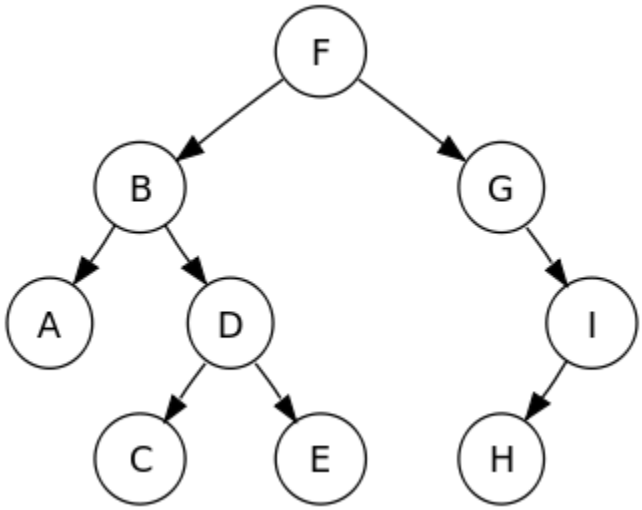
Tree traversal

	PAPER 1 - ⇑ Fundamentals of algorithms ⇑	
← Graph traversal	Tree traversal	

A tree is a special case of a graph, and therefore the graph traversal algorithms of the previous chapter also apply to trees. A graph traversal can start at any node, but in the case of a tree the traversal always starts at the root node. Binary trees can be traversed in three additional ways. The following tree will be used as the recurring example.



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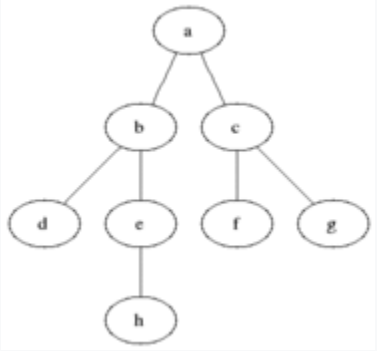
Breadth-first[edit]

Traversing a tree in breadth-first order means that after visiting a node X, all of X's children are visited, then all of X's 'grand-children' (i.e. the children's children), then all of X's 'great-grand-children', etc. In other words, the tree is traversed by sweeping through the breadth of a level before visiting the next level down, as shown in this animation:



Animated example of a breadth-first traversal

The children of a node may be visited in any order, but remember the algorithm uses a queue, so if a node X is enqueued (grey in the animation) before node Y, then X's children will be visited (black in the animation) before Y's children. For the example tree at the start of this chapter, two possible breadth-first traversals are F B G A D I C E H and F G B I D A H E C. In the second traversal, G is visited before B, so I is visited before A and D.

Exercise: Breadth First Traversal

List two other possible breadth-first traversals of the same tree.

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Answer:

Since F, B and D each have two children, there are in total 2*2*2=8 possible breadth-first traversals:

1. F B G A D I C E H
2. F B G A D I E C H
3. F B G D A I C E H
4. F B G D A I E C H
5. F G B I A D H C E
6. F G B I A D H E C
7. F G B I D A H C E
8. F G B I D A H E C

The first and last traversals were already given above, so you could have listed any other two.

Depth-first[edit]

As the name implies, a depth-first traversal will go down one branch of the tree as far as possible, i.e. until it stops at a leaf, before trying any other branch. The various branches starting from the same parent may be explored in any order. For the example tree, two possible depth-first traversals are F B A D C E G I H and F G I H B D E C A.

Exercise: Breadth First Traversal

List two other possible depth-first traversals of the same tree.

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Answer:

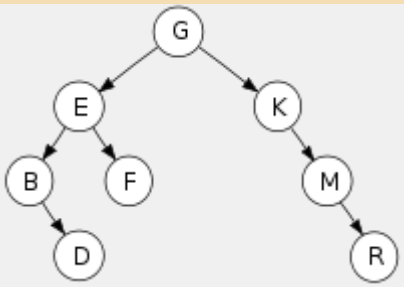
Since F, B and D each have two children, there are in total 2*2*2=8 possible depth-first traversals:

1. F B A D C E G I H
2. F B A D E C G I H
3. F B D C E A G I H
4. F B D E C A G I H
5. F G I H B A D C E
6. F G I H B A D E C
7. F G I H B D C E A
8. F G I H B D E C A

The first and last traversals were already given above, so you could have listed any other two.

