Electric Machines
Semester 6

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Contents

Chapter 1

Transformers

1.1 Introduction

- A transformer is a static device that transfers electrical energy from one circuit to another through inductively coupled conductors.
- It is used to increase or decrease the voltage level of an AC signal.
- The transformer is based on the principle of electromagnetic induction.
- The transformer is a passive device, meaning it does not require any external power source to operate.
- The transformer is used in power distribution systems to step up or step down the voltage level.
- The transformer is used in electronic circuits to isolate the input and output signals.

1.2 Ideal Transformer

- The primary and secondary windings have zero resistance.

$$R_1 = R_2 = 0.$$

- The core has infinite permeability.

$$\mu = \infty.$$

- The transformer has no leakage flux.
- The transformer has no hysteresis loss.
- The transformer has no eddy current loss.

$$\begin{aligned} \mathcal{R}_{P_{\text{core}}} &= 0 \\ \Delta P &= 0 \\ \Delta P_j &= 0 \end{aligned}$$

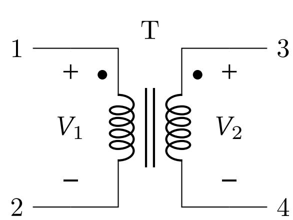

Figure 1.1: Ideal Transformer

$$\text{Turns Ratio:} \quad m = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

1.3 Real Transformer

We define the voltage equations of the transformer as follows:

$$\dot{V}_1 + \dot{E}_1 = \dot{I}_1 Z_1 \tag{1.1}$$
$$\dot{E}_2 - \dot{V}_2 = \dot{I}_2 Z_2 \tag{1.2}$$

$$E_1 = 4.44 f N_1 \Phi_m$$
$$E_2 = 4.44 f N_2 \Phi_m$$

If we consider $\dot{I}_1 Z_1 \ll \dot{V}_1$ and $\dot{I}_2 Z_2 \ll \dot{V}_2$, we can simplify the equations as follows:

$$\dot{V}_1 = -\dot{E}_1$$
$$\dot{V}_2 = \dot{E}_2$$

We define the magnetization current as the current required to produce the magnetic flux in the core.

$$I_m = \dot{I}_{10} = I_1 + \underbrace{mI_2}_{I_2'}.$$

We define the equivalent circuit of the transformer as follows:

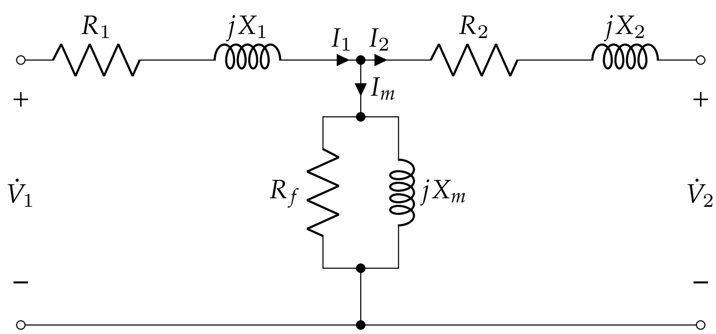

Figure 1.2: Transformer Equivalent Circuit

1.3.1 Γ Equivalent Circuit

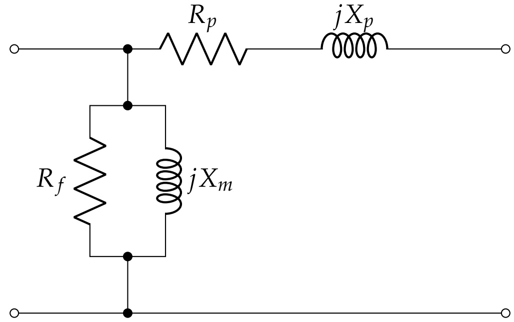

Figure 1.3: Γ Equivalent Circuit with the Primary Side as the Reference

$$
\begin{aligned}
R_p &= R_1 + R_2' = R_1 + \frac{R_2}{m^2} \\
X_p &= X_1 + X_2' = X_1 + \frac{X_2}{m^2} \\
Z_p &= R_p + jX_p
\end{aligned}
$$

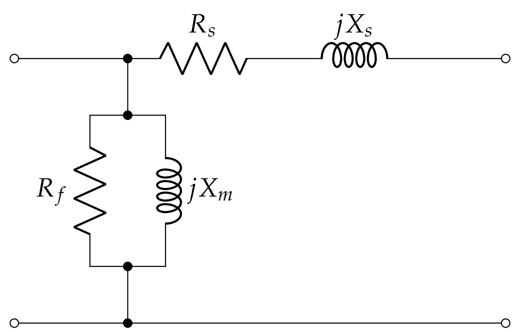

Figure 1.4: Γ Equivalent Circuit with the Secondary Side as the Reference

$$
\begin{aligned}
R_s &= m^2 R_1 + R_2 = m^2 R_1 + R_2 \\
X_s &= m^2 X_1 + X_2 = m^2 X_1 + X_2 \\
Z_s &= R_s + jX_s
\end{aligned}
$$

