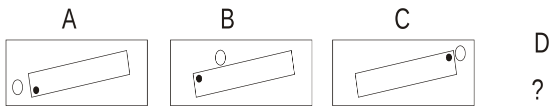
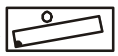
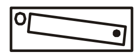
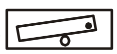
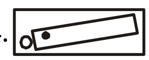
CSIR JUNE 2016 QUESTION PAPER

PART A

Q1. What will be the next figure in the following sequence?



1. 
2. 
3. 
4. 

Q2. For a certain regular solid , number of faces + number of vertices = number of edges +2. For three such distinct (not touching each other) objects, what is the total value of faces + vertices - edges?

1. Two 2. Four
3. Six 4. Zero

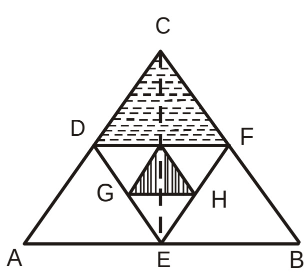
Q3. Abdul travels thrice the distance Catherine travels, which is also twice the distance that Binoy travels. Catherine's' speed is 1/3 of Abdul's speed , which is also 1/2 of Binoy's speed . If they start at the same time then who reaches first?

1. Both Abdul and Catherine 2. Binoy
3.Catherine 4. All three together

Q4. It takes 2 hours for Tiwari and Deo to do a job. Tiwari and Hari take 3 hours to do the same job. Deo and Hari take 6 hours to do the same job. Which of the following statements is incorrect?

1. Tiwari alone can do the job in 3 hours
2. Deo alone can do the job in 6 hours
3. Hari does not work at all
4. Hari is the fastest worker

Q5. Equilateral triangles are drawn one inside the other as shown. What is the ratio of the two shaded areas?



1. 2 : 1 2. $\sqrt{3}:4$
3. 4 : 1 4. 8 : 1

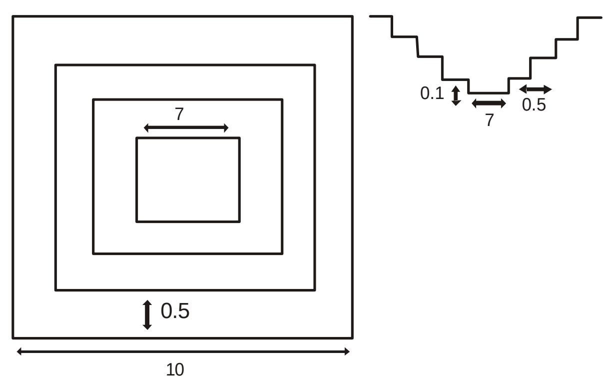
Q6. A train running at 36 km/h crosses a mark on the platform in 8 sec and takes 20 sec to cross the platform .What is the length of the platform?

1. 120 m 2. 280m
3. 40 m 4. 160m

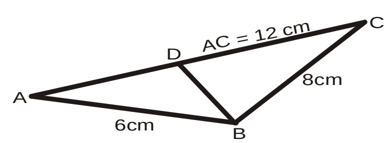
Q7. When a polynomial $f(x)$ is divided by $x-5$ or $x-3$ or $x-2$ it leaves a remainder of 1. Which of the following would be the polynomial?

1. $x^3-10x^2+31x+31$ 2. $x^3-10x^2+31x-29$
3. $x^3-10x^2+31x-31$ 4. $x^3-10x^2+31x+29$

Q8. The diagram (not to scale) shows the top view and cross section of a pond having a square outline and equal sized steps of 0.5 m width and 0.1m height. What will be the volume of water (in m^3) in the pond when it is completely filled?



Q9. D is a point on AC in the following triangle such that $\angle ADB = ABC$. . The BD (in cm) is



1. 8 2. 6
3. 3 4. 4

Q10. A notebook contains only hundred statements as under:

1. This notebook contains 1 false statement
2. This notebook contains 2 false statements.

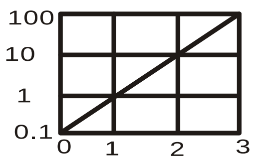
.
.
.

99. This notebook contains 99 false statements100. This notebook contains 100 false statements

Which of the statements is correct?

1. 100^{th} 2. 1^{st}
3. 99^{th} 4. 2^{nd}

Q11. The function $f(x)$ is plotted against x as shown. Extrapolate and find the value of the function at $x = -1$.



1. -0.01 2. -0.1
3. 0.01 4. 0.1

Q12. A frog hops and lands exactly 1 meter away at a time. What is the least number of hops required to reach a point 10 cm away?
1.1 2. 2 3. 3
4. It cannot travel such a distance

Q13. Choose the four digit number, in which the product of the first & fourth digits is 40 and thousand of the middle digits in 28. The digit as the hundreds digit is less than the tens digit.
1. 5478 2.5748
3. 8745 4.8475

Q14. A,B,C,D are points on a circle with AB= 5 cm, BC=12cm, AC=13cm and AD=7cm. Then the closet approximation of CD is
1. 9cm 2.10cm
3. 11cm 4.14cm

Q15. The difference between the square of the ages (in complete years) of a father and his son is 899. The age of the father when his son was born
1. cannot be ascertained due to inadequate data.
2.is 27 years
3.is 29 years
4.is 31 years

Q16. Water is slowly dripping out of a tiny hole at the bottom of a hollow metallic sphere initially full of water ignoring the water that has flowed away, the centre of mass of the system
1. remains fixed at the centre of the sphere
2. moves down steadily as the amount of water decreases
3. moves down for some time but eventually returns to the centre of the sphere
4. moves down until half of the water is lost and then moves up

Q17. A chocolate bar having $m \times n$ unit square title is given. calculate the number of cuts needed to break it completely, without stacking into individual titles.
1. $(m \times n)$ 2. $(m-1)\times(n-1)$
3. $(m \times n)-1$ 4. $(m \times n)+1$

18. An experiment leads to the following set of observations of the variable 'v' at different times't'

t	0	1	2	3	4	5	6
v	5	6.1	9.1	13.7	20.6	30.8	41.4

Allowing for experimental error, which of the following expressions best describes the relationship between t and v?

1. $v \ \alpha \ t^2$ 2. $(v-5) \ \alpha \ t^2$
3. $v = 5t + t^2$ 4. $(v-5) = (t+5)^2$

Q19. A bicycle tube has a mean circumference of 200 cm and a circular cross section of diameter 6 cm. What is the approximate volume of water (in cc) required to completely fill the tube assuming that it does not expand?
1. 600 π 2. 1200 π
3. 3600 π 4. 1800 π

Q20. A person paid income tax at the rate of R% for the first Rs 2 lakhs, and at the rate of (R+10)% for income exceeding Rs 2 lakhs.If the total tax paid is (R + 5)% of the annual imcome, then what is the annual income
1. Rs. 2.5 lakhs 2. Rs 3.0 lakhs
3. Rs.4.0 lakhs 4. Rs 5.0 lakhs

PART B

Q21. Consider the improper Riemann integral $\int_0^x y^{-1/2} dy$.

