Assignment 1

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Section 1.2

Exercise 1.m (p21)

The next term in the sequence is 18, according to how the elements in the sequence appear to be positive multiples of 3 in order. This feature can be described with the following closed formula:

$$a_n = 3n$$

The above closed formula can also be rewritten as a recursive formula:

$$\begin{aligned} a_1 &= 3 \\ a_n &= a_{n-1} + 3 \end{aligned}$$

Exercise 7.g (p21)

The first five terms:

$$\begin{aligned} a_1 &= 2^{2\times 1-1} - 1 = 1 \\ a_2 &= 2^{2\times 2-1} - 1 = 7 \\ a_3 &= 2^{2\times 3-1} - 1 = 31 \\ a_4 &= 2^{2\times 4-1} - 1 = 127 \\ a_5 &= 2^{2\times 5-1} - 1 = 511 \end{aligned}$$

Recursive formula:

$$\begin{aligned} a_1 &= 1 \\ a_n &= 4 \times a_{n-1} + 3 \end{aligned}$$

Exercise 9.a (p21)

The first five terms:

$$
\begin{aligned}
a_1 &= 5 \\
a_2 &= a_{2-1} + (2+4) = 11 \\
a_3 &= a_{3-1} + (3+4) = 18 \\
a_4 &= a_{4-1} + (4+4) = 26 \\
a_5 &= a_{5-1} + (5+4) = 35
\end{aligned}
$$

Closed formula:

$$a_n = \frac{n^2+n}{2} + 4n$$

Exercise 30 (p23)

Part (a)

- $a_1 = A$
- $a_2 = a_{2/2}B = a_1B = AB$
- $a_3 = a_{(3-1)/2}A = a_1A = AA$
- $a_4 = a_{4/2}B = a_2B = ABB$
- $a_5 = a_{(5-1)/2}A = a_2A = ABA$
- $a_6 = a_{6/2}B = a_3B = AAB$
- $a_7 = a_{(7-1)/2}A = a_3A = AAA$
- $a_8 = a_{8/2}B = a_4B = ABBB$
- $a_9 = a_{(9-1)/2}A = a_4A = ABBA$
- $a_{10} = a_{10/2}B = a_5B = ABAB$

Part (b)

- $a_{17} = a_{(17-1)/2}A = a_8A = ABBBA$
- $a_{21} = a_{(21-1)/2}A = a_{10}A = ABABA$

Part (c)

- $a_{630} = a_{630/2}B = a_{315}B = ABBAAABAAB$
- $a_{631} = a_{(631-1)/2}A = a_{315}A = ABBAAABAAA$

Section 1.3

Exercise 1.a (p37)

Let p represent "A is telling the truth" and q represent "B is telling the truth," according to the information given by the problem, we can generate the following truth table:

p	q	$p \wedge q$	$\neg p$
T	T	T	F
T	F	F	F
F	T	F	T
F	F	F	T

As seen from the truth table, only the third row provides a consistent result (where when $p = F$ and $q = T$, $p = p \wedge q$ and $q = \neg p$). Therefore, A is lying, and B is telling the truth.

Exercise 4.a (p37)

$$\neg p \wedge (q \vee r)$$

where p, q, and r represent A, B, and C being truthful, respectively.

Exercise 10.d (p38)

$$(x \leq 0) \wedge (y \leq 0)$$

Exercise 24.b (p40)

$$\begin{aligned}
(p \vee t) \wedge (p \vee c) &\equiv t \wedge (p \vee c) && \text{Universal bound} \\
&\equiv t \wedge p && \text{Identity} \\
&\equiv p \wedge t && \text{Commutative} \\
&\equiv p && \text{Identity}
\end{aligned}$$

Problem 7 (Java method)

Since `charIsNotLowerCaseLetter(c)` always produces exactly the opposite result of `charIsLowerCaseLetter(c)` and the definition of `charIsLowerCaseLetter(c)` is `'a' <= c && c <= 'z'`, according to De Morgan's Law,

```
    !('a' <= c && c <= 'z')
=== !('a' <= c) || !(c <= 'z')
=== 'a' > c || c > 'z'
```

Section 1.4

Exercise 14.c (p52)

$$\begin{aligned}\neg(\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, x = 2y) &\equiv \exists x \in \mathbb{Z}, \neg(\exists y \in \mathbb{R}, x = 2y)\\ &\equiv \exists x \in \mathbb{Z}, \forall y \in \mathbb{R}, \neg(x = 2y)\\ &\equiv \exists x \in \mathbb{Z}, \forall y \in \mathbb{R}, x \neq 2y\end{aligned}$$

Exercise 15.c (p52)

The original statement is true:

$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, x = 2y$$

Exercise 17.c (p52)

There exists a positive integer x, which for every positive integer y, y is greater than or equal to x, or y is not a factor of x.

Exercise 20.b (p52)

The negation of the original statement is true: "there exist two real numbers x and y, which for every integer n, $x \geq n$ or $n \geq y$."

Section 1.5

Exercise 11.c (p66)

This statement is true for elements in D.

Exercise 11.e (p66)

This statement is true for elements in D.

Exercise 14.d (p67)

$$\forall x \in \mathbb{R}^+, x < \sqrt{2} \to \frac{2}{x} > \sqrt{2}$$

Exercise 17.a (p67)

$$n \bmod 5 = 0 \to (n \bmod 10 = 5) \vee (n \bmod 10 = 0)$$

Exercise 25.e (p67)

If a triangle is not an isosceles triangle, then it has neither two equal sides nor two equal angles.

Exercise 26.e (p67)

If a triangle is an isosceles triangle, then it has either two equal sides or two equal angles.

Exercise 27.e (p67)

If a triangle has neither two equal sides nor two equal angles, then it is not an isosceles triangle.