UNIT 1 INTRODUCTION TO THE FORM OF ARGUMENTS IN MODERN LOGIC

Contents

1.0 OBJECTIVES

The purpose of this unit is to introduce the students to the importance of argument forms in modern logic. Arguments and argument forms are not the same. In our study of modern logic this distinction turns out to be crucial. This is so, mainly because all the Rules of Inference refer to argument forms rather than to arguments. This will become evident as we go along. In short, this block is designed to introduce the students of logic to the symbols and techniques of modern logic.

1.1 INTRODUCTION

Since this is the first unit dealing with symbolic logic, we confront a number of questions before we deal with the forms of argument in particular. What is the main concern of logic? What is the context in which symbolic logic was born? What is the history of logic? How do the old and new systems of logic differ? Why do we use symbols? What are the advantages of using symbols? What is the nature of arguments? We shall briefly deal with questions of this kind before we speak of argument forms proper.

The main question in logic, whether modern or ancient, has always been:

Does the conclusion *follow from, (= a consequence of)*, the premise(s)?

We need a general theory of inference to answer effectively this prime question of deductive reasoning. Explanation of the relation between the premises and the conclusions in deductive arguments, on the one hand and discovery of techniques to distinguish between valid and invalid deductive arguments, on the other, constitute such general theory. Two great logical theories have emerged in order to achieve these goals: The first is Aristotelian (classical) Logic, and the second is symbolic (modern) logic. The latter is an extension of the former. Though both these bodies of logical theory aim at the discovery of truth, they adopt different techniques. This is one difference. The second difference consists in the concept of proposition itself. Aristotle restricted the meaning of proposition to subject-predicate form, which is why it is called predicate logic whereas the second extends the concept of proposition to include simple and what is ordinarily understood in grammar as compound sentence. Inclusion of relational proposition brought logic and mathematics much closer. Since the technique invented by modern logic is much more advanced than the technique adopted by Aristotelian system in terms of performance, modern logic achieved what Aristotelian system could not. In this unit and in the units to follow a brief exposition of methods will find the place they richly deserve.

The answer to the question whether the conclusion follows from the premise(s) is not at all easy to deal with. This is so because of the linguistic fallacies such as equivocation, amphiboly, metaphorical styles, and so on. That is to say, there are peculiarities in natural language (English or any other) that make exact logical analysis rather difficult: Words may be vague or equivocal, the construction of arguments may be ambiguous, metaphors and idioms may be confusing or misleading, and emotional appeals may distract. Modern logic overcomes these difficulties with the introduction of an artificial language. The symbols themselves are artificial in the sense that they do not belong to any natural language. Symbolic logic achieved the breakthrough when it formulated sentences of arguments in this language. This is the context in which symbolic logic was born.

Symbols help us to get to the heart of the argument, unlike the natural language. It makes our logical operations mechanical and easy. (It is like using Indo-Arabic numbers 1, 2, 3, … instead of the Roman numerals I, II, III, etc. It is easier to multiply 113 by 9 than to multiply CXIII by IX). Symbols thus greatly facilitate our thinking about arguments and argument forms. Though it may sound paradoxical, symbolic language helps us to accomplish some intellectual tasks without even having any knowledge of the content of statements. Even if linguistic difficulties are thus overcome, the question of deciding the validity or invalidity of arguments remains, of course.

1.2 A SHORT STORY OF LOGIC

The second point of clarification has to do with the history of Logic. Logic was first systematized into a science by Aristotle, the Greek philosopher, in the fourth century BC. His contribution had been regarded as the last word in logic for several centuries. Aristotle's contemporaries and the medieval scholastic thinkers followed him with some cosmetic changes here and there. But nothing significant was added to the Aristotelian (Traditional) logic.

In the seventeenth century G. W. Von Leibniz, a mathematician and philosopher, however, felt that Aristotelian logic needed some modification. His suggestion was a prelude to the path which the development of logic took during subsequent centuries. It was only in the nineteenth century that the logicians began actualizing the ideas conceived by Leibniz. Then on, the development of logic has been unprecedented. This was due to the fast development in mathematics and its extensive dependence on logic, thanks to a host of philosophers of mathematics like Boole, Russell, Frege, etc. to name a few. It must be remembered that dissent voice was not absent. Philosophers like Poincare did oppose this influential school of thought.

