

1 Inference

Given the boolean random variables X , Y , and Z , and the following model:

$$P(+x) = 0.3$$

$$P(-x) = 0.7$$

$$P(+y|+x) = 0.8$$

$$P(+y|-x) = 0.3$$

$$P(+z|+x) = 0.8$$

$$P(+z|-x) = 0.4$$

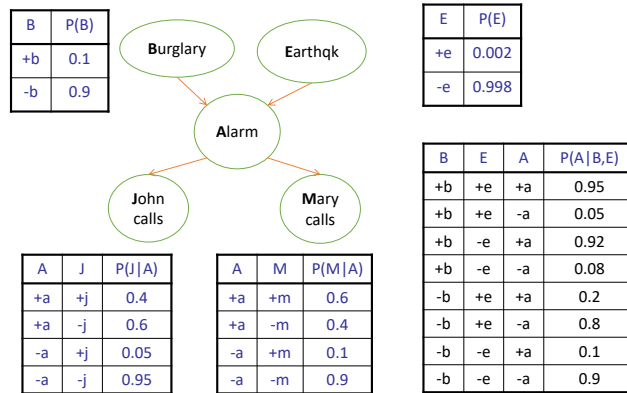
$Y \perp\!\!\!\perp Z \mid X$ (i.e., Y and Z are conditionally independent, given X)

Please compute (and show your work):

1. $P(-y|+x)$
2. $P(-z|-x)$
3. $P(-y|+x,+z)$
4. $P(+y,-z,-x)$
5. $P(-y)$
6. $P(+y|+z)$
7. $P(-y|+z)$

2 Joint Probabilities from BNs

Consider the following Bayes' Net:

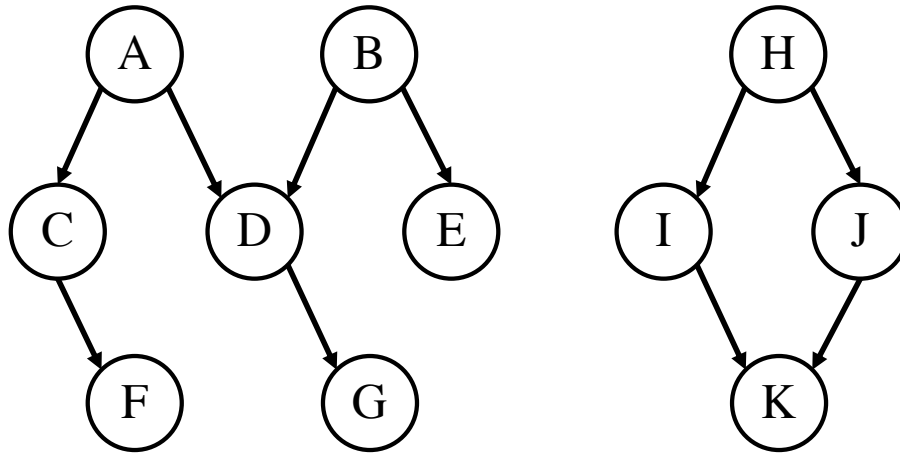


Please compute (and show your work):

1. $P(+b, -e, +a, +j, +m)$
2. $P(-b, -e, +a, -j, -m)$
3. $P(-b, +e, +a, +j, +m)$
4. $P(-b, -e, -a, -j, -m)$

3 D-Separation

Consider the following Bayes' Net:

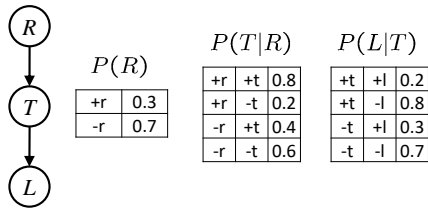


Please answer the following with 'yes', 'no', or 'not enough information'. **Hint: review the lectures for why I've included 'not enough information' as an option.**

1. Is A independent of B?
2. Is A independent of B given D?
3. Is A independent of I?
4. Is A independent of F?
5. Is A independent of B given G?
6. Is D independent of E?
7. Is D independent of E given B?
8. Is I independent of J?
9. Is I independent of J given H?

4 Variable Elimination

Consider the following Bayes' Net:

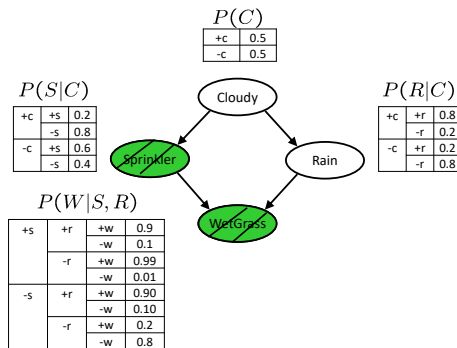


We are performing variable elimination in order to determine $P(L)$. Please do so using the following steps, showing your work and all intermediate factors.

1. Perform a join on R to determine $P(R, T)$.
2. Sum out R from $P(R, T)$ to determine $P(T)$.
3. Perform a join on T to determine $P(T, L)$.
4. Sum out T from $P(T, L)$ to determine $P(L)$.

5 Sampling

Consider the following Bayes' Net:



We are performing likelihood weighting sampling, in which we have fixed Sprinkler to +s and WetGrass to +w.

Please answer the following questions, showing your work.

What should the associate weight be for the following samples?

1. (+c, +s, +r, +w)
2. (-c, +s, +r, +w)
3. (+c, +s, -r, +w)
4. (-c, +s, -r, +w)

Let's assume we have drawn these 5 random samples:

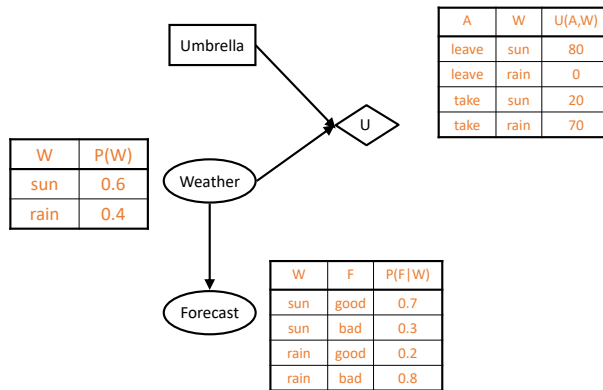
- (+c, +s, +r, +w)
- (-c, +s, +r, +w)
- (-c, +s, +r, +w)
- (-c, +s, -r, +w)
- (-c, +s, -r, +w)

What will our estimate be for

5. $P(+c|+s,+w)$?
6. $P(-c|+s,+w)$?

6 Decision Networks

Consider the following decision network:



Please answer the following questions, showing your work.

1. What is the expected utility of taking an umbrella, given no evidence?
2. What is expected utility of leaving the umbrella behind, given no evidence?
3. What is the optimal decision, taking or leaving the umbrella, given no evidence.
4. What is the expected utility of taking an umbrella, given that Forecast = good?
5. What is the expected utility of leaving the umbrella behind, given that Forecast = good?
6. What is the optimal decision, leave or take the umbrella, given that Forecast = good.