Homework 3

Qianlang Chen

Section 3.1 (p193)

1. Exercise 2

Part (a)

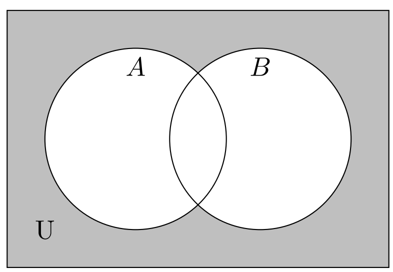
4, 8, 12, 16, 20.

Part (c)

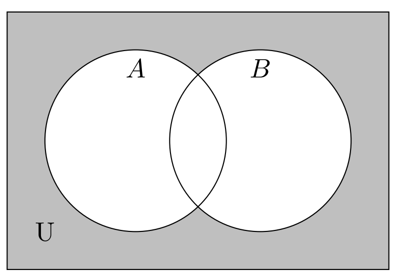
4, 8, 12, 16, 20.

2. Exercise 16

Part (b)



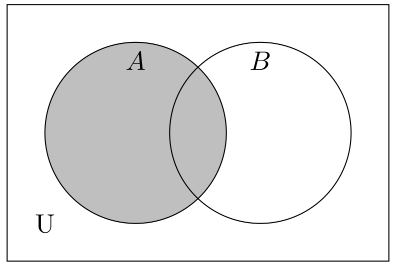
The Left-hand side: $(A \cup B)'$



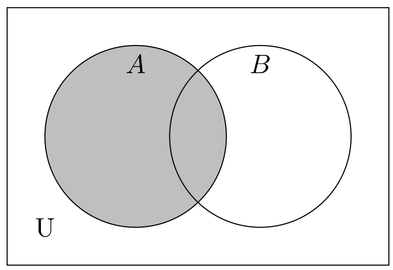
The Right-hand side: $A' \cap B'$

Therefore, since the Venn diagrams for both sides are the same, $(A \cup B)' = A' \cap B'$.

Part (d)



The Left-hand side: $A \cap (A \cup B)$

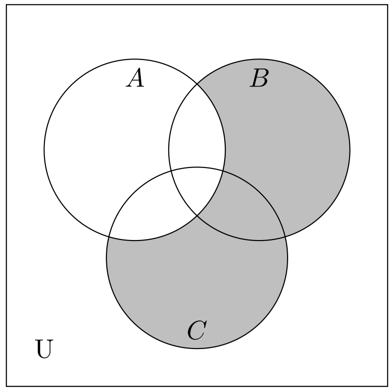


The Right-hand side: A

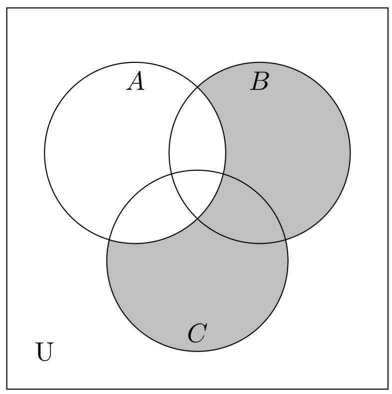
Therefore, since the Venn diagrams for both sides are the same, $A \cap (A \cup B) = A$.

3. Exercise 17

Part (b)



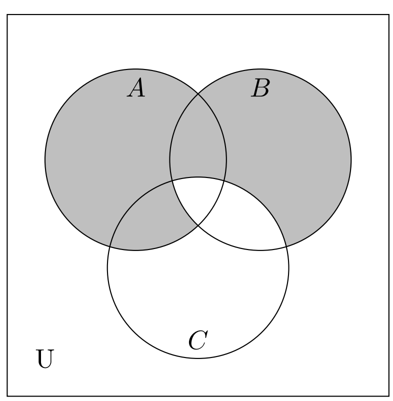
The Left-hand side: $(B \cup C) - A$



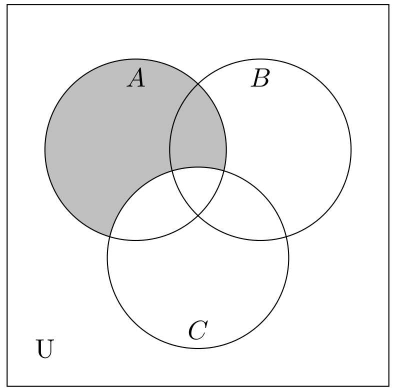
The Right-hand side: $(B - A) \cup (C - A)$

Therefore, since the Venn diagrams for both sides are the same, $(B \cup C) - A = (B - A) \cup (C - A)$.

