UNIT 1 HISTORY AND UTILITY OF SYMBOLIC LOGIC

Contents

1.0 OBJECTIVES

In this unit, an attempt is made to present a history of symbolic logic. You will be quick enough:

to notice that the moment you enter symbolic logic, you are confronted with mathematics as well.

to learn that development of logic and mathematics are inseparably related.

to know that logic and mathematics are two components of one enterprise.

to be familiar with conceptual developments with a brief description of what they are.

to set your priorities right, to identify the elements of logic in mathematical discussions.

1.1 INTRODUCTION

History of logic can safely be divided into three phases; ancient logic, medieval logic and modern logic. It is necessary to bear in mind that one is not just replacement for the other and that elements of later phase can be discerned in the earlier phase. Therefore development is significantly in terms of correction and improvements, but not total rejection. Therefore it is absolutely necessary to admit that the limitations of ancient and medieval systems of logic paved way for the rise of symbolic logic and its value in addition to pioneering work by some mathematicians.

1.2 EARLIEST CONTRIBUTION TO LOGIC

The greatest contribution of Aristotle to logic, undoubtedly, is his theory of syllogism in which the theory of classes and class relation is implicit. Another significant contribution of Aristotle is his notion of variables. Classes themselves are variables in the sense that in any proposition subject and predicate terms are not only variables but also they are the symbols of classes. Finally, the class relation, which is explicit in his four-fold analysis of categorical proposition, is understood as inclusion or exclusion - total or partial.

Theophrastus, a student of Aristotle, developed a theory of pure hypothetical syllogism. A hypothetical syllogism is said to be pure if all the three propositions are hypothetical propositions. Theophrastus showed that pure hypothetical inference (an inference which consists of only hypothetical propositions) could be constructed which corresponds to inference consisting of only categorical propositions (which Aristotle called syllogism).

A school of thought flourished during Socrates' period known as Megarians. The first generation of Megarians flourished in the 5th century B.C. onwards. In the 4th century B.C. one Megarian by name Eubulides of Miletus introduced now famous paradox – the paradox of liar. The last Greek logician, (who is also 'lost' because none of his writings is extant), who is worthy of consideration is Chrysippus of whom it is said that even gods would have used the logic of Chrysippus if they had to use logic.

Peter Abelard, who lived in the 11th Century, is generally regarded as the first important logician of medieval age followed by William of Sherwood and Peter of Spain in the 13th Century. They continued the work of Aristotle on categorical proposition and syllogism and other related topics. In reality, no vacuum was created in medieval age and hence there was continuity from Aristotelian logic to modern logic though no original contribution came from any logician. The most notable contribution to logic in this period consists in the developments, which took place in several important fields like analysis of syntax and semantics of natural language, theories of reference and application, philosophy of language, etc., the relevance of which was, perhaps realized only very recently. These are precisely some of the topics of modern logic. William of Sherwood and Peter of Spain were the first to make the distinction between descriptive and non-descriptive functions of language. They reserved the word 'term' only for descriptive function. Accordingly, only subject and predicate qualify for descriptive function and hence in categorical proposition we can find only two terms. These were called categorematic whereas other components of a sentence like 'all, some, and no', etc. were called syncategorematic. The former are terms whereas the latter are only words. Hence, terms were regarded as special words. It is in this context that the medieval logicians made semantic distinction of language levels. Categorematic term was divided into two classes, terms of first intension and terms of second intension. First class stands for things whereas the second stands not for a thing but for a language sign. In a limited sense, and at elementary level, it can be said that subject represents first class and predicate represents second class. Another field covered by medieval logicians was that of quantification which is of great importance in modern logic. again, this is another important topic of modern logic.

1.3 LIMITATIONS OF ARISTOTELIAN LOGIC

The very fact that Aristotle constructed an extraordinarily sound system of logic became its nemesis. Just as Newtonian Physics was held as infallible for a little more than two hundred years, Aristotle was held on similar lines for nearly two thousand years. However, neither of them anticipated this treatment to their systems. While this is one reason for the delayed beginning of modern logic, second and the most important reason is that mathematics also had not yet been developed.

