CHAPTER 9

DIFFERENTIABILITY

9.1 DIFFERENTIABILITY AT A POINT

DEFINITION *Let $f(x)$ be a real valued function defined on an open interval (a, b) and let $c \in (a, b)$. Then, $f(x)$ is said to be differentiable or derivable at $x = c$, iff* $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ *exists finitely.*

This limit is called the derivative or differential coefficient of the function $f(x)$ at $x = c$, and is denoted by $f'(c)$ or, $Df(c)$ or, $\left(\frac{df(x)}{dx}\right)_{x=c}$.

Thus, $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$.

Now,

$f(x)$ is differentiable at $x = c$

$$\Leftrightarrow \quad \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \text{ exists finitely}$$

$$\Leftrightarrow \quad \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

$$\Leftrightarrow \quad \lim_{h \to 0} \frac{f(c - h) - f(c)}{-h} = \lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$$

$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c}$ or, $\lim_{h \to 0} \frac{f(c - h) - f(c)}{-h}$ is called the left hand derivative of $f(x)$ at $x = c$ and is denoted by $f'(c^-)$ or, $Lf'(c)$.

$\lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$ or, $\lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$ is called the right hand derivative of $f(x)$ at $x = c$ and is denoted by $f'(c^+)$ or, $Rf'(c)$.

Thus, (f is differentiable at $x = c$) $\Leftrightarrow$ $Lf'(c) = Rf'(c)$.

If $Lf'(c) \neq Rf'(c)$, we say that $f(x)$ is not differentiable at $x = c$.

MEANING OF DIFFERENTIABILITY AT A POINT As we have seen in the chapter on continuity of a function that if a function $f(x)$ is continuous at a point $x = a$ (say), then its graph is an unbroken curve at $(a, f(a))$ and there are no holes and jumps in the graph of the function in the neighbourhood of point $x = a$. Now, a natural question arises: What do we mean when we say that a function $f(x)$ is differentiable at a point $x = c$? In the following discussion we shall try to answer this question.

Consider the function $f(x)$ defined on an open interval (a, b). Let $P\,(c, f(c))$ be a point on the curve $y = f\,(x)$, and let $Q\,(c-h, f(c-h))$, and $R\,(c+h, f(c+h))$ be two neighbouring points on the left and right hand side respectively of point P as shown in Fig. 9.1. Then,

$$\text{Slope of chord } PQ = \frac{f(c-h)-f(c)}{-h}$$

and, $$\text{Slope of chord } PR = \frac{f(c+h)-f(c)}{h}$$

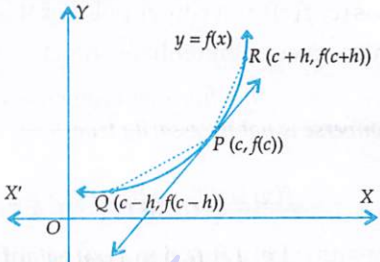

Fig. 9.1

We know that tangent to a curve at a point P (say) is the limiting position of chord PQ when Q tends to P. Therefore, as $h \to 0$ points Q and R both tend to P from left hand and right hand sides respectively. Consequently, chords PQ and PR become tangent(s) at point P.

Thus, $$\lim_{h\to 0}\frac{f\,(c-h)-f\,(c)}{-h} = \lim_{h\to 0}\ (\text{Slope of chord } PQ)$$

$$= \lim_{Q\to P}\ (\text{Slope of chord } PQ)$$

= Slope of the tangent at point P, which is the limiting position of the chords drawn on the left hand side of point P. ...(i)

and, $$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h} = \lim_{h\to 0}\ (\text{Slope of chord } PR)$$

$$= \lim_{R\to P}\ (\text{Slope of chord } PR)$$

= Slope of the tangent at point P, which is the limiting position of the chords drawn on the right hand side of point P ...(ii)

Now,

$f(x)$ is differentiable at $x = c$.

$\Leftrightarrow$ $$\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h} = \lim_{h\to 0}\frac{f\,(c+h)-f(c)}{h}$$

$\Leftrightarrow$ Slope of the tangent at point P, which is limiting position of the chords drawn on the left hand side of P is same as the slope of the tangent at point P, which is the limiting position of the chords drawn on the right hand side of P

$\Leftrightarrow$ There is a unique tangent at point P.

Thus, $f(x)$ is differentiable at point P, iff there exists a unique tangent at point P. In other words, $f(x)$ is differentiable at a point P iff the curve does not have P as a corner point.

Consider the function $f(x) = |\,x\,|$. This function is not differentiable at $x = 0$, because if we draw tangent at the origin as the limiting position of the chords on the left hand side of the origin, it is the line $y = -x$ whereas the tangent at the origin as the limiting position of the chords on the right hand side of the origin is the line $y = x$. Mathematically, left hand derivative at the origin is -1 (slope of the line $y = -x$) and the right hand derivative at the origin is 1 (slope of the line $y = x$).

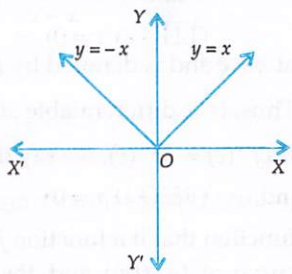

Fig. 9.2

Let $f(x)$ be a differentiable function at a point P. Then the curve $y = f(x)$ has a unique tangent at P. Since tangent at P is the limiting position of the chord PQ when $Q \to P$. So, if $f(x)$ is differentiable at a point P, then chords exist on both sides of point P. This means that the curve exists on both sides of P. Consequently $f(x)$ is continuous at P.

It follows from the above discussion that, if a function is not differentiable at $x = c$, then either it has $(c, f(c))$ as a corner point or it is discontinuous at $x = c$.

Also, every differentiable function is continuous as proved below.

