

Homework 03: Communication Time Complexity

Sunday, March 20, 2022 7:26 PM

① Given the problem (whose computational time complexity was solved in class):

DATA: $\vec{x} = (-1, 2, 4, 1, 6, 0, -1, 0) \rightarrow \text{size } 8$

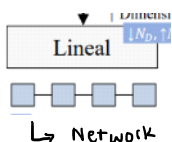
TASK: $\sum_{i=1}^{n=8} x_i = (-1) + 2 + 4 + 1 + 6 + 0 + (-1) + 0$

Calculate the communication time complexity $t_c(np, n)$ considering that the communication between two nodes takes 3 units of time as maximum, using the following communication network:

② Commuted - Direct - Linear

Solution

$$np = 2^{kp} \rightarrow \log_2 np = \log_2 2^{kp} \rightarrow \log_2 np = kp$$



Considering computation time complexity as:

$$t_c(np, n) = \left\lceil \frac{n}{np} + 1 \right\rceil + kp = \left\lceil \frac{n}{np} + 1 \right\rceil + \log_2 np$$

	Case	Data/Diagram	time	Tree (sums)
(serial)	$kp=0 \quad 2^{kp} = 2^0 = 1$	$\vec{x} = (-1, 2, 4, 1, 6, 0, -1, 0)$ 	$t = 0 (2)(3)$	
(parallel)	$kp=1 \quad 2^{kp} = 2^1 = 2$		$t = 6 = (1)(2)(3)$	
	$kp=2 \quad 2^{kp} = 2^2 = 4$		$t = 18 = 3(2)(3)$	
	$kp=3 \quad 2^{kp} = 2^3 = 8$		$t = 42 = 7(2)(3)$	
	$kp=4 \quad 2^{kp} = 16$		$15(2)(3)$	

In general form, we can say

$$t_c = (2^{kp} - 1)(2)(3)$$

$$= (2^{kp} - 1)(6)$$

$$= 6(2^{kp} - 1)$$

$$t_c = 6(2^{\log_2 np} - 1) \quad \text{and with } kp = \log_2 np$$

$$\text{with logarithm cancelling}$$

$$t_c(np, n) = \boxed{t_c = 6(np - 1)} \quad \text{for communication time complexity in a commuted - Direct - Linear network.}$$