UNIT 4 VALIDITY, INVALIDITY AND LIST OF VALID SYLLOGISMS

Contents

4.0 OBJECTIVES

This unit brings out the most important part of your study of categorical syllogism. You will be introduced to the rules which determine the validity of arguments. While this is the most important objective, the icing on the cake is the variety of the methods of determining the validity of arguments. Both traditional and modern methods of testing the validity receive due recognition in this unit. Therefore contribution of both John Venn and George Boole find place in this unit. This particular study enables you to grasp the relation between logic and set theory which is brought to the fore in this unit.

4.1 INTRODUCTION

In the second and third units we learnt two important aspects of categorical syllogism, viz., figures and moods. However, we did not develop the technique of distinguishing valid from invalid arguments. Consequently, we could not know under what conditions a mood becomes valid and what is still worse, we could not understand why a certain arrangement or configuration of propositions in one figure is legitimate (only a legitimate combination of propositions yields valid mood) and in some other figure illegitimate yielding only invalid moods, and conversely, why a certain configuration of propositions is illegitimate in some figures and legitimate in some other figure or figures. In other words, the question what makes an argument valid was not raised at all. The point is that the validity of an argument depends on whether or not the conclusion is a conclusion in the strict sense of the word, i.e. whether or not it logically follows from the premises. This brings us to the vital aspect of our study. Just as application or non-application of rules makes a game legitimate or illegitimate, mere application or non-application of rules makes an argument valid or invalid. Application of rules demands knowledge of rules. Therefore we must focus on the question what rules are there which determine the validity of syllogism.

4.2 THE RULES OF CATEGORICAL SYLLOGISM

Classical Logic lists eight rules of valid categorical syllogism; four of them concern the terms, and four of them concern the propositions. These rules are not provable. They have to be either accepted or rejected. If they are rejected, syllogism is not possible. Therefore what is given is only an explication of the rules. Classical logic classified these rules under *rules of structure, rules of distribution of terms, rules of quality,* and *rules of quantity.*

I) Rules of structure

1) Syllogism must Contain Three, and Only Three, Propositions

Syllogism is defined as a kind of mediate inference consisting of two premises which together determine the truth of the conclusion. This definition shows that if the number of propositions is more than two, then it ceases to be syllogism. Therefore by definition syllogism must consist of two premises and one conclusion. Therefore together they make up for three propositions.

2) Syllogism must Consist of Three Terms Only

A proposition consists of two terms. However, three propositions consist of only three terms because each term occurs twice. Suppose that there are four terms. Then there is no middle term, a term common to two premises. In such a case the violation of rule results in a fallacy called fallacy of four terms. Such a fallacy is never committed knowingly because knowing fully well the fixed number of terms, we do not choose four terms. But we do it unknowingly. It happens when an ambiguous word is used in two different senses on two different occasions. Then there are really four terms, not three terms. If an ambiguous word takes the place of middle term, then the fallacy committed is known as fallacy of ambiguous middle. Similarly, if an ambiguous term takes the place of the major or the minor term, then the fallacy of ambiguous major or ambiguous minor, as the case may be, is committed. The following argument illustrates the fallacy of ambiguous middle.

Fallacy of Ambiguous Middle

All *charged* particles are electrons.

Atmosphere in the college is *charged*.

∴ Atmosphere in the college is an electron.

The word in italics is ambiguous. The other two fallacies are hardly committed. Therefore there is no need to consider examples for them. The moral is that all sentences in arguments must be unambiguous. This is possible only when all terms are unambiguous in the given argument. We must also consider the inversion of ambiguous middle. Suppose that synonymous words are used in place of middle term. Then apparently there are four terms. But, in reality, there are three terms. For example *starry world* and *stellar world* are not two terms. Such usages also are uncommon. Hence they deserve to be neglected.

II) Rules of Distribution of Terms

1) Middle term must be distributed at least once in the premises. If this rule is violated, then the argument commits the fallacy of undistributed middle. One example will illustrate this rule.

2 All circles are geometrical figures.

All squares are geometrical figures.

∴ All circles are squares.

2) In the conclusion, no term may be taken in a more 'extensive' sense than in the premises. It also means that a term which is distributed in the conclusion must remain distributed in the respective premise. This rule can be stated this way also. A term which is undistributed in the premise must remain undistributed in the conclusion. However, it is not necessary that a term, which is distributed in the premise, must be distributed in the conclusion.

Suppose that the major term violates this rule. Then the argument commits the *fallacy of illicit major.* When the minor term violates this rule, *fallacy illicit minor is committed.* The following arguments illustrate these fallacies.

3) All philosophers are thinkers.

No ordinary men are philosophers.

∴ No ordinary men are thinkers.

4) All aquatic creatures are fish.

All aquatic creatures swim.

∴ All those which swim are fish.

First argument illustrates the fallacy of undistributed middle; second illustrates the fallacy of illicit major and the third illustrates the fallacy of illicit minor.

III) *Rules of Quality*

1) From two negative premises, no conclusion can be drawn. It only means that at least one premise must be affirmative.

2) If both premises are affirmative, the conclusion cannot be negative. Negatively, it only means that a negative conclusion is possible only when one premise is negative.

IV) *Rules of Quantity*

If both premises are particular, no conclusion can be drawn or the conclusion must always follow the weaker part. Here weaker part is particular. This rule shows that at least one premise must be universal.

If one premise is particular, then the conclusion must be particular only. It means that universal conclusion is possible only when both premises are universal. In practice, last three sets of rules play an important role in determining the validity of categorical syllogism.

Check Your Progress I

Note: Use the space provided for your answers.

Examine the following arguments.

1) All kings are thinkers.

Some ordinary men are not kings.

∴ No ordinary men are thinkers.

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2) All stars are bright.

All bright objects are attractive.

∴ All attractive objects are stars.

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3) Some radicals are good men.

Some good men are honest.

∴ Some radicals are honest.

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4) The monkey is nonhuman.

Some of those who are of capable of laughter are humans.

∴ The monkey is not capable of laughter.

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4.3 SPECIAL APPLICATIONS OF THE GENERAL RULES

In the previous unit we learnt that a certain arrangement of categorical propositions is legitimate in one figure and illegitimate in some other figure, the only exception being EIO. For the purpose of contrast we should recognize that its reversal, IEO, is invalid in all the figures. One way of recognizing valid or invalid arguments is the use of rules listed above. We have another method known as 'special rules of figures'.

