Digital Signal Processing

3rd Lab Exercise

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1. Design a lowpass **Butterworth** filter:

   - sampling frequency: 10 KHz
   - passband: 0–3 KHz
   - stopband: 4–5 KHz
   - passband rippling: 3 dB
   - attenuation: 30 dB

   First we need to determine the order of our filter. To do this, we first have to normalize the corner frequencies, and then we can use the function **buttord** with normalized arguments.

   $$f_{Nyq} = 10\ KHz \Rightarrow f = 5\ KHz$$

   $$\Omega_p(normalized) = \frac{f_p}{f} = \frac{3\ KHz}{5\ KHz} = 0.6$$

   $$\Omega_s(normalized) = \frac{f_s}{f} = \frac{4\ KHz}{5\ KHz} = 0.8$$

   After that, we can use the function **butter** to create the filter.

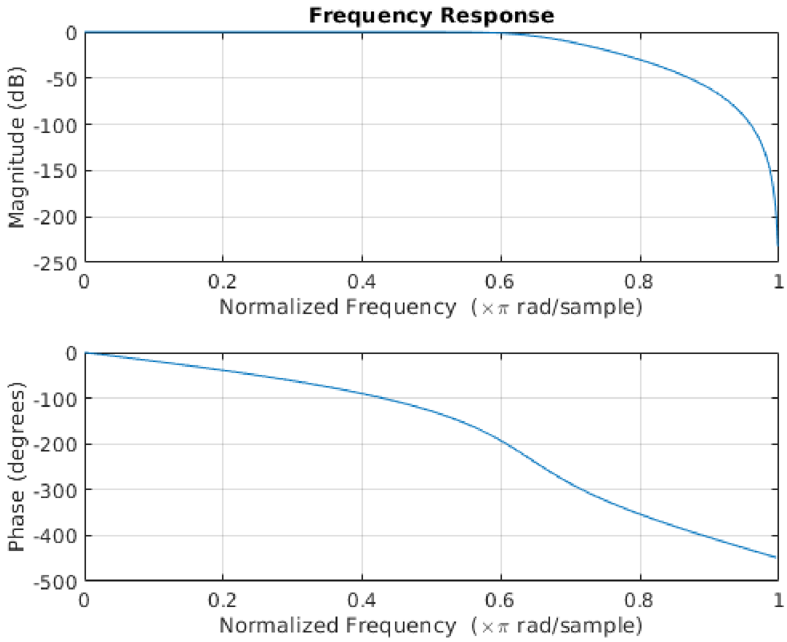

Figure 1: Frequency response of Butterworth lowpass filter

2. Design a highpass **Chebyshev** filter:

   - two orders: 2, 16
   - cutoff frequency: 2 rad/s
   - sampling period: 0.2 s
   - passband rippling: 3 dB

$$f_c = \frac{\Omega_c}{2\pi} = \frac{1}{\pi}$$

$$f_s = \frac{1}{T_s} = 5Hz$$

$$\Omega_c(normalized) = \frac{f_c}{f} = 0.127$$

We can create the filter using the **cheby1** function and argument “high” to create a highpass filter.

Observing the following graph, we can see that the 2^{nd} order filter has a more gradual ascent (wider transition band) than the 16^{th} order one. That means the 2^{nd} order filter is closer to the ideal filter. The 16^{th} order filter has more rippling.

