Causal Inference

→ So far, purely predictive questions!

→ If there are signs that there is correlation between features and target of interest, that's good enough

→ Causal Directionality is irrelevant. (Not completely true)

→ When there is dataset shift, or non-stationary data causality matters. Understanding the data deeply is very helpful.

→ In healthcare especially, causal questions are important to answer.

→ For example, it is more important to prevent Type 2 diabetes (causal) vs early diagnosis of Type 2 diabetes (predictive).

→ Naive way of inferring causality:

- Let's say we trained a DL model to predict onset of Type 2 diabetes
- Look at most negative feature (lowest weight), let's say it is Gastrogenic Bypass Surgery (yes or no)
- Then does that mean that if a patient underwent this surgery, he/she won't get diabetes?

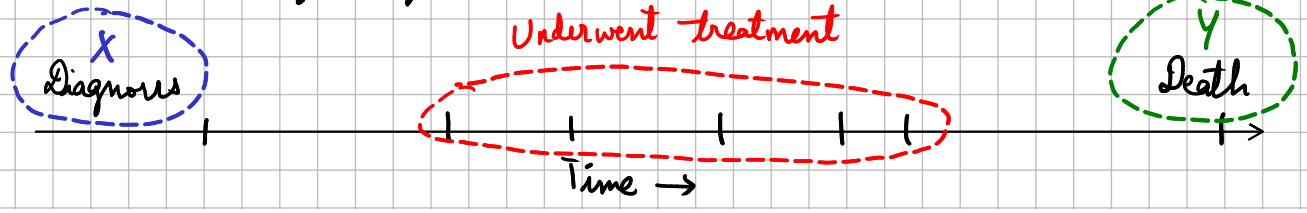
→ Look at predictive weights is not enough.

→ We need to come up with a mathematical model for causality

→ Another example:

- Let's say we train a DL model for predicting survival of breast cancer patient based on radiological mammogram and histopathological slides.
- Let's say one patient diagnosed with breast cancer survived for longer than 5 years.
- When a new patient, with similar diagnosis as the model's examples is given to the model, a higher survival is predicted.
- Does this mean we shouldn't treat the patient?

→ This is very dangerous!!



→ A longer survival time maybe because of treatment! Not solely because of diagnosis

→ But the model only learns the X (diagnosis) → Y (survival) mapping

Guiding Treatment Decisions

→ Another question that needs to be answered is: How do we guide treatment decisions?

→ How do we tell who is likely to be benifitted by a given treatment?

→ But people respond differently to treatments?

→ Also, data used to guide treatments is biased on existing treatment guidelines.

→ Naive way to guide treatment decisions:

Train a predictive model that learns to predict treatment decisions.

David → Treatment A

John → Treatment B

Juana → Treatment A

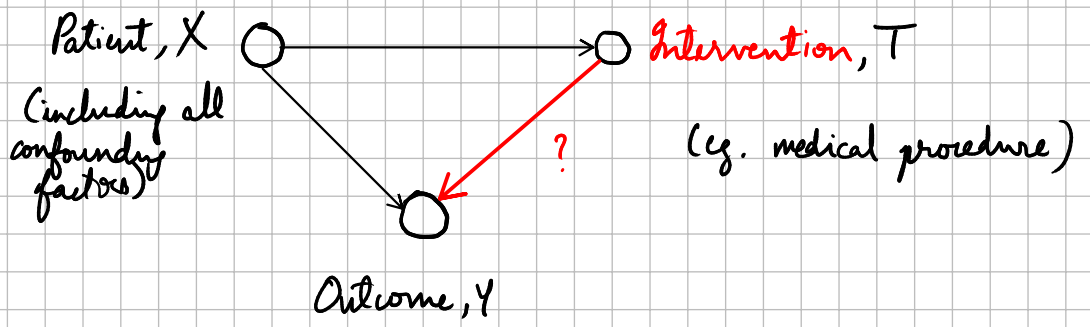
→ Best this can do is match current medical guidelines!

→ How do go we beyond this? We need to capture heterogeneity in treatment response. We need to change how we ask our question.

