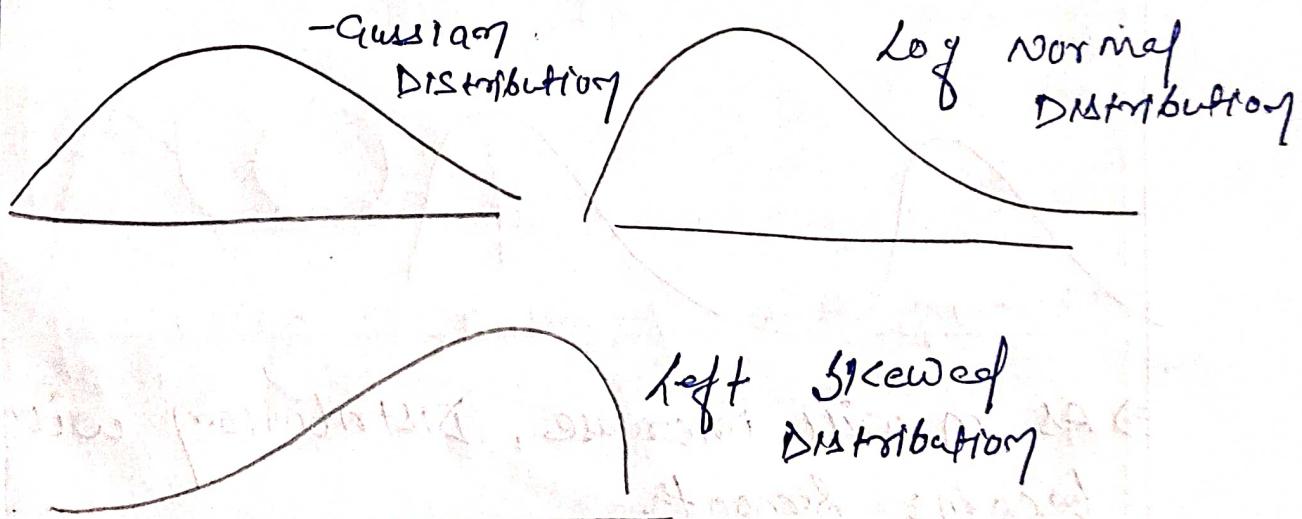


## DAY - 4

### Agenda

- ① Central limit theorem
2. Probability
3. Permutation & Combinations
4. Co-variance, Pearson correlation & Pearson of Rank correlation

### 1. Central Limit theorem



→ Let's take  $m$ , no. of samples of  $n$  size

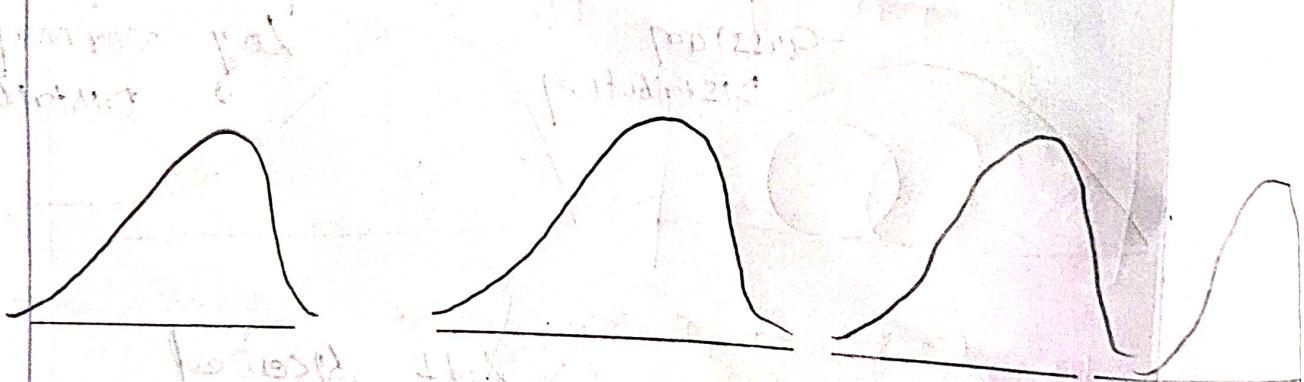
$$S_1 = \{x_1, x_{10}, x_1, \dots, x_m\} \rightarrow \text{mean } \bar{x}_1 = \bar{x}_1 = S_1$$

$$S_2 = \{x_3, x_4, x_5, \dots, x_m\} \rightarrow \text{mean } \bar{x}_2 = \bar{x}_2 = S_2$$

Suppose we have population Data which have Gaussian distribution or normal distribution or Any kind of Data :-

Central Limit theorem states that if you have a population with  $\mu$  and  $\sigma$  and take sufficiently large random sample from population where  $n > 30$  &  $m$  no. of sample & if we take all the mean of sample and plot in histogram then we will get Gaussian Distribution.

\* the more number of  $n$  will be there, the better the result are.



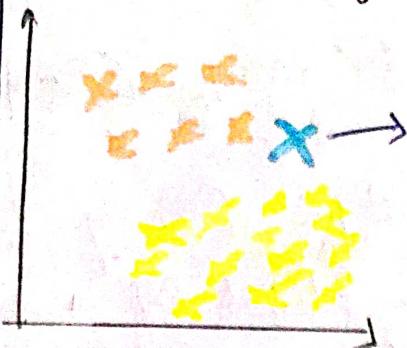
$\Rightarrow$  As  $n$  will increase, distribution will become smooth

### Probability

Probability is the measure of likelihood of an event.

- ① Probability of getting head in coin flip =  $\frac{1}{2}$
- ② Prob. of getting 1 in six face dice =  $\frac{1}{6}$

## Practical usage

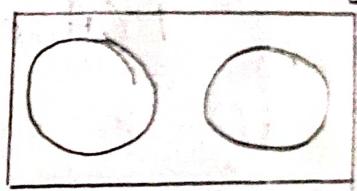


with the help of Probability, we will define  $X$  belongs to which collection.

### ① Mutual exclusive event

To event are mutually exclusive, if they can't occur at same time.

- Tossing coin
- Rolling dice



### ② Non Mutual exclusive event

Two events can occur at same time

- Picking a random card from deck, here King & ♦ heart come at same time.

### Mutual exclusive event

- ① What is the Probability of coin landing on head or tails?

$$\Rightarrow P(A \text{ or } B) = P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$$

- ② What is the Probability of getting 1 or 6 or 3 on Rolling a dice?

$$\Rightarrow P(A \text{ or } B \text{ or } C) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

## NON MUTUALLY EXCLUSIVE EVENTS

Bag of Marbles, 10 Red, 6 Green, 3 Red & 4 Green

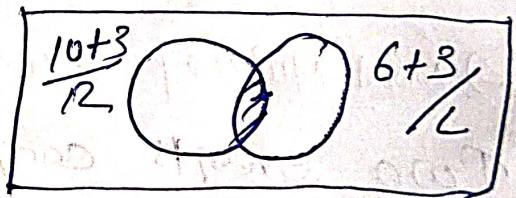
- ① When picking randomly from bag of marbles. What is the probability of choosing marble that is Red or Green?

### ADDITION RULE OF NON MUTUAL

$$\frac{5}{19} + \frac{9}{19}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{13}{19} + \frac{9}{19} - \frac{3}{19} = 1$$



- ② From deck of cards, Probability of choosing heart or queen at same time?

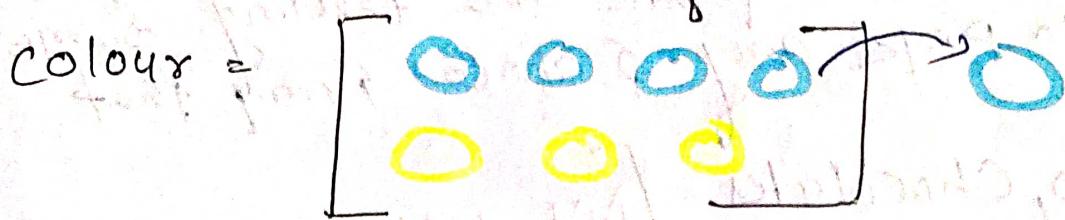
$$\Rightarrow P(\heartsuit \text{ or } Q) = P(\heartsuit) + P(Q) - P(\heartsuit \cap Q)$$

$$\Rightarrow \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52}$$

### MULTIPLICATION RULE

- ① Dependent events →  
Two events are dependent if they effect one another

## Conditional Probability



$\rightarrow$  Probability of taking out 1 blue ball =  $4/7$

\* Now we took one ball out & that ball is blue & now Probability of taking one green ball out is =  $1/3$

$1/3$

## Independent Event

$\Rightarrow$  Those events which does not effect another one

(Q) Probability of rolling a '5' & the '3' in dice

$$P(1) = 1/6, P(2) = 1/6, P(3) = 1/6$$

$$P(A \text{ } \& \text{ } B) = 1/6 \times 1/6 = 1/36$$

Mean Multiplication

$$P(A \text{ } \& \text{ } B) = P(A) \times P(B)$$

## Permutation

Suppose, we are having 5 different kinds of chocolates & we are telling some child to pick any 3 choco from that

So, Chocolates =  $\boxed{DM, KIC, MB, SNK, S\text{Star}}$   
3 chocolates can be picked

$\frac{5}{\text{Choices}} \times \frac{4}{\text{Choices}} \times \frac{3}{\text{Choices}}$   
→ they can choose any like

$[KIC, MB, SNK]$  or  $[SNK, S\text{Star}, DM]$

So, many ways to arrange

NOW.

$$\frac{5}{\text{1}^{st}} \times \frac{4}{\text{2}^{nd}} \times \frac{3}{\text{3}^{rd}}$$

1<sup>st</sup> Position - Any 5 can be chosen

2<sup>nd</sup> Position = One got deducted, 4 out of 5 can be chosen

3<sup>rd</sup> Position = two got deducted, 3 out of 5 can be chosen

there will be =  $5 \times 4 \times 3 = 60$  ways

It is a Mathematical technique that determines the numbers of possible arrangement in a set when order of the arrangement matters

Formula:-

$$m_{P_r} = \frac{m!}{(m-r)!}$$

$$\Rightarrow \frac{5!}{(5-3)!} = \frac{120}{2} = 60.$$

## COMBINATION

Combination is the way of selecting object in such a way that order of object is not repetitive

$$[DM, S\text{ star}, KK] \rightarrow [\cancel{DM}, KK, S\text{ star}, DM] - x$$

element are Repetitive not consider this

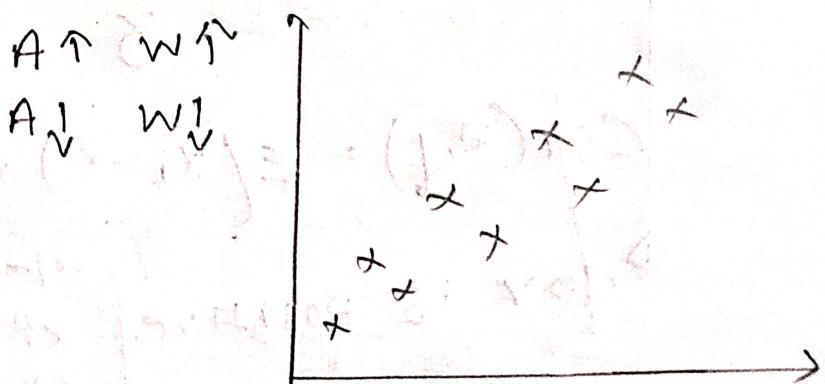
$$m_{C_r} = \frac{m!}{r!(m-r)!} = \frac{5!}{2!3!} = 2$$

Only unique combination should be there

## CO-variance

[Feature Selection]

Age	Weight
12	40
13	45
15	48
17	50
18	52



Covariance Measure the direction of relationship of two variables

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$\boxed{\text{Cov}(x, x) = \text{var}(x)}$$

$$\text{Variance} (\sigma^2) = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Question

Age weight

12	40
13	45
15	48
17	60
18	62
75	255

$$\bar{x} = 75/5 = 15$$

$$\bar{y} = 255/5 = 51$$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
-3	-11	+33
-2	-6	
0	-3	+12
2	9	0
3	11	18
		53
		96

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$\Rightarrow \text{Cov is positive} = 96/9 = 24$$

$x_1$	$y_1$	$x_2$
$x_2$	$y_2$	

When Cov is negative

$x_1$	$y_1$
$x_2$	$y_2$

When Cov is 0

No Relation between



Positive Covariance



Negative Covariance



No Relation

②

$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
10	4	2.25	-3	6.75
8	6	1.75	-1	-1.75
7	8	0.75	1	0.75
6	10	-1.75	3	-4.25

$$\bar{x} = 7.75 \quad \bar{y} = 7$$

$$\text{Cov} = -3.25$$

Negative Relation

## Pearson Correlation Coefficient

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$$\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}$$

$$r_{(x,y)} = \frac{\text{cov}(x,y)}{\sqrt{x} \sqrt{y}}$$

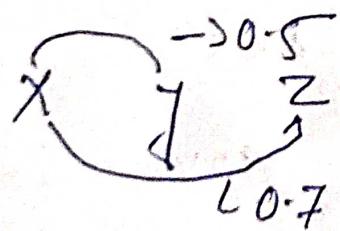
→ When we find co-variance?

There is no restriction in range of value of cov

→ The Comparisons between become difficult as value be like -3.25, -300.25, etc.

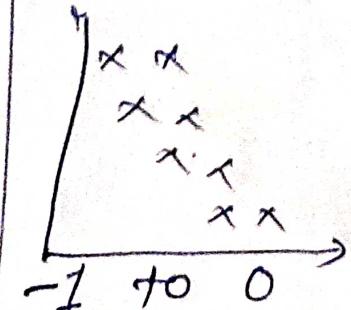
→ So, to restrict the number between range -1 to 1, we divide cov. by  $\sqrt{x} \cdot \sqrt{y}$

SUPPOSE

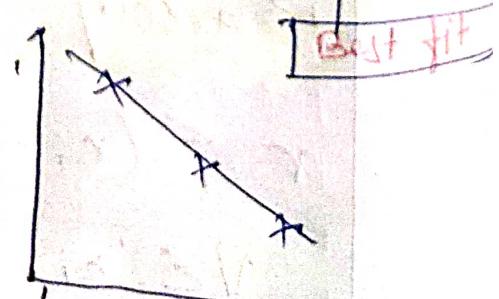


, 0.7 more co-related

→ More the value towards +1, more positive correlated it will be & vice-versa.



Because Data Points are not on same line



→ exactly (-1) because Data Points are on same line



+1

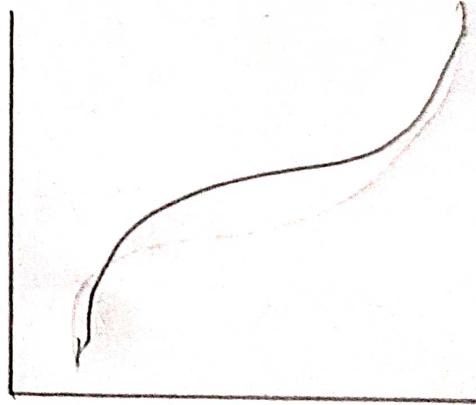


Between 0 to 1

## Spearman's Rank Correlation

- ⇒ For linear data only, Pearson correlation works well
- ⇒ So, for non-linear data, Spearman's Rank correlation works well.

$$\rho_s = \frac{\text{cov}(R(x), R(y))}{\sqrt{\text{var}(R(x))} \times \sqrt{\text{var}(R(y))}}$$



X	Y	R(X)	R(Y)
10	4	4	1
8	6	3	2
7	8	2	3
6	10	1	4

↳ Rank assigns 1 of ascending order.