UNIT 3 PROOFS OF VALIDITY

Contents

3.0 OBJECTIVES

The main objective of this unit is to give a perspective of the technique of proving validity. Evidently, this is in continuation of what we learnt in the previous unit. In order to reinforce what we have already learnt, arguments in verbal form are considered in detail in this unit in addition to symbolic expressions. Application of the technique of proving validity to arguments in verbal form is the second objective of this unit. At the end of this unit we should be able to apply the technique of proving validity to arguments in natural language. Secondly, we must learn to compare and evaluate different methods of proof of validity in terms of merits and demerits

3.1 INTRODUCTION

There are several ways of testing arguments. Every method is unique in the sense that it has its own advantages and disadvantages. Disadvantage means only limitation. Simplicity, economy, etc. are only matter of degree. Even when we consider complex methods, they have certain advantages. One such advantage is the development of insight into the inner structure of statements which are often hidden. Second most important advantage is that our ability to reason is sharpened. In philosophical discourse these two factors play decisive role. Therefore it is imperative that we explore all possible and plausible avenues to test the validity of arguments. Against this background, we must understand that the proliferation of plurality of methods of testing is imperative.

One such method is natural deduction method. Natural deduction method is also known as derivation by substitution or formal proof of validity. Formal proof of validity is relevant while proving the validity of a given argument. This is also an example for direct proof wherein we derive a given conclusion from a set of premises by using such accepted rules, such as Rules of Inference and Equivalence or Replacement. Arguments expressed in natural language can be subjected to the formal proof of validity when translated to symbolic language. In this context we should consider what is known as *decision procedure*. Decision procedure means the way of testing an expression in order to know whether it is a tautology or not. We say that an expression is a tautology when it is true in all instances. If an expression is false in all instances, then it is said to be a contradiction. Lastly, if an expression is true in some instances and false in one or more than one instance, then it is said to be contingent. An expression is nothing but an argument or argument form expressed in the form of a single statement which is necessarily a compound statement. Consider this argument and corresponding expression to understand what we mean by an expression and also what tautology, contradiction and contingent mean in terms of truth-value.

p => q

p / ∴ q

The argument is in accepted form. This is translated to the form of an expression in this manner.

{(p => q) ^ p} => q

Now we will construct truth-table for the aforementioned expression in the following manner.

p	¬ p	q	¬ q	{(p => q)	∧	p}	=>	q
1	0	1	0	1	1	1	1	1
1	0	0	1	0	0	1	1	1
0	1	1	0	1	0	0	1	1
0	1	0	1	1	0	0	1	0

How do we know that this expression is tautological? This is true in all instances. The truth-value in the main column is the standard of determination. Consider this argument.

p => q

∴ p ^ ¬ q

When we put it in the form of an expression, it becomes

{(p => q) ^ (p ^ ¬ q)}

We can construct truth-table using the same method (which is left as an exercise for the student) to find out that this expression is a contradiction. Likewise, the argument

p => q

¬ p

∴ ¬ q

becomes {(p => q) ^ (¬ p ^ ¬ q)} in the form of an expression and the construction of truth-table shows that it is contingent because it is true in some instances only.

Why do we need an alternate method? We already know that if there are two variables, we will have four rows according to the formula 2^n where n is the number of variables. Suppose that there are six variables. Then the number of rows is $2^6 = 64$. Evidently, construction of truth-table in this case is cumbersome. Naturally, we do not attempt to prove that the given expression is tautological using truth-table method. Therefore we search for alternate methods which are shorter. Only trial-and-error method helps us to know which is shorter and which is not. Rules of quantification help in discovering one such short method.

One important aspect to be remembered is that all valid arguments are tautological. Therefore when such an argument is put in the form of an expression, it must be true in all instances.

3.2 QUANTIFICATION AND EQUIVALENCE RELATION

In the previous unit we examined conversion. The same discussion is now extended to cover another important relation. Obversion is the one to be examined now.

1) All players are well-paid.

∴ No players are nonwell-paid.

As usual, transform this argument to symbolic form.