THEOREM *If a function is differentiable at a point, it is necessarily continuous at that point. But, the converse is not necessarily true.*

OR

$f(x)$ is differentiable at $x = c \Rightarrow f(x)$ is continuous at $x = c$

PROOF Let a function $f(x)$ be differentiable at $x = c$. Then, $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely.

Let $$\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = f'(c) \qquad \text{...(i)}$$

In order to prove that $f(x)$ is continuous at $x = c$, it is sufficient to show that $\lim_{x \to c} f(x) = f(c)$.

Now,

$$\lim_{x \to c} f(x) = \lim_{x \to c} \left\{ \left(\frac{f(x) - f(c)}{x - c} \right)(x - c) + f(c) \right\}$$

$$\Rightarrow \quad \lim_{x \to c} f(x) = \lim_{x \to c} \left\{ \left(\frac{f(x) - f(c)}{x - c} \right)(x - c) \right\} + f(c)$$

$$\Rightarrow \quad \lim_{x \to c} f(x) = \lim_{x \to c} \left(\frac{f(x) - f(c)}{x - c} \right) \times \lim_{x \to c} (x - c) + f(c) = f'(c) \times 0 + f(c) \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad \lim_{x \to c} f(x) = f(c)$$

Hence, $f(x)$ is continuous at $x = c$. Q.E.D.

REMARK *The converse of the above theorem is not necessarily true i.e., a function may be continuous at a point but may not be differentiable at that point. For example, the function $f(x) = |x|$ is continuous at $x = 0$ but it is not differentiable at $x = 0$ (See Example 1 below.)*

ILLUSTRATIVE EXAMPLES

BASED ON BASIC CONCEPTS (BASIC)

EXAMPLE 1 *Show that $f(x) = |x|$ is not differentiable at $x = 0$.*

SOLUTION We observe that:

$$(\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{h \to 0} \frac{f(0 - h) - f(0)}{0 - h - 0} = \lim_{h \to 0} \frac{f(-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{|-h| - |0|}{-h} = \lim_{h \to 0} \frac{|-h|}{-h} = \lim_{h \to 0} \frac{h}{-h} = \lim_{h \to 0} -1 = -1$$

and, $$(\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{|h| - |0|}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

$\therefore$ (LHD at $x = 0$) $\neq$ (RHD at $x = 0$). So, $f(x)$ is not differentiable at $x = 0$.

EXAMPLE 2 *Show that the function* $f(x) = \begin{cases} x-1, & \text{if } x < 2 \\ 2x-3, & \text{if } x \geq 2 \end{cases}$ *is not differentiable at* $x = 2$.

SOLUTION We observe that:

$$(\text{LHD at } x=2) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x \to 2} \frac{(x-1)-(4-3)}{x-2} \quad [\because f(x) = x-1 \text{ for } x < 2]$$

$$= \lim_{x \to 2} \frac{x-2}{x-2} = \lim_{x \to 2} 1 = 1$$

and, $$(\text{RHD at } x=2) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2} = \lim_{x \to 2} \frac{(2x-3)-(4-3)}{x-2} \quad [\because f(x) = 2x-3 \text{ for } x \geq 2]$$

$$= \lim_{x \to 2} \frac{2x-4}{x-2} = \lim_{x \to 2} 2 = 2$$

$\therefore$ (LHD at $x=2$) $\neq$ (RHD at $x=2$). So, $f(x)$ is not differentiable at $x = 2$.

EXAMPLE 3 *Show that* $f(x) = x^2$ *is differentiable at* $x = 1$ *and find* $f'(1)$.

SOLUTION We observe that:

$$(\text{LHD at } x=1) = \lim_{x \to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{h \to 0} \frac{f(1-h)-f(1)}{1-h-1}$$

$$= \lim_{h \to 0} \frac{(1-h)^2-1^2}{-h} = \lim_{h \to 0} \frac{-2h+h^2}{-h} = \lim_{h \to 0} (2-h) = 2.$$

and, $$(\text{RHD at } x=1) = \lim_{x \to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{h \to 0} \frac{f(1+h)-f(1)}{1+h-1}$$

$$= \lim_{h \to 0} \frac{(1+h)^2-1}{h} = \lim_{h \to 0} \frac{2h+h^2}{h} = \lim_{h \to 0} (2+h) = 2$$

$\therefore$ (LHD at $x=1$) = (RHD at $x=1$) = 2. So, $f(x)$ is differentiable at $x=1$ and $f'(1) = 2$.

EXAMPLE 4 *Discuss the differentiability of* $f(x) = x|x|$ *at* $x = 0$.

[CBSE 2020, NCERT EXEMPLAR]

SOLUTION We have, $f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

Now, $$(\text{LHD at } x=0) = \lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0} \frac{-x^2-0}{x-0} \quad [\text{Using definition of } f(x)]$$

$$= \lim_{x \to 0} -x = 0$$

and, $$(\text{RHD at } x=0) = \lim_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0} \frac{x^2-0}{x-0} \quad [\text{Using definition of } f(x)]$$

$$= \lim_{x \to 0} x = 0.$$

$\therefore$ (LHD at $x=0$) = (RHD at $x=0$). So, $f(x)$ is differentiable at $x = 0$.

