UNIT 4 TRUTH - FUNCTIONAL FORMS

Contents

4.0 OBJECTIVES

The aim of this unit is to introduce you to the concept of equivalence through two means; truth-table method and stroke and dagger function and contradiction through truth-table means. Though what you learn in this unit is much limited in terms of content, it forms the foundation of future learning. Hence this unit should prepare you to grasp the essence of the next block.

After you are thorough with this unit you should be in a position to:

- construct truth-tables for statements.
- identify propositions having different form but same content.
- reduce all verbal expression to non-verbal forms.
- discover that verbal form is more complex and not necessarily useful when compared with symbolic form, which is simpler and more useful in our logical enterprise.

4.1 INTRODUCTION

In Unit 2 we learnt that in our study of symbolic logic we replace propositions by variables. These variables may be called prepositional variables because they signify indifferently any statement. Therefore whenever a propositional variable is assigned any truth-value, then the same truth-value has to be assigned to any proposition signified by the respective variable. We also learnt that sentential connectives help us to obtain compound propositions. While statements are variables, various connectives like 'not', 'if…then', etc., which produce compound propositions, are logical constants. A study of symbolic logic starts with what is known as, 'calculus of propositions or propositional calculus'. There are different forms of truth-function, which constitute propositional calculus with which we have to familiarize. In other words, various relations between propositions require to be studied.

It is good to recapitulate what was discussed under compound statements. There are five kinds of compound propositions: implicative, conjunctive, disjunctive, negation and biconditional; each one defined by a definite form. An important aspect, which follows this discussion, is 'two kinds of relation which exist between these forms'. Contradiction and logical equivalence (equivalence in brief) are these forms with which we are concerned. The beginning of this study marks the beginning of the study of symbolic logic. Let us make a beginning with implication.

4.2 IMPLICATION AND ITS EQUIVALENT FORMS

Let p stands for 'there is increase in supply' and q stands for 'the prices will fall'. Then, as we know already, the statement, 'if there is increase in supply, then the price will fall' is an implication (material implication to be precise) in a standard form. Our task is to derive its various equivalent forms and contradiction. As usual, we shall construct truth-table and then go to verbal form:

Table: 1

					Implication	Disjunction		Negation
	p	**q**	**⌐p**	**⌐q**	**p=>q**	**⌐p v q**		**⌐(p Λ ⌐q)**
1	1	1	0	0	1	1	1	0
2	1	0	0	1	0	0	0	1
3	0	1	1	0	1	1	1	0
4	0	0	1	1	1	1	1	0

Under negation there are two columns which reflect truth-values. It must be remembered that the last but one column stands for equivalence relation. Therefore care should be taken to write the truth-value of negation exactly under the negation sign.The advantage of truth-value method is obvious. The equivalence relation, which exists between implication and disjunction, is self-explanatory. However, relation with negation requires some clarification. There are two columns under negation, which reflect truth-values. Suppose that we ignore negation sign and corresponding truth-values and consider the last column then we are not considering negation but conjunction. The last column is the same as the following one:

	p Λ #q
1	0
2	1
3	0
4	0

However, the required form is not conjunction but negation. The truth-value of negation, of course, truth-functionally depends upon the truth-value of conjunction form. Therefore while selecting the column, which corresponds to negation form, we should exercise a little caution.

Now we shall consider the verbal form of logical equivalence. Suppose that the given proposition is as follows:

1) 'If there is increase in supply, then the prices will fall'. The components of this proposition and their symbols are as follows.

a) There is increase in supply. p

b). The prices will fall. q

The form of given proposition is as follows:

$$p \Rightarrow q \quad — \quad (1)$$

If, instead of considering the form of any proposition, we symbolize propositions themselves, then we shall choose the first letter of the first term (in which case we ignore article, verb, etc.). In such a case we have to use upper-case letters. Then (1) is replaced by: I=>P. For some time let us use both the form of the proposition and symbols of given propositions.