The emphasis is not upon the defects of the system, but on the limitations because, ironically, the defects did not hinder the growth of logic. It may also be true that had the defects been detected very early, situation would not have been much different because time was not ripe for take-off of symbolic logic. One serious limitation of Aristotelian system is its narrow conception of proposition. He restricted it to subject-predicate form. Though class-relation is implicit in this theory of syllogism, Aristotle ignored it. There is little wonder that Aristotle did not think of any other relations. Consider these two examples:

All men are mortal.

All mortal beings are imperfect

∴ All men are imperfect.

Bangalore is to the east of Mangalore.

Madras is to the east of Bangalore.

Madras is to the east of Mangalore.

Both these arguments are valid in virtue of transitive relation. Aristotle recognized only the first example as valid and what is surprising is that he considered only the first type as an argument. The result is that most of the mathematical statements ceased to be propositions in his analysis. His narrow outlook eliminated any possibility of logic and mathematics interacting. Consequently, considerable types of arguments with much complicated structure fall outside the limits of Aristotelian logic and hence remain unexamined. Medieval logic, in spite of remarkable contributions to logic, did not take logic a step ahead because whatever research was done was only an in-house work, i.e., work within the system. What was required was transition from one system to another.

In what sense modern logic makes progress over Aristotelian logic? It is very important to answer this question. Modern logic did not supersede Aristotelian logic in the sense in which an amendment to constitution results in one act replacing another. Modern logic neither superseded nor succeeded Aristotelian logic. It only extended the boundaries of the system. Existing rules remained not only acceptable but also were augmented by new set of rules. Later we will learn that among nine rules of inference, six are from Aristotelian logic. And simple conversion and observation were retained but given 'extended meaning' in terms of the rules of commutation and double negation respectively. Meaning was extended because logic and mathematics mutually made inroads into one another's territory. In a similar fashion, the use of variables also underwent a change. While Aristotle used variables only to represent terms, modern logic extended the use to propositions as well. This inclusion had far reaching consequences. Lastly, quantification, which was introduced during medieval age, was further improvised.

The foregoing discussion should make one point clear. The tools used to test arguments or to construct arguments by Aristotelian system are insufficient. Modern logic further augmented the tools not only in number but also in variety. It should be remembered that the sky is the limit to improve and add.

Before we enter the modern era, one interesting question must be considered. How should we explain the relation between logic and mathematics? Two philosophers have differently described this relation. Raymond Wilder says that for Peano and his followers 'logic was the servant of mathematics'. Wilder put it in a more respectable and acceptable form, in connection with Frege's philosophy of mathematics, 'dependence (of mathematics) on logic… was more like that of child to parent than servant to master. Basson and O'connor have echoed more or less similar views while relating classical logic to modern logic. It is like embryo related to adult.

1.4 HISTORY AND UTILITY OF SYMBOLIC LOGIC

At this stage, two aspects must be made clear. Modern logic is also called symbolic logic because symbols replaced words to a great extent. Second, symbolic logic and mathematics do not stand sundered; so much so, modern logic is also called mathematical logic, which A.N. Prior terms 'loosely called.' However, Prior's remark has to be taken with a pinch of salt. Very soon, we realize that almost all people, whose names are associated with symbolic logic, are basically mathematicians. And at some stage it becomes extremely difficult to separate logic from mathematics and, if attempted, it will be an exercise in futility. However, a definite limitation must be considered. When we talk of mathematics we talk of pure mathematics only. So when we deal with history of a symbolic logic we deal with the history of pure mathematics.

Where exactly does symbolic logic score over classical logic? Language is, generally, ambiguous. It is so for two reasons. In the first place, a significant number of words are equivocal and secondly, many times the construction of sentences and their juxtaposition are misleading so much so they convey meaning very different from what the speaker or author intends. Replacement of words by symbols and application of logical syntax different from grammatical syntax completely eliminates ambiguity. The meaning of logical syntax becomes clear in due course when sentences are represented by symbols. It is possible to test the validity of arguments only when the statements are unambiguous. Further, use of symbols saves time and effort required to test the validity of arguments.

