UNIT 2 CONDITIONAL PROOF

Contents

2.0 OBJECTIVES

There are as many kinds of techniques as there are arguments. This unit is designed to introduce you to two new Rules which help you to compare the new technique or techniques with the earlier techniques. This comparative appraisal is possible only when you use both methods and consider suitable examples. This is the main objective of this unit. This unit enables you to understand that what is regarded as elementary valid argument forms or Rules of Inference and Rules of Replacement are also fundamental and hence indispensable. This is the second objective intended to be achieved.

2.1 INTRODUCTION

It is wrong to think that there is a single technique which is applicable on all occasions irrespective of the structure of arguments. On some occasions Rules of Inference and Replacement are useful and on some other occasions the technique known as Conditional Proof (C. P.) is useful. But there is no single method which is indispensable for all occasions. It is important to recognize this simple fact which prevents us from following a particular method blindly. When the advantage of new technique becomes clear, the significance of new Rule or Rules also becomes clear and it will be put into use. When we know that it is disadvantageous, we refrain from using that particular method. This means that a method is not to be used just because it has worked on earlier occasions. Instead, we try to apply a different method which is thought to be useful. The method of Conditional Proof (C.P.) is different in kind from the Rules of Inference or Replacements in one sense. There are a certain types of arguments, which can be tested with any of the Rules discussed in the previous chapters only with great difficulty or it may be practically impossible to test them at all. In all such cases C. P. comes to our rescue. Let us apply these methods and compare the results in order to understand the fact that the relevance of method is determined by the structure of argument.

1)

1	(A v B) => (C ^ D)	
2	(D v E) => F/ ∴ A => F	
3	¬ (A v B) v (C ^ D)	1, Impl.
4	(¬ A ^ ¬ B) v (C ^ D)	3, De. M.
5	(¬ A v C) ^ (¬ A v D) ^ (¬ B v C) ^ (¬ B v D)	4, Dist.
6	¬ (D v E) v F	2, Impl.
7	¬ A v D	5, Simp.
8	(¬D ^ ¬ E) v F	6, De. M.
9	(¬D v F) ^ (¬E v ¬ F)	8, Dist.
10	¬D v F	9, Simp.
11	A => D	7, Impl.
12	D => F	10, Impl.
13	∴ A => F `	11, 12, H. S.

It is obvious that we have used at present only Rules of Inference and Rules of Replacement. What is the position? It may be noted that from 3rd line to 13th line there are one hundred and twelve words and five Rules are used on eleven occasions. These figures become significant when we use new set of Rules and then calculate the length of proof construction which helps us to compare the length, number of words and others involved in these methods mentioned above. An important restriction is that the Rules of Inference are useful generally only when the arguments, have unconditional conclusions. So an argument, which has conditional conclusion, falls out of their purview on many occasions if simplicity is the yardstick. The most familiar example for conditional proposition is implicative proposition. Since implicative propositions have equivalent disjunctive and negation forms, they are also to be regarded as conditional propositions. Therefore if the conclusion is any one of these forms, then either the construction of proof may be very long and complex or may even be impossible. On such occasions we are likely to err. Therefore in order to insulate ourselves against highly probable errors we have to look for safer routes. C. P. is one such path. Again, C.P is not a system of proof, which is absolutely independent of nineteen Rules. Only, the number increases to twenty. Among them one Rule is called Rule of C. P. which is compulsorily used to test the validity when the conclusion is conditional. This Rule is unique in the sense that nowhere else it is used.

There are two Rules under this category. They are known as C. P., and the Strengthened Rule of C. P. This classification does not imply that the former is weak. The difference lies only in scope. We shall begin with the former first.

2.2 CONDITIONAL PROOF (C.P.)

Any deductive argument, whether valid or invalid, can be expressed in the form of a conditional proposition. What is more important to know is that the original argument is valid only when the corresponding conditional statement satisfies a condition known as 'tautology'. Otherwise, the argument is invalid. Consider this example:

2)

All A are B

All B are C / ∴ All A are C

Its corresponding conditional form is as follows:

"If all A are B and all A are C, then all A are C". (1)

Let the first premise be symbolized as P_1 and second as P_2. Conclusion is symbolized as C. Now (1) becomes:

(P1 ^ P2) => C (2)

(2) is said to be tautologous because its corresponding proposition form is tautologous. A proposition form is said to be tautologous when it has only true substitution. There are two conditions to be satisfied if C. P. should be used to show that the argument is valid.

