UNIT 3 UNCERTAINTY PRINCIPLE

Contents

3.0 OBJECTIVES

- To study the basics of the uncertainty principle.
- To see some of its implications, including philosophical implications.
- To have some basic ideas of the Copenhagen interpretation of quantum mechanics.

3.1 INTRODUCTION

This unit takes up one of the fundamental principles from quantum mechanics. It also studies the confusion and challenges created by it. Finally it studies some of the implications of this theory.

3.2 SIMPLE DEFINITION OF UNCERTAINTY PRINCIPLE

"The more precisely the position is determined, the less precisely the momentum is known." This is the simplest statement of the uncertainty principle in quantum mechanics. The position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision. There is a minimum for the product of the uncertainties of these two measurements. There is likewise a minimum for the product of the uncertainties of the energy and time. This is not a statement about the inaccuracy of measurement instruments, nor a reflection on the quality of experimental methods; it arises from the wave properties inherent in the quantum mechanical description of nature. Even with perfect instruments and technique, the uncertainty is inherent in the nature of things. Heisenberg formulated it in 1927 thus: "The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa." This is a succinct statement of the "uncertainty relation" between the position and the momentum (mass times velocity) of a subatomic particle, such as an electron. It asserts that the position and the velocity of an object cannot both be measured exactly, at the same time, EVEN IN THEORY. The very concepts of exact position and exact velocity together, in fact, have no meaning in nature.

Because of the scientific and philosophical implications of the seemingly harmless sounding uncertainty relations, physicists speak of an uncertainty principle, which is often called more descriptively the "principle of indeterminacy."

Check Your Progress I

Note: Use the space provided for your answer

1) Give a simple definition of the principle of uncertainty, explaining the various terms?

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2) Is it possible to measure the exact location of a particle, both practically and theoretically? Why?

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3.3 BEYOND STRONG OBJECTIVITY

Classical physics was caught completely off-guard with the discovery of the uncertainty principle. Ordinary experience provides no clue of this principle. It is easy to measure both the position and the velocity of, say, an automobile, because the uncertainties implied by this principle for ordinary objects are too small to be observed. The complete rule stipulates that the product of the uncertainties in position and velocity is equal to or greater than a tiny physical quantity, or constant (about 10^{-34} joule-second, the value of the quantity h (where h is Planck's constant). Only for the exceedingly small masses of atoms and subatomic particles does the product of the uncertainties become significant (Uorgaon 2011).

Any attempt to measure precisely the velocity of a subatomic particle, such as an electron, will knock it about in an unpredictable way, so that a simultaneous measurement of its position has no validity. This result has nothing to do with inadequacies in the measuring instruments, the technique, or the observer; it arises out of the intimate connection in nature between particles and waves in the realm of subatomic dimensions. Every particle has a wave associated with it;

each particle actually exhibits wavelike behavior. The particle is most likely to be found in those places where the undulations of the wave are greatest, or most intense. The more intense the undulations of the associated wave become, however, the more ill defined becomes the wavelength, which in turn determines the momentum of the particle. So a strictly localized wave has an indeterminate wavelength; its associated particle, while having a definite position, has no certain velocity. A particle wave having a well-defined wavelength, on the other hand, is spread out; the associated particle, while having a rather precise velocity, may be almost anywhere. A quite accurate measurement of one observable involves a relatively large uncertainty in the measurement of the other.

The uncertainty principle is alternatively expressed in terms of a particle's momentum and position. The momentum of a particle is equal to the product of its mass times its velocity. Thus, the product of the uncertainties in the momentum and the position of a particle equals h/(2) or more. The principle applies to other related (conjugate) pairs of observables, such as energy and time: the product of the uncertainty in an energy measurement and the uncertainty in the time interval during which the measurement is made also equals h/(2) or more. The same relation holds, for an unstable atom or nucleus, between the uncertainty in the quantity of energy radiated and the uncertainty in the lifetime of the unstable system as it makes a transition to a more stable state (Uorgaon 2011).

