UNIT 1 FORMAL PROOF OF VALIDITY: RULES OF INFERENCE

Contents

1.0 OBJECTIVES

The main objective of this unit is:

- to make explicit the art of testing arguments. This is being achieved in two ways; the limitations of traditional logic are exposed and at the same time the necessity of traditional logic is being demonstrated.
- to compare verbal form of arguments and its symbolic form.
- to assess the relative merits and demerits of two forms if any.
- to translate symbolic representation to verbal form and verbal form to symbolic form.

1.1 INTRODUCTION

The primary function of logic is to classify arguments into good and bad. This can be done by testing the validity of arguments. As we know only limited types of arguments are covered by classical logic. Even those arguments which are within the range of modern logic are not alike in all respects. Some are simple enough so that the truth-table technique is adequate for the purpose of testing. Now, what is this truth-table technique of determining the validity of arguments? Let us take an argument form, for example:

If p, the q

p

Therefore, q

Its truth table can be constructed as follows: we need the initial columns of the statement or proposition variable p and q; then we need a column for the first premise here which is an implicative statement (second premise and conclusion are the initial columns themselves); since there are only two variable in this argument we need only four rows, as we have learned earlier. The truth-table of the above argument is as follows:

	p	**q**	**p =>q**
1	1	1	1
2	1	0	0
3	0	1	1
4	0	0	1

Truth-table technique uses the principle that in a valid argument the conclusion is implied in the premises and so from true premises only true conclusions follow. In the above truth table, only in the first row the premises are true (see under p => q and p) and there under q we see the truth value as true. Hence we can say, in this truth-table no substitution instances (i.e., different rows) with true premises and false conclusion is seen and so it is a valid argument. This is a mechanical method; just construct a truth table for any given argument and see whether there are instances with true premises and false conclusions. Students can attempt the same for the following elementary arguments forms given below as rules of inference.

Generally, any argument, which consists of two or three simple but different propositions, is regarded as amenable to the truth-table method. But if the argument consists of more than three different propositions, then the truth-table method is of no avail. It is mainly because of its manoeuvrability, i.e., if there are three propositions, we need 8 rows in the truth-table; if four, then 16; if 5, the 32; if 6, the 64, and so on. In such circumstances we have to look for an alternative. Formal proof helps us here in quickly determining the validity of arguments. See the following example:

1. A => B
2. B => C
3. C => D
4. ⌐D
5. A v E / 4" E
6. A => C
7. A => D
8. ⌐A
9. E

In this argument we have five propositions like A, B, C, D, E; if we construct truth-table for it, we need 32 rows. Now we can prove its validity by applying certain rules of inference in just four lines. This is exactly the advantage of formal proofs.

An argument, which is complex in this sense, is nothing but an aggregate of several simple (by simple, in this context, we mean short) arguments. Examples make this point clear.

1) p => q
p
────────
∴q

2) q => r
q
────────
∴ r

3) p =>q
q => r
────────
∴ p => r

In classical logic also we have ‘complex’ type of argument in the form of sorites. (We should remember that the terms complex, simple, etc. are relative). An example for sorites is given:

1) All Indians are Asians.

All Hindus are Indians.

All Kannadigas are Hindus. .

∴ All Kannadigas are Asians.

There are three premises and a conclusion. Hence, it is a polysyllogistic argument. As a matter of fact, a sorites consists of at least two syllogistic arguments and therefore, two conclusions. So it is more complex than an ordinary syllogism. This point becomes clear when we break sorites into constituent syllogisms.

2a).

All Indians are Asians.
All Hindus are Indians.] → All Hindus are Asians.
All Kannadigas are Hindus. → All Kannadigas are Hindus.

∴All Kannadigas are Asians

Fortunately or unfortunately, we hardly encounter such stereotype arguments. So there is need to sharpen and augment the tools of testing. At this critical juncture, it is very important to remember that no rule stipulated by classical logic can be ignored or violated. It is the foundation on which the superstructure, i.e., modern logic is built. For the sake of convenience, let us restrict ourselves only to symbols and go to verbal form when we take up exercise.

