Exercise 4

(1) Since $10 \equiv 1 \mod 9$,

$$n = a_0 + 10a_1 + 10^2 a_2 + \cdots + 10^M a_M$$

$$\equiv a_0 + 1a_1 + 1^2 a_2 + \cdots + 1^M a_M \mod 9$$

$$\equiv a_0 + a_1 + a_2 + \cdots + a_M \mod 9$$

$$\equiv \sum_{i=0}^{M} a_i \mod 9$$

Therefore, $n \equiv 0 \mod 9$ iff $\sum_{i=0}^{M} a_i \equiv 0 \mod 9$,

meaning $9 \mid n$ iff $9 \mid \sum_{i=0}^{M} a_i$. $\square$

(2) Since $10 \equiv -1 \mod 11$,

$$n = a_0 + 10a_1 + 10^2 a_2 + \cdots + 10^M a_M$$

$$\equiv a_0 + (-1)a_1 + (-1)^2 a_2 + \cdots + (-1)^M a_M \mod 11$$

$$\equiv \sum_{i=0}^{M} (-1)^i a_i \mod 11$$

Therefore, $n \equiv 0 \mod 11$ iff $\sum_{i=0}^{M} (-1)^i a_i \equiv 0 \mod 11$,

meaning $11 \mid n$ iff $11 \mid \sum_{i=0}^{M} (-1)^i a_i$. $\square$

(3) In this case, $n =$ "21" $\times 11$, meaning that

sum of digits of $n = (2+1) \times 11 = 33$.

According to parts (1) and (2),

$$9 \nmid 33 \Rightarrow 9 \nmid n,$$

$$11 \mid 33 \Rightarrow 11 \mid n. \quad \square$$

Exercise 5

(1)

$$1979 = 15 \times 131 + 14$$
$$131 = 9 \times 14 + 5$$
$$14 = 2 \times 5 + 4$$
$$5 = 1 \times 4 + 1$$
$$4 = 4 \times 1 + 0$$

$$1979/131 = [15; 9, 2, \overset{1}{\cancel{4}}, 4].$$

Meanwhile,

$$[15; 9, 2, 1] = 423/28,$$

and $1979 \times (-28) + 131 \times 423 = 1$

$$\Rightarrow 131 \times 423 \equiv 1 \mod 1979$$

$$\Rightarrow \boxed{423}$$

(2) Given part (1),

$$131 x \equiv 11 \mod 1979$$
$$423 \cdot 131 x \equiv 423 \cdot 11 \mod 1979$$
$$x \equiv 4653 \mod 1979 \equiv \boxed{695} \mod 1979$$

(3)

$$1091 = 8 \times 127 + 75$$
$$127 = 1 \times 75 + 52$$
$$75 = 1 \times 52 + 23$$
$$52 = 2 \times 23 + 6$$
$$23 = 3 \times 6 + 5$$
$$6 = 1 \times 5 + 1$$

$$\Rightarrow [8; 1, 1, 2, 3, 1] = 189/22,$$

and $1091 \times (-22) + 127 \times 189 = 1$

$$\Rightarrow 127 \times \boxed{189} \equiv 1 \mod 1091$$

(4)

$$127 x \equiv 11 \mod 1091$$
$$189 \cdot 127 x \equiv 189 \cdot 11 \mod 1091$$
$$x \equiv 2079 \mod 1091$$
$$\equiv \boxed{988} \mod 1091$$

(5) $x \equiv 4 \bmod 55 \Rightarrow x = 55y + 4$

$x \equiv 11 \bmod 69 \Rightarrow 55y + 4 \equiv 11 \bmod 69$

$\Rightarrow 55y \equiv 7 \bmod 69$

Run Euclidean (tabular):

r:	66	55	14	13	1
q:		1	3	1	13

$\Rightarrow [1; 3, 1] = 5/4$

$\Rightarrow 69 \times 4 + 55 \times (-5) = 1$

$\Rightarrow 55 \times (-5) \equiv 1 \bmod 69$

$55y \equiv 7 \bmod 69 \Rightarrow y \equiv (-35) \bmod 69 \equiv 34 \bmod 69$

$\Rightarrow x = 55 \times 34 + 4 = \boxed{1874}$

(6) $x \equiv 5 \bmod 11 \Rightarrow x = 11y + 5 \Rightarrow 11y \equiv 2 \bmod 13$

r:	13	11	2	1
q:		1	5	2

$\Rightarrow [1; 5] = 6/5 \Rightarrow 11 \times 6 \equiv 1 \bmod 13$

$\Rightarrow y \equiv 6 \times 2 \bmod 13 \equiv 12 \bmod 13 \Rightarrow x = 11 \times 12 + 5 = \boxed{137}$

(7) $x \equiv 11 \bmod 16 \Rightarrow x = 16y + 11 \Rightarrow 16y \equiv 5 \bmod 27$

r:	27	16	11	5	1
q:		1	1	2	5

$\Rightarrow [1; 1, 2] = 5/3 \Rightarrow 16 \times (-5) \equiv 1 \bmod 27$

$\Rightarrow y \equiv (-5) \times 5 \bmod 27 \equiv 2 \bmod 27 \Rightarrow x = 16 \times 2 + 11 = \boxed{43}$