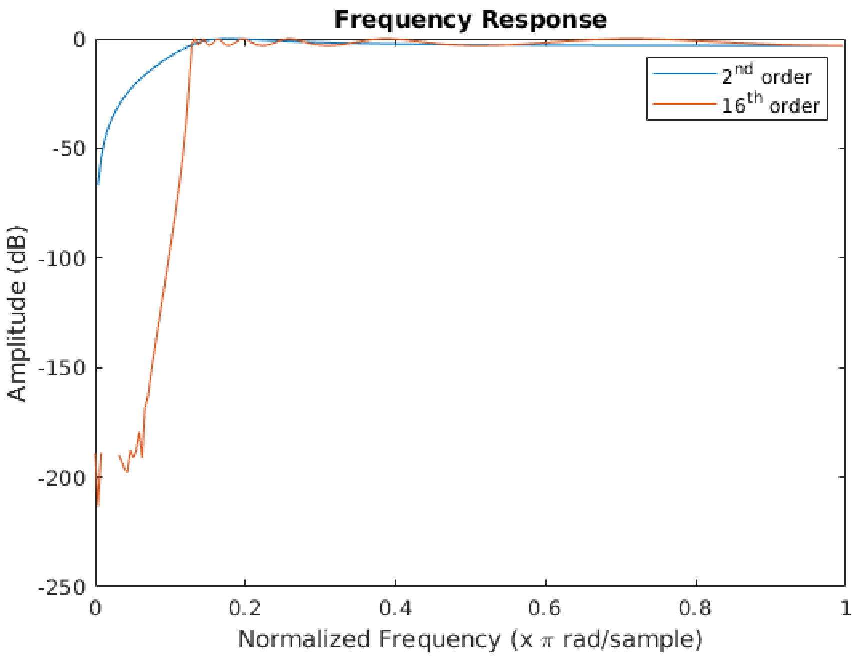

Figure 2: Frequency response of Chebyshev lowpass filter

3. Applications of filtering

   a. Apply the **Butterworth** lowpass filter created in 1. onto the following signal:

$$x(t) = 1 + \cos(1000t) + \cos(16000t) + \cos(30000t)$$

$$f_1 = \frac{1000}{2\pi} \approx 159\ Hz$$
$$f_2 = \frac{16000}{2\pi} \approx 2546\ Hz$$
$$f_3 = \frac{30000}{2\pi} \approx 4774\ Hz$$
$$f_{Nyq} = 2f_{\max} = 2 \cdot 4774 = 9548\ Hz$$

   i. Sample 500 points using $f_s = 10KHz$ as a sampling frequency and create $x[n]$ (No aliasing because $f_s \geq f_{Nyq}$)

   ii. Take the Fourier Transform of $x[n]$ to visualize the frequencies that make up $x[n]$

   iii. Use MATLAB's **filter** function to apply the filer onto a signal

   iv. Take the Fourier Transform of the filtered signal to visualize the filtered out frequencies.

v. Since the Cutoff frequency is 4 KHz any frequencies higher should be filtered out.

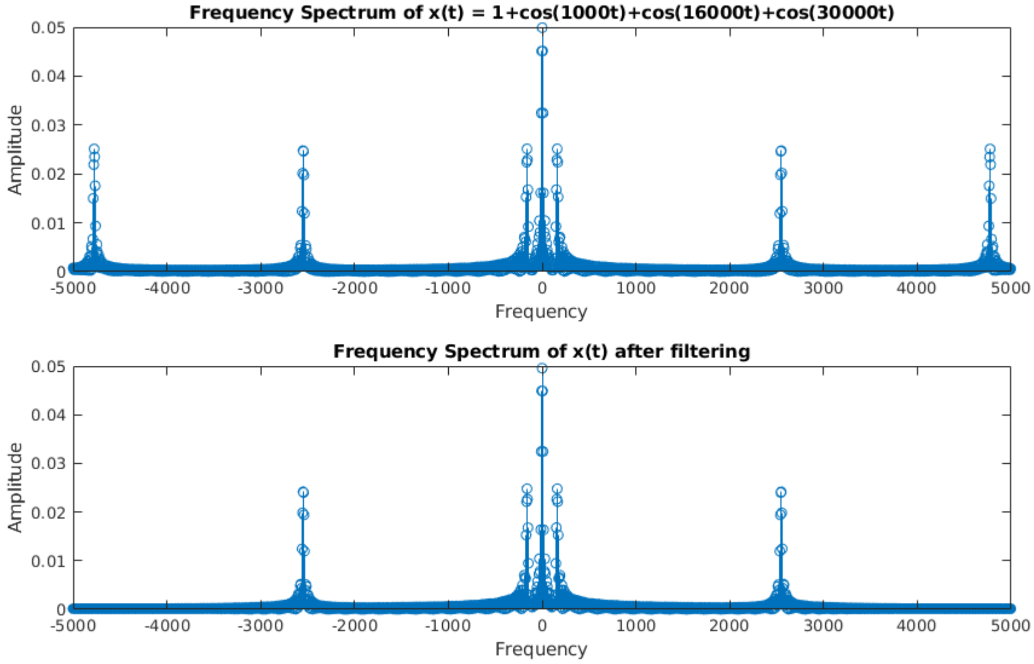

Figure 3: Frequency spectrum before and after filtering

Indeed we can see that $f_3 = 4774Hz$ has been filtered out while the rest of the signal remains intact.

b. Apply the **Chebyshev** highpass filter created in 2. onto the following signal:

$$x(t) = 1 + \cos(1.5t) + \cos(5t)$$

$$f_1 = \frac{1.5}{2\pi} \approx 0.238\ Hz$$

$$f_2 = \frac{5}{2\pi} \approx 0.795\ Hz$$

$$f_c = 0.318\ Hz$$

The same rules apply for this signal as well. Just as above, Fourier Transform the signal and the use the filter function. Observing the following graph we can see that the low frequencies have been filtered out.