1) (x) {Px => Wx}

2) ∴ (x) {Px => ¬ (¬ Wx)}

The negation within parentheses symbolizes the complement of predicate whereas the negation which precedes the parentheses symbolizes the quality of the conclusion. Now apply UI to the symbolic form.

3) Pa => Wa 1, UI

4) Pa => ¬ (¬ Wa) 3, DN

4 corresponds to the conclusion of 1. Applying UG we get

5) ∴ (x) {Px => ¬ (¬ Wx)}

Let us recall the method of obtaining contraposition. In the case of 'A' we convert obversion. In the language of modern logic it means that we have to commute double negation because double negation is another word for obversion. Now Commute 4.

6) (¬ Wa) => ¬ (Pa) 4, Com.

6 is partial contrapositive of 1. In natural language 6 means 'No nonwell-paid persons are players'. Next steps is self-explanatory.

7) (¬ Wa) => (¬ Pa)

7 is full contrapositive of 1 in virtue of the law of contradiction, the only survivor among opposition relations. If an element does not belong to a set, it ought to belong to its complement. In natural language it becomes 'All nonwell-paid persons are nonplayers'. The process of obtaining the conclusion stops at this

point since 'A' does not have conversion by limitation. Using the same technique, obversion and other relations for the remaining propositions can be determined by the student. It must be remembered that according to the stipulations laid down by modern logic, inversion, whether partial or full, is invalid since it is always particular whereas the premises (A or E) are universal.

Check Your Progress I

Note: Use the space provided for your answer.

1) Explain formal proof method.

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2) Examine the following arguments with the help of quantification rules.

   a) Some women are not sages.

      Therefore some non sages are not non women.

   b) Some scientists are mathematicians.

      Therefore some mathematicians are not nonscientists.

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3.3 RULES OF QUANTIFICATION AND NONSYLLOGISM

All arguments need not be syllogistic though they consist of two premises and a conclusion. Relational argument is one such example.

1) Bangalore is to the west of Chennai.

   Mangalore is to the west of Bangalore.

   ∴ Mangalore is to the west of Chennai.

Aristotelian system does not regard this class of arguments as syllogistic though this can be shown to be valid if the structure of propositions are modified, but it results in the distortion of the meaning of statements. If we try to retain the meaning, then it becomes impossible to demonstrate the validity or invalidity, as the case may be, of such argument.

Apart from relational arguments, there is another class of arguments which consists of more than three terms and propositions. Consider this argument.

1 2 3

Men are both stupid and dishonest.

4

Some men are irritable.

∴ Some dishonest persons are irritable.

Terms are numbered. So there is no confusion regarding the nonsyllogistic nature of arguments. However, the statements are misleading. If we regard a conjunctive proposition as one proposition, then in this argument there are three propositions. If we give priority to simple propositions then the first premise has two simple propositions. Then we will have four propositions. Therefore this type of argument is classified as nonsyllogistic. To test this kind of argument we do not require any additional rule. Proper symbolization of this class of argument is important. The symbolization is as follows:

1)	(x) [Mx => (Sx ^ Dx)]	
2)	(∃x) [Mx ^ Ix] / ∴ (∃x) (Ix ^ Sx)	
3)	[Ma ^ Ia]	2, E. I.
4)	Ma => (Sa ^ Da)	1, U. I.
5)	Ma	3, Simp.
6)	(Sa ^ Da)	4, 5, M. P.
7)	Sa	6, Simp.
8)	Ia	3, Simp.
9)	Ia Ë Sa	8, 7, Conj.
10)	(∃x) (Ix ^ Sx)	9, E.G.

The status of (1) calls for our attention. Had the first premise been regarded as a conjunctive proposition, then (1) ought to have been symbolized as

11) Sm ^ Dm

It is a well known fact that conjunction does not have any equivalent form. Therefore (1) is not equivalent to (11), which means that we have arrived at a proposition very different from the first premise.

Consider another statement, which has a very different structure.

Americans and Germans are pioneers in science.

