APPENDIX

CURVE SKETCHING

A.1 INTRODUCTION

In this chapter, we shall use the results of differential calculus to find an approximate shape of the curves from their equations without plotting a large number of points. For the said task we shall make use of various concepts of differential calculus such as continuity, monotonicity, maxima and minima, points of inflexion etc.

A.2 CURVE SKETCHING

Following points are very helpful to draw a rough sketch of a curve.

I SYMMETRY

(i) *Symmetry about x-axis*: If all powers of y in the equation of the given curve are even, then it is symmetric about x-axis i.e., the shape of the curve above x-axis is exactly identical to its shape below x-axis.

For example, $y^2 = 4\,ax$ is symmetric about x-axis.

(ii) *Symmetry about y-axis:* If all powers of x in the equation of the given curve are even, then it is symmetric about y-axis.

For example, $x^2 = 4ay$ is symmetric about y-axis.

(iii) *Symmetry in opposite quadrants:* If by putting $-x$ for x and $-y$ for y, the equation of a curve remains same, then it is symmetric in opposite quadrants.

For example, $x^2 + y^2 = a^2$ and $xy = a^2$ are symmetric in opposite quadrants.

(iv) *Symmetry about the line $y = x$*: If the equation of a given curve remains unaltered by interchanging x and y, then it is symmetric about the line $y = x$ which passes through the origin and makes an angle of 45° with the positive direction of x-axis.

II ORIGIN AND TANGENTS AT THE ORIGIN

See whether the curve passes through the origin or not. If the point (0, 0) satisfies the equation of the curve, then it passes through the origin and in such a case to find the equation(s) of the tangent(s) at the origin, equate the lowest degree term to zero.

For example, $y^2 = 4ax$ passes through the origin. The lowest degree term in this equation is $4ax$. Equating $4ax$ to zero, we get $x = 0$. So, $x = 0$ i.e. y-axis is tangent at the origin to $y^2 = 4ax$.

III POINTS OF INTERSECTION OF THE CURVE WITH THE COORDINATE AXES

By putting $y = 0$ in the equation of the given curve, find points where the curve crosses the x-axis. Similarly, by putting $x = 0$ in the equation of the given curve we can find points where the curve crosses the y-axis.

For example, to find points where the curve $xy^2 = 4a^2\,(2a - x)$ meets x-axis, we put $y = 0$ in the equation which gives $4a^2\,(2a - x) = 0$ or, $x = 2a$. So, the curve $xy^2 = 4a^2\,(2a - x)$, meets x-axis at $(2a, 0)$. This curve does not intersect y-axis, because by putting $x = 0$ in the equation of the given curve get an absurd result.

IV REGIONS WHERE THE CURVE DOES NOT EXIST

Determine the regions in which the curve does not exist. For this, find the value of y in terms of x from the equation of the curve and find the values of x for which y is imaginary. Similarly, find the value of x in terms of y and determine the values of y for which x is imaginary. The curve does not exist for these values of x and y.

For example, the values of y obtained from $y^2 = 4ax$ are imaginary for negative values of x. So, the curve does not exist on the left side of y-axis. Similarly, the curve $a^2 y^2 = x^2 (a - x)$ does not exist for $x > a$ as the values of y are imaginary for $x > a$.

V SPECIAL POINTS

Find the points at which $\frac{dy}{dx} = 0$. At these points the tangent to the curve is parallel to x-axis.

Find the points at which $\frac{dx}{dy} = 0$. At these points the tangent to the curve is parallel to y – axis.

VI SIGN OF $\frac{dy}{dx}$, AND POINTS OF MAXIMA AND MINIMA

Find the interval in which $\frac{dy}{dx} > 0$. In this interval, the function is monotonically increasing.

Find the interval in which $\frac{dy}{dx} < 0$. In this interval, the function is monotonically decreasing.

Put $\frac{dy}{dx} = 0$ and check the sign of $\frac{d^2y}{dx^2}$ at the points so obtained to find the points of maxima or minima.

Keeping the above facts in mind and plotting some points on the curve one can easily have a rough sketch of the curve.

Following examples illustrate the procedure.

ILLUSTRATIVE EXAMPLES

EXAMPLE 1 *Sketch the curve* $y = x^3$

SOLUTION We observe the following points about the given curve.