These rules are called special rules because they apply to only that particular figure but not to others. These rules are dependent upon general rules. Therefore it is possible to give proofs to these rules.

I) Figure

a) The minor must be affirmative.

b) The major must be universal.

$$\frac{M - P}{\begin{array}{c} S - M \\ S - P \end{array}}$$

a) If the minor is negative, then the conclusion must be negative. In negative conclusion P is distributed while it is undistributed in the major premise. This goes against the rule which asserts that a term undistributed in the premise should remain undistributed in the conclusion. Therefore minor must be affirmative.

b) That the major must be universal is clear from the fact that if the minor is affirmative, M in it is undistributed and therefore the major must be universal if M must be distributed in it.

Now we shall apply these special rules to know how or why a certain mood is valid and certain other moods invalid in a figure, a point which we discussed in the previous unit. Let us omit weakened moods.

The valid moods of I figure are listed below.

I) Fig:	AAA	BARBARA
	EAE	CELARENT
	AII	DARII
	EIO	FERIO

II) Figure:

a) One premise must be negative.

b) The major must be universal.

$$\frac{P - M}{\begin{array}{c} S - M \\ S - P \end{array}}$$

a) One premise must be negative. Otherwise, M remains undistributed in both the premises. b) The major must be universal because P is distributed in negative conclusion and hence it must be distributed in the major.

The valid moods of II figure are listed below.

EAE	CESARE
AEE	CAMESTRES
EIO	FESTINO
AOO	BAROCO

III) Figure:

a) The 'minor' must be affirmative.

b) The conclusion must be Particular.

$$\frac{\begin{array}{c} M - P \\ M - S \end{array}}{S - P}$$

a) Minor must be affirmative because negative minor gives only negative conclusion in which case P is distributed in the conclusion. P can be distributed in major only if it is negative. Negative minor results in negative major which is not allowed. Therefore minor must be affirmative.

b) The conclusion must be particular. Otherwise S becomes distributed in the conclusion while it remains undistributed in affirmative minor.

The valid moods are listed below.

III) Fig:

AAI	DARAPTI
IAI	DISAMIS
AII	DATISI
OAO	BOCARDO
EIO	FERISON
EAO	FELOPTON

IV) Figure:

a) If the 'major' is affirmative, the 'minor' must be universal.

b) If the minor is affirmative, the conclusion must be particular.

c) If the conclusion is negative, the major must be negative.

$$\frac{\begin{array}{c} P - M \\ M - S \end{array}}{S - P}$$

The valid moods are listed below.

AAI	DARAPTI
AEE	CAMENES
EAO	FESAPO
IAI	DIMARIS
EIO	FRESISON

It would be good logical exercise for the student to take up these special rules and try to deduce them from the general ones. This is the reason why we have left the special rules of figure 4 unexplained.

4.4 REDUCTION OF ARGUMENTS TO I FIGURE

Reducing arguments from other figures to the first figure is one of the techniques developed by Aristotle and one of his followers to test the validity of arguments. After reduction, if the argument is valid in the first figure, then it means that the original argument in the corresponding figure is valid. This technique is quite mechanical. So we are only required to know what exactly is involved in this method. We will learn this only by practice. Strengthened moods are included for the sake of exercise though they are not required from the point of view of modern logic. There is no need to consider weakened moods separately when the technique involved is reduction. What is required is replacement of universal by its corresponding subaltern in the conclusion.

II Figure

II Figure		I Figure
CESARE		CELARENT
P E M	→ Conversion→	M E P
S A M		S A M
S E P		S E P
No politicians are poets.	→ Conversion →	No poets are politicians.
All girls are poets.		All girls are poets.
∴No girls are politicians.		∴No girls are politicians

In CESARE 'S' after 'E' indicates simple conversion. It shows that 'E' (major premise) must undergo simple conversion.

II Figure

II Figure	I Figure
CAMESTRES	CELARENT
P A M	M E S
S E M → conversion M E S	P A M
∴S E P	∴P E S conversion→S E P

'S' and 'T' after 'E' show that 'E' (minor premise) should undergo simple conversion and both premises be transposed. 'S' after second 'E' shows that this 'E' (conclusion) also should undergo simple conversion. [The student is advised to construct arguments for this and subsequent reductions.]

II Figure

II Figure		I Figure
FESTINO		FERIO
P E M	→ Conversion →	M E P
S I M		S I M
S O P		S O P

FESTINO becomes FERIO when the major premise undergoes simple conversion. The kind of reduction of the above mentioned moods is known as direct reduction. BAROCO becomes FERIO through the process of indirect reduction. Indirect reduction includes, in addition to conversion, obversion also.

II Figure

II Figure		I Figure
BAROCO		FERIO
P A M → obversion $\overline{P}$ E M	→ Conversion →	$\overline{M}$ E P
S O M → obversion $\overline{S}$ I M		S I $\overline{M}$
S O P		S O P

III Figure

III Figure		I Figure
DARAPTI		DARII
M A P		M A P
M A S	→ Conversion →	S I M
S I P		S I P
DATISI		DARII
M A P		M A P
M I S	→ Conversion →	S I M
S I P		S I P
FELAPTON		FERIO
M E P		M E P
M A S	→ Conversion →	S I M
S O P		S O P

'P' which follows 'A' in DARAPTI and FELAPTON shows that conversion by limitation applies to 'A'.

FERISON		FERIO
M E P		M E P
M I S	→ Conversion →	S I M
S O P		S O P

DISAMIS		→ Conversion →	DARII
M I P	M A S		
M A S	M I P	→ Conversion →	P I M
S I P			P I S
			↓ Conversion
			S I P

While the reduction of the above-mentioned moods is direct, next one is indirect.

BOCARDO					
M O P	→ Obversion →	M I $\bar{P}$	→ Conversion →	$\bar{P}$ I M	M A S
M A S					
		M AS		M A S	$\bar{P}$ I M
S O P					$\bar{P}$ I S
					↓Conversion
					S I $\bar{P}$
					↓Obversion
					S O P

When BOCARDO undergoes reduction, conversion, obversion and transposition are required to complete the process. Here OAO becomes AII. Further, when we consider obverted conclusion of AII, we obtain AIO. This is, surely, a paradox.