1.2 FORMAL PROOF OF VALIDITY: ITS MEANING

In modern logic an argument is regarded as a sequence of statements. When proof is constructed to test the argument, the proof also takes the same form, which the argument takes. In this type of proof there is correspondence between the scheme of the given argument and the scheme of the proof. Every step, which is adduced while constructing proof, is the conclusion of the preceding statements, and in turn, becomes the premise for statements, which follow it (if not all, at least to some). Rules, which govern the process of deducing hidden conclusion, constitute what are known as ‘Rules of Inference’ in modern logic. Many of these rules have their origin in traditional logic.

There is a certain way of constructing proof in modern logic. More descriptive method, which consumes both space and time, has given way to much shorter and simpler method. Whatever conclusion can be drawn from any two given premises is written on left hand side (LHS) while the rule and the premises to which this particular rule applies to derive the conclusion used in further proof, are written on the right hand side (RHS). A rule of inference is applied to the whole line. This is an important point to note. As an economy measure, instead of premises, corresponding serial numbers are written. Thereby we save time. We must ensure that drawn conclusion, the respective premises and the rule applied are always juxtaposed. This procedure is the simplest and most economical in terms of time and effort to grasp the argument.

1.3 RULES OF INFERENCE

Our task, from now onwards, is very simple. Modern logic considers nine rules of inference. They are listed below.

1) Modus Ponens (M.P.)

$p \Rightarrow q$

p

$\therefore q$

2) Modus Tollens (M.T.)

$p \Rightarrow q$

$\neg q$

$\therefore \neg p$

3) Hypothetical Syllogism (H.S.)

$p \Rightarrow q$

$q \Rightarrow r$

$\therefore p \Rightarrow r$

4) Disjunctive Syllogism (D.S.)

$p \vee q$

$\neg p$

$\therefore q$

Or

$p \vee q$

$\neg q$

$\therefore p$

5) Constructive Dilemma (C.D.)

$(p \Rightarrow q) \wedge (r \Rightarrow s)$

$p \vee r$

$\therefore q \vee s$

6) Destructive Dilemma (D.D.)

$(p \Rightarrow q) \wedge (r \Rightarrow s)$

$\neg q \vee \neg s$

$\therefore \neg p \vee \neg r$

7) Simplification (Simp.)

$p \wedge q$

$\therefore p / q$

8) Conjunction (Conj.)

p

q

$\therefore p \wedge q$

9) Addition (Add.)

p

$\therefore p \vee q$

First six rules are standard rules of traditional logic. Last three rules need a little clarification. Consider, for example, simplification. Since $p \wedge q$ is given to us, we accept that p is true, and q is true as well. So there is no harm in dropping any of them. The case of conjunction is slightly different. p is given to us, so we take it as true; q is given to us. So we take q also as true. Since both are taken as true we can conveniently conjoin them. The case of addition, again, is different. Suppose that we have only p in the premises. Since it is a premise, we take it as true. Suppose that we require q to be added to p. We do not know whether q is true or not. There is no harm in adding q to p because even if q is false $p \vee q$ still remains true because p is true. After all, one true component can make disjunction true. But what is important is that conjunction does not mean addition. In logical language, addition means disjunction but not conjunction.

The rest of our job is very easy; just apply relevant rules for relevant pairs of lines. It needs only practice and detective's eyes to identify relevant lines and the rule applicable to those lines.

All arguments, which are required to be tested, are valid only because these are proofs only for validity but not for invalidity.

1.4 TESTING THE VALIDITY OF ARGUMENTS

Let us begin with the argument we have seen above:

1) $A \Rightarrow B$

2) $B \Rightarrow C$

3) $C \Rightarrow D$

4) ⌐D

5) A v E / ∴ E

6) A => C 1, 2, H.S.

7) A => D 6, 3, H.S.

8) ⌐A 7, 4, M.T.

9) E 5, 8, D.S.

