Homework 2

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Section 2.1

1. Exercise 11 (p97)

Part (a)

Let a, b be the two given odd integers. By definition of odd, we have

$$a = 2m + 1, b = 2n + 1 : m, n \in \mathbb{Z}$$

By substitution,

$$\begin{aligned} ab &= (2m + 1)(2n + 1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1 \\ &= 2k + 1, \text{ where } k = 2mn + m + n \end{aligned}$$

By closure under addition and multiplication, $k \in \mathbb{Z}$. By definition of odd, we have that ab is an odd number.

Part (b)

Let a be the given odd integer and b be the given even integer. By definition of odd and even, we have

$$a = 2m + 1, b = 2n : m, n \in \mathbb{Z}$$

By substitution,

$$\begin{aligned} ab &= (2m + 1)2n \\ &= 4mn + 2n \\ &= 2(2mn + n) \\ &= 2k, \text{ where } k = 2mn + n \end{aligned}$$

By closure under addition and multiplication, $k \in \mathbb{Z}$. By definition of even, we have that ab is an even number.

Part (c)

Let a be the given even integer and b be the given integer that is divisible by 3.

By definition of even, we have

$$a = 2m : m \in \mathbb{Z}$$

By definition of being divisible by 3, we have

$$b = 3n : n \in \mathbb{Z}$$

By substitution,

$$\begin{aligned} ab &= 2m \cdot 3n \\ &= 6mn \\ &= 6k, \text{ where } k = mn \end{aligned}$$

By closure under addition and multiplication, $k \in \mathbb{Z}$. By definition of being divisible by 6, we have that ab is divisible by 6.

2. Exercise 12.d (p97)

This pair of statements are *not* contrapositives of one another.

- Counterexample of case (i): a person that likes computers but does not like computer science.
- Counterexample of case (ii): a person that does not like computers but likes computer science.

3. Exercise 13.a (p97)

Contrapositive: "if $m = 0$ and $n = 0$, then $m^2 + n^2 = 0$."

Proof of the contrapositive: Since $m = 0$ and $n = 0$, by substitution, we have

$$m^2 + n^2 = 0^2 + 0^2 = 0$$

Since the contrapositive is true, the original statement "if $m^2 + n^2 \neq 0$, then $m \neq 0$ or $n \neq 0$" is true.

4. Exercise 14.c (p98)

Section 2.2

5. Exercise 7.e (p108)

6. Exercise 16 (p109)

7. Exercise 20 (p109)

8. ***Show that the square of an integer not divisible by 5 leaves a remainder of 1 or 4 when divided by 5.***

9. Exercise 26.c (p109)

Section 2.3

10. Exercise 2 (p121)

Part (a)

Part (b)

Part (c)

Part (d)

11. Exercise 3.f (p121)

12. Exercise 4.e (p122)

13. Exercise 8.f (p122)

14. Exercise 13 (p122)

Section 2.4

15. Exercise 1.d (p130)

16. Exercise 2.d (p130)

17. Exercise 4.b (p130)

18. Exercise 6 (p131)

Section 2.5

19. Exercise 12 (p147)

20. Exercise 20 (p148)

21. Exercise 32.b (p149)

22. Exercise 34.b (p149)