CSIR JUNE 2016 SOLUTION

1.-3

Sol.  ,anti clockwise rotation of dot is done

2. 3

Sol. Such regular solid is tetrahedron , so for three such objects 3(faces + vertices - edges) =
3(4 + 4 - 6) = 6

3. - 2

Abdul travels...reaches first?

Let distance Abdul travels $= 6x \ or \ 6$

$\Rightarrow$ " " Catherine " $= 2x \ or \ 2$

$\Rightarrow$ " " Binoy " $3x \ or \ 3.$

Let speed of Catherine $= y \ or \ 1$

$\Rightarrow$ " " Abdul $= 3y \ or \ 3$

$\Rightarrow$ " " Binoy $= 2y \ or \ 2$

then time $= \dfrac{dins\tan ce}{speed}$

time of Abdul $= 6/3 = 2$

" " Catherine $= 2/1 = 2$

" " Binoy $= 3/2 = 1.5$

minimum time is taken by Binoy

4.- 4

Sol. Let tiwari do work in $= x$ day

" deo " " $= y$ day

hari " " " = z day

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$$

$$\frac{1}{x}+\frac{1}{z}=1/3$$

$$\frac{1}{y}+\frac{1}{z}=1/6$$ ******************************

Solve these

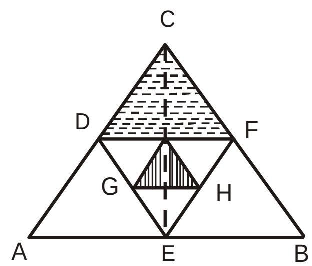
$$\frac{1}{x}=\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{6}\right)-\frac{1}{6}=\frac{1}{3} \;=>\; x=3 \text{ hours}$$

$$\frac{1}{y}=\frac{1}{6}\Rightarrow y=6 \text{ hours}$$

$\Rightarrow$ Hari does not work

$\Rightarrow$ incorrect option is hari is the fastest worker.

5. - 3 ; Ratio of the two shaded areas below is $4:1$, as length of line segment joining mid points of two sides of a triangle is half of the length of the third side and also it is parallel to it



6.-1

Sol. Speed $= 36\times\dfrac{5}{18}=10 \ m/s$

length of train $= 10\times 8 = 80m.$

total length (train and platform) $= 10\times 20 = 200m.$

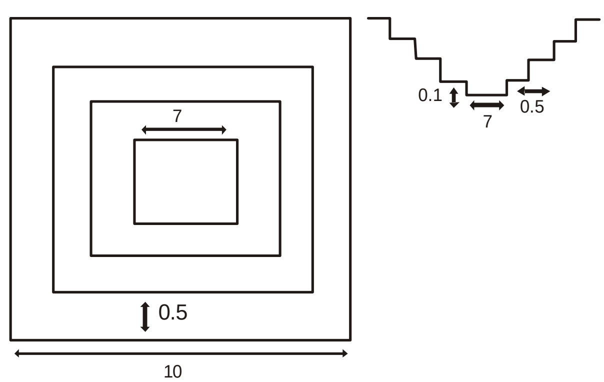
& plateform length $= 200-80=120$

7.- 2

Sol. Poly $= (x-5)(x-3)(x-2)+1$

$= x^3 - 10x^2 + 31x - 29$

8. - 2; The diagram has volume of water (in m^3) when it is completely filled is equal to
$(7^2 + 8^2 + 9^2 + 10^2)(0.1) = 29.4$



9.-4

Sol. $12/8 = \frac{6}{BD}$ => $BD = \frac{48}{12} = 4$

10. - 3 ; 99^{th} statement is correct in the notebook

11.-3

Sol. Functional value 100, 10, 1, 0.1 next term = 0.01

12.-2

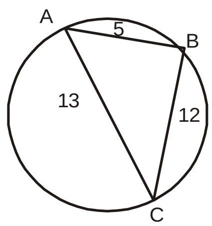
Sol. frog will take triangular root to get 10cm.

13.-1

Sol. all condition satisfy option 5478

14. -3

Sol.



Given , $AB = 5$, $BC = 12$

$AC = 13$ this is Right angle triagle

So, AD = 7 => minimum distance of $CD = \sqrt{13^2 - 7^2} \cong 11cm$ approximate

15.-3

Sol. $x^2 - y^2 = 899$ => $(30)^2 - (1)^2 = 899$

16.-3 ; If water is slowly dripping out of a tiny hole at the bottom of a hollow metallic sphere initially full of water ignoring the water that has flowed away, the centre of mass of the system moves down for some time but eventually returns to the centre of the sphere

17.- 3 ; A chocolate bar having $m \times n$ unit square tile is given. then the number of cuts needed to break it completely, without stacking into individual tiles is $(m) \times (n) - 1$, as each cut gives 1 smaller tile.

18.-2

Sol. option 2 satisfy all entries in

19. - 4 ; If a bicycle tube has a mean circumference of 200 cm and a circluar cross section of diameter 6 cm. , then the approximate volume of water (in cc) required to completely fill the tube assuming that it does not expand is equal to $\pi (6/2)^2 .200 = 1800\pi$

20.- 3

Sol. Let income is x

$$200000 \times \frac{R}{100} + \frac{(x - 200000)(R + 10)}{100}$$

$$= x\frac{(R+5)}{100}$$

PART B

Q21. (1)

Sol. $f(x) = \int_0^x \frac{1}{\sqrt{y}} dy = 2\left[\sqrt{y}\right]_0^x = 2\sqrt{x}$ which is continuous on $[0, \infty)$.

Q22. (1)

Sol. $f_n(x) = \frac{1}{n^2 + x^2}, n = 1, 2, ..., x \in [1/2, 1].$ clearly for each $x \in \left[\frac{1}{2}, 1\right]$ we have $\frac{1}{(n+1)^2 + x^2} \le \frac{1}{n^2 + x^2}$ and $\left|\frac{1}{n^2 + x^2}\right| \le \frac{1}{n^2} \to 0$ as $n \to \infty$ so the sequence $\left\langle \frac{1}{n^2 + x^2} \right\rangle$ is monotone and has 0 as the limit for all $x \in \left[\frac{1}{2}, 1\right]$ as $n \to \infty$

Q23. (1)

Sol. $\lim_{n\to\infty} \left(1 - \frac{1}{n^2}\right)^n = e^{\lim_{n\to\infty} -\frac{1}{n^2} \times n}$

$$= e^{\lim_{n\to\infty} \frac{1}{n}} = 1$$

Q24. (2)

Sol. If x is l;imit point of the set (-1,1) then there exits a sequence $\langle \alpha_n \rangle$ from the set (-1, 1) which converges to the point x. but the set of all limit points of the set (-1,1) i.e, (-1, 1) = [-1,1].