This statement actually means that a pioneer in science may be an American or a German. Surely, it does not mean that a pioneer in science is both an American and a German. Hence when this innocuous statement is translated into logical language, it becomes a disjunctive proposition with exclusive ‘Or’. Nor is it a conjunctive proposition of the form

Americans are pioneers in science and Germans are pioneers in science.

This is so because a conjunctive proposition of this form means the same as saying that a pioneer in science is both an American and a German, which is absurd. Consider this argument:

Americans and Germans are scientists.

Some white men are Americans.

Therefore, some white men are scientists.

This argument is symbolized as follows:

1)	(x) [(Ax v Gx) => Sx]	
2)	(∃x) [Wx ^ Ax]	/ ∴ (∃x)[Wx ^ Sx]
3)	Wa ^ Aa	2, E.I.
4)	Aa	3, Simp.
5)	(Aa v Ga)	4, Add.
6)	(Aa v Ga) => Sa	1, U.I.
7)	Sa	6, 5, M.P.
8)	Wa	3, Simp.
9)	Wa ^ Sa	8, 7, Conj.
10)	(∃x)[Wx ^ Sx]	9, E.G.

In one particular sense, nonsyllogistic arguments are more significant than traditional syllogism for the simple reason that in any debate, whether based in science or politics syllogism is seldom used. Application of nonsyllogistic arguments is widespread and more useful. Therefore there is greater need to become familiar with nonsyllogistic arguments.

3.4 EXERCISES

I) Construct formal proofs of validity.

1) (x)[Qx => Rx]
(∃x) (Qx)
∴ (∃x) Rx

2) (x)[Sx => (Tx =>Ux)]
(x)[Ux => (Vx ^ Wx)]
∴ (x) [Sx => (Tx => (Vx ^ Wx)]

3) (x)[Dx => ¬ Ex]
(x)[Fx => Ex]
∴ (x) [Fx => ¬ Dx]

4) (∃x) [Jx ^ Kx]
(x) [Jx => Lx]
∴ (∃x) [Lx ^ Kx]

5) (x) [(Ix => Ax)]
(∃x) [(Px ^ Ix)]
∴ (∃x) [(Px ^ Ax)]

6) (x) [(Mx => Nx)]
(∃x) [(Mx ^ Ox)]
∴ (∃x)[(Nx ^ Ox)]

7) (x) [(Dx => Cx)]
(∃x)[(Ax ^ Dx)]
∴ (∃x)[(Ax ^ Cx)]

8) (x) [(Px => Qx)]
(∃x) [(Rx ^ Px)]
∴ (∃x) [(Rx ^ Qx)]

9) (x) [(Tx => Ux)]
¬ Ut
∴ ¬ Tt

10) (x) [(Dx => Cx)]
(x) [(Ex => ¬ Cx)]
∴ (x) [(Dx => ¬ Ex)]

11) (∃x) [(Hx => Ix) ^ (Jx => Kx)]
(x) [(Ix ^ Kx) => Lx]
¬ Lx
∴ (∃x) (¬ Hx v ¬ Jx)

12) (x) (¬ Qx => ¬ Px)
(∃x) (Px ^ Rx)
∴ (∃x) (Qx ^ Rx)

13) (x) (¬ Px => ¬ Qx)
¬ Px
∴ ¬ Qx

14) (x) (¬ Ax => ¬ Bx)
(x) (Ex => ¬ Ax)
∴ (x) (¬ Ex => ¬ Bx)

15) (∃x) [(¬ Ax => ¬ Bx) ^ (¬ Kx => ¬ Jx)]
(x) [¬ Lx => ¬ (Kx ^ Ax)]
¬ Lx
∴ (∃x) (¬ Bx v ¬ Jx)

16) (x) [(Qx => Px)]
(∃x) [(¬ Px ^ ¬ Rx)]
∴ (∃x) [(¬ Qx ^ ¬ Rx)]

Check Your Progress II

Note: a) Use the space provided for your answer.

Apply quantification rules to test the validity of the following syllogistic arguments which were explained in the previous unit.

1) All fish swim.

No swimming creatures are mammals.

Therefore no mammals are fish.

2) Some historians are Marxists.

All historians are women.

Therefore some women are Marxists.

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3) No birds are harmful.