BASED ON LOWER ORDER THINKING SKILLS (LOTS)

EXAMPLE 5 *Show that the function* $f(x)=\begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , \text{ if } x \neq 0 \\ 0 & , \text{ if } x = 0 \end{cases}$ *is differentiable at* $x=0$ *and* $f'(0)=0$. **[NCERT EXEMPLAR]**

SOLUTION We observe that:

$$(\text{LHD at } x=0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0} = \lim_{h\to 0} \frac{f(-h)-f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{(-h)^2 \sin\left(\frac{1}{-h}\right)-0}{-h} = \lim_{h\to 0} h \sin\left(\frac{1}{h}\right)$$

$$= 0 \times (\text{an oscillating number between } -1 \text{ and } 1) = 0$$

and, $$(\text{RHD at } x=0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{0+h-0}$$

$$= \lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2 \sin\left(\frac{1}{h}\right)-0}{h} = \lim_{h\to 0} h \sin\left(\frac{1}{h}\right)$$

$$= 0 \times (\text{an oscillating number between } -1 \text{ and } 1) = 0$$

$\therefore$ (LHD at $x=0$) = (RHD at $x=0$) = 0.

So, $f(x)$ is differentiable at $x=0$ and $f'(0)=0$.

EXAMPLE 6 *Show that the function* $f(x) = \begin{cases} x \sin\frac{1}{x} & , \text{ when } x \neq 0 \\ 0 & , \text{ when } x = 0 \end{cases}$ *is continuous but not differentiable at* $x=0$.

SOLUTION For the continuity of the function refer Example 11 on page 8.8 of Chapter 8.

Now,

$$(\text{LHD at } x=0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0}$$

$$= \lim_{h\to 0} \frac{f(-h)-0}{-h} = \lim_{h\to 0} \frac{-h\sin\left(\frac{1}{-h}\right)}{-h} = -\lim_{h\to 0} \sin\left(\frac{1}{h}\right)$$

= A number which oscillates between -1 and 1

$\therefore$ (LHD at $x=0$) does not exist.

Similarly, it can be shown that RHD at $x=0$ does not exist. Hence, $f(x)$ is not differentiable at $x=0$.

EXAMPLE 7 *Show that the function* $f(x)=|x+1|+|x-1|$ *for all* $x \in R$, *is not differentiable at* $x=-1$ *and* $x=1$. **[CBSE 2015]**

SOLUTION We have,

$$f(x)=|x-1|+|x+1|=\begin{cases} -(x+1)-(x-1)=-2x, & \text{if } x<-1 \\ x+1-(x-1) \quad =2 & , \text{if } -1 \leq x < 1 \\ x+1+x-1 \quad =2x & , \text{if } x \geq 1 \end{cases}$$

Differentiability at $x=-1$: We find that

$$(\text{LHD at } x=-1) = \lim_{x\to -1^-} \frac{f(x)-f(-1)}{x-(-1)}$$

$$= \lim_{x\to -1^-} \frac{-2x-2}{x+1} = \lim_{x\to -1^-} \frac{-2(x+1)}{x+1} = \lim_{x\to -1^-} (-2) = -2$$

$$(\text{RHD at } x=-1) = \lim_{x\to -1^+} \frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\to -1^+} \frac{2-2}{x+1} = \lim_{x\to -1^+} \frac{0}{x+1} = \lim_{x\to -1^+} 0 = 0$$

$\therefore$ (LHD at $x=-1$) $\neq$ (RHD at $x=-1$). So, $f(x)$ is not differentiable at $x=-1$.

Differentiability at $x=1$: We find that

$$(\text{LHD at } x=1) = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{2-2}{x-1} = \lim_{x\to 1^-} \frac{0}{x-1} = \lim_{x\to 1^-} 0 = 0$$

$$(\text{RHD at } x=1) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{2x-2}{x-1} = \lim_{x\to 1^+} 2\left(\frac{x-1}{x-1}\right) = \lim_{x\to 1^+} 2 = 2$$

$\therefore$ (LHD at $x=1$) $\neq$ (RHD at $x=1$)

So, $f(x)$ is not differentiable at $x=1$. Hence, $f(x)$ is not differentiable at $x=-1$ and $x=1$.

EXAMPLE 8 *If $f(x)$ is differentiale at $x=a$, find* $\lim_{x\to a} \frac{x f(a)-a f(x)}{x-a}$.

SOLUTION It given that $f(x)$ is differentiable at $x=a$. Therefore, $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists finitely.

Let $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)$

Now, $$\lim_{x\to a} \frac{x f(a)-a f(x)}{x-a} = \lim_{x\to a} \frac{x f(a)-a f(a)+a f(a)-a f(x)}{x-a}$$

$$= \lim_{x\to a} \frac{(x-a) f(a)-a\{f(x)-f(a)\}}{x-a} = \lim_{x\to a} \frac{(x-a) f(a)}{x-a} - a \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} f(a) - a \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f(a) - a f'(a)$$

EXAMPLE 9 *If $f(2)=4$ and $f'(2)=1$, then find* $\lim_{x\to 2} \frac{x f(2)-2f(x)}{x-2}$.

SOLUTION Using definition of derivative, we obtain

$$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = f'(2) \Rightarrow \lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 1 \qquad [\because f'(2)=1] \qquad \ldots(\text{i})$$

Now,

$$\lim_{x\to 2} \frac{x f(2)-2f(x)}{x-2} = \lim_{x\to 2} \frac{x f(2)-2f(2)+2f(2)-2f(x)}{x-2}$$

$$= \lim_{x\to 2} \frac{(x-2) f(2)-2(f(x)-f(2))}{x-2} = \lim_{x\to 2} \frac{(x-2) f(2)}{x-2} - 2 \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$$

$$= f(2) - 2f'(2) = 4 - 2\times 1 = 2 \qquad [\text{Using (i) and } f(2)=4]$$

EXAMPLE 10 *If $f(x)$ is differentiable at $x=a$, find* $\lim_{x\to a} \frac{x^2 f(a)-a^2 f(x)}{x-a}$.

SOLUTION It is given that $f(x)$ is differentiable at $x = a$. Therefore, $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists finitely.