	Implication		Disjunction		Negation
a)	$p \Rightarrow q$	$\equiv$	$\neg p \vee q$	$\equiv$	$\neg (p \wedge \neg q)$
b)	$I \Rightarrow P$	$\equiv$	$\neg I \vee P$	$\equiv$	$\neg (I \wedge \neg P)$

Now we have to consider the negation of (a) and (b) mentioned above.

a') "There is no increase in supply" or

"It is not the case that there is increase in supply": $\neg p / \neg I$

b') "The prices will not fall" or

"It is not the case that the prices will fall": $\neg q / \neg p$

Disjunction: It is not the case that there is increase in supply or the prices will fall: $\neg p \vee q / \neg I \vee P$

Negation: It is not the case that both there is increase in supply and the prices do not fall: $\neg (p \wedge \neg q) / \neg (I \wedge \neg P)$

When negation is expressed in words, it is very important to observe that after 'it is not the case that' the word 'both' should invariably be used. Otherwise a mistake will be made. In fact, the word 'both' stands for the verbal expression of parentheses.

Implication has one more equivalent form called contraposition. Its structure is as follows:

Table: 2

					Implication	Contraposition
	p	**q**	**$\neg$ p**	**$\neg$ q**	**$p \Rightarrow q$**	**$\neg q \Rightarrow \neg p$**
1	1	1	0	0	1	1
2	1	0	0	1	0	0
3	0	1	1	0	1	1
4	0	0	1	1	1	1

Since sentential connective remains the same, the type proposition also remains the same. Hence its use is somewhat limited to the test of arguments.

4.3 DISJUNCTION AND ITS EQUIVALENT FORMS

If implication has equivalent disjunctive form, the converse also should hold good. The component proposition, 'there is increase in supply' and 'the prices will fall' are connected by the connective 'OR' and we obtain compound proposition as follows:

'There is increase in supply or the prices will fall'. Let us construct the truth table to be followed by verbal form.

Table: 3

	p	**q**	**⌐ p**	**⌐ q**	**Disjunction p v q**	**Implication ⌐ p => q**	**Negation ⌐ (⌐ p Λ ⌐q)**	
1	1	1	0	0	1	1	1	0
2	1	0	0	1	1	1	1	0
3	0	1	1	0	1	1	1	0
4	0	0	1	1	0	0	0	1

There is no difference in explanation for negation compared with implication. However, we shall repeat only truth-table form in order to eliminate any iota of doubt, if any. Accordingly, rewrite the truth-value of the last column.

	⌐ p Λ ⌐q
1.	0
2.	0
3.	0
4.	1

Evidently, what we have here is only conjunction, but what we want is negation. Therefore the set of relevant truth-values belong to the last but one column, which is truth-functionally dependent upon those of the last column.

Let us switch over to verbal form and begin from disjunction.

2. There is increase in supply or the price will fall.

   We shall rewrite the components and then append their negations.

   a) There is increase in supply. p/I

   b) The prices will fall. q/P

      a) There is no increase in supply or it is not the case that there is increase in supply. ⌐ p/⌐ I

      b) The prices will not fall. ⌐ q/⌐ P

   Implication: If it is not the case that there is increase in supply, then the prices will fall. ⌐ p => q or ⌐ I => P

Negation: It is not the case that both there is no increase in supply and the prices will not fall. ⌐ (⌐ p Λ ⌐ q) or ⌐ (⌐ I Λ ⌐ P)

As in the case of implication, in this case also:

Disjunction Implication Negation

$p \vee q \equiv \neg p \Rightarrow q \equiv \neg(\neg p \wedge \neg q)$

Unlike implication, disjunction allows simple transposition of disjunctions. $\therefore p \vee q \equiv q \vee p$. In this case also transposition has limited application in the test of arguments. The rule which governs such simple transposition is known as rule of commutation. Therefore when we construct disjunctive syllogism, we are free to choose any component.

The relation between $(p \vee q)$ and $\neg(\neg p \wedge \neg q)$ is explained by what is known as de Morgan's law. It says that equivalence of disjunction consists in the negation of the conjunction of the negation of components. It is very important to understand this law completely and clearly. Here negation and conjunction are algebraic functions. Conjunction is equivalent to multiplication. We know that in algebra parenthesis also is equivalent to multiplication. Therefore negation within parentheses goes and negation outside parentheses remains. It shows that it is inadmissible to cancel three negation signs. To put it symbolically, $\neg(\neg p \wedge \neg q) \neq (p \wedge q)$. The method of testing this inequality is very simple.

Table: 4

	p	**q**	**$\neg$ p**	**$\neg$ q**	**Negation** **$\neg(\neg p \wedge \neg q)$**		**Conjunction** **p $\wedge$ q**
1	1	1	0	0	1	0	1
2	1	0	0	1	1	0	0
3	0	1	1	0	1	0	0
4	0	0	1	1	0	1	0

Since the truth-value of these expressions is not the same in all instances, they are not identical.

