

Homework 03: Value Iteration

- Given the world defined by the following Transition Function $f_{M_T}(s, a)$, the Reward Function $f_R(s, a, sf)$ and $\gamma = 0.9$:

$$f_{M_T}(s, a) = \begin{matrix} & a_1 & a_2 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} & \begin{bmatrix} s_2 & s_2 \\ s_1 & s_3 \\ s_3 & s_1 \\ s_1 & s_4 \end{bmatrix} \end{matrix} \quad f_R(s_f) = \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ 10 \end{bmatrix}$$

Figure 1: Image

- Solve the Bellman Optimality Equations by Value Iteration for $V(s)$.

Solution

- It was solved in 45 iterations, if we assume that **a change of less than 0.1 between iterations** means it converged:

```

1  =====
2  Iteration 1:
3  fr: [2, 1, -1, 10]
4  s1      s2      s3      s4
5  V(s) current
6  0.00     0.00     0.00     0.00
7  V(s) new
8  1.00     2.90     2.90     10.00
9  =====
10 Iteration 2:
11 fr: [2, 1, -1, 10]
12 s1      s2      s3      s4
13 V(s) current
14 1.00     2.90     2.90     10.00
15 V(s) new
16 3.61     5.25     5.25     19.00
17 =====
18 Iteration 3:
19 fr: [2, 1, -1, 10]
20 s1      s2      s3      s4
21 V(s) current
22 3.61     5.25     5.25     19.00
23 V(s) new
24 5.72     7.15     7.15     27.10
25 =====
26 Iteration 4:
27 fr: [2, 1, -1, 10]
28 s1      s2      s3      s4
29 V(s) current
30 5.72     7.15     7.15     27.10
31 V(s) new
32 7.44     8.69     8.69     34.39
33 =====

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34 Iteration 5:
35 fr: [2, 1, -1, 10]
36 s1      s2      s3      s4
37 V(s) current
38 7.44     8.69     8.69     34.39
39 V(s) new
40 8.82     9.94     9.94     40.95
41 =====
42 Iteration 6:
43 fr: [2, 1, -1, 10]
44 s1      s2      s3      s4
45 V(s) current
46 8.82     9.94     9.94     40.95
47 V(s) new
48 9.95     10.95    10.95    46.86
49 =====
50 Iteration 7:
51 fr: [2, 1, -1, 10]
52 s1      s2      s3      s4
53 V(s) current
54 9.95     10.95    10.95    46.86
55 V(s) new
56 10.86    11.77    11.77    52.17
57 =====
58 Iteration 8:
59 fr: [2, 1, -1, 10]
60 s1      s2      s3      s4
61 V(s) current
62 10.86    11.77    11.77    52.17
63 V(s) new
64 11.59    12.43    12.43    56.95
65 =====
66 Iteration 9:
67 fr: [2, 1, -1, 10]
68 s1      s2      s3      s4
69 V(s) current
70 11.59    12.43    12.43    56.95
71 V(s) new
72 12.19    12.97    12.97    61.26
73 =====
74 Iteration 10:
75 fr: [2, 1, -1, 10]
76 s1      s2      s3      s4
77 V(s) current
78 12.19    12.97    12.97    61.26
79 V(s) new
80 12.68    13.41    13.41    65.13
81 =====
82 Iteration 11:
83 fr: [2, 1, -1, 10]
84 s1      s2      s3      s4
85 V(s) current

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86 12.68    13.41    13.41    65.13
87 V(s) new
88 13.07    13.76    13.76    68.62
89 =====
90 Iteration 12:
91 fr: [2, 1, -1, 10]
92 s1      s2      s3      s4
93 V(s) current
94 13.07    13.76    13.76    68.62
95 V(s) new
96 13.38    14.05    14.05    71.76
97 =====
98 Iteration 13:
99 fr: [2, 1, -1, 10]
100 s1      s2      s3      s4
101 V(s) current
102 13.38    14.05    14.05    71.76
103 V(s) new
104 13.64    14.28    14.28    74.58
105 =====
106 Iteration 14:
107 fr: [2, 1, -1, 10]
108 s1      s2      s3      s4
109 V(s) current
110 13.64    14.28    14.28    74.58
111 V(s) new
112 13.85    14.46    14.46    77.12
113 =====
114 Iteration 15:
115 fr: [2, 1, -1, 10]
116 s1      s2      s3      s4
117 V(s) current
118 13.85    14.46    14.46    77.12
119 V(s) new
120 14.02    14.62    14.62    79.41
121 =====
122 Iteration 16:
123 fr: [2, 1, -1, 10]
124 s1      s2      s3      s4
125 V(s) current
126 14.02    14.62    14.62    79.41
127 V(s) new
128 14.15    14.74    14.74    81.47
129 =====
130 Iteration 17:
131 fr: [2, 1, -1, 10]
132 s1      s2      s3      s4
133 V(s) current
134 14.15    14.74    14.74    81.47
135 V(s) new
136 14.27    14.84    14.84    83.32
137 =====

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138 Iteration 18:
139 fr: [2, 1, -1, 10]
140 s1      s2      s3      s4
141 V(s) current
142 14.27    14.84    14.84    83.32
143 V(s) new
144 14.35    14.92    14.92    84.99
145 =====
146 Iteration 19:
147 fr: [2, 1, -1, 10]
148 s1      s2      s3      s4
149 V(s) current
150 14.35    14.92    14.92    84.99
151 V(s) new
152 14.43    14.98    14.98    86.49
153 =====
154 Iteration 20:
155 fr: [2, 1, -1, 10]
156 s1      s2      s3      s4
157 V(s) current
158 14.43    14.98    14.98    86.49
159 V(s) new
160 14.49    15.04    15.04    87.84
161 =====
162 Iteration 21:
163 fr: [2, 1, -1, 10]
164 s1      s2      s3      s4
165 V(s) current
166 14.49    15.04    15.04    87.84
167 V(s) new
168 14.53    15.08    15.08    89.06
169 =====
170 Iteration 22:
171 fr: [2, 1, -1, 10]
172 s1      s2      s3      s4
173 V(s) current
174 14.53    15.08    15.08    89.06
175 V(s) new
176 14.57    15.12    15.12    90.15
177 =====
178 Iteration 23:
179 fr: [2, 1, -1, 10]
180 s1      s2      s3      s4
181 V(s) current
182 14.57    15.12    15.12    90.15
183 V(s) new
184 14.60    15.14    15.14    91.14
185 =====
186 Iteration 24:
187 fr: [2, 1, -1, 10]
188 s1      s2      s3      s4
189 V(s) current

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190 14.60    15.14    15.14    91.14
191 V(s) new
192 14.63    15.17    15.17    92.02
193 =====
194 Iteration 25:
195 fr: [2, 1, -1, 10]
196 s1      s2      s3      s4
197 V(s) current
198 14.63    15.17    15.17    92.02
199 V(s) new
200 14.65    15.18    15.18    92.82
201 =====
202 Iteration 26:
203 fr: [2, 1, -1, 10]
204 s1      s2      s3      s4
205 V(s) current
206 14.65    15.18    15.18    92.82
207 V(s) new
208 14.67    15.20    15.20    93.54
209 =====
210 Iteration 27:
211 fr: [2, 1, -1, 10]
212 s1      s2      s3      s4
213 V(s) current
214 14.67    15.20    15.20    93.54
215 V(s) new
216 14.68    15.21    15.21    94.19
217 =====
218 Iteration 28:
219 fr: [2, 1, -1, 10]
220 s1      s2      s3      s4
221 V(s) current
222 14.68    15.21    15.21    94.19
223 V(s) new
224 14.69    15.22    15.22    94.77
225 =====
226 Iteration 29:
227 fr: [2, 1, -1, 10]
228 s1      s2      s3      s4
229 V(s) current
230 14.69    15.22    15.22    94.77
231 V(s) new
232 14.70    15.23    15.23    95.29
233 =====
234 Iteration 30:
235 fr: [2, 1, -1, 10]
236 s1      s2      s3      s4
237 V(s) current
238 14.70    15.23    15.23    95.29
239 V(s) new
240 14.71    15.24    15.24    95.76
241 =====

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242 Iteration 31:
243 fr: [2, 1, -1, 10]
244 s1      s2      s3      s4
245 V(s) current
246 14.71    15.24    15.24    95.76
247 V(s) new
248 14.71    15.24    15.24    96.18
249 =====
250 Iteration 32:
251 fr: [2, 1, -1, 10]
252 s1      s2      s3      s4
253 V(s) current
254 14.71    15.24    15.24    96.18
255 V(s) new
256 14.72    15.25    15.25    96.57
257 =====
258 Iteration 33:
259 fr: [2, 1, -1, 10]
260 s1      s2      s3      s4
261 V(s) current
262 14.72    15.25    15.25    96.57
263 V(s) new
264 14.72    15.25    15.25    96.91
265 =====
266 Iteration 34:
267 fr: [2, 1, -1, 10]
268 s1      s2      s3      s4
269 V(s) current
270 14.72    15.25    15.25    96.91
271 V(s) new
272 14.72    15.25    15.25    97.22
273 =====
274 Iteration 35:
275 fr: [2, 1, -1, 10]
276 s1      s2      s3      s4
277 V(s) current
278 14.72    15.25    15.25    97.22
279 V(s) new
280 14.73    15.25    15.25    97.50
281 =====
282 Iteration 36:
283 fr: [2, 1, -1, 10]
284 s1      s2      s3      s4
285 V(s) current
286 14.73    15.25    15.25    97.50
287 V(s) new
288 14.73    15.26    15.26    97.75
289 =====
290 Iteration 37:
291 fr: [2, 1, -1, 10]
292 s1      s2      s3      s4
293 V(s) current

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294 14.73    15.26    15.26    97.75
295 V(s) new
296 14.73    15.26    15.26    97.97
297 =====
298 Iteration 38:
299 fr: [2, 1, -1, 10]
300 s1      s2      s3      s4
301 V(s) current
302 14.73    15.26    15.26    97.97
303 V(s) new
304 14.73    15.26    15.26    98.18
305 =====
306 Iteration 39:
307 fr: [2, 1, -1, 10]
308 s1      s2      s3      s4
309 V(s) current
310 14.73    15.26    15.26    98.18
311 V(s) new
312 14.73    15.26    15.26    98.36
313 =====
314 Iteration 40:
315 fr: [2, 1, -1, 10]
316 s1      s2      s3      s4
317 V(s) current
318 14.73    15.26    15.26    98.36
319 V(s) new
320 14.73    15.26    15.26    98.52
321 =====
322 Iteration 41:
323 fr: [2, 1, -1, 10]
324 s1      s2      s3      s4
325 V(s) current
326 14.73    15.26    15.26    98.52
327 V(s) new
328 14.73    15.26    15.26    98.67
329 =====
330 Iteration 42:
331 fr: [2, 1, -1, 10]
332 s1      s2      s3      s4
333 V(s) current
334 14.73    15.26    15.26    98.67
335 V(s) new
336 14.73    15.26    15.26    98.80
337 =====
338 Iteration 43:
339 fr: [2, 1, -1, 10]
340 s1      s2      s3      s4
341 V(s) current
342 14.73    15.26    15.26    98.80
343 V(s) new
344 14.73    15.26    15.26    98.92
345 =====

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346 Iteration 44:
347 fr: [2, 1, -1, 10]
348 s1      s2      s3      s4
349 V(s) current
350 14.73    15.26    15.26    98.92
351 V(s) new
352 14.74    15.26    15.26    99.03
353 =====
354 Iteration 45:
355 fr: [2, 1, -1, 10]
356 s1      s2      s3      s4
357 V(s) current
358 14.74    15.26    15.26    99.03
359 V(s) new
360 14.74    15.26    15.26    99.13
361 Optimal Politic:
362 s1 = a1,      s2 = a1,      s3 = a2,      s4 = a2,

