CHAPTER 13

DIFFERENTIALS, ERRORS AND APPROXIMATIONS

13.1 DIFFERENTIALS

In the chapter on differentiation we defined derivative of y with respect to x i.e. $\frac{dy}{dx}$ as the limit of the ratio $\frac{\Delta y}{\Delta x}$ as $\Delta x \to 0$ and considered $\frac{dy}{dx}$ as a symbol not as a quotient of two separate quantities dy and dx. In this chapter, we shall give a meaning to the symbols dx and dy in such a way that the original meaning of the symbol $\frac{dy}{dx}$ coincides with the quotient when dy is divided by dx.

Let $y = f(x)$ be a function of x, and let Δx be a small change in x. Let Δy be the corresponding change in y. Then,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x)$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = f'(x) + \in, \text{ where } \in \to 0 \text{ as } \Delta x \to 0$$

$$\Rightarrow \quad \Delta y = f'(x)\,\Delta x + \in \Delta x$$

$$\Rightarrow \quad \Delta y = f'(x)\,\Delta x, \text{ approximately}$$

$$\Rightarrow \quad \Delta y = \frac{dy}{dx}\,\Delta x\,, \text{ approximately} \qquad \left[\because f'(x) = \frac{dy}{dx}\right]$$

NOTE *This formula is very useful in the calculation of small changes (or errors) in dependent variable corresponding to small changes (or errors) in the independent variable and is of great importance in the theory of errors in Engineering, Physics, Statistics and several other branches of the science.*

SOME IMPORTANT TERMS

ABSOLUTE ERROR *The error Δx in x is called the absolute error in x.*

RELATIVE ERROR *If Δx is an error in x, then $\frac{\Delta x}{x}$ is called the relative error in x*

PERCENTAGE ERROR *If Δx is an error in x, then $\frac{\Delta x}{x} \times 100$ is called percentage error in x.*

REMARK 1 *We have, $\Delta y = f'(x) \cdot \Delta x + \in \cdot \Delta x$.*

Since $\in \cdot \Delta x$ is very small, therefore principal value of Δy is $f'(x)\,\Delta x$ which is called differential of y and is denoted by dy.

i.e. $\quad dy = f'(x)\,\Delta x \;$ or, $dy = \frac{dy}{dx} \cdot \Delta x$

So, the differential of x is given by

$$dx = \frac{dx}{dx} \cdot \Delta x = 1 \cdot \Delta x = \Delta x$$

$$\therefore \quad dy = \frac{dy}{dx}\,\Delta x \Rightarrow dy = \frac{dy}{dx}\,dx$$

GEOMETRICAL MEANING OF DIFFERENTIALS

In order to understand the geometrical meaning of differentials, let us take a point $P(x, y)$ on the curve $y = f(x)$, where $f(x)$ is a differentiable real function. Let $Q(x + \Delta x, y + \Delta y)$ be a neighbouring point on the curve, where Δx denotes a small change in x and Δy is the corresponding change in y. It is evident from the Fig. 13.1, that $\frac{\Delta y}{\Delta x}$ is the slope of secant PQ. But, as $\Delta x \to 0$, $\frac{\Delta y}{\Delta x}$ approaches the limiting value $\frac{dy}{dx}$ (slope of the tangent at P). Therefore, when $\Delta x \to 0$, $\Delta y\ (= QS)$ is approximately equal to $dy\ (= RS)$ as shown in Fig. 13.1.

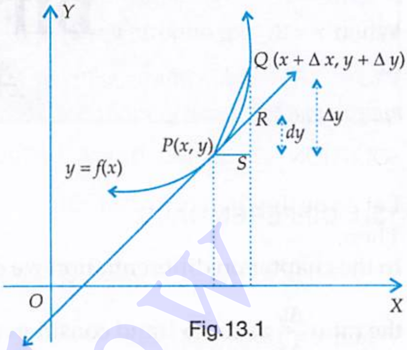

Fig.13.1

Geometrically the values of dx and dy are as shown in Fig. 13.1.