Exercise: Depth and Breadth First Traversal

List *one* breadth-first and *one* depth-first traversal for this tree:



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Answer:

The possible depth-first traversals are

- G E B D F K M R
- G E F B D K M R
- G K M R E B D F
- G K M R E F B D

The possible breadth-first traversals are

- G E K B F M D R
- G E K F B M D R
- G K E M B F R D
- G K E M F B R D

- Depth First traversal generally uses a Stack
- Breadth First generally uses a Queue

Pre-order[edit]

Binary trees are usually traversed from left to right, but right-to-left traversal is also possible and might appear in the exam questions. We will therefore make the direction always clear in the rest of this chapter.

If each node is visited *before* both of its subtrees, then it's called a *pre*-order traversal. The algorithm for left-to-right pre-order traversal is:

1. Visit the root node (generally output it)
2. Do a pre-order traversal of the left subtree
3. Do a pre-order traversal of the right subtree

which can be implemented as follows, using the tree data structure defined in the previous unit:

```
sub PreOrder(TreeNode)
   Output(TreeNode.value)
   If LeftPointer(TreeNode) != NULL Then
      PreOrder(TreeNode.LeftNode)
   If RightPointer(TreeNode) != NULL Then
      PreOrder(TreeNode.RightNode)
end sub
```

Since the algorithm completely determines the order in which nodes are visited, there is only one possible left-to-right pre-order traversal for each binary tree. For our example tree, which is a binary tree, it's F B A D C E G I H.

Because a pre-order traversal always goes down one branch (left or right) before moving on to the other branch, a pre-order traversal is always one of the possible depth-first traversals.

Post-order[edit]

If each node is visited *after* its subtrees, then it's a *post*-order traversal. The algorithm for left-to-right post-order traversal is:

1. Do a post-order traversal of the left subtree
2. Do a post-order traversal of the right subtree
3. Visit the root node (generally output this)

which can be implemented as:

```
sub PostOrder(TreeNode)
   If LeftPointer(TreeNode) != NULL Then
      PostOrder(TreeNode.LeftNode)
   If RightPointer(TreeNode) != NULL Then
      PostOrder(TreeNode.RightNode)
   Output(TreeNode.value)
```

```
end sub
```

There is only one left-to-right post-order traversal for each binary tree. For our example tree, it's A C E D B H I G F.

In-order[edit]

If each node is visited *between* visiting its left and right subtrees, then it's an *in*-order traversal. The algorithm for left-to-right in-order traversal is:

1. Do an in-order traversal of the left subtree
2. Visit root node (generally output this)
3. Do an in-order traversal of the right subtree

which can be implemented as:

```
sub InOrder(TreeNode)
   If LeftPointer(TreeNode) != NULL Then
      InOrder(TreeNode.LeftNode)
   Output(TreeNode.value)
   If RightPointer(TreeNode) != NULL Then
      InOrder(TreeNode.RightNode)
end sub
```

There is only one left-to-right in-order traversal for each binary tree. For our example tree, it's A B C D E F G H I. Note the nodes are visited in ascending order. That's no coincidence.

In binary search trees like our example tree, the values in the left subtree are smaller than the root and the values in the right subtree are larger than the root, so a left-to-right in-order traversal visits the nodes in ascending order.

Exercise: In-order Traversal

Is the statement "an in-order traversal always visits the nodes in ascending order" true or false?

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Answer:

It is false. An in-order traversal only visits the nodes in ascending order if it's a left-to-right traversal *and* the tree is a binary search

How would you change the algorithm above to visit the nodes of a binary search tree in descending order ?

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Answer:

A right-to-left in-order traversal would produce the desired order:

1. Do an in-order traversal of the right subtree
2. Visit root node (generally output this)
3. Do an in-order traversal of the left subtree

Traversal Tips[edit]

In the exam, you may be given some traversal pseudo-code and a binary tree, and asked to list the nodes in the order the code will visit them.

