Performance

jueves, 17 de marzo de 2022 06:59 a. m.

Amdahl and Gustafson's Law are a particular case of of the General Law:

$$S = \left[\frac{a + \frac{\beta}{\alpha}(1-a)}{a + \frac{\beta}{n_p \alpha}\left(\frac{1-a}{}\right)} \right]$$

β = factor that multiplies the time that can be parallelized

α = factor that multiplies the time that can't be parallel

$\gamma = \frac{\beta}{\alpha} = 1$ → means that β and α are equal

↓

the scaling and proportion of the two times are equal.

$\gamma = \frac{\beta}{\alpha} = n_p$ → The scaling factor is the amount of processors

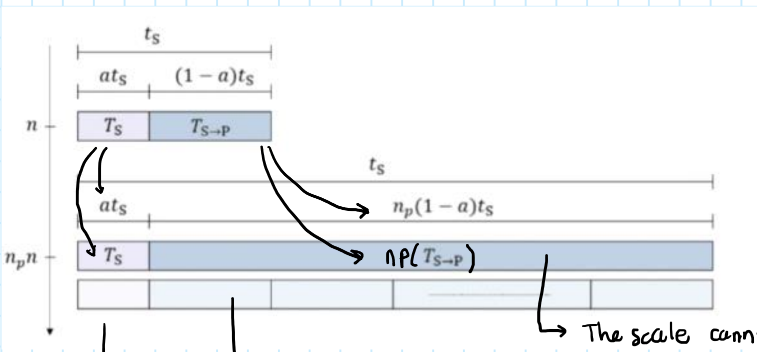
i.e. $np = 10$

$\gamma = \frac{\beta}{\alpha} = \frac{np}{1}$ → The parallel part is np times bigger than the serial part

doesn't change

But to obtain $\gamma = np$ can involve othe values:

$\gamma = \frac{\beta}{\alpha} = \frac{20}{2}$ → But still the proportion is 1:10, thus parallel time is 10 times bigger than serial time



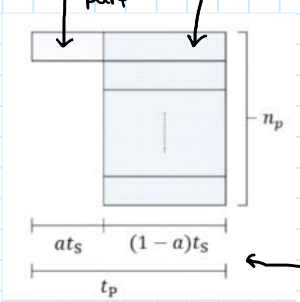
The scale cannot be arbitrary, np should always be the factor that grows the parallel time

↳ So that we can guarantee weak Scalability

all these are in parallel, thus can be stacked

no change part

in serial



Thus this is the total execution time:

$t_p = at_s + (1-a)t_s$

Exercises:

(1) The 95% of execution time of a program, occurs inside a loop that we can parallelize. What is the maximum acceleration we can expect from the parallel pogram, executed in 6 processors?

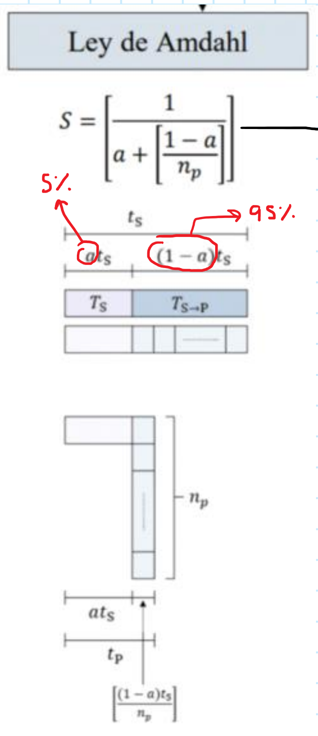
$95\%\ t_T \rightarrow t_T$ = Total execution time of the program, with 1 processor $= np = 1 = t_s$ (serial time)

$S_{max} = ?$

95% of t_T is parallelizable

$np = 6$

n = Constant $\longrightarrow$ We will use Amdahl's Law, since we only have 95% of proportion, and they don't tell us anything about weak scalability. If they tell you that the size of the problem is not constant: Gustafson' Law.



$(1-a) = 0.95$

$-a = 0.95 - 1$

$a = 1 - 0.95$

$a = 0.05$

Since we know a and np, we plug this in Amdahl's Law, this gives us the maximum scalability in a problem of said characteristics

$\downarrow$ Strong scalability

$$S_{MAX} = \left[\frac{1}{(0.05) + \left[\frac{1-0.05}{6}\right]}\right] = \boxed{4.8}$$

$$S(n_p, n) = \left[\frac{t(1,n)}{t(np,n)}\right]$$

it is a proportion that tells us that the program in parallel is 4.8 faster than the serial one.

