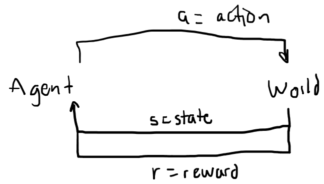
Week 3

Saturday, February 19, 2022 10:40 AM



World

Graph: nodes connected by lines/arrows.
↓
Circles

The world contains a set of States
↳ describes the configuration of the world.

→ states will be the nodes in the graph: $s_1, s_2, \dots s_n$
↳ the n states are visualized through the graph

→ Set of states : $S = \{s_1, \dots, s_n\}$ ie : the #th state in the game
↓
Capitalized S
each state: s_1, s_n.

→ if we can pass with an ACTION from a state to another, we connect the nodes with a line

→ To define the lines, we thus need to define the ACTIONS

$A = \{a_1, \dots, a_m\}$ → not the same as n (states)

→ We transit from a state to another through an action (a).

$$s_1 \xrightarrow{a} s_2$$

Write an arrow for each action you need to transition from a node to another

The set of States (S) and set of actions (A) are not enough to describe World, we need:

Transition Model : from which node to which node and with what action

1. Deterministic : defined by a two variable function

$$s_f = f_{MT}(s, a)$$

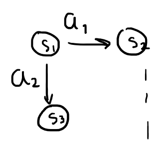
↳ final state ↓ current state ↳ action to do

This function can be seen in a matrix

$$\begin{array}{cc} & \xrightarrow{a} \\ s \downarrow & \begin{bmatrix} \boxed{s_f} \end{bmatrix} \end{array}$$

$s_i a_j = s_f$
↳ the state to which we transition

Example : we have a world:



a defined world

Thus

$s \in S = \{s_1, s_2, s_3\}$ → the domain of s is S

$a \in A = \{a_1, a_2\}$ → the domain of a is A

$s_f = f_{MT}(s, a)$
↳ Transition Model Function
↳ another variable whose domain is S also
$s_f \in S$

$$= \begin{array}{ccc} & a_1 & a_2 \\ s_1 \rightarrow & \boxed{s_2} & \boxed{s_3} \end{array}$$ since there's no $s_1 \xleftarrow{a} s_2$,

$s_f \in S$

=

	a_1	a_2
s_1	s_2	s_3
s_2	s_2	s_2
s_3	s_3	s_3

since there's no $s_1 \xleftarrow{a} s_2$, we stay in the current State

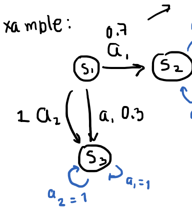
= Transition model function

2. Non-deterministic: it is modeled through a probability distribution that is conditioned.

$$S_f \sim P_{MT}(S_f \mid s, a)$$

you need a probab. distribution per pair of actions possible

Example:



the a_1 takes us from s_1 to s_2 with a probability of 0.7 (probability of transition)

$S_f = P_{MT}(S_f \mid s, a)$ = given s, a

	a_1	a_2
s_1	s_1a_1	s_1a_2
s_2	s_2a_1	s_2a_2
s_3	s_3a_1	s_3a_2

$P_{MT}(S_f \mid s_1, a_2)$

if s_1 and a_1 happens we have a probability Distribution of transitioning to all states

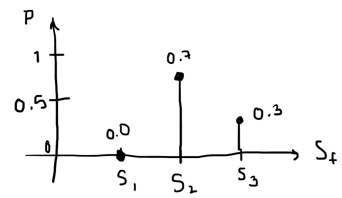
$P_{MT}(S_f \mid s_2, a_1)$

in total, we have 6 probability distribution function to define the world

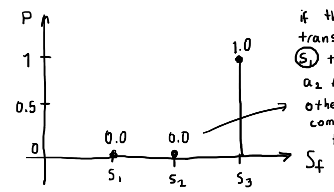
Let's see one of the six prob distrib func:

$P_{MT}(S_f \mid s_1, a_1)$

1)

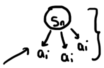


2)



$P_{MT}(S_f \mid s_1, a_2)$

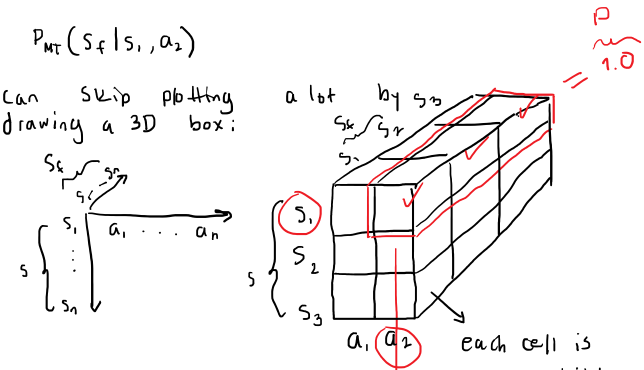
if there's no transition from s_1 to s_2 with a_2 AND s_1 has other a_2 arrows coming out of them, put its corresponding probability = 0.0



all arrows of a_i coming from s_n, must sum 1 (s_m can also have a_i's and they must sum 1).

