Homework 03: Value Iteration

1. Given the world defined by the following Transition Function `fmt(s, a)`, the Reward Function `fr(s, a, sf)` and `gamma = 0.9`:

$$f_{M_T}(s,a) = \begin{array}{c|cc|} & a_1 & a_2 \\ s_1 & s_2 & s_2 \\ s_2 & s_1 & s_3 \\ s_3 & s_3 & s_1 \\ s_4 & s_1 & s_4 \end{array} \qquad f_R(s_f) = \begin{array}{c|c|} s_1 & 2 \\ s_2 & 1 \\ s_3 & -1 \\ s_4 & 10 \end{array}$$

Figure 1: Image

a. Solve the Bellman Optimality Equations by Value Iteration for V(s).

Solution

- It was solved in 45 iterations, if we assume that **a change of less than 0.1 between iterations** means it converged:

```
==========================
Iteration 1:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
0.00    0.00    0.00    0.00
V(s) new
1.00    2.90    2.90    10.00
==========================
Iteration 2:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
1.00    2.90    2.90    10.00
V(s) new
3.61    5.25    5.25    19.00
==========================
Iteration 3:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
3.61    5.25    5.25    19.00
V(s) new
5.72    7.15    7.15    27.10
==========================
Iteration 4:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
5.72    7.15    7.15    27.10
V(s) new
7.44    8.69    8.69    34.39
==========================
Iteration 5:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
7.44    8.69    8.69    34.39
V(s) new
8.82    9.94    9.94    40.95
==========================
Iteration 6:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
8.82    9.94    9.94    40.95
V(s) new
9.95    10.95   10.95   46.86
==========================
Iteration 7:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
9.95    10.95   10.95   46.86
V(s) new
10.86   11.77   11.77   52.17
==========================
Iteration 8:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
10.86   11.77   11.77   52.17
V(s) new
11.59   12.43   12.43   56.95
==========================
Iteration 9:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
11.59   12.43   12.43   56.95
V(s) new
12.19   12.97   12.97   61.26
==========================
Iteration 10:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
```

```
V(s) current
12.19   12.97   12.97   61.26
V(s) new
12.68   13.41   13.41   65.13
==========================
Iteration 11:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
12.68   13.41   13.41   65.13
V(s) new
13.07   13.76   13.76   68.62
==========================
Iteration 12:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
13.07   13.76   13.76   68.62
V(s) new
13.38   14.05   14.05   71.76
==========================
Iteration 13:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
13.38   14.05   14.05   71.76
V(s) new
13.64   14.28   14.28   74.58
==========================
Iteration 14:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
13.64   14.28   14.28   74.58
V(s) new
13.85   14.46   14.46   77.12
==========================
Iteration 15:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
13.85   14.46   14.46   77.12
V(s) new
14.02   14.62   14.62   79.41
==========================
Iteration 16:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.02   14.62   14.62   79.41
V(s) new
```

```
14.15   14.74   14.74   81.47
==========================
Iteration 17:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.15   14.74   14.74   81.47
V(s) new
14.27   14.84   14.84   83.32
==========================
Iteration 18:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.27   14.84   14.84   83.32
V(s) new
14.35   14.92   14.92   84.99
==========================
Iteration 19:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.35   14.92   14.92   84.99
V(s) new
14.43   14.98   14.98   86.49
==========================
Iteration 20:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.43   14.98   14.98   86.49
V(s) new
14.49   15.04   15.04   87.84
==========================
Iteration 21:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.49   15.04   15.04   87.84
V(s) new
14.53   15.08   15.08   89.06
==========================
Iteration 22:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.53   15.08   15.08   89.06
V(s) new
14.57   15.12   15.12   90.15
==========================
Iteration 23:
```

```
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.57   15.12   15.12   90.15
V(s) new
14.60   15.14   15.14   91.14
==========================
Iteration 24:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.60   15.14   15.14   91.14
V(s) new
14.63   15.17   15.17   92.02
==========================
Iteration 25:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.63   15.17   15.17   92.02
V(s) new
14.65   15.18   15.18   92.82
==========================
Iteration 26:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.65   15.18   15.18   92.82
V(s) new
14.67   15.20   15.20   93.54
==========================
Iteration 27:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.67   15.20   15.20   93.54
V(s) new
14.68   15.21   15.21   94.19
==========================
Iteration 28:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.68   15.21   15.21   94.19
V(s) new
14.69   15.22   15.22   94.77
==========================
Iteration 29:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
```

```
14.69   15.22   15.22   94.77
V(s) new
14.70   15.23   15.23   95.29
============================
Iteration 30:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.70   15.23   15.23   95.29
V(s) new
14.71   15.24   15.24   95.76
============================
Iteration 31:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.71   15.24   15.24   95.76
V(s) new
14.71   15.24   15.24   96.18
============================
Iteration 32:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.71   15.24   15.24   96.18
V(s) new
14.72   15.25   15.25   96.57
============================
Iteration 33:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.72   15.25   15.25   96.57
V(s) new
14.72   15.25   15.25   96.91
============================
Iteration 34:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.72   15.25   15.25   96.91
V(s) new
14.72   15.25   15.25   97.22
============================
Iteration 35:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.72   15.25   15.25   97.22
V(s) new
14.73   15.25   15.25   97.50
```

```
===========================
Iteration 36:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.25   15.25   97.50
V(s) new
14.73   15.26   15.26   97.75
===========================
Iteration 37:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   97.75
V(s) new
14.73   15.26   15.26   97.97
===========================
Iteration 38:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   97.97
V(s) new
14.73   15.26   15.26   98.18
===========================
Iteration 39:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   98.18
V(s) new
14.73   15.26   15.26   98.36
===========================
Iteration 40:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   98.36
V(s) new
14.73   15.26   15.26   98.52
===========================
Iteration 41:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   98.52
V(s) new
14.73   15.26   15.26   98.67
===========================
Iteration 42:
fr: [2, 1, -1, 10]
```

```
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   98.67
V(s) new
14.73   15.26   15.26   98.80
==========================
Iteration 43:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   98.80
V(s) new
14.73   15.26   15.26   98.92
==========================
Iteration 44:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.73   15.26   15.26   98.92
V(s) new
14.74   15.26   15.26   99.03
==========================
Iteration 45:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
V(s) current
14.74   15.26   15.26   99.03
V(s) new
14.74   15.26   15.26   99.13
Optimal Politic:
s1 = a1,        s2 = a1,        s3 = a2,        s4 = a2,
```

Which gives out a result of:

	s1	s2	s3	s4
V(s)	14.74	15.26	15.26	99.13
f_pi(s)	a1	a1	a2	a2

Below is a convergence plot for all V(s) values for each of the states s = {s1, s2, s3, s4}, where x axis is the **amount of iterations** and y axis is the **V(s) result for that iteration**:

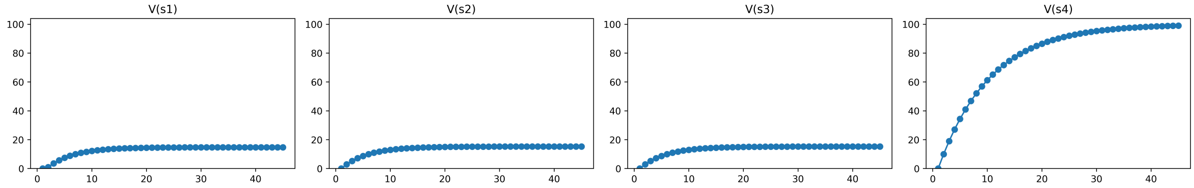


Figure 2: Image

b. Solve the Bellman Optimality Equations by Value Iteration for Q(s, a).

