Reading 1.1

1.17

(a) We use proof by cases. Case 1: $q(n)$ is non-constant
Let $q(n) = b_k n^k + b_{k-1} n^{k-1} + \dots + b_2 n^2 + b_1 n + c$
$b_k \neq 0$, $k \geq 1$
where k is the degree of the polynomial.
Then, $q(cm) = b_k c^k + b_{k-1} c^{k-1} + \dots + b_1 c + c$, $m \in \mathbb{Z}$
$= c(b_k c^{k-1} + b_{k-1} c^{k-2} + \dots + b_1 + 1)$
Hence $q(cm)$ is a multiple of c.

Case 2: $q(n)$ is constant. Then, $q(n) = c$; so
$\therefore q(cm) = c$ which is a multiple of c

Since the proposition is true for both cases, it is always true

(b) Consider all n of the form cm, where $n, m \in \mathbb{N}$
Since $q(n) = q(cm)$ is a multiple of c and because $q(n)=q(cm)$ grows unboundedly as m grows, there are infinitely many distinct $q(n)$ as ~~they~~ there are infinite $m \in \mathbb{N}$. The above argument is for $b_k > 0$. If $b_k < 0$ replace grows by falls.
Finally since $q(n)$ is a multiple of c by above lemma, and $c > 1$, $q(n)$ has twice factors c and $\frac{q(n)}{c}$ both greater than 1 for infinitely many n as $q(n)$ grows unboundedly a n grow.
Hence, all such $q(n) \in \mathbb{Z}$ are not prime.

(c) If $c \leq 1$, then there are two cases: $c = 1$ or 0 & $c < 0$
Case 1: $c = 1$ or 0 $q(0) = c = 1$ or 0 both not a prime
Case 2: $c < 0$ $q(0) = c < 0$ which cannot be a prime.
Hence, by lemma (b) and lemma (c), for every non-constant polynomial q, there must be an $n \in \mathbb{N}$ such that $q(n)$ is not prime