This particular revelation may be surprising, not just interesting for us. They established that logic is the foundation of mathematics. This is the principal thesis of Principia Mathematica written by Bertrand Russell and A. N. Whitehead. Emphasis on this relation between logic and mathematics may induce the feeling that one should have a mathematical background in order to understand symbolic logic. But this belief is not really well-founded. However, sound knowledge of mathematics is, surely, most useful.

1.3 CLASSICAL LOGIC AND SYMBOLIC LOGIC

Though these two systems differ with regard to the meaning of proposition which is of fundamental importance, the fact is that modern logic makes explicit what was implicit in Aristotelian logic. So, it may be said that the difference between the old and the new is one of degree rather than of kind. Some examples will easily make this disclosure clear. The difference between an adult and infant is a matter of degree. But the difference between a boy and a girl is in kind. But the difference in degree, between the ancient and modern logic, is enormous since particular class of symbols used by symbolic logic makes logic an immeasurably more powerful tool for analysis and deduction. This class of symbols is what is known as sentential connective. Of course, there are other symbols too which played crucial role in the development of logic.

How did it happen? Surely, it did not happen overnight. It is not revolution, but evolution that took place. As mentioned earlier, the performative ability of symbols achieved this feat. The special symbols in modern Logic permit us (i) to exhibit with greater clarity the logical structures of arguments obscured by their formulation in ordinary language, (ii) to divide more easily arguments into valid and invalid, for in it the peripheral problems of vagueness do not arise; and (iii) establish the nature of deductive argument. Though we have said that both symbolic logic and traditional logic are basically the same, these differences have to be taken note of. a) Traditional logic takes the terms (in a proposition) as the basic unit of analysis and is concerned with their relation. Symbolic logic takes proposition as the basic unit and is concerned with the relation between propositions. b) Symbolic logic, while dealing with argument forms, uses symbols instead of propositions which made the task much easier. It may be noted that we do not, for practical purposes, make a distinction between sentences, statements, and propositions. We use them interchangeably, though a clear distinction can be made between them since a sentence is always a part of language while a proposition is what a sentence in any language means.

1.4 WHY USE SYMBOLS?

As we have already said, the use of symbols is helpful i) to avoid peripheral linguistic difficulties, ii) to economize space and time needed for writing, iii) to restrict the attention needed for grasping the meaning of long sentences or equations, and so on. This distinct advantage explains why various sciences have developed their own symbolic language. Thus, for example, in mathematics the equation A x A x A A x A x A x A x A x A x A x A x A x A = B x B x B x B x B x B x B is expressed more briefly and intelligibly as A12 = B7. Logic too has evolved technical notations to achieve the goal. Aristotle used certain abbreviations to facilitate his own investigations. But then these are symbols which can perform only at elementary level. For that matter all terms are symbols only. Therefore what matters is the performative ability of symbols.

Modern Logic, however, introduced many more symbols. Such a step enabled logicians to simplify the most complex argument. Simplicity does not mean that something is devoid of content. It only means that an argument is capable of being tested with minimum number of Rules and within shortest possible time. In fact accomplishment of this task requires something like creativity. What is the value of all this exercise, it may be asked. The answer is simple. When mistakes are easily detected, they are less likely to be made.

