UNIT 3 THE NEO-CLASSICAL GROWTH MODEL

Structure

3.0 OBJECTIVES

After going through the unit, you will be able to:

- apply some essential techniques to study growth models in general;
- discuss the neo-classical growth model ;
- point out the difference between the Harrod-Domar model and neo-classical model(s);
- apply the neo-classical growth models in the analysis of certain economic topics like savings, fiscal policy, poverty, and so on;
- explain the comparative growth experiences of nations using the neo-classical model; and
- examine how the concepts of money and monetary processes have been brought into the neo-classical model.

3.1 INTRODUCTION

The previous two units of this block have introduced you to the concepts of economic growth and development, as well as the Harrod-Domar growth model. You saw how important economic growth is, how it differs from development, why we should study growth models and what the limitations of economic growth can be. The Harrod-Domar model was presented to you both as a unified model, as well as separately the models of Harrod and Domar. You became familiar with the model's assumptions, its structure, limitations as well as some applications, including application in Indian economic planning.

This unit presents for discussion a model which was put forward by Robert Solow and Trevor Swan, independently of each other, in 1956, although brief anticipations of the basic idea was carried out by James Tobin. However, it is commonly known as the Solow-Swan model, or even as the Solow model. Solow has done a lot to popularise the model through subsequent papers and books. The Solow model has proved to be one of the most used most robust and standard models in all of economic theory. Several economists of more than one generation have built upon, extended and refined the Solow model. Even when some have put forward new models, they did so referring to the Solow model as the base-line model that they critiqued and found limitations with. Empirical economists spent lots of time, effort and energy to examine data sets and use statistical tools to see if predictions which can be generated out of the Solow model, have actually been matched by the performances of group of countries. Solow justly received a Nobel Prize for his contribution to growth theory, and as he remarked with a touch of pride during the address he gave at the time of receiving the prize, his model started a "cottage industry [of model- building in growth theory]".

In this unit we study the neoclassical growth model. We begin by acquainting you with some tools and techniques you are going to need to study growth models, not only in this unit, but in subsequent units as well. Once you have familiarised yourself with these concepts, you will be introduced to the basic neoclassical model, with the assumptions stated and the structure spelled out and elaborated. Some applications and extensions of the model are presented, following which we bring in monetary factors into the basic neoclassical model. We close the discussion with a study of the very important and relevant topics of convergence and poverty traps.

3.2 SOME BASIC CONCEPTS AND TOOLS TO STUDY GROWTH MODELS

In this section we first set out some basic concepts regarding the meaning and measurement of economic growth that you will find useful not only for this unit but for the entire course.

Growth and change

The concept of the growth rate is very important and you thoroughly need to understand it.

Let x be an economic variable.

Let x_0 be the initial value

x_1 be the subsequent value.

The proportional change in going from x_0 to x_1 is simply

$$\frac{x_1 - x_0}{x_0} = \frac{\Delta x}{x_0}$$

The percentage change in going from x_0 to x_1 is simply 100 times the proportionate change:

$\% \Delta x = 100 \, (\Delta x / x_0)$

If *x* is itself in percentage one has to be careful in expressing the change.

For example, if unemployment decreases from 15% to 12% it is a reduction of 3 percentage points, but a $\frac{15-12}{15} \times 100 = 20\%$ fall in unemployment.

People sometimes erroneously express it as a 3% fall.

Calculation of growth rate assuming that the rate of growth is constant

If we assume a constant rate of growth, the formula used for calculating the growth rate for more than one year is as follows:

Let Y_0 be income in the initial year.

Let n be the number of years. Let r be the constant compound annual rate of growth. Then income in the final year, Y_t, is given by:

$Y_t = Y_o (1+r)^n$

=) $\log Y_t = \log Y_o + n \log (1+r)$ where log stands for logarithm.

$$\Rightarrow \log(1+r) = \frac{\log y_t - \log y_o}{n}$$

$$\text{or } \log(1+r) = \frac{\log(y_t/y_o)}{n}$$

$$\Rightarrow 1+r = \text{antilog}\left[\log\frac{(y_t/y_o)}{n}\right]$$

$$\therefore \quad r = \left\{anti\log\left[\frac{\log(y_t/y_o)}{n}\right] - 1\right\}$$

This assumes constant rate of growth. r (as in a constant rate of interest in compound interest formulations)

Calculation of growth when the rate of growth is not constant

When the rate of growth is *not* constant, it is say

$x_1, x_2, ...x_i ....x_t$ in say, years t=1,2,.......i,......t

then $r = (x_1, x_2, ...x_t)^{1/t}$

This is the geometric mean.

Exponential Growth

We know that growth can be written as

$\frac{\Delta x}{x}$. This is in general terms

Compound rates are the same as geometric growth. Continuous compound growth implies exponential growth

$$v_t = v_o(1+g)^t$$

$$\Rightarrow \left(\frac{v_t}{v_o}\right) = (1+g)^t$$

$$\Rightarrow \left(\frac{v_t}{v_o}\right)^{1/t} = 1+g$$

$$\therefore g = \left(\frac{v_t}{v_o}\right)^{1/t} - 1$$

Let v_o equal the value of the variable in year 0 (the fast year) & v_t equal the value of the variable t years later. Further, let g equal the average compound annual growth rate. Then

$$v_t = v_o(1+g)^t$$

$$\Rightarrow g = \left(\frac{v_t}{v_o}\right)^{1/t} - 1$$

If the growth rate is continuously compounded, then

$$v_o e^{gt} = v_t$$

$$e^{gt} = \frac{v_t}{v_o}$$

$$g = \frac{1}{t}\ln\left(\frac{v_t}{v_o}\right)$$

We saw above that when the compound rate of growth is constant,

$Y_t = Y_o\left(1+r\right)^t$. This means Y_o grows at 100r % If compounding takes place m times a year (you may think of money in a bank earning compound interest), then in this case,

$$Y_t = Y_o\left(1+\frac{r}{m}\right)^{mt}.$$

As m gets larger and larger, and approaches infinity (theoretically, but this is conceptually very important), the term within the parenthesis acquires a definite value. We can show it as follows:

We can write the above expression as

$$Y_t = Y_o\left[\left(1+\frac{r}{m}\right)^{\frac{m}{r}}\right]^{rt}$$

Let m/r be denoted h then

$$Y_t = Y_o\left[\left(1+\frac{1}{h}\right)^{h}\right]^{rt}$$

As m tends to infinity, so does h. A result in calculus states that

Limit of $\left(1+\frac{1}{x}\right)^{x}$ as $x \to \infty$ is equal to e where e = 2.71828 approximately. This e is the inverse of natural logarithm. That is, for any number w, $e^{\ln w} = w$

Hence as h→ ∞, $\left(1+\frac{1}{h}\right) \rightarrow$ e

Thus theoretically, $Y_t = Y_o e^{rt}$. This is called exponential growth. Exponential growth is the same as compound growth, where compounding is done instantaneously and constantly.

Depicting Growth in infinitesimal terms

Let $y = f\left(t\right)$ where t is time

Take natural logarithms of both sides

$$\ln y = \ln\left(f\left(t\right)\right)$$

Differentiating both sides with respect to time, we get

$$\frac{d}{dt}\left(\ln y\right) = \frac{d}{dt}\ln\left(f\left(t\right)\right)$$

$$= \frac{1}{f(t)} \bullet \frac{df(t)}{dt}$$

Using the fact of logarithmic differentiation and the chain rule (see course on quantitative methods MEC003)

The right hand side above we can recognise as the instantaneous proportionate rate of growth

$$\frac{\vartriangle x}{x} with \vartriangle x \rightarrow 0 = \frac{1dx}{xdt}$$

Usually $\frac{dx}{dt}$, the time derivative of a variable x, is denoted $\dot{x}$.

Time as an independent explanatory variable

You have studied in the course on quantitative methods that if a variable y depends on x, and we postulate a linear relation between y and x, we can depict a statistical relationship under uncertainty as

$y_i = \alpha + \beta x_i + \varepsilon_i$, where i is the observation and α and β are coefficients to be estimated. When events takes place over time, we can show this in two ways. The first way is that we take the usual regression method, and use observations over time, what is called time –series data. Then the subscript will indicate time and is usually denoted by t. The second way is to say that time itself causes the change and can thus be considered like the variable x above, that is, time is considered as an independent variable in regression, as a regressor. This is how we get the concept of trend.

A linear trend relationship can be denoted as:

$$y = \alpha + \beta T + \mu \qquad (1)$$

where T indicates time. One has to appropriately define the origin, that is, the starting year or time period.

Without disturbances, a constant growth series, as we have seen above, is given by the equation.

$$y_t = y_0(1+g)^t \qquad (2)$$

where g = $\frac{(y_t - y_{t-1})}{y_{t-1}}$ is the constant proportionate rate of growth per period

Taking longs of both sides of (2) given

$$\ln y = \alpha + \beta t \qquad (3)$$

where $\alpha = \ln y_0$ and $\beta = \ln(1+g)$

If we think that a series has a constant growth rate, plotting the log of the series against time provides an easy measure of growth. If the series is approximately linear, (3) can be fitted by least squares, regressing the log of y against time. The resultant slope coefficient then provides as estimates $\hat{g}$ of the growth rate.

The β coefficient of (1) represents the continuous rate of change $\partial \ln y_t / \partial t$, whereas g represents the discrete case.

Some Concepts Of Equilibrium Growth

An economy is said to be experiencing a **steady-state growth** if all variables are growing at a constant proportional rate or not at all. These rates may be different for different variables. An economy is said to be experiencing **balanced growth** if all variables are growing at the same constant proportional rate. Suppose in an economy, output per worker continues to grow at a constant 3%, savings rate continues to grow at 20%, capital per worker continues to grow at 8%, then this economy is showing steady state growth; if an economy is such where per-worker output, savings rate, capital per worker all grow at say 10 %, then this economy is experiencing balanced growth.

Kaldor's Stylised Facts about Economic Growth

Nicholas Kaldor, on the basis of his studies, presented in 1959 and 1961 works some "stylised facts" which he said could be observed with regularity, in the modern era, which we merely mention :

1) In the long run, per capita output grows at a positive rate which shows no signs of diminishing.

2) The capital-output ratio shows no perceptible upward or downward trend, that is, in the long run it remains roughly constant.

3) The return to physical capital, that is, the profit rate, shows no perceptible upward or downward trend, that is profit/capital remains largely constant.

4) There is a great variety of growth rates of per capita income income across the world, that is, the rates of per capita growth vary substantially across nations.

Facts 1 and 2 together imply that

5) In the long run the capital labour ratio grows at a positive rate which shows no signs of diminishing.

Facts 2 and 3 together imply that

6) The income shares of labour and physical capital remain constant in the long run, and this, in turn implies that wage rate shows a rising trend.

3.3 THE SOLOW MODEL

Let us begin our study with the basic neo-classical model. The model has certain assumptions which we state now and then set out the basic structure of the model.