So a sequence $\langle \alpha_n \rangle$ in (-1,1) can have limit point in [-1,1]

Q25. (3)

Sol. monotone functions has only jump discontinuous which are atmost countable for any monotone function

Q26. (1)

Sol. $f(x,y)=\frac{x^2}{y^2},(x,y)\in\left[\frac{1}{2},\frac{3}{2}\right]$ clearly f is differentiable on the given domain there for

$$D_{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)} f(1,1)=\left(f_x\cdot\frac{1}{\sqrt{2}}+f_y\cdot\frac{1}{\sqrt{2}}\right)$$

$$=\left(\frac{2x}{\sqrt{2}y^2}+\frac{-2x^2}{y^3}\frac{1}{\sqrt{2}}\right)$$

Q27. (1)

Sol. $e^B=\sum_{j=0}^{\infty}\frac{B^j}{J!}$. Since p be the characteristc ploynomial of B, p(B) = 0. Therefor $e^{p(B)}=e^0=I_{n\times n}$

Q28. (2)

Sol. $A_{n\times m}X=b$ admits a unique solution

$\Rightarrow$ column of $A_{n\times m}$ are linearly independent.

$\Rightarrow rank\left(A_{n\times m}\right)=m$

$\Rightarrow m\le n$

Q29. (2)

Sol. $T:V(\mathbb{R})\to V(\mathbb{R})$**:** $T\ \ p(x)=p(x)$

$\Rightarrow T(1)=0$

$\Rightarrow KerT\ne\{0\}$

$\Rightarrow$ T is singular

$\Rightarrow$ A i.e., matrix of T is singular

$\Rightarrow$ det A = 0

Q30. (1)

Sol. If $\delta=(\delta_1,-\delta_2,\delta_3)$ then we have $\delta=x\times y$ with X the cross product. Because of the properties of the cross product, δ is orthogonal to both x only.

i.e., δ is a normal vector of V if a plane has $(\delta_1,-\delta_2,\delta_3)$ as a normal vector then it satisfies the equation $\delta_1 u-\delta_2 v+\delta_2 w=0$

Q31. (3)

Sol. There exists $X\in\mathbb{R}^n$ such that $X'AX<0$. Then $(AX)'A^{-1}(AX)=Y'A^{-1}Y$ where $Y=AX$.

$\Rightarrow Y'A^{-1}Y=X'A^1A^{-1}AX$

$=X'AA^{-1}AX$ [Since A is symmetric]

$X'AX<0$

Therefor $Y'A^{-1}<0$ for some $Y\in\mathbb{R}^n$

Q32. (4)

Sol. $A=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$ $f:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ be defined by

$f(v,w)=w^TAv.$

$=[a,b]\begin{bmatrix}1&0\\0&-1\end{bmatrix}\begin{bmatrix}e\\d\end{bmatrix}$

$=[a-b]\begin{bmatrix}c\\d\end{bmatrix}$

$=ac-bd$

So for each $v=\begin{bmatrix}c\\d\end{bmatrix}$ we can choose $w=\begin{bmatrix}a\\b\end{bmatrix}$ such that $f(v,w)=ac-bd=0$.

Q33. (3)

Sol. $P(z)\in\mathbb{C}(z)$ and degree of $P(z)=m$

$Q(z)\in\mathbb{C}(z)$ and degree of Q(z) = n

Since $\mathbb{C}(z)$ is an integral domain then degree of $P(z)\cdot Q(z)=m+n$ then $P(z)\cdot Q(z)=0$ has exactly m + n roots in $\mathbb{C}(z)$

Q34 (2)

Sol. $f\in S\Rightarrow f\left(\frac{1}{2n}\right)=\frac{1}{2n}$

$\therefore$ By indentity theorem

$f(z)=Z,\ \therefore S=\{f:f(z)=z\}$

$f\in T\Rightarrow f\left(\frac{1}{2n}\right)=\frac{1}{2n}\Rightarrow f(z)=z$ and

$f\left(\frac{1}{2n+1}\right)=\frac{1}{2n}\Rightarrow f(z)=\frac{z}{1-z}$ which is not possible

$\therefore\qquad T=\phi$

Q35. (2)

Sol. $\sum_{n=0}^{\infty}a_nz^n=\sum_{n=0}^{\infty}P(n)z^n$ radius of convergence is given by $\frac{1}{R} = \lim_{n\to\infty} \frac{p(n+1)}{p(n)} = 1 \qquad \therefore R = 1$

Q36. (3)
Sol.

$$f(z) = e^{-e^{1/z}}$$
$$= e^{-\left[1+\frac{1}{z}+\frac{1}{z^2},\frac{1}{2!}+...\right]}$$
$$= e^{-1}e^{-\left[\frac{1}{z}+\frac{1}{z^2},\frac{1}{2!}\right]}$$
$$= e^{-1}\left[-\left(\frac{1}{z}+\frac{1}{z^2},\frac{1}{2!}+...\right)+...\right]$$

Residue of f at z = 0

co-efficient of $\frac{1}{z}$ in the above Laurent's series expansion of f at z = 0 = - e^{-1}.

Q37. (3)
Sol.

(i) $\gcd(1000, 6789) = 1$ then $x = 23(\text{mod } 1000)$ and $x \equiv 45(\text{mod } 6789)$ has solution by chinese reimainder theorem

(ii) $\gcd(1000, 6789) = 1$ then $x = 23(\text{mod } 1000)$ and $x \equiv 54(\text{mod } 6789)$ has solution by chinese reimainder theorem

iii $\gcd(1000, 9876) = 4$ and $4 \dagger 54 - 32$ then $x = 32(\text{mod } 1000)$ and $x \equiv 54(\text{mod } 9876)$ has no solution

(iv) $\gcd(1000, 9876) = 4$ and $4 \dagger 44 - 32$ then $x = 32(\text{mod } 1000)$ and $x \equiv 44(\text{mod } 9876)$ has solution

Q38. (1)

Sol. $G = \left(\frac{\mathbb{Z}}{25\mathbb{Z}}\right)^* \approx (\mathbb{Z}_{25})^* \approx U(25)$

$$\Rightarrow 0(G) = 0\left(\frac{\mathbb{Z}}{25\mathbb{Z}}\right)^* = 0(U(25)) = U(25) = 2\!\!3 \in \left(\frac{\mathbb{Z}}{25\mathbb{Z}}\right)^*$$

such that $0(3) = 20 = 0\left(\frac{\mathbb{Z}}{25\mathbb{Z}}\right)^*$ then $\left(\frac{\mathbb{Z}}{25\mathbb{Z}}\right)^*$ has elements of order 20 say 3. then $\left(\frac{\mathbb{Z}}{25\mathbb{Z}}\right)^* = \langle 3 \rangle$

Q39. (1) $G = F_b \times F_b$ number of elements of order p in $F_b \times F_b$ $F_b \times F_b = b^2 - 1$. total number subgroup of order p $= \frac{p^2-1}{\phi(p)} = \frac{(p-1)(p+1)}{(p-1)} = p+1$

if $p \geq 5$ then $F_p \times F_p$ has at least 5 +1 subgroup of order p.