4.4 NEGATION AND ITS EQUIVALENT FORMS

The equivalent forms of negation take implication and disjunction forms when suitably translated. We have two components, 'there is increase in supply' and 'the prices will fall'. When connected by 'not' we obtain,

3) It is not the case that both there is increase in supply and the prices will fall.

   Let us rewrite the components and append their negations.

   a) There is increase in supply. p/I

   b) The prices will fall. q/P

$\neg$ a) There is no increase in supply or it is not the case that there is increase in supply. $\neg$ p/$\neg$I

$\neg$ b) The prices will not fall or It is not the case that the prices will fall. #q/#P

   Now construct the truth-table for equivalent forms.

Table: 5

	P	q	⌐ p	⌐ q	Negation ⌐ (p Λ q)		Implication p => ⌐ q	Disjunction ⌐ p v ⌐ q
1	1	1	0	0	0	1	0	0
2	1	0	0	1	1	0	1	1
3	0	1	1	0	1	0	1	1
4	0	0	1	1	1	0	1	1

The verbal forms of relations are as follows:

Implication: If there is increase in supply, then the prices will not fall.

p => ⌐q / I => ⌐ P

Disjunction: There is no increase in supply or the prices will not fall:

⌐ p v ⌐ q / ⌐I v ⌐P

For negation also we do not consider transposition of components because it does not have any special significance. If the equivalent negation form of disjunction is given by de Morgan's law, then converse also naturally holds good. What is negated is negation of 'Conjunction'. That is, {# (p Λ q)} is negated. Therefore negation sign goes. Conjunction is replaced by disjunction and components are replaced by their negations. Hence, we get disjunction, which is equivalent to negation.

Before we pass on to check contradiction, it is good to challenge our own choice. Let us start with implication. How can we assert that only #p v q is equivalent to p=>q? Why cannot we say that p v #q is also equivalent? It is nearly impossible to give explanation in verbal form, as to how # p v q is equivalent to p => q, but not p v #q. If we compare the truth-values of # p v q and p v #q with p => q, then the solution becomes clear.

Table: 6

	p	q	⌐p	⌐q	p=>q	⌐p v q	p v ⌐q	⌐p v ⌐q
1	1	1	0	0	1	1	1	0
2	1	0	0	1	0	0	1	1
3	0	1	1	0	1	1	0	1
4	0	0	1	1	1	1	1	1

⌐ p v ⌐ q is added only to reinforce our position. An equivalent expression must be true in only those instances in which the original expression is true (and in all such instances) and it must be false in only those instances in which the original expression is false (and in all such instances). According to this criterion, only # p v q is equivalent disjunctive form to the original implication. The students are advised to test all other cases, like disjunctive proposition, using truth-table method to conclude that other than those mentioned are not equivalent to the original expression. At this stage, it should become clear that use of verbal expressions to determine their equivalent forms renders the task an uphill task and sometimes practically impossible. It is left to the students to verify the last statement which he can do by considering fairly a complex statement.

Now construct the scheme of equivalent expressions with truth- table.

Table: 7

	p	q	⌐p	⌐q		Implication	Disjunction	Negation
1	1	1	0	0	Implication: p => q	⌐ q => ⌐ p	⌐ p v q	⌐(p Λ ⌐q)
2	1	0	0	1	Disjunction p v q	⌐ p => q	(q v p)	⌐p Λ ⌐q
3	0	1	1	0	Negation ⌐ (p Λ q)	p=> ⌐q	⌐ p v ⌐q	⌐ (q Λ p)

It must be noted that while implication does not take equivalent converse form, disjunction and negation take.

4.5 CONJUNCTION AND BICONDITION

It is quite interesting to note that conjunction and bicondition do not have equivalent forms. Truth- table again comes to our rescue. It is sufficient if we consider any one-form, say, implication. If one equivalent form is absent, it is imperative that other forms are also absent.

Table: 8

	p	q	⌐p	⌐q	p Λ q	p=>q	⌐p=>q	p=>⌐q	⌐p=>⌐q
1	1	1	0	0	1	1	1	0	1
2	1	0	0	1	0	0	1	1	1
3	0	1	1	0	0	1	1	1	0
4	0	0	1	1	0	1	0	1	1

Except that truth-values of conjunction do not tally with any possible arrangement in implication form, no other explanation is conceivable for the absence of equivalent forms to conjunction [The students are advised to test other forms to convince themselves].