1.5 THE RISE OF SYMBOLIC LOGIC

Generally, bibliography of symbolic logic compiled by Alonzo Church is reckoned as authentic to determine the beginning of symbolic logic. In the year 1666, Leibniz published (or wrote) a thesis on a 'Theory of Combinations titled '*Dissertatio de Arte Combinatoria*.' It is said that the beginning of symbolic logic coincides with this work. If so, Chrysippus has to be heralded as the forerunner of symbolic logic because according to records long before Leibniz he showed some interest in Combinations. So he must have done some work on Combinations, which was, further, followed up by some logicians in the thirteenth century. In brief, let us describe the subject-matter of Combinations. Leibniz was more concerned with such issues as semantic interpretations of logical formulas. One example may clarify semantic consideration or considerations which engaged Leibniz. What does the statement 'All men are mortal' mean? Does it mean that every member of the class of men is also a member of the class of mortal beings? Or does it mean that every man possesses the attribute of being a mortal? Or does it mean that the attribute of 'being man' includes the 'attribute of being mortal'. In other words, the focus of this consideration is on the choice between extensional approach and intentional approach. Class-membership issue is extensional whereas attribute-inclusion or attribute – exclusion is intentional.

Another notable contribution of Leibniz was his work on logical algebra or logical calculus, which consists of several experimental sorts of studies. Some laws, which are features of his study, are laws of identity and explicit statement of transitive relation, which made Aristotelian syllogism significant. Consider these two rules:

a b is a

ab is b

These rules become intelligible when we substitute terms for a & b. suppose that a = intelligent; b = man

1) Intelligent man is a man

2) Intelligent man is intelligent

Likewise consider another rule:

if a is b and a is c then a is bc.

Again substitute of, b and c, a = Indian, b = Asian, c = Hindu. Then 3 becomes

If Indian is an Asian and Indian is a Hindu, then Indian is an Asian Hindu.

An important requirement of logical algebra is that substitution must be possible; this particular relation was explicitly recognized by Leibniz.

In the 18th century two mathematician, Euler and Lambert contributed to the development of logic. While Euler is known for geometrical representation of propositions through his circles, Lambert developed logical calculus on intensional lines. For example, if a and b are two concepts, then a + b becomes a complex concept and ab stands for conceptual element common to a and b. What applies to class membership applies also to attributes. Bolzano is another logician who contributed to logic in the 19th century. He regarded terms and propositions as fundamental constituents of logic. He is known for an extraordinary approach to the logical semantics of language. In this context, he regarded propositions as having universal application when certain conditions are satisfied and as universally inapplicable under certain other conditions and as consistent under certain other conditions. Bolzano in fact, modified Kant's definition of 'analytic judgment' using this particular criterion. Another important contribution of Bolzano was his conception of probability. He introduced some modifications into Laplace's conception of probability, which was widely held during his time. Laplace defined probability as equipossible while determining the probability value when only two possibilities are available as in the case of tossing of the coin. In fact, Bolzano's modification avoids this particular element. This is crucial because 'equipossible' involves circularity. By avoiding this term, Bolzano could avoid circularity, which was inherent in Laplace's theory.

In 1847 two mathematicians, de Morgan and George Boole published 'Formal Logic' and 'The Mathematical Analysis of Logic respectively. Symbolic logic actually took off from this point of time. De Morgan gave to the world of logic now famous notion of complement which was later exploited by John Venn to geometrically represent distribution of terms and test syllogistic arguments. De Morgan showed that if there are two classes, then there are four product classes and Jevons showed that if there are three classes, then there are eight product classes. So generalizing this relation, we can say that the relation between the number of classes and the number of product classes is given by the formula, n = 2x. Where, 'n' stands for the number of product-classes and x stands for the number of terms. This formula is only indicative of the type of relation, which holds good between classes (or sets) and product classes because there is no syllogism with more than three terms and no proposition (in traditional sense) has more than two terms. He also gave a formula known as de Morgan's law to write the contradiction for disjunctive and conjunctive propositions.

Boole's contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work 'An Investigation of the Laws of Thought!' It is in this work that the germs of the 20th century symbolic logic can be traced.

While Lambert invented union of concepts on intensional analysis. Boole invented union of sets on extensional basis. He used '1' to designate the universe. Following de Morgan, Boole called it the universe of discourse. He introduced the following laws, which play crucial role in mathematical logic.

