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Problem Set 1

1

We use proof by contradiction. Assume that $\log_4 6$ is rational. Hence $\log_4 6 = \frac{n}{d}$ for ~~n,d~~ some $n, d \in \mathbb{Z}$, $d \neq 0$ and in its lowest terms. Then $6 = 4^{\frac{n}{d}}$ and $6^d = 4^n$. Hence,

$$\boxed{2^d 3^d = 2^{2n}}$$

This is a contradiction because $3 \mid LHS$ but NOT $(3 \mid RHS)$. Therefore, $\log_4 6$ is irrational.

2

Let $C ::= \{ n \mid n > 3^{n/3} \}$ be the set of counterexamples. Assume C to be non-empty, for proof by contradiction. By WOP, C has a minimum element $m \in C$. This $m \geq 5$ because:

- $\underline{n=0}$ $\quad 3^{n/3} = 3^0 = 1 \geq 0$
- $\underline{n=1}$ $\quad (3^{1/3})^3 = 3 \geq 1^3 \Rightarrow 3^{1/3} \geq 1$
- $\underline{n=2}$ $\quad (3^{2/3})^3 = 9 \geq 2^3 \Rightarrow 3^{2/3} \geq 2$
- $\underline{n=3}$ $\quad 3^{3/3} = 3 \geq 3$
- $\underline{n=4}$ $\quad (3^{4/3})^3 = 3^4 \geq 4^3 \Rightarrow 3^{4/3} \geq 4$

Now, $m - 3 \leq 3^{\frac{m-3}{3}}$ as m is the minimum element of C

$$\Rightarrow m - 3 \leq \frac{3^{\frac{m}{3}}}{3} \qquad \text{But } 3^{m/3} < m$$

$$\Rightarrow m - 3 < \frac{m}{3} \Rightarrow 3m - 9 < m \Rightarrow \boxed{m < \frac{9}{2}}$$

This is a contradiction to $m \geq 5$. Hence C must be empty, proving $n \leq 3^{n/3}$ $\forall n \in \mathbb{N}$.

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$\boxed{3}$

(a)

P	Q	(P IMPLIES Q)	OR	(Q IMPLIES P)
T	T	T	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	T	T

Since the proposition is always true, it is valid.

(b) R: (P AND Q) OR (NOT(P) AND NOT(Q))

(c) ~~If P is true, P is valid, but~~

(c) If P is valid, all environment make it true, and hence NOT(P) is false in all environments, and hence not satisfiable.

If NOT(P) is not satisfiable, NOT(P) is false in all environments. Thus, P is true in all environments making it valid.

(d) ~~S := $\bar{P_1}$ OR $\bar{P_2}$ OR ... P_K because by part S by previous part.~~

(d) S is Consistent, P_1 AND P_2 ... AND P_K

So S := NOT (P_1 AND P_2 ... AND P_K)
= $\bar{P_1}$ OR $\bar{P_2}$ ... OR $\bar{P_K}$

by De-Morgan's law

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(a) $P_0 = \overline{a_0}$, $C = a_0$

(b) ~~No~~ If $b = 1$ then $O_i = P_i$ and
If $b = 0$, then $O_i = a_i$

Translating the above into logical statement
~~$O_i = (\text{NOT}(b) \text{ OR } P_i)$~~
~~$O_i = (b \text{ AND } P_i) \text{ OR } (\text{NOT}(B) \text{ AND } a_i)$~~

$\boxed{O_i = (\text{NOT}(b) \text{ OR } P_i) \text{ AND } (b \text{ OR } a_i)}$
because, If P then Q $\longleftrightarrow$ NOT(P) OR Q

(b) An equivalent form is $O_i = (b \text{ AND } P_i) \text{ OR } (\text{NOT}(b) \text{ AND } a_i)$

(c) If $c_{(1)} = 1$, then $C = c_{(2)}$, else $C = 0$
Translating into logic
$C = (\text{NOT}(c_1) \text{ OR } c_2) \text{ AND } (c_1 \text{ OR } 0)$
$0 = (\text{NOT}(c_1) \text{ AND } c_1) \text{ OR } (c_2 \text{ AND } c_1)$
$\boxed{C = C_1 \text{ AND } C_2}$

(d) $P_i = (\text{NOT}(c_{(1)}) \text{ OR } r_{i-(n+1)}) \text{ AND } (c_{(1)} \text{ OR } a_i)$
~~because~~, ~~if~~
because, ~~if~~ the ~~is~~ second half-adder is constructed from part (b) with $c_{(1)}$ replacing (b).

(e) For calculating P_i we need atmost 4 operations ~~to get~~ after obtaining ~~$c_{j(n+1)}$ and $c_{(1)}$~~ previous half-sized output bits. Hence, total $\boxed{4 \log_2 n}$ operations are required for any one bit providing exponential speedup over $O(n)$ ~~time~~ time.

2

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