② 10% of a program's execution time is spent within sequential code. What is the limit to the speedup achievable by a parallel version of the program. Suppose n is constant.

n = constant

$a = ?$

$np = ?$ $\succ$ so that we can use ambdahl's law:

$$S = \frac{1}{a + \left(\frac{1-a}{np}\right)}$$

$a = 0.1$ (10%)

np = since we don't have this, and we want the $\underbrace{\text{limit of speedup}}_{S_{max}}$, $np = \infty$

Therefore, $S_{MAX} = \lim_{np\to\infty} \left[\frac{1}{a + \underbrace{\left(\frac{1-a}{np}\right)}_{\approx 0}}\right] = \frac{1}{a} = \frac{1}{0.1} = \boxed{10}$

if we got $np \to \infty$ processors the fastest is 10 times faster than the program in sequential mode.

$S_{MAX} = 10 \rightarrow S$ usually will be smaller than 10.

③ **Example 1:** Suppose we have a parallel program that is 15% serial and 85% linearly parallelizable for a given problem size. Assume that the (absolute) serial time does not grow as the problem size is scaled.

(*i*) How much speedup can we achieve if we use 50 processors without scaling the problem?

(*ii*) Suppose we scale up the problem size by a factor of 100. How much speedup could we achieve with 50 processors?

this means that the size of the problem

$n \neq$ constant

Ts does not change

$n \neq$ constant

$\bar{n} = np(n)$ → as the processors grow, n grows by np(n)

Ts does not change → Gustafson's Law

this means that the size of the problem will grow linearly.

Gustafson's Law: used for problems were $n \neq$ constant

The parallelizable time is scaled by np, while t_s does not change

(i) if $n =$ constant and $np = 50$: Amdahl's Law

$$S = \frac{1}{a + \left(\frac{1-a}{np}\right)}$$

We know that 15% is serial and 85% parallel,

$a = 0.15$

We can now plug this into Amdahl's Law:

$$S = \frac{1}{0.15 + \left(\frac{1-0.15}{50}\right)} = \boxed{5.98}$$

(ii) Scale up the problem by a factor of 100. How much speedup (acceleration) can we have with 50 processors?

→ The scale does not follow the form

$n = np \times n$,

actually it grows as

$n = 100 \times n$

And therefore, since $np \times n$ is not used, we cannot use Gustafson's Law, thus we need the General Law:

$$S = \left[\frac{a + \frac{\beta}{\alpha}(1-a)}{a + \frac{\beta}{\alpha}\left(\frac{1-a}{np}\right)}\right]$$

α: multiplier for sequential part

β: multiplier for parallel part

$\alpha = 1$, since the problem says that t_s does not change

$\beta = 100$ → $\beta T_{s \to p} = 100 (T_{s \to p})$

$np = 50$

$a = 0.15$ } proportion that cant be parallelized

Thus,

$$S_{MAX} = \left[\frac{0.15 + \frac{100}{1}(1-0.15)}{0.15 + \frac{100}{1}\left(\frac{1-0.15}{50}\right)}\right] = \frac{85.65}{1.85} = \boxed{46.03}$$

4 **Example 2:** Assume that you want to write a program that should achieve a speedup of 100 on 128 processors.

(*i*) What is the maximum sequential fraction of the program when this speedup should be achieved under the assumption of strong scalability? → Amdahl's Law

We start with Amdahl's law and then isolate f as follows:

Thus, only less than 1% of your program can be serial in the strong scaling scenario!

(*ii*) What is the maximum sequential fraction of the program when this speedup should be achieved under the assumption of weak scalability whereby the ratio γ scales linearly? → Gustafson's Law

We now start with Gustafson's law and then isolate f as follows:

Thus, in this weak scaling scenario a significantly higher fraction can be serial! $\gamma = np$

(i) Strong scalability means as np grows, n is constant, thus, Amdahl's Law.

$S = 100$

$np = 128$

$S = 100$

$n_p = 128$

What is the maximum sequential fraction?

↳ we look for a : sequential fraction

$$100 = \left[\frac{1}{a + \left[\frac{1-a}{128}\right]} \right]$$

$$100 = \frac{1}{a + \left(\frac{1-a}{128}\right)}$$

$$100 \left(a + \left(\frac{1-a}{128} \right) \right) = 1$$

$$100a + 100 \left(\frac{1-a}{128} \right) = 1$$

$$\frac{100a}{1} + \frac{100 - 100a}{128} = 1$$

$$\frac{100a}{1} + \frac{100}{128} - \frac{100a}{128} = 1$$

$$\frac{128(100a)}{128} + \frac{100}{128} - \frac{100a}{128} = 1$$

$$\frac{1}{128} \left(12800a + 100 - 100a \right) = 1$$

$$12800a + 100 - 100a = 128$$

$$12700a + 100 = 128$$

$$12700a = 128 - 100$$

$$12700a = 28$$

$$a = \frac{28}{12700}$$

$$a = 0.0022$$

$$\boxed{a = 0.22\%}$$