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How Does the Reference Price Mechanism Affect Competition?

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ABSTRACT This paper proposes an operational strategy based on the theory of transaction utility, aiming to mitigate the negative impact of customers' free-riding behaviour. This operational strategy is named the reference price mechanism (RPM), in which the retailer gives a reference price that may be different from the retail price, and the customer's purchase decision is affected by the reference price. Based on the Hotelling model, we develop a duopoly game by considering the free-riding cost to systematically discuss the effectiveness of the RPM. The results show that the brick-and-mortar/online retailer can benefit from implementing the RPM separately, vice versa. If both retailers implement the RPM, only the retailer with the higher reference price can attain more profits. In addition, there is a threshold of the free-riding cost that can affect whether the retailers implement the RPM.

INDEX TERMS Operational strategy, reference price, free riding, duopoly.

I. INTRODUCTION

Confronted with online and offline dual-channel sales, some consumers enjoy using a brick-and-mortar (BM) retailer to learn about a product but purchase from an online retailer, a behaviour defined as "free-riding" [1], [2]. Because BM retailers can offer experiential services, while online retailers can provide lower-priced products due to their low cost of operation, customers prefer to purchase products with high digital attributes online [3]. With the rapid development of e-commerce technology, there is a growing number of people who have begun to adopt the free-riding mode of consumption. However, free-riding behaviour causes retailers to lose profits. Baal *et al.* [4] found that in the dual-channel supply chain consisting of BM retailers and online retailers, the proportion of consumers with free-riding behaviour is as high as 20%. Many BM stores, such as Target in the US and Su Ning in China, have become showrooms for online retailers [5]. Free-riding behaviour presents substantial costs to retailers without generating local sales [6], [7]. In addition, free-riding behaviour will inhibit the enthusiasm of BM retailers to provide services and lead to poor effects on

sales and lower levels of customer service while reducing the profits of retailers [8], [9].

Given that free-riding has a detrimental effect on revenues, some retailers have considered various sales mechanisms to prevent customers who first visit BM retailers from turning to online retailers. Except for the corporate-level strategy that requires no markdown pricing (see Aviv and Pazgal [10], Cachon and Swinney [11], and Su and Zhang [12] for discussion of such strategies), many companies in the retail industry provide price-matching guarantees, in which a retailer commits to compensating customers for any difference between the paid price and lower prices (see, for example, Liu [13], Chen and Chen [2]) offered by competitors. However, the price matching mechanism proposed by Chen and Chen [2] is only effective when the proportion of free-riding customers is low enough.

Another possible mechanism, which is also the focus of our paper, is to influence the customers' purchase behaviour by introducing a reference price. The underlying principle is that the reference price of the product can be used as a tool to influence the customer's judgement of product value. According to mental accounting (see, e.g., Kahneman and Tversky [14]) and transaction utility theory [15], consumers will produce two kinds of utility—"acquisition utility" and "transaction utility"—in the process of purchasing products.

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Acquisition utility is based on standard economic theory and is equivalent to what economists call “consumer surplus.” Transaction utility is the transaction quality that consumers perceive. The difference between the actual price paid and the “reference price” is one of the ways to describe transaction utility, and the reference price is the consumer’s expected price. To give a simple example, suppose you buy a sandwich in a scenic spot. This sandwich is the same as what you usually eat at noon, but the price twice as high as usual. There is nothing wrong with the sandwich itself, but the transaction is very unpleasant. This can result in a negative transaction utility because you feel like you have been cheated.

In contrast, if the paid price is lower than the reference price, the transaction utility is positive; it feels very cost-effective. In practice, the application of a reference price is very common. For example, before Double Eleventh Day in China, retailers may raise the price from the previous period, take the raised price as the reference price of the product, and then sell the product to customers using various promotional methods on that day. Thus, customers have a sense of greater transaction utility with smaller discounts. The method stimulates customers to buy more products on Double Eleventh Day.

In this paper, we develop a duopoly competition model to examine the price decision of retailers and the impact of the reference price mechanism (RPM) on retailers’ revenues under the free-riding behaviour of strategic customers. The Hotelling model is used to describe the competition between a BM retailer and an online retailer. We consider four types of RPM strategies. The first and second strategies are the “BM retailer only” (R) strategy and the “online retailer only” (O) strategy. In these strategies, only the BM/online retailers implement the RPM. The third strategy is the “BM & online” (RO) strategy in which both retailers implement the RPM. The last strategy is defined as Strategy F, where neither retailer implements the RPM. By analysing and comparing these strategies, we attempt to answer the following questions: (i) How does the reference price affect the purchase decision of consumers and the retailer’s profit by considering the free-riding cost and transaction utility? (ii) Can the BM retailer and the online retailer increase profits by implementing the RPM? (iii) For the two retailers, which is the optimal strategy among the four strategies in the face of free-riding behaviour?

This study contributes to the literature on online and offline retailing and operational strategies of customers’ strategic behaviour. First, a host of studies have focused on the impact of customers’ free-riding behaviour on the competition between online and offline retailers [16], [17]. However, none of these studies consider the cost of free-riding to study the impact of free-riding behaviour. In fact, customers need to spend more time and energy to learn about the product when they consume the service of the BM retailer for free, so there will be a free-riding cost. Therefore, this paper considers the free-riding cost based on Chen and Chen [2], which is more realistic. In addition, the present literature on operational

strategies to address the adverse effects of strategic customer behaviour mainly focuses on inventory control and pricing. This paper aims to consider the impact of transaction utility in price decision-making and use the difference between the reference price and the actual paid price to describe the transaction utility. Last, we explore the equilibrium pricing strategy of the two retailers when customers have free-riding costs and transaction utility.

Our analysis leads to several interesting main results. First, different from other studies, considering the cost of free riding, free riding behaviour has a negative impact on not only the BM retailer but also the online retailer. At the same time, it is surprising that free riding can benefit the BM retailer when the cost of free-riding is high. Second, by considering the transaction utility, the RPM can effectively alleviate the negative impact of free-riding behaviour, but it cannot benefit both retailers. If the BM (online) retailer implements the RPM separately, the profit of the BM (online) retailer may increase, but the online (BM) retailer’s profit will certainly decrease. Even if both retailers adopt the RPM, only the retailer with the higher reference price can increase its profits. Our results also show that BM retailers are more likely to achieve profit gains by adopting a higher reference price. Therefore, from the perspective of the whole market, it is optimal for BM retailers to implement RPM separately. Furthermore, the four strategies are likely to become the equilibrium strategy of the RPM, which should be determined according to the threshold of the free-riding cost. The equilibrium strategy of the RPM varies with the threshold of the free-riding cost.

The rest of the paper is organized as follows. We first review the related literature in Section 2. Section 3 introduces the model and its related assumptions. In Section 4, we analyse the optimal price, demand and profit of retailers under the operational strategy of the RPM. We explore the effectiveness of the RPM and obtain the equilibrium strategy of the RPM in Section 5. Section 6 provides concluding remarks and further research directions.

II. SURVEY OF EXISTING LITERATURE

This paper is related to three research streams: the impact of free-riding behaviour on online and offline retailers, the operational strategies of strategic customer behaviour, and the reference price effect.

A. LITERATURE ON THE IMPACT OF FREE-RIDING BEHAVIOUR

Research on the free-riding behaviour of strategic customers in academia began with Telser [18], and many scholars have made profound contributions to this field. Most of the early studies on the impact of free-riding behaviour asserted that consumers’ free-riding behaviour has a negative impact on service providers. Mittelstaedt [19] and Singley *et al.* [20] point out that free-riding behaviour reduces retailers’ service levels and leads to a drop in demand. Antia *et al.* [8] demonstrate that since consumers take the services of BM retailers without paying the corresponding fees, free-riding

behaviour would inhibit the enthusiasm of BM retailers to provide services, such as presale services and training of sales staff. However, some scholars believe that free-riding behaviour may have positive effects on service providers. Bernstein *et al.* [21] find that free-riding behaviour leads to further increases in product sales by spurring retailers to establish their own direct sales channels. Shin [1] shows that the differentiation of service provision under the free-riding behaviour of consumers will ease the price competition among retailers, thus leading to an increase in the profits of both competitors. Pun *et al.* [22] consider the differences in service capacity of different channels. They find that when customers exhibit free-riding behaviour, retailers bearing the free service costs are better compensated than the beneficiaries of free riding. Therefore, the impact of free-riding behaviour on retailers may depend on different scenarios, such as channel structure, purchase cost and different consumer preferences. In this paper, we consider that the strategic customers will incur a free-riding cost in the process of free riding on offline retailers' services. Under the influence of free-riding cost, whether the free-riding behaviour has a positive or negative impact on online and offline retailers needs to be reevaluated.

B. LITERATURE ON OPERATIONAL STRATEGIES OF STRATEGIC CUSTOMER BEHAVIOUR

Many scholars have proposed operational strategies to reduce the impact of strategic customer behaviour (including waiting and free-riding). The operational strategy is mainly presented from two perspectives: inventory control and pricing. In terms of inventory control, Levin *et al.* [23] find that the impact of strategic customer behaviour can be effectively reduced by controlling the initial inventory capacity. Su and Zhang [24] study the effect of inventory information on customer purchase behaviour. They demonstrate that the seller can improve his profits by combining inventory commitment and availability guarantees. Yin *et al.* [25] propose an operational strategy to control strategic behaviour via the inventory display format. They find a display format that conceals inventory information can be used to influence customers' perception of the risk of stockouts if they decide to wait. Aydinliyim *et al.* [26] describe an inventory level threshold within which inventory information should be disclosed, and it is optimal to ensure that the inventory exceeds this threshold. Cai *et al.* [27] propose a vendor-managed inventory decision model that considers strategic customer behaviour and, on this basis, introduce option contracts to achieve a Pareto improvement. In terms of pricing, some studies suggest price commitment to deter the strategic behaviour of customers. Perry *et al.* [28] research the operational strategy of a supplier's adoption of retail price maintenance to motivate retailers to improve the level of information service and reduce the negative impact of free riding. Li and Zhang [29] find that an effective way to attract strategic customers is to guarantee unchanged prices in the later period. Price matching is another operational strategy that can be used to discourage

strategic customers. Png [30] and Lai *et al.* [31] find that price matching is profitable when the firm sells to two groups of customers with different valuations. Xing and Liu [32] design a contract with a price match and selective compensation rebate to achieve sales effort coordination for a supply chain with one manufacturer and two retail channels, where an online retailer offers a lower price and free rides a BM retailer's sales effort. Chen and Chen [2] analyse the effectiveness of price matching on free-riding behaviour. They show that the BM retailer can effectively prevent strategic customers from transferring online when customers' shopping costs are moderate. Liu *et al.* [33] propose a two-way revenue sharing mechanism to coordinate the decentralized supply chain to reduce the free-riding behaviour of strategic customers. Unlike the above studies, this paper considers the transaction utility of consumers and tries to address the free-riding behaviour of consumers by introducing the reference price.

C. LITERATURE ON THE REFERENCE PRICE EFFECT

The purchase decision of customers is affected by transaction utility. Many scholars use the difference between the reference price and the real purchase price to describe the transaction utility, which is the reference price effect. The initial research on the reference price effect focused on the influence of reference price on consumer behaviour. According to the perspectives of prospect theory [34] and adaptation level theory [35], if the actual price observed by consumers at the point of purchase is inconsistent with the reference price, it will affect consumers' purchasing behaviour. Much of the current literature focuses on the application of reference price mechanisms to increase profits. Chen *et al.* [36] explore a dynamic pricing model that describes the reference price effect as a combination of current and past prices over a limited period. Dye *et al.* [37] use the reference price effect to model the pricing strategy of deteriorating commodities and prove that the initial reference price greatly influences the pricing and replenishment strategy. Courty and Nasiry [38] use the reference dependence model to analyse and explain the unified pricing of entertainment products with quality differences. Wang [39] incorporates the reference price into the consumer choice model to study product classification and pricing strategy. Crettez *et al.* [40] discuss the existence of the optimal dynamic pricing strategy when the demand depends on the reference price, and the authors give the optimal pricing strategy under different initial values of the reference price. Using the reference price effect, Li [41] analyses the multi-period pricing and inventory management issues of an omnichannel retailer and concludes that the convergence of pricing and ending inventory level to equilibrium depends on the relative position of the initial reference price relative to the unique equilibrium price. As the online sales environment matures, the reference price has an important impact on the price set by online retailers. Hardesty and Suter [42] use a two reference price environment (online, BM) \times two external reference price (low, high) between-subject experimental design

TABLE 1. Table of notation.

