Electricity 2

Contents

PART

I

Electric Current

Section 1

Current and Current Density

Electric charge in motion constitutes an electric current. In the steady flow of charge in a wire of cross-sectional area A, the total charge passing through this area in unit time is defined to be the electric current I at this place. If a total charge q flows through this area in time t, the current I is given by:

$$I = \frac{q}{t}. \tag{1.1}$$

where I is measured in Ampere. Sometimes it is also convenient to look at the density of the current we're observing over a certain area A called *current density*

$$J = \frac{I}{A}. \tag{1.2}$$

If the area in which the current is running through changes at points 1 and 2, the current between those 2 points remain constant ($I_1 = I_2$) while the current density changes to become

$$\frac{J_1}{J_2} = \frac{A_2}{A_1}.$$

If a positively charged cloud of n particles moves through a space with speed v and charge q, we find that[1]

[1] q can be positive or negative

$$\vec{\mathbf{J}} = nq\vec{\mathbf{v}} = \varrho_\tau \vec{\mathbf{v}}. \tag{1.3}$$

The total current through a *slice* of space mentioned prior it calculated using

$$I = \iint_A \vec{\mathbf{J}}\, \mathrm{d}A\,. \tag{1.4}$$

The volume element here is $\vec{\mathbf{v}}\, \mathrm{d}A\, \mathrm{d}t = \mathrm{d}\tau$

Section 2

Continuity Law

The continuity law for current is

$$\boldsymbol{\nabla} \cdot \left(\vec{\mathbf{J}} + \vec{\mathbf{J}}_c\right) = -\frac{\partial \varrho_\tau}{\partial t}. \tag{2.1}$$

Where
$\vec{\mathbf{J}}$ is the conduction current density.
$\vec{\mathbf{J}}_c = \varrho_\tau \vec{\mathbf{v}}_\tau$ is the convection current density.
$\vec{\mathbf{v}}_\tau$ is the velocity of the volume containing the particles.
There are 2 particular cases for the equation above

1. The volume is not moving ($\vec{\mathbf{v}}_\tau = 0 \implies \vec{\mathbf{J}}_c = 0$). The continuity law of static structures is

$$\boldsymbol{\nabla} \cdot \vec{\mathbf{J}} = -\frac{\partial \varrho_\tau}{\partial t}. \tag{2.2}$$

2. The volume is not moving and we are in a steady state, so the charge density of the particles doesn't change much as they are static so $\partial \varrho_\tau / \partial t = 0$ so

$$\oiint \vec{\mathbf{J}} \, \mathrm{d}A = 0. \tag{2.3}$$