Initial Summary of Results

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1 Summary of Initial Results and Methods

The following is an application of research on probability interference (derived from quantum cognition) by [Busemeyer and Bruza, 2014] and [Khrennikov and Haven, 2009] on projection bias. In particular, we study these effects on data on projection bias from [Read and Leeuwen, 1998]. To help the reader understand how I extend probability interference to projection bias, I am going to parallel my results with that of Khrennikov and Haven's on [Tversky and Shafir, 1992].

In [Tversky and Shafir, 1992], participants are asked about a series of two gambles (in which they have an equal chance to win \$200 or lose \$100) with three possible scenarios:

- they are informed they have won the first gamble, then chose whether to gamble
- they are informed they have lost the first gamble, then chose whether to gamble
- they are not informed about the outcome of the first gamble, then chose whether to gamble

In [Read and Leeuwen, 1998], participants are asked to predict whether they want a healthy or unhealthy snack one week in the future. In particular, the researchers have subjects make a decision when they are either in a hungry or satiated state. Then, one week later, the researchers returned with the snacks. Again, they ensure that subjects are in either a hungry or satisfied state. When subjects came for their snack, the researchers then gave them the option to break their earlier decision and change the snacks.

We can think of projecting one's hunger state as a 'gamble.' Contrasting it to [Tversky and Shafir, 1992], we can again get 3 possible scenarios:

- subjects were hungry when they made their snack decision, then receive chosen snack one week later
- subjects were satiated when they made their snack decision, then receive chosen snack one week later

- subjects pick a snack irrespective of their past hunger state

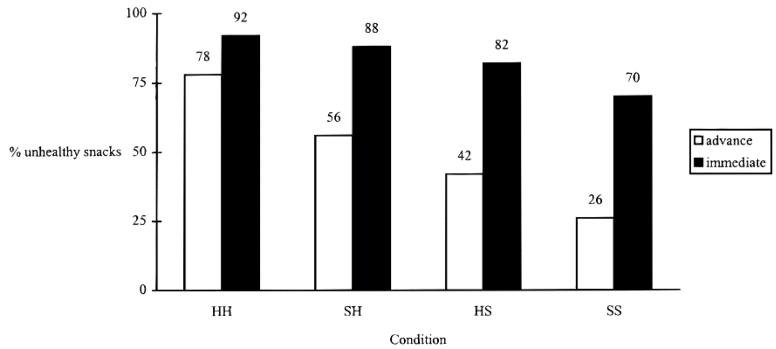

FIG. 1. Percentage of unhealthy snacks chosen in all conditions

Some notation:

- H means a subject was in a hungry state.
- S means a subject was in a satisfied state. Complement of H.
- unh means subject picked a healthy snack
- h means subject picked an unhealthy snack. Complement of unh.

Using data from the HH and SS groups, we have $P(b = unh|C) = \frac{0.92+0.88}{2} = 0.90$ and $P(b = h|C) = 1 - 0.90 = 0.10$. This the probability a subject who is hungry picks an unhealthy snack. Note I take the average here as we are considering subjects from both the HH and SH groups. I chose the HH and SH groups because the state in which the immediate decision is made is the same (Hungry). The transition probabilities are:

$$P = \begin{pmatrix} P_{H,unh} & P_{H,h} \\ P_{S,unh} & P_{S,h} \end{pmatrix} = \begin{pmatrix} .78 & .56 \\ .22 & .44 \end{pmatrix} \tag{1}$$

For example, the $P_{H,unh}$ term in the matrix above is the probability that someone who was hungry a week ago picked an unhealthy snack.

We also have the prior probabilities of $P(a = S|C) = P(a = H|C) = 0.50$. *Note: is this correct? I think so since there was likely a 50/50 chance of being in either group?*

Recall the difference δ between the classical and quantum laws of total probability is:

$$\pm\delta = P(b = (h \vee unh)|C) - P(a = H|C) \cdot P_{H\vee S,unh} - P(a = S|C)P_{H\vee S,h} \tag{2}$$

Thus, we have:

$$\delta_{unh} = P(b = unh|C) - P(a = H|C)P_{H,unh} - P(a = S|C)P_{H,h} \quad (3)$$
$$\delta_{unh} = 0.90 - 0.50 \cdot 0.78 - 0.56 \cdot 0.44 \quad (4)$$
$$\delta_{unh} = 0.23 \quad (5)$$

Likewise,

$$\delta_h = -0.23 \quad (6)$$
$$(7)$$

Now, for $\Pi_{h \vee unh}$ we have:

$$\Pi_{unh} = P(a = unh|C)P(a = h|C)P_{H,unh}P_{H,h} \quad (8)$$
$$= 0.50 \cdot 0.50 \cdot 0.78 \cdot .56 \quad (9)$$
$$= 0.109 \quad (10)$$

and

$$\Pi_h = P(a = unh|C)P(a = h|C)P_{S,unh}PS,h \quad (11)$$
$$= 0.50 \cdot 0.50 \cdot .22 \cdot .44 \quad (12)$$
$$= .0242 \quad (13)$$

From this, we can get the normalized coefficient of incompatibility:

$$\lambda_{unh} = \frac{\delta_{unh}}{2\sqrt{\Pi_{unh}}} \quad (14)$$
$$\lambda_{unh} = 0.348 \quad (15)$$

and

$$\lambda_h = \frac{\delta_h}{2\sqrt{\Pi_h}} \quad (16)$$
$$\lambda_h = -0.739 \quad (17)$$

Lastly, from the normalized coefficient of incompatibility of contexts we can get the phases:

$$\theta_{unh} = \arccos(\lambda_{unh}) = 1.215 \ radians \quad (18)$$
$$\theta_h = \arccos(\lambda_h) = 2.402 \ radians \quad (19)$$

References

[Busemeyer and Bruza, 2014] Busemeyer, J. R. and Bruza, P. D. (2014). *Quantum models of cognition and decision*. Cambridge Univ. Press.

[Khrennikov and Haven, 2009] Khrennikov, A. Y. and Haven, E. (2009). Quantum mechanics and violations of the sure-thing principle: The use of probability interference and other concepts. *Journal of Mathematical Psychology*, 53(5):378388.

[Read and Leeuwen, 1998] Read, D. and Leeuwen, B. V. (1998). Predicting hunger: The effects of appetite and delay on choice. *Organizational Behavior and Human Decision Processes*, 76(2):189205.

[Tversky and Shafir, 1992] Tversky, A. and Shafir, E. (1992). The disjunction effect in choice under uncertainty. *Psychological Science*, 3(5):305310.