Numerical Analysis
Semester 4
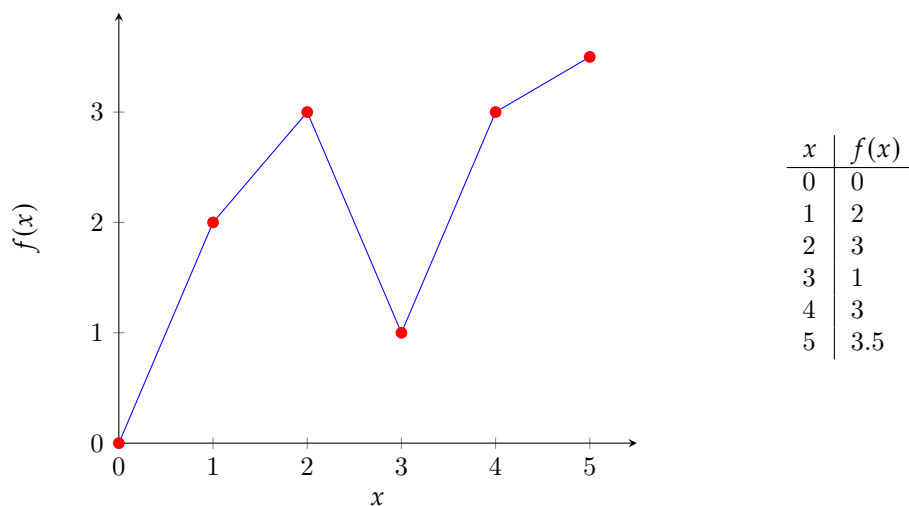
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Chapter 1

Interpolation

1.1 Linear Interpolation

x	$f(x)$
0	0
1	2
2	3
3	1
4	3
5	3.5

Linear interpolation is just drawing lines between the data points.

> **Definition 1.1.1: Linear Interpolation(lerp) equation**
>
> The equation of the lines between data points is
>
> $$y = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i) + y_i.$$
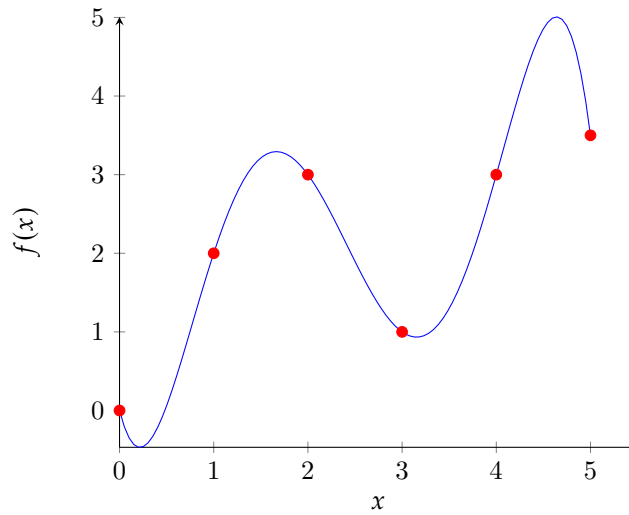
> **Theorem 1.1.1** Error due to linear interpolation
>
> Let f be a continuous and differentiable on $[a, b]$. We define the error $z(x)$ to be
>
> $$|z(x)| \leqslant \frac{(b-a)^2}{8} \sup_{a \leqslant x \leqslant b} |f''(x)|.$$

1.2 Polynomial Interpolation

1.2.1 Lagrange Polynomials

Really nice video here explaining Lagrange polynomials.

Theorem 1.2.1 Lagrange polynomial equation

Consider a set of n points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. The Lagrange polynomial for this set of data is

$$L(x) = \sum_{k=0}^{n} y_k \ell_k(x).$$

where

$$\ell_k(x) = \prod_{\substack{i=1 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}.$$

1.2.2 Newton Polynomial

Definition 1.2.1: Newton Polynomial equation

Consider a set of n points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. The Newton polynomial for this set of data is

$$p_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n \prod_{i=0}^{n-1}(x - x_i).$$

where

$$a_i = f[x_0, x_1, \ldots, x_i].$$

Here $f[\ldots]$ is the divided difference of the inputted data.

The divided difference has 2 formulas, the recurrence formula

$$f[x_0, x_1, \ldots, x_{n+1}] = \frac{f[x_1, x_2, \ldots, x_{n+1}] - f[x_0, x_1, \ldots, x_n]}{x_{n+1} - x_0}.$$

and a general formula

$$f[x_0, x_1, \ldots, x_n] = \sum_{i=1}^{n} \frac{y_i}{\prod_{\substack{k=0 \\ k \neq i}}^{n}(x_i - x_k)}.$$

Now forget you ever saw those cause there is an easier method to finding the divided difference.

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Divided Difference Table

$$x_0 \quad y_0$$

$$\frac{y_1-y_0}{x_1-x_0} = f[x_0,x_1]$$

$$x_1 \quad y_1 \qquad\qquad \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$$

$$\frac{y_2-y_1}{x_2-x_1} = f[x_1,x_2] \qquad \cdots$$

$$x_2 \quad y_2 \qquad\qquad \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1} \qquad \cdots$$

$$\frac{y_3-y_2}{x_3-x_2} = f[x_2,x_3] \qquad \cdots$$

$$x_3 \quad y_3 \qquad\qquad \frac{f[x_3,x_4]-f[x_2,x_3]}{x_4-x_2}$$

$$\frac{y_4-y_3}{x_4-x_3} = f[x_3,x_4]$$

$$x_4 \quad y_4$$

After we have constructed the table we can find the divided difference we want by looking at the top diagonal

$$x_0 \quad y_0$$

$$f[x_0,x_1]$$

$$x_1 \quad y_1 \qquad\qquad f[x_0,x_1,x_2]$$

$$\cdots \qquad\qquad f[x_0,x_1,x_2,x_3]$$

$$x_2 \quad y_2 \qquad\qquad \cdots \qquad\qquad f[x_0,x_1,x_2,x_3,x_4]$$

$$\cdots \qquad\qquad \cdots$$

$$x_3 \quad y_3 \qquad\qquad \cdots$$

$$\cdots$$

$$x_4 \quad y_4$$

Example 1.2.1 (Yes)

I wanna do an example for the data I used at the top but it's 1AM so cowabunga it is

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