Since there are 13 spade cards including an ace of spade and three aces other than an ace of spade.

$\therefore$ Favourable number of elementary events = ${}^{16}C_1 = 16$

So, $P(A) = \frac{16}{52} = \frac{4}{13}$.

Hence, odds against A are $P(\overline{A}) : P(A) = \frac{9}{13} : \frac{4}{13} = 9:4$

EXAMPLE 37 *The odds in favour of an event are 3 : 5. Find the probability of occurrence of this event.*

SOLUTION It is given that the odds in favour of an event are 3:5. Therefore,

Favourable number of elementary events = $3x$

Unfavourable number of elementary events = $5x$.

So, total number of elementary events = $3x + 5x = 8x$.

Hence, probability of the occurrence of the event $= \frac{3x}{8x} = \frac{3}{8}$

LEVEL-2

Type V MIXED PROBLEMS ON PROBABILITY

EXAMPLE 38 *A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) a consonant.* **[NCERT]**

SOLUTION There are 13 letters in the word 'ASSASSINATION' out of which there are 6 vowels viz., A, A, I, A, I, O and 7 consonants.

Total number of ways of selecting a word from 13 letters is $= 13$.

Number of ways of selecting a vowel out of 6 vowels = 6

Number of ways of selecting a consonant out of 7 consonants = 7.

$\therefore$ P (Selecting a vowel) $= \frac{6}{13}$ and, P (Selecting a consonant) $= \frac{7}{13}$

EXAMPLE 39 *If the letters of the word ASSASSINATION are arranged at random. Find the probability that*

(i) *Four S's come consecutively in the word.* (ii) *Two I's and two N's come together.*

(iii) *All A's are not coming together.* (iv) *No two A's are coming together.*

[NCERT EXEMPLAR]

SOLUTION There are 13 letters in the word ASSASSINATION out of which there are $3A$'s $4S$'s $2I$'s $2N$'s, one O and one T. These 13 letters can be arranged in a row in $\frac{13!}{3!\,4!\,2!\,2!\,1!\,1!}$ ways.

(i) Considering $4S$'s as one letter there are 10 letters ($3A$'s, $2I$'s, 2 N's, one O; one T and one letter formed by $4S$'s). These 10 letters can be arranged in $\frac{10!}{3!\,2!\,2!\,1!\,1!\,1!}$ ways.

$$\therefore \quad P\,(4S\text{'s come consecutively}) = \frac{\dfrac{10!}{3!\,2!\,2!\,1!\,1!\,1!}}{\dfrac{13!}{3!\,4!\,2!\,2!\,1!\,1!}} = \frac{4! \times 10!}{13!} = \frac{2}{143}$$

(ii) Two I's and two N's can be put together in $\frac{4!}{2!\,2!}$ ways. Considering these 4 letters as one, there are 10 letters which can be arranged in a row in $\frac{10!}{3!\,4!}$ ways.

$\therefore$ Number of arrangements in which two I's and two N's come together $=\dfrac{10!}{3!\,4!}\times\dfrac{4!}{2!\,2!}$

$$=\frac{10!}{3!\,2!\,2!}$$

Hence, P (Two I's and two N's come together) $=\dfrac{\dfrac{10!}{3!\,2!\,2!}}{\dfrac{13!}{3!\,4!\,2!\,2!}}=\dfrac{2}{143}$

(iii) Considering all A's as one letter, there are 11 letters which can be arranged in a row in $\dfrac{11!}{4!\,2!\,2!}$ ways.

$\therefore$ P (All A's come together) $=\dfrac{\dfrac{11!}{4!\,2!\,2!}}{\dfrac{13!}{3!\,4!\,2!\,2!}}=\dfrac{1}{26}$

Hence, P (All A's are not coming together) $=1-\dfrac{1}{26}=\dfrac{25}{26}$

(iv) Other than 3 A's there are 10 letters (4S's, 2I's 2N's, one O and one T). These 10 letters can be arranged in a row in $\dfrac{10!}{4!\,2!\,2!}$ ways. In each arrangement of these 10 letters there are 11 places which can be filled by 3 A's in ${}^{11}C_3$ ways.

$\therefore$ Number of arrangements in which no two A's come together $=\dfrac{10!}{4!\,2!\,2!}\times{}^{11}C_3$

$$=\frac{10!}{4!\,2!\,2!}\times\frac{11!}{8!\,3!}$$

Hence, P (No two A's are coming together) $=\dfrac{\dfrac{10!}{4!\,2!\,2!}\times\dfrac{11!}{8!\,3!}}{\dfrac{13!}{3!\,4!\,2!\,2!}}=\dfrac{15}{26}$.

EXAMPLE 40 *In a lottery, a person choses six different natural numebrs at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?*

SOLUTION Out of 20 numbers six numbers can be chosen in ${}^{20}C_6$ ways.

$\therefore$ Total number of elementary events $={}^{20}C_6=38760$

It is given that a persons wins the prize if six selected numbers match with the six numbers already fixed by the committee.

$\therefore$ Favourable number of ways = 1

Hence, required probability $=\dfrac{1}{38760}$

EXAMPLE 41 *A typical PIN (personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and the ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?*

SOLUTION It is given that a PIN is a sequence of four symbols selected from 36 (26 letters and 10 digits) symbols. Therefore,

Total numbers of PINs $=36\times36\times36\times36=36^4=1{,}679,616$

Total number of PINs with distinct symbols $= 36 \times 35 \times 34 \times 33 = 1,413,720$.

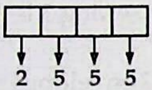
$\therefore$ The number of PINs that contain at least one repeated symbol $= 1,679,616 - 1,413,720$
$= 2,65,896$

Hence,

The probability that a randomly chosen PIN contains a repeated symbol $= \dfrac{2,65,896}{1,679,616}$
$= 0.1583$

EXAMPLE 42 *If 4-digit numbers greater than or equal to 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probabilty of forming number divisible by 5 when*
(i) the digits may be repeated (ii) the repetition of digits is not allowed. **[NCERT]**

SOLUTION (i) Total number of 4-digit numbers formed from the digits 0, 1, 3, 5 and 7 and greater than or equal to 5000 is $2 \times 5 \times 5 \times 5 = 250$.



A number is divisible by 5, if units digit is 0 to 5. Therefore, number of 4 digit numbers formed from the digits 0, 1, 3, 5 and 7, divisible by 5 and greater than or equal to 5000 is $2 \times 5 \times 5 \times 2 = 100$

$\therefore$ Probability of forming a number divisible by $5 = \dfrac{100}{250} = \dfrac{2}{5}$

(ii) If repetition of digits is not allowed, then the total number of 4 digit numbers formed from the digits 0, 1, 3, 5 and 7 is $2 \times 4 \times 3 \times 2 = 48$.

Now,

Number of 4 digit numbers divisible by 5 having 0 at one's place $= 2 \times 3 \times 2 \times 1 = 12$
Number of 4 digit numbers divisible by 5 having 5 at one's place $= 1 \times 3 \times 2 \times 1 = 6$

$\therefore$ Number of 4 digit numbers with distinct digits and divisible by $5 = 12 + 6 = 18$

Hence, probability of forming a number divisible by $5 = \dfrac{18}{48} = \dfrac{3}{8}$

EXAMPLE 43 *A fair coin is tossed four times, and a person wins Re 1 for each head and lose Rs. 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.* **[NCERT]**

SOLUTION The sample space associated with the random experiment of tossing four coins is given by

$$S = \{HHHH, HHHT, HHTH, THHH, HTHH, HHTT, TTHH, HTHT, HTTH, THTH, THHT, TTHT, HTTT, THTT, TTTH, TTTT\}$$

Let Rs X be the amount won by the person in a single throw of four coins.

If the person gets 4 heads in a throw of four coins, then amount won $=$ ₹ $(4 \times 1) =$ ₹ 4 i.e. $X = 4$.

If the person gets 3 heads and one tail in a throw of four coins, then amount won
$=$ ₹ $(3 \times 1 - 1.50 \times 1) =$ ₹ 1.50 i.e., $X = 1.50$

If the person gets 2 heads and 2 tails in a throw of four coins, then amount won
$=$ ₹ $(2 \times 1 - 1.50 \times 2) = -$₹ 1 i.e. amount lost $= -1$

If the person gets 1 head and 3 tails in a throw of four coins, then amount won/lost
$=$ ₹ $(1 \times 1 - 3 \times 1.50) = -$₹ 3.50. So, $X = -3.50$

If the person gets all tails in a throw of 4 coins, then amount lost
$=$ ₹ $(0 - 4 \times 1.50) = -$₹ 6 i.e., $X = -6$

Now, $P(X = 4) =$ Probability of getting all heads $= \dfrac{1}{16}$

$P(X = 1.50) =$ Probability of getting 3 heads and 1 tail $= \frac{4}{16} = \frac{1}{4}$

$P(X = -1) =$ Probability of getting 2 heads and 2 tails $= \frac{6}{16} = \frac{3}{8}$

$P(X = -3.50) =$ Probability of getting 1 head and 3 tails $= \frac{4}{16} = \frac{1}{4}$

and, $P(X = -6) =$ Probability of getting all tails $= \frac{1}{16}$

EXAMPLE 44 *Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.* **[NCERT]**

SOLUTION Three letters can be inserted in three envelopes in $3! = 6$ ways.

Let us now find the number of ways of inserting 3 letters in three envelopes so that no letter is put in proper envelope.

Number of ways of inserting 2 letters in 2 envelopes so that no letter is in proper envelope = 1

$\therefore$ Number of ways of inserting 3 letters in 3 envelopes so that no letter is in proper envelope

= Total number of ways of inserting 3 letters in 3 envelopes

– Number of ways in which one letter is in proper envelope and remaining two are in wrong envelopes

– Number of ways in which all are in proper envelopes

$= 3! - {}^3C_1 \times 1 - 1 = 6 - 3 - 1 = 2$

$\therefore$ Probability that at least one letter is in its proper envelope

$= 1 -$ Probability that no letter is in its proper envelope $= 1 - \frac{2}{6} = \frac{2}{3}$

EXAMPLE 45 *A word consists of 9 letters; 5 consonants and 4 vowels. Three letters are chosen at random. What is the probability that more than one vowel will be selected?*

SOLUTION Three letters can be chosen out of 9 letters in ${}^9C_3$ ways.

$\therefore$ Total number of elementary events $= {}^9C_3$

More than one vowels can be chosen in one of the following ways:

(i) 2 vowels and one consonant or, (ii) 3 vowels.

So, favourable number of elementary events $= {}^4C_2 \times {}^5C_1 + {}^4C_3$

Hence, required probability $= \frac{{}^4C_2 \times {}^5C_1 + {}^4C_3}{{}^9C_3} = \frac{17}{42}$

EXAMPLE 46 *A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). Find the probability that $x_3 = 30$.*

SOLUTION Five tickets out of 50 can be drawn in ${}^{50}C_5$ ways.

$\therefore$ Total number of elementary events $= {}^{50}C_5$

Since $x_1 < x_2 < x_3 < x_4 < x_5$ and $x_3 = 30$. Therefore, $x_1, x_2 < 30$ i.e., x_1 and x_2 should come from tickets numbered 1 to 29 and this may happen in ${}^{29}C_2$ ways. Remaining two i.e., $x_4, x_5 > 30$, should come from 20 tickets numbered from 31 to 50 in ${}^{20}C_2$ ways.

$\therefore$ Favourable number of elementary events $= {}^{29}C_2 \times {}^{20}C_2$

Hence, required probability $= \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5} = \frac{551}{15134}$

EXAMPLE 47 *A bag contains tickets numbered 1 to 30. Three tickets are drawn at random from the bag. What is the probability that the maximum number on the selected tickets exceeds 25?*

SOLUTION It is given that the maximum number on the selected tickets exceeds 25. This means that at least one of the selected tickets should bear a number that exceeds 25. Note that the negation of 'at least one' is none and in this case it will be easier for us to find the probability that none of selected tickets bear number exceeding 25.

Let A be the event that none of the selected tickets bear number exceeding 25. Then, $\overline{A}$ denotes the event that at least one of the selected tickets bears a number that exceeds 25.

$\therefore$ Required probability $= P(\overline{A}) = 1 - P(A)$

Now, we calculate $P(A)$.

The total number of ways of drawing three tickets out of 30 is ${}^{30}C_3$.

$\therefore$ Total number of elementary events $= {}^{30}C_3$

Since none of the selected tickets bear number exceeding 25. Therefore, three tickets are drawn from tickets bearing number 1 to 25. This can be done in ${}^{25}C_3$ ways.

$\therefore$ Favourable number of elementary events $= {}^{25}C_3$

So, $P(A) = \dfrac{{}^{25}C_3}{{}^{30}C_3} = \dfrac{115}{203}$

Hence, required probability $= P(\overline{A}) = 1 - \dfrac{115}{203} = \dfrac{88}{203}$

EXAMPLE 48 *Twelve balls are distributed among three boxes, find the probability that the first box will contain three balls.*

SOLUTION Since each ball can be put into any one of the three boxes. So, the total number of ways in which 12 balls can be put into three boxes in 3^{12}. Out of 12 balls, 3 balls can be chosen in ${}^{12}C_3$ ways. Put these three balls in the first box and put remaining 9 balls in the remaining two boxes, which can be done in 2^9 ways.

So, the total number of ways in 3 balls can be put in the first box and the remaining 9 in other two boxes is ${}^{12}C_3 \times 2^9$.

Hence, required probability $= \dfrac{{}^{12}C_3 \times 2^9}{3^{12}}$.

EXAMPLE 49 *Find the probability that the birth days of six different persons will fall in exactly two calender months.*

SOLUTION Since each person can have his birth day in any one of the 12 calender months. So, there are 12 options for each person.

$\therefore$ Total number of ways in which 6 persons can have their birth days

$$= 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^6$$

Out of 12 months, 2 months can be chosen in ${}^{12}C_2$ ways.

Now, birth days of six persons can fall in these two months in 2^6 ways. Out of these 2^6 ways, there are two ways when all six birth days fall in one month. So, there are $(2^6 - 2)$ ways in which six birth days fall in the chosen 2 months.

$\therefore$ Number of ways in which six birth days fall in exactly two calender month $= {}^{12}C_2 \times (2^6 - 2)$

Hence, required probability $= \dfrac{{}^{12}C_2 \times (2^6 - 2)}{12^6} = \dfrac{341}{12^5}$

EXAMPLE 50 *Three dice are thrown simultaneously. Find the probability that:*
(i) *all of them show the same face.* (ii) *all show distinct faces.* (iii) *two of them show the same face.*

SOLUTION The toal number of elementary events associated to the random experiment of throwing three dice simultaneously is $6 \times 6 \times 6 = 6^3$

(i) All dice show the same face in one of the following mutually exclusive ways :

$$(1, 1, 1), (2, 2, 2), (3, 3, 3,), (4, 4, 4), (5, 5, 5), (6, 6, 6)$$

So, favourable number of elementary events = 6.

Hence, required probability $= \dfrac{6}{6^3} = \dfrac{1}{36}$

(ii) The total number of ways in which all dice show different faces is equal to the number of ways of arranging 6 distinct objects by taking three at a time i.e. ${}^6C_3 \times 3!$.

So, favourable number of elementary events $= {}^6C_3 \times 3!$

Hence, required probability $= \dfrac{{}^6C_3 \times 3!}{6^3} = \dfrac{5}{9}$

(iii) Select a number which occurs on two dice out of the six numbers 1, 2, 3, 4, 5, 6 marked on the six faces of a die. This can be done in ${}^6C_1$ ways. Now, select a number from the remaining 5 numbers which occurs on the remaining one die. This can be done in ${}^5C_1$ ways. Now, we have three numbers like 1, 1, 2 ; 2, 2, 5 etc. These three digits can be arranged in $\dfrac{3!}{2!}$ ways.

So, the favourable number of elementary events $= {}^6C_1 \times {}^5C_1 \times \dfrac{3!}{2!}$

Hence, required probability $= \dfrac{90}{6^3} = \dfrac{5}{12}$

EXAMPLE 51 *What is the probability that in a group of*
(i) *2 people, both will have the same birth-day?* (ii) *3 people, at least two will have the same birth-day? assuming that there are 365 days in a year and no one has his/her birth day on 29th February.*

SOLUTION (i) First person may have any one of the 365 days of the year as a birth day. Similarly, second person may have any one of 365 days of the year as a birth day.

So, the total number of ways in which two persons may have their birth days $= 365 \times 365 = 365^2$

The number of ways in which two persons have the same birth-day = 365.

Hence, required probability $= \dfrac{365}{365^2} = \dfrac{1}{365}$

(ii) Let A be the event "At least two people have the same birth day". Then,

$\overline{A}$ = No two or more people have the same birth day = All the three persons have distinct birth-days.

$$\therefore \quad P(\overline{A}) = \frac{365 \times 364 \times 363}{365^3} = \frac{364 \times 363}{365^2}$$

Hence, required probability $= 1 - P(\overline{A}) = 1 - \dfrac{364 \times 363}{365^2}$

EXAMPLE 52 *If n biscuits are distributed among N beggars, find the chance that a particular beggar will get r (< n) biscuits.*

SOLUTION Since a biscuit can be given to any one of N beggars. Therefore, each biscuit can be distributed in N ways.

So, total number of ways of distributing n biscuits among N beggars $= \underbrace{N \times N \times \ldots \times N}_{n-\text{ times}} = N^n$.

Now, r biscuits can be given to a particular beggar in ${}^nC_r$ ways and the remaining $(n-r)$ biscuits can be distributed to $(N-1)$ beggars in $(N-1)^{n-r}$ ways. Thus, the number of ways in which a particular beggar receives r biscuits is ${}^nC_r \times (N-1)^{n-r}$.

Hence, required probability $= \dfrac{{}^nC_r \times (N-1)^{n-r}}{N^n}$

EXAMPLE 53 *The letters of word 'SOCIETY' are placed at random in a row. What is the probability that three vowels come together?*

SOLUTION There are 7 letters in the word 'Society'. These 7 letters can be arranged in a row in 7! ways.

$\therefore$ Total number of elementary events = 7!

There are 3 vowels viz. O, I, E in word 'SOCIETY'. Considering these three vowels as one letter we have 5 letters which can be arranged in a row in 5! ways. But, three vowels O, I, E can be put together in 3! ways. Therefore, the total number of arrangements in which three vowels come together is $5! \times 3!$.

So, favourable number of elementary events $= 5! \times 3!$

Hence, required probability $= \dfrac{5! \times 3!}{7!} = \dfrac{1}{7}$

EXAMPLE 54 *Find the probability that in a random arrangement of the letters of the word 'UNIVERSITY' the two I's come together.*

SOLUTION The total number of words which can be formed by permuting the letters of the word 'UNIVERSITY' is $\dfrac{10!}{2!}$ as there are two I's.

$\therefore$ Total number of elementary events $= \dfrac{10!}{2!}$

Regarding 2I's as one letter, number of ways of arrangement in which both I's are together = 9!

$\therefore$ Favourable number of elementary events = 9!

Hence, required probability $= \dfrac{9!}{10!/2!} = \dfrac{1}{5}$

EXAMPLE 55 *Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?* **[NCERT EXEMPLAR]**

SOLUTION Six employees can be seated in row in six desks in 6! ways. Married couple can occupy adjacent seats in the following 5 ways.

$1-2, 2-3, 3-4, 4-5, 5-6,$

Also, they can interchange their seats and the remaining 4 seats can be occupied by remaining 4 employees in 4! ways.

$\therefore$ Number of ways in which married couple will have adjacent seats $= 5 \times 2! \times 4!$

So, Number of ways in which married couple will have non-adjacent seats $= 6! - 5 \times 2! \times 4!$
$= 480.$

Hence, required probability $= \dfrac{480}{720} = \dfrac{2}{3}$

EXAMPLE 56 *In how many ways, can three girls and nine boys be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats?*

SOLUTION Each van has 7 seats. So, there are 14 numbered seats in two vans.

The total number of ways in which 3 girls and 9 boys can sit on these seats is ${}^{14}C_{12} \times 12!$

So, total number of seating arrangements = ${}^{14}C_{12} \times 12!$

In a van 3 girls can choose adjacent seats in the back row in two ways (1, 2, 3, or 2, 3, 4). So, the number of ways in which 3 girls can sit in the back row on adjacent seats is 2 (3!) ways. The number of ways in which 9 boys can sit on the remaining 11 seats is ${}^{11}C_9 \times 9!$ ways.

So, the number of ways in which 3 girls and 9 boys can sit in two vans

$$= 2(3!) \times {}^{11}C_9 \times 9! + 2(3!) \times {}^{11}C_9 \times 9!$$

Hence, required probability $= \dfrac{4 \times 3! \times {}^{11}C_9 \times 9!}{{}^{14}C_{12} \times 12!} = \dfrac{1}{91}$

EXAMPLE 57 *Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first. Find out the probability of all five persons leaving at different floors.*

SOLUTION Besides ground floor, there are 7 floors. Since a person can leave the cabin at any of the seven floors, therefore there are 7 ways for a person to leave the lift cabin. There are five persons in the cabin and each can leave the cabin in 7 ways. Therefore,

The total number of ways in which 5 persons can leave the cabin $= 7 \times 7 \times 7 \times 7 \times 7 \times = 7^5$

$\therefore$ Total number of elementary events $= 7^5$

The total number of ways in which five persons can leave the lift cabin at different floors is same as the number of arrangements of 7 by taking 5 at a time i.e., ${}^7C_5 \times 5!$

$\therefore$ Favourable number of elementary events $= {}^7C_5 \times 5!$

Hence, required probability $= \dfrac{{}^7C_5 \times 5!}{7^5}$

EXAMPLE 58 *If n persons are seated on a round table, what is the probability that two named individuals will be neighbours?*

SOLUTION Total number of ways in which n persons can sit on a round table is $(n-1)!$.

$\therefore$ Total number of elementary events $= (n-1)!$

Considering two named individuals as one person there are $(n-1)$ persons who can sit on a round table in $(n-2)!$ ways. But, two named individual can be seated together in 2! ways.

$\therefore$ Favourable number of elementary events $= (n-2)! \times 2!$

So, required probability $= \dfrac{(n-2)! \times 2!}{(n-1)!} = \dfrac{2}{n-1}$

EXAMPLE 59 *There are 4 letters and 4 addressed envelopes. Find the probability that all the letters are not dispatched in right envelopes.*

SOLUTION Four letters can be put in four addressed envelopes in 4! ways.

$\therefore$ Total number of elementary events = 4!

All four letters can be put in correct envelopes in exactly one way.

$\therefore$ Probability that all four letters are put in correct envelopes $= \dfrac{1}{4!}$

Hence, required probability $= 1 - \dfrac{1}{4!} = \dfrac{23}{24}$

EXAMPLE 60 *Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary six faced die. Find the probability that the equation will have real roots.*

SOLUTION Since each of the coefficients a, b and c can take the values from 1 to 6.

$\therefore$ Total number of equations $= 6 \times 6 \times 6 = 216$.

The roots of the equation $ax^2 + bx + c = 0$ will be real if $b^2 - 4\,ac \geq 0$ i.e. $b^2 \geq 4\,ac$.

The favourable number of elementary events can be enumerated as follows:

ac	*a*	*c*	*4ac*	*b* (so that $b^2 \geq 4\,ac$)	*No. of ways*
1	1	1	4	2, 3, 4, 5, 6	$1 \times 5 = 5$
2	1 2	2 1	8	3, 4, 5, 6	$2 \times 4 = 8$
3	1 3	3 1	12	4, 5, 6	$2 \times 3 = 6$
4	1 4 2	4 1 2	16	4, 5, 6	$3 \times 3 = 9$
5	1 5	5 1	20	5, 6	$2 \times 2 = 4$
6	1 6 2 3	6 1 3 2	24	5, 6	$4 \times 2 = 8$
7	*ac* is not possible				0
8	2 4	4 2	32	6	$2 \times 1 = 2$
9	3	3	36	6	1
					Total $= 43$

Since $b^2 \geq 4\,ac$ and since the maximum value of b^2 is 36, therefore ac = 10, 11, 12 ... etc. is not possible.

$\therefore$ Total number of favourable elementary events = 43.

Hence, required probability $= \dfrac{43}{216}$.

EXAMPLE 61 *Two numbers b and c are chosen at random with replacement from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9. Find the probability that $x^2 + bx + c > 0$ fro all $x \in R$.*

SOLUTION Since *b* and *c* both can assume value from 1 to 9. So, total numbers of ways of choosing *b* and *c* is $9 \times 9 = 81$.

Now, $x^2 + bx + c > 0$ for all $x \in R$

$\Rightarrow$ Disc < 0 *i.e.* $b^2 - 4c < 0$

The following table shows the possible values of *b* and *c* for which $b^2 - 4c < 0$

c	b	*Total*
1	1	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
		32

So, favourable number of favourable elementary events = 32

Hence, required probability = 32/81.

EXAMPLE 62 *Three squares of Chess board are selected at random. Find the probability of getting 2 squares of one colour and other of a different colour.* **[NCERT EXEMPLAR]**

SOLUTION In a Chess board, there are 64 squares of which 32 are white and 32 are black. Out of 64 squares 3 square can be chosen in ${}^{64}C_3$. Since 2 of one colour and 1 ways of other colour can be 2W, 1B or 1W, 2B. Therefore, the number of ways of selecting 2 squares of one colour and one other colour is ${}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_1 \times {}^{32}C_2 = 2\,({}^{32}C_2 \times {}^{32}C_1)$

$\therefore$ Favourable number of ways $= 2\,({}^{32}C_2 \times {}^{32}C_1)$

Hence, required probability $= \dfrac{2\,({}^{32}C_2 \times {}^{32}C_1)}{{}^{64}C_3} = \dfrac{16}{21}$.

EXERCISE 33.3

LEVEL-1

1. Which of the following cannot be valid assignment of probability for elementary events or outcomes of sample space $S = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$:

Elementary events:	w_1	w_2	w_3	w_4	w_5	w_6	w_7
(i)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(ii)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(iii)	0.7	0.6	0.5	0.4	0.3	0.2	0.1
(iv)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

2. A die is thrown. Find the probability of getting:
   (i) a prime number (ii) 2 or 4 (iii) a multiple of 2 or 3.
3. In a simultaneous throw of a pair of dice, find the probability of getting:
   (i) 8 as the sum (ii) a doublet
   (iii) a doublet of prime numbers (iv) a doublet of odd numbers
   (v) a sum greater than 9 (vi) an even number on first
   (vii) an even number on one and a multiple of 3 on the other
   (viii) neither 9 nor 11 as the sum of the numbers on the faces
   (ix) a sum less than 6 (x) a sum less than 7
   (xi) a sum more than 7 (xii) neither a doublet nor a total of 10

(xiii) odd number on the first and 6 on the second
(xiv) a number greater than 4 on each die
(xv) a total of 9 or 11
(xvi) a total greater than 8.

4. In a single throw of three dice, find the probability of getting a total of 17 or 18.
5. Three coins are tossed together. Find the probability of getting:
   (i) exactly two heads
   (ii) at least two heads
   (iii) at least one head and one tail.
6. What is the probability that an ordinary year has 53 Sundays?
7. What is the probability that a leap year has 53 Sundays and 53 Mondays?
8. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that:
   (i) All the three balls are white.
   (ii) All the three balls are red.
   (iii) One ball is red and two balls are white. **[NCERT EXEMPLAR]**
9. In a single throw of three dice, find the probability of getting the same number on all the three dice.
10. Two unbiased dice are thrown. Find the probability that the total of the numbers on the dice is greater than 10.
11. A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is:
    (i) a black king
    (ii) either a black card or a king
    (iii) black and a king
    (iv) a jack, queen or a king
    (v) neither a heart nor a king
    (vi) spade or an ace
    (vii) neither an ace nor a king
    (viii) a diamond card
    (ix) not a diamond card
    (x) a black card
    (xi) not an ace
    (xii) not a black card.
12. In shuffling a pack of 52 playing cards, four are accidently dropped; find the chance that the missing cards should be one from each suit.
13. From a deck of 52 cards, four cards are drawn simultaneously, find the chance that they will be the four honours of the same suit.
14. Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7?
15. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that one is red, one is white and one is blue.
16. A bag contains 7 white, 5 black and 4 red balls. If two balls are drawn at random, find the probability that: (i) both the balls are white (ii) one ball is black and the other red (iii) both the balls are of the same colour.
17. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that: (i) one is red and two are white (ii) two are blue and one is red (iii) one is red.
18. Five cards are drawn from a pack of 52 cards. What is the chance that these 5 will contain:
    (i) just one ace
    (ii) at least one ace?
19. The face cards are removed from a full pack. Out of the remaining 40 cards, 4 are drawn at random. What is the probability that they belong to different suits?
20. There are four men and six women on the city councils. If one council member is selected for a committee at random, how likely is that it is a women? **[NCERT]**

21. A box contains 100 bulbs, 20 of which are defective. 10 bulbs are selected for inspection. Find the probability that: (i) all 10 are defective (ii) all 10 are good (iii) at least one is defective (iv) none is defective

22. Find the probability that in a random arrangement of the letters of the word 'SOCIAL' vowels come together.

23. The letters of the word 'CLIFTON' are placed at random in a row. What is the chance that two vowels come together?

24. The letters of the word 'FORTUNATES' are arranged at random in a row. What is the chance that the two 'T' come together.

25. A committee of two persons is selected from two men and two women. What is the probability that the committee will have (i) no man ? (ii) one man? (iii) two men? **[NCERT]**

26. If odds in favour of an event be 2 : 3, find the probability of occurrence of this event.

27. If odds against an event be 7 : 9, find the probability of non-occurrence of this event.

28. Two balls are drawn at random from a bag containing 2 white, 3 red, 5 green and 4 black balls, one by one without, replacement. Find the probability that both the balls are of different colours.

29. Two unbiased dice are thrown. Find the probability that:
    (i) neither a doublet nor a total of 8 will appear
    (ii) the sum of the numbers obtained on the two dice is neither a multiple of 2 nor a multiple of 3

30. A bag contains 8 red, 3 white and 9 blue balls. If three balls are drawn at random, determine the probability that (i) all the three balls are blue balls (ii) all the balls are of different colours.

31. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both balls are red or both are black?

32. If a letter is chosen at random from the English alphabet, find the probability that the letter is (i) a vowel (ii) a consonant

33. In a lottery, a person chooses six different numbers at random from 1 to 20, and if these six numbers match with six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? **[NCERT]**

34. 20 cards are numbered from 1 to 20. One card is drawn at random. What is the probability that the number on the cards is: (i) a multiple of 4? (ii) not a multiple of 4? (iii) odd? (iv) greater than 12? (v) divisible by 5? (vi) not a multiple of 6?

35. Two dice are thrown. Find the odds in favour of getting the sum
    (i) 4 (ii) 5 (iii) What are the odds against getting the sum 6?

36. What are the odds in favour of getting a spade if a card is drawn from a well-shuffled deck of cards? What are the odds in favour of getting a king?

37. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn at random. From the box, what is the probability that: (i) all are blue? (ii) at least one is green?

38. A box contains 6 red marbles numbered 1 through 6 and 4 white marbles numbered from 12 through 15. Find the probability that a marble drawn is (i) white (ii) white and odd numbered (iii) even numbered (iv) red or even numbered.

39. A class consists of 10 boys and 8 girls. Three students are selected at random. What is the probability that the selected group has (i) all boys? (ii) all girls? (iii) 1 boy and 2 girls? (iv) at least one girl? (v) at most one girl?

40. Five cards are drawn from a well-shuffled pack of 52 cards. Find the probability that all the five cards are hearts.

41. A bag contains tickets numbered from 1 to 20. Two tickets are drawn. Find the probability that (i) both the tickets have prime numbers on them (ii) on one there is a prime number and on the other there is a multiple of 4.

42. An urn contains 7 white, 5 black and 3 red balls. Two balls are drawn at random. Find the probability that (i) both the balls are red (ii) one ball is red and the other is black (iii) one ball is white.

LEVEL-2

43. A and B throw a pair of dice. If A throws 9, find B's chance of throwing a higher number.

44. In a hand at Whist, what is the probability that four kings are held by a specified player?

45. Find the probability that in a random arrangement of the letters of the word 'UNIVERSITY', the two I's do not come together.

ANSWERS

1. (i), (ii) **2.** (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$ (iii) $\frac{2}{3}$ **3.** (i) $\frac{5}{36}$ (ii) $\frac{1}{6}$ (iii) $\frac{1}{12}$ (iv) $\frac{1}{12}$ (v) $\frac{1}{6}$ (vi) $\frac{1}{2}$ (vii) $\frac{11}{36}$ (viii) $\frac{5}{6}$ (ix) $\frac{5}{18}$ (x) $\frac{5}{12}$ (xi) $\frac{5}{12}$ (xii) $\frac{7}{9}$ (xiii) $\frac{1}{12}$ (xiv) $\frac{1}{9}$ (xv) $\frac{1}{6}$ (xvi) $\frac{5}{18}$ **4.** $\frac{1}{54}$ **5.** (i) $\frac{3}{8}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{4}$

6. $\frac{1}{7}$ **7.** $\frac{1}{7}$ **8.** (i) $\frac{5}{143}$ (ii) $\frac{28}{143}$ (iii) $\frac{40}{143}$ **9.** $\frac{1}{36}$ **10.** $\frac{1}{12}$

11. (i) $\frac{1}{26}$ (ii) $\frac{7}{13}$ (iii) $\frac{1}{26}$ (iv) $\frac{3}{13}$ (v) $\frac{9}{13}$ (vi) $\frac{4}{13}$ (vii) $\frac{11}{13}$ (viii) $\frac{1}{4}$ (ix) $\frac{3}{4}$ (x) $\frac{1}{2}$ (xi) $\frac{12}{13}$ (xii) $\frac{1}{2}$

12. $\frac{2197}{20825}$ **13.** $\frac{4}{270725}$ **14.** $\frac{2}{5}$ **15.** $\frac{4}{17}$

16. (i) $\frac{7}{40}$ (ii) $\frac{1}{6}$ (iii) $\frac{37}{120}$ **17.** (i) $\frac{3}{68}$ (ii) $\frac{7}{34}$ (iii) $\frac{33}{68}$

18. (i) $\frac{3243}{10829}$ (ii) $\frac{18472}{54145}$ **19.** $\frac{1000}{9139}$ **20.** $\frac{3}{5}$ **21.** (i) $\frac{^{20}C_{10}}{^{100}C_{10}}$ (ii) $\frac{^{80}C_{10}}{^{100}C_{10}}$ (iii) $1-\frac{^{80}C_{10}}{^{100}C_{10}}$ (iv) $\frac{^{80}C_{10}}{^{100}C_{10}}$ **22.** $\frac{1}{5}$ **23.** $\frac{2}{7}$

24. $\frac{1}{5}$ **25.** (i) $\frac{1}{6}$ (ii) $\frac{2}{3}$ (iii) $\frac{1}{6}$ **26.** $\frac{2}{5}$ **27.** $\frac{7}{16}$ **28.** 0.78 **29.** (i) $\frac{13}{18}$ (ii) $\frac{1}{3}$ **30.** (i) $\frac{7}{95}$ (ii) $\frac{18}{95}$ **31.** $\frac{31}{153}$ **32.** (i) $\frac{5}{26}$ (ii) $\frac{21}{26}$

33. $\frac{1}{38760}$ **34.** (i) $\frac{1}{4}$ (ii) $\frac{3}{4}$ (iii) $\frac{1}{2}$ (iv) $\frac{2}{5}$ (v) $\frac{1}{5}$ (vi) $\frac{17}{20}$ **35.** (i) 1 : 11 (ii) 1 : 8 (iii) 31 : 5 **36.** (i) 1 : 3, 1 : 12

37. (i) $\frac{^{20}C_5 \times {}^{40}C_0}{^{60}C_5} = \frac{34}{11977}$ (ii) $\frac{4367}{4484}$ **38.** (i) $\frac{2}{5}$ (ii) $\frac{1}{5}$ (iii) $\frac{1}{2}$

(iv) $\frac{4}{5}$ **39.** (i) $\frac{5}{34}$ (ii) $\frac{7}{102}$ (iii) $\frac{35}{102}$ (iv) $\frac{29}{34}$ (v) $\frac{10}{17}$

40. $\frac{^{13}C_5}{^{52}C_5} = \frac{33}{66640}$ **41.** (i) $\frac{14}{95}$ (ii) $\frac{4}{19}$ **42.** (i) $\frac{1}{35}$ (ii) $\frac{1}{7}$ (iii) $\frac{8}{15}$

43. $\frac{1}{6}$ **44.** $\frac{11}{4165}$ **45.** $\frac{4}{5}$

HINTS TO NCERT & SELECTED PROBLEMS

12. Required probability $= \frac{^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{^{52}C_4}$

13. Four honours means king, queen, jack and ace.
So required probability $= \frac{^{4}C_4 + {}^{4}C_4 + {}^{4}C_4 + {}^{4}C_4}{^{52}C_4}$

14. Total number of elementary events $= {}^{20}C_1 = 20$.
A multiple of 3 or 7 can be obtained as follows: 3, 6, 9, 12, 15, 18, 7, 14.
So, favourable number of elementary events $= {}^{8}C_1 = 8$.

15. Total number of elementary events $= {}^{18}C_3$.
Favourable number of elementary events $= {}^{6}C_1 \times {}^{4}C_1 \times {}^{8}C_1$

20. Out of 4 men and 6 women one person can be chosen in ${}^{10}C_1 = 10$ ways.
The number of ways of selecting 1 women out of 6 women $= {}^{6}C_1 = 6$.
$\therefore$ Required probability $= \frac{6}{10} = \frac{3}{5}$

25. A committee of two persons can be formed from two men and two women in ${}^{4}C_2 = 6$ ways.
(i) Number of committees having no man $= {}^{2}C_2 = 1$
$\therefore$ Probability that a committee has no man $= \frac{1}{6}$
(ii) Number of committees having one man $= {}^{2}C_1 \times {}^{2}C_1 = 4$
$\therefore$ Probability that the committee will have one man $= \frac{4}{6} = \frac{2}{3}$
$\therefore$ Probability that the committee has two men $= \frac{1}{6}$

33. Total number of ways of selecting six numbers from numbers 1 to 20 $= {}^{20}C_6$
$\therefore$ Total number of elementry events $= {}^{20}C_6 = 38760$
Favourable number of elementary events $= 1$
$\therefore$ Required probability $= \frac{1}{38760}$

44. Total number of elementary events $= {}^{52}C_{13}$
Favourable number of elementary events $= {}^{48}C_9 \times {}^{4}C_4$

33.8 ADDITION THEOREMS ON PROBABILITY

Uptill now we have been computing the probability of occurrence or non-occurrence of an event by using favourable and total number of elementary events. But it is not always convenient to compute favourable number of elementary events to a given event. In such cases, we express the given event as the union of two or more events and the probability of the given event is expressed in terms of the probabilities of these events. Theorems which express the probability of an event in terms of the probabilities of those events whose union is the given event are known as addition theorems on probability. In this section, we shall discuss addition theorems for two or more events.

THEOREM 1 *(Addition Theorem for two events) If A and B are two events associated with a random experiment, then* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

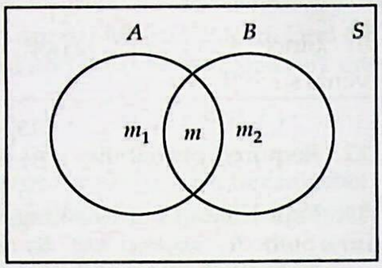


Fig. 33.3

PROOF Let S be the sample space associated with the given random experiment. Suppose the random experiment results in n mutually exclusive ways. Then, S contains n elementary events.

Let m_1, m_2 and m be the number of elementary events favourable to A, B and $A \cap B$ respectively. Then,

$$P(A) = \frac{m_1}{n}, \quad P(B) = \frac{m_2}{n} \text{ and } P(A \cap B) = \frac{m}{n}.$$

The number of elementary events favourable to A only is $m_1 - m$. Similarly, the number of elementary events favourable to B only is $m_2 - m$. Since m elementary events are favourable to both A and B. Therefore, the number of elementary events favourable to A or B or both i.e. $A \cup B$ is $m_1 - m + m_2 - m + m = m_1 + m_2 - m$.

So, $$P(A \cup B) = \frac{m_1 + m_2 - m}{n} = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m}{n}$$

$$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ Q.E.D.

COROLLARY *If A and B are mutually exclusive events, then* $P(A \cap B) = 0$.

$$\therefore \quad P(A \cup B) = P(A) + P(B)$$

This is the addition theorem for mutually exclusive events.

THEOREM 2 *(Addition Theorem for three events) If A, B, C are three events associated with a random experiment, then*

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

PROOF Let $D = B \cup C$. Then,

$$P(A \cup B \cup C) = P(A \cup D) = P(A) + P(D) - P(A \cap D) \quad \text{...(i)}$$

But, $$A \cap D = A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\begin{aligned} \therefore \quad P(A \cap D) &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \quad \text{...(ii)} \end{aligned}$$

and, $$P(D) = P(B \cup C) = P(B) + P(C) - P(B \cap C) \quad \text{...(iii)}$$

From (i), (ii) and (iii), we get

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)]$$

$$\Rightarrow P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Q.E.D.

COROLLARY *If A, B, C are mutually exclusive events, then*

$$P(A \cap B) = P(B \cap C) = P(A \cap C) = P(A \cap B \cap C) = 0.$$

$$\therefore \quad P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

This is the addition theorem for three mutually exclusive events.

THEOREM 3 *Let A and B be two events associated to a random experiment. Then,*

(i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

(ii) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(iii) $P((A \cap \overline{B}) \cup (\overline{A} \cap B)) = P(A) + P(B) - 2P(A \cap B)$

<u>PROOF</u> (i) Since $A \cap B$ and $\overline{A} \cap B$ are mutually exclusive events such that

$$(A \cap B) \cup (\overline{A} \cap B) = B$$

$$\therefore \quad P(A \cap B) + P(\overline{A} \cap B) = P(B)$$

$$\Rightarrow \quad P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

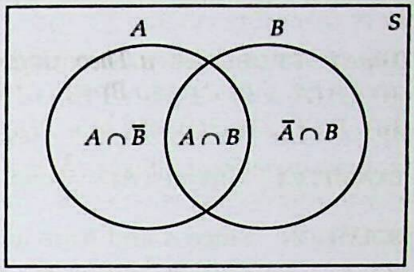


Fig. 33.4

(ii) Since $A \cap B$ and $A \cap \overline{B}$ are mutually exclusive events such that

$$(A \cap B) \cup (A \cap \overline{B}) = A$$

$$\therefore \quad P(A \cap B) + P(A \cup \overline{B}) = P(A)$$

$$\Rightarrow \quad P(A \cap \overline{B}) = P(A) - P(A \cap B).$$

(iii) Since $A \cap \overline{B}$ and $\overline{A} \cap B$ are mutually exclusive events. Therefore,

$$P\{(A \cap \overline{B}) \cup (\overline{A} \cap B)\} = P(A \cap \overline{B}) + P(\overline{A} \cap B)$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B) \qquad \text{[Using (i) and (ii)]}$$

$$= P(A) + P(B) - 2P(A \cap B)$$

Q.E.D.

<u>REMARK 1</u> $P(\overline{A} \cap B)$ is known as the probability of occurrence of B only.

<u>REMARK 2</u> $P(A \cap \overline{B})$ is known as the probability of occurrence of A only.

<u>REMARK 3</u> $P\{(A \cap \overline{B}) \cup (\overline{A} \cap B)\}$ is known as the probability of occurrence of exactly one of two events A and B.

<u>REMARK 4</u> *If A and B are two events associated to a random experiment such that $A \subset B$, then $\overline{A} \cap B \neq \phi$.*

$$\therefore \quad P(\overline{A} \cap B) \geq 0 \Rightarrow P(B) - P(A \cap B) \geq 0 \Rightarrow P(B) - P(A) \geq 0 \Rightarrow P(A) \leq P(B).$$

THEOREM 4 *For any two events A and B, prove that*

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B).$$

<u>PROOF</u> Since $A \cap B \subset A$. Therefore, we have

$$P(A \cap B) \leq P(A). \qquad \text{...(i)}$$

Also, $A \subset A \cup B \Rightarrow P(A) \leq P(A \cup B)$ ...(ii)

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad P(A \cup B) \leq P(A) + P(B) \qquad \text{...(iii)}$$

From (i), (ii) and (iii), we get

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B).$$

Q.E.D.

THEOREM 5 *For any two events A and B, prove that the probability that exactly one of A, B occurs is given by* $P(A) + P(B) - 2P(A \cup B) = P(A \cup B) - P(A \cap B)$.

<u>PROOF</u> We have,

$$P(\text{Exactly one of } A, B \text{ occurs}) = P\{(A \cap \overline{B}) \cup (\overline{A} \cap B)\}$$

$$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B)$$

$$= [P(A) + P(B) - P(A \cap B)] - P(A \cap B)$$

$$= P(A \cup B) - P(A \cap B).$$

Q.E.D.

ILLUSTRATIVE EXAMPLES

LEVEL-1

Type I PROBLEMS BASED UPON FORMULAE

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

EXAMPLE 1 *Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find $P(A$ or $B)$, if A and B are mutually exclusive events.*

SOLUTION Since A and B are mutually exclusive events.

$\therefore \quad P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

EXAMPLE 2 *A and B are two mutually exclusive events of an experiment. If P ('not A') = 0.65, $P(A \cup B) = 0.65$ and $P(B) = p$, find the value of p.*

SOLUTION By addition theorem for mutually exclusive events, we have

$$P(A \cup B) = P(A) + P(B)$$

$\Rightarrow \quad P(A \cup B) = 1 - P(\text{'not } A\text{'}) + P(B) \qquad [\because P(A) = 1 - P(\overline{A})]$

$\Rightarrow \quad 0.65 = 1 - 0.65 + p \Rightarrow p = 0.30$

EXAMPLE 3 *A and B are two non-mutually exclusive events. If $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$, find the values of $P(A \cap B)$ and $P(A \cap \overline{B})$.*

SOLUTION We have, $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$

By addition theorem, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\Rightarrow \quad \frac{1}{2} = \frac{1}{4} + \frac{2}{5} - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = \frac{1}{4} + \frac{2}{5} - \frac{1}{2} = \frac{3}{20}$

$\therefore \quad P(A \cap \overline{B}) = P(A) - P(A \cap B) = \frac{1}{4} - \frac{3}{20} = \frac{1}{10}$

EXAMPLE 4 *If E and F are two events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E$ and $F) = \frac{1}{8}$, find (i) $P(E$ or $F)$ (ii) $P($not E and not F$)$.*

SOLUTION We have,

$$P(E) = \frac{1}{4},\ P(F) = \frac{1}{2} \text{ and } P(E \cap F) = \frac{1}{8}$$

(i) $\quad P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$

(ii) $\quad P(\text{not } E \text{ and } not\ F) = P(\overline{E} \cap \overline{F})$

$$= P(\overline{E \cup F})$$

$$= 1 - P(E \cup F) = 1 - \{P(E) + P(F) - P(E \cap F)\}$$

$$= 1 - \left\{\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right\} = 1 - \frac{5}{8} = \frac{3}{8}$$

EXAMPLE 5 *The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\overline{A}) + P(\overline{B})$.*

SOLUTION We have,

P (At least one of the events A and B occurs) $= 0.6$ i.e. $P(A \cup B) = 0.6$

and, P (A and B occur simultaneously) $= 0.2$ i.e. $P(A \cap B) = 0.2$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow \quad 0.6 = P(A) + P(B) - 0.2$$
$$\Rightarrow \quad 0.6 = 1 - P(\overline{A}) + 1 - P(\overline{B}) - 0.2$$
$$\Rightarrow \quad 0.6 = 2 - 0.2 - [P(\overline{A}) + P(\overline{B})]$$
$$\Rightarrow \quad 0.6 = 1.8 - [P(\overline{A}) + P(\overline{B})]$$
$$\Rightarrow \quad P(\overline{A}) + P(\overline{B}) = 1.8 - 0.6 = 1.2$$

EXAMPLE 6 *Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined:*

(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$ (ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

SOLUTION (i) We have, $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

We know that $P(A \cap B) \le P(A)$ and $P(A \cap B) \le P(B)$. But, for the given probabilities $P(A \cap B) \nleq P(A)$. So, given probabilities are not consistently defined.

(ii) We have,

$$P(A) = 0.5,\ P(B) = 0.4 \text{ and } P(A \cup B) = 0.8$$
$$\therefore \quad P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.4 - 0.8 = 0.1$$

Clearly, $P(A \cap B) \le P(A)$ and $P(A \cap B) \le P(B)$. Hence, the given probabilities are consistently defined.

EXAMPLE 7 *Events E and F are such that P (not E or not F) = 0.25. State whether E and F are mutually exclusive.*

SOLUTION We have, P (not E or not F) $= 0.25$

i.e. $P(\overline{E} \cup \overline{F}) = 0.25 \Rightarrow P(\overline{E \cap F}) = 0.25 \Rightarrow 1 - P(E \cap F) = 0.25 \Rightarrow P(E \cap F) = 1 - 0.25 = 0.75 \ne 0$

Hence, E and F are not mutually exclusive.

EXAMPLE 8 *A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A), it being given that $P(B) = \frac{3}{2} P(A)$ and $P(C) = \frac{1}{2} P(B)$.*

SOLUTION Let $P(A) = p$. Then,

$$P(B) = \frac{3}{2} P(A) \Rightarrow P(B) = \frac{3}{2} p \text{ and } P(C) = \frac{1}{2} P(B) \Rightarrow P(C) = \frac{3}{4} p$$

Since A, B, C are mutually exclusive and exhaustive events associated with a random experiment.

$$\therefore \quad A \cup B \cup C = S$$
$$\Rightarrow \quad P(A \cup B \cup C) = P(S)$$
$$\Rightarrow \quad P(A \cup B \cup C) = 1 \qquad [\because P(S) = 1]$$
$$\Rightarrow \quad P(A) + P(B) + P(C) = 1 \qquad \text{[By addition Theorem]}$$
$$\Rightarrow \quad p + \frac{3}{2} p + \frac{3}{4} p = 1 \Rightarrow p = \frac{4}{13} \Rightarrow P(A) = \frac{4}{13}$$

EXAMPLE 9 *Four candidates A, B, C, D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that (i) C will be selected? (ii) A will not be selected?* **[NCERT EXEMPLAR]**

SOLUTION Let A_1, A_2, A_3 and A_4 be the events that candidates A, B, C and D respectively are selected as school cricket team coach. Then, It is given that

$$P(A_3) = 2\,P(A_4),\ P(A_2) = P(A_3) \text{ and } P(A_1) = 2P(A_2)$$

$\Rightarrow$ $P(A_1) = 4P(A_4),\ P(A_2) = P(A_3) = 2\,P(A_4)$

Clearly, A_1, A_2, A_3 and A_4 are mutually exclusive and exhaustive events. Therefore,

$$A_1 \cup A_2 \cup A_3 \cup A_4 = S$$

$\Rightarrow$ $P(A_1 \cup A_2 \cup A_3 \cup A_4) = P(S)$

$\Rightarrow$ $P(A_1) + P(A_2) + P(A_3) + P(A_4) = 1$

$\Rightarrow$ $4P(A_4) + 2P(A_4) + 2P(A_4) + P(A_4) = 1$

$\Rightarrow$ $9P(A_4) = 1$

$\Rightarrow$ $P(A_4) = \dfrac{1}{9}.$

(i) Required probability $= P(A_3) = 2\,P(A_4) = \dfrac{2}{9}$

(ii) Required probability $= P(\overline{A}_1) = 1 - P(A_1) = 1 - 4\,P(A_4) = 1 - \dfrac{4}{9} = \dfrac{5}{9}$

EXAMPLE 10 *Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes and or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty brakes as well as badly worn tires?* **[NCERT EXEMPLAR]**

SOLUTION Let B be the event that a truck stopped at the roadblock will have faulty brakes and T be the event that it will have badly worn tires.

It is given that $P(B) = 0.23$, $P(T) = 0.24$ and $P(B \cup T) = 0.38$. We have to find $P(B \cap T)$.

We know that

$P(B \cup T) = P(B) + P(T) - P(B \cap T)$ [By addition theorem]

$\Rightarrow$ $P(B \cap T) = P(B) + P(T) - P(B \cup T) = 0.23 + 0.24 - 0.38 = 0.09$

EXAMPLE 11 *The probability of two events A and B are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither A nor B occurs.*

SOLUTION We have, $P(A) = 0.25$, $P(B) = 0.50$ and $P(A \cap B) = 0.14$

$\therefore$ Required probability $= P(\overline{A} \cap \overline{B})$

$= P(\overline{A \cup B})$ $[\because\ (\overline{A \cup B}) = \overline{A} \cap \overline{B}]$

$= 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - (0.25 + 0.50 - 0.14) = 0.39$

Type II PROBLEMS BASED UPON ADDITION THEOREMS OF PROBABILITY BUT CAN BE SOLVED INDEPENDENTLY BY USING THE DEFINITION ONLY

NOTE *Following problems will be solved by using addition theorems but these problems can be solved otherwise also. Students are advised to do these problems without using addition theorem.*

EXAMPLE 12 *Find the probability of getting an even number on the first die or a total of 8 in a single throw of two dice.*

SOLUTION Let S be the sample space associated with the experiment of throwing a pair of dice. Then, $n(S) = 36$.

$\therefore$ Total number of elementary events $= 36$

Let A and B be two events given by

A = Getting an even number on first die, B = Getting a total of 8.

Then, $A \cap B$ = Getting an even number on first die and a total of 8.

Clearly, $A = \{(2, 1), ..., (2, 6), (4, 1), (4, 2), ..., (4, 6), (6, 1), (6, 2), ..., (6, 6)\}$,

$B = \{(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)\}$ and, $A \cap B = \{(2, 6), (6, 2), (4, 4)\}$.

$\therefore \quad P(A) = \frac{18}{36}, P(B) = \frac{5}{36}$ and $P(A \cap B) = \frac{3}{36}$

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

EXAMPLE 13 *A die is thrown twice. What is the probability that at least one of the two throws comes up with the number 4?*

SOLUTION Let S be the sample space associated with the random experiment of throwing a die twice. Then, $n(S) = 36$.

$\therefore$ Total number of elementary events = 36

Consider the events: A = First throw shows 4, B = Second throw shows 4

$\therefore$ $A \cap B$ = First and Second throw show 4 i.e. getting 4 in each throw.

Clearly,

$A = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}, B = \{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)\}$

and, $A \cap B = \{(4, 4)\}$

$\therefore \quad P(A) = \frac{6}{36}, P(B) = \frac{6}{36}$ and $P(A \cap B) = \frac{1}{36}$

$\therefore$ Required probability = Probability that at least one of the two throws shows 4.

$= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$

EXAMPLE 14 *One number is chosen from numbers 1 to 200. Find the probability that it is divisible by 4 or 6?*

SOLUTION Let S be the sample space. Then, $n(S) = 200$.

$\therefore$ Total number of elementary events = 200

Let A be the event that the number selected is divisible by 4 and B be the event that the number selected is divisible by 6. Then,

$A = \{4, 8, 12, ..., 200\}, B = \{6, 12, ..., 198\}$ and $A \cap B = \{12, 24, ..., 192\}$

Clearly,

$n(A) = \frac{200}{4} = 50, \; n(B) = \frac{198}{6} = 33$ and $n(A \cap B) = \frac{192}{12} = 16$

$\therefore \quad P(A) = \frac{50}{200}, \; P(B) = \frac{33}{200}$ and $P(A \cap B) = \frac{16}{200}$

Required probability = P (a number is divisible by 4 or 6)

$= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{50}{200} + \frac{33}{200} - \frac{16}{200} = \frac{67}{200}$

EXAMPLE 15 *A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.*

SOLUTION Consider the following events:

A = Getting a king, B = getting a heart card, C = Getting a red card.

Clearly,

$P(A) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52}, \; P(B) = \frac{^{13}C_1}{^{52}C_1} = \frac{13}{52}, \; P(C) = \frac{^{26}C_1}{^{52}C_1} = \frac{26}{52}$

$P(A \cap B) = P$ (Getting a king of heart) $= \frac{1}{52}$, $P(B \cap C) = P$ (Getting a heart card) $= \frac{13}{52}$

$P(C \cap A) = P$ (Getting a red king) $= \frac{2}{52}$, $P(A \cap B \cap C) = P$ (Getting a king of heart) $= \frac{1}{52}$

Required Probability $= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$

EXAMPLE 16 *A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or a bolt?*

SOLUTION Let A be the event that the item chosen is rusted and B be the event that the item chosen is a bolt. Clearly, there are 200 items in all, out of which 100 are rusted.

$\therefore \quad P(A) = \frac{100}{200}$, $P(B) = \frac{50}{200}$ and $P(A \cap B) = \frac{25}{200}$

Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = \frac{100}{200} + \frac{50}{200} - \frac{25}{200} = \frac{125}{200} = \frac{5}{8}$

EXAMPLE 17 *Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same suit.*

SOLUTION Since 4 cards can be drawn at a time from a pack of 52 cards in ${}^{52}C_4$ ways. Therefore,

Total number of elementary events $= {}^{52}C_4$

Consider the following events:

A = Getting all spade cards; B = Getting all club cards;

C = Getting all diamond cards, and D = Getting all heart cards.

Clearly, A, B, C and D are mutually exclusive events such that

$P(A) = \frac{{}^{13}C_4}{{}^{52}C_4}$, $P(B) = \frac{{}^{13}C_4}{{}^{52}C_4}$, $P(C) = \frac{{}^{13}C_4}{{}^{52}C_4}$ and $P(D) = \frac{{}^{13}C_4}{{}^{52}C_4}$

Required probability $= P(A \cup B \cup C \cup D)$

$= P(A) + P(B) + P(C) + P(D)$ [By addition Theorem]

$= 4\left(\frac{{}^{13}C_4}{{}^{52}C_4}\right) = \frac{44}{4165}$

EXAMPLE 18 *An integer is chosen at random from the numbers ranging from 1 to 50. What is the probability that the integer chosen is a multiple of 2 or 3 or 10?*

SOLUTION Out of 50 integers an integer can be chosen in ${}^{50}C_1$ ways.

$\therefore$ Total number of elementary events $= {}^{50}C_1 = 50$.

Consider the following events:

A = Getting a multiple of 2, B = Getting a multiple of 3 and, C = Getting a multiple of 10.

Clearly,

$A = \{2, 4, ..., 50\}$, $B = \{3, 6, ..., 48\}$, $C = \{10, 20, ..., 50\}$

$A \cap B = \{6, 12, ..., 48\}$, $B \cap C = \{30\}$, $A \cap C = \{10, 20, ..., 50\}$ and, $A \cap B \cap C = \{30\}$

$\therefore \quad P(A) = \frac{25}{50}$, $P(B) = \frac{16}{50}$, $P(C) = \frac{5}{50}$, $P(A \cap B) = \frac{8}{50}$, $P(B \cap C) = \frac{1}{50}$,

$P(A \cap C) = \frac{5}{50}$ and $P(A \cap B \cap C) = \frac{1}{50}$

Required probability $= P(A \cap B \cap C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap A) + P(A \cap B \cap C)$

$$= \frac{25}{50} + \frac{16}{50} + \frac{5}{50} - \frac{8}{50} - \frac{1}{50} - \frac{5}{50} + \frac{1}{50} = \frac{33}{50}$$

EXAMPLE 19 *In an essay competition, the odds in favour of competitors P, Q, R, S are 1 : 2, 1 : 3, 1 : 4, and 1 : 5 respectively. Find the probability that one of them wins the competition.*

SOLUTION Let A, B, C, D be the events that the competitors P, Q, R and S respectively win the competition. Then,

$$P(A) = \frac{1}{3},\ P(B) = \frac{1}{4},\ P(C) = \frac{1}{5} \text{ and } P(D) = \frac{1}{6}$$

Since only one competitor can win the competition. Therefore, A, B, C, D are mutually exclusive events.

$\therefore$ Required probability $= P(A \cup B \cup C \cup D)$

$= P(A) + P(B) + P(C) + P(D)$ [By addition Theorem]

$$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{114}{120}$$

EXAMPLE 20 *(i) Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4? (ii) What is the probability that the sum of the numbers on the two faces is divisible by 3 or 4?*

SOLUTION (i) Let S be the sample space associated with the experiment of throwing a pair of dice. Then, $n(S) = 36$.

$\therefore$ Total number of elementary events = 36

Consider the following events.

A = The sum of the numbers on two faces in divisible by 3

B = The sum of the numbers on two faces is divisible by 4.

Then, $A = \{(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)\}$

$B = \{(2, 2), (1, 3), (3, 1), (2, 6), (6, 2), (4, 4), (3, 5), (5, 3), (6, 6)\}$ and, $A \cap B = \{(6, 6)\}$

$$\therefore \quad P(A) = \frac{12}{36} = \frac{1}{3},\ P(B) = \frac{9}{36} = \frac{1}{4} \text{ and } P(A \cap B) = \frac{1}{36}$$

(i) Required probability $= P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$$= 1 - \{P(A) + P(B) - P(A \cap B)\} = 1 - \left\{\frac{1}{3} + \frac{1}{4} - \frac{1}{36}\right\} = \frac{4}{9}$$

(ii) Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{36} = \frac{5}{9}$

EXAMPLE 21 *An urn contains twenty white slips of paper numbered from 1 through 20, ten red slips of paper numbered from 1 through 10, forty yellow slips of paper numbered from 1 through 40 and ten blue slips of paper numbered from 1 through 10. If these 80 slips of paper are thoroughly shuffled so that each slip has the same probability of being drawn. Find the probabilities of drawing a slip of paper that is (i) blue or white (ii) number 1, 2, 3, 4 or 5 (iii) red or yellow and numbered 1, 2, 3, or 4. (iv) numbered 5, 15, 25 or 35. (v) white and numbered higher than 12 or yellow and numbered higher than 26.*

[NCERT EXEMPLAR]

SOLUTION There are 80 slips of paper out of which one slip can be chosen in ${}^{80}C_1 = 80$ ways.

So, total number of elementary events = 80

(i) There are 10 blue and 20 white slips out of which one slip can be chosen in ${}^{30}C_1 = 30$ ways.

$\therefore$ Favourable number of ways = 30

Hence, P (Drawing a blue or white slip) $= \frac{30}{80} = \frac{3}{8}$.

(ii) Consider the following events:

W = Drawing a white slip numbered 1, 2, 3, 4 or 5,

R = Drawing a red slip numbered 1, 2, 3, 4 or 5,
Y = Drawing a yellow slip numbered 1, 2, 3, 4 or 5,
B = Drawing a blue slip numbered 1, 2, 3, 4 or 5

Clearly, these events are mutually exclusive.

Required probability $= P(W \cup R \cup Y \cup B)$

$$= P(W) + P(R) + P(Y) + P(B) = \frac{5}{80} + \frac{5}{80} + \frac{5}{80} + \frac{5}{80} = \frac{20}{80} = \frac{1}{4}$$

(iii) Consider the following events:

R = Drawing a red slip numbered 1, 2, 3 or 4,
Y = Drawing a yellow slip numbered 1, 2, 3, or 4

Clearly, R and Y are mutually exclusive events such that $P(R) = \frac{4}{80}$ and $P(Y) = \frac{4}{80}$.

Required probability $= P(R \cup Y) = P(R) + P(Y) = \frac{4}{80} + \frac{4}{80} = \frac{1}{10}$

(iv) Consider the following events:

A = Drawing a slip numbered 5, B = Drawing a slip numbered 15
C = Drawing a slip numbered 25, D = Drawing a slip numbered 35

We observe that A, B, C and D are mutually exclusive events such that

$P(A) = \frac{4}{80}$ [$\because$ There are 4 tickets, one of each colour numbered 5]

$P(B) = \frac{2}{80}$ [$\because$ There is one white and one yellow ticket each numbered 15]

$P(C) = \frac{1}{80}$ [$\because$ There is just one yellow ticket numbered 25]

and, $P(D) = \frac{1}{80}$ [$\because$ There is just one yellow ticket numbered 35]

Required probability $= (A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D) = \frac{4}{80} + \frac{2}{80} + \frac{1}{80} + \frac{1}{80} = \frac{1}{10}$

(v) Consider the following events:

A = Drawing a white slip numbered higher than 12
B = Drawing a yellow slip numbered higher than 26

We observe that A and B are mutually exclusive events such that $P(A) = \frac{8}{80}$ and $P(B) = \frac{14}{80}$.

Required probability $= P(A \cup B) = P(A) + P(B) = \frac{8}{80} + \frac{14}{80} = \frac{11}{40}$.

Type III **PROBLEMS WHICH CAN BE SOLVED BY USING ADDITION THEOREMS ONLY**

EXAMPLE 22 *Two cards are drawn from a pack of 52 cards. What is the probability that either both are red or both are kings?*

SOLUTION Out of 52 cards, two cards can be drawn in ${}^{52}C_2$ ways.

So, total number of elementary events $= {}^{52}C_2$.

Consider the following events:

A = Two cards drawn are red cards, B = Two cards drawn are kings.

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [By addition Theorem] ...(i)

Let us now find $P(A)$, $P(B)$ and $P(A \cap B)$.

There are 26 red cards, out of which 2 red cards can be drawn in ${}^{26}C_2$ ways.

$$\therefore \quad P(A) = \frac{^{26}C_2}{^{52}C_2}$$

Since there are 4 kings, out of which 2 kings can be drawn in $^4C_2$ ways.

$$\therefore \quad P(B) = \frac{^{4}C_2}{^{52}C_2}$$

There are 2 cards which are both red and kings.

$\therefore \quad P(A \cap B) =$ Probability of getting 2 cards which are both red and kings.

$=$ Probability of getting 2 red kings $= \dfrac{^{2}C_2}{^{52}C_2}$

Required probability $= P(A) + P(B) - P(A \cap B)$

$$= \frac{^{26}C_2}{^{52}C_2} + \frac{^{4}C_2}{^{52}C_2} - \frac{^{2}C_2}{^{52}C_2} = \frac{325}{1326} + \frac{1}{221} - \frac{1}{1326} = \frac{55}{221}$$

EXAMPLE 23 *A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are defective. If a person takes out 2 at random what is the probability that either both are apples or both are good?*

SOLUTION Out of 30 items, two can be selected in $^{30}C_2$ ways.
So, total number of elementary events $= {^{30}C_2}$.
Consider the following events:

$A =$ Getting two apples, $B =$ Getting two good items

There are 20 apples, out of which 2 can be drawn in $^{20}C_2$ ways.

$$\therefore \quad P(A) = \frac{^{20}C_2}{^{30}C_2}$$

There are 8 defective pieces and the remaining 22 are good. Out of 22 good pieces, two can be selected in $^{22}C_2$ ways.

$$\therefore \quad P(B) = \frac{^{22}C_2}{^{30}C_2}$$

Since there are 15 pieces which are good apples out of which 2 can be selected in $^{15}C_2$ ways.

$\therefore \quad P(A \cap B) =$ Probability of getting 2 pieces which are good apples $= \dfrac{^{15}C_2}{^{30}C_2}$

$$\text{Required probability} = P(A) + P(B) - P(A \cap B) = \frac{^{20}C_2}{^{30}C_2} + \frac{^{22}C_2}{^{30}C_2} - \frac{^{15}C_2}{^{30}C_2} = \frac{316}{435}$$

EXAMPLE 24 *A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If the die is rolled once, determine*

(i) $P(1)$ (ii) $P(1 \text{ or } 3)$ (iii) $P(\text{not } 3)$

SOLUTION Let A, B, C be three events defined by

$A =$ Getting a face with number '1', $B =$ Getting a face with number '2',
$C =$ Getting a face with number '3'

Then, $P(A) = \dfrac{2}{6} = \dfrac{1}{3}$, $P(B) = \dfrac{3}{6} = \dfrac{1}{2}$ and $P(C) = \dfrac{1}{6}$

(i) $P(2) = P(A) = \dfrac{1}{3}$

(ii) $P(1 \text{ or } 3) = P(A \cup C) = P(A) + P(C)$ [$\because$ A and C are mutually exclusive]

$\Rightarrow \quad P(1 \text{ or}, 3) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

(iii) $\quad P(\text{not } 3) = P(\overline{C}) = 1 - P(C) = 1 - \frac{1}{6} = \frac{5}{6}$

EXAMPLE 25 *The probability that a student will receive A, B, C or D grade are 0.40, 0.35, 0.15 and 0.10 respectively. Find the probability that a student will receive*

(i) *B or C grade* (ii) *at most C grade.*

SOLUTION Let E_1, E_2, E_3 and E_4 denote respectively the events that a student will receive A, B, C and D grades. Then,

$$P(E_1) = 0.40,\ P(E_2) = 0.35,\ P(E_3) = 0.15 \text{ and } P(E_4) = 0.10$$

(i) Required probability $= P(E_2 \cup E_3)$

$= P(E_2) + P(E_3)$ [$\because$ E_2 and E_3 are mutually exclusive events]

$= 0.35 + 0.15 = 0.50$

(ii) Required probability = Probability that the student receives C or D grade

$= P(E_3 \cup E_4)$

$= P(E_3) + P(E_4)$ [$\because$ E_3 and E_4 are mutually exclusive]

$= 0.15 + 0.10 = 0.25$

EXAMPLE 26 *The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get plumbing contract is $\frac{4}{7}$. If the probability of getting at least one contract is $\frac{2}{3}$, what is the probability that he will get both?*

SOLUTION Consider the following events:

A = Person gets an electric contract, B = Person gets plumbing contract

Clearly,

$$P(A) = \frac{2}{5},\ P(\overline{B}) = \frac{4}{7} \text{ and } P(A \cup B) = \frac{2}{3}$$

We have to find $P(A \cap B)$. By addition theorem, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad \frac{2}{3} = \frac{2}{5} + \left(1 - \frac{4}{7}\right) - P(A \cap B) \Rightarrow P(A \cap B) = \frac{2}{5} + \frac{3}{7} - \frac{2}{3} = \frac{17}{105}$$

EXAMPLE 27 *If a person visits his dentist, suppose the probability that he will have his teeth cleaned is 0.48, the probability that he will have cavity filled is 0.25, probability that he will have a tooth extracted is 0.20, the probability that he will have a teeth cleaned and cavity filled is 0.09, the probability that he will have his teeth cleaned and a tooth extracted is 0.12, the probability that he will have a cavity filled and tooth extracted is 0.07, and the probability that he will have his teeth cleaned, cavity filled, and tooth extracted is 0.03. What is the probability that a person visiting his dentist will have at least one of these things done to him?* **[NCERT EXEMPLAR]**

SOLUTION Consider the following events:

C = The person will have his teeth cleaned, F = The person will have cavity filled

E = The person will have a tooth extracted

It is given that $P(C) = 0.48$, $P(F) = 0.25$, $P(E) = 0.20$,

$$P(C \cap F) = 0.09,\ P(C \cap E) = 0.12,\ P(E \cap F) = 0.07 \text{ and } P(C \cap F \cap E) = 0.03.$$

$\therefore$ Required probability $= P(C \cup F \cup E)$

$= P(C) + P(F) + P(E) - P(C \cap F) - P(F \cap E) - P(C \cap E) + P(C \cap F \cap E)$

$= 0.48 + 0.25 + 0.20 - 0.09 - 0.12 - 0.07 + 0.03 = 0.68$

EXAMPLE 28 *The probability that a patient visiting a dentist will have a tooth extracted is 0.06, the probability that he will have a cavity filled is 0.2 and the probability that he will have a tooth extracted as well as cavity filled is 0.03. What is the probability of that a patient has either a tooth extracted or a cavity filled?*

SOLUTION Let A be the event that the patient will have his tooth extracted, B the event that he will have a cavity filled.

We have, $P(A) = 0.06,\ P(B) = 0.2$ and $P(A \cap B) = 0.03$

$\therefore$ Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.06 + 0.2 - 0.03 = 0.23$

EXAMPLE 29 *The probability that a person visiting a dentist will have his teeth cleaned is 0.44, the probability that he will have a cavity filled is 0.24. The probability that he will have his teeth cleaned or a cavity filled is 0.60. What is the probability that a person visiting a dentist will have his teeth cleaned and cavity filled?*

SOLUTION Let A be the event that the patient will have his teeth cleaned and B be the event that he will have cavity filled.

We have, $P(A) = 0.44,\ P(B) = 0.24$ and $P(A \cup B) = 0.60$

$\therefore$ Required probability $= P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.44 + 0.24 - 0.60 = 0.08$

EXAMPLE 30 *Probability that Hameed passes in Mathematics is $\frac{2}{3}$ and the probability that he passes in English is $\frac{4}{9}$. If the probability of passing both courses is $\frac{1}{4}$, what is the probability that Hameed will pass in at least one of these subjects?*

SOLUTION Let A be the event that Hameed passes in Mathematics and B be the event that he passes in English.

We have, $P(A) = \frac{2}{3},\ P(B) = \frac{4}{9}$ and $P(A \cap B) = \frac{1}{4}$

$\therefore$ Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{3} + \frac{4}{9} - \frac{1}{4} = \frac{31}{36}$

EXAMPLE 31 *Find the probability of at most two tails or at least two heads in a toss of three coins.*

SOLUTION Consider the following events:

A = Getting at most two tails in a toss of three coins.

B = Getting at least two heads in a toss of three coins.

We have, $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$,

$A = \{HHH, HHT, HTH, THH, TTH, HTT, THT\}$, $B = \{HHT, HTH, THH, HHH\}$

$\therefore$ $P(A) = \frac{7}{8},\ P(B) = \frac{4}{8} = \frac{1}{2}$ and $P(A \cap B) = \frac{4}{8} = \frac{1}{2}$

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{8} + \frac{1}{2} - \frac{1}{2} = \frac{7}{8}$

EXAMPLE 32 *In a town of 6000 people 1200 are over 50 years old and 2000 are female. It is known that 30% of the females are over 50 years. What is the probability that a random chosen individual from the town either female or over 50 years?*

SOLUTION Consider the following events :

A = A randomly chosen individual is a female

B = A randomly chosen individual is over 50 years old.

Clearly,

$$P(A) = \frac{2000}{6000} = \frac{1}{3},\ P(B) = \frac{1200}{6000} = \frac{1}{5}$$

and, $P(A \cap B) = P$ (An individual is a female over 50 years old) $= \frac{30\% \text{ of } 2000}{6000} = \frac{600}{6000} = \frac{1}{10}$

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{10} = \frac{13}{30}$

EXAMPLE 33 *From the employees of a company, 5 persons are elected to represent them in the managing committee of the company. Particulars of the five persons are as follows:* **[NCERT]**

S. No.	*Person*	*Age (in years)*
1	*Male*	30
2	*Male*	33
3	*Female*	46
4	*Female*	28
5	*Male*	41

A person is selected at random from this group to act as a spokespersons. What is the probability that a spokespersons will be either male or over 35 years?

SOLUTION Consider the following events:

A = The spokespersons is a male, B = The spokespersons is over 35 years.

Clearly, $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{5}$

$\therefore$ Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}$

EXAMPLE 34 *In class XI of a school, 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology or both.* **[NCERT]**

SOLUTION Consider the following events:

M = A student studies Mathematics, B = A student studies Biology.

We have,

$$P(M) = \frac{40}{100},\ P(B) = \frac{30}{100} \text{ and } P(M \cap B) = \frac{10}{100}$$

Required probability $= P(M \cup B)$

$$= P(M) + P(B) - P(M \cap B) = \frac{40}{100} + \frac{30}{100} - \frac{10}{100} = \frac{60}{100} = \frac{3}{5}$$

EXAMPLE 35 *Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that:*

(i) *both Anil and Ashima will not qualify the exam.* **[NCERT]**

(ii) *at least one of them will not qualify the exam.*

(iii) *only one of them will qualify the exam.*

SOLUTION Let E and F denote the events that Anil and Ashima will qualify the examination. Then, $P(E) = 0.05$, $P(F) = 0.10$ and $P(E \cap F) = 0.02$

(i) Required probability $= P(\overline{E} \cap \overline{F})$

$$= P(\overline{E \cup F}) = 1 - P(E \cup F)$$

$$= 1 - \{P(E) + P(F) - P(E \cap F)\} = 1 - (0.05 + 0.10 - 0.02) = 0.87$$

(ii) Required probability $= P$ (At least one of them will not qualify the exam)

$= 1 - P$ (Both of them will qualify the exam)

$= 1 - P(E \cap F) = 1 - 0.02 = 0.98$

(iii) P (Only one of them will qualify the exam) $= P(E) + P(F) - 2P(E \cap F)$

$= 0.05 + 0.10 - 2 \times 0.02 = 0.15 - 0.04 = 0.11$

EXAMPLE 36 *In a class of 60 students 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that:*

(i) *the student opted for NCC or NSS* (ii) *the student has opted neither NCC nor NSS*

(iii) *the student has opted NSS but not NCC.* **[NCERT]**

SOLUTION Consider the following events:

A = A student opted NCC, B = A student opted NSS

We have,

$$P(A) = \frac{30}{60},\ P(B) = \frac{32}{60} \text{ and } P(A \cap B) = \frac{24}{60}$$

(i) Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \dfrac{30}{60} + \dfrac{32}{60} - \dfrac{24}{60} = \dfrac{38}{60} = \dfrac{19}{30}$

(ii) Required probability $= P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cap B) = 1 - \dfrac{19}{30} = \dfrac{11}{30}$

(iii) Required probability $= P(\overline{A} \cap B) = P(B) - P(A \cap B) = \dfrac{32}{60} - \dfrac{24}{60} = \dfrac{8}{60} = \dfrac{2}{15}$

EXAMPLE 37 *One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently the sample space S consists of four elementary outcomes as given below.*

S = { John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}

You are told that the chances of John's promotion is same as that of Gurpreet. Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(i) *Determine P (John promoted), P (Rita promoted), P (Aslam promoted), P (Gurpreet promoted)*

(ii) *If A = {John promoted or Gurpreet promoted }, find P(A)* **[NCERT EXEMPLAR]**

SOLUTION (i) Let P (John promoted) $= p$. Then, by hypothesis

P (Gurpreet promoted) $= p$, P (Rita promoted) $= 2p$ and, P (Aslam promoted) $= 4p$.

We have,

S = { John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}

$\therefore$ P (John promoted) $+P$(Rita promoted) $+P$(Aslam promoted) $+P$ (Gurpreet promoted) $= 1$

$\Rightarrow p + 2p + 4p + p = 1$

$\Rightarrow p = \dfrac{1}{8}$

Hence, P(John promoted) $= \dfrac{1}{8}$, P (Rita promoted) $= 2p = \dfrac{2}{8} = \dfrac{1}{4}$,

P (Aslam promoted) $= 4p = \dfrac{4}{8} = \dfrac{1}{2}$ and, P (Gurpreet promoted) $= p = \dfrac{1}{8}$.

(ii) A = { John promoted or Gurpreet promoted }

$\therefore$ $P(A) = P$ (John promoted) $+ P$ (Gurpreet promoted) $= p + p = 2p = \dfrac{2}{8} = \dfrac{1}{4}$

LEVEL-2

EXAMPLE 38 *A, B, C are events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then show that $P(B \cap C)$ lies in the interval (0.23, 0.48).*

SOLUTION We know that the probability of occurrence of an event is always less than or equal to 1 and it is given that $P(A \cup B \cup C) \geq 0.75$

$\therefore \quad 0.75 \leq P(A \cup B \cup C) \leq 1$

$\Rightarrow \quad 0.75 \leq P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \leq 1$

$\Rightarrow \quad 0.75 \leq 0.3 + 0.4 + 0.8 - 0.08 - P(B \cap C) - 0.28 + 0.09 \leq 1$

$\Rightarrow \quad 0.75 \leq 1.59 - 0.36 - P(B \cap C) \leq 1$

$\Rightarrow \quad 0.75 \leq 1.23 - P(B \cap C) \leq 1$

$\Rightarrow \quad -0.48 \leq -P(B \cap C) \leq -0.23 \Rightarrow 0.23 \leq P(B \cap C) \leq 0.48$

EXAMPLE 39 *If A and B are any two events such that* $P(A \cup B) = \frac{1}{2}$ *and* $P(\overline{A}) = \frac{2}{3}$, *find* $P(\overline{A} \cap B)$.

SOLUTION Clearly, $\overline{A} \cap B$ and A are mutually exclusive events such that

$$A \cup B = A \cup (\overline{A} \cap B)$$

$\Rightarrow \quad P(A \cup B) = P(A) + P(\overline{A} \cap B)$

$\Rightarrow \quad \frac{1}{2} = 1 - P(\overline{A}) + P(\overline{A} \cap B)$

$\Rightarrow \quad \frac{1}{2} = 1 - \frac{2}{3} + P(\overline{A} \cap B)$

$\Rightarrow \quad P(\overline{A} \cap B) = \frac{1}{6}$

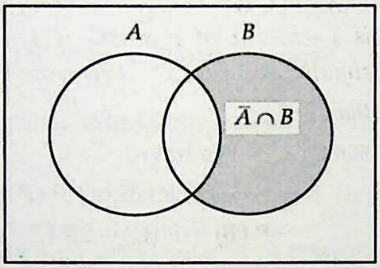


Fig. 33.5

EXAMPLE 40 *Figure 33.6 shows three events A, B and C and also the probabilities of the various intersections (for instance* $P(A \cap B) = 0.07$*) determine*

(i) $P(A)$ (ii) $P(B \cap \overline{C})$ (iii) $P(A \cup B)$

(iv) $P(A \cap \overline{B})$ (v) $P(B \cap C)$ (vi) *Probability of exactly one of the three events.*

[NCERT EXEMPLAR]

SOLUTION (i) We have, $P(A \cap \overline{B}) = 0.13$ and $P(A \cap B) = 0.07$

Since $A \cap \overline{B}$ and $A \cap B$ are mutually exclusive events such that

$$A = (A \cap \overline{B}) \cup (A \cap B).$$

$\therefore \quad P(A) = P(A \cap \overline{B}) + P(A \cap B)$

$\Rightarrow \quad P(A) = 0.13 + 0.07 = 0.20$

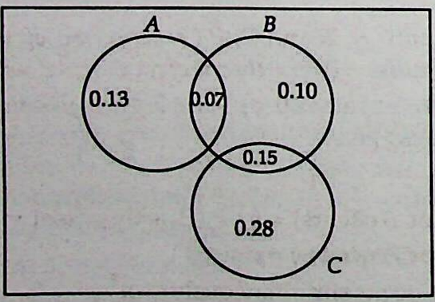


Fig. 33.6

(ii) It is evident from the Figure 33.6 that

$$P(A \cap B) = 0.07, P(B \cap C) = 0.15 \text{ and } P(\overline{A} \cap B \cap \overline{C}) = 0.10$$

Also, $A \cap B$, $B \cap C$ and $\overline{A} \cap B \cap \overline{C}$ are mutually exclusive events such that

$$B = (A \cap B) \cup (B \cap C) \cup (\overline{A} \cap B \cap \overline{C})$$

$\therefore \quad P(B) = P(A \cap B) + P(B \cap C) + P(\overline{A} \cap B \cap \overline{C})$

$\Rightarrow \quad P(B) = 0.07 + 0.15 + 0.10 = 0.32$

Now, $P(B \cap \overline{C}) = P(B) - P(B \cap C)$

$\Rightarrow \quad P(B \cap \overline{C}) = 0.32 - 0.15 = 0.17$

(iii) We have, $P(A) = 0.20$, $P(B) = 0.32$ and $P(A \cap B) = 0.07$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.20 + 0.32 - 0.07 = 0.45$

(iv) We have, $P(A) = 0.20$ and $P(A \cap B) = 0.07$

$\therefore \quad P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.20 - 0.07 = 0.13$

(v) It is evident from the Figure 33.6 that $P(B \cap C) = 0.15$.

(vi) Probability that exactly one of three events A, B and C occurs

$= P(A \cap \overline{B} \cap \overline{C}) + P(\overline{A} \cap B \cap \overline{C}) + P(\overline{A} \cap \overline{B} \cap C) = 0.13 + 0.10 + 0.28 = 0.51$

EXAMPLE 41 *Let A, B, C be three events. If the probability of occuring exactly one event out of A and B is $1 - x$, out of B and C is $1 - 2x$, out of C and A is $1 - x$, and that of occuring three events simultaneously is x^2, then prove that the probability that atleast one out of A, B, C will occur is greater than $1/2$.*

SOLUTION We have,

$$P(A) + P(B) - 2P(A \cap B) = 1 - x \quad \text{...(i)}$$

$$P(B) + P(C) - 2P(B \cap C) = 1 - 2x \quad \text{...(ii)}$$

$$P(C) + P(A) - 2P(C \cap A) = 1 - x \quad \text{...(iii)}$$

and, $$P(A \cap B \cap C) = x^2 \quad \text{...(iv)}$$

Adding (i), (ii) and (iii), we get

$$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3 - 4x}{2} \quad \text{...(v)}$$

$\therefore$ Probability that at least one out of A, B, C will occur

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \dfrac{3 - 4x}{2} + x^2$ [Using (iv) and (v)]

$= x^2 - 2x + \dfrac{3}{2} = (x - 1)^2 + \dfrac{1}{2} > \dfrac{1}{2}$

EXAMPLE 42 *For the three events A, B and C, P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P (exactly one of the events C and A occurs) = p and P (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then, find the probability of occurrence of at least one of the three events A, B, and C.*

SOLUTION It is given that

P (Exactly one of the events A or B occurs) $= p$, P (Exactly one of the events B or C occurs) $= p$

P (Exactly one of the events C or A occurs) $= p$

and, P (All the three events occur simultaneously) $= p^2$

i.e. $$P(A) + P(B) - 2P(A \cap B) = p \quad \text{...(i)}$$

$$P(B) + P(C) - 2P(B \cap C) = p \quad \text{...(ii)}$$

$$P(C) + P(A) - 2P(A \cap B) = p \quad \text{...(iii)}$$

and, $$P(A \cap B \cap C) = p^2 \quad \text{...(iv)}$$

Adding (i), (ii) and (iii), we get

$$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) = \frac{3p}{2} \quad ...(v)$$

Required probability $= P(A \cup B \cup C)$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{3p}{2} + p^2 = \frac{3p + 2p^2}{2}$$

EXAMPLE 43 *For a post three persons A, B and C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is thrice that of C. What are the individual probabilities of A, B, C being selected?*

SOLUTION Let A_1, A_2 and A_3 be three events as defined below:

A_1 = Person A is selected, A_2 = Person B is selected, A_3 = Person C is selected.

We have,

$P(A_1) = 2P(A_2)$ and $P(A_2) = 3P(A_3) \Rightarrow P(A_1) = 6P(A_3)$ and $P(A_2) = 3P(A_3)$.

Since A_1, A_2, A_3 are mutually exclusive and exhaustive events.

$\therefore \quad A_1 \cup A_2 \cup A_3 = S$

$\Rightarrow \quad P(A_1 \cup A_2 \cup A_3) = P(S)$

$\Rightarrow \quad P(A_1) + P(A_2) + P(A_3) = 1 \qquad [\because A_1, A_2, A_3$ are mutually exclusive$]$

$\Rightarrow \quad 6P(A_3) + 3P(A_3) + P(A_3) = 1$

$\Rightarrow \quad 10\,P(A_3) = 1$

$\Rightarrow \quad P(A_3) = \frac{1}{10}$

$\therefore \quad P(A_1) = \frac{6}{10}$ and $P(A_2) = \frac{3}{10}$

EXAMPLE 44 *P and Q are two candidates seeking admission in I.I.T. The probability that P is selected is 0.5 and the probability that both P and Q are selected is at most 0.3. Prove that the probability of Q being selected is at most 0.8.*

SOLUTION Let A_1 and A_2 be two events defined as follows:

$A_1 = P$ is selected, $A_2 = Q$ is selected.

We have, $P(A_1) = 0.5$ and $P(A_1 \cap A_2) \leq 0.3$

Now, $\quad P(A_1 \cup A_2) \leq 1$

$\Rightarrow \quad P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1$

$\Rightarrow \quad 0.5 + P(A_2) - P(A_1 \cap A_2) \leq 1$

$\Rightarrow \quad P(A_2) \leq 0.5 + P(A_1 \cap A_2) \Rightarrow P(A_2) \leq 0.5 + 0.3 \Rightarrow P(A_2) \leq 0.8$

EXAMPLE 45 *A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is atleast one ball of each colour.*

SOLUTION We observe that at least one ball of each colour can be drawn in one of the following mutually exclusive ways :

(i) 1 red, 1 white and 2 black balls. (ii) 2 red, 1 white and 1 black balls.

(iii) 1 red, 2 white and 1 black balls.

Thus, if we define three events A, B and C as follows:

A = Drawing 1 red, 1 white and 2 black balls, B = Drawing 2 red, 1 white and 1 black balls

C = Drawing 1 red, 2 white and 1 black balls

We observe that A, B, C are mutually exclusive events.

$$\therefore \quad \text{Required probability} = P(A \cup B \cup C)$$
$$= P(A) + P(B) + P(C) \quad \text{[By addition Theorem]}$$
$$= \frac{{}^6C_1 \times {}^4C_1 \times {}^5C_2}{{}^{15}C_4} + \frac{{}^6C_2 \times {}^4C_1 \times {}^5C_1}{{}^{15}C_4} + \frac{{}^6C_1 \times {}^4C_2 \times {}^5C_1}{{}^{15}C_4}$$
$$= \frac{6 \times 4 \times 10 + 15 \times 4 \times 5 + 6 \times 6 \times 5}{{}^{15}C_4} = \frac{24 \times 720}{15 \times 14 \times 13 \times 12} = \frac{48}{91}$$

EXAMPLE 46 *A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.*

SOLUTION Let A_i denote the event "Getting number i on the upper face of the die", $i = 1, 2, 3, 4, 5, 6$. Clearly, A_i; $i = 1, 2, \ldots, 6$ are mutually exclusive and exhaustive events.

It is given that $P(A_2) = P(A_4) = P(A_6) = p$ (*say*) and, $P(A_1) = P(A_3) = P(A_5) = 2p$.

Now, $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 = S$

$\Rightarrow \quad P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6) = P(S)$

$\Rightarrow \quad P(A_1) + P(A_2) + P(A_3) + P(A_4) + P(A_5) + P(A_6) = 1$

$\Rightarrow \quad 2p + p + 2p + p + 2p + p = 1 \Rightarrow 9p = 1 \Rightarrow p = \frac{1}{9}$

Now, $G = A_4 \cup A_5 \cup A_6$

$\Rightarrow \quad P(G) = P(A_4 \cup A_5 \cup A_6)$

$\Rightarrow \quad P(G) = P(A_4) + P(A_5) + P(A_6)$ $\quad$ [$\because$ A_4, A_5, A_6 are mutually exclusive]

$\Rightarrow \quad P(G) = p + 2p + p = 4p = \frac{4}{9}$

EXAMPLE 47 *A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple and very simple are respectively 0.15, 0.20, 0.31, 0.26, 0.08. Find the probabilities that a particular surgery will be rated (i) Complex or very complex (ii) neither very complex nor very simple (iii) routine or complex (iv) routine or simple.* **[NCERT EXEMPLAR]**

SOLUTION Consider the following events:

A = Surgery is very complex, B = Surgery is complex, C = Surgery is routine,
D = Surgery is simple, E = Surgery is very simple.

It is given that $P(A) = 0.15$ $P(B) = 0.20$, $P(C) = 0.31$, $P(D) = 0.26$ and $P(E) = 0.08$.

Clearly, A, B, C, D and E are mutually exclusive events.

(i) Required probability $= P(A \cup B) = P(A) + P(B) = 0.15 + 0.20 = 0.35$

(ii) Required probability $= P(\overline{A} \cap \overline{E})$
$$= P(\overline{A \cup E})$$
$$= 1 - P(A \cup E) = 1 - \{P(A) + P(E)\} = 1 - (0.15 + 0.08) = 0.77$$

(iii) Required probability $= P(C \cup B) = P(C) + P(B) = 0.31 + 0.20 = 0.51$

(iv) Required probability $= P(C \cup D) = P(C) + P(D) = 0.31 + 0.26 = 0.57$

EXERCISE 33.4

LEVEL-1

1. (a) If A and B be mutually exclusive events associated with a random experiment such that $P(A) = 0.4$ and $P(B) = 0.5$, then find:
   (i) $P(A \cup B)$ (ii) $P(\overline{A} \cap \overline{B})$ (iii) $P(\overline{A} \cap B)$ (iv) $P(A \cap \overline{B})$.

(b) A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find (i) $P(A \cup B)$ (ii) $P(\overline{A} \cap \overline{B})$ (iii) $P(A \cap \overline{B})$ (iv) $P(B \cap \overline{A})$

(c) Fill in the blanks in the following table:

	$P(A)$	$P(B)$	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	......
(ii)	0.35	....	0.25	0.6
(iii)	0.5	0.35	.....	0.7

2. If A and B are two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, find $P(A \cap B)$.
3. If A and B are two events associated with a random experiment such that $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, find $P(A \cup B)$.
4. If A and B are two events associated with a random experiment such that $P(A \cup B) = 0.8$, $P(A \cap B) = 0.3$ and $P(\overline{A}) = 0.5$, find $P(B)$.
5. Given two mutually exclusive events A and B such that $P(A) = 1/2$ and $P(B) = 1/3$, find P (A or B).
6. There are three events A, B, C one of which must and only one can happen, the odds are 8 to 3 against A, 5 to 2 against B, find the odds against C.
7. One of the two events must happen. Given that the chance of one is two-third of the other, find the odds in favour of the other.

<u>NOTE</u> *Students are advised to do the following exercises by using addition theorems and also by using the definition only i.e. by calculating exhaustive number of cases and favourable number of cases.*

8. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a spade or a king.
9. In a single throw of two dice, find the probability that neither a doublet nor a total of 9 will appear.
10. A natural number is chosen at random from amongst first 500. What is the probability that the number so chosen is divisible by 3 or 5?
11. A die is thrown twice. What is the probability that at least one of the two throws come up with the number 3?
12. A card is drawn from a deck of 52 cards. Find the probability of getting an ace or a spade card.
13. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75. What is the probability of passing the Hindi examination? **[NCERT]**
14. One number is chosen from numbers 1 to 100. Find the probability that it is divisible by 4 or 6?
15. From a well shuffled deck of 52 cards, 4 cards are drawn at random. What is the probability that all the drawn cards are of the same colour.
16. 100 students appeared for two examinations. 60 passed the first, 50 passed the second and 30 passed both. Find the probability that a student selected at random has passed at least one examination.
17. A box contains 10 white, 6 red and 10 black balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either white or red?
18. In a race, the odds in favour of horses A, B, C, D are $1:3, 1:4, 1:5$ and $1:6$ respectively. Find probability that one of them wins the race.
19. The probability that a person will travel by plane is $3/5$ and that he will travel by train is $1/4$. What is the probability that he (she) will travel by plane or train?
20. Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that either both are black or both are kings.

21. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both? **[NCERT]**
22. A box contains 30 bolts and 40 nuts. Half of the bolts and half of the nuts are rusted. If two items are drawn at random, what is the probability that either both are rusted or both are bolts?
23. An integer is chosen at random from first 200 positive integers. Find the probability that the integer is divisible by 6 or 8.
24. Find the probability of getting 2 or 3 tails when a coin is tossed four times.
25. Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9. **[NCERT EXEMPLAR]**
26. In a large metropolitan area, the probabilities are 0.87, 0.36, 0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either any one or both kinds of sets? **[NCERT EXEMPLAR]**
27. If A and B are mutually exclusive events such that $P(A) = 0.35$ and $P(B) = 0.45$, find
(i) $P(A \cup B)$ (ii) $P(A \cap B)$ (iii) $P(A \cap \overline{B})$ (iv) $P(\overline{A} \cap \overline{B})$ **[NCERT EXEMPLAR]**
28. A sample space consists of 9 elementary event $E_1, E_2, E_3, \ldots, E_8, E_9$ whose probabilities are
$P(E_1) = P(E_2) = 0.08,\ P(E_3) = P(E_4) = 0.1,\ P(E_6) = P(E_7) = 0.2,\ P(E_8) = P(E_9) = 0.07$
Suppose $A = \{E_1, E_5, E_8\}$, $B = \{E_2, E_5, E_8, E_9\}$ **[NCERT EXEMPLAR]**
(i) Compute $P(A)$, $P(B)$ and $P(A \cap B)$.
(ii) Using the addition law of probability, find $P(A \cup B)$.
(iii) List the composition of the event $A \cup B$, and calculate $P(A \cup B)$ by adding the probabilities of the elementary events.
(iv) Calculate $P(\overline{B})$ from $P(B)$, also calculate $P(\overline{B})$ directly from the elementary events of $\overline{B}$.

ANSWERS

1. (a) (i) 0.9 (ii) 0.1 (iii) 0.5 (iv) 0.4 1. (b) (i) 0.88 (ii) 0.12 (iii) 0.19 (iv) 0.34
1. (c) (i) $\frac{7}{15}$ (ii) 0.5 (iii) 0.15 2. 0.2 3. 0.6 4. 0.6 5. 5/6
6. 43 : 34 7. 3 : 2 8. 4/13 9. 13/18 10. 233/500 11. $\frac{11}{36}$
12. $\frac{4}{13}$ 13. 0.65 14. $\frac{33}{100}$ 15. $\frac{92}{833}$ 16. $\frac{4}{5}$ 17. $\frac{8}{13}$
18. $\frac{171}{420}$ 19. $\frac{17}{20}$ 20. $\frac{55}{221}$ 21. 0.556 22. $\frac{185}{483}$ 23. $\frac{1}{4}$
24. $\frac{5}{8}$ 25. 0.556 26. 0.93 27. (i) 0.8 (ii) 0 (iii) 0.35 (iv) 0.2
28. (i) 0.25, 0.32, 0.17 (ii) 0.40 (iii) 0.40 (iv) 0.68

HINTS TO NCERT & SELECTED PROBLEMS

1. (ii) Use : $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$
(iii) Use : $P(\overline{A} \cap B) = P(B) - P(A \cap B)$
(iv) Use : $P(A \cap \overline{B}) = P(A) - P(A \cap B)$
6. It is given that A, B, C are mutually exclusive and exhaustive.
$\therefore\ A \cup B \cup C = S \Rightarrow P(A \cup B \cup C) = P(S) \Rightarrow P(A) + P(B) + P(C) = 1$

7. Let A, B be two events. Then A, B are mutually exclusive and exhaustive.

$\therefore \quad A \cup B = S$

$\Rightarrow \quad P(A \cup B) = 1$

$\Rightarrow \quad P(A) + P(B) = 1$

$\Rightarrow \quad \frac{2}{3} P(B) + P(B) = 1 \qquad \left[\because P(A) = \frac{2}{3} P(B)\right]$

$\Rightarrow \quad P(B) = \frac{3}{5}$

Odds in favour of B are $P(B) : P(\overline{B})$ i.e. $3/5 : 2/5$ or $3 : 2$

9. Let A = Getting a doublet, B = Getting a total of 9. Then,

$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

12. A = Getting an ace, B = Getting a spade card.

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

13. Let E and H denote the events that the student will pass in English and Hindi examination respectively. Then, we have

$P(E \cap H) = 0.5$, $P(\overline{E} \cap \overline{H}) = 0.1$ and $P(E) = 0.75$

Now,

$P(\overline{E} \cap \overline{H}) = 0.1$

$\Rightarrow \quad P(\overline{E \cup H}) = 0.1$

$\Rightarrow \quad 1 - P(E \cup H) = 0.1$

$\Rightarrow \quad P(E \cup H) = 0.9$

$\Rightarrow \quad P(E) + P(H) - P(E \cap H) = 0.9 \Rightarrow 0.75 + P(H) - 0.5 = 0.9 \Rightarrow P(H) = 0.65.$

15. Let A = 4 cards drawn are red, B = 4 cards drawn are black. Then, A, B are mutually exclusive events.

So, required probability $= P(A \cup B) = P(A) + P(B)$

21. Let A and B denote the events that a randomly chosen student passes first and second examinations respectively. Then, $P(A) = 0.8$, $P(B) = 0.7$ and $P(A \cup B) = 0.95$

Required probability $= P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.8 + 0.7 - 0.95 = 0.55$

25. Consider the following events:

A = Integer chosen is a multiple of 2, B = Integer chosen is a multiple of 9.

We have,

$$P(A) = \frac{500}{1000} = \frac{1}{2},\ P(B) = \frac{111}{1000} \text{ and } P(A \cap B) = \frac{55}{1000}$$

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{111}{1000} - \frac{55}{1000} = \frac{555}{1000}$

26. Consider the following events:

A = Family owns colours television set, B = Family owns black and white television set

It is given that $P(A) = 0.87$, $P(B) = 0.36$ and $P(A \cap B) = 0.30$

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.87 + 0.36 - 0.30 = 0.93$

27. It is given that A and B are mutually exclusive events. Therefore, $P(A \cap B) = 0$.

(i) $P(A \cup B) = P(A) + P(B) = 0.35 + 0.45 = 0.8$

(ii) $P(A \cap B) = 0$

(iii) $P(A \cap \overline{B}) = P(A) - P(A \cap B) = P(A) = 0.35$

(iv) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cap B}) = 1 - P(A \cup B) = 1 - \{P(A) + P(B)\} = 1 = 0.8 = 0.2$

28. We have, $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) + P(E_7) + P(E_8) = 1 \Rightarrow P(E_5) = 0.1$

(i) $P(A) = P(E_1) + P(E_5) + P(E_8) = 0.08 + 0.1 + 0.07 = 0.25$

$$P(B) = P(E_2) + P(E_5) + P(E_8) + P(E_9) = 0.08 + 0.1 + 0.07 + 0.07 = 0.32$$

$$P(A \cap B) = P(E_5) + P(E_8) = 0.1 + 0.07 = 0.17$$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 025 + 0.32 - 0.17 = 0.40$

VERY SHORT ANSWER QUESTIONS (VSAQs)

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

1. Three numbers are chosen at random from numbers 1 to 30. Write the probability that the chosen numbers are consecutive.
2. $n (> 3)$ persons are sitting in a row. Two of them are selected. Write the probability that they are together.
3. A single letter is selected at random from the word 'PROBABILITY'. What is the probability that it is a vowel?
4. What is the probability that a leap year will have 53 Fridays or 53 Saturdays?
5. Three dice are thrown simultaneously. What is the probability of getting 15 as the sum?
6. If the letters of the word 'MISSISSIPPI' are written down at random in a row, what is the probability that four S's come together.
7. What is the probability that the 13th days of a randomly chosen month is Friday?
8. Three of the six vertices of a regular hexagon are chosen at random. What is the probability that the triangle with these vertices is equilateral.
9. If E and E_2 are independent evens, write the value of $P\left((E_1 \cup E_2) \cap (\bar{E} \cap \bar{E}_2)\right)$.
10. If A and B are two independent events such that $P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, then write the values of $P(A)$ and $P(B)$.

ANSWERS

1. $\frac{144}{145}$ 2. $\frac{2}{n}$ 3. $\frac{4}{11}$ 4. $\frac{3}{7}$ 5. $\frac{13}{216}$ 6. $\frac{4}{165}$ 7. $\frac{1}{84}$ 8. $\frac{1}{10}$

9. 0 10. $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. One card is drawn from a pack of 52 cards. The probability that it is the card of a king or spade is
   (a) 1/26 (b) 3/26 (c) 4/13 (d) 3/13
2. Two dice are thrown together. The probability that at least one will show its digit greater than 3 is
   (a) 1/4 (b) 3/4 (c) 1/2 (d) 1/8
3. Two dice are thrown simultaneously. The probability of obtaining a total score of 5 is
   (a) 1/18 (b) 1/12 (c) 1/9 (d) none of these
4. Two dice are thrown simultaneously. The probability of obtaining total score of seven is
   (a) 5/36 (b) 6/36 (c) 7/36 (d) 8/36
5. The probability of getting a total of 10 in a single throw of two dice is
   (a) 1/9 (b) 1/12 (c) 1/6 (d) 5/36

6. A card is drawn at random from a pack of 100 cards numbered 1 to 100. The probability of drawing a number which is a square is
   (a) 1/5 (b) 2/5 (c) 1/10 (d) none of these.
7. A bag contains 3 red, 4 white and 5 blue balls. All balls are different. Two balls are drawn at random. The probability that they are of different colour is
   (a) 47/66 (b) 10/33 (c) 1/3 (d) 1
8. Two dice are thrown together. The probability that neither they show equal digits nor the sum of their digits is 9 will be
   (a) 13/15 (b) 13/18 (c) 1/9 (d) 8/9
9. Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is
   (a) 11/21 (b) 9/21 (c) 10/21 (d) none of these
10. The probabilities of happening of two events A and B are 0.25 and 0.50 respectively. If the probability of happening of A and B together is 0.14, then probability that neither A nor B happens is
    (a) 0.39 (b) 0.25 (c) 0.11 (d) none of these
11. A die is rolled, then the probability that a number 1 or 6 may appear is
    (a) 2/3 (b) 5/6 (c) 1/3 (d) 1/2
12. Six boys and six girls sit in a row randomly. The probability that all girls sit together is
    (a) 1/122 (b) 1/112 (c) 1/102 (d) 1/132
13. The probabilities of three mutually exclusive events A, B and C are given by 2/3, 1/4 and 1/6 respectively. The statement
    (a) is true (b) is false
    (c) nothing can be said (d) could be either
14. If $\frac{(1-3p)}{2}, \frac{(1+4p)}{3}, \frac{(1+p)}{6}$ are the probabilities of three mutually exclusive and exhaustive events, then the set of all values of p is
    (a) (0,1) (b) (−1/4, 1/3) (c) (0,1/3) (d) $(0, \infty)$
15. A pack of cards contains 4 aces, 4 kings, 4 queens and 4 jacks. Two cards are drawn at random. The probability that at least one of them is an ace is
    (a) 1/5 (b) 3/16 (c) 9/20 (d) 1/9
16. If three dice are throw simultaneously, then the probability of getting a score of 5 is
    (a) 5/216 (b) 1/6 (c) 1/36 (d) none of these
17. One of the two events must occur. If the chance of one is 2/3 of the other, then odds in favour of the other are
    (a) 1 : 3 (b) 3 : 1 (c) 2 : 3 (d) 3 : 2
18. The probability that a leap year will have 53 Fridays or 53 Saturdays is
    (a) 2/7 (b) 3 / 7 (c) 4/7 (d) 1/7
19. A person write 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is
    (a) 1/4 (b) 11/24 (c) 15/24 (d) 23/24
20. A and B are two events such that $P(A) = 0.25$ and $P(B) = 0.50$. The probability of both happening together is 0.14. The probability of both A and B not happening is
    (a) 0.39 (b) 0.25 (c) 0.11 (d) none of these

21. If the probability of A to fail in an examination is $\frac{1}{5}$ and that of B is $\frac{3}{10}$. Then, the probability that either A or B fails is

(a) 1/2 (b) 11/25 (c) 19/50 (d) none of these

22. A box contains 10 good articles and 6 defective articles. One item is drawn at random. The probability that it is either good or has a defect, is

(a) 64/64 (b) 49/64 (c) 40/64 (d) 24/64

23. Three integers are chosen at random from the first 20 integers. The probability that their product is even is

(a) 2/19 (b) 3/29 (c) 17/19 (d) 4/19

24. Out of 30 consecutive integers, 2 are chosen at random. The probability that their sum is odd, is

(a) 14/29 (b) 16/29 (c) 15/29 (d) 10/29

25. A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected randomwise, the probability that it is black or red ball is

(a) 1/3 (b) 1/4 (c) 5/12 (d) 2/3

26. Two dice are thrown simultaneously. The probability of getting a pair of aces is

(a) 1/36 (b) 1/3 (c) 1/6 (d) none of these

27. An urn contains 9 balls two of which are red, three blue and four black. Three balls are drawn at random. The probability that they are of the same colour is

(a) 5/84 (b) 3/9 (c) 3/7 (d) 7/17

28. Five persons entered the lift cabin on the ground floor of an 8 floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first, then the probability of all 5 persons leaving at different floor is

(a) $\frac{{}^7P_5}{7^5}$ (b) $\frac{7^5}{{}^7P_5}$ (c) $\frac{6}{{}^6P_5}$ (d) $\frac{{}^5P_5}{5^5}$

29. A box contains 10 good articles and 6 with defects. One item is drawn at random. The probability that it is either good or has a defect is

(a) 64/64 (b) 49/64 (c) 40/64 (d) 24/64

30. A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, the probability that it is rusted or is a nail is

(a) 3/16 (b) 5/16 (c) 11/16 (d) 14/16

31. If S is the sample space and $P(A) = \frac{1}{3} P(B)$ and $S = A \cup B$, where A and B are two mutually exclusive events, then $P(A) =$

(a) 1/4 (b) 1/2 (c) 3/4 (d) 3/8

32. One mapping is selected at random from all the mappings of the set $A = \{1, 2, 3, \ldots, n\}$ into itself. The probability that the mapping selected is one to one is

(a) $\frac{1}{n^n}$ (b) $\frac{1}{n!}$ (c) $\frac{(n-1)!}{n^{n-1}}$ (d) None of these

33. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that $3P(A) = 2P(B) = P(C)$, then $P(A)$ is equal to

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{5}{11}$ (d) $\frac{6}{11}$

34. If A and B are mutually exclusive events then

(a) $P(A) \leq P(\overline{B})$ (b) $P(A) \geq P(\overline{B})$ (c) $P(A) < P(\overline{B})$ (d) None of these

35. If $P(A \cup B) = P(A \cap B)$ for any two events A and B, then

(a) $P(A) = P(B)$ (b) $P(A) > P(B)$ (c) $P(A) < P(B)$ (d) None of these

36. Three numbers are chosen from 1 to 20. The probability that they are not consecutive is

(a) $\frac{186}{190}$ (b) $\frac{187}{190}$ (c) $\frac{188}{190}$ (d) $\frac{18}{^{20}C_3}$

37. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is

(a) $\frac{1}{432}$ (b) $\frac{12}{431}$ (c) $\frac{1}{132}$ (d) None of these

38. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{30}$ (d) $\frac{5}{9}$

39. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

(a) > 0.5 (b) 0.5 (c) ≤ 0.5 (d) 0

40. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers what is the probability that this number has the same digits?

(a) $\frac{1}{16}$ (b) $\frac{16}{25}$ (c) $\frac{1}{645}$ (d) $\frac{1}{25}$

ANSWERS

1. (c)	2. (b)	3. (c)	4. (b)	5. (b)	6. (c)	7. (a)	8. (b)
9. (c)	10. (a)	11. (c)	12. (d)	13. (b)	14. (b)	15. (c)	16. (c)
17. (d)	18. (b)	19. (d)	20. (a)	21. (c)	22. (a)	23. (c)	24. (c)
25. (d)	26. (a)	27. (a)	28. (a)	29. (a)	30. (c)	31. (a)	32. (c)
33. (b)	34. (a)	35. (a)	36. (b)	37. (c)	38. (d)	39. (c)	40. (d)

SUMMARY

1. An experiment whose outcomes cannot be predicted or determined in advance is called a random experiment.
2. Each outcome of a random experiment is known as an elementary event.
3. The set of all possible outcomes (elementary events) of a random experiment is called the sample space associated with it.
4. A subset of the sample space associated with a random experiment is called an event.
5. An event is said to occur if any one of the elementary events belonging to it is an outcome.
6. An event associated with a random experiment is called a certain event if it always occurs whenever the experiment is performed.
   The sample space associated with a random experiment defines a certain event.
7. The null set of the sample space defines an impossible event.
8. An event associated with a random experiment is a compound event, if it is the disjoint union of two or more elementary events.

9. Two or more events associated with a random experiment are said to be mutually exclusive or incompatible events if the occurrence of any one of them prevents the occurrence of all others i.e. no two or more of them can occur simultaneously in the same trial.
If A and B are mutually exclusive events, then $A \cap B = \phi$.

10. Events $A_1, A_2, A_3, ..., A_n$ associated with a random experiment with sample space S are exhaustive if $A_1 \cup A_2 \cup ... \cup A_n = S$.

11. Let S be the sample space associated with a random experiment. A set of events $A_1, A_2, ..., A_n$ is said to form a set of mutually exclusive and exhaustive system of events if
(i) $A_1 \cup A_2 \cup ... \cup A_n = S$ (ii) $A_i \cap A_j = \phi$ for $i \neq j$

12. *Probability function:* Let $S = \{w_1, w_2, ..., w_n\}$ be the sample space associated with a random experiment. Then a function P which assigns every event $A \subset S$ to a unique non-negative real number $P(A)$ is called the probability function if the following axioms hold :
A - 1 : $0 \leq P(w_i) \leq 1$ for all $w_i \in S$
A - 2 : $P(S) = 1$ i.e. $P(w_1) + P(w_2) + ... + P(w_n) = 1$
A - 3 : For any event $A \subset S$, $P(A) = \sum_{w_k \in A} P(w_k)$, the number $P(w_k)$ is called probability of elementary event w_k.

13. *Probability of an event:* If there are n elementary events associated with a random experiment and m of them are favourable to an event A, then the probability of occurrence of A is defined as:
$$P(A) = \frac{m}{n} = \frac{\text{Favourable number of elementary events}}{\text{Total number of elementary events}}$$
The odds in favour of occurrence of the event A are defined by $m : (n - m)$
The odds against the occurrence of A are defined by $(n - m) : m$.
The probability of non-occurrence of A is given by $P(\overline{A}) = 1 - P(A)$.

14. If A and B are two events associated with a random experiment, then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

15. If A, B, C are three events associated with a random experiment, then
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C$$

16. If A and B are two events associated with a random experiment, then
(i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$ i.e. probability of occurrence of B only $= P(B) - P(A \cap B)$
(ii) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$ i.e. probability of occurrence of A only $= P(A) - P(A \cap B)$
(iii) Probability of occurrence of exactly one of A and B is $P(A) + P(B) - 2P(A \cap B)$
$= P(A \cup B) - P(A \cap B)$