3.3.1 Assumptions of the Solow Model

1) The economy produces one composite good which can either be consumed or accumulated as a stock of capital. This is a simplified picture of reality. while we do not deny that lots of goods are produced in the economy, we consider, for purposes of building a model – a model, after all, is a parable or a fable-- only one 'composite' or 'aggregated' good.
2) Labour supply is homogeneous. In other words, we do not distinguish between workers with different skills or between say, blue- and white-collared workers.
3) There is a stock of capital which has been accumulated from the past. This capital and the labour are the factors of production, inputs to the production process.
4) The production function exhibits constant returns to scale. This means that if labour and capital are increased by a certain proportion, say λ, output increases by the same proportion λ. If labour and capital are doubled, output is doubled. Thus there is an aggregate production function which is continuous and which displays constant returns to scale.
5) The labour force grows at an exogenously given growth rate $g_L = n$. Thus labour force at time t is equal to $L_t = L_0e^{nt}$.
6) People save a constant proportion of Income. If S denotes saving then
7) S = sY. This assumption is the same as in the Harrod-Domar model. Some people feel that Solow made this deliberately to make a comparison with Harrod-Domar model.
8) There is no foreign trade
9) The government does not intervene in the economy; there are no taxes or government purchase

3.3.2 Structure of the Model

Since we have assumed there is no depreciation, we consider the model here in the absence of depreciation. Later on, we shall discuss the case when there is depreciation of capital. To begin with consider the aggregate production function.

$$Y=F(K,L)$$

We have assumed that there are constant returns to scale. This means that if K & L are increases by a proportion λ , Y increases by the same proportion. The function F is homogenous of degree one.

$$Y=F(\lambda K,\lambda L) \text{ for all } \lambda>1$$

For simplicity, let $\lambda = {}^{1}\!/\!_{L}$

Then $\dfrac{Y}{L} = F\left(\dfrac{K}{L}, \dfrac{L}{L}\right)$

Or $\dfrac{Y}{L} = F\left(\dfrac{K}{L}, 1\right)$

Let in denote quantities divided by L, by lower-case letters

So y=F(k,1)

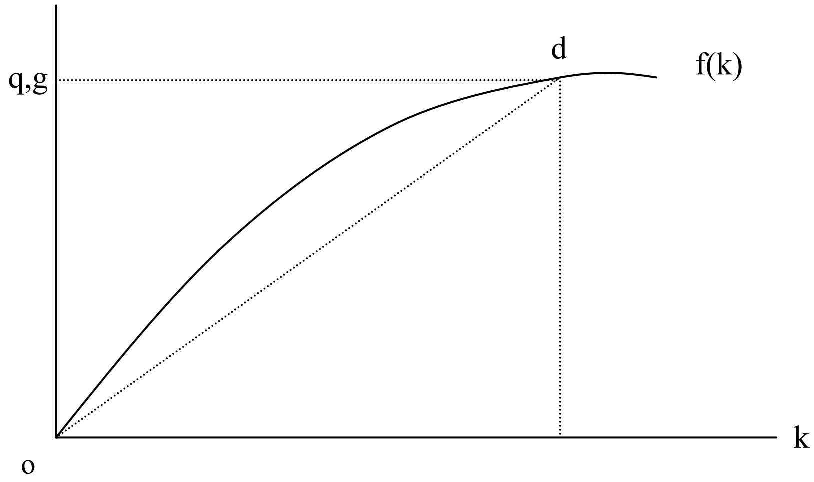
Or y=f (k)

This gives output per person as a function of capital labour ratio.

Sometimes income is used synonymously with output. Denoting output by Q and output-labour ratio by q, we have

q = f(k)

If we draw a picture of above the relationship we can show it as follows:



A ray od to any point d on the curve has a slobe that gives the ratio of output to capital. This is because the of this ray is $= \dfrac{Q/L}{K/L} = \dfrac{Q}{K}$

This is the is verse of capital output ratio v. Each point on the production function in the slow model is that ………….Harrad-Domar model, l is not fixed or exogenous different points on the production function show different, capital output ratios.

Equilibrium Growth

We have $k = K/L$

Taking natural logarithms (denoted by Ln), we get

$\ln(k) = \ln\left(\dfrac{K}{L}\right)$

$$\ln k = \ln \frac{K}{L}$$

or, $\ln k = \ln K - \ln L$

Differentiating with respect to figure, gives the proportional growth ratio:

$$\frac{d}{dt}(\ln k) = \frac{d}{dt}(\ln k) - \frac{d}{dt}(\ln L)$$

$$\frac{dk}{dt}.\frac{1}{k} = \frac{dK}{dt}.\frac{1}{K} - \frac{dL}{dt}.\frac{1}{L}$$

$$or\ \hat{k} = \hat{K} - \hat{L}$$

Now $\frac{dK}{dt}$ on the right hand side is equal to investment I

Investment = facing in equilibrium. So

$$\frac{dK}{dt} = \dot{K} = S$$

$S = SQ_t$ (as we have assumed)

$So \frac{dK}{dt} = sQ_t$ (t is the subscript denotes Q at a point of time)

The Second term on the right hand side is $\hat{L} = \frac{dL}{dt}.\frac{1}{L}$ which shows the proportional growth rate of labour. Which we have denoted n. So our equation

$$\hat{k} = \hat{K} - \hat{L}$$

can be written

$$\hat{k} = \frac{sQ}{K} - n$$

Dividing Q and K by L we get

$$\hat{k} = \frac{sq}{k} - n = \frac{sf(k)}{k} - n \text{ ....(A)}$$

This gives $\hat{k}$, the rate of growth of k, is term of k itself.

Equation (A) is the fundamental equation of the Solow model.