IV Figure

IV Figure	I Figure
BRAMANTIP	Weakened mood
P A M	M A S
M A S	P A M
S I P	P A S → Conversion →S I P

CAMENES	CELARENT
P A M	M E S
MES	PAM
S E P	S E P

DIMARIS	DARII
P I M	M A S
M A S	P I M
S I P	S I P

FESAPO		FERIO
P E M	→ Conversion→	M E P
M A S	→ Conversion→	S I M
S O P		S O P

As usual 'S' stands for simple conversion of 'E' (major Premise) and 'P' stands for conversion by limitation of 'A' (minor premise). This process is similar to the one applied for first and third moods of III figure.

FRESISON		FERIO
P E M	→ Conversion→	M E P
M I S	→ Conversion→	S I M
S O P		S O P

From reduction technique one point becomes clear. Originally, there were twenty-four valid moods. Later weakened and strengthened moods were eliminated on the ground that particular proposition (existential quantifier) cannot be deduced from universal propositions (universal quantifier) only, and the number was reduced to fifteen. Now after reduction to first figure the number came down to four. Strawson argues that reduction technique is superior to axiomatic technique to which he referred in the beginning of his work 'Introduction to Logical Theory'. He regards the moods as inference-patterns. He argues that the path of reduction should be an inverted pyramid. Strawson also maintains that in addition to equivalence relation, we require opposition relation also to effect reduction. What we gain in the process is economy in the number of moods.

4.5 ANTILOGISM OR INCONSISTENT TRIAD

This technique was developed by one lady by name, Christin Lad Franklin. This technique applies only to fifteen moods. The reason is, again, impropriety of deriving existential from universals only. The method is very simple. Consider Venn's results for all propositions. Replace the conclusion by its contradiction. This arrangement constitutes antilogism. If the corresponding argument should be valid, then antilogism should conform to certain structure. It must possess two equations and one inequation. A term must be common to equations. It should be positive in one equation and negative in another. Remaining two terms ought to appear only in inequation. Consider one example for a valid argument.

	Venn's Results	**Antilogism**
All Indians are Asians.	I Â = ¨	I Â = ¨
All Hindus are Indians.	H Î = ¨	H Î = ¨
∴ All Hindus are Asians.	H Â = ¨	H Â = ¨

In this case, antilogism satisfies all the requirements. 'I' is common to equations; in one equation it is positive and in another negative. There is only one inequation. Remaining terms appear in inequation. In all cases, this is the method to be followed. If any one of these characteristics is absent in antilogism, then the corresponding mood is invalid.

Now antilogism can be easily constructed for the remaining fourteen moods.

I Fig.

1) CELARENT

	Contradiction	
M E P		$MP = \varnothing$
S A M		$S\overline{M} = \varnothing$
S E P →	SIP	$SP \neq \varnothing$

2) DARII

M A P		$M\overline{P} = \varnothing$
S I M		$SM \neq \varnothing$
S I P →	SEP	$SP = \varnothing$

3) FERIO

M E P		$MP = \varnothing$
S I M		$SM \neq \varnothing$
S O P →	SAP	$S\overline{P} = \varnothing$

II Fig.

4) CESARE

P E M		$PM = \varnothing$
S A M		$S\overline{M} = \varnothing$
S E P →	SIP	$SP \neq \varnothing$

5) CAMESTRES

P A M		$P\overline{M} = \varnothing$
S E M		$SM = \varnothing$
S E P $\rightarrow$	SIP	$SP \neq \varnothing$

6) FESTINO

P E M		$PM = \varnothing$
S I M		$SM \neq \varnothing$
S O P $\rightarrow$	SAP	$S\overline{P} = \varnothing$

7) BAROCO

P A M		$P\overline{M} = \varnothing$
S O M		$S\overline{M} \neq \varnothing$
S O P $\rightarrow$	SAP	$S\overline{P} = \varnothing$

III Fig.

8) DISAMIS

M I P		$MP \neq \varnothing$
M A S		$M\overline{S} = \varnothing$
S I P $\rightarrow$	SEP	$SP = \varnothing$

9) DATISI

M A P		$M\overline{P} = \varnothing$
M I S		$MS \neq \varnothing$
S I P $\rightarrow$	SEP	$SP = \varnothing$

10) BOCARDO

M O P		$M\overline{P} \neq \varnothing$
M A S		$M\overline{S} = \varnothing$
S O P $\rightarrow$	SAP	$S\overline{P} = \varnothing$

11) FERISON

M E P		$MP = \varnothing$
M I S		$MS \neq \varnothing$
S O P $\rightarrow$	SAP	$S\overline{P} = \varnothing$

IV Fig.

12) CAMENES

PAM		$P\overline{M} = \varnothing$
MES		$MS = \varnothing$
SEP $\rightarrow$	SIP	$SP \neq \varnothing$

13) DIMARIS

PIM		$PM \neq \varnothing$
MAS		$M\overline{S} = \varnothing$
SIP $\rightarrow$	SEP	$SP = \varnothing$

14) FRESISON

PEM		$PM = \varnothing$
MIS		$MS \neq \varnothing$
SOP $\rightarrow$	SAP	$S\overline{P} = \varnothing$

Now consider a weakened mood.

II Fig.

Weakened mood:

PAM		$P\overline{M} = \varnothing$
SEM		$SM = \varnothing$
SOP $\rightarrow$	SAP	$S\overline{M} = \varnothing$

There is no inequation in this antilogism. Hence, corresponding argument is invalid. It can be shown that any other strengthened or weakened mood is invalid.

4.6 VENN DIAGRAM TECHNIQUE

Let us extend our knowledge of Venn diagram to the testing of arguments. If two terms yield four product classes, then three terms should yield eight product classes according to the formula $2^x = n$, where x stands for the number of terms and n stands for the number of product classes. Since syllogism consists of three terms, we have eight product classes. Let us begin with a valid mood and list these product classes.