Let $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a} = f'(a)$ ...(i)

Now, $\lim\limits_{x \to a} \dfrac{x^2 f(a) - a^2 f(x)}{x - a} = \lim\limits_{x \to a} \dfrac{x^2 f(a) - a^2 f(a) + a^2 f(a) - a^2 f(x)}{x - a}$

$$= \lim_{x \to a} \frac{(x^2 - a^2) f(a) - a^2 \{f(x) - f(a)\}}{x - a} = \lim_{x \to a} \left\{ \frac{(x^2 - a^2) f(a)}{x - a} - a^2 \left(\frac{f(x) - f(a)}{x - a} \right) \right\}$$

$$= \lim_{x \to a} (x + a) f(a) - a^2 \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = 2a f(a) - a^2 f'(a) \quad \text{[Using (i)]}$$

BASED ON HIGHER ORDER THINKING SKILLS (HOTS)

EXAMPLE 11 *Discuss the differentiability of* $f(x) = \begin{cases} x\, e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ *at* $x = 0$.

SOLUTION We observe that:

$$f(x) = \begin{cases} x\, e^{-\left(\frac{1}{x} + \frac{1}{x}\right)} = x\, e^{-2/x}, & x > 0 \\ 0 & , x = 0 \\ x\, e^{-\left(\frac{-1}{x} + \frac{1}{x}\right)} = x & , x < 0 \end{cases}$$

Now, $(\text{LHD at } x = 0) = \lim\limits_{x \to 0^-} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0} \dfrac{x - 0}{x - 0} = 1$ $[\because f(x) = x \text{ for } x < 0 \text{ and } f(0) = 0]$

and, $(\text{RHD at } x = 0) = \lim\limits_{x \to 0^+} \dfrac{f(x) - f(0)}{x - 0}$

$= \lim\limits_{x \to 0} \dfrac{x\, e^{-2/x} - 0}{x}$ $[\because f(x) = x\, e^{-2x} \text{ for } x > 0 \text{ and } f(0) = 0]$

$= \lim\limits_{x \to 0} e^{-2/x} = 0.$

$\therefore$ (LHD at $x = 0$) $\neq$ (RHD at $x = 0$). So, $f(x)$ is not differentiable at $x = 0$.

EXAMPLE 12 *For what choice of* a *and* b *is the function* $f(x) = \begin{cases} x^2 & , x \leq c \\ ax + b, & x > c \end{cases}$ *is differentiable at* $x = c$.

SOLUTION It is given that $f(x)$ is differentiable at $x = c$ and every differentiable function is continuous. So, $f(x)$ is continuous at $x = c$.

$$\therefore \quad \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

$\Rightarrow \quad \lim\limits_{x \to c} x^2 = \lim\limits_{x \to c} (ax + b) = c^2$ [Using definition of $f(x)$]

$\Rightarrow \quad c^2 = ac + b$ ...(i)

Now, $f(x)$ is differentiable at $x = c$

$\Rightarrow$ (LHD at $x = c$) = (RHD at $x = c$)

$$\Rightarrow \lim_{x \to c^-} \frac{f(x)-f(c)}{x-c} = \lim_{x \to c^+} \frac{f(x)-f(c)}{x-c}$$

$$\Rightarrow \lim_{x \to c} \frac{x^2-c^2}{x-c} = \lim_{x \to c} \frac{(ax+b)-c^2}{x-c} \qquad \text{[Using definition of } f(x)]$$

$$\Rightarrow \lim_{x \to c} \frac{x^2-c^2}{x-c} = \lim_{x \to c} \frac{ax+b-(ac+b)}{x-c} \qquad \text{[Using (i)]}$$

$$\Rightarrow \lim_{x \to c} (x+c) = \lim_{x \to c} \frac{a(x-c)}{x-c} \Rightarrow \lim_{x \to c} (x+c) = \lim_{x \to c} a \Rightarrow 2c = a \qquad \text{...(ii)}$$

From (i) and (ii), we get : $c^2 = 2c^2 + b \Rightarrow b = -c^2$. Hence, $a = 2c$ and $b = -c^2$.

EXAMPLE 13 *A function $f : R \to R$ satisfies that equation $f(x+y) = f(x) f(y)$ for all $x, y \in R$, $f(x) \neq 0$. Suppose that the function $f(x)$ is differentiable at $x = 0$ and $f'(0) = 2$. Prove that $f'(x) = 2 f(x)$.*

[NCERT EXEMPLAR]

SOLUTION We have, $f(x+y) = f(x) f(y)$ for all $x, y \in R$

$\therefore \quad f(0+0) = f(0) f(0)$ [Putting $x = 0, y = 0$]

$\Rightarrow \quad f(0) = f(0) f(0)$

$\Rightarrow \quad f(0)\{1 - f(0)\} = 0$ [$\because f(x) \neq 0$ for any $x \therefore f(0) \neq 0$]

$\Rightarrow \quad 1 - f(0) = 0 \Rightarrow f(0) = 1$

It is given that $f(x)$ is differentiable at $x = 0$ and $f'(0) = 2$.

$$\therefore \quad f'(0) = \lim_{h \to 0} \frac{f(0+h)-f(0)}{h} \qquad \left[\text{Putting } c = 0 \text{ in } f'(c) = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}\right]$$

$$\Rightarrow \quad 2 = \lim_{h \to 0} \frac{f(h)-f(0)}{h} \Rightarrow 2 = \lim_{h \to 0} \frac{f(h)-1}{h} \qquad \text{...(i)}$$

Now,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \quad f'(x) = \lim_{h \to 0} \frac{f(x)f(h)-f(x)}{h} \qquad [\because f(x+y) = f(x) f(y) \text{ for all } x, y \in R \;\therefore\; f(x+h) = f(x) f(h)]$$

$$\Rightarrow \quad f'(x) = f(x) \lim_{h \to 0} \frac{f(h)-1}{h} \Rightarrow f'(x) = 2f(x) \qquad \text{[Using (i)]}$$

Hence, $f'(x) = 2 f(x)$.