The equilibrium value of k is the one for which $\frac{dk}{dt}.\frac{1}{k} is - 0,\ i.e.\ \frac{dk}{dt} = 0$ or where k, once it reaches that value, does not change. Setting $\hat{k} = 0$ in the equation above, we get

$$\hat{k} = 0 = \frac{sf(k^*)}{k^*} - n$$

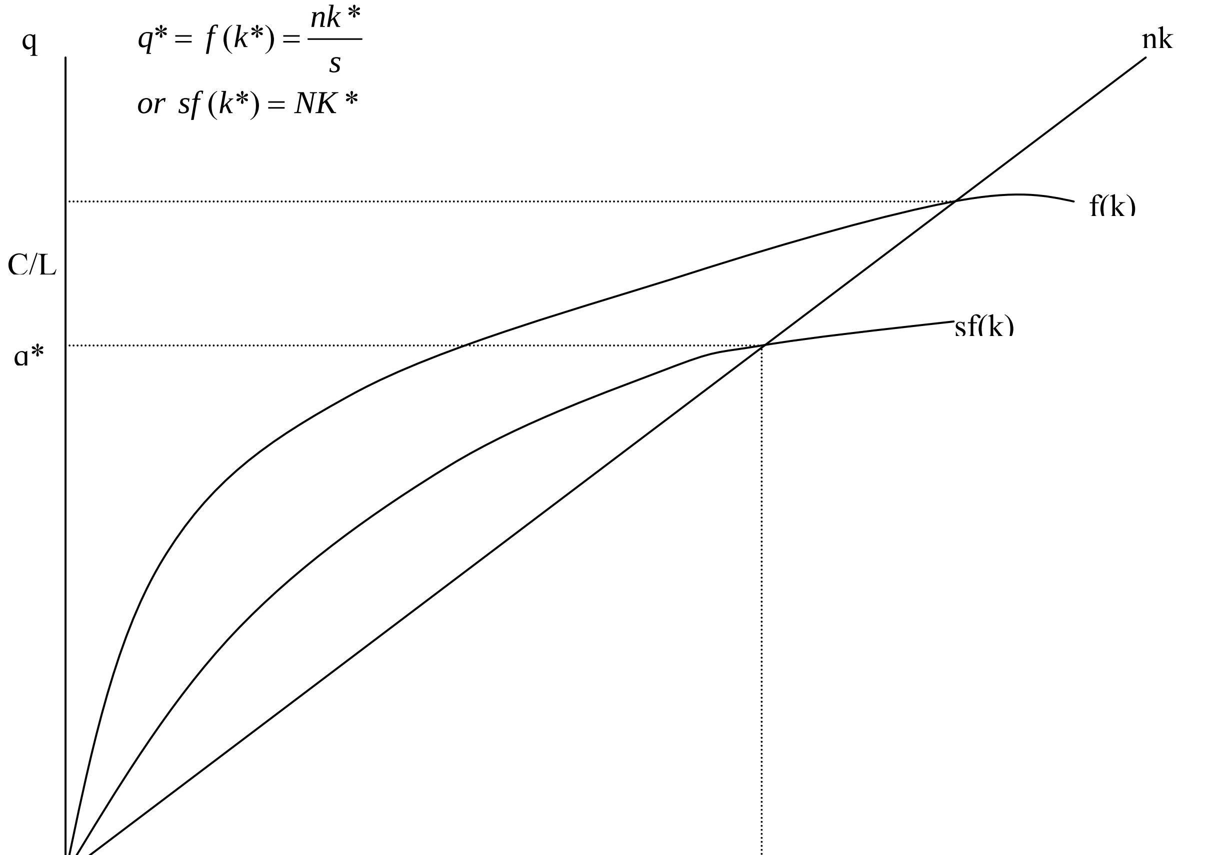
$$or\ \frac{sf(k^*)}{k^* = n}$$

where an * above k denotes its equilibrium value.

The equilibrium q value is obtained as

$$q^* = f(k^*) = \frac{nk^*}{s}$$

$$or\ sf(k^*) = NK^*$$



At any point to the left of k*, where k<k*,

$$f(k) \rangle \left[(n/s)k \right]$$

This implies

$$\frac{sf(k)}{k} \rangle n$$

And from equation (A), we can sea that in this case $\hat{k} \rangle 0,$ which mean that when $k \langle k^*, \hat{k}\, increase$

Similarly we can show that whenever k>k* (to the right of k*), $\hat{k}$ is rising and where k>k*, $\hat{k}$ is falling. Thus k* is a stable equilibrium point.

In equilibrium, when k equals k*, q reaches an equilibrium q*. As q* is a constant,

$$\frac{d\theta}{dt}.\frac{1}{t} = \frac{dL}{dt}.\frac{1}{t} = n$$

The economy thus converges to a steady state growth where $\hat{\theta} = \hat{K}$ and capital output ratio v is constant. However, $\hat{\theta}\, and\, \hat{K}$ are not greater than $\hat{L}$ but equal to it.

The equilibrium condition in the Solow model

$$\frac{sf(k^*)}{k^*} = n$$

can be written as

$$\frac{n}{s} = \frac{f(k)}{k} = \frac{q}{k} = \frac{Q/L}{K/L} = \frac{Q}{K} = \frac{1}{V^*}$$

Where V is the capital output ratio.

So we have $n = \frac{S}{V^*}$, the Harrod-Domar condition for balanced full employment growth. However the Solow model allows V to vary and explains how the economy will turn toward a growth bath along with the Harrod-Domar condition is and there in the Solow model, the capital output ratio V* emerges as an equilibrium value, and not as a necessary technology assumption.

Consumption in the Solow Model

We know that in a closed economy with no government intervention in equilibrium

$$Y = C + I$$

Where Y is aggregate output , C is aggregate consumption and I is investment.