→ One last example:

- Traditiond, does X cause Y?
- Does smoking cause cancer?
- Doing a randomized controlled trial is unethical.
- Could we just compare P(lung cancer | smoker) vs P(lung cancer | non smoker)?
- No because of confounding factors (see below)

→ To properly answer, we need to formulate as causal questions.



High dimensional Observational data

Causal Graphs

→ Instead of just $D \in \{X^{(i)}, Y^{(i)}\}$, we need to think in terms of triplets: $D \in \{X^{(i)}, T^{(i)}, Y^{(i)}\}$ → Interventions

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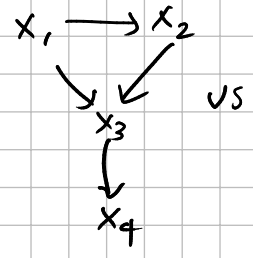
Earlier
→ Causal Inference might take a form like so: (Can Skip! Won't be discussed further)

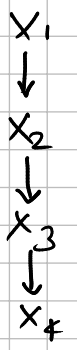
Data

X_1	X_2	X_3	X_4
0	1	0	0
0	0	1	0
⋮			
1	1	1	0

we Try to find right graph →



vs



What is the underlying causal graph?

For Two random variables and X_1 and X_2

$X_1 \rightarrow X_2$ i.e., $P(X_1)\,P(X_2|X_1)$

$X_2 \rightarrow X_1$ i.e, $P(X_2)\,P(X_1|X_2)$

Are indistinguible (because of conditional probability and bayes theorem)

Then we might wanna add interventions to $X_1 \rightarrow X_2$ and $X_2 \rightarrow X_1$ to disentangle the causalities.

→ This is the simplest possible case!

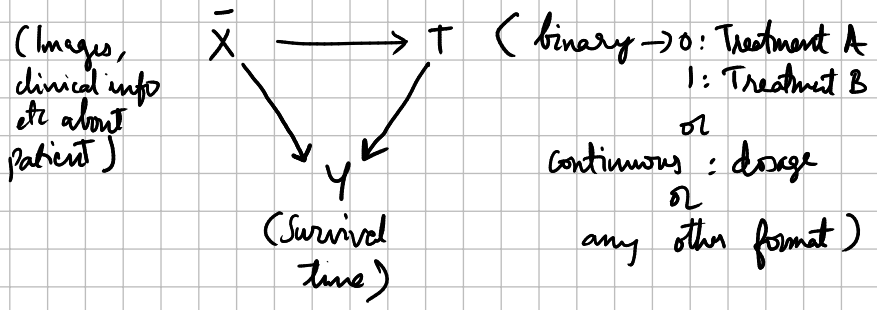
→ X is highdimensional, T and Y and single random variable.

→ The causal graph is assumed here.

→ We just don't know the connection weights' strength!

Modelling Breast Cancer example as a causal graph

X → Patient information (features) at Diagnosis
T → Choose between treatment A or treatment B. Also treatment plans only depend on what we knows about the patient at diagnosis
Y → Survival time



→ We will not change treatment in between (Dynamic treatment regimes or off policy reinforcement learning)

→ Causal Inference questions have been asked for decades in political science, economics and statistics but we can't intervene (can only use observational data)

→ In tradional statistics, domain knowledge was required to determine factor that determine the treatment to be taken. These factors are called confounding factors.

→ Now causal inference questions are asked on high dimensional data (images) and research on leveraging machine learning algorithms, by mapping a causal inference problem down to a machine learning model.

→ To formalize these notions concretely, we need a mathematical model

Potential Outcomes Framework (Rubin-Neyman Causal Model)

→ Each unit (individual) x_i has two potential outcomes:

- $Y_0(x_i)$ is the potential outcome had the unit not been treated: "control outcome"
- $Y_1(x_i)$ is the potential outcome had the unit been treated: "treated outcome"

→ Conditional average treatment effect for unit i:

$$CATE(x_i) = E_{Y_1 \sim P(Y_1|x_i)}[Y_1|x_i] - E_{Y_0 \sim P(Y_0|x_i)}[Y_0|x_i]$$

Given x_i (such as age = 56), mean all outcomes where treatment is given

Given x_i (such as age = 56) mean all outcomes where treatment is not given.