Part (d)

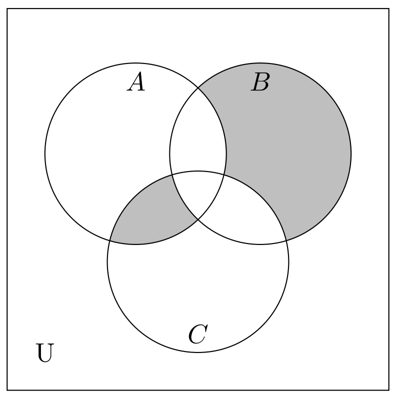


The Left-hand side: $(A - B) \cup (B - C)$



The Right-hand side: $A - C$

As seen from the Venn diagrams, there are some areas covered by the left-hand side but not the right-hand side. Specifically, these are the mentioned areas:



Therefore, the left-hand side may not be a subset of the right-hand side. $(A - B) \cup (B - C) \nsubseteq A - C$. As an example, if

$$A = \{1, 2, 3\},$$
$$B = \{4, 5, 6\},$$
$$C = \{7, 8, 9\},$$

then

$$(A - B) \cup (B - C) = \{1, 2, 3, 4, 5, 6\},$$
$$A - C = \{1, 2, 3\};$$
$$\{1, 2, 3, 4, 5, 6\} \nsubseteq \{1, 2, 3\}$$

4. Exercise 18

Part (b)

$$A - B = \{k : k \in \mathbb{Z}, k \bmod 6 \neq 0\}$$

Part (e)

$$\mathbb{Z} - A = \{2k + 1 : k \in \mathbb{Z}\}$$

Section 3.2 (p208)

5. Exercise 1

Part (b)

$$\begin{aligned}(A \times B) - (A \times A) &= \{2,4\} \times \{1,2,8\} - \{2,4\} \times \{2,4\} \\ &= \{(2,1),(2,2),(2,8),(4,1),(4,2),(4,8)\} - \{(2,2),(2,4),(4,2),(4,4)\} \\ &= \boxed{\{(2,1),(2,8),(4,1),(4,8)\}}\end{aligned}$$

Part (d)

$$\begin{aligned}\wp(B \cap C) &= \wp(\{1,2,8\} \cap \{1,2,5,6,10\}) \\ &= \wp(\{1,2\}) \\ &= \boxed{\{\ \{\}, \{1\}, \{2\}, \{1,2\}\ \}}\end{aligned}$$

6. Exercise 9

According to the division theorem, every integer falls into one of the following categories:

- $4q$,
- $4q+1$,
- $4q+2$,
- $4q+3$,

where $q \in \mathbb{Z}$. In other words, $\mathbb{Z} = B \cup C \cup S \cup T$, where

- $B = \{4q+1 : q \in \mathbb{Z}\}$ (given by the problem statement),
- $C = \{4q+3 : q \in \mathbb{Z}\}$ (given by the problem statement),
- $S = \{4q : q \in \mathbb{Z}\}$,
- $T = \{4q+2 : q \in \mathbb{Z}\}$.

Also, to show that $A = \{2k : k \in \mathbb{Z}\} = S \cup T$, since every integer is either even or odd,

- when k is even, $2k = 2(2q) = 4q$, where q is an integer by definition of even;
- when k is odd, $2k = 2(2q+1) = 4q+2$, where q is an integer by definition of odd.

Therefore, we showed that $A = S \cup T$. Now, we have $\mathbb{Z} = B \cup C \cup S \cup T = A \cup B \cup C$.

According to the definition of a partition, and

- $A \neq B \neq C \neq \emptyset$,
- $A \cap B = A \cap C = B \cap C = \emptyset$,
- $A \cup B \cup C = \mathbb{Z}$,

$\{A, B, C\}$ is a partition of $\mathbb{Z}$.

7. Exercise 11

Part (a)

True.

Part (b)

False. One counter-example: if $A = \{1\}$ and $B = \{2\}$,

$$\begin{aligned}
(A \cup B) \times (A - B) &= \{1, 2\} \times \{1\} \\
&= \{(1, 1), (2, 1)\}; \\
A^2 - B^2 &= \{(1, 1)\} - \{(2, 2)\} \\
&= \{(1, 1)\}; \\
(A \cup B) \times (A - B) &\neq A^2 - B^2
\end{aligned}$$

Part (c)

True.

8. Exercise 22

According to *Theorem 1*, $|A \times B| = |A|\ |B|$. Therefore,

$$\begin{aligned}
|S_1 \times S_2 \times \cdots \times S_{k-1} \times S_k| &= |S_1|\ |S_2|\ \cdots\ |S_{k-1}|\ |S_k| \\
&= (|S_1|\ |S_2|\ \cdots\ |S_{k-1}|)\ |S_k| \\
&= |S_1 \times S_2 \times \cdots \times S_{k-1}|\ |S_k| \\
&= \boxed{|(S_1 \times S_2 \times \cdots \times S_{k-1}) \times S_k|}
\end{aligned}$$

Section 3.3 (p219)

9. Exercise 2.e

10. Exercise 3.b

11. Exercise 11.d

12. Exercise 13.c

13. Exercises 14.e and 15.e

Exercise 14.e

Exercise 15.e

14. Exercise 19.b

15. Exercies 22.a

Section 3.4 (p227)

16. Exercise 1.d

17. Exercise 2.f

18. Exercise 6.b

19. Exercise 14