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Source: *The Journal of Finance*, Jun., 1985, Vol. 40, No. 2 (Jun., 1985), pp. 455-480

Published by: Wiley for the American Finance Association

Stable URL: https://www.jstor.org/stable/2327895

THE JOURNAL OF FINANCE • VOL. XL, NO. 2 • JUNE 1985

Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 through August 31, 1978

MARK RUBINSTEIN*

ABSTRACT

The tests reported here differ in several ways from those of most other papers testing option pricing models: an extremely large sample of observations of both trades and bid-ask quotes is examined, careful consideration is given to discarding misleading records, nonparametric rather than parametric statistical tests are used, reported results are not sensitive to measurement of stock volatility, special care is taken to incorporate the effects of dividends and early exercise, a simple method is developed to test several option pricing formulas simultaneously, and the statistical significance and consistency across subsamples of the most important reported results are unusually high. The three key results are: (1) short-maturity out-of-the-money calls are priced significantly higher relative to other calls than the Black-Scholes model would predict, (2) striking price biases relative to the Black-Scholes model are also statistically significant but have reversed themselves after long periods of time, and (3) no single option pricing model currently developed seems likely to explain this reversal.

ALTHOUGH WIDELY USED AMONG option traders, the Black-Scholes option pricing formula is often reported to produce model values which differ in systematic ways from market prices. An example might be the overpricing of out-of-the-money near-maturity calls relative to at-the-money middle-maturity calls on the same underlying stock.

These reports have stimulated interest in alternative option pricing formulas. While several assumptions underlying the Black-Scholes analysis can be questioned, emphasis has focused on the properties of the stochastic process followed by the underlying stock. Black and Scholes made two important assumptions about the stock price dynamics: first, the stock price follows a continuous path through time and second, the instantaneous volatility of the stock rate of return is nonstochastic (at most, a function of time).

* Graduate School of Business Administration, University of California at Berkeley. The author would like to thank the Chicago Board Options Exchange and Interactive Data Corporation for supplying the data necessary to undertake this study, and the Institute for Quantitative Research in Finance for funding. The original version of this paper contained only an empirical analysis of the *statistical* significance of deviations of market prices from Black-Scholes values. Special thanks are due to Kamal Duggirala who, while he was a student at Berkeley, independently replicated the results of the original version of the paper and went on to provide the supplemental analysis of *economic* significance.

Both assumptions have been relaxed in the subsequent literature which has produced five competing alternatives:

1. Pure jump model (Cox and Ross [4])
2. Mixed diffusion-jump model (Merton [11])
3. Constant elasticity of variance diffusion model (Cox and Ross [5])
4. Compound option diffusion model (Geske [7])
5. Displaced diffusion model (Rubinstein [14])

The first two relax the continuity assumption, and the last three relax the volatility constancy assumption. Other models, particularly those combining jump movements with stochastic volatility, can easily be constructed.

Indeed, theoretical ingenuity has long since outrun definitive empirical knowledge. Most empirical work designed to test these alternatives[1] has suffered from a number of deficiencies including failure to provide a separate test of the mathematical structure of option pricing formulas disjoint from problems of input measurement, severe limitations created by use of closing option and stock prices, and limited samples of calendar time or underlying stocks. As a result, despite the theoretical advances, the state of our empirical knowledge is in considerable disarray. The reports of biases from Black-Scholes values both from traders and from academic studies are conflicting. At present, no consensus exists.

Measurement of differences between Black-Scholes values and market prices requires knowledge of:

1. Option market price
2. Simultaneous underlying stock price
3. Time to expiration
4. Striking price
5. Rate of interest through expiration date
6. Future cash dividends prior to expiration
7. Future ex-dividend dates prior to expiration
8. Volatility of underlying stock

In this study, I shall argue that measurement problems associated with variables 3 through 8 can usually be easily overcome. The only important difficulties lie with variables 1 and 2. For many applications of option pricing formulas, measurement of the stock price volatility is the most significant difficulty. This study does not have this problem since it only draws conclusions about pairs of options that differ only by striking price or time to expiration. To do this, it is only necessary to compare the implied volatilities of each pair of options. The implied volatility, using the Black-Scholes formula, can be determined from variables 1 through 7. While time-to-expiration bias can be explained by compensating shifting stock volatility, a time-period consistent time to expiration bias (which is observed here) is unlikely to have this explanation.

The use of even daily closing prices poses several empirical problems including nonsimultaneity of option and stock closes, a bid-ask spread which can be a

[1] See Phillips and Smith [12] for a detailed criticism of this empirical work.

significant portion of the option's price, and artificial trading behavior at the close by floor traders to influence their margin requirements.

To surmount these and other difficulties, this study uses the CBOE's own record of all reported trades and quotes over a two-year period. This information, as originally received from the CBOE at Berkeley, occupies about 1,000 megabytes. The data are a time-stamped record to the nearest second, including option trade prices, quotes and volume, coupled with the stock price at the corresponding time during the day. In IBM, for example, the records are so dense that one is available for about each four-second interval during the day. The data were found to be of extraordinary quality primarily because of the CBOE computerized error check procedures immediately following input.

To put the great frequency of observations to best advantage, a subset of the data was created comprised only of those trades and quotes which indicated:

1. Depth of the market at reported prices
2. Narrow spread between bid and ask
3. Time period one of liquidity and short-run stability of stock price
4. Record was not reported near the beginning or ending of the trading day
5. Record was not the result of a late reported trade or a trade based on a contingency order
6. The corresponding option and stock prices associated in a record were, in fact, contemporaneous

The empirical tests reported here were performed only on this reduced set of data.

To circumvent possible objections to statistical methodology, nonparametric tests were used to identify significant signs of differences between Black-Scholes values and market prices. Unlike the more common (in the finance literature) parametric tests, these tests are "distribution-free" since they assume nothing about the population from which the observed sample is drawn. Correspondingly, these tests have much less power than their parametric cousins, but nevertheless may be sufficient to reject hypotheses with the large sample available for this study. In addition, parametric tests typically give greatest weight to outliers. If it is suspected that these may be noise, then nonparametric tests have the further advantage of symmetric weighting: all observations are weighted equally.

The null hypothesis is that option market prices and Black-Scholes values exhibit no systematic differences. If the null hypothesis is rejected by the nonparametric test, then we must either conclude that:

1. Inputs have been incorrectly measured, or
2. The options market is inefficient, or
3. The mathematical structure of the Black-Scholes formula is incorrect

We argue that the first possibility is extremely unlikely given the data base available and the way it has been used in this study. If, further, we suppose that the options market, although possibly sporadically inefficient, exhibits no systematic inefficiencies over periods of several months, then we will be forced to conclude that the trouble lies with the form of the Black-Scholes formula. Finally, by comparing observed biases with the predicted biases of alternative models, we may be able to reject the Black-Scholes formula in favor of a single competing theory.

In Section I, the biases from Black-Scholes values predicted by alternative models are examined. In Section II, the trade and quote data base and the criteria used to create a subset are described. Section III describes how this data was used to measure the eight variables necessary to perform the nonparmetric tests. In Section VI, the results of these tests, which reveal statistically significant biases of option prices from Black-Scholes values, are reported. Section V concludes with a comparison of the observed biases with those predicted by alternative models. Unfortunately, none of the alternative models seems capable of explaining the reversal of the striking price bias which occurred in October 1977.

I. Alternative Option Pricing Formulas

Table I gives an indication of the comparative properties of alternative option pricing formulas. All call values are based on an underlying current stock price of $50 and a discrete annualized rate of interest of 7 percent. The remaining parameters in each formula were chosen to yield an at-the-money ($50 striking price), middle maturity (120 days to expiration) call value of $3.97. For example, for the *Black-Scholes Formula*, this implied an annualized instantaneous volatility of stock rate of return of 30 percent. In every case, no cash dividends were assumed to be paid prior to expiration.

In addition to the current stock price, interest rate, striking price, and time to expiration, the *Displaced Diffusion Formula* requires two parameters: the current proportion α of the assets of the firm invested in risky assets and the instantaneous volatility of the rate of return on these assets. In one case, α has been set equal to 0.5 and in another to 1.5. In the latter case, the firm can be interpreted as financing additional investments in risky assets by borrowing.[2] To produce an at-the-money, middle-maturity call value of $3.97, it was necessary to set the annualized instantaneous volatility of stock rate of return equal to 61 percent when $\alpha = 0.5$ and equal to 19.93 percent when $\alpha = 1.5$.