This integral is:
1. continuous in $[0, \infty)$
2. continuous only in $(0, \infty)$
3. discontinuous in $(0, \infty)$
4. discontinuous only in $\left(\frac{1}{2}, \infty\right)$

Q22. Which one of the following statements is true for the sequence of functions.

$$f_n(x) = \frac{1}{n^2 + x^2}, n = 1, 2, ..., x \in [1/2, 1]?$$

1. The sequence is monotonic and has 0 as the limit for all $x \in [1/2, 1]$ as $n \to \infty$.

2. The sequence is not monotonic but has $f(x)=\frac{1}{x^2}$ as the limit as $n\to\infty$
3. The sequence is monotonic and has $f(x)=\frac{1}{x^2}$ as the limit as $n\to\infty$
4. The sequence is not monotonic but has 0 as the limit

Q23. $\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^n$ equal

1. 1 2. $e^{-1/2}$ 3. e^{-2} 4. e^{-1}

Q24. Consider the interval (-1, 1) and a sequence $\{\alpha_n\}_{n=1}^{\infty}$ of elements in it . Then,

1. Every limit point of $\{\alpha\}$ is in (-1, 1)
2. Every limit point of $\{\alpha\}$ is in [-1,1]
3. The limit points of $\{\alpha\}$ can only be in $\{-1,0,1\}$
4. The limit points of $\{\alpha\}$ cannot be in $\{-1,0,1\}$

Q25. Let $F:\mathbb{R}\to\mathbb{R}$ be a monotone function . Then
1. F has no discontinuities
2. F has only finitely many discontinuities.
3. F can have at most countabley many discontinuities.
4. F can have uncountably many discontinuities.

Q26. Consider the function

$$f(x,y)=\frac{x^2}{y^2},(x,y)\in[1/2,3/2]\times[1/2,3/2]$$

The derivative of the function at (1, 1) along the direction (1,1) is:
1. 0 2. 1 3.2 4. -2

Q27. Given a $n\times n$ matrix B, define e^B by $e^B=\sum_{j=0}^{\infty}\frac{B^j}{j!}$

Let p be the characteristic polynomial of B Then the matrix $e^{p(B)}$ is :

1. $I_{n\times n}$ 2. $0_{n\times n}$ 3. $eI_{n\times n}$ 4. $\pi I_{n\times n}$

Q28. Let A be a $n\times m$ matrix and b be a $n\times 1$ vector (with real entries). Suppose the equation $Ax=b,\ x\in\mathbb{R}^m$ admits a unique solution .Then we can conclude that

1. $m\geq n$ 2. $n\geq m$ 3. $n=m$ 4. $n>m$

Q29. Let V be the vector space of all real polynomials of degree ≤ 10 . Let $Tp(x)=p'(x)$ for $p\in V$ be a linear transformation from V to V. Consider the basis $\{1,x,x^2,...,x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then
1. Trace A = 1 2. det A= 0
3.there is no $m\in\mathbb{N}$, such that , $A^m=0$
4. A has a nonzero eigenvalue

Q30. Let $x=(x_1,x_2,x_3),\ y=(y_1,y_2,y_3)\in\mathbb{R}^3$ be linearly independent.
Let $\delta_1=x_2y_3-y_2x_3,\ \delta_2=x_1y_3-y_1x_3$, $\delta_3=x_1y_2-y_1x_2$. If V is the span of *x, y,* then
1. $V=\{(u,v,w):\delta_1u-\delta_2v+\delta_3w=0\}$
2. $V=\{(u,v,w):-\delta_1u+\delta_2v+\delta_3w=0\}$
3. $V=\{(u,v,w):\delta_1u+\delta_2v-\delta_3w=0\}$
4. $V=\{(u,v,w):\delta_1u+\delta_2v+\delta_3w=0\}$

Q31. Let A be a $n\times n$ real symmetric non-singular matrix . Suppose there exists $x\in\mathbb{R}^n$ such that $x'Ax<0$. Then we can conclude that
1. det (A) < 0. 2. B = – A is positive definite
3. $\exists y\in\mathbb{R}^n:y'A^{-1}y<0$ 4. $\forall y\in\mathbb{R}^n:y'A^{-1}y<0$

Q32. Let $A=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$. Let $f:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ be defined by $f(v,w)=w^TAv.$. Pick the correct statement from below:
1. There exists an eigenvector *v* of *A* such that *Av* is perpendicular to *v*
2. The set $\{v\in\mathbb{R}^2\mid f(v,v)=0\}$ is a nonzero subsapce of $\mathbb{R}^2$
3. If $v,w\in\mathbb{R}^2$ are nonzero vector such that $f(v,v)=0=f(w,w)$ then *v* is a scalar multiple of w.
4. For every $v\in\mathbb{R}^2$, there exists a nonzero $w\in\mathbb{R}^2$ such that $f(v,w)=0$

Q33. Let $P(z),Q(z)$ be two complex non-constant polynomials of degree *m,n* respectively. The number of roots of P(z)=P(z)Q(z) counted with multiplicity is equal to:

1. $\min\{m,n\}$
2. $\max\{m,n\}$
3. $m+n$
4. $m-n$

Q34. Let D be the open unit disc in $\mathbb{C}$ and H(D) be the collection of all holomorphic function on it.Let

$$S=\left\{f\in H(D): f\left(\frac{1}{2}\right)=\frac{1}{2}, f\frac{1}{4},..., f\left(\frac{1}{2n}\right)\right\}=\frac{1}{2_n},...\}$$

and

$$T=\left\{f\in H(D): f\left(\frac{1}{2}\right)=f\left(\frac{1}{3}\right)=\frac{1}{2}, f\left(\frac{1}{4}\right)=f\left(\frac{1}{5}\right)\right.$$

$$\left.=\frac{1}{4},..., f\left(\frac{1}{2n}\right)=f\left(\frac{1}{2n+1}\right)=\frac{1}{2n},..\right\}$$ .Then

1. Both S, T are singleton sets
2. S is a singleton set but $T=\phi$
3. T is a singleton set but $S=\phi$
4. Both S,T are empty

Q35. Let $P(x)$be a polynomial of degree $d\geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty}P(n)z^n$ is

1. 0
2. 1
3. ∞
4. dependent on d

Q36. The residue of the function $f(z)=e^{-e^{1/z}}$ at $z=0$ is

1. $1+e^{-1}$
2. e^{-1}
3. $-e^{-1}$
4. $1-e^{-1}$

Q37. Which of the following statements is FALSE. There exists an integer x such that

1. $x\equiv 23$ mod 1000 and $x\equiv$ 45 mod 6789
2. $x\equiv$ 23 mod 1000 and $x\equiv$ 54 mod 6789
3. $x\equiv$ 32 mod 10000 and $x\equiv$ 54 mod 9876
4. $x\equiv$ 32 mod 1000 and $x\equiv$ 44 mod 9876]

Q38. Let $G=(\mathbb{Z}/25\mathbb{Z})^*$ be the group of units (i.e. the elements that have a multiplicative inverse) in the ring ($(\mathbb{Z}/25\mathbb{Z})$. Which of the following is a generator of G?

1. 3
2. 4
3. 5
4. 6

Q39. Let $p\geq 5$ be a prime. Then

1. $F_p\times F_p$ has at least five subgroups of order p.
2. Every subgroup of $F_p\times F_p$ is of the form $H_1\times H_2$ where H_1,H_2 are subgroups of F_p.
3. Every subgroup of $F_p\times F_p$ is an ideal of the ring $F_p\times F_p$
4. The ring $F_p\times F_p$ is a field

Q40. Let p be a prime number. How many distinct sub-ring(with unity) of cardinality p does the field F_{p^2} have?