Indexes	
j	Subscript, index of the BM and the online retailer, $j = r, o$
W	Retailers' implementation strategy of RPM, where $W = \{R, O, RO, F\}$
F	Superscript, index of retailers' strategy when neither of the two retailers implements the RPM
R	Superscript, index of retailers' strategy with only the BM retailer implements the RPM
O	Superscript, index of retailers' strategy when the online retailer unilaterally implements the RPM
RO	Superscript, index of retailers' strategy when both retailers implement the RPM
Parameters	
v	Customer's valuation of the product
t	Transportation cost per unit distance of the customer
x	Distance between the customer and the BM retailer
s	Shopping cost of a customer purchasing from an online retailer
β	Service differentiation factor
θ	Free-riding cost (the smaller the θ is, the higher the cost of free riding)
δ	Price-sensitive coefficient of consumers to the reference price
P_j	Retailer j 's reference price
Decision variables	
p_j^W	Retailer j 's retail price under Strategy W
Other notation	
U_j^W, U_{ro}^W	Utility of customer: under Strategy W , purchase the product from a retailer j and for free-riding customers, respectively
d_j^W	Retailer j 's demand under Strategy W
π_j^W	Retailer j 's profit under Strategy W

with a single control condition to show that consumers expect to pay less in online e-tail settings than in BM retail settings. Zheng [43] find that online customers' willingness to pay will rise as reference prices increase by designing two surveys with different reference prices. However, these papers do not discuss the attempt to prevent free-riding behaviour of strategic customers through the establishment of a reference price, nor do they determine the optimal strategy to determine whether retailers should provide a reference price, which we will discuss.

III. MODEL

The notations that will be used in this paper are shown in TABLE 1.

In practice, when selling products at different prices through BM retailers and online retailers, some customers prefer BM retailers (to experience products before purchasing), some prefer online retailers (for convenience), and others may take strategic action by experiencing products at BM retailers but purchasing them from online retailers at lower prices.

Consider the market where online and offline channels coexist and two channels sell the same product. Based on the Hotelling model framework [44]–[48], it is assumed that there are two different types of retailers in the market, where r represents the BM retailer located in the corner (0) of the

Hotelling line and o represents the online retailer located in the corner (1). The two retailers are risk-neutral and pursue maximum profits by setting the prices (denoted by $p_j, j = r, o$). Customers are uniformly distributed on the Hotelling line, and each customer can buy at most one product. If a customer visits the BM retailer, the customer whose located at x will incur a transportation cost of tx , where t ($0 < t < 1$) represents the transportation cost per unit distance. Since customers purchase goods from online retailers by “clicking,” we assume that customers' online purchases will incur a uniform shopping cost (s), as described by Liu and Zhang [45].

The BM retailer educates and guides customers to learn about products in the store. When purchasing online, the customer cannot experience the service of the salesperson; touch, feel or see the product; or check it personally to determine whether it is appropriate, thus, the valuation will be discounted by the service differentiation factor β to distinguish between the BM retailer and the online retailer. Each customer's valuation of the product sold by the BM and online retailers is v and βv , respectively, as in Shum *et al.* [49], where $\beta \in (0, 1]$. The smaller the service difference factor (β) is, the higher the service level of the BM retailer is.

Customers experience the product at the BM retailer first and then purchase the product from the online retailer. This behaviour is called free-riding. A free-riding customer will visit the offline store to take advantage of the service provided by the BM retailer before purchasing the product online, so that there is no service difference between online and offline products, and the free-riding customer's valuation is v , as in Chen and Chen [2]. Compared with purchasing directly from BM retailers or online retailers, free-riding behaviour may cause customers to spend more time and energy to perceive, experience and understand the products and thus delay the delivery time. Accordingly, customers will have a free-riding cost, which is characterized by a parameter θ ($0 < \theta < 1$). The greater the free-riding cost is, the less utility the product brings to the customer; for example, delaying receiving the product in the process of free-riding reduces the customer utility. Therefore, the higher the value of θ is, the smaller free-riding cost is and leading to an increasing probability of the behaviour that customers will free ride.

Considering the free-riding behaviour of strategic customers, this paper focuses on the impact of the reference price mechanism (RPM) on retailer's decision-making and profits. The RPM specifically refers to the reference price of a product given by the retailer to the customer, which is not necessarily the price paid by the customer for the product. The RPM is widely used in daily life, which can be exemplified that with the historical transaction price displayed by online shopping platform, the suggested retail price on products, and the tag price of clothes are prevailing. The final transaction price is not necessarily equal to these reference prices, but the reference price will have a psychological impact on consumers (which may make consumers feel that their spending has been cost-effective or that they have been cheated). According to the transaction utility theory [15], [50]–[52],

these reference prices ultimately affect customer demand. When the actual purchase price is lower than the reference price, the transaction utility is positive, implying that customers think the transaction is a great deal, thus increasing their desire to buy products. For example, almost everyone has unworn clothes in their closets, but the sense that a purchase is cost-effective leads them to buy additional clothes. The transaction utility may also be negative. When the actual price paid is higher than the reference price, it will reduce the utility of consumers to buy the product, and even cause some customers not to buy it. This paper uses the reference price sensitivity coefficient (δ) and the difference between the reference price (P_j) and the retail price (p_j) to characterize the customer's transaction utility. Similar to Özer and Zheng [53], the customer's transaction utility is $\delta(P_j - p_j)$, where $\delta \in [0, 1]$. Customers have greater reference price sensitivity (δ) to products for which quality is difficult to evaluate, such as carpets and mattresses [51]. In the case of the situation that the quality of the product is difficult to evaluate, then the reference price will be used as an important indicator for customers to evaluate the quality of the product. At the same time, customers are more convinced of the authenticity of the reference price and thus have greater reference price sensitivity. The larger the δ is, the more sensitive the customer is to the reference price. The size of δ depends on the difficulty of product quality assessment, and the present paper assumes that retailers sell the same products, so δ for the BM retailer and the online retailer are the same. To facilitate the discussion, we also assume that all customers have the same δ .

We discuss four strategies related to the RPM. With Strategy R, only the BM retailer offers a reference price to strategic customers. The strategy in which the online retailer unilaterally provides a reference price is called Strategy O. We define the strategy in which both the BM retailer and the online retailer provide reference prices to strategic customers as Strategy RO. The strategy in which neither of the two retailers implements the RPM is denoted Strategy F. We denote these strategies as W , where $W = \{R, O, RO, F\}$.

Under Strategy R, since only the BM retailer implements the RPM, customers' utility to the BM retailer will be added, with $\delta(P_r - p_r^R)$. The utility of customers who buy products directly from the BM retailer is:

$$U_r^R = v - p_r^R - tx + \delta(P_r - p_r^R) \quad (3.1)$$

Free-riding customers will eventually buy products from the online retailer. Therefore, the BM retailer's reference price does not affect the transaction utility of customers who purchase products directly from the online retailer or free-riding customers. The utility of customers who buy products directly from online retailers is:

$$U_o^R = \beta v - p_o^R - s \quad (3.2)$$

The utility of free-riding customers is:

$$U_{ro}^R = \theta(v - p_o^R - tx - s) \quad (3.3)$$

Similar to Strategy R, when the online retailer provides a reference price to the customer, the customer who finally purchases from the online retailer will obtain an additional utility $\delta(P_o - p_o^O)$ in Strategy O. The customer's utility functions are:

$$\begin{cases} U_r^O = v - p_r^O - tx \\ U_o^O = \beta v - p_o^O - s + \delta(P_o - p_o^O) \\ U_{ro}^O = \theta[v - p_o^O - tx - s + \delta(P_o - p_o^O)] \end{cases} \quad (3.4)$$

Under Strategy RO, the utility functions become:

$$\begin{cases} U_r^{RO} = v - p_r^{RO} - tx + \delta(P_r - p_r^{RO}) \\ U_o^{RO} = \beta v - p_o^{RO} - s + \delta(P_o - p_o^{RO}) \\ U_{ro}^{RO} = \theta[v - p_o^{RO} - tx - s + \delta(\max(P_r, P_o) - p_o^{RO})] \end{cases} \quad (3.5)$$

Under Strategy F, the utility functions are:

$$\begin{cases} U_r^F = v - p_r^F - tx \\ U_o^F = \beta v - p_o^F - s \\ U_{ro}^F = \theta(v - p_o^F - tx - s) \end{cases} \quad (3.6)$$

IV. REFERENCE PRICING MECHANISM (RPM)

According to the transaction utility theory, the reference price may effectively enhance the transaction utility of customers. Therefore, this paper discusses the impact of reference price on the free-riding behaviour of strategic customers. The RPM is introduced to respond to free-riding behaviour by considering the reference price effect of customers. In this section, we analyse the influence of the RPM on strategic customer free-riding behaviour from the perspective of the BM retailer and the online retailer.

A. STRATEGY R

Considering the free-riding behaviour, the customers in the market can be divided into three categories: purchase product directly from BM retailers, purchase product after free-riding and purchase product directly from online retailers. A customer who is x units away from the BM retailer will purchase from the BM retailer if and only if $U_r^R > 0$ and $U_r^R > U_{ro}^R$, will purchase from the online retailer directly if and only if $U_o^R > 0$ and $U_o^R > U_{ro}^R$ and will go to the BM retailer before buying from the online retailer if and only if $U_{ro}^R > 0$ and $U_{ro}^R > \max(U_r^R, U_o^R)$. The market is segmented at the point where $U_r^R = U_{ro}^R$, which gives $x = x^R = \frac{\theta p_o^R + \theta s + (1-\theta)v - (1+\delta)p_r^R + \delta P_r}{t(1-\theta)}$. The customer who is x units away from the BM retailer prefers to purchase from the BM retailer rather than from the online retailer if $x \leq x^R$; otherwise, the customer prefers the online retailer. Therefore, the demands for the two retailers are:

$$\begin{cases} d_r^R = \frac{\theta p_o^R + \theta s + (1-\theta)v - (1+\delta)p_r^R + \delta P_r}{t(1-\theta)}, \text{ and} \\ d_o^R = 1 - d_r^R \\ = \frac{(1-\theta)(t-v) - \theta s + (1+\delta)p_r^R - \delta P_r - \theta p_o^R}{t(1-\theta)} \end{cases} \quad (4.1)$$

Correspondingly, the profits for the two retailers are:

$$\pi_r^R = p_r^R d_r^R, \quad s.t. \quad p_r^R < \frac{v - tx^R + \delta P_r}{1 + \delta} \quad (4.2)$$

$$\pi_o^R = p_o^R d_o^R, \quad s.t. \quad p_o^R < \min(v - tx^R - s, \beta v - s) \quad (4.3)$$

We define

$$\begin{aligned} \xi &= P_r^2 \delta^4 + 6(s + \frac{t}{3} - \frac{2v}{3} + \frac{2p_1}{3}) P_r \delta^3 + (9s^2 \\ &\quad + (6t - 12v + 8P_r) \delta^2 \\ &\quad + (12s^2 + (2t - 16v + 4P_r)s \\ &\quad + 2t^2 - (8v + 8P_r)t + 8v^2 + 4P_r v) \delta + 4s^2 \\ &\quad - (4t + 4v)s + (t - 2v)^2, \\ \theta_1^R &= \frac{(2v - 2P_r - 3s - t)\delta - P_r \delta^2 - 2s + 3t - \sqrt{\xi}}{(4\delta + 2)(v - s) + 2t(1 - \delta)} \\ \text{and } \theta_2^R &= \frac{(2v - 2P_r - 3s - t)\delta - P_r \delta^2 - 2s + 3t + \sqrt{\xi}}{(4\delta + 2)(v - s) + 2t(1 - \delta)}, \end{aligned}$$

where $\theta_1^R < \theta_2^R$. With the demands in Equation (4.1) and profits in Equation (4.2) and Equation (4.3), the pricing decisions can be summarized in Proposition 1.