1.5 THE NATURE OF ARGUMENTS

Having clarified some of the issues raised at the beginning of this unit, we now pass on to the nature of arguments. This is better understood when it is contrasted with argument forms. What is an Argument? An argument is a group of sentences where one sentence is claimed to follow from others, which are regarded as supplying conclusive evidences for its truth. Every argument has a structure, viz. premises and conclusion. Premises provide support to the conclusion. Therefore premises can be regarded as evidences based on which conclusion is accepted. All arguments involve the claim that their premises provide evidence for the truth of conclusions. But it is important to note that only deductive argument claims that the premises provide absolutely conclusive evidences for the truth of the conclusion. This is the reason why deductive arguments are characterized as 'valid' or 'invalid.' However, inductive argument claims that the premises constitute some evidences for the conclusion. Therefore, the characterization 'valid' & 'invalid' cannot properly be applied to inductive arguments. Here our main concern is with deductive arguments. A deductive argument is valid when the premises and the conclusion are so related as to make it absolutely impossible for the premises to be true unless the conclusion is true too. The task of deductive logic is a) to clarify the nature of the relation which holds between premises and the conclusion in a valid argument, and b) to provide techniques for distinguishing valid from invalid arguments.

1.6 TRUTH AND VALIDITY

Truth and falsity are properties of propositions whereas validity and invalidity are properties of arguments. This leads us to an important question; what is the relation between the validity or invalidity of an argument and the truth or falsity of its premises and the conclusion? The answer to this question has two parts.

A) Valid arguments with true propositions:

Here is an example.

All mammals have lungs.

All bats are mammals.

∴ All bats have lungs.

An argument may contain only false propositions and be still valid.

All mammals have wings.

All trout are mammals.

∴ All trout have wings.

This is valid. For, what it affirms is; if the premises are true, then the conclusion must be true, even though, as a matter of fact, they are all false.

Note: These two examples of arguments show that valid arguments may or may not have true conclusions. Therefore the validity of an argument does not guarantee the truth of its conclusion. However, the truth of the conclusion does guarantee the validity.

B) Invalid arguments with true propositions:

If I am the prime minister of India, then I am famous.

I am not the prime minister of India.

∴ I am not famous.

Here it is clear that although both premises and conclusion are true the argument is invalid. This can be shown to be invalid by comparing it with another argument of the same form.

If Amitabh Bachchan is the prime minister of India, then he is famous.

Amitabh Bachchan is not the prime minister of India.

∴ Amitabh Bachchan is not famous.

This is invalid since its premises are true but its conclusion is false.

Last two examples show that although some invalid arguments have false conclusions, not all of them are so. The falsehood of its conclusion does not guarantee the invalidity of an argument. However, the invalidity of an argument does guarantee the falsehood of the conclusion. But the falsehood of conclusion does guarantee that either the argument is invalid or at least one of its premises is false. There is asymmetry involved between validity and invalidity which must be noticed. Hence there are two conditions that an argument must satisfy to establish the truth of its conclusion: (a) it must be valid. In this case logician is concerned with validity even for arguments whose premises might be false; (b) all premises must be true. (It is the task of scientific inquiry to determine the last condition).

Check Your Progress I

Note: Use the space provided for your answers.

Examine the following arguments.

1) Analyze the features of traditional logic and modern logic.

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2) Distinguish between truth and validity.

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1.7 ARGUMENT FORMS

Argument forms are important in modern logic. For the Rules provided by the modern logicians to test the validity of arguments are based primarily on the form of the argument and not on its content. As we have already said, the primary task of deductive logic is to distinguish valid arguments from invalid ones. We have already shown above that if the premises of a valid argument are true, then the conclusion must be true. We have also seen that if the conclusion of a valid argument is false, at least one of the premises must be false. In other words, the premises of a valid argument give incontrovertible proof of the conclusion drawn. Now we need to make this formal account of validity more precise. In order to do this we introduce the concept of argument form. Let us consider two examples, which evidently have the same form:

1) If Tagore wrote the plays attributed to Shakespeare, then Tagore is a great writer.

   Tagore is a great writer.

   Therefore Tagore wrote the plays attributed to Shakespeare.

Even if we agree with the premises of this hypothetical syllogism, we cannot agree with its conclusion. For, we can clearly see that this argument is invalid. One of the ways of proving its invalidity is to use the method of logical analogy. For we can as well argue:

2) If Nehru was assassinated, then Nehru is dead.

   Nehru is dead.

   Therefore Nehru was assassinated.