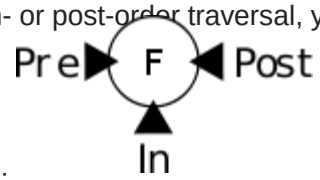
The first thing is to look carefully at the code and check:

- is the node visited before (pre-order), between (in-order) or after (post-order) visiting the subtrees?
- is the left subtree visited before the right subtree or the other way round?

For example, the following code does a left-to-right traversal and the comments show where the visit of the root might take place.

```
sub Traversal(TreeNode)
   'Output(TreeNode.value) REM Pre-Order
   If LeftPointer(TreeNode) != NULL Then
      Traversal(TreeNode.LeftNode)
   'Output(TreeNode.value) REM In-Order
   If RightPointer(TreeNode) != NULL Then
      Traversal(TreeNode.RightNode)
   'Output(TreeNode.value)  REM Post-Order
end sub
```

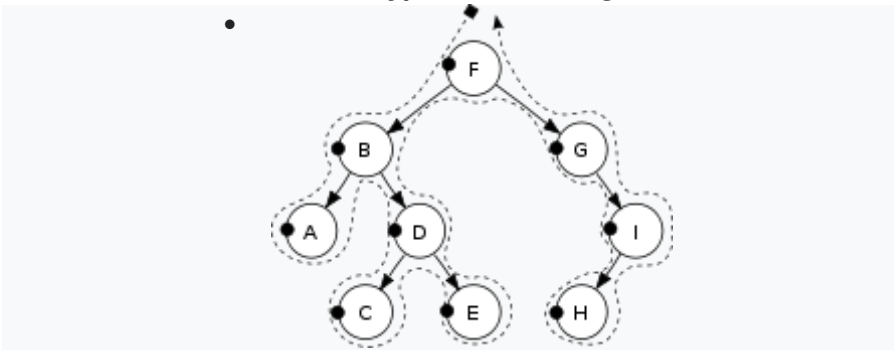
Let's assume it's left-to-right traversal. Once you know, from the position of the node visit, if it's a pre-, in- or post-order traversal, you can annotate each node of the binary tree as



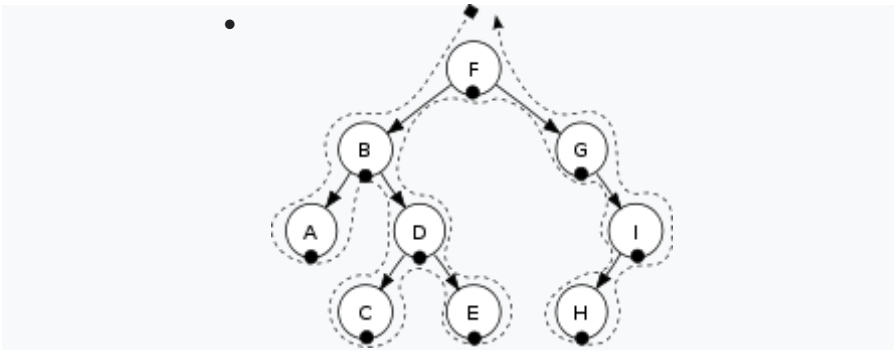
follows:

Type of Traversal	Position of Output code	Where to put your mark
Pre	Top	Left
In	Middle	Bottom
Post	Bottom	Right

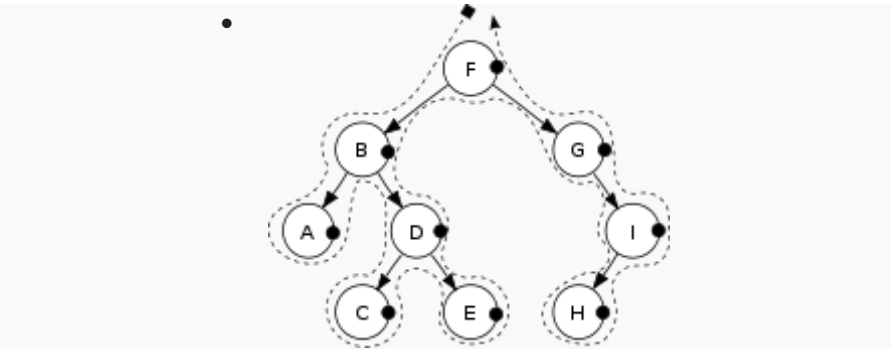
Finally, draw a line going anticlockwise around the tree, connecting the marks. Follow the line and write down each node as you meet a mark: that will be the order in which the code will visit the nodes. Here are the three possible left-to-right traversals:

- **The 3 different types of left-to-right traversal**
- 

Pre-order
FBADCEGIH

- 

In-order
ABCDEFGHI



Post-order
ACEDBHIGF

As you can see, following this tip you obtain the same answers as in the sections above.

If the traversal is right to left, you have to draw a clockwise line and swap the position of the pre-order or post-order mark.

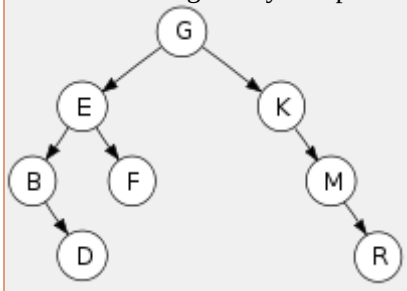
Type of Traversal	Position of Output code	Mark for left-to-right traversal	Mark for right-to-left traversal
Pre	Top	Left	Right
In	Middle	Bottom	Bottom
Post	Bottom	Right	Left

If the tree is a binary search tree and you're asked for an in-order traversal, you should have visited the nodes in ascending order (for left-to-right traversal) or descending order (for right-to-left traversal). If it's not a binary search tree, the in-order traversal won't visit the nodes in neither ascending nor descending order, but a pre-order or post-order traversal might, it will all depend where the nodes are placed in the tree.

Although in this chapter we have always made explicit whether it was a left-to-right or right-to-left traversal for clarity, the typical usage of the terms pre-order, in-order and post-order implies a left-to-right traversal, if nothing to the contrary is said.

Exercise: Binary Tree Traversal

For the following binary tree perform a pre-order, an in-order and a post-order traversal.



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Answer:

When nothing else is said, a left-to-right traversal is assumed.

- Pre-order traversal: GEBDFKMR
- In-order traversal: BDEFGKMR
- Post-order traversal: DBFERMKG

What traversal does the following code describe:

```
sub Traverse(TreeNode)
   If LeftPointer(TreeNode) != NULL Then
      Traverse(TreeNode.LeftNode)
   Output(TreeNode.value)
   If RightPointer(TreeNode) != NULL Then
      Traverse(TreeNode.RightNode)
end sub
```

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Answer:

An in-order traversal, because the node visit is in the centre of the code and the left subtree is traversed before the right subtree.

What does the following code do?

```
sub P(TreeNode)
   If RightPointer(TreeNode) != NULL Then
      P(TreeNode.RightNode)
   If LeftPointer(TreeNode) != NULL Then
      P(TreeNode.LeftNode)
   Output(TreeNode.value)
end sub
```

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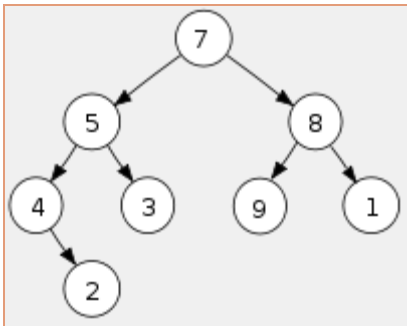
Answer:

It does a right-to-left post-order traversal, because it first visits the right subtree, then the left subtree and finally the node.

Using the following binary tree:



what would be the outputs for right-to-left:

- Pre-order traversal
- In-order traversal
- Post-order traversal

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Answer:

- right-to-left pre-order traversal: 7 8 1 9 5 3 4 2
- right-to-left in-order traversal: 1 8 9 7 3 5 2 4
- right-to-left post-order traversal: 1 9 8 3 2 4 5 7