- It was solved in 45 iterations as well, if we assume that **a change of less than 0.1 between iterations** means convergence:

```
    ==========================
Iteration 1:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
0.00    0.00    0.00    0.00
0.00    0.00    0.00    0.00
Q(s,a) new
1.00    2.90    -1.00   2.90
3.61    -1.00   5.25    12.61
==========================
Iteration 2:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
1.00    2.90    -1.00   2.90
3.61    -1.00   5.25    12.61
Q(s,a) new
3.61    5.25    3.72    5.25
5.72    3.72    7.15    21.35
==========================
Iteration 3:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
3.61    5.25    3.72    5.25
5.72    3.72    7.15    21.35
Q(s,a) new
5.72    7.15    5.44    7.15
7.44    5.44    8.69    29.21
==========================
Iteration 4:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
5.72    7.15    5.44    7.15
7.44    5.44    8.69    29.21
```

```
Q(s,a) new
7.44    8.69    6.82    8.69
8.82    6.82    9.94    36.29
==========================
Iteration 5:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
7.44    8.69    6.82    8.69
8.82    6.82    9.94    36.29
Q(s,a) new
8.82    9.94    7.95    9.94
9.95    7.95    10.95   42.66
==========================
Iteration 6:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
8.82    9.94    7.95    9.94
9.95    7.95    10.95   42.66
Q(s,a) new
9.95    10.95   8.86    10.95
10.86   8.86    11.77   48.40
==========================
Iteration 7:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
9.95    10.95   8.86    10.95
10.86   8.86    11.77   48.40
Q(s,a) new
10.86   11.77   9.59    11.77
11.59   9.59    12.43   53.56
==========================
Iteration 8:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
10.86   11.77   9.59    11.77
11.59   9.59    12.43   53.56
Q(s,a) new
11.59   12.43   10.19   12.43
12.19   10.19   12.97   58.20
==========================
Iteration 9:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
11.59   12.43   10.19   12.43
12.19   10.19   12.97   58.20
Q(s,a) new
```

```
12.19    12.97    10.68    12.97
12.68    10.68    13.41    62.38
==========================
Iteration 10:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
12.19    12.97    10.68    12.97
12.68    10.68    13.41    62.38
Q(s,a) new
12.68    13.41    11.07    13.41
13.07    11.07    13.76    66.14
==========================
Iteration 11:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
12.68    13.41    11.07    13.41
13.07    11.07    13.76    66.14
Q(s,a) new
13.07    13.76    11.38    13.76
13.38    11.38    14.05    69.53
==========================
Iteration 12:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
13.07    13.76    11.38    13.76
13.38    11.38    14.05    69.53
Q(s,a) new
13.38    14.05    11.64    14.05
13.64    11.64    14.28    72.58
==========================
Iteration 13:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
13.38    14.05    11.64    14.05
13.64    11.64    14.28    72.58
Q(s,a) new
13.64    14.28    11.85    14.28
13.85    11.85    14.46    75.32
==========================
Iteration 14:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
13.64    14.28    11.85    14.28
13.85    11.85    14.46    75.32
Q(s,a) new
13.85    14.46    12.02    14.46
```

```
14.02   12.02   14.62   77.79
==========================
Iteration 15:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
13.85   14.46   12.02   14.46
14.02   12.02   14.62   77.79
Q(s,a) new
14.02   14.62   12.15   14.62
14.15   12.15   14.74   80.01
==========================
Iteration 16:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.02   14.62   12.15   14.62
14.15   12.15   14.74   80.01
Q(s,a) new
14.15   14.74   12.27   14.74
14.27   12.27   14.84   82.01
==========================
Iteration 17:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.15   14.74   12.27   14.74
14.27   12.27   14.84   82.01
Q(s,a) new
14.27   14.84   12.35   14.84
14.35   12.35   14.92   83.81
==========================
Iteration 18:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.27   14.84   12.35   14.84
14.35   12.35   14.92   83.81
Q(s,a) new
14.35   14.92   12.43   14.92
14.43   12.43   14.98   85.43
==========================
Iteration 19:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.35   14.92   12.43   14.92
14.43   12.43   14.98   85.43
Q(s,a) new
14.43   14.98   12.49   14.98
14.49   12.49   15.04   86.88
```

```
==========================
Iteration 20:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.43   14.98   12.49   14.98
14.49   12.49   15.04   86.88
Q(s,a) new
14.49   15.04   12.53   15.04
14.53   12.53   15.08   88.19
==========================
Iteration 21:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.49   15.04   12.53   15.04
14.53   12.53   15.08   88.19
Q(s,a) new
14.53   15.08   12.57   15.08
14.57   12.57   15.12   89.38
==========================
Iteration 22:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.53   15.08   12.57   15.08
14.57   12.57   15.12   89.38
Q(s,a) new
14.57   15.12   12.60   15.12
14.60   12.60   15.14   90.44
==========================
Iteration 23:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.57   15.12   12.60   15.12
14.60   12.60   15.14   90.44
Q(s,a) new
14.60   15.14   12.63   15.14
14.63   12.63   15.17   91.39
==========================
Iteration 24:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.60   15.14   12.63   15.14
14.63   12.63   15.17   91.39
Q(s,a) new
14.63   15.17   12.65   15.17
14.65   12.65   15.18   92.25
==========================
Iteration 25:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.63   15.17   12.65   15.17
14.65   12.65   15.18   92.25
Q(s,a) new
14.65   15.18   12.67   15.18
14.67   12.67   15.20   93.03
==========================
Iteration 26:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.65   15.18   12.67   15.18
14.67   12.67   15.20   93.03
Q(s,a) new
14.67   15.20   12.68   15.20
14.68   12.68   15.21   93.73
==========================
Iteration 27:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.67   15.20   12.68   15.20
14.68   12.68   15.21   93.73
Q(s,a) new
14.68   15.21   12.69   15.21
14.69   12.69   15.22   94.35
==========================
Iteration 28:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.68   15.21   12.69   15.21
14.69   12.69   15.22   94.35
Q(s,a) new
14.69   15.22   12.70   15.22
14.70   12.70   15.23   94.92
==========================
Iteration 29:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.69   15.22   12.70   15.22
14.70   12.70   15.23   94.92
Q(s,a) new
14.70   15.23   12.71   15.23
14.71   12.71   15.24   95.43
==========================
Iteration 30:
```

```
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
14.70    15.23    12.71    15.23
14.71    12.71    15.24    95.43
Q(s,a) new
14.71    15.24    12.71    15.24
14.71    12.71    15.24    95.88
==========================
Iteration 31:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
14.71    15.24    12.71    15.24
14.71    12.71    15.24    95.88
Q(s,a) new
14.71    15.24    12.72    15.24
14.72    12.72    15.25    96.30
==========================
Iteration 32:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
14.71    15.24    12.72    15.24
14.72    12.72    15.25    96.30
Q(s,a) new
14.72    15.25    12.72    15.25
14.72    12.72    15.25    96.67
==========================
Iteration 33:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
14.72    15.25    12.72    15.25
14.72    12.72    15.25    96.67
Q(s,a) new
14.72    15.25    12.72    15.25
14.72    12.72    15.25    97.00
==========================
Iteration 34:
fr: [2, 1, -1, 10]
s1       s2       s3       s4
Q(s,a) current
14.72    15.25    12.72    15.25
14.72    12.72    15.25    97.00
Q(s,a) new
14.72    15.25    12.73    15.25
14.73    12.73    15.25    97.30
==========================
Iteration 35:
fr: [2, 1, -1, 10]
```

```
s1      s2      s3      s4
Q(s,a) current
14.72   15.25   12.73   15.25
14.73   12.73   15.25   97.30
Q(s,a) new
14.73   15.25   12.73   15.25
14.73   12.73   15.26   97.57
==========================
Iteration 36:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.25   12.73   15.25
14.73   12.73   15.26   97.57
Q(s,a) new
14.73   15.26   12.73   15.26
14.73   12.73   15.26   97.81
==========================
Iteration 37:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.26   12.73   15.26
14.73   12.73   15.26   97.81
Q(s,a) new
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.03
==========================
Iteration 38:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.03
Q(s,a) new
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.23
==========================
Iteration 39:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.23
Q(s,a) new
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.41
==========================
Iteration 40:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
```

```
Q(s,a) current
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.41
Q(s,a) new
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.56
==========================
Iteration 41:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.56
Q(s,a) new
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.71
==========================
Iteration 42:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.71
Q(s,a) new
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.84
==========================
Iteration 43:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.26   12.73   15.26
14.73   12.73   15.26   98.84
Q(s,a) new
14.73   15.26   12.74   15.26
14.74   12.74   15.26   98.95
==========================
Iteration 44:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
14.73   15.26   12.74   15.26
14.74   12.74   15.26   98.95
Q(s,a) new
14.74   15.26   12.74   15.26
14.74   12.74   15.26   99.06
==========================
Iteration 45:
fr: [2, 1, -1, 10]
s1      s2      s3      s4
Q(s,a) current
```

```
14.74    15.26    12.74    15.26
14.74    12.74    15.26    99.06
Q(s,a) new
14.74    15.26    12.74    15.26
14.74    12.74    15.26    99.15
Optimal Politic:
s1 = a1,          s2 = a1,          s3 = a2,          s4 = a2,
```

Which give out a result of:

	s1	s2	s3	s4
Q(s, a1)	**14.74**	**15.26**	12.74	15.26
Q(s, a2)	14.74	12.74	**15.26**	**99.15**
f_pi(s)	a1	a1	a2	a2

Below is a similarly plotted convergence plot for Q(s, a) values:

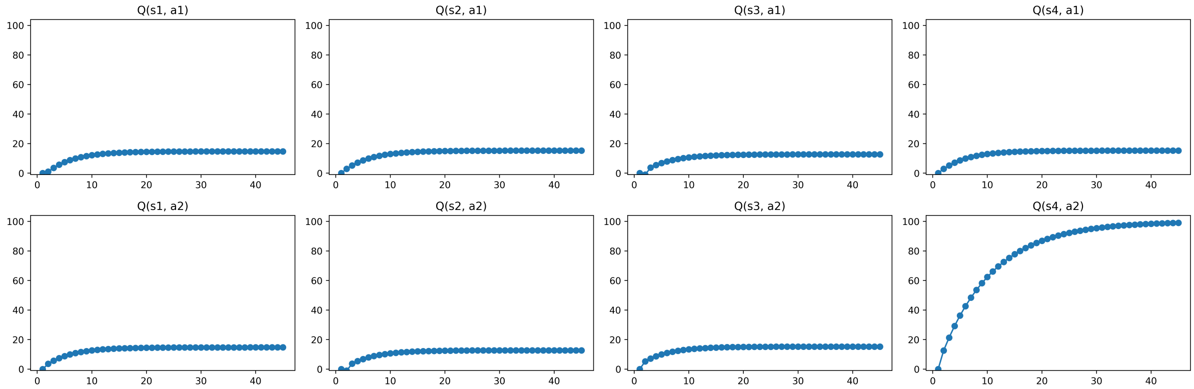


Figure 3: Image

2. Given the world defined by the Transition Function `Pmt(sf|s, a)`, the Reward Function `fr(s, a, sf)= fr(sf)` and `gamma = 0.6`:

$$f_{M_T}(s,a) = \begin{array}{c|cc|} & \multicolumn{2}{c}{s_f = s_1} \\ & a_1 & a_2 \\ s_1 & 0.4 & 0.2 \\ s_2 & 0.5 & 0 \\ s_3 & 1 & 0.3 \end{array} \quad \begin{array}{c|cc|} & \multicolumn{2}{c}{s_f = s_2} \\ & a_1 & a_2 \\ s_1 & 0.5 & 0.8 \\ s_2 & 0 & 0 \\ s_3 & 0 & 0.6 \end{array} \quad \begin{array}{c|cc|} & \multicolumn{2}{c}{s_f = s_3} \\ & a_1 & a_2 \\ s_1 & 0.1 & 0 \\ s_2 & 0.5 & 1 \\ s_3 & 0 & 0.1 \end{array} \qquad f_R(s_f) = \begin{array}{c|c|} s_1 & 2 \\ s_2 & 1 \\ s_3 & -1 \end{array}$$

Figure 4: Image

a. Solve the Bellman Optimality Equations by Value Iteration for V(s).

Solution

It was solved in 9 iterations, if we assume that **a change of less than 0.01 between iterations** means it converged:

```
===========================
Iteration 1:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
0.00    0.00    0.00
V(s) new
1.20    0.86    2.72
===========================
Iteration 2:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
1.20    0.86    2.72
V(s) new
1.91    1.89    3.15
===========================
Iteration 3:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
1.91    1.89    3.15
V(s) new
2.41    2.17    3.45
===========================
Iteration 4:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
2.41    2.17    3.45
V(s) new
2.64    2.33    3.58
===========================
Iteration 5:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
2.64    2.33    3.58
V(s) new
2.75    2.40    3.65
===========================
Iteration 6:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
2.75    2.40    3.65
```

```
V(s) new
2.80    2.43    3.68
==========================
Iteration 7:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
2.80    2.43    3.68
V(s) new
2.82    2.45    3.69
==========================
Iteration 8:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
2.82    2.45    3.69
V(s) new
2.83    2.46    3.70
==========================
Iteration 9:
fr: [2, 1, -1]
s1      s2      s3
V(s) current
2.83    2.46    3.70
V(s) new
2.84    2.46    3.70
Optimal Politic:
s1 = a1,        s2 = a1,        s3 = a1
```

Which gives out a result of:

	s1	s2	s3
V(s)	2.84	2.46	3.70
f_pi(s)	a1	a1	a1

The convergence plot for each V(s) value is given below:

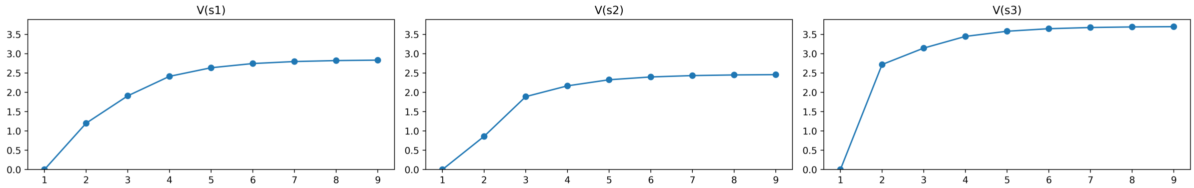


Figure 5: Image

b. Solve the Bellman Optimality Equations by Value Iteration for Q(s, a).

Solution

It was solved in 8 iterations, assuming **a change of less than 0.01 between iterations** means convergence:

```
===========================
Iteration 1:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
0.00    0.00    0.00
0.00    0.00    0.00
Q(s,a) new
1.20    0.86    2.72
1.76    0.63    1.89
===========================
Iteration 2:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
1.20    0.86    2.72
1.76    0.63    1.89
Q(s,a) new
2.04    1.93    3.23
2.37    0.94    2.41
===========================
Iteration 3:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
2.04    1.93    3.23
2.37    0.94    2.41
Q(s,a) new
2.54    2.23    3.52
2.58    1.11    2.58
===========================
Iteration 4:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
2.54    2.23    3.52
2.58    1.11    2.58
Q(s,a) new
2.70    2.37    3.62
2.66    1.17    2.66
===========================
Iteration 5:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
2.70    2.37    3.62
```

```
2.66    1.17    2.66
Q(s,a) new
2.77    2.42    3.66
2.69    1.20    2.69
==========================
Iteration 6:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
2.77    2.42    3.66
2.69    1.20    2.69
Q(s,a) new
2.81    2.44    3.69
2.71    1.21    2.71
==========================
Iteration 7:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
2.81    2.44    3.69
2.71    1.21    2.71
Q(s,a) new
2.83    2.45    3.70
2.72    1.22    2.71
==========================
Iteration 8:
fr: [2, 1, -1]
s1      s2      s3
Q(s,a) current
2.83    2.45    3.70
2.72    1.22    2.71
Q(s,a) new
2.84    2.46    3.70
2.72    1.22    2.72
Optimal Politic:
s1 = a1,        s2 = a1,        s3 = a1
```

Which gives out a result of:

	s1	s2	s3
Q(s, a1)	**2.84**	**2.46**	**3.70**
Q(s, a2)	2.72	1.22	2.72
f_pi(s)	a1	a1	a1

The convergence plot for each Q(s,a) value is given below:

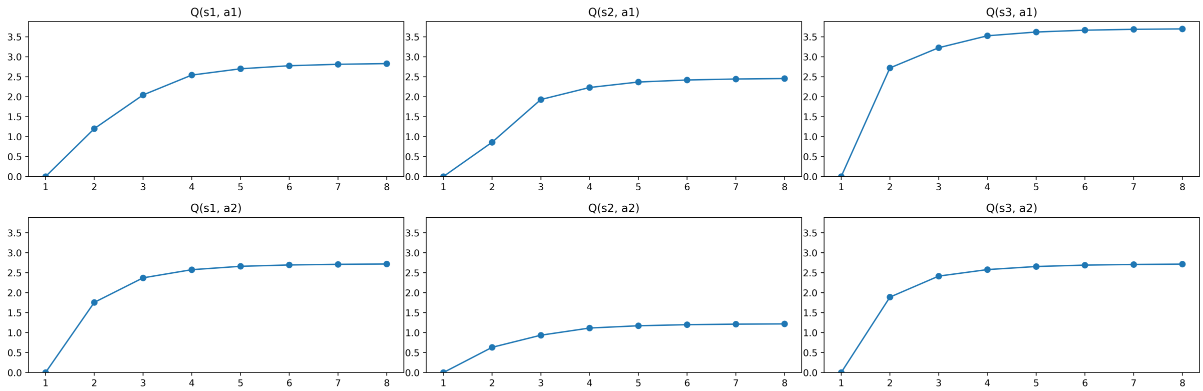
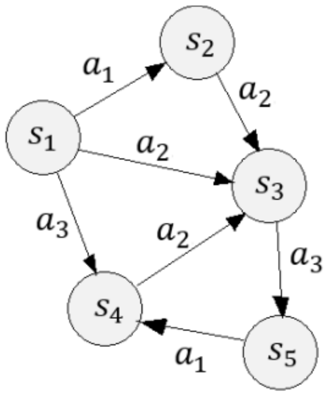