For the *Compound Option Formula*, in addition to the current stock price, riskless interest rate, striking price, and time to expiration, it is necessary to specify the time to maturity, T, of the firm's pure discount debt, the debt-to-equity ratio (B/S), where the debt is measured by its face value, and the current instantaneous volatility of the stock rate of return. To allow the compound option formula to produce noticeably different values than the Black-Scholes formula, T was set equal to three years and B/S equal to 0.8. Again, to produce an at-the-money, middle-maturity call value of $3.97, it was necessary to choose a current annualized instantaneous stock[3] volatility of 29.85 percent.

The *Absolute Diffusion Formula* is the simplest case, other than the Black-

[2] Technically, α cannot be greater than one in the displaced diffusion model because the firm would then consist of assets which could fall in value arbitrarily close to zero and a fixed obligation of riskless debt. The inconsistency is surmounted by the compound option model which allows for default of the debt.

[3] As originally developed, the compound option formula requires as input the current *value of the firm, V*, and the instantaneous volatility of the rate of growth of the value of the firm, σ_V. Instead,

Table I

Comparative Option Values ($S = 50$, $\dot{r} = 1.07$, $\sigma = 0.3$)

Striking Price	Time to Expiration (days)														
	10	40	120	200	270	10	40	120	200	270	10	40	120	200	270
	Displaced Diffusion ($\alpha = 0.5$)					Black-Scholes					Displaced Diffusion ($\alpha = 1.5$)				
40	10.07	10.30	11.02	11.78	12.43	10.07	10.31	11.14	12.03	12.79	10.07	10.32	11.19	12.12	12.90
45	5.09	5.59	6.93	8.04	8.89	5.10	5.63	7.06	8.26	9.19	5.10	5.65	7.11	8.34	9.29
50	1.05	2.19	3.97	5.27	6.23	1.04	2.16	3.97	5.31	6.32	1.03	2.16	3.97	5.33	6.35
55	0.04	0.62	2.14	3.38	4.33	0.03	0.54	1.98	3.21	4.17	0.03	0.51	1.93	3.16	4.13
60	—	0.14	1.10	2.14	3.00	—	0.09	0.88	1.84	2.66	—	0.07	0.82	1.74	2.55
	Pure Jump ($\sigma = 0.3$, $\lambda = 1$)					True Formula					Compound Option ($T = 3$, $B/S = 0.8$)				
40	10.07	10.30	10.88	11.47	11.95						10.07	10.32	11.20	12.15	12.95
45	5.08	5.33	5.99	7.47	8.88						5.10	5.65	7.12	8.36	9.32
50	0.43	1.59	3.97	5.58	6.55			?			1.03	2.16	3.97	5.33	6.36
55	0.30	1.12	2.71	3.69	4.22						0.03	0.50	1.91	3.14	4.11
60	0.18	0.65	1.44	2.14	3.17						—	0.07	0.79	1.71	2.52
	Diffusion-Jump ($\lambda = 1$, $\gamma = 0.8$)										Absolute Diffusion				
40	10.14	10.53	11.58	12.58	13.42						10.07	10.33	11.28	12.28	13.11
45	5.19	5.78	7.33	8.71	9.79						5.10	5.69	7.20	8.45	9.42
50	0.76	1.88	3.97	5.60	6.83						1.04	2.16	3.97	5.32	6.34
55	0.13	0.59	2.09	3.54	4.70						0.02	0.47	1.83	3.01	3.95
60	0.09	0.38	1.29	2.35	3.32						—	0.05	0.68	1.51	2.26

Scholes formula, of a constant elasticity of variance diffusion model. Since it presumes the stock rate of return to be normally distributed, the formula requires the same inputs as the Black-Scholes formula, except that the discrete volatility of the stock rate of return replaces the instantaneous volatility. To yield an at-the-money, middle-maturity call value of \$3.97, the discrete volatility was set equal to 30 percent.

The *Pure Jump Formula*, in addition to the current stock price, interest rate, striking price, and time to expiration, requires the instantaneous stock volatility, σ, the expected number of jumps per year, λ, and the downward drift of the constant component of stock return. Setting $\sigma = 30$ percent and $\lambda = 1$ necessitated a drift of -25.1 percent.

The *Diffusion Jump Formula* (with zero drift of the continuous component) requires, in addition to the current stock price, interest rate, striking price, and time to expiration, the total instantaneous stock volatility, σ, the proportion, γ, of σ^2 attributable to the jump component, and the expected number of jumps per year, λ. Setting $\lambda = 1$ and $\gamma = 0.8$ necessitated setting $\sigma = 37.2$ percent to produce the \$3.97 value for the at-the-money, middle-maturity call.

To provide an easier visual comparison and since this study measures Black-Scholes volatilities implied from observed prices, a more useful way to compare the formula values in Table I is given in Table II. For each value in Table I, Table II reports its Black-Scholes implied volatility divided by 0.3. For example, the 120-day call with a striking price of 55 is worth \$1.98 according to the Black-Scholes formula and \$2.71 according to the Pure Jump formula. The Black-Scholes implied volatility for this option given a market price of \$1.98 is, of course, 0.3. However, the Black-Scholes implied volatility would be 0.366 if instead the market price were \$2.71. Dividing each of these by 0.3 yields the respective entries 1.00 and 1.22 in Table II.

We can use Table II to make loose, qualitative statements about the values of options produced by alternative formulas *relative to the values of those same options according to the Black-Scholes formula.*[4] To do this we will ask two types of questions:

1. Given a fixed striking price, how does the time to expiration affect the relative value of a call?
2. Given a fixed time to expiration, how does the striking price affect the relative value of a call?

For example, given a fixed striking price, reducing the time to expiration has little effect on the relative values of compound options, while it causes out- and

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one can input the current stock price, S, and the current instantaneous volatility of the stock rate of return, σ, and then solve numerically the following two simultaneous equations for V and σ_V:

$$S = VN(x) - Br^{-T}N(x - \sigma_V\sqrt{T})$$

$$\sigma = N(x)V/S\sigma_V$$

where $x \equiv \log(V/Br^{-T})/\sigma_V + \frac{1}{2}\sigma_V$. For empirically realistic parameter values, a multivariate Newton-Raphson search converges quickly to the solution.

[4] Here we use the fact that, other things equal, the higher the value of an option, the higher will be its implied volatility.

Table II

Comparative Implied Volatilities, ($S = 50$, $r = 1.07$, $\sigma = 0.3$)

Striking Price	Time to Expiration (days)														
	10	40	120	200	270	10	40	120	200	270	10	40	120	200	270
	Displaced Diffusion ($\alpha = 0.5$)					Black-Scholes					Displaced Diffusion ($\alpha = 1.5$)				
40	0.99	0.89	0.87	0.86	0.85	1.00	1.00	1.00	1.00	1.00	1.01	1.04	1.04	1.04	1.05
45	0.96	0.96	0.94	0.93	0.92	1.00	1.00	1.00	1.00	1.00	1.02	1.02	1.02	1.02	1.03
50	1.02	1.01	1.00	0.99	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
55	1.06	1.06	1.05	1.04	1.03	1.00	1.00	1.00	1.00	1.00	0.98	0.98	0.99	0.99	0.99
60	1.13	1.10	1.09	1.08	1.07	1.00	1.00	1.00	1.00	1.00	0.97	0.97	0.97	0.97	0.98
	Pure Jump ($\sigma = 0.3$, $\lambda = 1$)					True Formula					Compound Option ($T = 3$, B/S = 0.8)				
40	0.98	0.58	0.41	0.35	0.32						1.02	1.05	1.05	1.06	1.06
45	0.55	0.34	0.24	0.75	0.92						1.02	1.02	1.03	1.03	1.03
50	0.38	0.71	1.00	1.06	1.05			?			1.00	1.00	1.00	1.00	1.00
55	1.71	1.37	1.22	1.11	1.01						0.98	0.98	0.98	0.98	0.99
60	*	1.64	1.22	1.08	1.11						0.96	0.96	0.96	0.97	0.97
	Diffusion-Jump ($\lambda = 1$, $\gamma = 0.8$)										Absolute Diffusion				
40	*	1.61	1.31	1.25	1.23						1.07	1.12	1.12	1.12	1.12
45	1.48	1.14	1.11	1.13	1.15						1.05	1.05	1.05	1.06	1.06
50	0.72	0.86	1.00	1.07	1.10						1.00	1.00	1.00	1.00	1.00
55	1.35	1.04	1.04	1.08	1.10						0.95	0.95	0.95	0.96	0.96
60	*	1.40	1.16	1.13	1.14						0.92	0.91	0.91	0.91	0.91

* Extremely high.

in-the-money pure jump relative values to increase and at-the-money pure jump relative values to decrease. Given a fixed time to expiration, reducing the striking price causes $\alpha = 1.5$ displaced diffusion relative values to increase and $\alpha = 0.5$ displaced diffusion relative values to decrease. Notice also that the relative values of the $\alpha = 1.5$ displaced diffusion and the compound option models are almost identical. This is what one would expect if corporate leverage but not the chance of default has a significant influence on option prices.

