Derivatives and Risk Management
Group D

Exotic options, Binomial tree option-pricing and Delta-hedging

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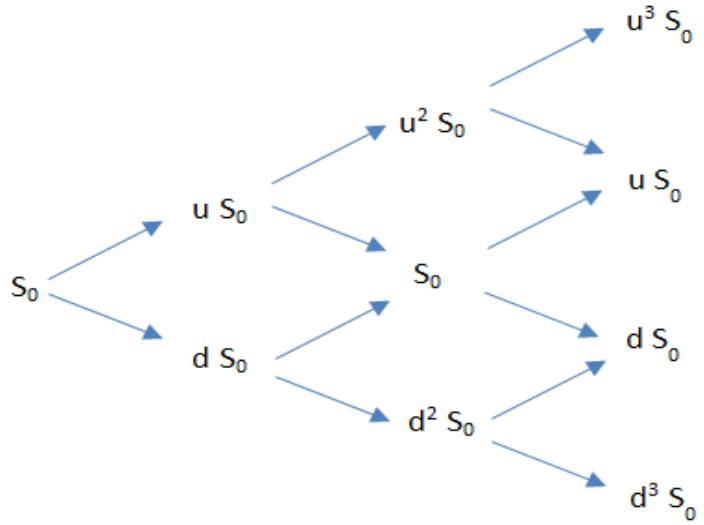
Agenda

- Binomial option-pricing model
- 2-periods' world i.e. Discrete time; T=1
- The underlying asset is a stock
- Current/Spot price = $100
- The underlying price can increase or decrease 5% per period

  i.e. u = 1.05; d = 0.95
- RFR = 0.5% per period.
- Strike price of the derivative (option) = $100
- Firm/Investor has a long position on the underlying asset:

Probability (Up) = $\pi = \frac{1+r-d}{u-d}$

Probability (Down) = *1* - π



- Hedge ratios: How many assets per option?
- Dynamic hedging?
- Buy or Sell options?
- Hedged portfolio: Guaranteed payoffs (RFR), irrespective of increase/decrease?

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Exotic options?

Plain Vanilla	Exotic
Standard	Non-standard
Actively traded on exchanges	OTC market

Reasons for developing exotic options:

- To meet a particular hedging requirement
- Because of tax/legal/regulatory reasons
- To intentionally make the product complicated and esoteric

Compound options

- Option on options
- 2 strike prices
- 1st strike price = $ 2 (in our case)
- 2 exercise dates
- Will be exercised on the first exercise date (T = 1), IF the option is 'in-the-money' at that time
- The (1st) option matures at T = 1.

  At T = 1, if you exercise, you buy/sell another (2nd) Call/Put option maturing at T = 2.
- Types:

1) Call on Call
2) Call on Put
3) Put on Call
4) Put on Put

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Asian options

- Payoff depends on the arithmetic average of the price of the underlying asset during the life of the option
- Also called “average price options”
- Relatively cheaper
- ‘Non-recombining’ in nature

Average Price option premiums:

Call: $\max(S_{ave} - K, 0)$

Put: $\max(K - S_{ave}, 0)$

Look-back options

- Depend on the maximum (or minimum) asset price reached during the life of the Put (or Call) option
- Holder can sell (or buy) the underlying asset at the highest (or lowest) price achieved during the life of the Put (or Call) option.
- Price of option, is sensitive to frequency with which the asset price is observed for computing maximum/minimum
- Relatively expensive
- 'Non-recombining' in nature
- Types:

1) Floating
2) Fixed
   - Strike price is specified
   - Final asset price is replaced by minimum (or maximum) asset price achieved during the life of the Put (or Call) option

Floating Lookback Call pays $S_T - S_{\min}$ at time T
Allows buyer to buy stock at lowest observed price in some interval of time

Floating Lookback Put pays $S_{\max} - S_T$ at time T
Allows buyer to sell stock at highest observed price in some interval of time

