Homework 3

Qianlang Chen

Problem 1

Part (a)

- **Upper bound:** when all four coins land with heads up: 4 heads − 0 tails = $\boxed{4}$;
- **Lower bound:** when all four coins land with tails up: 0 heads − 4 tails = $\boxed{-4}$.

Part (b)

$$\text{pmf}(X=k)=\begin{cases}\binom{4}{0}\cdot(1-\frac{1}{2})^4=\frac{1}{16}, k=-4\\ \binom{4}{1}\cdot\frac{1}{2}\cdot(1-\frac{1}{2})^3=\frac{1}{4}, k=-2\\ \binom{4}{2}\cdot(\frac{1}{2})^2\cdot(1-\frac{1}{2})^2=\frac{3}{8}, k=0\\ \binom{4}{3}\cdot(\frac{1}{2})^3\cdot(1-\frac{1}{2})=\frac{1}{4}, k=2\\ \binom{4}{4}\cdot(\frac{1}{2})^4=\frac{1}{16}, k=4\\ 0, k\notin\{-4,-2,0,2,4\}\end{cases}$$

Part (c)

$$\text{cdf}(X\le k)=\begin{cases}0, k<-4\\ \text{pmf}(X=-4)=\frac{1}{16}, -4\le k<-2\\ \text{pmf}(X=-4)+\text{pmf}(X=-2)=\frac{5}{16}, -2\le k<0\\ \text{pmf}(X=-4)+\text{pmf}(X=-2)+\text{pmf}(X=0)=\frac{11}{16}, 0\le k<2\\ \text{pmf}(X=-4)+\text{pmf}(X=-2)+\text{pmf}(X=0)+\text{pmf}(X=2)=\frac{15}{16}, 2\le k<4\\ 1, k\ge 4\end{cases}$$

Problem 2

todo haha

Problem 3

Part (a)

- **Upper bound:** all n coins land with heads up: $\boxed{n}$;
- **Lower bound:** all n coins land with tails up; tossing them all again, all tails up again: $\boxed{0}$.

Part (b)

Let us consider the outcome X for just a single coin. There are two possibilities:

- The coin lands with heads up. This can happen in one of two ways: landing heads on the first toss (probability $\frac{1}{2}$), or landing tails on the first toss but heads on the second toss (probability $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$). Total probability for heads: $\dfrac{1}{2} + \dfrac{1}{4} = \boxed{\dfrac{3}{4}}$.
- The coin lands with tails up. This can only happen in one way: landing tails on both the first and second tosses. Total probability for tails: $\dfrac{1}{2} \cdot \dfrac{1}{2} = \boxed{\dfrac{1}{4}}$.

Now, when there are n coins, since each coin has a certain chance of producing heads – incrementing the random variable X, we have

$$\begin{aligned}
\text{pmf}(X = k) &= \text{Prob of } k \text{ heads and } (n-k) \text{ tails} \times \text{Number of ways this can happen} \\
&= (\frac{3}{4})^k \cdot (\frac{1}{4})^{(n-k)} \cdot \binom{n}{k}
\end{aligned}$$

This fits the form of a **binomial distribution.**

Problem 4

Since the recognition system can only either pass or fail and the pass rate is $\frac{9}{10}$, it means that the probability of failing is $1 - \frac{9}{10} = \frac{1}{10}$.

Mathematical Expressions:

a. $\binom{40}{6} \cdot (\frac{1}{10})^6 \cdot (\frac{9}{10})^{(40-6)} \approx 11\%$

b. $\sum_{n=0}^{2} \binom{40}{n} \cdot (\frac{1}{10})^n \cdot (\frac{9}{10})^{(40-n)} \approx 22\%$

c. $\sum_{n=8}^{40} \binom{40}{n} \cdot (\frac{1}{10})^n \cdot (\frac{9}{10})^{(40-n)} \approx 4.2\%$

R Expressions:

a. `dbinom(6, 40, 1/10)`

b. `pbinom(2, 40, 1/10)`

c. `pbinom(8, 40, 1/10, FALSE) + dbinom(8, 40, 1/10)`