1 Union of any set and universal set is a universal set. Let X be a set. Then 1+X = 1

2 Product of a universal set and any non-null set X is X itself.

3 Product of null-set and any non-null set (universal set included) is a null-set itself.

If X is a non-null set, the 1 – X is its complementary.

5 It is self-evident that product of any non-null set and its complementary is a null-set.

5 Stands for Boole's definition of contradiction. He also showed that if X, Y, Z,…etc. stand for non- null sets, then all laws of algebra hold good. Most important among them are what are known as distributive and commutative laws. For the sake of brevity, these laws are stated as follows:

1 Distributive Law: a(b+c) = ab + ac

2 Commutative Law: ab= ba

or a+b=b+a

Using the concept of complementary class, Boole also showed that 'A, E, I and O' of traditional logic can be reinterpreted. His suggestion was geometrically represented by Venn.

In this interpretation, Boole actually considered what is called class logic, which later became the cornerstone of set theory. In logic, there is another topic called calculus of propositions. Boole integrated these two and defined the truth-value of what are called compound propositions which also consist of variables. While in the first interpretation the variables represent the sets or terms, in the second interpretation they represent the propositions. Consequently, products of classes, here, become conjunction and union or addition of classes becomes disjunction. Complement of a set becomes negation of a proposition.

Boolean analysis of logic is also called Boolean algebra for two reasons. In the first place, he freely used variables to explain various aspects of logic. Extensive use of variables characterizes algebra. Secondly, he defined all four operations of algebra; addition, multiplication, subtraction and division and extended the same to logic.

Venn's contribution to logic was partially mentioned earlier. Therefore the remaining part requires to be mentioned. Venn is well-known for making qualitative distinction, in addition to traditionally held quantitative distinction between universal and existential (particular) which has far reaching consequences. The distinction is that while universal proposition (in modern logic universal quantifier) denies the existence of membership in a class, existential quantifier affirms the same. Secondly, a large number of deductive inferences became invalid as a result of this description. The irony is that in this situation, progress is marked not by augmentation but by depletion in the number of inferences.

There were certain anomalies in Boolean system. Consider two identical sets, say X and Y where every member of X is a member of Y and every member of Y is a member of X; for example, the class of bachelors and the class of unmarried men. The product class should yield X Y. Since Y = X, XY=X2 or Y2. In algebra it makes sense, but surely not in logic. Similarly X+Y, the union of two sets ought to become 2X. Again, it holds good in algebra but not in logic. Jevons, a student of de Morgan, succeeded in eliminating these anomalies; according to his interpretation, the union of two identical sets does not double the strength, say from n to 2n. The reason is simple; every member is present in both the sets. We cannot count one individual as two just because he or it is present in two sets simultaneously. The same reasoning applies to product of identical sets. If there are 100 bachelors and 100 unmarried men then the product of these two sets does not produce 1002 = 10,000 bachelors who are also unmarried men, but 100 only. C.S. Peirce resolved this anomaly in a different way. He identified logical addition with *inclusive or* instead of *exclusive or* (either p or q but not both is an example for exclusive or and either p or q or both is an example for inclusive or).

Peirce introduced a symbol $\supset$ for class inclusion. He strangely argued that there is no difference between a proposition and inference or implication. In the ultimate analysis only implication survives. Secondly, all implications have quantifiers, which may be explicit or implicit. While Peirce thought that implication is the primary constituent of logic, at a later stage, there were attempts to eliminate implication and retain only negation and conjunction. While introducing symbols in a set of formulas Peirce was driven by a definite motive. He believed that symbols should resemble what they represent say thoughts. To achieve his aim, Peirce used, what he called, ‘existential graphs’. They were not graphs in geometrical sense. He regarded parentheses themselves as graphs. For example, ‘if p, then q’was represented graphically, by Peirce by using parentheses. He inserted p and q within parentheses and represented as (p (q)).

Christine Ladd Franklin invented a new technique of testing syllogism called antilogism or inconsistent triad. In addition to, Venn’s diagram, antilogism also eliminated weakened and strengthened moods on the ground that particular propositions cannot be deduced from universal propositions only.

