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# UNIT 1 INTRODUCTION TO THE THEORY OF CHAOS

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## 1.0 OBJECTIVES

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This primary objective of this unit is to introduce the chaos theory by analysing it from the scientific and philosophical perspectives. In chaos theory you will find a perfect blending of, once considered irreconcilable extremes of linearity and non-linearity, determinism and indeterminism which were the characteristic features of Newtonian mechanics and quantum mechanics respectively. The developments in scientific fields do not remain aloof from the real life settings instead they trickle down to various fields, including philosophy leading to the formation and transformation of human thought pattern. This can very well be seen in mechanistic deterministic vision of the universe that assured future to be present before us like past on one hand and the indeterministic vision that led to submission to randomness and chance on the other. Chaos theory brings in an alternative to these extremes views in both scientific and philosophical realm. The essential scientific features of chaos theory are discussed without going into the complex equations. The philosophical analysis of chaos theory will showcase the paradigm shift that chaos theory has initiated.

Thus by the end of this Unit, you should be able to:

- have a basic understanding of the chaos theory and its concepts.
- analyse the chaos theory as an alternative to Newtonian and quantum mechanics.
- understand how chaos theory initiates a paradigm shift
- reflect on the relevance of chaos theory in the contemporary world.

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## 1.1 INTRODUCTION

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The word chaos in everyday life would mean a totally disorganized random state. The term chaos used in chaos theory must be taken as a term of art and does not go along with the ordinary connotations. To be more specific, the noun chaos

and the adjective chaotic are used to describe the time and behaviour of a system when the behaviour of the system is aperiodic. The word ‘chaos’ first entered physics in the physicist James Clerk Maxwell’s phrase, ‘state of molecular chaos’ in the nineteenth century.

The sudden and dramatic changes in non-linear dynamic systems may give rise to complex chaotic behaviour. Underlying this aperiodic and apparent random behaviour, there is a determined order waiting to be revealed. The chaos that is being discussed here is deterministic chaos. Associating determinism with chaos may sound paradoxical but it is not the kind of determinism outlined by Newtonian laws but it depicts the uncertainty consistent with determinism. This is due to sensitive dependence on initial condition which results in the amplification of small scale uncertainties over a period of time resulting in unpredictability in long term behaviour even though the behaviour is predictable in the short term.

After Newtonian mechanics, relativity and quantum theory, the scientists have pinned their hopes on chaos theory as they believe in its potential to make significant contribution cutting across scientific disciplines. As chaos theory brought together thinkers from fields that had been widely separated, it is a science of the global nature of systems. The new alternative which chaos theory has opened incorporates both linearity in Newtonian mechanics and non-linearity in quantum mechanics. Chaos theory has not looked back since it made its first appearance in the scientific arena; the juggernaut still rolls on with lot of promises and surprises in store.

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## 1.2 CHAOS IN HISTORY

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Down the lanes of history, the concept of chaos can be looked at from both philosophical and scientific perspectives. Chaos was relatively a new entrant in the scientific field. Chaos began making appearance a century ago when scientists started bringing in the concepts of spontaneous change, irregularity and disorder, constructing a new epistemology which had the potential to call in to question the classical laws of physics. The philosophical roots of chaos are very deep with references dating back to 2500 BCE in the Asian, Egyptian, and Mayan cultures.

The focal point of human attempt during these times was to tame chaos and put it in an order to bring about harmony. Around the globe, people of various cultures and civilizations developed diverse techniques to master chaos and bring about an order. There were calendars to predict seasons, astronomical rules to judge the position of planets, markings in the river bank to note the rise and fall of water level in river Nile and the like. All these were done to infuse order to the chaotic world and were adorned with attractive myths. Beginning with myth and belief in magical powers, human beings moved forward into scientific thought process keeping in mind the intense desire to bring about order and harmony, or, in other words, to have a deterministic vision in order to have predictability. It was pure chaos that ruled in the age of early human history. Slowly, but steadily, the underlying rules in nature were brought in, the natural science as well as mathematics developed hand in hand.

