Probability Notes

$$\mathbb{E}[X] = \mu = \sum_{x_i \in \Omega} x_i \, \mathbb{P}\, x_i$$
$$\mathbb{E}[g(X)] = \sum_{x_i \in \Omega} g(x) \; \mathbb{P}\, x_i$$
$$\mathbb{E}[aX + b\, g(X) + c] = a\,\mathbb{E}[X] + b\,\mathbb{E}[g(X)] + c$$
$$\operatorname{Var}[X] = \sigma^2 = \mathbb{E}\left[(X-\mu)^2\right]$$
$$\operatorname{Var}[aX + b] = a^2 \operatorname{Var}[X]$$
$$\operatorname{Var}[X] = \mathbb{E}\left[X^2\right] - \mu^2$$

Common Probability Distributions

- Binomial Distribution:

$$X \sim \operatorname{Bin}(n,p) \Rightarrow \begin{cases} \mathbb{P}(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \\ \mathbb{E}[X] = np \\ \operatorname{Var}[X] = np(1-p) \end{cases}$$

- Poisson Distribution:

$$X \sim \operatorname{Poi}(\lambda) \Rightarrow \begin{cases} \mathbb{P}(X=x) = e^{-\lambda} \frac{\lambda^x}{x!} \\ \mathbb{E}[X] = \lambda \\ \operatorname{Var}[X] = \lambda \end{cases}$$

- Geometric Distribution:

$$X \sim \operatorname{Geo}(p) \Rightarrow \begin{cases} \mathbb{P}(X=x) = p(1-p)^{x-1} \\ \mathbb{E}[X] = \frac{1}{p} \\ \operatorname{Var}[X] = \frac{1-p}{p^2} \end{cases}$$

Transforms

- MGF:

$$M(s) = \mathbb{E}\left[e^{sX}\right]$$
$$\mathbb{E}[X^n] = M^{(n)}(0)$$
$$\operatorname{Var}[X] = M''(0) - M'(0)^2$$

- PGF:

$$G(z) = \mathbb{E}\left[z^X\right]$$
$$\mathbb{E}[X] = G'(1)$$
$$\operatorname{Var}[X] = G''(1) + G'(1) - G'(1)^2$$

Conditional Probability

$$\mathbb{P}_{Y|x}(y) = \frac{\mathbb{P}_{XY}(x,y)}{\mathbb{P}_X(x)}$$

Template by Michael Müller, source code can be found at `https://github.com/cmichi/latex-template-collection`