Proposition 1: For $\theta \in (\max(0, \theta_1^R), \min(\frac{2t-v-\delta P_r}{2t-v+s}, \theta_2^R))$, BM and online retailers have unique optimal pricing decisions (p_r^{R*}, p_o^{R*}) , which are given by:

$$\begin{cases} p_r^{R*} = \frac{(1 - \theta)(t + v) + \delta P_r + s\theta}{3(1 + \delta)}, & \text{and} \\ p_o^{R*} = \frac{(1 - \theta)(2t - v) - \delta P_r - s\theta}{3\theta}. \end{cases} \quad (4.4)$$

The demands for the two retailers are:

$$\begin{cases} d_r^R = \frac{(1 - \theta)(t + v) + \delta P_r + s\theta}{3t(1 - \theta)}, \\ d_o^R = \frac{(1 - \theta)(2t - v) - \delta P_r - s\theta}{3t(1 - \theta)} \end{cases} \quad (4.5)$$

The profits of the two retailers are:

$$\begin{cases} \pi_r^R = \frac{[(1 - \theta)(t + v) + \delta P_r + s\theta]^2}{9t(1 - \theta)(1 + \delta)} \\ \pi_o^R = \frac{[(1 - \theta)(2t - v) - \delta P_r - s\theta]^2}{9t\theta(1 - \theta)} \end{cases} \quad (4.6)$$

The proof of Proposition 1 is given in the appendix.

With Proposition 1, we have the following results:

Corollary 1:

- 1) If $P_r < (1 - \theta)(v + t) + s\theta$, the prices of the two retailers $(p_r^R \& p_o^R)$ decrease with reference price-sensitive coefficient of strategic customers (δ);
- 2) If $P_r > (1 - \theta)(v + t) + s\theta$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the price of the BM retailer (p_r^R) increases and the price of the online retailer (p_o^R) decreases.

The proof of Corollary 1 is given in the appendix.

Corollary 1 shows that under Strategy R, if the customer is more concerned about the reference price, the online retailer should lower the retail price, and the retail price of the BM

retailer will be decreased or increased according to the reference price. Supposing that the reference price is low, the BM retailer should decrease the retail price; otherwise, it is the same as that of the online retailer.

Corollary 2:

1) If $P_r < \frac{(1-\theta)(v+t)+s\theta}{\delta+2}$, the profits of the two retailers ($\pi_r^R \& \pi_o^R$) decrease with the reference price-sensitive coefficient of strategic customers (δ);

2) If $P_r > \frac{(1-\theta)(v+t)+s\theta}{\delta+2}$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the profit of the BM retailer (π_r^R) increases and the profit of the online retailer (π_o^R) decreases.

The proof of Corollary 2 is given in the appendix.

Corollary 2 indicates that under Strategy R, as the customer's price sensitivity to the reference price increases, the RPM can only increase the profit of the BM retailer if the reference price is high enough. The reason is that, when the reference price is sufficiently high, the BM retailer has enough range for a premium to increase its profit performance.

B. STRATEGY O

This subsection considers the case where only the online retailer implements the RPM. The reference price provided by the online retailer will affect the transaction utility of customers who purchase products online.

According to the consumer utility function (3.4), we find that d_r^O and d_o^O are:

$$\begin{cases} d_r^O = \frac{(1 - \theta)v + (1 + \delta)\theta p_o^O - p_r^O - \theta \delta P_o + \theta s}{t(1 - \theta)}, & \text{and} \\ d_o^O = 1 - d_r^O \\ = \frac{(1 - \theta)(t - v) - (1 + \delta)\theta p_o^O + p_r^O + \theta \delta P_o - \theta s}{t(1 - \theta)} \end{cases} \quad (4.7)$$

Correspondingly, π_r^O and π_o^O are:

$$\pi_r^O = p_r^O d_r^O, \quad s.t. \quad p_r^O < v - tx^O \quad (4.8)$$

$$\pi_o^O = p_o^O d_o^O, \quad s.t. \quad p_o^O < \min(\frac{\beta v - s + \delta P_o}{1 + \delta}, \frac{v - tx^O + \delta P_o}{1 + \delta}) \quad (4.9)$$

We define θ_1^O and θ_2^O , as shown at the bottom of the next page.

With the demands in Equation (4.7) and the profits in Equation (4.8) and Equation (4.9), the pricing decisions of the two retailers can be summarized in Proposition 2 as follows.

Proposition 2:

For $\theta \in (\max(0, \theta_1^O), \min(\frac{2t-v}{2t-v+s-\delta P_o}, \frac{t+v}{t+v-s+\delta P_o}, \theta_2^O))$, both of the two retailers have unique optimal pricing decisions (p_r^{O*}, p_o^{O*}) , which are given by:

$$\begin{cases} p_r^{O*} = \frac{(1 - \theta)(t + v) - \theta \delta P_o + s\theta}{3(1 + \delta)}, & \text{and} \\ p_o^{O*} = \frac{(1 - \theta)(2t - v) + \theta \delta P_o - s\theta}{3(1 + \delta)}. \end{cases} \quad (4.10)$$

The demands for the two retailers are:

$$\begin{cases} d_r^O = \frac{(1-\theta)(t+v) - \theta\delta P_o + s\theta}{3t(1-\theta)} \\ d_o^O = \frac{(1-\theta)(2t-v) + \theta\delta P_o - s\theta}{3t(1-\theta)} \end{cases} \quad (4.11)$$

The profits of the two retailers are:

$$\begin{cases} \pi_r^O = \frac{[(1-\theta)(t+v) - \theta\delta P_o + s\theta]^2}{9t(1-\theta)} \\ \pi_o^O = \frac{[(1-\theta)(2t-v) + \theta\delta P_o - s\theta]^2}{9t\theta(1-\theta)(1+\delta)} \end{cases} \quad (4.12)$$

With Proposition 2, we have the following results:

Corollary 3:

1) If $P_o < \frac{(1-\theta)(2t-v)+s\theta}{\theta}$, the prices of the two retailers (p_r^O & p_o^O) decrease with the reference price-sensitive coefficient of strategic customers (δ);

2) If $P_o > \frac{(1-\theta)(2t-v)+s\theta}{\theta}$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the price of the BM retailer (p_r^O) increases and the price of the online retailer (p_o^O) decreases.

Corollary 4:

1) If $P_o < \frac{(1-\theta)(2t-v)+s\theta}{\theta(\delta+2)}$, the profits of the two retailers (π_r^O & π_o^O) decrease with reference price-sensitive coefficient of strategic customers (δ);

2) If $P_o > \frac{(1-\theta)(2t-v)+s\theta}{\theta(\delta+2)}$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the profit of the BM retailer (π_r^O) increases and the profit of the online retailer (π_o^O) decreases.

The proofs of Proposition 2, Corollary 3, Corollary 4 and some of the following propositions and corollaries are similar to those of Proposition 1, Corollary 1 and Corollary 2, which will not be repeated in this paper.

Corollary 3 and Corollary 4 demonstrate again that under Strategy O, as customers' price sensitivity to the reference price increases, the RPM can only increase the profits of the mechanism implementer when the reference price is high enough.

C. STRATEGY RO

We discuss the strategy in which both retailers choose the option of reference price (Strategy RO) in this section. Similar to the previous form, we determine the two retailers' demand (d_r^{RO} & d_o^{RO}) from Equation (3.5), (4.13), as shown at the bottom of the next page.

Correspondingly, the profits of the BM (π_r^{RO}) and online (π_o^{RO}) retailers are:

$$\pi_r^{RO} = p_r^{RO} d_r^{RO} \quad s.t. \quad p_r^{RO} < \frac{v - tx^{RO} + \delta P_r}{1 + \delta} \quad (4.14)$$

$$\pi_o^{RO} = p_o^{RO} d_o^{RO} \quad s.t. \quad p_r^{RO} < \min\left(\frac{\beta v - s + \delta P_o}{1 + \delta}, \frac{v - tx^{RO} - s + \delta(\max(P_r, P_o))}{1 + \delta}\right) \quad (4.15)$$

We define θ_1^{RO} and θ_2^{RO} , as shown at the bottom of the next page. With the demands in Equation (4.13) and profits in Equation (4.14) and Equation (4.15), the pricing decisions of the two retailers can be summarized in Proposition 3 as follows.

Proposition 3: For

$$\theta \in (\max(0, \theta_1^{RO}), \min\left(\frac{2t - v - \delta P_r}{2t - v + s - \delta P_o}, \frac{t + v + \delta P_r}{t + v - s + \delta P_o}, \theta_2^{RO}\right)),$$

the BM and online retailers have unique optimal pricing decisions (p_r^{RO*} , p_o^{RO*}), which are given by:

$$\begin{cases} p_r^{RO*} = \frac{(1-\theta)(t+v) + \delta(P_r - \theta P_o) + s\theta}{3(1+\delta)}, \quad \text{and} \\ p_o^{RO*} = \frac{(1-\theta)(2t-v) - \delta(P_r - \theta P_o) - s\theta}{3(1+\delta)\theta}. \end{cases} \quad (4.16)$$

The demands for two retailers are:

$$\begin{cases} d_r^{RO} = \frac{(1-\theta)(t+v) + \delta(P_1 - \theta P_2) + s\theta}{3t(1-\theta)} \\ d_o^{RO} = \frac{(1-\theta)(2t-v) - \delta(P_1 - \theta P_2) - s\theta}{3t(1-\theta)} \end{cases} \quad (4.17)$$

The profits of the two retailers are:

$$\begin{cases} \pi_r^{RO} = \frac{[(1-\theta)(t+v) + \delta(P_r - \theta P_o) + s\theta]^2}{9t(1-\theta)(1+\delta)} \\ \pi_o^{RO} = \frac{[(1-\theta)(2t-v) - \delta(P_r - \theta P_o) - s\theta]^2}{9t\theta(1-\theta)(1+\delta)} \end{cases} \quad (4.18)$$

Proposition 3 shows that p_r^{RO} , d_r^{RO} and π_r^{RO} increase with P_r and decrease with P_o ; p_o^{RO} , d_o^{RO} and π_o^{RO} decrease with P_r and increase with P_o .

Corollary 5:

1) If $P_r - \theta P_o < (1-\theta)(v-2t) + s\theta$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the price of the BM retailer (p_r^{RO}) decreases and the price of the online retailer (p_o^{RO}) increases;

$$\theta_1^O = \frac{2\delta P_o - 2s + 3t - \sqrt{4v^2 + (4\delta P_o - 4s - 4t)v + 4(-\delta P_o + s - \frac{t}{2})^2}}{2(\delta P_o - s + t + v)}$$

$$\text{and } \theta_2^O = \frac{2\delta P_o - 2s + 3t + \sqrt{4v^2 + (4\delta P_o - 4s - 4t)v + 4(-\delta P_o + s - \frac{t}{2})^2}}{2(\delta P_o - s + t + v)}$$

2) If $(1 - \theta)(v - 2t) + s\theta < P_r - \theta P_o < (1 - \theta)(v + t) + s\theta$, the prices of the two retailers (p_r^{RO} & p_o^{RO}) decrease with the reference price-sensitive coefficient of strategic customers (δ);

3) If $(1 - \theta)(v + t) + s\theta < P_r - \theta P_o$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the price of the BM retailer (p_r^{RO}) increases and the price of the online retailer (p_o^{RO}) decreases.

Corollary 6:

1) If $P_r - \theta P_o < ((1 - \theta)(v - 2t) + s\theta)/(2 + \delta)$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the profit of the BM retailer (π_r^{RO}) decreases and the profit of the online retailer (π_o^{RO}) increases;

2) If $\frac{(1-\theta)(v-2t)+s\theta}{2+\delta} < P_r - \theta P_o < \frac{(1-\theta)(v+t)+s\theta}{2+\delta}$, the profits of the two retailers (π_r^{RO} & π_o^{RO}) decrease with the reference price-sensitive coefficient of strategic customers (δ);

3) If $((1 - \theta)(v + t) + s\theta)/(2 + \delta) < P_r - \theta P_o$, with the increase in the reference price-sensitive coefficient of strategic customers (δ), the profit of the BM retailer (π_r^{RO}) increases and the profit of the online retailer (π_o^{RO}) decreases.

Corollary 5 and Corollary 6 show that under Strategy RO, as the customer's price sensitivity to the reference price increases, the RPM can only increase the profits of the retailer that sets the higher reference price when there is a large difference between the reference prices set by the two retailers.

D. STRATEGY F

Under Strategy F, the demands for the two retailers are obtained from Equation (3.6):

$$\begin{cases} d_r^F = \frac{\theta p_o^F + \theta s + (1 - \theta)v - p_r^F}{t(1 - \theta)}, & \text{and} \\ d_o^F = 1 - d_r^F = \frac{(1 - \theta)(t - v) - \theta p_o^F - \theta s + p_r^F}{t(1 - \theta)} \end{cases} \quad (4.19)$$

The profit (π_r^F) of the BM retailer and the online retailer's profit (π_o^F) are:

$$\pi_r^F = p_r^F d_r^F, \quad \text{s.t. } p_r^F < v - tx^F \quad (4.20)$$

$$\pi_o^F = p_o^F d_o^F, \quad \text{s.t. } p_o^F < \min(v - tx^F - s, \beta v - s) \quad (4.21)$$

We define

$$\begin{aligned} \theta_1^F &= \frac{3t - 2s - \sqrt{4s^2 - 4st - 4sv + t^2 - 4tv + 4v^2}}{2(v + t - s)} \quad \text{and} \\ \theta_2^F &= \frac{3t - 2s + \sqrt{4s^2 - 4st - 4sv + t^2 - 4tv + 4v^2}}{2(v + t - s)} \end{aligned}$$

for the following exploration. With the demands in Equation (4.19) and the profits of the two retailers in Equation (4.20) and Equation (4.21), the pricing decisions of the two retailers can be summarized in Proposition 4 as follows.