→ also called Individual Treatment effect

→ Average treatment effect:

$$ATE: E[Y_1 - Y_0] = E_{x \sim p(x)}[CATE(x)]$$

→ Observed factual Outcome (You can only Y_0 or Y_1 at a time but not both):

$$y_i = t_i Y_1(x_i) + (1 - t_i) Y_0(x_i)$$

→ Unobserved counterfactual outcome (What if the opposite treatment was given?)

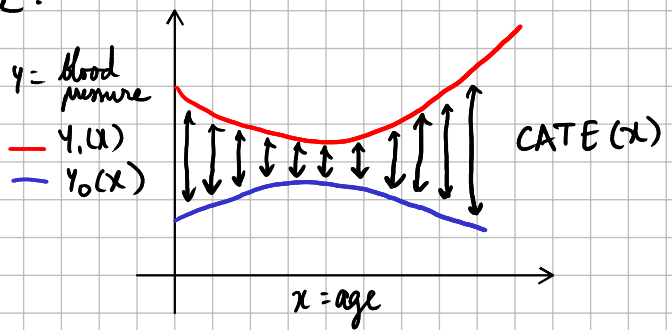
$$y_i^{CF} = (1 - t_i) Y_1(x_i) + t_i Y_0(x_i)$$

→ Since we observe only factual outcomes, we need to impute the counter factual outcomes.

"We only ever observe one of the two outcomes"

→ Fundamental problem in Causal Inference.

Example:



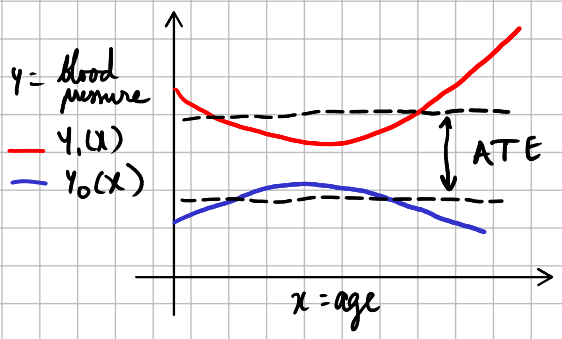
For middle aged people, difference between treatments 0 and 1 is significantly lower than younger and old aged people.

If treatment 1 is much cheaper than 0, we could use it for middle aged people.

→ We wanna predict CATE value using our data.

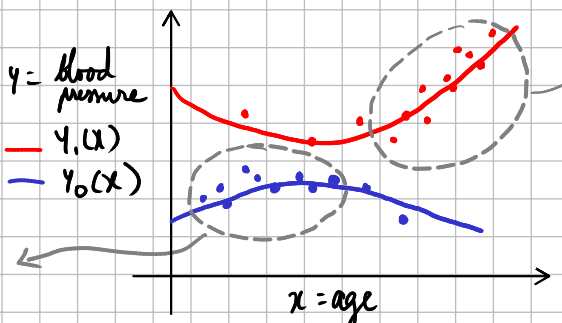
→ Sometimes we don't have the luxury of having personalized treatment effects (CATE). For example, the govt may start a policy that all men above 50 should receive prostate cancer screening.

→ Then you would wanna see the amount of decrease in prostate cancer related deaths. Such a policy is very broad. Mathematically captured by ATE.



Treatment 0 on average is better than treatment 1.

→ But IRL we don't observe curves, rather we observe data points.