No one will seriously defend this argument because its premises are known to be true and the conclusion is known to be false. Therefore, this argument is obviously invalid. The *form* of this argument is the same as that of the first argument which is invalid. This way of refuting an argument is very effective. This way of refutation is known as *refutation by logical analogy*. This method points to an excellent technique of testing arguments. That is, to prove the invalidity of an argument it is enough to formulate an argument that (i) has exactly the same form as the first and (ii) has true premises and a false conclusion. This method is based on the fact that validity and invalidity are purely *formal* characteristics of arguments. In other words, any two arguments that have exactly the same form are either both valid or both invalid, no matter what the differences are in the subject matter with which they are concerned.

A given argument becomes clear when the simple sentences in it are abbreviated by capital letters. We may thus abbreviate the statements, 'Tagore wrote the plays attributed to Shakespeare,' 'Tagore was a great writer,' 'Nehru was assassinated,' and 'Nehru is dead,' by the letters, T, G, A, and D respectively. In this manner we can easily symbolize the two sample arguments given above as

$$\begin{array}{rcr} T => G & \qquad & A => D \\ G & & D \\ \therefore T & & \therefore A \end{array}$$

Comparison of these two examples illustrates what we mean by argument form.

How do we obtain argument forms? We need a method to obtain them because we base our study on the forms of arguments only rather than on particular arguments having those forms. What applies to the form applies equally well to what conforms to such form. This is a sort of generalization very much akin to mathematical induction. This is made feasible by way of introducing the notion of variables. Modern logicians use, to avoid confusion, small or lowercase letters from the middle part of the alphabet, p, q, r, s, … as statement variables. A statement variable is simply a letter for which, we substitute a statement. Not only simple sentences but compound sentences also can be substituted for sentence (statement) variables. In the light of these considerations, we can now define what an argument form is: An argument form is any array of symbols which contains sentence variables (p, q, r, s, t) such that when sentences are substituted for the sentence variables - the same sentence replacing the same sentence variable throughout - the result is an argument. The form of two arguments is as follows:

$$\begin{array}{r} p => q \\ q \\ \therefore p \end{array}$$

This is an example for argument form. For, when the sentences T and G are substituted for the sentence variables p and q respectively, the result is the first argument given above. Similarly, if we substitute A and D for the sentence variables p and q, the result is the second argument given above. This leads us to the idea of what logicians mean by a substitution instance: Any argument that results from the substitution of statements for statement variables in an argument form is called a *substitution instance* of that argument form. Therefore if the form is invalid, then any argument which subscribes to this form is invalid.

We have become familiar with one argument form. It corresponds to what traditional logic calls mixed hypothetical syllogism. Since we have already considered invalid argument form, it is desirable that we consider now a valid argument form.

3) If there is nuclear warfare, then humanity perishes.

There is nuclear warfare.

Therefore the humanity perishes.

It is obvious that this argument is valid. If this aspect is not clear, then we shall resort to logical analogy and consider another example.

4) If there is inflation, then the cost of living goes up.

There is inflation.

Therefore the cost of living will go up.

A cursory look at these arguments shows that they have identical structure. Therefore if one of them is valid, then the other one also must be valid. It is obvious that (4) is valid. It means that (3) also must be valid. Logical analogy is helpful only for a beginner who is not familiar with the irrelevance of matter of argument in a study of deductive logic.

This is not the only argument form we come across. Traditional logic considered several other types of arguments; categorical syllogism, mixed disjunctive syllogism and pure hypothetical syllogism are the other types. All these types correspond to argument forms. Since in the last unit we had studied exhaustively several aspects of categorical syllogism, we can restrict ourselves to conditional arguments. Let us recall these arguments.

Mixed Disjunctive Syllogism:

a) $p \vee q$

$\neg p$

$\therefore q$

A disjunctive syllogism which conforms to this form is valid. Otherwise, it is invalid. So we shall consider the form of an invalid argument.

b) $p \vee q$	c) $p \vee q$
p	q
$\therefore q$	$\therefore p$

In our study of logic we must have a clear perception of what we have to do. We may have to examine several arguments and arguments with plurality of structures. In such a situation we have to reduce every argument to the form to which it corresponds. Proper identification or matching of argument and argument form is necessary. Any mismatch will lead us astray. Therefore this is an important step in our endeavour. This will also explain why we should be familiar with argument forms.