(i) The equation of the curve remains unchanged if x is replaced by $-x$ and y by $-y$. So, it is symmetric in opposite quadrants. Consequently the shape of the curve is similar in the first and the third quadrants.

(ii) The curve passes through the origin. Equating the lowest degree term y to zero, we get $y = 0$ i.e. x-axis is the tangent at the origin.

(iii) Putting $y = 0$ in the equation of the curve, we get $x = 0$. Similarly, when $x = 0$, we get $y = 0$. So, the curve meets the coordinate axes at (0, 0) only.

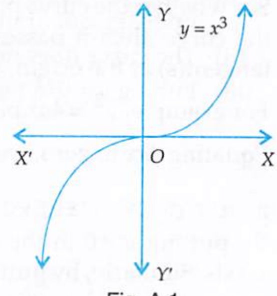


Fig. A.1

(iv) We have,

$$y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2, \ \frac{d^2y}{dx^2} = 6x \ \text{and} \ \frac{d^3y}{dx^3} = 6.$$

We observe that at the origin $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ changes its sign from negative to positive as x increases through O. So, the curve changes its nature from concave down to concave up as x increases through O. Hence, O (0, 0) is the point of inflexion.

Clearly, $\frac{dy}{dx} = \frac{d^2 y}{dx^2} = 0$ at the origin but $\frac{d^3 y}{dx^3} = 6 \neq 0$. So, origin is a point of inflexion.

(v) As x increases from 0 to ∞, y also increases from 0 to ∞. Keeping all the above points in mind, we obtain a sketch of the curve as shown in Fig. A.1.

EXAMPLE 2 *Sketch the curve $y = x^3 - 4x$.*

SOLUTION We make the following observations about the given curve.

(i) The equation of the curve remains same if x is replaced by $-x$ and y by $-y$. So, it is symmetric in opposite quadrants. Consequently, the curve in the first quadrant is identical to the curve in third quadrant and the curve in second quadrant is similar to the curve in fourth quadrant.

(ii) The curve passes through the origin. Equating the lowest degree term $y + 4x$ to zero, we get $y + 4x = 0$ or $y = -4x$. So, $y = -4x$ is tangent to the curve at the origin.

(iii) Putting $y = 0$ in the equation of the curve, we obtain $x^3 - 4x = 0 \Rightarrow x = 0, = \pm 2$. So, the curve meets x-axis at (0, 0) (2, 0) and (– 2, 0).
Putting $x = 0$ in the equation of the curve, we get $y = 0$. So, the curve meets y-axis at (0, 0) only.

(iv) We have, $y = x^3 - 4x \Rightarrow \frac{dy}{dx} = 3x^2 - 4$

$$\therefore \quad \frac{dy}{dx} > 0$$

$$\Rightarrow 3x^2 - 4 > 0$$

$$\Rightarrow x^2 - \frac{4}{3} > 0 \Rightarrow \left(x - \frac{2}{\sqrt{3}}\right)\left(x + \frac{2}{\sqrt{3}}\right) > 0 \Rightarrow x < -\frac{2}{\sqrt{3}} \text{ or } x > \frac{2}{\sqrt{3}}$$

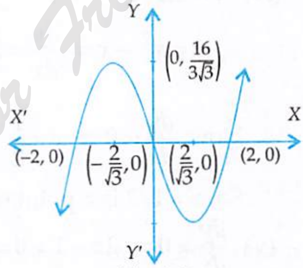


Fig. A.2

$$\text{and, } \frac{dy}{dx} < 0 \Rightarrow -\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$$

So, the curve is decreasing in the interval $(-2/\sqrt{3}, 2/\sqrt{3})$ and increasing for $x > 2/\sqrt{3}$ or $x < -2/\sqrt{3}$. $x = -2/\sqrt{3}$ is a point of local maximum and $x = 2/\sqrt{3}$, is a point of local minimum. Keeping the above points in mind, we obtain the rough sketch the curve as shown in Fig. A.2.

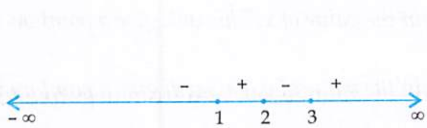
EXAMPLE 3 *Sketch the curve $y = (x-1)(x-2)(x-3)$.*

SOLUTION We note the following points about the given curve.