1. 0
2. 1
3. p
4. p^2

Q41. Let y_1 and y_2 be two solutions of the problem

$$\left.\begin{matrix} y''(t)+ay'(t)+by(t)=0, t\in\mathbb{R} \\ y(0)=0 \end{matrix}\right\}$$

where a and b are real constants. Let w be the Wronskian of y_1 and y_2. Then

1. $w(t)=0, \forall t\in\mathbb{R}$
2. $w(t)=c, \forall t\in\mathbb{R}$ for some positive constant c
3. w is a nonconstant positive function
4. There exists $t_1,t_2\in\mathbb{R}$ such that $w(t_1)<0<w(t_2)$

Q42. For the Cauchy problem

$$u_t-uu_x=0,\quad x\in\mathbb{R}, t>0$$
$$u(x,0)=x,\quad x\in\mathbb{R},$$

which of the following statements is true?

1. The solution u exists for all t > 0.
2. The solution u exists for $t<\frac{1}{2}$ and breaks down at $t=\frac{1}{2}$.
3. The solution u exists for t < 1 and breaks down at t =1
4. The solution u exists for t < 2 and breaks down t = 2.

Q43. Let $f(x)=x^2+2x+1$ and the derivative of f at $x=1$ is approximated by using the central difference formula

$$f'(1)\approx\frac{f(1+h)-f(1-h)}{2h}\text{ with } h=\frac{1}{2}.$$

Then the absolute value of the error in the approximation of $f'(1)$ is equal to

1. 1
2. 1/2
3. 0
4. 1/12

Q44. Consider the equations of motion for a system

$$\frac{d}{dt}\left(\frac{\partial t}{\partial q_i}\right)-\frac{\partial t}{\partial q_i}=0,\quad i=1,2,3,..,n$$ where

$$L = T - V\left[\begin{array}{l}\text{with } T(t,q_i,\dot{q}_i) \text{ as kinetic energy} \\ \text{and } V(t,q_i) \text{ as potential energy}\end{array}\right], q_i$$

the generalized coordinates, and q_i the generalized velocities. Then the equations of motion in the form as above are

1. necessarily restricted to a conservative system but there is no unique choice of L
2. not necessarily restricted to a conservative system and there is a unique choice of L.
3. necessarily restricted to a conservative system and there is a unique choice of L.
4. not necessarily restricted to a conservative system and there is no unique choice of L

Q45. Let $A=\begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix}, x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$

and $|x(t)| = (x_1^2(t)+x_2^2(t)+x_3^2(t))^{1/2}$

Then any solution of the first order system of the ordinary differential equation $\left.\begin{array}{c} x'(t)=Ax(t) \\ x(0)=x_0 \end{array}\right\}$ satisfies

1. $\lim_{t\to\infty}|x(t)|=0$
2. $\lim_{t\to\infty}|x(t)|=\infty$
3. $\lim_{t\to\infty}|x(t)|=2$
4. $\lim_{t\to\infty}|x(t)|=12$

Q46. Let a,b,c,d be four differentiable function defined on $\mathbb{R}^2$. Then the partial differential equation

$$\left(a(x,y)\frac{\partial}{\partial x}+b(x,y)\frac{\partial}{\partial y}\right)\left(c(x,y)\frac{\partial}{\partial x}+d(x,y)\frac{\partial}{\partial y}\right)u=0$$

is

1. always hyperbolic
2. always parabolic
3. never parabolic
4. never elliptic

Q47. The curve of fixed length *l*, that joins the points (0,0) and (1, 0), lies above the x-axis, and encloses the maximum area between itself and the x-axis is a segment of

1. a straight line
2. a parabola
3. an ellipse
4. a circle

Q48. Consider the integral equation $y(x) = x^3 + \int_0^x Sin(x-t)y(dt, x \in [0,\pi]$. Then the value of y (1) is

1. 19/20
2. 1
3. 17/20
4. 21/20

Q49. Let X and Y be independent and identically distributed random variables such that $P(X=0)=P(X=1)=\frac{1}{2}$. Let $Z = X+Y$ and $W = |X-Y|$. Then which statement is not correct?

1. X and W are independent.
2. Y and W are independent.
3. Z and W are uncorrelated.
4. Z and W are independent.

Q50. Let $X_1 \sim N(0,1)$ and let

$$X_2 = \begin{cases} -X_1, -2 \le X_1 \le 2 \\ X_1, \quad otherwise \end{cases}.$$

Then identify the correct statement.

1. corr $(X_1, X_2)=1$.
2. X_2 does not have N(0,1) distribution
3. (X_1, X_2) has a bivariate normal
4. (X_1, X_2) does not have a bivariate normal distribution

Q51. Let $X_1,..., X_n$ denote a random sample from a $N(\mu,\sigma^2)$ distribution. Let $\mu\in\mathbb{R}$ be known and $\sigma^2(>0)$ be unknown. Let $\chi^2_{n,\alpha/2}$ be an upper $(\alpha/2)^{th}$ percentile point of a χ^2_n distribution. Then a $100(1-\alpha)\%$ confidence interval for σ^2 is given by

1. $\left(\frac{(\sum_1^n X_i^2-\mu^2)}{n\chi^2_{n,\alpha/2}}, \frac{(\sum_1^n X_i^2-\mu^2)}{n\chi^2_{n,1-\alpha/2}}\right)$
2. $\left(\frac{\sum_1^n (X_i-\mu)^2}{(n-1)\chi^2_{(n-1),\alpha/2}}, \frac{(\sum_1^n X_i^2-\mu^2)}{(n-1)\chi^2_{(n-1),1-\alpha/2}}\right)$
3. $\left(\frac{\sum_1^n (X_i-\overline{X})^2}{n\chi^2_{n,\alpha/2}}, \frac{(\sum_1^n X_i-\overline{X})^2}{n\chi^2_{n,1-\alpha/2}}\right)$
4. $\left(\frac{\sum_1^n (X_i-\mu)^2}{n\chi^2_{n,\alpha/2}}, \frac{(\sum_1^n X_i^2-\mu)^2}{n\chi^2_{n,1-\alpha/2}}\right)$

Q52. Let Y_1, Y_2, Y_3 be uncorrelated observations with common variance, σ^2 and expectations given by $E(Y_1)=\beta_1, E(Y_2)=\beta_2$ and $E(Y_3)=\beta_1+\beta_2$, where

$\beta_1+\beta_2$ are unknwon parameters. The best linear unbiased estimator of $\beta_1+\beta_2$ is

1. Y_3
2. Y_1+Y_2
3. $\frac{1}{3}(Y_1+Y_2+2Y_3)$
4. $\frac{1}{2}(Y_1+Y_2+Y_3)$

Q53. Consider a series system with two independent component .Let the component lifespan have exponential distribution with density

$$f(x)=\begin{cases}\lambda e^{-\lambda x}, \lambda>0, x>0\\ 0, \quad otheriwise\end{cases}$$

If n observations $X_1, X_2, ..., X_n$ on lifespan of this component are available and $\overline{X}=\frac{1}{2}\sum_{1}^{n} X_i$, then the maximum likelihood estimator of the reliability of the system is given by

1. $\left(1-e^{-t/\overline{X}}\right)^2$
2. $1-\left(1-e^{-t/\overline{X}}\right)^2$
3. $e^{-2t/\overline{X}}$
4. $1-e^{-2t/\overline{X}}$

Q54. Customers arrive at an ice cream parlour according to a Poisson process with rate 2. Service time distribution has density function $f(x)=\begin{cases}3e^{-3x}, & x>0\\ 0, & x\le 0.\end{cases}$

Upon being served a customer may rejoin the queue with probability 0.4, independently of new arrivals;also a returning customer's service time is the same as that of a new arriving customer. Customers behave independetly of each other .Let X(t) = number of customer in the queue at time . Which among the following is correct

1. $\{(X(t)\}$ grows without bound with probability 1.
2. $\{(X(t)\}$ has stationary distribution given by $\pi_k=\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^k, k=0,1,2,...$
3. $\{(X(t)\}$ has stationary distribution given by $\pi_k=(0.1)(0.9)^k, k=0,1,2,...$
4. $\{(X(t)\}$ has stationary distribution given by $\pi_k=(0.4)(0.6)^k, k=0,1,2,...$

Q55. Hundred (100) tickets are marked 1, 2,...,100 and are arraged at random. Four tickets are picked from these tickets and are given to four persons A, B, C and D. What is the probablity that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with the smallest value (among A,B,C,D)?