Against this background, we shall examine examples. Though this is a repetition of what we studied earlier, considering the importance of argument form it is useful to do so.

5) Voters are either indifferent or ignorant.

Voters are not indifferent.

∴ Voters are ignorant.

6) Men are either humane or boorish.

Men are boorish.

∴ Men are humane.

How should we match? A little care reveals that (a) matches 9 (it is unimportant whether first alternative is denied or second alternative is denied) whereas (b) and 10 match. Any argument is examined in this fashion only.

Consider the form of pure hypothetical syllogism.

a) $p => q$	b) $p => q$	c) $p => q$
$q => r$	$\neg p => r$	$p => r$
$\therefore p => r$	$\therefore \neg q => r$	$\therefore p => r$

We have three argument forms. Among them first two are valid whereas the last form is invalid. In fact except first two forms any other form is invalid. Let us construct arguments which match these forms.

7) If atmosphere is polluted, then life on this planet becomes extinct.

If life on this planet becomes extinct, then God does not exist.

∴ If atmosphere is polluted, then God does not exist.

8) If scientists are honest, then science will progress.

If scientists are not honest, then religion is strong.

∴ If science does not progress, then religion is strong.

9) If politicians are patriots, then the country becomes prosperous.

If politicians are patriots, then democracy does not fail.

∴ If the country prospers, then democracy does not fail.

7 and 8 are valid. Therefore corresponding argument forms are also valid. 9 is invalid. Therefore corresponding argument form also is invalid. In other words, every argument has matter (content) and form. But it is the form that is fundamental from the point of view of validity. Modern logicians usually classify arguments according to the forms the arguments exhibit. Since it is possible, theoretically, to construct any number of arguments it is impossible to consider matter as the theme of deductive logic. Therefore form is the parameter to examine arguments.

As is clear from the above description, variables are *symbols which can be replaced*. There are three types of replaceable variables: class, individual, and sentential.

Class variables are those replaceable by names of classes. For example, cats, horses, mangoes etc. are class variables.

Individual variables are those that are replaceable by the names of individuals. Newton, the tallest animal in the world, the most massive star, etc. are individual names. In other words, general terms signify class variables whereas singular terms signify individual variables. Later we will understand that the difference between these variables contributes to the difference in notations, a point which becomes clear when we undertake a study of quantification.

Sentential variables are those replaceable by the names of sentences. x is regarded as sentential variable and when a proposition consists of 'x', it is called propositional function. Propositional function is neither true nor false. It takes truth-value only when it is given some value. Value is given when the propositional variable is replaced by a proposition. All propositions are constants in contrast with propositional function. It is an accepted practice to represent constants with upper case letters while propositional function is always represented with lower case x.

Similarly, we have already seen what is meant by a substitution instance. Let us clarify it a little more now. A substitution instance is any argument, which results from the substitution of sentences for the sentential variables of an argument form, is said to have that form, or to be a substitution instance of that form. For example, the argument 'U v W and -U; ∴ W' has the form 'p v q and -p, ∴ q'.

1.8 TRUTH-TABLE

The simplest way of understanding argument forms is through truth-table. This is important for one more reason. Construction of truth-table is basic to our study of symbolic logic. We shall construct truth-tables to distinguish between valid and invalid forms.

Mixed hypothetical syllogism: p => q; p; ∴ q

p	q	p => q	p	q
1	1	1	1	1
1	0	0	1	0
0	1	1	0	1
0	0	1	0	0

5th column stands for the conclusion. This table discloses that the conclusion is false only when one of the premises is false. In all other circumstances the conclusion is true. This is the condition of any valid inference. The same explanation holds good for the remaining argument forms.

