CHAPTER 25

PARABOLA

25.1 CONIC SECTIONS

A conic section, as the name implies, is a section cut-off from a circular (not necessary a right circular) cone by a plane in various ways. The shape of the section depends upon the position of the cutting plane.

Consider a double right circular cone of semi vertical angle α and let it be cut by a plane inclined at an angle θ to the axis of the cone. We will get different sections (curves) as follows:

<u>CASE I</u> *If the plane passes through the vertex O*

The curve of intersection is a pair of straight lines passing through the vertex which are

(i) real and distinct for $\theta < \alpha$.

(ii) coincident for $\theta = \alpha$ i.e. the plane touches the cone.

(iii) imaginary for $\theta > \alpha$.

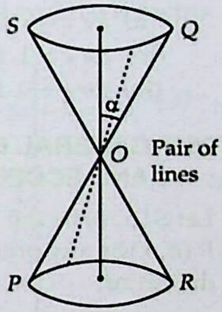

Fig. 25.1

<u>CASE II</u> *If the plane does not pass through the vertex O*

The curve of intersection is called

(i) a circle if $\theta = \dfrac{\pi}{2}$.

(ii) a parabola for $\theta = \alpha$ i.e. if the plane is parallel to the generator PQ.

(iii) an ellipse for $\theta > \alpha$ $(\theta \neq \pi / 2)$ i.e. if the plane cuts both the generating lines PQ and RS.

(iv) a hyperbola for $\theta < \alpha$ i.e. if the plane cuts both the cones.

Thus, we may get the section either as a pair of straight lines, a circle, a parabola, an ellipse or a hyperbola depending upon the different positions of the cutting plane. These curves of intersection are called the *conic sections.*

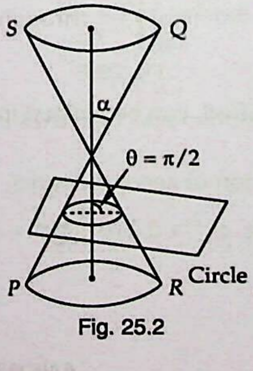

Fig. 25.2

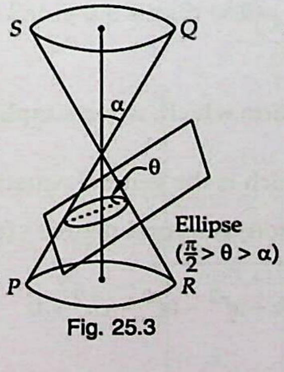

Fig. 25.3

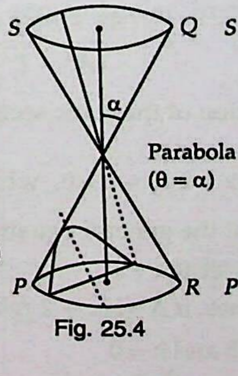

Fig. 25.4

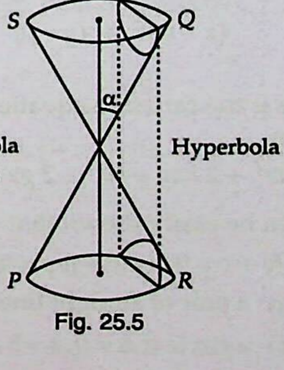

Fig. 25.5

25.2 ANALYTICAL DEFINITION OF CONIC SECTION

CONIC SECTION *A conic section or conic is the locus of a point P which moves in such a way that its distances from a fixed point S always bears a constant ratio to its distance from a fixed line, all being in the same plane.*

FOCUS *The fixed point is called the focus of the conic section.*

DIRECTRIX *The fixed straight line is called the directrix of the conic section.*

In general, every conic has four foci, two of them are real and the other two are imaginary. Due to two real foci, every conic has two directrices corresponding to each real focus.

ECCENTRICITY *The constant ratio is called the eccentricity of the conic section and is denoted by e.*

AXIS *The straight line passing through the focus and perpendicular to the directrix is called the axis of the conic section.*

VERTEX *The points of intersection of the conic section and the axis are called vertices of the conic section.*

CENTRE *The point which bisects every chord of the conic passing through it, is called the centre of the conic.*

LATUS-RECTUM *The latus-rectum of a conic is the chord passing through the focus and perpendicular to the axis.*

NOTE *As mentioned above the eccentricity of a conic is generally denoted by e and*

(i) *for $e < 1$, the conic obtained is an ellipse;*

(ii) *for $e = 1$, the conic obtained is a parabola;*

(iii) *for $e > 1$, the conic is a hyperbola;*

(iv) *for $e = 0$, the conic is a circle.*

25.3 GENERAL EQUATION OF A CONIC SECTION WHEN ITS FOCUS, DIRECTRIX AND ECCENTRICITY ARE GIVEN

Let $S(\alpha, \beta)$ be the focus, $Ax + By + C = 0$ be the directrix and e be the eccentricity of a conic. Let $P(h, k)$ be any point on the conic. Let PM be the perpendicular from P, on the directrix. Then, by definition

$$SP = e \cdot PM$$

$$\Rightarrow \quad SP^2 = e^2 PM^2$$

$$\Rightarrow \quad (h-\alpha)^2 + (k-\beta)^2 = e^2 \left(\frac{Ah + Bk + C}{\sqrt{A^2 + B^2}} \right)^2$$

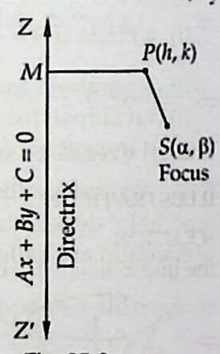

Fig. 25.6

Thus, the locus of (h, k) is

$$(x-\alpha)^2 + (y-\beta)^2 = e^2 \frac{(Ax + By + C)^2}{(A^2 + B^2)}$$

This is the cartesian equation of the conic section which, when simplified, can be written in the form

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, which is the general equation of second degree.

It can be easily shown that the general equation of second degree viz. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ always represents:

(i) a pair of straight lines, if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

(ii) a circle if $\Delta \neq 0$, $a = b$ and $h = 0$

(iii) a parabola if $\Delta \neq 0$ and $h^2 = ab$

(iv) an ellipse if $\Delta \neq 0$ and $h^2 < ab$

(v) a hyperbola if $\Delta \neq 0$ and $h^2 > ab$

(vi) a rectangular hyperbola if $\Delta \neq 0$, $h^2 > ab$ and $a + b = 0$.

25.4 THE PARABOLA

ANALYTICAL DEFINITION *A parabola is the locus of a point which moves in a plane such that its distance from a fixed point in the plane is always equal to its distance from a fixed straight line in the same plane.*

As defined in section 25.2, the fixed point is called the focus and the fixed straight line is called the directrix of the parabola. The line through the focus and perpendicular to the directrix is the axis of the parabola. The point on the axis midway between the focus and directrix is called the vertex of the parabola.

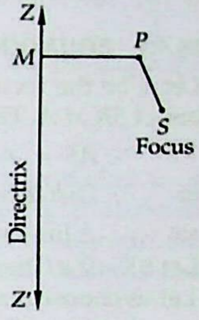

Fig. 25.7

Let S be the focus, ZZ' be the directrix and let P be any point on the parabola. Then, by definition

$$SP = PM$$

where PM is the length of the perpendicular from P on the directrix ZZ'.

ILLUSTRATION 1 *Find the equation of the parabola whose focus is $(-3, 2)$ and the directrix is $x + y = 4$.*

SOLUTION Let $P(x, y)$ be any point on the parabola whose focus is $S(-3, 2)$ and the directrix $x + y - 4 = 0$. Draw PM perpendicular to $x + y - 4 = 0$. Then,

$$SP = PM \quad \text{[By definition]}$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x+3)^2 + (y-2)^2 = \left|\frac{x+y-4}{\sqrt{1+1}}\right|^2$$

$$\Rightarrow \quad 2\left(x^2 + y^2 + 6x - 4y + 13\right) = \left(x^2 + y^2 + 16 + 2xy - 8x - 8y\right)$$

$$\Rightarrow \quad x^2 + y^2 - 2xy + 20x + 10 = 0$$

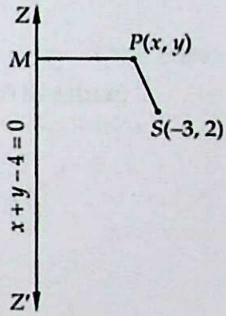

Fig. 25.8

Thus, the required equation of the parabola is $x^2 + y^2 - 2xy + 20x + 10 = 0$.

ILLUSTRATION 2 *Find the equation of the parabola whose focus is $(-3, 0)$ and the directrix is $x + 5 = 0$.*

SOLUTION Let $P(x, y)$ be any point on the parabola having its focus at $S(-3, 0)$ and directrix as the line $x + 5 = 0$. Then,

$SP = PM$, where PM is the length of the perpendicular from P on the directrix

$$\Rightarrow \quad SP^2 = PM^2$$

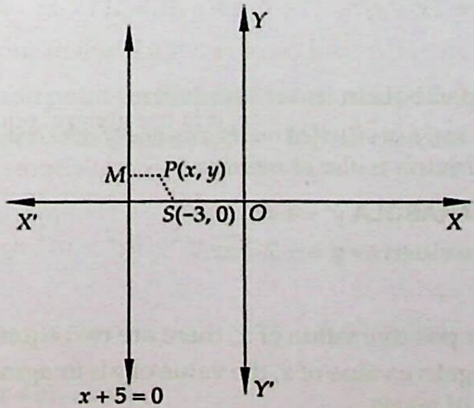

Fig. 25.9

$$\Rightarrow \quad (x+3)^2 + (y-0)^2 = \left|\frac{x+0y+5}{\sqrt{1+0}}\right|^2$$

$\Rightarrow \quad y^2 = 4x + 16$, which is the required equation of the parabola.

25.4.1 EQUATION OF THE PARABOLA IN ITS STANDARD FORM

Let S be the focus, $Z\,Z'$ be the directrix. Draw SK perpendicular from S on the directrix and bisect SK at A. Then,

$AS = AK$

$\Rightarrow$ Distance of A from the focus = Distance of A from the directrix

$\Rightarrow$ A lies on the parabola [By def.]

Let $SK = 2\,a$. Then, $AS = AK = a$.

Let us choose A as the origin, AS as x-axis and AY a line perpendicular to AS as y-axis. Then, the coordinates of S are $(a, 0)$ and the equation of the directrix ZZ' is $x = -a$.

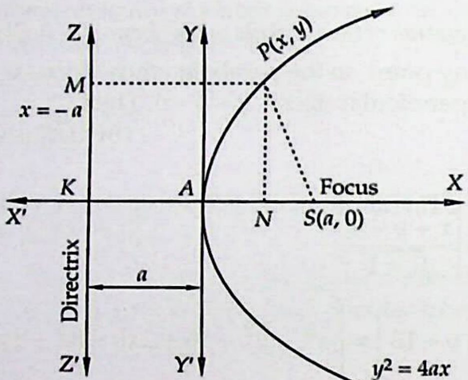

Fig. 25.10

Let $P\,(x, y)$ be any point on the parabola. Join SP and draw PM and PN perpendiculars on the directrix $Z\,Z'$ and X-axis. Then,

$PM = NK = AN + AK = x + a.$

Since P lies on the parabola. Therefore,

$SP = PM$ [By definition of parabola]

$\Rightarrow \quad SP^2 = PM^2$

$\Rightarrow \quad (x-a)^2 + (y-0)^2 = (x+a)^2$

$\Rightarrow \quad y^2 = 4\,ax$

This is the equation of the parabola in its standard form.