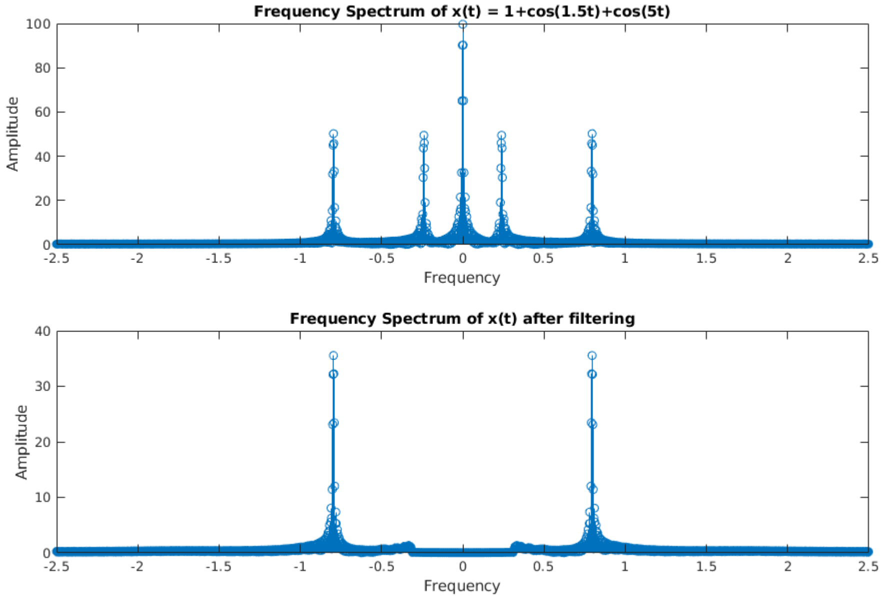

Figure 4: Frequency spectrum before and after filtering

4. Design an FIR filter:

$$\Omega_c = 0.5\pi$$

$$F_s = 0.1KHz$$

With lengths:

$$N = 21, 41 \qquad (Hamming, Hanning)$$

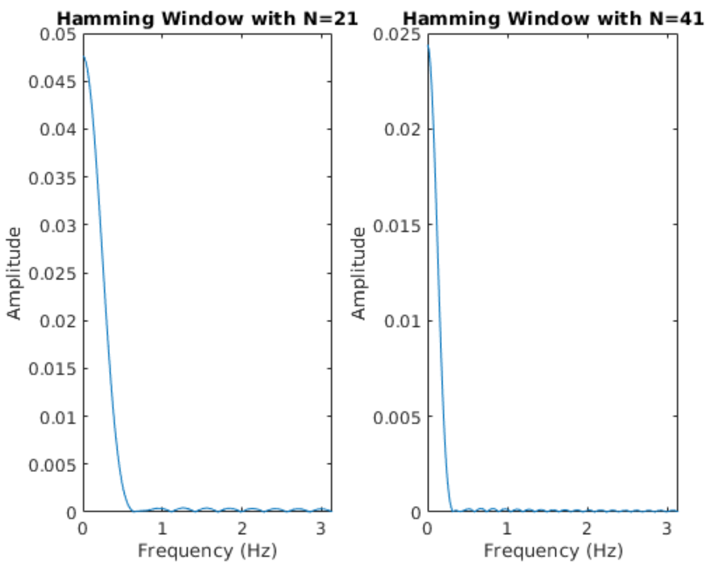

Figure 5: Hamming Windows

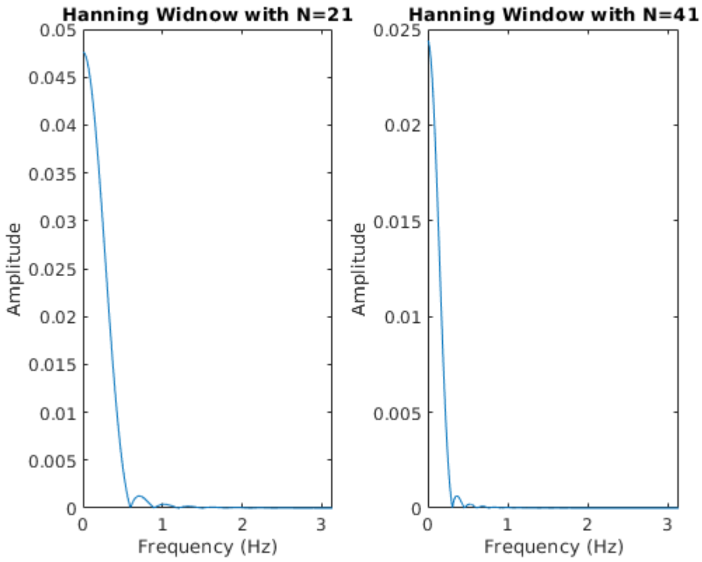

Figure 6: Hanning Windows

- The bigger the window length, the narrower the transition (steeper) band and significally less rippling, thus increasing the window length we can achieve a more effective filter.
- The hanning window has an even more stepper transition band and less ripling so it's more effective.
- The above differences are the result of the slight difference of their coefficients in their respective equations.

Applying the above mentioned windows onto:

$$x(t) = \sin(15t) + 0.25\sin(200t), \qquad F_s = 100Hz$$

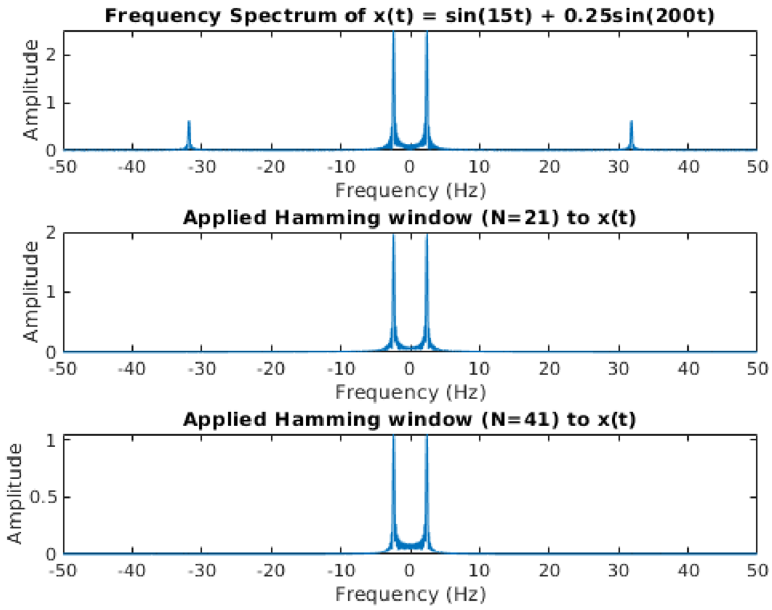

Figure 7: Applied Hamming windows onto $x(t)$

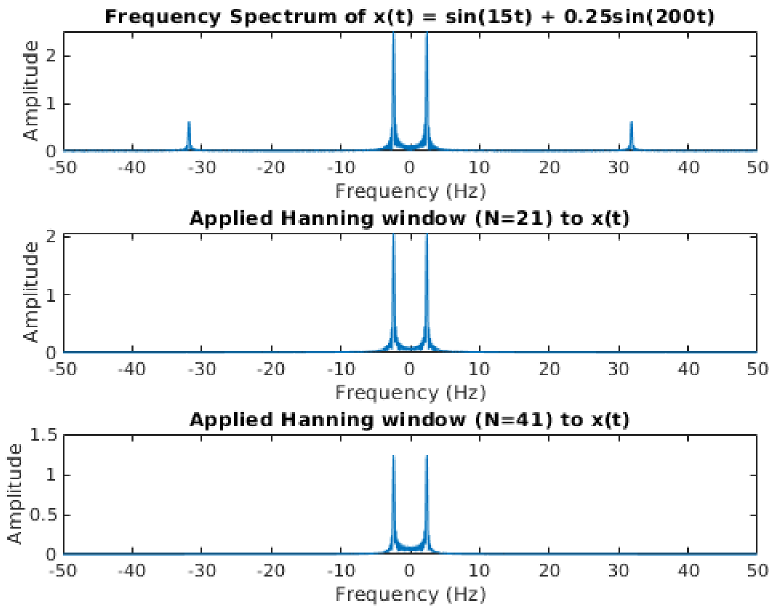

Figure 8: Applied Hanning windows onto $x(t)$

- The windows with length 41 clip the amplitude more than their counterparts. This is due to the fact that a window's attenuation increases proportional to their length (and ultimately to their band length).
- The Hanning window clips reduces the amplitude even more than the Hamming (in both lengths) due to its abrupt transition band.