The uncertainty principle, developed by W. Heisenberg, is a statement of the effects of wave-particle duality on the properties of subatomic objects. Consider the concept of momentum in the wave-like microscopic world. The momentum of wave is given by its wavelength. A wave packet like a photon or electron is a composite of many waves. Therefore, it must be made of many momentums. But how can an object have many momentums? Of course, once a measurement of the particle is made, a single momentum is observed. But, like fuzzy position, momentum before the observation is intrinsically uncertain. This is what is known as the uncertainty principle, that certain quantities, such as position, energy and time, are unknown, except by probabilities. In its purest form, the uncertainty principle states that accurate knowledge of complementarity pairs is impossible. For example, you can measure the location of an electron, but not its momentum (energy) at the same time (Uorgaon 2011).

A characteristic feature of quantum physics is the principle of complementarity, which "implies the impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear." As a result, "evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects." This interpretation of the meaning of quantum physics, which implied an altered view of the meaning of physical explanation, gradually came to be accepted by the majority of physicists during the 1930's.

Mathematically we describe the uncertainty principle as the following, where 'x' is position and 'p' is momentum:

$$\Delta\chi\Delta\rho \geq \frac{\hbar}{2}$$

Here *h = planks constant (h) / ð*

This is perhaps the most famous equation next to $E=mc^2$ in physics. It basically says that the combination of the error in position times the error in momentum must always be greater than Planck's constant. So, you can measure the position of an electron to some accuracy, but then its momentum will be inside a very large range of values. Likewise, you can measure the momentum precisely, but then its position is unknown. Notice that this is not the measurement problem in another form, the combination of position, energy (momentum) and time are actually undefined for a quantum particle until a measurement is made (then the wave function collapses).

Also notice that the uncertainty principle is unimportant to macroscopic objects since Planck's constant, h, is so small (10^{-34}). For example, the uncertainty in position of a thrown baseball is 10^{-30} millimeters (Uorgaon 2011).

The depth of the uncertainty principle is realized when we ask the question; is our knowledge of reality unlimited? The answer is no, because the uncertainty principle states that there is a built-in uncertainty, indeterminacy, unpredictability to Nature. The field of quantum mechanics concerns the description of phenomenon on small scales where classical physics breaks down. The biggest difference between the classical and microscopic realm, is that the quantum world can be not be perceived directly, but rather through the use of instruments. And a key assumption to an quantum physics is that quantum mechanical principles must reduce to Newtonian principles at the macroscopic level (there is a continuity between quantum and Newtonian mechanics).

Quantum mechanics was capable of bringing order to the uncertainty of the microscopic world by treatment of the wave function with new mathematics. Key to this idea was the fact that relative probabilities of different possible states are still determined by laws. Thus, there is a difference between the role of chance in quantum mechanics and the unrestricted chaos of a lawless Universe. Every quantum particle is characterized by a wave function. In 1925 Erwin Schrodinger developed the differential equation which describes the evolution of those wave functions. By using Schrodinger equation, scientists can find the wave function which solves a particular problem in quantum mechanics. Unfortunately, it is usually impossible to find an exact solution to the equation, so certain assumptions are used in order to obtain an approximate answer for the particular problem (Uorgaon 2011).

The difference between quantum mechanics and Newtonian mechanics is the role of probability and statistics. While the uncertainty principle means that quantum objects have to be described by probability fields, this doesn't mean that the microscopic world fails to conform to deterministic laws. In fact it does. And measurement is an act by which the measurer and the measured interact to produce a result, although this is not simply the determination of a pre-existing property. The quantum description of reality is objective (weak form) in the sense that everyone armed with a quantum physics education can do the same experiments and come to the same conclusions. Strong objectivity, as in classical physics, requires that the picture of the world yielded by the sum total of all experimental results to be not just a picture or model, but identical with the objective world, something that exists outside of us and prior to any measurement we might have of it. Quantum physics does not have this characteristic due to its built-in indeterminacy.

For centuries, scientists have gotten used to the idea that something like strong objectivity is the foundation of knowledge. So much so that we have come to believe that it is an essential part of the scientific method and that without this most solid kind of objectivity science would be pointless and arbitrary. However, the Copenhagen interpretation of quantum physics (see below) denies that there is any such thing as a true and unambiguous reality at the bottom of everything. Reality is what you measure it to be, and no more. No matter how uncomfortable science is with this viewpoint, quantum physics is extremely accurate and is the foundation of modern physics (perhaps then an objective view of reality is not essential to the conduct of physics). And concepts, such as cause and effect, survive only as a consequence of the collective behavior of large quantum systems (Uorgaon 2011).