NOTE *The parabola has two real foci situated on its axis one of which is the focus S and the other lies at infinity. The corresponding directrix is also at infinity.*

25.4.2 TRACING OF THE PARABOLA $y^2 = 4\,ax, \ a > 0$

The given equation can be written as $y = \pm\, 2\sqrt{ax}$.

We observe the following:

(i) *Symmetry:* For every positive value of x, there are two equal and opposite values of y.

(ii) *Region:* For every negative value of x, the value of y is imaginary. Therefore, no part of the curve lies to the left of y-axis

(iii) *Origin:* The curve passes through the origin and the tangent at the origin is $x = 0$ i.e., y-axis

(iv) *Intersection with the axes:* The curve meets the coordinate axes only at the origin.

(v) *Portion Occupied:* As $x \to \infty$, $y \to \infty$. Therefore the curve extends to infinity to the right of axis of y.

With the help of the above facts and by joining some convenient points on the parabola the general shape of the parabola $y^2 = 4ax$ is as shown in Fig. 25.10.

25.4.3 VARIOUS RESULTS RELATED TO THE PARABOLA

As discussed in section 25.4, the focus of the parabola $y^2 = 4ax$ is at $(a, 0)$ and the directrix is $x = -a$. The axis is a line passing through the focus and perpendicular to the directrix. In Fig. 25.10 x-axis i.e., $y = 0$ is the axis of the parabola $y^2 = 4ax$. The axis meets the curve $y^2 = 4ax$ at A, the origin. So, the coordinates of the vertex are $(0, 0)$. Clearly, the vertex A is the midway between the focus and the directrix i.e., the vertex is equidistant from the focus and the directrix.

DOUBLE ORDINATE *Let P be any point on the parabola $y^2 = 4ax$. A chord passing through P perpendicular to the axis of the parabola is called the double ordinate through the point P.*

In Fig. 25.10, PP' is the double ordinate of point P.

LATUS-RECTUM *A double ordinate through the focus is called the latusrectum i.e. the latusrectum of a parabola is a chord passing through the focus perpendicular to the axis.*

In Fig. 25.10, LSL' is the latusrectum of the parabola $y^2 = 4ax$. By the symmetry of the curve $SL = SL' = \lambda$ (say). So, the coordinates of L are (a, λ). Since L lies on $y^2 = 4ax$. Therefore,

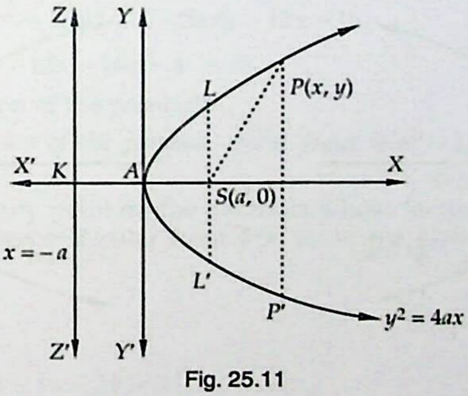

Fig. 25.11

$$\lambda^2 = 4a^2 \Rightarrow \lambda = 2a \Rightarrow LL' = 2\lambda = 4a.$$

$\therefore$ Latusrectum $= 4a$.

The coordinates of L and L', end points of the latusrectum, are $(a, 2a)$ and $(a, -2a)$ respectively.

FOCAL DISTANCE OF ANY POINT *The distance of $P(x, y)$ from the focus S is called the focal distance of the point P.*

Clearly, $SP = \sqrt{(x-a)^2 + (y-0)^2}$

$\Rightarrow \quad SP = \sqrt{(x-a)^2 + y^2}$

$\Rightarrow \quad SP = \sqrt{(x-a)^2 + 4ax}$ $\quad$ [$\because$ $P(x, y)$ lies on the parabola $\therefore y^2 = 4ax$]

$\Rightarrow \quad SP = \sqrt{(x+a)^2} = |x+a| = a + x$ $\quad$ [$\because x > 0, a > 0 \therefore x + a > 0$]

Hence, $a + x$ is the focal distance of any point $P(x, y)$ on the parabola $y^2 = 4ax$.

FOCAL CHORD *A chord of the parabola is a focal chord, if it passes through the focus.*

25.4.4 SOME OTHER STANDARD FORMS OF PARABOLA

Proceeding as in section 25.4, we find that there are three other standard forms of parabola viz. $y^2 = -4ax$, $x^2 = 4ay$ and $x^2 = -4ay$ depending upon the choice of the axes. Thus, in all there are four standard forms. The shapes of the curves in these four standard forms and their corresponding results are as follows:

	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Coordinates of vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Coordinates of focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of the directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of the axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Length of the Latusrectum	$4a$	$4a$	$4a$	$4a$
Focal distance of a point $P(x, y)$	$a + x$	$a - x$	$a + y$	$a - y$

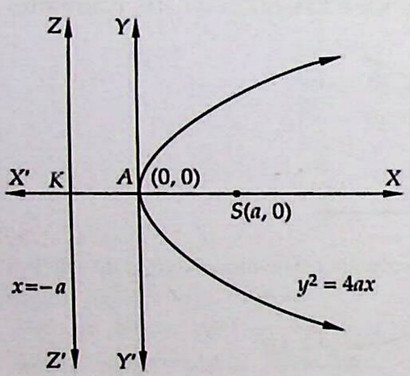

Fig. 25.12

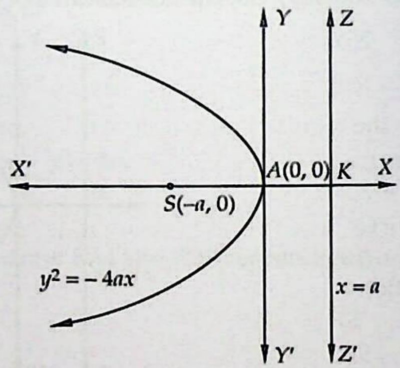

Fig. 25.13

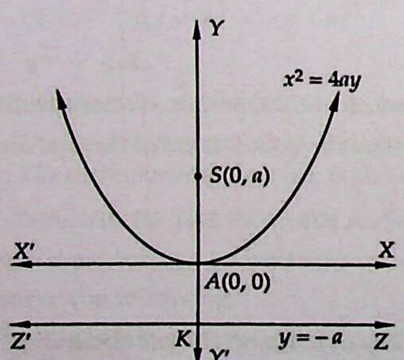

Fig. 25.14

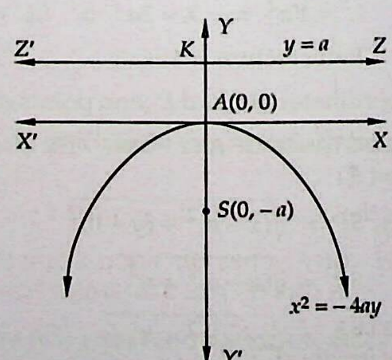

Fig. 25.15

REMARK *If the vertex of the parabola is at the point A (h, k) and its latusrectum is of length $4a$, then its equation is*

(i) $(y-k)^2 = 4a(x-h)$ or, $(y-k)^2 = -4a(x-h)$ according as its axis is parallel to OX or OX'.

(ii) $(x-h)^2 = 4a(y-k)$ or, $(x-h)^2 = -4a(y-k)$ according as its axis is parallel to OY or OY'.

ILLUSTRATIVE EXAMPLES

LEVEL-1

Type I **ON FINDING THE EQUATION OF A PARABOLA WHEN ITS FOCUS AND DIRECTRIX ARE GIVEN**

EXAMPLE 1 *Find the equation of the parabola whose focus is the point (0, 0) and whose directrix is the straight line $3x - 4y + 2 = 0$.*

SOLUTION Let $P(x, y)$ be any point on the parabola whose focus is $S(0, 0)$ and the directrix $3x - 4y + 2 = 0$. Draw PM perpendicular from P on the directrix. Then, by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-0)^2 + (y-0)^2 = \left|\frac{3x-4y+2}{\sqrt{3^2+(-4)^2}}\right|^2$$

$$\Rightarrow \quad x^2 + y^2 = \frac{(3x-4y+2)^2}{25}$$

$$\Rightarrow \quad 25(x^2+y^2) = (3x-4y+2)^2$$

$$\Rightarrow \quad 25x^2 + 25y^2 = 9x^2 + 16y^2 + 4 - 24xy + 12x - 16y$$

$$\Rightarrow \quad 16x^2 + 9y^2 + 24xy - 12x + 16y - 4 = 0$$

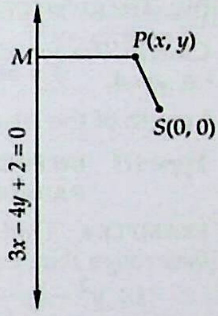

Fig. 25.16

This is the required equation of the parabola.

EXAMPLE 2 *Find the equation of the parabola whose focus is at $(-1, -2)$ and the directrix the line $x - 2y + 3 = 0$.*

SOLUTION Let $P(x, y)$ be any point on the parabola whose focus is $S(-1, -2)$ and the directrix $x - 2y + 3 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x - 2y + 3 = 0$. Then, by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x+1)^2 + (y+2)^2 = \left|\frac{x-2y+3}{\sqrt{1+4}}\right|^2$$

$$\Rightarrow \quad 5\left\{(x+1)^2 + (y+2)^2\right\} = (x-2y+3)^2$$

$$\Rightarrow \quad 5(x^2+y^2+2x+4y+5) = (x^2+4y^2+9-4xy+6x-12y)$$

$$\Rightarrow \quad 4x^2 + y^2 + 4xy + 4x + 32y + 16 = 0$$

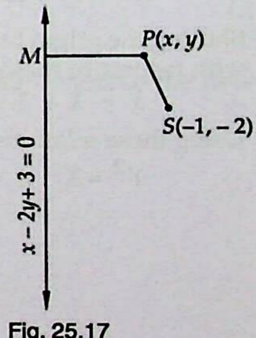

Fig. 25.17

This is the equation of the required parabola.

Type II **ON FINDING THE FOCUS, DIRECTRIX, LATUS-RECTUM, AXIS ETC. FOR A GIVEN PARABOLA IN ONE OF THE STANDARD FORMS**

EXAMPLE 3 *For the following parabolas find the coordinates of the foci, the equations of the directrices and the lengths of the latus-rectum:*

(i) $y^2 = 8x$ (ii) $x^2 = 6y$ (iii) $y^2 = -12x$ (iv) $x^2 = -16y$

SOLUTION (i) The given parabola $y^2 = 8x$ is of the form $y^2 = 4ax$, where $4a = 8$ i.e. $a = 2$.

The coordinates of the focus are $(a, 0)$ i.e. $(2, 0)$ and the equation of the directrix is $x = -a$ i.e. $x = -2$.

Length of the latus-rectum $= 4a = 8$.

(ii) The given parabola $x^2 = 6y$ is of the form $x^2 = 4ay$, where $4a = 6$ i.e. $a = 3/2$.

Clearly, the coordinates of the focus are $(0, a) = (0, 3/2)$ and the equation of the directrix is $y = -a$ i.e. $y = -3/2$.

Length of the latus-rectum $= 4a = 6$.

(iii) The given parabola $y^2 = -12x$ is of the form $y^2 = -4ax$, where $4a = 12$ i.e. $a = 3$.

