Digital Signal Processing

1st Lab Exercise

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1.

A. Convolution of 2 discrete signals with and without the conv() function. The 2 signals chosen were 2 discrete pulses x_1 and x_2:

$$x_1 = u[n-1] - u[n-3]$$

$$x_2 = u[n-3] - u[n-5] + u[n-7] - u[n-9]$$

$$n \in [0, 9.8] \;\; with\ a\ step\ of\ 0.2$$

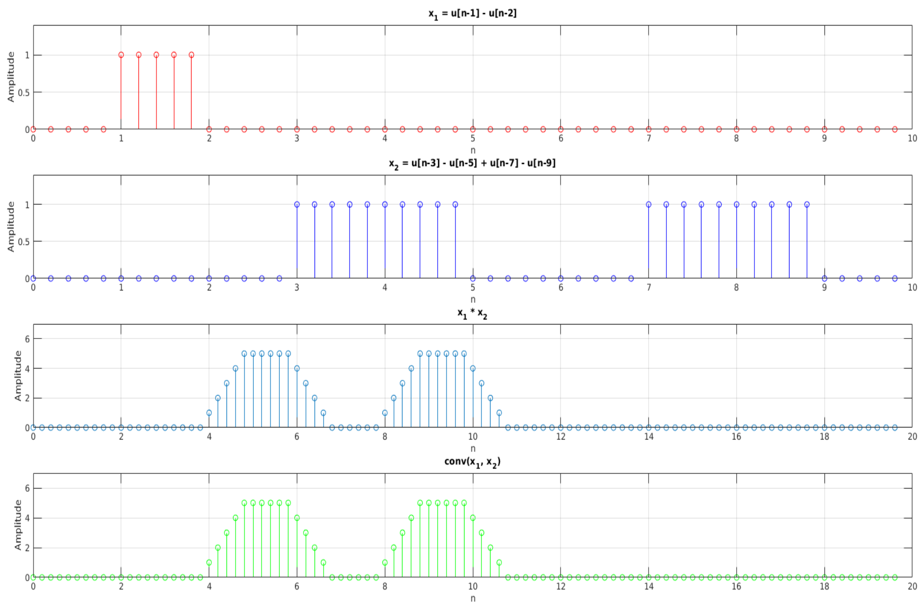

Figure 1: The 2 signals x_1 and x_2 and their convolution, from scratch and by using MATLAB's built in function, respectively

B. Proof of properties of Convolution and the Fourier Transform

$$x_1[n] \circledast x_2[n] = X_1[N] \cdot X_2[N]$$

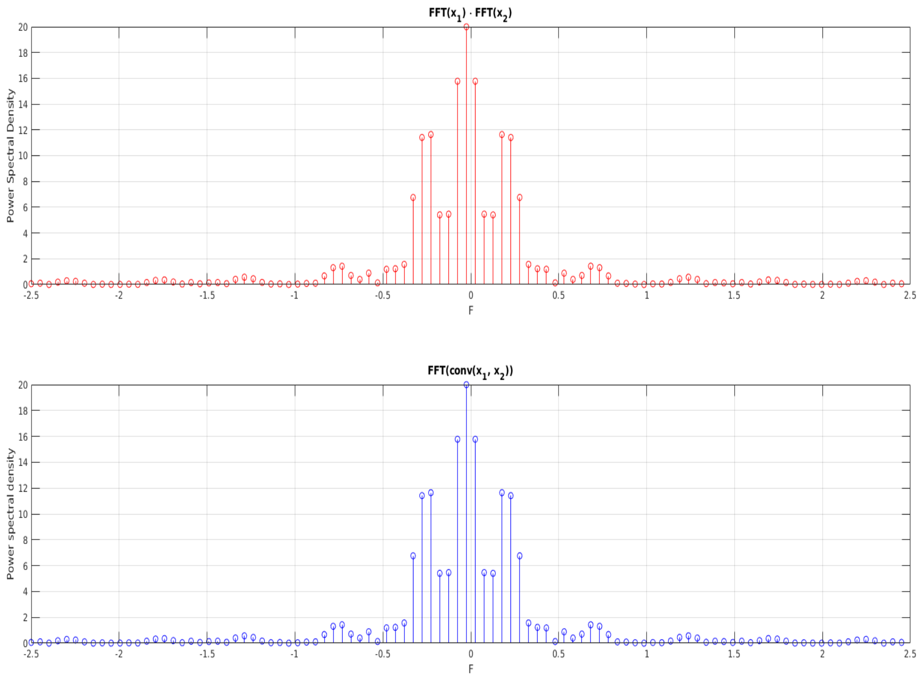

Figure 2: Multiplying the FFTs of both signals vs. taking the FFT of their convolution from earlier