Subsequently, we will make pairwise comparisons of the Black-Scholes implied volatilities inferred from observed option prices. Some matched pairs will hold the striking price fixed and isolate the effects of differing time to expiration, and other matched pairs will do the reverse. We will then be able to compare this pattern of relative values to those produced by alternative formulas.

II. Options Data Base[5]

A. MDR Data Base

The options data used for this study are taken from the *Market Data Report* of the CBOE. The data consist of almost all reported trades and quotes on the floor of the CBOE from August 23, 1976 through August 31, 1978.[6] A minimal loss of reported trades and quotes occurred due to computer failure at the exchange.

B. Data Consolidation

To reduce the size of the data base with minimum sacrifice of useful information, records were aggregated for each option series into a single consolidated record during consecutive time intervals within a day during which the recorded stock price remained unchanged. Table III is an example (somewhat reorganized for each for reading) of typical records in the consolidated data base.

C. Data Screening

For this study, a subset of the Berkeley Options Data Base was chosen to reduce various sources of statistical noise that may be present in the entire data base. Records were chosen only if they simultaneously satisfied the following eight criteria:

1. Previous and following different stock prices either both went up or both went down by 1/8
2. Time after the last occurrence of the previous different stock price is less than 500 seconds
3. At least 300 seconds within constant stock price interval

[5] A comprehensive description of the data base can be found in Bhattacharya and Rubinstein [1] and Rubinstein [13].

[6] A detailed description of floor trading procedures on the CBOE is available in Cox and Rubinstein [6].

Table III
Berkeley Options Data Base: Consolidated Format

Constant Stock Price Record	Date	Stock	Time Period Beginning	Time Period End	Stock Price	Surrounding Stock Price Code
	01/05/77	PRD	12:12:30	12:20:25	$37\frac{1}{2}$	23253535

Consolidated Options Data Record	Type	Expiration Month	Striking Price	Trade/Quote Low	Trade/Quote High	Volume Low	Volume High	Volume Total	Records 5 Min	Records Total
	Call	July	40	$3\frac{1}{4}$	$3\frac{1}{2}$	1	1	10	2	5

Note: Surrounding stock price code indicates: five immediately surrounding stock prices; approximate elapsed time after last different stock price; approximate elapsed time before next different stock price; and approximate number of records consolidated for *all* options to the stock during the constant stock price interval.

4. Record did not occur in the first 1,000 seconds after 9:00 or in the last 1,000 seconds before 3:00
5. Spread between high and low option prices less than or equal to $\frac{1}{4}$
6. Total contract volume at least 3
7. At least one bid-ask raw data record occurred in the first five minutes of constant stock price interval
8. Total number of raw data records consolidated is three or more

The first four relate to the stock price information and the second four to the option price. The first criterion assures that option records will be selected only for periods of relative stock price stability. In addition, the bid-ask prices in the stock can be estimated by observing whether the concurrent stock price is above or below the adjoining stock prices. If it is above, it is likely to be an ask price. If it is below, it is probably a bid price. The second criterion not only reduces the probability that changes in the stock price will not be missed but also assures that option records will only be selected during times of reasonable liquidity in the stock. The third criterion ensures that the option market has had ample time to adjust to the new stock price. The fourth criterion eliminates problems with artificial pricing that are most likely to occur at the beginning (reflecting execution of limit orders held over from the previous day) and ending (reflecting trades to influence market maker margin) of the day.

The fifth criterion reduces the uncertainty surrounding the "equilibrium" option price during the constant stock price interval. The sixth and eighth criteria assure a reasonable depth to the market in the option series both in terms of trading volume and the number of records during the time interval. The seventh criterion helps alleviate two possible problems: (i) all reported option prices during the interval, in fact, are late reports negotiated on the basis of the previous different stock price and (ii) all reported option prices during the interval relate not to the corresponding traded stock price, but rather to a bid-ask spread in the stock which has moved outside the last trade.

III. Measurement of Variables

Of the eight variables needed to be measured, the time to expiration and the striking price of an option can be measured with perfect accuracy.

The rate of interest associated with each option is measured as a compromise between probable default-free borrowing and lending opportunities of well-situated investors. The lending rate was assumed to be the yield to maturity on a T-bill maturing as close as possible to the option expiration date. The borrowing rate was estimated to be equal to this rate, plus ½ percent, plus the difference between the current broker call loan money rate and the yield to maturity on a two-week T-bill. The interest rate used in option value calculations was the average of this lending and borrowing rate.

Interest rates so calculated during the two-year period ranged from 4.9 to 9.9 percent. Compared to the other variables estimated, small measurement errors in interest rates should have little effect on computed implied volatilities. Moreover, since the nonparametric tests only ask whether the implied volatility of one option is greater than another, inaccurate interest rates will only be a problem if they reverse the ordering of paired implied volatilities. This should occur very rarely.

Future cash dividends prior to expiration were estimated in two ways. First, they were estimated as a naive extrapolation from the past. For example (except for adjustment for stock splits and stock dividends), the cash dividend paid in the third quarter last year is assumed to be equal to the cash dividend paid in the third quarter this year. Second, they were measured as though they had been forecast perfectly. For example (except for adjustment for stock splits and stock dividends), the cash dividend is assumed to be equal to the cash dividend actually paid.

An implied volatility estimate was then calculated for each estimate of future dividends for each consolidated option record. In the nonparametric tests, the implied volatility of one option is only considered greater than the implied volatility of a second option, if the ordering of their implied volatilities is the same irrespective of the cash dividend estimate used to calculate the implied volatility.

Future ex-dividend dates prior to expiration were measured as though they had been forecast perfectly. The only significant problem that is likely to arise from this procedure arises from option classes with expiration dates near ex-dividend dates. Honeywell, for example, occasionally has an ex-dividend date on the same Friday its options expire. Stocks, such as Honeywell, for which there was doubt about the number of ex-dividend dates prior to the expiration date were simply eliminated from the sample. Of the 32 underlying stocks with a sufficient number of postscreened records to be analyzed, only two stocks were eliminated on this account.

To handle ambiguities from optimal early exercise, all puts were eliminated from the sample and all calls, for which early exercise (based on perfectly forecasted cash dividends) was likely, were also eliminated.

To identify these calls, two European values were calculated for each call: one value assuming exercise only at the expiration date, and one value assuming exercise only just before the last ex-dividend date prior to expiration. In the former case, call values would tend to be greater because of the longer holding period, but values would tend to be less because of the increased number of dividends prior to the exercise date. Whenever the value based on early exercise exceeded the value based on holding the call to expiration, the option was eliminated from the sample. This method for reducing the influence of early exercise is imperfect. A recent paper by Geske and Roll [8] draws attention to the importance of exact attribution of value from the potential for early exercise.[7] In response to their concerns, I will report separate results for call options with no ex-dividend dates prior to expiration.

In my sample of stocks, the dividend effect creating a value to early exercise is reduced by the typically substantial separation between ex-dividend dates and corresponding option expiration dates. Only Boeing (*BA*) had ex-dividend dates in its option expiration months preceding its option expiration dates. In all other cases, there was at least a three-week separation between the last ex-dividend date prior to expiration and the corresponding option expiration date.

The underlying stock price given in the consolidated option price record was assumed to be the correct contemporaneous stock price with a minor correction for variation across the bid-ask spread. It has been frequently documented that transaction-to-transaction price reversals are more likely than continuations. To reflect this observation, the stock price in each record was adjusted upward (downward) by three cents, if the different stock prices in the surrounding constant stock price intervals were higher (lower). This is 50 percent of half the difference in the bid-ask spread to reflect uncertainty as to whether the $\frac{1}{8}$ change is a true change or just variation across the bid-ask spread.

The most difficult variable to measure with sufficient accuracy is the option market price. This itself is an indication of the minimal level of measurement error associated with this study. During a constant stock price interval, option trade prices are typically observed at two or three different levels. For example, for Polaroid in our earlier example, while the stock price apparently remained constant at $37\frac{1}{2}$, one July/40 call contract was traded at $3\frac{1}{4}$, eight at $3\frac{3}{8}$, and, one at $3\frac{1}{2}$. Will the true equilibrium option price please stand up? Unfortunately, even dollar measurement errors on the order of $\frac{1}{8}$ can make a significant difference in the implied volatility.