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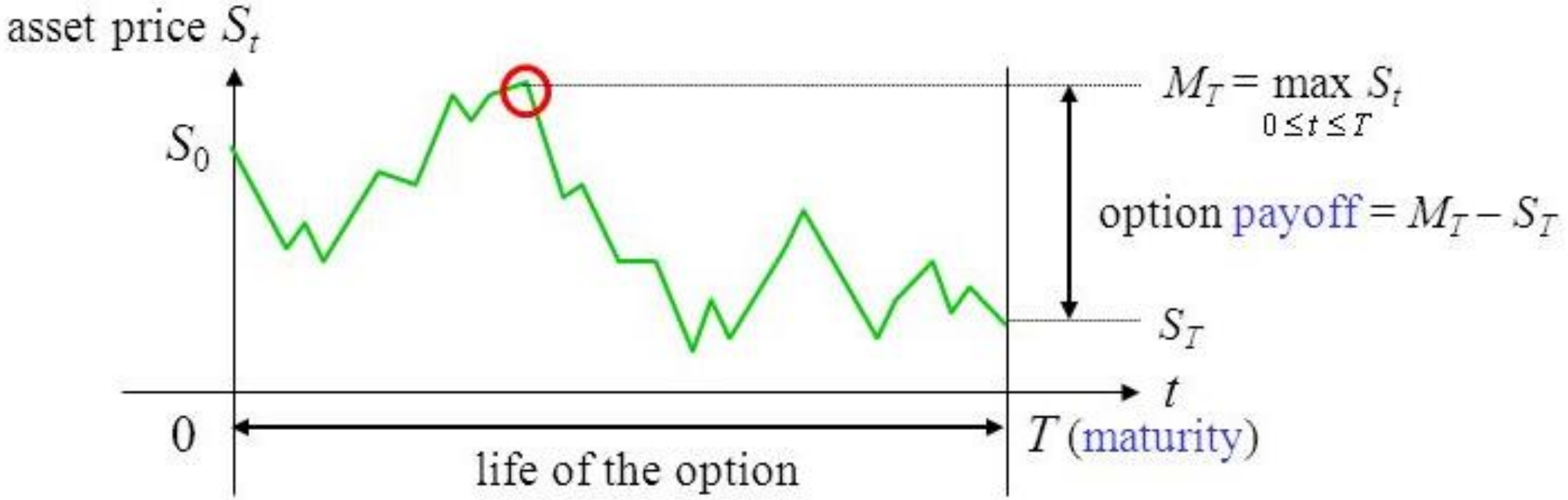
e.g. Floating Lookback Put option:



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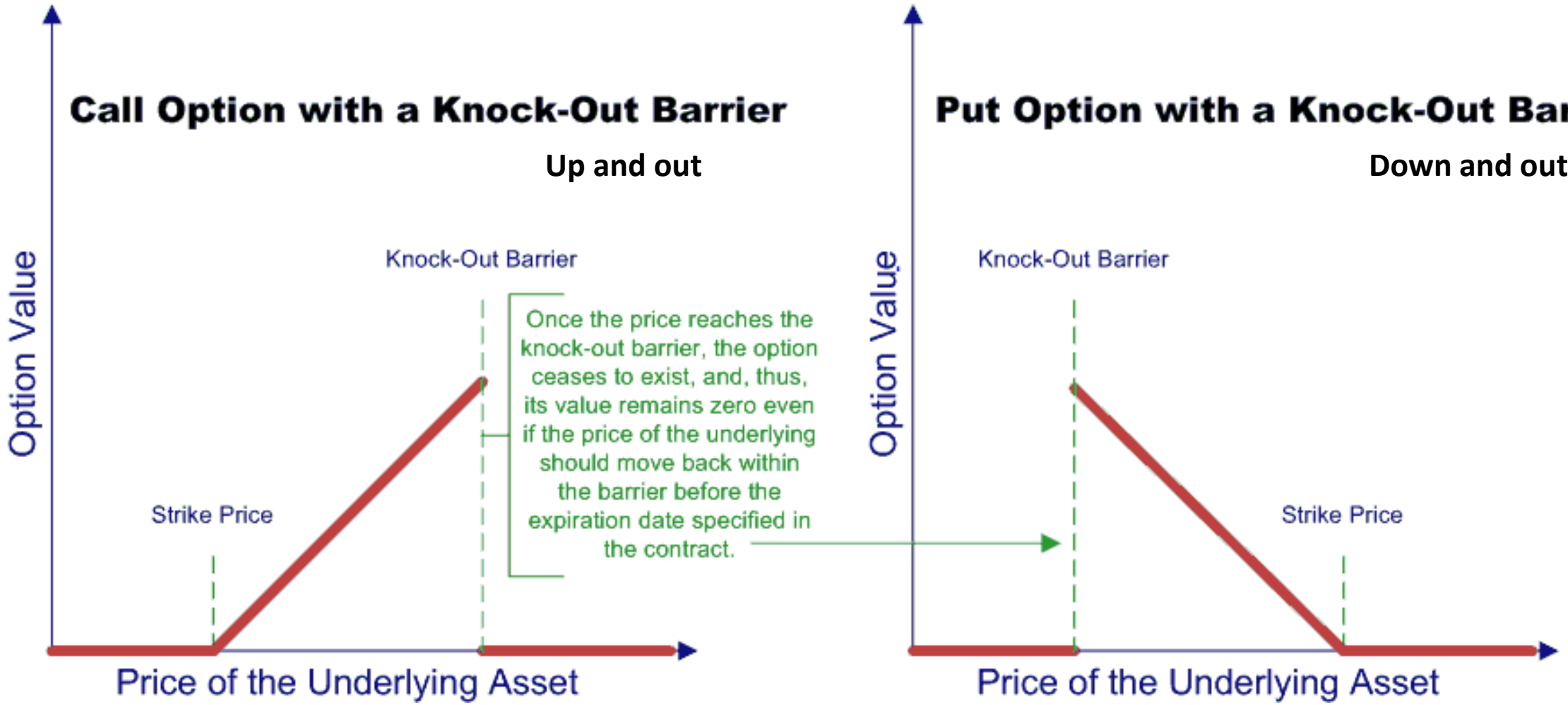
Barrier options

