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$$(a) \begin{aligned} A \text{ IFF } B &\leftrightarrow (A \text{ NOT}(A) \text{ OR } B) \text{ AND } (\text{NOT}(B) \text{ OR } A) \\ A \text{ XOR } B &\leftrightarrow \text{NOT}(A) \text{ OR } B \text{ AND } (\text{NOT}(A) \text{ OR } \text{NOT}(B)) \end{aligned}$$

(b) Since all operators can be constructed out of AND, OR, NOT every propositional formula can be made out of them.

$$(b) \begin{aligned} A \text{ AND } B &\leftrightarrow \text{NOT}(\text{NOT}(A \text{ AND } B)) \quad \text{Double Negation} \\ &\leftrightarrow \text{NOT}(\bar{A} \text{ OR } \bar{B}) \quad \text{De-Morgan's law} \end{aligned}$$

Hence, AND can be constructed from NOT and OR. So AND is not needed.

$$(c) \begin{aligned} \text{NOT}(A) &\leftrightarrow \text{NOT}(A \text{ AND } A) \quad \text{idempotence} \\ &\leftrightarrow \text{NAND}(A) \text{ NAND } A \end{aligned}$$

$$\begin{aligned} A \text{ OR } B &\leftrightarrow \text{NOT}(\bar{A} \text{ AND } \bar{B}) \quad \text{De-Morgan's law} \\ &\leftrightarrow \bar{A} \text{ NAND } \bar{B} \\ &\leftrightarrow (A \text{ NAND } A) \text{ NAND } (B \text{ NAND } B) \end{aligned}$$

Since NOT and OR can be constructed from NAND, all operators can be constructed from it.