

# State of Quantum Machine Learning: Analysis of Quantum Neural Networks and Linear Regression

Ryan Dougherty

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## Introduction

Quantum machine learning (QML) is a new approach to typical machine learning algorithms that takes advantage of superposition and entanglement to provide upwards of exponential speedup. QML is still in very early stages of its development. The goal of this paper is to discuss the current state of QML and its potential for the future. It will also go over the upsides as well as the downsides of QML. I will go over the classical approach to machine learning and how QML differs from it. I will also go over some examples of QML in action and how it is being used today. Finally, I will conclude with my personal thoughts on the subject and provide some recommendations for further reading.

## Classical Approach To Machine Learning

Classical approaches to machine learning are based on the idea of using a generalized function that maps a set of input features to outputs. The output can then be used to make predictions about new inputs. The overarching goal of machine learning is to find the best function that accurately guesses the output for a given input with a desired accuracy. There are many approaches to machine learning. In this paper I will go over two specific approaches:

- **Neural Networks:** Neural networks work similar to neurons in the human brain. They are made up of layers of nodes that are connected

to each other. Each node has a weight that is multiplied by the input and then added to the sum of the other nodes. The sum is then passed through an activation function to produce a desired output. The weights are then adjusted to minimize the error between the output and the desired output. Neural networks are often used in natural language processing and image recognition.

- **Linear Regression:** Linear regression is a supervised learning algorithm that is used to predict a continuous output. Supervised learning is when your algorithm is attempting to find a specific output based on its given input; it will continually refine its decision and prediction process to ensure its outputs match their desired results. Linear regression is based on the idea of a linear function that maps the input to the output. The goal is to find the best linear function that accurately predicts the output for a given input. Linear regression is mainly create prediction models, for example, predicting future stock and housing prices.

## Quantum Approach

Now that I've gone over the classical approach to machine learning, let's discuss the quantum approach to machine learning. In a nutshell, QML algorithms are developed by projecting their classical counterparts into a hilbert space representable by quantum states. This allows for the use of quantum superposition and entanglement to provide significant speedup not possible on classical computers.

I want to dissect two different approaches to quantum machine learning done in recent years. One such approach is quantum neural networks (QNN), which uses a quantum circuit to create a neural network similar to its classical counterpart. The other is quantum linear regression (QLR) which takes advantage of various algorithms we've learned throughout the course - such as the HHL algorithm, quantum fourier transform, and phase estimation.

## Quantum Neural Networks (QNN)

In 2018 Google and MIT demonstrated the creation of a small quantum neural network on a quantum simulator. [3] This algorithm was also designed

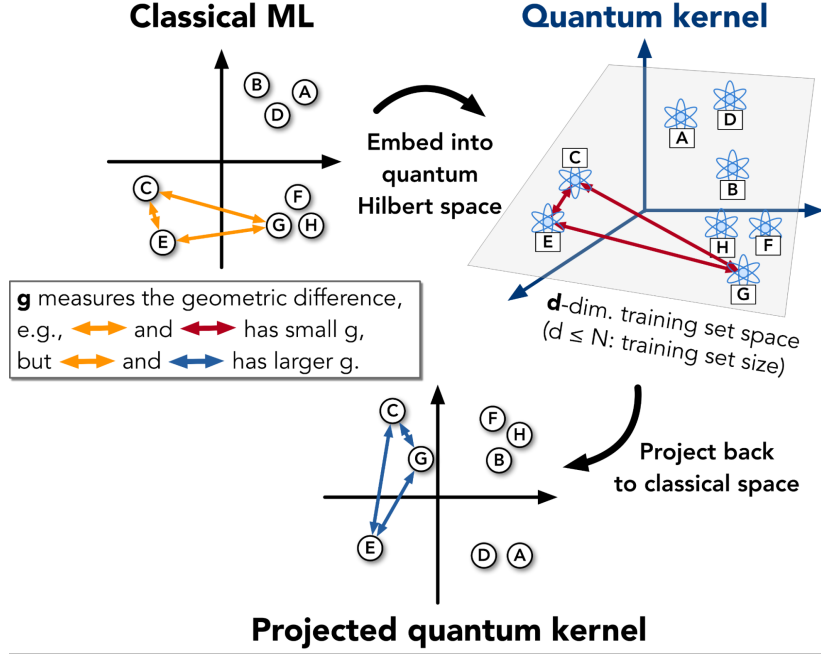


Figure 1: Projection of classical machine learning methods into a quantum hilbert space. Where  $g$  is a geometric quantity which quantifies the potential for quantum advantage.[5]

with near-term quantum computers in mind, being able to run on super conducting chips once their error rate and fidelity is further improved.