these 3 cubes are this plot

We can skip plotting a lot by drawing a 3D box:



each cell is a probability value

we can draw this 3D box like:

- Plot 1)
- Plot 2)
- Implicit probability

we say 1 since there's no

	a_1	a_2
s_1	0	✓
s_2	0	0
s_3	0	0

= ∅

$S_f = s_1$

	a_1	a_2
s_1	0.7	✓
s_2	1	1
s_3	0	0

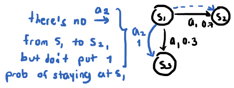
= ∅

$S_f = s_2$

	a_1	a_2
s_1	0.3	✓
s_2	0	0
s_3	1	1

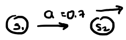
= 1

$S_f = s_3$



transition to S_f with a_n, thus we stay in current state s with probability = 1, if and only if current state has no other an arrow coming out: in that case, put its remaining probability

→ there's a probability of transition since in a NON-DETERMINISTIC world there is a randomness assumed



→ th is transition may not happen even though action a happens.

After defining S, A, f_{MT}, we need to define a prize and punish.

normally ⟨ (+) prices / (−) punishes } the program aims to accomplish the highest price possible

→ Price: 3 variable function f_R (can also be deterministic / non deterministic)

the common — not so useful and makes it very complex

$r_f = f_R(s, a, s_f)$

current state ← ; action done ; ↳ arrival state after action a

We can draw a 4D Plot by grouping 3D boxes

We now have defined The World, but how to define the AGENT?

↳ we define how the agent performs the action.

ACTIONS:

NON DET: helps model uncertainty

1. Deterministic $a = f_\pi(s)$ — π = Politic, since given a state s it tells which action a to do.

one variable defined as a vector

$$\begin{array}{c} s_1 \\ \vdots \\ s_n \end{array} \begin{bmatrix} \tilde{a}_1 \\ \vdots \\ \tilde{a}_n \end{bmatrix}$$

→ if s_1, we perform $\tilde{a}_1$

2. NON Deterministic:

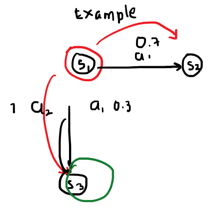
$a \sim P_\pi(a \mid s)$ ↳ given (or conditioned by) s state

$$s \left\{ \begin{array}{c} \\ s_1 \\ \vdots \\ s_n \end{array} \right. \begin{array}{c} a_1 \quad \cdots \quad a_m \\ \begin{bmatrix} P_{11} & \cdots & P_{1m} \\ & \ddots & \\ P_{n1} & \cdots & P_{nm} \end{bmatrix} \end{array} \begin{array}{c} = 1 \\ \\ = 1 \end{array}$$

→ given s_1, we have m probabilities since we have m possible actions

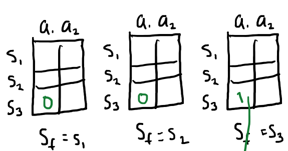
} each row sums 1

Example



Deterministic action

Transition model: $P_{MT}(S_f \mid s, a)$



30% s3
70% s2

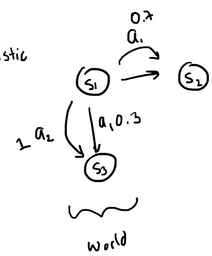
deterministic (its results is nondet)

$a = f_\pi(s) = \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} \begin{bmatrix} a_1 \\ a_2 \\ a_1 \end{bmatrix}$

we defined arbitrarily what actions are done given a state

we stay in s_3 if we perform a_1 after seeing the tables

Non deterministic action



Same $P_{MT}(s_f \mid s, a)$

$P_\pi(a \mid s_1) = 1$

$a \sim P_\pi(a \mid s)$ = Probability conditioned to s

$$\begin{array}{c|cc} & a_1 & a_2 \\ \hline s_1 & 0.6 & 0.4 \\ s_2 & 0.5 & 0.5 \\ s_3 & 1 & 0 \end{array}$$

= 1

These are the probabilities that define the stochastic action a

$P_\pi(a \mid s_3) = 1$

These are

world

$P_\pi(a|S_3) = 1$

action a

These are the probabilities of the agent (inside the world)

→ How does the agent acts inside the world?

↳ Suppose we first position agent in S_1, thus the probability of the agent taking action $\underbrace{a_1 \text{ is } 60\%}_{\text{you can go either to state } S_2 \text{ or } S_3}$. and $\underbrace{a_2}_{\text{directly goes to } S_3}$ is 40% (decided by a random generator)

→ The probability of arriving to S_2 from S_1 is $= (0.6)(0.7) = \boxed{0.42}$

probability of taking action a_1

prob given a_1 of arriving to S_2