Quiz 5 Solutions

Qianlang Chen

Problem 1

Part (a)

Ways to have 3 uppercase letters: 26^3

Ways to have 4 digits: 10^4

Total ways to have 3 uppercase letters followed by 4 digits: $\boxed{26^3 \times 10^4}$

Part (b)

Number of outcomes when selecting a single-digit even positive integer: $|\{2, 4, 6, 8\}| = 4$

Number of outcomes of a coin flip: 2

Number of outcomes of drawing a card from a standard deck: 52

Total number of outcomes: $\boxed{4 \times 2 \times 52}$

Problem 2

Part (a)

Same as the number of ways to get a 10-item permutation out of 200 items:

$$P(200, 10) = \frac{200!}{(200 - 10)!} = \boxed{\frac{200!}{190!}}$$

Part (b)

Since given four distinct digits, there is only one way to write them in strictly decreasing order, the answer is the same as the number of ways to get a set of 4 digits out of 10 digits:

$$C(10, 4) = \frac{10!}{(10 - 4)! \times 4!} = \boxed{\frac{10!}{6! \times 4!}}$$

Problem 3

Ways to arrange the 5 math books: $P(5,5) = \dfrac{5!}{(5-5)!} = 5!$

Ways to arrange the 4 history books: $P(4,4) = \dfrac{4!}{(4-4)!} = 4!$

Ways to arrange the 3 chemistry books: $P(3,3) = \dfrac{3!}{(3-3)!} = 3!$

Ways to arrange the 2 sociology books: $P(2,2) = \dfrac{2!}{(2-2)!} = 2!$

Ways to arrange those 4 subjects: $P(4,4) = \dfrac{4!}{(4-4)!} = 4!$

Total ways to arrange: $\boxed{5! \times 4! \times 3! \times 2! \times 4!}$

Problem 4

Part (a)

The answer is the same as the number of ways of putting 2 "R"s, 2 "A"s, and 1 "D" into 5 slots.

- Ways to select 2 slots for the 2 "R"s: $C(5,2) = \frac{5!}{(5-2)! \times 2!} = 10$
- Ways to select 2 slots for the 2 "A"s from the remaining 3 slots:

$$C(3,2) = \frac{3!}{(3-2)! \times 2!} = 3$$

- The "D" goes into the remaining 1 slot.

Number of total ways: $10 \times 3 = \boxed{30}$

Part (b)

The answer is the same as the number of ways of putting 3 "U"s, 1 "N", 1 "S", 1 "A", and 1 "L" into 7 slots.

- Ways to select 3 slots for the 3 "U"s: $C(7,3) = \frac{7!}{(7-3)! \times 3!} = \frac{7!}{4! \times 3!}$
- Ways to select a slot for the "N" from the remaining 4 slots: $C(4,1) = \frac{4!}{(4-1)! \times 1!} = \frac{4!}{3!} = 4$
- Ways to select a slot for the "S" from the remaining 3 slots: $C(3,1) = \frac{3!}{(3-1)! \times 1!} = \frac{3!}{2!} = 3$
- Ways to select a slot for the "A" from the remaining 2 slots: $C(2,1) = \frac{2!}{(2-1)! \times 1!} = \frac{2!}{1!} = 2$
- The "L" goes into the remaining 1 slot.

Number of total ways: $\boxed{\frac{7!}{4! \times 3!} \times 4 \times 3 \times 2}$

Problem 5

The answer is the same as the total number of subsets of 12 people, but excluding the subsets with only 0, 1, or 2 people.

- Total number of subsets formed out of 12 people: 2^{12}
- Number of subsets with 0 people: $C(12, 0) = \dfrac{12!}{(12-0)! \times 0!} = 1$
- Number of subsets with 1 person: $C(12, 1) = \dfrac{12!}{(12-1)! \times 1!} = 12$
- Number of subsets with 2 people: $C(12, 2) = \dfrac{12!}{(12-2)! \times 2!} = \dfrac{12!}{10! \times 2}$

Final answer: $\boxed{2^{12} - 1 - 12 - \dfrac{12!}{10! \times 2}}$

Problem 6

Total number of ways to promote 2 people out of $(8+4)$ people: $C(8{+}4, 2) = \dfrac{12!}{(12-2)! \times 2!} = \dfrac{12!}{10! \times 2}$

Total number of ways to promote 2 men out of 8 men: $C(8, 2) = \dfrac{8!}{(8-2)! \times 2!} = \dfrac{8!}{6! \times 2}$

Assuming that each person in the office has an equal chance of getting promoted, the probability that 2 promotions being men is

$$\frac{8!}{6! \times 2} \div \frac{12!}{10! \times 2} = \boxed{\frac{8! \times 10!}{6! \times 12!}}$$

Problem 7

Total number of possible codes: 10^4

Total number of codes having two "6"s and one "3": same as the number of ways of putting two "6"s, one "3", and one other digit into 4 slots.

