The Impact of Market Concentration

***Abstract*— The S&P 500 has long served as a benchmark for overall stock market performance. However, its capitalization-weighted structure has led to an increasing concentration in a handful of mega-cap stocks, particularly the Magnificent Seven, which have disproportionately driven index returns in recent years. This paper examines the implications of this concentration for using the S&P 500 as a representative market proxy within the investment process. We further analyze the consequences of such concentration on index options pricing and volatility from the perspective of a dispersion trader. Lastly, we explore the challenges this dynamic poses for institutional equity long/short investors and propose potential adjustments to mitigate concentration risk while preserving desired market exposure.**

I. INTRODUCTION

In this paper, we examine the impact of market concentration on the investment process, focusing on its implications for different types of investors. We structure our analysis into three main sections which detail our methodologies, empirical findings, and practical implications.

First, we empirically assess the extent to which the "Magnificent Seven" stocks drive overall market returns and risk, commonly referred to as beta. Our findings raise key concerns about the effectiveness of beta in accurately measuring market risk in a highly concentrated environment.

Next, we explore the implications of this concentration for index option pricing. We show how this increased concentration can be related to increases in implied correlation among the index, and further how this concentration leads to excess kurtosis under a simplified return model. We also highlight how current inefficiencies can be exploited by dispersion traders.

Finally, we analyze the effects of market concentration on institutional equity long/short investors, particularly those implementing long small-cap, short large-cap strategies. Through empirical backtesting, we provide insights and adjustments to enhance strategy performance and re-align expectations with the investors goals.

II. MARKET BETA AND INVESTMENT IMPLICATIONS

The S&P 500, along with other market-capitalization-weighted indices such as the NASDAQ 100 and MSCI ACWI, have become increasingly concentrated in recent years, driven by consistently higher valuations in the tech sector. Much of this concentration originates from a select group of seven stocks, dubbed the "Magnificent Seven" which include NVIDIA, Meta, Tesla, Amazon, Alphabet, Microsoft, and Apple. Unsurprisingly, these companies have been among the biggest beneficiaries of the rise of AI. By 2024, the Magnificent Seven accounted for 31.5% of the S&P 500, highlighting the extent of their dominance.

This level of concentration introduces significant risks and challenges for traditional financial models, as we will explore in this section.

A. Decomposing Market Beta

A natural question to pose in examining the impact of concentration on utilizing the S&P500 as a measure of the market is to first decompose the "risk" contribution that each of its constituents contributes to the overall index. Beta is perhaps the most common measure of market risk used in the financial industry. First introduced by William Sharpe (1964) [1] in his paper which introduced the Capital Asset Pricing model and then later further extended to multi-factor models such as the Fama-French 3 and 5 factor models. Importantly, beta is a linear function of the asset weights in a portfolio. That is, we can compute the beta of a portfolio as the weighted sum of the beta's of our portfolios constituents.

$$\beta_{\text{Index}} = \sum_{i=0}^{500} w_i \beta_i$$

where w_i is the weight of stock i, and β_i is its corresponding beta. If we select our "market" to be the S&P500, then we notice immediately that.

$$\beta_{\text{Index}} = \sum_{i=0}^{500} w_i \beta_i = 1$$

Finally, we can isolate the magnificent 7 to obtain our final equation:

$$\beta_{\text{Index}} = \sum_{i=0}^{7} w_i \beta_i + \sum_{i=8}^{500} w_i \beta_i = 1$$

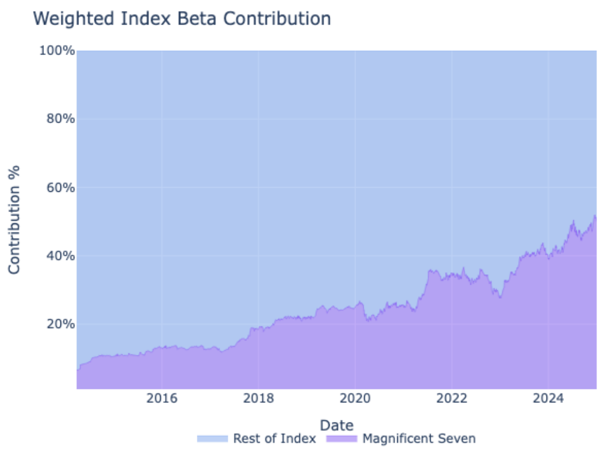

Fig. 1. Weighted Index Beta

Figure 1 shows the time series weighted index beta contribution of the magnificent 7 stocks, and the other constituents of the index. The results are clear - the mag 7 stocks, which are only 7/500 of stocks in the index, account for over 30% of the weight and nearly 50% of the weighted index beta, which has been growing steadily over the past few years.

B. Implications

The increasing concentration in the market suggests that investors should reconsider, or at the very least be aware of, how the representation of beta is evolving. We examine two distinct ways in which this concentration can negatively impact the use of beta.

1) Evaluating/Forecasting Beta/CAPM Issues: First, we consider perhaps the simplest application of beta. In simplified terms, the Capital Asset Pricing Model (CAPM) states that expected asset returns are driven by their risk relative to the market. Mathematically, this is expressed as:

$$E[r_i] - r_f = \alpha_i + \beta_i(E[r_m] - r_f)$$

where $E[r_i]$ represents the expected return of stock i, β_i is the stock's beta relative to the market, $E[r_m]$ is the market return, α_i is the stock's alpha, and r_f is the risk-free rate. While CAPM is a foundational asset pricing model and may not always be used in practice, it serves as the basis for more advanced models and thus offers a useful benchmark for evaluation.