Figure 6: Image

3. Given the world defined by the graph, the Reward Function `fr(s,a,sf)` and `gamma = 0.9`:



$$
f_R(s,a,s_f) =
\begin{array}{c}
s_f = s_1 \\
\begin{array}{c|ccc}
 & a_1 & a_2 & a_3 \\
\hline
s_1 & 0 & 0 & 0 \\
s_2 & 0 & 0 & 0 \\
s_3 & 0 & 0 & 0 \\
s_4 & 0 & 0 & 0 \\
s_5 & 0 & 0 & 0
\end{array}
\end{array}
\quad
\begin{array}{c}
s_f = s_2 \\
\begin{array}{c|ccc}
 & a_1 & a_2 & a_3 \\
\hline
s_1 & -2 & 0 & 0 \\
s_2 & 0 & 0 & 0 \\
s_3 & 0 & 0 & 0 \\
s_4 & 0 & 0 & 0 \\
s_5 & 0 & 0 & 0
\end{array}
\end{array}
\quad
\begin{array}{c}
s_f = s_3 \\
\begin{array}{c|ccc}
 & a_1 & a_2 & a_3 \\
\hline
s_1 & 0 & 5 & 0 \\
s_2 & 0 & 4 & 0 \\
s_3 & 0 & 0 & 0 \\
s_4 & 0 & -1 & 0 \\
s_5 & 0 & 0 & 0
\end{array}
\end{array}
\quad
\begin{array}{c}
s_f = s_4 \\
\begin{array}{c|ccc}
 & a_1 & a_2 & a_3 \\
\hline
s_1 & 0 & 0 & -3 \\
s_2 & 0 & 0 & 0 \\
s_3 & 0 & 0 & 0 \\
s_4 & 0 & 0 & 0 \\
s_5 & 1 & 0 & 0
\end{array}
\end{array}
\quad
\begin{array}{c}
s_f = s_5 \\
\begin{array}{c|ccc}
 & a_1 & a_2 & a_3 \\
\hline
s_1 & 0 & 0 & 0 \\
s_2 & 0 & 0 & 0 \\
s_3 & 0 & 0 & -6 \\
s_4 & 0 & 0 & 0 \\
s_5 & 0 & 0 & 0
\end{array}
\end{array}
$$

Figure 7: Image

a. Solve the Bellman Optimality Equations by Value Iteration for V(s).

Solution

The Transition Function can therefore be defined as:

$$f_{MT}(s,a) = \begin{array}{c|ccc|} & a_1 & a_2 & a_3 \\ s_1 & s_2 & s_3 & s_4 \\ s_2 & s_2 & s_3 & s_1 \\ s_3 & s_3 & s_3 & s_5 \\ s_4 & s_4 & s_3 & s_4 \\ s_5 & s_4 & s_5 & s_5 \end{array}$$

Figure 8: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```
=========================
Iteration 1:
fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
   0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0,
    0, 0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0],
    [0, 0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0,
    0], [1, 0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6],
   [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
V(s) current
0.00    0.00    0.00    0.00    0.00
V(s) new
5.00    4.00    0.00    0.00    1.00
=========================
Iteration 2:
fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
   0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0,
    0, 0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0],
    [0, 0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0,
    0], [1, 0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6],
   [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
V(s) current
5.00    4.00    0.00    0.00    1.00
V(s) new
5.00    4.00    0.00    0.00    1.00
Optimal Politic:
```

```
s1 = a2,          s2 = a2,          s3 = a1,          s4 = a1,
          s5 = a1
```

Which gives out a result of:

	s1	s2	s3	s4	s5
V(s)	5.00	4.00	0.00	0.00	1.00
f_pi(s)	a2	a2	a1	a1	a1

The convergence plot for each V(s) value is given below:

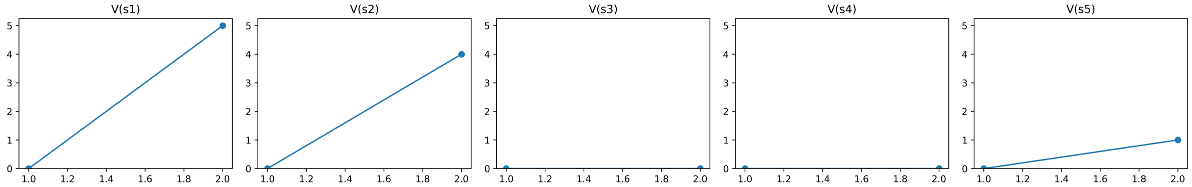
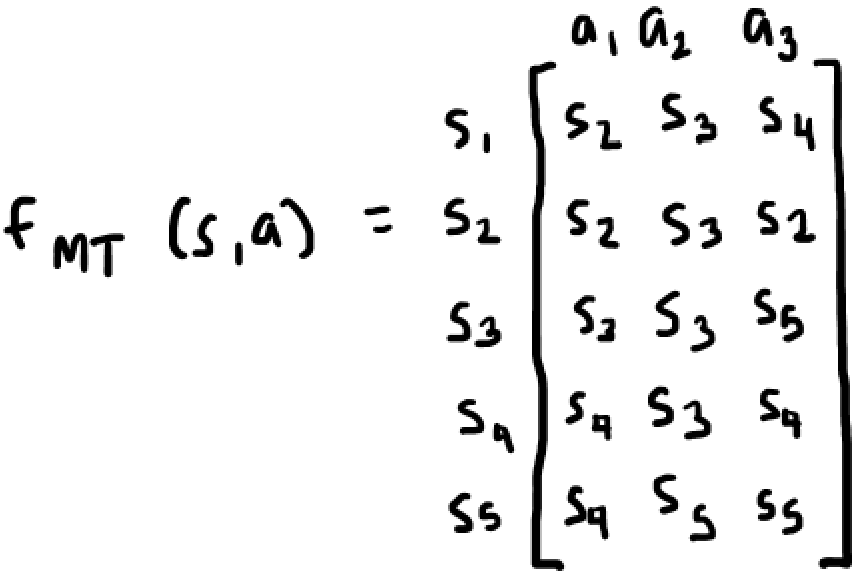


Figure 9: Image

b. Solve the Bellman Optimality Equations by Value Iteration for Q(s,a).

Solution

The Transition Function can therefore be defined as:



$$f_{MT}(s,a) = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline s_2 & s_3 & s_4 \\ s_2 & s_3 & s_2 \\ s_2 & s_3 & s_5 \\ s_4 & s_3 & s_4 \\ s_4 & s_5 & s_5 \end{array}$$

Figure 10: Image

Thus, the Bellman Optimality Equations were solved in 3 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```
===========================
Iteration 1:
fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0, 0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1, 0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
Q(s,a) current
0.00    0.00    0.00    0.00    0.00
0.00    0.00    0.00    0.00    0.00
0.00    0.00    0.00    0.00    0.00
Q(s,a) new
-2.00   0.00    0.00    0.00    1.00
5.00    4.00    0.00    -1.00   0.90
-3.00   3.60    -5.10   0.00    0.90
===========================
Iteration 2:
fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0, 0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1, 0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
Q(s,a) current
-2.00   0.00    0.00    0.00    1.00
5.00    4.00    0.00    -1.00   0.90
-3.00   3.60    -5.10   0.00    0.90
Q(s,a) new
1.60    3.60    0.00    0.00    1.00
5.00    4.00    0.00    -1.00   0.90
-3.00   3.60    -5.10   0.00    0.90
===========================
Iteration 3:
fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0, 0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1, 0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
Q(s,a) current
1.60    3.60    0.00    0.00    1.00
5.00    4.00    0.00    -1.00   0.90
-3.00   3.60    -5.10   0.00    0.90
Q(s,a) new
1.60    3.60    0.00    0.00    1.00
5.00    4.00    0.00    -1.00   0.90
```

```
-3.00    3.60     -5.10    0.00     0.90
Optimal Politic:
s1 = a2,         s2 = a2,         s3 = a1,         s4 = a1,
        s5 = a1,
```

Which gives out a result of:

	s1	s2	s3	s4	s5
Q(s, a1)	1.60	3.60	**0.00**	**0.00**	**1.00**
Q(s, a2)	**5.00**	**4.00**	0.00	-1.00	0.90
Q(s, a3)	-3.00	3.60	-5.10	0.00	0.90
f_pi(s)	a2	a2	a1	a1	a1

The convergence plot for each Q(s,a) value is given below:

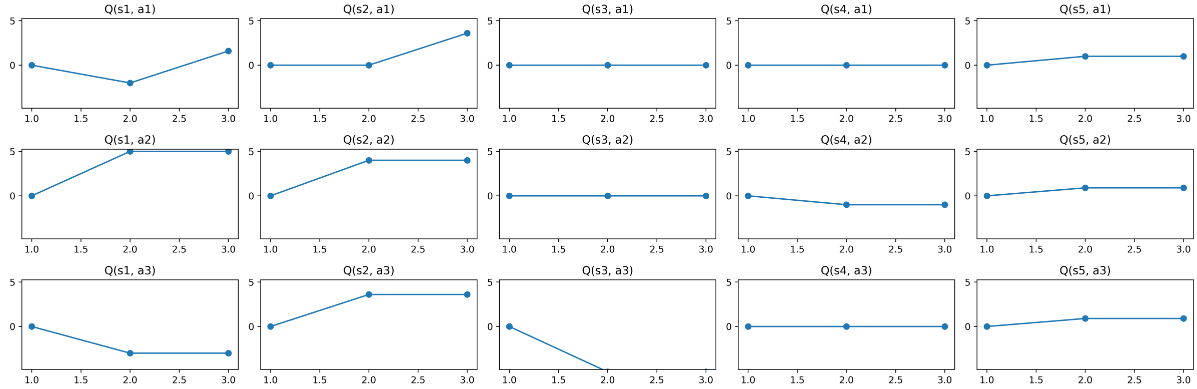
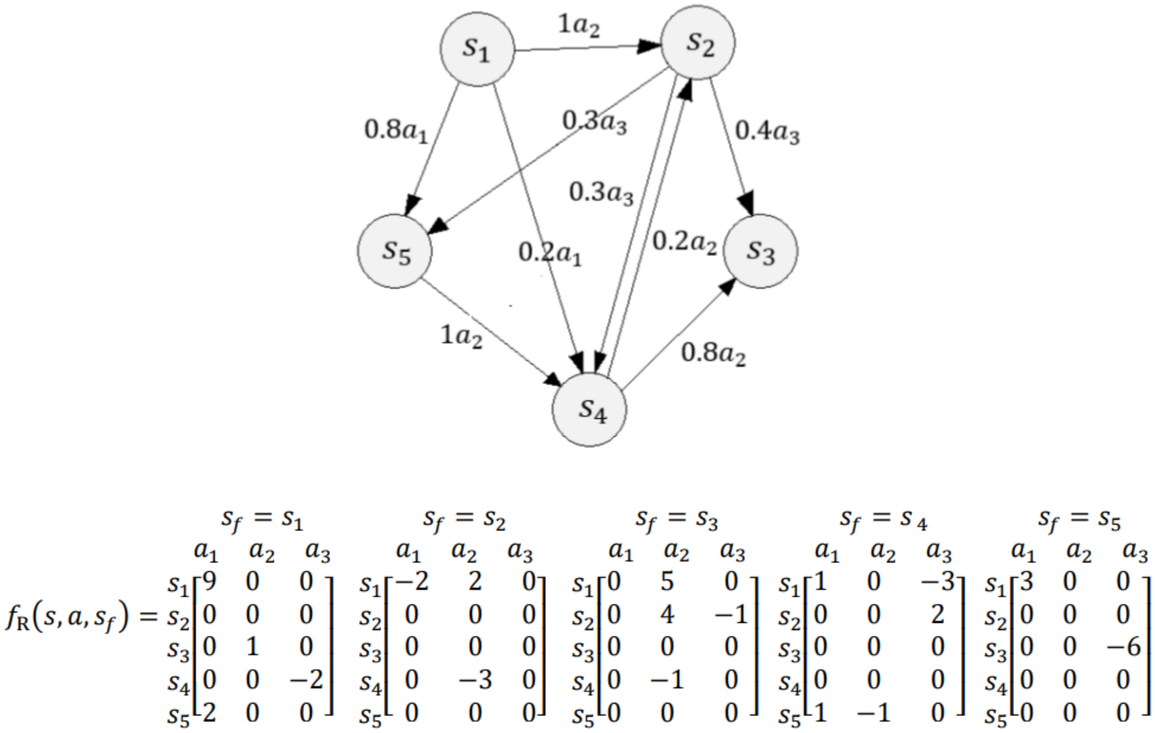


Figure 11: Image

4. Given the world defined by the graph and with the following Reward Function `fr(s,a,sf)` and `gamma = 0.7`:



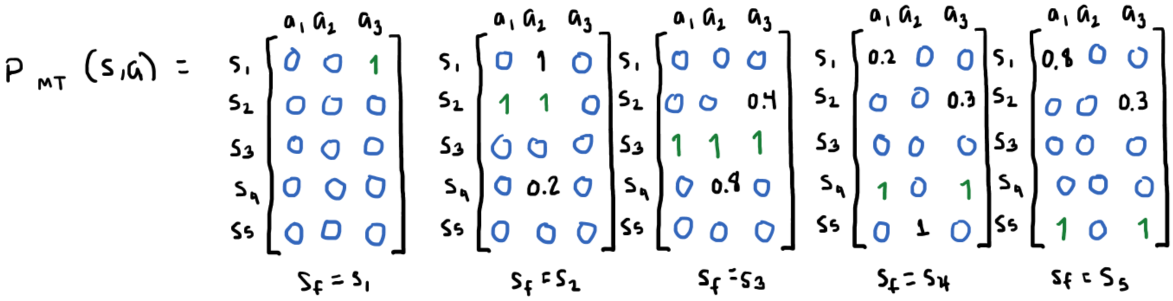
$$f_R(s,a,s_f) = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array}\overset{s_f = s_1}{\begin{array}{ccc} a_1 & a_2 & a_3 \\ 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 2 & 0 & 0 \end{array}} \quad \overset{s_f = s_2}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & -2 & 2 & 0 \\ s_2 & 0 & 0 & 0 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & -3 & 0 \\ s_5 & 0 & 0 & 0 \end{array}} \quad \overset{s_f = s_3}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 0 & 5 & 0 \\ s_2 & 0 & 4 & -1 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & -1 & 0 \\ s_5 & 0 & 0 & 0 \end{array}} \quad \overset{s_f = s_4}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 1 & 0 & -3 \\ s_2 & 0 & 0 & 2 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & 0 & 0 \\ s_5 & 1 & -1 & 0 \end{array}} \quad \overset{s_f = s_5}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 3 & 0 & 0 \\ s_2 & 0 & 0 & 0 \\ s_3 & 0 & 0 & -6 \\ s_4 & 0 & 0 & 0 \\ s_5 & 0 & 0 & 0 \end{array}}$$

Figure 12: Image

a. Solve the Bellman Optimality Equations by Value Iteration for V(s).

Solution

The Transition Model would be then:



$$P_{MT}(s,a) = \underset{s_f = s_1}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 0 & 0 & 1 \\ s_2 & 0 & 0 & 0 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & 0 & 0 \\ s_5 & 0 & 0 & 0 \end{array}} \quad \underset{s_f = s_2}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 0 & 1 & 0 \\ s_2 & 1 & 1 & 0 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & 0.2 & 0 \\ s_5 & 0 & 0 & 0 \end{array}} \quad \underset{s_f = s_3}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 0 & 0 & 0 \\ s_2 & 0 & 0 & 0.4 \\ s_3 & 1 & 1 & 1 \\ s_4 & 0 & 0.8 & 0 \\ s_5 & 0 & 0 & 0 \end{array}} \quad \underset{s_f = s_4}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 0.2 & 0 & 0 \\ s_2 & 0 & 0 & 0.3 \\ s_3 & 0 & 0 & 0 \\ s_4 & 1 & 0 & 1 \\ s_5 & 0 & 1 & 0 \end{array}} \quad \underset{s_f = s_5}{\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ s_1 & 0.8 & 0 & 0 \\ s_2 & 0 & 0 & 0.3 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & 0 & 0 \\ s_5 & 1 & 0 & 1 \end{array}}$$

