Telecommunication Systems I

Report of the first project

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Th.1

$$\phi(t) = \begin{cases} \frac{1}{\sqrt{T}}, & \text{if } |t| \leq \frac{T}{2} \\ 0, & \text{else.} \end{cases}$$

1. For:

$$t + \frac{T}{2} < -\frac{T}{2} \Rightarrow t < -T$$

Then the convolution is equal to zero:

$$R\phi\phi(t) = 0$$

2. For:

$$-\frac{T}{2} \leq t + \frac{T}{2} \leq \frac{T}{2} \Rightarrow -T \leq t \leq 0$$

$$R\phi\phi(t) = \int_{-\frac{T}{2}}^{t+\frac{T}{2}} \phi(\tau) \cdot \phi(\tau + t) d\tau = \int_{-\frac{T}{2}}^{t+\frac{T}{2}} \frac{1}{\sqrt{T}} \cdot \frac{1}{\sqrt{T}} d\tau = \frac{1}{T} \cdot (t + T) = \boxed{\frac{t}{T} + 1}$$

3. For:

$$-\frac{T}{2} \leq t - \frac{T}{2} \leq \frac{T}{2} \Rightarrow 0 \leq t \leq T$$

$$R\phi\phi(t) = \int_{t-\frac{T}{2}}^{\frac{T}{2}} \phi(\tau) \cdot \phi(\tau + t) d\tau = \frac{1}{T} \cdot (-t + T) = \boxed{1 - \frac{t}{T}}$$

4. For:

$$t - \frac{T}{2} > \frac{T}{2} \Rightarrow t > T$$

The convolution is equal to zero:

$$R\phi\phi(t) = 0$$

$$R\phi\phi(t) = \begin{cases} 1 + \frac{t}{T}, & -T \leq t \leq 0 \\ 1 - \frac{t}{T}, & 0 \leq t \leq T \\ 0, & \text{else.} \end{cases}$$

Th.2

$$\phi(t-10) = \begin{cases} \frac{1}{\sqrt{T}}, & \text{if } -\frac{T}{2} + 10 \leq t \leq \frac{T}{2} + 10 \\ 0, & \text{else.} \end{cases}$$

1.

$$t + \frac{T}{2} + 10 < -\frac{T}{2} + 10 \Rightarrow t < -T$$

Then the convolution is equal to zero:

$$R\phi\phi(t) = 0$$

2.

$$-\frac{T}{2} + 10 \leq t + \frac{T}{2} + 10 \leq \frac{T}{2} + 10 \Rightarrow -T \leq t \leq 0$$

$$R\phi\phi(t) = \int_{-\frac{T}{2}+10}^{t+\frac{T}{2}+10} \phi(\tau) \cdot \phi(\tau + t) d\tau = \frac{1}{T} \cdot (t + T) = \boxed{\frac{t}{T} + 1}$$

3.

$$-\frac{T}{2} + 10 \leq t - \frac{T}{2} + 10 \leq \frac{T}{2} + 10 \Rightarrow 0 \leq t \leq T$$

$$R\phi\phi(t) = \int_{t-\frac{T}{2}+10}^{\frac{T}{2}+10} \phi(\tau) \cdot \phi(\tau + t) d\tau = \frac{1}{T} \cdot (-t + T) = \boxed{1 - \frac{t}{T}}$$

4.

$$t - \frac{T}{2} + 10 > \frac{T}{2} + 10 \Rightarrow t > T$$

The convolution is equal to zero:

$$R\phi\phi(t) = 0$$

$$R\phi\phi(t-10) = \begin{cases} 1 + \frac{t}{T}, & -T \leq t \leq 0 \\ 1 - \frac{t}{T}, & 0 \leq t \leq T \\ 0, & \text{else.} \end{cases}$$

Th.3

$$\phi(t) = \begin{cases} \frac{1}{\sqrt{T}}, & \text{if } 0 \leq t < \frac{T}{2}, \\ -\frac{1}{\sqrt{T}}, & \text{if } \frac{T}{2} \leq t \leq T, \\ 0, & \text{else.} \end{cases}$$

1.

$$T + t < 0 \Rightarrow t < -T$$

The convolution is equal to zero:

$$R\phi\phi(t) = 0$$

2.

$$t < T$$

The convolution is equal to zero:

$$R\phi\phi(t) = 0$$

3.

$$0 \leq T + t \leq \frac{T}{2} \Rightarrow -T \leq t \leq -\frac{T}{2}$$

$$R\phi\phi(t) = \int_0^{T+t} \phi(\tau) \cdot \phi(\tau + t) d\tau = \int_0^{T+t} \frac{1}{\sqrt{T}} \cdot -\frac{1}{\sqrt{T}} d\tau = -\frac{1}{T}(T+t) = \boxed{-1 - \frac{t}{T}}$$

4.