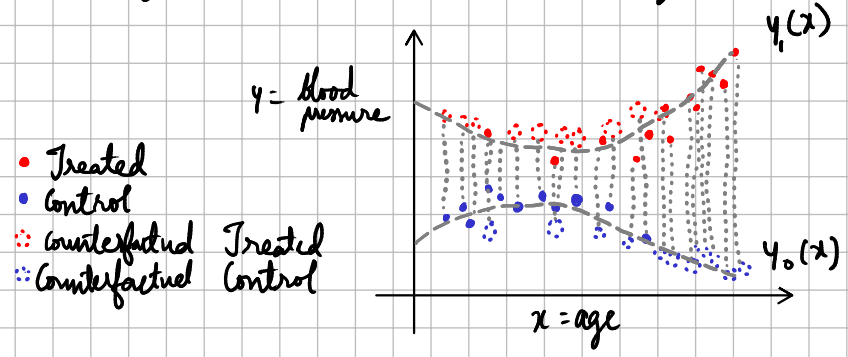


→ Most people given treatment 1 are older

Most people given treatment 0 are younger.

Can be due to a variety of reasons. This is modelled by $X \rightarrow T$ in causal graph

→ What if the other treatment had been given to the patient?



An Example:

(age, gender, exercise)	Sugar levels had they received medication A	Sugar levels had they received medication B	Observed Sugar levels
A (45, F, 0)	6	5.5	6
B (45, F, 1)	7	6.5	6.5
A (55, M, 0)	7	6	7
B (55, M, 1)	9	8	8
B (65, F, 0)	8.5	8	8
A (65, F, 1)	7.5	7	7.5
B (75, M, 0)	10	9	9
A (75, M, 1)	8	7	8

→ only observed factuals. Medication B is worse than medication A

mean(sugar | medication B) − mean(sugar | medication A)

= 7.875 − 7.125 = 0.75

mean(sugar | had they received B) − mean(sugar | had they received A)

= 7.125 − 7.875 = −0.75

→ along with unobserved counter factuals. Medication B is better than A

→ Without counterfactuals, we get the opposite reasoning for treatment B.

→ How do we solve without counterfactuals?

→ We need to make a ton of assumptions.

Assumptions in Causal Inference

No unmeasured confounders (Ignorability)

Y_0, Y_1 : potential outcomes for control and treated

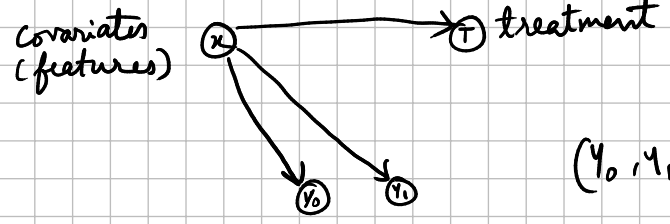
X : unit covariates (features)

T : treatment assignment.

We assume,

$$(Y_0, Y_1) \perp\!\!\!\perp T \mid X$$

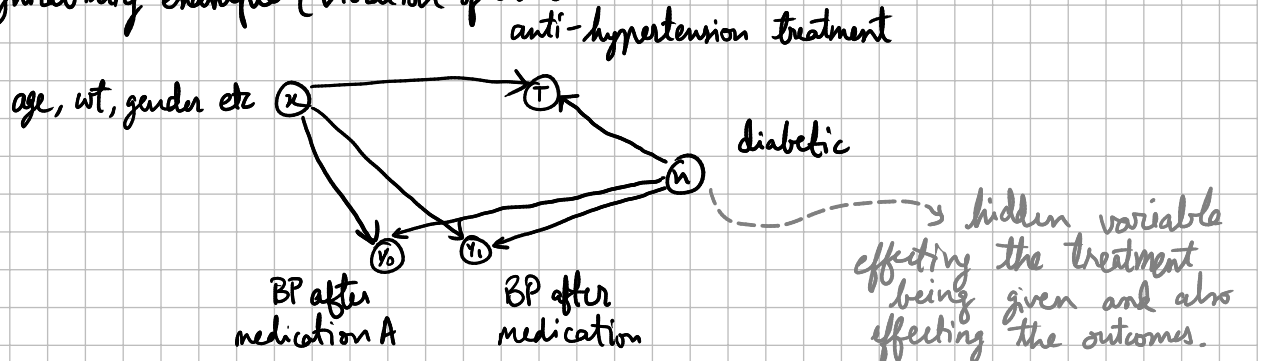
→ The potential outcomes are independent of treatment assignment, conditioned on covariates.