Q40. (2)

Sol. F_{p^2} is a field then $F_{p^2} \approx G\mp(p^2)$ number of sub field in $GF(p^2) = \tau(2) = 2$

i) $GF(p^1)$ and (ii) $GF(p^2)$ subring p is $GF(p^1)$ where $0\left(GF\left(p^1\right)\right) = p$

Q41. (1)

Sol. y_1 and y_2 are two solution of the given I.V.P. So, $y_1(0) = y_2(0) = 0$

$\Rightarrow y_1(0)$ and y_2 has common zero

$\Rightarrow W(t) = 0, \quad \forall t \in \mathbb{R}$ as y_1 and y_2 are lin early dependent solution

Q42. (3)

Sol. $u_t = uu_x = 0, \qquad x \in \mathbb{R}, t > 0$

$u(x,0) = x, \qquad x \in \mathbb{R} \; \frac{dx}{-u} = \frac{dt}{1} = \frac{du}{0}$

$\Rightarrow du = 0 \Rightarrow u = c_1$

Q43 (3)

Sol. $f(x) = x^2 + 2x + 1, f'(x) = 2x + 2$.

True value , $f'(1) = 4$.

measure value $= \frac{f\left(1+\frac{1}{2}\right) - f\left(1-\frac{1}{2}\right)}{1}$

$$= \frac{\frac{9}{4} + 4 - \frac{1}{4} - 2}{1} = 4$$

absolute error = True value - Measure value = 4-4 =0 again

$$\frac{dx}{-u} = dt \Rightarrow \frac{dx}{-c_1} = dt \Rightarrow \frac{-x}{x} = t + c_2$$

Now $u(x,0) = x \Rightarrow -\frac{x}{x} = 0 + c_2 \Rightarrow c_2 = -1$

Therefore $\frac{-x}{u} = t-1. \Rightarrow \frac{-u}{x} = \frac{1}{t-1}. \Rightarrow u = \frac{-x}{t-1}$.

Therefore solution u exist for all t < 1 and breaks down at t = 1.

Q44.* None are correct

Sol.

Q45. (1)

Sol. As eigenvalue of the matrix A are -2, -2, -2 there is term e^{-2t} outside of the solution

$\therefore \quad \lim_{t\to\infty} |x(t)| = 0$

Q46. (4)

Sol.

$$\left(a(x,y)\frac{\partial}{\partial x} + b(x,y)\frac{\partial}{\partial y}\right)\left(c(x,y)\frac{\partial}{\partial x} + d(x,y)\frac{\partial}{\partial y}\right)u = 0$$

$$\Rightarrow ac\frac{\partial^2 u}{\partial x^2} + ad\frac{\partial^2 u}{\partial x \partial y} + bc\frac{\partial^2 u}{\partial y \partial x} + bd\frac{\partial^2 u}{\partial y^2} = 0$$

$$\Rightarrow ac\frac{\partial^2 u}{\partial x^2} + (ad+bc)\frac{\partial^2 u}{\partial x \partial y} + bd\frac{\partial^2 u}{\partial y^2} = 0$$

$$\begin{aligned} S^2 - 4RT &= (ad+bc)^2 - 4(ac)(bd) \\ &= a^2d^2 + b^2c^2 + 2abcd - 4abcd \\ &= a^2d^2 + b^2c^2 - 2abcd \\ &= (ad-bc)^2 \ge 0 \end{aligned}$$

So it is never elliptic

Q47.(4)

Sol. The cicle $\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$ passes throught (0,0) and (1,0) and encloses maximum area above the x-axis

Q48. (4)

Sol. $y(x) = x^3 + \int_0^x \sin(x-t)y(t)dt \qquad x \in [0,\pi]$

$$y(x) = 3x^2 + \int_0^x \cos(x-t)y(t)dt + y''(x) = 6x + x^3$$

A.E. is $x^2 = 0 \Rightarrow x = 0, 0$

$$y = (c_1 + c_2 x)e^{0x} = c_1 + c_2 x$$

P.I. is $\frac{1}{D^2}\left(6x + x^3\right) = \frac{1}{D}\left(3x^2 + \frac{x^4}{4}\right) = x^3 + \frac{x^5}{20}$

$\therefore \quad y(x) = c_1 + c_2 x + x^3 + \frac{x^5}{20}$, $y(0) = 0 \Rightarrow c_1 = 0$

$\therefore \quad y'(x) = c_2 + 3x^2 + \frac{x^4}{20}$

$y'(0) = c_2$, also $y'(0) = 0 \Rightarrow c_2 = 0$. Therefore

$y(x) = x^3 + \frac{x^5}{20} \Rightarrow y(1) = \frac{21}{20}$

49.-3 ; Let X and Y be independent and identically distributed random variables such that $P(X=0) = P(X=1) = \frac{1}{2}$. Let $Z = X + Y$ and $W = |X - Y|$. Then Z and W are not uncorrelated.

50. - 4 ; Let $X_1 \sim N(0,1)$ and let

$$X_2 = \begin{cases} -X_1, -2 \le X_1 \le 2 \\ X_1, \quad otherwise \end{cases}.$$

Then (X_1, X_2) does not have a bivariate normal distribution

51. - 4 ; Let $X_1, ..., X_n$ denote a random sample from a $N(\mu, \sigma^2)$ distribution. Let $\mu \in \mathbb{R}$ be known and $\sigma^2 (> 0)$ be unknown. Let $\chi^2_{n,\alpha/2}$ be an upper $(\alpha/2)^{th}$ percentile point of a χ^2_n distribution. Then a $100(1-\alpha)\%$ confidence interval for σ^2 is given by

$$\left(\frac{\sum_1^n (X_i - \mu)^2}{n\chi^2_{n,\alpha/2}}, \frac{(\sum_1^n X_i^2 - \mu)^2}{n\chi^2_{n,1-\alpha/2}}\right)$$

52. - 3 Let Y_1, Y_2, Y_3 be uncorrelated obsevations with common variance σ^2 and expectations given by **Q62.**

$E(Y_1) = \beta_1, E(Y_2) = \beta_2$ and $E(Y_3) = \beta_1 + \beta_2$, where $\beta_1 + \beta_2$ are unknwon parameters. The best linear unbiased estimator of $\beta_1 + \beta_2$ is $\frac{1}{3}(Y_1 + Y_2 + 2Y_3)$

53.- 3; A series system with two independent component .If the component lifespan have exponential distribution with density

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, \lambda > 0, x > 0 \\ 0, \ otheriwise \end{cases}$$

If *n* observations $X_1, X_2, ..., X_n$ on lifespan of this component are available and $\overline{X} = \frac{1}{2}\sum_1^n X_i$, then the maximum likelihood estimator of the reliability of the system is given by $e^{-2t/\overline{X}}$

54. - 1 ; Customers arrive at an ice cream parlour according to a Poisson process with rate 2. Service time distribution has density function

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & x \le 0. \end{cases}$$

Upon being served a customer may rejoin the queue with probability 0.4, independently of new arrivals;also a returning customer's service time is the same as that of a new arriving customer. Customers behave independetly of each other .Let X(t) = number of customer in the queue at time . Then$\{(X(t)\}$ grows without bound with probability 1.

55. - 4 ; Hundred (100) tickets are marked 1, 2,...,100 and are arraged at random. Four tickets are picked from these tickets and are given to four persons A, B, C and D. Then the probablity that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with the smallest value (among A,B,C,D) is

$$= \frac{1 \times 2! \times 1}{4!} = \frac{1}{12}$$

56. - 2 ; If $\{X_t\}$ and $\{Y_t\}$ be two independent pure birth processes with birth rates λ_1 and λ_2 respectively , then if $Z_t = X_t + Y_t$. Then $\{Z_t\}$ is a pure birth process with birth rate $\lambda_1 + \lambda_2$.

