Hypothesis Testing Simulation worksheet

The objective of this worksheet is to demonstrate hypothesis testing using simulation.

Table of Contents

Hypothesis Testing Simulation worksheet 1

Table of Contents 1

Overview of Hypothesis Testing simulation panel 2

Drawing samples 3

Hypothesis testing (not reject the null hypothesis, two-sided): 5

Hypothesis testing (reject the null hypothesis, two-sided): 7

Hypothesis testing (not reject the null hypothesis, one-sided): 8

Hypothesis testing (reject the null hypothesis, one-sided): 9

Overview of Hypothesis Testing simulation panel

Hypothesis Testing:

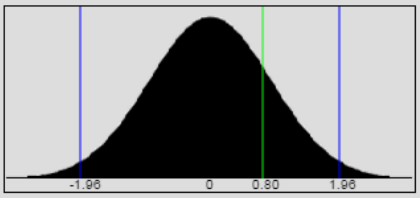
Sampling Parameter selection (Normally distributed):
μ = 0, σ² = 1 ▾

Generate Samples:
+1 +5 +10
Sample values: Clear Samples

```
0.91,-0.96,1.39,-0.49,-0.31,1.19,-0.15,0.06,0.56,-1.51,1.
05,-0.85,-0.43,1.29,0.07,1.5,1.33,0.32,1.3,1.48,-1.29,-0.
65,2.56,0.87,0.79,-0.49,-0.35,0.18,1.04,-1,-1.59,-0.18,-0
.29,1.59,-1.35,-0.02,-0.47,0.95,-0.6,1.41,0.3,0.62,-0.43,
0.96,0.64,-1.34,0.57,1.53,-0.99,0.36,-0.01,-1.2,0.63,2.04
,-0.87,0.37,0.75,2.33,-1.35,-1.93,0.97,1.08,1.74,-0.18,1.
```

Statistics: Show statistics Hide statistics
Number of Samples: 100
Sample Mean: 0.08
Sample SD: 1.00

Hypothesis test:
H_0: μ = μ₀ = 0 ▾ H_1: μ ≠ μ₀ ▾ Significance value: α = 0.05 ▾



Acceptance boundaries are show in blue lines.

Test Statistics (Assumption: Normal Distributed with unknown mean and known variance.):
Test Variance: σ² = 1 ▾ Test
Z-statistcs: 0.80
Test type: Two-tailed Test
Test result: We cannot reject null hypothesis

Population parameter panel

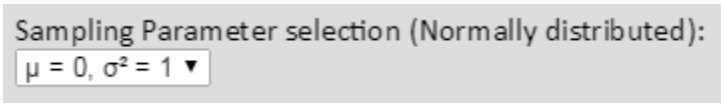
Sampling and sample statistics panel

Test null and alternative hypothesis, significance level

Test results

Drawing samples

1. First, choose population parameter $\mu = 0, \sigma^2 = 1$:

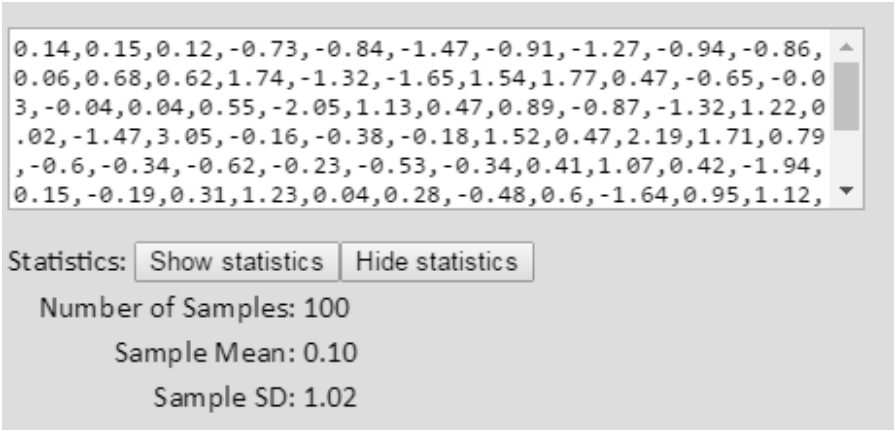


2. Click on +1 once, you will find that the text box shows one number (from the population distribution)

Noted that you may not get "-0.1" as it is supposed to be a random sample from the normal distribution with mean = 0 and variance = 1