1) Conclusion must be a conditional proposition.

2) It should be possible to deduce a conditional proposition from a conjunction of premises by a sequence of elementary valid arguments, which satisfy the relevant Rules of Inference. That is, all premises in C.P. should be supported by these Rules. The additional premise, which is a characteristic mark of C. P., is always the antecedent of the conclusion and the construction of proof always begins with antecedent of the conclusion as the premise. This premise itself is called C.P. An example of argument, which requires C.P., is given below.

P => (A => B) (3)

When P stands for the conjunction of premises, one of the Rules of Replacement, i.e., Exportation Rule permits us to rewrite (3) as:

(P ^ A) => B (4)

It is obvious that the conclusion of (4) is the consequent of the conclusion of (3). Since you start with an assumed premise, the proof is known as C.P. Here is the difference. All other premises are taken as true. The assumption should not really matter. Even if the assumed premise is false, it is possible to deduce a valid conclusion. If B can be validly drawn from P Λ A, then not only (4) is valid its corresponding original argument (3) also must be valid because (3) and (4) are logically equivalent.

Now consider the argument considered above.

3)

1.	(A v B) => (C ^ D)		
2.	(D v E) => F / ∴ A => F		
3.	A /∴F		C. P.
4.	A v B	3,	Add.
5.	C ^ D	1, 4,	M.P.
6.	D	5,	Simp.
7.	D v E	6,	Add.
8.	∴ F	2, 7,	M.P.

Now compare the lengths of 1 and 3. In 3 there are thirty five words whereas in 1 there are one hundred and twelve words and in 3 four Rules are used on six occasions whereas in 1 five Rules are used on eleven occasions. This comparison helps us to know the advantage of new technique.

A brief explanation of steps involved in proof construction in this method is necessary. You should start from assuming A which is the antecedent of the conclusion. Always the first line must have this structure in C. P. Slash against line 3 in, ∴ and (C.P) indicate that the method of C. P. is being used.

If there is only one condition in the conclusion, then C.P is used once. If there are two conditions in the consequent component of the conclusion, then C.P. is used twice. It means that the number of times the C. P. has to be used is equivalent to the number of conditions that appear in the consequent of the conclusion. Now it is plain that the complexity of argument increases with the increase in the number of conditions in the conclusion. The following example illustrates the procedure to be followed in such cases.

4)

1	A => (B => C)			
2	B => (C => D)	/∴ A => (B => D)		
3	A	/∴ B => D	(C.P.)	
4	B	/∴D	(C.P.)	
5	B => C		1, 3,	M.P.
6	C		5, 4,	M.P.
7	C => D		2, 4,	M.P.
8	∴D		7, 6,	M.P.

The student is advised to use the Rules of Inference or Replacement or both depending upon the need and compare the lengths of proof construction.

Consider an argument with a disjunctive conclusion and use both the methods without making any presumption to compare the lengths and complexity of proof construction. we shall begin with earlier method.

5)

1	A = > B	
2	C = > D	
3	(¬B v ¬D) ^ (A v B) /∴(¬A v ¬C)	
4	(A = > B) ^ (C = > D)	1, 2, Conj.
5	¬B v ¬D	3, Simp.
6	∴ ¬A v ¬C	4, 5, D. D.

A = > ¬C is equivalent to the original conclusion. Therefore if C. P. method has to be used, then A = > ¬C must replace ¬A v ¬C. Now you shall construct proof using C. P. method.

4	A	/ ∴ ¬C (C.P.)
5	¬B v ¬D	3, Simp.
6	B	1, 4, M. P.
7	¬D	5, 6, D.S.
8	∴ ¬C	2, 7, M.T.

In the former method thirty three words, three lines and three Rules are involved whereas in the latter thirty four words, five lines and four Rules are involved. This comparison illustrates the fact that just because the conclusion is conditional the method of C. P. is not necessarily preferable.