Figure 13: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```
===========================
Iteration 1:
fr: [[[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0, 0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0], [0, 0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1, 0], [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0],
   [0, 0, 0], [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0,
   -6], [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
V(s) current
0.00    0.00    0.00    0.00    0.00
V(s) new
2.60    0.00    0.00    0.00    0.00
==========================
Iteration 2:
fr: [[[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0,
    0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0],
   [0, 0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1,
    0], [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0],
   [0, 0, 0], [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0,
   -6], [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
V(s) current
2.60    0.00    0.00    0.00    0.00
V(s) new
2.60    0.00    0.00    0.00    0.00
Optimal Politic:
s1 = a1,        s2 = a1,        s3 = a1,        s4 = a1,
          s5 = a1
```

Which gives out a result of:

	s1	s2	s3	s4	s5
V(s)	2.60	0.00	0.00	0.00	0.00
f_pi(s)	a1	a1	a1	a1	a1

The convergence plot for each V(s) value is given below:

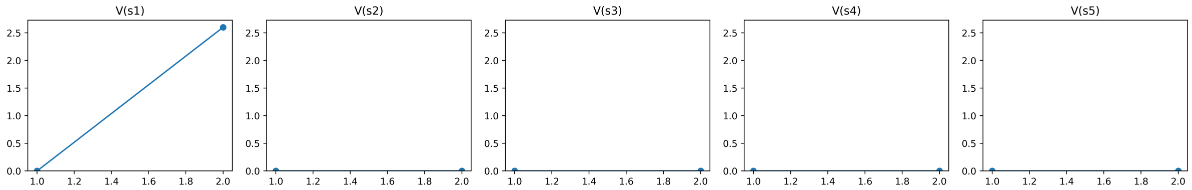
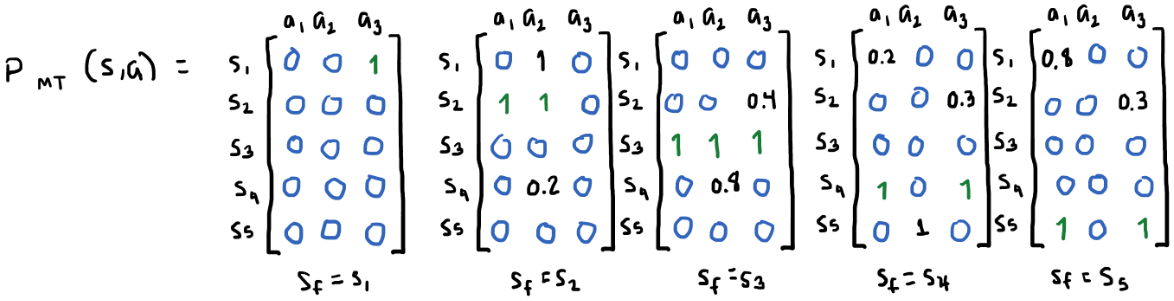


Figure 14: Image

b. Solve the Bellman Optimality Equations by Value Iteration for Q(s,a).

Solution

The Transition Function can therefore be defined as:



$$P_{MT}(s,a) = \begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ \hline s_1 & 0 & 0 & 1 \\ s_2 & 0 & 0 & 0 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & 0 & 0 \\ s_5 & 0 & 0 & 0 \end{array} \quad \begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ \hline s_1 & 0 & 1 & 0 \\ s_2 & 1 & 1 & 0 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & 0.2 & 0 \\ s_5 & 0 & 0 & 0 \end{array} \quad \begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ \hline s_1 & 0 & 0 & 0 \\ s_2 & 0 & 0 & 0.4 \\ s_3 & 1 & 1 & 1 \\ s_4 & 0 & 0.9 & 0 \\ s_5 & 0 & 0 & 0 \end{array} \quad \begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ \hline s_1 & 0.2 & 0 & 0 \\ s_2 & 0 & 0 & 0.3 \\ s_3 & 0 & 0 & 0 \\ s_4 & 1 & 0 & 1 \\ s_5 & 0 & 1 & 0 \end{array} \quad \begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ \hline s_1 & 0.8 & 0 & 0 \\ s_2 & 0 & 0 & 0.3 \\ s_3 & 0 & 0 & 0 \\ s_4 & 0 & 0 & 0 \\ s_5 & 1 & 0 & 1 \end{array}$$

$$s_f = s_1 \qquad s_f = s_2 \qquad s_f = s_3 \qquad s_f = s_4 \qquad s_f = s_5$$

Figure 15: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```
============================
Iteration 1:
fr: [[[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0,
     0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0],
    [0, 0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1,
     0], [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0],
    [0, 0, 0], [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0,
    -6], [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
Q(s,a) current
0.00    0.00    0.00    0.00    0.00
0.00    0.00    0.00    0.00    0.00
0.00    0.00    0.00    0.00    0.00
Q(s,a) new
2.60    0.00    0.00    0.00    0.00
2.00    0.00    0.00    -1.40   -1.00
1.82    -0.40   0.00    0.00    0.00
============================
Iteration 2:
fr: [[[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0,
     0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0],
    [0, 0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1,
     0], [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0],
    [0, 0, 0], [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0,
    -6], [0, 0, 0], [0, 0, 0]]]
s1      s2      s3      s4      s5
Q(s,a) current
2.60    0.00    0.00    0.00    0.00
2.00    0.00    0.00    -1.40   -1.00
1.82    -0.40   0.00    0.00    0.00
Q(s,a) new
2.60    0.00    0.00    0.00    0.00
2.00    0.00    0.00    -1.40   -1.00
1.82    -0.40   0.00    0.00    0.00
Optimal Politic:
s1 = a1,        s2 = a1,        s3 = a1,        s4 = a1,
            s5 = a1,
```

Which gives out a result of:

	s1	s2	s3	s4	s5
Q(s, a1)	**2.60**	**0.00**	**0.00**	**0.00**	**0.00**
Q(s, a2)	2.00	0.00	0.00	-1.40	-1.00
Q(s, a3)	1.82	-0.40	0.00	0.00	0.00
f_pi(s)	a1	a1	a1	a1	a1

The convergence plot for each Q(s,a) value is given below:

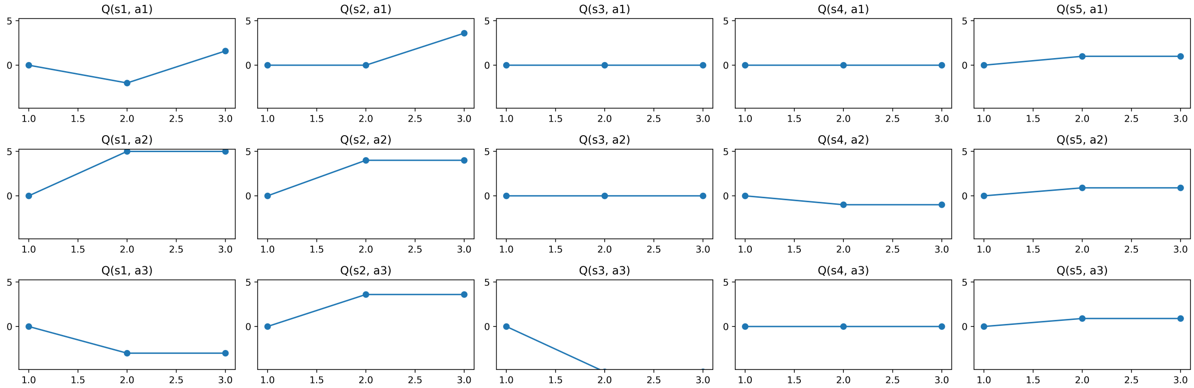


Figure 16: Image

5. The world has the states set S = {sf1, s1, s2, s3, sf2} where s1 = initial state and, sf1 and sf2 are terminal states:

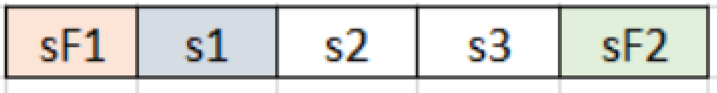
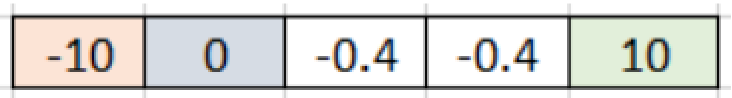


Figure 17: Image

The world has the set of actions A = {->, <-} where:

- -> = agent moves to the right one cell
- <- = agent moves to the left one cell

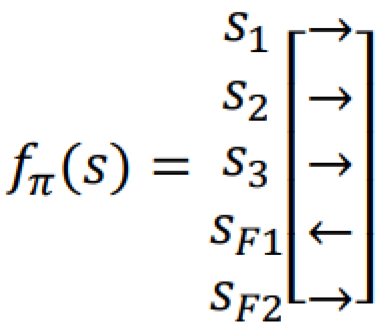
The Reward Function `fr(s,a,sf)= fr(sf)` only depends on the state to which the agent arrives:



-10	0	-0.4	-0.4	10

Figure 18: Image

The agent has the Action Function `f_pi(s)`:



$$f_\pi(s) = \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_{F1} \\ s_{F2} \end{array} \begin{bmatrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \leftarrow \\ \rightarrow \end{bmatrix}$$

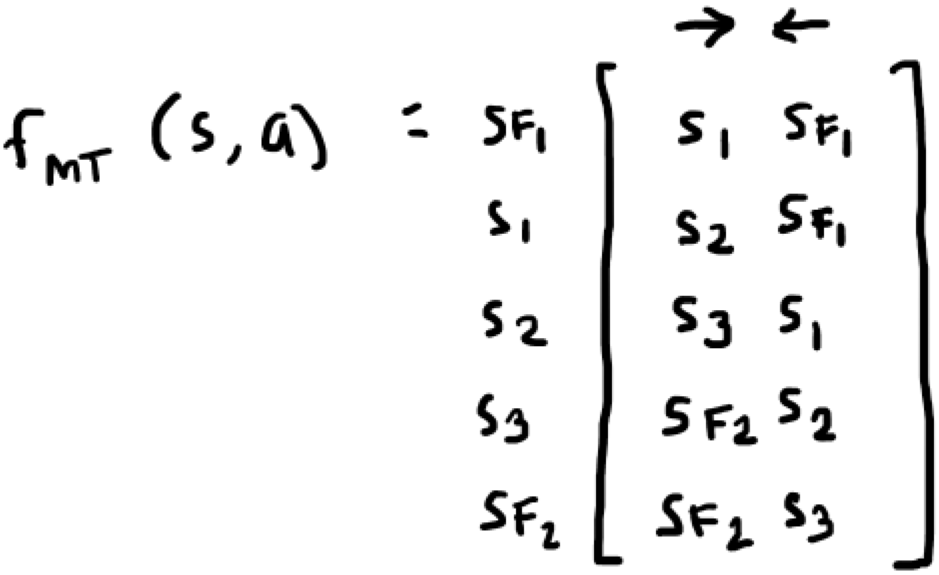
Figure 19: Image

Do the following:

a. Solve the Bellman Optimality Equations by Value Iteration for V(s).