Proposition 4: For $\theta \in (\max(0, \theta_1^F), \min(\frac{2t-v}{2t-v+s}, \theta_2^F))$, the optimal pricing decisions (p_r^{F*}, p_o^{F*}) for the BM and online retailers are:

$$\begin{cases} p_r^{F*} = \frac{1}{3}[(1 - \theta)(t + v) + s\theta], & \text{and} \\ p_o^{F*} = \frac{1}{3\theta}[(1 - \theta)(2t - v) - s\theta]. \end{cases} \quad (4.22)$$

According to Equation (4.22) and Equation (4.19), the demands of these two retailers are:

$$\begin{cases} d_r^F = \frac{(1 - \theta)(t + v) + s\theta}{3t(1 - \theta)} \\ d_o^F = \frac{(1 - \theta)(2t - v) - s\theta}{3t(1 - \theta)} \end{cases} \quad (4.23)$$

With prices in Equation (4.22) and demands in Equation (4.23), the profits of these two retailers are:

$$\begin{cases} \pi_r^F = \frac{[(1 - \theta)(t + v) + s\theta]^2}{9t(1 - \theta)} \\ \pi_o^F = \frac{[(1 - \theta)(2t - v) - s\theta]^2}{9t\theta(1 - \theta)} \end{cases} \quad (4.24)$$

Proposition 4 indicates that in the case of strategic customers with free-riding behaviour, BM retailers and online retailers will attract more customers through fierce price competition to achieve the purpose of maximizing their profits. We can see from Equation (4.22) and Equation (4.23) that as θ increases, p_r^F , p_o^F and d_o^F decrease and d_r^F increases. The practical explanation is that with the reduction in the free-riding cost, all retailers in the market will reduce their retail prices due to fierce price competition.

$$\begin{cases} d_r^{RO} = \frac{(1 - \theta)v + (1 + \delta)\theta p_o^{RO} - (1 + \delta)p_r^{RO} + \delta(P_r - \theta P_o) + \theta s}{t(1 - \theta)}, & \text{and} \\ d_o^{RO} = 1 - d_r^{RO} \\ = \frac{(1 - \theta)(t - v) - (1 + \delta)\theta p_o^{RO} + (1 + \delta)p_r^{RO} - \delta(P_r - \theta P_o) - \theta s}{t(1 - \theta)} \end{cases} \quad (4.13)$$

$$\theta_1^{RO} = \frac{1}{2(\delta P_o - s + t + v)}[2\delta P_o - 2s + 3t - \sqrt{4\delta^2 P_o^2 - 8\delta P_o s + 4\delta P_o t + 4\delta P_o v + 4s^2 - 4st - 4sv + t^2 - 4tv + 4v^2}]$$

$$\text{and } \theta_2^{RO} = \frac{1}{2(\delta P_o - s + t + v)}[2\delta P_o - 2s + 3t + \sqrt{4\delta^2 P_o^2 - 8\delta P_o s + 4\delta P_o t + 4\delta P_o v + 4s^2 - 4st - 4sv + t^2 - 4tv + 4v^2}]$$

V. COMPARATIVE ANALYSIS

In this section, the impact of free-riding behaviour is analysed by comparing the profit of the retailers in the two cases of strategic customers and non-strategic customers under Strategy F. Based on the situation where the RPM is not implemented, i.e., Strategy F, the profit changes of the three implementation situations of Strategy R, Strategy O and Strategy RO are analysed separately to explore the effectiveness of the RPM in response to free-riding behaviour. According to the profit comparison under the four strategies, the equilibrium strategy of the RPM is obtained.

A. IMPACT OF FREE-RIDING BEHAVIOUR

For convenience, we define ς , γ_1^F , γ_2^F , σ_1^F , and σ_2^F , as shown at the bottom of the page.

We have:

Proposition 5:

$$\left\{ \begin{array}{ll} \pi_r^F > \pi_r, & \text{if } \theta \in (\min\{\max(0, \theta_1^F), \gamma_1^F\}, \gamma_1^F) \\ & \cup (\gamma_2^F, \max\{\gamma_2^F, \min(\frac{2t-v}{2t-v+s}, \theta_2^F)\}); \\ \pi_r^F < \pi_r, & \text{if } \theta \in (\max(0, \theta_1^F, \gamma_1^F), \\ & \min(\frac{2t-v}{2t-v+s}, \theta_2^F, \gamma_2^F)); \\ \pi_o^F > \pi_o, & \text{if } \theta \in (\min\{\max(0, \theta_1^F), \sigma_1^F\}, \sigma_1^F) \\ & \cup (\sigma_2^F, \max\{\sigma_2^F, \min(\frac{2t-v}{2t-v+s}, \theta_2^F)\}); \\ \pi_o^F < \pi_o, & \text{if } \theta \in (\max(0, \theta_1^F, \sigma_1^F), \\ & \min(\frac{2t-v}{2t-v+s}, \theta_2^F, \sigma_2^F)). \end{array} \right.$$

where π_r and π_o represent the profit of the BM retailer and the profit of the online retailer without free-riding behaviour, respectively.

The proof of Proposition 5 is given in the appendix.

We attempt to use a numerical example to determine the conclusion. We set $v = 6$, $t = 5$, $\beta = 0.8$, and $s = 0.1$. The value range of θ should satisfy

$\theta \in (\max(0, \theta_1^F), \min(\frac{2t-v}{2t-v+s}, \theta_2^F))$ to ensure that the market is fully covered and that there is demand at both retailers. Therefore, we find that the range of θ is $\theta \in (0.38, 0.9)$. We define $\Delta\pi_r^F = \pi_r^F - \pi_r$, $\Delta\pi_o^F = \pi_o^F - \pi_o$. The changes in $\Delta\pi_r^F$, $\Delta\pi_o^F$ and $\Delta\pi_r^F + \Delta\pi_o^F$ with free-riding cost (θ) are illustrated in Figure 1.

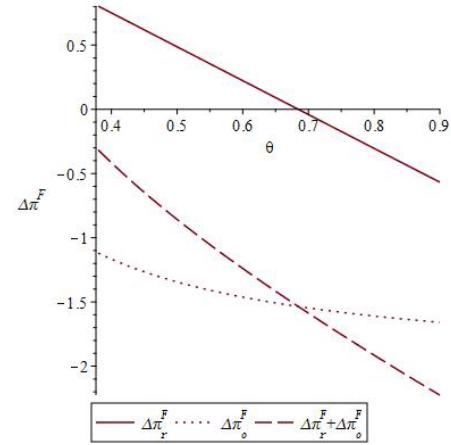


FIGURE 1. Change in profit margin $\Delta\pi^F$ with free-riding cost θ .

Figure 1 shows that the free-riding behaviour will reduce the total profit of the market, but, surprisingly, it may increase the profit of the BM retailer. If the free-riding cost is large (smaller θ), the profit of the BM retailer will increase under Strategy F ($\Delta\pi_r^F > 0$). However, the profit of the online retailer and the whole market will always decrease under Strategy F ($\Delta\pi_o^F < 0$, $\Delta\pi_r^F + \Delta\pi_o^F < 0$). This result is different from the conclusion of Chen and Chen (2019) that the free-riding behaviour of strategic customers will adversely impact BM retailers. The reason for this difference is that the free-riding cost is considered in this paper. It indicates that if customers care too much about experiencing the products and spend more time and energy to understand the products, customers' free-riding behaviour will positively impact

$$\begin{aligned} \varsigma &= \sqrt{5} \sqrt{(s^2 - (\frac{2}{5}\beta v + \frac{2}{5}t + \frac{2}{5}v)s + \frac{(t+v(1-\beta))^2}{5})(v+s+t-\beta v)^2} \\ \gamma_1^F &= \frac{1}{2(v+t-s)^2} [(-\beta^2 + 2\beta + 1)v^2 + ((2s+2t)\beta - 4s+2t)v - s^2 - 4st + t^2 - \varsigma] \\ \gamma_2^F &= \frac{1}{2(v+t-s)^2} [(-\beta^2 + 2\beta + 1)v^2 + ((2s+2t)\beta - 4s+2t)v - s^2 - 4st + t^2 + \varsigma] \\ \sigma_1^F &= \frac{1}{(2\beta^2 - 4\beta + 4)v^2 + ((-4s+8t)\beta - 16t)v + 4s^2 + 16t^2} [(\beta^2 - 2\beta + 3)v^2 + ((-2s+4t)\beta - 12t)v + s^2 + 12t^2 \\ &\quad - \sqrt{(-\beta v + s - 2t + v)^2(s^2 + (-2\beta v + 4t - 2v)s + 4(t + \frac{v(-1+\beta)}{2})^2)} \\ \sigma_2^F &= \frac{1}{(2\beta^2 - 4\beta + 4)v^2 + ((-4s+8t)\beta - 16t)v + 4s^2 + 16t^2} [(\beta^2 - 2\beta + 3)v^2 + ((-2s+4t)\beta - 12t)v + s^2 + 12t^2 \\ &\quad + \sqrt{(-\beta v + s - 2t + v)^2(s^2 + (-2\beta v + 4t - 2v)s + 4(t + \frac{v(-1+\beta)}{2})^2)} \end{aligned}$$

BM retailers. In contrast, if the customers have a good service experience without spending too much time and energy, the free-riding behaviour of strategic customers has a negative impact on BM retailers. Therefore, many BM retailers have established complicated processes of consumer experience, such as paying a deposit, the purpose of which is to increase the cost of free-riding.

TABLE 2. Optimal prices and demands of the two retailers with a change in θ ($v = 6$, $t = 5$, and $s = 0.1$).

θ	p_r^{F*}	p_o^{F*}	d_r^F	d_o^F	π_r^F	π_o^F
0.5	1.850	1.300	0.740	0.260	1.369	0.338
0.6	1.487	0.856	0.743	0.257	1.105	0.220
0.7	1.123	0.538	0.749	0.251	0.841	0.135
0.8	0.760	0.300	0.760	0.240	0.578	0.072

TABLE 2 shows that as the cost of free-riding decreases (θ increases), the profits of both the BM retailer and the online retailer decrease. When the cost of free riding is high, the emergence of strategic customers may have a positive impact on the BM retailer. With the reduction in the cost of free riding, customers have more incentive to free ride on the BM retailer's service and to switch to the online retailer for the seek of buying the product at a lower price, resulting in fierce price competition among retailers.

A large number of studies show that online shopping customers are more strategic than offline customers. Therefore, most online shopping customers are more inclined to engage in free riding. However, many strategic customers in the process of free riding on the BM retailer's service may end up buying a product from the BM retailer since BM retailers try their best to provide customers with satisfactory service. In this way, customers' free-riding behaviour converts some online shoppers into customers of the BM retailer. Therefore, in the market with a high free-riding cost, the free-riding behaviour of customers increases the profit of the BM retailer. Take Double Eleventh Day in China, for example. Before that day, many online shopping customers will first go to physical stores to find and experience their favourite products. However, while visiting physical stores, many strategic customers will buy offline because the sense of acquisition from buying in physical stores is higher than that of free riding, thus increasing the demand of the BM retailer. In recent years, before Double Eleventh Day, BM retailers have seized the opportunity of such traffic and carried out promotional activities, enabling more customers who want to experience the service of the BM retailer for free to purchase products directly from the BM retailer. With the reduction in the cost of free riding, increasingly, customers prefer to buy their favourite products by taking advantage of the service of the BM retailer for free. To maximize its profits, the BM retailer should reduce the price to retain customers.

For online retailers, when there is free-riding behaviour in the market, the demand for online retailers decreases. In addition, in the face of the price reduction of the BM retailer, the online retailer has to reduce its retail price to ensure its

profit maximization. Therefore, the free-riding behaviour of customers has an adverse effect on the online retailer.