This QNN algorithm operates on  $N+1$  qubits of input, where a single additional qubit is used to measure output. It creates a neural network from a basic set of  $L$  unitary gates which depend on some continuous set of parameters  $\theta$ . The goal is to have the measured output from a prepared state  $|\psi\rangle$  match a given label  $l(z)$ . The output is measured in the Pauli Y basis (denoted as  $Y_{n+1}$ ), yeilding a value of  $+1$  or  $-1$ . A loss function can be generated from the measured results in the form:

$$\text{loss}(\theta, z) = 1 - l(z) \langle z, 1 | U^\dagger(\theta) Y_{n+1} U(\theta) | z, 1 \rangle \quad (1)$$

Different values of thetas can be sampled from neighboring unitaries and the loss function can be minimized. This process is known as gradient descent. The algorithm is summarized in the figure above.

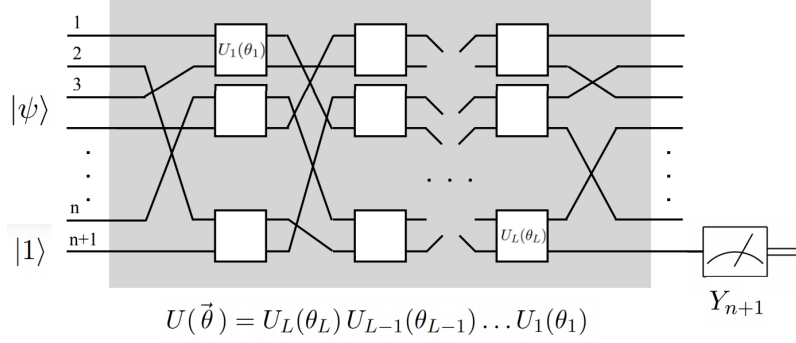


Figure 2: Quantum neural network circuit built from a set of  $L$  unitaries which depend on some continuous set of parameters  $\theta$ .

## Quantum Linear Regression (QLR)

Quantum linear regression proves to provide a polynomial speedup over its classical counterpart. This was achieved by a 2017 paper by Guoming Wang from University of Maryland. [7] The algorithm is based mostly on the use of the HHL algorithm, which we learned about in class. This is due to the fact that linear regression relies on solving a system of linear equations to best fit the input data.

The HHL algorithm works for a linear system in the form of  $Ax = b$ , where  $A$  is a Hermitian matrix,  $x$  is a vector of unknowns, and  $b$  is the output state we're trying to solve for. Since  $A$  is Hermitian, it can be decomposed into a sum of its eigenvalues and eigenvectors like so:

$$A = \sum_{i=1}^n \lambda_i |v_i\rangle \langle v_i| \quad (2)$$

Since  $A$  is also invertible, we can write  $A^{-1}$  as:

$$A^{-1} = \sum_{i=1}^n \frac{1}{\lambda_i} |v_i\rangle \langle v_i| \quad (3)$$

$|b\rangle$  can be written as a linear combination of the eigenvectors of  $A$ :

$$|b\rangle = \sum_{i=1}^n \alpha_i |v_i\rangle \quad (4)$$

Therefore we can simply solve for  $|x\rangle$  by multiplying  $A^{-1}$  by  $|b\rangle$ :

$$|x\rangle = \sum_{i=1}^n \frac{\langle u_i | b \rangle}{\lambda_i} |v_i\rangle \quad (5)$$

Thus, we can construct a quantum circuit that solves for  $|x\rangle$  by first preparing the state  $|b\rangle$  and extracting its eigenvalues with the quantum phase estimation algorithm; This will encode the eigenvalues into a quantum state. Applying eigenvalue inversion gives us the inverted eigenvalues, which can be converted back into amplitudes for measurement using an inverted form of the phase estimation algorithm. This can be done exponentially faster than a classical algorithm and provides a polynomial speedup for QLR.[6][7] The HHL algorithm is summarized in the figure below.

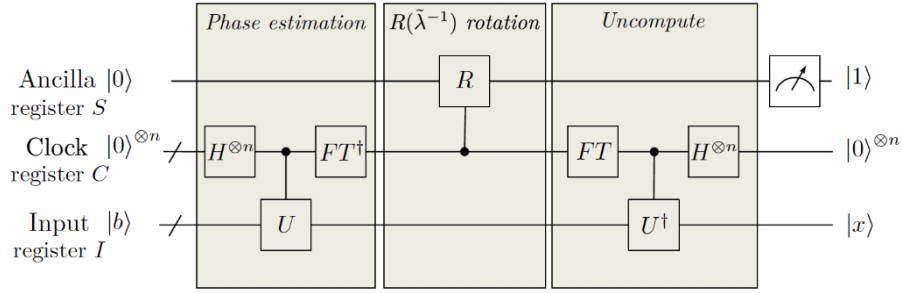


Figure 3: HHL algorithm for solving a linear system of equations. [6]

