

Electric Machines

Semester 6

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Chapter 1

Transformers

1.1 Introduction

- A transformer is a static device that transfers electrical energy from one circuit to another through inductively coupled conductors.
- It is used to increase or decrease the voltage level of an AC signal.
- The transformer is based on the principle of electromagnetic induction.
- The transformer is a passive device, meaning it does not require any external power source to operate.
- The transformer is used in power distribution systems to step up or step down the voltage level.
- The transformer is used in electronic circuits to isolate the input and output signals.

1.2 Ideal Transformer

- The primary and secondary windings have zero resistance.

$$R_1 = R_2 = 0.$$

- The core has infinite permeability.

$$\mu = \infty.$$

- The transformer has no leakage flux.
- The transformer has no hysteresis loss.
- The transformer has no eddy current loss.

$$\begin{aligned}\mathcal{R}_{p_{\text{core}}} &= 0 \\ \Delta P &= 0 \\ \Delta P_j &= 0\end{aligned}$$

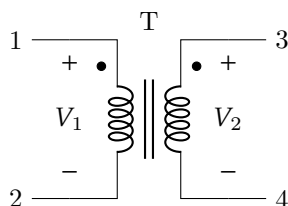


Figure 1.1: Ideal Transformer

$$\text{Turns Ratio: } m = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

1.3 Real Transformer

We define the voltage equations of the transformer as follows:

$$\dot{V}_1 + \dot{E}_1 = \dot{I}_1 Z_1 \quad (1.1)$$

$$\dot{E}_2 - \dot{V}_2 = \dot{I}_2 Z_2 \quad (1.2)$$

$$E_1 = 4.44fN_1\Phi_m$$

$$E_2 = 4.44fN_2\Phi_m$$

If we consider $\dot{I}_1 Z_1 \ll \dot{V}_1$ and $\dot{I}_2 Z_2 \ll \dot{V}_2$, we can simplify the equations as follows:

$$\dot{V}_1 = -\dot{E}_1$$

$$\dot{V}_2 = \dot{E}_2$$

We define the magnetization current as the current required to produce the magnetic flux in the core.

$$I_m = \dot{I}_{10} = I_1 + \underbrace{mI_2}_{I'_2}.$$

We define the equivalent circuit of the transformer as follows:

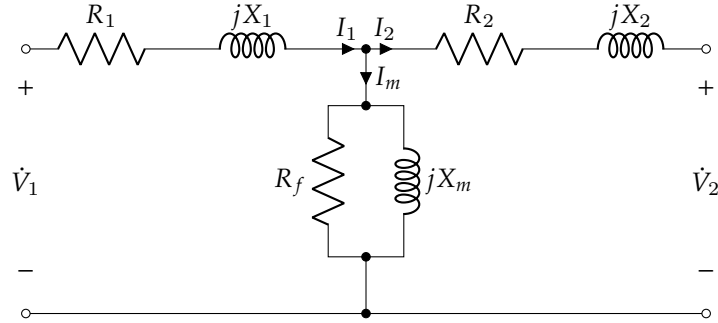


Figure 1.2: Transformer Equivalent Circuit

1.3.1 Γ Equivalent Circuit

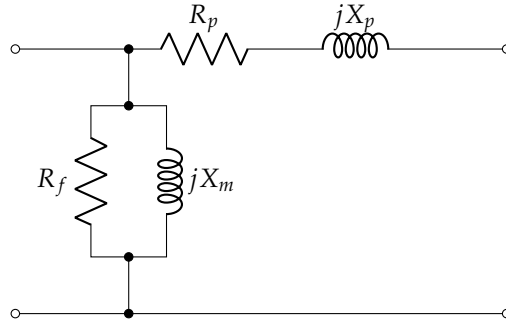


Figure 1.3: Γ Equivalent Circuit with the Primary Side as the Reference

$$R_p = R_1 + R'_2 = R_1 + \frac{R_2}{m^2}$$

$$X_p = X_1 + X'_2 = X_1 + \frac{X_2}{m^2}$$

$$Z_p = R_p + jX_p$$

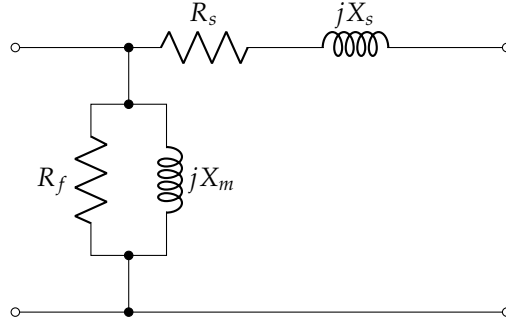


Figure 1.4: Γ Equivalent Circuit with the Secondary Side as the Reference

$$R_s = m^2 R_1 + R_2 = m^2 R_1 + R_2$$

$$X_s = m^2 X_1 + X_2 = m^2 X_1 + X_2$$

$$Z_s = R_s + jX_s$$

1.3.2 κ Equivalent Circuit

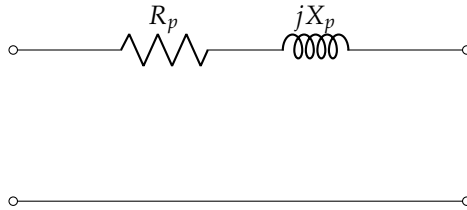


Figure 1.5: κ Equivalent Circuit with the Primary Side as the Reference

1.4 Transformer Tests

1.4.1 Open Circuit Test

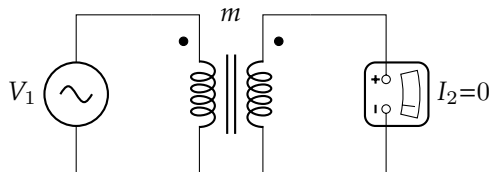


Figure 1.6: Open Circuit Test

We measure V_{10} , I_{10} , P_{10} and V_{20} .

We can calculate the following:

$$\begin{aligned}
Q_{10} &= V_{10} I_{10} \sin \varphi_o \\
P_{10} &= V_{10} I_{10} \cos \varphi_o \\
\cos \varphi_o &= \frac{P_{10}}{V_{10} I_{10}} \\
S_{10} &= V_{10} I_{10} \\
R_f &= \frac{V_{10}^2}{P_{10}} \\
X_m &= \frac{V_{10}^2}{Q_{10}}
\end{aligned}$$

1.4.2 Short Circuit Test

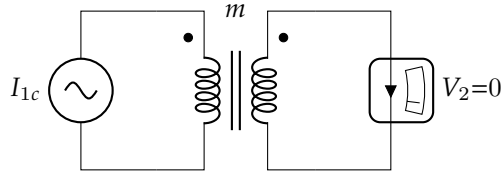


Figure 1.7: Short Circuit Test

Typically we run the short circuit test at 5 – 15% of the nominal current.

$$\begin{aligned}
I_{2c} &\leq I_{2n} \\
V_{1c} &= (5 - 15)\% V_{1n} \\
I_{2c} &\ll I_{2n} \\
I_{1c} &\ll I_{1n}
\end{aligned}$$

We measure V_{1c}, I_{1c}, P_{1c} and I_{2c} .

We can calculate the following:

$$\begin{aligned}
R_p &= \frac{P_{1c}}{I_{1c}^2} \\
X_p &= \frac{Q_{1c}}{I_{1c}^2}
\end{aligned}$$

1.5 Variation of the Voltage of Conduction

1. Exact Calculation:

$$\begin{aligned}
V_2 &< V_{20} \\
m_c &= \frac{V_2}{V_1} < m
\end{aligned}$$

$$V_1 = V_2' + \Delta V_p = V_2' + I_2' (R_p + jX_p) .$$

2. Approximate Calculation (Variation of the Secondary Voltage):

$$\Delta V_s = V_{20} - V_2.$$

$$\Delta V_p = \frac{\Delta V_s}{m}.$$

$$\begin{aligned}\Delta V_s \% &= \frac{V_{20} - V_2}{V_{20}} \times 100 \\ &= \frac{mV_1 - V_2}{mV_1} \times 100 \\ &= \frac{V'_1 - V_2}{V'_1} \times 100\end{aligned}$$

The KAPP formula states

$$\Delta V_s = I_2 (R_s \cos \varphi_2 \pm X_s \sin \varphi_2).$$

The \pm sign depends on the nature of the load (inductive or capacitive).