$$\frac{T}{2} \leq T + t \leq T \Rightarrow -\frac{T}{2} \leq t \leq 0$$

$$R\phi\phi(t) = \int_0^{T+t} \phi(\tau) \cdot \phi(\tau + t) d\tau$$

$$= \int_0^{\frac{T}{2}+t} \phi(\tau)\cdot\phi(\tau+t)d\tau + \int_{\frac{T}{2}+t}^{\frac{T}{2}} \phi(\tau)\cdot\phi(\tau+t)d\tau + \int_{\frac{T}{2}}^{T+t} \phi(\tau)\cdot\phi(\tau+t)d\tau$$

$$= \int_0^{\frac{T}{2}+t} \frac{1}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}d\tau + \int_{\frac{T}{2}+t}^{\frac{T}{2}} -\frac{1}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}d\tau + \int_{\frac{T}{2}}^{T+t} -\frac{1}{\sqrt{T}}\cdot -(\frac{1}{\sqrt{T}})d\tau$$

$$= \frac{1}{T}\cdot(\frac{T}{2}+t) + \frac{t}{T} + \frac{1}{T}\cdot(\frac{T}{2}+t)$$

$$= \frac{1}{T}\cdot(T+3t) = \boxed{1+\frac{3t}{T}}$$

5.

$$0 \leq t \leq \frac{T}{2}$$

$$R\phi\phi(t) = \int_t^{\frac{T}{2}} \phi(\tau)\cdot\phi(\tau+t)d\tau + \int_{\frac{T}{2}}^{\frac{T}{2}+t} \phi(\tau)\cdot\phi(\tau+t)d\tau + \int_{\frac{T}{2}+t}^{T} \phi(\tau)\cdot\phi(\tau+t)d\tau$$

$$= \int_t^{\frac{T}{2}} \frac{1}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}d\tau + \int_{\frac{T}{2}}^{\frac{T}{2}+t} -\frac{1}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}d\tau + \int_{\frac{T}{2}+t}^{T} -\frac{1}{\sqrt{T}}\cdot -(\frac{1}{\sqrt{T}})d\tau$$

$$= \frac{1}{T}\cdot(\frac{T}{2}-t) - \frac{t}{T} + \frac{1}{T}\cdot(\frac{T}{2}-t)$$

$$= \frac{1}{T}\cdot(T-3t) = \boxed{1-\frac{3t}{T}}$$

6.

$$\frac{T}{2} \leq t \leq T$$

$$R\phi\phi(t) = \int_t^{T} \phi(\tau)\cdot\phi(\tau+t)d\tau = \int_t^{T} -\frac{1}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}d\tau = -\frac{1}{T}(T-t) = \boxed{-1+\frac{t}{T}}$$

$$R\phi\phi(t) = \begin{cases} -1-\frac{t}{T}, & -T \leq t < -\frac{T}{2}, \\ 1+\frac{3t}{T}, & -\frac{T}{2} \leq t < 0, \\ 1-\frac{3t}{T}, & 0 \leq t < \frac{T}{2}, \\ -1+\frac{t}{T}, & \frac{T}{2} \leq t < T, \\ 0, & \text{else.} \end{cases}$$

By observing each ϕ function convoluted with its mirrored self, we can conclude that the more overlapped area they share with each other the higher the value of the self-similarity function $R\phi\phi$ with a maximum value of 1.

Transposing ϕ, and then finding its $R\phi\phi$ results in the same output function.

A.1 By increasing the roll-off value (a), the further away the signal travels from its center, the more the amplitude is attenuated. Also, the higher the roll-off value the larger the maximum amplitude at the center of the pulse.

A.2 Plotting all 3 of them on the same plot was simple enough with the use of a 2D Matrix "phi", and a simple for loop:

```
for i=1:3
    PHI_f(i,:) = fftshift(fft(phi(i,:),N));
    PHI_F(i,:) = PHI_f(i,:).*Ts;
    plot(f_axis,(abs(PHI_F)).^2);
    hold on;
end
```

A.3 a. For each roll-off value, the theoretical Bandwidth ($=\frac{1+a}{2T}$)is:

i. $BW = 500$, for ϕ with $a = 0$

ii. $BW = 750$, for ϕ with $a = 0.5$

iii. $BW = 1000$, for ϕ with $a = 1.0$

b. The most efficient pulse is the one with the shortest Bandwidth, in this case the pulse with the roll-off value of 0. It is worth noting that we define the cut-off point of each pulse.