This difficulty was handled by assuming that the true "equilibrium" option price was a weighted average of the traded prices during the interval where each traded price was weighted by its relative volume reported during the constant stock price interval. In the above example, this would have been:

$$\left(\tfrac{1}{10} \times 3\tfrac{1}{4}\right) + \left(\tfrac{8}{10} \times 3\tfrac{3}{8}\right) + \left(\tfrac{1}{10} \times 3\tfrac{1}{2}\right) = 3\tfrac{3}{8}$$

[7] Geske and Roll center their criticism around procedures that value options by the larger of two values: the Black-Scholes value assuming exercise at expiration and the Black-Scholes value assuming exercise just prior to the last ex-dividend date. They argue that deviation of these pseudo-American values from true American values could explain the reversal of the striking price bias later documented in this study. However, it is important to realize that this valuation procedure was *not* followed here. Instead, calls were *eliminated* from the sample with pseudo-American values greater than their Black-Scholes values based on holding to expiration. Thus, the remaining calls used in the study were all valued at their Black-Scholes values based on holding to expiration.

It is likely that this procedure still leaves a significant amount of noise in the estimated option price. However, it is unlikely to cause statistical bias. To test for the sensitivity of the results to this weighting procedure, other reasonable methods were used to infer a single option price during a constant stock price interval. The results reported in this paper, while not quite as strong, would have changed little using these alternatives.

IV. Results of Nonparametric Tests

To conduct the nonparametric tests, each consolidated record remaining after the screening procedure was placed in one of 25 categories depending on the time to expiration and ratio of the stock price to the striking price for the associated option. Five ranges of time to expiration were distinguished:

1. Very near maturity (21 to 70 days)
2. Near maturity (71 to 120 days)
3. Middle maturity (121 to 170 days)
4. Far maturity (171 to 220 days)
5. Very far maturity (221+ days)

Options with less than 21 days to expiration were omitted. The ratio of the current stock price to the striking price is also divided into five ranges:

1. Deep out-of-the-money (0.75 to 0.85)
2. Out-of-the-money (0.85 to 0.95)
3. At-the-money (0.95 to 1.05)
4. In-the-money (1.05 to 1.15)
5. Deep in-the-money (1.15+)

Options with a ratio less than 0.75 were omitted. In addition, deep in-the-money options selling below parity[8] were also eliminated from the sample.

For the time to expiration comparison, all possible nonoverlapping matched pairs of option records were selected. That is, no option record appeared in more than one matched pair. In this case, to be matched, two records must belong to the same underlying stock, be observed on the same day during the same constant stock price interval, and have the same striking price. Altogether for the sample, there were 13,114 such matched pairs. For the striking price comparison, two records are considered matched if they belong to the same underlying stock, are observed on the same day during the same constant stock price interval, and have the same time to expiration. Altogether for the sample, there were 9,704 such matched pairs.

For example, consider matched pairs for options with the same striking price falling in the range 0.75 to 0.85, but with different times to expiration such that one call falls in the range 71 to 120 days and the other call falls into the range 171 to 220 days. The sample contained a total of 343 of these matched pairs. In 325 of these, the option with the shorter maturity had the higher Black-Scholes implied volatility.

[8] For options selling below parity, a Black-Scholes implied volatility does not exist.

Our null hypothesis is that the Black-Scholes formula produces unbiased values. If this were true, then (ignoring ties) for any matched pair, the probability is ½ that the option with the shorter maturity should have the higher implied volatility. Thus, in a sample of 343 matched pairs, we would expect 171.5 cases where the shorter maturity option has the higher implied volatility. In fact, in our sample we found 325 cases out of 343. The probability that a result this extreme could have occurred given the truth of the null hypothesis is well approximated by:

$$1 - N\left[\frac{(325 + ½) - (½ \times 343)}{½ \times \sqrt{343}}\right]$$

where N is the standard normal distribution function. This probability is less than 0.000001. Thus, for these matched pairs we can safely reject the null hypothesis.

To complement the sign-test measure of *statistical* significance, we will adopt the following nonparametric measure of *economic* significance. If the null hypothesis were true, then if the matched pairs were ranked by the difference in implied volatilities of the options in each matched pair, the median difference would be expected to be zero. Unfortunately, the median difference in implied volatilities is not readily translated into a dollar or percentage pricing bias from Black-Scholes values. Therefore, a similar but more meaningful measure of economic significance has been adopted. For each matched pair, we calculated the implied volatility which equalizes the percentage difference of the price of each call from its value. This is the σ which satisfies

$$\frac{C_1(\sigma) - P_1}{P_1} = \frac{P_2 - C_2(\sigma)}{P_2} \equiv 0.01\alpha$$

where C_i is the Black-Scholes value of a call at volatility σ, and P_i is the market price of the corresponding call for $i = 1, 2$. Because $\delta C/\delta\sigma > 0$, α can also be interpreted as the *minimum percentage deviation of the market prices of the options in the matched pair from their corresponding Black-Scholes values, over all estimates of volatility.* In other words, we can say that, irrespective of the level of stock volatility, one of the options in the matched pair must be "mispriced" relative to Black-Scholes values by at least α percent. This lower bound measure of economic bias from the Black-Scholes formula is consistent with the methodology used in this paper since it does not require an estimate of stock volatility. The alphas reported in the tables are the median values for each subset of matched pairs.

The results of these sign and median tests for each sample of matched pairs are presented in Tables IV to VII. For reasons which will become clear, the sample was divided into two time periods: August 23, 1976 through October 21, 1977 and October 24, 1977 through August 31, 1978. Tables IV and VI summarize the first period and Tables V and VII summarize the second. For both periods, the numbers in Tables IV and V corresponding to our example are 1.00 and 1.00 (fifth row, sixth column). In the first period, of 214 matched pairs, in 199 cases the shorter maturity option had the higher implied volatility. The numbers in

Table IV

Pairwise Comparisons of Otherwise Identical Calls with Different Time to Expiration (First Period: 08/23/76–10/21/77)

Stock Price ÷ Striking Price	Time at Expiration (days) 21–70 vs. 71–120	21–70 vs. 121–170	21–70 vs. 171–220	21–70 vs. 221+	71–120 vs. 121–170	71–120 vs. 171–220	71–120 vs. 221+	121–170 vs. 171–220	121–170 vs 221+	171–120 vs. 221+
1.15+ (deep-in)	NA* (14, 10, 4)	−0.3% 0.27 (66, 30, 36)	NA (6, 4, 2)	NA (15, 9, 6)	−0.9% 0.11 (23, 8, 15)	−0.4% 0.98 (58, 36, 21)	NA (14, 7, 7)	NA (8, 2, 6)	0.2% 0.87 (20, 12, 8)	NA (6, 3, 3)
1.05–1.15 (in)	−1.4% 0.01 (61, 21, 39)	−1.2% 0 (352, 134, 218)	−1.7% 0.06 (32, 11, 21)	0.1% 0.64 (69, 35, 33)	−1.5% 0 (49, 10, 39)	−0.2% 0.19 (193, 89, 102)	0.5% 0.73 (43, 23, 20)	−1.0% 0.06 (41, 15, 26)	0.4% 0.93 (60, 34, 24)	NA (6, 2, 4)
0.95–1.05 (at)	−3.1% 0 (260, 83, 173)	−1.1% 0 (1226, 508, 696)	−5.1% 0 (113, 30, 82)	−1.4% 0.13 (226, 103, 121)	−1.7% 0 (199, 72, 123)	−0.8% 0 (862, 377, 465)	−1.1% 0.22 (107, 49, 58)	−0.5% 0.42 (93, 44, 47)	0.2% 0.90 (237, 126, 107)	−0.6% 0.44 (52, 25, 27)
0.85–0.95 (out)	3.8% 1.00 (101, 68, 30)	4.0% 1.00 (626, 478, 141)	5.6% 1.00 (46, 37, 8)	5.1% 1.00 (117, 100, 17)	2.7% 1.00 (155, 110, 42)	1.3% 1.00 (692, 448, 231)	3.1% 1.00 (82, 65, 12)	1.0% 1.00 (106, 70, 33)	1.6% 1.00 (256, 185, 60)	1.3% 1.00 (75, 48, 25)
0.75–0.85 (deep-out)	NA (12, 12, 0)	13.1% 1.00 (129, 125, 4)	NA (6, 6, 0)	20.3% 1.00 (26, 26, 0)	9.1% 1.00 (27, 26, 1)	5.3% 1.00 (214, 199, 12)	NA (14, 14, 0)	2.3% 1.00 (25, 19, 5)	2.8% 1.00 (86, 71, 15)	NA (16, 13, 3)

* NA indicates a sample of less than 20 matched pairs.