Biconditional proposition also does not have any equivalent form. The reason is very simple. Biconditional is, in reality, conjunction only and both the conjuncts are implicative. First we shall know why it is regarded as conjunction.

Table: 9

	p	q	⌐p	⌐q	p<=>q	(p=>q)	Λ	(q=>p)
1	1	1	0	0	1	1	1	1
2	1	0	0	1	0	0	0	1
3	0	1	1	0	0	1	0	0
4	0	0	1	1	1	1	1	1

The method of computing is as follows; first, we shall compute the truth-values of implication (p => q) and then we will compute the truth-values of q => p. These two sets of truth-values together determine the truth-value of conjunction. When we compare columns 1 and 3, we will come to know that these two expressions have identical truth-values in all instances. It shows that bicondition is also a conjunctive proposition where the conjuncts themselves are compound propositions. Therefore what applies to conjunction naturally, applies to bicondition also.

4.6 FORM OF CONTRADICTION

When arguments are to be tested, quite frequently, we look for contradiction. Therefore it is necessary that we should know the contradiction of compound propositions so that with ease we can detect contradiction in arguments. The rule of contradiction is as follows: Whenever p is true its contradiction is false and whenever p is false its contradiction is true. That is to say contradiction and negation are same. The truth-table for contradiction is as follows.

Table: 10

	p	**q**	**⌐p**	**⌐q**	**Implication p=>q**	**Contradiction p Λ ⌐q**
1	1	1	0	0	1	0
2	1	0	0	1	0	1
3	0	1	1	0	1	0
4	0	0	1	1	1	0

It is not difficult to express or understand contradiction in verbal form. We shall consider the components and their negation mentioned earlier.

Implication: If there is increase in supply, then the prices will fall.

a) There is increase in supply: p/I

b) The prices will fall: q/P

a') There is no increase in supply: ⌐ p /⌐ I

b') The prices will not fall ⌐ q /⌐ P

Contradiction: There is increase in supply and the prices will not fall:

p Λ ⌐q or I Λ ⌐P

As we challenged earlier conclusion, we shall again challenge this conclusion also. How can we say that p Λ #q is the only contradiction? How do we know that this is the only form of contradiction permissible? Contradiction, in this case, does not have equivalent relation because p Λ #q is a conjunction and conjunction does not have equivalent forms. As a rule, for any given proposition there is only one form of contradiction. We shall consider one disjunction form:

Table: 11

	p	**q**	**⌐p**	**⌐q**	**Implication p=>q**	**Disjunction p v ⌐q**
1	1	1	0	0	1	1
2	1	0	0	1	0	1
3	0	1	1	0	1	0
4	0	0	1	1	1	1

In order to test the conclusion, we effected only one change; we replaced conjunction by disjunction. In first and fourth instances, we notice that the truth-value remained the same whereas it should have been different. Therefore p v ⌐q is not a contradiction of implication.

Contradiction of disjunction is, again, determined in accordance with de Morgan's law; replace disjunction by conjunction and disjuncts by their negations. Therefore the removal of negation prefixed to equivalent form of disjunction results in contradiction. The truth table is as follows:

Table: 12

					Disjunction	**Contradiction**
	p	**q**	**⌐p**	**⌐q**	**p v q**	**⌐p Λ ⌐q**
1	1	1	0	0	1	0
2	1	0	0	1	1	0
3	0	1	1	0	1	0
4	0	0	1	1	0	1

The verbal form is as follows:

Disjunction: There is increase in supply or the prices will fall: p v q / I v P

Contradiction: There is no increase in supply and the prices will not fall:

⌐p Λ ⌐q / ⌐I Λ ⌐P

[In this case also contradiction does not have equivalent forms. If the student wishes to test other alternatives, he or she can follow the method suggested earlier.]

Contradiction of conjunction also is determined in accordance with de Morgan's law; replace conjunction by disjunction and the conjuncts by their contradictions.