For cut-off at $c_1 = \frac{T}{10^{-3}}$

i. $BW = 569.2$, for ϕ with $a = 0$

ii. $BW = 754.6$, for ϕ with $a = 0.5$

iii. $BW = 986.2$, for ϕ with $a = 1.0$

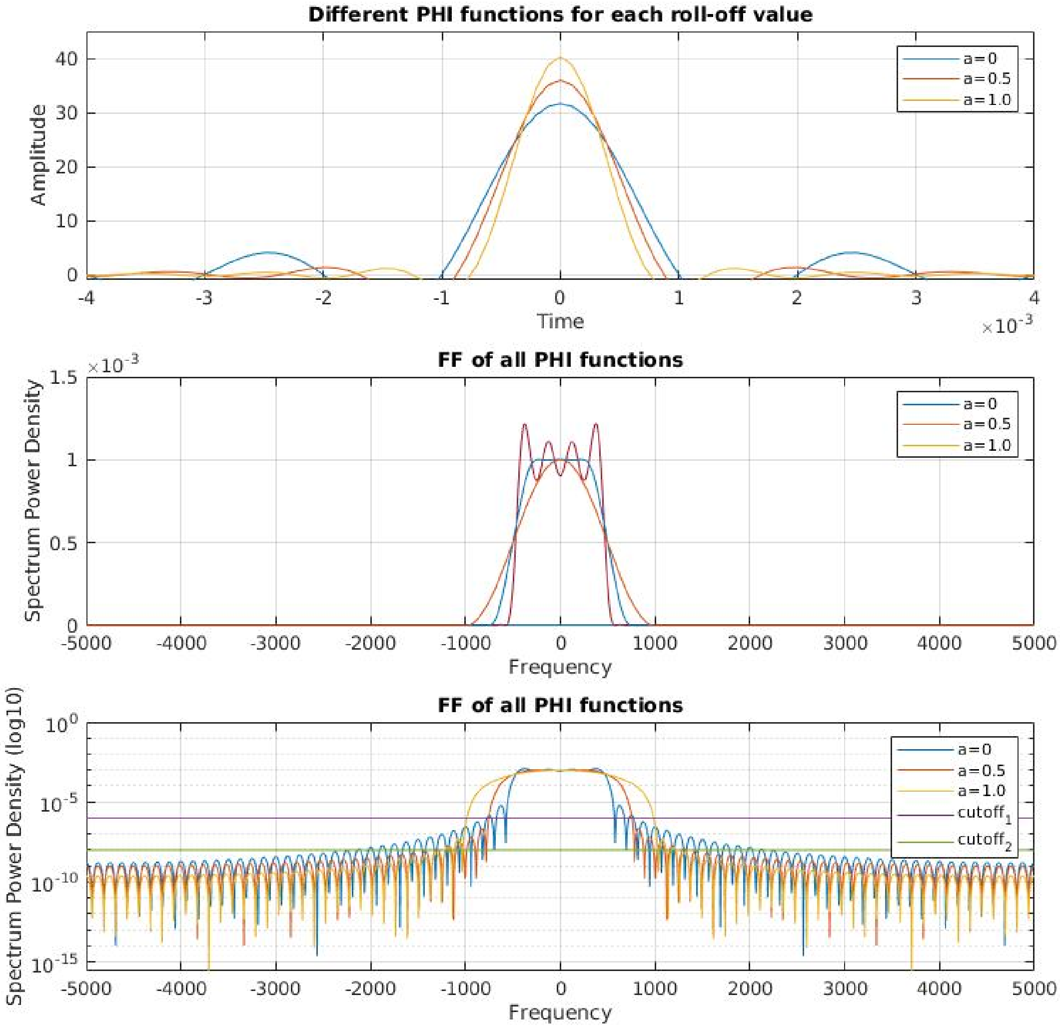
For cut-off at $c_2 = \frac{T}{10^{-5}}$