(i) The curve does not have any type of symmetry about the coordinate axes and also in the opposite quadrants.

(ii) The curve does not pass through the origin.

(iii) Putting $y = 0$ in the equation of the curve, we get $(x-1)(x-2)(x-3) = 0 \Rightarrow x = 1, 2, 3$.



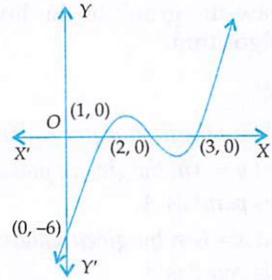


Fig. A.3

So, the curve meets x-axis at (1, 0), (2, 0) and (3, 0).

Putting $x = 0$ in the equation of the curve, we get $y = -6$. So, the curve crosses y-axis at $(0, -6)$. The signs of y for different values of x are shown below:

Clearly, y decreases as x decreases for all $x < 1$ and y increases as x increases for $x > 3$.

Keeping all the above points in mind, we obtain the rough sketch of the curve as shown in Fig. A.3.

EXAMPLE 4 *Sketch the curve $y = x^2 - x$.*

SOLUTION We note the following points about the curve.

(i) The curve does not have any kind of symmetry.

(ii) The curve passes through the origin and the tangent at the origin is obtained by equating the lowest degree term to zero. The lowest degree term is $x + y$. Equating it to zero, we get $x + y = 0$ as the equation of the tangent at the origin.

(iii) Putting $y = 0$ in the equation of the curve, we get $x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$.

So, the curve crosses x-axis at $(0, 0)$ and $(1, 0)$. Putting $x = 0$, in the equation of the curve, we obtain $y = 0$. So, the curve meets y-axis at $(0, 0)$ only.

(iv) We have,

$$y = x^2 - x \Rightarrow \frac{dy}{dx} = 2x - 1 \text{ and } \frac{d^2y}{dx^2} = 2.$$

Now, $\frac{dy}{dx} = 0 \Rightarrow x = \frac{1}{2}$. At $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = 2 > 0$.

So, $x = 1/2$ is a point of local minima.

(v) $\frac{dy}{dx} > 0 \Rightarrow 2x - 1 > 0 \Rightarrow x > 1/2$

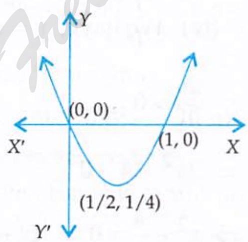


Fig. A.4

So, y is increasing for all $x > 1/2$ and decreasing for all $x < 1/2$. Keeping the above points in mind, we obtain the sketch of the curve as shown in Fig. A.4.

A.3 SKETCHING OF SOME STANDARD CURVES

In section A.1, we have learnt about sketching of curves using calculus. In this section, we shall discuss sketching of some standard curves viz. straight line, circle, parabola, ellipse and hyperbola, without taking the help of various points learnt in the previous section. Sketching of these curves will be very helpful in finding the areas of bounded regions.

A.3.1 STRAIGHT LINE

As we know that every first degree equation in x, y represents a straight line. The general equation of a straight line is $ax + by + c = 0$. If $c = 0$, then the line passes through the origin. In order to draw the graph of the line represented by $ax + by + c = 0$, $c \neq 0$, we may follow the following algorithm.

ALGORITHM

Step I *Obtain the first degree equation, say, $ax + by + c = 0$, $c \neq 0$.*

Step II *Put $y = 0$ in the given equation and obtain the value of x. This will give a point on x-axis. Mark this point as A.*

Step III *Put $x = 0$ in the given equation and obtain the value of y. This will give a point on y-axis. Mark this point as B.*

Step IV *Draw line passing through A and B. The line so obtained will be the graph of the line represented by the given equation.*

ILLUSTRATION 1 *Draw a rough sketch of the line represented by the equation* $3x + 2y - 6 = 0$.

SOLUTION We have, $3x + 2y - 6 = 0$ ...(i)

Putting $y = 0$ in (i) we get $x = 2$. So, the line (i) meets x-axis at $A\,(2, 0)$.

Now, putting $x = 0$ in (i), we get $y = 3$. So, the line (i) cuts y-axis at $B\,(0, 3)$.

