Electricity 2

Contents

PART

I

Electric Current

Section 1

Current and Current Density

Electric charge in motion constitutes an electric current. In the steady flow of charge in a wire of cross-sectional area A, the total charge passing through this area in unit time is defined to be the electric current I at this place. If a total charge q flows through this area in time t, the current I is given by:

$$I = \frac{q}{t}. \tag{1.1}$$

where I is measured in Ampere. Sometimes it is also convenient to look at the density of the current we're observing over a certain area A called *current density*

$$J = \frac{I}{A}. \tag{1.2}$$

If the area in which the current is running through changes at points 1 and 2, the current between those 2 points remain constant ($I_1 = I_2$) while the current density changes to become

$$\frac{J_1}{J_2} = \frac{A_2}{A_1}.$$

If a positively charged cloud of n particles moves through a space with speed v and charge q, we find that[1]

[1] q can be positive or negative

$$\vec{\mathbf{J}} = nq\vec{\mathbf{v}} = \varrho_\tau \vec{\mathbf{v}}. \tag{1.3}$$

The total current through a *slice* of space mentioned prior it calculated using

$$I = \iint_A \vec{\mathbf{J}}\,\mathrm{d}A\,. \tag{1.4}$$

The volume element here is $\vec{\mathbf{v}}\,\mathrm{d}A\,\mathrm{d}t = \mathrm{d}\tau$

Section 2

Continuity Law

The continuity law for current is

$$\boldsymbol{\nabla} \cdot \left(\vec{\mathbf{J}} + \vec{\mathbf{J}}_c\right) = -\frac{\partial \varrho_\tau}{\partial t}. \tag{2.1}$$

Where
$\vec{\mathbf{J}}$ is the conduction current density.
$\vec{\mathbf{J}}_c = \varrho_\tau \vec{\mathbf{v}}_\tau$ is the convection current density.
$\vec{\mathbf{v}}_\tau$ is the velocity of the volume containing the particles.
There are 2 particular cases for the equation above

1. The volume is not moving ($\vec{\mathbf{v}}_\tau = 0 \implies \vec{\mathbf{J}}_c = 0$). The continuity law of static structures is

$$\boldsymbol{\nabla} \cdot \vec{\mathbf{J}} = -\frac{\partial \varrho_\tau}{\partial t}. \tag{2.2}$$

2. The volume is not moving and we are in a steady state, so the charge density of the particles doesn't change much as they are static so $\partial \varrho_\tau / \partial t = 0$ so

$$\oiint \vec{\mathbf{J}} \, \mathrm{d}A = 0. \tag{2.3}$$

PART
II

Ohm's and Joule's Law

Section 3

Electrical Mobility and Conductivity

The velocity of a charge carrier in in an electric field $\vec{\mathbf{E}}$ is

$$\vec{\mathbf{v}} = \mu \vec{\mathbf{E}}. \tag{3.1}$$

where μ is the mobility of the charge carrier.

The current density vector in a conductor with conductivity σ is said to be

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}. \tag{3.2}$$

Ohm's law(doesn't really need an introduction)

$$U = RI. \tag{3.3}$$

where we can find R using

$$R = \frac{\rho l}{A}. \tag{3.4}$$

where A is the surface area of the resistor and l is the length of the conductor[2].

[2] $\sigma = \dfrac{1}{\rho}$

Section 4

Resistance

The symbol of a resistive conductor is represented by

$$P = UI \tag{4.1}$$
$$W = Pt \tag{4.2}$$

Power loss due to Joule's law

$$P = I^2R = \frac{U^2}{R} \tag{4.3}$$
$$W = I^2Rt \tag{4.4}$$

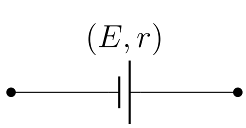
Section 5

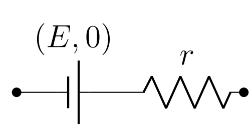
Electric Ciruits

Subsection 5.1

Power Sources/Generators

A DC source is characterized by their electromotive force E and internal resistance r





