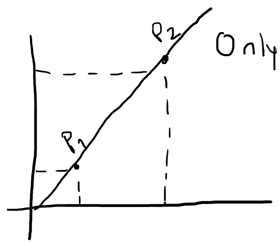
Week2

Saturday, February 12, 2022 12:30 PM

Deep learning



Only one line passes through 2 points

$f(x) = mx + b$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-1}{5-1} = \frac{9}{4}$

$1 = \frac{9}{4} \cdot 1 + b$

$1 - \frac{9}{4} = b$

$b = -\frac{5}{4}$

What happens when we have 2 or more points?

Infinite lines pass through N points

↳ the best line is the one that minimizes the error

minimize with respect to m,b

$$\min_{m,b} \sum_{i=1}^{N} e^2$$

$$= \min_{m,b} \sum_{i=1}^{N} (y_i - f(x_i))^2$$

$$\frac{\partial}{\partial m}\left(\sum_{i=1}^{N} (y_i - mx_i - b)^2\right) = 0 \qquad \text{Eq (1)}$$

$$\frac{\partial}{\partial m}\left(\sum_{i=1}^{N} (y_i - mx_i - b)^2\right) = 0 \qquad \text{Eq. (2)}$$

If we differentiate Eq (1) with respect to m

$$\frac{\partial}{\partial m} \sum_{i=1}^{N} \left((y_i - mx_i - b)^2\right) = 0$$

$$2 \sum_{i=1}^{N} (y_i - mx_i - b)(-x_i) = 0 \Rightarrow$$

Similar to

$2w = 0$

$w = \frac{0}{2}$

$w = 0$ if $w = \sum_{i=1}^{N} (y_i - mx_i - b)(-x_i) = 0$

$$\sum_{i=1}^{N} (-y_i x_i + m x_i^2 + x_i b) = 0$$

$$-\sum_{i=1}^{N} x_i y_i + m \sum x_i^2 + b \sum x_i = 0$$

And solve for b, $b = \dfrac{\sum_{i=1}^{N} x_i y_i - m \sum_{i=1}^{N} x_i^2}{\sum_{i=1}^{N} x_i}$

Thus Eq. 1 $= b = \dfrac{\sum x_i y_i - m \sum x_i^2}{2 x_i}$

If we differentiate Eq. 2

$$\frac{\partial}{\partial b} \sum (y_i - mx_i - b)^2 = 0$$

$$\sum 2(y_i - mx_i - b)\frac{\partial}{\partial b}(y_i - mx_i - b) = 0$$

$$\sum (y_i - mx_i - b)(-1) = 0$$

$$\sum (y_i - mx_i - b) = 0$$

$$\sum y_i - m\sum x_i - \underbrace{\sum b}_{Nb} = 0$$

$$Nb = m\sum x_i - \sum y_i$$

$$b = \frac{-m\sum x_i + \sum y_i}{N}$$

thus, Eq (2) $\Rightarrow$ $b = \dfrac{-m\sum x_i + \sum y_i}{N}$

Now if we perform Eq(1) = Eq(2)

$$\boxed{(1)\ b = \frac{\sum x_i y_i - m\sum x_i^2}{\sum x_i}} \quad \boxed{(2)\ b = \frac{\sum y_i - m\sum x_i}{N}}$$

$$\frac{\sum x_i y_i - m\sum x_i^2}{\sum x_i} = \frac{\sum y_i - m\sum x_i}{N}$$

$$N(\sum x_i y_i - m\sum x_i^2) = \sum x_i (\sum y_i - m\sum x_i)$$

$$N\sum x_i y_i - N\sum x_i^2 = \sum x_i \sum y_i - \underbrace{\sum x_i\, m\sum x_i}_{m(\sum x_i)^2}$$

$$m((\sum x_i)^2 - N\sum x_i^2) = \sum x_i \sum y_i - N\sum x_i y_i$$

$$\boxed{m = \frac{\sum x_i \sum y_i - N\sum x_i y_i}{(\sum x_i)^2 - N\sum x_i^2}}$$