Sunday, September 5, 2021
4:49 PM

Predicates. $f(x)$ $P(x): x+1=3.$

$4+1=3$

$P(4) \to F$

$P(2) \to T$

$2+1=3.$

the value of prop. fn. at x

$x+y=2$

$x-y+z=2.$

$x \in$ Domain

Universe of discourse

$P(x,y): x+y=2.$

$P(1,2) \to F$

$P(1,1) \to T$

$P(0,1,2)$, $P(4,2,0)$

$P(x,y,z): x-y+z=2$

EXAMPLE: Let A(c, n) denote the statement "Computer c is connected to network n," where c is a variable representing a computer and n is a variable representing a network. Suppose that the computer MATH1 is connected to network CAMPUS2, but not to network CAMPUS1. What are the values of A(MATH1, CAMPUS1) and A(MATH1, CAMPUS2)?

T

c belongs to the set of all computers in the campus.

n " " ——— networks " " "

$P(x,y)$ A(c,n): Computer c is connected to network n

A(MATH1, CAMPUS2) $\to$ T

Quantifiers.

Universal. If $P(x)$ holds for every elt. x of the domain

Existential.

$\forall$ for all

$\forall x\, P(x)$ holds.

EXAMPLE: Express the statement "Every student in this class has studied calculus" using predicates and quantifiers.

$C(x)$: x has studied calculus.

$\forall x\, C(x)$. $x \in$ this class

$C(x)$: x has studied calculus.

$\forall x\, C(x)$ $x \in$ this class

Domain = every student in this world.

$S(x)$: x is in this class

$\forall x\,[S(x) \to C(x)]$

$\forall x\,[S(x) \wedge C(x)]$ ✗

EXAMPLE: Express the statements "Some student in this class has visited Mexico" and "Every student in this class has visited either Canada or Mexico" using predicates and quantifiers.

Existential Quantifier: $P(x)$

$\exists x\, M(x)$

Domain = This class

$M(x)$: x has visited Mexico

Atleast one student in this class has visited Mexico.

Domain = the set of all students across the ~~x~~ globe.

$S(x)$: x is a student of this class.

(a) $\forall x\,[S(x) \to M(x)]$ (b) $\exists x\,[S(x) \to M(x)]$ (c) $\exists x\,[S(x) \wedge M(x)]$

$f(x) = x^2$ $x \in$ Domain

$x \in \mathbb{R}^+$, $x \in \mathbb{R}$

EXAMPLE: Express the statements "Some student in this class has visited Mexico" and "Every student in this class has visited either Canada or Mexico" using predicates and quantifiers.

Domain = This class.

$C(x)$: x has visited Canada

$M(x)$: " " " Mexico

$\forall x\,[C(x) \vee M(x)]$

Domain = The set of all students across the globe.

$S(x)$: x is a student of this class.

$\forall x\,[S(x) \to C(x) \vee M(x)]$

De Morgan's Law for Quantifiers.

De Morgan's Law for quantifiers.

(1) $\sim \forall x\, P(x) \equiv \exists x \sim P(x)$

(2) $\sim \exists x\, P(x) \equiv \forall x \sim P(x)$.

Every student in this class has studied calculus

$C(x)$: x has studied calculus.

Dom. = this class.

$\sim [\forall x\, C(x)]$ = Not every student in this class has studied Cal.

$\equiv \exists x (\sim C(x))$ There exists atleast one student in this class who has not studied calculus.

"Some student in this class has visited Mexico"

$M(x)$: x has visited Mexico

$\sim \exists x\, M(x) \equiv \forall x \sim M(x)$ Dom. = This class.

EXAMPLE: What are the negations of the statements "There is an honest politician" and "All Americans eat cheeseburgers"?

Dom. = the set of all Politicians.

$\sim [\exists x\, H(x)]$ $H(x)$: x is honest.

$\equiv \forall x (\sim H(x))$ Every polit. is dishonest.

No polit. is honest.