Fama and French (2004) [2] demonstrate that the CAPM model fails to explain stock return variance as effectively as the Fama-French model. We show that this explanatory power has further deteriorated as the market has become increasingly concentrated. To illustrate this, we conduct rolling linear regressions based on the CAPM framework and analyze the rolling R^2.

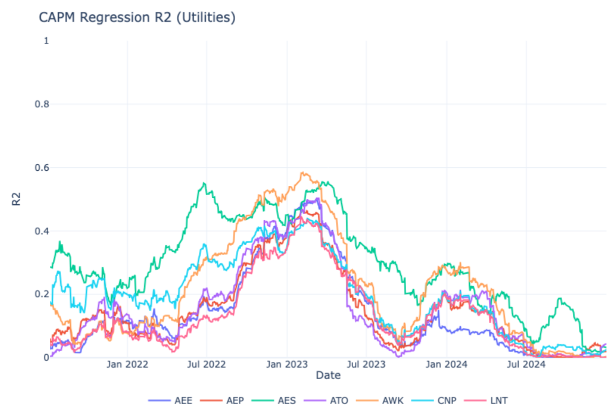

Fig. 2. CAPM R^2 Utilities

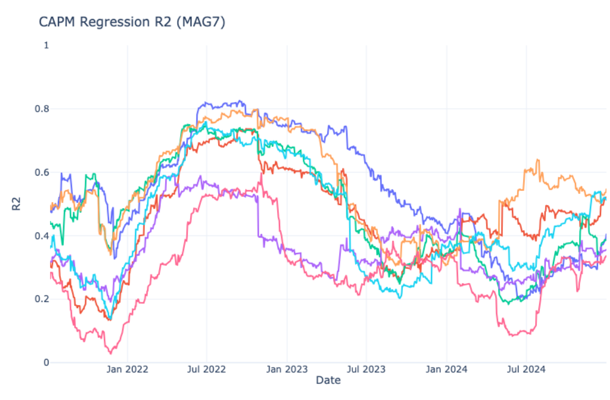

Fig. 3. CAPM R^2 Mag 7

It is evident that explainability has significantly declined for index constituents that are less represented, such as utilities, due to this increased concentration. This poses substantial financial risks for investors who rely on CAPM or its variants in evaluating and forecasting returns, as is often required in traditional mean-variance optimization.

2) Investment Decisions and Hedging: Beyond the challenges posed to traditional mean-variance optimization and the declining effectiveness of market beta as a risk measure, we also examine how increasing market concentration impacts the ability to hedge portfolios effectively.

Risk management is a fundamental aspect of portfolio construction, and hedging unwanted exposures plays a crucial role in this process. While portfolio managers employ various techniques to mitigate risk, one common approach is hedging against "market risk"—ensuring the portfolio is insulated from broad market movements. We consider the case where a portfolio manager seeks to fully hedge their market exposure, aiming for a portfolio beta of zero. The hedging relationship is given by:

$$\beta_{\text{portfolio}} w_{\text{portfolio}} + \beta_{\text{hedge}} w_{\text{hedge}} = 0$$

From this, it follows that the optimal hedging ratio is:

$$w_{\text{hedge}} = -\frac{\beta_{\text{portfolio}}}{\beta_{\text{hedge}}} w_{\text{portfolio}}$$

To evaluate the effectiveness of this hedging strategy, we measure the reduction in portfolio return variance. We define an R^2-type measure of hedge effectiveness as:

$$\text{Hedge Effectiveness} = 1 - \frac{Var(\pi_{\text{hedged}})}{Var(\pi_{\text{unhedged}})}$$

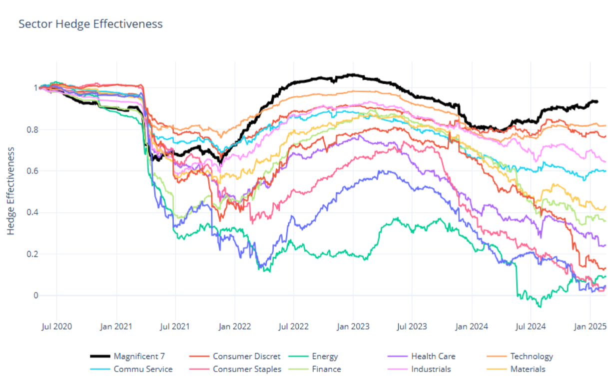

Fig. 4. Hedge Effectiveness

Figure 4 illustrates that the variance reduction achieved through this hedging approach has declined significantly over time. This effect is particularly pronounced for portfolios composed of securities that constitute a smaller portion of the index. Notably, hedging a MAG 7 portfolio using this method has become more effective in recent years, whereas the same strategy applied to sector portfolios has seen diminishing results.

These findings highlight the limitations of relying solely on market beta for risk management in an increasingly concentrated market. As a result, portfolio managers may consider incorporating additional risk factors such as sector exposures, style factors, etc. to construct more effective hedges. Again, as we we showed this shift is particularly relevant for portfolios that are underrepresented in the broader market index.