In the Greek tradition, the concept of chaos made its first appearance in Hesiod’s *Theogony*, dated back to 800 BCE. The mythical idea of chaos as immense and

creative with an uneasy tension between chaos and order was injected upon with scientific attitude by the Greeks and also was subject to religious interpretations. With the arrival of Galileo, Kepler, Descartes and Newton on the scene, linearity and non-linearity or the order and chaos which once co-existed were separated. The reductionist science called the shots and sidelined the concept of chaos. Francis Bacon and Descartes were instrumental in reducing the four causes of Aristotle into two namely material cause and efficient cause. The formal cause and final cause were skipped to philosophy of religion. Thus Aristotelian holistic and organic vision of the universe was overthrown and was replaced by mechanical vision of universe. The laws and concepts of Newton gave a strong deterministic assurance to the whole world. Stating that the non-linear system as imperfect, only the linear systems were studied and the scientists even thought that non-linearity could be incorporated into linearity.

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### 1.3 NEWTONIAN DETERMINISM AND QUANTUM INDETERMINISM

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The scientific community and the world at large believed that on April 29, 1686, the day when Sir Issac Newton presented his research – *Principia Mathematica* to Royal Society of London, would mark the death of chaos and herald the birth of determinism and predictability. Newton for the first time provided a unified quantitative explanation for a wide range of physical phenomena with his laws of motion, law of universal gravitation, the concepts of absolute space and absolute time together with mathematical techniques of calculus. Thus he placed the stamp of approval on the rigorous deterministic philosophical vision of Descartes and Bacon. Determinism, predictability and reversibility were the characteristics of Newtonian mechanics.

In the Newtonian deterministic vision, the whole world was conceived to be a gigantic machine. When each part of the machine namely, mechanics, heat waves, sound, light, magnetism, electricity- were all brought under control, the wild excitement resulted in extreme deterministic views like Laplace Demon. It stated that there would be nothing in the world which would be uncertain and the future would be present before our eyes like past.

The Newtonian world view trickled down to various fields. Just like physicists reduce the property of gases to motion of their atoms or molecules, John Locke reduced the patterns observed in society to the behavior of individuals and stated that there were natural laws governing human societies as we had laws in the physical universe. He also stated that the function of government was not to impose laws but to discover and enforce the natural laws inherent to mankind which became the corner stone of the value systems of the enlightenment and influenced the development of modern economic and political thought. Science and Philosophy which were once wedded to each other were separated by the influence of Newtonian Physics. Man was considered to be the master of the gigantic machine called universe which was given to him to exploit and this led to the plundering of nature. Yet another consequence of Newtonian mechanics was that the present became a sum of the past events or a passage to determine future events resulting in the oblivion of the present. Chaotic or the non-linear phenomena were considered to be imperfections that could be appropriated to linearity and thus, linearity ruled the stage.

The Newtonian vision of a clock work universe with linearity, determinism and predictability remained at the helm of scientific affairs for years. Contrary to the expectation of the time, new theories came up. As Illya Prigogine says, “Classical science, the mythical science of a simply passive world, belongs to the past, was killed not by philosophical criticism or empirical resignation but by the internal development of science itself.” As the time unfolded, relativity eliminated the Newtonian illusion of absolute space and time and Einstein said that everyone would have his or her own personal timings. Quantum theory eliminated the Newtonian dream of a controllable measurement process. The matrix and wave mechanics represented reality through mathematical symbols and manipulated them to get results. Three important philosophical principles associated with quantum mechanics were complimentality, uncertainty and probability. Complimentality suggests that particle and wave are complimentary while uncertainty explains that there is a certain amount of uncertainty when we are precisely measuring speed and position, time and energy. The probability determines the result. Thus uncertainty and probability together described nature as indeterministic.

**Check Your Progress I**

**Note:** Use the space provided for your answers.s.

1)

Trace the significant developments in the evolution of the concept of chaos till the time of Newton.

2)

What are the philosophical consequences of deterministic Newtonian mechanics?