~~Not all politic. are honest.~~

EXAMPLE: Consider these statements. "All lions are fierce." "Some lions do not drink coffee." "Some fierce creatures do not drink coffee." Let P (x), Q(x), and R(x) be the statements "x is a lion," "x is fierce," and "x drinks coffee," respectively. Assuming that the domain consists of all creatures, express the statements in the argument using quantifiers and P (x), Q(x) and R(x).

$\forall x\, [P(x) \to Q(x)]$

Creatures = $\{L_1, L_2, E_1, E_2, E_3\}$

~~$\exists x\, [P(x) \to \sim R(x)]$~~ 2

$L_1 \quad L_2$

$\exists x\, [P(x) \wedge \sim R(x)] \equiv \sim \forall x\, [P(x) \to R(x)]$

$$\exists x[P(x) \wedge \sim R(x)] \equiv \sim \forall x[P(x) \rightarrow R(x)]$$

$$\sim \forall x[P(x) \rightarrow R(x)] \equiv \exists x \sim [P(x) \rightarrow R(x)]$$

$a \rightarrow b \equiv \sim a \vee b$

$$\equiv \exists x \sim [\sim P(x) \vee R(x)]$$

$$\equiv \exists x[P(x) \wedge \sim R(x)]$$

Some Lions don't drink coffee $\equiv$ Negation of all lions drink coffee)

$$\exists x[P(x) \wedge \sim R(x)] \equiv \sim \forall x[P(x) \rightarrow R(x)]$$

EXAMPLE: Consider these statements. "All lions are fierce." "Some lions do not drink coffee." "Some fierce creatures do not drink coffee." Let P (x), Q(x), and R(x) be the statements "x is a lion," "x is fierce," and "x drinks coffee," respectively. Assuming that the domain consists of all creatures, express the statements in the argument using quantifiers and P (x), Q(x) and R(x).

$a \rightarrow b \equiv \sim a \vee b$

$$\forall x[P(x) \rightarrow Q(x)] \equiv \forall x[\sim P(x) \vee Q(x)]$$

$$\exists x[Q(x) \wedge \sim R(x)] \equiv \sim \forall x[Q(x) \rightarrow R(x)]$$

EXAMPLE: Consider these statements "All hummingbirds are richly colored." "No large birds live on honey." "Birds that do not live on honey are dull in color." "Hummingbirds are small." Let P (x), Q(x), R(x), and S(x) be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and P (x), Q(x), R(x) and S(x).

$$\forall x[P(x) \rightarrow S(x)] \equiv \forall x[\sim P(x) \vee S(x)]$$

$$\equiv \forall x \sim [P(x) \wedge \sim S(x)]$$

$$\equiv \sim \exists x[P(x) \wedge \sim S(x)]$$

$$\sim \exists x[Q(x) \wedge R(x)]$$

$$\equiv \forall x[Q(x) \rightarrow \sim R(x)]$$

All hummingbirds are richly colored $\equiv$ Negation of Some hummingbirds are not richly colored.

EXAMPLE: Consider these statements "All hummingbirds are richly colored." "No large birds live on honey." "Birds that do not live on honey are dull in color." "Hummingbirds are small." Let P (x), Q(x), R(x), and S(x) be the statements

EXAMPLE: Consider these statements "All hummingbirds are richly colored." "No large birds live on honey." "Birds that do not live on honey are dull in color." "Hummingbirds are small." Let P (x), Q(x), R(x), and S(x) be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and P (x), Q(x), R(x) and S(x).

$$\forall x\,[\sim R(x) \rightarrow \sim S(x)] \equiv \forall x\,[S(x) \rightarrow R(x)]$$

$$\equiv \forall x\,[\sim S(x) \vee R(x)] \equiv \sim \exists x\,[S(x) \wedge \sim R(x)]$$

$$\forall x\,[P(x) \rightarrow \sim Q(x)] \equiv \forall x\,[\sim P(x) \vee \sim Q(x)] \equiv \sim \exists x\,[P(x) \wedge Q(x)]$$

Hummingbirds are small. ≡ Negation of Some hummingbirds are large.