Press Clear Samples, to clear all sample(s) in the text box as follow:

3. Click Show Statistics first and draw 100 samples by clicking +10 ten times and you will get your statistics roughly as follow:



4. Clear the samples and try mean = 1 and variance = 2 with 100 samples
   Repeat the process with mean = 2 and variance = 3

Sampling Parameter selection (Normally distributed):
μ = 1, σ² = 2 ▾

Generate Samples:
+1 +5 +10
Sample values: Clear Samples

-0.62,-0.06,0.58,-1.62,2.79,-0.44,1.62,-3.29,2.71,2.45,-0
.98,-1.43,0.5,4.85,1.94,1.05,0.51,1.81,1.39,-3.12,0.91,0.
33,1.63,-1.45,0.43,-0.12,2.06,-2.46,0.97,2.26,1.84,-0.48,
0.45,3.62,1.9,1.69,0.08,0.82,0.55,-2.43,3.71,0.6,-2.98,0.
48,0.86,2.96,-0.1,0.45,4.33,4.53,1.57,-0.03,1.74,1.86,2.6
4,1.13,1.32,-0.08,2.33,1.3,-0.02,-0.61,0.7,2.12,-3.34,1.3

Statistics: Show statistics Hide statistics
Number of Samples: 100
Sample Mean: 0.78
Sample SD: 1.65

Sampling Parameter selection (Normally distributed):
μ = 2, σ² = 3 ▾

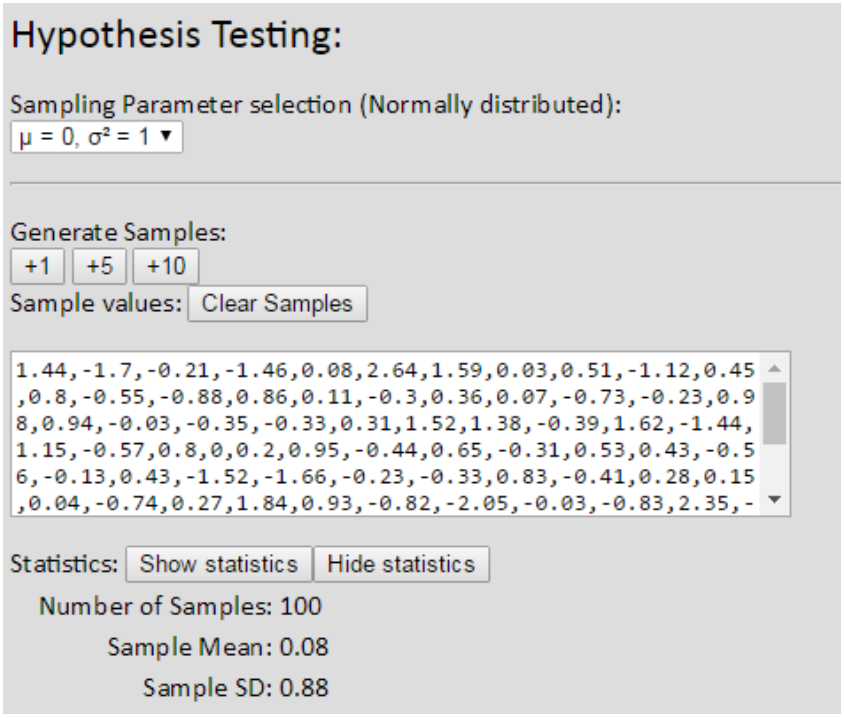
Generate Samples:
+1 +5 +10
Sample values: Clear Samples

