

# Homework 5

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## Problem 1

### Part (a)

Since a person can only either have a heart attack or not have one,

$$\begin{aligned}P(S = 0) &= P(S = 0 \cap P = 0) + P(S = 0 \cap P = 1) \\&= 0.50 + 0.03 \\&= \boxed{0.53}\end{aligned}$$

### Part (b)

Similarly, since a person is either a smoker or a non-smoker,

$$\begin{aligned}P(H = 0) &= P(H = 0 \cap S = 0) + P(H = 0 \cap S = 1) \\&= 0.50 + 0.44 \\&= \boxed{0.94}\end{aligned}$$

### Part (c)

$$\begin{aligned}P(H = 1|S = 0) &= \frac{P(H = 1 \cap S = 0)}{P(S = 0)} \\&= \frac{0.03}{0.53} \\&\approx \boxed{0.057}\end{aligned}$$

$$\begin{aligned}P(H = 1|S = 1) &= \frac{P(H = 1 \cap S = 1)}{P(S = 1)} \\&= \frac{0.03}{1 - 0.53} \\&\approx \boxed{0.064}\end{aligned}$$

These two numbers indicate the probability of drawing a non-smoker having heart attack and drawing a smoker having heart attack, respectively. The probability of the latter is higher, which matches our intuition.

**Part (d)**

No. According to the results of the calculations done above,  $P(H = 1) \neq P(H = 1|S = 1)$ . By definition of independence, the variables  $H$  and  $S$  are not independent.

## Problem 2

### Part (a)

$$p_{X,Y}(s, t) = \begin{cases} \frac{1}{\pi}, & s^2 + t^2 \leq 1 \\ 0, & s^2 + t^2 > 1 \end{cases}$$

### Part (b)

Intuitively,  $p_X(0) > p_X(1)$ . The reason is if  $X = 1$ , then the only value  $Y$  can be is 0 for the point to still be inside the unit disk; the probability of  $Y = 0$  for a continuous random variable  $Y$  is infinitely small. On the other hand, when  $X = 0$ ,  $Y$  can be anything between -1 and 1, which happens 100% of the time. Therefore,  $p_X(0)$  should be greater.

### Part (c)

$\mathbb{E}[X + Y] = 0$  since  $X$  and  $Y$  are both symmetric about the origin.

### Part (d)

This is not a valid way to sample uniformly. For example, as described in *Part (b)*, we have  $p_X(0) > p_X(1)$ , meaning that  $X$  is “denser” around 0 and it is more likely to have an  $X$  value close to 0 than one close to 1. Therefore,  $X$  cannot be sampled uniformly between  $[-1, 1]$  in the first step.

### Problem 3

Part (a)

Part (b)

Part (c)