C. Interdependence

So far we have examined the effects of market concentration on a long term basis, irrespective of the period of time being examined. However, it has to be noted that market variables at times break the assumptions of a Gaussian Copula and correlation can heavily increase during times of market stress. For example: Consider a stock S that plays an important role in the market, such as one of the Magnificent 7. When discussing the effect of a movement in the stock on the index as a whole, it could be trivial to discuss the weighting of this stock as a percentage of the market and extrapolate the market will move with the same ratio. However, this approach misses the interdependence of the stocks. This has become especially prevalent, due to the dominance of technology sector, and the emergence of the Magnificent 7, which are all driven by similar factors. To illustrate this, consider the PCA decomposition in Figure 5, which has a positive loading in the eigenvector with the largest eigenvalue for all of the Magnificent 7. This analysis shows how the majority of the covariance structure is driven by a factor which has a positive relationship with each of the magnificent 7, implying that a move in one of the stocks is likely to be associated with a movement of the others in the same direction, hence a disproportionally large move in the index.

1) Unconditional Beta: A trivial measure to display this effect of inter-correlation would be to use an OLS beta, derived from a regression of the

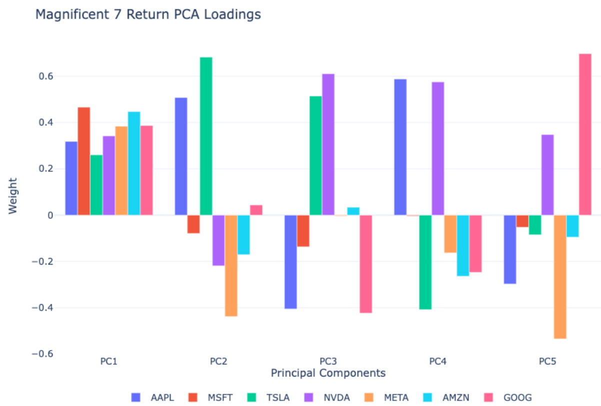

Fig. 5. Magnificent Seven PCA Loadings

index on a given stock. The formula for beta of the index to a stock S is seen below, and it has an affine relation to the correlation of the stock with the other 499 stock in the index.

$$\beta_{M,S} = \frac{w_S\,\sigma_S \;+\; \sum_{\substack{i=1 \\ i\neq S}}^{N} w_i\,\rho\big(i,S\big)\,\sigma_i}{\sigma_S}$$

The results of this base case are seen below.

Stock	Beta
AAPL	0.53
TSLA	0.15
GOOG	0.49
AMZN	0.38
MSFT	0.56
NVDA	0.27
META	0.28
Mag 7	0.62

TABLE I
UNCONDITIONAL BETA (BASE CASE)

We see that the market is most sensitive to moves in Microsoft and Apple, and least sensitive to Tesla.

2) Conditional Beta and Market Stress: A more sophisticated approach to hedging and risk management altogether would be to consider the beta conditioned on a significant market dislocation. This is a natural step from the naive model, as a varying assumption of correlation implies a varying beta, as seen in the formula above. For our purposes, a given day is considered as having a significant move if at least one of the Mag 7 absolute returns surpasses a multiple of it's own 1-year rolling standard deviation. Therefore, a beta conditioned on a stock making an x standard deviation move can be defined as follows:

$$\beta_{SP500,S|M_t>x} = \frac{\mathrm{Cov}\big(SP500_t,\, S_t \,\big|\, M_t > x\big)}{\mathrm{Var}\big(S_t \,\big|\, M_t > x\big)}.$$

Where M_t is the max of the absolute returns divided by the respective standard deviation.

$$M_t = \max\left(\frac{r_i}{\sigma_i}\right)$$

Table II displays the index betas, as calculated for standard deviation thresholds of 1 and 2. Through these charts we can see that the effect of a significant move in a stock on the market is highly dependent on the level of significance. Furthermore, it would be difficult to continue this study to higher levels of significance due to increasingly smaller sample sizes, however, by extrapolating, we can assume that the usage of the naive beta at significance 0 greatly underestimates the effect of Mag 7 movements under market stress, and that beta under heavy market stress could be as high as 0.7 for several of Mag 7.

Stock	$x=0$ (unconditional)	$x=1$	$x=2$
AAPL	0.53	0.55	0.61
TSLA	0.15	0.16	0.18
GOOG	0.49	0.51	0.54
AMZN	0.38	0.40	0.42
MSFT	0.56	0.58	0.62
NVDA	0.27	0.29	0.33
META	0.28	0.29	0.29
Mag 7	0.62	0.63	0.67

TABLE II
BETA VALUES BY STANDARD DEVIATION THRESHOLD (X)

As an example, in Figure 6, we show the increase in Microsoft beta as we increase the standard deviation threshold x.

III. PRICING INDEX OPTIONS AND DISPERSION

Given the interdependence examined in the previous section, particularly the impact of concentration on index volatility, a natural next step is to explore this effect on index options. In this section, we analyze this impact both analytically and empirically using SPX options data.

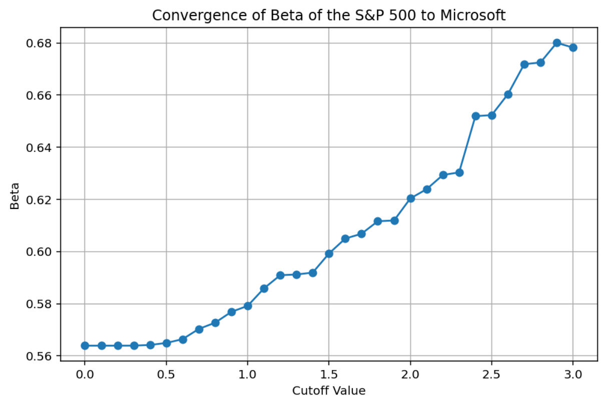

Fig. 6. Convergence of Microsoft Beta by X values

Index options, especially those on broad-market indices like the S&P 500, serve various purposes. For instance, investors may buy deep out-of-the-money SPY put options as protection against market downturns or deep in-the-money call options to chase upside returns. This strong demand for index options creates a distinct pricing environment compared to individual equity options.