Table V

Pairwise Comparisons of Otherwise Identical Calls with Different Time to Expiration
(Second Period: 10/24/77–08/31/78)

Stock Price ÷ Striking Price	Time at Expiration (days) 21–70 vs. 71–120	21–70 vs. 121–170	21–70 vs. 171–220	21–70 vs. 221+	71–120 vs. 121–170	71–120 vs. 171–220	71–120 vs. 221+	121–170 vs. 171–220	121–170 vs. 221+	171–120 vs. 221+
1.15+ (deep-in)	NA* (10, 7, 3)	0.8% 1.00 (58, 42, 16)	NA (4, 3, 1)	NA (9, 5, 4)	−0.1% 0.29 (30, 13, 17)	0.8% 1.00 (57, 39, 17)	NA (2, 2, 0)	NA (1, 1, 0)	NA (15, 10, 5)	NA (2, 1, 1)
1.05–1.15 (in)	0.9% 1.00 (41, 30, 11)	0.8% 1.00 (287, 186, 99)	3.6% 1.00 (27, 23, 4)	0.8% 0.99 (49, 33, 6)	0.5% 0.92 (76, 43, 32)	1.7% 1.00 (174, 150, 23)	0.9% 0.94 (34, 21, 13)	NA (14, 7, 7)	0.6% 1.00 (49, 33, 15)	NA (11, 5, 6)
0.95–1.05 (at)	3.1% 1.00 (84, 71, 13)	2.2% 1.00 (821, 640, 170)	1.9% 1.00 (51, 35, 16)	3.8% 1.00 (151, 117, 33)	2.4% 1.00 (304, 242, 56)	2.2% 1.00 (681, 551, 124)	4.3% 1.00 (115, 102, 13)	1.1% 1.00 (45, 36, 9)	1.8% 1.00 (148, 127, 19)	0.9% 1.00 (55, 42, 9)
0.85–0.95 (out)	9.0% 1.00 (65, 59, 5)	6.7% 1.00 (634, 615, 15)	9.5% 1.00 (29, 28, 1)	8.9% 1.00 (86, 84, 2)	7.6% 1.00 (173, 163, 8)	4.2% 1.00 (610, 580, 27)	8.1% 1.00 (105, 103, 2)	4.0% 1.00 (64, 63, 1)	3.3% 1.00 (184, 171, 10)	2.6% 1.00 (63, 57, 5)
0.75–0.85 (deep-out)	NA* (12, 12, 0)	14.9% 1.00 (116, 115, 1)	NA (10, 10, 0)	NA (19, 19, 0)	9.8% 1.00 (44, 44, 0)	7.7% 1.00 (129, 126, 2)	12.9% 1.00 (32, 32, 0)	5.1% 1.00 (22, 22, 0)	5.7% 1.00 (56, 53, 2)	3.7% 1.00 (23, 23, 0)

* NA indicates a sample of less than 20 matched pairs.

Table VI

Pairwise Comparisons of Otherwise Identical Calls with Different Striking Prices (First Period: 08/23/76–10/21/77)

Stock Price ÷ Striking Price	Time to Expiration (days) 21–70	71–120	121–170	171–220	221+
1.05–1.15	0.4%	1.1%	0.8%		
vs.	0	0	0	NA*	NA
1.15+	(78, 52, 26)	(45, 31, 14)	(22, 17, 5)	(10, 8, 2)	(4, 2, 2)
0.95–1.05	0.8%	1.8%	1.7%	2.5%	2.3%
vs.	0	0	0	0	0
1.15+	(200, 154, 43)	(178, 130, 48)	(138, 102, 36)	(60, 47, 13)	(53, 38, 15)
0.95–1.05	1.4%	1.4%	2.0%	2.6%	4.1%
vs.	0	0	0	0	0
1.05–1.15	(275, 194, 80)	(257, 158, 97)	(133, 102, 31)	(66, 61, 5)	(32, 28, 4)
0.85–0.95	0.6%	1.8%	3.3%	3.6%	
vs.	0.06	0	0	0	NA
1.15+	(50, 30, 20)	(101, 73, 28)	(43, 37, 6)	(37, 27, 10)	(18, 11, 7)
0.85–0.95	0.2%	1.3%	1.4%	2.2%	3.1%
vs.	0.29	0	0	0	0
1.05–1.15	(319, 163, 154)	(297, 187, 110)	(243, 157, 84)	(126, 94, 31)	(49, 41, 8)
0.85–0.95	−1.9%	1.0%	1.7%	1.7%	2.6%
vs.	1.00	0	0	0	0
0.95–1.05	(311, 111, 196)	(442, 254, 179)	(297, 204, 84)	(193, 139, 52)	(117, 99, 17)
0.75–0.85					
vs.	NA	NA	NA	NA	NA
1.15+	(6, 4, 2)	(13, 12, 1)	(9, 8, 1)	(19, 18, 1)	(14, 12, 2)
0.75–0.85		1.0%	3.6%	3.8%	
vs.	NA	0.01	0	0	NA
1.05–1.15	(8, 4, 4)	(28, 20, 8)	(34, 29, 5)	(38, 33, 5)	(19, 14, 5)
0.75–0.85	−7.5%	−1.2%	1.2%	1.5%	2.9%
vs.	1.00	0.99	0	0	0
0.95–1.05	(99, 22, 77)	(185, 75, 106)	(210, 129, 80)	(172, 108, 61)	(84, 69, 14)
0.75–0.85	−19.1%	1.3%	4.2%	2.3%	3.5%
vs.	1.00	0.5	0	0	0
0.85–0.95	(34, 8, 26)	(117, 66, 49)	(111, 97, 12)	(97, 74, 22)	(29, 25, 3)

* NA indicates a sample of less than 20 matched pairs.

the tables indicate the probability of 199 or more cases occurring, given the null hypothesis, is less than 0.005. By convention, 1.00 also means it was the *shorter* maturity options that tended to have the higher implied volatilities, while an entry of 0 indicates at the same level of significance that the *longer* maturity options have the higher implied volatility. If the Black-Scholes formula showed no bias, the expected value of all entries in the tables would be 0.5.

Table VII

Pairwise Comparisons of Otherwise Identical Calls with Different Striking Prices (Second Period: 10/24/77–08/31/78)

Stock Price ÷ Striking Price	Time to Expiration (days)				
	21–70	71–120	121–170	171–220	221+
1.05–1.15	0.6%	−0.5%			
vs.	0	1.00	NA*	NA	NA
1.15+	(63, 44, 19)	(49, 15, 33)	(11, 2, 9)	(3, 1, 2)	(0, 0, 0)
0.95–1.05	0.4%	−0.3%	−0.6%	−0.2%	
vs.	0.01	0.94	0.95	0.86	NA
1.15+	(167, 97, 69)	(178, 77, 99)	(62, 24, 38)	(31, 12, 19)	(10, 6, 4)
0.95–1.05	−1.2%	−1.8%	−1.5%	−0.7%	
vs.	1.00	1.00	1.00	0.96	NA
1.05–1.15	(155, 37, 117)	(177, 50, 125)	(57, 4, 52)	(28, 9, 19)	(10, 7, 3)
0.85–0.95	0.8%	−0.6%	−0.7%	−0.5%	
vs.	0.04	0.98	1.00	0.58	NA
1.15+	(52, 32, 20)	(64, 23, 40)	(38, 9, 28)	(26, 12, 14)	(12, 4, 8)
0.85–0.95	−2.3%	−2.1%	−2.0%	−2.5%	−0.5%
vs.	1.00	1.00	1.00	1.00	1.00
1.05–1.15	(291, 64, 225)	(282, 62, 216)	(214, 29, 181)	(119, 24, 95)	(40, 8, 30)
0.85–0.95	−5.3%	−3.2%	−2.0%	−0.6%	−0.0%
vs.	1.00	1.00	1.00	0.98	1.00
0.95–1.05	(198, 27, 167)	(384, 94, 285)	(183, 47, 133)	(120, 46, 69)	(41, 11, 29)
0.75–0.85					
vs.	NA	NA	NA	NA	NA
1.15+	(6, 3, 3)	(12, 5, 7)	(11, 3, 8)	(2, 0, 2)	(2, 0, 2)
0.75–0.85		−2.6%			
vs.	NA	0.99	NA	NA	NA
1.05–1.15	(14, 3, 11)	(20, 4, 16)	(17, 6, 11)	(8, 1, 7)	(12, 5, 7)
0.75–0.85	−6.2%	−5.4%	−1.6%	−1.3%	0.1%
vs.	1.00	1.00	1.00	1.00	0.38
0.95–1.05	(115, 16, 98)	(182, 26, 156)	(142, 45, 95)	(101, 25, 76)	(44, 22, 21)
0.75–0.85	−9.3%	−2.0%	1.5%	−0.4%	2.4%
vs.	1.00	1.00	0.22	0.73	0
0.85–0.95	(34, 0, 33)	(52, 14, 37)	(84, 45, 39)	(44, 19, 24)	(39, 26, 10)

* NA indicates a sample of less than 20 matched pairs.