The original argument's premises are symbolized up to the fifth row. After the slant line is written the conclusion, which is to be proved through the formal proof. The way we have found out the sub-conclusion six (A => C) is written in the justification on right hand side, i.e., by applying the hypothetical syllogism (see above the rules of inference and their abbreviations) to the first and second premises (1, 2), we got the sixth premise or sub-conclusion. The rest of the steps can be easily understood through the justification written on the right hand side. Thus by applying elementary valid argument forms, we could derive the conclusion of the original argument and so this is a valid argument; we have established its validity by constructing a formal proof. Let us see certain formal proofs already constructed:

2

1	(B v N) => (K Λ L)	
2	⌐K	
3	⌐M / ∴ ⌐B Λ ⌐M	
4	⌐K v ⌐L	2, Add.
5	⌐(B v N)	1, 4, M.T.
6	⌐B Λ⌐ N	5, De.M.
7	⌐B	6, Simpl.
8	⌐B Λ ⌐M	7,3, Conj.

3

1	(K => A) Λ (M => D)	
2	⌐A	
3	⌐D / ∴ ⌐K Λ ⌐M	
4	K=> A	1, Simpl.
5	⌐K	4,2, M.T.
6	M => D	1, Simpl.
7	⌐M	6,3, M.T.
8	⌐K Λ ⌐M	5,7, Conj.

4

1	(M v N) => (P Λ Q)	
2	N /∴ P	
3	M v N	2, Add.
4	P Λ Q	1, 3, M.P.
5	P	4, Simpl.

5

1	(A Λ B) => (C v D)	
2	A	
3	B / ∴C v D	
4	A Λ B	2, 3, Conj.
5	C v D	1, 4, M.P.

6

1	(T => K) Λ (R => S)	
2	S => D	
3	D => T	
4	R / ∴ T	
5	R => S	1 Simp.
6	S	5,4, M. P.
7	D	2, 6, M. P.
8	T	3, 7, M.P.

7

1	(A v B) Λ(⌐D ΛE)	
2	A v B =>K /∴ K Λ(⌐D ΛE)	
3	A v B)	1, Simp.
4	K	2, 3, M.P.
5	⌐D ΛE	1, Simp.
6	K Λ(⌐D ΛE)	4, 5, Conj.

8

1	(P =>Q) Λ(R=>S)	
2	⌐A=> ⌐Q	
3	A => ⌐B	
4	B /∴ ⌐P v ⌐S	
5	⌐A	3,4, M.T.
6	⌐ Q	2,5, M.P.
7	P=>Q	1, Simp.
8	⌐P	7, 6, M.T.
9	⌐P v ⌐S	8, Add.

9

1	Av(B ΛC)	
2	A=>P	
3	⌐P / ∴C	
4	⌐A	2,3, M.T.
5	B ΛC	1,4, D.S.
6	C	5, Simp.

10

1	A Λ (B v C)	
2	A=>P	
3	Q / ∴PΛQ	
4	A	1, Simp.
5	P	2,4, M.P.
6	P Λ Q	5,3, Conj.

11

1	⌐B	
2	⌐D	
3	(A=>B) Λ(C=>D)	
4	K / ∴ C (K Λ⌐A)	
5	A=>B	3, Simp.
6	⌐A	5, 1, M.T.
7	C=>D	3, Simp.
8	⌐C	7, 2, M.T.

12

1	(B≡K)=>(Z ΛD)	
2	⌐(Z ΛD) /∴⌐(B≡K)	
3	⌐ (B≡K)	1,2, M.T.

13

1	(K ΛT)=>(A v B)	
2	(A v B)=>(P Λ⌐L)	
3	(P Λ⌐L)=>D	
4	⌐(D) /∴⌐(K ΛT)	
5	(KΛT)=>(P Λ⌐L)	1,2, H.S.
6	(K ΛT)=>D	5,3, H.S.

14

1	(K Λ A)=>(⌐ B v C)	
2	M=>(K Λ A)	
3	M /∴⌐B v C	
4	M=> (⌐B v C)	2,1. H.S.
5	⌐B v C	4,3, M.P.

15

1	A=>D	
2	B=>C	
3	A v B /∴ D v C	
4	(A=>D)Λ(B=>C)	1,2,Conj.
5	D v C	4,3, C.D.

16

1	A=>D	
2	B=>C	
3	⌐D v ⌐C/∴⌐A v ⌐B	
4	(A=>D)Λ(B=>C)	1,2,Conj.
5	⌐Av ⌐B	4,3, D.D.

17

1	(A=>G)=>(K v ⌐D)	
2	⌐(K v ⌐ D) /∴⌐(A=>G)	
3	⌐(A=>G)	1,2, M.T.