1. $\frac{1}{4}$
2. $\frac{1}{6}$
3. $\frac{1}{2}$
4. $\frac{1}{12}$

Q56. Let $\{X_t\}$ and $\{Y_t\}$ be two independent pure birth processes with birth rates λ_1 and λ_2 respectively. Let $Z_t=X_t+Y_t$. Then

1. $\{Z_t\}$ is not a pure birth process.
2. $\{Z_t\}$ is a pure birth process with birth rate $\lambda_1+\lambda_2$.
3. $\{Z_t\}$ is a pure birth process with birth rate $\min(\lambda_1,\lambda_2)$.
4. $\{Z_t\}$ is a pure birth process with birth rate $\lambda_1\lambda_2$

Q57. Let $X_1,..., X_n$ be a random sample from $N(\theta,1)$, where $\theta\in\{1,2\}$. Then which of the following statements about the maximum likelihood estimator (MLE) of θ is correct?

1. MLE of θ does not exist.
2. MLE of θ is $\overline{X}$
3. MLE of θ exist but it not $\overline{X}$
4. MLE of θ is an unbiased estimator of θ

Q58. In the context of testing of statistical hypotheses, which one of the following statements is true?

1. When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1 the likelihood ratio principle leads to the most powerful test
2. When testing a simple hypothesis H_0 against an alternative simple hypothesisis H_1, P[rejecting $H_0 \mid H_0$ is true] $+P$[accepting $H_0 \mid H_1$ is true]=1
3. For testing a simple hypothesis H_0 against an alternative simple hypothesis H_1 ,randomized test is used to achieve the desired level of the power of the test.
4. UMP tests for testing a simple hypothesis H_0 against an alternative composite H_1, always exist.

Q59. Let $\underline{X}\sim N_3\left(\underline{\mu},\Sigma\right)$ where $\underline{\mu}=(1,1,1)$ and

$$\Sigma\begin{pmatrix}1 & 1 & 1\\ 1 & 3 & c\\ 1 & c & 2\end{pmatrix}.$$

The value of c such that X_2 and $-X_1 + X_2 - X_3$ are independent is

1. -2 2. 0 3. 2 4. 1

Q60. A sample fo size $n(\geq 2)$ is drawn without replacement from a finite population of size N, using an arbitrary sampling scheme. Let π_i denote the inclusion probability of the i-th unit and π_{ij} the joint inclusion probability of units i and j, $1 \leq i < j < N$. Which of the following statements is always true?

1. $\sum_{i=1}^{N} \pi_i = n$
2. $\sum_{j=1}^{N} \pi_{ij} = n\pi, 1 \leq i \leq N$
3. $\pi_{ij} > 0$ for all $i, j, 1 \leq i \leq j \leq N$
4. $\pi_i \pi_j - \pi_{ij} > 0$ for all $i, j, 1 \leq i \leq j \leq N$

PART C

Q61. Let $\{x_n\}$ be an arbitrary sequence of real numbers. Then

1. $\sum_{n=1}^{\infty} |x_n|^p < \infty$ for some $1 < p < \infty$ implies $\sum_{n=1}^{\infty} |x_n|^q < \infty$ for any $q > p$.
2. $\sum_{n=1}^{\infty} |x_n|^p < \infty$ for some $1 < p < \infty$ implies $\sum_{n=1}^{\infty} |x_n|^1 < \infty$ for any $1 \leq q < p$.
3. Given any $1 < p < q < \infty$, there is a real sequence $\{x_n\}$ such that $\sum_{n=1}^{\infty} |x_n|^p < \infty$ but $\sum_{n=1}^{\infty} |x_n|^q = \infty$
4. Given any $1 < q < p < \infty$, there is a real sequence $\{x_n\}$ such that $\sum_{n=1}^{\infty} |x_n|^p < \infty$ but $\sum_{n=1}^{\infty} |x_n|^q = \infty$

Q62. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function and $f(x+1) = f(x)$ for all $x \in \mathbb{R}$. Then

1. f is bounded above ,but not bounded below
2. f is bounded above and below but may not attain its bounds
3. f is bounded above and below and f attain its bounds
4. f is uniformly continuous

Q63. Which of the following is/are true?

1. (0,1) with the usual topology admits a metric which is complete
2. (0,1)with the usual topology admits a metric which is not complete
3. [0,1] with the usual topology admits metric which is not complete
4. [0,1] with the usual topology admits metric which is complete

Q64. Let $x_1 = 0, x_2 = 1,$ and for $n \geq 3,$ define $x_n = \frac{x_{n-1} + x_{n-2}}{2}$. Which of the following is/are true?

1. $\{x_n\}$ is a monotone sequence.
2. $\lim_{n\to\infty} x_n = \frac{1}{2.}$
3. $\{x_n\}$ is a Cauchy sequence
4. $\lim_{n\to\infty} x_n = \frac{2}{3}.$

65. Take the closed interval [0,1] and open interval (1/3,2/3). Let K=[0,1]\(1/3,2/3). For $x \in [0,1]$ define $f(x) = d(x, K)$ where $d(x, K) = \inf\{|x - y| \,|\, y \in K\}$. Then

1. $f : [0,1] \to \mathbb{R}$ is differentiable at all points of (0,1)
2. $f : [0,1] \to \mathbb{R}$ is not differentiable at 1/3 and 2/3
3. $f : [0,1] \to \mathbb{R}$ is not differentiable at 1/2
4. $f : [0,1] \to \mathbb{R}$ is not continuous

Q66. Let V be the span of (1,1,1,) and $(0,1,1) \in \mathbb{R}^2$. Let $u_1 = (0,0,1), u_2 = (0,1,0)$ and $u_3 = (1,0,0)$. Which of the following are correct?

1. $(\mathbb{R}^3 \setminus V) \cup \{(0,0,0)\}$ is not connected.
2. $(\mathbb{R}^3 \setminus V) \cup \{tu_1 + (1-t)u_3 : 0 \leq t \leq 1\}$ is connected
3. $(\mathbb{R}^3 \setminus V) \cup \{tu_1 + (1-t)u_2 : 0 \leq t \leq 1\}$ is connected
4. $(\mathbb{R}^3 \setminus V) \cup \{(t, 2t, 2t) : t \in \mathbb{R}\}$ is connected

Q67. Which of the following function is/are uniformly continuous on the interval (0,1)?

1. $\frac{1}{x}$ 2. $\sin\frac{1}{x}$ 3. $x\sin\frac{1}{x}$ 4. $\frac{\sin x}{x}$

Q68. Let V be the vector space of all complex polynomials p with deg $p \leq n.$ Let $T : V \to V$ be the map $(Tp)(x) = p'(1), x \in \mathbb{C}.$ Which of the folloing are correct?

