ECE-GY 6143 INTRO TO MACHINE LEARNING

HOMEWORK-5

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Question 1:

1/1/21 HW5

1) Let $D \in \{1,2,3\}$ be the event 'Car behind this door'

Let $H \in \{1,2,3\}$ be the event 'Door opened by host'

Let $I \in \{1,2,3\}$ be the event 'Initial chosen door'

Opening a door behind which the car is present is equally likely.

$$P(D=1) = P(D=2) = P(D=3) = \frac{1}{3}$$

Let us assume we choose door 1 and the host opened door 3.

The events D & I are independent.

$$P(H=3 \mid D=1, I=1) = \frac{1}{2}$$

$$P(H=3 \mid D=2, I=1) = 1$$

$$P(H=3 \mid D=3, I=1) = 0$$

Applying Baye's Theorem,

$$P(D=1 \mid H=3, I=1) = \frac{P(H=3 \mid D=1, I=1)\, P(D=1 \mid I=1)}{\sum_{k=1}^{3} P(H=3 \mid D=k, I=1)\, P(D=k \mid I=1)}$$

$$= \frac{P(H=3 \mid D=1, I=1)\, P(D=1)}{\sum_{k=1}^{3} P(H=3 \mid D=k, I=1)\, P(D=k)}$$

$$= \frac{P(H=3 \mid D=1, I=1)}{\sum_{k=1}^{3} P(H=3 \mid D=k, I=1)}$$

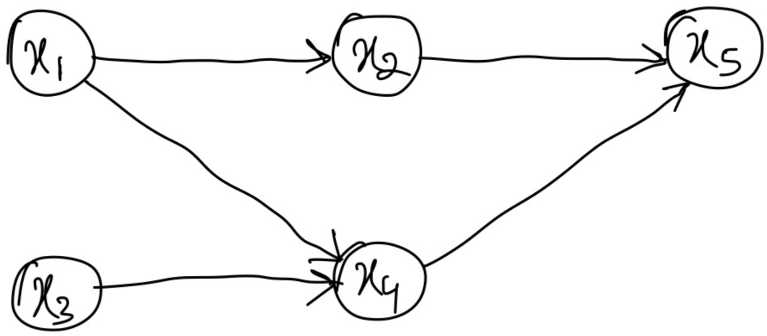
$$P(D=1 \mid H=3, I=1) = \frac{1/2}{1/2+1+0} = \frac{1}{3}$$

$$P(D=2 \mid H=3, I=1) = 1 - P(D=1 \mid H=3, I=1) = \frac{2}{3}$$

Therefore, the chances are higher if door 2 is chosen to maximize the winning.

Question 2:

2)



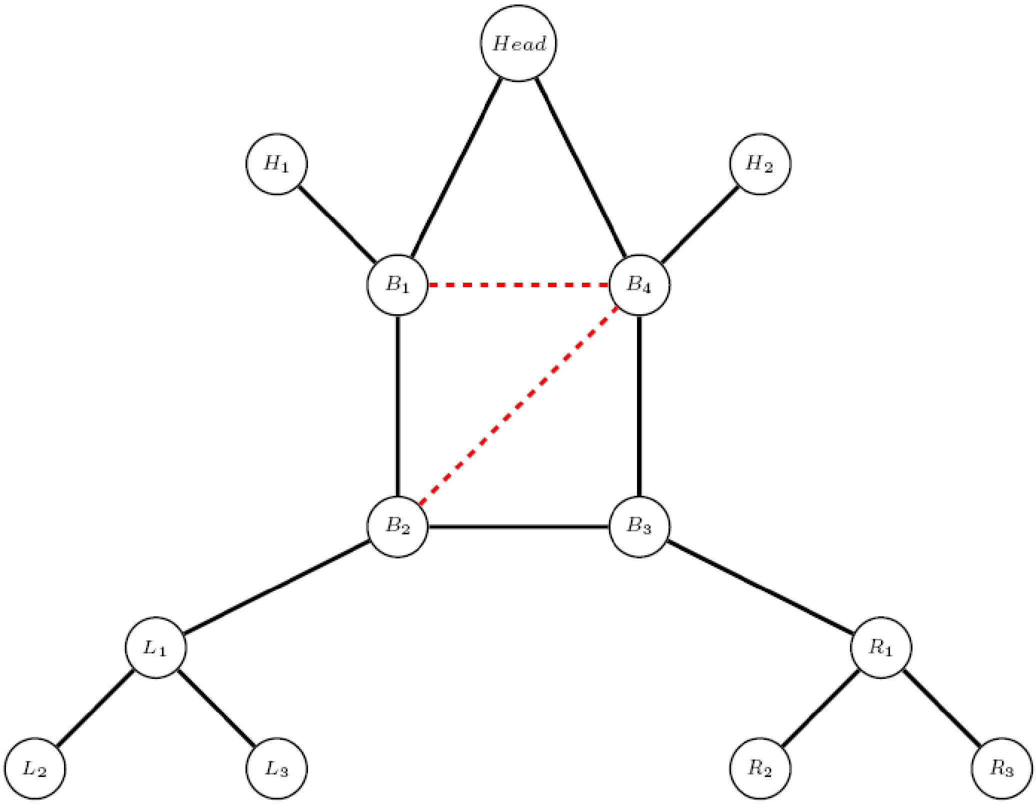
$$p(x_1, x_2, \dots, x_n) = \prod_{i=1}^{5} p(x_1 \mid \text{parents})$$