-0.31,-0.2,3.82,5.07,0.68,3.08,2.69,-0.02,5.02,0.28,0.4,0
.61,0.79,0.01,3.02,4.39,4.22,0.42,2.17,4.16,0.22,1.95,-0.
05,1.67,1.76,1.9,1.05,4.35,0.49,2.48,-0.02,2.3,1.96,4.23,
0.65,-0.45,4.47,1.61,2.28,3.46,-1.52,2.37,0.93,2.59,4.68,
0.49,1.28,4.94,4.33,0.54,3.25,1.45,1.06,2.36,1.95,1.8,3.8
3,1.68,1.08,-1.98,2.84,2.49,-0.94,1.42,2.12,2.66,4.88,-0.

Statistics: Show statistics Hide statistics
Number of Samples: 100
Sample Mean: 1.85
Sample SD: 1.82

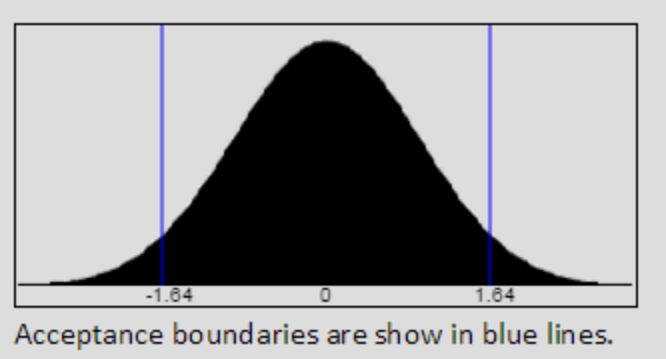
Hypothesis testing (not reject the null hypothesis, two-sided):

1. Clear the samples and Draw 100 samples with mean = 0 and variance = 1



2. Set
   Null Hypothesis (H_0) to population mean μ_0 = sample mean μ = 0
   Alternate Hypothesis (H_1) to population mean $\mu_0 \neq$ sample mean μ
   Significant value (α) to 0.1
   As follows:

   You will find that there are two boundaries marked with two blue vertical lines specifying the critical value (-1.64 and 1.64) and critical region (the area to the left of -1.64 and the area to the right of 1.64), the sum of the total area under the curve in these two critical regions is 0.1



3. Set the Test Variance to the same as the variance of the population which we drew our sample from, which is Test Variance: $\sigma^2 = 1$ and press Test

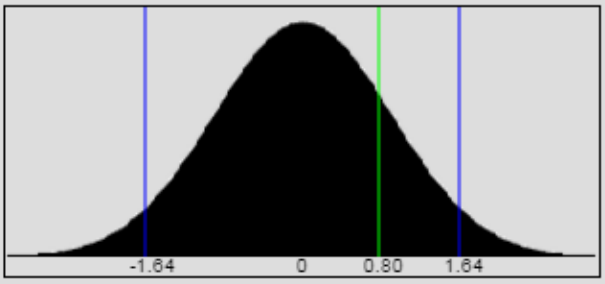
Test Statistics (Assumption: Normal Distributed with unknown mean and known variance.):
Test Variance: $\sigma^2 = 1$ Test

Z-statistcs: 0.80

Test type: Two-tailed Test

Test result: We cannot reject null hypothesis

Notice that the green line shows the Z-statistics of the sample mean (0.8). As the green line is within the non-critical region (in between two critical values), we cannot reject the null hypothesis



Acceptance boundaries are show in blue lines.