In a market with low free-riding cost, the BM retailer and the online retailer tend to compete on price in pursuit of profit maximization. However, the fierce price competition will depress the whole market to a certain extent. This paper proposes the RPM to address the adverse effects of free-riding behaviour. The effectiveness of this mechanism is studied through comparative analysis. The specific research process is presented in the next section.

B. EFFECTIVENESS OF THE RPM

1) STRATEGY R

We obtain Proposition 5 by comparing π_j^R and π_j^F and then discuss the impact of free-riding behaviour on the profits of the two retailers under Strategy R. We define $\Delta\pi_r^R = \pi_r^R - \pi_r^F$, $\Delta\pi_o^R = \pi_o^R - \pi_o^F$, and $\gamma_1^R = \frac{-t-v+P_r+\sqrt{\delta P_r^2+P_r^2}}{s-t-v}$.

Proposition 6: Under Strategy R, the comparison of the retailers' profits is as follows:

$$\begin{cases} \pi_r^R > \pi_r^F, & \text{if } \theta \in (\max(0, \theta_1^R, \gamma_1^R), \\ & \min(\frac{2t-v-\delta P_r}{2t-v+s}, \theta_2^R)), \\ \pi_r^R < \pi_r^F, & \text{if } \theta \in (\min(\max(0, \theta_1^R, \gamma_1^R), \gamma_1^R), \\ \pi_o^R < \pi_o^F, & \text{if } \theta \in (\max(0, \theta_1^R), \min(\frac{2t-v-\delta P_r}{2t-v+s}, \theta_2^R)). \end{cases}$$

The proof of Proposition 6 is given in the appendix.

Proposition 6 illustrates the results in which under Strategy R, the RPM is effective for the BM retailer when $\theta \in (\max(0, \theta_1^R, \gamma_1^R), \min(\frac{2t-v-\delta P_r}{2t-v+s}, \theta_2^R))$, which can alleviate the negative impact of the free-riding behaviour of strategic customers on the BM retailer. However, when the cost of free riding is high (smaller θ), the retailer can set a higher retail price, and the reference price is not enough to bring premium space to the BM retailer. Therefore, the RPM is ineffective for the BM retailer when the free-riding cost is small. Whether the RPM will eventually become invalid to BM retailers depends on the reference price of the BM retailer (P_r). If the reference price of the BM retailer is high enough, the RPM is always effective for the BM retailer under Strategy R. However, the RPM will aggravate the negative impact of free-riding behaviour on the online retailer under Strategy R.

Considering that the reference price is generally set around the value of the goods in the market, we take the reference price as the customer's valuation in a numerical example. We set $v = P_r = 6$, $t = 5$, $\theta = 0.6$ and $\delta = s = 0.1$. The value range of θ in Figure 2(a) satisfies $\theta \in (\max(0, \theta_1^R), \min(\frac{2t-v-\delta P_r}{2t-v+s}, \theta_2^R))$. The changes in $\Delta\pi_r^R$, $\Delta\pi_o^R$ and $\Delta\pi_r^R + \Delta\pi_o^R$ with θ and δ are illustrated in Figure 2.

We observe from Figure 2 that the unilateral implementation of the RPM by the BM retailer increases the profit of the BM retailer (π_r^R) and decreases the profit of the online retailer (π_o^R). For the whole market, it is only in the high range of free-riding cost that the RPM can increase the gross profit. If customers are more likely to take a free ride and are more

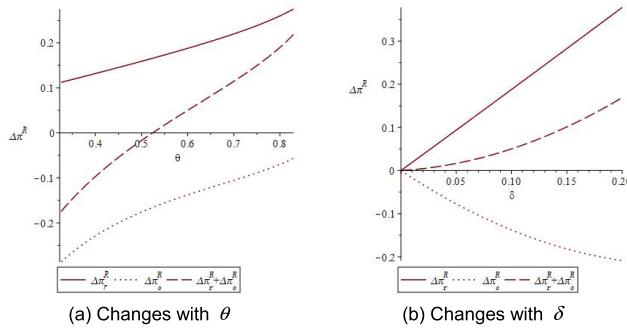


FIGURE 2. Changes in $\Delta\pi_r^R$, $\Delta\pi_o^R$ and $\Delta\pi_r^R + \Delta\pi_o^R$ with θ and δ .

sensitive to the reference price, the RPM is more effective for the BM retailer under Strategy R.

TABLE 3. Optimal prices and demands of the two retailers with a change in θ ($v = P_r = 6$, $t = 5$, and $\delta = s = 0.1$).

θ	P_r^{*}	P_o^{*}	d_r^R	d_o^R	π_r^R	π_o^R
0.5	1.864	0.818	0.820	0.180	1.528	0.162
0.6	1.533	0.475	0.843	0.157	1.293	0.082
0.7	1.203	0.229	0.882	0.118	1.061	0.030
0.8	0.873	0.045	0.960	0.040	0.838	0.002

TABLE 3 shows that under Strategy R, as the free-riding cost decreases (θ increases), the profits of both the BM retailer and the online retailer decrease. Compared with TABLE 2, in the market with low free-riding cost (higher θ), Strategy R effectively alleviates the adverse impact of customers' free-riding behaviour on BM retailers. According to the above example results, we can draw two conclusions:

- Under Strategy R, the RPM is beneficial to BM retailers and unfavourable to online retailers.
- Under Strategy R, when the cost of free riding is low, the RPM can effectively alleviate the market downturn caused by the free-riding behaviour of strategic customers.

2) STRATEGY O

In this section, we explore the impact of the unilateral implementation of the RPM by the online retailer on free-riding behaviour, and Proposition 7 is obtained. We define $\Delta\pi_r^O = \pi_r^O - \pi_r^F$, $\Delta\pi_o^O = \pi_o^O - \pi_o^F$, $\sigma_1^O = \frac{(2t+s-v+P_o-\sqrt{P_o^2(1+\delta)(2t-v)}}{-P_o^2\delta+(4t-2v+2s)P_o+(s+2t-v)^2}$ for convenience.

Proposition 7: Under Strategy O, the comparison of retailers' profits is:

$$\left\{ \begin{array}{ll} \pi_r^O < \pi_r^F, & \text{if } \theta \in (\max(0, \theta_1^O), \\ & \min(\frac{2t-v}{2t-v+s-\delta P_o}, \frac{t+v}{t+v-s+\delta P_o}, \theta_2^O)); \\ \pi_o^O < \pi_o^F, & \text{if } \theta \in (\min(\max(0, \theta_1^O), \sigma_1^O), \sigma_1^O); \\ \pi_o^O > \pi_o^F, & \text{if } \theta \in (\max(0, \theta_1^O, \sigma_1^O), \\ & \min(\frac{2t-v}{2t-v+s-\delta P_o}, \frac{t+v}{t+v-s+\delta P_o}, \theta_2^O)). \end{array} \right.$$

The proof of Proposition 7 is given in the appendix.

TABLE 4. Optimal prices and demands of the two retailers with a change in θ ($v = P_o = 6$, $t = 5$, and $\delta = s = 0.1$).

θ	p_r^{O*}	p_o^{O*}	d_r^O	d_o^O	π_r^O	π_o^O
0.5	1.591	1.364	0.700	0.300	1.225	0.409
0.6	1.242	0.960	0.683	0.317	0.934	0.304
0.7	0.894	0.671	0.656	0.344	0.645	0.231
0.8	0.545	0.455	0.600	0.400	0.360	0.182

Proposition 7 shows that when the online retailer implements the RPM (Strategy O), the profit of the BM retailer (π_r^O) will decrease and the profit of the online retailer (π_o^O) will increase. However, the unilateral implementation of the RPM by the online retailer may also reduce the profit of the online retailer (π_o^O). Higher reference price (P_o) gives customers a positive transaction utility in markets with low free-riding cost and low retail price.

We set $v = 6$, $P_o = 6$, $t = 5$, $\delta = 0.1$, $\theta = 0.6$ and $s = 0.1$. The value range of θ in Figure 3(a) satisfies $\theta \in (\max(0, \theta_1^O), \min(\frac{2t-v}{2t-v+s-\delta P_o}, \frac{t+v}{t+v-s+\delta P_o}, \theta_2^O))$. The changes in $\Delta\pi_r^O$, $\Delta\pi_o^O$ and $\Delta\pi_r^O + \Delta\pi_o^O$ with θ and δ are illustrated in Figure 3.

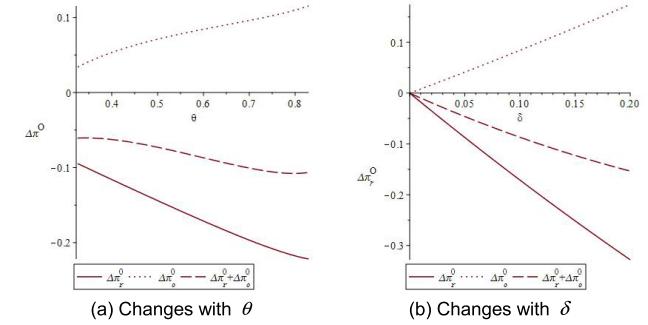


FIGURE 3. Changes in $\Delta\pi_r^O$, $\Delta\pi_o^O$ and $\Delta\pi_r^O + \Delta\pi_o^O$ with θ and δ .

In Figure 3, we demonstrate that the unilateral implementation of the RPM by the online retailer (Strategy O) can effectively improve the profit of the online retailer ($\Delta\pi_o^O > 0$) and mitigate the adverse impact of the free-riding behaviour of strategic customers on the online retailer. However, the BM retailer and the market's profits will decrease under Strategy O ($\Delta\pi_r^O < 0$, $\Delta\pi_r^O + \Delta\pi_o^O < 0$). The increase in profit of the online retailer is not enough to make up for the loss of profit of the BM retailer. As a result, the unilateral implementation of the RPM by the online retailer leads to an overall decrease in market profits, and Strategy O is only effective in improving the profit of the online retailer.

TABLE 4 shows that as the free-riding cost decreases (θ increases), the demand of the online retailer increases, and other indicators decrease. This result indicates that under Strategy O, with the reduction in the free-riding cost, the online retailer can gain more market share to mitigate the adverse impact of free riding. However, Strategy O cannot solve the problem that free-riding behaviour intensifies the price competition between the two retailers. With the reduction in free-riding cost, the trend of retailers' retail price and profit reduction remains unchanged.

Similar to Strategy R, the online retailer can increase its profit by implementing the RPM because the reference price is provided to satisfy the customer's utility. The difference between Strategy R and Strategy O is the change in the whole market's profits. Compared to Strategy F, the profits of the market will decrease in Strategy O. We obtain two interesting conclusions:

- Unilateral implementation of the RPM can increase the profit of implementers.
- From the perspective of the whole market, Strategy R is better than Strategy O.

3) STRATEGY RO

How does the simultaneous implementation of the RPM by the BM retailer and the online retailer affect the competition? Are the profits of both sides increased, or will this strategy lead to price competition? This problem is discussed below.

As it is challenging to measure the profit difference under Strategy RO analytically, we conduct a comprehensive numerical study to derive additional management insights. We define $\Delta\pi_r^{RO} = \pi_r^{RO} - \pi_r^F$, $\Delta\pi_o^{RO} = \pi_o^{RO} - \pi_o^F$, $v = 6$, $t = 5$ and $\delta = s = 0.1$. The value range of θ in Figure 4 satisfies $\theta \in (\max(0, \theta_1^{RO}), \min(\frac{2t-v-\delta P_r}{2t-v+s-\delta P_o}, \frac{t+v+\delta P_r}{t+v-s+\delta P_o}, \theta_2^{RO}))$. The changes in $\Delta\pi_r^{RO}$, $\Delta\pi_o^{RO}$ and $\Delta\pi_r^{RO} + \Delta\pi_o^{RO}$ with θ are illustrated in Figure 4.

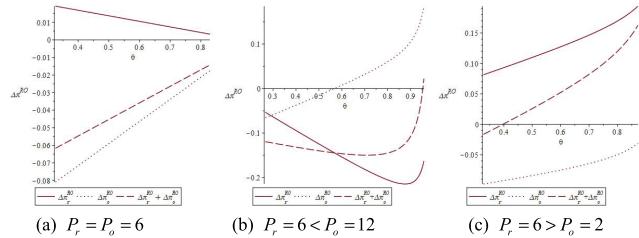


FIGURE 4. Changes in $\Delta\pi_r^{RO}$, $\Delta\pi_o^{RO}$ and $\Delta\pi_r^{RO} + \Delta\pi_o^{RO}$ with θ .