Clearly, the coordinates of the focus are $(-a, 0) = (-3, 0)$ and the equation of the directrix is $x = a$ i.e. $x = 3$.

Length of the latus-rectum $= 4a = 12$.

(iv) The given parabola is of the form $x^2 = -4ay$, where $4a = 16$ i.e. $a = 4$.

Clearly, the coordinates of its focus are $(0, -a) = (0, -4)$ and the equation of the directrix is $y = a$ i.e. $y = 4$.

Length of the latus-rectum $= 4a = 16$.

Type III **ON FINDING THE VERTEX, FOCUS, AXIS, DIRECTRIX, LATUS-RECTUM ETC. OF THE PARABOLAS REDUCIBLE TO ONE OF THE FOUR STANDARD FORMS**

EXAMPLE 4 *Find the vertex, axis, focus, directrix, latus-rectum of the following parabolas. Also, draw their rough sketches:*

(i) $y^2 - 8y - x + 19 = 0$ (ii) $4y^2 + 12x - 20y + 67 = 0$

(iii) $y = x^2 - 2x + 3$ (iv) $x^2 + 2y - 3x + 5 = 0$

SOLUTION (i) The given equation is

$$y^2 - 8y - x + 19 = 0$$

$$\Rightarrow \quad y^2 - 8y = x - 19$$

$$\Rightarrow \quad y^2 - 8y + 16 = x - 19 + 16 \Rightarrow (y-4)^2 = (x-3) \qquad \text{...(i)}$$

Shifting the origin to the point (3, 4) without rotating the axes and denoting the new coordinates with respect to these new axes by X and Y, we have

$$x = X + 3, \quad y = Y + 4 \qquad \text{...(ii)}$$

Using these relations, equation (i) reduces to

$$Y^2 = X \qquad \text{...(iii)}$$

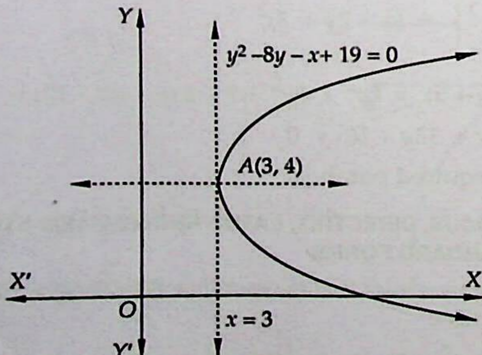

Fig. 25.18

This is of the form $Y^2 = 4aX$. On comparing, we get $4a = 1$ or, $a = \frac{1}{4}$.

Vertex: The coordinates of the vertex with respect to the new axes are $(X = 0, Y = 0)$.

So, the coordinates of the vertex with respect to the old axes are

(3, 4) [Putting $X = 0, Y = 0$ in (ii)]

Axis: The equation of the axis of the parabola (iii) with respect to the new axes is $Y = 0$.

So, the equation of the axis with respect to the old axes is $y = 4$. [Putting $Y = 0$ in (ii)]

Focus: The coordinates of the focus with respect to the new axes are $(X = a, Y = 0)$

i.e., $(X = 1/4, Y = 0)$

So, the coordinates of the focus with respect to the old axes are

(13/4, 4) [Putting $X = 1/4$ and $Y = 0$ in (ii)]

Directrix: The equation of the directrix with respect to the new axes is $X = -a$ i.e., $X = -1/4$.

So, the equation of the directrix with respect to the old axes is

$$x = -\frac{1}{4} + 3 \Rightarrow x = \frac{11}{4} \qquad \text{[Putting } X = -\frac{1}{4} \text{ in (ii)]}$$

Latus-rectum: The length of the latus-rectum of the parabola (iii) is equal to $4a = 1$.

(ii) The given equation is

$$4y^2 + 12x - 20y + 67 = 0$$

$$\Rightarrow \quad y^2 + 3x - 5y + \frac{67}{4} = 0$$

$$\Rightarrow \quad y^2 - 5y = -3x - \frac{67}{4}$$

$$\Rightarrow \quad y^2 - 5y + \left(\frac{5}{2}\right)^2 = -3x - \frac{67}{4} + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow \quad \left(y - \frac{5}{2}\right)^2 = -3x - \frac{42}{4}$$

$$\Rightarrow \quad \left(y - \frac{5}{2}\right)^2 = -3\left(x + \frac{7}{2}\right) \qquad \text{...(i)}$$

Shifting the origin to the point (–7/2, 5/2) without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we have

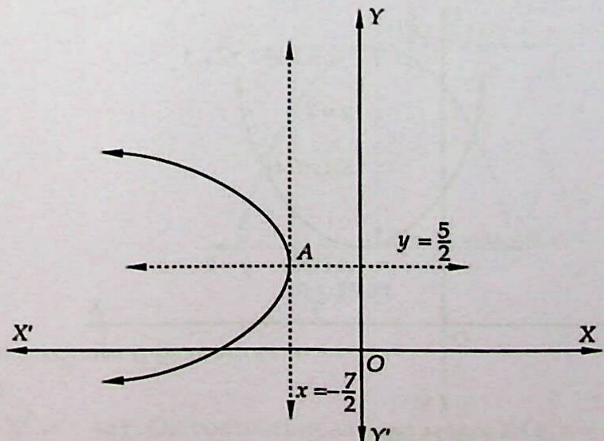

Fig. 25.19

$$x = X - \frac{7}{2}, \quad y = Y + \frac{5}{2} \qquad \text{...(ii)}$$

Using these relations, equation (i) reduces to

$$Y^2 = -3X \qquad \text{...(iii)}$$

This is of the form $Y^2 = -4aX$.

On comparing, we get: $4a = 3 \Rightarrow a = 3/4$.

Vertex: The coordinates of the vertex with respect to the new axes are $(X = 0, Y = 0)$.

So, the coordinates of the vertex with respect to the old axes are

$(-7/2, 5/2)$ [Putting $X = 0$, $Y = 0$ in (ii)]

Axis: The equation of the axis of the parabola with respect to the new axis is $Y = 0$. So, the equation of the axis with respect to the old axes is

$y = 5/2$ [Putting $Y = 0$ in (ii)]

Focus: The coordinates of the focus with respect to the new axes are $(X = -a,\ Y = 0)$ i.e. $(X = -3/4,\ Y = 0)$. So, the coordinates of the focus with respect to the old axes are

$(-17/4, 5/2)$ [Putting $X = -3/4$ and $Y = 0$ in (ii)]

Directrix: The equation of the directrix with respect to the new axes is $X = a$ *i.e.* $X = \frac{3}{4}$.

So, the equation of the directrix with respect to the old axes is $x = -\frac{11}{4}$ [Putting $X = 3/4$ in (ii)]

Latus-rectum: The length of the latus-rectum of the given parabola is $4a = 3$.

(iii) The given equation is

$$y = x^2 - 2x + 3$$

$$\Rightarrow \quad x^2 - 2x = y - 3$$

$$\Rightarrow \quad x^2 - 2x + 1 = y - 3 + 1$$

$$\Rightarrow \quad (x-1)^2 = y - 2 \qquad \text{...(i)}$$

Shifting the origin to the point (1, 2) without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we obtain

$$x = X + 1, \quad y = Y + 2 \qquad \text{...(ii)}$$

Using these relations, equation (i) reduces to

$$X^2 = Y \qquad \text{...(iii)}$$

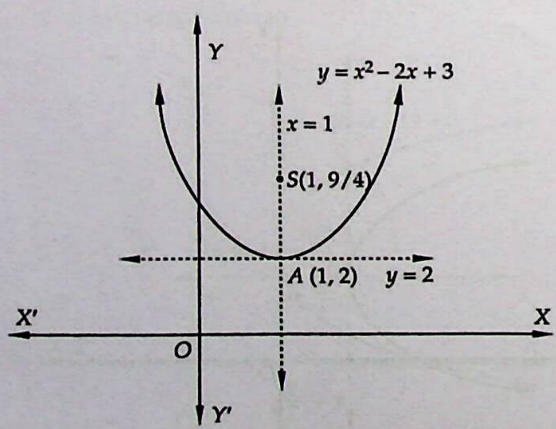

Fig. 25.20

This is of the form $X^2 = 4aY$. On comparing, we get

$4a = 1$ i.e. $a = 1/4$.

Vertex: The coordinates of the vertex with respect to the new axes are $(X = 0, Y = 0)$. So, the coordinates of the vertex with respect to the old axes are $(1, 2)$ [Putting $X = 0, Y = 0$ in (ii)]

Axis: The equation of the axis of the parabola with respect to the new axes is $X = 0$.

So, the equation of the axis with respect to the old axes is $x = 1$. [Putting $X = 0$ in (ii)]

Focus: The coordinates of the focus with respect to the new axes are $(X = 0, Y = a)$ i.e.

$(X = 0, Y = 1/4)$

So, the coordinates of the focus S with respect to the old axes are

$(1, 9/4)$ [Putting $X = 0, Y = \frac{1}{4}$ in (ii)]

Directrix: The equation of the directrix with respect to the new axes is $Y = -a$ i.e. $Y = -1/4$.

So, the equation of the directrix with respect to the old axes is

$y = -\frac{1}{4} + 2$ or $y = \frac{7}{4}$ [Putting $Y = -\frac{1}{4}$ in (ii)]

Latus-rectum: The length of the latus-rectum of the given parabola is equal to $4a = 1$.

(iv) The given equation is

$$x^2 + 2y - 3x + 5 = 0$$

$$\Rightarrow \quad x^2 - 3x = -2y - 5$$

$$\Rightarrow \quad x^2 - 3x + \frac{9}{4} = -2y - 5 + \frac{9}{4}$$

$$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 = -2\left(y + \frac{11}{8}\right) \quad \text{...(i)}$$

Shifting the origin to the point $(3/2, -11/8)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we obtain

$$x = X + \frac{3}{2}, \quad y = Y - \frac{11}{8} \quad \text{...(ii)}$$

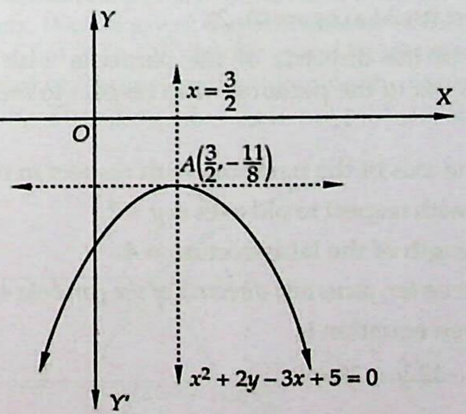

Fig. 25.21

Using these relations, equation (i) reduces to

$$X^2 = -2Y \quad \text{...(iii)}$$

This is of the form $X^2 = -4aY$. On comparing, we get : $4a = 2$ i.e. $a = 1/2$.

Vertex: The coordinates of the vertex with respect to the new axes are ($X = 0$, $Y = 0$). So, the coordinates of the vertex with respect to the old axes are:

$(3/2, -11/8)$ [Putting $X = 0$, $Y = 0$ in (ii)]

Axis: The equation of the axis of the parabola with respect to the new axes is $X = 0$.