A. Black-Scholes Model and Index Volatility

We begin by examining the impact of concentration through traditional options pricing framework. Perhaps the most well-known options pricing model is the Black-Scholes-Merton formula, introduced in Black & Scholes and Merton (1973) [4]. This model provides a closed-form solution for European option prices under key assumptions of constant volatility and no arbitrage. The formula for a European call option is given by:

$$C = S_0 N(d_1) - K e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Here, C represents the call option price, S_0 is the current stock price, K is the strike price, r is the risk-free rate, T is the time to maturity, σ is the volatility, and $N(\cdot)$ denotes the cumulative distribution function of the standard normal distribution.

As previously mentioned, volatility is a critical input into the options pricing model. It is both well understood and easily verifiable that, all else being equal, an increase in volatility leads to an increase in the theoretical price of an option contract. This relationship presents a unique dynamic when dealing with index options. While the volatility input for equity options is typically based on historical volatility, the calculation for index options is more nuanced. Specifically, index volatility is calculated as follows:

$$\sigma_{\text{Index}}^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i<j} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Where:

- σ_{Index} is the volatility of the index.
- w_i is the weight of the i-th stock in the index.
- σ_i is the volatility of the i-th stock.
- ρ_{ij} is the correlation between the returns of the i-th and j-th stocks.
- n is the number of stocks in the index.

From the formula, it is fairly clear to reason that index volatility is generally lower than the average volatility of its constituents due to diversification effects. Although individual components may exhibit high volatility, their combined impact at the index level is mitigated by less-than-perfect correlations. We now turn to how the pairwise correlation between individual options influences option volatility.

B. Implied Correlation and Concentration

To understand the impact of concentration on index option pricing, we can analyze the implied correlation among the index components. This is derived from the index volatility formula and reflects the market's expectation of how the individual stocks will move together.

1) Math Framework for Implied Correlation: We can define the implied correlation as:

$$\rho_{\text{implied}} = \frac{\sigma_I^2 - \sum_{i=1}^{n} w_i \sigma_i^2}{2 \sum_{i<j} w_i w_j \sigma_i \sigma_j} \tag{1}$$

2) Two-Asset Setting: To simplify the analysis, we first consider a portfolio with only two assets ($n = 2$). This allows us to clearly visualize the relationship between weight concentration and implied correlation. *(See full argument in Appendix).*

Figure 7 illustrates this relationship, highlighting how the relative magnitudes of individual asset volatilities (σ_1, σ_2) compared to the portfolio volatility (σ_I) influence the implied correlation.

Key observations from this analysis include:

- **U-Shaped Curves:** For most scenarios, the implied correlation curves exhibit a U-shape. This signifies that the implied correlation tends to be higher when the portfolio is heavily concentrated in either Asset 1 or 2 (i.e., w_1 is close to 0 or 1).
- **Impact of Volatility:** The relative volatilities of the assets influence the shape of the curve and the level of implied correlation. When both $\sigma_1, \sigma_2 < \sigma_I$ (blue or green region), the curves are U-shaped, confirming that the implied correlation increases as the portfolio becomes more concentrated in either asset. When one of the volatility is greater than the index volatility (red region), we see that the implied correlation can be negative. However, the case with a negative correlation is less of our interest as we rarely observe negative implied correlation.

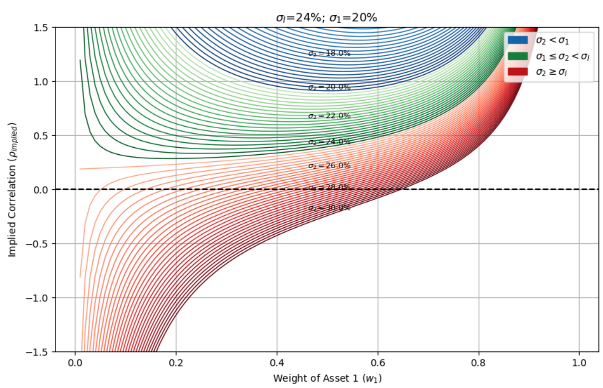

Fig. 7. The relationship between concentration and correlation in a two asset setting - $\sigma_1 > \sigma_{\text{Index}}$, varying σ_2.

3) Multi-Asset Setting: Extending the analysis to a multi-asset setting, we use simulations to investigate how the concentration of portfolio weights and the skewness of asset volatilities affect the implied correlation. *(See full argument in Appendix).*

The heatmap in Figure 8 reveals, as expected, greater weight concentration corresponds to higher implied correlation, particularly pronounced when volatility is skewed towards lower values. This suggests that in portfolios dominated by a few assets, the interrelationships between these dominant assets heavily influence the overall portfolio volatility, driving a stronger implied correlation.

Moreover, implied correlation tends to be lower when volatilities cluster on the lower side. This could be attributed to the fact that when most assets exhibit low volatility, their individual contributions to portfolio risk are diminished, leading to a weaker overall interconnectedness and thus a lower implied correlation.

