Homework 01

Monday, March 7, 2022 7:28 AM

① Build the graph of the world defined by the following transition function $f_{MT}(s,a)$:

$$f_{MT}(s,a) = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{cc} a_1 & a_2 \\ \hline \end{array}\! \begin{array}{c} \\ \begin{bmatrix} s_2 & s_2 \\ s_1 & s_3 \\ s_3 & s_1 \\ s_1 & s_4 \end{bmatrix} \end{array}$$

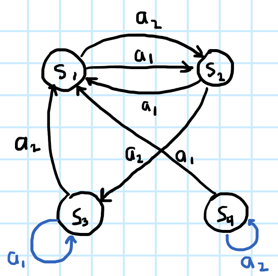
<u>Solution</u>

$$f_{MT}(s,a) = \begin{array}{c|cc} & a_1 & a_2 \\ \hline s_1 & s_2 & s_2 \\ s_2 & s_1 & s_3 \\ s_3 & s_3 & s_1 \\ s_4 & s_1 & s_4 \end{array} \rightarrow \text{deterministic transition function} \longrightarrow s_f = f_{MT}(s,a)$$

Thus,

$$S = \{s_1, s_2, s_3, s_4\} \qquad A = \{a_1, a_2\}$$

The world:



with $f_{MT}(s,a) = \begin{array}{c|cc} & a_1 & a_2 \\ \hline s_1 & s_2 & s_2 \\ s_2 & s_1 & s_3 \\ s_3 & s_3 & s_1 \\ s_4 & s_1 & s_4 \end{array}$

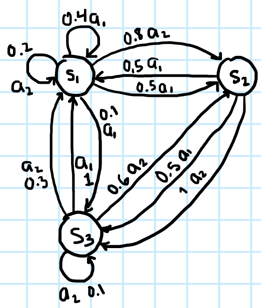
② Build the graph of the world defined by the following transition function $P_{MT}(s_f|s,a)$:

$$f_{MT}(s,a): \quad \begin{array}{c|cc} s_f = s_1 & a_1 & a_2 \\ \hline s_1 & 0.4 & 0.2 \\ s_2 & 0.5 & 0 \\ s_3 & 1 & 0.3 \end{array} \quad \begin{array}{c|cc} s_f = s_2 & a_1 & a_2 \\ \hline s_1 & 0.5 & 0.8 \\ s_2 & 0 & 0 \\ s_3 & 0 & 0.6 \end{array} \quad \begin{array}{c|cc} s_f = s_3 & a_1 & a_2 \\ \hline s_1 & 0.1 & 0 \\ s_2 & 0.5 & 1 \\ s_3 & 0 & 0.1 \end{array}$$

<u>Solution</u>

Non-deterministic transition function $\quad s_f \sim P_{MT}(s_f | s, a)$

Thus, the world looks like below:



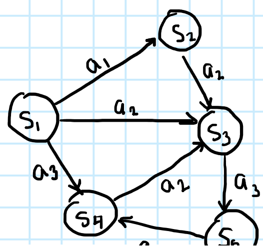
with $f_{MT}(s,a): \quad \begin{array}{c|cc} s_f = s_1 & a_1 & a_2 \\ \hline s_1 & 0.4 & 0.2 \\ s_2 & 0.5 & 0 \\ s_3 & 1 & 0.3 \end{array} \quad \begin{array}{c|cc} s_f = s_2 & a_1 & a_2 \\ \hline s_1 & 0.5 & 0.8 \\ s_2 & 0 & 0 \\ s_3 & 0 & 0.6 \end{array} \quad \begin{array}{c|cc} s_f = s_3 & a_1 & a_2 \\ \hline s_1 & 0.1 & 0 \\ s_2 & 0.5 & 1 \\ s_3 & 0 & 0.1 \end{array}$

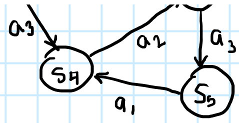
sum = 1

this means no self transition

no a_1 action makes S_3 stay in itself

③ Build the transition function $f_{MT}(s,a)$ of the following world:





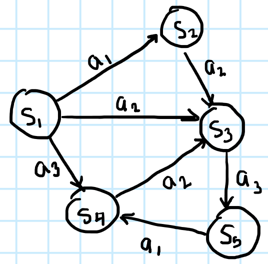
Solution

We can define $S = \{s_1, s_2, s_3, s_4, s_5\}$ and $A = \{a_1, a_2, a_3\}$, and by looking at the world, the transition function is deterministic:

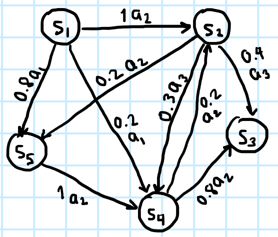
$$s_f = f_{MT}(s, a)$$

Therefore,

$$s_f = f_{MT}(s,a) = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} s_2 & s_3 & s_4 \\ s_2 & s_3 & s_2 \\ s_3 & s_3 & s_5 \\ s_4 & s_3 & s_4 \\ s_4 & s_5 & s_5 \end{bmatrix} \end{array} \quad \text{for:}$$



4. Build the transition function $P_{MT}(s_f | s, a)$ for the world:

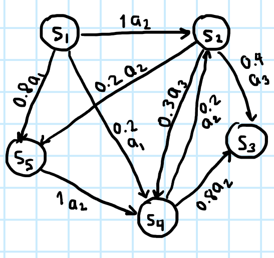


Solution:

By looking at the world, it needs a non-deterministic transition function:

$S = \{s_1, s_2, s_3, s_4, s_5\}$ and $A = \{a_1, a_2, a_3\}$

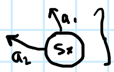
$$s_f \sim P_{MT}(s_f | s, a) = \overset{s_f = s_1}{\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.2 & 0.1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}} \overset{s_f = s_2}{\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0.2 & 0.1 \\ 0 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}} \overset{s_f = s_3}{\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.2 & 0.4 \\ 1 & 1 & 1 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}} \overset{s_f = s_4}{\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0.3 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}} \overset{s_f = s_5}{\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.2 & 0.1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}}$$



Notes to myself:

Steps:

1. Fill all blacks by going matrix by matrix: Which ones arrive to this matrix?
1. fill every $s_i a_j$ remaining with its probability left, two cases:
   ↓ if they got blacks
   a) No prob left: zeros
   b) Prob left: i.e. blacks are $0.4 + 0.3 \rightarrow 0.3$ left to 3 matrizes, 0.1 for each
3. If all matrices' row x is empty: go to S_x row for matrix $S_f = S_x$ and:



$\Big\}$ no arriving arrow = fill S_x row for $S_f = S_x$ with 1's.

4. For all remaining $s_i a_j$ cells in all its matrices, look for $\overbrace{S_f = S_i \text{ and row } S_i}^{\text{row you're checking}}$, put 1. For ex, S_5 for $S_f = S_5$