3.4 THE HISTORICAL ORIGIN OF UNCERTAINTY PRINCIPLE

The origins of uncertainty involve almost as much personality as they do physics. Heisenberg's route to uncertainty lies in a debate that began in early 1926 between Heisenberg and his closest colleagues on the one hand, who espoused the "matrix mechanics" form of quantum mechanics, and Erwin Schrödinger and his colleagues on the other, who espoused the new "wave mechanics." (Cassidy 2011)

Most physicists were slow to accept "matrix mechanics" because of its abstract nature and its unfamiliar mathematics. They gladly welcomed Schrödinger's alternative wave mechanics when it appeared in early 1926, since it entailed more familiar concepts and equations, and it seemed to do away with quantum jumps and discontinuities. French physicist Louis de Broglie had suggested that not only light but also matter might behave like a wave. Drawing on this idea, to which Einstein had lent his support, Schrödinger attributed the quantum energies of the electron orbits in the old quantum theory of the atom to the vibration frequencies of electron "matter waves" around the atom's nucleus. Just as a piano string has a fixed tone, so an electron-wave would have a fixed quantum of energy. This led to much easier calculations and more familiar visualizations of atomic events than did Heisenberg's matrix mechanics, where the energy was found in an abstruse calculation.

In May 1926 Schrödinger published a proof that matrix and wave mechanics gave equivalent results: mathematically they were the same theory. He also argued for the superiority of wave mechanics over matrix mechanics. This provoked an angry reaction, especially from Heisenberg, who insisted on the existence of discontinuous quantum jumps rather than a theory based on continuous waves. Heisenberg had just begun his job as Niels Bohr's assistant in Copenhagen when Schrödinger came to town in October 1926 to debate the alternative theories with Bohr. The intense debates in Copenhagen proved inconclusive. They showed only that neither interpretation of atomic events could be considered satisfactory. Both sides began searching for a satisfactory physical interpretation of the quantum mechanics equations in line with their own preferences (Cassidy 2011).

Studying the papers of fellow scientists, Dirac and Jordan, while in frequent correspondence with Wolfgang Pauli, Heisenberg discovered a problem in the way one could measure basic physical variables appearing in the equations. His analysis showed that uncertainties, or imprecisions, always turned up if one tried to measure the position and the momentum of a particle at the same time. (Similar uncertainties occurred when measuring the energy and the time variables of the particle simultaneously.) These uncertainties or imprecisions in the measurements were not the fault of the experimenter, said Heisenberg, they were inherent in quantum mechanics. Heisenberg presented his discovery and its consequences in a 14-page letter to Pauli in February 1927. The letter evolved into a published paper in which Heisenberg presented to the world for the first time what became known as the uncertainty principle (Cassidy, 2011).

3.5 SOME IMPLICATIONS OF UNCERTAINTY

Heisenberg realized that the uncertainty relations had profound implications. First, if we accept Heisenberg's argument that every concept has a meaning only in terms of the experiments used to measure it, we must agree that things that cannot be measured really have no meaning in physics. Thus, for instance, the path of a particle has no meaning beyond the precision with which it is observed. But a basic assumption of physics since Newton has been that a "real world" exists independently of us, regardless of whether or not we observe it. (This assumption did not go unchallenged, however, by some philosophers.) Heisenberg now argued that such concepts as orbits of electrons do not exist in nature unless and until we observe them (Jammer 1974). There were also far-reaching implications for the concept of causality and the determinacy of past and future events. These are discussed on the page about the origins of uncertainty. Because the uncertainty relations are more than just mathematical relations, but have profound scientific and philosophical implications, physicists sometimes speak of the "uncertainty principle."

In the sharp formulation of the law of causality, Heisenberg in 1927 asserted: "If we know the present exactly, we can calculate the future -it is not the conclusion that is wrong but the premise." This implies that we can never know the present reality exactly. Heisenberg also drew profound implications for the concept of causality, or the determinacy of future events. Schrödinger had earlier attempted to offer an interpretation of his formalism in which the electron waves represent the density of charge of the electron in the orbit around the nucleus. Max Born, however, showed that the "wave function" of Schrödinger's equation does not represent the density of charge or matter. It describes only the probability of finding the electron at a certain point. In other words, quantum mechanics cannot give exact results, but only the probabilities for the occurrence of a variety of possible results. (Cassidy, 2011a)

Heisenberg took this one step further: he challenged the notion of simple causality in nature, that every determinate cause in nature is followed by the resulting effect. Translated into "classical physics," this had meant that the future motion of a particle could be exactly predicted, or "determined," from a knowledge of its present position and momentum and all of the forces acting upon it. The uncertainty principle denies this, Heisenberg declared, because one cannot know the precise position and momentum of a particle at a given instant, so its future cannot be determined. One cannot calculate the precise future motion of a particle, but only a range of possibilities for the future motion of the particle. (However, the probabilities of each motion, and the distribution of many particles following these motions, could be calculated exactly from Schrödinger's wave equation.)

