

Reading 1.2

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1.10

(a) We use proof by contradiction. Suppose n is not even, then $n = 2k+1$ for some $k \in \mathbb{N}$. Then,

$$n^2 = (2k+1)^2$$

$$= 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2l + 1$$

Since $n^2 = 2l + 1$, where $l = 2k^2 + 2k$, it is not even. This is a contradiction. Therefore, n must be even.

(b) ~~Suppose~~ We use proof by contradiction. Suppose n is not a multiple of 3. Then $n = 3k + l$ for some $k, l \in \mathbb{N}$ and $l \in \{1, 2\}$.

$$\text{Then, } n^2 = (3k + l)^2$$

$$= 9k^2 + 6kl + l^2$$

There are 2 cases:

$$l=1: n^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$$

$$= 3p + 1 \text{ where } p = 3k^2 + 2k \in \mathbb{N}$$

Since $n^2 = 3p + 1$, it is not a multiple of 3.

$$l=2: n^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1$$

$$= 3q + 1 \text{ where } q = 3k^2 + 4k + 1$$

Since $n^2 = 3q + 1$, it is not a multiple of 3.

By case analysis, n^2 is not a multiple of 3. This is a contradiction. Therefore n is a multiple of 3.

(c)

1.11

$n=6, m=9$ is an example such that n^2 is a multiple of m , but n is not a multiple of m . If $m < n$, then we can have $n=6, m=4$ as an example.