Week 4

Saturday, February 19, 2022 10:40 AM

$$\text{Agent} \xrightarrow{a = \text{action}} \text{World}$$

World

Grafo: nodes connected by lines / arrows.
↓
Circles

The world contains a set of States
↳ describes the configuration of the world.

→ states will be the nodes in the graph: $s_1, s_2, \dots s_n$
↳ the n states are visualized through the graph

→ Set of states : $S = \{s_1, \dots, s_n\}$ ie : the #° state in the game
↓
Capitalized S
each State: s_1, s_n.

→ if we can pass with an ACTION from a state to another, we connect the nodes with a line

→ To define the lines, we thus need to define the ACTIONS
$A = \{a_1, \dots, a_m\}$ → not the same as n (states)

→ We transit from a state to another through an action (a).

$$s_1 \xrightarrow{a} s_2$$

Write an arrow for each action you need to transition from a node to another

The set of States (S) and set of actions (A) are not enough to describe World, we need:

Transition Model : from which node to which node and with what action

1. Deterministic : defined by a two variable function

$$s_f = f_{MT}(s, a)$$
↳ final State ↓ current state ↳ action to do

This function can be seen in a matrix

$$S \downarrow \overset{a \rightarrow}{\begin{bmatrix} \boxed{s_f} \end{bmatrix}}$$

$s_i a_j = s_f$
↳ the state to which we transition

Example : we have a world:

$$\begin{array}{ccc} s_1 & \xrightarrow{a_1} & s_2 \\ \downarrow a_2 & & \\ s_3 & & \end{array}$$

a defined world

Thus
$s \in S = \{s_1, s_2, s_3\}$ → the domain of s is S
$a \in A = \{a_1, a_2\}$ → the domain of a is A
$s_f = f_{MT}(s, a)$
↳ Transition Model Function
↳ another variable whose domain is S also
$s_f \in S$

$$= \begin{array}{c|cc} & a_1 & a_2 \\ \hline s_1 & s_2 & s_3 \\ s & & \end{array}$$

→ since there's no $s_1 \xleftarrow{a} s_2$,

	a_1	a_2
s_1	s_2	s_3
s_2	s_2	s_2
s_3	s_3	s_3

since there's no $s_1 \xleftarrow{a} s_2$, we stay in the current state

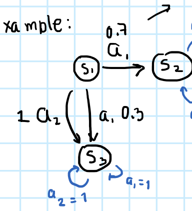
= Transition model function

2. Non-deterministic: it is modeled through a probability distribution that is conditioned.

$$S_f \sim P_{MT}(S_f \mid s, a)$$

you need a probab. distribution per pair of actions possible x

Example:



the a_1 takes us from S_1 to S_2 with a probability of 0.7 (probability of transition)

$S_f = P_{MT}(S_f \mid s, a)$ =

given s, a

	a_1	a_2
s_1	s_1a_1	s_1a_2
s_2	s_2a_1	s_2a_2
s_3	s_3a_1	s_3a_2

$P_{MT}(S_f \mid s_1, a_2)$

if s_1 and a_1 happens we have a probability Distribution of transitioning to all states

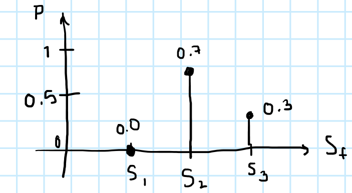
$P_{MT}(S_f \mid s_2, a_1)$

in total, we have 6 probability distribution function to define the world

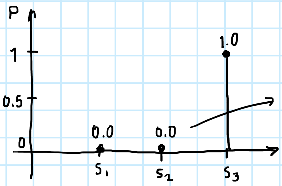
Let's see one of the six prob distrib func:

$P_{MT}(S_f \mid s_1, a_1)$

1)



2)



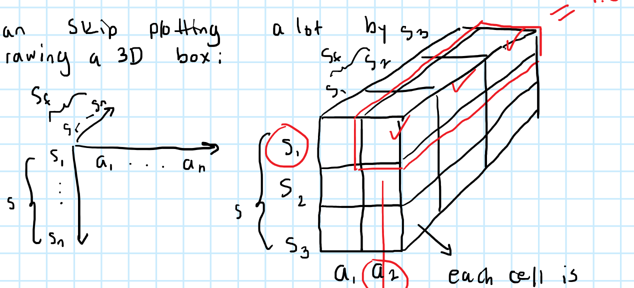
if there's no transition, $p = 0.0$

these 3 cubes are this plot

$p = 1.0$

$P_{MT}(S_f \mid s_1, a_2)$

We can skip plotting a lot by drawing a 3D box:



each cell is a probability value

we can draw this 3D box like:

$S_f = S_1$

	a_1	a_2	
s_1	0	✓	$= 0$
s_2	0	0	
s_3	0	0	

$S_f = S_2$

	a_1	a_2	
s_1	0.7	✓	$= 0$
s_2	1	1	
s_3	0	0	

$S_f = S_3$

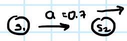
	a_1	a_2	
s_1	0.3	✓	$= 1$
s_2	0	0	
s_3	1	1	

- Plot 1)
- Plot 2)
- Implicit probability

we say 1 since there's no

transition to S_f with a_n,
thus we stay in current state s
with probability $= 1$.

→ there's a probability of transition since in a NON-DETERMINISTIC world there is a randomness assumed



→ th is transition may not happen even though action a happens.

After defining S, A, f_{MT}, we need to define a prize and punish.

normally (+) prices, (−) punishes } the program aims to accomplish the highest price possible

→ Price: 3 variable function f_R (can also be deterministic / non deterministic)

deterministic → the common

non deterministic → not so useful and makes it very complex

$r_f = f_R(s, a, s_f)$

s → current state; a → action done; s_f → arrival state after action a

We can draw a 4D Plot by grouping 3D boxes

We now have defined The World, but how to define the AGENT?

↳ we define how the agent performs the action.

ACTIONS:

1. Deterministic $a = f_\pi(s)$

π = Politic, since given a state s it tells which action a to do.

NON DET: helps model uncertainty

one variable defined as a vector

$$\begin{array}{c} s_1 \\ \vdots \\ s_n \end{array} \begin{bmatrix} \tilde{a}_1 \\ \vdots \\ \tilde{a}_n \end{bmatrix}$$

→ if s_1, we perform $\tilde{a}_1$

2. NON Deterministic:

$a \sim P_\pi(a \mid s)$

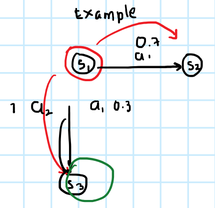
↳ given (or conditioned by) s state

$$S \left\{ \begin{array}{c} \\ s_1 \\ \vdots \\ s_n \end{array} \begin{bmatrix} a_1 & \cdots & a_m \\ P_{11} & \cdots & P_{1m} \\ & \ddots & \\ P_{n1} & \cdots & P_{nm} \end{bmatrix} \begin{array}{c} \\ =1 \\ \\ =1 \end{array} \right.$$

→ given s_1, we have m probabilities since we have m possible actions

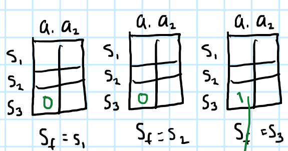
} each row sums 1

Example



Deterministic action

Transition model:
$P_{MT}(s_f \mid s, a)$:



$S_f = s_1$ $S_f = s_2$ $S_f = s_3$

$$a = f_\pi(s) = \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} \begin{bmatrix} a_1 \\ a_2 \\ a_1 \end{bmatrix}$$

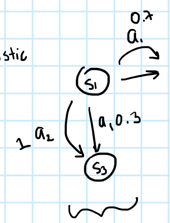
} we defined arbitrarily what actions are done given a state

deterministic (its results is nondet)

30% s3
70% s2

we stay in s_3 if we perform a_1 after seeing the tables

Non deterministic action



Same $P_{MT}(s_f \mid s, a)$

$P_\pi(a \mid s_1) = 1$

$$a \sim P_\pi(a \mid s) = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{bmatrix} a_1 & a_2 \\ 0.6 & 0.4 \\ 0.5 & 0.5 \\ 1 & 0 \end{bmatrix}$$

Probability conditioned to s

$= 1$

These are the probabilities that define the stochastic action a

These are

$P_\pi(a \mid s_3) = 1$

world

$P_\pi(a|S_3) = 1$

action a

These are the probabilities of the agent (inside the world)

→ How does the agent acts inside the world?

↳ Suppose we first position agent in S_1, thus the probability of the agent taking action a_1 is 60%. and a_2 is 40% (decided by a random generator)

a_1: you can go either to state S_2 or S_3

a_2: directly goes to S_3

→ The probability of arriving to S_2 from S_1 is $= \underset{\text{probability of taking action } a_1}{(0.6)}\underset{\text{prob given } a_1 \text{ of arriving to } S_2}{(0.7)} = \boxed{0.42}$