

Homework 4: Bayes' Nets

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Problem I: Inference

Part I.1

$$P(-y \mid +x) = 1 - P(+y \mid +x) = 1 - .8 = \boxed{.2}$$

Part I.2

$$P(-z \mid -x) = 1 - P(+z \mid -x) = 1 - .4 = \boxed{.6}$$

Part I.3

$$P(-y \mid +x, +z) = P(-y \mid +x) = \boxed{.2}$$

Part I.4

$$\begin{aligned} P(+y, -z, -x) &= P(+y, -z \mid -x) P(-x) \\ &= P(+y \mid -x) P(-z \mid -x) P(-x) \\ &= .3 \times .6 \times .7 \\ &= \boxed{.126} \end{aligned}$$

Part I.5

$$\begin{aligned}P(-y) &= 1 - P(+y) \\&= 1 - (P(+y, +x) + P(+y, -x)) \\&= 1 - (P(+y \mid +x) P(+x) + P(+y \mid -x) P(-x)) \\&= 1 - (.8 \times .3 + .3 \times .7) \\&= \boxed{.55}\end{aligned}$$

Part I.6

$$\begin{aligned}P(+y, +z) &= P(+y, +z, +x) + P(+y, +z, -x) \\&= P(+y \mid +z, +x) P(+z, +x) + P(+y \mid +z, -x) P(+z, -x) \\&= P(+y \mid +x) P(+z \mid +x) P(+x) + P(+y \mid -x) P(+z \mid -x) P(-x) \\&= .8 \times .8 \times .3 + .3 \times .4 \times .7 \\&= .276\end{aligned}$$

$$\begin{aligned}P(+z) &= P(+z, +x) + P(+z, -x) \\&= P(+z \mid +x) P(+x) + P(+z \mid -x) P(-x) \\&= .8 \times .3 + .4 \times .7 \\&= .52\end{aligned}$$

$$\begin{aligned}P(+y \mid +z) &= \frac{P(+y, +z)}{P(+z)} \\&= \frac{.276}{.52} \\&\approx \boxed{.5308}\end{aligned}$$

Part I.7

$$\begin{aligned}P(-y \mid +z) &= 1 - P(+y \mid +z) \\&\approx 1 - .5308 \\&\approx \boxed{.4692}\end{aligned}$$

Problem 2: Joint Probabilities

Part 2.1

$$\begin{aligned}P(+b, -e, +a, +j, +m) &= P(+b) P(-e) P(+a \mid +b, -e) P(+j \mid +a) P(+m \mid +a) \\&= .1 \times .998 \times .92 \times .4 \times .6 \\&\approx \boxed{.02204}\end{aligned}$$

Part 2.2

$$\begin{aligned}P(-b, -e, +a, -j, -m) &= P(-b) P(-e) P(+a \mid -b, -e) P(-j \mid +a) P(-m \mid +a) \\&= .9 \times .998 \times .1 \times .6 \times .4 \\&\approx \boxed{.02156}\end{aligned}$$

Part 2.3

$$\begin{aligned}P(-b, +e, +a, +j, +m) &= P(-b) P(+e) P(+a \mid -b, +e) P(+j \mid +a) P(+m \mid +a) \\&= .9 \times .002 \times .2 \times .4 \times .6 \\&= \boxed{8.64 \times 10^{-5}}\end{aligned}$$

Part 2.4

$$\begin{aligned}P(-b, -e, -a, -j, -m) &= P(-b) P(-e) P(-a \mid -b, -e) P(-j \mid -a) P(-m \mid -a) \\&= .9 \times .998 \times .9 \times .95 \times .9 \\&\approx \boxed{.6912}\end{aligned}$$

Problem 3: D-Separation

Part 3.1

Yes

Part 3.2

Not enough information

Part 3.3

Yes

Part 3.4

Not enough information

Part 3.5

Not enough information

Part 3.6

Not enough information

Part 3.7

Yes

Part 3.8

Not enough information

Part 3.9

Yes

Problem 4: Variable Elimination

Part 4.1

R	T	$P(R, T)$
$+r$	$+t$	$= P(+r) \times P(+t \mid +r) = .3 \times .8 = .24$
$+r$	$-t$	$= P(+r) \times P(-t \mid +r) = .3 \times .2 = .06$
$-r$	$+t$	$= P(-r) \times P(+t \mid -r) = .7 \times .4 = .28$
$-r$	$-t$	$= P(-r) \times P(-t \mid -r) = .7 \times .6 = .42$

Part 4.2

T	$P(T)$
$+t$	$= P(+r, +t) + P(-r, +t) = .24 + .28 = .52$
$-t$	$= P(+r, -t) + P(-r, -t) = .06 + .42 = .48$