Although Einstein and others objected to Heisenberg's and Bohr's views, even Einstein had to admit that they are indeed a logical consequence of quantum mechanics. For Einstein, this showed that quantum mechanics is "incomplete." Research has continued to the present on these and proposed alternative interpretations of quantum mechanics. One should note that Heisenberg's uncertainty principle does not say "everything is uncertain." Rather, it tells us very exactly where the limits of uncertainty lie when we make measurements of sub-atomic events. Heisenberg's uncertainty principle constituted an essential component of the broader interpretation of quantum mechanics known as the Copenhagen Interpretation (Cassidy, 2011).

Check Your Progress II

Note: Use the space provided for your answers.

1) How does Heisenberg relate the principle of uncertainty to the principle of causality?

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2) Where lies the basic difference between classical and quantum mechanics?

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3.6 TRIUMPH OF COPENHAGEN INTERPRETATION

The Copenhagen interpretation is a commonly taught interpretation of quantum mechanics. Classical physics draws a distinction between particles and energy, holding that only the latter exhibit waveform characteristics, whereas quantum mechanics is based on the fact that matter has both wave and particle aspects and postulates that the state of every subatomic particle can be described by a wave-function—a mathematical representation used to calculate the probability that the particle, if measured, will be in a given location or state of motion (Cassidy, 2011).

The Copenhagen interpretation of quantum mechancis is an attempt to explain the results of the experiments and their mathematical formulations, as formulated by Bohr, Werner Heisenberg and others in the years 1924–27. They theorised a new world of energy quanta, entities which fit neither the classical idea of particles nor the classical idea of waves. They thereby stepped beyond the world of empirical experiments and pragmatic predictions of such phenomena as the frequencies of light emitted under various conditions. According to their interpretation, the act of measurement causes the calculated set of probabilities to "collapse" to the value defined by the measurement. This feature of the mathematics is known as wave-function collapse. The Copenhagen interpretation is, in form, a composite of those statements which can be legitimately made in natural language to complement the statements and predictions made in the language of instrument readings and mathematical operations (Cassidy, 2011). Essentially, it attempts to answer the question, "What do these amazing experimental results really mean?" The concept that quantum mechanics does not yield an objective description of microscopic reality but deals only with probabilities, and that measurement plays an ineradicable role, is the most significant characteristic of the Copenhagen interpretation.

Because it consists of the views developed by a number of scientists and philosophers during the second quarter of the 20th Century, there is no definitive statement of the Copenhagen Interpretation. Thus, various ideas have been associated with it; Some of its key notions are as follows (CI 2011):

1) A system is completely described by a wave function ø, representing an observer's subjective knowledge of the system. (Heisenberg)

2) The description of nature is essentially probabilistic, with the probability of an event related to the square of the amplitude of the wave function related to it. (The Born rule, after Max Born)

3) It is not possible to know the value of all the properties of the system at the same time; those properties that are not known with precision must be described by probabilities. (Heisenberg's uncertainty principle)

4) Matter exhibits a wave-particle duality. An experiment can show the particle-like properties of matter, or the wave-like properties; in some experiments both of these complementary viewpoints must be invoked to explain the results, according to the complementarity principle of Niels Bohr.

5) Measuring devices are essentially classical devices, and measure only classical properties such as position and momentum.

6) The quantum mechanical description of large systems will closely approximate the classical description. (The correspondence principle of Bohr and Heisenberg.)