Marking these points on the coordinate axes and drawing a line passing through them, we obtain the graph of the line represented by equation (i) as shown in Fig. A.5.

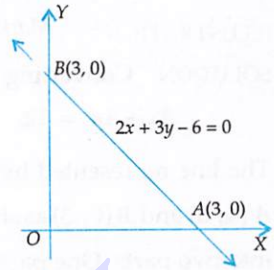


Fig. A.5

A first degree equation of the form $ax + by = 0$ or $y = mx$ always represents a straight line passing through the origin. In order to draw a rough sketch of a line passing through the origin, we may follow the following algorithm:

ALGORITHM

Step I *Obtain the first degree equation. Let the equation be* $ax + by = 0$.

Step II *Find the slope m of the line represented by the given equation.*
If $m > 0$, *then the line makes an acute angle with x-axis. If* $m < 0$, *then the line makes an obtuse angle with x-axis.*

Step III *Draw a line passing through the origin and making an angle* $\tan^{-1}(m)$ *with the positive direction of x-axis.*

The line so obtained represents the graph of the line represented by the given equation.

ILLUSTRATION 2 *Draw the rough sketches of the following lines:*

(i) $y = x$ (ii) $y = -x$ (iii) $y - 2x = 0$

SOLUTION The equations $y = x$, $y = -x$ and $y - 2x = 0$ represent lines passing through the origin and making angles $\tan^{-1}(1) = 45°$, $\tan^{-1}(-1) = 135°$ and $\tan^{-1}(2)$ respectively with the positive directions of x-axis. So, their graphs are as shown in Fig. A.6.

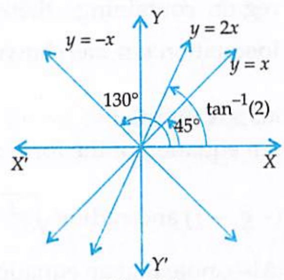


Fig. A.6

REGION REPRESENTED BY A LINEAR INEQUATION

Every straight line divides the xy-plane into two parts (or regions), one lying below it and the other lying above it. These two regions are represented by the two inequations obtained from the equation of the given line. In order to determine the region represented by a given linear inequation, we follow the following algorithm.

ALGORITHM

Step I *Obtain the inequation and convert the inequation into an equation by replacing the inequality sign by equality sign.*

Step II *Draw the straight line represented by the linear equation obtained in step I.*

Step III *Choose a convenient point, e.g., origin, or some point on the coordinate axes.*

Step IV *Substitute the coordinates of the point, chosen in step III, in the given inequation and see whether it holds true or not.*

Step V *If the inequation holds good, then the region containing the chosen point will be the region represented by the given inequation. Otherwise, the region on the other side of the line will be the required region.*

ILLUSTRATION 3 *Mark the region represented by* $3x + 4y \leq 12$.

SOLUTION Converting the given inequation into equation, we obtain

$$3x + 4y = 12.$$

The line represented by this equation meets the coordinate axes at $A(4, 0)$ and $B(0, 3)$ as shown in Fig. A.7. Clearly, it divides the plane into two parts. One part containing the origin and other part on the other side of the line. We observe that $O(0, 0)$ satisfies the inequation $3x + 4y \leq 12$. So, the region represented by the given inequation is the region containing the origin as shown in Fig. A.7.

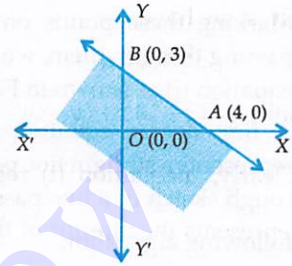


Fig. A.7

ILLUSTRATION 4 *Mark the region represented by* $y \leq x$.

SOLUTION We have, $y \leq x$.

Converting the inequation into equation, we get $y = x$.

Clearly, it represents a straight line passing through the origin and making 45° with x-axis. We observe that the points $(2, 0), (3, 1), (0, -5), (0, -2)$ etc. satisfy the inequation $y \leq x$. So, the region containing these points is the region represented by the inequation $y \leq x$ as shown in Fig. A.8.