From the KAPP formula, we define the voltage regulation as

$$\Sigma = \frac{\Delta V_s}{V_2} \times 100.$$

$$\Delta V_p = I_1 (R_p \cos \varphi_2 \pm X_p \sin \varphi_2).$$

1.6 Vector Diagram of the Transformer

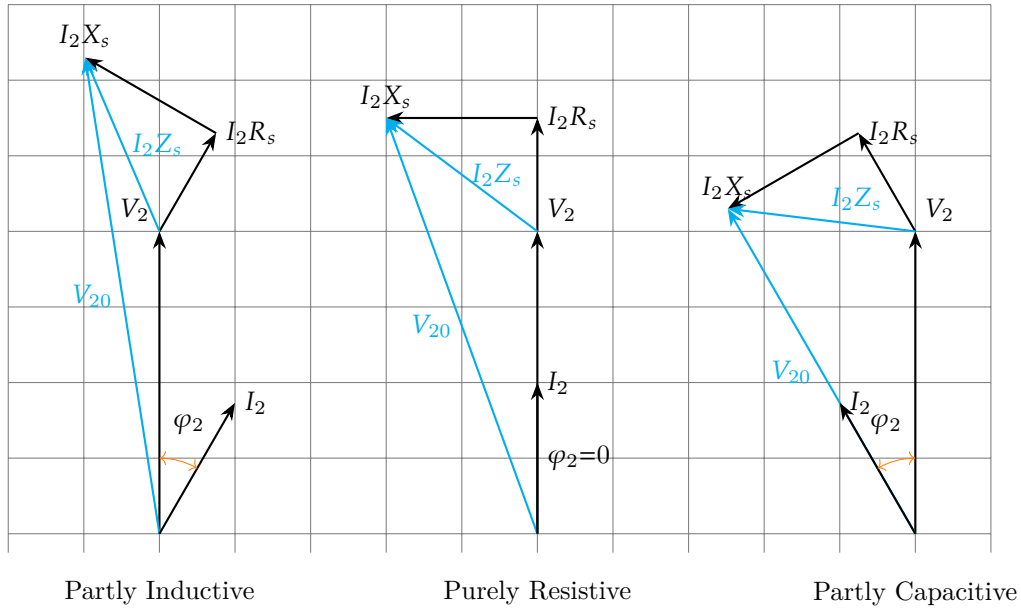


Figure 1.8: Vector Diagram of a Transformer

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + \Delta P_{Jt} + \Delta P_f}.$$

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Where

$$\Delta P_{jt} = \Delta P_{j1} + \Delta P_{j2} = I_{11}^2 R_p = I_{21}^2 R_s$$

$$\Delta P_f = P_{10} = P_{20}$$

Example 1.6.1

Given a single phase transformer 200/400V - 50 Hz. By tests, we measure the following:

- Open Circuit Test: $V_{10} = 200 \text{ V}$, $I_{10} = 0.7 \text{ A}$, $P_{10} = 70 \text{ W}$.
 - Short Circuit Test: $V_{2c} = 15 \text{ V}$, $I_{1c} = 10 \text{ A}$, $P_{1c} = 85 \text{ W}$.
1. Calculate the equivalent circuit parameters.
 2. Calculate V_2 if the charge is 5 kW, $\cos \varphi = 0.8$, and $V_1 = 200 \text{ V}$. Deduce the voltage regulation (Σ).
 3. Deduce the efficiency of the transformer.

Solution: $m = 2$

1.

$$R_f = \frac{V_{10}^2}{P_{10}} = \frac{200^2}{70} = 571.43 \Omega$$

$$S_{10} = V_{10} I_{10} = 200 \times 0.7 = 140 \text{ V A}$$

$$Q_{10} = \sqrt{S_{10}^2 - P_{10}^2} = \sqrt{140^2 - 70^2} = 121.6 \text{ VAR}$$

$$X_m = \frac{V_{10}^2}{Q_{10}} = \frac{200^2}{121.6} = 328.95 \Omega$$

To find R_p and X_p , we need to find R_s and X_s first.