- Ways to select 2 slots for the 2 "6"s: $C(4,2) = \dfrac{4!}{(4-2)! \times 2!} = 6$
- Ways to select 1 slot for the "3" from the remaining 2 slots: $C(2,1) = \dfrac{2!}{(2-1)! \times 1!} = 2$
- Ways to select a digit (other than "3" or "6") for the remaining slot: $10 - 2 = 8$
- Total: $6 \times 2 \times 8 = 96$

Probability of a code having two "6"s and one "3": $\boxed{\dfrac{96}{10^4}}$

Problem 8

Let E be the event that the prize winner comes from the winning team and F be the event that the prize winner is a boy.

Now, the event $(E \cap F)$ means that the prize winner is a boy from the winning team. There are 4 ways for this event to happen since there are 4 boys in the winning team.

Meanwhile, there are 10 ways the event F can happen since there are 10 boys $(4 + 6)$ in total.

Therefore,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{|E \cap F|}{|F|} = \boxed{\frac{4}{10}}$$

Problem 9

Part (a)

(1) 0010100001000

(2) 0001001000001

(3) 1110000000000

Part (b)

Four types of fruit: $n = 4$

Ten total pieces of fruit: $r = 10$

Total number of ways to buy: $C(r{+}n{-}1, r) = C(10{+}4{-}1, 10) = \dfrac{13!}{(13-10)! \times 10!} = \boxed{\dfrac{13!}{3! \times 10!}}$

Part (c)

Ways to buy only apples and pears (no oranges or lemons): $n = 2, r = 10$

$$C(r+n-1, r) = C(10+2-1, 10) = \frac{11!}{(11-10)! \times 10!} = 11$$

Ways to buy apples, pears, and one or more oranges: $n = 3, r = 10 - 1 = 9$

$$C(r+n-1, r) = C(9+3-1, 9) = \frac{11!}{(11-9)! \times 9!} = \frac{11!}{2 \times 9!}$$

Ways to buy apples, pears, and one or more lemons: $n = 3, r = 10 - 1 = 9$

$$C(r+n-1, r) = C(9+3-1, 9) = \frac{11!}{(11-9)! \times 9!} = \frac{11!}{2 \times 9!}$$

Total number of ways to buy:

$$11 + \frac{11!}{2 \times 9!} + \frac{11!}{2 \times 9!} = \boxed{11 + \frac{11!}{9!}}$$

Problem 10

Part (a)

Chance of getting a King in one draw: $\dfrac{1}{13}$

Expected number of Kings in one draw: $1 \times \dfrac{1}{13} + 0 \times (1 - \dfrac{1}{13}) = \dfrac{1}{13}$

Expected number of Kings in six draws: $6 \times \dfrac{1}{13} = \boxed{\dfrac{6}{13}}$

Part (b)

Probability that team A wins with 3-to-0:

$$C(3+0-1,0) \times (\frac{6}{10})^3 = \frac{2!}{(2-0)! \times 0!} \times \frac{6^3}{10^3} = \frac{6^3}{10^3}$$

Probability that team A wins with 3-to-1:

$$C(3+1-1,1) \times (\frac{6}{10})^3 \times (1 - \frac{6}{10}) = \frac{3!}{(3-1)! \times 1!} \times \frac{6^3}{10^3} \times \frac{4}{10} = \frac{3 \times 6^3 \times 4}{10^4}$$

Probability that team A wins with 3-to-2:

$$C(3+2-1,2) \times (\frac{6}{10})^3 \times (1 - \frac{6}{10})^2 = \frac{4!}{(4-2)! \times 2!} \times \frac{6^3}{10^3} \times \frac{4^2}{10^2} = \frac{4 \times 6^3 \times 4^2}{10^5}$$

Total probability that team A wins:

$$\boxed{\frac{6^3}{10^3} + \frac{3 \times 6^3 \times 4}{10^4} + \frac{4 \times 6^3 \times 4^2}{10^5}}$$