1.3.2 κ Equivalent Circuit

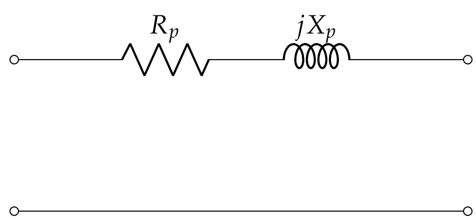

Figure 1.5: κ Equivalent Circuit with the Primary Side as the Reference

1.4 Transformer Tests

1.4.1 Open Circuit Test

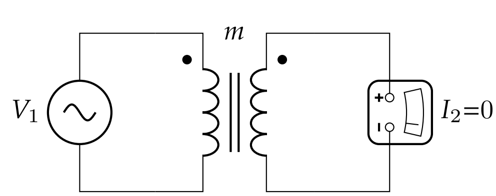

Figure 1.6: Open Circuit Test

We measure V_{10}, I_{10}, P_{10} and V_{20}.

We can calculate the following:

$$\begin{aligned} Q_{10} &= V_{10} I_{10} \sin \varphi_o \\ P_{10} &= V_{10} I_{10} \cos \varphi_o \\ \cos \varphi_o &= \frac{P_{10}}{V_{10} I_{10}} \\ S_{10} &= V_{10} I_{10} \\ R_f &= \frac{{V_{10}}^2}{P_{10}} \\ X_m &= \frac{{V_{10}}^2}{Q_{10}} \end{aligned}$$

1.4.2 Short Circuit Test

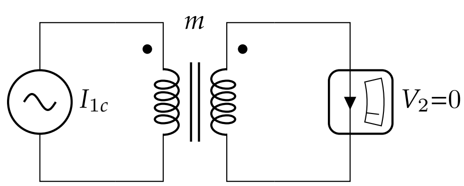

Figure 1.7: Short Circuit Test

Typically we run the short circuit test at $5 - 15\%$ of the nominal current.

$$\begin{aligned} I_{2c} &\leqslant I_{2n} \\ V_{1c} &= (5 - 15)\% V_{1n} \\ I_{2c} &\ll I_{2n} \\ I_{1c} &\ll I_{1n} \end{aligned}$$

We measure V_{1c}, I_{1c}, P_{1c} and I_{2c}.

We can calculate the following:

$$\begin{aligned} R_p &= \frac{P_{1c}}{{I_{1c}}^2} \\ X_p &= \frac{Q_{1c}}{{I_{1c}}^2} \end{aligned}$$

1.5 Variation of the Voltage of Conduction

1. **Exact Calculation:**

$$\begin{aligned} V_2 &< V_{20} \\ m_c = \frac{V_2}{V_1} &< m \end{aligned}$$

$$V_1 = V_2' + \Delta V_p = V_2' + I_2' \left(R_p + j X_p \right).$$

2. **Approximate Calculation (Variation of the Secondary Voltage):**

$$\Delta V_s = V_{20} - V_2.$$

$$\Delta V_p = \frac{\Delta V_s}{m}.$$

$$\begin{aligned}\Delta V_s\% &= \frac{V_{20} - V_2}{V_{20}} \times 100\\ &= \frac{mV_1 - V_2}{mV_1} \times 100\\ &= \frac{V_1' - V_2}{V_1'} \times 100\end{aligned}$$

The KAPP formula states

$$\Delta V_s = I_2 \left(R_s \cos\varphi_2 \pm X_s \sin\varphi_2\right).$$

The $\pm$ sign depends on the nature of the load (inductive or capacitive).