```

Which gives out a result of:

	s1	s2	s3	s4
V(s)	14.74	15.26	15.26	99.13
f_pi(s)	a1	a1	a2	a2

Below is a convergence plot for all $V(s)$ values for each of the states $s = \{s1, s2, s3, s4\}$, where x axis is the **amount of iterations** and y axis is the **$V(s)$ result for that iteration**:

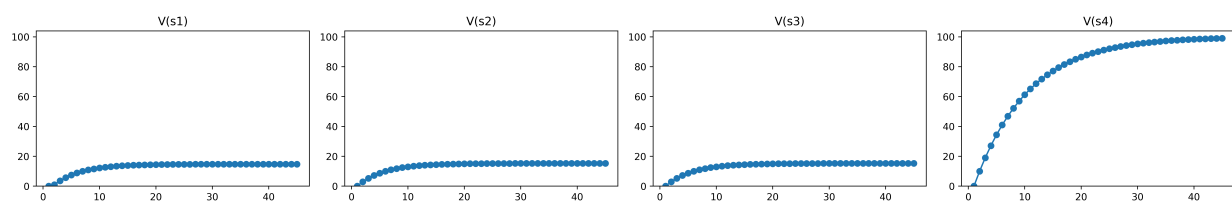


Figure 2: Image

b. Solve the Bellman Optimality Equations by Value Iteration for $Q(s, a)$.

- It was solved in 45 iterations as well, if we assume that **a change of less than 0.1 between iterations** means convergence:

```

1      =====
2 Iteration 1:
3 fr: [2, 1, -1, 10]
4 s1      s2      s3      s4
5 Q(s,a) current
6 0.00    0.00    0.00    0.00

```



```

7  0.00    0.00    0.00    0.00
8  Q(s,a) new
9  1.00    2.90   -1.00    2.90
10 3.61    -1.00    5.25   12.61
11 =====
12 Iteration 2:
13 fr: [2, 1, -1, 10]
14 s1      s2      s3      s4
15 Q(s,a) current
16 1.00    2.90   -1.00    2.90
17 3.61    -1.00    5.25   12.61
18 Q(s,a) new
19 3.61    5.25    3.72    5.25
20 5.72    3.72    7.15   21.35
21 =====
22 Iteration 3:
23 fr: [2, 1, -1, 10]
24 s1      s2      s3      s4
25 Q(s,a) current
26 3.61    5.25    3.72    5.25
27 5.72    3.72    7.15   21.35
28 Q(s,a) new
29 5.72    7.15    5.44    7.15
30 7.44    5.44    8.69   29.21
31 =====
32 Iteration 4:
33 fr: [2, 1, -1, 10]
34 s1      s2      s3      s4
35 Q(s,a) current
36 5.72    7.15    5.44    7.15
37 7.44    5.44    8.69   29.21
38 Q(s,a) new
39 7.44    8.69    6.82    8.69
40 8.82    6.82    9.94   36.29
41 =====
42 Iteration 5:
43 fr: [2, 1, -1, 10]
44 s1      s2      s3      s4
45 Q(s,a) current
46 7.44    8.69    6.82    8.69
47 8.82    6.82    9.94   36.29
48 Q(s,a) new
49 8.82    9.94    7.95    9.94
50 9.95    7.95   10.95   42.66
51 =====
52 Iteration 6:
53 fr: [2, 1, -1, 10]
54 s1      s2      s3      s4
55 Q(s,a) current
56 8.82    9.94    7.95    9.94
57 9.95    7.95   10.95   42.66
58 Q(s,a) new

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59 9.95      10.95      8.86      10.95
60 10.86     8.86      11.77     48.40
61 =====
62 Iteration 7:
63 fr: [2, 1, -1, 10]
64 s1      s2      s3      s4
65 Q(s,a) current
66 9.95     10.95     8.86     10.95
67 10.86     8.86     11.77     48.40
68 Q(s,a) new
69 10.86     11.77     9.59     11.77
70 11.59     9.59     12.43     53.56
71 =====
72 Iteration 8:
73 fr: [2, 1, -1, 10]
74 s1      s2      s3      s4
75 Q(s,a) current
76 10.86     11.77     9.59     11.77
77 11.59     9.59     12.43     53.56
78 Q(s,a) new
79 11.59     12.43     10.19     12.43
80 12.19     10.19     12.97     58.20
81 =====
82 Iteration 9:
83 fr: [2, 1, -1, 10]
84 s1      s2      s3      s4
85 Q(s,a) current
86 11.59     12.43     10.19     12.43
87 12.19     10.19     12.97     58.20
88 Q(s,a) new
89 12.19     12.97     10.68     12.97
90 12.68     10.68     13.41     62.38
91 =====
92 Iteration 10:
93 fr: [2, 1, -1, 10]
94 s1      s2      s3      s4
95 Q(s,a) current
96 12.19     12.97     10.68     12.97
97 12.68     10.68     13.41     62.38
98 Q(s,a) new
99 12.68     13.41     11.07     13.41
100 13.07     11.07     13.76     66.14
101 =====
102 Iteration 11:
103 fr: [2, 1, -1, 10]
104 s1      s2      s3      s4
105 Q(s,a) current
106 12.68     13.41     11.07     13.41
107 13.07     11.07     13.76     66.14
108 Q(s,a) new
109 13.07     13.76     11.38     13.76
110 13.38     11.38     14.05     69.53

