,, \qquad G(X,\phi) = X\,. \tag{1.3}$$

This identification allows us to directly use all necessary formulas derived in [4]. The quadratic action for curvature perturbations ζ is written by

$$S_2 = \int dt\, d^3x\, a^3 \left(A(t)\,\dot{\zeta}^2 - \frac{B(t)}{a^2}\left(\partial_i\zeta\right)^2\right)\,. \tag{1.4}$$

The formula for the normalization of the curvature perturbations is given by (A.9) page 36, [4]

$$A = \frac{2XD}{\left(H - \dot{\phi}XG_{,X}\right)^2}\,, \tag{1.5}$$

where

$$D = K_{,X} + 2XK_{,XX} - 2G_{,\phi} - 2XG_{,X\phi} + 6\dot{\phi}H\left(G_{,X} + XG_{,XX}\right) + 6X^2G_{,X}^2\,. \tag{1.6}$$

The sound speed is given by the formula (A.11) page 36, [4]

$$c_s^2 = \frac{B(t)}{A(t)} = \frac{\dot{\phi}XG_{,X}\left(H - \dot{\phi}XG_{,X}\right) - \partial_t\left(H - \dot{\phi}XG_{,X}\right)}{XD}\,, \tag{1.7}$$

which can be written in terms of γ introduced in arXiv:1606.08880 [gr-qc]. An equivalent expression (A.12) on the same page is more useful to check

$$c_s^2 = \frac{K_{,X} - 2G_{,\phi} + 2XG_{,\phi X} + 2\ddot{\phi}\left(G_{,X} + XG_{,XX}\right) + 4\dot{\phi}HG_{,X} - 2X^2G_{,X}^2}{D}\,. \tag{1.8}$$

The action for *curvature perturbations* from arXiv:1008.0048 [hep-th] is:

$$S = \frac{1}{2}\int dtd^3x\, a^3 A\left(\dot{\zeta}^2 - \frac{c_s^2}{a^2}\left(\partial_i\zeta\right)^2\right)\,, \tag{1.9}$$

and is different from the definition (11) from arXiv:1606.08880 [gr-qc] by factor 1/2.

For the Lagrangian given by (**??**) and (**??**) we have

$$D = k + 6Xq + 6\dot{\phi}H + 6X^2\,, \tag{1.10}$$

and consequently

$$A = \frac{2X\left(k + 6Xq + 6\dot{\phi}H + 6X^2\right)}{\left(H - \dot{\phi}X\right)^2} = \frac{\dot{\phi}^2\left(k + 3\dot{\phi}^2q + 6\dot{\phi}H + {}^3\!/\!_2\dot{\phi}^4\right)}{\left(H - {}^1\!/\!_2\dot{\phi}^3\right)^2}\,. \tag{1.11}$$

Clearly this formula is *exactly the same* (with the notational difference because of 1/2) as (12) from arXiv:1606.08880 [gr-qc]. Note again that arXiv:1008.0048 [hep-th] defines the action with factor 1/2 in front, contrary to arXiv:1606.08880 [gr-qc].

[2]At the beginning the authors of [1] also used $G(X,\phi) = b(\phi)\,X$, however this additional free function $b(\phi)$ can be eliminated by a simple field-redefinition.

The coefficient B from arXiv:1606.08880 [gr-qc] can be obtained as

$$B = \frac{1}{2} A c_s^2 = \frac{X\left(K_{,X} - 2G_{,\phi} + 2XG_{,\phi X} + 2\ddot{\phi}\left(G_{,X} + XG_{,XX}\right) + 4\dot{\phi}HG_{,X} - 2X^2G_{,X}^2\right)}{\left(H - \dot{\phi}XG_{,X}\right)^2} \,. \tag{1.12}$$

We have

$$B = \frac{\dot{\phi}^2\left(k + q\dot{\phi}^2 + 2\ddot{\phi} + 4\dot{\phi}H - \frac{1}{2}\dot{\phi}^4 b^2\right)}{2\left(H - {}^1\!/_2\dot{\phi}^3\right)^2} \,, \tag{1.13}$$

which is exactly the formula (13) from arXiv:1606.08880 [gr-qc].