EXERCISE 9.1

BASIC

1. Show that $f(x) = |x-3|$ is continuous but not differentiable at $x = 3$. [CBSE 2012, 2013]

2. Show that $f(x) = \begin{cases} 12x - 13, & \text{if } x \leq 3 \\ 2x^2 + 5, & \text{if } x > 3 \end{cases}$ is differentiable at $x = 3$. Also, find $f'(3)$.

3. Show that the function f defined by $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$ is continuous at $x = 2$, but not differentiable thereat. [CBSE 2010]

4. Find whether the following function is differentiable at $x=1$ and $x=2$ or not :

$$f(x) = \begin{cases} x &, \quad x \le 1 \\ 2-x &, \quad 1 \le x \le 2 \\ -2+3x-x^2 &, \quad x > 2 \end{cases}$$

[CBSE 2015]

BASED ON LOTS

5. Show that $f(x) = x^{1/3}$ is not differentiable at $x=0$.

6. Discuss the continuity and differentiability of the function $f(x) = |x| + |x-1|$ in the interval $(-1, 2)$. [CBSE 2015]

7. Find the values of a and b so that the function $f(x) = \begin{cases} x^2+3x+a &, \text{ if } x \le 1 \\ bx+2 &, \text{ if } x > 1 \end{cases}$ is differentiable at each $x \in R$. [NCERT EXEMPLAR, CBSE 2016]

8. If $f(x) = \begin{cases} ax^2 - b &, \text{ if } |x| < 1 \\ \dfrac{1}{|x|} &, \text{ if } |x| \ge 1 \end{cases}$ is differentiable at $x=1$, find a, b.

BASED ON HOTS

9. Show that the function $f(x) = \begin{cases} x^m \sin\left(\dfrac{1}{x}\right) &, x \ne 0 \\ 0 &, x = 0 \end{cases}$ is

(i) differentiable at $x=0$, if $m>1$

(ii) continuous but not differentiable at $x=0$, if $0<m<1$

(iii) neither continuous nor differentiable, if $m \le 0$

10. Show that the function $f(x) = \begin{cases} |2x-3|\,[x] &, \quad x \ge 1 \\ \sin\left(\dfrac{\pi x}{2}\right) &, \quad x < 1 \end{cases}$ is continuous but not differentiable at $x=1$.

ANSWERS

3. 12 5. Continuous on $(-1, 2)$ but not differentiable at $x=0, 1$.

6. Not differentiable at $x=1$, but differentiable at $x=2$. 7. $a=3, b=5$

8. $a=-1/2, b=-3/2$

HINTS TO SELECTED PROBLEMS

7. Use (LHD at $x=1$) = (RHD at $x=1$) and, $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1)$

8. $f(x)$ can be re-written as : $f(x) = \begin{cases} -\dfrac{1}{x} &, \text{ if } x \le -1 \\ (ax^2-b) &, \text{ if } -1 < x < 1 \\ \dfrac{1}{x} &, \text{ if } x \ge 1 \end{cases}$

Now, check continuity and differentiability of $f(x)$.

10. $f(x)$ can be re-written as : $f(x) = \begin{cases} (2x-3)[x] & , x \geq \frac{3}{2} \\ -(2x-3) & , 1 \leq x < \frac{3}{2} \\ \sin\left(\frac{\pi x}{2}\right) & , x < 1 \end{cases}$

Now, check continuity and differentiablility of $f(x)$.

9.2 DIFFERENTIABILITY IN A SET

A function $f(x)$ defined on an open interval (a, b) is said to be differentiable or derivable in open interval (a, b) if it is differentiable at each point of (a, b).

A function $f(x)$ defined on $[a, b]$ is said to be differentiable or derivable at the end points a and b if it is differentiable from the right at a and from the left at b.

In other words, $\lim\limits_{x \to a^+} \frac{f(x)-f(a)}{x-a}$ and $\lim\limits_{x \to b^-} \frac{f(x)-f(b)}{x-b}$ both exist.

If f is derivable in the open interval (a, b) and also at the end points a and b, then f is said to be derivable in the closed interval $[a, b]$.

A function f is said to be a differentiable function if it is differentiable at every point of its domain.

If a function is differentiable at each $x \in R$, then it is said to be every where differentiable.

For example, a constant function, a polynomial function, $\sin x$, $\cos x$ etc. are everywhere differentiable.

SOME USEFUL RESULTS ON DIFFERENTIABILITY

(i) Every polynomial function is differentiable at each $x \in R$.

(ii) The exponential function a^x, $a > 0$ is differentiable at each $x \in R$.

(iii) Every constant function is differentiable at each $x \in R$.

(iv) The logarithmic function is differentiable at each point in its domian.

(v) Trigonometric and inverse-trigonometric functions are differentiable in their respective domains.

(vi) The sum, difference, product and quotient of two differentiable functions is differentiable.

(vii) The composition of differentiable function is a differentiable function.

(viii) If a function $f(x)$ is differentiable at every point in its domain, then

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \text{ or, } \lim_{h \to 0} \frac{f(x-h)-f(x)}{-h}$$

is called the derivative or differentiation of f at x and is denoted by $f'(x)$ or, $\frac{d}{dx}(f(x))$.

