Homework: Drag Forces

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Problem 2

Assuming the mass of the ball is m.

Part (a)

$$
\begin{aligned}
F_{\text{net}x} &= -F_{\text{drag}x} \\
m\left(\frac{\mathrm{d}v_x}{\mathrm{d}t}\right) &= -kv_x \\
\frac{1}{v_x}\,\mathrm{d}v_x &= -\frac{k}{m}\,\mathrm{d}t \\
\int \frac{1}{v_x}\,\mathrm{d}v_x &= \int -\frac{k}{m}\,\mathrm{d}t \\
\ln v_x - \ln v_{0x} &= -\frac{kt}{m} \\
\frac{v_x}{v_{0x}} &= \exp\left(\frac{kt}{m}\right) \\
v_x &= \boxed{v_{0x}\exp\left(\frac{kt}{m}\right)}
\end{aligned}
$$

Part (b)

$$
\begin{aligned}
F_{\text{net}y} &= F_g - F_{\text{drag}y} \\
m\left(\frac{\mathrm{d}v_y}{\mathrm{d}t}\right) &= mg - kv_y \\
\frac{\mathrm{d}v_y}{\mathrm{d}t} &= g - \frac{kv_y}{m} \\
\mathrm{d}v_y &= -\frac{k}{m}\left(-\frac{mg}{k} + v_y\right)\,\mathrm{d}t \\
\frac{1}{v_y - mg/k}\,\mathrm{d}v_y &= -\frac{k}{m}\,\mathrm{d}t \\
\int \frac{1}{v_y - mg/k}\,\mathrm{d}v_y &= \int -\frac{k}{m}\,\mathrm{d}t \\
\ln\left(v_y - \frac{mg}{k}\right) - \ln\left(v_{0y} - \frac{mg}{k}\right) &= -\frac{kt}{m} \\
\left(v_y - \frac{mg}{k}\right) / \left(v_{0y} - \frac{mg}{k}\right) &= \exp\left(-\frac{kt}{m}\right) \\
v_y &= \boxed{\frac{mg}{k} + \left(v_{0y} - \frac{mg}{k}\right)\exp\left(-\frac{kt}{m}\right)}
\end{aligned}
$$

Problem 4

Part (b)

$$
\begin{aligned}
F_{\text{net}} &= -F_{\text{drag}} \\
m\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right) &= -\frac{k}{m}\,\mathrm{d}t \\
\frac{1}{\sqrt{v}}\,\mathrm{d}v &= -\frac{k}{m}\,\mathrm{d}t \\
\int \frac{1}{\sqrt{v}}\,\mathrm{d}v &= \int -\frac{k}{m}\,\mathrm{d}t \\
2\sqrt{v} - 2\sqrt{v_0} &= -\frac{kt}{m} \\
v &= \boxed{\left(\sqrt{v_0} - \frac{kt}{2m}\right)^2}
\end{aligned}
$$