$$R_s = \frac{P_{2c}}{I_{1c}^2} = \frac{85}{100} = 0.85 \Omega$$

$$S_{2c} = V_{2c} I_{1c} = 15 \times 10 = 150 \text{ V A}$$

$$Q_{2c} = \sqrt{S_{2c}^2 - P_{1c}^2} = \sqrt{150^2 - 85^2} = 123 \text{ VAR}$$

$$X_s = \frac{Q_{2c}}{I_{1c}^2} = \frac{123}{100} = 1.23 \Omega$$

$$R_p = \frac{R_s}{m^2} = \frac{0.85}{4} = 0.2125 \Omega$$

$$X_p = \frac{X_s}{m^2} = \frac{1.23}{4} = 0.3075 \Omega$$

2.

$$I_2 = \frac{P_2}{V_2 \cos \varphi} = \frac{5000}{400 \times 0.8} = 15.625 \text{ A}$$

$$\Delta V_s = I_2 (R_s \cos \varphi \pm X_s \sin \varphi) = 15.625 (0.85 \times 0.8 + 1.23 \times 0.6) = 22.2 \text{ V}$$

$$\Sigma = \frac{\Delta V_s}{V_2} \times 100 = \frac{22.2}{400} \times 100 = 5.55\% \quad \text{If } V_{20} = 400 \text{ V.}$$

To obtain $V_2 = 400 \text{ V}$, we need to increase the primary voltage to

$$\Delta V_p = \frac{\Delta V_s}{m} = \frac{22.2}{2} = 11.1 \text{ V}$$

$$V_1 = V_2' + \Delta V_p = \frac{400}{2} + 11.1 = 211.1 \text{ V}$$

3.

$$\Delta P_{Jt} = I_{11}^2 R_p = 15.6^2 \times 0.85 = 206 \text{ W}$$

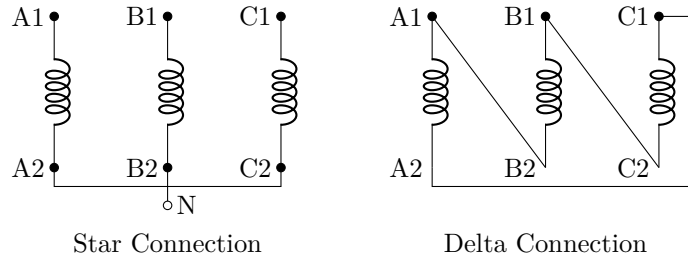
$$\Delta P_f = P_{10} = P_{20} = 70 \text{ W}$$

$$\eta = \frac{5000}{5000 + 206 + 70} = 0.947$$

Chapter 2

Three Phase Transformers

A three phase transformer is a transformer that consists of three single phase transformers connected together. The three single phase transformers are connected in a delta or star configuration. They can have 4 different connection configurations: λ/λ , λ/Δ , Δ/λ , and Δ/Δ



For a λ/λ connection

$$\lambda/\lambda : \quad M = \frac{U_{2L}}{U_{1L}} = \frac{\sqrt{3}V_{2ph}}{\sqrt{3}V_{1ph}} = m = \frac{n_2}{n_1}$$

$$\lambda/\Delta : \quad M = \frac{U_{2L}}{U_{1L}} = \frac{V_{2ph}}{\sqrt{3}V_{1ph}} = \frac{m}{\sqrt{3}} = \frac{n_2}{\sqrt{3}n_1}$$

$$\Delta/\lambda : \quad M = \frac{U_{2L}}{U_{1L}} = \frac{\sqrt{3}V_{2ph}}{V_{1ph}} = \sqrt{3}m = \sqrt{3}\frac{n_2}{n_1}$$

$$\lambda : \quad I_{ph} = I_1 \quad ; \quad V_{ph} = \frac{U_L}{\sqrt{3}}$$

$$\Delta : \quad I_{ph} = \frac{I_1}{\sqrt{3}} \quad ; \quad V_{ph} = U_L$$

$$P = 3V_{ph}I_{ph} \cos \varphi = \sqrt{3}V_{ph}I_{ph} \cos \varphi$$

$$Q = 3V_{ph}I_{ph} \sin \varphi = \sqrt{3}V_{ph}I_{ph} \sin \varphi$$

$$S = 3V_{ph}I_{ph} = \sqrt{3}V_{ph}I_{ph}$$