→ Since we already knows what treatment is given to patient, it doesn't make sense to have an edge from T to Y_0 and Y_1.

→ We are using a potential outcomes notation.

No ignorability example (Violation of above)



$$(Y_0, Y_1) \not\perp\!\!\!\perp T \mid X$$

→ h is confounding factor.

→ If h only effects treatments but not outcomes, then h is not a confounding

factor.

→ We need to observe all hidden confounding factors for causal inference to work, unlike in machine learning, we the only downside would be lower prediction accuracy.

→ To access the robustness to violations of our assumed causal model, we can perform sensitivity analysis. "How much do my conclusions change if there were a confounding factor that is W amount strong?"

Common Support

Y_0, Y_1 : potential outcomes for control and treated

x : unit covariates (features)

T : treatment assignment

We assume, ----→ Propensity Score

$$P(T=t \mid X=x) > 0 \quad \forall\ t, x$$

→ It means that for a given set of patients $(X=x)$, there has to be some amount of patients who have received treatment $T=t$.

→ Essentially, causal inference will be meaningless if no patients are given a certain treatment $T=t$, as we won't be able to determine the counterfactuals.

Framing the question for Causal Inference

1. Where could we go to for data to answer these questions?
2. What should X, T and Y be to satisfy ignorability? → Cannot be tested using observational data.
3. What is the specific causal inference question that we are interested in?
4. Are you worried about common support? ----→ Can be tested using observational data.

→ In the table, check if (75, M, 1) B ovurs, if not common support assumption is violated. But we can make approximations. For example age 74 instead of 75 exactly might be good enough.

→ Now that we know the assumptions and confirmed that they are held, how do we then do causal inference?

→ How do we actually compute CATE and ATE for observational data?

Average Treatment Effect - The adjustment formula

→ Also called G-formula

$$ATE := E[Y_1 - Y_0]$$

$$E[Y_1] = E_{x \sim p(x)}\left[E_{y_1 \sim p(y_1|x)}[Y_1|x]\right]$$

$$= E_{x \sim p(x)}\left[E_{y_1 \sim p(y_1|x)}[Y_1|x, T=1]\right]$$

$$= E_{x \sim p(x)}\left[E[Y_1|x, T=1]\right]$$

$$E[Y_0] = E_{x \sim p(x)}\left[E_{y_0 \sim p(y_0|x)}[Y_0|x]\right]$$

$$= E_{x \sim p(x)}\left[E_{y_1 \sim p(y_0|x)}[Y_0|x, T=0]\right]$$

$$= E_{x \sim p(x)}\left[E[Y_0|x, T=0]\right]$$

Quantities we can estimate from our data

$$ATE = E[Y_1 - Y_0] = E\left[E[Y_1|x, T=1]\right] - E\left[Y_0|x, T=0\right]$$

$E[Y_0|x, T=1]$

$E[Y_1|x, T=0]$

$E[Y_0|x]$

$E[Y_1|x]$

Quantities we cannot directly estimate from our data.

→ Empirically we have samples from $p(x|T=1)$ & $p(x|T=0)$. Extrapolate to $p(x)$

How to extrapolate to get counterfactuals?

→ Covariate adjustment

→ Propensity Reweighting } Discussed

→ Doubly robust estimators

→ Matching

...

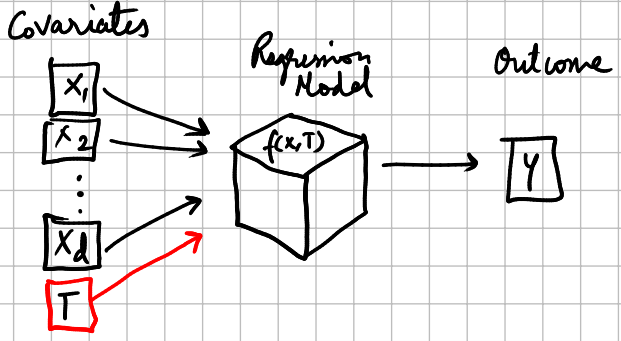
Covariate Adjustment

→ Explicitly model the relationships between treatment, confounders and outcome.

