Hamiltonian Simulation

Monday, February 6, 2023 3:52 PM

- lets do Hamiltonian sim for a qubit!

$$H = \underbrace{\frac{\delta}{2}\sigma_z}_{H_1} + \underbrace{\frac{\Omega}{2}\sigma_x}_{H_2}$$

let $H_1 = \frac{\delta}{2}\sigma_z$

$H_2 = \frac{\Omega}{2}\sigma_x$

- therefore our matrix exponential is:

$$U(t) = e^{-i(H_1 + H_2)t}$$

$$= \underbrace{e^{-iH_1 t}}_{U_1} \underbrace{e^{-iH_2 t}}_{U_2}$$

- lets take a look at these terms individually

$$U_1 = e^{-i\left(\frac{\delta}{2}\sigma_z\right)t}$$

$$= \cos\left(\frac{\delta}{2}t\right) I - i\sin\left(\frac{\delta}{2}t\right)\sigma_z$$

$$\boxed{= R_z(\delta t)}$$

$$U_2 = e^{-i\left(\frac{\Omega}{2}\sigma_x\right)t}$$

$$= \cos\left(\frac{\Omega}{2}t\right) I - i\sin\left(\frac{\Omega}{2}t\right)\sigma_x$$

$$\boxed{= R_x(\Omega t)}$$

$$\boxed{U(t) = R_z(\delta t)\, R_x(\Omega t)}$$

$$t^2/r < \epsilon$$

lets run for $t = 10$ sec

$$2^2/r < \epsilon$$

let $r = 1000$

$\Downarrow$

$\epsilon = 1/10$ (or 10%)

$$\frac{100}{1000} = 1/10$$