Table: 13

					Conjunction	**Contradiction**
	p	**q**	**⌐p**	**⌐q**	**p Λ q**	**⌐p v ⌐q**
1	1	1	0	0	1	0
2	1	0	0	1	0	1
3	0	1	1	0	0	1
4	0	0	1	1	0	1

The verbal form is as follows:

Conjunction: There is increase in supply and the prices will fall: p Λ q / I Λ P

Contradiction: There is no increase in supply or the prices will not fall:

⌐ p v ⌐ q / ⌐I v ⌐P

Since contradiction is in disjunctive form it has equivalent implicative form. I =>⌐P, Ooviously, is its equivalent form.

The contradiction of biconditional proposition is indirectly found and it is in accordance with de Morgan's law since its conjunctive feature is only concealed (i.e., the biconditional is a conjunction of two conditionals, as we see under 2,3,4 in the table) . Let us start with truth-table. [The verbal form is left out so that the student can attend the same.]

Table: 14

							Contradiction
					1	2 3 4	5 6 7
	P	q	⌐p	⌐q	p<=>q	(p=>q)Λ(q=>p)	(pΛ⌐q) v (qΛ⌐p)
1	1	1	0	0	1	1 1 1	0 0 0
2	1	0	0	1	0	0 0 1	1 1 0
3	0	1	1	0	0	1 0 0	0 1 1
4	0	0	1	1	1	1 1 1	0 0 0

Compare columns 3 and 6. It becomes clear that A and B are contradictories.

For the sake of clarity let us consider contradiction in more than one step.

Step: 1 Contradiction components

	Given expression	Contradiction
a)	p=>q	p Λ ⌐ q
b)	q=>p	q Λ ⌐ p

Replace given expression by their contradictions, we obtain: (pΛ⌐q)Λ(qΛ⌐ p)

Step: 2 In Step1, we apply one aspect of de Morgan's law, i.e., replacing conjunct by their negation. In step 2 we apply second aspect of de Morgan's law; i.e., replace conjunction by disjunction. We get: (pΛ⌐q) V (qΛ⌐p)

We are only required to compare columns 3 and 6 to assure ourselves that the chosen and tested form is the contradiction of the original expression.

We shall tabulate the results and at this stage we can omit the basic columns, i.e., truth-values of p, q, ⌐ p & ⌐q since we are familiar with the process involved.

			Contradiction Form
a)	Implication	(p=>q)	p Λ ⌐q
b)	Disjunction	(p v q)	⌐ p Λ ⌐q
c)	Conjunction	(p Λ q)	⌐ p v ⌐q
d)	Bicondition	(p<=>q)	(p Λ ⌐q) Λ (q Λ ⌐p)

It may be noted that when we compute equivalent forms we can do away with implication. We are at liberty to retain disjunction or conjunction. Only negation is constant. Since we can derive from negation and disjunction all other sentential connectives, these two one are called primitive connectives. (However, bicondition is an exception). Such a process results in a sort of simplification since the number of connectives we require comes down as a result of this process. In order to further reduce the number of connectives, a different technique was introduced. This is known as stroke and dagger operation.

4.7 THE STROKE FUNCTION (%)

Though the stroke function was introduced by C.S. Peirce, it is better known as the Sheffer-function after H.M. Sheffer, a mathematician. This function has negative force. The stroke function also is called stroke operator. This is also a connective because its use determines the truth-value of compound proposition, given the truth-value of its components.The definition of this function can be attempted in this fashion:

"When a stroke connects any two statements, then it has to be construed that at least one of them is false, if the function itself must be true." Suppose that p and q are statements forms. Then p%q means that either p is false or q is false when p%q is true. This definition does not rule out the possibility of both p and q being false. It can be depicted in the following manner:

Table: 15

	p	q	p | q
1	1	1	0
2	1	0	1
3	0	1	1
4	0	0	1

Accordingly, compound propositions can be expressed in stroke form in the following manner:

1) Negation:

Truth-table method		Stroke Method
p	⌐p	p | p
1	0	0
0	1	1

2) Conjunction

Table: 16

Truth-table method Stroke method

					1	2 3 4
	p	q	⌐p	⌐q	p∧q	(p|q) | (p|q)
1	1	1	0	0	1	0 1 0
2	1	0	0	1	0	1 0 1
3	0	1	1	0	0	1 0 1
4	0	1	1	1	0	1 0 1

This process needs some explanation and explanation is in terms of truth-value. Consider p | q and apply the definition of stroke function. p | q is false only when both p and q are true, i.e. in the first instance only. In all other instances, from Table 14, we understand that at least one of them is false. So the stroke function is true. Now consider columns 2 and 4. Only in the first instance '0' appears in these two columns and nowhere else '0' appears in columns 2 and 4. Therefore in accordance with the definition of stroke function column 3 takes the value 1 only in the first instance. When we compare column (1) and (3) we learn that there is agreement in terms of the truth-value in all the instances. Therefore (p Λ q) a (p | q) | (p | q), i.e., they are logically equivalent.

