Homework 4: Bayes' Nets

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Problem 1: Inference

Part 1.1

$$P(-y \mid +x) = 1 - P(+y \mid +x) = 1 - .8 = \boxed{.2}$$

Part 1.2

$$P(-z \mid -x) = 1 - P(+z \mid -x) = 1 - .4 = \boxed{.6}$$

Part 1.3

$$P(-y \mid +x, +z) = P(-y \mid +x) = \boxed{.2}$$

Part 1.4

$$\begin{aligned} P(+y, -z, -x) &= P(+y, -z \mid -x)\, P(-x) \\ &= P(+y \mid -x)\, P(-z \mid -x)\, P(-x) \\ &= .3 \times .6 \times .7 \\ &= \boxed{.126} \end{aligned}$$

Part 1.5

$$
\begin{aligned}
P(-y) &= 1 - P(+y) \\
&= 1 - (P(+y, +x) + P(+y, -x)) \\
&= 1 - (P(+y \mid +x)\, P(+x) + P(+y \mid -x)\, P(-x)) \\
&= 1 - (.8 \times .3 + .3 \times .7) \\
&= \boxed{.55}
\end{aligned}
$$

Part 1.6

$$
\begin{aligned}
P(+y, +z) &= P(+y, +z, +x) + P(+y, +z, -x) \\
&= P(+y \mid +z, +x)\, P(+z, +x) + P(+y \mid +z, -x)\, P(+z, -x) \\
&= P(+y \mid +x)\, P(+z \mid +x)\, P(+x) + P(+y \mid -x)\, P(+z \mid -x)\, P(-x) \\
&= .8 \times .8 \times .3 + .3 \times .4 \times .7 \\
&= .276
\end{aligned}
$$

$$
\begin{aligned}
P(+z) &= P(+z, +x) + P(+z, -x) \\
&= P(+z \mid +x)\, P(+x) + P(+z \mid -x)\, P(-x) \\
&= .8 \times .3 + .4 \times .7 \\
&= .52
\end{aligned}
$$

$$
\begin{aligned}
P(+y \mid +z) &= \frac{P(+y, +z)}{P(+z)} \\
&= \frac{.276}{.52} \\
&\approx \boxed{.5308}
\end{aligned}
$$

Part 1.7

$$
\begin{aligned}
P(-y \mid +z) &= 1 - P(+y \mid +z) \\
&\approx 1 - .5308 \\
&\approx \boxed{.4692}
\end{aligned}
$$

Problem 2: Joint Probabilities

Part 2.1

$$\begin{aligned}
\mathrm{P}(+b, -e, +a, +j, +m) &= \mathrm{P}(+b)\,\mathrm{P}(-e)\,\mathrm{P}(+a \mid +b, -e)\,\mathrm{P}(+j \mid +a)\,\mathrm{P}(+m \mid +a) \\
&= .1 \times .998 \times .92 \times .4 \times .6 \\
&\approx \boxed{.02204}
\end{aligned}$$

Part 2.2

$$\begin{aligned}
\mathrm{P}(-b, -e, +a, -j, -m) &= \mathrm{P}(-b)\,\mathrm{P}(-e)\,\mathrm{P}(+a \mid -b, -e)\,\mathrm{P}(-j \mid +a)\,\mathrm{P}(-m \mid +a) \\
&= .9 \times .998 \times .1 \times .6 \times .4 \\
&\approx \boxed{.02156}
\end{aligned}$$

Part 2.3

$$\begin{aligned}
\mathrm{P}(-b, +e, +a, +j, +m) &= \mathrm{P}(-b)\,\mathrm{P}(+e)\,\mathrm{P}(+a \mid -b, +e)\,\mathrm{P}(+j \mid +a)\,\mathrm{P}(+m \mid +a) \\
&= .9 \times .002 \times .2 \times .4 \times .6 \\
&= \boxed{8.64 \times 10^{-5}}
\end{aligned}$$

Part 2.4

$$\begin{aligned}
\mathrm{P}(-b, -e, -a, -j, -m) &= \mathrm{P}(-b)\,\mathrm{P}(-e)\,\mathrm{P}(-a \mid -b, -e)\,\mathrm{P}(-j \mid -a)\,\mathrm{P}(-m \mid -a) \\
&= .9 \times .998 \times .9 \times .95 \times .9 \\
&\approx \boxed{.6912}
\end{aligned}$$

Problem 3: D-Separation

Part 3.1

Yes

Part 3.2

Not enough information

Part 3.3

Yes

Part 3.4

Not enough information

Part 3.5

Not enough information

Part 3.6

Not enough information

Part 3.7

Yes

Part 3.8

Not enough information

Part 3.9

Yes

Problem 4: Variable Elimination

Part 4.1

R	T	$P(R,T)$
$+r$	$+t$	$= P(+r) \times P(+t \mid +r) = .3 \times .8 = .24$
$+r$	$-t$	$= P(+r) \times P(-t \mid +r) = .3 \times .2 = .06$
$-r$	$+t$	$= P(-r) \times P(+t \mid -r) = .7 \times .4 = .28$
$-r$	$-t$	$= P(-r) \times P(-t \mid -r) = .7 \times .6 = .42$

Part 4.2

T	$P(T)$
$+t$	$= P(+r,+t) + P(-r,+t) = .24 + .28 = .52$
$-t$	$= P(+r,-t) + P(-r,-t) = .06 + .42 = .48$