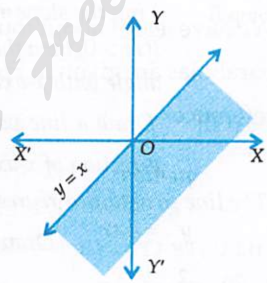


Fig. A.8

A.3.2 CIRCLE

An equation of the form $x^2 + y^2 + 2gx + 2fy + c = 0$ always represents a circle with centre at $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

Also, note that an equation of the form $(x - a)^2 + (y - b)^2 = r^2$ represents a circle with centre at (a, b) and radius r.

The inequation $(x - a)^2 + (y - b)^2 \leq r^2$ represents the interior of the circle $(x - a)^2 + (y - b)^2 = r^2$ and its exterior is represented by the inequation $(x - a)^2 + (y - b)^2 \geq r^2$

ILLUSTRATION 1 *Mark the region represented by* $x^2 + y^2 \leq 9$.

SOLUTION We have, $x^2 + y^2 \leq 9$

Clearly, it represents the interior of the region lying inside the circle $x^2 + y^2 = 9$ as shown in Fig. A.9.

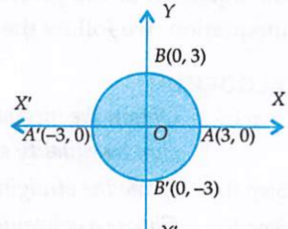


Fig. A.9

ILLUSTRATION 2 *Mark the region represented by the inequations*

$$x^2 + y^2 \leq 9 \quad \text{and,} \quad x^2 + y^2 - 6x \leq 0.$$

SOLUTION We have,

$$x^2 + y^2 \le 9 \quad \text{and,} \quad x^2 + y^2 - 6x \le 0$$

Now,

$$x^2 + y^2 - 6x \le 0 \Rightarrow (x-3)^2 + (y-0)^2 \le 3^2.$$

Thus, we have

$$x^2 + y^2 \le 9 \qquad \text{...(i)}$$

and, $$(x-3)^2 + (y-0)^2 \le 3^2 \qquad \text{...(ii)}$$

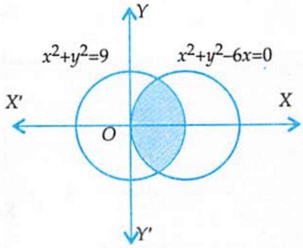


Fig. A.10

Clearly, inequation (i) represents the interior of the circle $x^2 + y^2 = 9$ and inequation (ii) represents the interior of the circle $(x-3)^2 + (y-0)^2 = 3^2$. The common region is shaded in Fig. A.10. It is the region Represented by both the inequations.

A.3.3 PARABOLA

We have learnt about four standard forms of parabola in earlier classes. Rough sketches of these parabolas are given in Fig. A.14 and various terms associated to them are given below for ready reference.

Equation	*Vertex*	*Focus*	*Latustrectum*	*Directrix*
$y^2 = 4ax$	(0, 0)	$(a, 0)$	$4a$	$x = -a$
$y^2 = -4ax$	(0, 0)	$(-a, 0)$	$4a$	$x = a$
$x^2 = 4ay$	(0, 0)	$(0, a)$	$4a$	$y = -a$
$x^2 = -4ay$	(0, 0)	$(0, -a)$	$4a$	$y = a$

REMARK *The inequation $y^2 \le 4ax$ represents the region lying inside the parabola $y^2 = 4ax$ and the region lying outside the parabola $y^2 = 4ax$ is represented by the inequation $y^2 \ge 4ax$. Similarly, the inequations $y^2 \le -4ax$, $x^2 \le 4ay$ and $x^2 \le -4ay$ represent the regions lying inside the parabolas $y^2 = -4ax$, $x^2 = 4ay$ and $x^2 = -4ay$ respectively.*