Test Statistics (Assumption: Normal Distributed with unknown mean and known variance.):
Test Variance: $\sigma^2 = 1$ Test

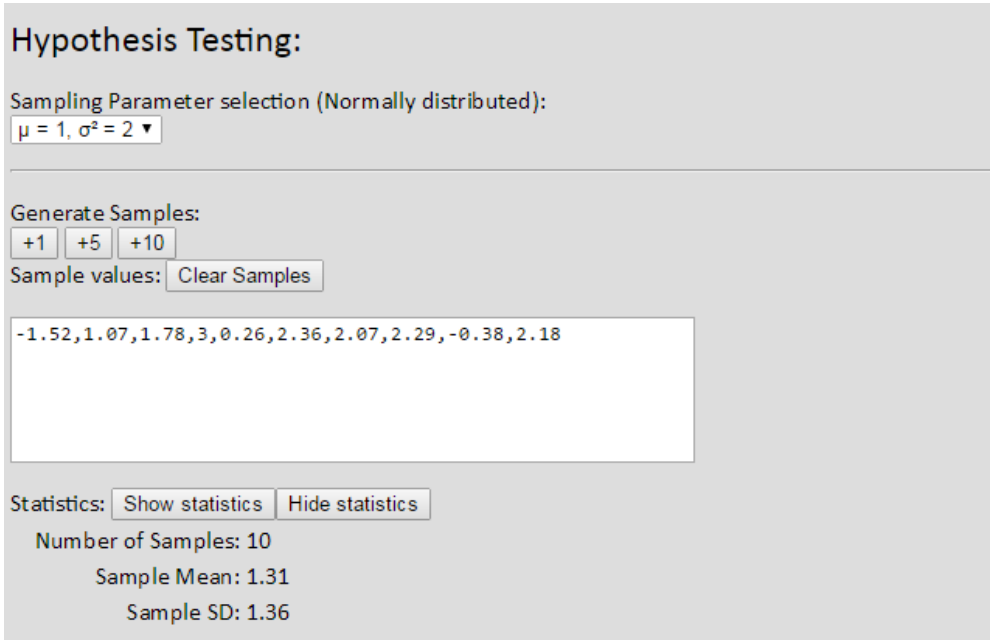
Z-statistcs: 0.80

Test type: Two-tailed Test

Test result: We cannot reject null hypothesis

Hypothesis testing (reject the null hypothesis, two-sided):

1. Take a sample of size 10 from a population with mean 1 and variance 2



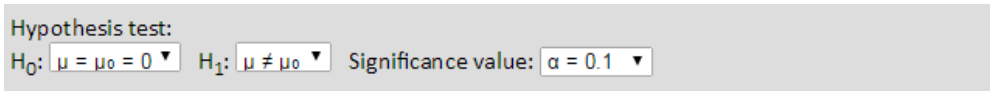
2. Set

   Null hypothesis (H_0) as population mean μ_0 = sample mean μ = 0

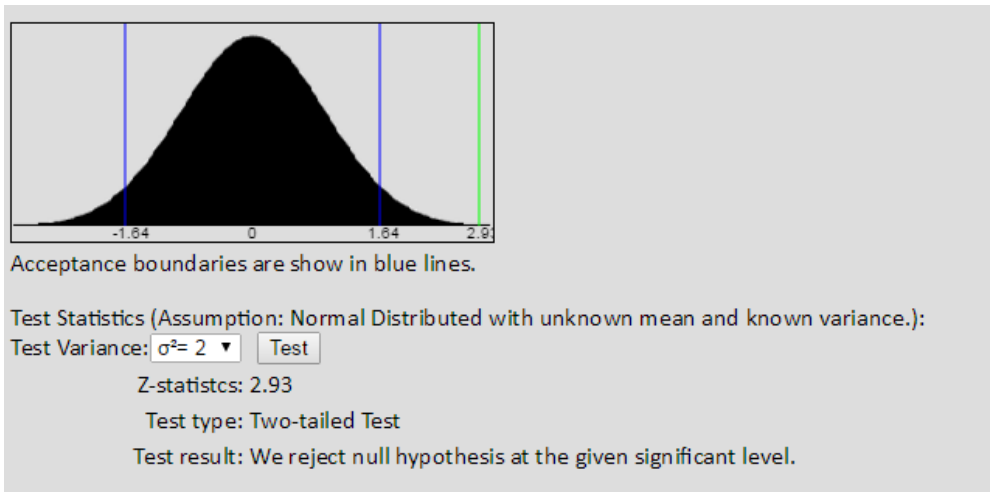
   Alternative hypothesis (H_1) as population mean $\mu_0 \neq$ sample mean μ

   Significance level (α) at 0.1

   Please mind that the population for null hypothesis here which μ_0 belongs to is a different population from the one we drew our sample from, this population has a mean of 0.