Now let us consider a more complex example with multiple variables and more number of premises. Again we will begin with the Rules of Inference.

1	H => (I => J)	`
2	K => (I => J)	
3	(¬ H ^ ¬ K) => (¬ L v ¬ M)	
4	(¬ L => ¬ N) ^ (¬ M => ¬ O)	
5	(P => N) ^ (Q => O)	
6	¬ (I => J)	/∴ ¬ P v ¬ Q
7	¬ H	1,6, M.T.
8	¬ K	2,6, M.T.
9	¬ H ^ ¬ K	7,8, Conj.
10	¬ L v ¬ M	3,9, M.P.
11	¬ N v ¬ O	4,10, C.D.
12	∴ ¬ P v ¬ Q	5,11, D.D.

In this method there are forty three words, six lines and five Rules. Now apply the method of C.P. Before we do so the conclusion must be transformed into implicatory form. Hence the conclusion is P => ¬Q.

6	¬ (I => J) / P => ¬ Q	
7	P / ∴ ¬ Q (C. P.)	
8	P => N	5, Simp.
9	N	8, 7, M. P.
10	¬ L => ¬ N	4, Simp.
11	L	10, 9, M.T.
12	¬H	1, 6, M.T.
13	¬K	2, 6, M.T.
14	¬H ^ ¬K	12, 13, Conj.
15	¬ L v ¬ M	3, 14, M.P.
16	L => ¬ M	14, Impl.
17	¬ M	16, 11, M.P.
18	¬ M => ¬ O	4, Simp.
19	¬ O	18, 17, M.P.
20	Q => O	5, Simp.
21	∴ ¬ Q	20, 19, M.T.

We know prima facie that this method is very long with ninety seven words, fifteen lines and five Rules and we know that with the exception of Conjunction other Rules recur again and again. Therefore C.P. must be used only when it is economical in terms of space and effort.

2.3 EXERCISES I

I. Some arguments are considered below which are tested using the method of C. P.

1)

1.	P => (Q => R)	/∴ (P => Q) => (P => R)	
2.	P => Q	/∴ P => R	(C.P.)
3.	P	/∴ R	(C.P.)
4.	(P ^ Q) => R	1,	Exp.
5.	∴R	4, 2,	M.P.

2)

1.	P => (Q => R) / ∴ Q => (P => R)		
2.	Q	/∴ P => R	(C.P.)
3.	P	/∴ R	(C.P.)
4.	Q => R	1, 3,	M.P.
5.	∴R	4, 2,	M.P.

3)

1.	P => Q	/ ∴ ¬ Q => ¬ P	
2.	¬ Q	/ ∴ ¬ P (C.P.)	
3.	∴¬P	2, 1,	M.T.

4)

1.	P => ¬ ¬ P		
2.	¬ ¬ P	/∴ P	(C.P.)
3.	∴ P	2,	D.N.

5)

1.	A => B / (B => C) => (A => C)		
2.	B => C	/∴ A => C	(C.P.)
3.	A	/∴C	(C.P.)
4.	B	1, 3,	M.P.
5.	∴C	2, 4,	M.P.

6)

1.	(A => B) ^ (A => C)	/∴ A => (B v C)	
2.	A	/∴B v C	(C.P.)
3.	A => B	1,	Simp.
4.	B	3, 2,	M.P.
5.	∴B v C	4,	Add.

7)

1.	(A => B) ^ (A => C)	/ ∴ A => (B ^ C)	
2.	A	/∴B ^ C	(C.P.)
3.	A => B	1, Simp.	
4.	B	3, 2,	M.P.
5.	A => C	1,Simp.	
6.	C	5, 2,	M.P.
7.	∴B ^ C	4, 6,	Conj.

8)

1.	A => B	/∴ (¬ A => B) => B		
2.	(¬ A => B)	/∴ B	(C.P.)	
3.	¬ A	/∴ B	(C.P.)	
4.	∴ B		2, 3,	M.P.

9)

1.	(A => B)	/∴ (A ^ C) => (B ^ C)		
2.	A ^ C	/∴ B ^ C		(C.P.)
3.	A		2,	Simp.
4.	B		1, 3,	M.P.
5.	C		2,	Simp.
6.	∴ B ^ C		4, 5,	Conj.