- Payoff depends on whether the underlying asset's price reaches a certain level ("barrier")
- Relatively cheaper
- Important factor: frequency with which the asset price is observed, to determine whether the barrier has been reached
- Types:
  1) Knock-in: comes into existence only when the underlying asset price reaches the barrier
  2) Knock-out: ceases to exist, when the underlying asset price reaches the barrier
     - Up-and-in
     - Up-and-out
     - Down-and-in
     - Down-and-out
- High volatility (higher chance of hitting the barrier) can decrease price of the option
- "Parisian options"?



Chooser options

- After a specified period of time T_1 $(0 < T_1 < T_2)$, the holder can choose whether the option is a Call or a Put
- "As-you-like-it" options
- Relatively very expensive
- Possible sharp change in value (and sign) of Delta

Delta-hedging: (Dynamic)

- Need to rebalance: How frequently?
- Cost of rebalancing?
- Some exotic options are easier to delta-hedge (than plain vanilla options) while others are more difficult
- E.g. Asian options?
- Or Barrier options?

Delta Hedging

Procedure

With Delta changing we buy and sell the Underlying by issuing or purchasing bonds to minimise changes in the Hedge Portfolio.

Assumption

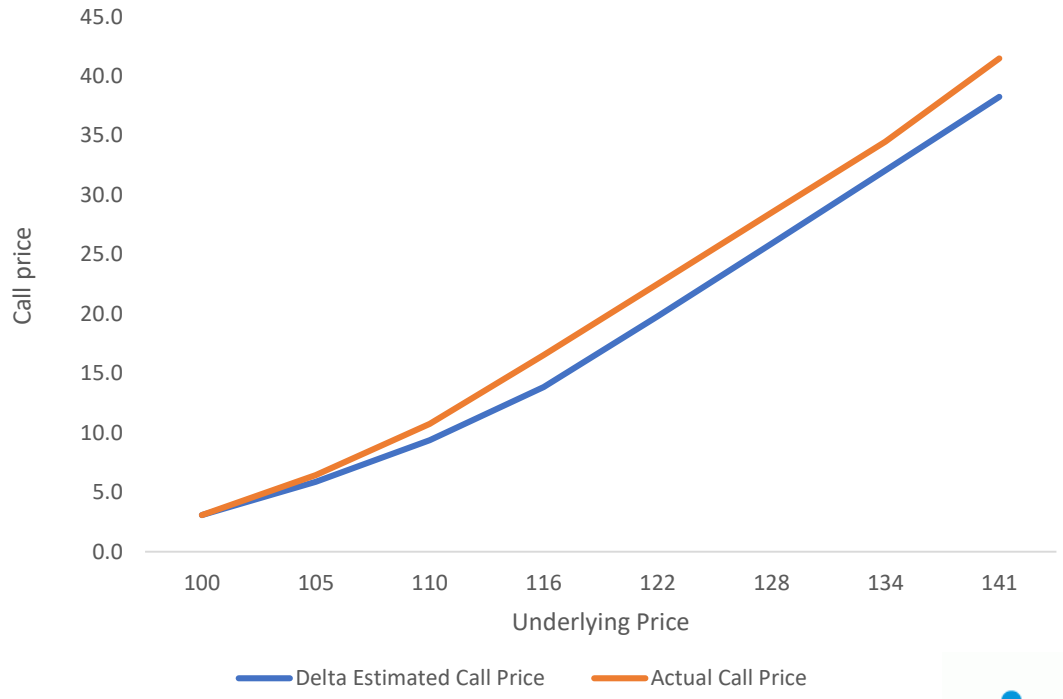
We assume the Company can issue Bonds at the risk free rate.