→ Also called "Response Surface Modelling"

→ Used for both ITE and ATE

→ A regression problem.



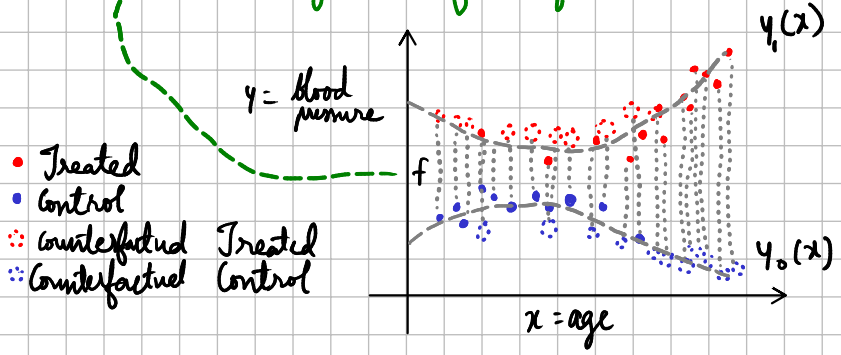
→ Fit a model $f(x,t) \approx E[Y_t | T=t, x]$

$$\widehat{ATE} = \frac{1}{n}\sum_{i=1}^{n} f(x_i,1) - f(x_i,0)$$

→ No observed data here, only model predictions. You can use observed data when available and imputed model prediction for the unobserved counter factual as well. It is also a consistent estimator.

$$\widehat{CATE}(x_i) = f(x_i,1) - f(x_i,0)$$

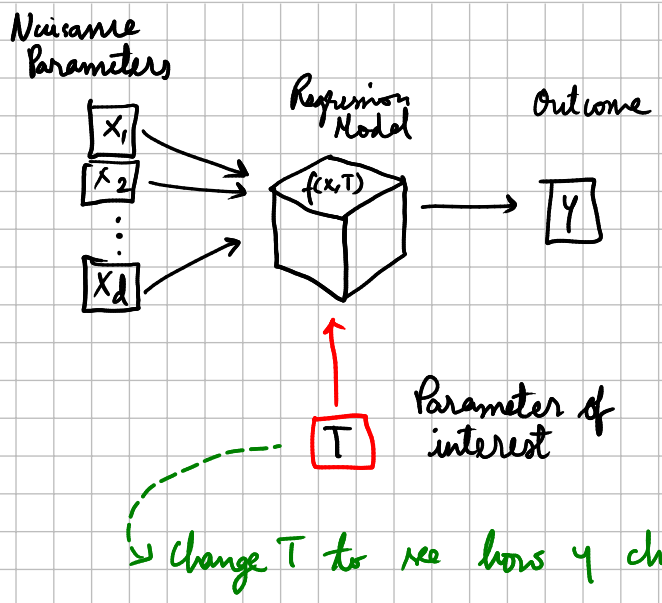
→ Get counterfactuals from $f(x,T)$



→ How do we know that $y \approx f(x,T)$ is modelled correctly? It can so happen that the ML may completely ignore the effect of T (might consider it an insignificant feature) and essentially learn $f(x)$ instead of $f(x,T)$

→ This is an area of active research!

→ Instead of treating treatment T as a covariate, we need to use the treatment T as a parameter for the model



→ This problem becomes prevalent when x is high dimensional and regularization (l_1 or l_2) is necessary

→ When does the problem of common support (Overlap) show up?

$$\widehat{CATE}(x_i) = f(x_i, 1) - f(x_i, 0)$$

Example:
→ If there is nobody in the dataset with x_i like features and received treatment 0, then the model will output a garbage value. CATE will then be incorrect.

→ As we approach infinite data, CATE or ATE by covariate adjustment will get more accurate.

→ More samples → Better estimate!

→ Your model family to fit the observational + treatment data should be complex enough to actually fit it properly

Example of how covariate adjustment fails when there is no overlap