Gottlob Frege is one of the pioneers, who gave a new dimension to mathematical logic. In 1879 ‘Begriffsschrift’ the first of his most important works was published followed by Die Grudlagan der Arithamatik in 1884. His first work dealt with proper symbolization with the help of rules of quantification. His intention was to codify logical principles used in mathematical reasoning like substitution, modus ponens, etc. In this work he introduced the notion of function, which was later renamed as propositional function. He also introduced a system of basic formulas for propositions in terms of implication and negation. In his second work, Frege made the most crucial attempt to trace the roots of mathematics to logic. He himself regarded arithmetic as simply a development of logic. Consequently, every proposition of arithmetic became merely a law of logic. History has recorded that Frege’s thesis would not have got what it deserved but for Russell’s discovery of Frege. Hence the relation between arithmetic and logic is known as Frege-Russell thesis.

It is said that modern logic began with Frege. It means that in one sense the history of symbolic logic stops before Frege. Whatever development that took place after Frege’s period characterize contemporary logic. Even in this period, there were remarkable changes with new theses being presented regularly. Giuseppe Peano tried to establish the relation between logic and mathematics in a slightly different manner. Instead of tracing the roots of mathematics to logic, Peano tried to express mathematical methods in a different form similar to that of logical calculus. For example, the successor of ‘a’ was designated by the symbol ‘a+’; also in addition to the symbol ⊃ he introduced another symbol ∈. This shows that implication or class inclusion (⊃) is distinct from ‘element of’ or ‘belongs to’. In Peano’s system there is no interpretation of any symbol and hence mathematics becomes a formal system.

In the beginning of the 20th century Zermelo proposed his theory of sets known as Axiomatic Set theory. He intended his theory to be free from contradictions. He regarded it as well ordered because it was axiomatized. His claim was totally rejected by Poincare. Perhaps only two mathematicians disputed the theory that mathematics has its foundations in logic. Opposition to this approach developed first in the 19th century. Kronecker, a professor of mathematics at the University of Berlin in 1850s, was the first mathematician to oppose this dominant trend. He disagreed with Cantor’s theory of sets which included the concept of infinity. Kronecker went to the extent of arguing that integers are made by God, but everything else is the work of man. After Kronecker, it was Poincare who believed that mathematics does not have its base in logic. His main thesis is that in the first place, mathematical induction cannot be reduced to logic; secondly, according to him, even mathematics proceeds from particular to universal only; a clear opposition to deductive logic.

1.6 THE AGE OF *PRINCIPIA MATHEMATICA*(PM)

In 1910 Bertrand Russell in association with A.N. Whitehead published *Principia Mathematica*. What was referred to as the Frege-Russel thesis in the previous section found exposition in this work. Only a few aspects of this great work can be dealt here. The principal thesis remains the same, that mathematics is an extension of logic. Jevons, earlier, remarked that ‘algebra’ is nothing but highly developed logic’ to which Frege added: ‘inferences….. are based on general laws of logic.’ Frege was actually referring to mathematical induction. In the preface itself the authors admitted that ‘thanks to Peano and his followers symbolic logic… acquired the technical and the logical comprehensiveness that are essential to a mathematical instrument’. Clearly, the new age mathematicians bypass Poincare and Kronecker in this regard.

PM makes a clear distinction between proposition and propositional function. While variables constitute propositional function, substitutions to variables constitute propositions. The former is neither true nor false. But the latter is either true or false. For example, X is the husband of Y is neither true nor false. But Rama (X) is the husband of Sita (Y) is true.

A key logical term, which finds place in PM is material implication. Russell and Whitehead used ‘⊃’ to designate implication. Material implication is defined as follows:

$p \supset q \equiv \sim p \; g \; q$

Truth-values were assigned by PM as follows. Both p and q can be true together, or when p is false, q may be false or true. But when p is true q cannot be false. Implication, therefore, does not imply necessary connection. To distinguish implication from prohibited possibility Russell and Whitehead used material implication instead of mere ‘implication’.