Part 4.3

T	L	$P(T, L)$
$+t$	$+l$	$= P(+t) \times P(+l \mid +t) = .52 \times .2 = .104$
$+t$	$-l$	$= P(+t) \times P(-l \mid +t) = .52 \times .8 = .416$
$-t$	$+l$	$= P(-t) \times P(+l \mid -t) = .48 \times .3 = .144$
$-t$	$-l$	$= P(-t) \times P(-l \mid -t) = .48 \times .7 = .336$

Part 4.4

L	$P(L)$
$+l$	$= P(+t, +l) + P(-t, +l) = .104 + .144 = .248$
$-l$	$= P(+t, -l) + P(-t, -l) = .416 + .336 = .752$

Problem 5: Sampling

Part 5.1

$$w(+s, +w, +c, +r) = P(+c) P(+r \mid +c) = .5 \times .8 = \boxed{.4}$$

Part 5.2

$$w(+s, +w, -c, +r) = P(-c) P(+r \mid -c) = .5 \times .2 = \boxed{.1}$$

Part 5.3

$$w(+s, +w, +c, -r) = P(+c) P(-r \mid +c) = .5 \times .2 = \boxed{.1}$$

Part 5.4

$$w(+s, +w, -c, -r) = P(-c) P(-r \mid -c) = .5 \times .8 = \boxed{.4}$$

Part 5.5

The total value of the samples is

$$.4 + .1 + .1 + .4 + .4 = 1.4$$

Therefore,

$$P(+s, +w, +c, +r) = \frac{1 \times .4}{1.4} \approx .2857$$

$$P(+s, +w, +c, -r) = \frac{0 \times .1}{1.4} = 0$$

$$P(+s, +w, +c) = P(+s, +w, +c, +r) + P(+s, +w, +c, -r) \approx .2857 + 0 \approx \boxed{.2857}$$

Part 5.6

$$P(+s, +w, -c, +r) = \frac{2 \times .1}{1.4} \approx .1429$$

$$P(+s, +w, -c, -r) = \frac{1 \times .4}{1.4} \approx .2857$$

$$P(+s, +w, -c) = P(+s, +w, -c, +r) + P(+s, +w, -c, -r) \approx .1429 + .2857 \approx \boxed{.4286}$$

Problem 6: Decision Networks

Part 6.1

$$\begin{aligned}
 EU(\text{take}) &= U(\text{take, sun}) P(\text{sun}) + U(\text{take, rain}) P(\text{rain}) \\
 &= 20 \times .6 + 70 \times .4 \\
 &= \boxed{40}
 \end{aligned}$$

Part 6.2

$$\begin{aligned}
 EU(\text{leave}) &= U(\text{leave, sun}) P(\text{sun}) + U(\text{leave, rain}) P(\text{rain}) \\
 &= 80 \times .6 + 0 \times .4 \\
 &= \boxed{48}
 \end{aligned}$$

Part 6.3

Since $EU(\text{leave}) > EU(\text{take})$, it is better to leave the umbrella behind.

Part 6.4

F	$P(F)$
good	$= P(\text{good} \text{sun}) P(\text{sun}) + P(\text{good} \text{rain}) P(\text{rain}) = .6 \times .7 + .4 \times .2 = .5$
bad	$= P(\text{bad} \text{sun}) P(\text{sun}) + P(\text{bad} \text{rain}) P(\text{rain}) = .6 \times .3 + .4 \times .8 = .5$

W	F	$P(W F)$
sun	good	$= \frac{P(\text{sun}) P(\text{good} \text{sun})}{P(\text{good})} = \frac{.6 \times .7}{.5} = .84$
sun	bad	$= \frac{P(\text{sun}) P(\text{bad} \text{sun})}{P(\text{bad})} = \frac{.6 \times .3}{.5} = .36$
rain	good	$= \frac{P(\text{rain}) P(\text{good} \text{rain})}{P(\text{good})} = \frac{.4 \times .2}{.5} = .16$
rain	bad	$= \frac{P(\text{rain}) P(\text{bad} \text{rain})}{P(\text{bad})} = \frac{.4 \times .8}{.5} = .64$

$$\begin{aligned}
 EU(\text{take} | \text{good}) &= U(\text{take, sun}) P(\text{sun} | \text{good}) + U(\text{take, rain}) P(\text{rain} | \text{good}) \\
 &= 20 \times .84 + 70 \times .16 \\
 &= \boxed{28}
 \end{aligned}$$

Part 6.5

$$\begin{aligned}
 EU(\text{leave} | \text{good}) &= U(\text{leave, sun}) P(\text{sun} | \text{good}) + U(\text{leave, rain}) P(\text{rain} | \text{good}) \\
 &= 80 \times .84 + 0 \times .16 \\
 &= \boxed{67.2}
 \end{aligned}$$

Part 6.6

Since $EU(\text{leave} \mid \text{good}) > EU(\text{take} \mid \text{good})$, it is better to leave the umbrella behind.