57. - 3 ; If $X_1, ..., X_n$ be a random sample from $N(\theta, 1)$, where $\theta \in \{1, 2\}$. Then the maximum likelihood esti mator (MLE) of θ exist but it not $\overline{X}$

58. - 1 ; When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1 the likelihood ratio principle leads to the most powerful test

59.- 3 ; Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (1,1,1)$ and

$$\Sigma \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & c \\ 1 & c & 2 \end{pmatrix}.$$

The value of c such that X_2 and $-X_1 + X_2 - X_3$ are independent is c = 2

60. - 1 ; A sample fo size $n(\le 2)$ is drawn without replacement from a finite population of size N, using an <u>arbitrary</u> sampling scheme. If π_i denote the inclusion probability of the *i*-th unit and π_{ij} the joint inclusion probability of units *i* and *j*, $1 \le i < j < N$, *then*

$$\sum_{i=1}^{N} \pi_i = n$$

<u>PART C</u>

Q61.

Sol. (1,4)

(i) $\sum |x_n|^p < \infty \Rightarrow |x_n|^p \to 0$

$\Rightarrow |x_n|^p < \in \quad (0, < \in < 1) \quad \forall n \ge m.$

$\Rightarrow |x_n|^q < |x_n|^p \quad \forall n \ge m, q > p.$

$\Rightarrow \sum |x_n|^q$ is cgt by comparision tests

(ii) $\sum \left|\frac{1}{n}\right|^2$ is cgt but $\sum \left|\frac{1}{n}\right|^1$ is not convergent

(iii) Not true by first option

(iv) $\sum |x_n| = \sum \frac{1}{n^{2/3}}$ then $\sum |x_n|^3$ is convegent $\sum |x_n|^3$ is convergent by $\sum |x_n|^{3/2}$ is divergent

Q62. (3,4)

Sol. $f:\mathbb{R}\to\mathbb{R}$ be continuous such that $f(x)=f(x+1)$ that is f continuous and periodical on $\mathbb{R}$. Therefore nature of *f* on $\mathbb{R}$ is same as on the interval [0,1]. So, *f* bounded and uniformly continuous on $\mathbb{R}$,
f also attain its bounded as any continuous function on [0,1] will attains its bounds

Q63. (1,2,4)
Sol.

(i) $f:(0,1)\to\mathbb{R}$ $x\to\tan\pi\left(x-\frac{1}{2}\right)$ is a bijection which allows you to define a complete metric $d(x,y)=|f(x)-f(y)|$ on (0,1) induced by usual topology

(ii) ((0,1),d) $d(x,y)=|x-y|$ induced by usual topology is not complete as not closed

(iv) $([0,1]d),d(x,y)=|x-y|$ induced by usual topology is complete as closed subsed of complete metric space is complete

Q64. (3, 4)

Sol. $x_1=0,x_2=1,x_n=\frac{x_{n-1}+x_{n-2}}{2}$ $0,1,\frac{1}{2},\frac{1}{2},\frac{3}{4},\frac{5}{8},\ldots$

Clearly $\langle x_n\rangle$ is not a monotone seqeunce of differences is $1,-\frac{1}{2},\frac{1}{4},-\frac{1}{8},\frac{1}{16},\ldots$ which is G.P. with common ratio -1/2 and given seqeunce is seqeunce of partial sume of this G.P

$$\therefore \quad x_n=\frac{\left(\frac{-1}{2}\right)^{n-1}-1}{\left(\frac{-1}{2}-1\right)}=\frac{2}{3}\left(1-\left(\frac{1}{2}\right)^{n-1}\right)$$. Therfore

$\langle x_n\rangle$ is Cauchy seqeunce and convergent to $\frac{2}{3}$.

Q65. (2, 3)

Sol. $f:[0,1]\to\mathbb{R}$ is deinfed as

$$f(x)=\begin{cases} 0 & \text{if } x\in\left[0,\frac{1}{3}\right] \\ x-\frac{1}{3} & \text{if } x\in\left[\frac{1}{3},\frac{1}{2}\right] \\ \frac{2}{3}-x & \text{if } x\in\left[\frac{1}{2},\frac{2}{3}\right] \\ 0 & \text{if } x\in\left[\frac{2}{3},1\right] \end{cases}$$

Clearly f is continuous on [0,1] but not differentiable at $\frac{1}{3},\frac{1}{2}$ and $\frac{2}{3}$ as L.H.D. and R.H.D. are different at these points

Q66. (3,4)

Sol. $V\langle(1,1,1)(0,1,1)\rangle$

$\Rightarrow V=\{(a,a+b,a+b):a,b\in\mathbb{R}\}$, which is a plane passing through origin therfore $\mathbb{R}^3/V$ is disconnected

i.e., if we add any point of V to $\mathbb{R}^3/V$ we get a connected space

therefore $\left(\mathbb{R}^3/V\right)\cup(0,0,0)$ is connected

$\left(\mathbb{R}^3/V\right)\cup\{tu,+(1-t)u_3:0\le t\le 1\}$

$=\left(\mathbb{R}^3/V\right)\cup\{(1-t,0,1):0\le t\le 1\}$ is not connected as a + b=0,a + b = 1 which is not possible

$\left(\mathbb{R}^3/V\right)\cup\{tu,+(1-t)u_2:0\le t\le 1\}$

$=\left(\mathbb{R}^3/V\right)\cup\{(t,1-t,t):0\le t\le 1\}$ is connected as put $t=\frac{1}{2}$ we get a point $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)=(a,a+b,a+b)$ a point of V correcponding to

$a=\frac{1}{2},b=0$ $\left(\mathbb{R}^3/V\right)\cup\{(1,2t,2t):t\in\mathbb{R}\}$ is connected as by t = 0, we get a point (0,0,0) which is a point of V.

Q67.(3,4)

Sol. $\lim_{x\to 0}\frac{1}{x}$ does't exists $\Rightarrow\frac{1}{x}$ is not U.C

$\lim_{x \to 0} Sin\frac{1}{x}$ does't exist $\Rightarrow$ $Sin\left(\frac{1}{x}\right)$ is not U.C. on (0,1)

$\lim_{x \to 0,1} x\ Sin\frac{1}{x}$ exist $\Rightarrow$ $x\ Sin\frac{1}{x}$ is U.C on (0,1)

$\lim_{x \to 0,1} \frac{Sin\ x}{x}$ exist $\frac{Sin\ x}{x}$ is U.C. on (0,1)

Q68. (1, 2)

Sol. $T: V \to V, \dim V = n+1$

$(Tp)(x) = p'(1), \forall x \in \mathbb{C}$

Rank (T) = $\{p'(x) : p \in V\} = \mathbb{C}$

$\Rightarrow \dim\ Rank\ T = 1$

$\Rightarrow \dim\ Ker\ T = n.$

Q69. (3, 4)

Sol.