So, the equation of the axis with respect to the old axes is $x = \frac{3}{2}$ [Putting $X = 0$ in (ii)]

Focus: The coordinates of the focus with respect to the new axes are $(X = 0, Y = -a)$ i.e. $(X = 0, Y = -1/2)$. So, the coordinates of the focus with respect to the old axes are

$(3/2, -15/8)$ [Putting $X = 0$, $Y = -1/2$ in (ii)]

Directrix The equation of the directrix with respect to the new axes is $Y = a$ i.e. $Y = 1/2$. So, the equation of the directrix with respect to the old axes is $y = -\frac{7}{8}$ [Putting $Y = 1/2$ in (ii)]

Latus-rectum The length of the latus-rectum of the given parabola is equal to $4a = 2$.

EXAMPLE 5 *Find the vertex, focus, directrix, axis and latus-rectum of the parabola* $y^2 = 4x + 4y$.

SOLUTION The given equation is

$$y^2 = 4x + 4y$$

$$\Rightarrow \quad y^2 - 4y = 4x \Rightarrow y^2 - 4y + 4 = 4x + 4 \Rightarrow (y-2)^2 = 4(x+1) \qquad ...(i)$$

Shifting the origin to the point $(-1, 2)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we have

$$x = X + (-1), \; y = Y + 2 \qquad ...(ii)$$

Using these relations equation (i), reduces to

$$Y^2 = 4X \qquad ...(iii)$$

This is of the form $Y^2 = 4aX$. On comparing, we get: $4a = 4 \Rightarrow a = 1$.

Vertex: The coordinates of the vertex with respect to new axes are $(X = 0, Y = 0)$. So, coordinates of the vertex with respect to old axes are $(-1, 2)$. [Putting $X = 0$, $Y = 0$ in (ii)]

Focus: The coordinates of the focus with respect to new axes are $(X = 1, Y = 0)$. So, coordinates of the focus with respect to old axes are $(0, 2)$. [Putting $X = 1$, $Y = 0$ in (ii)]

Directrix: Equation of the directrix of the parabola with respect to new axes is $X = -1$. So, equation of the directrix of the parabola with respect to old axes is

$x = -2$ [Putting $X = -1$ in (ii)]

Axis: Equation of the axis of the parabola with respect to new axes is $Y = 0$.

So, equation of axis with respect to old axes is $y = 2$. [Putting $Y = 0$ in (ii)]

Latus-rectum: The length of the latus-rectum = 4.

EXAMPLE 6 *Find the vertex, focus and directrix of the parabola* $4y^2 + 12x - 12y + 39 = 0$.

SOLUTION The given equation is

$$4y^2 + 12x - 12y + 39 = 0$$

$$\Rightarrow \quad 4y^2 - 12y = -12x - 39$$

$$\Rightarrow \quad 4(y^2 - 3y) = -12x - 39$$

$$\Rightarrow \quad 4\left(y^2 - 3y + \frac{9}{4}\right) = -12x - 39 + 9$$

$$\Rightarrow \quad 4\left(y-\frac{3}{2}\right)^2 = -12\left(x+\frac{5}{2}\right)$$

$$\Rightarrow \quad \left(y-\frac{3}{2}\right)^2 = -3\left(x+\frac{5}{2}\right) \qquad \text{...(i)}$$

Shifting the origin to the point $(-5/2, 3/2)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we obtain

$$x = X + \left(-\frac{5}{2}\right), \; y = Y + \frac{3}{2} \qquad \text{...(ii)}$$

Using these relations equation (i), reduces to

$$Y^2 = -3X \qquad \text{...(iii)}$$

This is of the form $Y^2 = -4aX$. On comparing, we get: $a = \frac{3}{4}$.

Vertex: The coordinates of the vertex with respect to new axes are $(X = 0, Y = 0)$. So, coordinates of the vertex with respect to old axes are $(-5/2, 3/2)$. [Putting $X = 0$, $Y = 0$ in (ii)]

Focus: The coordinates of the focus of the parabola with respect to new axes are

$$\left(X = -\frac{3}{4}, Y = 0\right).$$

So, coordinates of the focus with respect to old axes are

$$\left(-\frac{13}{4}, \frac{3}{2}\right) \qquad \left[\text{Putting } X = -\frac{3}{4}, Y = 0 \text{ in (ii)}\right]$$

Directrix: The equation of the directrix of the parabola with respect to new axes is $X = \frac{3}{4}$. So, equation of the directrix of the parabola with respect to old axes is

$$x = -\frac{7}{4} \qquad [\text{Putting } X = 3/4 \text{ in (ii)}]$$

Type IV **ON FINDING THE EQUATION OF A PARABOLA WHEN ITS FOCUS AND VERTEX ARE GIVEN**

EXAMPLE 7 *Find the equation of the parabola with vertex (2, – 3) and focus (0, 5).*

SOLUTION In order to find the equation of a parabola, we need to know the coordinates of its focus and the equation of the directrix. We are given the coordinates of focus and vertex. So, we require the equation of the directrix. Let $Z(x_1, y_1)$ be the point of intersection of axis and the directrix. The vertex A is the mid-point of the line segment joining the focus S and the point Z of intersection of the axis and directrix. Therefor, $(2, -3)$ is the mid-point of the line segment joining $S(0, 5)$ and $Z(x_1, y_1)$.

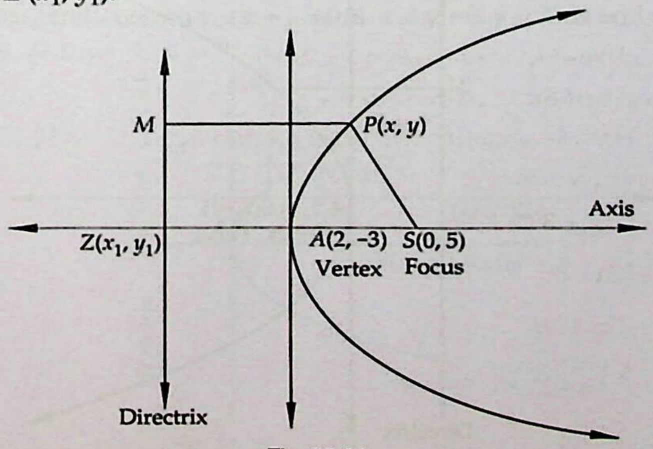

Fig. 25.22

$\therefore \quad \dfrac{x_1 + 0}{2} = 2$ and $\dfrac{y_1 + 5}{2} = -3 \Rightarrow x_1 = 4,\ y_1 = -11.$

Thus, the directrix meets the axis at $Z(4, -11)$.

Let m_1 be the slope of AS. Then,

$$m_1 = \frac{5+3}{0-2} = -4$$

Let m_2 be the slope of the directrix. Since directrix is perpendicular to AS.

$$\therefore \quad m_1 m_2 = -1 \Rightarrow m_2 = -\frac{1}{m_1} = \frac{1}{4}.$$

Thus, the directrix passes through the point $Z(4, -11)$ and has slope $1/4$. Therefore, the equation of the directrix is

$$y + 11 = \frac{1}{4}(x-4) \text{ or, } x - 4y - 48 = 0.$$

Let $P(x, y)$ be any point on the required parabola, and let PM be the length of the perpendicular from P on the directrix. Then,

$$SP = PM \qquad \text{[By definition]}$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-0)^2 + (y-5)^2 = \left|\frac{x - 4y - 48}{\sqrt{1^2 + (-4)^2}}\right|^2$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 170y + 425 = x^2 + 16y^2 + 2304 - 8xy - 96x + 384y$$

$\Rightarrow \quad 16x^2 + y^2 + 8xy + 96x - 554y - 1879 = 0$ which is the required equation of the parabola.

EXAMPLE 8 *Find the equation of the parabola whose focus is $(1, -1)$ and whose vertex is $(2, 1)$. Also, find its axis and latus-rectum.*

SOLUTION In order to find the equation of a parabola, we require the coordinates of its focus and the equation of the directrix. Here, we are given the coordinates of the focus and vertex. So, we require the equation of the directrix. Let $Z(x_1, y_1)$ be the coordinates of the point of intersection of the axis and the directrix. Then, the vertex $A(2, 1)$ is the mid-point of the line segment joining $Z(x_1, y_1)$ and the focus $S(1, -1)$.

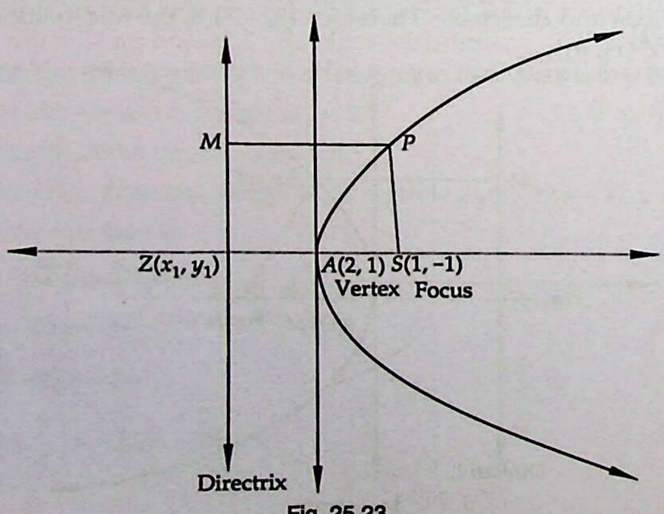

Fig. 25.23

$\therefore \quad \dfrac{x_1 + 1}{2} = 2$ and $\dfrac{y_1 + (-1)}{2} = 1 \Rightarrow x_1 = 3,\ y_1 = 3.$

Thus, the directrix meets the axis at $Z(3, 3)$.

Let m_1 be the slope of the axis. Then,

$$m_1 = \text{(Slope of the line joining the focus } S \text{ and the vertex } A) = \frac{1+1}{2-1} = 2 \quad \text{...(i)}$$

$\therefore$ Slope of the directrix $= -\dfrac{1}{2}$ [$\because$ Directrix is perpendicular to the axis]

Thus, the directrix passes through (3, 3) and has slope $-1/2$. So its equation is

$$y - 3 = -\frac{1}{2}(x - 3) \text{ or, } x + 2y - 9 = 0$$

Let $P(x, y)$ be a point on the parabola. Then,

Distance of P from the focus = Distance of P from the directrix

$$\Rightarrow \quad \sqrt{(x-1)^2 + (y+1)^2} = \left| \frac{x + 2y - 9}{\sqrt{1^2 + 2^2}} \right|$$

$$\Rightarrow \quad (x-1)^2 + (y+1)^2 = \frac{(x + 2y - 9)^2}{5}$$

$$\Rightarrow \quad 5x^2 + 5y^2 - 10x + 10y + 10 = x^2 + 4y^2 + 81 + 4xy - 18x - 36y$$

$$\Rightarrow \quad 4x^2 + y^2 - 4xy + 8x + 46y - 71 = 0,$$ which is the required equation of the parabola.

