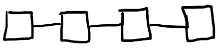
Homework 03: Solution

jueves, 24 de marzo de 2022 06:51 a. m.

Linear



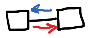
kp : how many processors

Time

$kp \to \emptyset \longrightarrow np = 2^{kp} = 2^0 = 1$

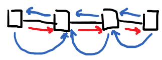
$\emptyset(3) + \emptyset(3)$

$kp \to 1 \longrightarrow np = 2^1 = 2$



$\underbrace{1(3)}_{\text{distribution}} + \overbrace{1(3)}^{\text{partial sums}}$

$kp = 2 \longrightarrow np = 2^2 = 4$

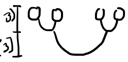


sums

$\underbrace{3(3)}_{\text{read}} + \underbrace{1(3) + 3}_{\text{partial sums}}$

last

3(np-1)

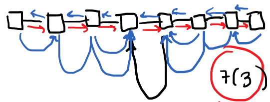


In time complexity, we often take the biggest or more important term

the one that increases faster

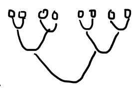
i.e. $t_c = 3(np-1) + np^2 \sim O(np^2)$

$kp = 3 \longrightarrow np = 2^3 = 8$



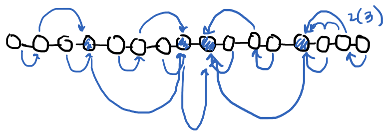
7(3)

3(np-1)

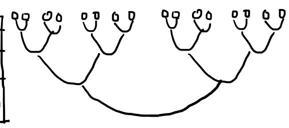


$\hookrightarrow 3(1+2+1)$

$kp = 4 \longrightarrow np = 2^4 = 16$



15(3)



$3(1+2+4+1)$

$= 3\left[\underbrace{\sum_{i=0}^{kp-2} (2^i)}_{\text{geometric sum}} + 2^0\right]$

geometric sum

$$\sum_{n=0}^{N} a x^n = a\left[\frac{1-x^{N+1}}{1-x}\right], \quad x \neq 1$$

In this case $a=1$, $x=2$, $N = kp-2$

$$= 3 + (1)\left[\frac{1-(2)^{(kp-2)+1}}{1-(2)}\right]$$

$$= 3 + \left[\frac{1-2^{kp-1}}{-1}\right]$$

$$= 3 + \left[2^{kp-1} - 1\right]$$

$$= 3 + 2^{kp-1} - 1$$

$$= 3 + 2^{kp}2^{-1} - 1$$

$$= 2 + \frac{2^{kp}}{2} = \boxed{2 + \frac{1}{2}np} + 3(np-1)$$

Thus,

$$t_c = 3(np-1) + 2 + \frac{np}{2}$$

$$t_c = \underbrace{\frac{7}{2}np - 1}_{\text{the constants may change due to the way we return partial sums}} \Big\} O(np)$$

} The important thing is that it is linear growth

the constants may change due to the way we return partial sums