Please enter your name and uID below.

Name:
uID:

Submission notes

- Due at 11:59 pm on Friday, October 2.
- Solutions must be typeset using one of the template files. For each problem, your answer must fit in the space provided (e.g. not spill onto the next page) *without* space-saving tricks like font/margin/line spacing changes.
- Upload a PDF version of your completed problem set to Gradescope.
- Teaching staff reserve the right to request original source/tex files during the grading process, so please retain these until an assignment has been returned.
- Please remember that for problem sets, collaboration with other students must be limited to a high-level discussion of solution strategies. If you do collaborate with other students in this way, you must identify the students and describe the nature of the collaboration. You are not allowed to create a group solution, and all work that you hand in must be written in your own words. Do not base your solution on any other written solution, regardless of the source.

1. (Antimicrobial Coins BT) To combat the pandemic, your country has introduced a new currency, whose physical representation consists entirely of antimicrobial coins with values $v_1 = 1, v_2, \ldots, v_n \in \mathbb{Z}^+$. Since these are expensive to produce, the bank always wants to dispense the fewest possible coins for a given withdrawal request $W \in \mathbb{Z}^+$. The bank has access to an unlimited number of each denomination of coin, but wants to always make change using the minimum total number of coins.

   Give a recursive backtracking algorithm (specified by a recurrence relation with 1 or more equations, along with necessary base cases) for computing the minimum number of coins needed to satisfy a withdrawal W in this currency and *argue* its correctness (this does not need to rise to the level of a formal proof, but should be logically complete and written in complete sentences).

2. (Antimicrobial Coins DP)

   a. Convert your backtracking algorithm for (from Problem 1) into pseudo-code for a dynamic programming algorithm. For full credit, your algorithm should run in $O(nW)$ time.

   b. Argue that your DP algorithm is correct. You must describe (1) the data structure the algorithm uses to memoize recursive calls, (2) the order in which the algorithm should fill in the data structure to guarantee that values have been properly computed prior to lookup, and (3) which value in the data structure gives the final solution to the problem.

   c. State the runtime and space complexity of your algorithm.

3. (COVID Classrooms Greedy)

   An unnamed large state university in the West has a scheduling problem. They have n incoming freshman who all want to take CS 1410, but their classrooms have reduced capacities to accommodate social distancing. The scheduling office has provided a list of all m classrooms that can be used, each of which has a capacity C_i. The university would like to use as few classrooms as possible for 1410, since (apparently) other departments also need to teach on campus this semester. Since students are all wearing masks, you can assume they are indistinguishable (it does not matter which student is assigned to which room).

   Give a greedy algorithm for finding a minimum size set S of classrooms which can accommodate all n students safely and *prove* its correctness.

4. (Cost-Controlled COVID Classrooms BT)

   A few weeks into the semester, the university has realized that they're unhappy with your greedy algorithm from Problem 3 because not all classrooms cost the same to maintain/keep clean – and they really need to minimize operating costs given the circumstances. They have updated the list of (m) COVID-compatible classrooms so that now each room has both a capacity C_i and a cleaning expense E_i. Note that E_i depends on many factors (e.g. type of furniture, flooring, location within a building, distance from center campus), and thus cannot be assumed to be correlated with the capacity.

   Give a recursive backtracking algorithm (specified by a recurrence relation with 1 or more equations, along with necessary base cases) for computing the minimum cost of a set of classrooms that can accommodate all n enrolled students and *argue* its correctness (this does not need to rise to the level of a formal proof, but should be logically complete and written in complete sentences).

5. (Cost-Controlled COVID Classrooms DP)

   a. Convert your backtracking algorithm for Cost-Controlled COVID Classrooms (from Problem 4) into pseudo-code for a dynamic programming algorithm. For full credit, your algorithm should run in $O(mn)$ time.

   b. Argue that your DP algorithm is correct. You must describe (1) the data structure the algorithm uses to memoize recursive calls, (2) the order in which the algorithm should fill in the data structure to guarantee that values have been properly computed prior to lookup, and (3) which value in the data structure gives the final solution to the problem.

   c. State the runtime and space complexity of your algorithm.