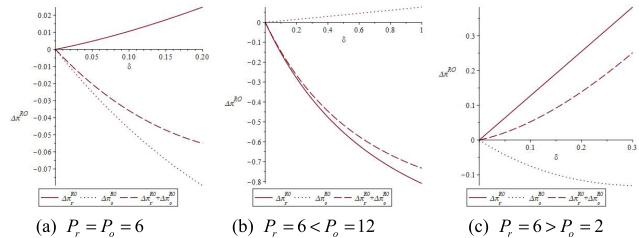


FIGURE 5. Changes in $\Delta\pi_r^{RO}$, $\Delta\pi_o^{RO}$ and $\Delta\pi_r^{RO} + \Delta\pi_o^{RO}$ with δ .

We set $v = 6$, $t = 5$, $\theta = 0.6$ and $s = 0.1$. The changes in $\Delta\pi_r^{RO}$, $\Delta\pi_o^{RO}$ and $\Delta\pi_r^{RO} + \Delta\pi_o^{RO}$ with δ are illustrated in Figure 5.

Figure 4 and Figure 5 show that the RPM may increase the profit of a retailer who sets a reference price sufficiently higher than the competitor's price. In any case, when one retailer's profit increases, the other retailer's profit will decrease. Each retailer must set a higher reference price to maximize its profits. Due to the existence of free-riding cost (θ), the influence of the reference price of the BM retailer

(P_r) is greater than that of the online retailer (P_o). In the case where the reference prices of the two retailers are equal ($P_r = P_o = 6$), the profit of the BM retailer increases ($\Delta\pi_r^{RO} > 0$), while that of the online retailer decreases ($\Delta\pi_o^{RO} < 0$). Since the reference price has a greater impact on BM retailers, the most likely way to increase the market's profits is for the reference price set by the BM retailer to be higher than that of the online retailer ($P_r > P_o$). We can see from Figures 4(c) and 5(c) that when $P_r > P_o$, the overall market profit is most likely to increase. The reason is that the higher reference price of BM retailers prevents free-riding customers from switching to online purchases. The RPM effectively restrains the free-riding behaviour of strategic customers and alleviates the retail price competition between the BM retailer and the online retailer when P_r is sufficiently greater than P_o . We can observe the failure of the RPM from Figure 4(b). When the cost of free riding is low and the reference price of the BM retailer is low, again, as shown in Figure 4(b), the RPM will reduce the profits of both retailers. This effect occurs because the RPM not only reduces the price of the BM retailer but also reduces the price and demand of the online retailer. As the reference price-sensitive coefficient (δ) increases, the RPM is more effective.

By exploring the above three strategies of the RPM (Strategy R, Strategy O and Strategy RO) and performing a comparative analysis with the non-implementation of the RPM (Strategy F), we obtain the following conclusions:

- The RPM may have a positive impact on a retailer that sets the reference price much higher than the competitor.
- The introduction of the RPM will enable retailers to enter into a new round of price competition, bidding up their reference price.
- From the whole market perspective, the BM retailer should provide a higher reference price.

Although in our analysis, Strategy R is the best strategy for the whole market, retailers are profit-oriented and will not change their strategies to benefit the profit of the whole market. They will do their best to improve their own profits or reduce their own losses. In such a situation, what is the equilibrium strategy for implementing the RPM for the BM and the online retailer? We discuss this issue in the next section.

C. THE EQUILIBRIUM STRATEGY OF THE RPM

In this section, we derive the equilibrium strategy of implementing the RPM for the BM retailer and the online retailer by discussing the four situations where the online retailer and the BM retailer either implement the RPM or not. We define

$$\begin{aligned}\Delta\pi_r^{ROO} &= \pi_r^{RO} - \pi_r^O, \quad \Delta\pi_o^{ROR} = \pi_o^{RO} - \pi_o^R, \\ \theta_1^{ROO} &= \frac{t+v-P_r(\sqrt{\delta+1}+1)}{\delta P_o - s + t + v}, \\ \theta_2^{ROO} &= \frac{t+v+P_r(\sqrt{\delta+1}-1)}{\delta P_o - s + t + v},\end{aligned}$$

$$\begin{aligned}\theta_1^{ROR} &= \frac{(2t+s-v+P_o-\sqrt{P_o^2(1+\delta)})(-\delta P_r+2t-v)}{-P_o^2\delta+(2s+4t-2v)P_o+(s+2t-v)^2}, \\ \theta_2^{ROR} &= \frac{(2t+s-v+P_o+\sqrt{P_o^2(1+\delta)})(-\delta P_r+2t-v)}{-P_o^2\delta+(2s+4t-2v)P_o+(s+2t-v)^2}, \\ \theta_L &= \max(0, \theta_1^F, \theta_1^R, \theta_1^O, \theta_1^{RO}), \\ \theta_H &= \min\left(\frac{2t-v-\delta P_r}{2t-v+s}, \frac{t+v}{t+v-s+\delta P_o}, \theta_2^F, \theta_2^R, \theta_2^O, \theta_2^{RO}\right)\end{aligned}$$

and their intervals in TABLE 5.

TABLE 5. Definitions of the intervals.

Interval	Range of interval
U	(θ_L, θ_H)
D_1	$(\max(\theta_L, \theta_1^{RO}), \min(\theta_2^{RO}, \theta_H))$
D_2	$(\max(\theta_L, \gamma_1^R), \theta_H)$
D_3	$(\max(\theta_L, \theta_1^{ROR}), \min(\theta_2^{ROR}, \theta_H))$
D_4	$(\max(\theta_L, \sigma_1^O), \theta_H)$

Whether these two retailers should implement the RPM is discussed in the range of the value of θ . $\theta \in U$ is assumed to ensure that the market is fully covered and that the two retailers are not going out of business.

According to the comparative study on the profit of retailers, we have the following proposition:

Proposition 8: The equilibrium strategy of implementing RPM is:

*Strategy**

$$= \begin{cases} \text{Strategy RO} & \text{if } \theta \in C_{D_1 \cap D_3} (D_1 \cap D_3 \cap C_U D_2 \cap C_U D_4) \\ \text{Strategy R} & \text{if } \theta \in C_{D_2 \cap C_U D_3} (D_2 \cap C_U D_3 \cap C_U D_1 \cap D_4) \\ \text{Strategy O} & \text{if } \theta \in C_{C_U D_1 \cap D_4} (C_U D_1 \cap D_4 \cap D_2 \cap C_U D_3) \\ \text{Strategy F} & \text{if } \theta \in C_{C_U D_2 \cap C_U D_4} (C_U D_2 \cap C_U D_4 \cap D_1 \cap D_3) \end{cases}$$

The proof of Proposition 8 is given in the appendix.

Proposition 8 shows that when θ is in a different range for the BM and online retailers, the two retailers have different equilibrium strategies for determining whether to implement the RPM. In this subsection, we attempt to utilize numerical examples to verify the results obtained from Proposition 8 and determine conclusions that are difficult to draw directly from the theoretical analysis. We set $v = 6, t = 5, \delta = 0.1, P_1 = 6, P_2 = 6$, and $s = 0.1$.

Figure 6 shows

$$\pi_r^{RO} > \pi_r^O, \quad \pi_r^R > \pi_r^F, \quad \pi_o^{RO} > \pi_o^R, \quad x\pi_o^O > \pi_o^F. \quad (4.25)$$

According to Equation (4.25), we can obtain a game matrix between the BM retailer and the online retailer, as shown in Figure 7.

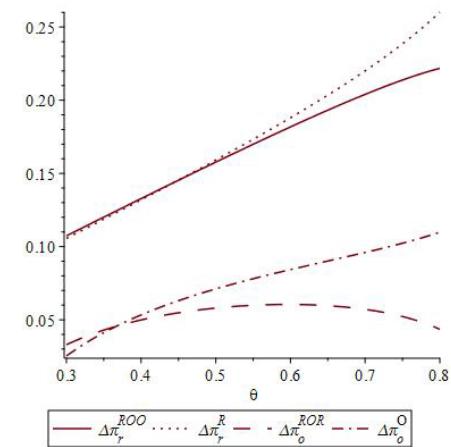


FIGURE 6. Changes in $\Delta\pi_r^{ROO}$, $\Delta\pi_r^R$, $\Delta\pi_o^{ROR}$, and $\Delta\pi_o^O$ with θ .

		The online retailer	
		Not Implement	Implement
The BM retailer	Implement	(π_r^{RO}, π_o^{RO})	(π_r^R, π_o^R)
	Not Implement	(π_r^O, π_o^O)	(π_r^F, π_o^F)

FIGURE 7. Game matrix of RPM.

The game matrix in Figure 7 shows that the equilibrium strategy is Strategy RO (i.e., both retailers implement the RPM) when $\theta \in C_{D_1 \cap D_3}$ ($D_1 \cap D_3 \cap C_U D_2 \cap C_U D_4$). In this situation, the BM retailer will implement the RPM regardless of whether the online retailer implements the RPM. Moreover, the online retailer will also implement RPM if the BM retailer implements RPM. Therefore, the implementation of the RPM, as a tacit agreement and collusion between retailers, is an equilibrium strategy for the two retailers.

According to Proposition 8, other strategies may also be used as an equilibrium strategy of the two retailers in other thresholds of free-riding cost. Take $\pi_r^{RO} > \pi_r^O, \pi_r^R > \pi_r^F, \pi_o^{RO} > \pi_o^R, \pi_o^O > \pi_o^F$ as an example. It is advantageous for the BM retailer to implement the RPM regardless of whether the online retailer implements the RPM. Given that the BM retailer implements the RPM, it is optimal for the online retailer not to implement the RPM, so Strategy R is the equilibrium strategy in this case. Similarly, we can also identify cases in which Strategy O or Strategy R is the equilibrium strategy. Therefore, the equilibrium strategy in which the two retailers implement the RPM will be different with different free-riding costs.

VI. CONCLUSION

Based on the Hotelling model and transaction utility theory, this paper develops a duopoly model that describes

free-riding behaviour and the reference price effect. This research provides management with new insights for addressing the free-riding behaviour of strategic customers in the highly competitive retail environment of online and offline retailers. Our research provides (i) suggestions on how to adjust prices for BM retailers and online retailers according to different strategies, (ii) a better understanding of whether BM retailers and online retailers should implement reference pricing strategies to respond to strategic customers, and (iii) a method to mitigate the adverse effects of strategic customers with free-riding behaviour.

According to the research of this paper, we obtain the following main conclusions:

(i) The impact of customers' free-riding behaviour on the BM retailer depends on the free-riding cost, but it has a negative impact on the online retailer and the overall market. We show that when the free-riding cost is low, the existence of strategic customers leads to fierce competition, which decreases the profit of the BM retailer. However, when the free-riding cost is high, there is no need for fierce price competition between the BM retailer and the online retailer to fight for market share. Therefore, the increase in demand from the BM retailer leads to an increase in its own profit. The free-riding behaviour of strategic customers leads to price competition between the two retailers, which reduces the retail prices of both retailers and the demand of the online retailer, resulting in the reduction of profits of the online retailer and the whole market.

(ii) The RPM may only have a positive impact on one retailer in the market. The retailer who unilaterally implements the RPM will increase its profit and reduce the profit of the competitor when the reference price is high enough. Even if the two retailers implement the RPM simultaneously, it is only possible to increase the profit of the retailer who sets higher reference prices.

(iii) The equilibrium strategy for implementing the RPM of the two retailers varies with free-riding cost. Strategy R, Strategy O, Strategy RO, and Strategy F are all likely to become the equilibrium strategy of the RPM under different free-riding costs.

The reference price is widely used in practice. For BM retailers, examples include tag prices in many brand-name clothing stores and official guidance prices in the automobile industry. The marking of these reference prices can indeed bring product awareness to customers. Moreover, if the retail price is lower than the reference price, it can enhance the transaction utility of customers and reduce the possibility of free riding by customers. Online retailers can also use the reference price to improve the transaction utility of customers and their competitiveness in the market to cope with the free-riding behaviour of strategic customers. The historical transaction price, offer price and original price displayed on the online shopping platform can be taken as examples. However, the effectiveness of this mechanism must be based on the retailer's credibility. That is, the customer must believe that the reference price reflects the real value of the product.

Therefore, it can be seen in some small commodity markets that the reference price often induces customers to engage in a greater degree of bargaining, which may strengthen customers' determination to free ride.