All cheats are harmful.

Therefore no birds are cheats.

3.5 MULTIPLY GENERAL PROPOSITIONS

There are two types of general proposition; singly general and multiply general. If a general proposition has only one quantifier, then it is called singly general. Up till now, we considered only propositions of former kind. If a general proposition consists of two or more than two quantifiers, then such a proposition is called multiply general propositions. Consider, for example, this proposition:

"If all Indians play cricket, then there are at least some Asians who play cricket."

Its symbolization is as follows:

1) All Indians play cricket: $(x)\{Ix \Rightarrow Px\}$
2) There are at least some Asians who play cricket: $(\exists x)\{Ax \wedge Px\}$

Now the symbolization of the whole sentence is as follows:

$$(x)[\{Ix \Rightarrow Px\}] \Rightarrow (\exists x)\{Ax \wedge Px\}$$

Depending upon the complexity of the given statement quantifiers may occur any number of times.

3.6 THE STRENGTHENED RULE OF C.P. AND QUANTIFICATION

In an earlier unit, we learnt that assumption is different from C. P. and that assumption does not include the conclusion. It depends solely on the premise. A few examples will illustrate how an argument can be tested using these techniques.

1) 1) (x)[Cx => Dx]

2) (x)[Ex => ¬ Dx]

∴ (x)[Ex => ¬ Cx]

The argument is written in standard form;

1)	(x)[Cx => Dx]		
2)	(x)[Ex => ¬ Dx]	/ (x)[Ex => ¬ Cx]	
3)	Ey		
4)	Cy => Dy	1,	U.I.
5)	Ey => ¬ Dy	2,	U.I.
6)	¬Dy	4, 3,	M.P.
7)	¬Cy	4, 6,	M.T.
8)	Ey => ¬ Cy	3, 7,	C.P.
9)	(x)[Ex => ¬ Cx]	8,	U.G.

From (1) two aspects become clear. The limit of assumption ends, when CP is used. So it does not depend upon the assumption. Second, since we are making an assumption, in place of 'x' only 'y'; an arbitrary chosen symbol can be used. This explanation holds good whenever the strengthened rule of CP is used.

1)	1) (x)[Nx => Ox]		
2)	(x) [Px => ¬ Ox]	/ (x) (Nx ^ ¬Px) => Ox	
3)	Ny		
4)	Ny => Oy	1,	U.I.
5)	Py => ¬ Oy	2,	U.I.
6)	Oy	4, 3,	M.P.
7)	¬ Py	5, 6,	M.T.
8)	Ny ^ ¬ Py	3, 8,	Conj.
9)	(Ny ^ ¬ Px) => Oy	8, 6,	C.P.
10)	(x) (Nx ^ ¬ Px) => Ox	9,	U.G.

Check Your Progress III

Note: Use the space provided for your answer.

Using the method of derivation by substitution prove the following arguments.

1) (∃x) [(¬ Hx => Ix) ^ (Mx => Rx)]

(x) [(Ix ^ Rx) => Lx]

¬ Lx / ∴ (∃x) ¬ (¬ Hx ^ ¬ Mx)

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2) (∃x) [(¬ Ax => Bx) ^ (Cx => Dx)]

(x) [(Bx ^ Dx) => ¬Lx]

Lx / ∴ (∃x) ¬ (¬Ax ^ Cx)

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3.7 LET US SUM UP

The most important vehicle for a formal system is its component for 'derivation' or 'proof'. The derivation procedure that looked into this unit is called formal derivation because it relies on the valid argument forms, and not on the invalid arguments. On the whole, function of formal derivation is to infer the logical consequences from the premises. It uses the valid argument forms as the logical rules to determine which consequences can be correctly or validly drawn from the premises. The logical rules, being valid argument forms themselves, have a special quality. They are all truth-preserving.

3.8 KEY WORDS

Formal derivation : Deductive proof procedures that establish the validity of an argument by inferring the logical consequences from given premises by valid argument forms.

Nonsyllogism : It is a kind of mediate inference which consists of propositions with complex structure.

3.9 FURTHER READINGS AND REFERENCES

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