1. dim Ker $T = n$ 2. dim range $T = 1$.

3 dim Ker T = 1. 4. dim range $T = n + 1$

Q69. Let A be an $n\times n$ real matrix. Pick the correct answer(s) from the following

1. A has at least one real eigenvalue
2. For all nonzero vectors $v, w, \in \mathbb{R}^n, (Aw)^T (Av) > 0.$
3. Every eigenvalue of $A^T A$ is a nonnegative real number.
4. $I + A^T A$ is invertible

Q70. Let T be a $n\times n$ matrix with the property $T^n = 0.$ Which of the following is/are true?

1. T has n distinct eigenvalues
2. T has one eigenvalue of multyplicity n
3. 0 is an eigenvalue of T
4. T is similar to a diagonal matrix

Q71. Let $V = \{f : [0,1] \to \mathbb{R} |\ f$ is a polynomial of degree less than or equal to n}.

Let $f_j(x) = x^j$ for $0 \le j \le n$ and let A be the $(n+1)\times(n+1)$ matrix given by $a_{ij} = \int_0^1 f_i(x) f_j(x)dx.$.

Then which of the following is/are true?

1. dim V = n 2.dim V > n.
3. A is nonenegative definite, i.e, for all $v \in \mathbb{R}^n, \langle Av, v\rangle > 0.$
4. det A>0

Q72. Let A be any set. Let $P(A)$ be the power set of A , that is the set of all subsets of $A; P(A) = (B : B \subseteq A\}.$. Then which of the following is/are true about the set P(A)?

1.P(A) = Φ for some A.
2. P(A) is a finite set for some A.
3. P(A) is a countable set for some A.
4. P(A) is a uncountable set for some A.

73. Define f on [0,1] by $f(x) = \begin{cases} x^2 & \text{if x is rational} \\ x^3 & \text{if x is irrational} \end{cases}$.

Then

1. f is not riemann integrable on [0,1]
2. f is riemann integrable and $\int_0^1 f(x)dx = \frac{1}{4}$
3. f is riemann integrable and $\int_0^1 f(x)dx = \frac{1}{3}$
4. $\frac{1}{4} = \int_{0}^{1}{}_{-} f(x)dx < \int_0^{\bar{1}} f(x)dx = \frac{1}{3,}$ where $\int_0^1{}_{-} f(x)dx$ and $\int_0^{\bar{1}} f(x)dx$ are the lower and upper riemann integrals of f

Q74. Consider the real vector space V of polyniomial of degree less than or equal to d . *For* $p \in V$ *define* $\|p\|_k = \max\{| p(0)|,| p^{(1)}(0)|,...,| p^{(k)}(0)|\}.$ *where* $p^{(i)}(0)$ *is the* i^{th} derivative of p evaluated at 0. Then $\|p\|_k$ defines a norm on V if and only if

1. $k \ge d-1$ 2. $k < d$
3. $k \ge d$ 4. $k < d-1$

Q75. Let A,B be $n\times n$ real matrices such that det $A > 0$ and det B<0. For $0 \le t \le 1.$ consider $C(t) = tA + (1-t)B.$ Then

1. C(t) is invertible for each $t \in [0,1].$
2. There is a $t_0 \in (0,1)$ such that $C(t_0)$ is not invertiable
3. C(t) is not invertible for each $t \in [0,1]$
4. $C(t)$ invertible for only finitely many $t \in [0,1]$

Q76. Let $\{a_1,...,a_n\}$ and $\{b_1,...,b_n\}$ be two bases of $\mathbb{R}^n$. Let P be an $n\times n$ matrix with real entries such that $Pa_i = b_i\ i = 1,2,...,n.$ suppose that every eigenvalue of P is either -1 or 1. Let $q = I + 2P$. Then which of the following statements are true?

1. $\{a_i + 2b_i \,|i = 1,2,...,n|\}$ is also a basis of V
2. Q is invertible.
3.Every eigenvalue of Q is either 3 or -1.
4. det Q>0 if det P>0

Q77. Let A be an $n\times n$ matrix with real entries .define $\langle x, y\rangle_A := \langle Ax, Ay\rangle, x, y \in \mathbb{R}^n$. Then $\langle x, y\rangle_A$ defines an inner-product if and only if

1. Ker A = {0}. 2.rank A=n.
3. All eigenvalues of A are positive
4. All eigenvalues of A are non-negative

Q78. Suppose $\{v_1,...v_n\}$ are unit vectors in $\mathbb{R}^n$ such that $\|v\|^2 = \sum_{i=1}^{n} |\langle v_i, v\rangle|^2, \forall v \in \mathbb{R}^n$. Then decide the cor rect statements in the following

1. $v_1 ... v_n$ are mutually orthogonal.
2. $\{v_1 ... v_n\}$ is a basis for $\mathbb{R}^n$
3. $v_1, ..., v_n$ are not mutually orthogonal
4. Atmost n-1 of the elements in the set $\{v_1, ..., v_n\}$ can be orthogonal.

Q79. Let $H = \{z = x + iy \in \mathbb{C} : y > 0\}$ be the upper half plane and $D = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc. Suppose that f is a Mobius transformation which maps H conformally onto D . Suppose that $f(2i) = 0$ Pick each correct statement from below

1. f has a simple pole at z = -2i
2. f satisfies $f(i)\overline{f(-i)} = 1$.
3. f has an essential singularity at z=-2i
4. $|f(2+2i)| = \frac{1}{\sqrt{5}}$

Q80. Consider the function

$$F(z) = \int_1^2 \frac{1}{(x-z)^2} dx, \qquad \text{Im}(z) > 0.$$

Then there is a meromorphic function G(z) on C that agrees with F(z) when Im (z) >0, such that

1. 1, ∞ are poles of G(z)
2. 0,2, ∞ are poles of G(z)
3. 1,2 are poles of G(z)
4. 1,2 are simple poles of G(z)

Q81. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function . Suppose that f = u + iv where u,v are the real and imaginary parts of f respectively. Then f is constant if

1. $\{u(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded .
2. $\{v(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded
3. $\{u(x, y) + v(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded
4. $\{u^2(x, y) + v^2(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded

Q82. Consider the symmetric group S_{20} and it's subgroup A_{20} consisting of all even permutations. Let D be a 7-Sylow subgroup of A_{20}. Pick each correct statement from below:

1. |H| = 49
2. H must be cyclic.
3. H is a normal subgroups of A_{20}
4. Any Sylow subgroup of S_{20} is a subset of A_{20}

Q83. Let R be a commutative ring with unity such that R[X] is a UFD. Denote the ideal (X) of R[X] by I . Pick each correct statement from below :

1. I is prime
2. If I is maximum then R[X] is a PID
3. If R[X] is a euclidean domain, then I is maximal.
4. If R[X] is a PID then it is a Euclidean

84. Consider the smallest topology τ on $\mathbb{C}$ in which all the singleton sets are closed . Pick each correct statements from below.