Table 17

3). Disjunction:

Truth-table method Stroke method

					1	2 3 4
	p	q	⌐p	⌐q	p v q	(p|p) | (q|q)
1	1	1	0	0	1	0 1 0
2	1	0	0	1	1	0 1 1
3	0	1	1	0	1	1 1 0
4	0	0	1	1	0	1 0 1

Considering the fact that stroke function is somewhat subtle, explanation is desirable. Apply the definition of stroke function to p | p and q | q. p | p is true only in 3rd and 4th instances where p is false. According to the definition of stroke functions, stroke function is true only when at least one component is false. p | p is false 1st and 2nd instances when p is true. Similarly, q | q is true in 2nd and 4th instances when q is false. Now apply stroke function to column 3. It takes the value 1in the first three instances since 0 appears either in column 2 or column 4 in these instances. It can take the value '0' only in the fourth instance since only in this instance the columns 2 and 4 take the value1. When we compare columns (1) and (3) we learn that there is agreement in terms of the truth-value in all the instances. Therefore, (p v q) a (p | p) | (q | q), i.e., they are logically equivalent.

3) Implication:

Table 18

Truth-table method Stroke method

					1	2	3	4
	p	q	¬p	¬q	p => q	p (q|q)		|
1	1	1	0	0	1	1	1	0
2	1	0	0	1	0	1	0	1
3	0	1	1	0	1	0	1	0
4	0	0	1	1	1	0	1	1

The truth-value, which appears in column 3 is truth-functionally dependent on truth-values, which appear in columns 2 and 4. Column 3 takes the value 1 in instances 1, 3 and 4.Since in these instances '0' appears in one or the other column. Only in second instance column 3 takes the value 0 since columns 2 and 4 both take the value 1.This is in accordance with the definition of stroke function. Columns 1 and 3 agree in all the instances in terms of truth-value. Therefore, (p=>q) a p | (q | q), they are logically equivalent.

4.8 THE DAGGER FUNCTION ("!)

The dagger version can be regarded as stronger variation of the stroke function. When a compound proposition is expressed in terms of stroke function, the rule is that at least one of the components must be false if the stroke function must be true, though the possibility of both being false to make stroke function true is allowed. However, in dagger function, both the components must be false to make it true. This statement is regarded as the definition of dagger function.

Suppose p and q are statements forms, then, p"!q is true if and only if both p and q are false. Otherwise, it is false. Accordingly, compound propositions can be expressed in dagger form in the following manner.

Table: 19

	p	**q**	**p ↓ q**
1	1	1	0
2	1	0	0
3	0	1	0
4	0	0	1

1) Negation:

Truth- table method

p	⌐ p
1	0
0	1

Dagger method

p↓ p
0
1

It is clear that ⌐ p ≡ p↓ p. In this respect, the stroke and dagger functions concurr.

2. Conjunction:

Table: 20

					1	2	3	4
	p	q	¬p	¬q	pΛq	(p↓p)	↓	(q↓q)
1	1	1	0	0	1	0	1	0
2	1	0	0	1	0	0	0	1
3	0	1	1	0	0	1	0	0
4	0	0	1	1	0	1	0	1

Columns (2) and (4) determine the truth-values in column (3) in accordance with the definition of the dagger function. Column (1) and (3) horizontally agree in terms of truth-values in all the instances. Therefore, (pΛq) a" (p↓ p)"!(q↓ q)

3) Disjunction:

Table: 21

					1	2	3	4
	p	q	⌐ p	¬q	p v q	(p↓q)	↓	(p↓q)
1	1	1	0	0	1	0	1	0
2	1	0	0	1	1	0	1	0
3	0	1	1	0	1	0	1	0
4	0	0	1	1	0	1	0	1

Explanation is left out so that the student can attempt the same.