18

1	JV(K Λ L)	
2	J=>D	
3	⌐D/∴K Λ L	
4	⌐J	2,3, M.T.
5	(K ΛL)	1,4 D.S.

19

1	D v (A=>B)	
2	(A=>B)=>(C v K)	
3	⌐(C v K) /∴D	
4	⌐(A=>B)	2,3, M.T.
5	D	1,4, D.S.

20

1	A Λ(B=>C)	
2	B /∴C	
3	B=>C	1, Simp.
4	C	3,2, M.P.

21

1	(A=>B) Λ(C=>D)	
2	A/∴B v D	
3	A v C	2, Add.
4	B v D	1,3, C.D.

22

1	A v (B ΛC)	
2	A=>D	
3	⌐D /∴B	
4	⌐A	2,3, M.T.
5	B ΛC	1,4 , D.S.
6	B	5, Simp.

23

1	A=>B	
2	B=>C	
3	⌐C /∴⌐A	
4	⌐B	2,3, M.T.
5	∴⌐A	1,4, M.T.

24

1	(A v B)=> C	
2	D=> ⌐ C	
3	D /∴⌐(A v B)	
4	⌐C	2,3, M.P.
5	⌐(A v B)	1,4, M.T.

25

1	(A=>C) Λ(B=>D)	
2	K=>A	
3	K /□C v D	
4	A	2,3, M.P.
5	A v B	4, Add.
6	C v D	1,5, C.D.

26

1	A v (B=>C)	
2	A=>D	
3	⌐D /□B=>C	
4	⌐A	2,3, M.T.
5	B=>C	1,4, D.S.

27

1	(A=>B)=>(C=>D)	
2	(E=>F)=>(A=>B)	
3	⌐(C=>D) /□⌐ (E=>F)	
4	(E=>F)=>(C=>D)	2,1, H.S.
5	⌐(E=>F)	4,3, M.T.

28

1	(K≡L) => (A Λ B)	
2	D => (K≡L)	
3	D	/ □A
4	D => (A Λ B)	1,2,H.S.
5	A Λ B	4,3,M.P.
6	A	5, Simp.

29

1	A Λ B	
2	(A v C)=>D/□(A ΛD)	
3	A	1, Simp.
4	A v C	3, Add.
5	D	2,4, M.P.
6	(AΛD)	3,5, Conj.

30

1	I=>J	
2	J=>K	
3	L=>M	
4	I v L /□ K v M	
5	I=>K	1,2, H.S.
6	(I=>K)Λ(L=>M)	5,3,Conj.
7	K v M	6,4, C.D.

31

1	(E v F) Λ(G v H)	
2	(E=>G) Λ(F=>H)	
3	⌐G /□H	
4	E v F	1, Simp.
5	G v H	1, Simp.
6	H	5,3, D.S.

32

1	N=>(O ΛN)	
2	(OΛ N)=>P	
3	⌐P /□⌐N	
4	⌐ (OΛ N)	2, 3, M.T.
5	⌐N	1,4, M.T.

33

1	(W=>Y)=>Z	
2	W=>Y /□(Z v X)	
3	Z	1,2,M.P.
4	Z v X	3, Add.

34

1	(A v B)=>C	
2	(C v B)=>(A=> D)	
3	A ΛD /□ (D v F)	
4	A	3, Simp.
5	(A v B)	4, Add.
6	C	1,5, M.P.
7	(C v B)	6, Add.
8	A=>D	2,7, M.P.
9	D	8,4, M.P.
10	(D v F)	9, Add.

35

1	F=> ⌐ G	
2	⌐F=> (H=> G)	
3	(⌐I v ⌐H) => G	
4	⌐I /□H => G	
5	⌐I v ⌐ H	4, Add.
6	G	3,5, M.P.
7	⌐F	1,6, M.T.
8	H=>G	2,7, M.P.