i. $BW = 2110$, for ϕ with $a = 0$

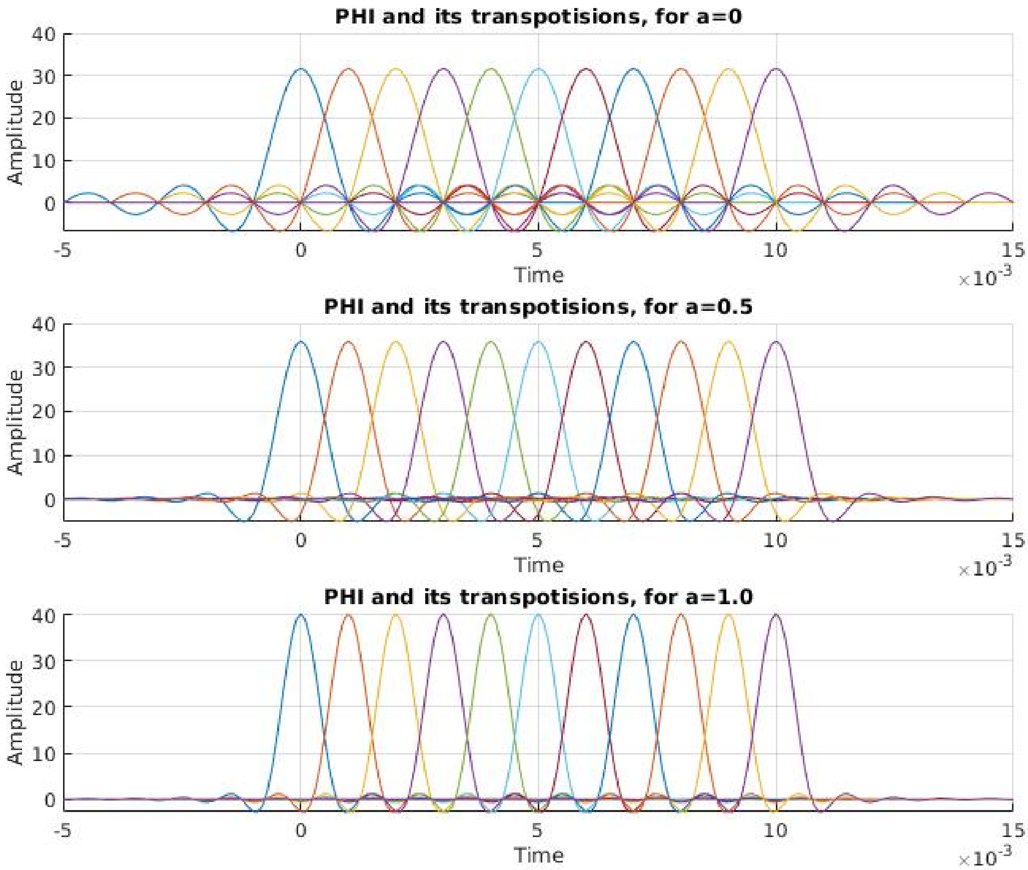
ii. $BW = 1285$, for ϕ with $a = 0.5$

iii. $BW = 1168.5$, for ϕ with $a = 1.0$

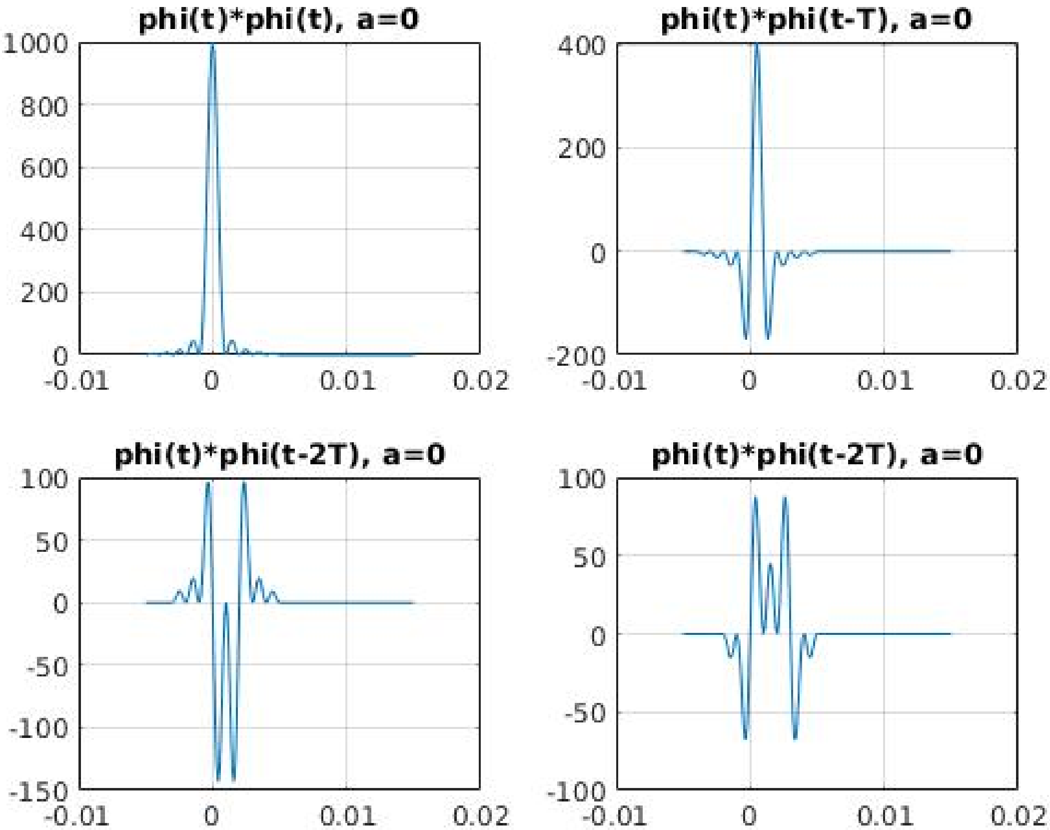
After defining a second cutoff point the most efficient pulse become the one with roll-off $a = 1$, due to its steepest decent.