REMARK 2 *Let $y = f(x)$ be a function of x, and let Δx be a small change in x. Let the corresponding change in y be Δy. Then ,*

$$y + \Delta y = f(x + \Delta x)$$

But , $$\Delta y = \frac{dy}{dx} \cdot \Delta x = f'(x)\, \Delta x, \text{ approximately}$$

$$\therefore \quad f(x + \Delta x) = y + \Delta y$$

$$\Rightarrow \quad f(x + \Delta x) = y + f'(x) \cdot \Delta x, \text{ approximately}$$

$$\Rightarrow \quad f(x + \Delta x) = y + \frac{dy}{dx} \cdot \Delta x, \text{ approximately}$$

Let x be the independent variable and y be the dependent variable connected by the relation $y = f(x)$. We use the following algorithm to find an approximate change Δy in y due to a small change Δx in x.

ALGORITHM

Step I *Choose the initial value of the independent variable as x and the changed value as $x + \Delta x$.*

Step II *Find Δx and assume that $dx = \Delta x$.*

Step III *Find $\frac{dy}{dx}$ from the given relation $y = f(x)$.*

Step IV *Find the value of $\frac{dy}{dx}$ at (x, y).*

Step V *Find dy by using the relation $dy = \frac{dy}{dx}\, dx$.*

Step VI *Put $\Delta y \cong dy$ to obtain an approximate change in y.*

ILLUSTRATIVE EXAMPLES

BASED ON BASIC CONCEPTS (BASIC)

EXAMPLE 1 *If $y = x^4 - 10$ and if x changes from 2 to 1.99, what is the approximate change in y? Also, find the changed value of y.*

SOLUTION Let $x = 2$, $x + \Delta x = 1.99$. Then, $\Delta x = 1.99 - 2 = -0.01$. Let $dx = \Delta x = -0.01$

We have,

$$y = x^4 - 10 \Rightarrow \frac{dy}{dx} = 4x^3 \Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 4(2)^3 = 32$$

$\therefore \quad dy = \frac{dy}{dx} dx \Rightarrow dy = 32(-0.01) = -0.32 \Rightarrow \Delta y = -0.32$ approxiamtely $\quad [\because \Delta y \cong dy]$

So, approximate change in y is -0.32.

When $x = 2$, we obtain: $y = 2^4 - 10 = 6$. So, changed value of y is $y + \Delta y = 6 + (-0.32) = 5.68$.

EXAMPLE 2 *A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate if the radius of the plate before heating is 10 cm.*

SOLUTION Let at any time, x be the radius and y be the area of the plate. Then, $y = \pi x^2$.

Let Δx be the change in the radius and let Δy be the corresponding change in the area of the plate. Then,

$$\frac{\Delta x}{x} \times 100 = 2 \text{ (given)}$$

When $x = 10$,

$$\frac{\Delta x}{x} \times 100 = 2 \Rightarrow \frac{\Delta x}{10} \times 100 = 2 \Rightarrow \Delta x = \frac{2}{10} \Rightarrow dx = \frac{2}{10} \quad [\because dx \cong \Delta x] \quad ...(i)$$

Now, $\quad y = \pi x^2 \Rightarrow \frac{dy}{dx} = 2\pi x \Rightarrow \left(\frac{dy}{dx}\right)_{x=10} = 20\pi$

$\therefore \quad dy = \frac{dy}{dx} dx \Rightarrow dy = 20\pi \times \frac{2}{10} = 4\pi \Rightarrow \Delta y = 4\pi \quad [\because dy \cong \Delta y]$

Hence, the approximate change in the area of the plate is 4π.

EXAMPLE 3 *Find the percentage error in calculating the volume of a cubical box if an error of 1% is made in measuring the length of edges of the cube.*

SOLUTION Let x be the length of an edge of the cube and y be its volume. Then, $y = x^3$. Let Δx be the error in x and Δy be the corresponding error in y. Then,

$$\frac{\Delta x}{x} \times 100 = 1 \text{ (given)} \Rightarrow \frac{dx}{x} \times 100 = 1 \quad [\because dx \cong \Delta x] \quad ...(i)$$

We have to find $\frac{\Delta y}{y} \times 100$.