```

```

111 =====
112 Iteration 12:
113 fr: [2, 1, -1, 10]
114 s1      s2      s3      s4
115 Q(s,a) current
116 13.07    13.76    11.38    13.76
117 13.38    11.38    14.05    69.53
118 Q(s,a) new
119 13.38    14.05    11.64    14.05
120 13.64    11.64    14.28    72.58
121 =====
122 Iteration 13:
123 fr: [2, 1, -1, 10]
124 s1      s2      s3      s4
125 Q(s,a) current
126 13.38    14.05    11.64    14.05
127 13.64    11.64    14.28    72.58
128 Q(s,a) new
129 13.64    14.28    11.85    14.28
130 13.85    11.85    14.46    75.32
131 =====
132 Iteration 14:
133 fr: [2, 1, -1, 10]
134 s1      s2      s3      s4
135 Q(s,a) current
136 13.64    14.28    11.85    14.28
137 13.85    11.85    14.46    75.32
138 Q(s,a) new
139 13.85    14.46    12.02    14.46
140 14.02    12.02    14.62    77.79
141 =====
142 Iteration 15:
143 fr: [2, 1, -1, 10]
144 s1      s2      s3      s4
145 Q(s,a) current
146 13.85    14.46    12.02    14.46
147 14.02    12.02    14.62    77.79
148 Q(s,a) new
149 14.02    14.62    12.15    14.62
150 14.15    12.15    14.74    80.01
151 =====
152 Iteration 16:
153 fr: [2, 1, -1, 10]
154 s1      s2      s3      s4
155 Q(s,a) current
156 14.02    14.62    12.15    14.62
157 14.15    12.15    14.74    80.01
158 Q(s,a) new
159 14.15    14.74    12.27    14.74
160 14.27    12.27    14.84    82.01
161 =====
162 Iteration 17:

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163 fr: [2, 1, -1, 10]
164 s1      s2      s3      s4
165 Q(s,a) current
166 14.15    14.74    12.27    14.74
167 14.27    12.27    14.84    82.01
168 Q(s,a) new
169 14.27    14.84    12.35    14.84
170 14.35    12.35    14.92    83.81
171 =====
172 Iteration 18:
173 fr: [2, 1, -1, 10]
174 s1      s2      s3      s4
175 Q(s,a) current
176 14.27    14.84    12.35    14.84
177 14.35    12.35    14.92    83.81
178 Q(s,a) new
179 14.35    14.92    12.43    14.92
180 14.43    12.43    14.98    85.43
181 =====
182 Iteration 19:
183 fr: [2, 1, -1, 10]
184 s1      s2      s3      s4
185 Q(s,a) current
186 14.35    14.92    12.43    14.92
187 14.43    12.43    14.98    85.43
188 Q(s,a) new
189 14.43    14.98    12.49    14.98
190 14.49    12.49    15.04    86.88
191 =====
192 Iteration 20:
193 fr: [2, 1, -1, 10]
194 s1      s2      s3      s4
195 Q(s,a) current
196 14.43    14.98    12.49    14.98
197 14.49    12.49    15.04    86.88
198 Q(s,a) new
199 14.49    15.04    12.53    15.04
200 14.53    12.53    15.08    88.19
201 =====
202 Iteration 21:
203 fr: [2, 1, -1, 10]
204 s1      s2      s3      s4
205 Q(s,a) current
206 14.49    15.04    12.53    15.04
207 14.53    12.53    15.08    88.19
208 Q(s,a) new
209 14.53    15.08    12.57    15.08
210 14.57    12.57    15.12    89.38
211 =====
212 Iteration 22:
213 fr: [2, 1, -1, 10]
214 s1      s2      s3      s4

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215 Q(s,a) current
216 14.53    15.08    12.57    15.08
217 14.57    12.57    15.12    89.38
218 Q(s,a) new
219 14.57    15.12    12.60    15.12
220 14.60    12.60    15.14    90.44
221 =====
222 Iteration 23:
223 fr: [2, 1, -1, 10]
224 s1      s2      s3      s4
225 Q(s,a) current
226 14.57    15.12    12.60    15.12
227 14.60    12.60    15.14    90.44
228 Q(s,a) new
229 14.60    15.14    12.63    15.14
230 14.63    12.63    15.17    91.39
231 =====
232 Iteration 24:
233 fr: [2, 1, -1, 10]
234 s1      s2      s3      s4
235 Q(s,a) current
236 14.60    15.14    12.63    15.14
237 14.63    12.63    15.17    91.39
238 Q(s,a) new
239 14.63    15.17    12.65    15.17
240 14.65    12.65    15.18    92.25
241 =====
242 Iteration 25:
243 fr: [2, 1, -1, 10]
244 s1      s2      s3      s4
245 Q(s,a) current
246 14.63    15.17    12.65    15.17
247 14.65    12.65    15.18    92.25
248 Q(s,a) new
249 14.65    15.18    12.67    15.18
250 14.67    12.67    15.20    93.03
251 =====
252 Iteration 26:
253 fr: [2, 1, -1, 10]
254 s1      s2      s3      s4
255 Q(s,a) current
256 14.65    15.18    12.67    15.18
257 14.67    12.67    15.20    93.03
258 Q(s,a) new
259 14.67    15.20    12.68    15.20
260 14.68    12.68    15.21    93.73
261 =====
262 Iteration 27:
263 fr: [2, 1, -1, 10]
264 s1      s2      s3      s4
265 Q(s,a) current
266 14.67    15.20    12.68    15.20

```

```

267 14.68    12.68    15.21    93.73
268 Q(s,a) new
269 14.68    15.21    12.69    15.21
270 14.69    12.69    15.22    94.35
271 =====
272 Iteration 28:
273 fr: [2, 1, -1, 10]
274 s1      s2      s3      s4
275 Q(s,a) current
276 14.68    15.21    12.69    15.21
277 14.69    12.69    15.22    94.35
278 Q(s,a) new
279 14.69    15.22    12.70    15.22
280 14.70    12.70    15.23    94.92
281 =====
282 Iteration 29:
283 fr: [2, 1, -1, 10]
284 s1      s2      s3      s4
285 Q(s,a) current
286 14.69    15.22    12.70    15.22
287 14.70    12.70    15.23    94.92
288 Q(s,a) new
289 14.70    15.23    12.71    15.23
290 14.71    12.71    15.24    95.43
291 =====
292 Iteration 30:
293 fr: [2, 1, -1, 10]
294 s1      s2      s3      s4
295 Q(s,a) current
296 14.70    15.23    12.71    15.23
297 14.71    12.71    15.24    95.43
298 Q(s,a) new
299 14.71    15.24    12.71    15.24
300 14.71    12.71    15.24    95.88
301 =====
302 Iteration 31:
303 fr: [2, 1, -1, 10]
304 s1      s2      s3      s4
305 Q(s,a) current
306 14.71    15.24    12.71    15.24
307 14.71    12.71    15.24    95.88
308 Q(s,a) new
309 14.71    15.24    12.72    15.24
310 14.72    12.72    15.25    96.30
311 =====
312 Iteration 32:
313 fr: [2, 1, -1, 10]
314 s1      s2      s3      s4
315 Q(s,a) current
316 14.71    15.24    12.72    15.24
317 14.72    12.72    15.25    96.30
318 Q(s,a) new

```

```

319 14.72    15.25    12.72    15.25
320 14.72    12.72    15.25    96.67
321 =====
322 Iteration 33:
323 fr: [2, 1, -1, 10]
324 s1      s2      s3      s4
325 Q(s,a) current
326 14.72    15.25    12.72    15.25
327 14.72    12.72    15.25    96.67
328 Q(s,a) new
329 14.72    15.25    12.72    15.25
330 14.72    12.72    15.25    97.00
331 =====
332 Iteration 34:
333 fr: [2, 1, -1, 10]
334 s1      s2      s3      s4
335 Q(s,a) current
336 14.72    15.25    12.72    15.25
337 14.72    12.72    15.25    97.00
338 Q(s,a) new
339 14.72    15.25    12.73    15.25
340 14.73    12.73    15.25    97.30
341 =====
342 Iteration 35:
343 fr: [2, 1, -1, 10]
344 s1      s2      s3      s4
345 Q(s,a) current
346 14.72    15.25    12.73    15.25
347 14.73    12.73    15.25    97.30
348 Q(s,a) new
349 14.73    15.25    12.73    15.25
350 14.73    12.73    15.26    97.57
351 =====
352 Iteration 36:
353 fr: [2, 1, -1, 10]
354 s1      s2      s3      s4
355 Q(s,a) current
356 14.73    15.25    12.73    15.25
357 14.73    12.73    15.26    97.57
358 Q(s,a) new
359 14.73    15.26    12.73    15.26
360 14.73    12.73    15.26    97.81
361 =====
362 Iteration 37:
363 fr: [2, 1, -1, 10]
364 s1      s2      s3      s4
365 Q(s,a) current
366 14.73    15.26    12.73    15.26
367 14.73    12.73    15.26    97.81
368 Q(s,a) new
369 14.73    15.26    12.73    15.26
370 14.73    12.73    15.26    98.03

```

```

371 =====
372 Iteration 38:
373 fr: [2, 1, -1, 10]
374 s1      s2      s3      s4
375 Q(s,a) current
376 14.73    15.26    12.73    15.26
377 14.73    12.73    15.26    98.03
378 Q(s,a) new
379 14.73    15.26    12.73    15.26
380 14.73    12.73    15.26    98.23
381 =====
382 Iteration 39:
383 fr: [2, 1, -1, 10]
384 s1      s2      s3      s4
385 Q(s,a) current
386 14.73    15.26    12.73    15.26
387 14.73    12.73    15.26    98.23
388 Q(s,a) new
389 14.73    15.26    12.73    15.26
390 14.73    12.73    15.26    98.41
391 =====
392 Iteration 40:
393 fr: [2, 1, -1, 10]
394 s1      s2      s3      s4
395 Q(s,a) current
396 14.73    15.26    12.73    15.26
397 14.73    12.73    15.26    98.41
398 Q(s,a) new
399 14.73    15.26    12.73    15.26
400 14.73    12.73    15.26    98.56
401 =====
402 Iteration 41:
403 fr: [2, 1, -1, 10]
404 s1      s2      s3      s4
405 Q(s,a) current
406 14.73    15.26    12.73    15.26
407 14.73    12.73    15.26    98.56
408 Q(s,a) new
409 14.73    15.26    12.73    15.26
410 14.73    12.73    15.26    98.71
411 =====
412 Iteration 42:
413 fr: [2, 1, -1, 10]
414 s1      s2      s3      s4
415 Q(s,a) current
416 14.73    15.26    12.73    15.26
417 14.73    12.73    15.26    98.71
418 Q(s,a) new
419 14.73    15.26    12.73    15.26
420 14.73    12.73    15.26    98.84
421 =====
422 Iteration 43:

```



```

423 fr: [2, 1, -1, 10]
424 s1      s2      s3      s4
425 Q(s,a) current
426 14.73    15.26    12.73    15.26
427 14.73    12.73    15.26    98.84
428 Q(s,a) new
429 14.73    15.26    12.74    15.26
430 14.74    12.74    15.26    98.95
431 =====
432 Iteration 44:
433 fr: [2, 1, -1, 10]
434 s1      s2      s3      s4
435 Q(s,a) current
436 14.73    15.26    12.74    15.26
437 14.74    12.74    15.26    98.95
438 Q(s,a) new
439 14.74    15.26    12.74    15.26
440 14.74    12.74    15.26    99.06
441 =====
442 Iteration 45:
443 fr: [2, 1, -1, 10]
444 s1      s2      s3      s4
445 Q(s,a) current
446 14.74    15.26    12.74    15.26
447 14.74    12.74    15.26    99.06
448 Q(s,a) new
449 14.74    15.26    12.74    15.26
450 14.74    12.74    15.26    99.15
451 Optimal Politic:
452 s1 = a1,      s2 = a1,      s3 = a2,      s4 = a2,

```

Which give out a result of:

	s1	s2	s3	s4
Q(s, a1)	14.74	15.26	12.74	15.26
Q(s, a2)	14.74	12.74	15.26	99.15
f_pi(s)	a1	a1	a2	a2

Below is a similarly plotted convergence plot for $Q(s, a)$ values:

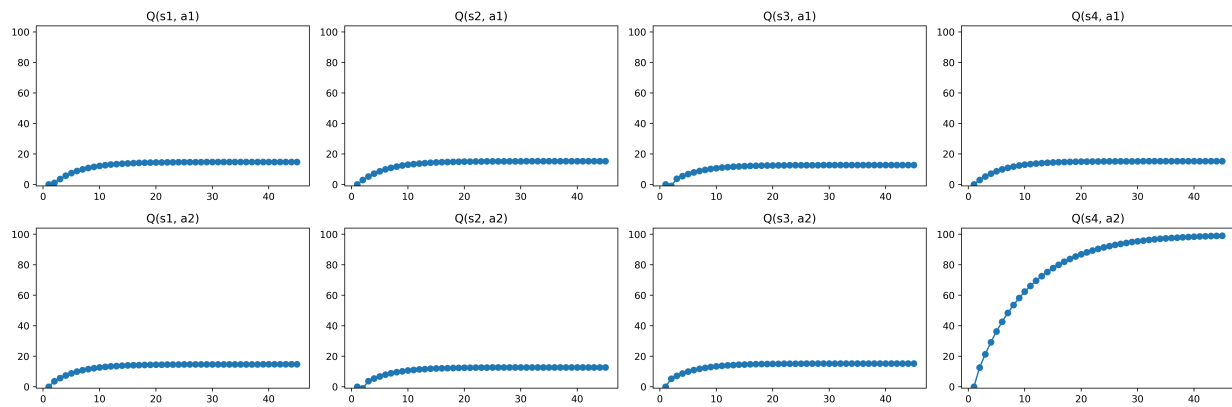


Figure 3: Image

2. Given the world defined by the Transition Function $\text{Pmt}(sf|s, a)$, the Reward Function $\text{fr}(s, a, sf) = \text{fr}(sf)$ and $\text{gamma} = 0.6$:

$$f_{M_T}(s, a) = \begin{matrix} & \begin{matrix} s_f = s_1 \\ a_1 & a_2 \end{matrix} & \begin{matrix} s_f = s_2 \\ a_1 & a_2 \end{matrix} & \begin{matrix} s_f = s_3 \\ a_1 & a_2 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} & \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 0 \\ 1 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.5 & 0.8 \\ 0 & 0 \\ 0 & 0.6 \end{bmatrix} & \begin{bmatrix} 0.1 & 0 \\ 0.5 & 1 \\ 0 & 0.1 \end{bmatrix} \end{matrix} \quad f_R(s_f) = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Figure 4: Image

- a. Solve the Bellman Optimality Equations by Value Iteration for $V(s)$.

Solution

It was solved in 9 iterations, if we assume that **a change of less than 0.01 between iterations** means it converged:

```

1  =====
2  Iteration 1:
3  fr: [2, 1, -1]
4  s1      s2      s3
5  V(s) current
6  0.00    0.00    0.00
7  V(s) new
8  1.20    0.86    2.72
9  =====
10 Iteration 2:
11 fr: [2, 1, -1]
12 s1      s2      s3
13 V(s) current
14 1.20    0.86    2.72
15 V(s) new
16 1.91    1.89    3.15
17 =====
18 Iteration 3:
19 fr: [2, 1, -1]
20 s1      s2      s3

```

```

21 V(s) current
22 1.91      1.89      3.15
23 V(s) new
24 2.41      2.17      3.45
25 =====
26 Iteration 4:
27 fr: [2, 1, -1]
28 s1      s2      s3
29 V(s) current
30 2.41      2.17      3.45
31 V(s) new
32 2.64      2.33      3.58
33 =====
34 Iteration 5:
35 fr: [2, 1, -1]
36 s1      s2      s3
37 V(s) current
38 2.64      2.33      3.58
39 V(s) new
40 2.75      2.40      3.65
41 =====
42 Iteration 6:
43 fr: [2, 1, -1]
44 s1      s2      s3
45 V(s) current
46 2.75      2.40      3.65
47 V(s) new
48 2.80      2.43      3.68
49 =====
50 Iteration 7:
51 fr: [2, 1, -1]
52 s1      s2      s3
53 V(s) current
54 2.80      2.43      3.68
55 V(s) new
56 2.82      2.45      3.69
57 =====
58 Iteration 8:
59 fr: [2, 1, -1]
60 s1      s2      s3
61 V(s) current
62 2.82      2.45      3.69
63 V(s) new
64 2.83      2.46      3.70
65 =====
66 Iteration 9:
67 fr: [2, 1, -1]
68 s1      s2      s3
69 V(s) current
70 2.83      2.46      3.70
71 V(s) new
72 2.84      2.46      3.70

```

```

73 Optimal Politic:
74 s1 = a1,          s2 = a1,          s3 = a1

```

Which gives out a result of:

	s1	s2	s3
V(s)	2.84	2.46	3.70
f_pi(s)	a1	a1	a1

The convergence plot for each $V(s)$ value is given below:

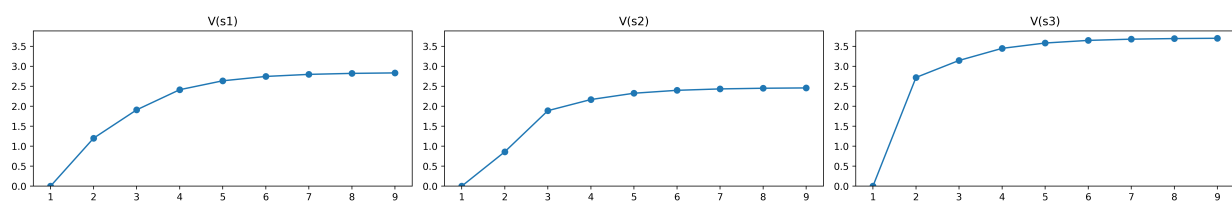


Figure 5: Image

b. Solve the Bellman Optimality Equations by Value Iteration for $Q(s, a)$.

Solution

It was solved in 8 iterations, assuming **a change of less than 0.01 between iterations** means convergence:

```

1  =====
2  Iteration 1:
3  fr: [2, 1, -1]
4  s1      s2      s3
5  Q(s,a) current
6  0.00    0.00    0.00
7  0.00    0.00    0.00
8  Q(s,a) new
9  1.20    0.86    2.72
10 1.76    0.63    1.89
11 =====
12 Iteration 2:
13 fr: [2, 1, -1]
14 s1      s2      s3
15 Q(s,a) current
16 1.20    0.86    2.72
17 1.76    0.63    1.89
18 Q(s,a) new
19 2.04    1.93    3.23
20 2.37    0.94    2.41
21 =====
22 Iteration 3:

```

```

23 fr: [2, 1, -1]
24 s1      s2      s3
25 Q(s,a) current
26 2.04     1.93     3.23
27 2.37     0.94     2.41
28 Q(s,a) new
29 2.54     2.23     3.52
30 2.58     1.11     2.58
31 =====
32 Iteration 4:
33 fr: [2, 1, -1]
34 s1      s2      s3
35 Q(s,a) current
36 2.54     2.23     3.52
37 2.58     1.11     2.58
38 Q(s,a) new
39 2.70     2.37     3.62
40 2.66     1.17     2.66
41 =====
42 Iteration 5:
43 fr: [2, 1, -1]
44 s1      s2      s3
45 Q(s,a) current
46 2.70     2.37     3.62
47 2.66     1.17     2.66
48 Q(s,a) new
49 2.77     2.42     3.66
50 2.69     1.20     2.69
51 =====
52 Iteration 6:
53 fr: [2, 1, -1]
54 s1      s2      s3
55 Q(s,a) current
56 2.77     2.42     3.66
57 2.69     1.20     2.69
58 Q(s,a) new
59 2.81     2.44     3.69
60 2.71     1.21     2.71
61 =====
62 Iteration 7:
63 fr: [2, 1, -1]
64 s1      s2      s3
65 Q(s,a) current
66 2.81     2.44     3.69
67 2.71     1.21     2.71
68 Q(s,a) new
69 2.83     2.45     3.70
70 2.72     1.22     2.71
71 =====
72 Iteration 8:
73 fr: [2, 1, -1]
74 s1      s2      s3

```

```

75 Q(s,a) current
76 2.83      2.45      3.70
77 2.72      1.22      2.71
78 Q(s,a) new
79 2.84      2.46      3.70
80 2.72      1.22      2.72
81 Optimal Politic:
82 s1 = a1,      s2 = a1,      s3 = a1

```

Which gives out a result of:

	s1	s2	s3
$Q(s, a1)$	2.84	2.46	3.70
$Q(s, a2)$	2.72	1.22	2.72
$f_{\pi}(s)$	a1	a1	a1

The convergence plot for each $Q(s,a)$ value is given below:

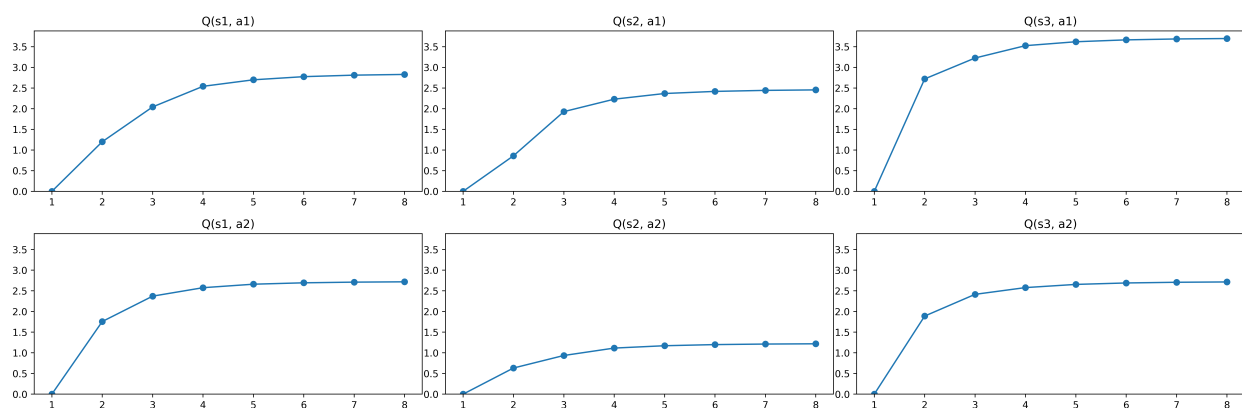


Figure 6: Image

- Given the world defined by the graph, the Reward Function $fr(s,a,sf)$ and $\gamma = 0.9$:

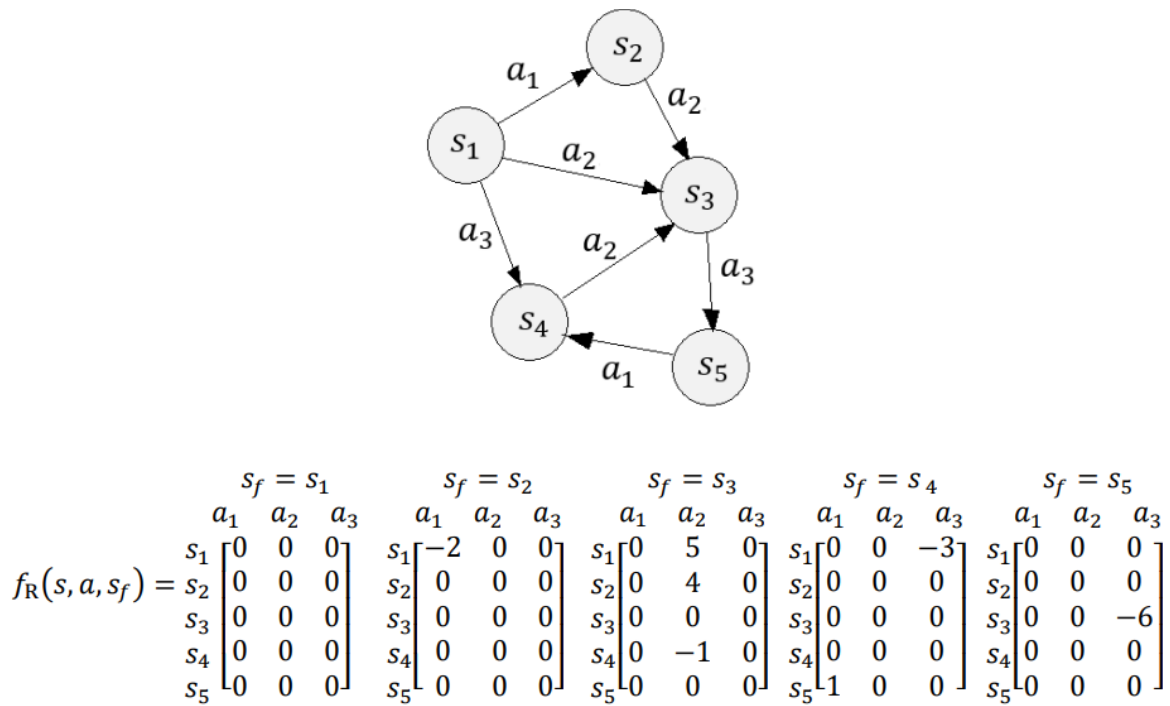


Figure 7: Image

a. Solve the Bellman Optimality Equations by Value Iteration for $V(s)$.

Solution

The Transition Function can therefore be defined as:

$$f_{MT}(s, a) = \begin{matrix} a_1 & a_2 & a_3 \\ s_1 & \begin{bmatrix} s_2 & s_3 & s_4 \end{bmatrix} \\ s_2 & \begin{bmatrix} s_2 & s_3 & s_4 \end{bmatrix} \\ s_3 & \begin{bmatrix} s_2 & s_3 & s_5 \end{bmatrix} \\ s_4 & \begin{bmatrix} s_4 & s_3 & s_4 \end{bmatrix} \\ s_5 & \begin{bmatrix} s_4 & s_5 & s_5 \end{bmatrix} \end{matrix}$$

Figure 8: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```

1 =====
2 Iteration 1:
3 fr: [[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
      0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
      0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0,
      0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1,
      0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0],
      [0, 0, 0]]
4 s1      s2      s3      s4      s5
5 V(s) current

```

```

6  0.00    0.00    0.00    0.00    0.00
7  V(s) new
8  5.00    4.00    0.00    0.00    1.00
9  =====
10 Iteration 2:
11 fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
      0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
      0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0,
      0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1,
      0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0],
      [0, 0, 0]]]
12 s1      s2      s3      s4      s5
13 V(s) current
14 5.00    4.00    0.00    0.00    1.00
15 V(s) new
16 5.00    4.00    0.00    0.00    1.00
17 Optimal Politic:
18 s1 = a2,      s2 = a2,      s3 = a1,      s4 = a1,
      s5 = a1

```

Which gives out a result of:

	s1	s2	s3	s4	s5
V(s)	5.00	4.00	0.00	0.00	1.00
f_pi(s)	a2	a2	a1	a1	a1

The convergence plot for each $V(s)$ value is given below:

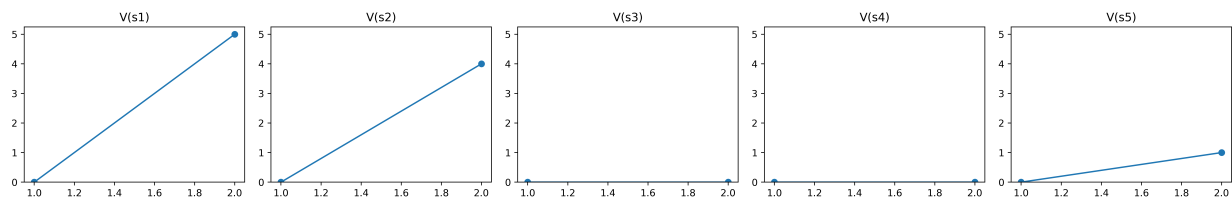


Figure 9: Image

b. Solve the Bellman Optimality Equations by Value Iteration for $Q(s,a)$.

Solution

The Transition Function can therefore be defined as:

$$f_{MT}(s,a) = \begin{matrix} & a_1 & a_2 & a_3 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} & \begin{bmatrix} s_2 & s_3 & s_4 \\ s_2 & s_3 & s_2 \\ s_2 & s_3 & s_5 \\ s_4 & s_3 & s_4 \\ s_4 & s_5 & s_5 \end{bmatrix} \end{matrix}$$

Figure 10: Image

Thus, the Bellman Optimality Equations were solved in 3 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```

1  =====
2  Iteration 1:
3  fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
        0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
        0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0,
        0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1,
        0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0],
        [0, 0, 0]]]
4  s1      s2      s3      s4      s5
5  Q(s,a) current
6  0.00     0.00     0.00     0.00     0.00
7  0.00     0.00     0.00     0.00     0.00
8  0.00     0.00     0.00     0.00     0.00
9  Q(s,a) new
10 -2.00     0.00     0.00     0.00     1.00
11  5.00     4.00     0.00    -1.00     0.90
12 -3.00     3.60    -5.10     0.00     0.90
13  =====
14 Iteration 2:
15 fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
        0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
        0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0,
        0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1,
        0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0],
        [0, 0, 0]]]
16 s1      s2      s3      s4      s5
17 Q(s,a) current
18 -2.00     0.00     0.00     0.00     1.00
19  5.00     4.00     0.00    -1.00     0.90
20 -3.00     3.60    -5.10     0.00     0.90
21 Q(s,a) new
22  1.60     3.60     0.00     0.00     1.00
23  5.00     4.00     0.00    -1.00     0.90
24 -3.00     3.60    -5.10     0.00     0.90
25  =====
26 Iteration 3:
27 fr: [[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
        0]], [[-2, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0,
        0]], [[0, 5, 0], [0, 4, 0], [0, 0, 0], [0, -1, 0], [0,