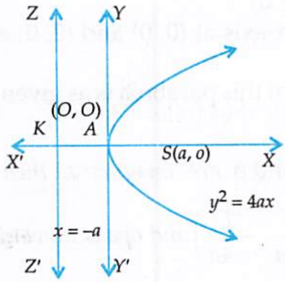


Fig. A.11

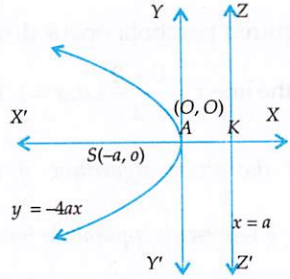


Fig. A.12

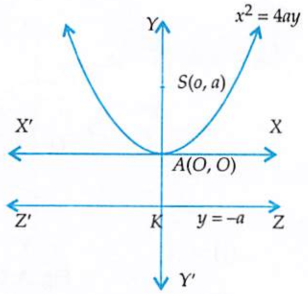


Fig. A.13

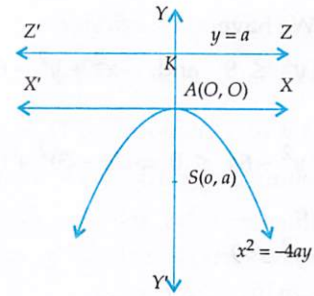


Fig. A.14

SKETCHING OF CURVES REPRESENTED BY $y = ax^2 + bx + c$: The equation $y = ax^2 + bx + c$ always represents a parabola having vertex at

$$\left(\frac{-b}{2a}, \frac{-D}{4a}\right) \text{ and axis } x = -\frac{b}{2a}.$$

The parabola opens upward or downward according as $a > 0$ or < 0. It meets x-axis at $(\alpha, 0)$ and $(\beta, 0)$, where α and β are the roots of the equation $ax^2 + bx + c = 0$. If the roots of this equation are not real, then the parabola does not cross x-axis. In order to draw rough sketch of the parabolas given by the equations of the form $y = ax^2 + bx + c$, we may follow the following algorithm.

ALGORITHM

Step I *Obtain the equation and observe the sign of the coefficient of x^2 in it.*

Step II *Put $y = 0$ in the given equation and get the values of x. Let the values be α and β.*

Step III *Mark the points $A(\alpha, 0)$ and $B(\beta, 0)$ on x-axis.*

Step IV *Draw a parabola passing through points A and B having its vertex on $x = \frac{-b}{2a} = \frac{\alpha + \beta}{2}$ and opening upward or downward according as the coefficient of x^2 in the given equation is positive or negative.*

ILLUSTRATION 1 *Draw a rough sketch of $x^2 - 2x + y = 0$.*

SOLUTION We have,

$$x^2 - 2x + y = 0 \Rightarrow y = -x^2 + 2x \qquad \text{...(i)}$$

Clearly, coefficient of x^2 is negative. So, the given equation represents a parabola opening downward.

Putting $y = 0$ in (i), we get

$$-x^2 + 2x = 0 \Rightarrow x = 0, 2.$$

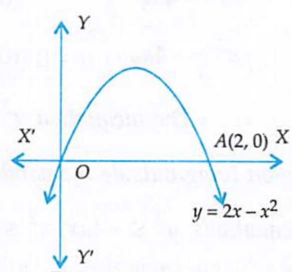


Fig. A.15

Therefore, the parabola cuts x-axis at (0, 0) and (2, 0).

Thus, the required parabola opens downward, crosses x-axis at (0, 0) and (2, 0) and its axis of symmetry is the line $x = \frac{0+2}{2}$ i.e. $x = 1$. The rough sketch of this parabola is as given in Fig. A.15.

REMARK *In the above algorithm, if the values of α and β are imaginary, then the equation $y = ax^2 + bx + c$ represents a parabola having vertex at $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ and opens upward or downward according as $a > 0$ or $a < 0$.*

ILLUSTRATION 2 *Draw a rough sketch of the curve $y = x^2 + 2$.*

SOLUTION We have, $y = x^2 + 2$

Since coefficient of x^2 is positive. Therefore, $y = x^2 + 2$ represents a parabola opening upward. We observe that $x^2 + 2 = 0$ gives imaginary values of x. So, the parabola does not cross x-axis. The coordinates of the vertex are (0, 2). The rough sketch of the curve is as shown in Fig. A.16.

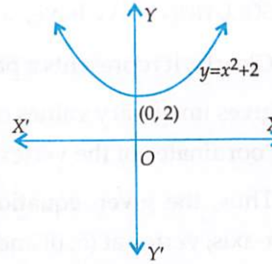


Fig. A.16

SKETCHING OF CURVES REPRESENTED BY $x = ay^2 + by + c$

The equation $x = ay^2 + by + c$ also represents a parabola having vertex at $\left(\frac{-D}{4a}, \frac{-b}{2a}\right)$ axis $y = \frac{-b}{2a}$ and it opens leftward or rightward according as $a < 0$ or $a > 0$. It crosses y-axis at $(0, \alpha)$ and $(0, \beta)$, where α and β are the roots of the equation $ay^2 + by + c = 0$. If α, β are not real, then the parabola does not cross y – axis and it opens rightward if $a > 0$ and leftward if $a < 0$. In order to draw a rough sketch of the parabolas given by the equations of the form $x = ay^2 + by + c$, we may follow the following algorithm.