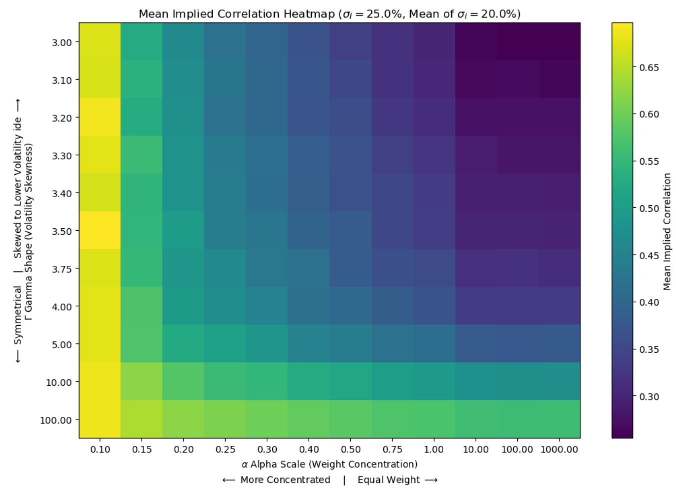

Fig. 8. Heatmap of Implied Correlation under different combinations of individual volatility and weight distributions.

C. Volatility Shocks and Dispersion

Figure 9 shows historical data highlighting similarities between patterns of MAG7 basket volatility and S&P500 index implied volatility, for a period between 2020 and 2023. For example, see the period between 2022-03-01 and 2022-07-19, where a decline in both index option volatility and basket volatility occurs.

Similar timing and magnitude of drifts in the MAG7 and index volatility highlights the tendency we have observed: as the S&P500 becomes more dominated by MAG7 stocks, its volatility aligns more with that of the MAG7 basket.

This has deleterious effects for use of such ETFs for tail-risk hedging. Indeed, greater concentration of indices in few stocks negates diversification opportunities.

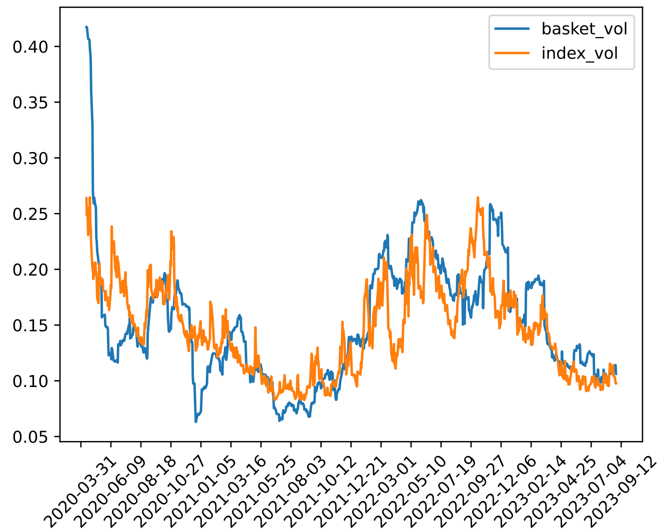

Fig. 9. Implied correlation of SPY500 option, alongside annualized basket volatility of MAG7 stocks.

IV. Extensions higher moments of the index distribution

Our discussion so far has focused on the impact of concentration on volatility. However, it's important to consider the effect on higher moments of the index distribution, such as skewness and kurtosis, which can also affect option pricing.

Empirical evidence shows that stock returns often exhibit excess kurtosis, leading to a volatility smile in option prices. We can argue that higher concentration can lead to a preservation of this excess kurtosis in the index.

A. Modeling Kurtosis

To illustrate this, consider a factor model where stock returns are driven by common factors and idiosyncratic residuals:

$$r_i(t) \;=\; \sum_{k=1}^{m} \beta_{i,k}\, F_k(t) \;+\; \epsilon_i(t),$$

where

- Each factor $F_k(t)$ is normally distributed with mean 0 and variance σ^2,
- The residuals $\epsilon_i(t)$ are i.i.d. with *excess kurtosis* k.

Then assume each stock has a weight w_i in the index. The portfolio return is:

$$r_p(t) = \sum_{i=1}^{n} w_i r_i(t).$$

By substituting this formula into the factor model, and defining:

$$\beta_{p,k} = \sum_{i=1}^{n} w_i \beta_{i,k},$$

we arrive at:

$$r_p(t) = \sum_{k=1}^{m} \beta_{p,k} F_k(t) + \sum_{i=1}^{n} w_i \epsilon_i(t).$$

Since the factors $F_k(t)$ are normally distributed, they have zero excess kurtosis, so only the residual term contributes to excess kurtosis. As a result, by applying the linearity of the kurtosis formula, we see that the variance of the portfolio residual term is:

$$\sigma^2_{\epsilon_p} = \sum_{i=1}^{n} w_i^2 \sigma_\epsilon^2.$$

Thus, the excess kurtosis of the portfolio return is:

$$\text{Excess Kurt}[r_p] = \frac{\sum_{i=1}^{n} w_i^4 k}{\left(\sum_{i=1}^{n} w_i^2\right)^2}.$$

Through these formulas we see how diversification reduces Kurtosis, and concentration increases kurtosis. This is of course an expected outcome, since by the Central Limit Theorem we would expect a sum of well diversified residuals to tend to a Gaussian distribution (since residuals are independent).

B. Simulation of Kurtosis and Concentration

To visualize this relationship (Figure 10), we simulate a portfolio with varying levels of concentration and calculate the corresponding kurtosis.

Therefore, as concentration increases, we not only expect to pay a higher premium for options in general, but also a relatively higher premium for options that are heavily in or out of the money.

