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7:22 PM

How can this English sentence be translated into a logical expression? "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

p: You can ride the RC.

q: You are under 4 feet tall.

r: " " older than 16 years.

$q \wedge (\sim r) \to \sim p$

$p \to \sim q \vee r$

ⓐ $q \wedge r \to p$

ⓑ $\sim q \wedge r \to p$

ⓒ $\sim q \wedge \sim r \to p$

A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology. A compound proposition that is always false is called a contradiction. A compound proposition that is neither a tautology nor a contradiction is called a contingency

$p \vee \sim p \equiv T$

$p \wedge \sim p \equiv F$

| $p$ | $\sim p$ | $p \vee \sim p$ | $p \wedge \sim p$ |
|---|---|---|---|
| T | F | T | F |
| F | T | T | F |

The compound propositions p and q are called logically equivalent if p ↔ q is a tautology. The notation p ≡ q denotes that p and q are logically equivalent

$a \to b \equiv \sim b \to \sim a$

$p \equiv q$

$p \leftrightarrow q$ ⓐ F ⓑ Tautology

Contradiction

(c) Contingency

| $a$ | $b$ | $a \to b$ | $\sim b$ | $\sim a$ | $\sim b \to \sim a$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|---|
| T | T | | | | | T |
| T | F | | | | | T |
| F | T | | | | | T |
| F | F | | | | | T |

① $\sim(p \vee q) \equiv \sim p \wedge \sim q$

② $\sim(p \wedge q) \equiv \sim p \vee \sim q$

| $p$ | $q$ | $p \to q$ | $\sim p$ | $\sim p \vee q$ |
|---|---|---|---|---|
| T | T | T | F | T |

② $\sim(p \wedge q) \equiv \sim p \vee \sim q$

(3) $\boxed{p \rightarrow q \equiv \sim p \vee q}$

(4) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(5) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

| | | | | |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

Distributive Laws.

$\boxed{a \times (b + c) = a \times b + a \times c}$

| p | q | r | $q \wedge r$ | $p \vee (q \wedge r)$ | $\overset{a}{(p \vee q)}$ | $\overset{b}{p \vee r}$ | $a \wedge b$ |
|---|---|---|---|---|---|---|---|
| T | T | T | | | | | |
| T | T | F | | | | | |
| T | F | T | | | | | |
| F | T | T | | | | | |
| T | F | F | | | | | |
| F | T | F | | | | | |
| F | F | T | | | | | |
| F | F | F | | | | | |

**TABLE 6 Logical Equivalences.**

| *Equivalence* | *Name* |
|---|---|
| $p \wedge \mathbf{T} \equiv p$<br>$p \vee \mathbf{F} \equiv p$ | Identity laws |
| $p \vee \mathbf{T} \equiv \mathbf{T}$<br>$p \wedge \mathbf{F} \equiv \mathbf{F}$ | Domination laws |
| $p \vee p \equiv p$<br>$p \wedge p \equiv p$ | Idempotent laws |
| $\neg(\neg p) \equiv p$ | Double negation law |
| $p \vee q \equiv q \vee p$<br>$p \wedge q \equiv q \wedge p$ | Commutative laws |
| $(p \vee q) \vee r \equiv p \vee (q \vee r)$<br>$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | Associative laws |
| $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$<br>$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | Distributive laws |
| $\neg(p \wedge q) \equiv \neg p \vee \neg q$<br>$\neg(p \vee q) \equiv \neg p \wedge \neg q$ | De Morgan's laws |
| $p \vee (p \wedge q) \equiv p$<br>$p \wedge (p \vee q) \equiv p$ | Absorption laws |
| $p \vee \neg p \equiv \mathbf{T}$<br>$p \wedge \neg p \equiv \mathbf{F}$ | Negation laws |

$\boxed{A^2 = A}$

$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

$$x + y = y + x$$
$$xy = yx$$

$$\boxed{\begin{aligned} x+(y+z) &= (x+y)+z \\ x(yz) &= (xy)z \end{aligned}}$$

$$x - y \neq y - x \qquad x-(y-z) \neq (x-y-z)$$