

Further to the example\*

$$(123)(231) = (132) \text{ order } 3$$

$(123456) * (123) = (231456)$  and  $(123456) * (231) = (231456)!$  It looks like  $(123) = (231)$ . Yes, they are cycles and therefore  $(123) = (231) = (312)$ .

Order!  $(123456) * (123) = (231456)$  and  $(231456) * (123) = (312456)$ ; now the third application of the permutation  $(123)$  gives  $(312456) * (123) = (123456)$ , back to the initial, standard, permutation. Three applications of  $(123)$  return to the initial state. The *order* of  $(123)$  is 3.

$$\begin{pmatrix} 123456 \\ 231456 \end{pmatrix}, \begin{pmatrix} 231456 \\ 312456 \end{pmatrix}, \begin{pmatrix} 312456 \\ 123456 \end{pmatrix}.$$

Going backwards!  $(123)$  the other way around is  $(321)$ . A canonical cycle would have the smallest element first, and  $(321)$  written in its cononical form is  $(132)$ .

Parity! The permutation 3-cycle  $(123)$  can also be written as a two 2-cycle permutation  $(12)(13)$ , which shows that the *parity* of  $(123)$  is *even*.

$$(123456) * (123) = (231456);$$

$$(123456) * (12) = (213456),$$

$$(213456) * (13) = (231456).$$

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\* <http://web.mit.edu/sp.268/www/rubik.pdf>