The axis passes through the focus $(1, -1)$, and its slope is $m_1 = 2$. Therefore, equation of the axis is

$$y + 1 = 2(x - 1) \text{ or, } 2x - y - 3 = 0$$

Now,

Latus-rectum = 2 (Length of the perpendicular from the focus on the directrix)

$= 2$ [Length of the perpendicular from $(1, -1)$ on $x + 2y - 9 = 0$]

$$= 2\left| \frac{1 - 2 - 9}{\sqrt{1 + 4}} \right| = 2 \times \frac{10}{\sqrt{5}} = 4\sqrt{5}.$$

Type V ON FINDING THE EQUATION OF A PARABOLA WHEN ITS VERTEX AND DIRECTRIX ARE GIVEN

EXAMPLE 9 *Find the equation of the parabola whose vertex is at (2, 1) and the directrix is $x = y - 1$.*

SOLUTION In order to find the equation of a parabola, its focus and directrix are required. Here, we are given it directrix and vertex. So, we first find its focus which lies on the axis. The axis of the parabola is a line perpendicular to the directrix and passing through the vertex. The equation of a line perpendicular to $x - y + 1 = 0$ is $x + y + \lambda = 0$. This will pass through (2, 1), if

$$2 + 1 + \lambda = 0 \Rightarrow \lambda = -3.$$

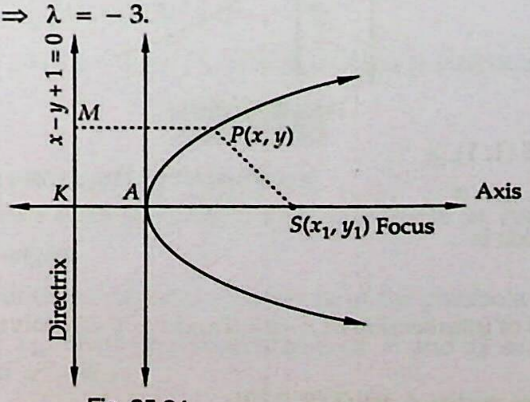

Fig. 25.24

Putting $\lambda = -3$ in $x + y + \lambda = 0$, we obtain

$$x + y - 3 = 0 \quad \text{...(i)}$$

as the axis of the parabola.

The equation of the directrix is

$$x - y + 1 = 0 \quad \text{...(ii)}$$

Solving (i) and (ii), we get $x = 1, y = 2$. So, the coordinates of K are $(1, 2)$.

Let (x_1, y_1) be the coordinates of the focus S. Then, A is the mid-point of KS.

$$\therefore \quad \frac{x_1 + 1}{2} = 2 \text{ and } \frac{y_1 + 2}{2} = 1 \Rightarrow x_1 = 3 \text{ and } y_1 = 0$$

So, the coordinates of the focus S are $(3, 0)$.

Let $P(x, y)$ be a point on the parabola. Then,

$$PS = PM$$

$$\Rightarrow \quad PS^2 = PM^2$$

$$\Rightarrow \quad (x-3)^2 + (y-0)^2 = \left|\frac{x - y + 1}{\sqrt{1^2 + (-1)^2}}\right|^2$$

$$\Rightarrow \quad 2(x^2 + y^2 - 6x + 9) = x^2 + y^2 + 1 - 2xy + 2x - 2y$$

$\Rightarrow \quad x^2 + y^2 - 14x + 2y + 2xy + 17 = 0$, which is the required equation of the parabola.

EXAMPLE 10 *Find the equation of the parabola whose focus is (1, 1) and tangent at the vertex is* $x + y = 1$.

SOLUTION Here, we are given the coordinates of the focus and the equation of the tangent at the vertex. To find the equation of a parabola, we require the coordinates of its focus and the equation of the directrix. So, we first find the equation of the directrix of the parabola from the given components. Let S be the focus and A be the vertex of the parabola. Let K be the point of intersection of the axis and directrix. Since axis is a line passing through S (1, 1) and perpendicular to $x + y = 1$. So, let the equation of the axis be $x - y + \lambda = 0$.

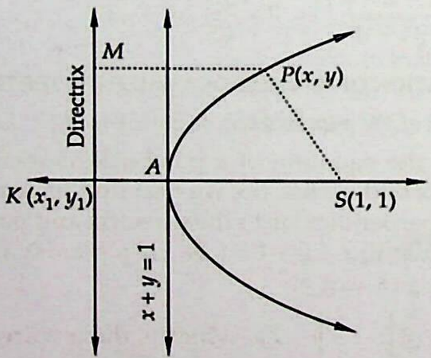

Fig. 25.25

This will pass through S (1, 1), if

$$1 - 1 + \lambda = 0 \Rightarrow \lambda = 0$$

So the equation of the axis is

$$x - y = 0 \quad \text{...(i)}$$

The vertex A is the point of intersection of $x - y = 0$ and $x + y = 1$. Solving these two equations, we get $x = 1/2$ and $y = 1/2$.

So, the coordinates of the vertex A are $(1/2, 1/2)$.

Let (x_1, y_1) be the coordinates of K. As A is the mid-point of SK.

$$\therefore \quad \frac{x_1+1}{2} = \frac{1}{2}, \ \frac{y_1+1}{2} = \frac{1}{2} \Rightarrow x_1 = 0, y_1 = 0$$

So, the coordinates of K are $(0, 0)$. Since directrix is a line passing through $K(0, 0)$ and parallel to $x + y = 1$. Therefore, equation of the directrix is

$$y - 0 = -1(x - 0) \text{ or, } x + y = 0. \quad \text{...(ii)}$$

Let $P(x, y)$ be any point on the parabola. Then,

Distance of P from the focus S = [Distance of P from the directrix $x + y = 0$]

$$\Rightarrow \quad \sqrt{(x-1)^2 + (y-1)^2} = \left| \frac{x+y}{\sqrt{1^2+1^2}} \right|$$

$$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 2xy$$

$$\Rightarrow \quad x^2 + y^2 - 2xy - 4x - 4y + 4 = 0,$$ which is the required equation of the parabola.

EXAMPLE 11 *Find the equation of the parabola whose latus-rectum is 4 units, axis is the line $3x + 4y - 4 = 0$ and the tangent at the vertex is the line $4x - 3y + 7 = 0$.*

SOLUTION Let $P(x, y)$ be any point on the parabola and let PM and PN be perpendiculars from P on the axis and tangent at the vertex respectively. Then,

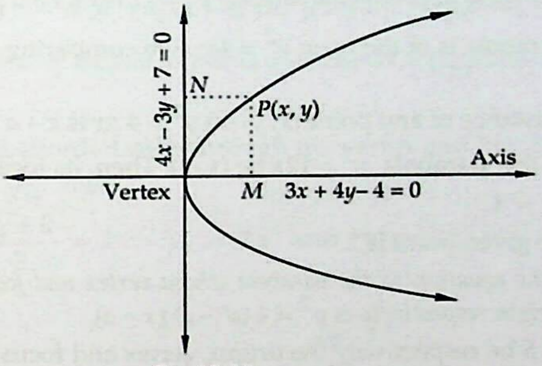

Fig. 25.26

$$PM^2 = (\text{Latusrectum})(PN)$$

$$\Rightarrow \quad \left| \frac{3x+4y-4}{\sqrt{3^2+4^2}} \right|^2 = 4 \left| \frac{4x-3y+7}{\sqrt{4^2+(-3)^2}} \right|$$

$$\Rightarrow \quad (3x+4y-4)^2 = 20(4x-3y+7),$$ which is the required equation of the parabola.

LEVEL-2

Type VI **MISCELLANEOUS PROBLEMS ON PARABOLA**

EXAMPLE 12 *A double ordinate of the parabola $y^2 = 4ax$ is of length $8a$. Prove that the lines from the vertex to its ends are at right angles.*

SOLUTION Let PQ be the double ordinate of length $8a$ of the parabola $y^2 = 4ax$. Then, $PR = QR = 4a$. Let $AR = x_1$. Then, the coordinates of P and Q are $(x_1, 4a)$ and $(x_1, -4a)$ respectively. Since P lies on $y^2 = 4ax$.

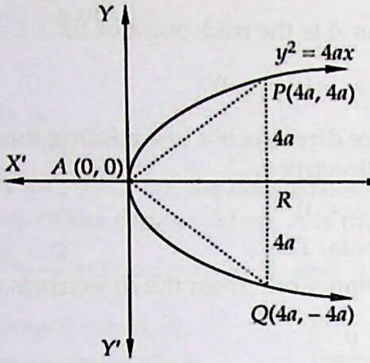

Fig. 25.27

$\therefore \qquad (4a)^2 = 4ax_1 \Rightarrow x_1 = 4a.$

So, coordinates of P and Q are $(4a, 4a)$ and $(4a, -4a)$ respectively. Also, the coordinates of the vertex A are $(0, 0)$.

$\therefore \qquad m_1 = \text{Slope of } AP = \dfrac{4a-0}{4a-0} = 1, \text{ and, } m_2 = \text{Slope of } AQ = \dfrac{-4a-0}{4a-0} = -1$

Clearly, $m_1 m_2 = -1$. Hence, AP is perpendicular to AQ.

EXAMPLE 13 *The focal distance of a point on the parabola $y^2 = 12x$ is 4. Find the abscissa of this point.*

SOLUTION The given parabola is of the form $y^2 = 4ax$. On comparing, we obtain $4a = 12$ i.e. $a = 3$.

We know that the focal distance of any point (x, y) on $y^2 = 4\,ax$ is $x + a$.

Let the given point on the parabola $y^2 = 12x$ be (x, y). Then, its focal distance is $x + 3$.

$\therefore \qquad x + 3 = 4 \Rightarrow x = 1.$

Hence, the abscissa of the given point is 1.

EXAMPLE 14 *Prove that the equation to the parabola whose vertex and focus are on the x-axis at a distance a and a′ from the origin respectively is $y^2 = 4\,(a' - a)\,(x - a)$.*

SOLUTION Let O, A and S be respectively the origin, vertex and focus of the parabola. Then, $OA = a$, $OS = a'$. Therefore, the coordinates of S are $(a', 0)$. Let KK' be the directrix of the required parabola. Suppose SA produced meets the directrix at Z. Let the coordinates of Z be (x_1, y_1). Then,

$$\frac{x_1 + a'}{2} = a \text{ and } \frac{y_1 + 0}{2} = 0 \qquad [\because A \text{ is the mid-point of } SZ]$$

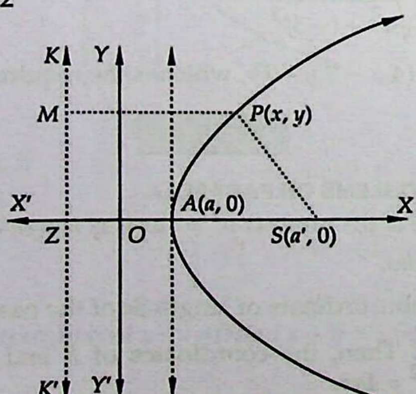

Fig. 25.28

$\Rightarrow \quad x_1 = 2a - a'$ and $y_1 = 0$

So, the equation of the directrix KK' is $x = x_1$ i.e. $x = 2a - a'$.

Let $P(x, y)$ be any point on the parabola. Then,

$SP = PM$ [By def.]

$$\Rightarrow \sqrt{(x-a')^2 + (y-0)^2} = \left|\frac{x - 2a + a'}{\sqrt{1+0}}\right|$$

$$\Rightarrow (x-a')^2 + y^2 = (x - 2a + a')^2$$

$$\Rightarrow (x-a')^2 + y^2 = [(x-a') - 2(a-a')]^2$$

$$\Rightarrow (x-a')^2 + y^2 = (x-a')^2 + 4(a-a')^2 - 4(x-a')(a-a')$$

$$\Rightarrow y^2 = 4(a-a')\{(a-a') - (x-a')\}$$

$$\Rightarrow y^2 = 4(a'-a)(x-a).$$

ALITER The parabola has its vertex at $(a, 0)$ and the length of its Latus-rectum = 4 (Distance between foucs and vertex) = $4(a' - a)$. The axis is along OX.