1.4 SCIENTIFIC ANALYSIS OF CHAOS THEORY

Torn up between the extremes of rigorous deterministic Newtonian mechanics and indeterministic quantum mechanics, Chaos theory shows the way out. Chaos theory has shown that chaos in non-linear dynamical systems is not random but it has got a hidden order that helps us in making short term predictions as well as long term trends. Thus, the alternative which Chaos theory opens up reconciling both determinism and indeterminism, yields meaningful and scientific output. The shift from mechanical worldview can be characterized by words like organic,

holistic and ecological. Philosophically, the Aristotelian holistic vision of blending the abstract (mathematical), tangible (imaginative) and the metaphysical into a single whole is making a grand come back. Joseph Ford of Georgia Institute of Technology, terms chaos as, “Dynamics freed at last from the shackles of order and predictability ... systems liberated to randomly explore their every dynamical possibility ... exciting variety, richness of choice, of cornucopia of opportunity.”

Henri Poincare had outlined the central features of deterministic Chaos Theory a century ago in his book *Science and Method*, “a very small cause which escapes our notice, determines a considerable effect that we cannot fail to see, and then we say the effect is due to chance.” It was not well acknowledged during that time because of the emphasis on linearity and outright rejection on non-linearity. Edward Lorenz, a meteorologist at the Massachusetts Institute of Technology, presented a paper titled, “Predictability: Does the flap of a butterfly’s wing in Brazil set off a Tornado in Texas?” and is credited with the renewed interest in Chaos theory. He coined the term Butterfly Effect. He came across this interesting concept while working on the problem of weather prediction in 1961. While trying to make weather prediction possible, Lorenz entered 0.506 instead of 0.506127 in his computer in order to save space, thinking that the shortened rounded off number which was once part in a thousand would make no effect. This very small difference made a great diversion in the outcome. This effect came to be known as the butterfly effect. Scientifically it is called sensitive dependence on initial condition.

Chaotic Systems are deterministic, non-linear, sensitive to initial conditions and bounded. Determinism in Chaos would mean that chaos does not arise from same lawless behaviour governed by chance but it is the stochastic behaviour occurring in deterministic systems or the lawless behaviour governed entirely by law. In linearity the output is directly or inversely proportional to the input but for a non-linear system, a small change in parameter can lead to sudden and dramatic changes in both qualitative and quantitative behaviour of the system. It will exhibit bifurcations and also have multiple basins of attractions. Boundedness in the chaotic dynamics would mean that even though chaotic systems have sensitive dependence to initial condition, the trajectories confine themselves to a bound region, which will have maximum and minimum parameter values beyond which they won’t wander, unless perturbed.

Chaotic Systems are not random, or periodic. Random events have equal probability of being in any state they can be in, from one moment to the next, independent of the previous state. In the midst of the apparent random behaviour in a chaotic system, there is an underlying structure revealed in the phase space that sometimes allows us to make prediction about its long-term trend and very short term behaviour. Edward Lorenz called this ‘orderly disorder’ in normal space. The state space or phase space is the mathematical space where each point represents a possible state of the system and helps us to study the geometric properties of the trajectories of the target system without knowing the exact solutions to the dynamical equations. Chaotic systems usually possess strange attractors, often with fractal dimensions. They are called strange attractors because, unlike fixed point, limit cycle and torus attractor; they don’t exhibit properties like seemingly random behaviour, sensitivity to initial conditions and mixing in finite time. Fractal image contain infinite detail when we zoom in. The advantage of fractal image is that the extraordinary detail present in fractal images can be



generated by very simple recipes. The attractors not only provide the standard statistical observation but also the 'directional' information which show how the system tends to change from its current state. This helps us make predictions about long term trend and short term behaviour. The properties of attractors are key sign posts at the junction where Chaos theory matures past mere metaphor and offers opportunities for practical applications.

When chaotic, complex non linear system is perturbed, the system loses its stability. The farther the system moves from equilibrium the more unstable it gets. The system then makes changes so as to regain the lost equilibrium. These changes are linear, gradual, segmental, predictable, moderate and incremental. But when this doesn't work, the continued perturbation will result in bifurcation, the diverging point, where the possibilities are many. One fork is chosen where the bifurcation occurs at a certain point which leads to second order change, characterized as turbulent, chaotic, non-linear, sudden, dramatic, transformative and unpredictable.

