Homework 1

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Fri 01/17/20

Problem 1

Part (a)

No, it is not possible to have $P(B) = 0.7$, given that A and B disjoint events and that $P(A) = 0.4$.

According to the second rule in the definition of a probability function, when A and B are disjoint events, we have

$$P(A \cup B) = P(A) + P(B)$$

Therefore, in this scenario when $P(A) = 0.4$ and $P(B) = 0.7$, since A and B disjoint,

$$P(A \cup B) = P(A) + P(B) = 0.4 + 0.7 = 1.1$$

However, the definition also states that a probability function must assign every event in a sample space Ω a number within $[0, 1]$. Since the event $A \cup B$ has a probabilty of $1.1 > 1$, it is not a valid event. Therefore, when $P(A) = 0.4$, it is not possible to have $P(B) = 0.7$.

Part (b)

According to the definition of the difference between sets, we have

$$A \cap B^c = A - B$$

However, A and B are disjoint events ($A \cap B = \emptyset$), all elements in A are not in B, meaning that

$$A - B = A - (A \cap B) = A - \emptyset = A$$

Therefore,

$$P(A \cap B^c) = P(A) = 0.4$$

Problem 2

Part (a)

$$\Omega^3 = \{H,T\}^3 = \{(H,H,H),(H,H,T),(H,T,H),(H,T,T),(T,H,H),(T,H,T),(T,T,H),(T,T,T)\}$$

Part (b)

1. $\{(H,T,T),(T,H,T),(T,T,H)\}$
2. $\{(H,T,T),(T,H,T),(T,T,H),(T,T,T)\}$
3. $\{(H,H,H),(H,H,T),(H,T,T),(T,T,T)\}$

Problem 3

a. $P(R^c)$

b. $P(R \cup F)$

c. $P(R^c \cup F^c)$

Problem 4

Part (a)

$$\begin{aligned} P(R^c) &= 1 - P(R) && \text{The 1}^{\text{st}}\text{ probability rule} \\ &= 1 - 0.1 \\ &= 0.9 \end{aligned}$$

Part (b)

$$\begin{aligned} P(R \cup F) &= P(R) + P(F) - P(R \cap F) && \text{The 2}^{\text{nd}}\text{ probability rule} \\ &= 0.1 + 0.07 - 0.03 \\ &= 0.14 \end{aligned}$$

Part (c)

$$\begin{aligned} P(R^c \cup F^c) &= P((R \cap F)^c) && \text{De Morgan's Law} \\ &= 1 - P(R \cap F) && \text{The 1}^{\text{st}}\text{ probability rule} \\ &= 1 - 0.03 \\ &= 0.97 \end{aligned}$$

Problem 5

Part (a)

Let:

- Ω_{bags} be the sample space of the 16 bags,
- B be the event that we choose a bag containing only **blue** balls in the beginning,
- M be the event that we choose a bag containing **mixed** blue and red balls in the beginning,
- R be the event that we choose a bag containing only **red** balls in the beginning, and
- E be the event that we choose a blue ball at the **end**.

After we have chosen a bag, since we pick a ball at random from the bag, the probabily of picking a blue ball is:

- $\frac{2}{2} = 1$, if we have chosen a bag with only blue balls [$P(E|B) = 1$];
- $\frac{1}{2}$, if we have chosen a bag with mixed blue and red balls [$P(E|M) = \frac{1}{2}$]; or
- $\frac{0}{2} = 0$, if we have chosen a bag with only red balls [$P(E|R) = 0$].

Since we first choose a one bag at random, and bags are independent from each other, we have

$$
\begin{aligned}
P(E) &= P(B) \times P(E|B) + P(M) \times P(E|M) + P(R) \times P(E|R) \\
&= \frac{|B|}{|\Omega_{\text{bags}}|} \times 1 + \frac{|M|}{|\Omega_{\text{bags}}|} \times \frac{1}{2} + \frac{|R|}{|\Omega_{\text{bags}}|} \times 0 \\
&= \frac{7}{16} \times 1 + \frac{5}{16} \times \frac{1}{2} + \frac{4}{16} \times 0 \\
&= \frac{19}{32}
\end{aligned}
$$

There is a $\frac{19}{32}$ (about 59%) chance that we have picked a blue ball at the end.

Part (b)

Let O be the event that the **other** ball is also blue.

Since O can only happen (and must happen) when we have chosen a bag containing only blue balls in the beginning, meaning that $P(O \cap E) = P(B)$, we have

$$
\begin{aligned}
P(O|E) &= \frac{P(O \cap E)}{P(E)} \\
&= \frac{P(B)}{P(E)} \\
&= \frac{7}{16} \div \frac{19}{32} \\
&= \frac{14}{19}
\end{aligned}
$$

There is a $\frac{14}{19}$ (about 74%) chance that the other ball in the bag is also blue.