NAME: _____ UID: _____

# CS 2100: Discrete Structures

Quiz 5

Spring 2020

**This is a take-home exam.**

Pleases read these instructions carefully and put your name and UID on the first page:

- The exam is take-home, open-book and open-notes. However, **please do not use the internet to search for solutions**. **Please solve the problem independently. Copying solutions from the internet or copying solutions from other students is considered academic misconduct. For academic misconduct in CS 2100, the sanction is to fail the course.**
- This exam is released online via Canvas at 8:00 a.m. on Thursday, April 9, it is due at 7:59 a.m. on Friday, April 10th, giving you 24 hours to complete it.
- Please submit the exam via Gradescope on the due date and time.
- The exam takes, on average, 80 minutes to complete. You can take as long as you want to work on the exam within the given time window (i.e., 24 hours).
- Students with CDS document can turn in exam after 48 hours, on or before Saturday, April 11, at 7:59 a.m. by emailing it as a PDF to TA Don Wang at comidon@outlook.com.
- The exam is to be done independently. Please do not discuss solutions with anyone.
- Please ask any clarifying question regarding the exam via Piazza under "quiz 5". Please do not post solutions via Piazza.
- To work on the exam, there are a few options (similar to solving homework problems):
  - You can write your answers by hand, using WORD, or using LATEX, then submit a scanned image or a PDF. We only need your solution to each problem.
  - You can also download the PDF and use Adobe or Preview (or similar tools) for editing your solutions directly on the PDF file.
  - If you are solving by hand (please use a black pen), pleases submit a scanned version in PDF or a photoed version.
  - **Please do not put more than one problem on each page**.
  - If you need more space to provide a solution to a problem, use a blank piece of paper. Do not use a single paper for more than one problem.
- **Important submission information**: the exam should be submitted via Gradescope as a PDF (preferred) or an image file. The submission process is very similar to the submission of a homework problem:
  - Please match each problem to the outline specified on Gradescope: the page number containing the answer to each individual question should be specified during submission.
  - If no pages are specified for each problem, then 20% is deducted from the final score.
  - **The submission should be legible; any problem that is not legible will not be graded and will not receive any credit.**
- **To be eligible for partial credit, you must show your work.**
- It is the student's responsibility to ensure the successful and timely submission of the exam via Gradescope— start early and follow the instructions carefully. Corrupted or missing files are not grounds for extensions — double-check your submissions and save a digital copy of all of your work in your CADE account.

- For all problems, express your solutions in terms of factorials, exponents, multiplication, division, addition, and/or subtraction. You need not simplify to the actual numerical solution. However, you may not give final solutions in terms of $P(n, r)$ and/or $C(n, r)$, please provide the actual mathematical formulation.
- For example, in stead of writing $P(10, 8)$ as a final answer, write $P(10, 8) = 10!/2! = 45$ (when the math is straightforward); instead of writing $P(100, 95)$, write at least $P(100, 95) = \frac{100!}{(100-95)!} = \frac{100!}{5!}$ (when the exact answer is hard to get without a calculator).
- For mathematical formulation you choose, please provide justifications to be eligible for partial/full credit.

1. **(10 points)**

   (a). **(5 points)** Suppose a license plate number is of length 7. The Department of Motor Vehicles is considering requiring that each license plate must consist of 3 upper case letters (A to Z) followed by 4 digits (0 to 9). How many different license plate numbers are possible?

   (b). **(5 points)** A game consists of selecting a single-digit even positive integer, flipping a coin, and drawing a card from a standard poker deck (52 cards). How many possible outcomes does this game have?

   **Solution:**

2. **(10 points)**

   (a). **(5 points)** In how many ways can a movie critic make a list of the top 10 movies (ranked 1 to 10) out of a set of 200 movies?

   (b). **(5 points)** How many 4-digits (base 10) numbers have strictly decreasing digits? For example, "5432" and "9863" satisfy the requirement, "2351" and "3321" does not.

   **Solution:**

3. **(10 points)** Find the number of ways that 5 math books, 4 history books, 3 chemistry books, and 2 sociology books can be arranged on a shelf so that all books of the same subject are together. Assume all books have different titles.

   **Solution:**

4. **(10 points)** Find the number of distinct permutations that can be formed from all the letters of each word.

   (a). **(5 points)** RADAR.

   (b). **(5 points)** UNUSUAL.

   **Solution:**

5. **(10 points)** In how many ways can 3 or more persons be selected from 12 persons?

   **Solution:**

6. **(10 points)** There are 8 men and 4 women that work in an office, and the 2 promotions that were recently announced both went to men. If the 2 promoted had been randomly selected, then what is the probability they both would have been men?

   **Solution:**

7. **(10 points)** A home security code consists of 4 digits (0-9). If all such codes are equally likely to be chosen, then what is the probability that a randomly chosen code will contain exactly one "3" and exactly two "6"s?
   Hint: Let $E$ and $F$ be two events, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$, where $P(E)$ denotes the probability of $E$ happening.

   **Solution:**

8. **(10 points)** At a picnic, two teams of kids played a soccer game, after which one player's name was randomly chosen to win a prize. There are 22 kids in total. The winning team in the soccer game consists of 7 girls and 4 boys, and the losing team consists of 5 girls and 6 boys. Given that the prize winner is a boy, what is the probability that he also comes from the winning soccer team?

   Hint: Let $E$ and $F$ be events in a sample space $S$ with $P(F) > 0$. The conditional probability of $E$ given $F$ is $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{n(E \cap F)}{n(F)}$.

   **Solution:**

9. **(10 points)**

(a) **(3 points)** Suppose we want to fill a bag with 10 pieces of fruit from 4 possible fruits: apples, pears, oranges, and lemons. Describe the binary sequence that would represent the following bags:

(1) 2 apples, 1 pear, 4 oranges, and 3 lemons

(2) 3 apples, 2 pears, and 5 oranges

(3) 10 lemons

(b) **(3 points)** How many ways are there to fill a bag with 10 pieces of fruit? You can have one to four kinds of fruit.

(c) **(4 points)** How many ways are there to fill this bag with 10 pieces of fruit if we have the constraint that there are either oranges or lemons in the bag but never both? In other words, there may be oranges or lemons or neither of them but never both.

**Solution:**

10. **(10 points)**

(a). **(5 points)** Suppose 6 cards are drawn from a standard deck. What is the expected number of kings among the 6 cards?
Hint: Expectation in Bernoulli trials.

(b). **(5 points)** Suppose that team $A$ has a 60% chance of winning any given game against team B. What is the probability that team A wins a best-of-five series against team B?
Hint: team A needs to win 3 games in a best-of-five series. Solve by case: A wins after 3, 4, and 5 games. Please provide justifications to your solution.

**Solution:**