The importance of bifurcation lies in its universality. The ratio of the successive spacing tends to a number  $\delta = 4.6692$  as  $n \rightarrow \infty$ . What is again a pleasant surprise is that this number is universal for most single hump maps. There is yet another universal number associated with the pitchfork bifurcation. The spacing gets smaller and smaller and the ratio converges to a universal number  $\alpha = 2.5029$ , rather rapidly. Feigenbaum discovered that both  $\delta$  and  $\alpha$  are universal numbers for a period doubling cascades. Experiments conducted by scientists over the years confirmed Feigenbaum's discovery regarding the universality of bifurcation was not confined to turbulent fluids alone, but in all kinds of physical systems like electronic, optical and even biological.

Chaos was once considered unreliable, uncontrollable and therefore unusable, but scientists have turned the situation topsy-turvy by making chaos manageable, exploitable and even invaluable. Three basic methods of chaos control are- Ott Gregboi and Yorke (OGY) method, Pyrages method and Bradley's method. The Lyapunov exponent, discovered by the Russian mathematician Aleksandr M. Lyapunov, is useful for evaluating a model's sensitivity to perturbation which is indeed the root of unpredictability of chaos. Glenn E. James states the important consequences of chaos control in his book *Chaos Theory: The Essentials for Military Application*, "no model was needed, minimal computation was required, parameter adjustments were quite small, different periodic behaviours were stabilized for the same system and control was possible even with feedback based on imprecise measurements."

The ability to control chaos in the dynamical systems would mean that chaos theory can be put to work in a wide variety of fields. From laser technology to management leadership, chaos theory has proved to be of great value. Some of the applications of chaos theory in mechanical systems include laser, encryption, chaotic circuit, engine systems and also in space and satellite mission. In natural systems the applications can be seen in fisheries, ecology and in living systems it ranges from economy, management leadership and military systems. Thus chaos theory in action has set the ball rolling not only with an array of scientific discoveries from mechanical, living and complex systems, but also with far reaching philosophical implications.

**Check Your Progress II**

**Note:** Use the space provided for your answers.s.

- 1) Explain chaos theory as an alternative to Newtonian and quantum mechanics?

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- 2) What is the significance of strange attractor in chaotic dynamics?

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**1.5 PHILOSOPHY OF CHAOS THEORY**

Chaos theory has brought in a paradigm shift in the philosophical deliberations especially in philosophy of science where the benchmark had been the classical mechanistic paradigm. Differing methodologically and epistemologically from the classical science, chaos theory can be considered as an anti-establishment and holistic science.

Mark Stone analyses the term determinism and makes a fourfold division outlining the various senses in which the term determinism is used in order to understand better the determinism associated with chaos theory. The fourfold division are differential dynamics, unique evolution, value determinateness and total predictability. The determinism which is associated with chaos theory is differential dynamics. Differential dynamics would state that the system is deterministic if the way it changes in time can be specified by a set of differential equations. The complex behaviour in a dynamical system must arise not from the fact that system is an approximation of a huge number of complicated interacting subsystems but from the internal mathematical structure. The deterministic approach towards chaos theory takes to the study of apparently disorderedly behaviour is thus in contrast to a statistical approach which focuses on the evolution of average values at many places in the system, or averages over many systems.

The unique evolution holds that the complete instantaneous description of a deterministic system fixes the past and future with no alternatives. As this forms the core of Laplacian vision of determinism, John Earman calls this the ‘Laplacian determinism.’ The third level of determinism namely value determinateness would mean that for deterministic systems the accuracy of a state description is infinitely refinable, even though any given state description will contain some error or in

other words, physical quantities have exact values. Total predictability is the idea that the universe is predictable, in principle, by an all-powerful intelligence or computational scheme, given complete information of instantaneous conditions and the complete set of physical laws. Karl Popper calls this scientific determinism.

Determinism associated with chaos theory is used in the sense of differential dynamics and not in the sense of unique evolution, value determinateness or total predictability. The non-linearity in chaos theory, thwarts any attempt to fix the past and future, infinitely refine physical values and the dream of total predictability. The existence of chaotic systems leads us to the conclusion that the world is not totally predictable and any definition of determinism that includes total predictability in determinism is false.