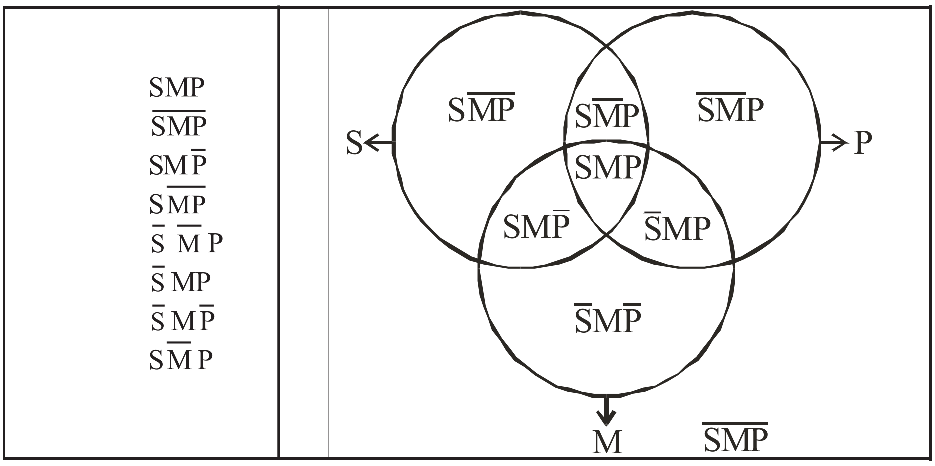
BARBARA

p1: All M are P. $\qquad M\overline{P} = \Phi$

p2: All S are M. $\qquad S\overline{M} = \Phi$

q: $\therefore$ All S are P

The product classes are as follows: -



While listing product classes, sufficient care should be taken to ensure that no product class is repeated. It is always advisable to make a list of product classes with diagrams and mark classes accurately to avoid confusion.

Now let us use diagram to represent the propositions. The procedure is as follows. null sets are shaded and non-null sets are starred. We should also note that product of null set and non-null set is a null set. It is like saying that 4 x 0 = 0. But the union, i.e., addition of a non-null set and null set is a non-null set. Remember 4 + 0 = 4.

Since M$\bar{P}$ is a null set, not only SM$\bar{P}$, but also $\bar{S}$ M $\bar{P}$ is a null set. It does not mean that there are two null sets. There is only one null set. S $\bar{M}$ is also a null set. Therefore not only the product of S $\bar{M}$ & P, but also S $\bar{M}$ and $\bar{P}$ is a null set. Now we shall shade relevant subsets, which are null.

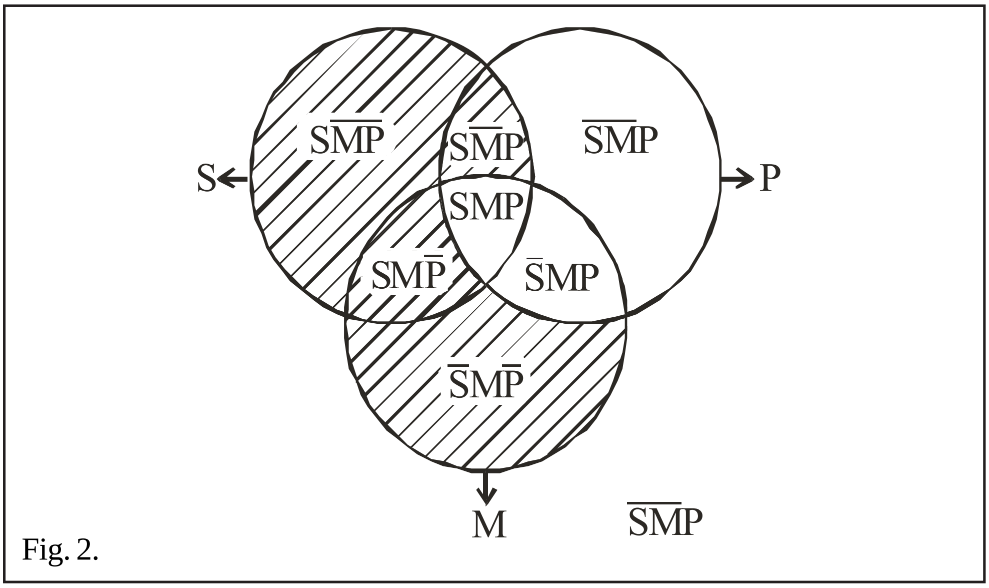
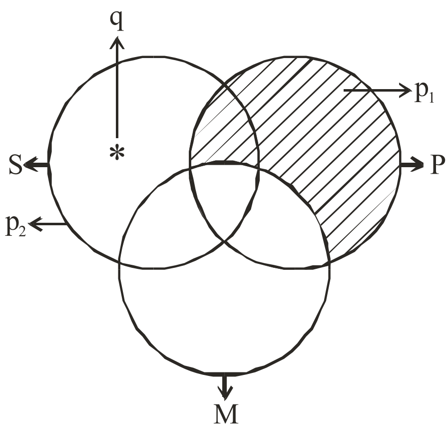


Fig. 2.

p_1 and p_2 show that: $\bar{S}M\bar{P}$ = SM= S= SP = Ø. The conclusion shows that S also is a null set. We did not specially shade S. Shading of M and S included naturally the shading of S segment. This is what actually happens in the case of valid arguments. Marking of premises naturally includes the conclusion. It is not marked separately. In other words marking, of conclusion is inclusive. When we adopt Venn diagram technique, this important condition should be borne in mind. Secondly, when any premise is particular, the segment, which corresponds to the universal premise, should be shaded first. This is the initial step to be followed. Now we shall consider some moods. Others are left for the student as an exercise. [In all cases all product classes should be identified by the student even if there is no need. This is a good exercise.]

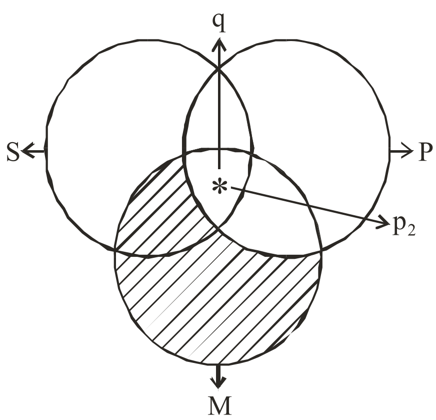
2 BAROCO

p1: All P are M.	$P\overline{M} = \emptyset$
p2: Some S are not M.	$S\overline{M} \neq \emptyset$
q: ∴ Some S are not P.	$S\overline{P} \neq \emptyset$



3 DATISI

p1: All M are P.	$M\overline{P} = \emptyset$
p2: Some M are S.	$M S \neq \emptyset$
q: ∴ Some S are P.	$S P \neq \emptyset$