This particular definition of material implication has a very important consequence. ‘Necessary relation’ was an unwanted metaphysical baggage, which was overthrown by Hume. But there was no way of interpreting implication in the absence of necessary relation. Fixation of truth-value by PM made a distinct advance in this case. And it is precisely this type of implication that is used in mathematics. Consider a very familiar example, 'If ABC is a plane triangle, then the sum of the three angles equals two right angles'. That there is no plane triangle at all does not affect the relation because even when the antecedent is false the consequent can continue to be true. Hence it comes to mean that a true premise can imply only true conclusion whereas a false premise can imply either true or false conclusion.

PM includes five axioms (Russell and Whitehead use the word 'principle'), which can be regarded as primitive logical truths. They are follows:

1	Tautology	(Taut)
2	Addition	(Add)
3	Permutation	(Perm)
4	Association	(Assoc)
5	Summation	(Sum)

Example provided here is taken from the text itself. The authors in all these cases use the symbol I- which is read 'it is asserted that' or it is true that' and the dots after assertion I– sign indicate range. 'v' is read 'or' and '⊃ is read 'if...then'.

Taut: I- : p g p. ⊃ .P It is true that p or p implies p.

Add: I-: q. ⊃ . p γ q It is true that if q, then p or q.

Perm: I-: p g q. ⊃ .q γ p If p or q, then q or p.

Assoc: I-: p g (q g r). ⊃. q γ (pg r) If p or q or r, then q or p or r.

Sum: I- :q ⊃ r. ⊃: p γ q. ⊃ .p g r If q implies r, then p or q implies p or r.

For 'Add' the example is 'if today is Wednesday (q), then today is either Tuesday or Wednesday. The examples can be constructed on similar lines for other axioms. For perm, the example read as follows; if today is Wednesday or Tuesday, then today is Tuesday or Wednesday. In all cases, the sentences are preceded by 'it is true that'. The colleen immediately after the assertion sign indicates range, but the dots which follow or precede variables are only customary.

PM also includes equivalence relation, which explains the equivalence of the law of the Excluded Middle and the Law of contradiction. In the beginning of the summary of *3 the authors say that 'it is false that either p is false or q is false, which is obviously true when and only when p and q are both true. Symbolically,

p.q = . ~ (~ pg ~ q)

Reductio ad absurdum is one method accepted by mathematics. It means that the contradiction of what has to be proved is assumed to be true and then the conclusion contradicting the assumption is deduced. This contradiction shows that the assumption is false in which case its contradiction must be true. This is again a primitive logical truth. The principle of double negative is another, which can be easily derived from the law of the Excluded Middle.

David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore, a consistent system, in Hilbert's analysis is an axiomatized system.

A distinguishing mark of Hilbert's analysis is his 'discovery' of 'ideal limit'. From the days of Cantor and Weirstrass who introduced the concept of 'infinity' or 'transfinite' the concept of ideal limit engaged the attention of mathematicians. While elementary number theory could be empirically interpreted, infinity could not be interpreted in that manner. So Hilbert chose to regard transfinite as limit.

There should not be break in history – circuit. Therefore another contribution of Hilbert secures a place in our discussion. Hilbert embarked upon his project to defend classical mathematics from one theory of mathematics known as intuitionism spearheaded by the Dutch mathematician Jan Brouwer, according to whom mathematics is not a system of formulas but is a sort of abstract activity, which abstracts the concept of 'numberness.' By any standard, 'intuitionist mathematics ceases to be a logical enterprise, but confines itself to the narrow domains of psychological activity at best and some sort of esoteric activity at worst.

Following the tradition of PM, Emil Post presented the method of truth-tables published as 'Introduction to a General Theory of Propositions' in the American Journal of Mathematics in 1921. In this paper, Post included not only classical logic, which allowed only two values but a system allowing many values. In the same year Wittgenstein's *Tractus logico-Philosophicus* was published, which also included this technique. Wittgenstein held the view that mathematics is nothing but a bundle of tautologies. While this is the view of earlier Wittgenstein, in later Wittgenstein the conception of mathematics underwent dramatic change. In 'Remarks on the Foundations of Mathematics' Wittgenstein argues that both logic and mathematics form parts of language games. At this point of time he became a conventionalist and argued that mathematical propositions are immune to falsification. This position of Wittgenstein is much closer to intuitionism than to anything else.