Mixed disjunctive syllogism: p v q; ¬ p, ∴ q

p	q	p v q	¬ p	q
1	1	1	0	1
1	0	1	0	0
0	1	1	1	1
0	0	0	1	0

Pure hypothetical syllogism: p => q; q => r; p => r

p	q	r	p => q	q => r	p => r
1	1	1	1	1	1
0	0	0	1	1	1
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	1	1	1
0	1	1	1	1	1
0	1	0	1	0	1
1	0	1	0	1	1

It is obvious that the conclusion is false only when at least one premise is false. In this particular instance the last but one column shows the truth-values of the conclusion. Therefore this form is a valid form.

Testing an argument form containing n distinct sentential variables requires a truth-table having 2^n rows. How do we construct such a truth-table? It is convenient to do it by following the cyclic method, i.e. practice of simply alternating pairs of 1s and 0s down the extreme right hand initial column, alternating pairs of 1s with the pairs of 0s down the column directly to its left, next alternating quadruples of 1s with quadruples of 0s… and so on. It is important to remember that truth-values do not belong to sentence forms (propositional function) but to sentences.

1.9 KINDS OF SENTENCE FORMS AND SENTENCES

These are of three kinds: tautology, contradiction, and contingent. First, let us consider

i) *Tautology:* It is a sentence form having only true substitution instance.

Example: p v ¬p

p	¬p	p v ¬p
1	0	1
0	1	1

This can be known to be true without empirical investigation. That is, it is necessarily true. Any sentence, which is a substitution instance of a tautologous sentence form, is *formally* true; and is itself said to be a tautology. Example: Balboa discovered the Pacific Ocean (B). In this case, B is to be known empirical, whereas the truth of 'B v ¬ B' is necessarily known.

(ii) *Contradiction*: It is a kind of sentence form having only false substitution instances. (Its substitution instances are also called 'contradictory' or a 'contradiction'). Example: Cortex discovered the pacific (C). This is known to be false empirically (*happens* to be) false whereas 'C and ¬C' is *formally* false.

'Contradiction' may mean (a) *relation between sentences* (impossible for two sentences both to be true); (b) *Self-contradictory* sentence: (logically impossible for a particular sentence to be true, the sense in which it is presently used by us).

p	¬ p	p and ¬ p
1	0	0
0	1	0

(iii) *Contingent*: These are sentences or sentence forms that are neither tautologous nor contradictory. p, ¬ p, p v q, p => q, are all contingent. Their truth-values are not formally determined but depend on what happens to be the case.

1.10 TESTING THE VALIDITY OF ARGUMENT FORMS

One of the methods of testing the validity of an argument is provided: An argument is valid if and only if the conclusion is a consequence of the premises. In other words, it is valid if and only if whenever the premises are true, so is the conclusion. So in order to determine whether a given argument is valid or not, we must reduce the sentences of the argument to their logical forms and run a joint truth - table for both the premises and the conclusion. If the truth - table shows that whenever the premises are true, the conclusion is also true, then the argument is valid, not otherwise.

Check Your Progress II

Note: Use the space provided for your answers.

1) Distinguish between tautology and contradiction with examples.

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2) Examine the role of truth-table in modern logic.

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1.11 EXERCISES

Use truth-table to determine the validity or invalidity of each of the following argument forms:

1) p ^ q / ∴ p
2) p / ∴ p ^ q
3) p v q / ∴ p
4) p / ∴ p v q
5) p / p => q

1.12 LET US SUM UP

Logic has its beginning in the works of Aristotle. Leibniz laid the foundation for modern logic. Aristotelian and modern logic differ with respect to method. Mathematicians contributed to the evolution of modern logic. Symbols play a major role in modern logic. So it is also called symbolic logic. Truth characterizes statements and validity characterizes argument. Argument form and arguments are different. Tautology, contradiction and contingent are the forme of sentences. Truth-table is the most convenient method of determining the equivalent forms of compound propositions.

1.13 KEY WORDS

Tautology: An expression is said to be tautologous if it is true in all circumstances.

Contradiction: An expression is said to be a contradiction if it is false in all circumstances.

1.14 FURTHER READINGS AND REFERENCES

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