Now, $\quad y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$

$\therefore \quad dy = \frac{dy}{dx} dx$

$\Rightarrow \quad dy = 3x^2\, dx \Rightarrow \frac{dy}{y} = \frac{3x^2}{y} dx \Rightarrow \frac{dy}{y} = \frac{3x^2}{x^3} dx = 3\frac{dx}{x} \quad [\because y = x^3]$

$\Rightarrow \quad \frac{dy}{y} \times 100 = 3\left(\frac{dx}{x} \times 100\right) = 3 \quad$ [Using (i)]

$\Rightarrow \quad \frac{\Delta y}{y} \times 100 = 3 \quad [\because dy \cong \Delta y]$

So, there is 3% error in calculating the volume of the cube.

EXAMPLE 4 *The time T of a complete oscillation of a simple pendulum of length l is given by the equation*

$$T = 2\pi\sqrt{\frac{l}{g}},$$

where g is constant. What is the percentage error in T when l is increased by 1%?

SOLUTION Let Δl be the change in l and ΔT be the corresponding error in T. Then,

$$\frac{\Delta l}{l}\times 100 = 1 \text{ (given)} \Rightarrow \frac{dl}{l}\times 100 = 1 \qquad [\because dl \cong \Delta l] \qquad \text{...(i)}$$

Now, $T = 2\pi\sqrt{\frac{l}{g}}$

$$\Rightarrow \log T = \log 2\pi + \frac{1}{2}\log l - \frac{1}{2}\log g \Rightarrow \frac{1}{T}\frac{dT}{dl} = \frac{1}{2}\cdot\frac{1}{l} \Rightarrow \frac{dT}{dl} = \frac{T}{2l}$$

$$\therefore \quad dT = \frac{dT}{dl}dl$$

$$\Rightarrow dT = \frac{T}{2l}dl \Rightarrow \frac{dT}{T} = \frac{1}{2}\frac{dl}{l} \Rightarrow \frac{dT}{T}\times 100 = \frac{1}{2}\left(\frac{dl}{l}\times 100\right) \Rightarrow \frac{dT}{T}\times 100 = \frac{1}{2} \text{ [Using (i)]}$$

$$\Rightarrow \frac{\Delta T}{T}\times 100 = \frac{1}{2}. \qquad [\because dT \cong \Delta T]$$

So, there is (1/2)% error in calculating the time period T.

EXAMPLE 5 *Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.*

SOLUTION Let Δx be the change in x and ΔV be the corresponding change in V. It is given that $\frac{\Delta x}{x}\times 100 = 2$.

We have, $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$

$$\therefore \quad \Delta V = \frac{dV}{dx}\Delta x = 3x^2\,\Delta x = 3x^2\times\frac{2x}{100} = 0.06x^3 \qquad \left[\because \frac{\Delta x}{x}\times 100 = 2 \Rightarrow \Delta x = \frac{2x}{100}\right]$$

Thus, the approximate change in volume is $0.06x^3\ \text{m}^3$.

BASED ON LOWER ORDER THINKING SKILLS (LOTS)

EXAMPLE 6 *If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximating error in calculating its volume.* **[CBSE 2011]**

SOLUTION Let r be the radius of the sphere and Δr be the error in measuring the radius. Then, $r = 9$ cm and $\Delta r = 0.03$ cm. Let V be the volume of the sphere. Then,

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 \Rightarrow \left(\frac{dV}{dr}\right)_{r=9} = 4\pi\times 9^2 = 324\,\pi$$

Let ΔV be the error in V due to error Δr in r. Then,

$$\Delta V = \frac{dV}{dr}\,\Delta r \Rightarrow \Delta V = 324\pi\times 0.03 = 9.72\,\pi\ \text{cm}^3.$$

Thus, the approximate error in calculating the volume is $9.72\,\pi\ \text{cm}^3$.

EXAMPLE 7 *Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.* **[NCERT, CBSE 2014]**

SOLUTION Let $y = f(x)$, $x = 3$ and $x + \Delta x = 3.02$. Then, $\Delta x = 0.02$.

For $x = 3$, we obtain: $y = f(3) = 3\times 3^2 + 5\times 3 + 3 = 45$.

Now,

$$y = f(x) \Rightarrow y = 3x^2 + 5x + 3 \Rightarrow \frac{dy}{dx} = 6x + 5 \Rightarrow \left(\frac{dy}{dx}\right)_{x=3} = 6\times 3 + 5 = 23$$

Let Δy be the change in y due to change Δx in x. Then,

$$\Delta y = \frac{dy}{dx}\ \Delta x \Rightarrow \Delta y = 23 \times 0.02 = 0.46$$

$$\therefore \quad f(3.02) = y + \Delta y = 45 + 0.46 = 45.46.$$

EXAMPLE 8 *Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.* **[NCERT EXEMPLAR]**

SOLUTION Let x be the radius and y be the volume of a solid sphere. Then,

$$y = \frac{4}{3}\pi x^3 \Rightarrow \frac{dy}{dx} = 4\pi x^2$$

We have, $x = 3$ cm, $x + \Delta x = 3.0005$ cm. Therefore, $\Delta x = 0.0005$ cm. Let $dx = \Delta x = 0.0005$. Then,

$$dy = \frac{dy}{dx}dx \Rightarrow dy = 4\pi x^2\ dx \Rightarrow dy = 4\pi(3)^2 \times 0.0005 = 0.018\pi \text{ cm}^3 \Rightarrow \Delta y = 0.018\pi$$

Hence, the approximate volume of the metal is 0.018π cm^3.

Type II ON FINDING THE APPROXIMATE VALUE USING DIFFERENTIALS

In order to find the approximate values by using differentials, we may use the following algorithm:

ALGORITHM

Step I *Define a functional relationship between the independent variable x and dependent variable y by observing the given expression. For example, if we have to find the approximate value of the square root or cube root of a number, then we define $y = x^{1/2}$ or $x^{1/3}$. If we have to find the approximate value of logarithmic of a given number, then we consider $y = \log x$.*

Step II *Choose a value of x nearest to the value for which we have to find y in such a way that either y is given for the chosen x or y can be easily computed for chosen x. For example, if we have to find an approximate value of $(65)^{1/3}$ we take x as 64, because cube root of 64 can be easily calculated.*

Step III *Denote the value of x at which we have to find y by $x + \Delta x$.*

Step IV *Find Δx and assume that $dx = \Delta x$.*

Step V *Find $\frac{dy}{dx}$ from the relation obtained in step I.*

Step VI *Find the value of $\frac{dy}{dx}$ by putting the value of x chosen in step II.*

Step VII *Find dy by using the relation $dy = \frac{dy}{dx}\,dx$*

Step VIII *Assume that $\Delta y \cong dy$.*

Step IX *Find the value of y by putting the value of x chosen in step II in the relation obtained in step I.*

Step X *The approximate value of y is $y + \Delta y$.*

EXAMPLE 9 *Use differentials to approximate $\sqrt{25.2}$.*

SOLUTION Consider the function $y = f(x) = \sqrt{x}$. Let $x = 25$ and $x + \Delta x = 25.2$. Then, $\Delta x = 25.2 - 25 = 0.2$

For $x = 25$, we obtain: $y = \sqrt{25} = 5$ [Putting $x = 25$ in $y = \sqrt{x}$]

Let $dx = \Delta x = 0.2$.

$$\text{Now,} \quad y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow \left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{2(5)} = \frac{1}{10}$$

$\therefore \quad dy = \frac{dy}{dx}dx \Rightarrow dy = \frac{1}{10}(0.2) = 0.02 \Rightarrow \Delta y = 0.02 \qquad [\because \Delta y \cong dy]$

Hence, $\sqrt{25.2} = y + \Delta y = 5 + 0.02 = 5.02.$

EXAMPLE 10 *Use differentials to approximate the cube root of 127.*

SOLUTION Since we have to find the approximate value of the cube root of 127. So, we consider the function $y = f(x) = x^{1/3}$. Let $x = 125$ and $x + \Delta x = 127$. Then, $\Delta x = 127 - 125 = 2$.

For $x = 125$, we obtain: $y = (125)^{1/3} = 5$. [Putting $x = 125$ in $y = x^{1/3}$]

Let $dx = \Delta x = 2$.

Now,

$$y = x^{1/3} \Rightarrow \frac{dy}{dx} = \frac{1}{3x^{2/3}} \quad \left(\frac{dy}{dx}\right)_{x=125} = \frac{1}{3\,(125)^{2/3}} = \frac{1}{3\,(5^3)^{2/3}} = \frac{1}{75}$$

$\therefore \quad dy = \frac{dy}{dx}dx \Rightarrow dy = \frac{1}{75}(2) = \frac{2}{75} \Rightarrow \Delta y = \frac{2}{75} \qquad [\because \Delta y \cong dy]$

Hence, $(127)^{1/3} = y + \Delta y = 5 + \frac{2}{75} = 5.026.$

EXAMPLE 11 *Use differentials to find the approximate value of $\sqrt{0.037}$.*

SOLUTION Let $y = f(x) = \sqrt{x}$, $x = 0.040$ and $x + \Delta x = 0.037$. Then, $\Delta x = 0.037 - 0.040 = -0.003$.

For $x = 0.040$, we obtain: $y = \sqrt{0.040} = 0.2$. [Putting $x = 0.040$ in $y = \sqrt{x}$]

Let $dx = \Delta x = -0.003$.

Now, $\quad y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow \left(\frac{dy}{dx}\right)_{x=0.040} = \frac{1}{2\sqrt{0.040}} = \frac{1}{0.4}$

$\therefore \quad dy = \frac{dy}{dx}dx \Rightarrow dy = \frac{1}{0.4}(-0.003) = -\frac{3}{400} \Rightarrow \Delta y = -\frac{3}{400} \qquad [\because \Delta y \cong dy]$

Hence, $\sqrt{0.037} = y + \Delta y = 0.2 - \frac{3}{400} = 0.2 - 0.0075 = 0.1925.$

EXAMPLE 12 *Use differentials to find the approximate value of $\log_e(4.01)$, having given that $\log_e 4 = 1.3863$.*

SOLUTION Let $y = f(x) = \log_e x$, $x = 4$ and $x + \Delta x = 4.01$. Then, $\Delta x = 0.01$.

For $x = 4$, we obtain: $y = f(4) = \log_e 4 = 1.3863$ [Given]

Let $dx = \Delta x = 0.01$

Now, $\quad y = \log_e x \Rightarrow \frac{dy}{dx} = \frac{1}{x} \Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{1}{4}$

$\therefore \quad dy = \frac{dy}{dx}dx \Rightarrow dy = \frac{1}{4} \times 0.01 = 0.0025 \Rightarrow \Delta y = 0.0025 \qquad [\because \Delta y \cong dy]$

Hence, $\log_e(4.01) = y + \Delta y = 1.3863 + 0.0025 = 1.3888.$

EXAMPLE 13 *Using differentials find the approximate value of tan 46°, if it is being given that 1° = 0.01745 radians.*

SOLUTION Let $y = f(x) = \tan x$, $x = 45° = (\pi/4)^c$ and $x + \Delta x = 46°$. Then, $\Delta x = 1° = 0.01745$ radians.

For $x = \pi/4$, we obtain: $y = f(\pi/4) = \tan \pi/4 = 1$. Let $dx = \Delta x = 0.01745$.

Now, $\quad y = \tan x \Rightarrow \frac{dy}{dx} = \sec^2 x \Rightarrow \left(\frac{dy}{dx}\right)_{x=\pi/4} = \sec^2\frac{\pi}{4} = 2$

$\therefore \quad dy = \frac{dy}{dx}dx \Rightarrow dy = 2(0.01745) = 0.03490 \Rightarrow \Delta y = 0.03490 \qquad [\because \Delta y \cong dy]$

Hence, $\tan 46° = y + \Delta y = 1 + 0.03490 = 1.03490.$

BASED ON HIGHER ORDER THINKING SKILLS (HOTS)

EXAMPLE 14 *If in a triangle ABC, the side c and the angle C remain constant, while the remaining elements are changed slightly, using differentials show that* $\frac{da}{\cos A}+\frac{db}{\cos B}=0.$

SOLUTION We are given that the side c and angle C remain constant.

$$\therefore \quad \frac{c}{\sin C} = k \text{ (constant)}$$

$$\Rightarrow \quad \frac{a}{\sin A} = \frac{b}{\sin B} = k \qquad \left[\because \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\right]$$

$$\Rightarrow \quad a = k\sin A \text{ and } b = k\sin B \Rightarrow \frac{da}{dA} = k\cos A \text{ and } \frac{db}{dB} = k\cos B$$

Now, $da = \frac{da}{dA}dA \Rightarrow da = k\cos A\, dA \Rightarrow \frac{da}{\cos A} = k\, dA$

and, $db = \frac{db}{dB}dB \Rightarrow db = k\cos B\, dB \Rightarrow \frac{db}{\cos B} = k\, dB$

$$\therefore \quad \frac{da}{\cos A}+\frac{db}{\cos B} = k\,dA + k\,dB = k\,d(A+B) = k\,d(\pi - C)$$

$$\Rightarrow \quad \frac{da}{\cos A}+\frac{db}{\cos B} = k(0) = 0 \qquad [\because \pi - C = \text{Constant} \therefore d(\pi - C) = 0]$$

Hence, $\frac{da}{\cos A}+\frac{db}{\cos B} = 0.$

EXAMPLE 15 *If a triangle ABC, inscribed in a fixed circle, be slightly varied in such away as to have its vertices always on the circle, then show that* $\frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C} = 0.$

SOLUTION We know that

$$a = 2R\sin A,\ b = 2R\sin B \text{ and } c = 2R\sin C$$

$$\Rightarrow \quad \frac{da}{dA} = 2R\cos A,\ \frac{db}{dB} = 2R\cos B \text{ and } \frac{dc}{dC} = 2R\cos C \qquad [\because R = \text{constant}]$$

But, $da = \frac{da}{dA}dA,\ db = \frac{db}{dB}dB$ and $dc = \frac{dc}{dC}dC$

$$\therefore \quad da = 2R\cos A\, dA,\ db = 2R\cos B\, dB \text{ and } dc = 2R\cos C\, dC$$

$$\Rightarrow \quad \frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C} = 2R\,(dA + dB + dC)$$

$$\Rightarrow \quad \frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C} = 2Rd\,(A+B+C) = 2R\,d\,(\pi) \qquad [\because A+B+C=\pi]$$

$$\Rightarrow \quad \frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C} = 2R\,(0) = 0$$

EXERCISE 13.1

BASIC

1. If $y = \sin x$ and x changes from $\pi/2$ to $22/14$, what is the approximate change in y?
2. The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.
3. A circular metal plate expends under heating so that its radius increases by $k\%$. Find the approximate increase in the area of the plate, if the radius of the plate before heating is 10 cm.

4. Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in measuring the lengths of edges of the cube.
5. If there is an error of 0.1% in the measurement of the radius of a sphere, find approximately the percentage error in the calculation of the volume of the sphere.
6. The pressure p and the volume v of a gas are connected by the relation $pv^{1.4}$ = const. Find the percentage error in p corresponding to a decrease of $1/2\%$ in v.
7. The height of a cone increases by $k\%$, its semi-vertical angle remaining the same. What is the approximate percentage increase (i) in total surface area, and (ii) in the volume, assuming that k is small ?
8. Show that the relative error in computing the volume of a sphere, due to an error in measuring the radius, is approximately equal to three times the relative error in the radius.

BASED ON LOTS

9. Using differentials, find the approximate values of the following:

(i) $\sqrt{25.3}$ [CBSE 2020] (ii) $(0.009)^{1/3}$ (iii) $(0.007)^{1/3}$ (iv) $\sqrt{401}$

(v) $(15)^{1/4}$ (vi) $(255)^{1/4}$ (vii) $\dfrac{1}{(2.002)^2}$ (viii) $\sqrt{49.5}$ [CBSE 2012]

(ix) $(3.968)^{3/2}$ [CBSE 2014] (x) $\sqrt{36.6}$ (xi) $25^{1/3}$ (xii) $\dfrac{1}{\sqrt{25.1}}$