10)

1.	B => C	/∴ (A v B) => (C v A)		
2.	A v B	/∴ C v A	(C.P.)	
3.	¬ A => B		2,	Impl.
4.	¬ A => C		3, 1,	H.S.
5.	A v C		4,	Impl.
6.	∴ C v A		5,	Com.

11)

1.	(A v B) => C	/∴ [(C v D) => E] => (A => E)		
2.	(C v D) => E	/ ∴ A => E	(C.P.)	
3.	A	/∴ E	(C.P.)	
4.	A v B		3,	Add.
5.	C		1, 4,	M.P.
6.	C v D		2,	Add.
7.	∴ E		2, 6,	M. P.

12)

1.	(P ^ Q) => P			
2.	P	/ ∴ Q => P		
3.	Q	/∴ P	(C.P.)	
4.	P ^ Q		2, 3,	Conj.
5.	∴ P		4,	Simp.

[Note: You can apply M. P. to 1 and 4 to obtain the same result.]

13)

1.	P => Q	/∴ (¬ Q v R) => ¬ (R ^ P)		
2.	(R ^ P) => (Q ^ R)		1,	Trans.
3.	R ^ P	/∴ Q ^ R		(C.P.)
4.	P		3,	Simp.
5.	Q		1, 4,	M.P.
6.	R		3	Simp.
7.	∴ Q ^ R		5, 6,	Conj.

Check your progress I.

Note: Use the space provided for your answer.

1) Explain the significance of Conditional Proof.

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2) What is the advantage of C. P. method?

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2.4 THE STRENGTHENED RULE OF C. P.

In Conditional Proof method, the conclusion depends upon the antecedent of the conclusion. There is another method, which is called the Strengthened Rule of Conditional Proof. In this method, the construction of proof does not necessarily assume the antecedent of the conclusion. The structure of this method needs some elaboration. An assumption is made initially. There is no need to know the truth-status of the assumption because an assumption may be false, but the conclusion can still be true. Further, the assumption can be any component of any premise or conclusion. This method is called the Strengthened Rule because we enjoy more freedom in making assumption or assumptions which means that plurality of assumptions is allowed. It strengthens our repertoire of testing equipments. In this sense, this method is called the Strengthened Rule of C.P. Another feature of this method is the limit of assumption. The last step is always outside the limits of assumption. If there are two or more than two assumptions in an argument, then there will be a separate last step for each assumption. This last step can be regarded as the conclusion relative to that particular assumption. It shows that the last step is deduced with the help of assumption in conjunction with the previous steps in such a way that the Rules of Inference permit such conjunction. Before the conclusion is reached the function of assumption also ceases. Then it will have no role to play. Then, automatically, the assumption is said to have been discharged. When the Strengthened Rule of C. P. is used adjacent to the line of assumption, the word assumption is not mentioned unlike as in the case of C.P. Here the head of the bent arrow points to 'assumption'. In case of the Strengthened Rule of C.P., the conclusion is always a conditional statement which consists of statements from the sequence itself.

Thus the range of the application of condition is defined. In order to easily identify the range of its application, a slightly different method is used. An arrow is used to indicate what is assumed and with the help of the same arrow its range also is defined. The application of C.P. is restricted to the space covered by the arrows. All steps, which are outside this arrow, are also independent of the condition. While the head of the arrow makes the assumption, its terminus separates the lines, which depend upon the condition from the line, which does not depend. Since the conclusion does not depend upon its own antecedent, it has to depend upon the first assumption only. In this sense, it is a strengthened condition. In this case there is no reason to mention C.P. because the arrow helps us to identify the assumption. Consider this example:

1)

1 (A v B) =>{(C v D) =>E} /∴ A=>[(C ^ D) => E]		
2 A		
3 A v B	2,	Add.
4 (C v D)=>E	1, 3,	M.P.
5 (C ^ D)		
6 C	5,	Simp.
7 C v D	6,	Add.
8 E	4, 7,	M.P.
9 (C v D)=>E	5, 8,	C.P.
10 ∴ A=> [C ^ D) =>E]	2, 9,	C.P.