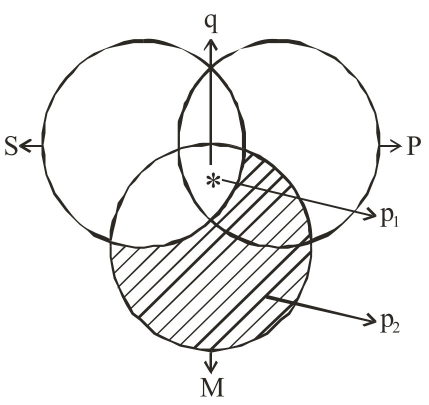


4 DISAMIS

p1: Some M are P. $MP \neq \emptyset$

p2: All M are S. $M\bar{S} = \emptyset$

q: ∴ Some S are P. $SP \neq \emptyset$

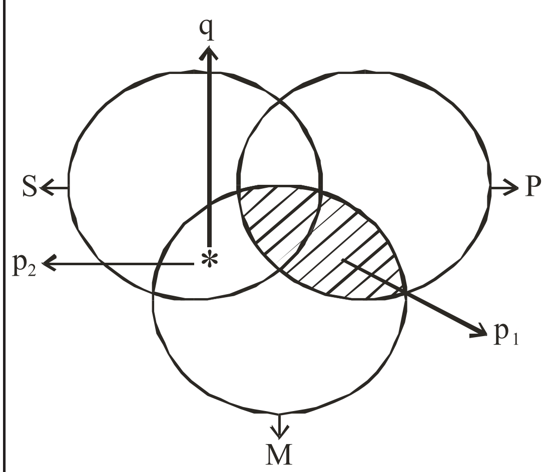


5 FERISON

p1: No M are P. $MP = \emptyset$

p2: Some M are S. $MS \neq \emptyset$

q: Some S are not P. $S\bar{P} \neq \emptyset$



6 BOCARDO

p1: Some M are not P. $M\bar{P} \neq \varnothing$

p2: All M are S. $M\bar{S} = \varnothing$

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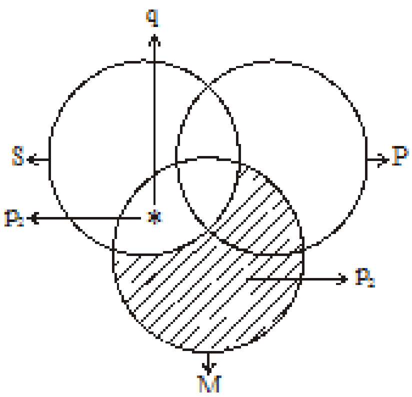
q: ∴ Some S are not P. $S\bar{P} \neq \varnothing$



7 CAMENES

p1: All P are M. $P\bar{M} = \varnothing$

p2: No M are S. $M S = \varnothing$

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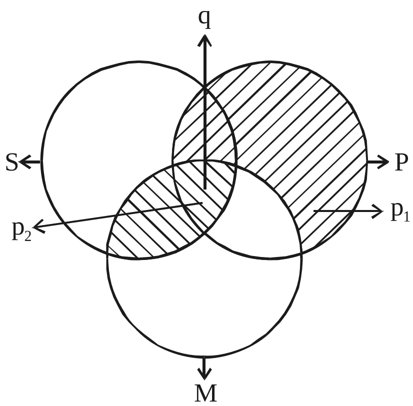
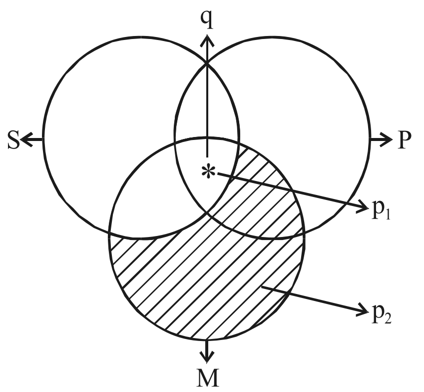
q: No S are P. $S P = \varnothing$



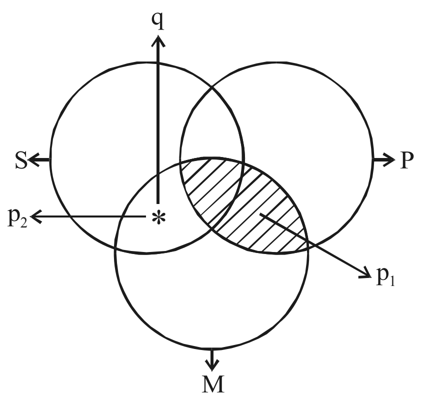
8 DIMARIS

p1: Some P are M.	$P\,M \neq \varnothing$
p2: All M are S.	$M\,\bar{S} = \varnothing$
q: ∴ Some S are P.	$S\,P \neq \varnothing$



9 FRESISON

p1: No P are M.	$P\,M = \varnothing$
p2: Some M are S.	$M\,S \neq \varnothing$
q: Some S are not P.	$S\,\bar{P} \neq \varnothing$



Let us examine a few weakened and strengthened moods using Venn's diagram.

10 BRAMANTIP

p1: All P are M. $P\overline{M} = Ø$

p2: All M are S. $M\overline{S} = Ø$

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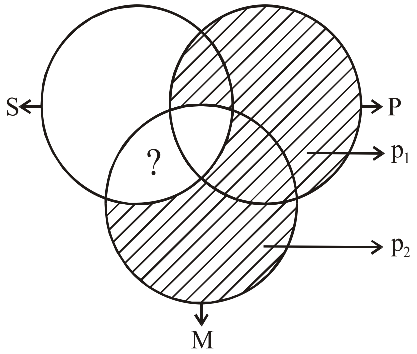
q: Some S are P. $S P \neq Ø$



No information on S M and SMP is available after the premises are diagrammed. Therefore BRAMANTIP is invalid. Now consider a weakened mood.

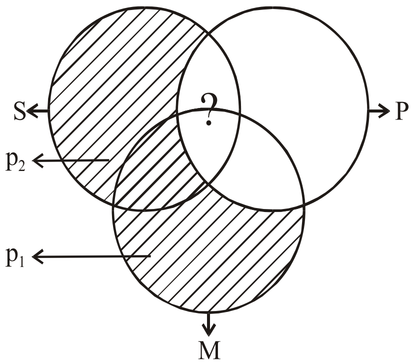
AAI

p1: M A P. $M\overline{P} = Ø$

p2: S A M. $S\overline{M} = Ø$

---

q: ∴ S I P. $S P \neq Ø$



In this case also no information is available on S P and S M P after the premises are diagrammed. Hence AAI is invalid.