We regard quantum mechanics as a complete theory for which the fundamental physical and mathematical hypotheses are no longer susceptible of modification. (Heisenberg and Max Born, paper delivered to Solvay Congress of 1927)

3.7 DIFFICULTIES AND CHALLENGES

Not everyone agreed with the new interpretation, or with Born and Heisenberg's statement about future work. Einstein and Schrödinger were among the most notable dissenters. Until the ends of their lives they never fully accepted the Copenhagen doctrine. Einstein was dissatisfied with the reliance upon probabilities. But even more fundamentally, he believed that nature exists independently of the experimenter, and the motions of particles are precisely determined. It is the job of the physicist to uncover the laws of nature that govern these motions, which, in the end, will not require statistical theories. The fact that quantum mechanics did seem consistent only with statistical results and could not fully describe every motion was for Einstein an indication that quantum mechanics was still incomplete. Alternative interpretations have since been proposed and are now under serious consideration. The objections of Einstein and others notwithstanding, Bohr, Heisenberg and their colleagues managed to ensure the acceptance of their interpretation by the majority of physicists at that time. They did this both by presenting the new interpretation on lecture trips around the world and by demonstrating that it worked. The successes of the theory naturally attracted many of the best students to institutes such as Heisenberg's, some coming from as far away as America, India, and Japan. These bright students, nurtured by the Copenhagen doctrine and educated into the new quantum mechanics, formed a new and dominant generation of physicists. Those in Germany and Central Europe carried the new ideas with them as they dispersed around the world during the 1930s and 1940s in the wake of Hitler's rise to power in Germany (Cassidy 2011).

Albert Einstein was not happy with the uncertainty principle, and he challenged Niels Bohr and Werner Heisenberg with a famous thought experiment we fill a box with a radioactive material which randomly emits radiation. The box has a shutter, which is opened and immediately thereafter shut by a clock at a precise time, thereby allowing some radiation to escape. So the time is already known with precision. We still want to measure the conjugate variable energy precisely. Einstein proposed doing this by weighing the box before and after. The equivalence between mass and energy from special relativity will allow you to determine precisely how much energy was left in the box. Bohr countered as follows: should energy leave, then the now lighter box will rise slightly on the scale. That changes the position of the clock. Thus the clock deviates from our stationary reference frame, and again by special relativity, its measurement of time will be different from ours, leading to some unavoidable margin of error. In fact, a detailed analysis shows that the imprecision is correctly given by Heisenberg's relation.

Within the widely but not universally accepted Copenhagen interpretation of quantum mechanics, the uncertainty principle is taken to mean that on an elementary level, the physical universe does not exist in a deterministic form—but rather as a collection of probabilities, or potentials (Cassidy 2011). For example, the pattern (probability distribution) produced by millions of photons passing through a diffraction slit can be calculated using quantum mechanics, but the exact path of each photon cannot be predicted by any known method. The Copenhagen interpretation holds that it cannot be predicted by any method.

It is this interpretation that Einstein was questioning when he said "I cannot believe that God would choose to play dice with the universe." Bohr, who was one of the authors of the Copenhagen interpretation responded, "Einstein, don't tell God what to do." Einstein was convinced that this interpretation was in error. His reasoning was that all previously known probability distributions arose from deterministic events. The distribution of a flipped coin or a rolled dice can be described with a probability distribution (50% heads, 50% tails). But this does *not* mean that their physical motions are unpredictable. Ordinary mechanics can be used to calculate exactly how each coin will land, if the forces acting on it are known. And the heads/tails distribution will still line up with the probability distribution (given random initial forces). Einstein did not believe that the theory of quantum mechanics was complete. But Heisenberg and Max Born asserted at the famous Solvay Congress of 1927: "We regard quantum mechanics as a complete theory for which the fundamental physical and mathematical hypotheses are no longer susceptible of modification." That is the basic difference between them.

3.8 PHILOSOPHICAL IMPLICATIONS OF UNCERTAINTY PRINCIPLE

The Uncertainty Principle is often presented as a manifestation of the fact that the act of measurement inevitably perturbs the state that is being measured. Thus, the smaller the particle being observed, the shorter the wavelength of light needed to observe it, and hence the larger the energy of this light and the larger the perturbation it administers to the particle in the process of measurement. This interpretation, while helpful for visualization, has its limitations. It implies that the particle being observed does have a precise position and a precise momentum which we are unable to ascertain because of the clumsiness of the measurement process. However, more correctly, we should view the Uncertainty Principle as telling us that the concepts of position and momentum cannot coexist without some ambiguity. There is no precise state of momentum and position independent of the act of measurement, as naïve realist philosophers had assumed. In large, everyday situations this quantum mechanical uncertainty is insignificant for all practical purposes. In the sub-atomic world it is routinely confirmed by experiment and plays a fundamental role in the stability of matter. Note that if we take the limit in which the quantum aspect of the world is neglected (so Planck's constant, h, is set to zero), then the Heisenberg Uncertainty would disappear and we would expect to be able to measure the position and momentum of any object with perfect precision using perfect instruments (of course in practice this is never possible) (Barrow 2006).