Rules mentioned on the RHS make it clear that all lines from 3 to 9 depend on A either directly or through lines which depend A. In lines 9 and 10 implication makes them C.P.

One advantage of C.P. in its strengthened form is that it has an extended application. It can be used in all those cases where conclusions are conditional but do not appear to be so. Using the strengthened Rule of C. P. let us solve some problems.

2)

1.	(E v F) = > G	
2.	H => (I ^ G) / ∴ (E => G) ^ (H ^ I)	
3.	E	
4.	E v F	3, Add
5.	G	4, 3, M. P.
6.	E => G	3, 5, C. P.
7.	H	
8.	I ^ G	2, 7, M. P.
9.	I	8, Simp.
10.	H => I	7, 9, C. P.
11.	∴ (E => G) ^ (H => I)	6, 10, Conj.

3)

1. Q v (R => S)
2. {R => (R ^ S)} => (T v U)
3. (T => Q) ^ (U => V) / ∴ Q v V

In this argument, in reality, the conclusion is conditional. We know that disjunction can be translated to implication form. When it so translated, the conclusion becomes

¬Q => V

Now let us construct proof for this argument.

4. ¬ Q	
5. R => S	1, 4, D. S.
6. R	
7. S	5, 6, M. P.
8. R ^ S	6, 7, Conj.
9. R => (R ^ S)	6, 8, C. P.
10. T v U	2, 9, M. P.
11. ∴Q v V	3, 10, C. D.

4) 1. (C v D) => (E => F)

2. {E => (E ^ F)} => G

3. G => {(¬H v ¬ ¬ H)} => (C ^ H)} / ∴ C ≡ G

4. C	
5. C v D	4, Add
6. E => F	1, 5, M. P.
7. E	
8. F	6, 7, M. P.
9. E ^ F	7, 8, Conj.
10. E => (E ^ F)	7, 9, C. P.
11. G	2, 10, M. P.
12. H	
13. ¬ ¬ H	12, D. N.
14. ¬ ¬ H v ¬ H	13, Add
15. ¬ H v ¬ ¬ H	14, Com.
16. G => (¬ H v ¬ ¬ H)	11, 15, C. P.
17. C ^ H	3, 16, M. P.
18. C	17, Simp.
19. C => G	18, 11, C. P.
20. G => C	11, 18, C. P.
21. (C => G) ^ (G => C)	19, 20, Conj.
22. ∴C ≡ G	21, Equiv.

5)

1 ¬ P∨ (Q => R)

¬ Q ∨ (¬ R ∨ D) / ∴P => (Q => D)

1.	¬ P ∨ (Q => R)	
2.	¬ Q ∨ (¬ R ∨ D)	/ ∴ P => (Q => D)
3.	P	/ ∴ (Q => D)
4.	Q	/∴ D
5.	¬ ¬ Q	D.N 4.
6.	¬ R ∨ D	D.S. 2 &5
7.	¬ ¬ P	D.N 3
8.	Q => R	D.S 1&7
9.	R	M.P. 8&4
10.	¬ ¬ R	D.N. 9
11.	D	D.S.6&10
12.	Q => D	C.P. 4-11
13.	∴P => (Q => D)	C.P. 3-12

6)

(P => ¬ Q) ∨ R

(R => S)

¬ S / ∴Q => ¬ P

1.	(P => ¬ Q) ∨ R	
2.	(R => S)	
3.	¬ S	/∴ Q => ¬ P
4.	Q	/∴ ¬ P
5.	¬ R	M.T 2&3.
6.	R ∨ (P => ¬ Q)	Com 1.
7.	(P => ¬ Q)	D.S 6&5
8.	¬ ¬ Q	D.N 4.
9.	¬ P	M.T. 7&8
10.	∴Q => ¬ P	C.P 4&9

7)

$\neg A \lor (B \Rightarrow C)$
$\neg B \lor (\neg C \lor D)$ $Q \Rightarrow \neg P$ / $\therefore$ $A \Rightarrow (B \Rightarrow D)$