```

```

0, 0]], [[0, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0], [1,
0, 0]], [[0, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0, 0],
[0, 0, 0]]]
28 s1      s2      s3      s4      s5
29 Q(s,a) current
30 1.60    3.60    0.00    0.00    1.00
31 5.00    4.00    0.00    -1.00   0.90
32 -3.00   3.60   -5.10    0.00   0.90
33 Q(s,a) new
34 1.60    3.60    0.00    0.00    1.00
35 5.00    4.00    0.00    -1.00   0.90
36 -3.00   3.60   -5.10    0.00   0.90
37 Optimal Politic:
38 s1 = a2,      s2 = a2,      s3 = a1,      s4 = a1,
      s5 = a1,

```

Which gives out a result of:

	s1	s2	s3	s4	s5
$Q(s, a1)$	1.60	3.60	0.00	0.00	1.00
$Q(s, a2)$	5.00	4.00	0.00	-1.00	0.90
$Q(s, a3)$	-3.00	3.60	-5.10	0.00	0.90
$f_{\pi}(s)$	a2	a2	a1	a1	a1

The convergence plot for each $Q(s,a)$ value is given below:

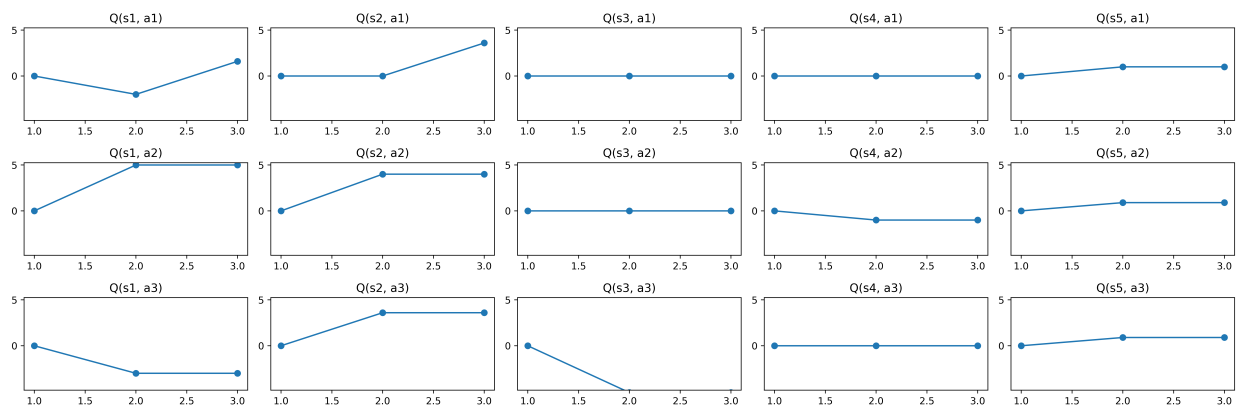
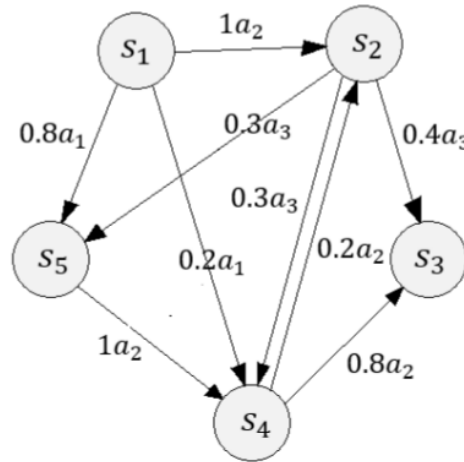


Figure 11: Image

- Given the world defined by the graph and with the following Reward Function $fr(s,a, sf)$ and $\gamma = 0.7$:



$$f_R(s, a, s_f) = \begin{matrix} & \begin{matrix} s_f = s_1 & s_f = s_2 & s_f = s_3 & s_f = s_4 & s_f = s_5 \end{matrix} \\ \begin{matrix} a_1 & a_2 & a_3 \end{matrix} & \begin{matrix} s_1 & s_2 & s_3 & s_4 & s_5 \end{matrix} \end{matrix} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 5 & 0 \\ 0 & 4 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 12: Image

a. Solve the Bellman Optimality Equations by Value Iteration for $V(s)$.

Solution

The Transition Model would be then:

$$P_{MT}(s, \omega) = \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.1 \\ 1 & 1 & 1 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} & \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0.3 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix} \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} & \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

Figure 13: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```

1 =====
2 Iteration 1:
3 fr: [[[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0,
      0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0], [0,
      0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1, 0],
      [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0],
      [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0,
      0], [0, 0, 0]]]
4 s1      s2      s3      s4      s5

```

```

5 V(s) current
6 0.00      0.00      0.00      0.00      0.00
7 V(s) new
8 2.60      0.00      0.00      0.00      0.00
9 =====
10 Iteration 2:
11 fr: [[[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0,
      0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0], [0,
      0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1, 0],
      [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0],
      [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0,
      0], [0, 0, 0]]]
12 s1      s2      s3      s4      s5
13 V(s) current
14 2.60      0.00      0.00      0.00      0.00
15 V(s) new
16 2.60      0.00      0.00      0.00      0.00
17 Optimal Politic:
18 s1 = a1,      s2 = a1,      s3 = a1,      s4 = a1,
      s5 = a1

```

Which gives out a result of:

	s1	s2	s3	s4	s5
V(s)	2.60	0.00	0.00	0.00	0.00
f_pi(s)	a1	a1	a1	a1	a1

The convergence plot for each $V(s)$ value is given below:

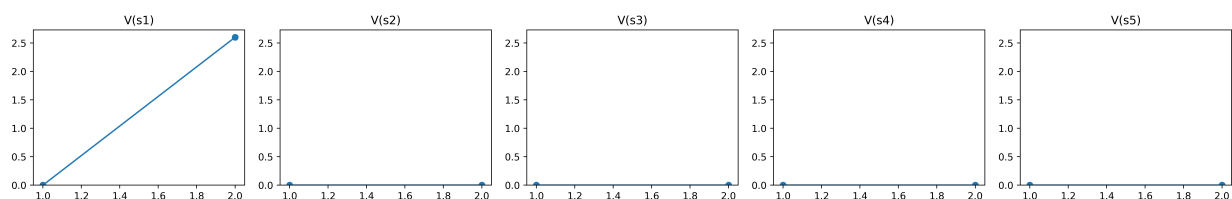


Figure 14: Image

b. Solve the Bellman Optimality Equations by Value Iteration for $Q(s,a)$.

Solution

The Transition Function can therefore be defined as:

$$P_{MT}(s_i, a) = \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.1 \\ 1 & 1 & 1 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0.3 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} & \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0 & 0.3 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \\ & s_f = s_1 & s_f = s_2 & s_f = s_3 & s_f = s_4 & s_f = s_5 \end{matrix}$$

Figure 15: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence.