$A^t AX = \lambda X, X \neq 0$

$\Rightarrow\ X^t A^t AX = \lambda X^t X$

$\Rightarrow\ (AX)^t\ AX = \lambda X^t X$

i.e., $\|Ax\| = \lambda\|X\|$ but $\|Ax\| \geq 0 \Rightarrow \lambda \geq 0$

$\therefore$ eigenvalues of $A^t A$ are non negative and hence $I + A^t A$ is invertiable

Q70.(2, 3)

Sol. $T^n = 0$ $\Rightarrow$ T is nilpotent matrix $\Rightarrow$ eigenvalues of T are 0,0,...0

If $T \neq 0$ and $T^n = 0$ then T cannot be diagonalizable in that case $A.M(0) \neq G.M(0)$]]

Q71. (2,3,4)

Sol. $f_j(x) = x^j$ for $0 \leq j \leq n$ $a_{ij} = \int_0^1 f_i(x) f_j(x)\,dx$

learly $\dim V(\mathbb{R}) = n+1 > n..$

Since $\langle f_i, f_j \rangle > \int_0^1 f_i(x) f_j(0)\,dx$ defines an inner-product and A being the matrix of this inner product spaces.

So. $\langle Av, v\rangle \geq 0$ asA is positive definte

A is positive definite $\Rightarrow \det A > 0$

72. (2,3,4)

Sol. For any set $|P(A)| \geq 1$ i.e., Power set P(A) is always non empty

For any finite set A with n elements $|P(A)| = 2^n$.

i.e., for any finte set A P(A) is also a finite set

If we consider the definition of a contable set as it is similar the set $\mathbb{N}$ then for any contable set

$A, |P(A)| = 2^{X_0} = |\mathbb{R}|$

i.e., for any set A, P(A) is never countably infinite

For any infinite set A, P(A) is always uncountable as above point

Q73. (1,4)

Sol. $f(x) = \begin{cases} x^2 & \text{if x is rational} \\ x^3 & \text{if x is irrational} \end{cases}$

on [0,1]

Clearly f is continuous only at x = 0 $\Rightarrow$ f is NOT Riemann's integrable

Also $\int_0^1 f(x) = \int_0^1 x^3 dx = \frac{1}{4}$ as x^3 given less values of f and

$\int_0^1 f(x) = \int_0^1 x^2 dx = \frac{1}{3}$ as x^2 given high values of f

Hence

$\frac{1}{4} = \underline{\int_0^1} f(x)\,dx < \overline{\int_0^1} f(x)\,dx = \frac{1}{3}$ and f is NOT Riemenn's integrable

Q74. (3)

Sol. $\|p\|_k = \max\{|p(0)|, |p'(0)|, ..., |p^k(0)|\}$ $\|p\|_k$ is a norm iff $\|p\|_k = 0 \Leftrightarrow p = 0$ (as all other properties of norm are obvious)

Now $\|p\|_k = 0 \Leftrightarrow$

$p(0) = 0, p'(0) = 0, ..., p^k(0) = 0$

$\therefore$ $\|p\|_k = 0 \Leftrightarrow p = 0 \Leftrightarrow k \geq d$ as if $k \geq d$ then 0 become a roots of p(x) of multiplicity $> d = \deg p(x)$

75. (2)

Sol.

$\det A > 0, \det B < 0$

$C(t) = tA + (1-t)B.$

$C(t):[0,1] \to M_n(R)$ is a continuous function

$\det : M_n(R) \to R$ is also a continuous function

Therefore $\det 0(t):[0,1] \to R$ is also a continuous function

Now

$|C(0)| = |B| < w0$
$|(1)| = |A| > 0$ .Thereofore by Intermediate property of continuous function there exist $\exists$ to $\in (0,1)$ such that $|C(t_0)| = 0$ that is matrix $C(t_0)$ s is not invertible

Again $\det(c(t))$ is continuous on $[0,1]$ so it can't be zero for al but finitely many values of [0,1] i.e., c(t) is invertible for only finitely many $t \in [0,1]$

Q76. (1,2,3,4)

Sol. $Q = I + 2P,$ and eigen values of P are -1 or 1

$\Rightarrow$ Eigen values of Q is either 3 or -1

$\Rightarrow$ Q is invertible and det Q > 0 if det P > 0 as number of positive and negative eigenvalues of P and Q are same in multiplicity

Again $Q = I + 2P$ is invertiable so it will map basis to basis

Hence $\{(I+2P)a'_i = a'_i + 2b_i \;\; | i = 1, 2...n\}$ is also a basis of V

Q 77. (1, 2)

Sol. $\langle x, y\rangle = \langle Ax, Ay\rangle, x, y \in \mathbb{R}^n$ .To form $\langle x, y\rangle$ an inner product all property except positive definiteness are trivial

$\langle x, y\rangle = \langle Ax, Ay\rangle$

$= (Ax)^t Ax.$

$= x^t A^t Ax \geq 0$

as $A^t A$ is always symmetric with non-negative eigenvalues. Therefore for positive definite property we must have eigenvalues of $A^t A$ are postive

i.e, A is invertiable

i.e., Ker $A = \{0\}$

i.e., $P(A) = n$

Q78.(1,2)

Sol. Take $\{v_1, v_2, ..., v_n\} = \{e_1, e_2, ..., e_n\}$ where $e_i = (0, 0, ..., 1, 0...0)$. Then clearly $u_1, u_2, ..., u_n$ are naturally orthogonal and form a bais for $\mathbb{R}^n$

Q79. (1,2,4)

Sol. The most genral Bilinear transformation f which maps upper half to unit disc with $f(2i) = 0$ is given by

$$f(z) = e^{i\theta} \frac{z - 2i}{z + 2i}$$

$\Rightarrow$ f has simple pole at $z = -2i$ with $f(i)\overline{f(-i)} = 1$ and others are also satisfied

Q80. (3, 4)

Sol. $f(x) = \int_1^2 \frac{1}{(x-z)^2} dx$, $\text{Im}(z) > 0$

$$= \frac{-1}{(x-z)^2}\Big|_1^2 = \frac{-1}{2-z} + \frac{1}{1-z}$$

So $F(z)$ has simple poles at $z = 1$ and 2.

Q81. (1,2,3,4)

Sol. f is an entire function, then

Re(f) is bounded $\Rightarrow$ f is constant

Im(f) is bounded $\Rightarrow$ f is constant

$\{u(x, y) + v(x, y) : x + iy \in \mathbb{C}\}$ is bounded

$\Rightarrow \{u^2(x, u) + v^2(x, u) : z = x + iy \in \mathbb{C}\}$

$\Rightarrow$ u and v are bounded

$\Rightarrow$ f is bounded.

Q82.(1,4)

Sol. $A_{20} \trianglelefteq S_{20}$

H $\to$ 7- sylow subgroup of A_{20} $|A_{20}| = \frac{20!}{2}$

(i) In $|S_{20}|$, the highest power of 7 which divides 20! is 7^2 so sylow-7 subgroup of S_2 is of order 49.

(ii) S_{20} has no element of 49 so H is not cyclic

(iii) Sylow 7-Subgroups are not unique so not normal

Q83. (1,2,3,4)

Sol.