So, its equation is $(y-0)^2 = 4(a'-a)(x-a)$ or, $y^2 = 4(a'-a)(x-a)$

EXAMPLE 15 *Find the locus of the middle points of all chords of the parabola $y^2 = 4ax$ which are drawn through the vertex.*

SOLUTION Let OA be a chord, drawn through the vertex and $P(h, k)$ be its mid-point. Let the coordinates of A be (x_1, y_1). Then,

$$\frac{x_1 + 0}{2} = h, \ \frac{y_1 + 0}{2} = k \Rightarrow x_1 = 2h \text{ and } y_1 = 2k$$

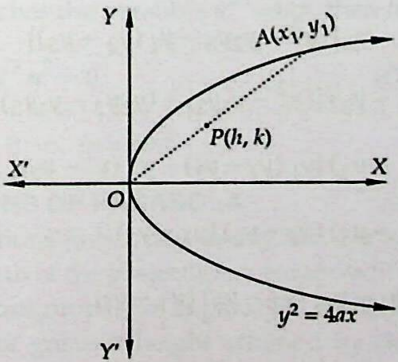

Fig. 25.29

So, the coordinates of A are $(2h, 2k)$. Since A lies on $y^2 = 4ax$.

$\therefore \quad (2k)^2 = 4a(2h) \Rightarrow k^2 = 2ah$

Hence, the locus of (h, k) is $y^2 = 2ax$.

EXAMPLE 16 *An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ whose vertex is at the vertex of the parabola. Find the length of its side.* **[NCERT EXEMPLAR]**

SOLUTION Let $AB = l$. Then,

$$AM = l \cos 30° = \frac{l\sqrt{3}}{2} \text{ and, } BM = l \sin 30° = \frac{l}{2}.$$

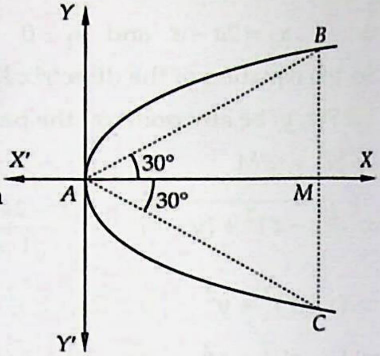

Fig. 25.30

So, the coordinates of B are $\left(\frac{\sqrt{3}l}{2}, \frac{l}{2}\right)$. Since, B lies on $y^2 = 4ax$. So, coordinates of B satisfy $y^2 = 4ax$.

$$\therefore \quad \frac{l^2}{4} = 4a\left(\frac{l\sqrt{3}}{2}\right) \Rightarrow l = 8a\sqrt{3}$$

EXAMPLE 17 *If y_1, y_2, y_3 be the ordinates of a vertices of the triangle inscribed in a parabola $y^2 = 4ax$, then show that the area of the triangle is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$.*

SOLUTION Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC. Since (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on the parabola. Therefore,

$$y_1^2 = 4ax_1, \ y_2^2 = 4ax_2 \text{ and } y_3^2 = 4ax_3 \Rightarrow x_1 = \frac{y_1^2}{4a}, \ x_2 = \frac{y_2^2}{4a} \text{ and } x_3 = \frac{y_3^2}{4a}$$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\left[\frac{y_1^2}{4a}(y_2 - y_3) + \frac{y_2^2}{4a}(y_3 - y_1) + \frac{y_3^2}{4a}(y_1 - y_2)\right]$$

$$= \frac{1}{8a}[y_1^2(y_2 - y_3) + (y_2^2 y_3 - y_2 y_3^2) - y_1(y_2^2 - y_3^2)]$$

$$= \frac{1}{8a}[y_1^2(y_2 - y_3) + y_2 y_3(y_2 - y_3) - y_1(y_2^2 - y_3^2)]$$

$$= \frac{1}{8a}(y_2 - y_3)[y_1^2 + y_2 y_3 - y_1(y_2 + y_3)]$$

$$= \frac{1}{8a}(y_2 - y_3)[(y_1^2 - y_1 y_2) + (y_2 y_3 - y_1 y_3)]$$

$$= \frac{1}{8a}(y_2 - y_3)[y_1(y_1 - y_2) - y_3(y_1 - y_2)]$$

$$= \frac{1}{8a}(y_2 - y_3)(y_1 - y_2)(y_1 - y_3)$$

$$= -\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$$

$$\text{Hence, Area of } \Delta ABC = \frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$$

EXAMPLE 18 *PQ is a double ordinate of a parabola $y^2 = 4ax$. Find the locus of its points of trisection.*

SOLUTION Let R and S be the points of trisection of the double ordinates PQ. Let (h, k) be the coordinates of R. Then, $L = h$ and $RL = k$.

$$\therefore \quad RS = RL + LS = k + k = 2k.$$

$$\Rightarrow \quad PR = RS = SQ = 2k$$

$$\Rightarrow \quad LP = LR + RP = k + 2k = 3k$$

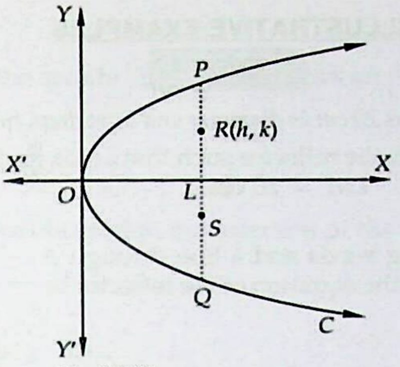

Fig. 25.31

Thus, the coordinates of P are $(h, 3k)$. Since $(h, 3k)$ lies on $y^2 = 4ax$.

$\therefore \quad 9k^2 = 4ah$

Hence, the locus of (h, k) is $9y^2 = 4ax$.

EXAMPLE 19 *If the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$, prove that $ln = am^2$*

[NCERT EXEMPLAR]

SOLUTION The x-coordinates of the points of intersection of the line $lx + my + n = 0$ or $y = -\left(\frac{lx+n}{m}\right)$ and the parabola $y^2 = 4ax$ are roots of the equation

$$\left\{-\left(\frac{lx+n}{m}\right)\right\}^2 = 4ax \qquad \left[\text{On eliminating } y \text{ between } y = -\left(\frac{lx+n}{m}\right) \text{ and } y^2 = 4ax\right]$$

or, $\quad l^2x^2 + 2x(ln - 2am^2) + n^2 = 0$

If the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$, then this equation has equal roots.

$$\therefore \quad 4\left(ln - 2am^2\right)^2 - 4l^2n^2 = 0 \qquad \text{[Putting discriminate equal to zero]}$$

$$\Rightarrow \quad -4a\,lm^2n + 4a^2m^4 = 0 \Rightarrow ln = am^2$$

25.9 SOME APPLICATIONS OF PARABOLA

Parabola has many applications in our day-to-day life. For example, if an object (projectile) is thrown in space, then the path of the projectle is a parabola. If we know the equation of the path of a projectile by using various properties of parabola studied in earlier sections, we can obtain many important results like greatest height attained by the projectile, its horizontal range reached etc.

Parabolic reflectors have the property that the light rays or sound waves coming parallel to its axis converge at the focus and then it reflects them parallel to the axis. Due to this property, parabolic reflectors are used in cars, automobiles, loudspeakers, solar cookers, telescopes etc.

If the roadway of a suspension bridge is loaded uniformly per horizontal metre, the suspension cable hangs in the form of arcs which closely approximate to parabolic arcs. Therefore, parabolic arcs are used in suspension cable bridge construction.

In this section, we shall discuss some examples on these applications

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *If a parabolic reflector is 20 cm in diameter and 5 cm deep, find its focus.* **[NCERT]**

SOLUTION Let LAM be the parabolic reflector such that LM is its diameter and AN is its depth. It is given that $AN = 5$ cm and $LM = 20$ cm.

$\therefore \quad LN = 10$ cm

Taking A as the origin, AX along x-axis and a line through A perpendicular to AX as y-axis, let the equation of the reflector be

$$y^2 = 4ax \quad \text{...(i)}$$

The point L has coordinates (5, 10) and lies on (i). Therefore,

$$10^2 = 4a \times 5 \Rightarrow a = 5$$

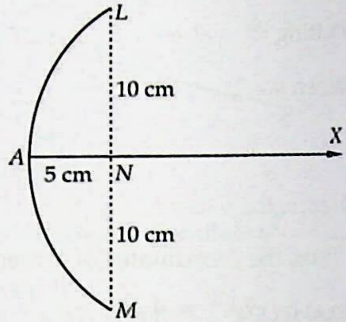

Fig. 25.32

So, the equation of the reflector is $y^2 = 20x$.

Its focus is at (5, 0) i.e. at point N.

Hence, the focus is at the mid-point of the given diameter.

EXAMPLE 2 *The focus of a parabolic mirror as shown in Fig. 25.33 is at a distance of 6 cm from its vertex. If the mirror is 20 cm deep, find the distance LM.* **[NCERT]**

SOLUTION Let the axis of the mirror be along the positive direction of x-axis and the vertex A be the origin.

Since the focus is at a distance of 6 cm from the vertex. Then, the coordinates of the focus are (6, 0). Therefore, the equation of the parabolic section is

$$y^2 = 24x \quad [\text{Putting } a = 6 \text{ in } y^2 = 4ax]$$

Since L (20, LN) lies on this parabola. Therefore,

$$LN^2 = 24 \times 20$$

$$\Rightarrow \quad LN = 4\sqrt{30}$$

$$\therefore \quad LM = 2LN = 8\sqrt{30} \text{ cm.}$$

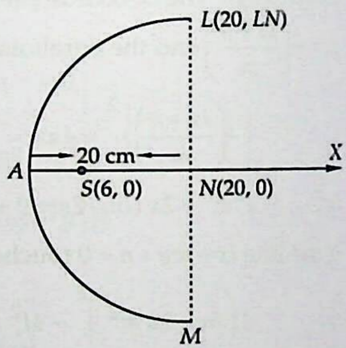

Fig. 25.33

EXAMPLE 3 *An arc is in the form of a parabola with its axis vertical. The arc is 10 m high and 5 m wide at the base. How wide is it 2m from the vertex of the parabola.* **[NCERT]**

SOLUTION Let the vertex of the parabola be at the origin and axis be along OY. Then, the equation of the parabola is

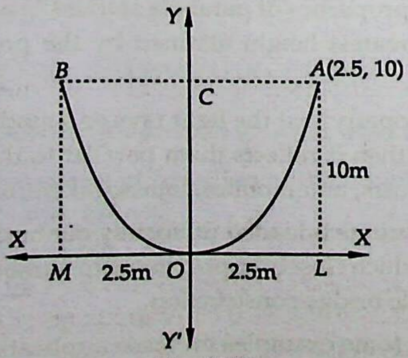

Fig. 25.34

$$x^2 = 4ay \quad \text{...(i)}$$

The coordinates of end A of the arc are (2.5, 10) and it lies on the parabola (i).

$$\therefore \quad (2.5)^2 = 4a \times 10$$

$$\Rightarrow \quad a = \frac{6.25}{40} = \frac{625}{4000} = \frac{5}{32}$$

Putting the value of a in (i), we obtain that the equation of the parabolic arc is $x^2 = \frac{5}{8}y$.

When $y = 2$, we obtain

$$x^2 = \frac{5}{8} \times 2 \Rightarrow x = \frac{\sqrt{5}}{2} \text{ m.}$$

Hence, the width of the arc at a height of 2 m from the vertex is $2 \times \frac{\sqrt{5}}{2}$ m $= \sqrt{5}$ m.

