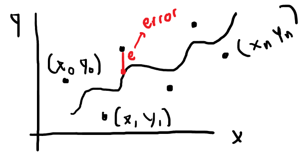
Week 3

Saturday, February 19, 2022 12:30 PM

If we have $N \to \infty$ points we can fit a polynomial

Polynomial Regression



Normally, it can be an interpolation, but only if points form a continuous function. Otherwise, is an approximation.

$$a_0 + a_1 x_0 + a_2 x_0^2 + a_3 x_0^3 + \dots + a_N x_0^N = y$$

$$a_0 + a_1 x_1 + a_2 x_1^2 + a_3 x_1^3 + \dots + a_N x_1^N = y_1$$

$$a_0 + a_1 x_N + a_2 x_1^2 + \dots + a_N x_N^N = y_N$$

¿$a_0, a_1, a_2, \dots a_N$?

To matrix notation

Vodemor Matrix

$$\underbrace{\begin{bmatrix} 1 & x_0 & x_0^2 & x_0^3 & \dots & x_0^N \\ 1 & x_1 & x_1^2 & \dots & & \\ 1 & & & & & \\ 1 & & & & & \\ \vdots & & & & & x_N^N \end{bmatrix}}_{\mathbb{X}} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_N \end{bmatrix}}_{\vec{a}} = \underbrace{\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{bmatrix}}_{\vec{y}}$$

To vector notation

$$\mathbb{X}\vec{a} = \vec{y}$$

The error

error vector

$$e = f(x_0) - y_0$$
$$\vec{e} = f(\vec{x}) - \vec{y}$$
$$\underbrace{\vec{e}^T\vec{e}}_{\text{"square"}} = \text{scalar}$$

Minimizing $\vec{e}$ with respect to a

↳ differentiate and equal to $\varnothing$

$$\frac{\partial}{\partial a}\left(\vec{e}^T\vec{e}\right) = 0$$

$$\frac{\partial}{\partial \vec{a}}\left((\mathbb{X}\vec{a} - \vec{y})^T(\mathbb{X}\vec{a} - \vec{y})\right) = 0$$

$$\frac{\partial}{\partial \vec{a}}\left((\mathbb{X}\vec{a})^T\mathbb{X}\vec{a} - (\mathbb{X}\vec{a})^T\vec{y} - (\vec{y})^T\mathbb{X}\vec{a} + (\vec{y})^T\vec{y}\right) = 0 \qquad \boxed{(\vec{y}^T)^T = \vec{y}}$$

↳ $((\vec{y})^T\mathbb{X}\vec{a})^T = \vec{a}\mathbb{X}^T\vec{y}$

Taking on account $(\mathbb{A}\mathbb{B})^T = \mathbb{B}^T\mathbb{A}^T$

$$\frac{\partial}{\partial \vec{a}}\left((\vec{a}^T\mathbb{X}^T)\mathbb{X}\vec{a} - \vec{a}^T\mathbb{X}^T\vec{y} - \vec{a}\mathbb{X}^T\vec{y} + \vec{y}^T\vec{y}\right) = 0$$

$$\frac{\partial}{\partial \vec{a}}\left((\underbrace{\vec{a}^T\mathbb{X}^T}_{[\text{vector}]})\underbrace{\mathbb{X}\vec{a}}_{[\text{vector}]} - 2(\vec{a}^T\mathbb{X}^T\vec{y}) + \vec{y}^T\vec{y}\right) = 0$$

$= \text{vector}^2 \to f' = 2\underbrace{\text{vector}}_{\mathbb{X}^T\mathbb{X}\vec{a}}$

$$\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1^2 + 2^2$$

↳ bilinear forms

$$2\mathbb{X}^T\mathbb{X}\vec{a} - 2\mathbb{X}^T\vec{y} = 0$$

$$\mathbb{X}^T\mathbb{X}\vec{a} = \mathbb{X}^T\vec{y}$$

$$\boxed{\vec{a} = (\mathbb{X}^T\mathbb{X})^{-1}\mathbb{X}^T\vec{y}} \to \text{least squares solution}$$

How can we guarantee the inverse matrix exists?

↳ if the system has solution

no solution: equal points, to few points