Part 4.3

T	L	$P(T,L)$
$+t$	$+l$	$= P(+t) \times P(+l \mid +t) = .52 \times .2 = .104$
$+t$	$-l$	$= P(+t) \times P(-l \mid +t) = .52 \times .8 = .416$
$-t$	$+l$	$= P(-t) \times P(+l \mid -t) = .48 \times .3 = .144$
$-t$	$-l$	$= P(-t) \times P(-l \mid -t) = .48 \times .7 = .336$

Part 4.4

L	$P(L)$
$+l$	$= P(+t,+l) + P(-t,+l) = .104 + .144 = .248$
$-l$	$= P(+t,-l) + P(-t,-l) = .416 + .336 = .752$

Problem 5: Sampling

Part 5.1

$$\mathrm{w}(+s, +w, +c, +r) = \mathrm{P}(+c)\,\mathrm{P}(+r \mid +c) = .5 \times .8 = \boxed{.4}$$

Part 5.2

$$\mathrm{w}(+s, +w, -c, +r) = \mathrm{P}(-c)\,\mathrm{P}(+r \mid -c) = .5 \times .2 = \boxed{.1}$$

Part 5.3

$$\mathrm{w}(+s, +w, +c, -r) = \mathrm{P}(+c)\,\mathrm{P}(-r \mid +c) = .5 \times .2 = \boxed{.1}$$

Part 5.4

$$\mathrm{w}(+s, +w, -c, -r) = \mathrm{P}(-c)\,\mathrm{P}(-r \mid -c) = .5 \times .8 = \boxed{.4}$$

Part 5.5

The total value of the samples is

$$.4 + .1 + .1 + .4 + .4 = 1.4$$

Therefore,

$$\begin{aligned}
\mathrm{P}(+s, +w, +c, +r) &= \frac{1 \times .4}{1.4} \approx .2857 \\
\mathrm{P}(+s, +w, +c, -r) &= \frac{0 \times .1}{1.4} = 0 \\
\mathrm{P}(+s, +w, +c) &= \mathrm{P}(+s, +w, +c, +r) + \mathrm{P}(+s, +w, +c, -r) \approx .2857 + 0 \approx \boxed{.2857}
\end{aligned}$$

Part 5.6

$$\begin{aligned}
\mathrm{P}(+s, +w, -c, +r) &= \frac{2 \times .1}{1.4} \approx .1429 \\
\mathrm{P}(+s, +w, -c, -r) &= \frac{2 \times .4}{1.4} \approx .5714 \\
\mathrm{P}(+s, +w, -c) &= \mathrm{P}(+s, +w, -c, +r) + \mathrm{P}(+s, +w, -c, -r) \approx .1429 + .5714 \approx \boxed{.7143}
\end{aligned}$$

Problem 6: Decision Networks

Part 6.1

$$\begin{aligned} \mathrm{EU(take)} &= \mathrm{U(take, sun)\ P(sun) + U(take, rain)\ P(rain)} \\ &= 20 \times .6 + 70 \times .4 \\ &= \boxed{40} \end{aligned}$$

Part 6.2

$$\begin{aligned} \mathrm{EU(leave)} &= \mathrm{U(leave, sun)\ P(sun) + U(leave, rain)\ P(rain)} \\ &= 80 \times .6 + 0 \times .4 \\ &= \boxed{48} \end{aligned}$$

Part 6.3

Since EU(leave) > EU(take), it is better to leave the umbrella behind.

Part 6.4

F	$\mathrm{P}(F)$
good	$= \mathrm{P(good \mid sun)\ P(sun) + P(good \mid rain)\ P(rain)} = .6 \times .7 + .4 \times .2 = .5$
bad	$= \mathrm{P(bad \mid sun)\ P(sun) + P(bad \mid rain)\ P(rain)} = .6 \times .3 + .4 \times .8 = .5$

W	F	$\mathrm{P}(W \mid F)$
sun	good	$= \frac{\mathrm{P(sun)\ P(good \mid sun)}}{\mathrm{P(good)}} = \frac{.6 \times .7}{.5} = .84$
sun	bad	$= \frac{\mathrm{P(sun)\ P(bad \mid sun)}}{\mathrm{P(bad)}} = \frac{.6 \times .3}{.5} = .36$
rain	good	$= \frac{\mathrm{P(rain)\ P(good \mid rain)}}{\mathrm{P(good)}} = \frac{.4 \times .2}{.5} = .16$
rain	bad	$= \frac{\mathrm{P(rain)\ P(bad \mid rain)}}{\mathrm{P(bad)}} = \frac{.4 \times .8}{.5} = .64$

$$\begin{aligned} \mathrm{EU(take \mid good)} &= \mathrm{U(take, sun)\ P(sun \mid good) + U(take, rain)\ P(rain \mid good)} \\ &= 20 \times .84 + 70 \times .16 \\ &= \boxed{28} \end{aligned}$$

Part 6.5

$$\begin{aligned} \mathrm{EU(leave \mid good)} &= \mathrm{U(leave, sun)\ P(sun \mid good) + U(leave, rain)\ P(rain \mid good)} \\ &= 80 \times .84 + 0 \times .16 \\ &= \boxed{67.2} \end{aligned}$$

Part 6.6

Since EU(leave | good) > EU(take | good), it is better to leave the umbrella behind.