The economic significance of deviations from Black-Scholes values is given by the numbers followed by percentage signs. For example, during the first period, for otherwise identical deep-out-of-the-money calls, Table VIII lists the matched pairs resulting from comparing calls with 71 to 120 days versus 171 to 220 days to expiration, sorted by the lower bound percentage deviation of their market prices from Black-Scholes values. The median deviation is 5.3 percent which

Table VIII

Matched Pairs of Otherwise Identical Deep-Out-of-the-Money Calls with 71–120 Versus 171–220 Days to Expiration Sorted by Lower Bound Percentage Deviation of Market Price from Black-Scholes Value (First Period: 08/23/76–10/21/77)

Stock	Month*	Yield	Deviation	Stock	Month*	Yield	Deviation	Stock	Month*	Yield	Deviation
GW	77/10	2.8	−8.4	HIA	76/11	1.9	3.9	HOI	77/10	1.4	6.9
XRX	76/10	1.0	−7.9	PRD	77/04	1.0	4.0	HIA	77/02	2.0	6.9
BS	77/07	2.1	−5.9	GW	77/01	2.0	4.0	HIA	77/02	1.9	7.0
HIA	77/05	1.9	−5.5	NSM	77/02	0.0	4.0	GW	77/01	2.0	7.0
PRD	77/03	0.1	−2.7	HOI	77/10	1.4	4.0	KN	76/10	1.0	7.0
HOI	77/04	1.2	−2.5	NSM	77/02	0.0	4.0	NSM	77/02	0.0	7.0
FNC	76/09	2.3	−2.4	HOI	77/09	1.2	4.1	KN	76/10	1.1	7.0
EK	77/07	2.2	−1.8	NSM	77/01	0.0	4.1	HIA	76/11	1.8	7.1
HOI	77/07	1.1	−1.7	NSM	76/08	0.0	4.1	KN	76/10	1.1	7.2
EK	77/07	2.2	−1.2	NSM	76/11	0.0	4.2	NSM	76/10	0.0	7.3
HOI	77/07	1.1	−0.9	PRD	76/10	0.7	4.2	NSM	77/05	0.0	7.3
BLY	77/05	0.1	−0.8	NSM	77/02	0.0	4.3	HOI	77/10	1.2	7.3
PRD	77/10	1.5	0.1	HOI	77/09	1.2	4.3	KN	77/01	1.0	7.7
NSM	76/11	0.0	0.2	NSM	76/11	0.0	4.3	HOI	77/10	1.3	7.8
NSM	76/08	0.0	0.2	HOI	77/10	1.3	4.4	HOI	77/10	1.2	7.8
PRD	77/03	1.0	0.3	HOI	77/10	1.2	4.4	NSM	77/05	0.0	7.9
SYN	76/12	1.2	0.4	NSM	76/11	0.0	4.4	NSM	77/02	0.0	7.9
PRD	76/10	0.7	0.6	HOI	77/10	1.2	4.4	HOI	77/09	1.2	7.9
RCA	76/10	2.2	0.6	NSM	77/02	0.0	4.5	HOI	77/10	1.2	7.9
NSM	76/12	0.0	0.7	NSM	76/08	0.0	4.5	HOI	77/10	1.2	7.9
HOI	77/04	1.2	0.7	HOI	77/10	1.1	4.6	OXY	77/08	2.6	8.0
NSM	77/01	0.0	0.9	HOI	77/04	1.2	4.6	NSM	77/05	0.0	8.2
OXY	77/08	2.6	1.1	HOI	77/10	1.3	4.6	PRD	76/10	0.7	8.3
HOI	77/04	1.3	1.2	OXY	77/08	2.5	4.6	XRX	77/04	1.7	8.4
NSM	76/11	0.0	1.2	HOI	77/09	1.2	4.6	BLY	77/05	0.1	8.7
NSM	76/12	0.0	1.2	NSM	76/10	0.0	4.7	KN	76/10	1.0	8.7
HOI	77/04	1.2	1.3	PRD	76/10	0.7	4.7	KN	77/03	1.1	8.9
NSM	77/02	0.0	1.4	NSM	77/02	0.0	4.8	HIA	77/02	2.0	9.2
HOI	77/09	1.2	1.4	RCA	76/10	2.2	4.9	UAL	77/02	1.5	9.3
UAL	76/11	1.2	1.4	HOI	77/10	1.2	5.1	HOI	77/10	1.2	9.3
NSM	77/02	0.0	1.4	HOI	77/10	1.4	5.1	NSM	77/02	0.0	9.3
HIA	76/10	1.9	1.4	HOI	77/10	1.2	5.1	HIA	77/05	1.9	9.5
HIA	77/01	1.9	1.6	HOI	77/10	1.4	5.1	NSM	77/01	0.0	9.6

HOI	77/07	1.1	1.6	PRD	77/04	1.0	5.3	BLY	77/05	0.1	9.8
HOI	77/04	1.2	1.8	NSM	77/02	0.0	5.4	BLY	77/08	0.1	9.9
NSM	77/01	0.0	1.8	EK	77/08	2.3	5.4	NSM	77/05	0.0	10.0
RCA	76/10	2.2	1.8	HOI	77/10	1.4	5.5	HIA	77/02	2.0	10.3
HOI	77/10	1.1	1.8	NSM	76/11	0.0	5.5	HOI	77/10	1.2	10.5
NSM	76/11	0.0	1.8	NSM	77/02	0.0	5.6	HOI	77/10	1.2	10.6
HOI	77/09	1.1	1.9	HOI	77/10	1.4	5.6	PRD	76/10	0.7	10.6
HOI	77/10	1.1	1.9	HOI	77/03	1.1	5.7	HOI	77/10	1.2	11.1
HOI	77/09	1.1	1.9	XRX	77/03	1.7	5.7	OXY	77/08	2.6	11.2
NSM	77/02	0.0	2.0	NSM	76/10	0.0	5.8	KN	76/10	1.1	11.3
HIA	77/02	1.9	2.2	HOI	77/10	1.1	5.8	HOI	77/10	1.3	11.4
NSM	76/11	0.0	2.2	NSM	77/02	0.0	5.9	HIA	77/02	1.9	11.7
CDA	77/08	0.0	2.3	HOI	77/10	1.2	6.0	HOI	77/10	1.3	12.1
HIA	76/11	1.9	2.5	NSM	77/01	0.0	6.0	NSM	77/10	0.0	12.6
MCD	76/10	0.1	2.5	HOI	77/10	1.1	6.1	HIA	77/02	1.9	12.8
NSM	77/02	0.0	2.6	NSM	76/10	0.0	6.1	CDA	77/05	0.0	13.1
HOI	77/03	1.1	2.6	HOI	77/10	1.4	6.1	HOI	77/04	1.1	13.3
OXY	77/08	2.5	2.6	SYN	76/10	1.1	6.1	NSM	77/02	0.0	13.8
NSM	77/02	0.0	2.6	RCA	76/10	2.3	6.1	HOI	77/05	1.0	14.2
HOI	77/09	1.2	2.7	GW	77/01	2.0	6.2	NSM	77/05	0.0	14.4
NSM	76/10	0.0	2.9	HOI	77/10	1.4	6.2	NSM	77/02	0.0	15.0
NSM	77/02	0.0	3.0	HOI	77/10	1.2	6.2	OXY	77/08	2.6	15.1
HOI	77/10	1.1	3.0	BLY	77/08	0.1	6.2	HOI	77/05	1.0	15.3
HOI	77/10	1.2	3.2	NSM	77/02	0.0	6.3	HIA	77/02	2.0	15.6
NSM	76/11	0.0	3.2	NSM	77/01	0.0	6.3	HOI	77/05	1.0	16.3
HOI	77/04	1.3	3.3	NSM	76/10	0.0	6.4	BS	77/10	2.6	16.8
NSM	77/02	0.0	3.3	KN	76/10	1.0	6.4	KN	77/09	1.3	17.3
HOI	77/10	1.4	3.3	EK	77/03	1.2	6.5	OXY	77/08	2.6	17.6
NSM	77/02	0.0	3.5	HOI	77/03	1.1	6.6	HOI	77/05	1.0	18.0
HOI	77/10	1.2	3.5	HIA	76/10	1.9	6.6	HIA	77/04	2.0	18.9
HOI	77/10	1.2	3.6	BLY	77/05	0.1	6.6	BS	77/07	2.2	19.8
HOI	77/10	1.2	3.7	NSM	77/02	0.0	6.6	DOW	77/10	2.0	20.0
NSM	76/11	0.0	3.7	BLY	77/05	0.1	6.7	CDA	77/05	0.0	23.4
BLY	77/06	0.1	3.8	NSM	76/10	0.0	6.7	EK	77/04	1.2	25.6
HOI	77/09	1.2	3.8	BLY	77/05	0.1	6.9	SYN	77/04	1.3	41.5
HOI	77/10	1.4	3.8	HOI	77/10	1.4	6.9	KN	77/10	1.3	51.6
HOI	77/10	1.2	3.8	OXY	77/08	2.6	6.9				
NSM	77/01	0.0	3.9	HOI	77/10	1.2	6.9				