4.7 BOOLEAN ANALYSIS

George Boole published his work *The Mathemaical Analysis of Logic* in 1847. This work provided not only the required breakthrough to logic but also a new direction to its development. This analysis is known as *The Boolean Algebra of Classes*. It is a rewarding exercise to understand this approach.

Boolean analysis presupposes some axioms. Basson and O'connor list thirteen axioms while Alexander considers seven. However, for our purpose only four of them are sufficient to understand this analysis. Let us begin with these axioms.

1) Law of multiplication: a) the product of a universal set and a non-null set(S) is a non-null set. b) The product of null set and a non-null set is null set.

$1 \times S = S$ (where 1 is the universal set.) 1a

$\Phi \times S = \Phi$ 1b

2) Law of addition: The addition of complementary sets is universal set.

$S + \overline{S} = 1$ 2

3) Law of Commutation for a) addition and b) multiplication: Transposition of two or more than two sets is equivalent to original structure.

a) (S+ P + M) = (S+ M +P) = (P + M + S) = (M+ S +P) ….. 3a

b) (S PM) = (SMP) = (MSP) = (MPS)……… 3b

(Instead of addition and multiplication we can also use union and product respectively.)

4) Law of distribution: The multiplication of a l set on the one hand and the addition of two non-null sets on the other is equivalent to the addition of the product of two sets.

S(P+M) = SP+PM……. 4

Some valid moods are worked out and the rest are left as exercises for the student.

1) BARBARA

p1: All M are P. $M\overline{P} = \Phi$

p2: All S are M. $S\overline{M} = \Phi$

q: ∴ All S are P $S\overline{P} = \Phi$

Boolean analysis begins with the expansion of statements. The first stage of the expansion of major premise is as follows.

$$\begin{aligned} M\overline{P} &= M\overline{P} \times 1 && \text{Rule 1b} \\ &= M\overline{P}(S + \overline{S}) = M\overline{P} && \text{Rule 2} \\ &= M\overline{P}S + M\overline{P}\,\overline{S} = M\overline{P} && \text{Rule 4} \end{aligned}$$

Now we shall pass on to the second stage.

$$a) \left.\begin{aligned} S \times M\overline{P} = SM\overline{P} = \Phi \\ \overline{S} \times M\overline{P} = \overline{S}M\overline{P} = \Phi \end{aligned}\right\} \quad \text{Rule 1}$$

$$M\overline{P} = SM\overline{P} + \overline{S}M\overline{P} = \Phi$$

The last line corresponds to the expansion of major premise. While expanding these lines, we must obtain the addition or union of the product of all relevant sets and their complements as well. On these lines, we shall expand remaining lines.

$$b) \left.\begin{aligned} P \times S\overline{M} = PS\overline{M} = \Phi \\ \overline{P} \times S\overline{M} = \overline{P}S\overline{M} = \Phi \end{aligned}\right\} \quad \text{Rule 1b}$$

$$\left.\begin{aligned} PS\overline{M} = S\overline{M}P = \Phi \\ \overline{P}S\overline{M} = S\overline{MP} = \Phi \end{aligned}\right\} \quad \text{Rule 3b}$$

$$S\overline{M} = S\overline{M}P + S\overline{MP} = \Phi$$

The last line corresponds to the expansion of minor premise.

$$
\text{c)} \quad \left.\begin{aligned} S\bar{P} \times M = S\bar{P}M = \Phi \\ S\bar{P} \times \bar{M} = S\bar{P}\bar{M} = \Phi \end{aligned}\right\} \quad \text{Rule 1b}
$$

$$
\left.\begin{aligned} S\bar{P}M = SM\bar{P} = \Phi \\ S\bar{P}\bar{M} = S\bar{M}\bar{P} = \Phi \end{aligned}\right\} \quad \text{Rule 3b}
$$

$$S\bar{P} = SM\bar{P} + S\bar{M}\bar{P} = \Phi$$

$$a + b = SM\bar{P} + \bar{S}M\bar{P} + S\bar{M}P + S\bar{M}\bar{P} = \Phi$$

Since the union of four product classes is null set any set in this group is null set. Consider the union of relevant sets.

$$SM\bar{P} + S\bar{M}\bar{P} = \Phi$$

Since this is equivalent to what we have obtained from the conclusion, the argument is valid. This shows that the expansion of conclusion must be equal to or less than the union of premises if the argument is valid. Hence this conclusion is not repeated further while dealing with some arguments which are valid. Since we follow this method throughout, we should bear in our mind all these details.

2) CELARENT

p1 : No M are P.	$MP = \Phi$
p2 : All S are M.	$S\bar{M} = \Phi$
q : $\therefore$ No S are P.	$SP = \Phi$

Expansion of major premise:

a : $SMP + \bar{S}MP = \Phi$

Expansion of minor premise:

b : $S\bar{M}P + S\bar{M}\bar{P} = \Phi$

Expansion of conclusion:

c : $SMP + S\bar{M}P = \Phi$

$$SP\bar{M} = S\bar{M}P$$

$$\therefore \quad SMP + SP\bar{M} = SMP + S\bar{M}P = \Phi$$

$$a + b \quad \Rightarrow \quad SMP + \bar{S}MP + S\bar{M}P + S\bar{M}\bar{P} = \Phi$$

$$c = SMP + S\bar{M}P = \Phi$$

$$a + b = c$$

3) DARII

p1 : All M are P.	$M\bar{P} = \Phi$
p2 : Some S are M.	$SM \neq \Phi$
q : $\therefore$ Some S are P.	$SP \neq \Phi$

Expansion of major premise:

Expansion of minor premise:

$$b: \quad S\overline{M}P + S\overline{MP} = \Phi$$

Expansion of conclusion:

$$c: \quad SMP + S\overline{M}P = \Phi$$

$$SP\overline{M} = S\overline{M}P$$

$$\therefore \quad SMP + SP\overline{M} = SMP + S\overline{M}P = \Phi$$

$$a + b \quad \Rightarrow \quad SMP + \overline{S}MP + S\overline{M}P + S\overline{MP} = \Phi$$

$$c = SMP + S\overline{M}P = \Phi$$

$$a + b = c$$

Since SMP is a non-null set, its union with null set yields a non-null set.