Solution

The Transition Function would be:



$$f_{MT}(s,a) = \begin{array}{c|cc} & \rightarrow & \leftarrow \\ \hline s_{F_1} & s_1 & s_{F_1} \\ s_1 & s_2 & s_{F_1} \\ s_2 & s_3 & s_1 \\ s_3 & s_{F_2} & s_2 \\ s_{F_2} & s_{F_2} & s_3 \end{array}$$

Figure 20: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume `gamma = 0.0`.

```
===========================
Iteration 1:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
V(s) current
0.00    0.00    0.00    0.00    0.00
V(s) new
0.00    -0.40   0.00    10.00   10.00
===========================
Iteration 2:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
V(s) current
0.00    -0.40   0.00    10.00   10.00
V(s) new
0.00    -0.40   0.00    10.00   10.00
Optimal Politic:
sf1 = ->,      s1 = ->,        s2 = ->,        s3 = ->,
          sf2 = ->
```

Which gives out a result of:

	sf1	s1	s2	s3	sf2
V(s)	0.00	-0.40	0.00	10.00	10.00
f_pi(s)	->	->	->	->	->

The convergence plot for each V(s) value is given below:

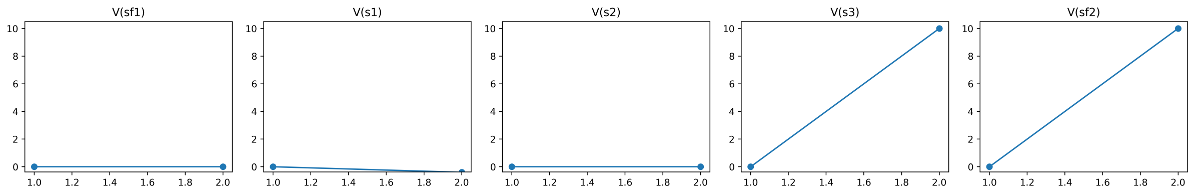


Figure 21: Image

b. Solve the Bellman Optimality Equations by Value Iteration for Q(s,a).

Solution

The Transition Function would be:

$$f_{MT}(s,a) = \begin{array}{c|cc} & \rightarrow & \leftarrow \\ \hline SF_1 & s_1 & SF_1 \\ s_1 & s_2 & SF_1 \\ s_2 & s_3 & s_1 \\ s_3 & SF_2 & s_2 \\ SF_2 & SF_2 & s_3 \end{array}$$

Figure 22: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume `gamma = 0.0`.

```
==========================
Iteration 1:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
Q(s,a) current
0.00    0.00    0.00    0.00    0.00
0.00    0.00    0.00    0.00    0.00
Q(s,a) new
0.00    -0.40   -0.40   10.00   10.00
-10.00  -10.00  0.00    -0.40   -0.40
==========================
Iteration 2:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
Q(s,a) current
0.00    -0.40   -0.40   10.00   10.00
-10.00  -10.00  0.00    -0.40   -0.40
Q(s,a) new
0.00    -0.40   -0.40   10.00   10.00
-10.00  -10.00  0.00    -0.40   -0.40
Optimal Politic:
sf1 = ->,       s1 = ->,        s2 = <-,        s3 = ->,
          sf2 = ->
```

Which gives out a result of:

	s1	s2	s3	s4	s5
Q(s, ->)	**0.00**	**-0.40**	-0.40	**10.00**	**10.00**
Q(s, <-)	-10.00	-10.00	**0.00**	-0.40	-0.40
f_pi(s)	->	->	<-	->	->

The convergence plot for each Q(s,a) value is given below:

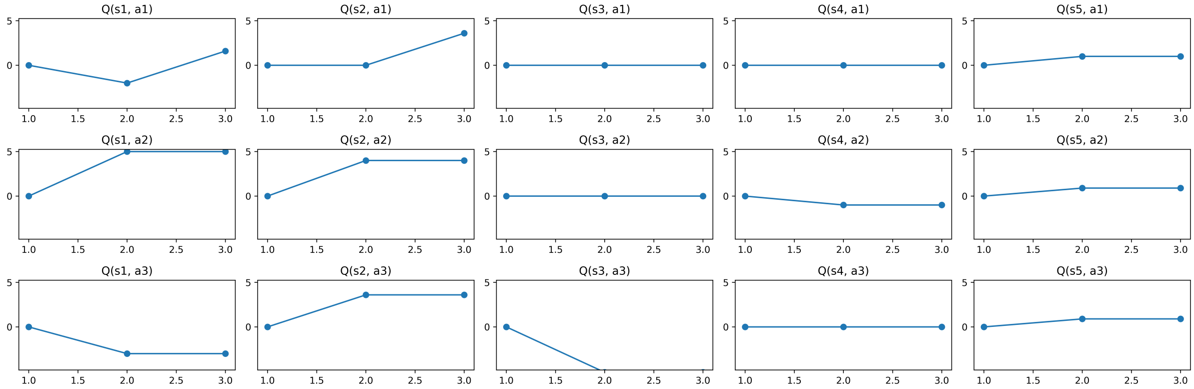
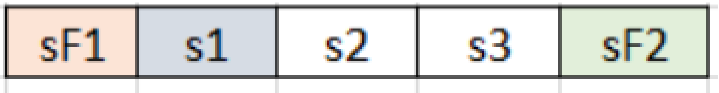


Figure 23: Image

6. The world has the states set S = {sf1, s1, s2, s3, sf2} where s1 = initial state and, sf1 and sf2 are terminal states:



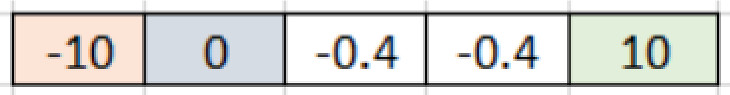
sF1	s1	s2	s3	sF2

Figure 24: Image

The world has the set of actions A = {->, <-} where:

- -> = agent moves to the right one cell with probability 0.8, and one cell to the left with probability 0.2.
- <- = agent moves to the left one cell with probability 0.8, and one cell to the right with probability 0.2.

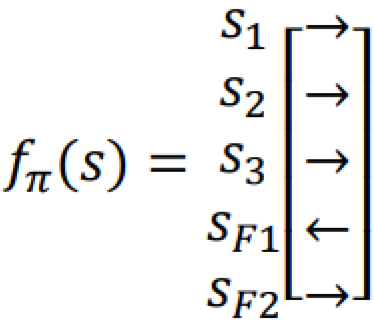
The Reward Function `fr(s,a,sf)= fr(sf)` only depends on the state to which the agent arrives:



-10	0	-0.4	-0.4	10

Figure 25: Image

The agent has the Action Function `f_pi(s)`:



$$f_\pi(s) = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_{F1} \\ S_{F2} \end{array} \begin{bmatrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \leftarrow \\ \rightarrow \end{bmatrix}$$

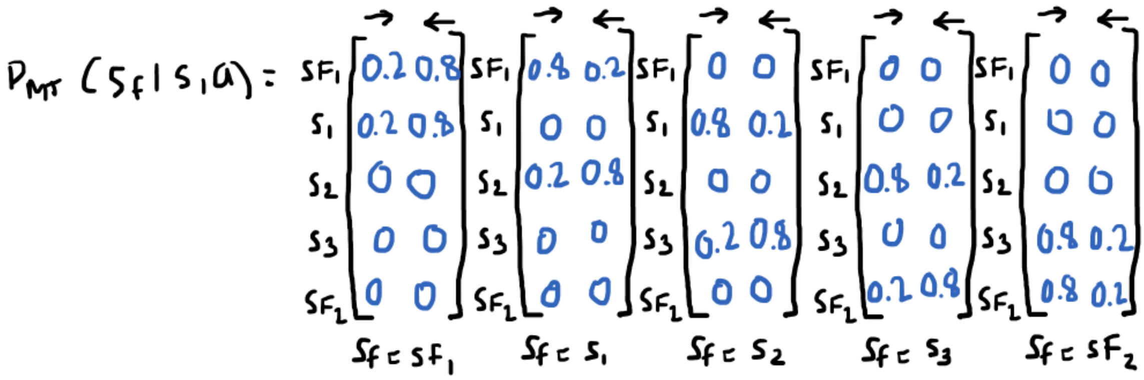
Figure 26: Image

Do the following:

a. Solve the Bellman Optimality Equations by Value Iteration for V(s).