Writing in per-worker ………., we have

$$\frac{Y}{L} = \frac{C}{L} + \frac{I}{L}$$

We know $\frac{Y}{L} = y = f(k)$

So $f(k) = \frac{C}{L} + \frac{I}{L}$ …………………………………..(B)

Now consider capital labour ratio

$k = K/L$

We have seen $\frac{dk}{dt}.\frac{1}{k} = \frac{dK}{dt}.\frac{1}{k} - \frac{dL}{dt}.\frac{1}{L}$

Or $\hat{k} = \hat{K} - \hat{L}$

Where $\wedge$ denotes proportional growth rate. We had already denoted $\hat{L}$ *by* n

So we have

$\hat{k} = \hat{K} - n$

or $\frac{\dot{k}}{k} = \frac{\dot{K}}{k} - n$ where $\dot{X}$ *denotes* $\frac{dX}{dt}$ Multiplying both sides by K/L we get

$\frac{\dot{k}}{k}.\frac{K}{L} = \frac{\dot{K}}{K}.\frac{K}{L} - \frac{nK}{L}$

or $\dot{k} = \frac{\dot{K}}{L} - nk$

Alternatively putting it;

$\frac{\dot{K}}{L} = \dot{k} + nk$ …………………………………………..(C)

Since one of the assumptions we had made was that there is no depreciation, hence

$\dot{k} = \frac{dK}{dt} = I$

So we may write equation (C) as

$\frac{I}{L} = \dot{k} + nK$

In equation (B) we can replace $\frac{I}{L}$ by the right hand side of the above equation. Equation (B) then becomes

$f(k) = \frac{C}{L} + \dot{k} + nK$ …………………………………………..(D)

This equation states the following: Output per worker (since we are taking θ as equal to Y and hence q=y) is put to three uses which are shown on the right hand side. First, consumption per worker $\frac{C}{L}$; a portion of investment *nk*, that maintains the capital labour ratio constant in the face of growing labour force; and a portion of investment, $\dot{k}$ which increases the capital labour ratio. When capital goods increase faster then the increase in labour, so that the capital-labour ratio rises, it is called capital deepening, while when capital goods rise merely to keep pace with the rise in labour force so that the capital labour ratio remains constant, it is called capital widening. Thus output per worker, in equation D, is divided among consumption per worker, capital deepening and capital widening.

We can arrive at equation D by a different route, from our fundamental equation of the Solow Model.

Recall that the fundamental equation, equation (A) is

$$\hat{k} = \frac{sf(k)}{k} - n = \frac{sq}{k} - n$$

Since $\hat{k} = \frac{\dot{k}}{k}$, hence multiplying the above equation throughout by k, we get

$$\dot{k} = sq = nk$$

Recall our assumption that q = y. We have

$$\dot{k} = sy - nk$$

Since $y = \frac{Y}{L}$, we have $\dot{k} = \frac{sY}{L} - nk$

We had made the assumption that S = sY

Now in equilibrium S =Y-C = I

Hence we have $\dot{k} = \frac{Y}{L} - \frac{C}{L} - nk$

Now switching back in rotation f(k) for $\frac{Y}{L = y}$, we have

$$\dot{k} = f(k) - \frac{C}{L} - nk$$

or $f(k) = \frac{C}{L} + \dot{k} + nk$

which is equation (D).

What are the basic proposition and conclusion that we get from the Solow model? Does it provide some guidelines for studying the growth trajectories of actual economies? We give below some theoretical conclusions that emerge from the Solow model.

First, given the assumptions as stated earlier, there exists a steady state (balanced growth) solution for the model. The balanced growth solution is stable. Stability is there in that whenever the initial values of all the variables, the economy eventually reverts to the steady state equilibrium value of y and k. We have already seen this from the diagram.

The second conclusion we get that the balanced rate of growth (all variables grow at the same rate) is the constant exogenous grow the of labour force, which is *n*.

3.3.3 A Comparison with the Harrod-Domar Model

The first basic difference that the Solow model has with the Harrod Domar model is that in the Solow model the capital output ratio is not exogenously fixed. It is endogenously determined. the second difference the process of adjustment in the variables is different in the two models. In the HD model, there are two knife-edges: the balance between the actual and warranted rates of growth, and between the warranted and natural rates of growth. Solow’s model does not address this question, and instead assumes that planned investment equals planned savings at all times. This is, of course, again because s, v, n were all fixed in the HD model, whereas in the Solow model, there is a spectrum of values that that v, that is, the capital output ratio can take, and adjust to that value at which the warranted rate of growth equals the natural rate of growth.

Check You Progress 1

1) State and discuss the assumptions of the Solow model.

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2) Describe the structure of the Solow model.

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3) Compare the Solow model with the Harrod-Domar model.

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3.4 SOME APPLICATIONS OF THE NEO-CLASSICAL MODEL

3.4.1 Depreciation of Capital Stock

We know that in absence of depreciation

$\dot{K} = I$ Here I denote grow investment.

Let us now answer that a certain proportion δK of capital depreciates (through wear and ………). We can now write

$$I = \dot{K} + \delta K$$

Where I now is net investment and δ is the constant rate of depreciation of capital stock.

Dividing by L, we obtain

$$\frac{I}{L} = \frac{\dot{K}}{L} + \frac{\delta K}{L} \quad \text{……………………………………………..(E)}$$

We know from equation (C), which we …………….earlier, that

$$\frac{\dot{K}}{L} = \dot{k} + nk$$

Substituting this expression for $\frac{\dot{K}}{L}$ into equation (E), we obtain

$$\frac{I}{L} = \dot{k} + nk + \delta k$$

$$or \frac{I}{L} = \dot{k} + (n+\delta)k$$

Writing $\frac{I}{L}$ as $\frac{S}{L}$ (in equation) and they as sy or sf(k)

We have $sf(k) = \dot{k} + (n+\delta)k \;\; or \;\; \dot{k} = sf(k) - (n+\delta)k$

Or $\dot{k} = \frac{sf(k)}{k} - (n+\delta)$

This is a modified form of the fundamental equation of the Solow model. The basic analysis that we studied, carrier over for the case of depreciating capital; we merely need to replace n by (n+δ).