B.1 The orthocanonical function ϕ has some special properties. Mainly the integral with its transpositions of integer multiples of T is 0! It is observed that the approximation of the integral gets better for $a > 0$.



For $a = 0$



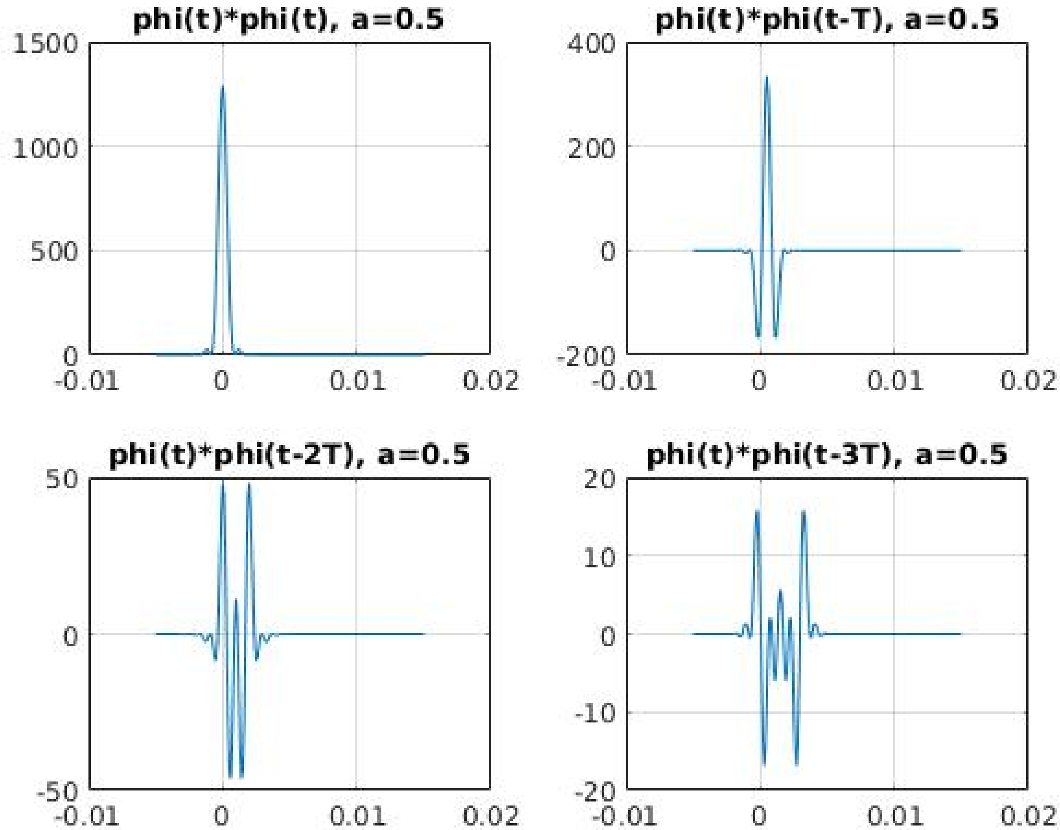
i. $\int_{-\infty}^{\infty} \phi(t)\phi(t)\,dt = 0.98$

