Electricity 2

Contents

PART I

Electric Current

SECTION 1

Current and Current Density

Electric charge in motion constitutes an electric current. In the steady flow of charge in a wire of cross-sectional area A, the total charge passing through this area in unit time is defined to be the electric current I at this place. If a total charge q flows through this area in time t, the current I is given by:

$$I = \frac{q}{t}. \tag{1.1}$$

where I is measured in Ampere. Sometimes it is also convenient to look at the density of the current we're observing over a certain area A called *current density*

$$J = \frac{I}{A}. \tag{1.2}$$

If the area in which the current is running through changes at points 1 and 2, the current between those 2 points remain constant ($I_1 = I_2$) while the current density changes to become

$$\frac{J_1}{J_2} = \frac{A_2}{A_1}.$$

If a positively charged cloud of n particles moves through a space with speed v and charge q, we find that[1]

[1] q can be positive or negative

$$\vec{\mathbf{J}} = nq\vec{\mathbf{v}} = \varrho_\tau \vec{\mathbf{v}}. \tag{1.3}$$

The total current through a *slice* of space mentioned prior it calculated using

$$I = \iint_A \vec{\mathbf{J}} \, \mathrm{d}A\,. \tag{1.4}$$

The volume element here is $\vec{\mathbf{v}} \, \mathrm{d}A \, \mathrm{d}t = \mathrm{d}\tau$

SECTION 2

Continuity Law

The continuity law for current is

$$\boldsymbol{\nabla} \cdot \left(\vec{\mathbf{J}} + \vec{\mathbf{J}}_c \right) = -\frac{\partial \varrho_\tau}{\partial t}. \tag{2.1}$$

Where
$\vec{\mathbf{J}}$ is the conduction current density.
$\vec{\mathbf{J}}_c = \varrho_\tau \vec{\mathbf{v}}_\tau$ is the convection current density.
$\vec{\mathbf{v}}_\tau$ is the velocity of the volume containing the particles.
There are 2 particular cases for the equation above

1. The volume is not moving ($\vec{\mathbf{v}}_\tau = 0 \implies \vec{\mathbf{J}}_c = 0$). The continuity law of static structures is

$$\boldsymbol{\nabla} \cdot \vec{\mathbf{J}} = -\frac{\partial \varrho_\tau}{\partial t}. \tag{2.2}$$

2. The volume is not moving and we are in a steady state, so the charge density of the particles doesn't change much as they are static so $\partial \varrho_\tau / \partial t = 0$ so

$$\oiint \vec{\mathbf{J}} \, \mathrm{d}A = 0. \tag{2.3}$$

PART
II

Ohm's and Joule's Law

Section 3

Electrical Mobility and Conductivity

The velocity of a charge carrier in in an electric field $\vec{\mathbf{E}}$ is

$$\vec{\mathbf{v}} = \mu \vec{\mathbf{E}}. \tag{3.1}$$

where μ is the mobility of the charge carrier.

The current density vector in a conductor with conductivity σ is said to be

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}. \tag{3.2}$$

Ohm's law(doesn't really need an introduction)

$$U = RI. \tag{3.3}$$

where we can find R using

$$R = \frac{\rho l}{A}. \tag{3.4}$$

where A is the surface area of the resistor and l is the length of the conductor[2].

[2] $\sigma = \frac{1}{\rho}$

Section 4

Resistance

The symbol of a resistive conductor is represented by

$$\begin{aligned} P &= UI \qquad (4.1) \\ W &= Pt \qquad (4.2) \end{aligned}$$

Power loss due to Joule's law

$$\begin{aligned} P &= I^2R = \frac{U^2}{R} \qquad (4.3) \\ W &= I^2Rt \qquad (4.4) \end{aligned}$$

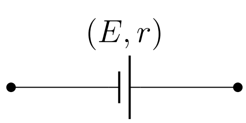
Section 5

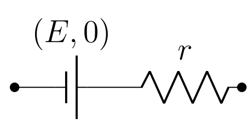
Electric Ciruits

Subsection 5.1

Power Sources/Generators

A DC source is characterized by their electromotive force E and internal resistance r





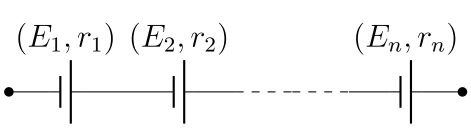
The voltage supplied by the source is

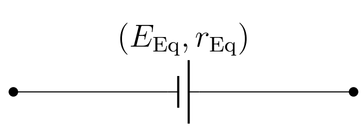
$$V_{\text{source}} = E - rI.$$

and it's efficiency is

$$\eta = 1 - \frac{rI}{E}.$$

5.1.1 In Series

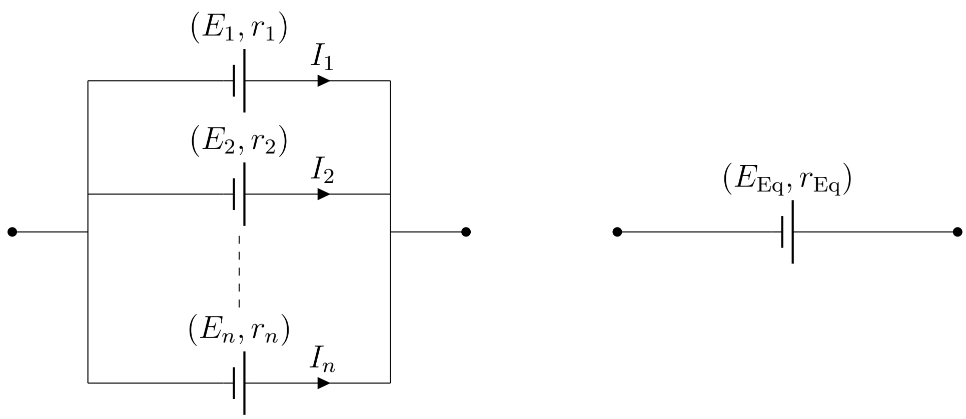




where

$$E_{\text{Eq}} = \sum_{i=1}^{n} E_i$$
$$r_{\text{Eq}} = \sum_{i=1}^{n} r_i$$

5.1.2 In Parallel



In case of *identical* sources:

$$I = \sum_{i=1}^{n} I_i$$
$$\frac{1}{r_{\text{Eq}}} = \sum_{i=1}^{n} \frac{1}{r_i}$$

and

$$V_{\text{source}} = E - r_{\text{Eq}} I.$$

Subsection 5.2

Loads

An electrical load is an electrical component or portion of a circuit that consumes electric power. Electric loads are represented by a counter electromotive force e and internal resistance r' (with the exception of resistors)

All formulas for generators are the same as loads with the exception of the efficiency

$$\eta = 1 - \frac{r'I}{U}.$$