Considerations on Delta Hedging

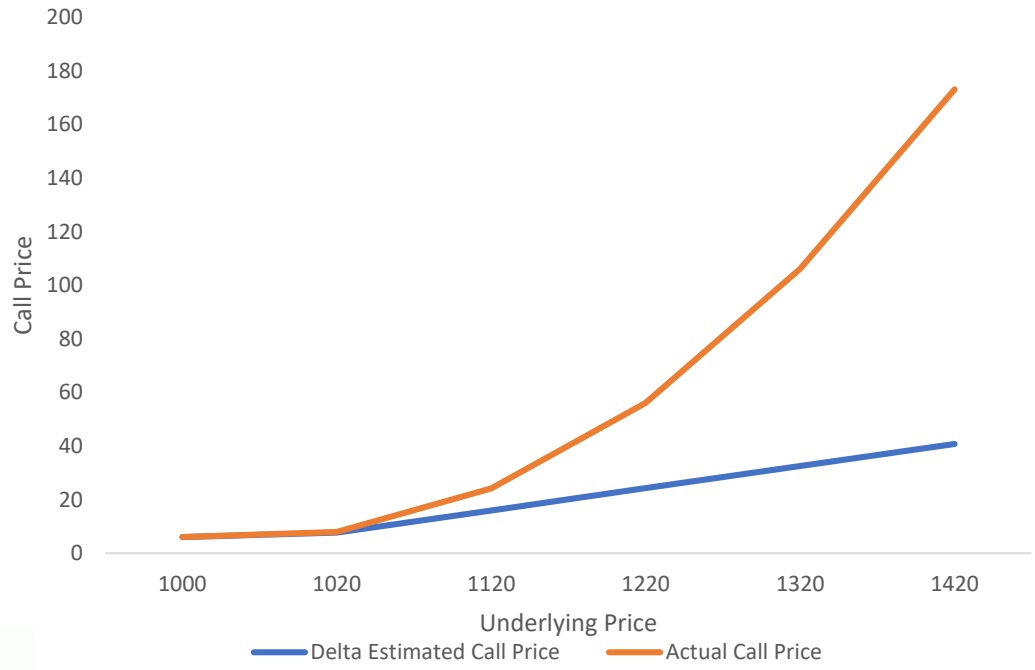
- Delta is only an approximation it changes with changes in the underlying and Time.
- The further we move away from the Underlying the worse the worse the delta approximation.
- The effects are asymmetric. for calls the delta under estimates the effects of increases in the underlying and overestimates the effects of the decreases in the underlying.
- Gamma Hedge - to hedge changes in delta combining other options to move the delta and gamma to zero.