2. Fourier Transform (on paper) the following signal and sample it using the following frequencies:

$$x(t) = 5\cos(2\pi 12t) - 2\sin(2\pi \frac{3}{4}t)$$

a. $f_s = 48Hz$

b. $f_s = 24Hz$

c. $f_s = 12Hz$

d. $f_s = 90Hz$

i. To calculate the Nyquist frequency we need to take the largest frequency ($f_{\max}$) of our two signals in this case $12Hz$. Nyquist's frequency is the lowest sampling frequency of a signal that can reconstruct the original. Sampling with any frequency lower than this, will render the reconstruction worthless.

$$f_{nyq} = 2 * f_{\max} = 2 * 12Hz = 24Hz$$

ii. Using the complex identities of the sine and cosine functions we can easily derive the Fourier Transform of the above mentioned signal:

$$\cos(t) = \frac{e^{jt} + e^{-jt}}{2}$$

$$\sin(t) = \frac{e^{jt} - e^{-jt}}{2j}$$

$$x(t) = \frac{5}{2}e^{2\pi j 12t} + \frac{5}{2}e^{-2\pi j 12t} + je^{-2\pi j \frac{3}{4}t} - je^{2\pi j \frac{3}{4}t}$$

iii. Transforming each term from above and using the time/frequency shift properties of the FT we end up with:

$$F\{1\} = \delta$$

$$X(F) = \frac{5}{2}\delta(F - 12) + \frac{5}{2}\delta(F + 12) + j\delta(F + \frac{3}{4}) - j\delta(F - \frac{3}{4})$$

iv. As seen below, sampling with $f_{\max}$ does not produce enough information for us to be able to reconstruct the signal, where as in sampling with twice the $f_{\max}$ yields just bearly enough information about the signal. Taking samples with any frequency $> f_{\max}$, will result in a better resolution and of course more samples. In my case the last sampling frequency is $90Hz$ which is more that enough.

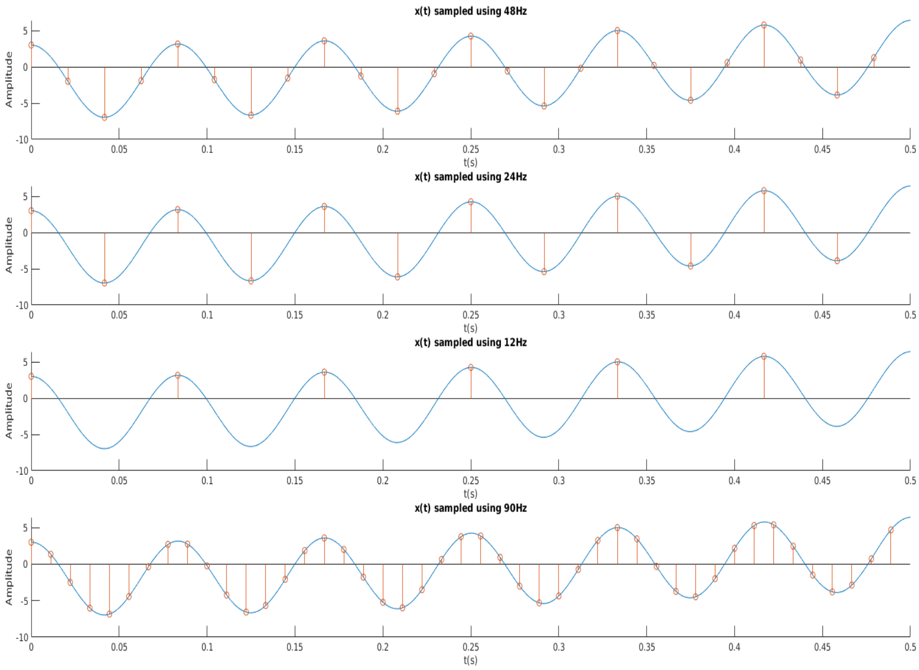

Figure 3: Sampling of $x(t)$ using different frequencies

3.