The deterministic chaos theory with the presence of differential equation helps in the short term predictions. Even though long term predictions are ruled out in the non linear chaotic system, that doesn't mean it is completely indeterministic. The presence of strange attractor and the fractal nature of this strange attractor help in predicting the long term trend. Thus, chaos theory shows an alternative to the scientific world torn apart by rigorous determinism of the Newtonian mechanism and indeterminism of quantum mechanics.

While the popular literature hails chaos theory as a revolution, an analysis based on Thomas Kuhn's perspective in the philosophy of science gives a different picture. He states three characteristics of a revolution: "1) Rejection of once accepted scientific theory by the new one. 2) Shift in problems available for scientific scrutiny 3. Transformation of world within which scientific work is done." Apart from the first criteria, the second and third fits in with regard to chaos theory.

Focusing on the second criteria that call for interesting problems and successful solution to bestow the status of revolution, Katherine Hayles makes a convincing case. She brings before the study of physicist Bernardo Huberman that shows the chaotic model of eye moments in schizophrenics. Thus second characteristic fits the bill to classify chaos theory as a revolution. As chaos theory has brought about a change in scientific imagination, the third criteria too doesn't contradict chaos theory.

But chaos theory loses out in the first criteria which state the 'rejection of one time honoured scientific theory in favour of another incompatible with it.' Chaos theory so far has not displaced any theories. The chaotic behaviour occurs in Newtonian systems too. Chaos theory involves no fundamental theoretical change, while a Kuhnian revolution involves 'reconstruction of the field from new fundamentals, a reconstruction that changes some of the fields' most elementary theoretical generalizations.' So chaos theory cannot be called a revolution in the Kuhnian sense as it doesn't fit into the characteristic spelt out by Thomas Kuhn.

The focus of chaos theory is on qualitative understanding rather than quantitative information. Stephen H. Kellert defines chaos theory as "the qualitative study of unstable aperiodic behaviour in deterministic non linear dynamical systems." In qualitative understanding, rather than giving a fixed accurate solution, the emphasis is on giving general information and the great classification, which is



obtained by dwelling to questions pertaining to periodicity and stability of orbits, the symmetric and asymptotic properties of behaviour, and the structure of the set of solutions. Thus chaos theory with its focus on qualitative understanding is a welcome change from the rigorous deterministic quantitative analysis of Newtonian mechanics. Thus, chaos theory initiates a paradigm shift.

Chaos theory can best suit Thomas Kuhn's analysis of scientific paradigms. According to Kuhn "a paradigm is not a theory as such, but a frame work of thought – a conceptual scheme – around which the data of experiment and observation are organized." The basic paradigm shift occurs from time to time in the history of ideas. So with this it's not only the scientific theories that change but also the scientist's conception of the world too undergoes a change. In the Kuhnian scientific understanding emphasis is not on the structure of the scientific theory but "theories are cohesive systematic bodies of knowledge defined mainly by the roles they play in normal science practice within a dominant paradigm."

The strict model-target approach too undergoes a change with the arrival of chaos theory. Following a strategy called piecemeal improvement there are two basic approaches to model a target system, invoking faithful model assumption in philosophical literature. In the first basic approach the model is kept fixed and initial data is successively refined so as to converge it to target systems behaviour. The second approach involves keeping the initial data fixed and making successive refinement in the model. As the principle of linear superposition no longer holds in non-linear models, the two approaches mentioned above are met with serious difficulties when applied to non-linear models. This is because a small change in the data quality of a small refinement in the model can result in huge divergence in system behaviour unlike linear systems. This non-linear approach from the perspective of chaos theory shows that in any non-linear system there can be mismatch between models and targets. Going behind the strict model oriented approaches that bank on rigorous deterministic predictability, without leaving space for creative response has all possibility to backfire.