36

1	(L=>M)=>(N≡O)	
2	(P=> ⌐ Q)=>(M=> ⌐ Q)	
3	[(P=> ⌐ Q)v(R≡S)} Λ(N v O)]=>{(R≡S)=>(L =>M)}	
4	(P=> ⌐ Q)v(R≡S)	
5	N v O /□ (M≡ ⌐ Q)v(N≡O)	
6	{(P=> ⌐ Q) v (R≡S)} Λ(N v O)	4,5, Conj.
7	(R≡S) => (L=>M)	3,6, M.P.
8	(R≡S) => (N≡O)	7,1 H. S.
9	{(P=> ⌐ Q) => (M≡ ⌐ Q)} Λ{(R≡S) => (N≡O)}	2, 8, Conj.
10	(M≡ ⌐ Q) v (N≡O)	9,4, C.D.

37

1	(F=>G) Λ(H=>I)	
2	J=>K	
3	(F v J) Λ(H v L) /□G v K	
4	F=>G	1, Simp.
5	(F=>G)Λ(J=>K)	4,2,Conj.
6	F v J	3, Simp.
7	G v K	5,6, C.D.

38

1	(⌐M Λ⌐N) => (O => N)	
2	N=>M	
3	⌐M /□⌐O	
4	⌐N	2,3, M.T.
5	(⌐M Λ⌐N)	3,4, Conj.
6	O => N	1,5, M.P.
7	⌐O	6,4, M.T.

39

1	(K v L)=>(M v N)	
2	(M v N)=>(O ΛP)	
3	K /□O	
4	K v L	3, Add.
5	M v N	1,4, M.P.
6	OΛP	2,5, M.P.
7	O	6, Simp.

40

1	W => (W ΛX)	
2	(W ΛX) => (W ΛY)	
3	(W ΛY) =>Z /□W=>Z	
4	W=> (W ΛY)	1,2 H.S.
5	W=> Z	4,3, H.S.

41

1	A=>B	
2	C=>D	
3	A v C /□B v D	
4	(A=>B) Λ (C=>D)	1,2, Conj.
5	B v D	4,3, C.D.

42

1	(E v F)=>(G ΛH)	
2	(G v H)=>I	
3	E /□I	
4	E v F	3, Add.
5	G Λ H	1,4, M.P.
6	G	5, Simp.
7	G v H	6, Add.
8	I	2,7, M.P.

43

1	J=>K	
2	K v L	
3	(L Λ⌐J)=>(M Λ⌐J)	
4	⌐K /□M	
5	L	2,4, D.S.
6	-J	1,4, M.T.
7	L Λ⌐J	5,6, Conj.
8	M Λ⌐J	3,7, M.P.
9	M	8, Simp.

44

1	(D v E) => (F ΛG)	
2	D /□F	
3	D v E	2, Add.
4	F Λ G	1,3, M.P.
5	F	4, Simp.

45

1	Q=>R	
2	R=>S	
3	⌐S /□⌐Q Λ⌐R	
4	⌐R	2,3, M.T.
5	⌐Q	1,4, M.T.
6	⌐Q Λ⌐R	5,4, Conj.

46

1	(T=>U) Λ(V=>W)	
2	(U=>X) Λ(W=>Y)	
3	T /□X v Y	
4	T=>U	1, Simp.
5	U	4,3, M.P.
6	U=>X	2, Simp.
7	X	6,5, M.P.
8	X v Y	7, Add.

47

1	T => U	
2	V v ⌐U	
3	(⌐V Λ⌐W) /□⌐T	
4	⌐V	3, Simp.
5	⌐U	2,4, D.S.
6	⌐T	1,5, M.T.

48

1	⌐X =>Y	
2	Z =>X	
3	⌐X /□Y Λ⌐Z	
4	⌐Z	2,3, M.T.
5	Y	1,3 , M.P.
6	Y Λ⌐Z	5,4, Conj.

49

1	(A v B) => ⌐ C	
2	C v D	
3	A /∴ D	
4	A v B	3, Add.
5	⌐ C	1,4, M.P.
6	D	2,5, D.S.

50

1	(H => I) Λ(J => K)	
2	K v H	
3	⌐K /∴ I	
4	H	2,3, D.S.
5	H=>I	1, Simp.
6	I	5, 4, M.P.