* The month is the year/month of the matched pair observation, and the yield is the dividend yield totaling all cash dividends to go ex-dividend prior to the expiration of the longest maturing option in the matched pair.

appears in Table IV, row 5, column 6. Table VIII also confirms that 12 matched pairs (those with negative percentage deviations) had lower prices than did values for the shorter maturity options, while 199 matched pairs (those with positive percentage deviations) had higher prices than did values for the shorter maturity options.

Examination of Tables IV and V gives us our first conclusion:

1. For an out-of-the money or deep-out-of-the-money call: the *shorter the time to expiration*, the higher its implied volatility. This conclusion is true to a very high level of significance across both time periods. It is also of interest to see how consistent this conclusion is across individual underlying stocks in our sample. For each underlying stock, Table IX lists the number of matched pairs remaining after the screening procedure described in Section II, the significance levels aggregated across both periods and maturity pairs for the time to expiration comparison when the stock price to striking price ratio is in the range 0.85 to 0.95. Conclusion 1 applies consistently across all the underlying stocks in the sample for which there were a sufficient number of observations.

A second, somewhat weaker conclusion, can also be drawn from Tables IV and V.

2. For an at-the-money call: in the *first period*, the *longer the time to expiration*, the higher the option's implied volatility; in the *second period*, the *shorter the time to expiration*, the higher the option's implied volatility.

Examination of Tables and VI and VII gives us our next conclusions:

3. In the *first period*, with the sole exception of very near maturity deep out-of-the money and out-of-the money calls, for calls with the same time to expiration: the *lower the striking price*, the higher an option's implied volatility.
4. In the *second period*, for the most part, for calls with the same time to expiration: the *higher the striking price*, the higher an option's implied volatility.

The difference between Tables VI and VII is striking. Indeed, let us divide the first period into five subperiods, each ending on an expiration date in the January–April–July–October expiration cycle, and divide the second period similarly into four subperiods. Conclusion 3 holds over each of the first five subperiods and conclusion 4 holds over each of the last four subperiods, at a significance level of at least 99.9 percent in *each* subperiod.

Examination of the behavior of options on specific stocks provides further indication of the pervasiveness of these results. Tables X and XI list the statistical and economic significance levels aggregated across all striking price pairs. We see that during the first period (Table X), for most stocks the implied volatilities of otherwise identical options were higher, the lower the striking price. In the second period (Table XI), for most firms this tendency was reversed, swinging from highly significant biases in the first period to opposite but highly significant biases in the second.

Table IX

Pairwise Comparisons of Otherwise Identical Calls with Different Times to Expiration (Both Periods 08/23/76–08/31/78 by Option Class)

		Stock Price/Striking Price			
		0.75–0.85		0.85–0.95	
Ticker Symbol	Underlying Stock	No. of Pairs	PROB ($X >$ Result)	No. of Pairs	PROB ($X >$ Result)
AVP	Avon	(0, 0, 0)	NA*	(71, 59, 12)	1.00
BA	Boeing	(3, 3, 0)	NA	(44, 35, 8)	1.00
BLY	Bally Manufacturing	(40, 36, 3)	1.00	(97, 85, 11)	1.00
BS	Bethlehem Steel	(10, 8, 2)	NA	(54, 31, 18)	0.98
CDA	Control Data	(28, 28, 0)	1.00	(167, 147, 19)	1.00
DEC	Digital Equipment	(9, 8, 1)	NA	(191, 173, 14)	1.00
DOW	Dow Chemical	(13, 13, 0)	NA	(128, 95, 31)	1.00
EK	Eastman Kodak	(48, 41, 6)	1.00	(389, 314, 70)	1.00
F	Ford	(0, 0, 0)	NA	(52, 32, 20)	0.96
FNC	Citicorp	(9, 8, 1)	NA	(63, 46, 17)	1.00
GE	General Electric	(1, 1, 0)	NA	(48, 45, 3)	1.00
GM	General Motors	(1, 1, 0)	NA	(97, 66, 30)	1.00
GW	Gulf & Western	(32, 30, 2)	1.00	(85, 55, 28)	1.00
HIA	Holiday Inns	(38, 34, 4)	1.00	(94, 72, 20)	1.00
HM	Homestake Mining	(7, 7, 0)	NA	(79, 70, 8)	1.00
HOI	Houston Oil & Mineral	(339, 333, 5)	1.00	(643, 605, 35)	1.00
IBM	IBM	(1, 1, 0)	NA	(91, 83, 7)	1.00
ITT	International Telephone & Telegraph	(2, 2, 0)	NA	(102, 85, 17)	1.00
KN	Kennecott	(51, 50, 1)	1.00	(174, 149, 23)	1.00
MCD	McDonald's	(2, 2, 0)	NA	(76, 65, 10)	1.00
NSM	National Semiconductor	(193, 179, 12)	1.00	(413, 331, 74)	1.00
OXY	Occidental Petroleum	(77, 77, 0)	1.00	(192, 135, 54)	1.00
PRD	Polaroid	(30, 28, 1)	1.00	(237, 209, 25)	1.00
RCA	RCA	(16, 14, 2)	NA	(188, 170, 16)	1.00
S	Sears	(19, 19, 0)	NA	(87, 69, 15)	1.00
SYN	Syntex	(23, 22, 1)	1.00	(116, 88, 28)	1.00
T	AT&T	(0, 0, 0)	NA	(12, 12, 0)	NA
UAL	United Airlines	(12, 12, 0)	NA	(51, 48, 3)	1.00
XON	Exxon	(0, 0, 0)	NA	(46, 40, 3)	1.00
XRX	Xerox	(14, 10, 4)	NA	(182, 119, 56)	1.00

* NA indicates a sample of less than 20 matched pairs.

Table X

Pairwise Comparisons of Otherwise Identical Calls with Different Striking Prices (First Period 08/23/76–10/21/77 by Option Class)

Approximate Dividend Yield	Ticker Symbol	Underlying Stock	No. of Pairs	PROB ($X >$ Result)	Median Minimum % Deviation
4.4	AVP	Avon	(167, 94, 72)	0.04	0.6
2.6	BA	Boeing	(44, 30, 14)	0.01	3.0
0.6	BLY	Bally Manufacturing	(188, 134, 54)	0	1.4
6.7	BS	Bethlehem Steel	(31, 21, 10)	0.02	1.8
0.7	CDA	Control Data	(160, 123, 37)	0	3.0
0.0	DEC	Digital Equipment	(209, 112, 92)	0.07	0.2
2.8	DOW	Dow Chemical	(146, 101, 45)	0	2.9
3.2	EK	Eastman Kodak	(326, 217, 102)	0	1.8
6.0	F	Ford	(37, 17, 19)	0.57	−0.1
3.7	FNC	Citicorp	(39, 25, 14)	0.03	3.3
3.7	GE	General Electric	(65, 33, 30)	0.31	0.4
4.8	GM	General Motors	(130, 86, 44)	0	6.3
4.7	GW	Gulf & Western	(54, 41, 13)	0	6.1
3.5	HIA	Holiday Inns	(85, 83, 2)	0	4.7
3.1	HM	Homestake Mining	(107, 56, 51)	0.28	0.1
2.1	HOI	Houston Oil & Mineral	(935, 620, 307)	0	1.4
3.8	IBM	IBM	(140, 81, 55)	0.01	0.9
5.6	ITT	International Telephone & Telegraph	(132, 101, 31)	0	2.7
2.2	KN	Kennecott	(131, 88, 40)	0	2.1
0.4	MCD	McDonald's	(105, 50, 53)	0.58	−0.2
0.0	NSM	National Semiconductor	(790, 433, 347)	0	0.4
5.2	OXY	Occidental Petroleum	(312, 169, 140)	0.04	0.3
1.9	PRD	Polaroid	(478, 393, 82)	0	2.1
4.3	RCA	RCA	(229, 154, 72)	0	2.1
3.4	S	Sears	(40, 16, 24)	0.87	−1.5
2.2	SYN	Syntex	(71, 44, 27)	0.02	1.1
6.8	T	AT&T	(91, 80, 11)	0	5.4
2.7	UAL	United Airlines	(25, 16, 9)	0.05	2.1
5.8	XON	Exxon	(44, 18, 26)	0.85	−1.7
2.7	XRX	Xerox	(348, 233, 112)	0	2.1

