

Assignment: Dimensionality Reduction

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```
import numpy

A = numpy.loadtxt('./data/A.csv', delimiter=',')
print(A.shape)
```

(3500, 20)

Problem 1

Part A

```
from scipy import linalg

U, S, Vt = linalg.svd(A)
S = numpy.identity(len(S)) * S

def Ak(k):
    Uk = U[:, :k]
    Sk = S[:k, :k]
    Vtk = Vt[:k, :]
    return Uk @ Sk @ Vtk

K = tuple(range(1, 11))
```

```
for k in K:
    print(f'k = {k}: {linalg.norm(A - Ak(k), 2)}')
```

```
k = 1: 100.00246446709691
k = 2: 92.11139537014573
k = 3: 87.43842799887248
k = 4: 70.84270143876726
k = 5: 58.89263665883072
k = 6: 57.77458152758134
k = 7: 25.86135552441823
k = 8: 24.871609868369216
k = 9: 24.395000417106157
k = 10: 22.997273663616927
```

Part B

```
norm_A = linalg.norm(A, 2)

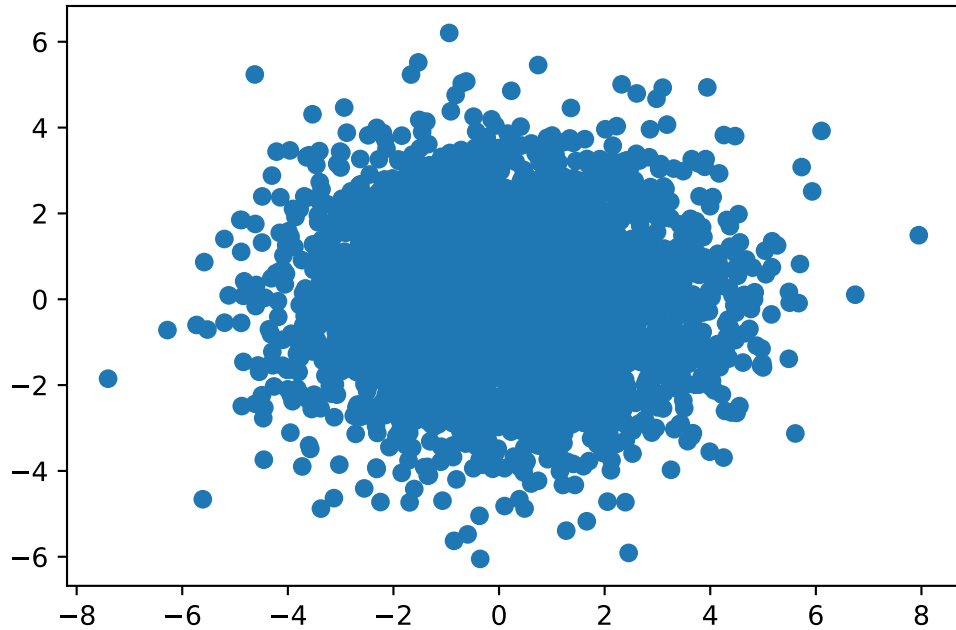
K = tuple(range(1, 21))
for k in K:
    if linalg.norm(A - Ak(k), 2) < .2 * norm_A:
        print(k)
        break
```

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Part C

```
from matplotlib import pyplot
from sklearn import decomposition

A2 = decomposition.PCA(2).fit_transform(A)
_ = pyplot.scatter(A2[:, 0], A2[:, 1])
```



I applied the PCA method from the lecture that found a subspace B minimizing $\text{SSE}(A, B)$. The algorithm first centered the data points and then performed SVD. I made use of an existing implementation for this method from SciKit-Learn (<https://scikit-learn.org/stable/modules/generated/sklearn.decomposition.PCA.html>).

Problem 2

Part A

- How large does l need to be for the above error to be at most $\|A\|_F^2/20$?

```
from lib import FD

def find_min_l(A, err_bound):
    n, d = A.shape
    for l in range(1, n // 2):
        B = FD.freq_dir(A, l)
        err = linalg.norm(A.T @ A - B.T @ B)
        if err <= err_bound:
            return l, err

l, err = find_min_l(A, linalg.norm(A)**2 / 20)
print(l)
```

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- How does this compare to the theoretical bound (e.g. for $k = 0$)?

```
k = 0
print(f' ||Ax||^2 - ||Bx||^2: \t\t\t\t{err}')
print(f' ||A - Ak||_F^2 / (1 - k) [with l = {l}, k = {k}]: '
      f' \t\t{linalg.norm(A - Ak(k))**2 / (1 - k)}')
```

```
||Ax||^2 - ||Bx||^2:                2467.226247591852
||A - Ak||_F^2 / (1 - k) [with l = 7, k = 0]:  8075.627550160504
```

- How large does l need to be for the above error to be at most $\|A - Ak\|_F^2/20$ (for $k = 2$)?

```
k = 2
l, err = find_min_l(A, linalg.norm(A - Ak(k))**2 / 2)
print(l)
```

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Appendix

lib/FD.py

```
import numpy
from scipy import linalg

def freq_dir(A, l):
    n, d = A.shape
    B = numpy.zeros((2 * l, d))
    zero_index = 0
    for i in range(n):
        B[zero_index] = A[i]
        zero_index += 1
    if zero_index == 2 * l: # full
        U, S, Vt = linalg.svd(B)
        delta = S[-1]**2
        S_prime = numpy.zeros((2 * l, d))
        numpy.fill_diagonal(S_prime,
                             [(sigma**2 - delta)**.5 for sigma in S])
        B = S_prime @ Vt
    for zero_index in range(2 * l):
        if not B[zero_index].any(): break # the first empty row
    return B
```