ILLUSTRATIVE EXAMPLES

BASED ON BASIC CONCEPTS (BASIC)

EXAMPLE 1 *If $f(x) = x^2 + 2x + 7$, find $f'(3)$.*

SOLUTION We know that a polynomial function is everywhere differentiable. Therefore, $f(x)$ is differentiable at $x = 3$.

$$\therefore \quad f'(3) = \lim_{h \to 0} \frac{f(3+h)-f(3)}{h}$$

$$\Rightarrow \quad f'(3) = \lim_{h \to 0} \frac{\{(3+h)^2 + 2(3+h) + 7\} - \{9+6+7\}}{h} = \lim_{h \to 0} \frac{8h + h^2}{h} = \lim_{h \to 0} (8+h) = 8.$$

EXAMPLE 2 *Find $f'(2)$ and $f'(5)$ when $f(x) = x^2 + 7x + 4$.*

SOLUTION We know that a polynomial function is everywhere differentiable. Therefore, $f(x)$ is everywhere differentiable. The derivative of f at x is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\{(x+h)^2 + 7(x+h) + 4\} - \{x^2 + 7x + 4\}}{h}$$

$$= \lim_{h \to 0} \frac{2hx + 7h + h^2}{h} = \lim_{h \to 0} (2x + 7 + h) = 2x + 7$$

Putting $x = 2$ and $x = 5$ respectively in $f'(x) = 2x + 7$, we get

$\therefore \quad f'(2) = 2 \times 2 + 7 = 11$ and $f'(5) = 2 \times 5 + 7 = 17$.

EXAMPLE 3 *For the function f given by $f(x) = x^2 - 6x + 8$, prove that $f'(5) - 3f'(2) = f'(8)$.*

SOLUTION Clearly, $f(x)$ being a polynomial function, is everywhere differentiable. The derivative of f at x is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\{(x+h)^2 - 6(x+h) + 8\} - \{x^2 - 6x + 8\}}{h}$$

$$= \lim_{h \to 0} \frac{2hx - 6h + h^2}{h} = \lim_{h \to 0} (2x - 6 + h) = 2x - 6$$

$\therefore \quad f'(5) - 3f'(2) = (2 \times 5 - 6) - 3(2 \times 2 - 6) = 4 + 6 = 10$ and, $f'(8) = 2 \times 8 - 6 = 10$.

Hence, $f'(5) - 3f'(2) = f'(8)$.

BASED ON LOWER ORDER THINKING SKILLS (LOTS)

EXAMPLE 4 *Discuss the continuity and differentiability of* $f(x) = \begin{cases} 1 - x &, \ x < 1 \\ (1-x)(2-x) &, \ 1 \le x \le 2 \\ 3 - x &, \ x > 2 \end{cases}$

SOLUTION When $x < 1$, we have $f(x) = 1 - x$. We know that a polynomial function is everywhere continuous and differentiable. So, $f(x)$ is continuous and differentiable for all $x < 1$.

Similarly, $f(x)$ is continuous and differentiable for all $x \in (1, 2)$ and $x > 2$.

Thus, the possible points where we have to check the continuity and differentiability of $f(x)$ are $x = 1$ and $x = 2$.

Continuity at $x = 1$: We find that:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (1 - x) = 1 - 1 = 0 \qquad [\because f(x) = 1 - x \text{ for } x < 1]$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (1-x)(2-x) = 0 \qquad [\because f(x) = (1-x)(2-x), \text{ for } 1 \le x \le 2]$$

and, $f(1) = (1-1)(2-1) = 0$.

$\therefore \quad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$. So, $f(x)$ is continuous at $x = 1$.

Continuity at $x = 2$: We find that:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (1-x)(2-x) = (2-1)(2-2) = 0 \qquad [\because f(x) = (1-x)(2-x) \text{ for } 1 \le x \le 2]$$

and, $$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (3 - x) = 3 - 2 = 1 \qquad [\because f(x) = 3 - x \text{ for } x > 2]$$

$\therefore \quad \lim\limits_{x \to 2^-} f(x) \neq \lim\limits_{x \to 2} f(x)$. So, $f(x)$ is not continuous at $x = 2$.

Differentiability at $x = 1$: We observe that:

$$(\text{LHD at } x = 1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{(1 - x) - 0}{x - 1} \quad [\text{Using definition of } f(x)]$$

$$= -\lim_{x \to 1} \frac{x - 1}{x - 1} = -1$$

and, $$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{(1 - x)(2 - x) - 0}{x - 1} \quad [\text{Using definition of } f(x)]$$

$$= \lim_{x \to 1} \frac{(x - 1)(x - 2)}{x - 1} = \lim_{x \to 1} x - 2 = 1 - 2 = -1$$

Clearly, (LHD at $x = 1$) = (RHD at $x = 1$). So, $f(x)$ is differentiable at $x = 1$.

Differentiability at $x = 2$: Since $f(x)$ is not continuous at $x = 2$. So, it is not differentiable at $x = 2$.

EXAMPLE 5 *Discuss the differentiability of* $f(x) = |x - 1| + |x - 2|$.

SOLUTION We have,

$$f(x) = |x - 1| + |x - 2|$$

$$\Rightarrow \quad f(x) = \begin{cases} -(x - 1) - (x - 2) & \text{for } x < 1 \\ x - 1 - (x - 2) & \text{for } 1 \le x < 2 \\ (x - 1) + (x - 2) & \text{for } x \ge 2 \end{cases} \Rightarrow f(x) = \begin{cases} -2x + 3 & , \ x < 1 \\ 1 & , \ 1 \le x < 2 \\ 2x - 3 & , \ x \ge 2 \end{cases}$$

When $x < 1$, we have $f(x) = -2x + 3$ which, being a polynomial function is continuous and differentiable.

When $1 \le x < 2$, we have $f(x) = 1$ which, being a constant function, is differentiable on $(1, 2)$.

When $x \ge 2$, we have $f(x) = 2x - 3$ which, being a polynomial function, is differentiable for all $x > 2$.

Thus, the possible points of non-differentiability of $f(x)$ are $x = 1$ and $x = 2$. So, let us check the differentiability of $f(x)$ at these points.