(i) $\frac{R[X]}{I} \cong R \Rightarrow I$ is prime

(ii) I is maximal $\Rightarrow \frac{R[X]}{I} \cong R$ is a field.

$\Rightarrow R[X]$ is a P.I.D

(iii) R[X} is E.D $\Rightarrow$ R is a field

$\Rightarrow$ I is maximal as $\frac{R[X]}{I} \cong R$.

(iv) R[X] isa P.I.D. $\Rightarrow$ R is a field

$\Rightarrow$ R[X] is E.D

Q84.(2,3,4)

Sol. Smallest topology on $\mathbb{C}$ in which all singletion sets are closed is the co-finite topology on $\mathbb{C}$.

Co-finite topology $(\mathbb{C},\tau)$ is the not Hausdorff as any two open sets in $(\mathbb{C},\tau)$ intersects.

In co-finite topology $(\mathbb{C},\tau)$ any infinite set is dense.

So $\mathbb{Z}$ is dense in $(\mathbb{C},\tau)$

Q85. (1, 2, 3)

Sol. $\{X_\alpha\}_{\alpha \in I}$ be discrete topological spaces. Then the product space

$X = \prod_{\alpha \in I} X_\alpha$ is discrete iff for all but a finitely many value of α, X_α is singleton or trivial space

Q86. (1,2,4)

Sol. $A = \{z \in \mathbb{C} \mid |z| > 1\}$ $B = \{z \in \mathbb{C} \mid z \neq 1\}$

(i) $e^z : A \to B$ is a continuous onto function

(ii) $f : B \to A$

$f\left(re^{i\theta}\right) = (r+1)e^{i\theta}$

continuous one-one function

(iii) If $f : B \to A$ is non-constant analytic

$\Rightarrow \left|\frac{1}{f}\right| < 1$ and $\frac{1}{f}$ is analytic having removeable sin gularity at z = 0

$\Rightarrow \frac{1}{f}$ is constant, which is a contracdiction

(iv) $f : A \to B$ $f(z) = e^z$ is non constant analytic function

Q87. (2,4)*

Sol. $A = \int_0^1 x^n (1-x)^n \, dx$ is Beta function with value as $\frac{n!n!}{(2n+1)!}$ which is a rational number and

$A^{-1} = \frac{(2n+1)!}{n!n!}$ which is a natural number

Again

$0 < \frac{1}{x(1-x)} \le 4$

$\Rightarrow 0 < x^n (1-x)^n \le 4^{-n}$

$\Rightarrow 0 < A \le 4^{-n}$

Q88. (1,4)

Sol.

(i) If G be a finite abelian group and $d \mid 0 \mid G$ then G has subgroup of order d

(ii) $G = \mathbb{Z}_2 \times \mathbb{Z}_4$ is abelian group and $8 \mid 0(G)$ but G has no element of order 8.

(iii) $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ is abelian group and every proper subgroup of G is cyclic but G is not cyclic.

(iv) Since G is abelian then all subgroup of G is normal and if d | 0|G then G has subgroup of order d which is normal.

If H is subgroup of G then $0(0) \mid 0 \mid (G)$.Let $\frac{0(G)}{0(H)} = K$ then G has normal subgroup of order K such that .Let N is normal subgroup of G of then $0\left(\frac{G}{H}\right) \approx 0(H)\ \frac{G}{\infty} \approx H$

Q89.(1,2,4)

Sol.

(i) Upto isomorphism only Abelian group of order p^2 are $\mathbb{Z}_{p^2}$ and $\mathbb{Z}_p \times \mathbb{Z}_p$

(ii) Upto isomorphism group of order p^2 are $\mathbb{Z}_{p^2}$ and

$\mathbb{Z}_p \times \mathbb{Z}_p$

(iii) Finite I.D. are fields and any field of order p^2 isomorphic

Q90.(1,2)

Sol. If $f(x) \in \mathbb{Z}[x]$ and $f(x)$ is irreducible in then $f(x)$ is irreducible in $Q[x]$ $Z(x)$ because every reducible polynomial in $Q(x)$ is also reducible polynomial in $Z(x)$

91. 2 , 4 ; For the Cauchy problem for the Eikonal equation $p^2 + q^2 = 1;\qquad p \equiv \frac{\partial u}{\partial x},\ q \equiv \frac{\partial u}{\partial y}$

$u(x, y) = 0$ on $x + y = 1,\ (x, y) \in \mathbb{R}^2$.

The Charpit's equations for the differential equa tion are $\frac{dx}{dt} = 2p;\ \frac{dy}{dt} = 2q;\ \frac{du}{dt} = 2;\ \frac{dp}{dt} = 0;\ \frac{dq}{dt} = 0$..

and $u\left(1, \sqrt{2}\right) = 1$

92.- 1 ,2 ; let H(x) be the cubic Hermite interpolation of $f(x) = x^4 + 1$ on the interval $I = [0,1]$ interpolating at $x = 0$ and $x = 1$. Then

$\max_{x\in I} |f(x) - H(x)| = \frac{1}{16}$ and

The maximum of $|f(x) - H(x)|$ is attained at

$x = \frac{1}{2}$.

93. (1,2)

Sol $2y'' + 3y' + y = e^{-3x}$ with $\lim_{x\to\infty} e^x y(x) = 0$

A.E. is given by $2x^2 + 3x + 1 = 0$

i.e., $(x+1)(2x+1) = 0$

i.e., $x = -1, -\frac{1}{2}$

$C.F. = y(x) = c_1 e^{-x} + c_2 e^{-\frac{1}{2}x}$

$P.I. = \frac{e^{-3x}}{2D^2 + 3D + 1} = \frac{e^{-3x}}{10}$, so complete solution is $y(x) = c_1 e^{-x} + c_2 e^{-3x} + \frac{e^{-3x}}{10}$

$\lim_{x\to\infty} e^x y(x) = 0$

if $c_1 = c_2 = 0$

$\because\quad y(x) = \frac{e^{-3x}}{10}$

$\Rightarrow\ y(1) = \frac{1}{10}, \lim_{x\to\infty} e^{2x} y(x) = 0$

Q94. (2,3)

Sol. $y'(x) = \lambda Sin(x+y), y(0) = 1.$

$f(x,y) = \lambda Sin(x+y)$

$\frac{\partial f}{\partial y} = \lambda Cos(x+y)$ which is bounded so f is lipschitz function on whole $\mathbb{R}$. Then given O.D.E has a unique solution in a neighbourhood of 0.

For $\lambda = 0,\ y = 1$ is a solution of ODE on $\mathbb{R}$ .Therefore for $|\lambda| < 1$ there is a solution on $\mathbb{R}$

Q95.(3,4)

Sol. $-y'' + (1+x) = \lambda y, x \in (0,1)$

$y(0) = y(1) = 0$

$\Rightarrow\ y'' - (1 + x - \lambda) y = 0,\ y(0) = y(1) = 0$.