The Uncertainty Principle has had a major effect upon the philosophy of science and belief in determinism. It means that it is impossible to determine the present state of the world (or any small part of it) with perfect precision. Even though we may be in possession of the mathematical laws that predict the future from the present with complete accuracy we would not be able to use them to predict the future. The Uncertainty Principle introduces an irreducible indeterminacy, or graininess, in the state of the world below a particular level of observational scrutiny. It is believed that this inevitable level of graininess in the state of matter in the universe during the first moments of its history led to the production of irregularities that eventually evolved into galaxies (Barrow 2006). Experiments are underway in space to test the detailed predictions about the variations left over in the temperature of the universe that such a theory makes. Of the other pairs of physical quantities that Heisenberg showed cannot be measured simultaneously with arbitrarily high precision, the most frequently discussed pair is energy and time. Strictly, this pair is not a true indeterminate pair like position and momentum because time is not an observable in the way that energy, position, and momentum are in quantum mechanics. By using a time defined externally to the system being observed (rather than intrinsically by it), it would be possible to beat the requirement that the product of the uncertainty in energy times the uncertainty in time be always greater than Planck's constant divided by 4 ð (Barrow 2006).

The physicist Niels Bohr (1885–1962) called quantities, like position and momentum, whose simultaneous measurement accuracy was limited by an uncertainty principle *complementary pairs*. The limitation on simultaneous knowledge of their values is called *complementarity.* Bohr believed that the principle of complementarity had far wider applicability than as a rigorous deduction in quantum mechanics. This approach has also been adopted in some contemporary religious apologetics, notably by Donald Mackay and Charles Coulson. There has also been an interest in using quantum uncertainty, and the breakdown of rigid determinism that it ensures, to defend the concept of free will and to provide a channel for divine action in the world in the face of unbreakable laws of nature. The Uncertainty Principle also changes our conception of the vacuum. Quantum uncertainty does not allow us to say that a volume of space is empty or contains nothing. Such a statement has no operational meaning. The quantum vacuum is therefore defined differently, as the lowest energy state available to the system locally. This may not characterize the vacuum uniquely and usually a physical system will have more than one possible vacuum state. Under external changes it may be possible to change from one to another. It is therefore important to distinguish between the non-scientific term, "nothing" and the quantum mechanical conception of "nothing" when discussing creation out of nothing in modern cosmology (Barrow 2006).

Check Your Progress III

Note: Use the space provided for your answers.

1) Why did Einstein object to some of the elements of quantum mechanics?

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2) What are some of the philosophical implications of uncertainty principle?

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3.9 LET US SUM UP

In this unit we tried to see the uncertainty principle, its difficulties and implications. It has changed the way we normally looked at the nature of particle, velocity, etc., and so opened a totally new way of understanding reality.

3.10 KEYWORDS

Decoherence : quantum decoherence (is how quantum systems interact with their environments to exhibit probabilistically additive behavior. Quantum decoherence gives the appearance of wave function collapse (the reduction of the physical possibilities into a single possibility as seen by an observer) and justifies the framework and intuition of classical physics as an acceptable approximation (See also the previous unit keywords).

Matrix mechanics : Matrix mechanics is a formulation of quantum mechanics created by Werner Heisenberg, Max Born, and Pascual Jordan in 1925. Here physical quantities are represented by matrices and matrix algebra is used to predict the outcome of physical measurements.

Momentum : he quantity of motion of a moving body, measured as a product of its mass and velocity.

Wave function collapse : The reduction of the physical possibilities in quantum mechancis into a single possibility as seen by an observer.

Wave mechanics : A method of analysis of the behavior of atomic phenomena with particles represented by wave equations. It is based on Schrodinger's equation; atomic events are explained as interactions between particle waves

3.11 FURTHER READINGS AND REFERENCES

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