HW7_Code

March 17, 2023

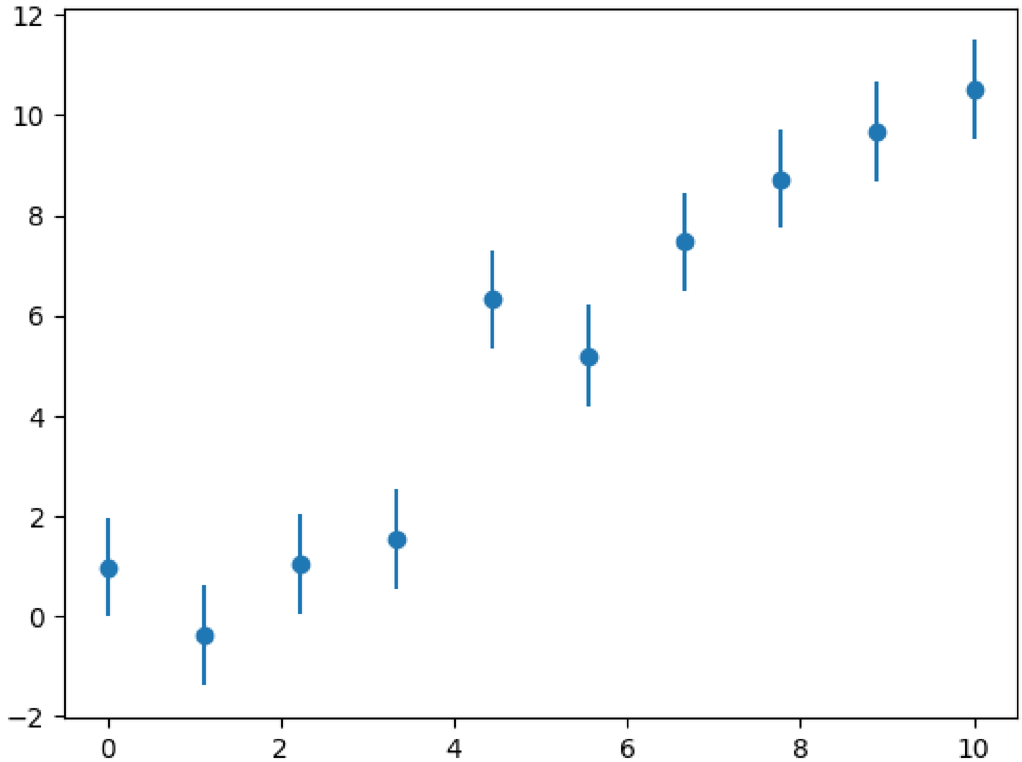
[ ]:
```
import random

from qutip import *
from qiskit.visualization import array_to_latex
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import seaborn as sns
from scipy.optimize import curve_fit
from scipy.stats import chisquare
```

1 1)

1.1 1.A)

[ ]:
```
slope = 1
offset = 0.25
time = np.linspace(0,10,10)
data = np.random.normal(slope*time + offset,1)
sigma = 1*np.random.normal(np.ones(len(data)),0.01)

plt.errorbar(time,data,yerr=sigma,fmt='o')
```

[ ]:
```
<ErrorbarContainer object of 3 artists>
```



1.2 1.B)

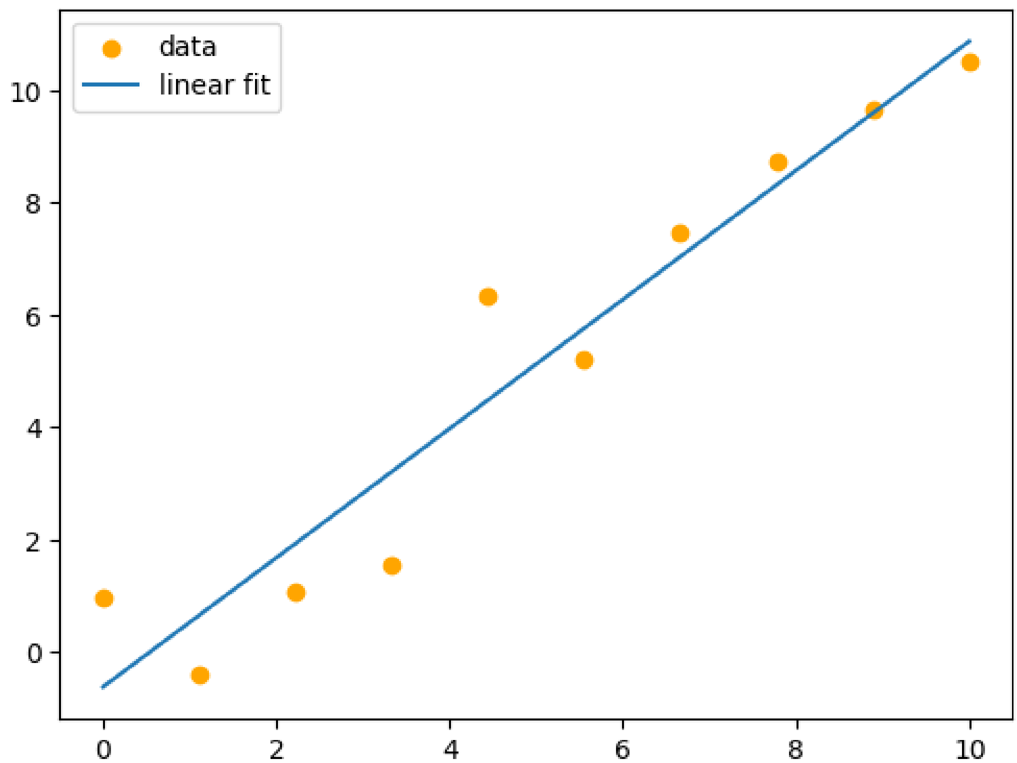
First, lets take a look at the linear fit

[ ]:
```
def func_linear(x, a, b):
    return a*x + b
```

[ ]:
```
lin_fit = curve_fit(func_linear, time, data, sigma=sigma)
print("Linear fit parameters: a={:0.4f}, b={:0.4f}".format(lin_fit[0][0],
 ↪lin_fit[0][1]))
```

```
Linear fit parameters: a=1.1479, b=-0.6148
```

[ ]:
```
plt.scatter(time, data, label='data', color='orange')
plt.plot(time, func_linear(time, *lin_fit[0]), label='linear fit')
plt.legend()
plt.show()
```



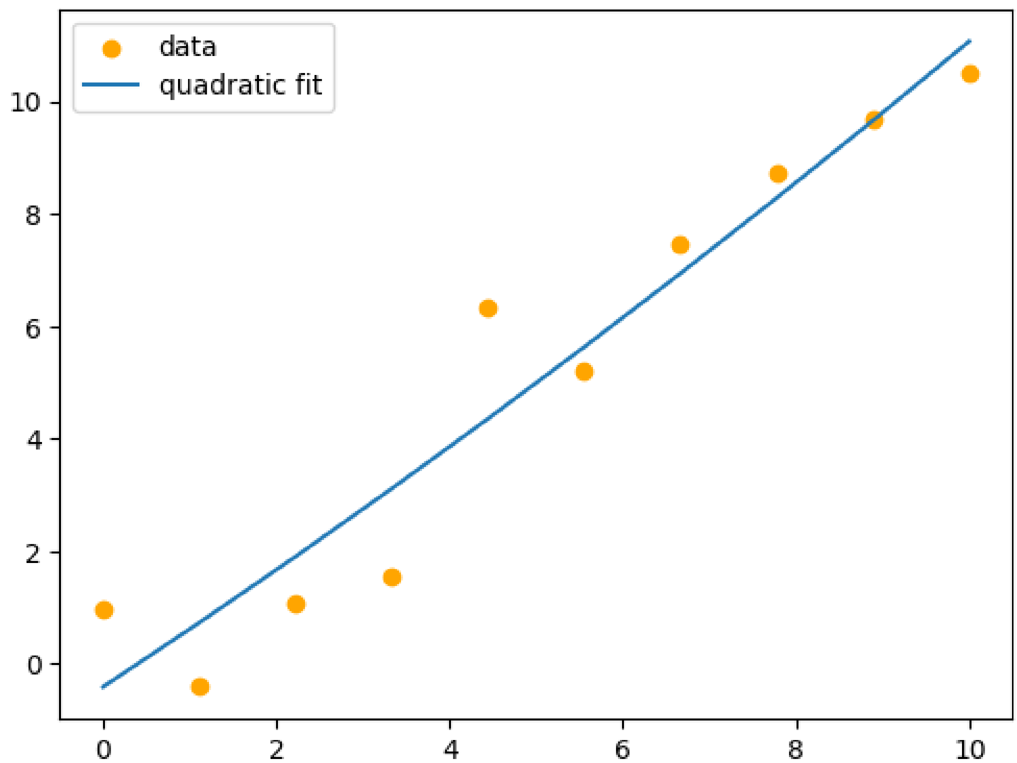
Next, lets take a look at the quadtatic fit.

```
[ ]: def func_quadratic(x, a, b, c):
         return a * x**2 + b * x + c
```

```
[ ]: quad_fit = curve_fit(func_quadratic, time, data, sigma=sigma)
     print("Quadratic fit parameters: a={:0.4f}, b={:0.4f}, c={:0.4f}".
      ↪format(quad_fit[0][0], quad_fit[0][1], quad_fit[0][2]))
```

```
Quadratic fit parameters: a=0.0135, b=1.0132, c=-0.4167
```

```
[ ]: plt.scatter(time, data, label='data', color='orange')
     plt.plot(time, func_quadratic(time, *quad_fit[0]), label='quadratic fit')
     plt.legend()
     plt.show()
```



2 1.C)

```
[ ]: def chi_squared(data, model, sigma):
         sum = 0
         for i in range(len(data)):
             sum += ((data[i] - model[i])/sigma[i])**2
         return sum
```

```
[ ]: chi_square_linear = chi_squared(data, func_linear(time, *lin_fit[0]), sigma)
     v_linear = len(data) - len(lin_fit[0])
     print(v_linear)
     print("X^2 / V for linear fit: {:0.4f}".format(chi_square_linear/v_linear))
```

```
8
X^2 / V for linear fit: 1.4412
```

```
[ ]: chi_square_quadratic = chi_squared(data, func_quadratic(time, *quad_fit[0]),␣
      ↪sigma)
     v_quadratic = len(data) - len(quad_fit[0])
     print(v_quadratic)
```

```
print("X^2 / V for quadratic fit: {:0.4f}".format(chi_square_quadratic/
 ↪v_quadratic))
```

```
7
X^2 / V for quadratic fit: 1.6257
```

Therefore, we can conclude that the linear fit is slightly better for fitting this data than the quadratic

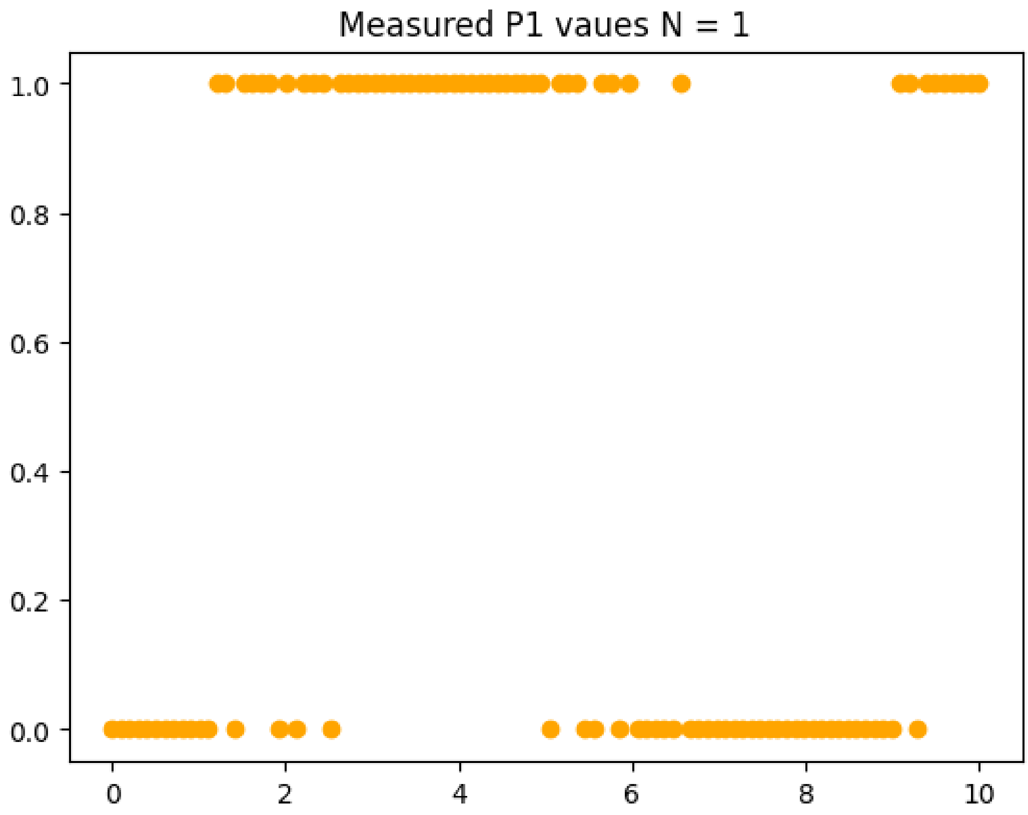
3 3)