This is S.L.P. with eigen values as

$\lambda_n = \left(\frac{n\pi}{\lambda}\right)^2, n = 1, 2, \ldots$

$\ell = 1 \Rightarrow \lambda_n = (n\pi)^2,\ n = 1, 2, \ldots$

$\Rightarrow \lambda_n$ are countable and $\lambda_n < 2$ for $n \in \mathbb{N}$

Q96. (3, 4)

Sol. DAmbert's solution of the wave equation

$u_{tt} - u_{xx} = 0,$ for $(x,t) \in \mathbb{R} \times (0, \infty)$

$u(x,0) = 0 f(x), x \in \mathbb{R}$

$u_t(x,0) = g(x), x \in \mathbb{R}$ is given by

$u(x,t) = \frac{1}{2}\left[f(x+t) + f(x-t)\right] + \frac{1}{2}\int_{x-t}^{x+t} f(z)\,dz$

Now, it is given that

$f(x) = g(x) = 0$ for $x \notin [0,1]$

Now

If $x+t<0 \Rightarrow f(x+t)=f(x-t)=0$

and $\int_{x-t}^{x+t} g(z)=0$

Again if $x-t>0$ then $x+t>0$

$$\Rightarrow f(x+t)=f(x-t)=0 \text{ and } \int_{x-t}^{x+t} g(z)=0$$

Q97.(1 , 3)

Sol. Laplace equation

$u_{xx}+u_{yy}=0, \qquad 0<x,y<\pi$

$u(x,0)=0=u(x,\pi)$ for $0\le x\le\pi$.

$u(0,y)=0, u(\pi,y)=Siny+\sin 2y$ for $0\le y\le\pi$

solution is given by

$$u(x,y)=\sum_{n=1}^{\infty}\mathbb{C}_n Sin\left(\frac{n\pi y}{b}\right)Sin\left(\frac{n\pi x}{b}\right), b=\pi$$

i.e., $u(x,y)=\sum_{n=1}^{\infty} D_n Sin(xy)Sinh(nx)$

Now $u(\pi,y)=Siny+Sin2y$ gives

$$\sum_{n=1}^{\infty} D_n Sin(xy)Sinh(n\pi)=Siny+Sin2y$$

$$\Rightarrow D_1=\frac{1}{Sinh\pi}, D_2=\frac{1}{Sinh(2\pi)},$$

$D_{n=0}$ for $n>2$.

$\therefore$ solution is given by

$$u(x,y)=\frac{Sin(y)Sinhx}{Sinh\pi}+\frac{Sin(2y)Sinh(2x)}{Sinh(2\pi)}$$

$$\Rightarrow u\left(1,\frac{\pi}{2}\right)\frac{Sin\left(\frac{\pi}{2}\right)Sinh(1)}{Sinh\pi}+\frac{\sin\pi.Sinh(2)}{Sin2(2+1)}$$

$$=\left[Sinh(\pi)\right]Sinh(1)$$

Again

$$u\left(1,\frac{\pi}{4}\right)=\frac{Sin\left(\frac{\pi}{4}\right)Sinh(1)}{Sinh\pi}+\frac{Sin\left(\frac{\pi}{2}\right)Sinh(2)}{Sinh(2\pi)}$$

$$=\frac{\left[Sinh(\pi)\right]^{-1}Sinh(1)}{\sqrt{2}}+\left[Sinh(2\pi)\right]^{-1}Sin(2)$$

Q98. (1,3)

Sol. $y_{n+1}=y_n+w_1k_1+w_2k_2$

$k_1=h\ f(x_n,y_n)$

$k_2=h\ f(x_n+c_2h, y_2+a_{12}k_1)$

with

$w_1+w_2=1$

$c_2w_2=1/2$

$a_{12}w_2=1/2$

(i) take $w_1=w_2=\frac{1}{2}, c_2=1, a_{12}=1$

(ii) take $w_1=\frac{1}{4}, w_2=\frac{3}{4}, c_2=\frac{2}{3}, a_{12}=\frac{2}{3}$

Q99. (1,3)

Sol. $I(y(x))=\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx$

We know that shortest distance between two point is given by straight lines. Therefore the general solution of Euler's equation for the above functional is $y=c_1x+c_2$.

Also, since both the end points lies on the extremals $y=c_{y=c_1x+c_2}x+c_2$.

therefor $c_1x_1+c_2=x_1^2$ and $c_1x_2+c_2=x_2-5$

Transversality conditions are

$$\left[\sqrt{1+y'^2}+(2x-y')y'\left(1+y'_2\right)^{-\frac{1}{2}}\right]x=x_1=0 \qquad \ldots(1)$$

$$\left[\sqrt{1+y'^2}+(1-y')y'\left(1+y'^2\right)^{-\frac{1}{2}}\right]x=x_2=0 \qquad \ldots(2)$$

Put $y^1=c_1$ in (1) and (2) we get

$$c_1=-1, c_2=\frac{3}{4}, x_1=\frac{1}{2}, x_2=\frac{23}{8}$$

Therefore extremal is $y=-x+\frac{3}{4}$ and the shortest distance between the given parabola and the strainght line is given by $\int_{1/2}^{23/8}(1+1)^{1/1}a_n=\frac{19\sqrt{2}}{8}$.

100. - 2 , 3 , 4 ; A particle of unit mass moves in the direction of x-axis such that it has the Lagrangian

$L = \frac{1}{12}\dot{x}^4 + \frac{1}{2}x\dot{x}^2 - x^2$

If $Q = \dot{x}^2 \ddot{x}$ represent a force acting on the particle in the x-direction. If $x(0) = 1$ and $\dot{x}(0) = 1,$ then the value of x is 1 at x =1 ; $\sqrt{5}$ *at* $x = \frac{1}{2}$ and 0 *at* $x = \sqrt{\frac{3}{2}}$

Q101. (1, 2)

Sol. Since $\left|1 - |x-2|\right|$ is linear polynomial in $[0,1],[1,2],[2,3]$ and in subinterval [0,2], [2,3] and Trapezoid rule give exact value of integration for polynomial of degree ≤ 1

So only option (i) and (2) are correct one

102. - 1 ; The curve $y = y(x),$ passing throught the point $(\sqrt{3},1)$ and defined by $\int_0^y \frac{f(v)dv}{\sqrt{y-v}} = 4\sqrt{y},$ where

$f(y) = \sqrt{1 + \frac{1}{y'^2}}$ is the part of a straight line

103.- 4 ; If (Ω, F, P) be a probability space and A be an event with P(A)>0 , then Q define a probability measure on (Ω, F) if $Q(D) = P(D \mid A) \quad \forall\ D \in F$

104. - 1 ,4 ; If X and Y are independent and indentically distributed random variable and Z=X+Y , then the distribution of Z is in the same family as that of X and Y if X is normal or binomial.