3. Set Test Variance at 2 which is the same as the sample population.

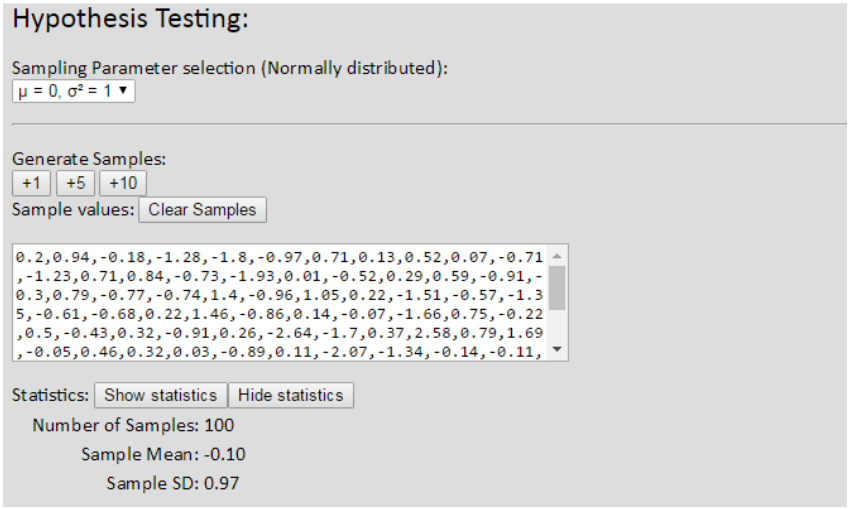


   You will find the Z-statistics equal to 2.93 which falls in the critical region, so we reject our null hypothesis and conclude that sample mean μ 1.31 is not equal to population mean μ_0 0 (which is right).

Try to increase the sample size to 100 and repeat the process. You will find the Z-statistics get larger and the green line will be out of sight, try to find out why is this.

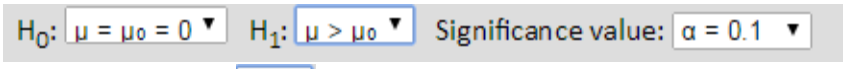
Hypothesis testing (not reject the null hypothesis, one-sided):

1. Set the population to mean = 0, variance = 1 and draw 100 sample

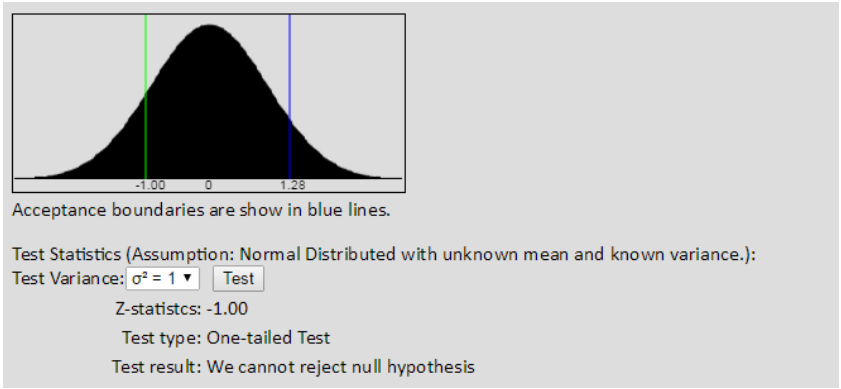


2. Set

   Null hypothesis (H_0) as population mean μ_0 = sample mean μ = 0
   Alternative hypothesis (H_1) as population mean μ_0 > sample mean μ
   Significance level (α) at 0.1
   As follows:



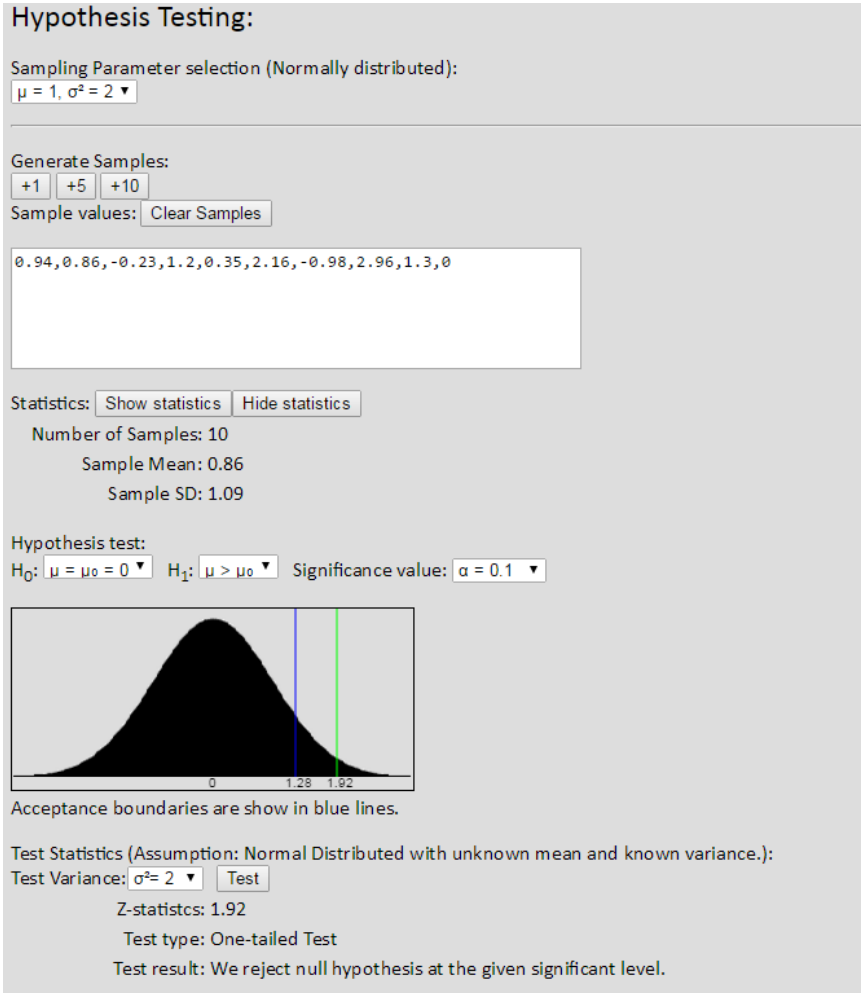
3. Set Test Variance to 1 and press Test and you will find the Z-statistics fall within the non-critical region.



The rule of thumb is to set a two-sided alternative hypothesis rather than a one-sided alternative hypothesis unless you have specific reasons to, for example, test the height of a subject at age of 15 and age of 20, null hypothesis is there is no difference between these two heights and alternative hypothesis is height at age of 20 is **higher** than age of 15, since height can only increase with age so it is optimal for a one-sided alternative than a two-sided one (the height at age of 20 is **different** from age of 15)

Hypothesis testing (reject the null hypothesis, one-sided):

1. Set the population mean to 1 and variance to 2
   Draw 10 samples
   Keep: Null hypothesis (H_0) as population mean μ_0 = sample mean μ = 0
   Alternative hypothesis (H_1) as population mean μ_0 > sample mean μ
   Significance level (α) at 0.1
   Set the Test Variance to 2
   As follows:



   You will find that we reject the null hypothesis as Z-statistics is equal to 1.92 which falls in the critical region (which is right because sample mean 0.86 is bigger than population mean 0