1. $(\mathbb{C}, \tau)$ is Hausdorff.
2. $(\mathbb{C}, \tau)$ is compact.
3. $(\mathbb{C}, \tau)$ is connected
4. $\mathbb{Z}$ is dense in $(\mathbb{C}, \tau)$

Q85. Let $\{X_\alpha\}_{\alpha \in I}$ be discrete topological spaces and let $X = \prod_{\alpha \in I} X_\alpha$. from the statements given below pick each statement that implies that the product topology on X equals the discrete topology on X

1. I is finite
2. I is countably infinite and X_α are singletons for all but finitely many a
3. I is uncountably infinite and X_α are singletons for all but finitely may a
4. I is infinite and X_α are finite for all a

Q86. Let $A = \{z \in \mathbb{C} \,\|\, |z| > 1\}$ $B = \{z \in \mathbb{C} \,|\, z \neq 0\}$. Which of the following are true?

1. There is a continuous onto function $f : A \to B$
2. There is a continuous one to one function $f : B \to A$
3. There is a nonconstant analytic function $f : B \to A$
4. There is a nonconstant analytic function $f : A \to B$

Q87. Consider the integral $A = \int_0^1 x^n (1-x)^n dx$. Pick each correct statement from below

1. A is not a rational number
2. 0 < A $\leq 4^{-n}$
3. A is a natural number
4. A^{-1} is a natural numBer

Q88. Let G be a finite abelian group of order n . Pick each correct statement from below.

1. If d divides n, there exists a subgroup of G of order d.
2. If d divides n, there exists an element of order d in G
3. If every proper subgroup of G is cyclic, then G is cyclic
4. If H is a subgroup of G there exists a subgroup N

of G such that G/N $\cong$ H.

Q89. Let p be a prime . Pick each correct statement from below. Upto isomorphism

1. there are exactly two abelian subgroup of order p^2
2. there are exactly two group of order p^2
3. there are exactly two commutative rings of order p^2
4. there is exactly one integral domain of order p^2

Q90. Let $f(x) \in \mathbb{Z}[x]$ be a polynomial of degree ≥ 2. Pick each correct statement from below

1. If f(x) is irreducible in $\mathbb{Z}[x]$ then it is irreducible in $\mathbb{Q}[x]$
2. If f(x) is irreducible in $\mathbb{Q}[x]$, then it is irreducible in $\mathbb{Z}[x]$.
3. If f(x) is irreducible $\mathbb{Z}[x]$, then for all primes p the reduction $\overline{f(x)}$ of $f(x)$ modulo p is irreducible in $F_p[x]$
4. If $f(x)$ is irreducible $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{R}[x]$

Q91. Consider the Cauchy problem for the Eikonal equation $p^2 + q^2 = 1; \quad p \equiv \dfrac{\partial u}{\partial x}, \; q \equiv \dfrac{\partial u}{\partial y}$

$u(x, y) = 0$ on $x + y = 1, \; (x, y) \in \mathbb{R}^2$. Then

1. The Charpit's equations for the differential euation are $\dfrac{dx}{dt} = 2p; \; \dfrac{dy}{dx} = 2q; \; \dfrac{du}{dt} = 2; \; \dfrac{dp}{dt} = -p; \; \dfrac{dq}{dt} = -q.$
2. The Charpit's equations for the differential equa tion are $\dfrac{dx}{dt} = 2p; \; \dfrac{dy}{dt} = 2q; \; \dfrac{du}{dt} = 2; \; \dfrac{dp}{dt} = 0; \; \dfrac{dq}{dt} = 0..$
3. $u\left(1, \sqrt{2}\right) = \sqrt{2}.$
4. $u\left(1, \sqrt{2}\right) = 1$

Q92. let H(x) be the cubic Hermite interpolation of $f(x) = x^4 + 1$ on the interval $I = [0,1]$ interpolation at $x = 0$ and $x = 1$. Then

1. $\max_{x \in I} | f(x) - H(x) | = \dfrac{1}{16}$
2. The maximum of $| f(x) - H(x) | = \dfrac{1}{16}$
3. $\max_{x \in I} | f(x) - H(x) | = \dfrac{1}{21}$
4. The maximum of $| f(x) - H(x) | = \dfrac{1}{21}$ is attained at $x = \dfrac{1}{4}$.

Q93. Let $y : \mathbb{R} \to \mathbb{R}$ be a solution of the ordinary differential equation $2y'' + 3y' + y = e^{-3x}, \; x \in \mathbb{R}$ satisfying $\lim_{x \to \infty} e^x y(x) = 0$ .Then

1. $\lim_{x \to \infty} e^{2x} y(x) = 0$
2. $y(0) = \dfrac{1}{10}$
3. y is bounded function on $\mathbb{R}$.
4. y(1) = 0.

Q94. For $\lambda \in \mathbb{R}$, consider the differential equation $y'(x) = \lambda \sin(x + y(x)), y(0) = 1$. Then this initial value problem has:

1. no solution in any neighbourhood of 0.
2. a solution in $\mathbb{R}$ if $|\lambda| < 1$.
3. a solutin in a neighbourhood of 0.
4. a solution in $\mathbb{R}$ only if $|\lambda| > 1$.

Q95. The problem , $\left.\begin{array}{ll} -y''(1+x)y = \lambda y, & x \in (0,1) \\ & y(0) = y(1) = 0 \end{array}\right\}$

has a non zero solution

1. for all $\lambda < 0$.
2. for all $\lambda \in [0,1]$.
3. for some $\lambda \in [2, \infty]$.
4. for a countable number of λ ' 's

Q96. Let $u : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ be a solution of the initial value problem

$$\left.\begin{array}{r} u_{tt} - u_{xx} = 0, \text{ for } (x,t) \in \mathbb{R} \times (0, \infty) \\ u(x, 0) = f(x), x \in \mathbb{R} \\ u_t(x, 0) = g(x), x \in \mathbb{R} \end{array}\right\}$$

Suppose $f(x) = g(x) = 0$ for $x \notin [0,1]$. then we always have

1. $u(x,t) = 0$ for all $(x,t) \in (-\infty, 0) \times (0, \infty)$.
2. $u(x,t) = 0$ for all $(x,t) \in (1, \infty) \times (0, \infty)$
3. $u(x,t) = 0$ for all (x,t) satisfying $x + t < 0$.
4. $u(x,t) = 0$ for all (x,t) satisfying $x - t > 1$.

Q97. Let u be the solution of the boundary value problem

$u_{xx} + u_{yy} = 0$ for $0 < x, y < \pi$

$u(x,0) = 0 = u(x,\pi)$ for $0 \le x \le \pi$

$u(0,y) = 0, u(\pi, y) = \sin y + \sin 2y$ for $0 \le y \le \pi$

Then

1. $u\left(1, \frac{\pi}{2}\right) = (\sinh(\pi))^{-1}\sinh(1)$

2. $u\left(1, \frac{\pi}{2}\right) = (\sinh(1))^{-1}\sinh(\pi)$

3. $u\left(1, \frac{\pi}{4}\right) = (\sinh(\pi))^{-1}\sinh(1))\frac{1}{\sqrt{2}} + (\sinh(2\pi))^{-1}\sinh(2).$

4. $u\left(1, \frac{\pi}{4}\right) = (\sinh(1))^{-1}\sinh(\pi)\frac{1}{\sqrt{2}} + (\sinh(2\pi))^{-1}\sinh(2\pi).$

Q98. Consider the Runge-Kutta method of the form $y_{n+1} = y_n + ak_1 + bk_2$

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + ah, y_n + \beta k_1)$ to approximate the solution of the initial value problem

$y'(x) = f(x, y(x)), y(x_0) = y_0.$

Which of the following choice of a,b, α and β yield a second order method?