ALGORITHM

Step I *Obtain the equation and observe the sign of coefficient of y^2 i.e. of a.*

Step II *Put $x = 0$ in the given equation and get the values of y. Let the values of y be α and β.*

Step III *Mark points $A(0, \alpha)$ and $B(0, \beta)$ on y-axis.*

Step IV *Draw a parabola passing through points A and B having its vertex on the line $y = \frac{-b}{2a} = \left(\frac{\alpha + \beta}{2}\right)$ and opening rightward or leftward according as $a > 0$ or, $a < 0$.*

ILLUSTRATION 1 *Draw a rough sketch of the curve $x = y^2 + 4y - 5$.*

SOLUTION We have, $x = y^2 + 4y - 5$ ...(i)

Clearly, coefficient of $y^2 > 0$. So, the given equation represents a parabola opening rightward.

Putting $x = 0$ in (i), we get

$$y^2 + 4y - 5 = 0 \Rightarrow (y+5)(y-1) = 0 \Rightarrow y = -5, 1.$$

So, the parabola cuts y-axis at $A(0, -5)$ and $B(0, 1)$. The vertex of the parabola is on the line $y = \frac{-5+1}{2}$ i.e. $y = -2$. The coordinates of the vertex are $(-9, -2)$. Thus, the given equation represents a parabola opening rightward having its vertex at $(-9, -2)$ and cuts y-axis at $(0, -5)$ and $(0, 1)$. The rough sketch is as given in Fig. A.17.

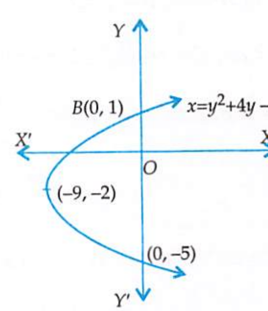


Fig. A.17

ILLUSTRATION 2 *Draw a rough sketch of the curve $x = 2y^2 + 5$.*

SOLUTION We have, $x = 2y^2 + 5$

Clearly, it represents a parabola opening rightward. Since $2y^2 + 5 = 0$ gives imaginary values of y. So, the curve does not cut y-axis. The coordinates of the vertex are (5, 0) and axis is x-axis.

Thus, the given equation represents a parabola having its axis as x-axis, vertex at (5, 0) and it opens rightward. The rough sketch of the parabola is as shown in Fig. A.18.

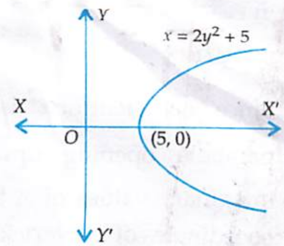


Fig. A.18

A.3.4 ELLIPSE

The equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents an ellipse having the following properties:

Centre	*Vertices*	*Major axis*	*Minor axis*	*Directrices*	*Foci*
(0, 0)	$(a, 0), (-a, 0)$	$2a$	$2b$	$x = \pm \frac{a}{e}$, where $e = \sqrt{1 - \frac{b^2}{a^2}}$	$(\pm ae, 0)$

The rough sketch of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is as shown in Fig. A.19.

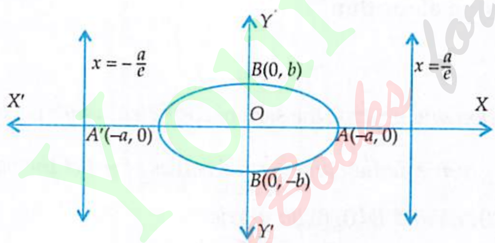


Fig. A.19

REMARK 1 *The inequation* $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$ *represents the region enclosed by the ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ *whereas the inequation* $\frac{x^2}{a^2} + \frac{y^2}{b^2} \geq 1$ *represents the region lying outside the ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

REMARK 2 *The equation* $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ *also represents an ellipse having its centre at* (h, k) *and major and minor axes parallel to the coordinate axes.*