105. - 2 , 3 ; The method of moment estimators of α exists and it is a consistent estimator of α and also the method of moment estimators of μ exist and it is a consistent estimator of μ

106. - 1 , 3 ; The hazard function of X is a constant function for $p = 0$ and p=1 and decreasing function for all 0<p<1

107. - 1,2,3 ; If Y_1, Y_2, Y_2, Y_4 be uncorrelated obsercvations such that

$E(Y_1) = \beta + \beta + \beta = E(Y_2), E(Y_3) = \beta_1 - \beta_2 = E(Y_4)$ and $Var(Y_i) = \sigma^2$ for $i = 1, 2, 3, 4.$ Then, $p_1\beta_1 + p_2\beta_2 + p_3\beta_3$ is estimable if and only if $p_1 + p_2 = 2p_3.,$

An unbiased estimator of σ^2 is

$[Y_1 - Y_2)^2 + (Y_3 - Y_4)^2]/4$ and

The best linear unbiased estimator of

$\beta_1 - \beta_2$ *is* $\frac{1}{2}(Y_3 + Y_4).$

108.- 1,2,3,4 ; For the given linear regression model If $h_{ij} = 0$ or 1 for some i, then h_{ij} =0 for all $j \neq i.$

The variance covariance matrix of the vector of the predicted values $\underline{\tilde{Y}}$ (of Y) is $\sigma^2 H$

and , For $1 \leq i \leq n,$ if e, is the residual corresponding to $Y_i, i.e,\ e_i = Y_i - \tilde{Y}_i, \tilde{Y}_i,$ being the predicted values of Y_i then the variance of e_i equals $\sigma^2(1 - h_{ii})$

(Here, Y_i is the ith component of $\underline{Y}$)

109.- 1,3 ; If (X,Y) follow a bivariate normal distribution with mean vector (0.0) and dispersion matrix

$\sum = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \rho \neq 0.$ Suppose $Z = \frac{X - Y}{X + Y}\sqrt{\frac{1+\rho}{1-\rho}}$. Then

$\sqrt{\frac{1+\rho}{1-\rho}} \times \frac{X - Y}{\sqrt{X^2 + Y^2 + 2XY}}$ has a student-t distribution.

& Z is symmetric about 0.

110.- 1,2,3 ; A sample of size two is drawn from a population of 4 units using probability propotional to size sampling with replacement. The selection probabilities are $p_1 = 0.2, p_2 = 0.3, p_3 = 0.1$ and $p_4 = 0.4$ for units 1,2,3 and 4 in the population, respectively. If the value of a study variance for the i-th unit be $y_i, i = 1, 2, 3, 4.$ Let π_i denote the inclusion probability of the i-th unit and π_{ij} the joint inclusion probability of units i and $j,\ i < j, i, j = 1, 2, 3, 4.$ Then

$T = \left(\frac{1}{2}\right)\sum \frac{y_i}{p_i}$ is an unbiased estimator of the population total, where the sum is over the units in the sample , also

$\pi_1 = 0.36, \pi_2 = 0.51., \quad \pi_{12} = 0.12.$

111.- 1,2,3,4 ; For a balanced incomplete block design d with v treatment , b blocks, replication r, block size k and pairwise concurrence parameter λ ,and the standard fixed effect model for the data obtained throught d. then

The design is connected if $k \geq 2$.

The inequality $b \geq v$ holds for d.

The variance of the best linear unbiased estimator (BLUE) of a normalized treatment contrast is a constant.

The covariance between the BLUE of two or thogonal treatment consteasts is zero.

112. - 1 ; $q_3 > q_1$.

113.-2,3 ;

Extreme points of the convex region enclosed by

$$x \geq 0, \quad y \geq 0, \quad x \leq 3, \frac{1}{2}x + y \leq 4, \quad x + y \leq 5.$$

are (0 , 0) , (3 , 2) , (2 , 3) and (0 , 4)

So , $3x + 4y$ at these points are 0 , 17 , 18 & 16 respectively , so the optiomal value is 18.and (3,2) is an extreme point of the feasible region

114.- 1,3 ; If X_1, X_2, X_{2n+1} be a random sample from a uniform distribution on the interval $(\theta - 1, \theta + 1)$. and $T_1 = \overline{X}$, the sample mean, $T_2 = \overline{X}$, the sample median and $T_3 = \frac{T_1 + T_2}{2}$ be three estimators of θ. Then T_1 is consistent for θ and All the three estimators are unbiased for θ.

115.- 1, 2 ;

The joint probability density function of (X,Y) is

$$f(x, y) = \begin{cases} 3(y-1)^2, & 0 < y < 1 \\ 0, & otherwise \end{cases}$$

then

X and Y are not independent

and $f_Y(y) = \begin{cases} 3(y-1)^2, & 0 < y < 1 \\ 0, & otherwise \end{cases}$

116. - 1,2,4 ; If X_n be the result of the n-th roll of a fair die, $n \geq 1$. $S_n = \sum_{i=1}^{n} X_i$ and Y_n be the last digit of S_n for $n \geq 1$ and $Y_0 = 0$. Then

$\{Y_n : n \geq 0\}$ is an irreducible markov chain ,

$\{Y_n : n \geq 0\}$ is an aperiodic Markov chain

and $P(Y_n = 5) \to \frac{1}{10}$ as $n \to \infty$

117. - 1,3,4 ; If $\{X_i\}$ is a sequence of independent and identically distribution random variables with common density function , $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & otherwise \end{cases}$

and $\{Y_i\}$ is a sequence of independent identically distribution random variables with common density function , $g(x) = \begin{cases} 4e^{-4y}, & y > 0 \\ 0, & otherwise \end{cases}$

also $\{X_i\}, \{Y_j\}$ are independent families and $Z_k = Y_k - 3X_k, k = 1, 2, \ldots$, then , $P(Z_k > 0) > 0$

, $\sum_{k=1}^{n} Z_k \to -\infty$ with probability 1 , and $P(Z_k < 0) > 0$.

118. - 2 ; If $X_1, ..., X_n$ be a random sample from

$$f(x; y) = \begin{cases} 2\lambda x e^{-\lambda x^2}; & x > 0 \\ 0, & otherwise \end{cases}$$

where $\lambda > 0$ is an unknown parameter. To test the following hypothesis at level $\alpha > 0$. We want to test $H_0 : \lambda \leq 1$ vs $H_1 : \lambda > 1$. Then UMP test is of the form $\sum_{i=1}^{n} x_i^2 < d_n$ wirh $d_n < d_{n+1}$,for all n.

119.- 1,2,3 ; If $X_1, \ldots X_n$ be i.i.d. $N(\mu, 1)$. .. , to test $H_0 : \mu = 0$ versus of the UMP test at μ of size α based on sample size n.

$\lim_{n \to \infty} p_n(\mu, \alpha) = 1 \ \forall \mu > 0, \forall \alpha > 0.$,

$\lim_{\mu \to 0} p_n(\mu, \alpha) = \alpha \ \forall n \geq 1, \forall \alpha > 0.$ and

$\lim_{\alpha \to 0} p_n(\mu, \alpha) = 0 \ \forall n \geq 1, \forall \mu > 0.$

120.- 1,2,3 ; If X be a random sample from a Poisson distribution with parameter λ .Then parameter λ has a prior distribtion $f(z)$; where

$$f(z) = \begin{cases} e^{-z}; & z > 0 \\ 0, & otherwise. \end{cases}$$

under the squared error loss function

The Bayes' estimator of e^{λ} is 2^{X+1},

The posterior means of λ is $\frac{X+1}{2}$ &

The posterior distribution of λ is gamma.