ii. $\int_{-\infty}^{\infty} \phi(t)\phi(t-T)\,dt = 0.02$

iii. $\int_{-\infty}^{\infty} \phi(t)\phi(t-2T)\,dt = -0.03$

iv. $\int_{-\infty}^{\infty} \phi(t)\phi(t-3T)\,dt = 0.03$

For $a = 0.5$



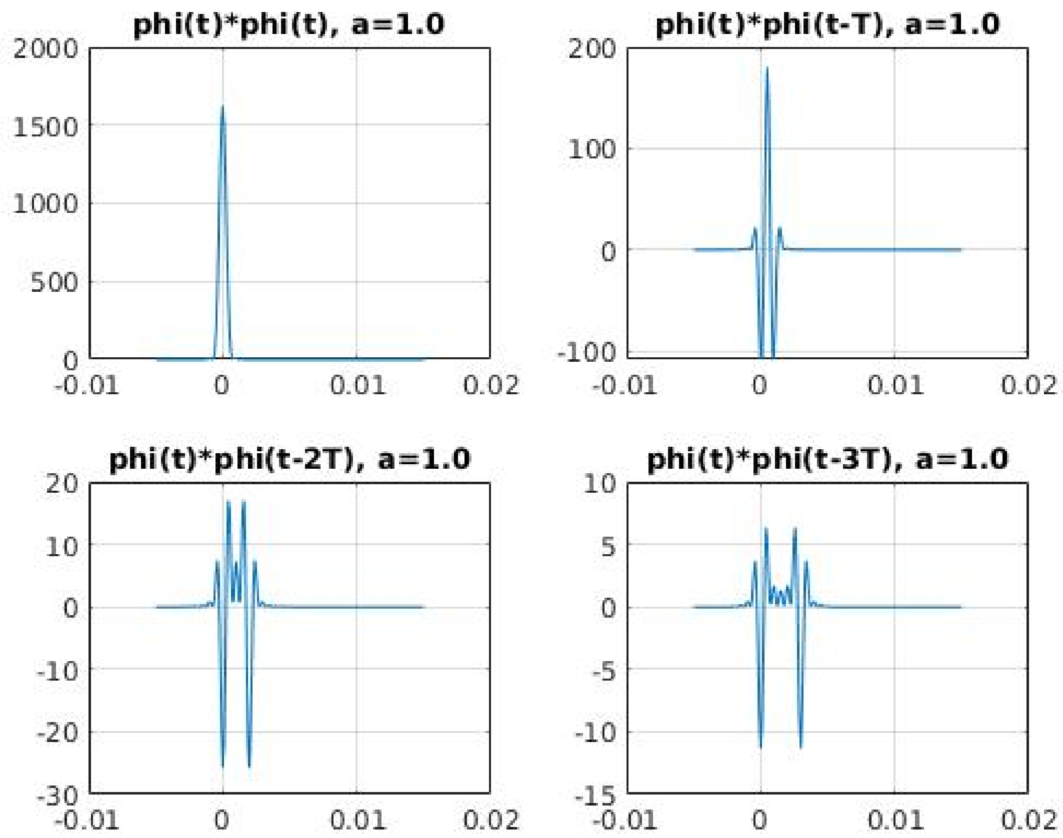
i. $\int_{-\infty}^{\infty} \phi(t)\phi(t)\,dt = 1$

ii. $\int_{-\infty}^{\infty} \phi(t)\phi(t-T)\,dt = 0$

iii. $\int_{-\infty}^{\infty} \phi(t)\phi(t-2T)\,dt = 0$

iv. $\int_{-\infty}^{\infty} \phi(t)\phi(t-3T)\,dt = 0$

For $a = 1.0$



i. $\int_{-\infty}^{\infty} \phi(t)\phi(t)\, dt = 1$

ii. $\int_{-\infty}^{\infty} \phi(t)\phi(t-T)\, dt = 0$

iii. $\int_{-\infty}^{\infty} \phi(t)\phi(t-2T)\, dt = 0$

iv. $\int_{-\infty}^{\infty} \phi(t)\phi(t-3T)\, dt = 0$

For creating the transpositions of $\phi(t)$ a new function called **phi_kT** was created that returns a *length*(k) x *length*(*time*) matrix. It (pre)appends zeroes to $\phi(t-kT)$ so that all ϕ's have the same time vector. Then the products and integrals can be computed by taking the requested rows from that matrix. Function Code:

```
function [phi_K] = phi_kT(phi, time, T, Ts, A)
    phi_K = zeros(11,length(time));
    for k=0:1:2*A
        offset = uint64(k*T/Ts);
        padL = zeros(1,offset);
        padR = zeros(1,length(time) - offset - length(phi));
        t_phi_k = [padL, phi, padR];
        phi_K(k+1,:)= t_phi_k;
    end
end
```

C.1 Create N (50) random bits using the function

```
(sign(randn(bits,1)) + 1)/2
```

C.2 The simple 2-PAM system

a. The function **bits_to_2PAM()** maps the value of each bit to a specific symbol using the following table:

$$0 \longrightarrow +1,$$

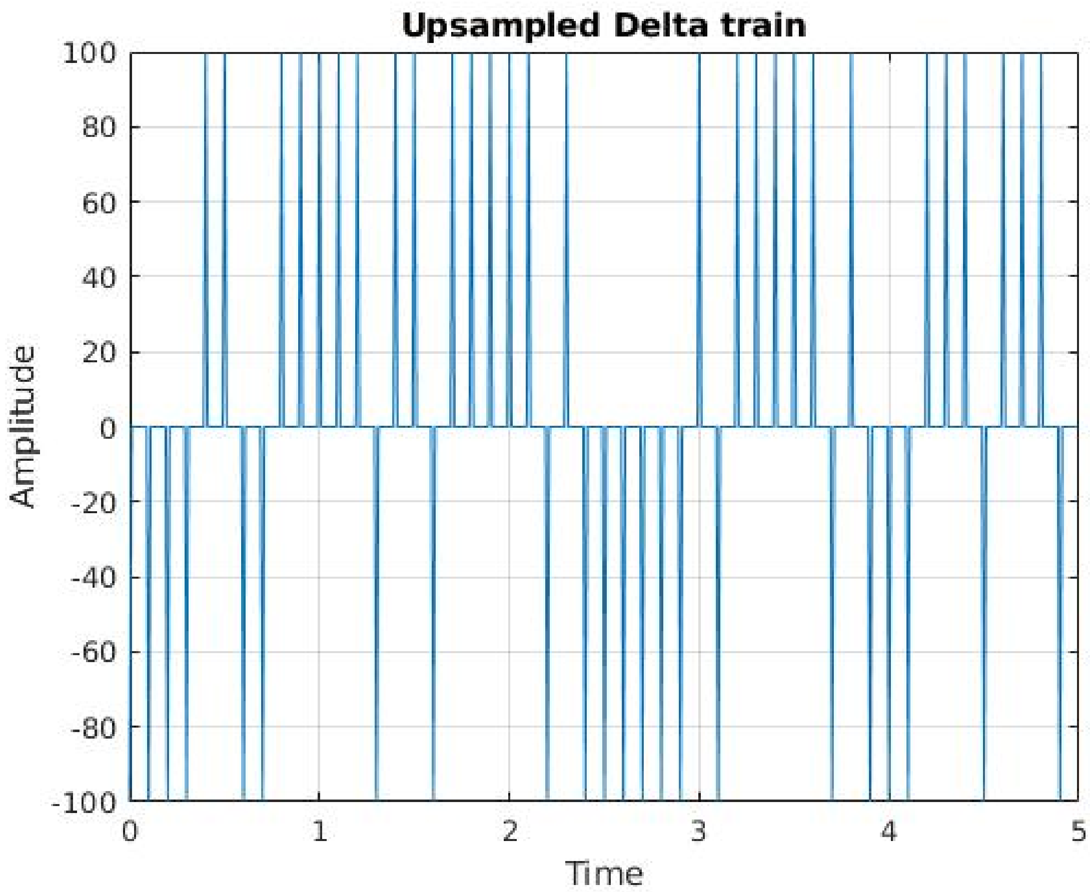
$$1 \longrightarrow -1.$$

Function Code:

```
function X = bits_to_2PAM(bits)
    X = zeros(1, length(bits));
    X(bits==1) = -1;
    X(bits==0) =  1;
end
```

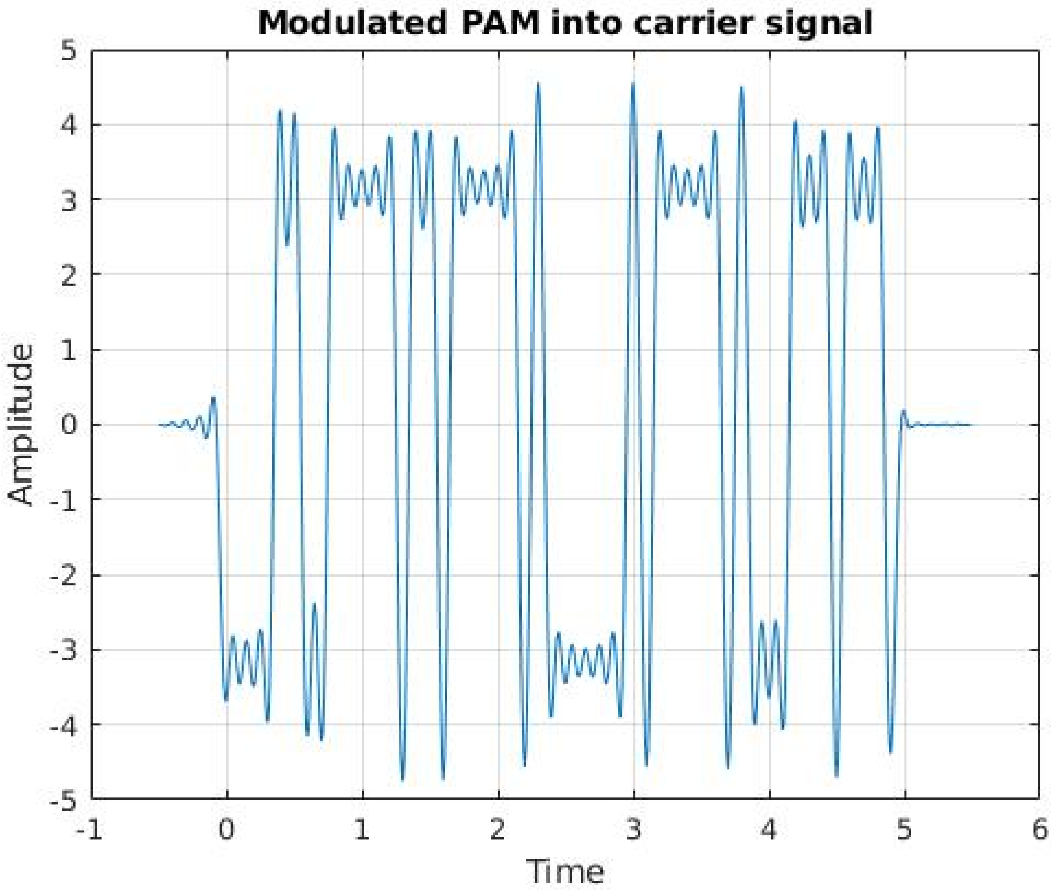
b. After creating the bit stream, it gets upsampled and multiplied by $1/T_s$ so that on every sample at kT a delta pulse is captured (The time vector needs readjustment to accommodate the injected zeroes):

```
X_delta = 1/Ts * upsample(X, over)
T_delta = 0:Ts:bits/over-Ts;
```