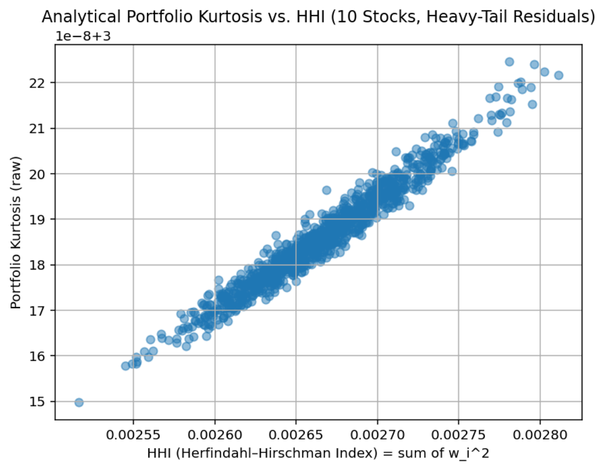

Fig. 10. Return Kurtosis vs HHI

V. Extensions to Equity Long/Short Investing

Index concentration has not only impacted the effectiveness of market beta and index options pricing but also introduced real financial challenges for institutional investors. As major indices have become increasingly dominated by a handful of mega-cap stocks—particularly in the technology sector—their strong returns have made it difficult for investors without significant exposure to these stocks to outperform the market.

In this section, we examine how this shift has been particularly detrimental to investors seeking exposure to the size factor but fail to account for the shift in additional factor exposures when constructing their portfolios. We will particularly focus on how index concentration can exacerbates the challenges in constructing these portfolios.

The size factor, formally introduced by Fama and French (1992) [3] in their three-factor model, which captures the historical tendency for small-cap stocks to generate a return premium over large-cap stocks. This premium was initially observed by Banz (1981) [5] and later formalized in asset pricing models. Historically, small-cap stocks have delivered higher returns over longer periods, reflecting compensation for additional risk. However, in recent years, with the exception of a brief rebound during the COVID-19 recovery, large-cap stocks have significantly outperformed. Table III shows annualized performance for the size factor in recent years.

2018	2019	2020	2021	2022	2023	2024
-3.08	-4.57	12.02	-2.39	-6.29	-1.25	-8.19

TABLE III
Size Factor Returns

A. Implications

We start by assuming the role of an equity long/short hedge fund, who wishes to gain exposure to the size factor. A naive implementation of this strategy would be to simply take long positions in small cap stocks, and short positions in large cap stocks. We represent this investor with a benchmark portfolio which is constructed with a long position in the S&P600 Small Cap Index and a short position in the S&P500 Index. We additionally assume that the portfolio is constructed to be dollar neutral.

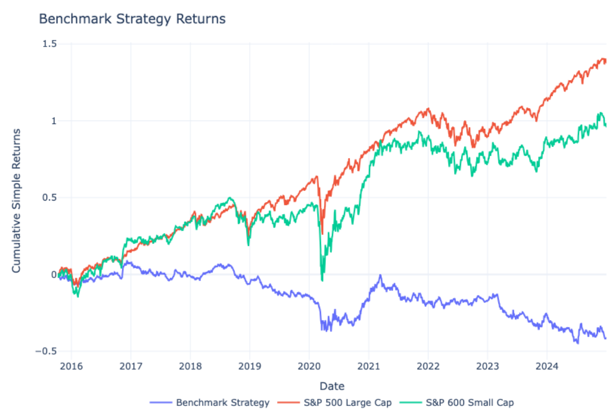

Fig. 11. Benchmark Strategy Performance

Figure 11 and our previous discussions of index concentration highlight key issues in the construction of this portfolio. Firstly, while the S&P600 Small Cap Index has performed well over the past few years, it has underperformed relative to the S&P500. Secondly, we know that the technology sector, which represents a large portfolio of the S&P500 has performed exceedingly well in recent years. It's no surprise, that when this strategy is constructed naively at the sector level, technology is the worst performing, seen clearly in Figure 12.

Given that our benchmark portfolio can be viewed as a weighted average of the above sector level returns, the dominance of technology in the

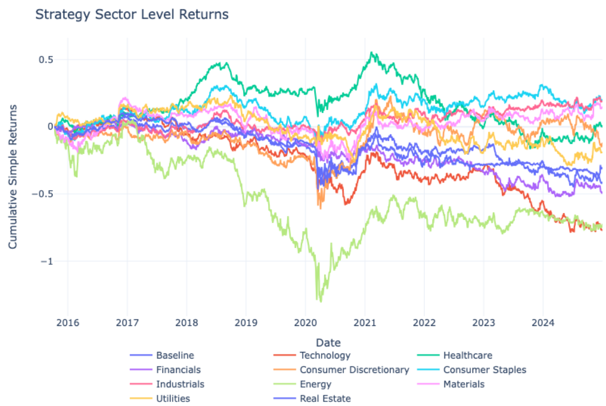

Fig. 12. Sector: Small Cap - Large Cap

S&P500, and it's clear under performance when the strategy is constructed at the sector level highlights a clear direction to improving the strategy - limiting portfolio exposures at the sector level.

Additionally, given the rise of technology in the S&P500, the index has seen a shift in factor exposures. More specifically, the index has become increasingly concentrated in growth style stocks. This is contradictory to the goals of an investor who is seeking exposure to the size factor, who, traditionally would seek complementary exposure to factors such as value, quality, and sentiment.

B. *Strategy Improvements: Sector Level*

Our first improvement of the strategy is to limit the sector level exposure, and also limit exposures sectors which are relatively over-performing on our short side. This improvement to the initial strategy is similar in nature to a traditional momentum strategy. We construct and re-balance the portfolios at constant intervals of 30 days, with a lookback period of 120 days. The portfolio construction process is as follows:

1) Compute sector-level returns r and volatility σ over the lookback window for small-cap and large-cap ETFs.
2) Define the momentum factor as:
$$m = \frac{r}{\sigma^2}$$
where r is the cumulative return over the lookback period and σ is the volatility over the same period.
3) Rank small-cap and large-cap stocks based on their momentum factor m.
4) Construct the short portfolio using an equal weight of 5 worst ranking large-cap stocks.
5) Construct the long portfolio using an equal weight of 5 top ranking small-cap stocks.