[ ]:
```
t_p = 1
delta = np.pi / 4
Omega = 2 * np.pi
```

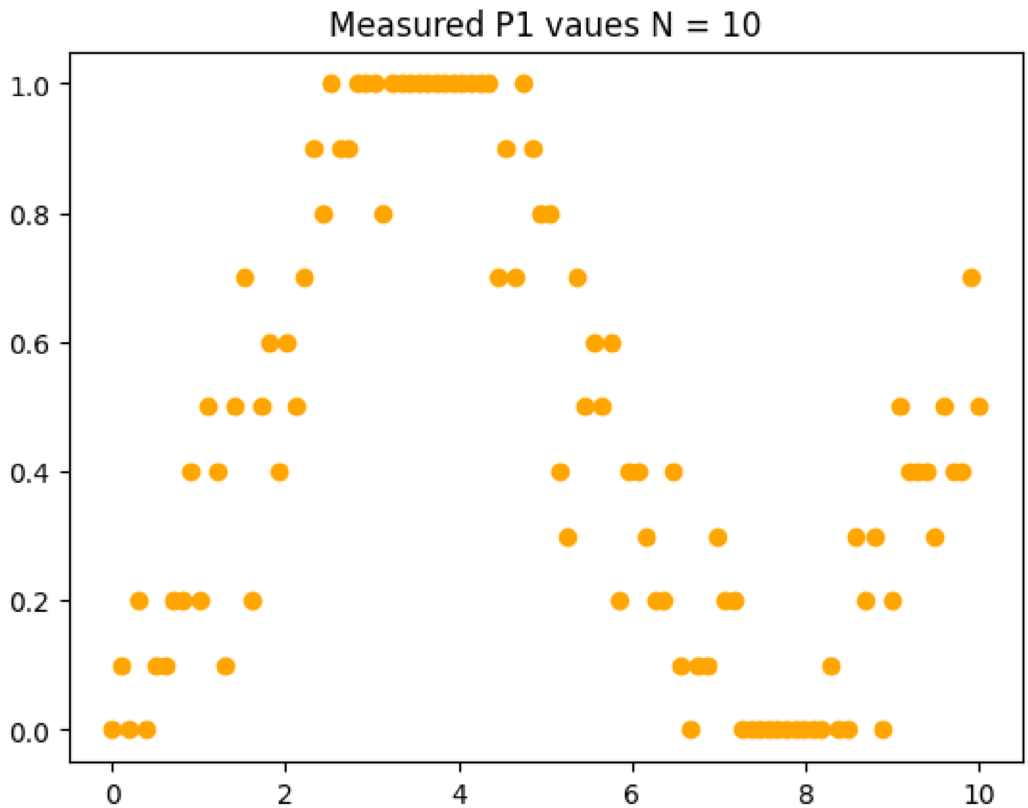
[ ]:
```
def P1(t_w, Omega, delta):
    Omega_p = np.sqrt(Omega**2 + delta**2)
    return 1 - (Omega**2) / (Omega_p**4) * (Omega_p * np.cos(delta * t_w / 2) *␣
 ↪np.sin(np.pi/2) - 2 * delta * np.sin(delta * t_w / 2) * np.sin(np.pi/4)**2␣
 ↪)**2
```

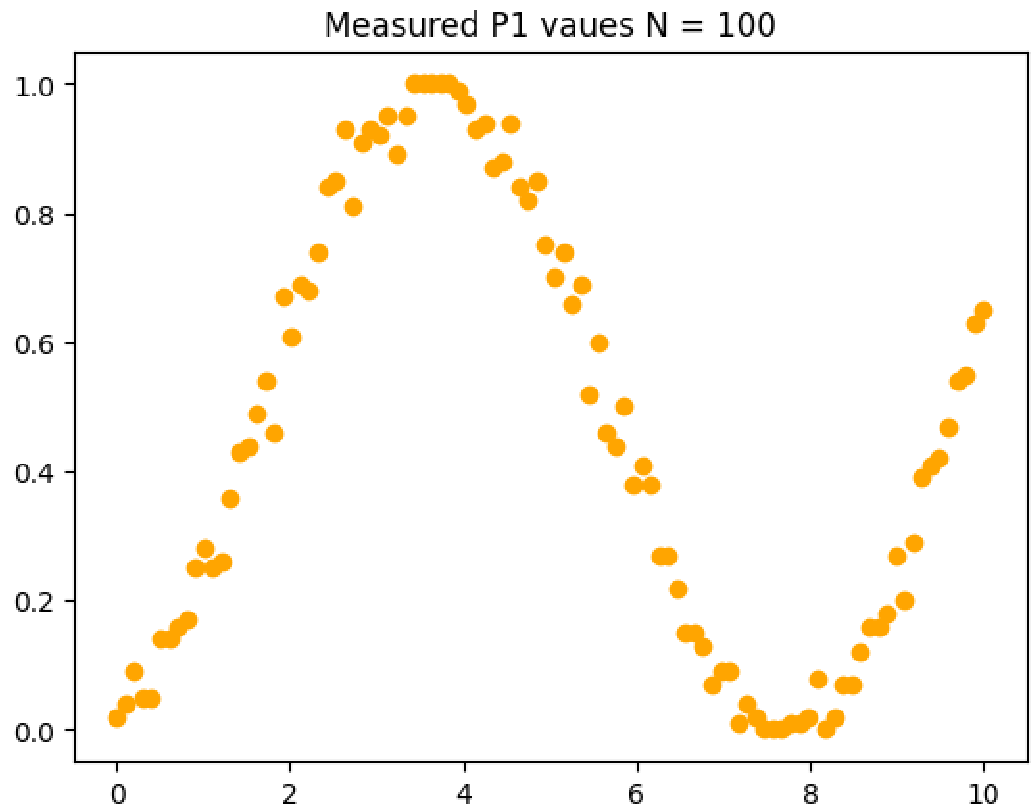
3.1 3.A)

[ ]:
```
for N in [1, 10, 100, 1000]:
    t_w = np.linspace(0, 10, 100)
    measured_results = []
    for t in t_w:
        true_p1 = P1(t, Omega, delta)
        p1_vals = np.sum(np.random.binomial(1, true_p1, N)) / N
        measured_results.append(p1_vals)

    plt.scatter(t_w, measured_results, label='data', color='orange')
    plt.title("Measured P1 vaues N = {}".format(N))
    plt.show()
```

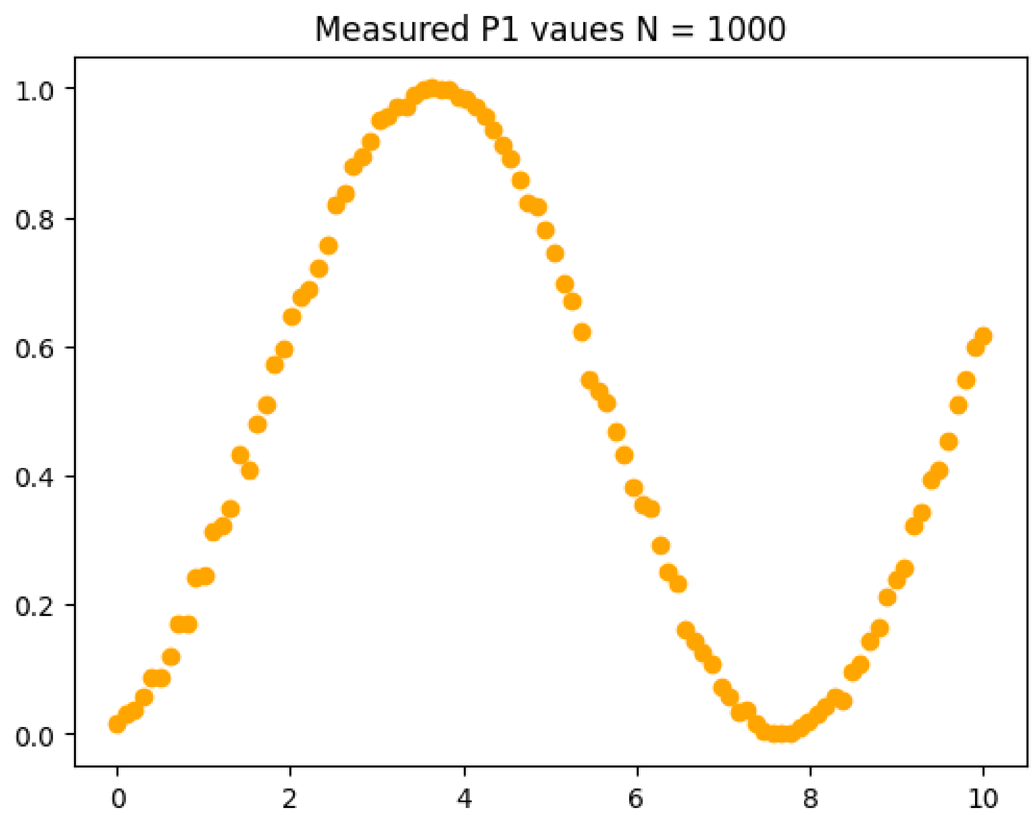


Measured P1 vaues N = 1
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10


Measured P1 vaues N = 10
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10



Measured P1 vaues N = 100
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10



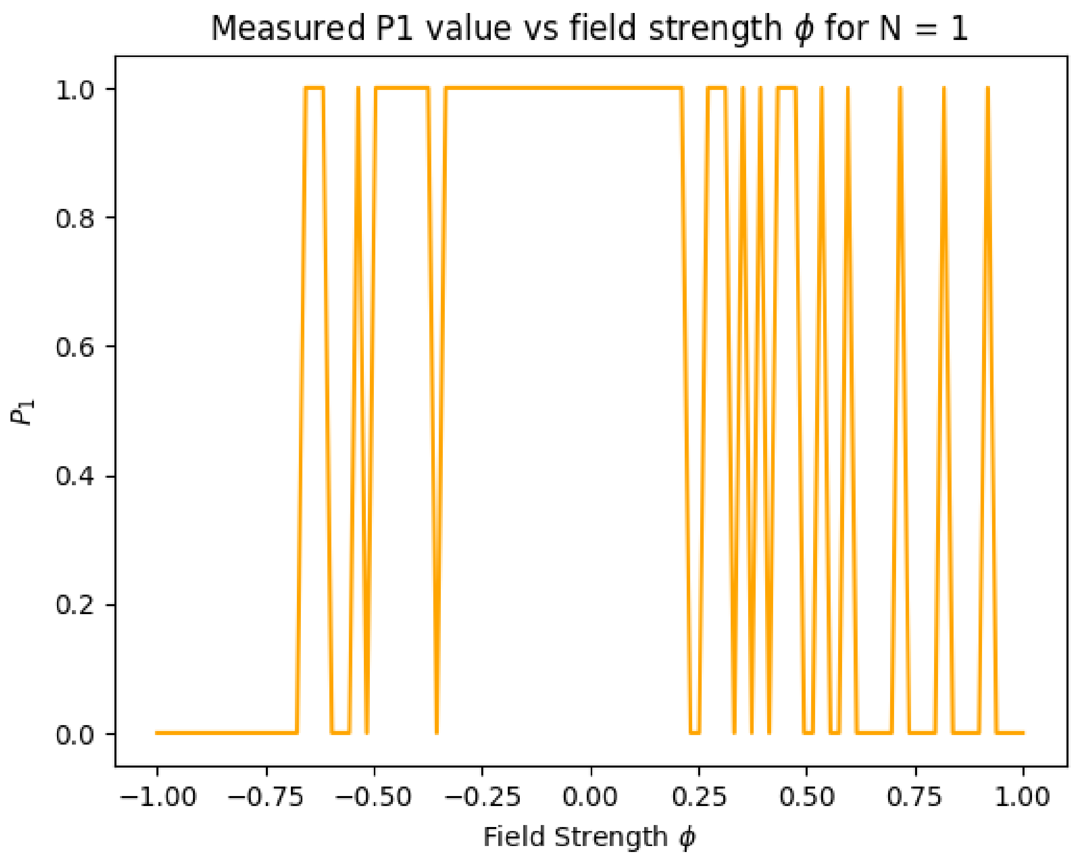
3.2 3.B)