Table XI

Pairwise Comparisons of Otherwise Identical Calls with Different Striking Prices (Second Period 10/24/77–08/78/31 by Option Class)

Approximate Dividend Yield	Ticker Symbol	Underlying Stock	No. of Pairs	PROB ($X >$ Result)	Median Minimum % Deviation
4.9	AVP	Avon	(33, 6, 27)	1.00	−3.1
3.2	BA	Boeing	(97, 56, 40)	0.04	1.6
0.5	BLY	Bally Manufacturing	(103, 45, 58)	0.88	−0.3
4.6	BS	Bethlehem Steel	(65, 19, 46)	1.00	−2.0
0.9	CDA	Control Data	(280, 92, 186)	1.00	−1.2
0.0	DEC	Digital Equipment	(403, 142, 254)	1.00	−1.0
3.3	DOW	Dow Chemical	(34, 3, 31)	1.00	−6.8
3.5	EK	Eastman Kodak	(434, 54, 370)	1.00	−2.1
7.6	F	Ford	(11, 2, 9)	NA	NA
5.0	FNC	Citicorp	(36, 5, 31)	1.00	−6.0
4.8	GE	General Electric	(27, 3, 24)	1.00	−7.7
6.3	GM	General Motors	(162, 83, 78)	0.32	0.5
5.7	GW	Gulf & Western	(22, 14, 8)	0.07	4.2
3.1	HIA	Holiday Inns	(34, 1, 33)	1.00	−3.7
2.8	HM	Homestake Mining	(81, 21, 59)	1.00	−3.1
0.8	HOI	Houston Oil & Mineral	(857, 285, 558)	1.00	−1.0
4.2	IBM	IBM	(107, 5, 99)	1.00	−3.8
6.6	ITT	International Telephone & Telegraph	(41, 5, 36)	1.00	−7.9
2.7	KN	Kennecott	(140, 53, 85)	1.00	−0.8
0.6	MCD	McDonald's	(44, 7, 36)	1.00	−3.1
0.0	NSM	National Semiconductor	(317, 85, 229)	1.00	−2.3
5.5	OXY	Occidental Petroleum	(144, 46, 96)	1.00	−2.9
2.5	PRD	Polaroid	(168, 34, 133)	1.00	−1.7
5.0	RCA	RCA	(35, 4, 31)	1.00	−5.4
4.2	S	Sears	(104, 14, 90)	1.00	−4.6
2.8	SYN	Syntex	(67, 5, 62)	1.00	−2.8
7.5	T	AT&T	(3, 0, 3)	NA	NA
3.1	UAL	United Airlines	(95, 48, 47)	0.42	0.6
6.6	XON	Exxon	(33, 0, 31)	1.00	−6.0
3.8	XRX	Xerox	(68, 23, 44)	0.99	−2.1

V. Summary

Recalling Table II of comparative implied volatilities, we can ask which option pricing formula seems to move us closest to the observed biases from Black-Scholes values. Below we list the formulas compatible with each of our four conclusions (in Section IV):

1. $\alpha = 0.5$ displaced diffusion, pure jump, diffusion-jump
2. First period: pure jump, diffusion jump
   Second period: $\alpha = 0.5$ displaced diffusion
3. First period: $\alpha = 1.5$ displaced diffusion, compound option, absolute diffusion
4. Second period: $\alpha = 0.5$ displaced diffusion, pure jump

No one model seems to capture them all. Even during the first period alone, the models that pick up the time-to-expiration biases are disjoint from those that pick up the striking price biases. However, during the second period, the $\alpha = 0.5$ displaced diffusion model can explain all the biases documented by our four conclusions.

Similar nonstationary striking price biases have been reported elsewhere in the academic literature. Black [2] reports that in the early years of trading on the CBOE, implied volatilities tended to rise with increasing striking price, other things being equal. MacBeth and Merville [10] document a reversal of this bias for the year 1976, when implied volatilities tended to rise with decreasing striking price. This study confirms their findings for the last four months of 1976 and shows that the new bias continued until about October 1977. Subsequently, during the last two months of 1977 and into 1978, the original bias observed by Black reappeared. This is also confirmed by MacBeth [9] for the year 1978. In total, this evidence is consistent with the hypothesis that striking price biases from Black-Scholes values are statistically significant, the direction of bias tends to be the same for most underlying stocks at any point in time, but the direction of bias changes from period to period.

In response to the concerns of Geske and Roll [8], Table XII shows results of the striking price tests, with matched pairs aggregated for those with no cash dividends prior to expiriation and those with cash dividends prior to expiration. The former consist of matched pairs of other stocks sufficiently close to expiration that no ex-dividend dates prior to expiration remained. The reversal of the sign of the bias is the same for both dividend and nondividend groups, and the

Table XII

Pairwise Comparisons of Otherwise Identical Calls with Different Striking Prices Aggregated with and without Dividends

	Matched Pairs	Median Minimum % Deviation
First Period: 08/23/76–10/21/77		
Calls with Dividends	3,608	2.14
Calls without Dividends	2,051	0.3
Second Period: 10/24/77–08/31/78		
Calls with Dividends	2,451	−1.6
Calls without Dividends	1,594	−1.6

economic magnitude of the bias during the second period is the same for both groups. Only in the first period is there some evidence that the dividend effect may be important for the results.

Other evidence on the importance of dividends can be taken from Tables X and XI by focusing on zero-dividend (DEC, NSM) or low-dividend (BLY, CDA, MCD) stocks. In each of these cases, except the equivocating MCD in the first period, the evidence confirms the striking price bias reversal between the two time periods (conclusions 3 and 4).

The effect of inadequate consideration of the effects of dividends on early exercise should only serve to weaken the time-to-expiration bias (conclusion 1). The shorter the maturity of an option, the fewer the number of ex-dividend dates remaining and hence the lower the value of the potential for early exercise. In the extreme case, with no ex-dividend dates remaining, the value of early exercise is zero. Thus, if the dividend effect were important, proper consideration would only serve to increase the time-to-expiration bias already observed. Again, Table IX shows that zero-dividend (DEC, NSM) and low-dividend (BLY, CDA, MCD) show statistically significant time-to-expiration bias. Table VIII shows specific evidence on NSM indicating the economic significance of the bias.

Although the statistical significance of the biases we have observed seems unquestionably strong, their economic significance is arguable. Is a 2 percent deviation of option price from value economically significant? This will depend upon what you plan to do with options. If you are a scalper (market maker who trades frequently with high turnover), the answer may be yes; if you are a positioner holding for the longer term, the answer may be no. In either case, remember that the measure of economic significance I have used, the median lower bound percentage deviation of market price from Black-Scholes values, is designed to give the benefit of every doubt to the Black-Scholes formula. Not only will all other estimates of stock volatility yield a higher deviation for at least one of the options in each matched pair, but also even at the stock volatility used, *both* options, not just one, are mispriced by the same percentage. Finally, as Table VIII illustrates, despite the care taken to screen out potentially misleading option records from the original data base, the dispersion of lower bound percentage deviations around the median is suggestive of even larger transitory differences of market prices from Black-Scholes values.

Since one formula does not seem to explain all of our observations across time, what are we to conclude? Perhaps we need to build a composite model. However, even if we do so, it may be difficult to capture in our formula the cross-over striking price bias we have observed. One additional analysis might help us: perhaps we can correlate the bias observed in any period with the level of some macroeconomic variables. Given the structure of the alternative formulas, natural candidates are the level of stock market prices, the level of stock market volatility, and the level of interest rates.

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