The improved strategy performance is shown in Figure 13, and summary statistics in Table IV.

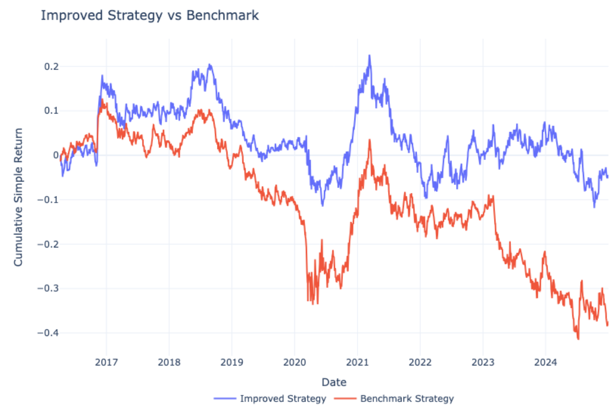

Fig. 13. Improved Strategy vs Benchmark

Metric	Benchmark Strategy	Improved Strategy
Annualized Return	-0.0497	-0.0136
Annualized Volatility	0.1282	0.1292
Sharpe Ratio	-0.3337	-0.0414
Sortino Ratio	-0.0021	-0.0002
Maximum Drawdown	-0.4551	-0.3161

TABLE IV
SUMMARY STATISTICS

While our improved strategy has clearly outperformed the initial one, it is also important to assess how it better aligns with the investor's objectives relative to the risks they are exposed to. To gain further insight, we examine the Fama-French factor betas for each strategy, shown in Table V

Factor	Benchmark (P-value)	Improved (P-value)
SMB	1.0481 (0.000)	0.9095 (0.000)
HML	0.1740 (0.000)	0.1175 (0.000)
RMW	0.1032 (0.000)	0.0210 (0.584)
CMA	0.0437 (0.032)	-0.0231 (0.673)
UMD	-0.0023 (0.768)	0.2392 (0.000)

TABLE V
FAMA-FRENCH REGRESSION BETAS

The above summary table shows that, while we were able to maintain our desired risk exposure to the size factor, the improvement allowed us to materially reduce our exposure to the HML factor. In other words, our portfolio's risks are now less correlated with growth stocks. This was a clear issue with the benchmark approach, which had a tech bias and, consequently, a growth bias that was negatively impacting our portfolio performance.

While the strategy presented above outperformed the benchmark and improved upon key characteristics of the strategy, a clear weakness is that the signal generation was limited to returns and volatility. As a result, this makes it difficult to construct portfolios that are well aligned with factors that compliment an investor who is seeking exposure to the size factor. These complementary style factors might include value, quality etc. As a result, future extensions to this strategy could additionally capture quantitative measurements of these complementary factors at the firm level, such as price-to-book for value or return on equity for quality. These could then be used during signal generation and thus better align the portfolio with complementary exposures to size.

VI. Conclusion

In summary, we have identified significant impacts across the investment landscape due to market concentration. Specifically, we examined how this concentration affects traditional risk models like the CAPM and the effectiveness of single-factor hedging. We also discussed its influence on the prices of index options and the underlying pricing mechanics. In the final section, we explored a clear example of how market concentration affects an investor seeking exposure to the size factor, which has been notably impacted by large-cap dominance. Finally, we empirically showed how such an investor might mitigate these effects and re-emphasized the importance of incorporating additional risk factors when constructing and analyzing portfolios.

References

[1] Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. The Journal of Finance, 19(3), 425–442.

[2] Fama, E. F., & French, K. R. (2004). The capital asset pricing model: Theory and evidence. Journal of Economic Perspectives, 18(3), 25-46.

[3] Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. Journal of Finance, 47(2), 427-465.

[4] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. Journal of Political Economy, 81(3), 637-654.

[5] Banz, R. W. (1981). The relationship between return and market value of common stocks. Journal of Financial Economics, 9(1), 3-18.

VII. Appendix

A. Analysis of Implied Correlation in a Two-Asset Portfolio ($n = 2$)

We analyze the implied correlation (ρ_{implied}) in a two-asset portfolio, where the portfolio variance (σ_I^2) is related to individual asset variances (σ_1^2, σ_2^2) and the weight of asset 1 (w_1) by:

$$\rho_{\text{implied}} = \frac{\sigma_I^2 - w_1^2\sigma_1^2 - (1-w_1)^2\sigma_2^2}{2w_1(1-w_1)\sigma_1\sigma_2}$$

We consider two main cases:

Case 1: Equal Asset Volatilities ($\sigma_1 = \sigma_2 = \sigma$)
The implied correlation simplifies to:

$$\rho_{\text{implied}} = \frac{\sigma_I^2 - \sigma^2}{2w_1(1-w_1)\sigma^2}$$

- **Trivial Case ($\sigma_I = \sigma$):** If the portfolio volatility equals the individual asset volatility, then $\rho_{\text{implied}} = 0$.
- **Portfolio Volatility Higher ($\sigma_I > \sigma$):**
  - $\rho_{\text{implied}} > 0$ for all $w_1 \in (0, 1)$.
  - ρ_{implied} is minimized at $w_1 = 0.5$, indicating the lowest implied correlation when the portfolio is equally weighted.
  - As w_1 deviates from 0.5 (increased concentration), ρ_{implied} increases.
- **Portfolio Volatility Lower ($\sigma_I < \sigma$):**
  - $\rho_{\text{implied}} < 0$ for all $w_1 \in (0, 1)$.
  - $|\rho_{\text{implied}}|$ is minimized at $w_1 = 0.5$.
  - As w_1 deviates from 0.5, $|\rho_{\text{implied}}|$ increases.