## Quantum Annealing

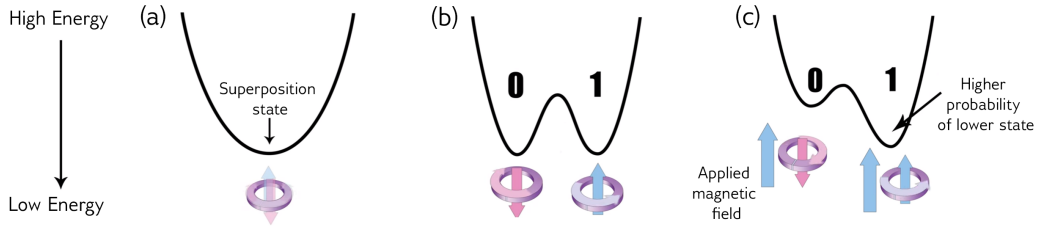


Figure 4: Quantum

An additional approach to quantum machine learning is quantum annealing. Quantum annealing takes advantage of the fact that quantum systems

follow the laws of thermodynamics. Overtime, a quantum system will decay into its lowest state of energy - known as the ground state. This means that the global minima of a function can be mapped to the ground state of a quantum system. This is done by encoding the function as a Hamiltonian and then running the system through a quantum annealer. The company D-Wave Systems has worked to develop quantum annealers that can be used to do linear regression and solve specific optimization problems.[1] [2]

## **Advantages of QML**

### **Computational Complexity**

We've seen that quantum approaches to machine learning are able to provide polynomial and exponential runtime speedups to the classical algorithms of neural networks and linear regression. As mentioned earlier, the HHL algorithm is an example of this computational speedup. Quantum systems naturally work in hilbert spaces decomposable by a set of eigenvalues and eigenvectors. Therefore, we can take advantage of this fact to quickly estimate the solution of a system of linear equations. Since linear regression relies heavily on solving a system of linear equations, the HHL algorithm proves very promising for quantum linear regression.

### **Data Efficiency**

Machine learning needs a significant sample of training data in order to perform optimally. Even if a machine learning algorithm runs quickly, it may still take a while to achieve good results due to the fact that it needs a lot of training data. QML has the ability to perform more efficiently with less data than its classical counterpart. This is key in machine learning, as the more data you have, the more accurate your model typically will be. This suggests that with scale, quantum machine learning could possibly outperform classical machine learning purely due to the fact that it needs to process less data to achieve the comparable results. [4]

## Disadvantages of QML

### HHL can only be used for specific problems

As we noticed in class, the HHL algorithm comes with quite a few limitations. It requires that the eigenvalues of the matrix to be known ahead of time in order to perform proper eigenvalue decomposition. It also requires that the matrix be Hermitian, which is not always the case. HHL also cannot output negative solutions. This means that additional time must be spent classically to find which parts of the output vector are positive and negative.

### Hardware limitations

Quantum computers are still in their infancy; they are still noisy and error-prone. This of course means that quantum algorithms are not nearly as reliable as their classical counterparts when run on a quantum system. QML algorithms can be run with complete accuracy on quantum simulators, however, simulators still need lots of classical processing power and can only scale to a handful of qubits before they run out of memory. It will probably be awhile before we see quantum computers that can run QML algorithms with the same accuracy as classical computers.

Additionally, some methods of QML require the use of qRAM, which has not yet been developed. QRAM allows for loading in of additional quantum states later on into an a circuit. This is particularly useful for dealing with large amounts of data, which QNN and QLR both require.

## Conclusion

Quantum neural networks and quantum linear regression are two promising approaches to QML. They both provide polynomial speedups over their classical counterparts. Quantum annealing is another approach to QML that is still in its infancy. It is able to provide exponential speedups over classical algorithms, but it is still limited by the fact that it can only be used for specific problems. In general, there are quite a few limitations to QML, but it is still a very promising field of research. As quantum computers become more reliable and more qubits are added to quantum systems, we will see more and more applications of QML in the real world.

## References

- [1] Steven H Adachi and Maxwell P Henderson. What is quantum annealing?, 2021.
- [2] Prasanna Date and Thomas Potok. Adiabatic quantum linear regression. *Scientific Reports*, 11(1), 2021.
- [3] Edward Farhi and Hartmut Neven. Classification with quantum neural networks on near term processors, 2018.
- [4] Hsin-Yuan Huang, Michael Broughton, Masoud Mohseni, Ryan Babbush, Sergio Boixo, Hartmut Neven, and Jarrod R. McClean. Power of data in quantum machine learning. *Nature Communications*, 12(1):2631, May 2021.
- [5] Jarrod McClean and Hsin-Yuan Huang. Quantum machine learning and the power of data, Jun 2021.
- [6] Jens Palsberg. Quantum algorithms solving linear equations. *Quantum Algorithms Solving Linear Equations*, Apr 2022.
- [7] Guoming Wang. Quantum algorithm for linear regression. *Physical Review A*, 96(1), jul 2017.