Differentiability at $x = 1$:

$$(\text{LHD at } x = 1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{(-2x + 3) - 1}{x - 1} \quad [\because f(x) = -2x + 3 \text{ for } x < 1]$$

$$= \lim_{x \to 1} \frac{-2(x - 1)}{x - 1} = \lim_{x \to 1} = -2 = -2$$

and, $$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{1 - 1}{x - 1} = 0. \quad [\because f(x) = 1 \text{ for } 1 \le x < 2]$$

$\therefore \quad$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$). So, $f(x)$ is not differentiable at $x = 1$.

Differentiability at $x = 2$:

$$(\text{LHD at } x = 2) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{1 - (2 \times 2 - 3)}{x - 2} \quad [\because f(x) = 1 \text{ for } 1 \le x < 2 \ \& \ f(2) = 2 \times 2 - 3]$$

$$= \lim_{x \to 2} \frac{1 - 1}{x - 2} = 0.$$

and, $\quad (\text{RHD at } x = 2) = \lim\limits_{x \to 2^+} \dfrac{f(x) - f(2)}{x - 2}$

$= \lim\limits_{x \to 2} \dfrac{(2x-3) - (2 \times 2 - 3)}{x - 2} \qquad [\because\ f(x) = 2x - 3 \text{ for } x \geq 2]$

$= \lim\limits_{x \to 2} \dfrac{2x - 4}{x - 2} = \lim\limits_{x \to 2} \dfrac{2(x-2)}{x - 2} = 2$

$\therefore \quad (\text{LHD at } x = 2) \neq (\text{RHD at } = 2)$. So, $f(x)$ is not differentiable at $x = 2$.

REMARK *The function f(x) given by* $f(x) = |x - a_1| + |x - a_2| + |x - a_3| + \ldots + |x - a_n|$ *is not differentiable at* $x = a_1, a_2, a_3, \ldots, a_n$. *However, it is continuous at these points.*

BASED ON HIGHER ORDER THINKING SKILLS (HOTS)

EXAMPLE 6 *If* $f(x) = \begin{cases} x^2 + 3x + a, & \text{for } x \leq 1 \\ bx + 2, & \text{for } x > 1 \end{cases}$ *is everywhere differentiable, find the values of a and b.*

SOLUTION For $x \leq 1$, we have $f(x) = x^2 + 3x + a$ which is a polynomial.

For $x > 1$, we have $f(x) = bx + 2$ which is also a polynomial. Since a polynomial function is everywhere differentiable. Therefore, $f(x)$ is differentiable for all $x > 1$ and also for all $x < 1$. Thus, we have to use the differentiability of $f(x)$ at $x = 1$ to find the values of a and b.

Now,

$f(x)$ is differentiable at $x = 1$

$\Rightarrow \quad f(x)$ is continuous at $x = 1$

$\Rightarrow \quad \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1)$

$\Rightarrow \quad \lim\limits_{x \to 1} x^2 + 3x + a = \lim\limits_{x \to 1} bx + 2 = 1 + 3 + a \Rightarrow 1 + 3 + a = b + 2 \Rightarrow a - b + 2 = 0 \qquad \ldots(\text{i})$

Again, $f(x)$ is differentiable at $x = 1$.

$\Rightarrow \quad (\text{LHD at } x = 1) = (\text{RHD at } x = 1)$

$\Rightarrow \quad \lim\limits_{x \to 1^-} \dfrac{f(x) - f(1)}{x - 1} = \lim\limits_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$

$\Rightarrow \quad \lim\limits_{x \to 1} \dfrac{x^2 + 3x + a - (4 + a)}{x - 1} = \lim\limits_{x\ 1} \dfrac{(bx + 2) - (4 + a)}{x - 1}$

$\Rightarrow \quad \lim\limits_{x \to 1} \dfrac{x^2 + 3x - 4}{x - 1} = \lim\limits_{x \to 1} \dfrac{bx - (2 + a)}{x - 1}$

$\Rightarrow \quad \lim\limits_{x \to 1} \dfrac{(x+4)(x-1)}{x - 1} = \lim\limits_{x \to 1} \dfrac{bx - b}{x - 1} \qquad [\text{From (i)}, 2 + a = b]$

$\Rightarrow \quad \lim\limits_{x \to 1} (x + 4) = \lim\limits_{x \to 1} b \Rightarrow 5 = b.$

Putting $b = 5$ in (i), we get $a = 3$. Hence, $a = 3$ and $b = 5$.

EXAMPLE 7 *Discuss the differentiability of* $f(x) = |\log_e x|$ *for* $x > 0$.

SOLUTION We have,

$$f(x) = |\log_e x| = \begin{cases} -\log_e x, & \text{for } 0 < x < 1 \\ \log_e x, & \text{for } x \geq 1 \end{cases}$$

Clearly, $f(x)$ is differentiable for all $x > 1$ as well as for all $x < 1$. So, we have to check its differentiability at $x = 1$.

We have,

$$(\text{LHD at } x=1) = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1}$$

$$= \lim_{x\to 1^-} \frac{-\log x - \log 1}{x-1} \qquad [\because f(x) = -\log_e x \text{ for } 0<x<1]$$

$$= -\lim_{x\to 1^-} \frac{\log x}{x-1} = -\lim_{h\to 0} \frac{\log(1-h)}{1-h-1} = -\lim_{h\to 0} \frac{\log(1-h)}{-h} = -1$$

and, $$(\text{RHD at } x=1) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$$

$$= \lim_{x\to 1^+} \frac{\log x - \log 1}{x-1} = \lim_{h\to 0} \frac{\log(1+h)}{1+h-1} = \lim_{h\to 0} \frac{\log(1+h)}{h} = 1$$

Clearly, (LHD at $x = 1$) $\neq$ (RHD at $x = 1$). So, $f(x)$ is not differentiable at $x = 1$.