Changing the sampling frequency F_s to $50Hz$:

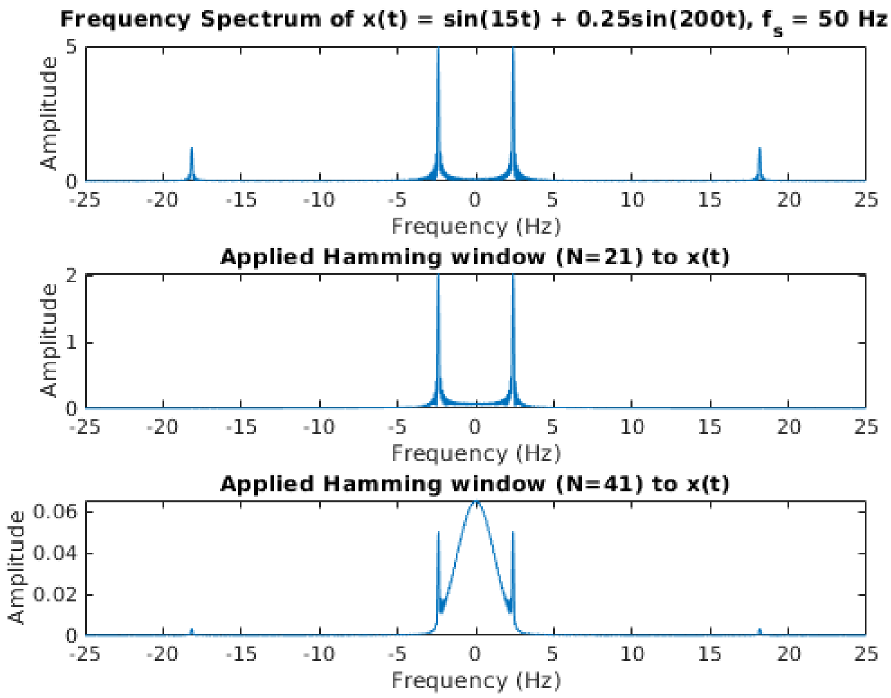

Figure 9: Applied Hamming windows onto $x(t), F_s = 50Hz$

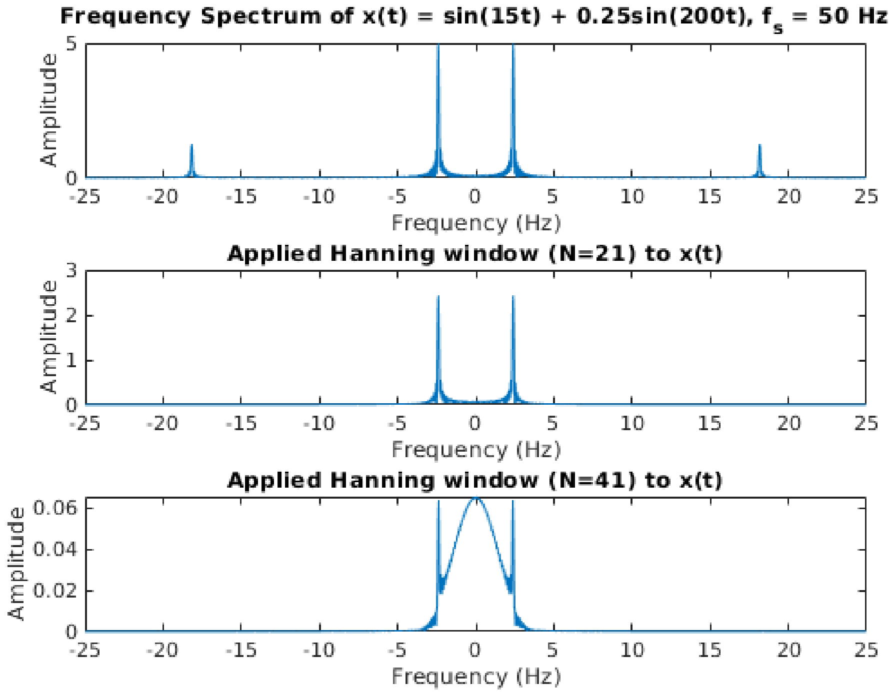

Figure 10: Applied Hanning windows onto $x(t), F_s = 50Hz$

Chaning the sampling frequency to $f_s = 50Hz$ creates the phenomenon of aliasing:

$$f_{\max} = \frac{200}{2\pi} = 31.83Hz \Rightarrow f_{Nyq} \geq 2f_{\max} = 63.66Hz$$

As confirmed by the artifacts created.