Mechanics of Materials
Semester 4

Contents

Chapter 1

Mathematical Concepts

1.1 Tensors

Definition 1.1.1: Einstein Notation

Also known as summation notation, says that if we have a repeated index then we are summing over that index. For example

$$y = c_i \hat{\mathbf{e}}_i.$$

implies that

$$y = \sum_{i=1}^{3} c_i \hat{\mathbf{e}}_i = c_1 \hat{\mathbf{e}}_1 + c_2 \hat{\mathbf{e}}_2 + c_3 \hat{\mathbf{e}}_3.$$

same thing with

$$a_i \cdot b_i = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3.$$

Definition 1.1.2

Kronecker delta is defined to be

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}.$$

and the permutation symbol

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{if } (i,j,k) \text{ is } (1,2,3),(2,3,1), \text{ or } (3,1,2), \\ -1 & \text{if } (i,j,k) \text{ is } (3,2,1),(1,3,2), \text{ or } (2,1,3), \\ 0 & \text{if } i = j, \text{ or } j = k, \text{ or } k = i \end{cases}.$$

And they appear in

$$\hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j = \delta_{ij}$$
$$\hat{\mathbf{e}}_i \times \hat{\mathbf{e}}_j = \varepsilon_{ijk} \hat{\mathbf{e}}_k$$

Definition 1.1.3: Tensors

In an m-dimensional space, a tensor of rank n is a mathematical object that has n indices, m^n components, and obeys certain *transformation rules*

Note:-

Typically $m = 3$ corresponding to the 3D space.

Example 1.1.1

- A rank 0 tensor is a scalar
$$A.$$
- A rank 1 tensor is a vector
$$A\hat{\mathbf{x}} = A_i x_i = A_1 x_1 + A_2 x_2 + A_3 x_3 = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}.$$
- A rank 2 tensor is a matrix
$$A(\hat{\mathbf{x}}, \hat{\mathbf{y}}) = A_{ij} x_i y_j = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}.$$

Some notable tensors are:

1. Symmetric tensors
$$A_{ij} = A_{ji}.$$
2. Anti-symmetric tensors
$$A_{ij} = -A_{ji}.$$
3. General tensor. It can be represented using a symmetric and an anti symmetric tensor
$$A = A^S + A^A.$$
where
$$A^S = \frac{1}{2}(A + A^T)$$
$$A^A = \frac{1}{2}(A \cdot A^T)$$