

(a)

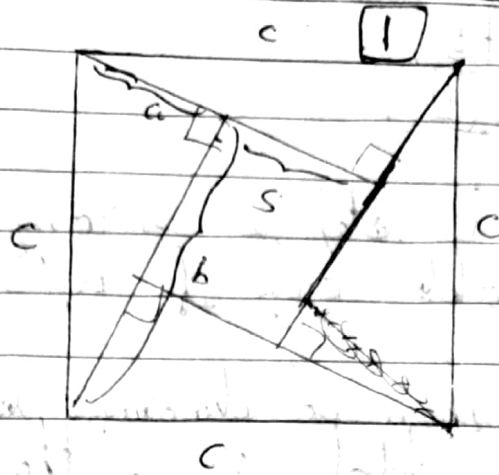
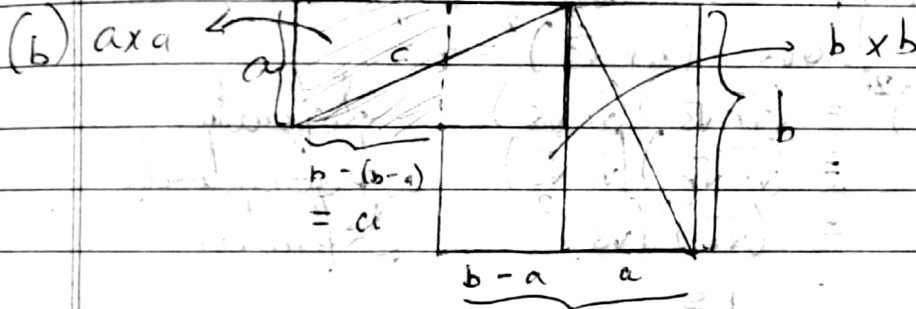


Figure not to scale.
and angles shown may be incorrect.

Since $s = a + s = b \Rightarrow s = b + a$
 s is the side of the square



(c) The rearrangement is justified because we have only used facts like the sum of non-right angles of a right triangle is 90° . The case $a=b$ has no problem, in which the square would have side 0 or, i.e. just a point.

(d)

- Sum of non-right angles of a right triangle is 90° .
- Length of a line segment is equal to sum of lengths of two parts in which it is divided.
- Sum of angles on a line is 180° .

2

(a) $\sqrt{(-1)(-1)} = \sqrt{-1} \sqrt{-1}$ step is wrong.
 $\sqrt{ab} = \sqrt{a} \sqrt{b}$ holds only if $a, b \in \mathbb{R}^+ \cup \{0\}$

(b) Assume $1 = -1$. Then get dividing both sides by 2,
 $\frac{1}{2} = -\frac{1}{2}$ Add $\frac{3}{2}$, then
 $2 = 1$. ⊗

(c) Since $(\sqrt{rs})^2 = rs$ by definition, we
 need to prove $(\sqrt{r} \sqrt{s})^2 = rs$
 $(\sqrt{r} \sqrt{s})^2 = (\sqrt{r} \sqrt{s})(\sqrt{r} \sqrt{s})$ Definition
 $= (\sqrt{r} \sqrt{r})(\sqrt{s} \sqrt{s})$ Associativity
 $= (\sqrt{r})^2 (\sqrt{s})^2$ Definition
 $= rs$

~~(b)~~

3

(a) The error is $3 \log_{10}(1/2) > 2 \log_{10}(1/2)$
 because $\log_{10}(1/2) < 0$ as $\frac{1}{2} < 1$, and multiplying
 by negative numbers inverts the inequality.

(b) The error is $\$0.01 = (\$0.1)^2$ as the units
 of $(\$0.1)^2$ are $\2 but the LHS has units $\$$.
 Also $(10¢)^2 = 100¢$ is wrong.

~~(c)~~

(c) The step ~~(a-b)~~ $a+b=b$ is wrong because
 since $a=b$, $a-b=0$ and 0 can't be cancelled
 from both sides of inequality.

4

The problem is that the proof proves that if $\frac{a+b}{2} \geq \sqrt{ab}$ then $(a-b)^2 \geq 0$ while this

is not what we wanted. In fact, the proof should be in reverse order.

$$(a-b)^2 \geq 0 \quad \text{so,}$$

$$a^2 - 2ab + b^2 \geq 0 \quad \text{so,}$$

$$a^2 + 2ab + b^2 \geq 4ab \quad \text{so, (as } a, b \geq 0)$$

$$(a+b)^2 \geq (2\sqrt{ab})^2 \quad \text{so}$$

$$a+b \geq 2\sqrt{ab} \quad \text{so,}$$

$$\frac{a+b}{2} \geq \sqrt{ab}.$$

5

The problem is with the word "surprise" and its precise mathematical implication.

For a good discussion, refer:

The surprise examination or unexpected hanging paradox, by Imre Y. Loh.