Delta Hedging is not efficient

Delta Estimated Call VS Actual Call at 7% Volatility



Delta Estimated Call Vs Actual Call at 30% Volatility



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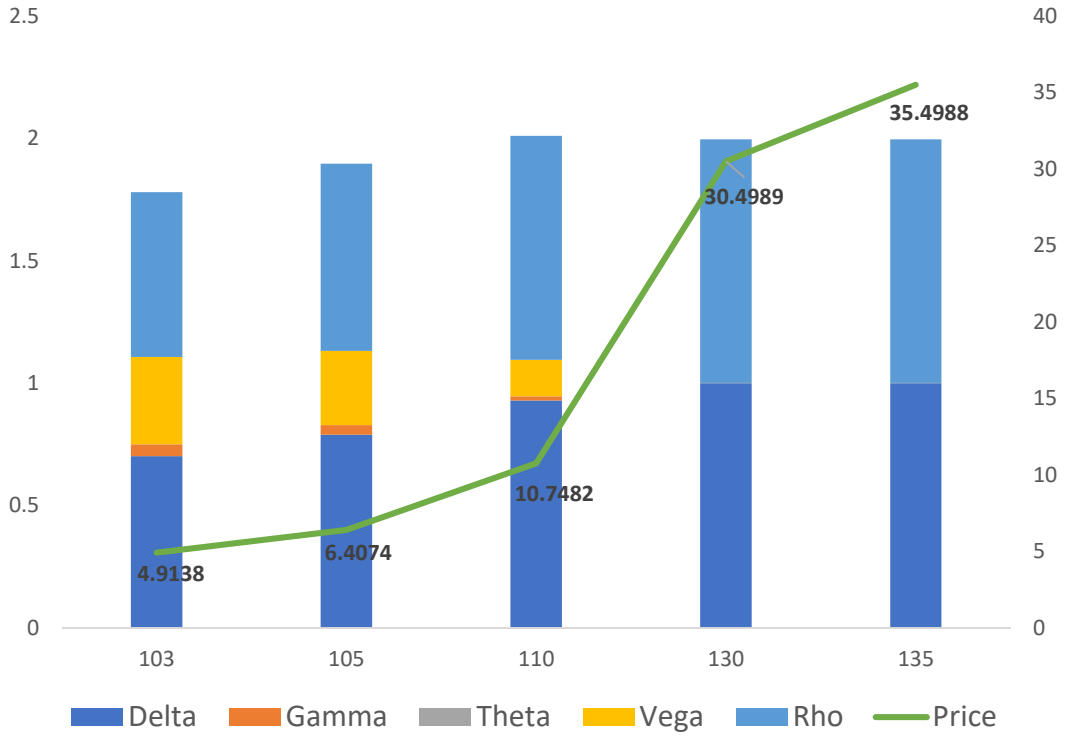
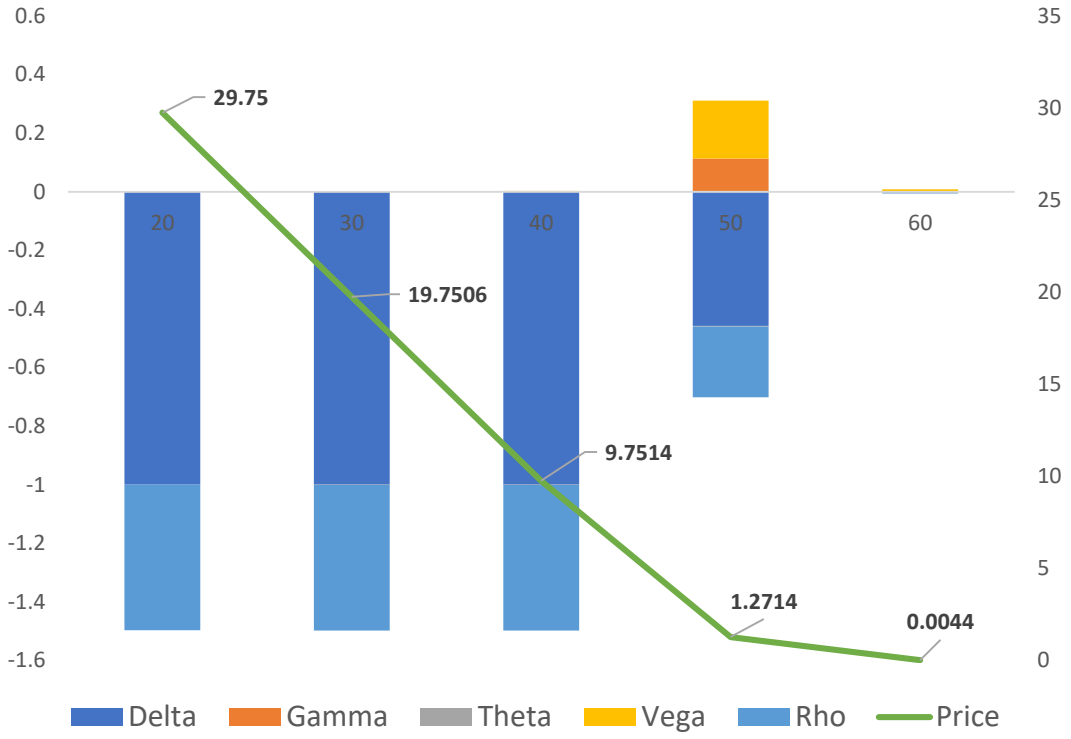
Greeks

CALL GREEKS COMPOSITION PER PRICE



PUT GREEKS WITH PRICE CHANGES



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Observations

- GAMMA highest for ATM puts and calls
- VEGA highest for ATM puts and calls
- DELTA Puts and Calls Increase with Increases in Underlying price
- RHO is Positive for calls and negative for Puts
- RHO is larger for Options that are in the Money and near to expiration.
- SPECULATIVE PREMIUM OR TIME VALUE highest for ATM puts and calls
- BSM model assumes constant volatility so the measure of VEGA is an artificial Construction.