3.4.2 Variable Savings Rate

We can use the Solow model to look at the situation where the saving rate is variable. Suppose the saving rate increases from s_1 to s_2. this means that the *sf(k)* curve shifts upwards. This means the sf(k) curve will cut the nk curve at a new point which is higher and to the right of the earlier point of intersection. This means equilibrium k and y will both rise. So an increase in the rate of savings does push up the output per man in the Solow model. But it is also important to note that in the Solow model, the savings rate is a key determinant of the level of capital per person . if the saving rate is high the economy will have a large capital stock and output. But saving has only a temporary effect on the *growth* in output per person. The economy will grow only till the economy reaches a new equilibrium capital per person level.

The above observation that a *permanent* change in the savings rate has only a *temporary* effect on the economy's *growth rate* is called the Solowian paradox of thrift. This point is important to note since it is often suggested that developing nations should raise their savings rate as far as possible. this is supposed to push up the growth rates of per capita income. This proposition is present in the theories of development economists like Nurkse, Rostow and Lewis. You will study these theories in block 5. Here you should keep in mind that increasing the saving rate would push up growth rates only temporarily. The basic lesson of the Solow model is that permanent increase in the growth rate of per capita income comes about only through a change or improvement in technology.

3.4.3 Population Growth

We can now examine in the Solow model the effect of a growth in population. We have seen that capital accumulation by itself cannot explain long-term growth. This can come about through technical progress. The other source is a growth in population.

Let us suppose that the population increase at a rate n. what effect does this have on the steady state growth? Once the number of workers rises, this will cause capital per worker k to fall.

We know that $\triangle k = sf\left(k\right) - nk$. So increase in population (not the rate, but the *level*) reduces k. of course, we had seen this while studying the basic model in section 3.2. the steady state level of k is determined from the pint where the curve sf(k) cuts the line nk. Dropping a perpendicular from this point to the horizontal axis (which measures k) gives us the steady state value k^*. of course, you have come across this diagram in section 3.3.

Let us now see what happens when the rate of population itself changes, that is, n itself undergoes a change. Let the rate of population growth increase from n_1 to n_2. This means that the nk line will tilt upwards. If the sf(k) curve remains the same the new nk line will cut the sf(k) curve to the left of the earlier intersection point. This results in the steady state capital per worker k^* to fall. Since y = f(k), a reduction in k^* results in y falling. Thus in the Solow model, if the *rate of growth* of population rises, output per worker will fall. This has lessons for developing nations. These nations should not allow the rate of population to increase as this may have detrimental effect on the per-worker output.

3.5 MONEY IN THE NEO-CLASSICAL GROWTH MODEL

James Tobin, in a paper published in 1965, presented a simple model of monetary growth which follows the Solowian model. Tobin had of course, earlier, independently developed a growth model in l955 which was similar to the Solow model but had the existence of government debt (net "outside" wealth). For our analysis, we assume that there is only one such type of outside wealth: money. Let money yield a certain rate of return, which we shall denote as R (we are concerned with a broad aggregate for money).

In the original Solow model, physical capital was the only form of wealth that existed. However, money is an important alternative store of wealth. Tobin extended this to bring in outside government wealth in the form of money. How does this change the Solow model?

Tobin used the simple quantity theory of money MV = PY. Assuming the velocity of money is constant, the growth in money equals the difference between the inflation rate and growth in nominal income.

$$\hat{M} = \hat{P} - \hat{Y}$$

where ^ represents the proportional growth rate of a variable x. Rewriting this, we see that the rate of inflation is merely the difference between the rate of money growth and the rate of output growth. The greater the difference, the higher will be the inflation rate. Note the important implication that if money stock is constant ($M^{\wedge}$ = 0), then prices will fall at the rate of growth of output $Y^{\wedge}$ and real money balances, M/P, will grow at that same rate.

Let real wealth (A) be held in two assets, real money (M/P) and physical capital (K), in a proportion such that:

$A = \alpha(M/P) + (1-\alpha)K$

where α lies between 0 and 1. The value of α is determined through portfolio selection by households. The important thing Tobin did was to consider the real rate of return froom money as the difference between the nominal rate of interest and inflation rate. He suggested that people make a comparison between R and the marginal product of K. People do this comparing and in the economy it is determined how much wealth will be held and how much physical capital. This has implications thrugh the function f(k) on output growth. Thus in situations of inflation, people may hold less M/P because real value of money falls, people might hold more K. this leads to a fall inmarginal productivity of capital

D. Levhari and Don Patinkin (1968) and Harry. Johnson (1967) pointed out that a limitation of Tobin's model is the fact that it treats money solely as a store of value and, in effect, does not consider its use in overcoming transactions costs, etc. they suggested extensions to the Tobin model where money directly enters the utility function.

We have discussed the Tobin model. An extension of the Tobin model was provided by Sidrauski. However, we do not present the Sidrauski model here as it beyond the scope of our discussion at this point. It uses concepts and tools that you will learn only in block 3 in the unit on optimal growth. You will meet Sidrauski's model when you come to that unit.

Check Your Progress 2

1) Discuss the effect of a change in the savings rate using the Solow model.

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2) Discuss the effect of a change in (a) the level of population and (b) the rate of growth ogf population using the Solow model.

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3) Briefly state how Tobin brings in money in a growth model.

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3.6 CONVERGENCE AND POVERTY TRAPS

In this section we look at two more important applications of the neoclassical model. These have some bearing on development economics, which means studying the development of the poorer nations. So you can read these, and when you come to block 4, and unit 18 in block 5

3.6.1 Convergence

If you look at the Solow model carefully, it might seem to you that the model suggests that the growth rates of different nations would tend to converge over time, that is, towards the same rate of growth. We can think of three reasons why the Solow model suggests this behaviour. First, the Solow model suggests the rate of return on capital is higher in countries with less capital per worker. Hence capital ought to flow from the rich nations to the poorer nations. Secondly, the Solow model predicts that countries eventually would converge to their **balanced growth** path. Finally, while there may be initial differences in incomes among countries because some countries have better stock of knowledge, eventually, as the poorer countries gain access to the latest technologies, such differences in incomes will shrink.

