Performance

jueves, 17 de marzo de 2022 06:59 a. m.

Amdahl and Gustafson's Law are a particular case of of the General Law:

$$S = \left[\frac{a + \frac{\beta}{\alpha}(1-a)}{a + \frac{\beta}{\alpha}\left(\frac{1-a}{np}\right)} \right]$$

β = factor that multiplies the time that can be parallelized

α = factor that multiplies the time that can't be parallel

$\gamma = \frac{\beta}{\alpha} = 1 \rightarrow$ means that β and α are equal

$\downarrow$

the scaling and proportion of the two times are equal.

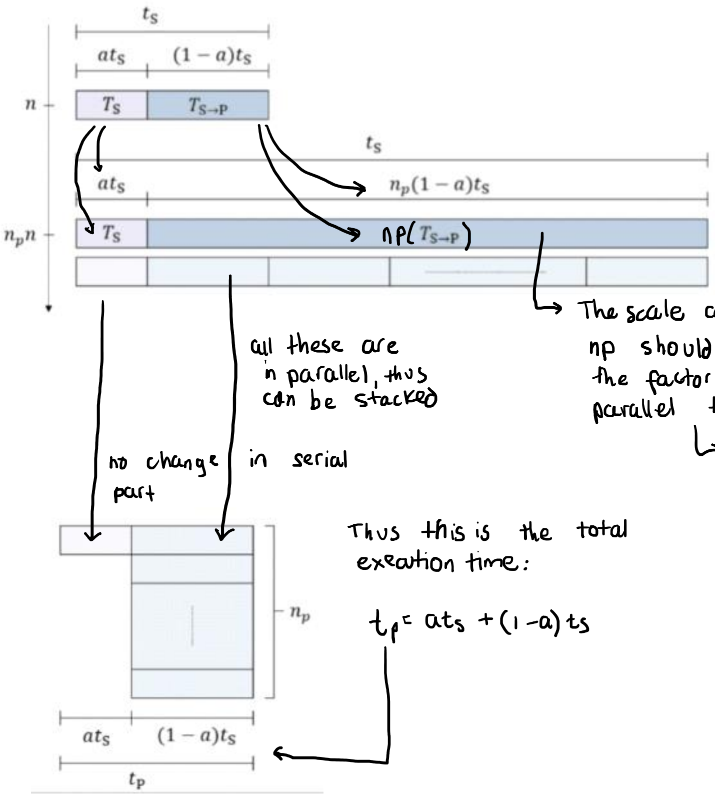
$\gamma = \frac{\beta}{\alpha} = n_p \rightarrow$ The scaling factor is the amount of processors

i.e. $np = 10$

$\gamma = \frac{\beta}{\alpha} = \frac{np}{1} \rightarrow$ The parallel part is np times bigger than the serial part doesn't change

But to obtain $\gamma = np$ can involve othe values:

$\gamma = \frac{\beta}{\alpha} = \frac{20}{2} \rightarrow$ But still the proportion is 1:10, thus parallel time is 10 times bigger than serial time



all these are in parallel, thus can be stacked

no change in serial part

The scale cannot be arbitrary, np should always be the factor that grows the parallel time

↳ So that we can guarantee weak Scalability

Thus this is the total execution time:

$$t_p = at_s + (1-a)t_s$$

Exercises:

① The 95% of execution time of a program, occurs inside a loop that we can parallelize. What is the maximum acceleration we can expect from the parallel program, executed in 6 processors?

95% t_T → t_T = Total execution time of the program, with 1 processor
$= np = 1 = t_s$ (serial time)

$S_{max} = ?$

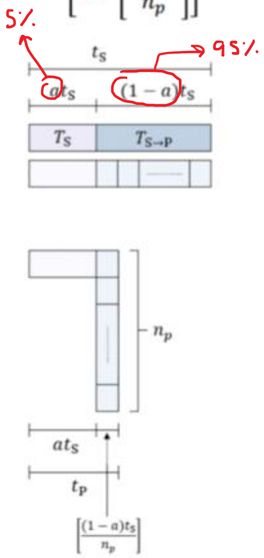
95% of t_T is parallelizable

$np = 6$

n = constant → We will use Amdahl's Law, since we only have 95% of proportion, and they don't tell us anything about weak scalability. If they tell you that the size of the problem is not constant: Gustafson' Law.

Ley de Amdahl

$$S = \left[\frac{1}{a + \left[\frac{1-a}{n_p}\right]}\right]$$



$(1-a) = 0.95$
$-a = 0.95 - 1$
$a = 1 - 0.95$
$a = 0.05$

Since we know a and np, we plug this in Amdahl's Law, this gives us the maximum scalability in a problem of said characteristics
↓
Strong scalability

$$S_{MAX} = \left[\frac{1}{(0.05) + \left[\frac{1-0.05}{6}\right]}\right] = \boxed{4.8}$$

$$S(n_p, n) = \left[\frac{t(1,n)}{t(n_p, n)}\right]$$

it is a proportion that tells us that the program in parallel is 4.8 faster than the serial one.

