

How can this English sentence be translated into a logical expression? "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

$p$ : You can ride the RC.

$q$ : You are under 4 feet tall.

$r$ :  $\text{older than } 16 \text{ years}$ .

$$\checkmark q \wedge (\neg r) \rightarrow \neg p$$

$$\neg p \rightarrow (\neg q \wedge r)$$

$$\begin{array}{l} a) q \wedge r \rightarrow p \\ b) \neg q \wedge r \rightarrow p \\ c) \neg q \wedge \neg r \rightarrow p \end{array}$$

A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology. A compound proposition that is always false is called a contradiction. A compound proposition that is neither a tautology nor a contradiction is called a contingency.

$$p \vee \neg p \equiv T$$

$$p \wedge \neg p \equiv F$$

	$p$	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	T	F	T	F
F	F	T	T	F

The compound propositions  $p$  and  $q$  are called logically equivalent if  $p \leftrightarrow q$  is a tautology. The notation  $p \equiv q$  denotes that  $p$  and  $q$  are logically equivalent.

$$a \rightarrow b \equiv \neg b \rightarrow \neg a$$

(a)  $F$  (b) Tautology  
Contradiction  
(c) Contingency

	$a$	$b$	$a \rightarrow b$	$\neg a$	$\neg b \rightarrow \neg a$	$p \rightarrow q$
T	T	T	T	F	T	T
T	F	T	F	F	T	F
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$$\textcircled{1} \quad \neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\textcircled{2} \quad \neg(p \wedge q) \equiv \neg p \vee \neg q$$

	$p$	$q$	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	T	F	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	F	T	T	T

$$② \sim(p \wedge q) \equiv \sim p \vee \sim q$$

$$(3) [p \rightarrow q] \equiv \sim p \vee q$$

$$(4) \cancel{p \vee (q \wedge r)} \equiv (p \vee q) \wedge (p \vee r)$$

$$(5) \cancel{p \wedge (q \vee r)} \equiv (p \wedge q) \vee (p \wedge r)$$

T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T
F	F	F	T	T

Distributive laws.

$$\text{Tax}(b+c) = axb + axc$$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r)$	$a$	$b$	$a \wedge b$
T	T	T	T	T	T	T			
T	T	F	F	T	F	F			
T	F	T	F	T	F	F			
F	T	T	T	T	T	T			
T	F	F	F	T	F	F			
F	T	F	F	F	F	F			
F	F	T	T	T	T	T			
F	F	F	F	F	F	F			

**TABLE 6** Logical Equivalences.

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De-Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

Handwritten notes on logical equivalences and algebraic properties:

- Identity laws:  $p \vee p \equiv p$ ,  $p \wedge p \equiv p$
- Domination laws:  $p \vee T \equiv T$ ,  $p \wedge F \equiv F$
- Idempotent laws:  $p \vee p \equiv p$ ,  $p \wedge p \equiv p$
- Double negation law:  $\neg(\neg p) \equiv p$
- Commutative laws:  $p \vee q \equiv q \vee p$ ,  $p \wedge q \equiv q \wedge p$
- Associative laws:  $(p \vee q) \vee r \equiv p \vee (q \vee r)$ ,  $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive laws:  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ ,  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- De-Morgan's laws:  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ ,  $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- Absorption laws:  $p \vee (p \wedge q) \equiv p$ ,  $p \wedge (p \vee q) \equiv p$
- Negation laws:  $p \vee \neg p \equiv T$ ,  $p \wedge \neg p \equiv F$
- Algebraic properties (grouped by curly braces):
  - $p \vee p = p$  (checkmark)
  - $p \wedge p = p$  (checkmark)
  - $x + y = y + x$  (checkmark)
  - $xy = yx$  (checkmark)
  - $x - y \neq y - x$  (checkmark)
  - $x - (y - z) \neq (x - y) - z$  (checkmark)