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## 1.6 RELEVANCE OF CHAOS THEORY

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Chaos theory with its popular appeal, far reaching implications and wide range of applications has brought in a 'new paradigm' both in scientific and social arena. Chaos theory has neither created nor destroyed any laws it but it has brought to the forefront the once ignored disorder, instability, diversity, disequilibrium, non-linearity and temporality concepts which once suffered under the excessive emphasis of non-linearity, determinism, predictability and reversibility. Taking strong exception to the mechanistic vision that emphasis on fixed framework structures, chaos theory focus on a context which is in a state of becoming in an open environment that receives and exchanges data or information and are sensitive to the changes happening around.

Chaos theory as a metaphor has proved its usefulness in diverse fields. But irrational application of this metaphor will seriously question the credibility of the theory. Ian Stewart would point to the dangerous use of the metaphor of chaos in the wrong places with absolutely wrong interpretations. Chaos theory is fast getting misinterpreted as that which glorifies randomness and there is every possibility that it might end up in false conception like the popular interpretation of Einstein's theory of relativity, where relativity was taken by the popular

interpreters as everything is relative, whereas Einstein meant relative to the speed of light. There are scientists argue that chaos is merely a subcomponent of non-linear dynamics which indeed is part of complex systems and so it can't be called a discipline in its own right.

The new generation in the globalized world is characterised by its reliance on randomness and chance and blind oppositions to dogmas of religions, structures, cultures, value systems and above all established systems. Chaos theory, by revealing the hidden order in the apparent disorder and by reconciling the extremes can turn out to be a powerful antidote in this regard.

Chaos theory also gives warning signals with the fact that the sensitive dependence on initial conditions which over the course of time result in bifurcations that can cause serious consequences. A recent example to such a phenomenon is the 2011 revolution in Egypt. It all began when a 26 year old woman wrote in the social networking site called facebook, 'People, I am going to Tahrir Square.' The message soon snowballed into a movement that ousted Egyptian president Hosni Mubarak. This would also mean that, even an uprising in a country or a war between two nations over the course of time may lead to a nuclear war between nations creating irrevocable harm to human beings.

Unveiling the mysteries of universe, sending out warning signals, providing breathing space in a world torn between extremes and seriously questioning the established theories, chaos theory has opened up innumerable possibilities of research. Chaos theory has heralded the dawn of a new era where the 'deterministic chaos' will have a greater say in the affairs of the world. Cutting across disciplines, chaos theory has the potential to permeate diverse streams and render intelligible the diverse phenomena, initiating a 'paradigm shift'.

**Check Your Progress III**

**Note:** Use the space provided for your answers.s.

1) According to Mark Stone, what kind of determinism is associated with chaos theory?

2) What is the relevance of chaos theory in the contemporary world?

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## 1.7 LET US SUM UP

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In this unit we have tried to give a general introduction to the dynamics of deterministic chaos theory. Beginning with the evolution of the concept of chaos in world history revealing the diverse way in which chaos and order was perceived. The scientific and philosophical implications of the extremes of Newtonian mechanics and quantum theory are discussed followed by chaos theory, which is presented as an alternative that brings together the opposing poles without making any rejection or addition of laws. Finally we conclude the unit by showing the significance of chaos theory as having the potential to initiate a paradigm shift in the state of affairs of the contemporary world.

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## 1.8 KEY WORDS

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<b>Attractor and Strange Attractor:</b>	Attractors are the set of points in the dynamical system that attracts the orbits and to which the system settles down after its transient dynamics die out. Strange Attractor is complicated, bounded orbits of trajectories of a chaotic system, exhibiting fractal nature, allowing us to make short term predictions and long term trends.
<b>Linearity and Non-linearity:</b>	In linearity the output is directly or inversely proportional to input. Non-linearity means that the output of a system is not directly or inversely proportional to its input. A small change in parameter can lead to sudden and dramatic changes in qualitative and quantitative behaviour of the system and give rise to complex behaviour called chaos.
<b>Bifurcation:</b>	The tendency of a non-linear system exhibiting the drastic appearance of a qualitatively different solution when one controlling parameter is varied.
<b>Fractals:</b>	A fractal is a rough geometric shape that can be split into parts, each of which is exactly, approximately or statistically a reduced-size copy of the whole, a property called self-similarity.

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## 1.9 FURTHER READINGS AND REFERENCES

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