[ ]:
```
def P1_field(t_w, Omega, gamma, phi):
    delta = (np.pi / 2 * t_w) + (2 * gamma * phi)
    Omega_p = np.sqrt(Omega**2 + delta**2)
    return 1 - (Omega**2) / (Omega_p**4) * (Omega_p * np.cos(delta * t_w / 2) *␣
 ↪np.sin(np.pi/2) - 2 * delta * np.sin(delta * t_w / 2) * np.sin(np.pi/4)**2␣
 ↪)**2
```

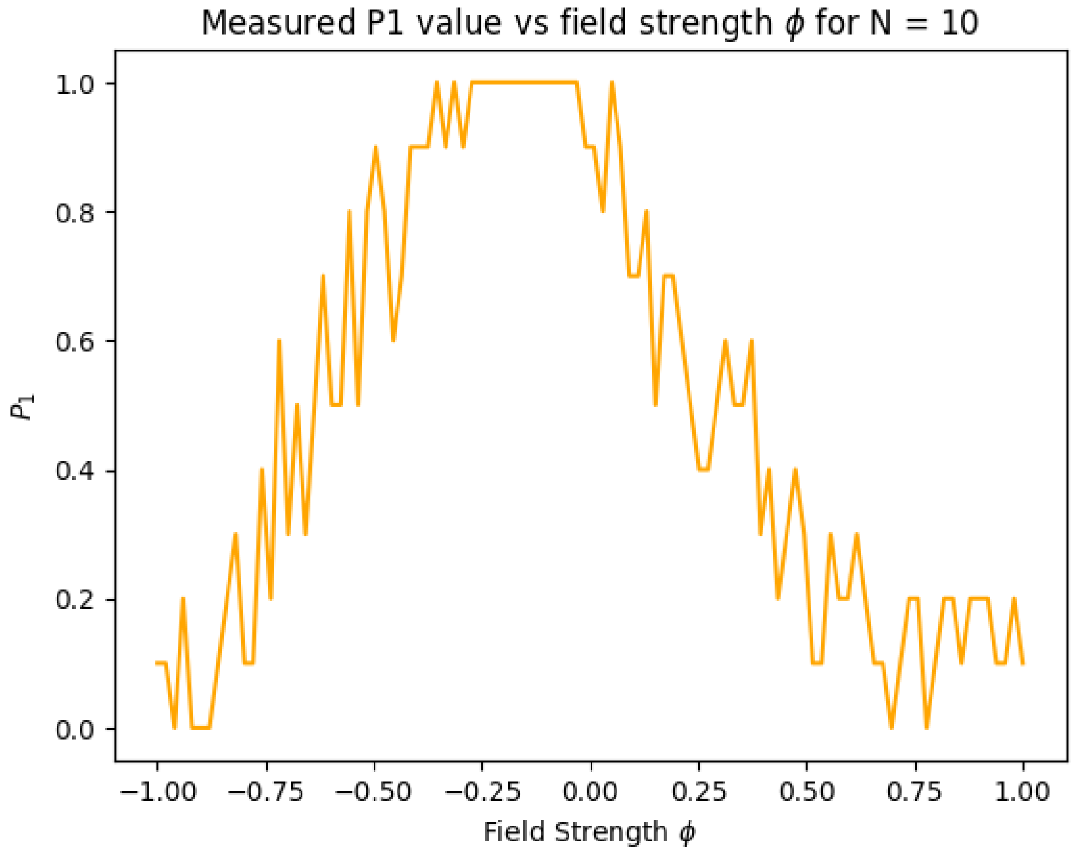
[ ]:
```
for N in [1, 10, 100, 1000]:
    t_w = np.linspace(1, 100, 1000)

    # First, find the best t by sampling around 2
    best_t = 0
    for t in t_w:
        true_p1 = P1(t, Omega, np.pi/4)
        p1_val = np.sum(np.random.binomial(1, true_p1, N)) / N
        if p1_val >= 0.5:
            best_t = t
            break
```

```python
    print("Best t for N = {} is {}".format(N, best_t))

    # Now, lets plot our results relative to the strengh of the field phi
    gamma = 1
    phis = np.linspace(-1, 1, 100)
    measured_p1 = []
    for phi in phis:
        true_p1 = P1_field(best_t, Omega, gamma, phi)
        p1_val = np.sum(np.random.binomial(1, true_p1, N)) / N
        measured_p1.append(p1_val)


    plt.plot(phis, measured_p1, label='data', color='orange')
    plt.title("Measured P1 value vs field strength $\phi$ for N = {}".format(N))
    plt.xlabel("Field Strength $\phi$")
    plt.ylabel("$P_1$")
    plt.show()
```

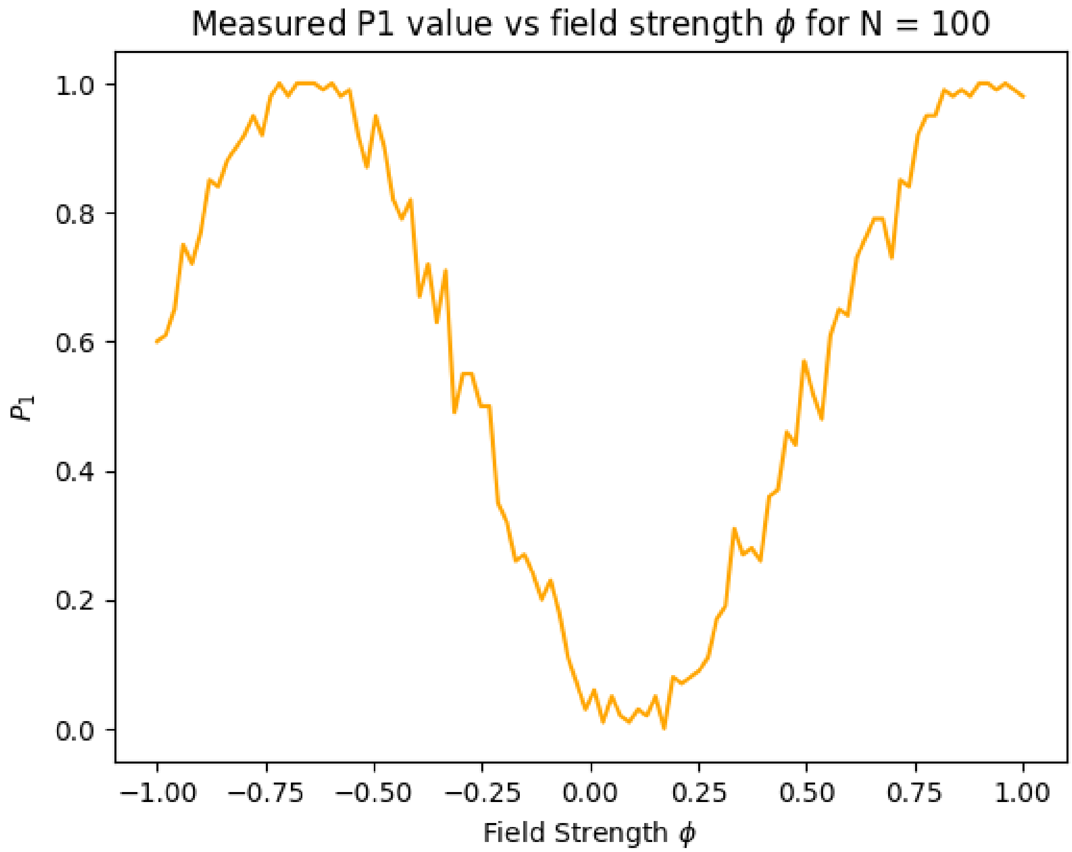
```
Best t for N = 1 is 1.2972972972972974
```



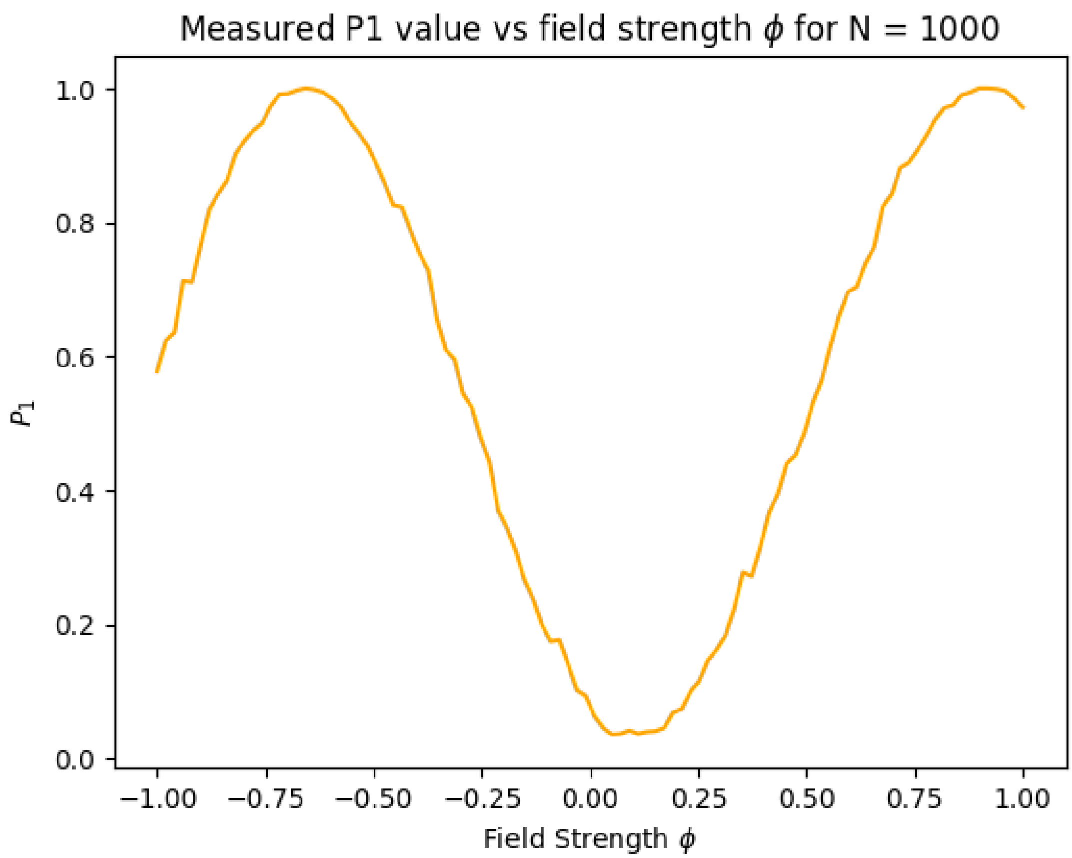
```
Best t for N = 10 is 1.3963963963963963
```




```
Best t for N = 100 is 1.7927927927927927
```




```
Best t for N = 1000 is 1.7927927927927927
```



As we can see, we get a linear relationship between our field strength and our shifted slope probability at $ P_1 1/2 $

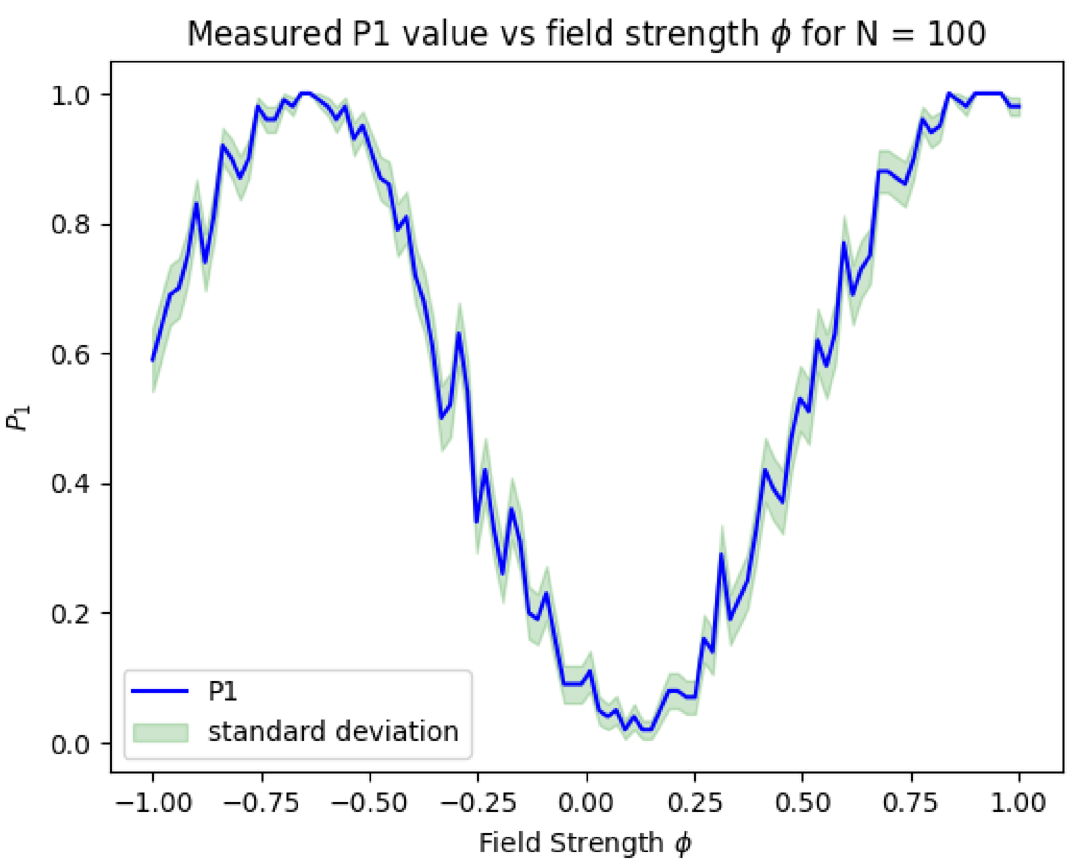
3.3 3.C)

Now, lets plot our N=100 solution with the std error bars

[ ]:
```
N = 100
t_w = np.linspace(1, 100, 1000)

# First, find the best t by sampling around 2
best_t = 0
for t in t_w:
    true_p1 = P1(t, Omega, np.pi/4)
    p1_val = np.sum(np.random.binomial(1, true_p1, N)) / N
    if p1_val >= 0.5:
        best_t = t
        break
print("Best t for N = {} is {}".format(N, best_t))
```

```
# Now, lets plot our results relative to the strengh of the field phi
gamma = 1
phis = np.linspace(-1, 1, 100)
measured_p1 = []
std_devs = []
for phi in phis:
    p1_field = P1_field(best_t, Omega, gamma, phi)
    p1_val = np.sum(np.random.binomial(1, p1_field, N)) / N
    measured_p1.append(p1_val)
    std_devs.append(np.sqrt(p1_val * (1 - p1_val) / N))

plt.plot(phis, measured_p1, label='P1', color='blue')
plt.fill_between(phis, np.array(measured_p1) - np.array(std_devs), np.
 ↪array(measured_p1) + np.array(std_devs), color='green', alpha=0.2,␣
 ↪label='standard deviation')
plt.title("Measured P1 value vs field strength $\phi$ for N = {}".format(N))
plt.xlabel("Field Strength $\phi$")
plt.ylabel("$P_1$")
plt.legend()
plt.show()
```

```
Best t for N = 100 is 1.7927927927927927
```



From this graph we can see that the standard error is quite small around the peak and valley of the distribution. This is expected because we expect when the slope is near zero, the error will be small. However, when the slope is near 1 or -1, the error will be larger.

4 4)

4.1 4.A)

```
[ ]: def U(n, t, omega):
         return (-1.j * omega * t * (create(n) * destroy(n) + 0.5 * identity(n)) ).
     ↪expm()
```

```
[ ]: n = 5
     m = 1
     alpha = 1
     omega = 2 * np.pi
     t_vals = np.linspace(0, 10, 100)
```

Here's what our evolved states look like from time 0 - 10 for 100 time steps.

```
[ ]: evolved_states = []
     for t in t_vals:
         state = U(n, t, omega) * coherent(n, alpha)
         state_p1_prob = np.abs((fock(n,0).dag() * state * fock(n,0))[0,0])**2

         evolved_states.append(state)

     print("Eveolved states:")
     print(evolved_states)
```

```
Eveolved states:
[Quantum object: dims = [[5], [1]], shape = (5, 1), type = ket
Qobj data =
[[0.60655682]
 [0.60628133]
 [0.4303874 ]
 [0.24104351]
 [0.14552147]], Quantum object: dims = [[5], [1]], shape = (5, 1), type = ket
Qobj data =
[[ 0.57627211-0.18926601j]
 [ 0.35167767-0.49386219j]
 [-0.00682851-0.43033322j]
 [-0.14597828-0.19181322j]
 [-0.13962682-0.04099813j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
```

```
Qobj data =
[[ 0.48844216-0.35963235j]
 [-0.1982952 -0.57293635j]
 [-0.43017072+0.0136553j ]
 [-0.06423178+0.23232789j]
 [ 0.12242045+0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.35183747-0.4940866j ]
 [-0.58172267-0.17080919j]
 [ 0.02047866+0.42989992j]
 [ 0.22377706-0.08958682j]
 [-0.09529629-0.10997779j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.18009908-0.57920246j]
 [-0.47656931+0.37477825j]
 [ 0.42952089-0.02729686j]
 [-0.20681133-0.1238186j ]
 [ 0.0604518 +0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.00962361-0.60648047j]
 [ 0.02884803+0.60559461j]
 [-0.03410818-0.42903373j]
 [ 0.02671683+0.23955831j]
 [-0.02070986-0.14404027j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.1983853 -0.57319669j]
 [ 0.51003631+0.32778043j]
 [-0.42843857+0.04091092j]
 [ 0.17445139-0.16633906j]
 [-0.02070986+0.14404027j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.36733668-0.48267477j]
 [ 0.56285214-0.22533201j]
 [ 0.04770336+0.42773555j]
 [-0.23801573-0.03808521j]
 [ 0.0604518 -0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.49960665-0.34395403j]
 [ 0.14293624-0.58919121j]
 [ 0.42692485-0.0544838j ]
 [ 0.11383788+0.21246861j]
 [-0.09529629+0.10997779j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
```

```
type = ket
Qobj data =
[[-0.581987  -0.1708868j ]
 [-0.39702983-0.45819685j]
 [-0.06125051-0.42600667j]
 [ 0.10013309-0.21926089j]
 [ 0.12242045-0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.60625144+0.01924479j]
 [-0.60353604+0.0576307j ]
 [-0.42498125+0.06800181j]
 [-0.23512102+0.05310442j]
 [-0.13962682+0.04099813j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.56997697+0.20745465j]
 [-0.30314066+0.52505503j]
 [ 0.07473599+0.42384885j]
 [ 0.18465004+0.15493978j]
 [ 0.14552147+0.j        ]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.47678586+0.37494855j]
 [ 0.25185836+0.55149289j]
 [ 0.42260974-0.08145135j]
 [ 0.01146931-0.24077049j]
 [-0.13962682-0.04099813j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.33598399+0.50500092j]
 [ 0.59532503+0.1147395j ]
 [-0.08814621-0.42126424j]
 [-0.19854189+0.1366861j ]
 [ 0.12242045+0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.16163151+0.58462503j]
 [ 0.43878643-0.41838202j]
 [-0.41981269+0.09481888j]
 [ 0.22900847+0.07521364j]
 [-0.09529629-0.10997779j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.02886114+0.60586979j]
 [-0.08628283-0.60011026j]
 [ 0.10146767+0.41825545j]
 [-0.07883761-0.22778631j]
```

```
 [ 0.0604518 +0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.21647177+0.56661375j]
 [-0.53888433-0.27781418j]
 [ 0.41659292-0.10809093j]
 [-0.13351883+0.20068556j]
 [-0.02070986-0.14404027j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.38246602+0.47077693j]
 [-0.53888433+0.27781418j]
 [-0.11468697-0.41482552j]
 [ 0.24055821-0.01528792j]
 [-0.02070986+0.14404027j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.51026807+0.32792937j]
 [-0.08628283+0.60011026j]
 [-0.41295368+0.12125414j]
 [-0.15784993-0.18216853j]
 [ 0.0604518 -0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.58711588+0.15233552j]
 [ 0.43878643+0.41838202j]
 [ 0.12779079+0.41097789j]
 [-0.04936732+0.23593397j]
 [-0.09529629+0.10997779j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[0.60533562-0.03847021j]
 [0.59532503-0.1147395j ]
 [0.40889864-0.13429526j]
 [0.21764458-0.10359927j]
 [0.12242045-0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.5631079 -0.2254344j ]
 [ 0.25185836-0.55149289j]
 [-0.14076593-0.40671644j]
 [-0.21424801-0.11045253j]
 [-0.13962682+0.04099813j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.46464948-0.38988721j]
 [-0.30314066-0.52505503j]
 [-0.40443186+0.14720116j]
```

```
 [ 0.04185677+0.23738151j]
 [ 0.14552147+0.j        ]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.3197922 -0.51540675j]
 [-0.60353604-0.0576307j ]
 [ 0.15359933+0.40204547j]
 [ 0.16355029-0.17706856j]
 [-0.13962682-0.04099813j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.14300119-0.58945893j]
 [-0.39702983+0.45819685j]
 [ 0.39955786-0.15995884j]
 [-0.23995204-0.02291264j]
 [ 0.12242045+0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.0480696 -0.60464906j]
 [ 0.14293624+0.58919121j]
 [-0.16627807-0.39696966j]
 [ 0.12708428+0.2048208j ]
 [-0.09529629-0.10997779j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.23434027-0.55946029j]
 [ 0.56285214+0.22533201j]
 [-0.39428154+0.17255545j]
 [ 0.08602511-0.22517028j]
 [ 0.0604518 +0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.39721024-0.45840506j]
 [ 0.51003631-0.32778043j]
 [ 0.17878939+0.39149415j]
 [-0.23127955+0.0679098j ]
 [-0.02070986-0.14404027j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.52041569-0.31157452j]
 [ 0.02884803-0.60559461j]
 [ 0.3886082 -0.18497831j]
 [ 0.19410517+0.14291661j]
 [-0.02070986+0.14404027j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.59165359-0.13363086j]
 [-0.47656931-0.37477825j]
```

```
 [-0.19112067-0.38562443j]
 [-0.00382439-0.24101317j]
 [ 0.0604518 -0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.60381028+0.05765689j]
 [-0.58172267+0.17080919j]
 [-0.38254358+0.19721492j]
 [-0.18947299+0.14900321j]
 [-0.09529629+0.10997779j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.55567183+0.24318715j]
 [-0.1982952 +0.57293635j]
 [ 0.20325952+0.37936642j]
 [ 0.23331775+0.06053759j]
 [ 0.12242045-0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[-0.45204523+0.40443328j]
 [ 0.35167767+0.49386219j]
 [ 0.37609376-0.20925295j]
 [-0.09312598-0.22232751j]
 [-0.13962682+0.04099813j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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 [ 0.60628133+0.j        ]
 [-0.2151937 -0.37272642j]
 [-0.12052175+0.2087498j ]
 [ 0.14552147+0.j        ]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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 [ 0.35167767-0.49386219j]
 [-0.36926525+0.22108028j]
 [ 0.23910427-0.03051429j]
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 [ 0.2269112 +0.36571112j]
 [-0.16908597-0.1717903j ]
 [ 0.12242045+0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.25197281+0.55174349j]
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```
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 [ 0.36206492-0.23268499j]
 [-0.03430407+0.23859003j]
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Qobj data =
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 [-0.47656931+0.37477825j]
 [-0.23840021-0.35832757j]
 [ 0.21063572-0.11719457j]
 [ 0.0604518 +0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
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 [ 0.02884803+0.60559461j]
 [-0.35450002+0.24405542j]
 [-0.22082199-0.0966417j ]
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Qobj data =
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 [ 0.51003631+0.32778043j]
 [ 0.24964918+0.35058322j]
 [ 0.05682816+0.23424887j]
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 [0.34657817-0.2551801j ]
 [0.15199063-0.18708507j]
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 [ 0.14293624-0.58919121j]
 [-0.26064678-0.34248587j]
 [-0.24092215-0.00764781j]
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 [-0.39702983-0.45819685j]
 [-0.33830735+0.26604784j]
 [ 0.13981895+0.19634824j]
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Qobj data =
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 [ 0.07157073-0.23017298j]
 [-0.13962682+0.04099813j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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 [ 0.32969587-0.27664769j]
 [-0.2265068 +0.08244173j]
 [ 0.14552147+0.j        ]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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Qobj data =
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 [ 0.25185836+0.55149289j]
 [-0.28184381-0.32526509j]
 [ 0.2027787 +0.13031796j]
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type = ket
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 [-0.32075243+0.28696898j]
 [-0.01910269-0.24028537j]
 [ 0.12242045+0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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 [ 0.43878643-0.41838202j]
 [ 0.2920219 +0.31615901j]
 [-0.17964116+0.16072034j]
 [-0.09529629-0.10997779j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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Qobj data =
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 [ 0.31148601-0.29700131j]
 [ 0.23668754+0.04561778j]
 [ 0.0604518 +0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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 [-0.02070986-0.14404027j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
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Qobj data =
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Qobj data =
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type = ket
Qobj data =
[[0.60167695+0.07678552j]
 [0.56285214+0.22533201j]
 [0.34657817+0.2551801j ]
 [0.15199063+0.18708507j]
 [0.0604518 +0.13237098j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
type = ket
Qobj data =
[[ 0.59559555-0.11479163j]
 [ 0.51003631-0.32778043j]
 [ 0.24964918-0.35058322j]
 [ 0.05682816-0.23424887j]
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type = ket
Qobj data =
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 [ 0.02884803-0.60559461j]
 [-0.35450002-0.24405542j]
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Qobj data =
[[ 0.41155451-0.44557161j]
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Qobj data =
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 [ 0.23910427+0.03051429j]
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Qobj data =
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 [ 0.23331775-0.06053759j]
 [ 0.12242045+0.07867484j]], Quantum object: dims = [[5], [1]], shape = (5, 1),
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 [-0.18947299-0.14900321j]
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```

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 [ 0.25185836+0.55149289j]
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 [ 0.43878643-0.41838202j]
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 [-0.06125051+0.42600667j]
 [ 0.10013309+0.21926089j]
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Qobj data =
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```
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Qobj data =
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 [ 0.42952089+0.02729686j]
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Qobj data =
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Qobj data =
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type = ket
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```
Qobj data =
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 [ 0.35167767+0.49386219j]
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 [-0.14597828+0.19181322j]
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Qobj data =
[[0.60655682]
 [0.60628133]
 [0.4303874 ]
 [0.24104351]
 [0.14552147]]]
```
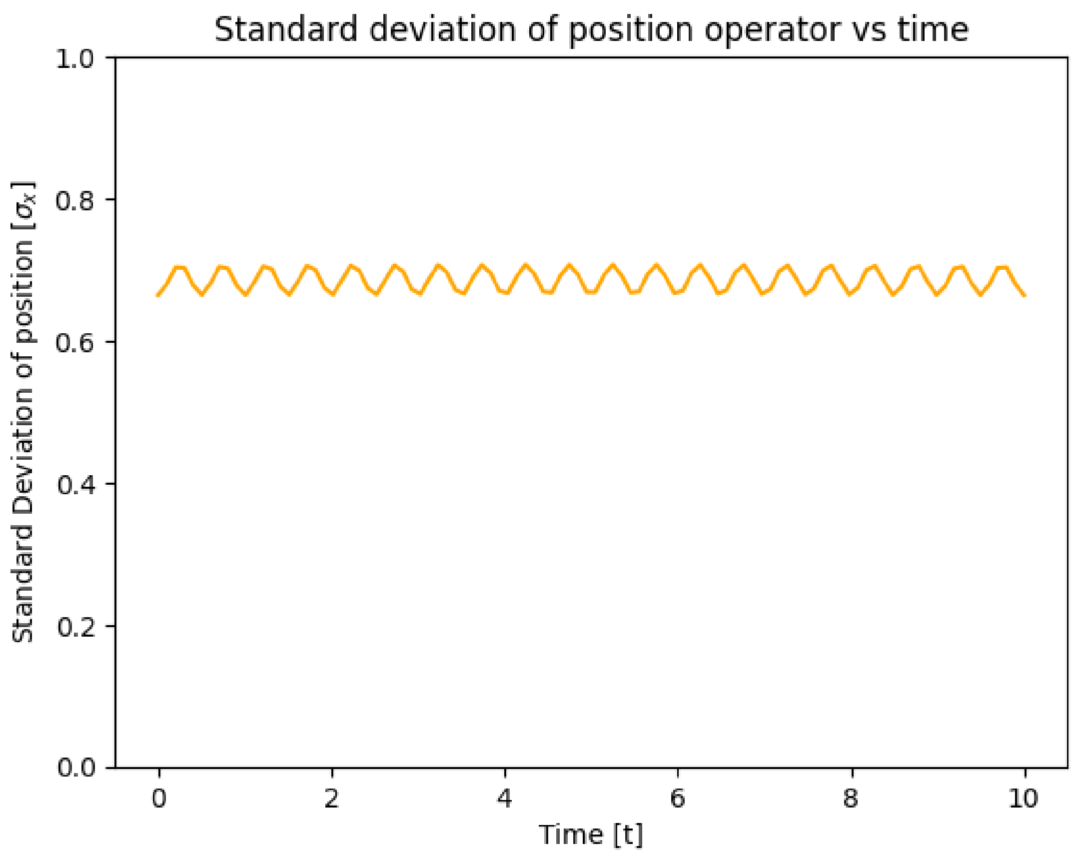

4.2 4.B)

First, lets create a helper function that gives us σ_x and σ_p for our given time evolved hamiltonian

[ ]:
```
def get_std_dev(op):
    std_devs = []
    for t in t_vals:
        state = U(n, t, omega) * coherent(n, alpha)

        # Calculate the standard deviation of the operator
        std_dev = np.sqrt(expect(op*op, state) - expect(op, state)**2)
        std_devs.append(std_dev)

    return std_devs
```

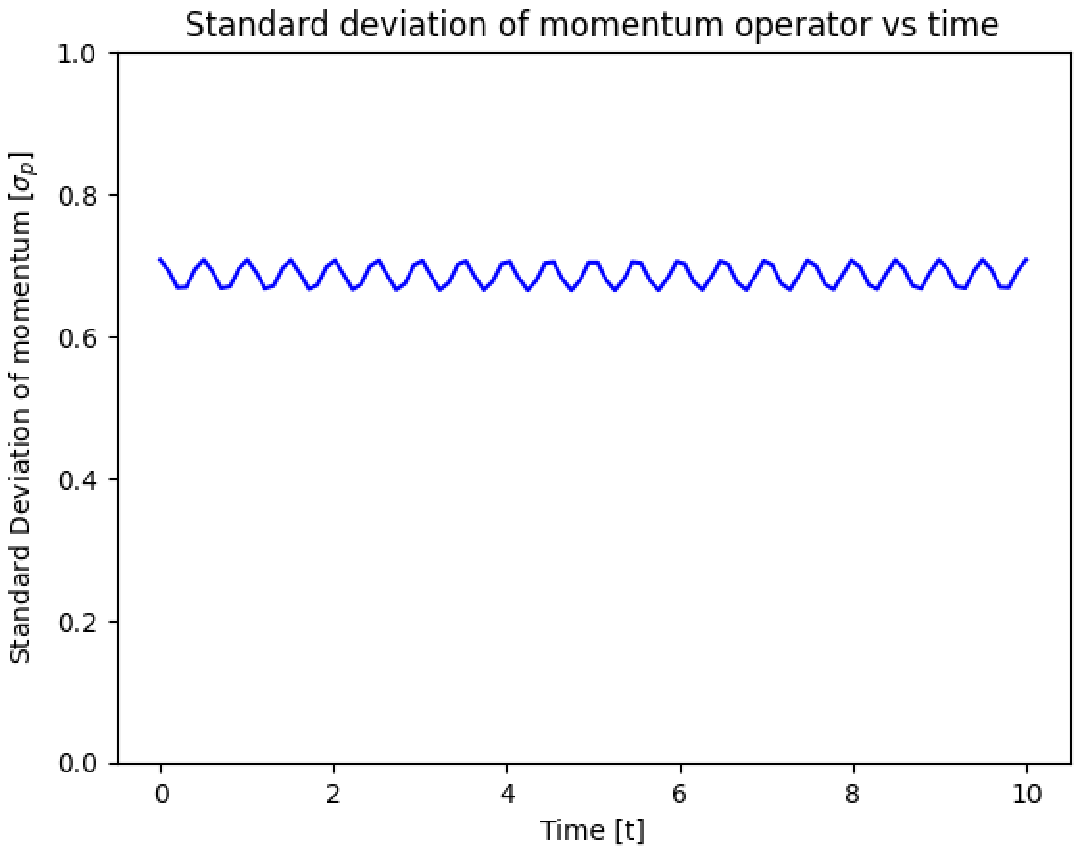
Now, lets see what σ_x vs time looks like:

[ ]:
```
std_devs_x = get_std_dev(position(n))

plt.plot(t_vals, std_devs_x, label='data', color='orange')
plt.title("Standard deviation of position operator vs time")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of position [$\sigma_x$]")
plt.ylim(0, 1)
plt.show()
```



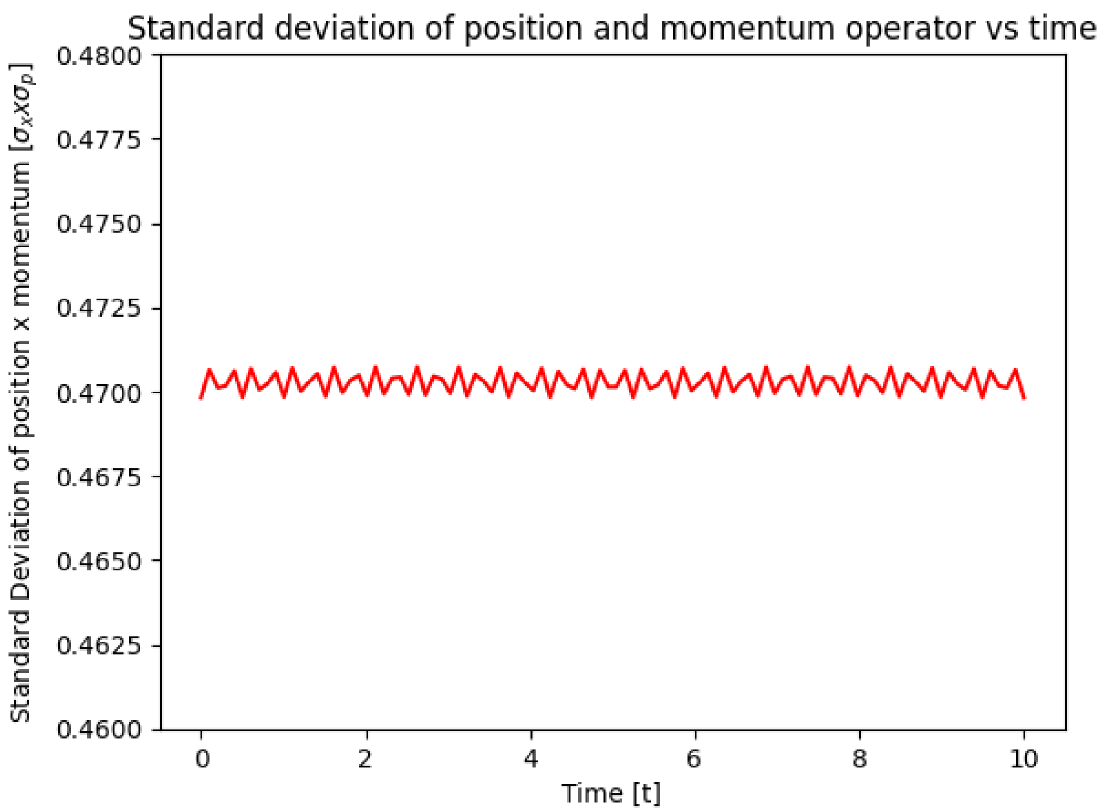
And here's see what σ_p vs time looks like:

[ ]:
```
std_devs_p = get_std_dev(momentum(n))

plt.plot(t_vals, std_devs_p, label='data', color='blue')
plt.title("Standard deviation of momentum operator vs time")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of momentum [$\sigma_p$]")
plt.ylim(0, 1)
plt.show()
```



Lastly, lets see what σ_x * σ_p looks like:

[ ]:
```
std_devs_xp = np.multiply(std_devs_x, std_devs_p)

plt.plot(t_vals, std_devs_xp, label='data', color='red')
plt.title("Standard deviation of position and momentum operator vs time")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of position x momentum [$\sigma_x x \sigma_p$]")
plt.ylim(0.46, 0.48)
plt.show()
```

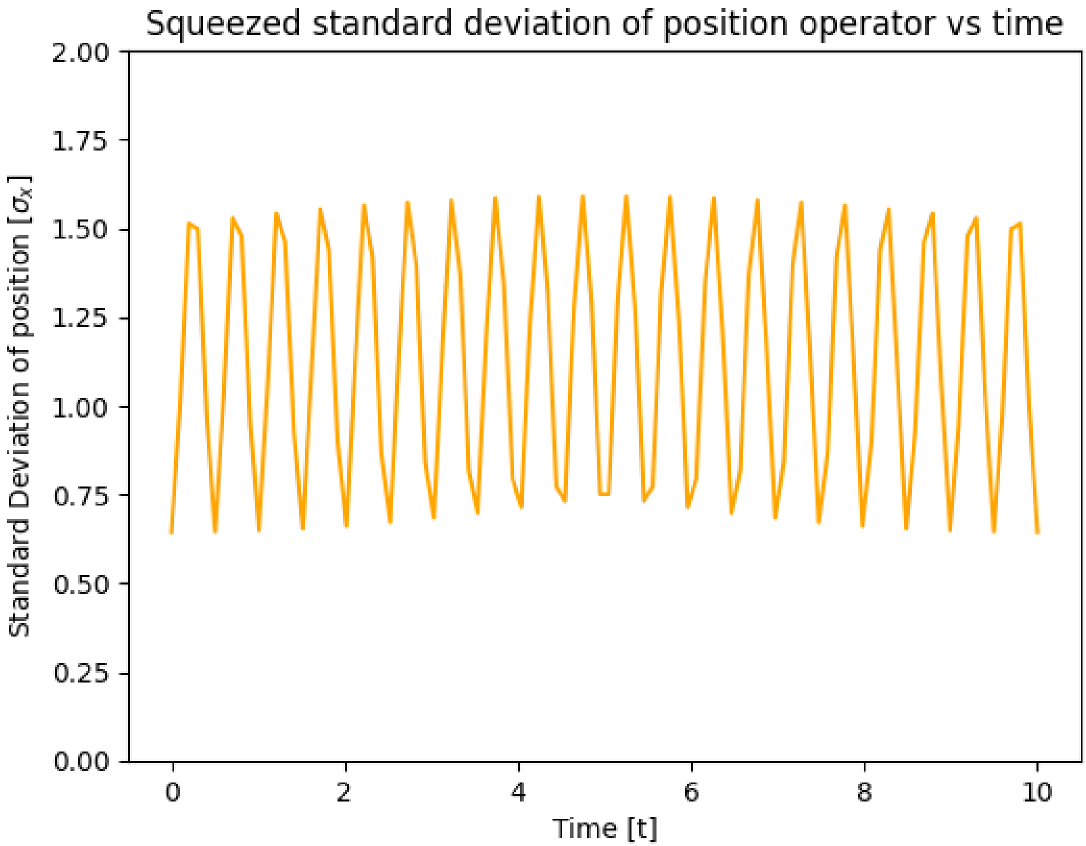


4.3 4.C)

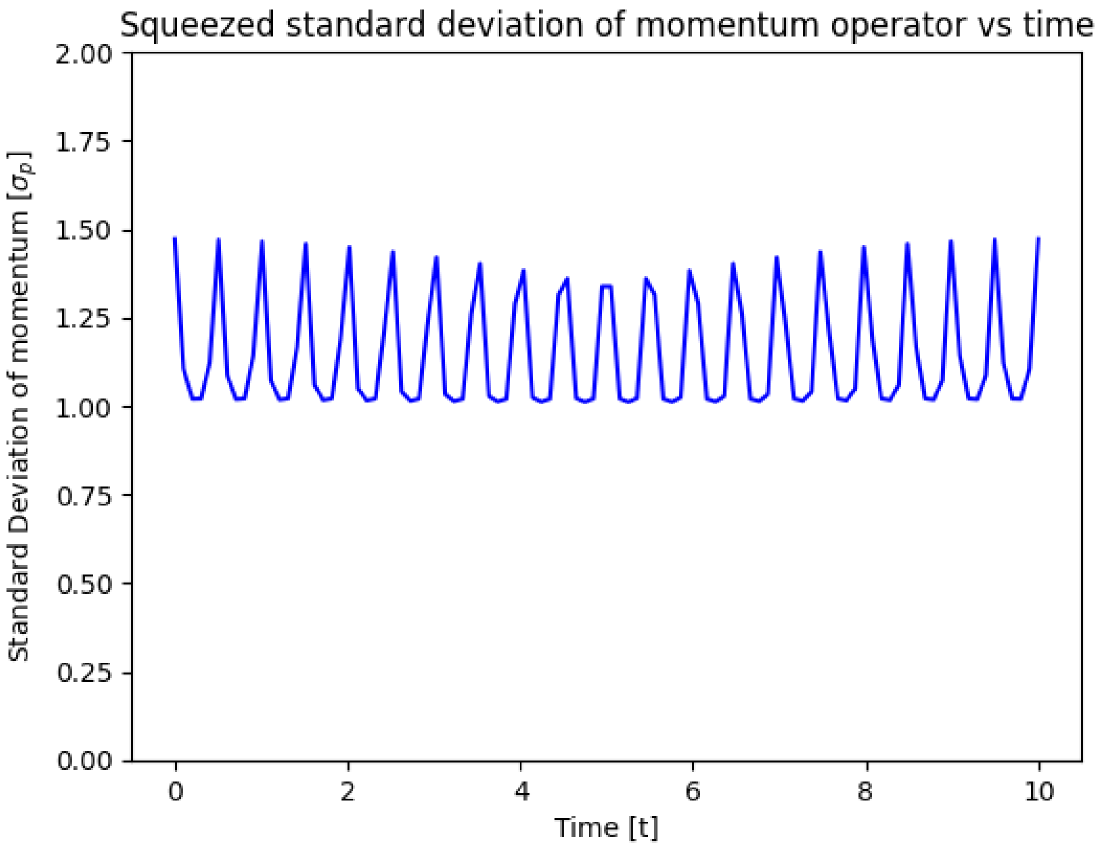
[ ]:
```python
def get_std_dev_squeezed(op):
    r = 1
    std_devs = []
    for t in t_vals:
        state = squeeze(n, r) * U(n, t, omega) * coherent(n, alpha)

        # Calculate the standard deviation of the operator
        std_dev = np.sqrt(expect(op*op, state) - expect(op, state)**2)
        std_devs.append(std_dev)

    return std_devs
```

[ ]:
```python
squeezed_std_devs_x = get_std_dev_squeezed(position(n))

plt.plot(t_vals, squeezed_std_devs_x, label='data', color='orange')
plt.title("Squeezed standard deviation of position operator vs time")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of position [$\sigma_x$]")
plt.ylim(0, 2)
```

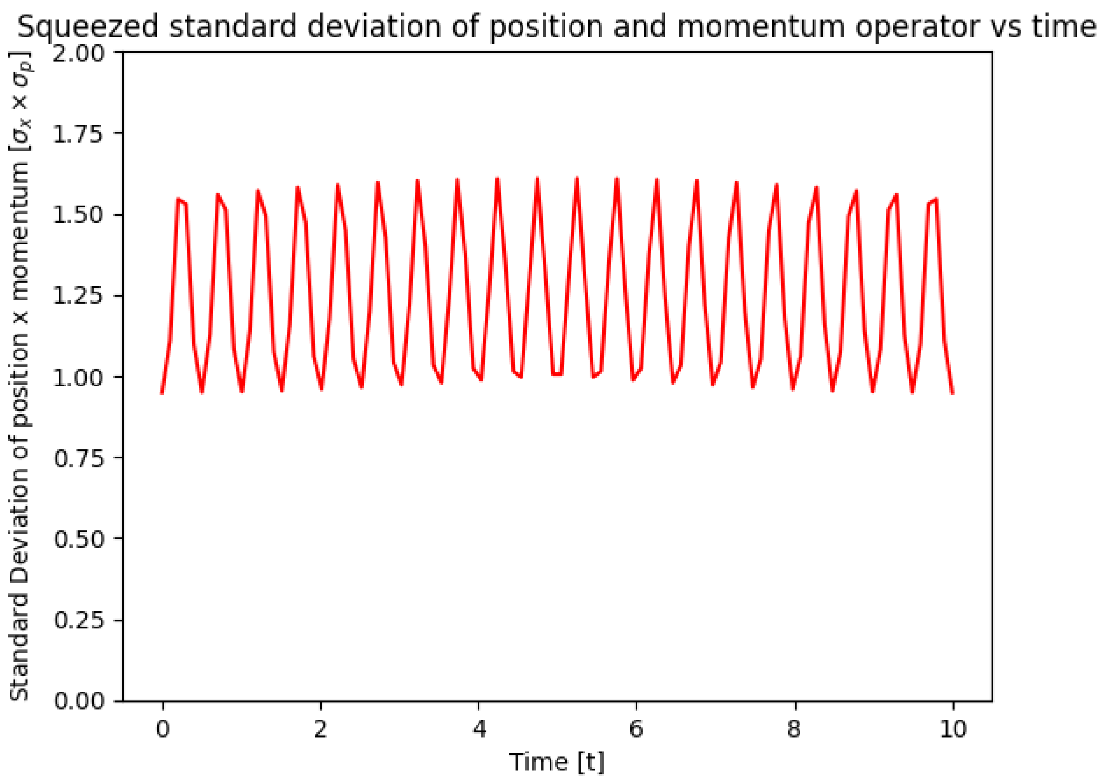
```
plt.show()
```




```
[ ]: squeezed_std_devs_p = get_std_dev_squeezed(momentum(n))

     plt.plot(t_vals, squeezed_std_devs_p, label='data', color='blue')
     plt.title("Squeezed standard deviation of momentum operator vs time")
     plt.xlabel("Time [t]")
     plt.ylabel("Standard Deviation of momentum [$\sigma_p$]")
     plt.ylim(0, 2)
     plt.show()
```



[ ]:
```
squeezed_std_devs_xp = np.multiply(squeezed_std_devs_x, squeezed_std_devs_p)

plt.plot(t_vals, squeezed_std_devs_xp, label='data', color='red')
plt.title("Squeezed standard deviation of position and momentum operator vs␣
 ↪time")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of position x momentum [$\sigma_x \\times␣
 ↪\sigma_p$]")
plt.ylim(0, 2)
plt.show()
```



We can clearly see that the values of σ_x and σ_p are no longer the same. This is due to the fact that we 'squeezed' the system. This means that as we evolve the system, the values of σ_x and σ_p will oscillate between the same and different. This allows us to get a better measurement for position or momentum at a given time.

4.4 4.D)

Lets create a fucntion that will give us the driven QHO with respect to the position operator:

[ ]:
```
def H_B1_drive_coeff(t, args):
    omega = args['omega']
    B1 = args['B1']

    return B1 * np.exp(np.sin(2 * omega * t))

def H_B2_drive_coeff(t, args):
    omega = args['omega']
    B2 = args['B2']

    return B2 * np.exp( np.sin(2 * omega * t))
```

Next, lets plug this into a solver starting at the ground state. Our drive will create a coherent state and squeeze our QHO - which will offset the values of σ_x and σ_p over time.

```
[ ]: psi0 = fock(n, 0)
t_vals = np.linspace(0, 100, 1000)
B1 = 1
B2 = 1

QHO_H = create(n) * destroy(n) + 0.5 * identity(n)
H_B1 = position(n)
H_B2 = position(n)**2

H = [QHO_H, [H_B1, H_B1_drive_coeff], [H_B2, H_B2_drive_coeff]]

res = sesolve(H, psi0, t_vals, args={'n': n, 'omega': omega, 'B1': B1, 'B2':␣
 ↪B2})
res_states = res.states
```

```
[ ]: def get_driven_QHO_exp_val(op, states):
    exp_vals = []
    for state in states:
        exp_val = expect(op, state)
        exp_vals.append(exp_val)

    return exp_vals
```

```
[ ]: def get_driven_QHO_std_dev(op, states):
    std_devs = []
    for state in states:
        # Calculate the standard deviation of the operator
        std_dev = np.sqrt(expect(op*op, state) - expect(op, state)**2)
        std_devs.append(std_dev)

    return std_devs
```

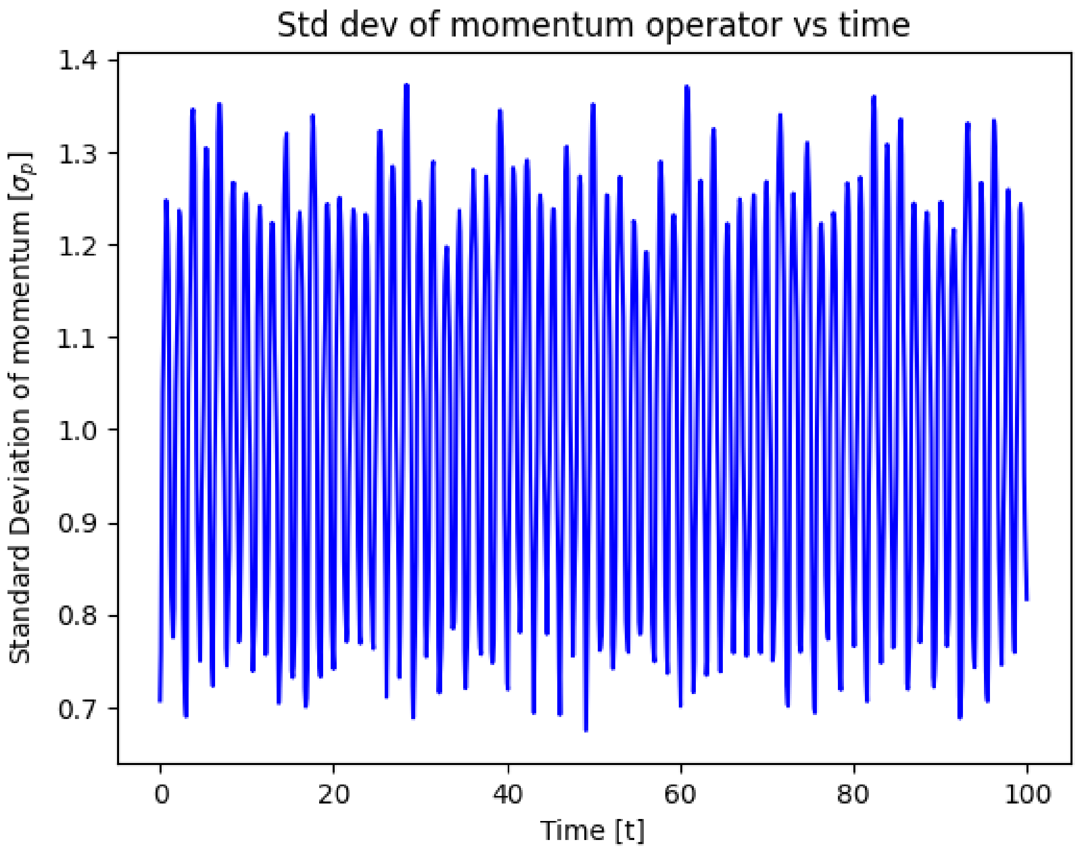
First, lets look at σ_x vs time:

```
[ ]: driven_x_std_dev = get_driven_QHO_std_dev(position(n), res_states)
plt.plot(t_vals, driven_x_std_dev, label='data', color='orange')
plt.title("Std dev of position operator vs time")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of position [$\sigma_x$]")
plt.show()
```

We see that our σ_x is oscillating between the same and different values. This is due to the fact that we are squeezing the system. This means that we are offsetting the values of σ_x and σ_p over time.

Next, lets look at σ_p vs time:

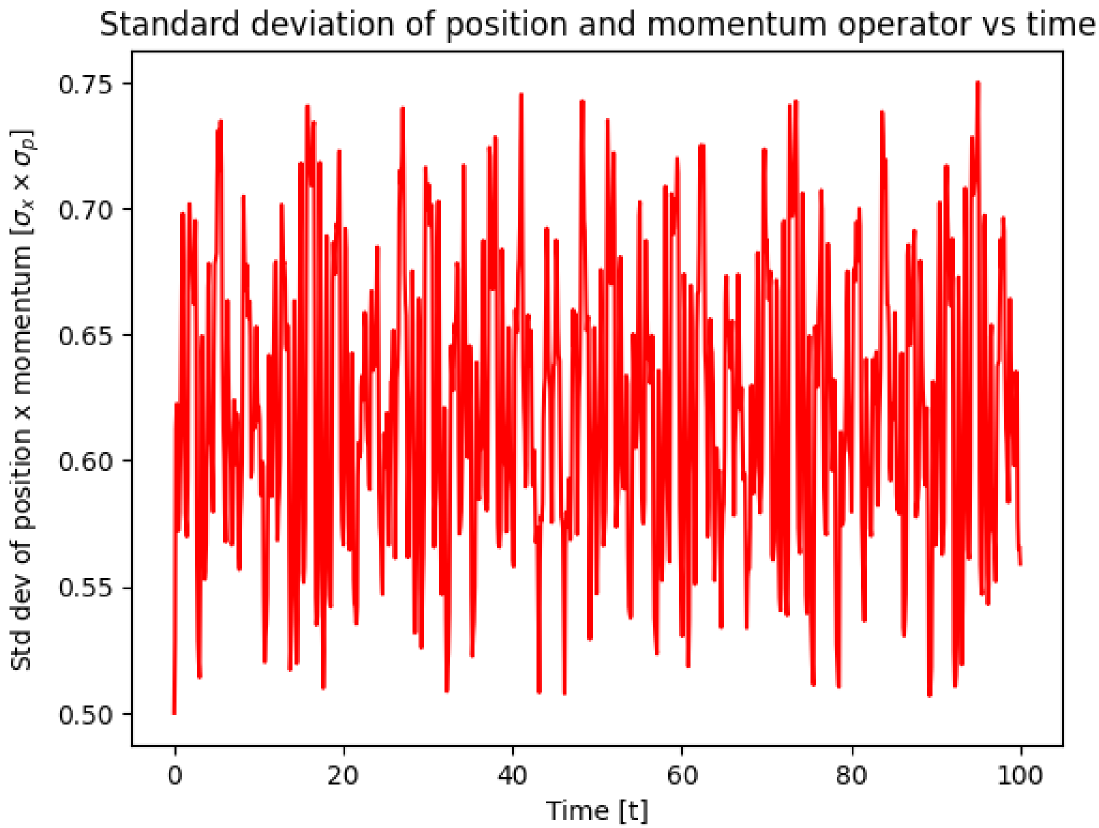
[ ]:
```
driven_p_std_dev = get_driven_QHO_std_dev(momentum(n), res_states)
plt.plot(t_vals, driven_p_std_dev, label='data', color='blue')
plt.title("Std dev of momentum operator vs time")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of momentum [$\sigma_p$]")
plt.show()
```



We can clearly see that σ_p also changes over time.

Lastly, lets look at the combined relation:

[ ]:
```
driven_xp_std_dev = np.multiply(driven_x_std_dev, driven_p_std_dev)
plt.plot(t_vals, driven_xp_std_dev, label='data', color='red')
plt.title("Standard deviation of position and momentum operator vs time")
plt.xlabel("Time [t]")
plt.ylabel("Std dev of position x momentum [$\sigma_x \\times \sigma_p$]")
plt.show()
```



From these results, we can clearly see that our system is in fact being 'squeezed'. This means that we are offsetting the values of σ_x and σ_p over time.

4.5 4.E)

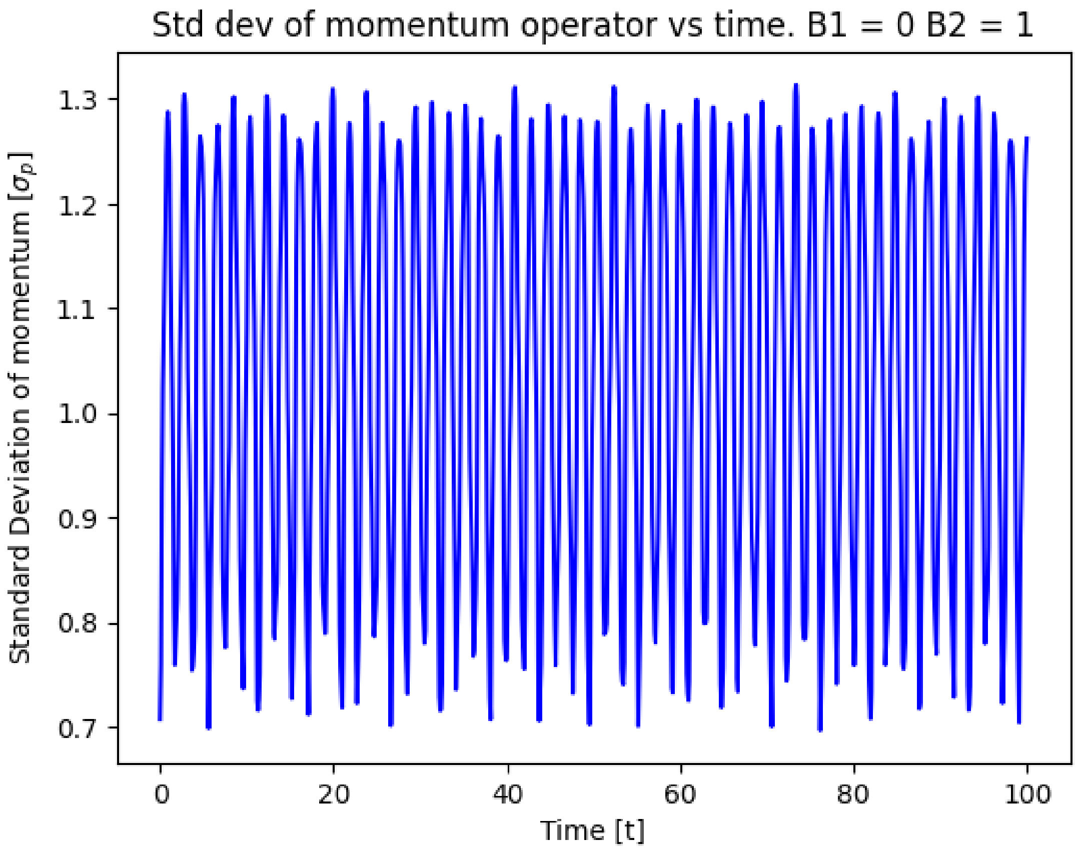
Lastly, lets see if we can make sense of these B1 and B2 values in our drive. Lets start by adjusting B1 and see how it varies our squeezing:

[ ]:
```
psi0 = fock(n, 0)
t_vals = np.linspace(0, 100, 1000)
B1 = 0
B2 = 1

QHO_H = create(n) * destroy(n) + 0.5 * identity(n)
H_B1 = position(n)
H_B2 = position(n)**2

H = [QHO_H, [H_B1, H_B1_drive_coeff], [H_B2, H_B2_drive_coeff]]

res_B_vals = sesolve(H, psi0, t_vals, args={'n': n, 'omega': omega, 'B1': B1,␣
↪'B2': B2})
```

```python
res_states_B_vals = res_B_vals.states
```

[ ]:
```python
driven_x_std_dev_B_vals = get_driven_QHO_std_dev(position(n), res_states_B_vals)
plt.plot(t_vals, driven_x_std_dev_B_vals, label='data', color='orange')
plt.title(f"Std dev of position operator vs time. B1 = {B1} Bz = {B2}")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of position [$\sigma_x$]")
plt.show()
```

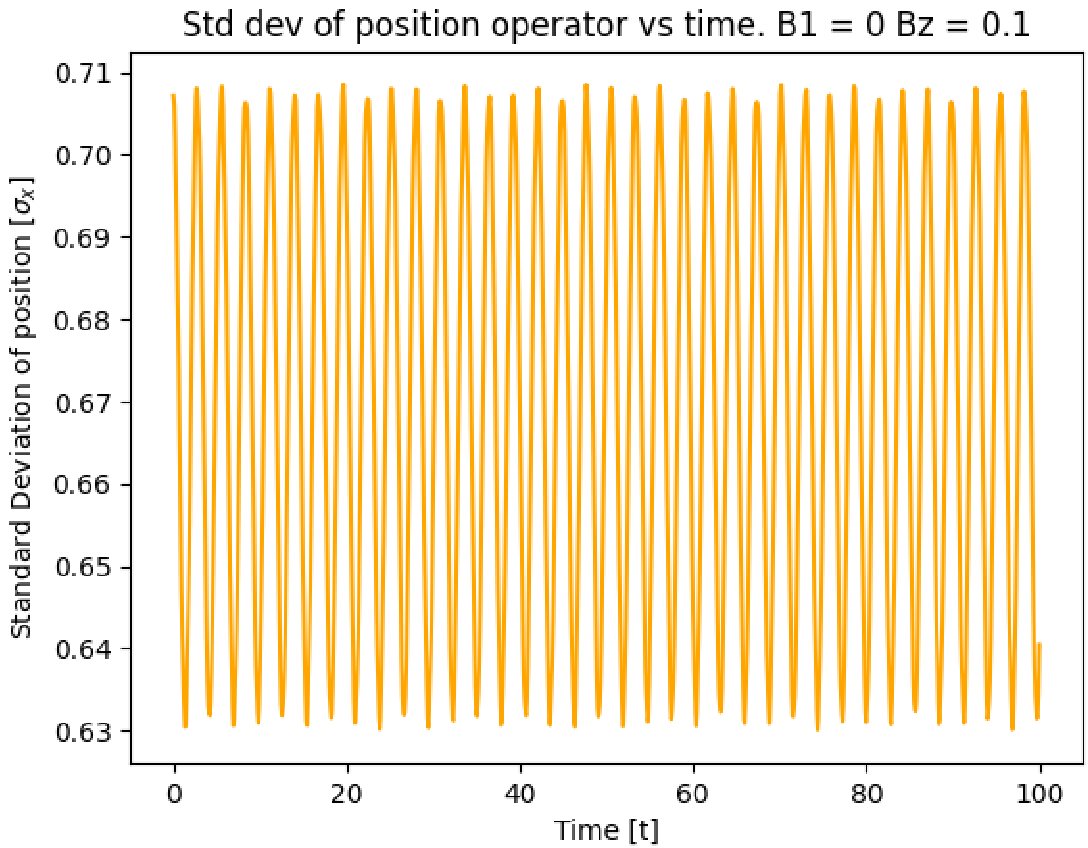
[ ]:
```python
driven_p_std_dev_B_vals = get_driven_QHO_std_dev(momentum(n), res_states_B_vals)
plt.plot(t_vals, driven_p_std_dev_B_vals, label='data', color='blue')
plt.title(f"Std dev of momentum operator vs time. B1 = {B1} B2 = {B2}")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of momentum [$\sigma_p$]")
plt.show()
```



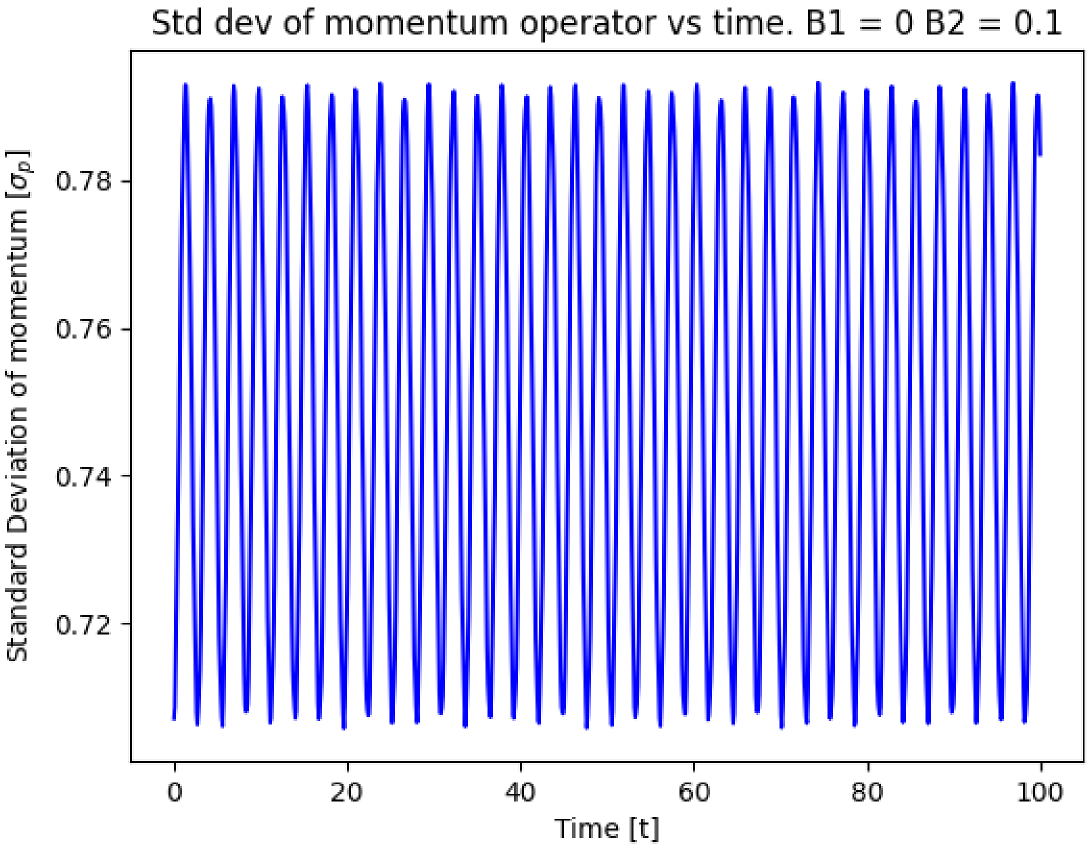
When we set B1 to 0, it appears that our position stay relatively constant with eachother. This imples that our B1 strength must be linked to the squeezing of our system.

[ ]:
```
B1 = 0
B2 = 0.1

res_B_vals = sesolve(H, psi0, t_vals, args={'n': n, 'omega': omega, 'B1': B1,␣
↪'B2': B2})
res_states_B_vals = res_B_vals.states
```

[ ]:
```
driven_x_std_dev_B_vals = get_driven_QHO_std_dev(position(n), res_states_B_vals)
plt.plot(t_vals, driven_x_std_dev_B_vals, label='data', color='orange')
plt.title(f"Std dev of position operator vs time. B1 = {B1} Bz = {B2}")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of position [$\sigma_x$]")
plt.show()
```



[ ]:
```
driven_p_std_dev_B_vals = get_driven_QHO_std_dev(momentum(n), res_states_B_vals)
plt.plot(t_vals, driven_p_std_dev_B_vals, label='data', color='blue')
plt.title(f"Std dev of momentum operator vs time. B1 = {B1} B2 = {B2}")
plt.xlabel("Time [t]")
plt.ylabel("Standard Deviation of momentum [$\sigma_p$]")
plt.show()
```



When we make B2 nearly zero, we see that our oscillations are much smaller. This implies that B2 is linked to the oscillations of our squeezing.

HW 7

Tuesday, March 14, 2023 5:16 PM

2) $\Phi_{\substack{slope \\ min}} = \frac{\sigma_p}{\gamma t}$ where σ_p is the binomial distrib:

$$\sigma_p = \frac{\sigma_{N_1}}{N} = \sqrt{\frac{P_1(1-P_1)}{N}}$$

A) slope detection

$P_1 \approx \frac{1}{2} + \delta p_1$

$\therefore$

$$\sigma_p = \sqrt{\frac{(\frac{1}{2}+\delta p_1)(1-\frac{1}{2}+\delta p_1)}{N}}$$

$$\sigma_p = \sqrt{\frac{\delta^2 p_1^2 + \delta p_1 + \frac{1}{4}}{N}}$$

$\Downarrow$ sub in for $\delta = \frac{\Omega}{2t} + (2\gamma\Phi)$ → for when our $P_1 \approx \frac{1}{2}$

$$\sigma_p = \sqrt{\frac{(\frac{\Omega}{2t}+2\gamma\Phi)^2 p_1^2 + (\frac{\Omega}{2t}+2\gamma\Phi)p_1 + \frac{1}{4}}{N}}$$

$$\sigma_p = \sqrt{\frac{\left[\frac{\Omega^2}{4t^2} + \frac{4\gamma\Phi\Omega}{t} + 4\gamma^2\Phi^2\right]p_1^2 + \frac{\Omega}{2t}p_1 + 2\gamma\Phi p_1 + \frac{1}{4}}{N}}$$

$$\sigma_p = \frac{1}{2}\sqrt{\frac{(4\gamma\Phi t p_1 + \Omega p_1 + t)^2}{t^2 N}}$$

$\therefore$

$$\boxed{\Phi_{slope\ min} = \frac{|4\gamma\Phi t p_1 + \Omega p_1 + t|}{2\gamma t\sqrt{N}}}$$

$$\boxed{\Phi_{variance\ min} = \frac{\sqrt{|4\gamma\Phi t p_1 + \Omega p_1 + t|}}{2\gamma t\sqrt{t}\, N^{\frac{1}{4}}}}$$

B)

$$\frac{\Phi_{slope\ min}}{\Phi_{variance\ min}} = \frac{\frac{|4\gamma\Phi t p_1 + \Omega p_1 + t|}{2\gamma t\sqrt{N}}}{\frac{\sqrt{|4\gamma\Phi t p_1 + \Omega p_1 + t|}}{2\gamma t\sqrt{t}\, N^{\frac{1}{4}}}}$$

→ let this $= \lambda$

$$= \frac{|4\gamma\Phi t p_1 + \Omega p_1 + t|}{\cancel{2\gamma t}\cancel{\sqrt{N}}} \cdot \frac{\cancel{2\gamma t}\sqrt{t}\, \cancel{N^{\frac{1}{4}}}}{\sqrt{|4\gamma\Phi t p_1 + \Omega p_1 + t|}} \quad = \frac{1}{2}$$

$$\boxed{= \frac{|\lambda|\sqrt{t}}{\sqrt{|\lambda|}}}$$

- From this relation, it seems that variance detection is never better than slope detection - since its $\propto \sqrt{|\lambda|}$

- However, when we look at $QPN = \sqrt{\frac{N p_1(1-p_1)}{N}}$ we are expanding about $P_1 = 0$ or 1 for variance detection.

- Therefore, there is no projection noise and only techical noise for variance detection. If we can make our detection noise $\sqrt{\sigma_{P_1\ var}} < \sigma_{P_1\ slope}$, then variance detection is better