Saturday, September 4, 2021
9:56 AM

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

$x + 0 = x$

$x(1) = x$

$x + (y + z) = (x + y) + z$

$x(yz) = (xy)z$

$x - (y - z) \neq (x - y) - z$

Q.1 $\sim(p \to q) \equiv ?$

(a) $\sim p \wedge q$ (b) $\sim p \vee q$ (c) $p \wedge \sim q$ (d) NOT

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Q.2 $(p \wedge q) \to (p \vee q) \equiv ?$

(a) Contingency (b) Tautology (c) Contradiction.

$\sim(p \to q) \equiv \sim(\sim p \vee q) \equiv \sim(\sim p) \wedge \sim q$
$\equiv p \wedge \sim q$

$(p \wedge q) \to (p \vee q)$
$\equiv \sim(p \wedge q) \vee (p \vee q)$
$\equiv (\sim p \vee \sim q) \vee (p \vee q) \equiv \sim p \vee \sim q \vee p \vee q$
$\equiv (\sim p \vee p) \vee (\sim q \vee q)$
$\equiv T \vee T \equiv T$

$\sim p \to q \equiv \sim(\sim p) \vee q$
$\equiv p \vee q$

$\sim(p \to \sim q) \equiv \sim(\sim p \vee \sim q) \equiv p \wedge q$

$p \to (q \wedge r) \equiv \sim p \vee (q \wedge r) \equiv (\sim p \vee q) \wedge (\sim p \vee r)$

$a \to b \equiv \sim a \vee b$

$\equiv (p \to q) \wedge (p \to r)$

$(p \vee q) \to r \equiv \sim(p \vee q) \vee r \equiv (\sim p \wedge \sim q) \vee r \equiv r \vee (\sim p \wedge \sim q)$

$$(p\vee q)\to r \equiv \sim(p\vee q)\vee r \equiv (\sim p\wedge \sim q)\vee r \equiv r\vee(\sim p\wedge\sim q)$$
$$\equiv (r\vee\sim p)\wedge(r\vee\sim q)$$
$$\equiv (\sim p\vee r)\wedge(\sim q\vee r)$$
$$\equiv (p\to r)\wedge(q\to r)$$

TABLE 8 **Logical Equivalences Involving Biconditional Statements.**

$p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$
$p\leftrightarrow q\equiv\neg p\leftrightarrow\neg q$
$p\leftrightarrow q\equiv(p\wedge q)\vee(\neg p\wedge\neg q)$
$\neg(p\leftrightarrow q)\equiv p\leftrightarrow\neg q$

$$p\leftrightarrow q\equiv (p\to q)\wedge(q\to p)$$
$$\equiv (\sim q\to\sim p)\wedge(\sim p\to\sim q)$$
$$\equiv \sim q\leftrightarrow\sim p$$

$$p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$$
$$\equiv(\sim p\vee q)\wedge(\sim q\vee p)$$
$$\equiv\{(\sim p\vee q)\wedge(\sim q)\}\vee\{(\sim p\vee q)\wedge p\}$$
$$\equiv\{(\sim q)\wedge(\sim p\vee q)\}\vee\{p\wedge(\sim p\vee q)\}$$
$$\equiv\{(\sim q\wedge\sim p)\vee(\sim q\wedge q)\}\vee\{(p\wedge\sim p)\vee(p\wedge q)\}$$
$$\equiv\{(\sim q\wedge\sim p)\vee F\}\vee\{F\vee(p\wedge q)\}$$

$a\vee F\equiv a$

$$\sim(p\to q)\equiv\sim[(\sim q\wedge\sim p)\vee(p\wedge q)]$$
$$\equiv\sim(\sim q\wedge\sim p)\wedge\sim(p\wedge q)$$
$$\equiv(q\vee p)\wedge(\sim p\vee\sim q)$$
$$\equiv(\sim q\to p)\wedge(p\to\sim q)$$

$\sim a\vee b\equiv a\to b$

$a\vee b\equiv\sim a\to b$

$$\equiv\{\sim p\to\sim(\sim q)\}\wedge\{\sim(\sim q)\to\sim p\}$$
$$\equiv(\sim p\to q)\wedge(q\to\sim p)$$

$$\sim(p\leftrightarrow q)\equiv\sim q\leftrightarrow p$$
$$\sim(p\leftrightarrow q)\equiv q\leftrightarrow\sim p$$

Generalised De Morgan Law

$$\sim(p_1 \wedge p_2 \wedge \text{——} \wedge p_n) \equiv \sim p_1 \vee \sim p_2 \vee \sim p_3 \text{ ——} \vee \sim p_n$$

$$\sim(p_1 \vee p_2 \vee \text{——} \vee p_n) \equiv \sim p_1 \wedge \sim p_2 \wedge \text{—— ——} \wedge \sim p_n$$

Predicates. $f(x)$ $\boxed{P(x) : x + 2 = 4}$

$3+2=4$

$P(3) \to F$

$P(2) \to T$

the value of prop. fn.

at x

$x + y = 3$

$x - y + z = 2$

$x \in$ Domain

Universe of discourse

$P(x, y) : x + y = 3$

$P(1, 2)$

$P(1, 1)$