Homework 3: RL and Probability

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Problem 1

Part 1.1

As follows *(note that if a state does not appear in the following table, we don't have enough information about it and its value is unknown/undefined)*:

s	$V(s)$
$(1,1)$	$(93+95+95)/3 \approx 94.33$
$(1,2)$	$(96+96)/2 = 96$
$(1,3)$	$(97+97)/2 = 97$
$(2,1)$	$94/1 = 94$
$(2,3)$	$(98+98)/2 = 98$
$(3,3)$	$(99+99)/2 = 99$
$(4,3)$	$(100+100)/2 = 100$

Part 1.2

As follows:

s	a	s'	$T(s, a, s')$
$(1, 1)$	right	$(2, 1)$	$1/1 = 1$
$(1, 1)$	up	$(1, 2)$	$2/2 = 1$
$(1, 2)$	up	$(1, 3)$	$2/2 = 1$
$(1, 3)$	right	$(2, 3)$	$1/1 = 1$
$(1, 3)$	up	$(2, 3)$	$1/1 = 1$
$(2, 1)$	left	$(1, 1)$	$1/1 = 1$
$(2, 3)$	right	$(3, 3)$	$2/2 = 1$
$(3, 3)$	right	$(4, 3)$	$2/2 = 1$
$(4, 3)$	exit	(done)	$2/2 = 1$

Part 1.3

By initializing all states with a value larger than the maximum theoretical value, we're essentially assuming that all unvisited states are *better* than any state we've ever reached. This forces the agent to try and explore all unvisited states until the values of all reachable states are properly updated, after which the values truly reflect what can theoretically happen, wiping out our initial assumption and revealing the best policy.

Problem 2

Part 2.1

Weather and Temperature.

Part 2.2

{cold, warm}

Part 2.3

0.4

Part 2.4

According to the Law of Total Probability,

$$\begin{aligned}
\mathrm{P}(\mathrm{Weather}=\mathrm{sun}) &= \mathrm{P}(\mathrm{Weather}=\mathrm{sun}, \mathrm{Temperature}=\mathrm{cold}) \\
&\quad + \mathrm{P}(\mathrm{Weather}=\mathrm{sun}, \mathrm{Temperature}=\mathrm{warm}) \\
&= 0.3 + 0.2 \\
&= \boxed{0.5}
\end{aligned}$$

Part 2.5

$$\begin{aligned}
\mathrm{P}(\mathrm{Weather}=\mathrm{snow}) &= \mathrm{P}(\mathrm{Weather}=\mathrm{snow}, \mathrm{Temperature}=\mathrm{cold}) \\
&\quad + \mathrm{P}(\mathrm{Weather}=\mathrm{snow}, \mathrm{Temperature}=\mathrm{warm}) \\
&= 0.1 + 0.4 \\
&= 0.5 \\
\mathrm{P}(\mathrm{Weather}=\mathrm{cold}) &= \mathrm{P}(\mathrm{Weather}=\mathrm{snow}, \mathrm{Temperature}=\mathrm{cold}) \\
&\quad + \mathrm{P}(\mathrm{Weather}=\mathrm{sun}, \mathrm{Temperature}=\mathrm{cold}) \\
&= 0.1 + 0.3 \\
&= 0.4
\end{aligned}$$

According to the Inclusion-Exclusion Principle,

$$
\begin{aligned}
&\mathrm{P}(\text{Weather} = \text{snow} \cup \text{Temperature} = \text{cold}) \\
&= \mathrm{P}(\text{Weather} = \text{snow}) + \mathrm{P}(\text{Temperature} = \text{cold}) - \mathrm{P}(\text{Weather} = \text{snow}, \text{Temperature} = \text{cold}) \\
&= 0.5 + 0.4 - 0.1 \\
&= \boxed{0.8}
\end{aligned}
$$

Problem 3

Part 3.1

$$\begin{aligned}\mathrm{P}(+x \mid +y) &= \frac{\mathrm{P}(+x, +y)}{\mathrm{P}(+y)} \\ &= \frac{0.4}{0.4 + 0.3} \\ &= \boxed{\frac{4}{7}}\end{aligned}$$

Part 3.2

$$\begin{aligned}\mathrm{P}(-x \mid +y) &= \frac{\mathrm{P}(-x, +y)}{\mathrm{P}(+y)} \\ &= \frac{0.3}{0.4 + 0.3} \\ &= \boxed{\frac{3}{7}}\end{aligned}$$

Part 3.3

$$\begin{aligned}\mathrm{P}(+x \mid -y) &= \frac{\mathrm{P}(+x, -y)}{\mathrm{P}(-y)} \\ &= \frac{0.1}{0.1 + 0.2} \\ &= \boxed{\frac{1}{3}}\end{aligned}$$

Part 3.4

$$\begin{aligned}\mathrm{P}(-x \mid -y) &= \frac{\mathrm{P}(-x, -y)}{\mathrm{P}(-y)} \\ &= \frac{0.2}{0.1 + 0.2} \\ &= \boxed{\frac{2}{3}}\end{aligned}$$

Part 3.5

$$\begin{aligned} \mathrm{P}(-y \mid +x) &= \frac{\mathrm{P}(-y, +x)}{\mathrm{P}(+x)} \\ &= \frac{0.1}{0.4 + 0.1} \\ &= \boxed{\frac{1}{5}} \end{aligned}$$

Part 3.6

???

Problem 4

First, let's calculate the probability/proportion of time that the dog is barking using the Law of Total Probability:

$$\begin{aligned}
\mathrm{P}(+b) &= \mathrm{P}(+b, m) + \mathrm{P}(+b, q) + \mathrm{P}(+b, n) \\
&= \mathrm{P}(+b \mid m)\ \mathrm{P}(m) + \mathrm{P}(+b \mid q)\ \mathrm{P}(q) + \mathrm{P}(+b \mid n)\ \mathrm{P}(n) \\
&= 0.8 \times 0.1 + 0.9 \times 0.01 + 0.4 \times 0.89 \\
&= 0.445
\end{aligned}$$

Therefore, the dog barks 44.5% of the time or in other words, the dog has a 44.5% chance of barking given any situation.

Now, let's use Bayes' Theorem to find the likelihood of events happening given the evidence of the dog barking:

$$\begin{aligned}
\mathrm{P}(m \mid +b) &= \frac{\mathrm{P}(+b \mid m)\ \mathrm{P}(m)}{\mathrm{P}(+b)} = \frac{0.8 \times 0.1}{0.445} \approx 18\% \\
\mathrm{P}(q \mid +b) &= \frac{\mathrm{P}(+b \mid q)\ \mathrm{P}(q)}{\mathrm{P}(+b)} = \frac{0.9 \times 0.01}{0.445} \approx 2\% \\
\mathrm{P}(n \mid +b) &= \frac{\mathrm{P}(+b \mid n)\ \mathrm{P}(n)}{\mathrm{P}(+b)} = \frac{0.8 \times 0.89}{0.445} \approx 80\%
\end{aligned}$$

Since 80% of the time there's really nothing happening when the dog is barking, you should just ignore the dog. Though, as 80% isn't very confident, you should also try getting more evidence to help narrow down the situation.