51

1	L v (M => N)	
2	⌐L=> (N => O)	
3	⌐L /∴ M => O	
4	N => O	2,3, M.P.
5	M=> N	1,3, D.S.
6	M => O	5,4, H.S.

52

1	K=>L	
2	M=>N	
3	(O=>N) Λ(P=>L)	
4	(⌐N v ⌐L) Λ(⌐M v ⌐ O)/ ∴ (⌐O v ⌐P) Λ(⌐M v ⌐ K)	
5	(M => N) Λ (K=>L)	2,1, Conj.
6	⌐N v ⌐L	4, Simp.
7	⌐M v ⌐ K	5,6,D.D.
8	⌐ O v ⌐P	3,6, D.D.
9	(⌐O v ⌐ P) Λ (⌐M v ⌐ K)	8,7.Conj.

53

1	(G=> H) => (I≡J)	
2	K v⌐ (L=> M)	
3	(G=> H) v ⌐K	
4	N=> (L=> M)	
5	⌐ (I≡J) / ∴ ⌐N	
6	⌐ (G => H)	1,5, M.T.
7	⌐K	3,6, D.S.
8	⌐(L=>M)	2,7, D.S.
9	⌐N	4,8, M.T.

54

1	(O => ⌐ P) Λ (⌐ Q => R)	
2	(S =>T) Λ (⌐U=> ⌐V)	
3	(⌐P => S) Λ (R => ⌐ U)	
4	(T v ⌐ V) => (W ΛX)	
5	O v ⌐ Q /∴ (W ΛX)	
6	⌐P v R	1,5, C.D.
7	S v ⌐ U	3,6, C.D.
8	T v ⌐ V	2,7, C.D.
9	W Λ X	4,8, M.P.

55

1	E => (F Λ ⌐ G)	
2	(F v G) => H	
3	E / ∴ H	
4	F Λ ⌐ G	1,3, M.P.
5	F	4, Simp.
6	F v G	5, Add.
7	H	2,6, M.P.

1.5 TESTING OF THE VALIDITY OF ARGUMENTS (VERBAL)

For change, let us start with verbal form of argument and symbolize the statements and logical constants before proceeding to test the validity of the arguments. (Problems are worked out at the end.)

I) If Rama joins, then the club's social prestige will rise; and if Krishna joins, then the club's financial position will be more secure. Either Rama or Krishna joins. If the club's social prestige rises, then Krishna will join; and if the club's financial position becomes more secure, then Govinda will join. Therefore either Krishna or Govinda will join.

II) If Vishnu received the wire, then he took the plane; and if he took the plane, then he will not be late for the meeting. If the telegram was incorrectly addressed, then Vishnu will be late for the meeting. Either Vishu received the wire or the telegram was incorrectly addressed. Therefore either Vishnu took the plane or he will be late for the meeting.

III) If Narayana buys the plot, then an office building will be constructed; whereas if Madhava buys the plot, then it quickly will be sold again. If Keshava buys the plot, then a store will be constructed; and if the store is constructed, then Lakshmi will offer to lease it. Either Narayana or Keshava will buy the lot. Therefore either an office building or a store will be constructed.

IV) If Jagannath goes to the meeting, then a complete report will be made; if Jagannath does not go to the meeting, then a special election will be required. If a complete report is made, then an investigation will be launched. If Jagannath going to the meeting implies that a complete report will be made, then if the making of a complete report implies that an investigation will be launched, then either Jagannath goes to the meeting and an investigation is launched or Jagannath does not go to the meeting and no investigation is launched. If Jagannath goes to the meeting and an investigation is launched, then some members will have to stand trial. But if Jagannath does not go to the meeting and no investigation is launched then the organization will disintegrate very rapidly. Therefore either some members will have to stand trial or the organization will disintegrate very rapidly.

Statements are symbolized in the following manner:

1)

1	Rama joins	=R
2	The club's social prestige will rise	= S
3	Krishna joins	= K
4	The club's financial position rises	= F
5	Govinda will join	= G

Now the argument becomes:

1	(R=> S) Λ (K=> F)	
2	R v K	
3	(S=> K) Λ (F=> G)/∴ K v G	
4	S v F	1, 2, C.D.
5	K v G	3, 4, C.D.