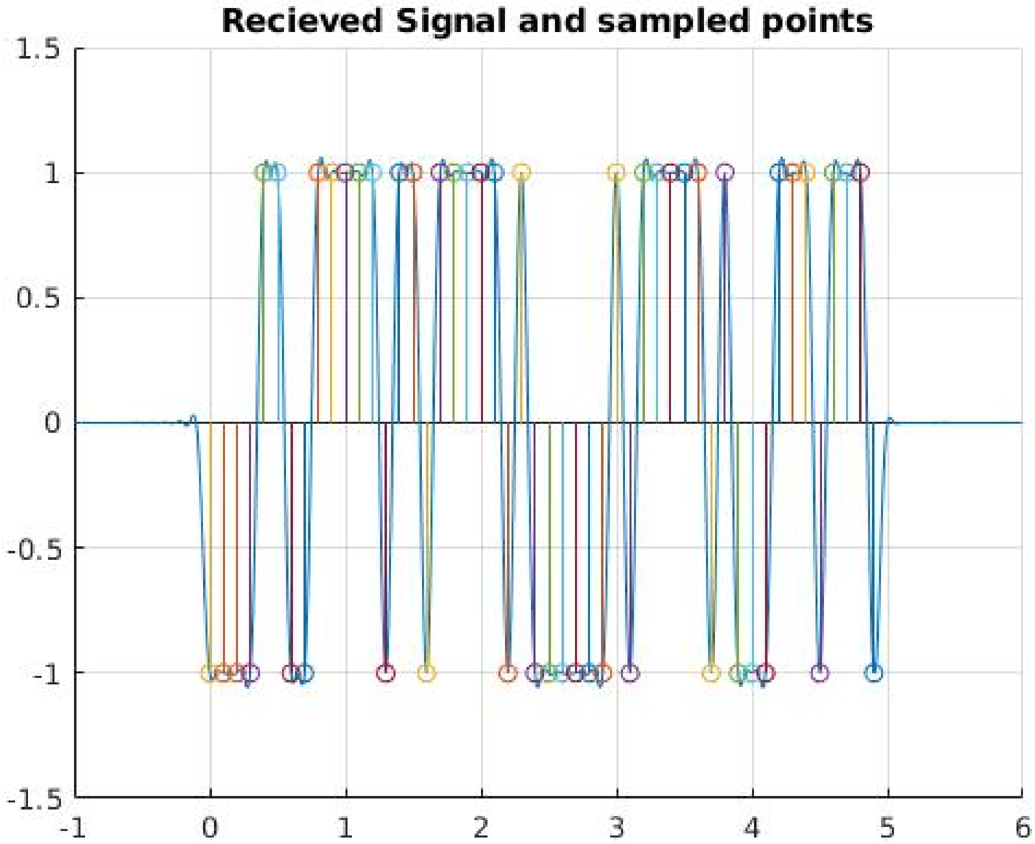
c. Convoluting the carrier wave $\phi(t)$ with X_delta results in the signal to be transmitted, $X(t)$:

```
t_phi_conv = (t(1) + T_delta(1):Ts:t(end) + T_delta(end));
X_t = conv(X_delta,phi)*Ts;
plot(t_phi_conv, X_t);
```



The receiving end:

d. Considering an ideal communications channel, the received signal should be the $X(t)$ without any additional noise. To recreate the sequence that was sent, a convolution with the matched filter ,$\phi(-t)$, needs to be performed. The output of the filter is the integral mentioned in [B.1]. If this output is sampled at specific times it can recreate the original symbol sequence. On kT multiples of T, ϕ should pass from 1 or -1.



The matched filter is created by mirroring ϕ and its time vector using:

```
t_phi_minus = -fliplr(t);
phi_minus = fliplr(phi);
```

After filtering, the convolution can take place and then the sampling

```
tconv = (t_phi_minus(1) + t_phi_conv(1):Ts:t_phi_minus(end) +
t_phi_conv(end));
X_z = conv(phi_minus, X_t)*Ts;
plot(tconv, X_z);
stem((0:49)*T,X_50);
for t=1:length(tconv)
    if mod(tconv(t), T) == 0 && tconv(t) >= 0
        && tconv(t) <= A(2)-Ts
        stem(tconv(t), X_z(t));
    end
end
```

Looking closely one would observe that there is a slight deviation from the sent symbol and the received one. All sent symbols line up perfectly on $y = 1$ and $y = -1$, however because ϕ is just an approximation a slight deviation is expected.