```

1  =====
2  Iteration 1:
3  fr: [[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0,
      0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0], [0,
      0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1, 0],
      [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0],
      [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0,
      0], [0, 0, 0]]
4  s1      s2      s3      s4      s5
5  Q(s,a) current
6  0.00     0.00     0.00     0.00     0.00
7  0.00     0.00     0.00     0.00     0.00
8  0.00     0.00     0.00     0.00     0.00
9  Q(s,a) new
10 2.60     0.00     0.00     0.00     0.00
11 2.00     0.00     0.00     -1.40    -1.00
12 1.82     -0.40    0.00     0.00     0.00
13 =====
14 Iteration 2:
15 fr: [[9, 0, 0], [0, 0, 0], [0, 1, 0], [0, 0, -2], [2, 0,
      0]], [[-2, 2, 0], [0, 0, 0], [0, 0, 0], [0, -3, 0], [0,
      0, 0]], [[0, 5, 0], [0, 4, -1], [0, 0, 0], [0, -1, 0],
      [0, 0, 0]], [[1, 0, -3], [0, 0, 0], [0, 0, 0], [0, 0, 0],
      [1, -1, 0]], [[3, 0, 0], [0, 0, 0], [0, 0, -6], [0, 0,
      0], [0, 0, 0]]
16 s1      s2      s3      s4      s5
17 Q(s,a) current
18 2.60     0.00     0.00     0.00     0.00
19 2.00     0.00     0.00     -1.40    -1.00
20 1.82     -0.40    0.00     0.00     0.00
21 Q(s,a) new
22 2.60     0.00     0.00     0.00     0.00
23 2.00     0.00     0.00     -1.40    -1.00
24 1.82     -0.40    0.00     0.00     0.00
25 Optimal Politic:
26 s1 = a1,      s2 = a1,      s3 = a1,      s4 = a1,
      s5 = a1,

```

Which gives out a result of:

	s1	s2	s3	s4	s5
$Q(s, a1)$	2.60	0.00	0.00	0.00	0.00
$Q(s, a2)$	2.00	0.00	0.00	-1.40	-1.00
$Q(s, a3)$	1.82	-0.40	0.00	0.00	0.00
$f_{\pi}(s)$	a1	a1	a1	a1	a1

The convergence plot for each $Q(s,a)$ value is given below:

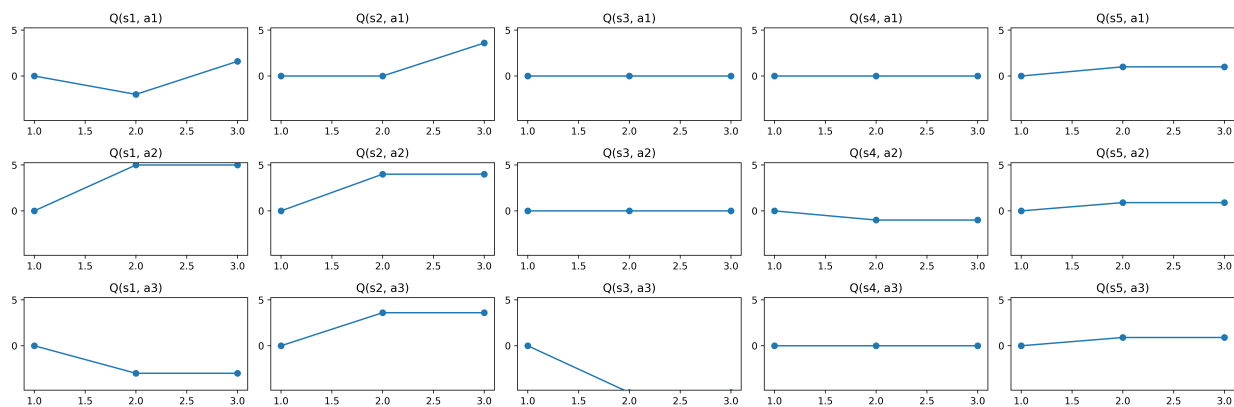


Figure 16: Image

5. The world has the states set $S = \{sf1, s1, s2, s3, sf2\}$ where $s1$ = initial state and, $sf1$ and $sf2$ are terminal states:

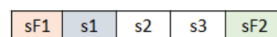


Figure 17: Image

The world has the set of actions $A = \{->, <- \}$ where:

- $->$ = agent moves to the right one cell
- $<-$ = agent moves to the left one cell

The Reward Function $fr(s,a,sf) = fr(sf)$ only depends on the state to which the agent arrives:

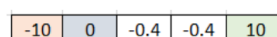


Figure 18: Image

The agent has the Action Function $f_{\pi}(s)$:

$$f_{\pi}(s) = \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_{F1} \\ s_{F2} \end{matrix} \begin{bmatrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \leftarrow \\ \rightarrow \end{bmatrix}$$

Figure 19: Image

Do the following:

- Solve the Bellman Optimality Equations by Value Iteration for $V(s)$.

Solution

The Transition Function would be:

$$f_{\pi}(s, a) = \begin{matrix} & \rightarrow & \leftarrow \\ \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F2} \end{matrix} & \begin{bmatrix} s_1 & s_{F1} \\ s_2 & s_{F1} \\ s_3 & s_1 \\ s_{F2} & s_2 \\ s_{F2} & s_3 \end{bmatrix} \end{matrix}$$

Figure 20: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume `gamma` = 0.0.

```

1  =====
2  Iteration 1:
3  fr: [-10, 0, -0.4, -0.4, 10]
4  sf1    s1      s2      s3      sf2
5  V(s) current
6  0.00    0.00    0.00    0.00    0.00
7  V(s) new
8  0.00    -0.40    0.00    10.00    10.00
9  =====
10 Iteration 2:
11 fr: [-10, 0, -0.4, -0.4, 10]
12 sf1    s1      s2      s3      sf2
13 V(s) current
14 0.00    -0.40    0.00    10.00    10.00
15 V(s) new
16 0.00    -0.40    0.00    10.00    10.00
17 Optimal Politic:
18 sf1 = ->,      s1 = ->,      s2 = ->,      s3 = ->,
      sf2 = ->

```

Which gives out a result of:

	sf1	s1	s2	s3	sf2
$V(s)$	0.00	-0.40	0.00	10.00	10.00
$f_{\pi}(s)$	->	->	->	->	->

The convergence plot for each $V(s)$ value is given below:

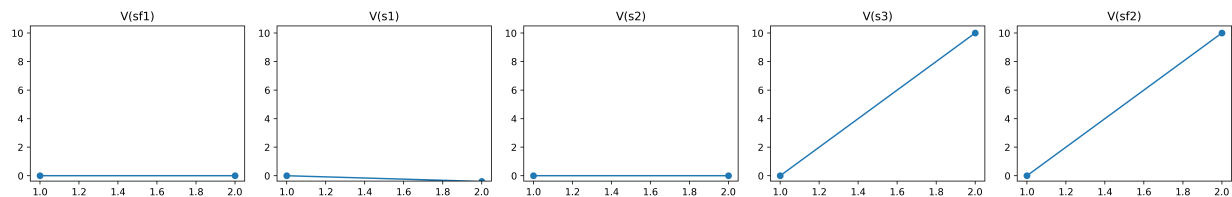


Figure 21: Image

b. Solve the Bellman Optimality Equations by Value Iteration for $Q(s,a)$.

Solution

The Transition Function would be:

$$f_{MT}(s,a) = \begin{matrix} \rightarrow & \leftarrow \\ s_1 & s_{F_1} \\ s_1 & s_2 & s_{F_1} \\ s_2 & s_3 & s_1 \\ s_3 & s_{F_2} & s_2 \\ s_{F_1} & s_{F_2} & s_3 \end{matrix}$$

Figure 22: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume $\gamma = 0.0$.

```

1  =====
2  Iteration 1:
3  fr: [-10, 0, -0.4, -0.4, 10]
4  sf1      s1      s2      s3      sf2
5  Q(s,a) current
6  0.00     0.00     0.00     0.00     0.00
7  0.00     0.00     0.00     0.00     0.00
8  Q(s,a) new
9  0.00     -0.40     -0.40     10.00     10.00
10 -10.00    -10.00     0.00     -0.40     -0.40
11 =====

```