Answer to the first argument makes one point very clear. Verbal expression is naturally very long and tedious, whereas symbolic representation is short and clear.

Subsequent examples are symbolized in a similar fashion

2)

1	Vishnu received the wire	=V
2	He took the plane	=P
3	He will not be late of the meeting	= ⌐L
4	Telegram was incorrectly addressed	= ⌐T

5 Vishnu will be late for the meeting = L

Now the arguments becomes:

1	(V=> P) Λ(P => ⌐ L)	
2	(⌐T=> L)	
3	V v ⌐T	/∴ P v L
4	V=> P	1, Simp.
5	(V=> P) Λ (⌐T=> L)	4, 2, Conj.
6	P v L	5, 3, C.D.

The students are advised to construct formal proof of validity for the remaining arguments.

Check Your Progress

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

There are 54 arguments, which have been examined. Just as you symbolized verbal form of argument, you substitute statements for symbols. Use your own statements to construct arguments for as many arguments as you can.

1.6 LET US SUM UP

Modern Logic is an extension of traditional logic. However, there is qualitative difference in testing. Difference consists in accuracy and clarity of proof. Nine rules of inference include many rules from traditional logic like modus ponens. Rulr of inference is applied to the whole line. All nine rules are not required always. Only some rules are required. There is no rule, which says that one line must be considered only once.

1.7 KEY WORDS

Modus Ponens (MP) and Modus Tollens (MT) : Modus ponens is a valid, simple argument form sometimes referred to as affirming the antecedent or the law of detachment. It is closely related to another valid form of argument, *modus tollens* or "denying the consequent."

Validity : An argument is valid if and only if the truth of its premises entails the truth of its conclusion. It would be self-contradictory to affirm the premises and deny the conclusion. The corresponding conditional of a valid argument states that logical truth and the negation of its corresponding conditional is a contradiction. The conclusion is a logical consequence of its premises.

Polysyllogism : it is a series of syllogisms in which the conclusion of the preceding one becomes the premise of the following syllogism. Sorites is one type of polysyllogism in which all the conclusions except the last are suppressed.

1.8 FURTHER READINGS AND REFERENCES

Basson, A.H. & O'connor, D.J. *Introduction to Symbolic Logic*. Calcutta: Oxford University Press, 1976.

Copi, I.M. *Symbolic Logic*. 4th Ed. New Delhi: Collier Macmillan International, 1973.

Copi, I.M . *Introduction to Logic*. 9th Ed. New Delhi: Prentice Hall of India, 1995.

Joseph, H.W.B. *An Introduction to Logic*. Oxford: 1906.

Lewis, C.I. & Longford, C.H. *Symbolic Logic*. New York: Dover Pub. Inc.,1959.

1.9 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress I

Clue: Symbolize each component using the first letter of first noun (proper or common) or verb.

3) Narayana buys the plot – N

Office building…… - O

Madhava buys the plot - M

It will be …….sold - S

Keshava buys…….. - K

…..store will be constructed –C

Lakshmi will ……. - L

Let us symbolize components accordingly:

1) N => O

2) M => S

3) K => C

4) C=> L

5) N v K / $\therefore$ O v C

6) (N=>O) Λ (K => C) 1, 3, Conj.

7) $\therefore$ O V C 6, 5, C.D.

It is quite obvious that lines 2 and 4 are not required to test the validity though they are parts of argument.

4) Jagannath goes to the meeting—— J
complete report——————— C
special election————————— S
investigation will be launched——I
some members will have to stand a trial——M
the organization will disintegrate————— D
Let us symbolize the components accordingly:

1) J => C

2) ⌐J => S

3) C => I

4) { (J => C) ? (C => I) } => (J ⌐ I) v (⌐J Λ ⌐I)

5) (J Λ I) => M

6) (⌐J Λ ⌐I) => D /∴ M v D

7)	(J => C) Λ (C => I)	1, 3, Conj.
8)	(J Λ I) v (⌐J Λ ⌐I)	4, 7, M. P.
9)	{(J Λ I) => M } Λ {(⌐J Λ ⌐I) => D}	5, 6, Conj
10)	∴ M v D	9, 8, C.D.