## 1. Plain T<sub>E</sub>Xnology

**Theorem T.** *All things are not necessarily the same\**

## 2. Permutations

TAoCP in chapter 1.2.5 gives two methods to generate all permutations of a given ordered set. Quantites of permutations are considered with relevance to computing efficiencies.

## 3. The Wide-Awake example Group

We re-think\*, re-word, and re-start with a set of attributes, elements or objects, $W = \{$ woozy, vacuous, sleepy, wide-awake $\}$. These elements are used to generate all possible arrangements $\eta$ which are orderd $n$-tuples with $1 \leq n \leq 4$. For example, $\eta = ($ woozy, wide-awake $)$ is a 2-tuple. Now the set $Woozy$ is the set of all permutations that jumble such elements like $\eta$.

Let $(Woozy, \circ, 0, -)$ be the group with the set $Woozy$, a binary operation $\circ$, a neutral elment 0, and for each element $\pi \in Woozy$ there is an inverse element $-\pi \in Woozy$ such that $\pi \circ -\pi = 0$.

For now, here, we call this group's binary operation *composition.* Given two elements $\pi, \eta \in Woozy$, then $\pi \circ \eta \in Woozy$ and $\eta \circ \pi \in Woozy$.

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\* *T<sub>E</sub>Xbook, texbook.tex, https://www.ctan.org/tex-archive/systems/knuth/dist/tex*
\* The Mathematics of the Rubiks Cube, http://web.mit.edu/sp.268/www/rubik.pdf

## 4. Creating the Woozy set

**Theorem X.** *An ordered set of $n$ elements has $n!$ arrangements.*

This had a little consideration. Here, we convey our understanding of the Permutations and Factorials section.*

Given a set of objects $W = \{a_1, a_2, ..., a_n\}$. $P_n$ is the set of arrangements given $n$ objects $a_1, ..., a_n \in W$, such as $\{(a_1, a_2, ...a_n), (a_2, a_1, ...), ...\}$. For example, with $W = \{1, 2, 3\}$, we have

$$P_3 = \{(123), (231), (312), (132), (321), (213)\}.$$

Method 1, now, moves from $n = 3$ to $n = 4$ as follows. For each element in $P_{n-1} = P_3$, place element $a_n$ in each possible vacuous position to arrive at $P_n = P_4$, that is

$$P_4 = \{(a_n a_1 a_2 a_3), (a_1 a_n a_2 a_3), (a_1 a_2 a_n a_3), (a_1 a_2 a_3 a_n), ..., (a_n a_2 a_1 a_3), (a_2 a_n a_1 a_3), (a_2 a_1 a_n a_3), (a_2 a_1 a_3 a_n)\}$$

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\* TAoCP chapter 1.2.5, https://www-cs-faculty.stanford.edu/%7Eknuth/taocp.html

## 5. Accounting for these Arrangements

Adding up all permutations that are so generated we have $p_n$ the number of all elements in $P_n$

And again, after some re-view, we sense a need to re-word. $P_{nn}$ is the set of permuted $n$-tuples, and $P_n$ is the, probably bigger, set of all the $k$-tuples with $k \in \{1, 2, .., n\}$.. In other words, $P_n$ may mean different things, or sets of things. This also applies to quantities that could be denoted like $p_{nk}$, and $p_{nn}$, and in case of our big wide-awake bean bag, which we sum up to $p_n$; probably.

First, we started with $p_n = \sum_{k=1}^{n} k!$ to be the quantity $p_n$ that accounts for all the elements of arrangements in set $P_n$, with $p_k = k!$ for $1 \le k \le n$.

However, on the back of some scrap paper, we jotted down $\{(1), (2), (3), (4)\}$ and saw that $\{(2), (3), (4)\}$ are not included in our sum, and $\{(12), (21), (13), (31), (14), (41), (23), (32), (24), (42), (34), (43)\}$ has 10 2-tuples unaccounted for, etc.)

So, for now, given that $p_{nk} = n(n-1)...(n-k+1)$*, combined with $p_n = \sum_{k=1}^{n} p_{nk}$, we count the number of arrangements of $n$ objects to be $p_n = \sum_{k=1}^{n} \frac{n!}{(n-k)!}$ or some such like.

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\* TAoCP chapter 1.2.5, https://www-cs-faculty.stanford.edu/%7Eknuth/taocp.html

## 6. Making concrete Space

We now look at the set $W$ that we enumerated above and apply the method 1 to arrange things.

Given $W$ as above, we have
$Woozy_{41} = \{(\text{ woozy }), (\text{ vacuus }), (\text{ sleepy }), (\text{ wide-awake })\}$
Then, taking one step at a time and with method 1, given the set
$Woozy_{21} = \{(\text{ sleepy }), (\text{ wide-away })\}$we make
$Woozy_{22} = \{(\text{ sleepy, wide-awake }), (\text{ wide-awake, sleepy })\}$.