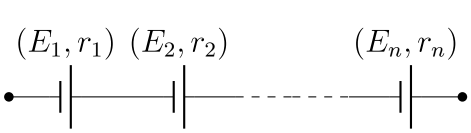
The voltage supplied by the source is

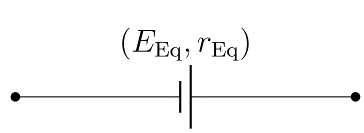
$$V_{\text{source}} = E - rI.$$

and it's efficiency is

$$\eta = 1 - \frac{rI}{E}.$$

5.1.1 In Series

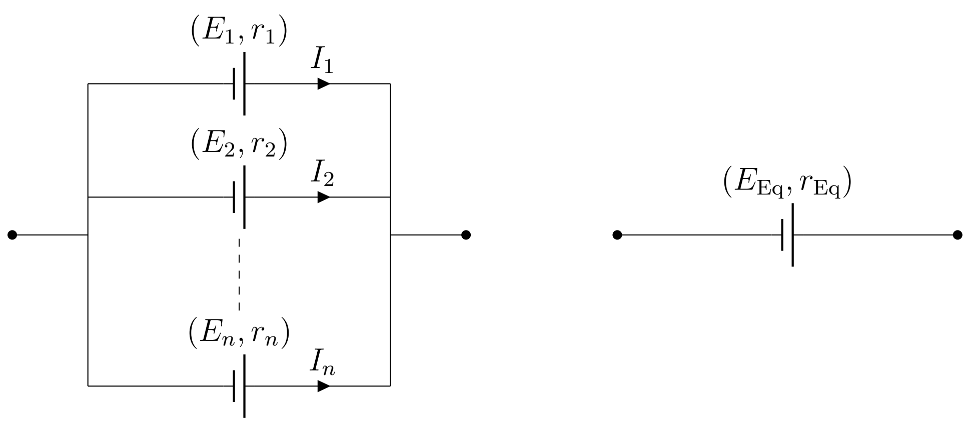




where

$$E_{\text{Eq}} = \sum_{i=1}^{n} E_i$$
$$r_{\text{Eq}} = \sum_{i=1}^{n} r_i$$

5.1.2 In Parallel



In case of *identical* sources:

$$I = \sum_{i=1}^{n} I_i$$
$$\frac{1}{r_{\text{Eq}}} = \sum_{i=1}^{n} \frac{1}{r_i}$$

and

$$V_{\text{source}} = E - r_{\text{Eq}} I.$$

Subsection 5.2

Loads

An electrical load is an electrical component or portion of a circuit that consumes electric power. Electric loads are represented by a counter electromotive force e and internal resistance r' (with the exception of resistors)

All formulas for generators are the same as loads with the exception of the efficiency

$$\eta = 1 - \frac{r' I}{U}.$$

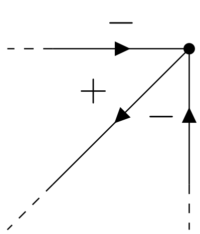
Circuits Analysis Techniques

PART III

Terminology:
Node A point where two or more circuit elements join.
Essential node A node where three or more circuit elements join.
Branch A path that connects two nodes
Loop A path whose last node is the same as the starting node
Mesh A loop that does not enclose any other loops

- **KVL**: the algebraic sum of all the voltages around any closed path in a circuit equals zero.
- **KCL**: the algebraic sum of all the currents at any node in a circuit equals zero.

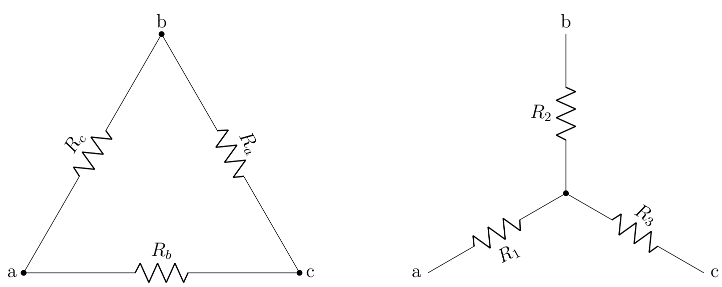
To use Kirchhoff's current law, an algebraic sign corresponding to a reference direction must be assigned to every current at the node. Assigning a positive sign to a current leaving a node requires assigning a negative sign to a current entering a node.



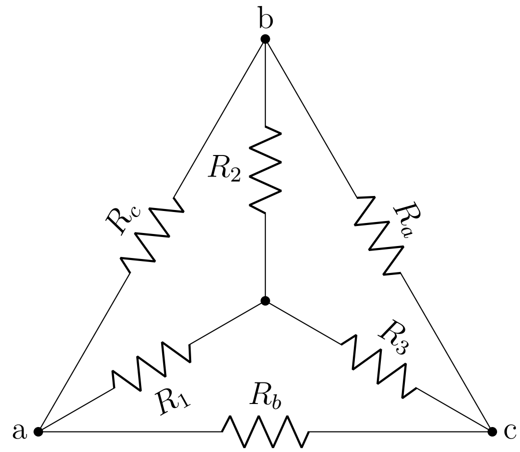
Section 6

Wye-Delta and Delta-Wye Transformations

This transformation of a set of resistors configured in the shape of the letter Δ to a configuration of a shape of the letter Y.