Solution

The Transition Function would be:



$$P_{M\pi}(S_f \mid s, a) = \begin{array}{c|cc} & \rightarrow & \leftarrow \\ SF_1 & 0.2 & 0.8 \\ S_1 & 0.2 & 0.8 \\ S_2 & 0 & 0 \\ S_3 & 0 & 0 \\ SF_2 & 0 & 0 \end{array}_{S_f = SF_1} \begin{array}{c|cc} & \rightarrow & \leftarrow \\ SF_1 & 0.8 & 0.2 \\ S_1 & 0 & 0 \\ S_2 & 0.2 & 0.8 \\ S_3 & 0 & 0 \\ SF_2 & 0 & 0 \end{array}_{S_f = S_1} \begin{array}{c|cc} & \rightarrow & \leftarrow \\ SF_1 & 0 & 0 \\ S_1 & 0.8 & 0.2 \\ S_2 & 0 & 0 \\ S_3 & 0.2 & 0.8 \\ SF_2 & 0 & 0 \end{array}_{S_f = S_2} \begin{array}{c|cc} & \rightarrow & \leftarrow \\ SF_1 & 0 & 0 \\ S_1 & 0 & 0 \\ S_2 & 0.8 & 0.2 \\ S_3 & 0 & 0 \\ SF_2 & 0.2 & 0.8 \end{array}_{S_f = S_3} \begin{array}{c|cc} & \rightarrow & \leftarrow \\ SF_1 & 0 & 0 \\ S_1 & 0 & 0 \\ S_2 & 0 & 0 \\ S_3 & 0.8 & 0.2 \\ SF_2 & 0.8 & 0.2 \end{array}_{S_f = SF_2}$$

Figure 27: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume `gamma = 0.0`.

```
===========================
Iteration 1:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
V(s) current
0.00    0.00    0.00    0.00    0.00
V(s) new
-2.00   -2.32   -0.08   7.92    7.92
===========================
Iteration 2:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
V(s) current
-2.00   -2.32   -0.08   7.92    7.92
V(s) new
-2.00   -2.32   -0.08   7.92    7.92
Optimal Politic:
sf1 = ->,       s1 = ->,        s2 = ->,        s3 = ->,
          sf2 = ->
```

Which gives out a result of:

	sf1	s1	s2	s3	sf2
V(s)	-2.00	-2.32	-0.08	7.92	7.92
f_pi(s)	->	->	->	->	->

The convergence plot for each V(s) value is given below:

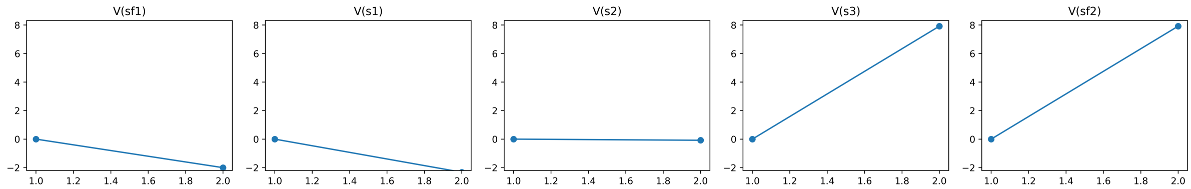


Figure 28: Image

b. Solve the Bellman Optimality Equations by Value Iteration for Q(s,a).

Solution

The Transition Function would be:

$$P_{MT}(s_f|s,a) = \underset{s_f = sf_1}{\begin{array}{c|cc} & \rightarrow & \leftarrow \\ \hline sf_1 & 0.2 & 0.8 \\ s_1 & 0.2 & 0.8 \\ s_2 & 0 & 0 \\ s_3 & 0 & 0 \\ sf_2 & 0 & 0 \end{array}} \quad \underset{s_f = s_1}{\begin{array}{c|cc} & \rightarrow & \leftarrow \\ \hline sf_1 & 0.8 & 0.2 \\ s_1 & 0 & 0 \\ s_2 & 0.2 & 0.8 \\ s_3 & 0 & 0 \\ sf_2 & 0 & 0 \end{array}} \quad \underset{s_f = s_2}{\begin{array}{c|cc} & \rightarrow & \leftarrow \\ \hline sf_1 & 0 & 0 \\ s_1 & 0.8 & 0.2 \\ s_2 & 0 & 0 \\ s_3 & 0.2 & 0.8 \\ sf_2 & 0 & 0 \end{array}} \quad \underset{s_f = s_3}{\begin{array}{c|cc} & \rightarrow & \leftarrow \\ \hline sf_1 & 0 & 0 \\ s_1 & 0 & 0 \\ s_2 & 0.8 & 0.2 \\ s_3 & 0 & 0 \\ sf_2 & 0.2 & 0.8 \end{array}} \quad \underset{s_f = sf_2}{\begin{array}{c|cc} & \rightarrow & \leftarrow \\ \hline sf_1 & 0 & 0 \\ s_1 & 0 & 0 \\ s_2 & 0 & 0 \\ s_3 & 0.8 & 0.2 \\ sf_2 & 0.8 & 0.2 \end{array}}$$

Figure 29: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume `gamma = 0.0`.

```
==========================
Iteration 1:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
Q(s,a) current
0.00    0.00    0.00    0.00    0.00
0.00    0.00    0.00    0.00    0.00
Q(s,a) new
-2.00   -2.32   -0.32   7.92    7.92
-8.00   -8.08   -0.08   1.68    1.68
==========================
Iteration 2:
fr: [-10, 0, -0.4, -0.4, 10]
sf1     s1      s2      s3      sf2
Q(s,a) current
-2.00   -2.32   -0.32   7.92    7.92
-8.00   -8.08   -0.08   1.68    1.68
Q(s,a) new
-2.00   -2.32   -0.32   7.92    7.92
-8.00   -8.08   -0.08   1.68    1.68
Optimal Politic:
sf1 = ->,       s1 = ->,        s2 = <-,        s3 = ->,
          sf2 = ->
```

Which gives out a result of:

	sf1	s1	s2	s3	sf2
Q(s, ->)	-2.00	-2.32	-0.32	7.92	7.92

	sf1	s1	s2	s3	sf2
Q(s, <-)	-8.00	-8.08	-0.08	1.68	1.68
f_pi(s)	->	->	<-	->	->

The convergence plot for each Q(s,a) value is given below:

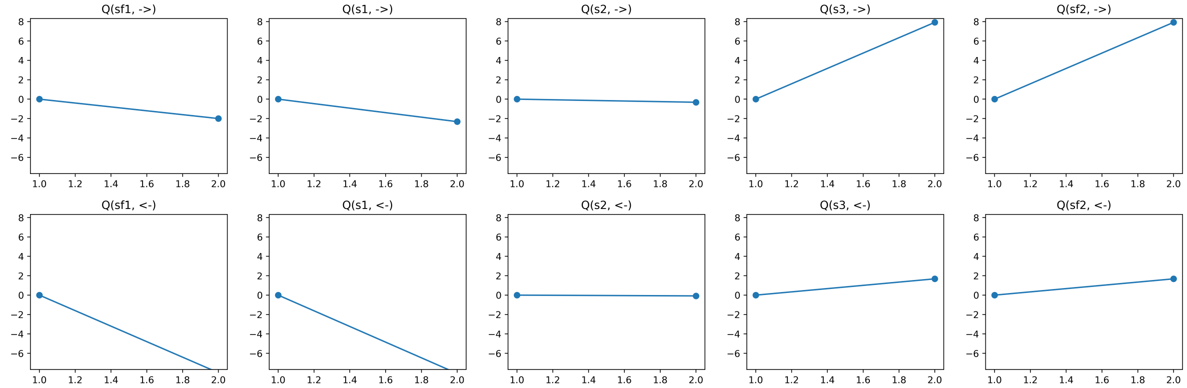


Figure 30: Image