LEVEL-2

EXAMPLE 4 *The towers of a bridge, hung in the form of a parabola, have their tops 30 m above the roadway and are 200 metres apart. If the cable is 5 m above the roadway at the centre of the bridge, find the length of the vertical supporting cable 30 metres from the centre.* **[NCERT]**

SOLUTION Let CAB be the bridge and $X'OX$ be the roadway. Let A be the centre of the bridge. Taking $X'OX$ as x-axis and y-axis along OA, we find that the coordinates of A are (0, 5). Clearly, the bridge is in the shape of a parabola having its vertex at A (0, 5). Let its equation be

$$x^2 = 4a(y-5) \quad \text{...(i)}$$

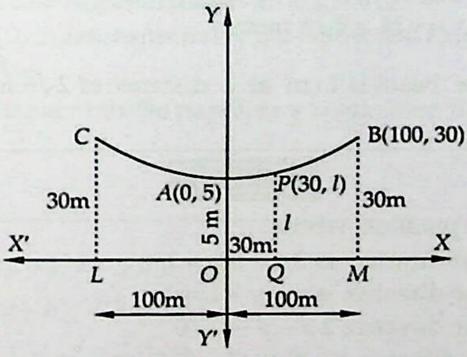

Fig. 25.35

It passes through B (100, 30).

$$\therefore \quad (100)^2 = 4a(30-5) \Rightarrow a = 100.$$

Putting the value of a in (i), we get

$$x^2 = 400(y-5) \quad \text{...(ii)}$$

Let l metres be the length of the vertical supporting cable 30 metres from the centre. Then, P (30, l) lies on (ii).

$$\therefore \quad 900 = 400(l-5) \Rightarrow l = \frac{9}{4} + 5 = \frac{29}{4} \text{ m.}$$

Hence, the length of the vertical supporting cable 30 metres from the centre of the bridge is $\frac{29}{4}$ m.

EXAMPLE 5 ***A beam is supported at its ends by supports which are 12 metres apart. Since the load is connected at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1 cm?*** **[NCERT]**

SOLUTION Let O be the centre of the beam in deflected position. Taking O as the origin OX as x-axis and OY as y-axis. The equation representing the parabolic shape of the beam is $x^2 = 4ay$. This passes through $Q\left(6, \frac{3}{100}\right)$.

$\therefore \quad 36 = 4a \times \frac{3}{100} \Rightarrow a = 300$ m

So, the equation of the curve representing deflected beam is $x^2 = 1200y$.

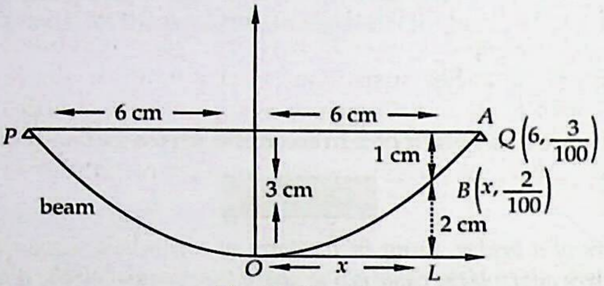

Fig. 25.36

Let the deflection of the beam be 1 cm $= \frac{1}{100}$ m at point B. Then, the coordinates of B are $\left(x, \frac{2}{100}\right)$, where $OL = x$. Since B lies on the parabola $x^2 = 1200\,y$.

$\therefore \quad x^2 = 1200 \times \frac{2}{100} \Rightarrow x = \sqrt{24} = 2\sqrt{6}$ metres.

Hence, the deflection of the beam is 1 cm at a distance of $2\sqrt{6}$ metres from the centre O.

EXERCISE 25.1

LEVEL-1

1. Find the equation of the parabola whose:
   (i) focus is (3, 0) and the directrix is $3x + 4y = 1$
   (ii) focus is (1, 1) and the directrix is $x + y + 1 = 0$
   (iii) focus is (0, 0) and the directrix $2x - y - 1 = 0$
   (iv) focus is (2, 3) and the directrix $x - 4y + 3 = 0$. [NCERT EXEMPLAR]
2. Find the equation of the parabola whose focus is the point (2, 3) and directrix is the line $x - 4y + 3 = 0$. Also, find the length of its latus-rectum.
3. Find the equation of the parabola, if
   (i) the focus is at $(-6, -6)$ and the vertex is at $(-2, 2)$
   (ii) the focus is at $(0, -3)$ and the vertex is at $(0, 0)$
   (iii) the focus is at $(0, -3)$ and the vertex is at $(-1, -3)$
   (iv) the focus is at $(a, 0)$ and the vertex is at $(a', 0)$
   (v) the focus is at (0, 0) and vertex is at the intersection of the lines $x + y = 1$ and $x - y = 3$.
4. Find the vertex, focus, axis, directrix and latus-rectum of the following parabolas
   (i) $y^2 = 8x$ (ii) $4x^2 + y = 0$ (iii) $y^2 - 4y - 3x + 1 = 0$
   (iv) $y^2 - 4y + 4x = 0$ (v) $y^2 + 4x + 4y - 3 = 0$ (vi) $y^2 = 8x + 8y$
   (vii) $4(y-1)^2 = -7(x-3)$ (viii) $y^2 = 5x - 4y - 9$ (ix) $x^2 + y = 6x - 14$
5. For the parabola $y^2 = 4px$ find the extremities of a double ordinate of length $8p$. Prove that the lines from the vertex to its extremities are at right angles.

6. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus-rectum. **[NCERT EXEMPLAR]**
7. Find the coordinates of the point of intersection of the axis and the directrix of the parabola whose focus is (3, 3) and directrix is $3x - 4y = 2$. Find also the length of the latus-rectum.
8. At what point of the parabola $x^2 = 9y$ is the abscissa three times that of ordinate ?
9. Find the equation of a parabola with vertex at the origin, the axis along x-axis and passing through (2, 3).
10. Find the equation of a parabola with vertex at the origin and the directrix, $y = 2$.
11. Find the equation of the parabola whose focus is (5, 2) and having vertex at (3, 2).
12. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest wire being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle. **[NCERT]**

LEVEL-2

13. Find the equations of the lines joining the vertex of the parabola $y^2 = 6x$ to the point on it which have abscissa 24. **[NCERT EXEMPLAR]**
14. Find the coordinates of points on the parabola $y^2 = 8x$ whose focal distance is 4. **[NCERT EXEMPLAR]**
15. Find the length of the line segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where the line-segment makes an angle θ to the x-axis. **[NCERT EXEMPLAR]**
16. If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola. **[NCERT EXEMPLAR]**
17. If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$, then find the value of m. **[NCERT EXEMPLAR]**

ANSWERS

1. (i) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$
   (ii) $x^2 + y^2 - 2xy - 6x - 6y + 3 = 0$
   (iii) $x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$
   (iv) $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$.
2. $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$, L.R. $= \dfrac{14}{\sqrt{17}}$.
3. (i) $(2x - y)^2 + 4(26x + 37y - 31) = 0$ (ii) $x^2 = -12y$
   (iii) $y^2 + 6y - 4x + 5 = 0$ (iv) $y^2 = -4(a' - a)(x - a')$
   (v) $(x + 2y)^2 + 40x - 20y - 100 = 0$.
4.

	vertex	*focus*	*axis*	*directrix*	*L.R.*
(i)	(0, 0)	(2, 0)	$y = 0$	$x = -2$	8
(ii)	(0, 0)	(0, −1/16)	x	$y = 1/16$	1/4
(iii)	(−1, 2)	(−1/4, 2)	$y = 2$	$x = -\frac{7}{4}$	3

(iv)	$(1, 2)$	$(0, 2)$	$y = 2$	$x = 2$	4
(v)	$(7/4, -2)$	$(3/4, -2)$	$y + 2 = 0$	$4x = 11$	4
(vi)	$(-2, 4)$;	$(0, 4)$	$y = 4$	$x + 4 = 0$	8
(vii)	$(3, 1)$	$(41/16, 1)$	$y = 1$	$x = 55/16$	7/4
(viii)	$(1, -2)$	$(9/4, -2)$	$y = -2$	$4x + 1 = 0.$	5
(ix)	$(3, -5)$	$(3, -21/4)$	$x = 3$	$4y + 19 = 0$	1

5. $(4p, 4\theta)$, $(4p, -4\theta)$ 6. 18 sq 7. $\left(\frac{18}{5}, \frac{11}{5}\right)$

8. $(3, 1)$ 9. $2y^2 = 9x$ 9. $2y^2 = 9x$ 10. $x^2 = -8y$

11. $y^2 - 4y - 8x + 28 = 0$ 12. 9.11 m (approx.) 13. $x \pm 2y = 0$

14. $(2, 4), (2, -4)$ 15. $4a \operatorname{cosec} \theta \cdot \cot \theta$ 16. $x^2 + 8y = 32$ 17. $m = 1$

HINTS TO SELECTED PROBLEMS

6. Required Area $= \frac{1}{2}\ (LL' \times OS) = \frac{1}{2} \times 12 \times 3 = 18$ sq. units

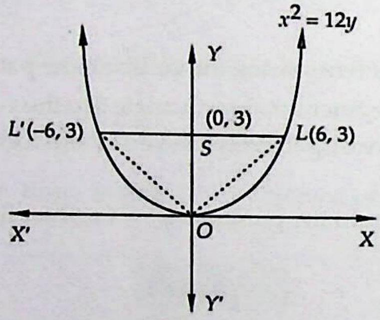

Fig. 25.37

12. Let $X'OX$ be the bridge and PAQ be the suspension cable. The suspension cable forms a parabola with vertex at $(0, 6)$. So, let the equation of the parabola formed by suspension cable be

$$(x - 0)^2 = 4a\,(y - 6) \quad \text{...(i)}$$

It passes through $P(-50, 30)$ and $Q(50, 30)$.

$$\therefore \quad 2500 = 4a\,(30 - 6) \Rightarrow 4a = \frac{2500}{24}$$

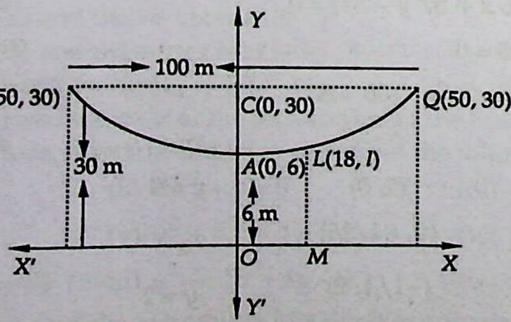

Fig. 25.38

Substituting this value of $4a$ in (i), we get

$$x^2 = \frac{2500}{24}(y-6) \qquad \text{...(ii)}$$

Let LM be the supporting wire attached at M which is 18 m from the middle O of the bridge. Let the coordinates of L be $(18, l)$. It lies on parabola (ii). Therefore,

$$18^2 = \frac{2500}{24}(l-6) \Rightarrow l-6 = 3.11 \Rightarrow l = 9.11 \text{ m.}$$

13. The parabola $y^2 = 6x$ is symmetric about x-axis. So, for a given abscissa there will be two points on the parabola as shown in Fig. 25.39. Let P and Q be two points on the parabola whose abscissa is 24. Let their coordinates be $(24, y_1)$ and $(24, -y_1)$ respectively.

