The Khrennikov-Haven Model: A Summary

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1 Theory

Khrennikov and Haven (2009) write the law of total probability for the classical case as

$$P(b = \pm|C) = P(a = +|C)P(b = \pm|C_+^A) + P(a = -|C)P(b = \pm|C_-^A) \tag{1}$$

or

$$P(b = \pm|C) = P(a = +|C)P_{\pm,+} + P(a = -|C)P_{\pm,-} \tag{2}$$

and for the quantum case as

$$P(b = \pm|C) = P(a = +|C)P_{\pm,+} + P(a = -|C)P_{\pm,-} + 2\cos\theta_\pm\sqrt{\Pi_\pm} \tag{3}$$

where $P_{\pm,+} \equiv P(b = \pm|C_+^A)$ and $P_{\pm,-} \equiv P(b = \pm|C_-^A)$. The difference between the classical and quantum laws, $2\cos\theta_\pm\sqrt{\Pi_\pm}$, is represented by the variable $\delta_\pm$:

$$\delta_\pm = 2\cos\theta_\pm\sqrt{\Pi_\pm} \tag{4}$$

where

$$\Pi_\pm = P(a = +|C)P(a = -|C)P_{\pm,+}P_{\pm,-}\ . \tag{5}$$

The normalized coefficient of incompatibility of contexts, $\lambda_\pm$, is

$$\lambda_\pm = \frac{\delta_\pm}{2\sqrt{\Pi_\pm}}\ , \tag{6}$$

and the phase $\theta_\pm$ is

$$\theta_\pm = \begin{cases} \arccos\lambda_\pm & \text{when } |\lambda_\pm| \leq 1 \\ \operatorname{arccosh}|\lambda_\pm| & \text{otherwise.} \end{cases} \tag{7}$$

Note that we have added $\pm$ to a few of the terms (e.g., Π is written $\Pi_\pm$) to emphasize the fact that they have more than one value and have been a bit more explicit about the context (e.g., writing $P(b = \pm|C)$ instead of $P(b = \pm)$) in some cases.

2 Tversky and Shafir (1992) Gambling Experiment

This experiment, discussed in Khrennikov and Haven (2009) and in Khrennikov (2014), maps onto the theory developed in Section 1 as follows. The results of Tversky and Shafir (1992) are expressed as $P(b=+|C)=0.36$ and $P(b=-|C)=1-P(b=+|C)=0.64$. The transition probabilities are

$$P=\begin{pmatrix} P_{+,+} & P_{+,-} \\ P_{-,+} & P_{-,-} \end{pmatrix}=\begin{pmatrix} 0.69 & 0.59 \\ 0.31 & 0.41 \end{pmatrix} \tag{8}$$

Give the equal *a prior* probabilities of

$$P(a=+|C)=P(a=-|C)=\tfrac{1}{2} \tag{9}$$

and writing the difference between the classical and quantum laws of total probability as

$$\delta\pm=P(b=\pm|C)-P(a=+|C)P_{\pm,+}-P(a=-|C)P_{\pm,-} \tag{10}$$

we have

$$\begin{aligned}
\delta_+ &= P(b=+|C)-P(a=+|C)P_{+,+}-P(a=-|C)P_{+,-} && (11)\\
&= 0.36-\tfrac{1}{2}\times 0.69-\tfrac{1}{2}\times 0.59 && (12)\\
&= -0.28\ , && (13)
\end{aligned}$$

$$\begin{aligned}
\delta_- &= P(b=-|C)-P(a=+|C)P_{-,+}-P(a=-|C)P_{-,-} && (14)\\
&= 0.64-\tfrac{1}{2}\times 0.31-\tfrac{1}{2}\times 0.41 && (15)\\
&= +0.28\ . && (16)
\end{aligned}$$

Proceeding similarly for $\Pi_\pm$ we have

$$\begin{aligned}
\Pi_+ &= P(a=+|C)P(a=-|C)P_{+,+}P_{+,-} && (17)\\
&= \tfrac{1}{2}\times\tfrac{1}{2}\times 0.69\times 0.59 = 0.101775\ , && (18)
\end{aligned}$$

and

$$\begin{aligned}
\Pi_- &= P(a=+|C)P(a=-|C)P_{-,+}P_{-,-} && (19)\\
&= \tfrac{1}{2}\times\tfrac{1}{2}\times 0.31\times 0.41 = 0.031775\ , && (20)
\end{aligned}$$

from which we see that the normalized coefficient of incompatibility of contexts are

$$\lambda_+=\frac{\delta_+}{2\sqrt{\Pi_+}}=\frac{-0.28}{2\sqrt{0.101775}}=-0.4388\approx-0.44 \tag{21}$$

and

$$\lambda_-=\frac{\delta_-}{2\sqrt{\Pi_-}}=\frac{+0.28}{2\sqrt{0.031775}}=+0.785\approx+0.79\ . \tag{22}$$

From the normalized coefficient of incompatibility of contexts the phases follow as

$$\theta_+=\arccos(\lambda_+)=2.025\text{ radians }\approx 2.03\text{ radians} \tag{23}$$

and

$$\theta_-=\arccos(\lambda_-)=0.667\text{ radians }\approx 0.67\text{ radians} \tag{24}$$

3 Status

I've used the approach illustrated above to successfully reproduce the results given in Section 7.6 of Khrennikov (2014) for both of the TverskyShafir gambling experiments and for the ShafirTversky PD experiment. I have not been able to get the parameters for the Tversky-Shafir Hawaii experiment in Section 7.4.3 of Khrennikov (2014) to generate the results in Section 7.6, nor have yet been able to get the parameters for the Croson (1999) experiment in Section 7.4.3 of Khrennikov (2014) to generate the results in Appendix F of Busemeyer and Bruza (2012).

References

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