The authors of IS-bounce pustulated a particular fairly simple time-dependence of the Hubble parameter

$$H\left(t\right) = H_0\, t\, \exp\left(-F\left(t - t_*\right)^2\right)\,, \tag{1.14}$$

where H_0, F and t_* are constants, and proposed an "inverse method" to find free functions $k\left(\phi\right)$ and $q\left(\phi\right)$ in (1.1) which can realize this cosmological evolution. The key observation of the "inverse method" is that one can can also independently postulate $\gamma\left(t\right)$ in

$$\frac{d}{dt}\gamma^{-1} + H\gamma^{-1} = B\left(t\right) + 1\,. \tag{1.15}$$

For the action (1.1) one obtains

$$\gamma\left(t\right) = H\left(\mathrm{t}\right) - \frac{1}{2}\dot{\phi}^3\,. \tag{1.16}$$

The IS-bounce postulates

$$\gamma = \gamma_0 \exp\left(3\theta t\right) + H\left(t\right)\,, \tag{1.17}$$

where γ_0 and θ are additional constants with respect to already introduced H_0, F and t_*. From (1.16) one can obtain

$$\phi\left(t\right) = \phi_0 + \int_{t_0}^{t} dt'\left(2\left(H - \gamma\right)\right)^{1/3}\,, \tag{1.18}$$

where $\phi\left(t_0\right) = \phi_0$. It is convenient to chose this initial value as $\phi_0 = \left(-2\gamma_0/\theta\right)^{1/3}\exp\left(3t_0\right)$, so that the particular solution postulated in IS-bounce is

$$\phi\left(t\right) = \left(\frac{-2\gamma_0}{\theta^3}\right)^{1/3}\exp\left(\theta t\right)\,. \tag{1.19}$$

For the later it is suitable to denote

$$\phi_\star = \left(\frac{-2\gamma_0}{\theta^3}\right)^{1/3}\,, \tag{1.20}$$

as a characteristic field range, and use the normalized scalar field

$$\Phi = \phi/\phi_\star\,, \tag{1.21}$$

so that

$$t = \frac{1}{\theta}\log\left(\phi/\phi_\star\right)\,. \tag{1.22}$$

Using the substitutions (1.14) and (1.17) one obtains the functions $k(\phi)$ and $q(\phi)$ as functions of time on the particular solution (1.19)

$$k(t) = -\frac{2\left(2\dot{H} + 3H^2 + \dot{\gamma} + 3H\gamma\right)}{\left(2\left(H-\gamma\right)\right)^{2/3}}, \tag{1.23}$$

and

$$q(t) = \frac{4\left(2\dot{H} + \dot{\gamma} + 9H\gamma\right)}{3\left(2\left(H-\gamma\right)\right)^{4/3}}. \tag{1.24}$$

For the later it is convenient to introduce functions

$$W(\phi) = \exp\left[\frac{F}{\theta^2}\left(\log\left(\frac{\phi}{\phi_\star}\right) - \theta t_*\right)^2\right], \tag{1.25}$$

and

$$\Omega(\phi) = W(\phi)\,\theta^3\left(\theta\phi\right)^3 + H_0\left[\log\left(\frac{\phi}{\phi_\star}\right)\left(4F\left[\log\left(\frac{\phi}{\phi_\star}\right) - \theta t_*\right] + 2\theta\left(\theta\phi\right)^3\right) - 2\theta^2\right], \tag{1.26}$$

in terms of which the defining functions are

$$k(\phi) = -\frac{12H_0^2\log^2\left(\phi/\phi_\star\right) - 3W(\phi)\left[\Omega(\phi) - H_0\left(\theta\phi\right)^3\log\left(\phi/\phi_\star\right)\right]}{W^2(\phi)\,\theta^2\left(\theta\phi\right)^2}, \tag{1.27}$$

and

$$q(\phi) = \frac{12H_0^2\log^2\left(\phi/\phi_\star\right) - 2W(\phi)\left[\Omega(\phi) + H_0\theta\left(\theta\phi\right)^3\log\left(\phi/\phi_\star\right)\right]}{W^2(\phi)\,\theta^2\left(\theta\phi\right)^4}. \tag{1.28}$$

These expressions defining the theory which should be related to the origins of the universe neither look well-motivated nor natural from any point of view. This is the prize for the chosen simple exact solution.

2 Other Solutions, Phase Space

3 Superluminality

4 Perturbations

5 Conclusions

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