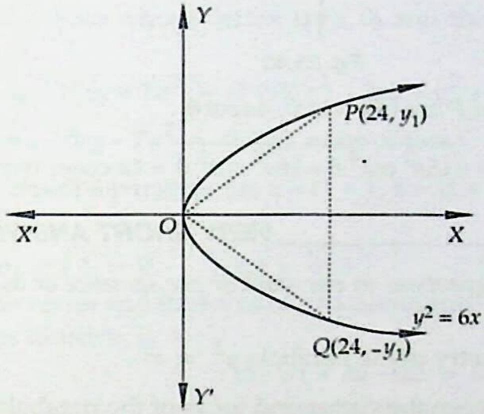

Fig. 25.39

Since $P(24, y_1)$ lies on $y^2 = 6x$.

$\therefore \quad y_1^2 = 6 \times 24 \Rightarrow y_1 = 12$

So, the coordinates of P and Q are $(24, 12)$ and $(24, -12)$ respectively.

The equations OP and OQ are

$$y - 0 = \frac{12-0}{24-0}(x-0) \text{ and } y - 0 = \frac{-12-0}{24-0}(x-0)$$

or, $\quad x = 2y$ and $x = -2y$ respectively.

14. Comparing $y^2 = 8x$ with $y^2 = 4ax$, we obtain $4a = 8$ or $a = 2$. The focal distance of any point $P(x, y)$ on $y^2 = 4ax$ is $a + x$. Therefore,

$$a + x = 4 \Rightarrow 2 + x = 4 \Rightarrow x = 2$$

Putting $x = 2$ in $y^2 = 8x$, we obtain $y = \pm 4$.

Hence, the coordinates of required points are $(2, 4)$ and $(2, -4)$.

15. Let $P(x, y)$ be a point on the parabola $y^2 = 4ax$ such that the segment OP makes an angle θ with x-axis. Then,

$$\tan\theta = \text{Slope of } OP \Rightarrow \tan\theta = \frac{y-0}{x-0} \Rightarrow y = x\tan\theta.$$

Since $P(x, y)$ lies on $y^2 = 4ax$. Therefore,

$$(x\tan\theta)^2 = 4ax \Rightarrow x = 4a\cot^2\theta \Rightarrow y = 4a\cot\theta$$

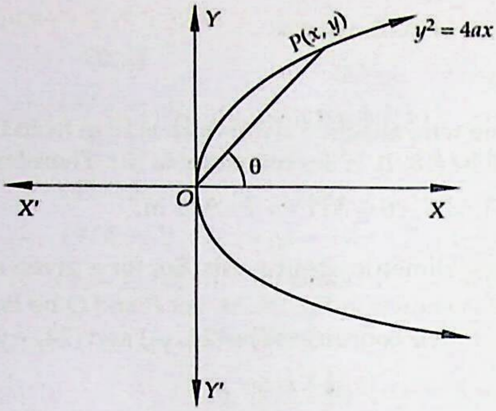

Fig. 25.40

Thus, the coordinates of P are $(4a\cot^2\theta, 4a\cot\theta)$.

Hence, $OP = \sqrt{x^2+y^2} = \sqrt{16a^2\cot^4\theta + 16a^2\cot^2\theta} = 4a \operatorname{cosec}\theta \cot\theta$

VERY SHORT ANSWER QUESTIONS (VSAQs)

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

1. Write the axis of symmetry of the parabola $y^2 = x$.
2. Write the distance between the vertex and focus of the parabola $y^2 + 6y + 2x + 5 = 0$.
3. Write the equation of the directrix of the parabola $x^2 - 4x - 8y + 12 = 0$.
4. Write the equation of the parabola with focus (0, 0) and directrix $x + y - 4 = 0$.
5. Write the length of the chord of the parabola $y^2 = 4ax$ which passes through the vertex and is inclined to the axis at $\frac{\pi}{4}$.
6. If b and c are lengths of the segments of any focal chord of the parabola $y^2 = 4ax$, then write the length of its latus-rectum.
7. PSQ is a focal chord of the parabola $y^2 = 8x$. If $SP = 6$, then write SQ.
8. Write the coordinates of the vertex of the parabola whose focus is at $(-2, 1)$ and directrix is the line $x + y - 3 = 0$.
9. If the coordinates of the vertex and focus of a parabola are $(-1, 1)$ and $(2, 3)$ respectively, then write the equation of its directrix.
10. If the parabola $y^2 = 4ax$ passes through the point (3, 2), then find the length of its latusrectum.
11. Write the equation of the parabola whose vertex is at $(-3, 0)$ and the directrix is $x + 5 = 0$.

ANSWERS

1. x-axis 2. 1/2 3. $y = -1$ 4. $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$ 5. $4\sqrt{2}a$

6. $\frac{4bc}{b+c}$ 7. 3 8. $(-1, 2)$ 9. $3x + 2y + 14 = 0$ 10. 4/3 11. $y^2 = 8(x+3)$

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. The coordinates of the focus of the parabola $y^2 - x - 2y + 2 = 0$ are
(a) $(5/4, 1)$ (b) $(1/4, 0)$ (c) $(1, 1)$ (d) none of these

2. The vertex of the parabola $(y + a)^2 = 8a(x - a)$ is
(a) $(-a, -a)$ (b) $(a, -a)$ (c) $(-a, a)$ (d) none of these

3. If the focus of a parabola is $(-2, 1)$ and the directrix has the equation $x + y = 3$, then its vertex is
(a) $(0, 3)$ (b) $(-1, 1/2)$ (c) $(-1, 2)$ (d) $(2, -1)$

4. The equation of the parabola whose vertex is $(a, 0)$ and the directrix has the equation $x + y = 3a$, is
(a) $x^2 + y^2 + 2xy + 6ax + 10ay + 7a^2 = 0$ (b) $x^2 - 2xy + y^2 + 6ax + 10ay - 7a^2 = 0$
(c) $x^2 - 2xy + y^2 - 6ax + 10ay - 7a^2 = 0$ (d) none of these

5. The parametric equations of a parabola are $x = t^2 + 1$, $y = 2t + 1$. The cartesian equation of its directrix is
(a) $x = 0$ (b) $x + 1 = 0$ (c) $y = 0$ (d) none of these

6. If the coordinates of the vertex and the focus of a parabola are $(-1, 1)$ and $(2, 3)$ respectively, then the equation of its directrix is
(a) $3x + 2y + 14 = 0$ (b) $3x + 2y - 25 = 0$
(c) $2x - 3y + 10 = 0$ (d) none of these.

7. The locus of the points of trisection of the double ordinates of a parabola is a
(a) pair of lines (b) circle (c) parabola (d) straight line

8. The equation of the directrix of the parabola whose vertex and focus are $(1, 4)$ and $(2, 6)$ respectively is
(a) $x + 2y = 4$ (b) $x - y = 3$ (c) $2x + y = 5$ (d) $x + 3y = 8$

9. If V and S are respectively the vertex and focus of the parabola $y^2 + 6y + 2x + 5 = 0$, then $SV =$
(a) 2 (b) 1/2 (c) 1 (d) none of these

10. The directrix of the parabola $x^2 - 4x - 8y + 12 = 0$ is
(a) $y = 0$ (b) $x = 1$ (c) $y = -1$ (d) $x = -1$

11. The equation of the parabola with focus $(0, 0)$ and directrix $x + y = 4$ is
(a) $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$ (b) $x^2 + y^2 - 2xy + 8x + 8y = 0$
(c) $x^2 + y^2 + 8x + 8y - 16 = 0$ (d) $x^2 - y^2 + 8x + 8y - 16 = 0$

12. The line $2x - y + 4 = 0$ cuts the parabola $y^2 = 8x$ in P and Q. The mid-point of PQ is
(a) $(1, 2)$ (b) $(1, -2)$ (c) $(-1, 2)$ (d) $(-1, -2)$

13. In the parabola $y^2 = 4ax$, the length of the chord passing through the vertex and inclined to the axis at $\pi/4$ is
(a) $4\sqrt{2}\,a$ (b) $2\sqrt{2}\,a$ (c) $\sqrt{2}\,a$ (d) none of these

14. The equation $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$ represents
(a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

15. The length of the latus-rectum of the parabola $y^2 + 8x - 2y + 17 = 0$ is

(a) 2 (b) 4 (c) 8 (d) 16

16. The vertex of the parabola $x^2 + 8x + 12y + 4 = 0$ is

(a) $(-4, 1)$ (b) $(4, -1)$ (c) $(-4, -1)$ (d) $(4, 1)$

17. The vertex of the parabola $(y-2)^2 = 16(x-1)$ is

(a) $(1, 2)$ (b) $(-1, 2)$ (c) $(1, -2)$ (d) $(2, 1)$

18. The length of the latus-rectum of the parabola $4y^2 + 2x - 20y + 17 = 0$ is

(a) 3 (b) 6 (c) 1/2 (d) 9

19. The length of the latus-rectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is

(a) 4 (b) 6 (c) 8 (d) 10

20. The focus of the parabola $y = 2x^2 + x$ is

(a) $(0, 0)$ (b) $(1/2, 1/4)$ (c) $(-1/4, 0)$ (d) $(-1/4, 1/8)$

21. Which of the following points lie on the parabola $x^2 = 4ay$?

(a) $x = at^2,\ y = 2at$ (b) $x = 2at,\ y = at^2$

(c) $x = 2at^2,\ y = at$ (d) $x = 2at,\ y = at^2$

22. The equation of the parabola whose focus is $(1, -1)$ and the directrix is $x + y + 7 = 0$ is

(a) $x^2 + y^2 - 2xy - 18x - 10y = 0$ (b) $x^2 - 18x - 10y - 45 = 0$

(c) $x^2 + y^2 - 18x - 10y - 45 = 0$ (d) $x^2 + y^2 - 2xy - 18x - 10y - 45 = 0$

ANSWERS

1. (a)	2. (b)	3. (c)	4. (b)	5. (a)	6. (a)	7. (c)	8. (a)
9. (b)	10. (c)	11. (a)	12. (c)	13. (a)	14. (b)	15. (c)	16. (b)
17. (a)	18. (c)	19. (c)	20. (c)	21. (d)	22. (d)		

SUMMARY

1. A parabola is the locus of a point which is equidistant from a fixed point (called focus) and a fixed line (called directrix).

   Thus, if (α, β) is the focus and $ax + by + c = 0$ is the equation of the directrix of a parabola, then its equation is

$$(x-\alpha)^2 + (y-\beta)^2 = \frac{(ax+by+c)^2}{a^2+b^2}$$

   This equation is of the form

   $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ satisfying the conditions

   $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$ and $h^2 - ab = 0$.

2. *Axis* : The straight line passing through the focus and perpendicular to the directrix is called the axis of the parabola.

3. *Vertex*: The point of intersection of the parabola and its axis is called the vertex of the parabola.

4. *Latus-rectum* : A chord passing through the focus and perpendicular to the axis is called the latus-rectum.
5. *Focal chord* : Any chord passing through the focus of a parabola is called its focal chord.
6. *Double ordinate* : Any chord perpendicular to the axis of a parabola is called double ordinate.
7. Following are four standard forms of parabola:

	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Coordinates of vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Coordinates of focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of the directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of the axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Length of the Latus-rectum	$4a$	$4a$	$4a$	$4a$
Focal distance of a point $P(x, y)$	$a + x$	$a - x$	$a + y$	$a - y$