From the KAPP formula, we define the voltage regulation as

$$\Sigma = \frac{\Delta V_s}{V_2} \times 100.$$

$$\Delta V_p = I_1 \left(R_p \cos\varphi_2 \pm X_p \sin\varphi_2\right).$$

1.6 Vector Diagram of the Transformer

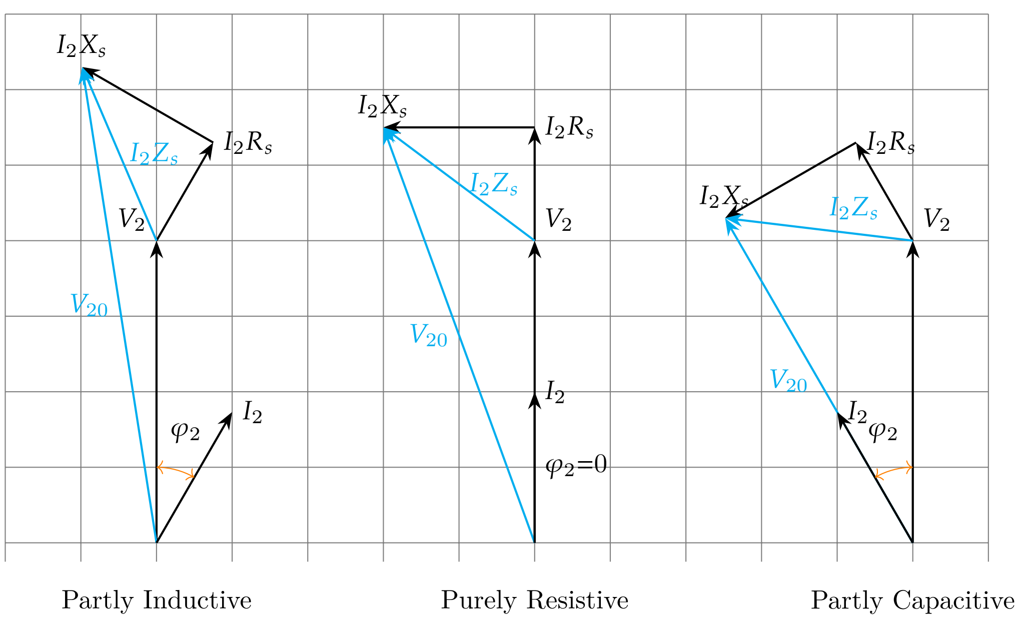

Figure 1.8: Vector Diagram of a Transformer

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + \Delta P_{Jt} + \Delta P_f}.$$

Where

$$\Delta P_{Jt} = \Delta P_{J1} + \Delta P_{J2} = {I_{11}}^2 R_p = {I_{21}}^2 R_s$$
$$\Delta P_f = P_{10} = P_{20}$$

Example 1.6.1

Given a single phase transformer 200/400V - 50 Hz. By tests, we measure the following:

- Open Circuit Test: $V_{10} = 200\,\mathrm{V}$, $I_{10} = 0.7\,\mathrm{A}$, $P_{10} = 70\,\mathrm{W}$.
- Short Circuit Test: $V_{2c} = 15\,\mathrm{V}$, $I_{1c} = 10\,\mathrm{A}$, $P_{1c} = 85\,\mathrm{W}$.

1. Calculate the equivalent circuit parameters.
2. Calculate V_2 if the charge is $5\,\mathrm{kW}$, $\cos\varphi = 0.8$, and $V_1 = 200\,\mathrm{V}$. Deduce the voltage regulation (Σ).
3. Deduce the efficiency of the transformer.

Solution: $m = 2$

1.

$$R_f = \frac{{V_{10}}^2}{P_{10}} = \frac{200^2}{70} = 571.43\,\Omega$$
$$S_{10} = V_{10} I_{10} = 200 \times 0.7 = 140\,\mathrm{V\,A}$$
$$Q_{10} = \sqrt{{S_{10}}^2 - {P_{10}}^2} = \sqrt{140^2 - 70^2} = 121.6\,\mathrm{VAR}$$
$$X_m = \frac{{V_{10}}^2}{Q_{10}} = \frac{200^2}{121.6} = 328.95\,\Omega$$

To find R_p and X_p, we need to find R_s and X_s first.

$$R_s = \frac{P_{2c}}{I_{1c}^2} = \frac{85}{100} = 0.85\,\Omega$$
$$S_{2c} = V_{2c} I_{1c} = 15 \times 10 = 150\,\mathrm{V\,A}$$
$$Q_{2c} = \sqrt{S_{2c}^2 - P_{1c}^2} = \sqrt{150^2 - 85^2} = 12.3\,\mathrm{VAR}$$
$$X_s = \frac{Q_{2c}}{I_{1c}^2} = \frac{12.3}{100} = 1.23\,\Omega$$
$$R_p = \frac{R_s}{m^2} = \frac{0.85}{4} = 0.2125\,\Omega$$
$$X_p = \frac{X_s}{m^2} = \frac{1.23}{4} = 0.3075\,\Omega$$

2.

$$I_2 = \frac{P_2}{V_2 \cos\varphi} = \frac{5000}{400 \times 0.8} = 15.625\,\mathrm{A}$$
$$\Delta V_s = I_2 \left(R_s \cos\varphi \pm X_s \sin\varphi\right) = 15.625\,(0.85 \times 0.8 + 1.23 \times 0.6) = 22.2\,\mathrm{V}$$

$$\Sigma = \frac{\Delta V_s}{V_2} \times 100 = \frac{22.2}{400} \times 100 = 5.55\% \quad \text{If } V_{20} = 400\,\mathrm{V}.$$

To obtain $V_2 = 400\,\mathrm{V}$, we need to increase the primary voltage to

$$\Delta V_p = \frac{\Delta V_s}{m} = \frac{22.2}{2} = 11.1\,\mathrm{V}$$
$$V_1 = V_2' + \Delta V_p = \frac{400}{2} + 11.1 = 211.1\,\mathrm{V}$$

3.

$$\Delta P_{Jt} = I_{11}^2 R_p = 15.6^2 \times 0.85 = 206\,\mathrm{W}$$
$$\Delta P_f = P_{10} = P_{20} = 70\,\mathrm{W}$$
$$\eta = \frac{5000}{5000 + 206 + 70} = 0.947$$