To find the equivalent resistances we apply



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Section 7

Divider Circuits

Subsection 7.1

Voltage Divider

At timesespecially in electronic circuitsdeveloping more than one voltage level from a single voltage supply is necessary. One way of doing this is by using a voltage-divider circuit.

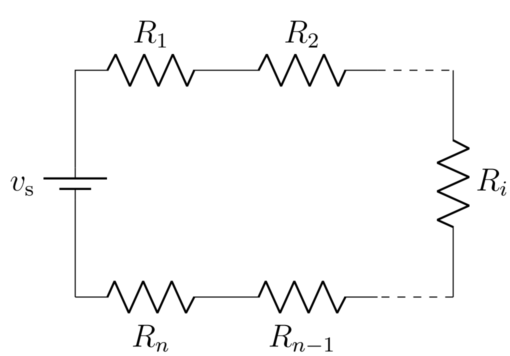


Figure 3. An n resistor voltage divider

$$v_i = v_s \frac{R_i}{\sum_{j=1}^{n} R_j}.$$

Subsection 7.2

Current Divider

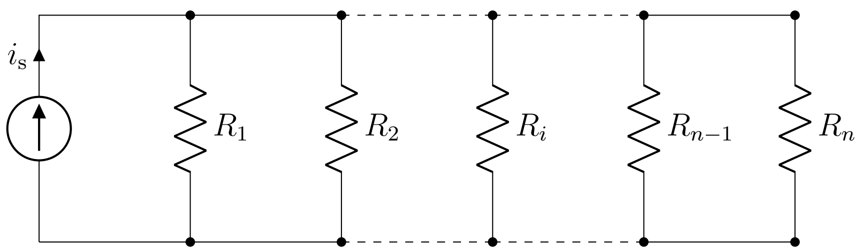


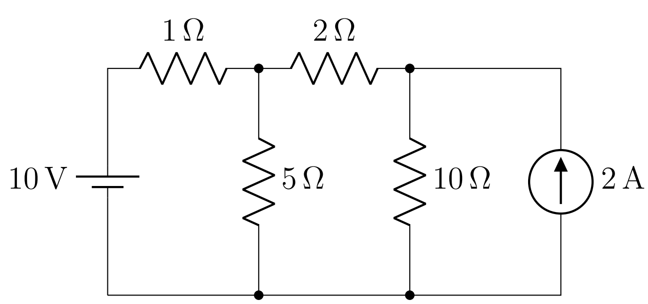
Figure 3. An n resistor current divider

$$i_i = i_s \frac{R_{Eq}}{R_i}.$$

Section 8

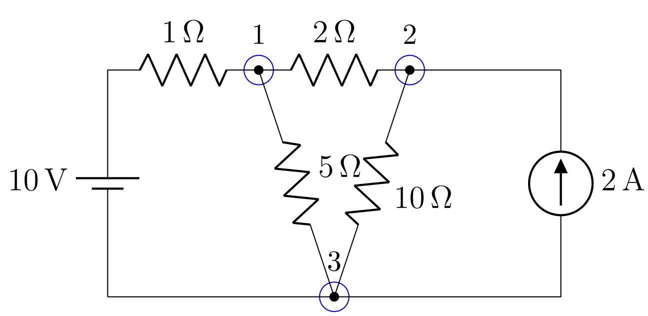
Node Voltage Method

We introduce the node-voltage method by using the essential nodes of the circuit.
To better understand node voltage method, we will apply directly on a circuit. Consider the following circuit:

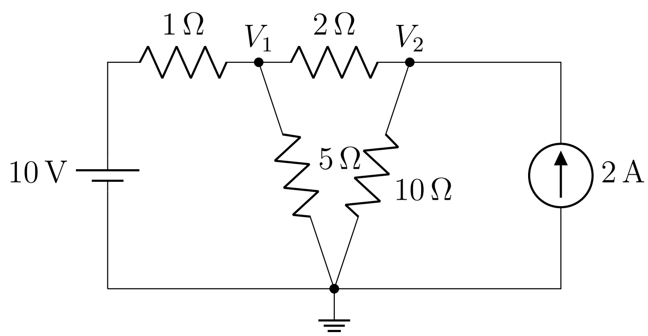


To find the voltages across the resistors are

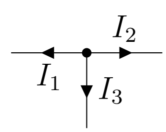
1. Assign the essential nodes (nodes with 3 or more branches):



2. Choose a reference node: select one of the essential nodes to be a reference. Usually we choose the lowest node. The assign voltages to the other nodes.