There are two types of convergence that is usually studied, absolute or unconditional convergence and conditional convergence. Absolute convergence states the following: suppose there is a group of countries. Suppose all these countries have access to the same technology *f(.)*, and have the same population growth rate n, have the same propensity to save, that is, the same saving rate s, but differ, and this is the *only* point of difference, in their initial capital-output ratio. Then the Solow model suggests that all countries would converge to the same equilibrium steady state capital labour ratio, output per capita and consumption per capita (k^*, y^*, c^*) and the same growth rate n.

To understand this further, suppose that there are two countries, one rich and one poor. Supoose the poor country has capital labour ratio k_1 and the rich country has k_2.. Since $k_1$is < k_2 the absolute convergence predicts that both countries will ultimately converge to a level k^*, between k_1 and k_2. This means the poor country will grow much faster as its capital and output grow faster than n, while in the rich nation the capital and output will grow slower than n. in other words, since $k_1 < k_2$, marginal product of k, that is, capital relative to labour is higher in the poor country than in the rich one, and thus the poor country will accumulate more capital and grow at a faster rate than the rich one.

Conditional convergence is invoked in light of the fact that in the real world, rich and poor nations do not seem to be converging to a steady state growth rate, as implied by the Solow model. Conditional convergence states that if nations are endowed with the same technological possibilities and population growth rates but differ in savings propensities that is, savings rate and the initial capital-labour ratio, then there will still be convergence to the same growth rate but not necessarily at the same equilibrium capital-labour ratio. This is due to the Solowian paradox of savings that we had outlined in sub-section 3.4.2. thus the conditional convergence hypothesis suggests that nations can differ in their capital-labour ratios and thus differ in their consumption per capita, but as long as they have the same population rate n, then all their *level* variables (as opposed to per capita) capital, output, consumption – will eventually grow at the same rate. Of course, we usually see the absence of even conditional convergence, but that may be because nations differ in their population growth rates.

Let us discuss the concept of convergence in some detail. If we consider unconditional convergence, it might appear that nothing much of great importance is being talked about. After all if we assume that parameters like savings rate, population growth, capital depreciation are similar. It might seem common sense to argue that this is only to be expected. But note that the hypothesis of unconditional convergence does not assume that the initial level of per capita income or the capital stock is the same. If the parameters governing the evolution of the economies are similar the initial levels do not matter.

A well-known study to examine the empirical validity of the unconditional convergence hypothesis was conducted by William Baumol and published in a 1986 paper. Baumol used data compiled by noted economic historian Angus Maddison. Baumol studied a set of 16 countries that are among the richest today. Baumol plotted the 1870 per capita incomes for these countries on the horizontal axis and plotted the growth rate of per capita incomes of these countries over the period 1870-1979 on the vertical axis. If the unconditional hypothesis were true, we would get an inverse relationship between initial per capita income *level* and *rate of growth* of per capita income. Baumol indeed found a downward sloping scatter plot, which showed a negative relationship. Baumol also ran a regression where the dependent variable was the difference of the natural logarithms of the per-capita incomes of 1979 and 1870 and the dependent variable was the natural logarithm of the per capita income of 1870. the regression can be represented as

$\ln\left[\left(\frac{Y}{N}\right)\right] \ln\left[\left(\frac{Y}{N}\right)_{i,1979}\right] - \ln\left[\left(\frac{Y}{N}\right)_{i,1870}\right] = a + b\ln\left[\left(\frac{Y}{N}\right)_{I,1870}\right] + \varepsilon_i$ where ln (Y/N) is log of income per person and ε is an error term. The estimated values of a and b were 8.457 and (minus)0.995 respectively. Since b is almost equal to –1, it shows almost perfect convergence.

Baumol's findings were challenged by Bradford De Long on two grounds. First, De Long pointed out that there were *sample selection bias* or, bias in the selection of the sample itself. When Baumol carried out the study, the countries chosen in his sample were already rich. In 1870, there was no way of knowing which countries were going to grow fast later. Ex-post knowledge cannot be a basis of ex-ante prediction. Historical data is constructed retrospectively. Those countries for which data is available over a long period are likely to be those that *have* grown rapidly in the past 100 years or so, and are industrialized today. On the other hand, countries that were already rich hundred years ago are likely to be included even if there subsequent growth has not been so fast.

The second point on which De Long challenged Baumol's findings is measurement error. Estimates of data for 1870 may not be precise. Those countries whose 1870 per capita income was overstated actually had grown faster than had been estimated. Those countries for which 1870 per capita income was understated had grown slower than had been measured. Either way, there was a bias towards showing convergence.

3.6.2 Poverty Traps

Empirically, the convergence hypothesis has not held up very well. The neo-classical model has not been very successful in showing why rates of growth differ across nations. One fact was staring everyone in the face: many rations of the world were poor. In a sense, what was going on was the exact opposite of what was suggested by the convergence hypothesis: some countries were showing stagnant growth, while others were progressing very fact. This idea that the poorer nations were actually caught is a trap has come to be called poverty trap. It is a trap because in spite of efforts, these nations stay at a low-level 'equilibrium'.

There are two types of poverty traps: technological-induced and population induced. They can both be demonstrated in the Solow system.

a) **Technological trap:**

If we consider the Solow production function y = f (k), or $Y/L = F\left(K/L\right)$, and suppose there is a certain range where for certain values of L, K, the production function exhibits increasing returns to scale, then there will be multiple equilibrium. For certain values of k, say k* if the economy grows at a level lower than k*, the economy will slump back towards a very low income and output level because the level k* may show a unstable equilibrium.