Case 2: Unequal Asset Volatilities ($\sigma_1 \neq \sigma_2$)
We express individual volatilities as $\sigma_1 = \beta_1\sigma_I$ and $\sigma_2 = \beta_2\sigma_I$, where $\beta_1, \beta_2 \neq 1$. The implied correlation becomes:

$$\rho_{\text{implied}} = \frac{1 - w_1\beta_1^2 - (1-w_1)\beta_2^2}{2w_1(1-w_1)\beta_1\beta_2}$$

- **Case 2.1: σ_I Between σ_1 and σ_2 (i.e., $\beta_1 < 1 < \beta_2$ or $\beta_2 < 1 < \beta_1$)**
  - There exists a weight $\hat{w}_1 = \frac{\beta_2^2 - 1}{\beta_2^2 - \beta_1^2}$ where $\rho_{\text{implied}} = 0$.
  - ρ_{implied} can be negative, but we focus on the positive side (as negative implied correlations are less common).
  - ρ_{implied} is minimized at $\hat{w}_1$.
  - As w_1 deviates from $\hat{w}_1$ (increased concentration), ρ_{implied} increases.
- **Case 2.2: $\sigma_1, \sigma_2 < \sigma_I$ (i.e., $\beta_1, \beta_2 < 1$)**
  - $\rho_{\text{implied}} > 0$ for all $w_1 \in (0, 1)$.
  - ρ_{implied} is concave upwards and has a minimum at some $\hat{w}_1 \in (0, 1)$.
  - As w_1 deviates from $\hat{w}_1$ (increased concentration), ρ_{implied} increases.
- **Case 2.3: $\sigma_1, \sigma_2 > \sigma_I$ (i.e., $\beta_1, \beta_2 > 1$)**
  - $\rho_{\text{implied}} < 0$ for all $w_1 \in (0, 1)$.
  - ρ_{implied} is concave downwards and has a maximum (minimum absolute value) at some $\hat{w}_1 \in (0, 1)$.
  - This case is less common in markets, as portfolio volatility is usually higher than the lowest individual asset volatility.

B. Implied Correlation Simulation

In our simulation, we modeled portfolios comprising 100 assets. We conducted 10,000 iterations for each combination of weight concentration and volatility skewness parameters. The target mean volatility for the assets was set to 20%. This extensive simulation allowed us to explore a wide range of scenarios and analyze the distribution of implied correlations under different conditions.

We use the Dirichlet distribution (Figure 14) to generate random portfolio weights with varying degrees of concentration. The scale (α) parameter controls the concentration, with smaller values leading to more concentrated weights (similar to the "Magnificent Seven" stocks dominating the S&P 500).

We use the Gamma distribution to generate random asset volatilities with different skewness levels. The γ parameter controls the skewness, with smaller values resulting in more skewed distributions (representing scenarios where most assets have low volatilities, but a few have high volatilities).

For each combination of generated weights and volatilities, we calculate the implied correlation.

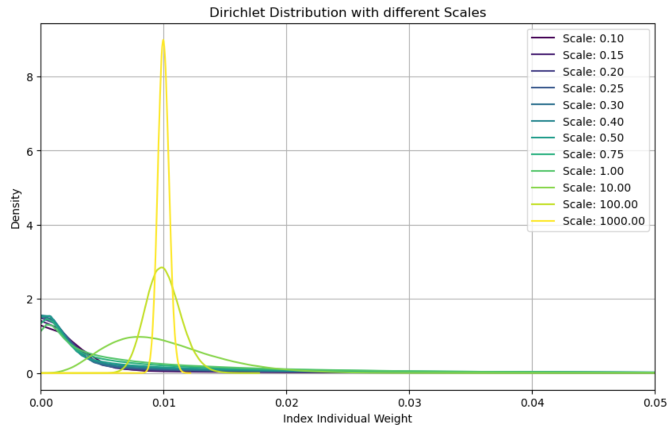

Fig. 14. Dirichlet Distribution with varying α (scale) parameter. The x-axis is the values of asset weights, while the y-axis is the density.

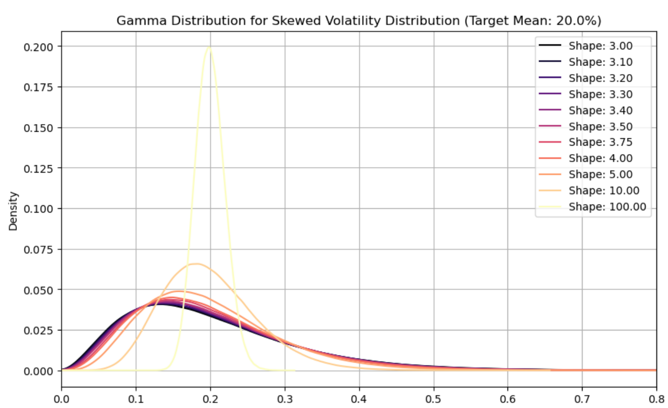

Fig. 15. Gamma Distribution with varying γ (scale) parameter. The x-axis is the values of asset volatility, while the y-axis is the density.