This line of research can be extended in several directions. First, we assume that the customer thinks that any reference price set by the retailer is real and that the customer can obtain transaction utility from it. However, customers may not necessarily trust the reference prices given by retailers, or they may have different levels of trust in different reference prices. It would be interesting to explore the effectiveness of reference price strategies in depth by introducing a variable to characterize customers' belief in different reference prices and thus obtain different levels of transaction utility. In addition, we believe that determining the reference price is of general interest and can deepen the insights provided here. Our initial attempt suggests that determining a reference price will often lead to considerable analytical challenges, and therefore, this issue also indicates future research directions. Finally, the reference price can also be regarded as the promotion of discount information. For example, retailers clearly tell the customers how much discount they are receiving instead of directly giving the sales price. The greater the discount of the product, the higher the transaction utility the customer will obtain but the lower the customer's trust in the price before the discount may be. Determining the appropriate discount intensity is also a future research direction.

APPENDIX

A. THE PROOF OF PROPOSITION 1

With demand (d_r^R) and profit function (π_r^R) of the BM retailer in Equation (4.1) and Equation (4.2), we have $\partial\pi_r^R/\partial p_r^R = \frac{(1-\theta)v+\theta p_o^R+\delta P_r+\theta s}{t(1-\theta)} - \frac{2(1+\delta)}{t(1-\theta)}p_r^R$. Thus, $\partial\pi_r^R/\partial p_r^R = 0$ gives:

$$p_r^{R*}(p_o^R) = \frac{(1-\theta)v+\theta p_o^R+\delta P_r+\theta s}{2(1+\delta)} \quad (A1)$$

With demand (d_o^R) and profit function (π_o^R) of the online retailer in Equation (4.1) and Equation (4.3), we have $\partial\pi_o^R/\partial p_o^R = \frac{(1-\theta)(t-v)+(1+\delta)p_r^R-\delta P_r-\theta s}{t(1-\theta)} - \frac{2\theta}{t(1-\theta)}p_o^R$. Thus, $\partial\pi_o^R/\partial p_o^R = 0$ gives:

$$p_o^{R*}(p_r^R) = \frac{(1-\theta)(t-v)+(1+\delta)p_r^R-\delta P_r-\theta s}{2\theta} \quad (A2)$$

We can prove that $\partial^2\pi_r^R/\partial p_r^R < 0$ and $\partial^2\pi_o^R/\partial p_o^R < 0$. According to Equations (A1) and (A2), we can solve $p_r^{R*} = \frac{(1-\theta)(t+v)+\delta P_r+s\theta}{3(1+\delta)}$ and $p_o^{R*} = \frac{(1-\theta)(2t-v)-\delta P_r-s\theta}{3\theta}$ jointly. To ensure that the whole market is covered (i.e., $U_r^R = U_{ro}^R > 0$), $\theta \in (\theta_1^R, \theta_2^R)$. In addition, we need to make sure that the online retailer will not exit the market (i.e., $p_o^{R*} > 0$); therefore, $\theta < \frac{2t-v-\delta P_r}{2t-v+s}$. In summary, the value range of $\theta \in (\max(0, \theta_1^R), \min(\frac{2t-v-\delta P_r}{2t-v+s}, \theta_2^R))$.

This method applies to Proposition 2-Proposition 4.

B. THE PROOF OF COROLLARIES 1 & 2

With the BM retailer's price in Equation (4.4), taking derivatives of p_r^{R*} w.r.t. δ , we have:

$$\frac{\partial p_r^{R*}}{\partial \delta} = \frac{P_r - (1 - \theta)(v - t) - \theta s}{3(1 + \delta)^2}. \quad (\text{A3-1})$$

According to (A3-1), when $P_r < (1 - \theta)(v + t) + s\theta$, then $\partial p_r^{R*}/\partial \delta < 0$; when $P_r > (1 - \theta)(v + t) + s\theta$, then $\partial p_r^{R*}/\partial \delta > 0$. Taking the derivatives of p_o^{R*} w.r.t. δ , we have: $\partial p_o^{R*}/\partial \delta = -P_r/3\theta < 0$.

Corollary 1 is proved.

With the BM retailer's profit in Equation (4.6), taking the derivatives of π_r^R w.r.t. δ , we have (A3-2), as shown at the top of the next page.

According to (A3-2), when $P_r < ((1 - \theta)(v + t) + s\theta)/(\delta + 2)$, then $\partial \pi_r^R/\partial \delta < 0$; when $P_r > ((1 - \theta)(v + t) + s\theta)/(\delta + 2)$, then $\partial \pi_r^R/\partial \delta > 0$. Taking the derivatives of π_o^R w.r.t. δ , we have: $\partial \pi_o^R/\partial \delta = -(2((1 - \theta)(2t - v) - \delta P_r - s\theta P_r)/(9t(1 - \theta)\theta)) < 0$.

Corollary 2 is proved.

This method applies to Corollary 3- Corollary 6.

C. THE PROOF OF PROPOSITION 5

In the case of non-strategic customers, only two types of customers are considered: those who purchase products directly from BM retailers or from online retailers. The market is divided at the point where $U_r^F = U_o^F$, which gives $x^* = [s + (1 - \beta)v + p_o - p_r]/t$. Therefore, the demands for the two retailers are:

$$\begin{cases} d_r = [s + (1 - \beta)v + p_o - p_r]/t, & \text{and} \\ d_o = 1 - d_r = [-s + t - (1 - \beta)v - p_o + p_r]/t. \end{cases} \quad (\text{A4})$$

The profit of the BM retailer (π_r) is $\pi_r = p_r d_r$; thus, we have $\partial \pi_r/\partial p_r = [s + (1 - \beta)v + p_o - 2p_r]/t$. Therefore, $\partial \pi_r/\partial p_r = 0$ gives:

$$p_r^*(p_o) = \frac{1}{2}[s + (1 - \beta)v + p_o] \quad (\text{A5})$$

The profit of the online retailer (π_o) is $\pi_o = p_o d_o$; thus, we have $\partial \pi_o/\partial p_o = [-s + t - (1 - \beta)v + p_r - 2p_o]/t$. Therefore, $\partial \pi_o/\partial p_o = 0$ gives:

$$p_o^*(p_r) = \frac{1}{2}[-s + t - (1 - \beta)v + p_r] \quad (\text{A6})$$

We can prove that $\partial^2 \pi_r/\partial p_r^2 < 0$ and $\partial^2 \pi_o/\partial p_o^2 < 0$. Simultaneously solving Equations (A5) and (A6), we can find that $p_r^* = [s + t + (1 - \beta)v]/3$, and $p_o^* = [-s + 2t - (1 - \beta)v]/3$.

To ensure coverage of the whole market (i.e., $U_r^F = U_o^F > 0$), $\beta > (2s + 2t - v)/2v$. At the same time, we need to make sure that the online retailer will not exit the market (i.e., $p_o^* > 0$); therefore, $\beta > (v + s - 2t)/v$. In summary, the value range of $\beta > \min((2s + 2t - v)/2v, (v + s - 2t)/v)$.

With Equation (A4)-Equation (A6), the demands for these two retailers are:

$$\begin{cases} d_r = [s + t + (1 - \beta)v]/3t \\ d_o = [2t - s - (1 - \beta)v]/3t \end{cases} \quad (\text{A7})$$

With Equation (A5)-Equation (A7), the profits of these two retailers are:

$$\begin{cases} \pi_r = [s + t + (1 - \beta)v]^2/9t \\ \pi_o = [-s + 2t - (1 - \beta)v]^2/9t \end{cases} \quad (\text{A8})$$

Compare π_r^F with π_r . When $\theta \in (\gamma_1^F, \gamma_2^F)$, π_r^F is less than π_r . Concurrently, when $\theta \in (0, \gamma_1^F) \cup (\gamma_2^F, 1)$, π_r^F may be larger than π_r . According to the value range of θ in Proposition 2, the final result is:

$$\begin{aligned} \pi_r^F &< \pi_r, & \text{if } \theta \in (\max(0, \theta_1^F, \gamma_1^F), \\ &\quad \min(\frac{2t - v}{2t - v + s}, \theta_2^F, \gamma_2^F)) \\ \pi_r^F &> \pi_r, & \text{if } \theta \in (\min\{\max(0, \theta_1^F), \gamma_1^F\}, \gamma_1^F) \\ &\quad \cup (\gamma_2^F, \max\{\gamma_2^F, \min(\frac{2t - v}{2t - v + s}, \theta_2^F)\}) \end{aligned}$$

Similarly, π_o^F and π_o can be compared.

D. THE PROOF OF PROPOSITION 6

Compare π_r^R with π_r^F , as shown at the top of the next page.

When $\theta \in (\gamma_1^R, (v + t + (\sqrt{1 + \delta} - 1)P_r)/(v + t - s))$, $\Delta \pi_r^R > 0$. Since $\theta \in (0, 1)$ and $(v + t + (\sqrt{1 + \delta} - 1)P_r)/(v + t - s) > 1$, we find that when $\theta \in (\max(0, \gamma_1^R), 1)$, $\Delta \pi_r^R > 0$. Moreover, when $\theta \in (0, \max(0, \gamma_1^R))$, $\Delta \pi_r^R < 0$. Compare π_o^R with π_o^F .

$$\begin{aligned} \Delta \pi_o^R &= \pi_o^R - \pi_o^F \\ &= -2P_r \delta((2t - v)(1 - \theta) - \theta s - \delta P_r/2)/9t(1 - \theta)\theta. \end{aligned}$$

$\Delta \pi_o^R$ is obviously less than 0. Considering the value range of θ in Proposition 4, the final comparison results are:

$$\begin{aligned} \pi_r^R &< \pi_r^F, & \text{if } \theta \in (\min(\max(0, \theta_1^R), \gamma_1^R), \gamma_1^R) \\ \pi_r^R &> \pi_r^F, & \text{if } \theta \in (\max(0, \theta_1^R, \gamma_1^R), \min(\frac{2t - v - \delta P_1}{2t - v + s}, \theta_2^R)) \\ \pi_o^R &< \pi_o^F, & \text{if } \theta \in (\max(0, \theta_1^R), \min(\frac{2t - v - \delta P_1}{2t - v + s}, \theta_2^R)) \end{aligned}$$

E. THE PROOF OF PROPOSITION 7

Compare π_o^O with π_o^F . When $\theta \in (0, \max(0, \sigma_1^O))$, π_o^O is less than π_o^F . Concurrently, when $\theta \in (\max(0, \sigma_1^O), 1)$, π_o^O is greater than π_o^F . Comparing π_r^O with π_r^F reveals that π_r^O is less than π_r^F . Considering the value range of θ in Proposition 6, the final comparison results are:

$$\begin{aligned} \pi_r^O &< \pi_r^F, & \text{if } \theta \in (\min(\max(0, \theta_1^O), \sigma_1^O), \sigma_1^O) \\ \pi_o^O &> \pi_o^F, & \text{if } \theta \in (\max(0, \theta_1^O, \sigma_1^O), \\ &\quad \min(\frac{2t - v}{2t - v + s - \delta P_2}, \frac{t + v}{t + v - s + \delta P_2}, \theta_2^O))) \end{aligned}$$

$$\frac{\partial \pi_r^R}{\partial \delta} = \frac{(\delta P_r + (s - t - v)\theta + t + v)((\delta + 2)P_r - (s - t - v)\theta - t - v)}{9t(1 - \theta)(1 + \delta)^2} \quad (\text{A3-2})$$

$$\Delta \pi_r^R = \pi_r^R - \pi_r^F = \frac{(2(t + v - P_r)(v + t - s)\theta - (v + t - s)^2\theta^2 - t^2 - 2(v - P_r)t + \delta P_r^2 - v^2 + 2P_r v) \delta}{9t(1 - \theta)(1 + \delta)}$$

$$\begin{aligned} \pi_r^O < \pi_r^F, \quad & \text{if } \theta \in (\max(0, \theta_1^O), \\ & \min(\frac{2t - v}{2t - v + s - \delta P_2}, \frac{t + v}{t + v - s + \delta P_2}, \theta_2^O)) \end{aligned}$$

F. THE PROOF OF PROPOSITION 8

Based on the previous analysis, we can obtain the value range of θ as (θ_L, θ_H) .