4). Implication:

(p => q) ≡ {(p↓ p)↓ q}↓ {(p↓ p)↓ q}

{

Explanation and truth-table are left out so that the student can attempt the same.

Biconditinal proposition can be expressed neither in stroke form nor in dagger form. So it does not have any form of equivalence. Negation of conjunction also does not have equivalence in these forms.

Check Your Progress

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

Read carefully the questions and instructions before you answer.

Six statements are given. Answer as per directions.

1. Crow is black.

   ....

2. Philosophy is the mother of all sciences.

   ....

3. Walter Scott is the author of King Waverly.

   ....

4. Japan is a small nation.

   ....

5. America is an ally of China.

   ....

6. Brutus killed Caeser.

   ....

Choose any two propositions. Make as many pairs as possible, like (1,2),(1,3),(2,6),(3,5), etc. Connect them using =>, Λ, v and <=>. The following assumptions have to be made.

	p_1	and	p_2
1	True		True
2	False		False
3	True		False
4	False		True

In this way assume the truth-values of components. After constructing compound proposition based on the truth-value you have assigned to components, determine their truth-value. Also, remember that for every assumption of truth-value there are four compound propositions. Two examples are given below:

(1). 1 – True 6 – True (2) 1 – False 6 – False

(3) 1 – True 6 – False (4) 1 – False 6 – True

For (1) you have to construct four compound propositions using four connectives given above. Likewise for (2) four compound propositions, for (3) four compound propositions etc. In all cases write equivalent forms, which include stroke and dagger functions and contradiction.

4.9 LET US SUM UP

Truth-function and variables are basic to propositional calculus. Symbolic logic begins with propositional calculus. Compound propositions are characterized by both variables and constants. Contradiction and equivalence are two important logical relations. While conjunction and bicondition do not have equivalent forms other compound propositions have equivalent forms. Equivalent forms eliminate all but two connectives, negation and disjunction, which are primitive connectives. Stroke and dagger operators reduce the number to one. Only bconditional remains unaffected.

4.10 KEY WORDS

Operator: In symbolic logic 'operator' means a tool with the help of which an action is performed. Here the act consists in determining the truth-value of a compound proposition.

Constant: A constant is a quantity that does not change, over time or otherwise. It has a fixed value.

Variable: A variable is a symbol for which there many suitable substitutions.

4.11 FURTHER READINGS AND REFERENCES

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4.12 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress I

If all combinations are formed the number runs to about three-hundred. Since it is neither feasible nor required to work out all possibilities only a few are worked out for illustration purpose. In your own interest you should make as many combinations as you can.

1) Crow is black-1; ; philosophy is the mother of all sciences.-1

   Crow is black and philosophy is the mother of all sciences.-1

   Since both components are conjunction is true.

2. Crow is black-0; philosophy is the mother of all sciences.-1

   Crow is black and philosophy is the mother of all sciences.- 0

   Since one component is false, the conjunction is false.

3. Crow is black-0; philosophy is the mother of all sciences.-1

   If Crow is black, then philosophy is the mother of all sciences.-1

   Since the antecedent is false and the consequent is true, the implication is true. Crow is black-1; philosophy is the mother of all sciences.-0

   If Crow is black, then philosophy is the mother of all sciences.-0

   Since the antecedent is true and and the consequent is false, the implication is false.

4. Crow is black-1; philosophy is the mother of all sciences.-1

   Crow is black or philosophy is the mother of all sciences.-1

   The disjunction is true because both components are true.

5. Crow is black-0; philosophy is the mother of all sciences.-0

   Crow is black or philosophy is the mother of all sciences.-0

   The disjunction is false because both the components are false.

6. Crow is black-0; ; philosophy is the mother of all sciences.-0

   Crow is black if and only if philosophy is the mother of all sciences.-1

   The bicondition is true because both the components possess the same truth-value.

7. Crow is black-1; philosophy is the mother of all sciences.-0

   Crow is black if and only if philosophy is the mother of all sciences.-0

   The bicondition is false because the two components have different truth- values.

8. Crow is black-1; ; philosophy is the mother of all sciences.-1

   It is not the case that both crow is black and philosophy is the mother of all sciences.-0

   Negation of conjunction is false because both components are true.

9. Crow is black-1 and philosophy is the mother of all sciences.-0

   It is not the case that both crow is black and philosophy is the mother of all sciences.-1

   Since one of the components is false, the negation of conjunction is true.