EXERCISE 9.2

BASIC

1. If f is defined by $f(x) = x^2$, find $f'(2)$.
2. If f is defined by $f(x) = x^2 - 4x + 7$, show that $f'(5) = 2f'\left(\frac{7}{2}\right)$
3. Show that the derivative of the function f given by $f(x) = 2x^3 - 9x^2 + 12x + 9$, at $x = 1$ and $x = 2$ are equal.
4. If for the function $\Phi(x) = \lambda x^2 + 7x - 4$, $\Phi'(5) = 97$, find λ.
5. If $f(x) = x^3 + 7x^2 + 8x - 9$, find $f'(4)$.
6. Find the derivative of the function f defined by $f(x) = mx + c$ at $x = 0$.

BASED ON LOTS

7. Examine the differentiability of the function f defined by

$$f(x) = \begin{cases} 2x+3, & \text{if } -3 \le x < -2 \\ x+1, & \text{if } -2 \le x < 0 \\ x+2, & \text{if } 0 \le x \le 1 \end{cases}$$ [NCERT EXEMPLAR]

8. Write an example of a function which is everywhere continuous but fails to be differentiable exactly at five points.

BASED ON HOTS

9. Discuss the continuity and differentiability of $f(x) = |\log|x||$.
10. Discuss the continuity and differentiability of $f(x) = e^{|x|}$.
11. Discuss the continuity and differentiability of $f(x) = \begin{cases} (x-c)\cos\left(\frac{1}{x-c}\right), & x \neq c \\ 0, & x = c \end{cases}$
12. Is $|\sin x|$ differentiable? What about $\cos|x|$?

ANSWERS

1. 4 4. 9 5. 112 6. m
7. Not differentiable at $x = 0$ and $x = -2$. 8. $f(x) = |x| + |x-1| + |x-2| + |x-3| + |x-4|$
9. Not differentiable at $x = \pm 1$ 10. Not differentiable at $x = 0$

11. Not differentiable at $x = c$
12. $|\sin x|$ is not differentiable at $x = n\pi, n \in Z$, $\cos|x|$ is everywhere differentiable.

FILL IN THE BLANKS TYPE QUESTIONS (FBQs)

1. The function $f(x) = |x+1|$ is not differentiable at $x =$ ....
2. The function $g(x) = |x-1| + |x+1|$ is not differentiable at $x =$ ....
3. The set of points where $f(x) = x - [x]$ not differentiable is ....
4. The number of points in $[-\pi, \pi]$ where $f(x) = \sin^{-1}(\sin x)$ is not differentiable is ....
5. The function $f(x) = \cos^{-1}(\cos x)$, $x \in (-2\pi, 2\pi)$ is not differentiable at $x =$ ....
6. The function $f(x) = |\sin x|$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is not differentiable at $x =$ ....
7. Let $f(x) = \begin{cases} ax^2 + 3, & x > 1 \\ x + \frac{5}{2}, & x \le 1 \end{cases}$. If $f(x)$ is differentiable at $x = 1$, then $a =$ ....
8. If $f(x) = x|x|$, then $f'(-1) =$ ....
9. If $f(x) = x|x|$, then $f'(2) =$ ....
10. The set of point where the function $f(x) = |2x-1|$ is differentiable, is ....
11. The set of points where the function $f(x) = \begin{cases} x+1, & x < 2 \\ 2x-1, & x \ge 2 \end{cases}$ is not differentiable, is ....
12. An example of a function which is everywhere continuous but fails to be differentiable exactly at two points is ....
13. The set of points where $f(x) = \cos|x|$ is differentiable, is ....
14. The set of points where $f(x) = |\sin x|$ is not differentiable, is ....
15. The set of points at which the function $f(x) = \frac{1}{\log|x|}$ is not differentiable, is ....
16. The greatest integer function $f(x) = [x]$, $0 < x < 2$ is not differentiable at $x =$...... **[CBSE 2020]**

ANSWERS

1. -1 2. ± 1 3. Z 4. 2 5. $\pm\pi$ 6. 0 7. $a = \frac{1}{2}$ 8. 2
9. 4 10. $R - \left\{\frac{1}{2}\right\}$ 11. $\{2\}$ 12. $f(x) = |x-1| + |x-2|$ 13. R
14. $\{0\}$ 15. $\{-1, 0, 1\}$ 16. $x = 1$

VERY SHORT ANSWERS QUESTIONS (VSAQs)

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

1. Define differentiability of a function at a point.
2. Is every differentiable function continuous?
3. Is every continuous function differentiable?
4. Give an example of a function which is continuous but not differentiable at a point.
5. If $f(x)$ is differentiable at $x = c$, then write the value of $\lim_{x \to c} f(x)$.
6. If $f(x) = |x-2|$ write whether $f'(2)$ exists or not.
7. Write the points where $f(x) = |\log_e x|$ is not differentiable.
8. Write the points of non-differentiability of $f(x) = |\log|x||$.

9. Write the derivative of $f(x) = |x|^3$ at $x = 0$.

10. Write the number of points where $f(x) = |x| + |x-1|$ is continuous but not differentiable.

11. If $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely, write the value of $\lim_{x \to c} f(x)$.

12. Write the value of the derivative of $f(x) = |x-1| + |x-3|$ at $x = 2$.

13. If $f(x) = \sqrt{x^2 + 9}$, write the value of $\lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$.

ANSWERS

2. Yes	3. No	4. $f(x) = \lvert x \rvert$ at $x = 0$	5. $f(c)$	6. Does not exist	7. 1
8. ± 1	9. 0	10. $x = 0, 1$	11. $f(c)$	12. 0	13. $\frac{4}{5}$