(xiii) $\sin\left(\dfrac{22}{14}\right)$ (xiv) $\cos\left(\dfrac{11\pi}{36}\right)$ (xv) $(80)^{1/4}$ (xvi) $(29)^{1/3}$

(xvii) $(66)^{1/3}$ (xviii) $\sqrt{26}$ [CBSE2000] (xix) $\sqrt{37}$ [CBSE 2000]

(xx) $\sqrt{0.48}$ [CBSE 2002C] (xxi) $(82)^{1/4}$ [CBSE 2005] (xxii) $\left(\dfrac{17}{81}\right)^{1/4}$

(xxiii) $(33)^{1/5}$ (xxiv) $\log_{10} 10.1$, it being given that $\log_{10} e = 0.4343$.

(xxv) $\cos 61°$, it being given that $\sin 60° = 0.86603$ and $1° = 0.01745$ radian.

(xxvi) $\log_e 4.04$, it being given that $\log_{10} 4 = 0.6021$ and $\log_{10} e = 0.4343$.

(xxvii) $\log_e 10.02$, it being given that $\log_e 10 = 2.3026$.

(xxviii) $(1.999)^5$ [NCERT EXEMPLAR] (xxix) $\sqrt{0.082}$ [NCERT EXEMPLAR]

10. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$. [NCERT]
11. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$. [NCERT]
12. Find the approximate value of $\log_{10} 1005$, given that $\log_{10} e = 0.4343$.
13. If the radius of a sphere is measured as 9 cm with an error of 0.03 m, find the approximate error in calculating its surface area. [NCERT]
14. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%. [NCERT]
15. If the radius of a sphere is measured as 7 m with an error of 0.02 m, find the approximate error in calculating its volume. [NCERT]
16. Find the approximate change in the volume of a cube of side x metres caused by increasing the side by 1%. [NCERT]

ANSWERS

1. No change 2. $80\pi \text{ cm}^3$ 3. $2k\pi$ 4. 2% 5. 0.3% 6. 0.7%

7. (i) $2k\%$ (ii) $3k\%$ 9. (i) 5.03 (ii) 0.208333 (iii) 0.1916667
(iv) 20.025 (v) 1.96875 (vi) 3.9961 (vii) 0.2495 (viii) 7.0357
(ix) 7.9041 (x) 6.05 (xi) 2.926 (xii) 0.1996 (xiii) 1
(xiv) 0.575575 (xv) 2.9907 (xvi) 3.074 (xvii) 4.0416 (xviii) 5.1
(xix) 6.083 (xx) 0.693 (xxi) 3.009 (xxii) 0.677 (xxiii) 2.0125
(xxiv) 1.004343 (xxv) 0.4849 (xxvi) 1.396368 (xxvii) 2.3046
(xxviii) 31.92 (xxix) 0.2867 10. 28.21 11. -34.99 12. 3.0021

13. $2.16\,\pi \text{ m}^2$ 14. $0.12\,x^2 \text{ m}^2$ 15. $3.92\,\pi \text{ m}^3$ 16. $0.03\,x^3 \text{ m}^3$

FILL IN THE BLANKS TYPE QUESTIONS (FBQs)

1. If $y = x^3 + 5$ and x changes from 3 to 2.99, then the approximate change in y is ....
2. The approximate change in the volume of a cube of side x metres caused by increasing the side by 2%, is ....

ANSWERS

1. -0.27 2. $0.06\,x^3 \text{ m}^3$

VERY SHORT ANSWER QUESTIONS (VSAQs)

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

1. For the function $y = x^2$, if $x = 10$ and $\Delta x = 0.1$. Find Δy.
2. If $y = \log_e x$, then find Δy when $x = 3$ and $\Delta x = 0.03$.
3. If the relative error in measuring the radius of a circular plane is α, find the relative error in measuring its area.
4. If the percentage error in the radius of a sphere is α, find the percentage error in its volume.
5. A piece of ice is in the form of a cube melts so that the percentage error in the edge of cube is a, then find the percentage error in its volume.
6. If $f(x) = x^4 - 10$, then find the approximate value of $f(2.1)$. [CBSE 2020]

ANSWERS

1. 2 2. 0.01 3. 2α 4. 3α 5. $3a$ 6. 9.2