1. $\neg A \lor (B \Rightarrow C)$
2. $\neg B \lor (\neg C \lor D)$ / $\therefore$ $A \Rightarrow (B \Rightarrow D)$
3. A /$\therefore$ $(B \Rightarrow D)$
4. B / $\therefore$ D
5. $\neg \neg B$ D.N 4.
6. $\neg C \lor D$ D.S. 2 &5
7. $\neg \neg A$ D.N 3
8. $B \Rightarrow C$ D.S 1&7
9. C M.P. 8&4
10. $\neg \neg C$ D.N. 9
11. D D.S.6&10
12. $B \Rightarrow D$ C.P. 4-11
13. $\therefore$ $\Rightarrow (B \Rightarrow D)$ C.P. 3-12

8)

$(\neg P \lor \neg M) \lor R$
$(\neg S \Rightarrow \neg R)$ /$\therefore$ $M \Rightarrow \neg P$

1. $(\neg P \lor \neg M) \lor R$
2. $(\neg S \Rightarrow \neg R)$
3. $\neg S$ /$\therefore$ $M \Rightarrow \neg P$
4. M / $\therefore$ $\neg P$
5. $\neg R$ M.P 2 & 3.
6. $R \lor (\neg P \lor \neg M$ Com 1.
7. $(\neg P \lor \neg M)$ D.S 6 & 5
8. $(\neg M \lor \neg P)$ Com 7
9. $\neg \neg M$ D.N. 4
10. $\neg P$ D.S. 8 & 9
11. $M \Rightarrow \neg P$ C.P.4 & 9

Check your progress II.

Note: Use the space provided for your answer.

1) Write a brief note on the salient aspects of the Rule of Strengthened Proof.

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2) What do you mean by 'discharging of assumption'?

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2.5 EXERCISES II

1) (A v B) => (C ^ D)
   (D v E) => F / ∴ (A => F)

2) [A => (B => C)]
   [B => (C => D)] / ∴ [A => (B => D)]

3) C => D / ∴ [(D => E) => (C => E)]

4) (N => P) ^ (B => S) / ∴ [(N ^ B) => (P ^ S)]

5) (E => F
   [E => (F => G)]
   [E => (G => H)] / ∴ (E => H)

6) [¬(P v Q) (R ^ S)]
   [¬(S v T) v U] / ∴ (P => U)

7) [P => (¬Q v R)]
   [Q => (¬S =>¬R)] / ∴ [P => (Q => S)]

8) ¬C v D / ∴ [(¬D v E) => (C => E)]

9) (M => Q) ^ (B => S) / ∴ [(M ^ B) => (Q ^ S)]

10) (p => q)
    [P => (Q => R)]
    [P => (R => S)]   / ∴ (P => S)

11) (¬A v B) => (C ^ D)
    (D v E) => ¬F   / ∴ (¬A => ¬F)

12) [A => (¬B => C)]
    [¬B => (C => D)]   / ∴ [A => (¬B => D)]

13) ¬X => ¬D   / ∴ [(¬D => E) => (¬X => E)]

14) (B => S) ^ (N => P)   / ∴ [(N ^ B) => (P ^ S)]

15) (X => F)
    [X => (F => G)]
    [X => (G => H)]   / ∴ (X => H)

16) [¬(V Q) (R ^ X)]
    [¬(X ^ T) v U]   / ∴ (V => U)

17) [P => (X R)]
    [X => (¬S => ¬R)]   / ∴ [¬P => (X => S)]

18) ¬Y v Z   / ∴ [(Z v E) => (Y => E)]

19) (B => S) ^ (M => Q)   /∴ [(M ^ B) => (Q ^ S)]

20) (P => M)
    [P => (M => R)]
    [P => (R => ¬S)]   / ∴ (P => ¬S)

2.6 LET US SUM UP

In this unit we have tried to understand the significance of Conditional Proof. Its advantages and limitations were assessed in comparison with Rules of Inference and Replacement. A distinction was made between C. P. and the Strengthened Rule of C. P. We have learnt that logicians allow us to assume or introduce any proposition and not necessarily the antecedent of a conditional as a conditional assumption. But all assumptions must be discharged by applying the Rule of conditional proof.

2.7 KEY WORDS

Discharging the assumption: Ending an assumption when its truth is no longer being assumed as a maneuver within the proof.

2.8 FURTHER READINGS AND REFERENCES

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