1. $a = \frac{1}{2}, b = \frac{1}{2}, a = 1, \beta = 1$

2. $a = 1, b =, a = \frac{1}{2}, \beta = \frac{1}{2}$

3. $a = \frac{1}{4}, b = \frac{3}{4}, \alpha = \frac{2}{3}, \beta = \frac{2}{3}$

4. $a = \frac{3}{4}, b = \frac{1}{4}, \alpha = 1\beta = 1$

Q99. Let $y = y(x)$ be the extremal of the functional

$$I[y(x)] = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx,$$

subject to the condition that the left end of the extremal moves along $y = x^2$, while the right end moves along $x - y = 5.$ Then the

1.shortest dinstance between the parabola and the straight line is $(\frac{19\sqrt{2}}{8})$.

2.slope of the extremal at (x, y) is $(-\frac{3}{2})$

3.point $\left(\frac{3}{5}, 0\right)$ lies on the extremal

4. extremal is orthogonal to the curve $y = \frac{x}{2}$

Q100. A particle of unit mass moves in the direction of x-axiz such that it has the Lagrangian $L = \frac{1}{12}x^4 + \frac{1}{2}xx^2 - x^2$.

Let $Q = x^2 x$ represent a force (not arising from a potential) acting on the particle in the x-direction. If $x(0) = 1$ and $x(0) = 1,$ then the value of x is

1. some non-zero finite value at x = 0
2. 1 at x =1
3. $\sqrt{5}$ at $x = \frac{1}{2}$
4. 0 at $x = \sqrt{\frac{3}{2}}$

Q101. Let $f : [0,3] \to \mathbb{R}$ be defined by $f(x) = |1 - |x - 2||$ where |.| denoteds the absolute value. Then for the numerical approximation of $\int_0^3 f(x)dx$, which of the following statements are true?

1. The composite trapezoid rule with three equal sub-interval is exact.
2. The composite midpoint rule with three equal sub-intervals is exact
3. The composite trapezoid rule with four equal sub-intervals in exact
4. The composite midpoint rule with four equal subintervals is exact.

Q102.The curve, $y = y(x),$ passing throught the point $(\sqrt{3}, 1)$ and defined by the following property (voltera integral equation of the first kind) $\int_0^y \frac{f(v)dv}{\sqrt{y-v}} = 4\sqrt{y},$

whre $f(y) = \sqrt{1 + \frac{1}{y'^2}}$ is the part of a

1. straight line
2. circle
3. parabola
4. cycloid

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Q103. Let (Ω, F, P) be a probability space and let A be an event with P(A)>0. In which of the following cases does Q define a probability measure on (Ω, F) ?

1. $Q(D) = P(A \cup D) \quad \forall\ D \in F$

2. $Q(D) = P(A \cap D) \quad \forall\ D \in F$

3. $Q(D) = \begin{cases} P(A|D), if \quad D \in F \text{ with } P(D) > 0 \\ 0, \quad if \quad P(D) = 0 \end{cases}$

4. $Q(D) = P(D|A) \quad \forall\ D \in F$

Q104. Suppoe X and Y are independent and indentically distributed random variable and let Z=X+Y. Then the distribution of Z is in the same family as that of X and Y if X is

1. normal
2. Exponential
3. uniform
4. binomial.

Q105. Let $X_1, ..., X_n$ be a random sample from the following probability density function

$$f(x;\mu,\alpha) = \begin{cases} \frac{1}{\Gamma(\alpha)}(x-\mu)^{\alpha-1}e^{-(x-\mu)}; & x \geq \mu. \\ 0, & otheriwise \end{cases}$$

Here $-\infty < \mu < \infty$ and α >0. Then which of the following statement are correct?

1. The method of moment estimators of neither α nor μ exist.
2. The method of moment estimators of α exist and it is a consistent estimator of α
3. The method of moment estimators of μ exist and it is a consistent estimator of μ
4. The method of moment estimator of both α and μ exist, but they are not consistent.

Q106. Suppose X is a random variable with following pdf, $f(x) = \begin{cases} pe^{-x} + 2(1-p)e^{-2x}; & x > 0, \\ 0, & otherwise \end{cases}$ and $0 \leq p \leq 1$. Then the hazard function of X is a

1. constant function for $p = 0$ and p=1
2. constant function for all $0 \leq p \leq 1$
3. decreasing function for all 0 < p < 1
4. non-monotone function for all 0< p <1

Q107. Let Y_1, Y_2, Y_2, Y_4 be uncorrelated observations such that $E(Y_1) = \beta + \beta + \beta = E(Y_2), E(Y_3) = \beta_1 - \beta_2 = E(Y_4)$ and $Var(Y_i) = \sigma^2$ for $i = 1,2,3,4$. Then, which of the following statement are true?

1. $p_1\beta_1 + p_2\beta_2 + p_3\beta_3$ is estimable if and only if $p_1 + p_2 = 2p_3$.
2. An unbiased estimator of σ^2 is $[Y_1 - Y_2)^2 + (Y_3 - Y_4)^2]/4$
3. The best linear unbiased estimator of $\beta_1 - \beta_2$ *is* $\frac{1}{2}(Y_3 + Y_4)$.
4. The variance of the best linear unbiased estimator of $\beta_1 + \beta_2 + \beta_3$ is σ^2

Q108.Consider a linear regression model $\underline{Y} = X\underline{\beta} + \underline{\varepsilon} = \underline{0}$. $D(\underline{\varepsilon}) = \sigma^2 I, E(\cdot)$ stands for expectation, $D(\cdot)$ denotes the variance covariance matrix and I is the n-th order identity matrix define the $n \times n$ matrix $H = (h_{ij}) = X(X'X)^{-1}X'$. Then which of the following are correct ?

1. $0 \leq h_{ij} \leq 1, \quad 1 \leq i \leq n.$
2. If $h_{ij} = 0$ or 1 for some i, then h_{ij} =0 for all $j \neq i$.
3. The varinace covariance matrix of the vector of the predicted values $\underline{\tilde{Y}}$ (of Y) is $\sigma^2 H$
4. For $1 \leq i \leq n$, if e_i, is the residual corresponding to Y_i, *i.e*, $e_i = Y_i - \tilde{Y}_i, \tilde{Y}_i$, being the predicted values of Y_i then the variance of e_i equals $\sigma^2(1-h_{ii})$ (Here, Y_i is the ith component of $\underline{Y}$)

Q109. Let (X,Y) follow a bivariate normal distribution with mean vector (0 , 0) and dispresion matrix $\sum = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \rho \neq 0.$ Suppose $Z = \frac{X-Y}{X+Y}\sqrt{\frac{1+\rho}{1-\rho}}$. Then which of the folowiwng statements are correct?

1. $\sqrt{\frac{1+\rho}{1-\rho}} \times \frac{X-Y}{\sqrt{X^2+Y^2+2XY}}$ has a student-t distribution.
2. $\sqrt{\frac{1+\rho}{1-\rho}} \times \frac{X-Y}{\sqrt{X^2+Y^2-2XY}}$ has a student-t distribution
3. Z is symmetric about 0.
4. E(Z) exists and equals zero

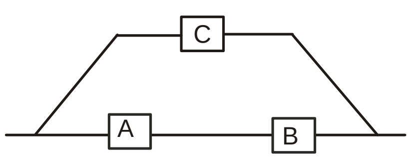
Q110. A sample of size two is drawn from a population of 4 units using probability propotional to size sampling with replacement. The selection probabilities are $p_1 = 0.2, p_2 = 0.3, p_3 = 0.1$ and $p_4 = 0.4$, for units 1,2,3 and 4 in the population, respectively. Let the value of a study variance for the i-th unit be $y_i, i = 1,2,3,4$. Let π_i denote the inclusion probability of the i-th unit and π_{ij} the joint inclusion probability of units i and j, $i < j, i, j = 1,2,3,4$. The which of the following statements are correct?