4) FERIO

p1 : No M are P.	$MP = \Phi$
p2 : Some S are M.	$SM \neq \Phi$
q : $\therefore$ Some S are not P.	$S\overline{P} \neq \Phi$

Expansion of major premise:

$$a: \quad SM\overline{P} + \overline{S}M\overline{P} = \Phi$$

Expansion of minor premise:

$$b: \quad SMP + SM\overline{P} \neq \Phi$$

Expansion of conclusion:

$$c: \quad SMP + S\overline{M}P \neq \Phi$$

$$a + b \quad \Rightarrow \quad SMP + \overline{S}MP + SMP + SM\overline{P} \neq \Phi$$

$$= \Phi + SM\overline{P} \neq \Phi$$

$$SM\overline{P} \neq \Phi$$

$$SM\overline{P} + S\overline{MP} \neq \Phi$$

$$c = SM\overline{P} + S\overline{MP} \neq \Phi$$

$$a + b = c$$

5) CESARE

p1: No P are M.	$PM = \Phi$
p2: All S are M.	$S\overline{M} = \Phi$
q: $\therefore$ No S are P.	$SP = \Phi$

Expansion of major premise:

$$a: \quad SMP + \overline{S}MP = \Phi$$

Expansion of minor premise:

$$b: \quad S\overline{M}P + S\overline{MP} = \Phi$$

Expansion of conclusion:

$$c: \quad SMP + S\overline{M}P = \Phi$$

$$a+b \quad \Rightarrow \quad SMP+\bar{S}MP+S\bar{M}P+S\overline{MP}=\Phi$$

$$\therefore\ SMP+S\bar{M}P=\Phi$$

$$c=SMP+S\bar{M}P=\Phi$$

$$a+b=c$$

6) CAMESTRES

p1 : All P are M.	$P\bar{M}=\Phi$
p2 : No S are M.	$SM=\Phi$
q : $\therefore$ No S are P.	$SP=\Phi$

Expansion of major premise:

$$a: \quad S\bar{M}P+\bar{S}\bar{M}P=\Phi$$

Expansion of minor premise:

$$b: \quad SMP+SM\bar{P}=\Phi$$

Expansion of conclusion:

$$c: \quad SMP+S\bar{M}P=\Phi$$

$$a+b \quad \Rightarrow \quad S\bar{M}P+\bar{S}\bar{M}P+SMP+SM\bar{P}=\Phi$$

$$=SMP+S\bar{M}P=\Phi$$

$$c=SMP+S\bar{M}P=\Phi$$

a+b = c

7) FESTINO

p1 : No P are M.	$PM=\Phi$
p2 : Some S are M.	$SM\neq\Phi$
q : $\therefore$ Some S are not P.	$S\bar{P}\neq\Phi$

Expansion of major premise:

$$a: \quad SMP+\bar{S}MP=\Phi$$

Expansion of minor premise:

$$b: \quad SMP+SM\bar{P}\neq\Phi$$

Expansion of conclusion:

$$c: \quad SM\bar{P}+S\bar{M}\bar{P}\neq\Phi$$

$$a+b \quad \Rightarrow \quad SMP+\bar{S}MP+SMP+SM\bar{P}\neq\Phi$$

$$=\Phi+SM\bar{P}\neq\Phi$$

$$\therefore SM\bar{P}\neq\Phi$$

$$c=SM\bar{P}+S\overline{MP}\neq\Phi$$

$$a+b=c$$

8) BAROCO

p1 : All P are M.	$P\overline{M} = \Phi$
p2 : Some S are not M.	$S\overline{M} \neq \Phi$
q : $\therefore$ Some S are not P.	$S\overline{P} \neq \Phi$

Expansion of major premise:

a : $S\overline{M}P + \overline{S}\,\overline{M}P = \Phi$

Expansion of minor premise:

b : $S\overline{M}P + S\overline{MP} \neq \Phi$

Expansion of conclusion:

c : $SM\overline{P} + S\overline{MP} \neq \Phi$

$a + b \Rightarrow S\overline{M}P + \overline{S}\,\overline{M}P + S\overline{M}P + S\overline{MP} \neq \Phi$

$= \Phi + S\overline{MP} \neq \Phi$

$\therefore S\overline{MP} \neq \Phi$

$\therefore SM\overline{P} + S\overline{MP} \neq \Phi$

$a + b = c$

9) DISAMIS

p1: Some M are P.	$MP \neq \Phi$
p2: All M are S.	$M\overline{S} = \Phi$
q: $\therefore$ Some S are P.	$SP \neq \Phi$

Expansion of major premise:

a : $SMP + \overline{S}MP \neq \Phi$

Expansion of minor premise:

b : $\overline{S}MP + \overline{S}M\overline{P} = \Phi$

Expansion of conclusion:

c : $SMP + S\overline{M}P \neq \Phi$

$a + b \Rightarrow SMP + \overline{S}MP + \overline{S}MP + \overline{S}M\overline{P} \neq \Phi$

$= SMP + \Phi \neq \Phi$

$\therefore SMP \neq \Phi$

$\therefore SMP + S\overline{M}P \neq \Phi$

$a + b = c$

10) DATISI

p1: All M are P.	$M\overline{P} = \Phi$
p2 : Some M are S.	$MS \neq \Phi$
q : $\therefore$ Some S are P.	$SP \neq \Phi$

Expansion of major premise:

Expansion of minor premise:

b : $SMP + SM\overline{P} \neq \Phi$

Expansion of conclusion:

c : $SMP + S\overline{M}P \neq \Phi$

$$a + b \Rightarrow SM\overline{P} + \overline{S}M\overline{P} + SMP + SM\overline{P} \neq \Phi$$
$$= \Phi + SMP \neq \Phi$$

$\therefore SMP \neq \Phi$

$\therefore SMP + S\overline{M}P \neq \Phi$

a+b = c

11) BOCARDO

p1 : Some M are not P.	$M\overline{P} \neq \Phi$
p2 : All M are S.	$M\overline{S} = \Phi$
q : $\therefore$ Some S are not P.	$S\overline{P} \neq \Phi$

Expansion of major premise:

a : $SM\overline{P} + \overline{S}M\overline{P} \neq \Phi$

Expansion of minor premise:

b : $\overline{S}MP + \overline{S}M\overline{P} = \Phi$

Expansion of conclusion.:

c : $SM\overline{P} + S\overline{MP} \neq \Phi$

$$a + b \Rightarrow SM\overline{P} + \overline{S}M\overline{P} + \overline{S}MP + \overline{S}M\overline{P} \neq \Phi$$
$$= \Phi + SM\overline{P} \neq \Phi$$