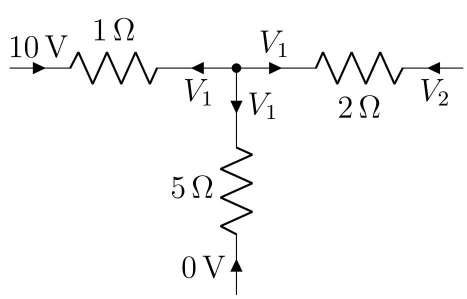


3. Apply KCL at each node to determine the voltage. For example let's look at node 1:
   Assume that there are currents leaving the node from all directions and that carry the voltage V_1 with them where by KCL



$$I_1 + I_2 + I_3 = 0.$$

   We let these currents carry our voltage V_1 with them, now we find the values of the currents. So the equation for node 1 is



$$\frac{V_1 - 10}{1} + \frac{V_2}{5} + \frac{V_1 - V_2}{2} = 0.$$

   Similarly for node 2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0.$$

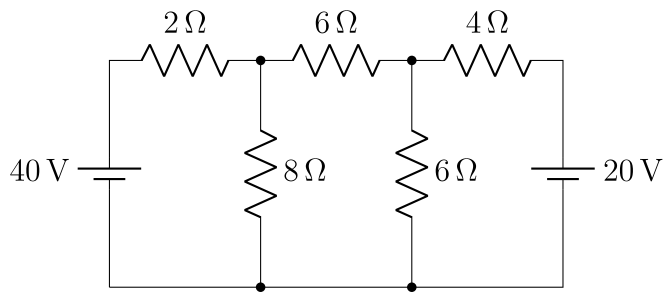
   By simply solving the equations we get

$$V_1 = 9.09\,\mathrm{V}$$
$$V_2 = 10.91\,\mathrm{V}$$

Section 9

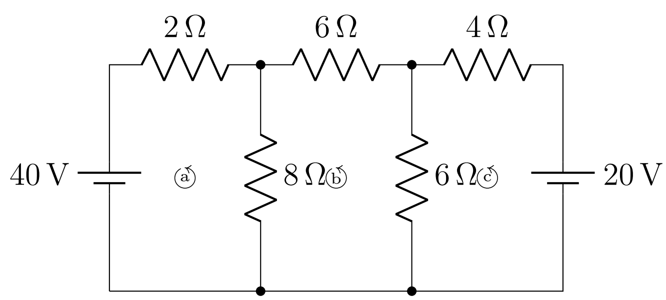
Mesh Current Method

We introduce the mesh current method by using the meshes of the circuit. To better understand mesh current method, we will apply directly on a circuit. Consider the following circuit:



Steps:

1. Assign the meshes (loop with no other loop inside): We have 3 meshes: a, b, and c
2. Define a current running in each mesh to be flowing in the counter clock wise direction.
3. Apply KVL in each mesh to find the currents.



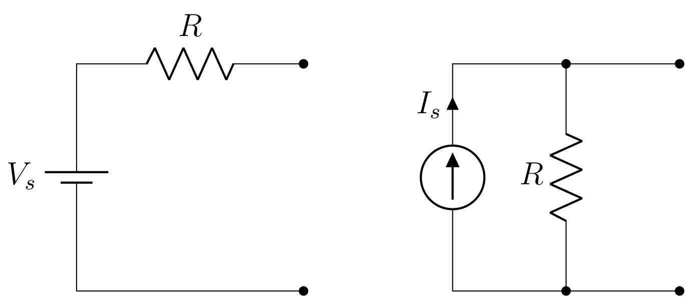
- Mesh a: $-40 + 2I_a + 8(I_a - I_b) = 0$
- Mesh b: $8(I_b - I_a) + 6I_b + 6(I_b - I_c) = 0$
- Mesh c:$6(I_c - I_b) + 4I_c + 20 = 0$

By solving the equations we get that

$$\begin{aligned} I_a &= 5.6\,\mathrm{A} \\ I_b &= 2\,\mathrm{A} \\ I_c &= -0.8\,\mathrm{A} \end{aligned} .$$

Section 10

Source Transformations


R
V_s
I_s
R