```

12 Iteration 2:
13 fr: [-10, 0, -0.4, -0.4, 10]
14 sf1      s1      s2      s3      sf2
15 Q(s,a) current
16 0.00     -0.40    -0.40    10.00    10.00
17 -10.00   -10.00    0.00     -0.40    -0.40
18 Q(s,a) new
19 0.00     -0.40    -0.40    10.00    10.00
20 -10.00   -10.00    0.00     -0.40    -0.40
21 Optimal Politic:
22 sf1 = ->,      s1 = ->,      s2 = <-,      s3 = ->,
      sf2 = ->

```

Which gives out a result of:

	s1	s2	s3	s4	s5
$Q(s, ->)$	0.00	-0.40	-0.40	10.00	10.00
$Q(s, <-)$	-10.00	-10.00	0.00	-0.40	-0.40
$f_{\pi}(s)$	->	->	<-	->	->

The convergence plot for each $Q(s,a)$ value is given below:

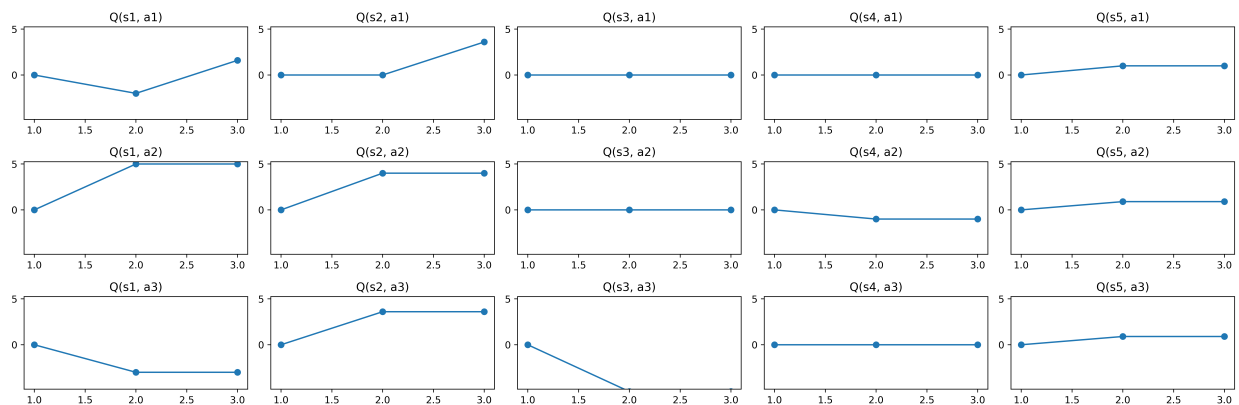


Figure 23: Image

6. The world has the states set $S = \{sf1, s1, s2, s3, sf2\}$ where $s1$ = initial state and, $sf1$ and $sf2$ are terminal states:

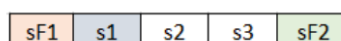


Figure 24: Image

The world has the set of actions $A = \{->, <- \}$ where:

- \rightarrow = agent moves to the right one cell with probability 0.8, and one cell to the left with probability 0.2.
- \leftarrow = agent moves to the left one cell with probability 0.8, and one cell to the right with probability 0.2.

The Reward Function $\text{fr}(s, a, sf) = \text{fr}(sf)$ only depends on the state to which the agent arrives:

-10	0	-0.4	-0.4	10
-----	---	------	------	----

Figure 25: Image

The agent has the Action Function $f_{\pi}(s)$:

$$f_{\pi}(s) = \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_{F1} \\ s_{F2} \end{matrix} \begin{bmatrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \leftarrow \\ \rightarrow \end{bmatrix}$$

Figure 26: Image

Do the following:

- Solve the Bellman Optimality Equations by Value Iteration for $V(s)$.

Solution

The Transition Function would be:

$$P_{\pi}(s_f | s, a) = \begin{matrix} \begin{matrix} \rightarrow & \leftarrow \end{matrix} \\ \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{matrix} \begin{bmatrix} \begin{matrix} 0.1 & 0.8 \\ 0.2 & 0.8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{bmatrix} \begin{matrix} \begin{matrix} \rightarrow & \leftarrow \end{matrix} \\ \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{matrix} \begin{bmatrix} \begin{matrix} 0.4 & 0.2 \\ 0 & 0 \\ 0.2 & 0.8 \\ 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{bmatrix} \begin{matrix} \begin{matrix} \rightarrow & \leftarrow \end{matrix} \\ \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{matrix} \begin{bmatrix} \begin{matrix} 0 & 0 \\ 0.8 & 0.1 \\ 0 & 0 \\ 0.2 & 0.8 \\ 0 & 0 \end{matrix} & \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{bmatrix} \begin{matrix} \begin{matrix} \rightarrow & \leftarrow \end{matrix} \\ \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{matrix} \begin{bmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0.4 & 0.2 \\ 0 & 0 \\ 0.1 & 0.9 \end{matrix} & \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{bmatrix} \begin{matrix} \begin{matrix} \rightarrow & \leftarrow \end{matrix} \\ \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \\ s_{F1} \end{matrix} \end{matrix} \begin{bmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0.4 & 0.2 \\ 0.8 & 0.1 \end{matrix} & \begin{matrix} s_{F1} \\ s_1 \\ s_2 \\ s_3 \end{matrix} \end{bmatrix}$$

Figure 27: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume $\gamma = 0.0$.

```

1  =====
2  Iteration 1:
3  fr: [-10, 0, -0.4, -0.4, 10]
4  sf1      s1      s2      s3      sf2
5  V(s) current
6  0.00      0.00      0.00      0.00      0.00
7  V(s) new
8  -2.00     -2.32     -0.08      7.92      7.92
9  =====
10 Iteration 2:
11 fr: [-10, 0, -0.4, -0.4, 10]
12 sf1      s1      s2      s3      sf2
13 V(s) current
14 -2.00     -2.32     -0.08      7.92      7.92
15 V(s) new
16 -2.00     -2.32     -0.08      7.92      7.92
17 Optimal Politic:
18 sf1 = ->,      s1 = ->,      s2 = ->,      s3 = ->,
      sf2 = ->

```

Which gives out a result of:

	sf1	s1	s2	s3	sf2
V(s)	-2.00	-2.32	-0.08	7.92	7.92
f_pi(s)	->	->	->	->	->

The convergence plot for each $V(s)$ value is given below:

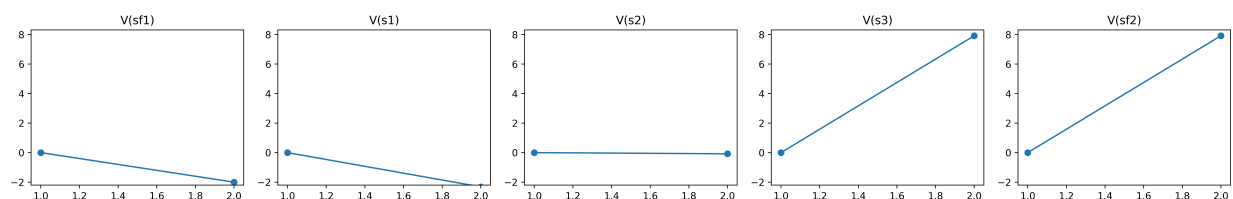


Figure 28: Image

b. Solve the Bellman Optimality Equations by Value Iteration for $Q(s,a)$.

Solution

The Transition Function would be:

$$P_{\text{M}}(s_f | s, a) = \begin{matrix} & \begin{matrix} \xrightarrow{\text{sf1}} & \xleftarrow{\text{sf1}} & \xrightarrow{\text{sf2}} & \xleftarrow{\text{sf2}} & \xrightarrow{\text{sf3}} & \xleftarrow{\text{sf3}} & \xrightarrow{\text{sf4}} & \xleftarrow{\text{sf4}} \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_{f1} \end{matrix} & \begin{bmatrix} 0.1 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0.8 & 0 & 0 \end{bmatrix} & \begin{matrix} s_{f1} \\ s_{f2} \\ s_{f3} \\ s_{f4} \end{matrix} \end{matrix}$$

Figure 29: Image

Thus, the Bellman Optimality Equations were solved in 2 iterations if we assume that **a change smaller than 0.01 between iterations** means convergence, and since gamma is not specified, we assume $\gamma = 0.0$.

```

1  =====
2  Iteration 1:
3  fr: [-10, 0, -0.4, -0.4, 10]
4  sf1      s1      s2      s3      sf2
5  Q(s,a) current
6  0.00      0.00      0.00      0.00      0.00
7  0.00      0.00      0.00      0.00      0.00
8  Q(s,a) new
9  -2.00     -2.32     -0.32      7.92      7.92
10 -8.00     -8.08     -0.08      1.68      1.68
11 =====
12 Iteration 2:
13 fr: [-10, 0, -0.4, -0.4, 10]
14 sf1      s1      s2      s3      sf2
15 Q(s,a) current
16 -2.00     -2.32     -0.32      7.92      7.92
17 -8.00     -8.08     -0.08      1.68      1.68
18 Q(s,a) new
19 -2.00     -2.32     -0.32      7.92      7.92
20 -8.00     -8.08     -0.08      1.68      1.68
21 Optimal Politic:
22 sf1 = ->,      s1 = ->,      s2 = <-,      s3 = ->,
      sf2 = ->

```

Which gives out a result of:

	sf1	s1	s2	s3	sf2
Q(s, ->)	-2.00	-2.32	-0.32	7.92	7.92
Q(s, <-)	-8.00	-8.08	-0.08	1.68	1.68
f_pi(s)	->	->	<-	->	->

The convergence plot for each Q(s,a) value is given below:

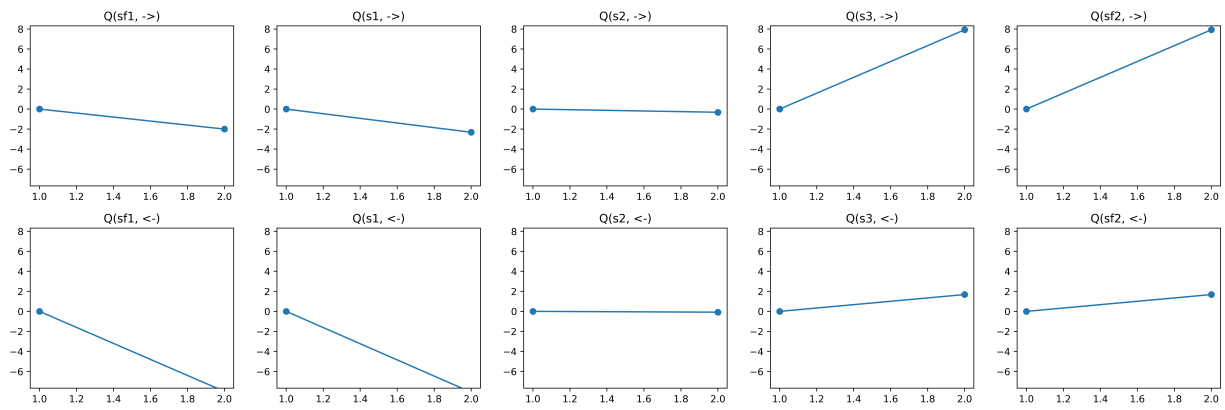


Figure 30: Image