$\therefore SM\overline{P} \neq \Phi$

$\therefore SM\overline{P} + S\overline{M}\overline{P} \neq \Phi$

$a + b = c$

12) FERISON

p1 : No M are P.	
p2 : Some M are S.	$MS \neq \Phi$
Some S are not P.	$S\overline{P} \neq \Phi$

Expansion of major premise:

a : $SMP + \overline{S}MP = \Phi$

Expansion of minor premise:

b : $SMP + SM\overline{P} \neq \Phi$

Expansion of conclusion:

c : $SM\overline{P} + S\overline{M}\overline{P} \neq \Phi$

$a + b \quad \Rightarrow \quad SMP + \overline{S}MP + SMP + SM\overline{P} \neq \Phi$

$\quad = \Phi + SM\overline{P} \neq \Phi$

$\therefore SM\overline{P} \neq \Phi$

$\therefore SM\overline{P} + S\overline{M}\,\overline{P} \neq \Phi$

a+b = c

13) CAMENES

p1 : All P are M. $\qquad P\overline{M} = \Phi$

p2 : No M are S. $\qquad MS = \Phi$

q : $\therefore$ No S are P. $\qquad SP = \Phi$

Expansion of major premise:

a : $\quad S\overline{M}P + \overline{S}\,\overline{M}P = \Phi$

Expansion of minor premise:

b : $\quad SMP + SM\overline{P} = \Phi$

Expansion of conclusion:

c : $\quad SMP + S\overline{M}P = \Phi$

$a + b \quad \Rightarrow \quad S\overline{M}P + \overline{S}\,\overline{M}P + SMP + SM\overline{P} = \Phi$

$\quad = S\overline{M}P + SMP = \Phi$

a+b =c

14) DIMARIS

p1 : Some P are M. $\qquad PM \neq \Phi$

p2 : All M are S. $\qquad M\overline{S} = \Phi$

q : $\therefore$ Some S are P. $\qquad SP \neq \Phi$

Expansion of major premise:

a : $\quad SMP + \overline{S}MP \neq \Phi$

Expansion of minor premise:

b : $\quad \overline{S}MP + \overline{S}M\overline{P} = \Phi$

Expansion of conclusion:

c : $\quad SMP + S\overline{M}P \neq \Phi$

$a + b \quad \Rightarrow \quad SMP + \overline{S}MP + \overline{S}MP + \overline{S}M\overline{P} \neq \Phi$

$\quad = SMP + \Phi \neq \Phi$

$\therefore SMP \neq \Phi$

$\therefore \; c = SMP + S\overline{M}P \neq \Phi$

a+b = c

15) FRESISON

p1 : No P are M. $\qquad PM = \Phi$

p2 : Some M are S. $\qquad MS \neq \Phi$

q : $\therefore$ Some S are not P. $\qquad S\overline{P} \neq \Phi$

Expansion of major premise:

$$a: \quad SMP + \bar{S}MP = \Phi$$

Expansion of minor premise:

$$b: \quad SMP + SM\bar{P} \neq \Phi$$

Expansion of conclusion:

$$c: \quad SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$$

$$a + b \quad \Rightarrow \quad SMP + \bar{S}MP + SMP + SM\bar{P} \neq \Phi$$

$$= \Phi + SM\bar{P} \neq \Phi$$

$$\therefore SM\bar{P} \neq \Phi$$

$$\therefore \; SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$$

a+b = c

Let us examine an invalid mood which is regarded as valid in traditional framework.

16) AAI

p1 : All M are P.	$M\bar{P} = \Phi$
p2 : All S are M.	$S\bar{M} = \Phi$
q: $\therefore$ Some S are P.	$SP \neq \emptyset$

Expansion of major premise:

$$a: \quad SM\bar{P} + \bar{S}M\bar{P} = \Phi$$

Expansion of minor premise:

$$b: \quad S\bar{M}P + S\bar{M}\bar{P} = \Phi$$

Expansion of conclusion:

$$c: \quad SMP + S\bar{M}P \neq \Phi$$

$$a + b = SM\bar{P} + \bar{S}M\bar{P} + S\bar{M}P + S\bar{M}\bar{P} = \Phi$$

a+b $\neq$ c

This is so because from equations alone it is not possible to obtain inequation. Antilogism, Venn Diagram Technique and Boolean Analysis have one distinct advantage. They do away with the concept of distribution of terms which is a cumbersome to apply. What is required is only the application of some elements of set theory.

Apply these techniques for the following arguments to test their validity.

1) All dogs have four legs.
All animals have four legs.
$\therefore$ All dogs are animals.

2) All dogs have four legs.
All chairs have four legs.
$\therefore$ All dogs are chairs.

3) No bats are cats.
No rats are bats.
$\therefore$ No rats are cats.

4) No fish are birds.
No golden plovers are fish.
∴ No golden plovers are birds.

5) All Indians are people.
John is a person.
∴ John is an Indian.

6) Some readers are philosophers.
Chanakya is a philosopher.
∴ Chanakya is a reader.

7) No human being is perfect.
Some human beings are presidents.
∴ Some presidents are not perfect.

8) All matter obeys wave equations.
All waves obey wave equations.
∴ All matter is waves.

9) All human action is conditioned by circumstances.
All human action involves morality.
∴ All that involves morality is conditioned by circumstances.

10) All that is good is pleasant.
All eating is pleasant.
∴ All eating is good.

11) All patriots are voters.
Some citizens are not voters.
∴ Some citizens are not patriots.

12) All potatoes have eyes.
John's head has eyes.
∴ John is a potato head.

Check Your Progress II

Note: Use the space provided for your answers.

1) EIO is valid and IEO is invalid in all the figures. Explain.

....

....

....

....

2) If both premises are universal, then the conclusion must also be universal. Explain.

....

....

....

....

4.8 LET US SUM UP

General rules apply to all figures whereas special rules apply to specific figures. Special rules indirectly depend upon general rules only. Antilogism, Venn diagram technique and Boolean analysis do away with the concept of distribution of terms. According to the last three methods weakened and strengthened moods become invalid though traditional logic regards them as valid.

4.9 KEY WORDS

Mood : By the 'mood' of a syllogism is meant that kind of a syllogism which is determined by the 'quantity' and 'quality' of the three propositions.

Figure : By the 'figure' of a syllogism is meant that kind of syllogism which is determined by the function the 'middle term' plays in the syllogism.

4.10 FURTHER READINGS AND REFERENCES

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