(2) 10% of a program's execution time is spent within sequential code. What is the limit to the speedup achievable by a parallel version of the program. Suppose n is constant.

n = constant
$a = ?$
$np = ?$ > so that we can use Amdahl's Law:

$$S = \frac{1}{a + \left(\frac{1-a}{np}\right)}$$

$a = 0.1$ (10%)

np = since we don't have this, and we want the $\underbrace{\text{limit of speedup}}_{S_{max}}$, $np = \infty$

Therefore, $$S_{MAX} = \lim_{np\to\infty}\left[\frac{1}{a + \underbrace{\left(\frac{1-a}{np}\right)}_{\approx 0}}\right] = \frac{1}{a} = \frac{1}{0.1} = \boxed{10}$$

→ if we got $np \to \infty$ processors the fastest is 10 times faster than the program in sequential mode.
$S_{MAX} = 10$ → S usually will be smaller than 10.

(3) **Example 1:** Suppose we have a parallel program that is 15% serial and 85% linearly parallelizable for a given problem size. Assume that the (absolute) serial time does not grow as the problem size is scaled.
(*i*) How much speedup can we achieve if we use 50 processors without scaling the problem?

(*ii*) Suppose we scale up the problem size by a factor of 100. How much speedup could we achieve with 50 processors?

$n \neq$ constant
$\bar{n} = np(n)$ → as the processors grow, n

T_s does not change

this means that the size of the problem

$n \neq$ constant
$\bar{n} = np(n) \rightarrow$ as the processors grow, n grows by $np(n)$

Ts does not change
Gustafson's Law

this means that the size of the problem will grow linearly.

Gustafson's Law: used for problems were $n \neq$ constant

The parallelizable time is scaled by np, while t_s does not change

(i) if $n =$ constant and $np = 50$: Amdahl's Law

$$S = \frac{1}{a + \left(\frac{1-a}{np}\right)}$$

We know that 15% is serial and 85% parallel,

$$a = 0.15$$

We can now plug this into Amdahl's Law:

$$S = \frac{1}{0.15 + \left(\frac{1-0.15}{50}\right)} = \boxed{5.98}$$

(ii) Scale up the problem by a factor of 100. How much speedup (acceleration) can we have with 50 processors?

→ The scale does not follow the form
$n = np \times n$,
actually it grows as
$n = 100 \times n$
And therefore, since $np \times n$ is not used, we cannot use Gustafson's Law, thus we need the General Law:

$$S = \left[\frac{a + \frac{\beta}{\alpha}(1-a)}{a + \frac{\beta}{\alpha}\left(\frac{1-a}{np}\right)}\right]$$

α: multiplier for sequential part
β: multiplier for parallel part

$\alpha = 1$, since the problem says that t_s does not change
$\beta = 100 \rightarrow \beta T_{s \rightarrow p} = 100 (T_{s \rightarrow p})$
$np = 50$
$a = 0.15$ } proportion that cant be parallelized

Thus,

$$S_{MAX} = \left[\frac{0.15 + \frac{100}{1}(1-0.15)}{0.15 + \frac{100}{1}\left(\frac{1-0.15}{50}\right)}\right] = \frac{85.65}{1.85} = \boxed{46.03}$$

4 **Example 2:** Assume that you want to write a program that should achieve a speedup of 100 on 128 processors.
(i) What is the maximum sequential fraction of the program when this speedup should be achieved under the assumption of strong scalability? → Amdahl's Law
We start with Amdahl's law and then isolate f as follows:

Thus, only less than 1% of your program can be serial in the strong scaling scenario!
(ii) What is the maximum sequential fraction of the program when this speedup should be achieved under the assumption of weak scalability whereby the ratio γ scales linearly?
We now start with Gustafson's law and then isolate f as follows:
→ Gustafson's Law
Thus, in this weak scaling scenario a significantly higher fraction can be serial! $\gamma = np$

(i) Strong scalability means as np grows, n is constant, thus, Amdahl's Law.

$S = 100$
$np = 128$

What is the maximum sequential fraction?

↳ we look for a: sequential fraction

$$100 = \left[\frac{1}{a + \left[\frac{1-a}{128}\right]}\right]$$

$$100 = \frac{1}{a + \left(\frac{1-a}{128}\right)}$$

$$100\left(a + \left(\frac{1-a}{128}\right)\right) = 1$$

$$100a + 100\left(\frac{1-a}{128}\right) = 1$$

$$\frac{100a}{1} + \frac{100 - 100a}{128} = 1$$

$$\frac{100a}{1} + \frac{100}{128} - \frac{100a}{128} = 1$$

$$\frac{128(100a)}{128} + \frac{100}{128} - \frac{100a}{128} = 1$$

$$\frac{1}{128}\left(12800a + 100 - 100a\right) = 1$$

$$12800a + 100 - 100a = 128$$

$$12700a + 100 = 128$$

$$12700a = 128 - 100$$

$$12700a = 28$$

$$a = \frac{28}{12700}$$

$$a = 0.0022$$

$$\boxed{a = 0.22\%}$$