Rudolph Carnap's contribution to symbolic logic consists in the extension of the same to epistemology and philosophy of science. He argued that all meaningful sentences belong to the language of science. He followed what is called the 'principle of tolerance' with which any form of expression could be defended if sufficient logical rules are there to determine the use of such expression. Under the influence of Alfred Tarski, he included such notions as truth and meaning in his analysis.

Kurt Goedel is another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous '*Incompleteness Theorem'*. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted.

Alonso Church is a noted historian of symbolic logic. Logicians and mathematicians alike are interested in questions related to the decidability of logical and mathematical theories. His main thesis is that there is no general technique to determine or discover the truth or proof of any proposition in arithmetic. In this respect, Church stands opposed to Hilbert who argued that classical mathematics is a consistent system. W.V.O Quine and Curry are two other prominent personalities. While Quine is known for his contribution to the development of set theory, Harkell B. Curry's name is associated with a new branch of logic called 'Combinatory Logic'. It had its birth in H.M.Shaffer's discovery of 'stroke' symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables. Curry proceeded further with Schonfinkel's works with set of operations different from stroke symbol. He introduced what is called the theory of λ – conversion (λ is read 'lamda'), where λ is known as binary operation. Church used this operation to analyze formal systems to which variables belong and to which arbitrary objects can be substituted. Here objects mean the functions in which they stand for arguments. It means that a variable in a system is substituted by an argument. λ – conversion is a theory proposed by Church in connection with such substitutions.

In short, symbolic logic is a system of algebraic combination and mechanical substitution of symbols for the purpose of inference. It is the study of symbolic abstractions that captures the formal features of logical inference. C.I. Lewis observes the following characteristics for symbolic logic: the use of ideograms (i.e., signs that stand directly for concepts) instead of phonograms (signs that depict sounds first and indirectly concepts); deductive method and use of variable having definite range of significance. It has mainly two parts: truth-functional or propositional or sentential logic and predicate logic. The former is a formal system in which propositions can be formed by combining simple propositions using sentential connectives, and a system of formal proof in determining the validity of arguments. Predicate logic provides an account of quantifiers in the symbolization of arguments and laws for the determination of their validity.

Check Your Progress I

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Examine Boole's contribution to modern logic.

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2. Examine the role played by PM in the 20th century logic.

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3. Contrast Hilbert's and Goedel's views on proofs in mathematics.

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4. What is the significance of Shaffer's and Schonfinkel's studies? Explain.

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1.7 LET US SUM UP

Logic has its roots in Greek civilization. Aristotle systematized the technique of thinking. During medieval ages, lot of research work was undertaken within the limits of Aristotelian system. Modern logic took its birth with Leibniz' work '*Dissertatio de Arte Combinatoria*'. Boole's works provided impetus to the growth of symbolic logic. Contemporary symbolic logic begins with de Morgan. Initially, Frege and Russell and later, Russell and Whitehead heralded a new era in symbolic logic. Combinatory logic has its beginning in H.M. Shaffer's work which was later developed by Haskell B. Curry. Today logic and mathematics have become two faces of the same coin.

1.8 KEY WORDS

Theorem: In mathematics, a theorem is a statement proved on the basis of previously accepted or established statements such as axioms.

1.9 FURTHER READINGS AND REFERENCES

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1.10 ANSWERS TO CHECK YOUR PRGRESSS

Check Your Progress I

1. Boole's contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work 'An Investigation of the Laws of Thought!' It is in this work that the germs of the 20th century symbolic logic can be traced.

2. The publication of Principia Mathematica by Russell and Whitehead heralded a new era in the history of mathematics and logic. In this work they established that logic is the foundation of mathematics. The term implication acquired a new meaning when new rules of inference were evolved. These rules of inference forced logicians to distinguish implication from entailment. Also this work influenced Emil Post to present the methods of truth-table which is the backbone of mathematical logic. The earlier Wittgenstein was also partly influenced by this work.

3. David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a

sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore a consistent system, in Hilbert's analysis is an axiomatized system. Kurt Godel is another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous '*Incompleteness Theorem*'. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted.

4. Combinatory logic had its birth in H.M.Shaffer's discovery of 'stroke' symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables.