HW 7

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2) $\Phi_{\text{slope min}} = \frac{\sigma_p}{\gamma t}$ where σ_p is the binomial distrib:

$$\sigma_p = \frac{\sigma_{N_1}}{N} = \sqrt{\frac{P_1(1-P_1)}{N}}$$

A) slope detection

$$P_1 \approx \frac{1}{2} + \delta_{P_1}$$

$$\therefore \quad \sigma_p = \sqrt{\frac{\left(\frac{1}{2}+\delta_{P_1}\right)\left(1-\frac{1}{2}+\delta_{P_1}\right)}{N}}$$

$$\sigma_p = \sqrt{\frac{\delta^2 P_1^2 + \delta_{P_1} + \frac{1}{4}}{N}}$$

$\Downarrow$ sub in for $\delta = \frac{n}{2t} + \left(2\gamma\Phi\right)$ → for when our $P_1 \approx \frac{1}{2}$

$$\sigma_p = \sqrt{\frac{\left(\frac{n}{2t} + 2\gamma\Phi\right)^2 P_1^2 + \left(\frac{n}{2t} + 2\gamma\Phi\right) P_1 + \frac{1}{4}}{N}}$$

$$\sigma_p = \sqrt{\frac{\left[\frac{n^2}{4t^2} + \frac{4\gamma I n}{t} + 4\gamma^2\Phi^2\right] P_1^2 + \frac{n}{2t} P_1 + 2\gamma\Phi P_1 + \frac{1}{4}}{N}}$$

$$\sigma_p = \frac{1}{2}\sqrt{\frac{\left(4\gamma\Phi t P_1 + n P_1 + t\right)^2}{t^2 N}}$$

$$\therefore \quad \Phi_{\text{slope min}} = \frac{\left|4\gamma\Phi t P_1 + n P_1 + t\right|}{2\gamma t\sqrt{N}}$$

$$\Phi_{\text{variance min}} = \frac{\sqrt{\left|4\gamma\Phi t P_1 + n P_1 + t\right|}}{2\gamma t \sqrt{t}\, N^{\frac{1}{4}}}$$

B)

$$\frac{\Phi_{\text{slope min}}}{\Phi_{\text{variance min}}} = \frac{\frac{\left|4\gamma\Phi t P_1 + n P_1 + t\right|}{2\gamma t\sqrt{N}}}{\frac{\sqrt{\left|4\gamma\Phi t P_1 + n P_1 + t\right|}}{2\gamma t\sqrt{t}\, N^{\frac{1}{4}}}}$$

→ let this = λ

$$= \frac{\left|4\gamma\Phi t P_1 + n P_1 + t\right|}{2\gamma t\sqrt{N}} \cdot \frac{2\gamma t\sqrt{t}\, N^{\frac{1}{4}}}{\sqrt{\left|4\gamma\Phi t P_1 + n P_1 + t\right|}}, \qquad N^{\frac{1}{4}} = \frac{1}{2}$$

$$= \frac{|\lambda|\sqrt{t}}{\sqrt{|\lambda|}}$$

- From this relation, it seems that variance detection is never better than slope detection - since its $\propto \sqrt{|\lambda|}$
- However, when we look at $QPN = \frac{\sqrt{N P_1(1-P_1)}}{N}$ we are expanding about $P_1 = 0$ or 1 for variance detection.
- Therefore, there is no projection noise and only techical noise for variance detection. If we can make our detection noise $\sqrt{\sigma_{P_1,\text{var}}} < \sigma_{P_1,\text{slope}}$, then variance detection is better