- Given that the online retailer implements the RPM, we compare the profit of the BM retailer that implements the RPM (π_r^{RO}) with the profit of the BM retailer that does not implement the RPM (π_r^O). When $\theta \in (\max(\theta_L, \theta_1^{RO}), \min(\theta_2^{RO}, \theta_H))$, π_r^{RO} is greater than π_r^O , and the BM retailer will implement the RPM.
- Given that the online retailer does not implement the RPM, we compare the profit of the BM retailer that implements the RPM (π_r^R) with the profit of the BM retailer that does not implement the RPM (π_r^F). We have made the same comparison in the exploration of Strategy R. Referring to the result in Proposition 5, we find that when $\theta \in (\max(\theta_L, \gamma_1^R), \theta_H)$, π_r^R is greater than π_r^F , and the BM retailer will implement the RPM.
- Given that the BM retailer implements the RPM, we compare the profit of the online retailer that implements the RPM (π_o^{RO}) with that of the online retailer that does not implement the RPM (π_o^R). When $\theta \in (\max(\theta_L, \theta_1^{ROR}), \min(\theta_2^{ROR}, \theta_H))$, π_o^{RO} is greater than π_o^R , and the online retailer will implement the RPM.
- Given that the BM retailer does not implement the RPM, we compare the profit of the online retailer that implements the RPM (π_o^R) with the profit of the online retailer that does not implement the RPM (π_o^F). We have made the same comparison in the exploration of Strategy O. Referring to the result in Proposition 7, we find that when $\theta \in (\max(\theta_L, \sigma_1^O), \theta_H)$, π_o^R is greater than π_o^F , and the online retailer will implement the RPM.

Now, we seek the game equilibrium between the BM retailer and the online retailer. Take Strategy RO as an example. When Strategy RO is the equilibrium strategy, we should ensure that both $\pi_r^{RO} > \pi_r^O$ and $\pi_o^{RO} > \pi_o^R$ are established simultaneously, and at most one of the two conditions $\pi_r^F > \pi_r^R$ and $\pi_o^F > \pi_o^R$ holds. Thus, the RPM will be implemented by both retailers when $\theta \in \mathbb{C}_{D_1 \cap D_3} (D_1 \cap D_3 \cap \mathbb{C}_U D_2 \cap \mathbb{C}_U D_4)$, and Strategy RO is the equilibrium strategy for implementing the RPM. The same method can be easily adapted to obtain Proposition 8.

REFERENCES

- J. Shin, "How does free riding on customer service affect competition," *Marketing Sci.*, vol. 26, no. 4, pp. 488–503, 2007.
- J. Chen and B. Chen, "When should the offline retailer implement price matching?" *Eur. J. Oper. Res.*, vol. 277, no. 3, pp. 996–1009, Sep. 2019.
- T.-P. Liang and J.-S. Huang, "An empirical study on consumer acceptance of products in electronic markets: A transaction cost model," *Decis. Support Syst.*, vol. 24, no. 1, pp. 29–43, Nov. 1998.
- S. van Baal and C. Dach, "Free riding and customer retention across retailers' channels," *J. Interact. Marketing*, vol. 19, no. 2, pp. 75–85, Jan. 2005.
- A. Zimmerman, "Showdown over 'showrooms,'" *Wall Street J.-Eastern Ed.*, vol. 259, no. 17, pp. B1–B5, 2012.
- F. Bernstein, J.-S. Song, and X. Zheng, "Free riding in a multi-channel supply chain," *Nav. Res. Logistics*, vol. 56, no. 8, pp. 745–765, Dec. 2009.
- A. Mehra, S. Kumar, and J. S. Raju, "Competitive strategies for brick-and-mortar stores to counter 'showrooming,'" *Manage. Sci.*, vol. 64, no. 7, pp. 3076–3090, Jul. 2018.
- K. D. Antia, M. Bergen, and S. Dutta, "Competing with gray markets," *Mit Sloan Manage. Rev.*, vol. 46, no. 1, pp. 63–69, 2004.
- S. U. Kucuk and R. C. Maddux, "The role of the internet on free-riding: An exploratory study of the wallpaper industry," *J. Retailing Consum. Services*, vol. 17, no. 4, pp. 313–320, Jul. 2010.
- Y. Aviv and A. Pazgal, "Optimal pricing of seasonal products in the presence of forward-looking consumers," *Manuf. Service Oper. Manage.*, vol. 10, no. 3, pp. 339–359, Jul. 2008.
- G. P. Cachon and R. Swinney, "Purchasing, pricing, and quick response in the presence of strategic consumers," *Manage. Sci.*, vol. 55, no. 3, pp. 497–511, Mar. 2009.
- X. Su and F. Zhang, "Strategic customer behavior, commitment, and supply chain performance," *Manage. Sci.*, vol. 54, no. 10, pp. 1759–1773, 2008.
- Q. Liu. (May 10, 2013). *Showrooming and Price-Matching Guarantee*. [Online]. Available: <https://ssrn.com/abstract=1862181>
- D. Kahneman and A. Tversky, "On the reality of cognitive illusions," *Psychol. Rev.*, vol. 103, no. 3, pp. 582–591, 1996.
- R. H. Thaler, "Transaction utility theory," *Adv. Consum. Res.*, vol. 10, no. 4, pp. 229–232, 1983.
- R. Tojo and N. Matsubayashi, "Competition between online and physical stores: The implications of providing product information by pure-play e-tailer," *Marketing Sci.*, vol. 26, no. 4, pp. 488–503, 2011.
- Y.-W. Zhou, J. Guo, and W. Zhou, "Pricing/service strategies for a dual-channel supply chain with free riding and service-cost sharing," *Int. J. Prod. Econ.*, vol. 196, pp. 198–210, Feb. 2018.
- L. G. Telser, "Why should manufacturers want fair trade?" *J. Law Econ.*, vol. 3, pp. 86–105, Oct. 1960.
- R. A. Mittelstaedt, "Sasquatch, the abominable snowman, free riders and other elusive beings," *J. Macromarketing*, vol. 6, no. 2, pp. 25–35, Dec. 1986.
- R. B. Singley and M. R. Williams, "Free riding in retail stores: An investigation of its perceived prevalence and costs," *J. Marketing Theory Pract.*, vol. 3, no. 2, pp. 64–74, Apr. 1995.
- F. Bernstein, J. Song, and X. Zheng, "'Bricks-and-mortar' vs. 'clicks-and-mortar': An equilibrium analysis," *Eur. J. Oper. Res.*, vol. 187, no. 3, pp. 90–671, 2008.
- H. Pun, J. Chen, and W. Li, "Channel strategy for manufacturers in the presence of service freeriders," *Eur. J. Oper. Res.*, vol. 287, no. 2, pp. 460–479, Dec. 2020.
- Y. Levin, J. McGill, and M. Nedjalk, "Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers," *Prod. Oper. Manage.*, vol. 19, no. 1, pp. 40–60, Jan. 2010.

- [24] X. Su and F. Zhang, "On the value of commitment and availability guarantees when selling to strategic consumers," *Manage. Sci.*, vol. 55, no. 5, pp. 713–726, May 2009.
- [25] R. Yin, Y. Aviv, A. Pazgal, and C. S. Tang, "Optimal markdown pricing: Implications of inventory display formats in the presence of strategic customers," *Manage. Sci.*, vol. 55, no. 8, pp. 1391–1408, Aug. 2009.
- [26] T. Aydinliyim, M. S. Pangburn, and E. Rabinovich, "Inventory disclosure in online retailing," *Eur. J. Oper. Res.*, vol. 261, no. 1, pp. 195–204, Aug. 2017.
- [27] J. Cai, Q. Zhou, J. Sun, X. Hu, C. Feng, and X. Li, "Competition model and coordination mechanism considering strategic customer behavior under vendor-managed inventory," *Int. Trans. Oper. Res.*, vol. 28, no. 5, pp. 2782–2809, Sep. 2021.
- [28] M. K. Perry and R. H. Porter, "Can resale price maintenance and franchise fees correct sub-optimal levels of retail service?" *Int. J. Ind. Org.*, vol. 8, no. 1, pp. 115–141, Jan. 1990.
- [29] C. Li and F. Zhang, "Advance demand information, price discrimination, and preorder strategies," *Manuf. Service Oper. Manage.*, vol. 15, no. 1, pp. 57–71, Feb. 2013.
- [30] I. P. L. Png, "Most-favored-customer protection versus price discrimination over time," *J. Political Economy*, vol. 99, no. 5, pp. 1010–1028, Oct. 1991.
- [31] G. Lai, L. G. Debo, and K. Sycara, "Buy now and match later: Impact of posterior price matching on profit with strategic consumers," *Manuf. Service Oper. Manage.*, vol. 12, no. 1, pp. 33–55, Jan. 2010.
- [32] D. Xing and T. Liu, "Sales effort free riding and coordination with price match and channel rebate," *Eur. J. Oper. Res.*, vol. 219, no. 2, pp. 264–271, Jun. 2012.
- [33] C. Liu, B. Dan, M. Zhou, and X. Zhang, "Supply chain coordination under information asymmetry considering customer's service free-riding behavior," in *Proc. Int. Conf. Logistics, Informat. Service Sci. (LISS)*, Jul. 2016, pp. 1–6.
- [34] E. A. Greenleaf, "The impact of reference price effects on the profitability of price promotions," *Marketing Sci.*, vol. 14, no. 1, pp. 82–104, Feb. 1995.
- [35] G. Kalyanaram and R. S. Winer, "Empirical generalizations from reference price research," *Marketing Sci.*, vol. 14, no. 3, pp. G161–G169, Aug. 1995.
- [36] X. Chen, P. Hu, and Z. Hu, "Efficient algorithms for the dynamic pricing problem with reference price effect," *Manage. Sci.*, vol. 63, no. 12, pp. 4389–4408, Dec. 2017.
- [37] C.-Y. Dye and C.-T. Yang, "Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects," *Omega*, vol. 62, pp. 52–67, Jul. 2016.
- [38] P. Courty and J. Nasiry, "Loss aversion and the uniform pricing puzzle for media and entertainment products," *Econ. Theory*, vol. 66, no. 1, pp. 105–140, Jul. 2018.
- [39] R. Wang, "When prospect theory meets consumer choice models: Assortment and pricing management with reference prices," *Manuf. Service Oper. Manage.*, vol. 20, no. 3, pp. 583–600, Jul. 2018.
- [40] B. Crettez, N. Hayek, and G. Zaccour, "Existence and characterization of optimal dynamic pricing strategies with reference-price effects," *Central Eur. J. Oper. Res.*, vol. 28, no. 2, pp. 441–459, Jun. 2020.
- [41] Y. Li, "Multiperiod dynamic pricing and inventory control decisions for an omnichannel BOPS retailer with reference price effects," *Math. Problems Eng.*, vol. 2021, no. 12, pp. 1–21, Feb. 2021.
- [42] D. M. Hardesty and T. A. Suter, "E-tail and retail reference price effects," *J. Product Brand Manage.*, vol. 14, no. 2, pp. 129–136, Mar. 2005.
- [43] Y. Zheng, "The effect of reference price on customers' decisions in online shopping," *Int. J. Psychol. Stud.*, vol. 11, no. 4, p. 26, Sep. 2019.
- [44] H. Hotelling, "Stability in competition," in *The Collected Economics Articles of Harold Hotelling*, A. C. Darnell, Ed. New York, NY, USA: Springer, 1990, pp. 50–63.
- [45] Y. Liu and Z. J. Zhang, "The benefits of personalized pricing in a channel," *Marketing Sci.*, vol. 25, no. 1, pp. 97–105, 2006.
- [46] Y. Liu and R. K. Tyagi, "The benefits of competitive upward channel decentralization," *Manage. Sci.*, vol. 57, no. 4, pp. 741–751, Apr. 2011.
- [47] J. Chen and R. Grewal, "Competing in a supply chain via full-refund and no-refund customer returns policies," *Int. J. Prod. Econ.*, vol. 146, no. 1, pp. 246–258, Nov. 2013.
- [48] X. Yang, G. Cai, Y.-J. Chen, and S.-J.-S. Yang, "Competitive retailer strategies for new market research, entry and positioning decisions," *J. Retailing*, vol. 93, no. 2, pp. 172–186, Jun. 2017.
- [49] S. Shum, S. Tong, and T. Xiao, "On the impact of uncertain cost reduction when selling to strategic customers," *Manage. Sci.*, vol. 63, no. 3, pp. 843–860, Mar. 2017.
- [50] R. H. Thaler, "Mental accounting matters," *J. Behav. Decis. Making*, vol. 12, no. 3, pp. 183–206, Sep. 1999.
- [51] R. H. Thaler, "Commentary—Mental accounting and consumer choice: Anatomy of a failure," *Marketing Sci.*, vol. 27, no. 1, pp. 12–14, Jan. 2008.
- [52] R. H. Thaler, "Misbehaving: The making of behavioral economics," *J. Real Estate Literature*, vol. 25, no. 6, pp. 570–587, 2015.
- [53] Ö. Özer and Y. Zheng, "Markdown or everyday low price? The role of behavioral motives," *Manage. Sci.*, vol. 62, no. 2, pp. 326–346, Feb. 2016.



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