A. Capture 128 samples of the following signal and display the signal into the frequency spectrum

$$x(t) = 10\cos(2\pi 20t) - 4\sin(2\pi 40t)$$

with a frequency such that, the aliasing effect is not visible.

Using:

$$f_{sampl} \geqslant f_{nyq} = 2 * f_{\max} \Rightarrow f_{sampl} \geqslant 80Hz$$

Chosen:

$$f_{sampl} = 500Hz$$

We can observe peaks in the frequency spectrum at the positive and negative frequencies of the two signals used to create $x(t)$.

Peaks at:

i. $-f_1 = -20Hz,\ f_1 = 20Hz$

ii. $-f_2 = -40Hz,\ f_2 = 40Hz$

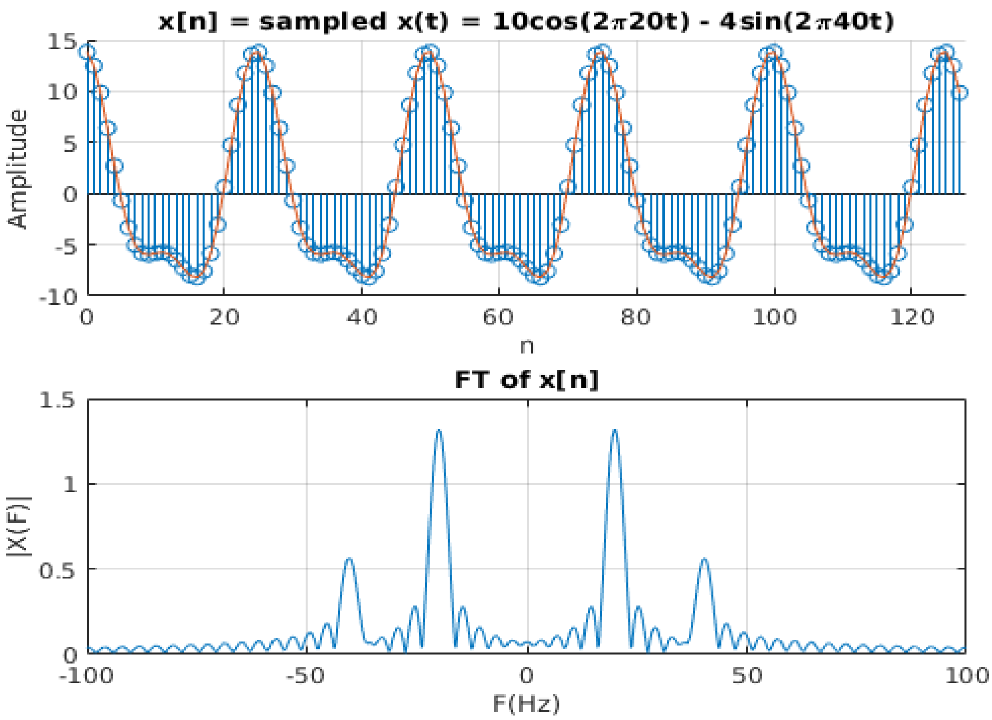

Figure 4: Sampling of $x(t)$ and its frequency spectrum

B. Define the following signal:

$$x(t) = \sin(2\pi f_0 t + \phi)$$

Sampling the above signal with a SF (f_s) of 8KHz and by using this property:

$$x[n] = x_a(nT_s) = x_a(n\frac{1}{f_s})$$

the resulting discrete signal is:

$$x[n] = \sin(2\pi\frac{f_0}{f_s}n + \phi)$$

i. Plotting for all 4 (low) different sinusodial frequencies we get the following spectrum for each:

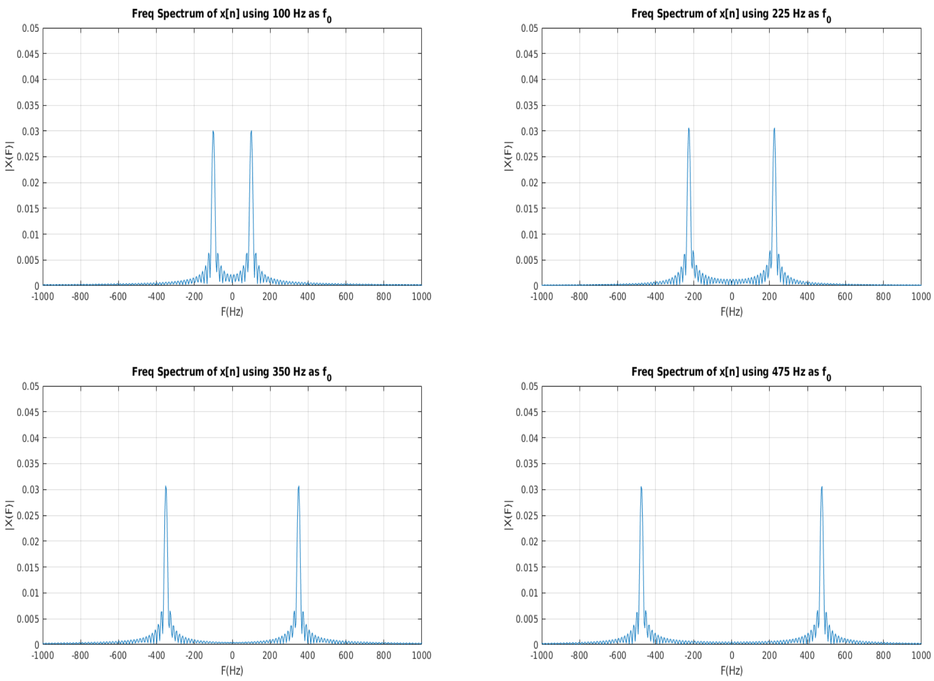

Figure 5: Frequency Spectrum of $x(t)$

We can observe that, because the SF, f_s, is so high in respect to f_0 the peaks are shown at every f_0 point on the Frequency Spectrum which that is expected, since that is the Frequency of our sinusodial wave.

ii. Plotting for all 4 (high) different sinusodial frequencies we get the following spectrum for each:

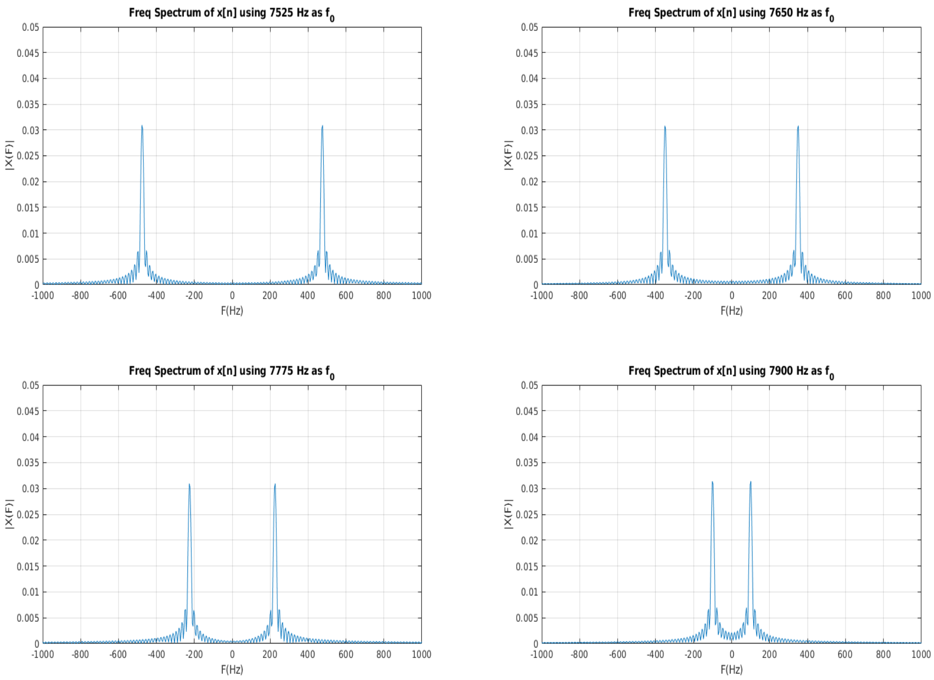

Figure 6: Frequency Spectrum of $x(t)$

However in this case the peaks exist at points $f_0 - f_s$. This is happening because the SF f_s is bellow the Nyquist limit. This is called the aliasing effect, because the sinusodial wave, clearly, has a way larger Frequency than before but, despite that, the signal that "seems" to be sampled is a signal with a Frequency of $f_0 - f_s$ and not f_0. Changing the starting phase ϕ will have no effect what so ever, because ϕ is just a starting offset, and does not mess with the Frequency.