1. $T = \left(\frac{1}{2}\right)\sum \frac{y_i}{p_i}$ is an unbiased estimator of the popu lation total where the sum is over the units in the sample
2. $\pi_1 = 0.36, \pi_2 = 0.51.$
3. $\pi_{12} = 0.12.$
4. $\pi_1 = \pi_2 + \pi_3 + \pi_4 = 2.$

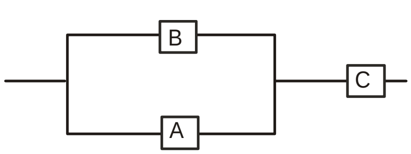
Q111. Consider a balanced incomplete block design d with v treatment , b blocks, replication r, block size k and pairwise concurrence parameter λ. Assume the standard fixed effect model for the data obtained throught d. Which of the following statement is (are) true?

1. The design is connected if $k \geq 2$.
2. The inequality $b \geq v$ holds for d.
3. The variance of the best linear unbiased estimator (BLUE) of a normalized treatment contrast is a constant.
4. The covariance between the BLUE of two or thogonal treatment contrasts is zero.

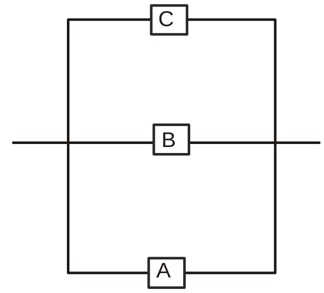
Q112. Three types of components are used in electrical circuits 1,2,3 as shown below



Circuit 1



Circuit 2



circuit 3

Suppose that each of the thrre components fail with probability p and independently of each other.Let Prob(Circuit i does not fail); i=1,2,3. For 0 < p < 1, we have

1. $q_3 > q_1$. 2. $q_1 > q_2$ 3. $q_2 > q_1$. 4. $q_2 > q_3$

Q113. Maximize $3x + 4y$ subject to

$$x \geq 0, \quad y \geq 0, \quad x \leq 3, \quad \frac{1}{2}x + y \leq 4, \quad x + y \leq 5.$$

Which among the following are correct
1. The optiomal value is 19.
2. The optiomal value is 18.
3.(3,2) is an extreme point of the feasible region
4. $\left(3, \frac{5}{2}\right)$ is an extreme point of the deasible region.

Q114. Let $X_1, X_2, ..., X_{2n+1}$ be a random sample from a uniform distribution on the interval $(\theta - 1, \theta + 1)$. Let $T_1 = \overline{X}$, the sample mean, $T_2 = \tilde{X}$, the sample median and $T_3 = \frac{T_1 + T_2}{2}$ be three estimators of θ. Then which of the following statements are correct

1. T_1 is consistent for θ
2. Both T_1 and T_2 are more efficient than T_3
3. All the three estimators are unbiased for θ.
4. T_2 is a sufficient statistic for θ

Q115. The joint probability density function of (X,Y)is

$$f(x, y) = \begin{cases} 3(y-1)^2, & 0 < y < 1 \\ 0, & otherwise \end{cases}$$

Which of the following are correct
1.X and Y are not independent

2. $f_Y(y) = \begin{cases} 3(y-1)^2, & 0 < y < 1 \\ 0, & otherwise \end{cases}$

3. X and Y are independent

4. $f_Y(y) = \begin{cases} 3\left((y - \frac{1}{2}y^2\right), & 0 < y < 1 \\ 0, & otherwise \end{cases}$

Q116. Let X_n be the result of the n-th roll of a fair die, $n \geq 1$. $S_n = \sum_{i=1}^{n} X_i$ and Y_n be the last digit of S_n for $n \geq 1$ and $Y_0 = 0$. Then which of the following statements are correct?

1. $\{Y_n : n \geq 0\}$ is an irreducible markov chain
2. $\{Y_n : n \geq 0\}$ is an aperiodic Markov chain
3. $P(Y_n = 0) \to \frac{1}{6}$ *as* $n \to \infty$
4. $P(Y_n = 5) \to \frac{1}{10}$ *as* $n \to \infty$

Q117. $\{X_i\}$ is an sequence of independent and identically distribution random variables with common density function $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & otherwise \end{cases}$ $\{Y_i\}$ is a sequence of independent identically distribution random variables with common density function $, g(x) = \begin{cases} 4e^{-4y,} & y > 0 \\ 0, & otherwise \end{cases}$

also $\{X_i\}, \{Y_j\}$ are independent families . Let $Z_k = Y_k - 3X_k, k = 1, 2, \ldots$ Which among the following are correct

1. $P(Z_k > 0) > 0$ 2. $\sum_{k=1}^{n} Z_k \to +\infty$ with probablity 1

3. $\sum_{k=1}^{n} Z_k \to -\infty$ with probability 1. 4. $P(Z_k < 0) > 0$.

Q118. Let $X_1, ..., X_n$ be a random sample from

$$f(x; y) = \begin{cases} 2\lambda x e^{-\lambda x^2}; & x > 0 \\ 0, & otheriwise \end{cases}$$

Here $\lambda > 0$ is an unknown parameter.It is desired to test the following hypothesis at level $\alpha > 0$. We want to test $H_0 : \lambda \leq 1$ vs $H_1 : \lambda > 1$. Then which of the following are true?

1. UMP test is of the form $\sum_{i=1}^{n} x_i < c_n$, with $c_n < c_{n+1}$ for all n.
2. UMP test is of the form $\sum_{i=1}^{n} x_i^2 < d_n$ with $d_n < d_{n+1}$ for all n.
3. UMP test is of the form $\sum_{i=1}^{n} x_i < c_n$, with $c_{n+1} < c_n$ for all n.
4. UMP test is of the form $\sum_{i=1}^{n} x_i^2 < d_n$ with $d_{n+1} < d_n$ for all n.

Q119. Let $X_1, ...X_n$ be i.i.d. $N(\mu, 1)$. .It is proposed to test $H_0 : \mu = 0$ versus of the UMP test at μ of size α based on sample size n.

1. $\lim_{n\to\infty} p_n(\mu, \alpha) = 1 \ \forall \mu > 0, \forall \alpha > 0.$
2. $\lim_{\mu\to 0} p_n(\mu, \alpha) = \alpha \ \forall n \geq 1, \forall \alpha > 0.$
3. $\lim_{\alpha\to 0} p_n(\mu, \alpha) = 0 \ \forall n \geq 1, \forall \mu > 0.$
4. $\lim_{\alpha\to 1} p_n(\mu, \alpha) = 0 \ \forall n \geq 1, \forall \mu > 0.$

Q120. Let X be a random sample from a Poisson distribution with parameter λ.Then parameter λ has a prior distribtion $f(z)$; where

$$f(z) = \begin{cases} e^{-z}; & z > 0 \\ 0, & otherwise. \end{cases}$$

under the squared error loss function which of the following statemets are correct?

1. The Bayes' estimator of e^{λ} is 2^{X+1}
2. The posterior means of λ is $\frac{X+1}{2}$
3. The posterior distribution of λ is gamma.
4. The Bayes' estimator of $e^{2\lambda}$ is $2^{2(X+1)}$.