$$p(x_1, x_2, \dots, x_n) = p(x_1)\,p(x_2 \mid x_1)\,p(x_3)\,p(x_4 \mid x_1, x_3)\,p(x_5 \mid x_2, x_4)$$

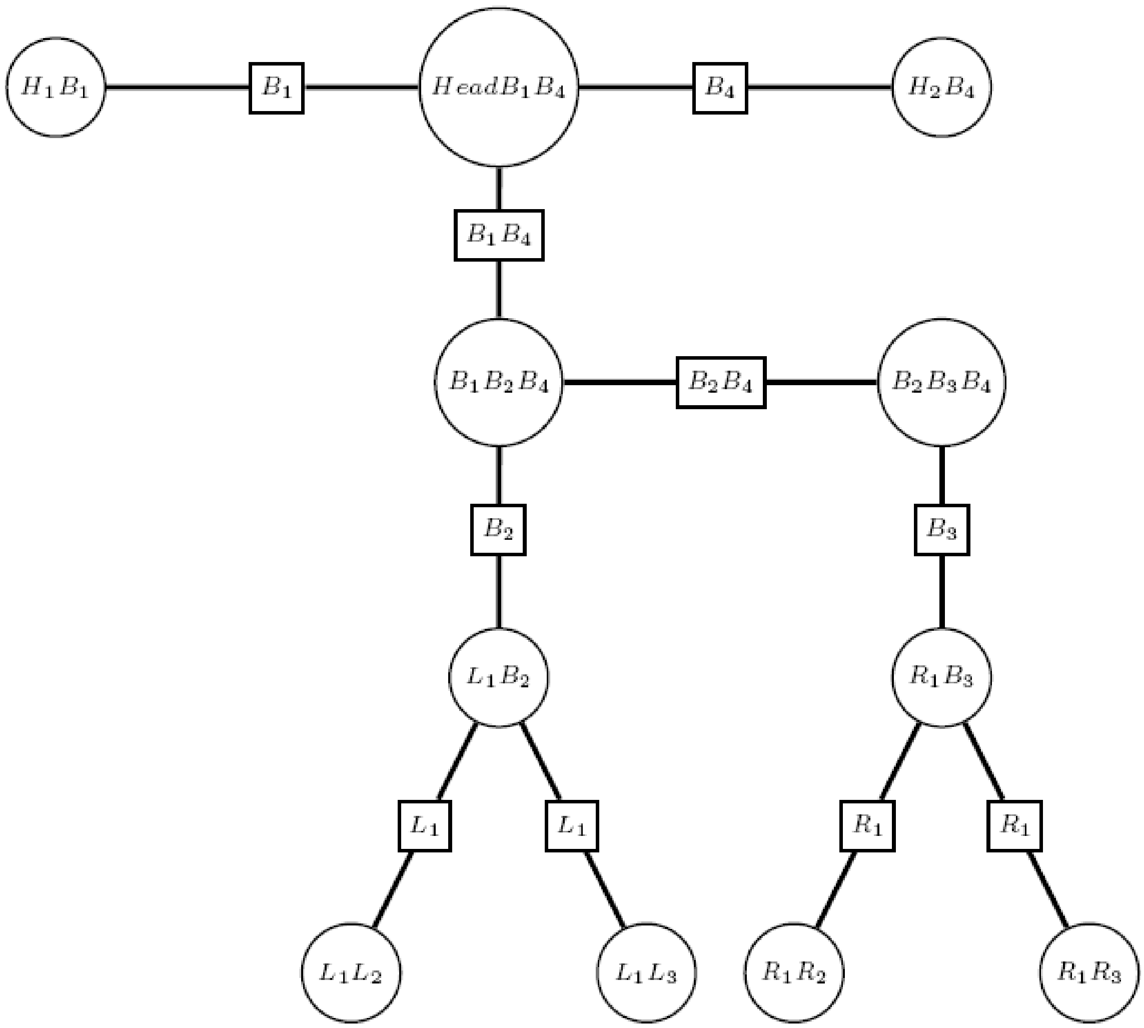
1. False
2. False
3. True
4. False
5. True
6. False
7. True
8. True
9. False
10. False

Question 3:

Graph after Moralization and Triangulation ->



Constructed Junction Tree ->



Question 4:

The Junction Tree Algorithm code is given below ->

```
clc;
clear variables;

clique_potential_initial = {[0.1, 0.7; 0.8, 0.3];
                            [0.5, 0.1; 0.1, 0.5];
                            [0.1, 0.5; 0.5, 0.1];
                            [0.9, 0.3; 0.1, 0.3]};

marginals = clique_potential_initial;
sepSize = size(marginals,1);
separators = ones(sepSize-1,2);

%Forward
for i=1:sepSize-1
    separators(i,:) = sum(marginals{i});
    marginals{i+1} = marginals{i+1}.*(separators(i,:)'*[1,1]);
end

%Backward
for i=1:sepSize-1
    separatorsPrevious = separators(sepSize-i,:);
    separators(sepSize-i,:) = sum(marginals{sepSize-i+1},2)';
    marginals{sepSize-i} = marginals{sepSize-i}.*([1;1]*(separators(sepSize-
i,:)./separatorsPrevious));
end

%Normalize
for i=1:sepSize
    marginals{i} = marginals{i}/sum(sum(marginals{i}));
end

disp('Marginals:');
for i=1:sepSize
    val = marginals{i};
    fprintf('\t\t X = 0   | X = 1\n');
    fprintf('X = 0 -> %.4f | %.4f\n',val(1,:));
    fprintf('X = 1 -> %.4f | %.4f\n\n',val(2,:));
end
```

The joint probability distribution over cliques is as follows ->

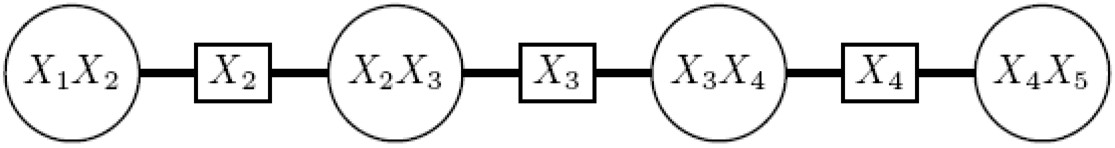
$P(x_1, x_2)$	$x_2 = 0$	$x_2 = 1$	$P(x_1)$
$x_1 = 0$	0.0405	0.4451	0.4856
$x_1 = 1$	0.3237	0.1908	0.5145
$P(x_2)$	0.3642	0.6359	1

$P(x_2, x_3)$	$x_3 = 0$	$x_3 = 1$	$P(x_2)$
$x_2 = 0$	0.2601	0.1040	0.3641
$x_3 = 1$	0.0578	0.5780	0.6358
$P(x_3)$	0.3179	0.6820	1

$P(x_3, x_4)$	$x_4 = 0$	$x_4 = 1$	$P(x_3)$
$x_3 = 0$	0.1192	0.1987	0.3179
$x_3 = 1$	0.6395	0.0426	0.6821
$P(x_4)$	0.7587	0.2413	1

$\mathbf{P}(x_4, x_5)$	$x_5 = 0$	$x_5 = 1$	$\mathbf{P}(x_4)$
$x_4 = 0$	0.5690	0.1897	0.7587
$x_4 = 1$	0.0603	0.1810	0.2413
$\mathbf{P}(x_5)$	0.6293	0.3707	1

Constructed Junction Tree ->



Question 5:

The ArgMax Junction Tree Algorithm code is given below ->

```
clc;
clear variables;

TransitionProbabilities = [0.8, 0.2; 0.2, 0.8];
EmissionProbabilities = [0.4, 0.1, 0.3, 0.2; 0.1, 0.4, 0.2, 0.3];
ObservedStates = [1, 4, 2, 2, 3];
InitialProbabilities = [1; 0];

tSize = size(TransitionProbabilities, 1);
nSize = size(ObservedStates, 2);
psi = zeros(tSize, tSize, nSize);
phi = zeros(tSize, nSize);
phi(:, 1) = InitialProbabilities;

% Forward
for i = 2:nSize
    k = ObservedStates(1, i);
    psi(:, :, i) = diag(phi(:, i - 1)) *TransitionProbabilities *
diag(EmissionProbabilities(:,k));
    phi(:, i) = max(psi(:, :, i));
end

% Backward
for i = nSize-1:-1:1
    phi_new = max(psi(:, :, i + 1), [], 2);
    psi(:, :, i) = psi(:, :, i) * diag(phi_new ./ phi(:, i));
    phi(:, i) = phi_new;
end

[~, HiddenStates] = max(phi);

disp('Emotional States of Mario:');
for i=1:size(HiddenStates,2)
    fprintf('Day %d: ', i);
    if(HiddenStates(i)==1)
        disp('Happy');
    else
        disp('Angry');
    end
end
```

SuperMario has the following emotional states on 5 days ->

Day	State
1	Happy
2	Angry
3	Angry
4	Angry
5	Angry