The idea behind technological trap is that had such a hypothetical country received an initial injection of capital so as to give a value of k larger k*, it would have been pushed over the level. This idea is sometimes called the 'Big push Theory' of which you will read later in block 4. The idea of poverty trap being caused by low savings, low technology is sometimes called the vicious circle of poverty, about which too, you will read in block 4.

b) **Population Trap**

The second way in which poverty traps can arise is induced by population growth. In the neo-classical model, the rate of population growth was given exogenously. However for classical writers population growth was given endogenously. Robert Malthus in his 1798 work, of which you will study in block 4, suggested that the rate of growth of population depends on per capita income. As per capita income rises, the rate of population growth rises ever faster. This has come to be knows as the theory of demographic transition.

We can bring in the idea of demographic transition into the neo classical model. Recall that in the neo classical model population grows exogenously at a rate n. Now suppose population growth rate is dependent on y, as suggested by the theory of demographic transition. We know that y is a function of capital per person. $y = f(k/l)$. Thus n, the population growth rate indirectly becomes a function of the capital labour ratio:

$$n = g\left(k/l\right)$$

We may think of some interval $k_2 - k_1$ where for values of k below k_1, n is < 0, but for values of k in the range $[k_2, k_1]$ n is >0; again for values of k > k_2, n may again become <0. Historically, in olden time in societies where k was below k_1, population was lost through wars, disease etc.

In our analysis, consider the range $k_2 - k_1$. Even here, n, although >0, can itself increase or fall. In other words, although the population is increasing, the rate of increase may itself vary. The idea is that, this leads to a situation where the saving (or investment) curve changes shape and may turn out to be S shaped. Then, we may have multiple equilibrium points of k, most of which are unstable: any movement from these points pushes the system farther away rather than bringing it back into equilibrium. We have considered the interval (or range of values between) $[k_2, K_1]$. Supposing we have two equilibrium points k_a and k_b. Let k_a lie between k_1 and k_2, i.e. $k_2>k_a>k_1$. Let k_b be greater than k_2. Here k_a is the stable equilibrium while k_b is the unstable one. n is the stable equilibrium which creates the problem here. For any value of k leather k_2, the economy is pulled back to equilibrium level k_a. Only if an injection of capital is given which pushed the k level above k_b (where the equilibrium is unstable) will the k level be given a "Big Push" and sent to higher and higher values and thus raising the level of y via the f-function.

We mentioned the demographic transition, which roughly says that n, depends on y and *k/l*. But in the last century, due to advances is health care and medicine, low y did not necessarily lead to low n. As death rates dropped, population increased. On the other hand sub-Saharan African nation like Ethiopia did see n very low (other nations are under populated) due to very low levels of y.

Check Your Progress 3

1) Explain the meaning of absolute and conditional convergence and distinguish between the two.

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2) Explain the concept of poverty trap. Discuss how it can come about.

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3.7 LET US SUM UP

The neo-classical growth model is the central, ‘base-line’ model which has saved on the spring board for almost all most researches into growth theory. The neo-classical model, associated with the name of Robert Solow, presented the first major extension to the Harrod-Domar model, by endogenising the capital output ratio.

In this unit, we learnt some concepts and tools useful for studying growth theory, and then we studied the structure of the neo-classical model, along with the assumptions. We extended the Solow model to consider depreciation and variable savings. We then proceeded to apply the Solow model to look at some applications, like convergence and poverty traps. We also looked at some implications of the Solow model namely that technical progress is more important than capital accumulation, and raising the savings ratio in the short run is not going to help. In the Solow model, technological progress and consequently, output growth is exogenous. When we get to unit 9 on endogenous growth, we will look at models that endogenous these.

3.8 KEY WORDS

Closed Economy: An economy which does not engage in any foreign trade.

Convergence: The basic idea that the natures or region with low level of per capita income will display higher rates of growth and later the levels of income of the different nations or regions will ‘converge’.

Depreciation: Wear and tear in the use of capital goods (machinery, plant, equipment).

Exogenous: Something, the value of which is given from outside. In modelling, a variable the values of which are not determined by solving the model, but is taken as given.

Poverty trap: The idea that low savings rate and low capital, labour ratio lead to an equilibrium situation where poor nations are trapped in poverty, under a ‘Big push’ raises capital. Labour ratio sufficiently.

Rate: Change in the values of a variable with respect to another variable, usually time.

Returns to scale: The proportion by which output changes, when all inputs are increased by a certain equal proportion.

Steady State: A condition in which the key variables are not changing.

3.9 SOME USEFUL BOOKS

Barro, Robert, and Sala- I-Martin. (2004), *Economic Growth* (2nd edition) McGraw-Hill Singapore

Jones, Charles I. (2002), *Introduction to Economic Growth* (2nd edition). W.W.Norton, New York.

Lucas, Robert E. (2002), *Lectures on Economic Growth*. Harvard University Press, Cambridge, Massachusetts.

Romer, David. (2001), *Advanced Macroeconomics* (2nd edition). McGraw-Hill, Singapore.

Solow, Robert M. (2000), *Growth Theory: An Exposition* (2nd edition). Oxford University Press, Oxford and New York.

3.10 ANSWERS/ HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) See Sub-section 3.3.1 and answer.
2) See Sub-section 3.3.2 and answer.
3) See Sub-section 3.3.3 and answer.

Check Your Progress 2

1) See Sub-section 3.4.2 and answer.
2) See Sub-section 3.4.3 and answer.
3) See Section 3.5 and answer.

Check Your Progress 3

1) See Sub-section 3.6.1 and answer.
2) See Sub-section 3.6.2 and answer.