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A SIMPLE MODEL OF NON-STATIONARITY OF SYSTEMATIC RISK

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I. INTRODUCTION; A SPECIFIC MODEL OF NON-STATIONARITY

In the Capital Asset Pricing Model (CAPM) developed and elaborated by Sharpe (1964), Lintner (1965), Mossin (1966), Fama (1971), and others, the equilibrium expected rate of return of a security is related to its systematic risk, (the beta coefficient). The model does not require that the beta coefficient of a security be stable over time. In theory market participants are presumed to know the beta coefficients of the available securities. But the model provides no explicit clues as to how this knowledge is to be obtained.

In empirical applications of the model, the almost universal practice has been to estimate the beta coefficient of a security (or portfolio) by regressing its realized rate of return on the contemporaneous realized rate of return on a market portfolio. This procedure assumes that the beta coefficient is stationary. Blume (1971) was the first to investigate the stationarity problem and found a product moment correlation of .6 for individual securities. Later empirical and theoretical studies (e.g., Blume (1975), Gonedes (1973), Fisher and Kamin (1972), Meyers (1973)) suggest that beta coefficients can not be considered stationary.

The empirical evidence, however, that the beta coefficients of individual securities are non-stationary has had little or no influence on the way beta coefficients are actually estimated, because no specific usable alternative to the assumption of stability has received general acceptance.

In this paper we are suggesting a specific model of non-stationarity that employs a rather simple approach, but is, nevertheless, consistent with similar ideas suggested in recent studies. The basic approach is to get at the sources of non-stationarity by investigating the relation between the risk of the security and the risk of the underlying real assets. Rubinstein's [1973] simplified model suggests the following relation between the β of a security and the value-V of the underlying asset

$$\beta = B/V \tag{1}$$

where B is the risk of the real asset.[1] β can be thought of as a measure of the amount of risk per dollar of value. For a non-zero β to remain constant when the value of the asset changes, there must be a proportional change in the value of B. The quantity B can be thought of as a measure of the absolute amount of risk associated with the asset. If the absolute amount of risk associated with the asset

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1. The model is derived in Brenner and Smidt (1975).

remains constant when the value of the asset changes, then variations in β will be inversely proportional to variations in the value of the asset. It can be shown that a non-zero β might be expected to remain constant if there are no changes in any of the underlying conditions, or if the firm acquires additional assets, and makes no changes in its capital structure.[2] Almost any other change in underlying conditions that might lead to a change in the price of a security is likely to lead also to a change in the beta coefficient of the security.

A number of other writers have recently presented theoretical models that involve similar ideas and imply patterns of non-stationarity that are very close to the above model.[3]

The basic question considered in this paper is which of the following two quantities is more nearly constant over time:

$$\mathrm{Cov}\left(\frac{\Delta P_t + D_t}{P_{t-1}}, \frac{R_m}{\sigma^2(R_m)}\right), \qquad \text{or} \qquad \mathrm{Cov}\left(\Delta P_t + D_t, \frac{R_m}{\sigma^2(R_m)}\right).$$

The first alternative represents the usual practice. The second alternative is in the spirit of Rubinstein and the other authors cited above.

II. Non-Stationarity in the Market Model Context

The capital asset pricing model describes a relationship that should exist in equilibrium between the expected rate of return on a security, the expected rate of return on the market portfolio, and the covariance between the two rates of return. Neither of these expected rates of return, nor their covariance are directly observable. Therefore to use the model in any empirical applications, (including tests of the validity of the hypothetical relationships) additional assumptions are needed. The purpose of these additional assumptions is to establish a relationship between some directly observable qualities, and the theoretical concepts specified by the model. These additional assumptions are discussed in Fama (1973, 1968), and Beja (1972).

Consider the following assumptions, which are assumed valid for all t:

$$\tilde{R}_t = R_{ft} + \beta_t\left[\tilde{R}_{mt} - R_{ft}\right] + \tilde{\mu}_t \tag{2a}$$

$$E(\tilde{\mu}_t) = 0 \tag{2b}$$

$$E\left(\tilde{\mu}_t \cdot \tilde{R}_{mt}\right) = 0 \tag{2c}$$

$$\mathrm{Var}\left(\tilde{R}_{mt}\right) = \sigma_m^2, \qquad \text{for all } t \tag{2d}$$

$$E\left(\tilde{R}_{mt}\right) = \overline{R}_m, \qquad \text{for all } t \tag{2e}$$

It follows that

$$E\left(\tilde{R}_t\right) = R_{ft} + \beta_t\left[E\left(\tilde{R}_{mt}\right) - R_{ft}\right] \tag{3}$$

2. See Brenner and Smidt (1975).

3. See Brennan (1973), p. 671, Black and Scholes (1973), p. 645, Galai and Masulis (1975), p. 9, and Stone (1974), p. 10.

For present purposes the essential point to note is that equations (2) do not necessarily require beta stability. In practice most investigators have made the additional assumption that

$$\beta_t = \beta, \qquad \text{for all } t \tag{4a}$$

which implies that beta coefficients are stable. Alternatively we could specify a market model conforming to (2) and consistent with the CAPM in which

$$\beta_t = \frac{B}{V_{t-1}}, \qquad \text{for all } t \tag{4b}$$

where V_{t-1} is the end-of-period $(t-1)$ value of the common stock of the firm. The exact specification of β_t in (4b) is derived from a set of assumptions about the process generating investors expectations and is given in Brenner and Smidt (1975). (4b) is just the empirical-observable counterpart of (1).

The point to be emphasized is that the CAPM by itself does not imply either beta stability, or any particular form of non-stationarity. If (4b) is valid then we could write (2a) as

$$\tilde{R}_t = R_{ft} + B\frac{\left(\tilde{R}_{Mt} - R_{ft}\right)}{V_{t-1}} + \tilde{\mu}_t \tag{5}$$

Previous theoretical models that suggested non-stationarity have not been subjected to empirical testing since no specific alternative was provided. With (4b) (or (5)) we can now test the stationarity of β against a specific alternative.

III. Empirical Tests: Methodology and Results

A. *Data and Market Models Used for Tests*

The population studied consisted of 762 New York Stock Exchange (NYSE) stocks for which data were available on the CRSP tapes for 120 consecutive months ending in June 1968.[4]

The following four market models were used to provide statistics for our different tests:

$$\tilde{R}_{jt} - R_{ft} = \beta_j\left(\tilde{R}_{mt} - R_{ft}\right) + \tilde{e}_{jt} \tag{6A}$$

$$\tilde{R}_{jt} - R_{ft} = B_j\frac{\left(\tilde{R}_{mt} - R_{ft}\right)}{V_{jt-1}} + \tilde{u}_{jt} \tag{6B}$$

$$\tilde{R}_{jt} - \tilde{R}_{zt} = \beta_j\left(\tilde{R}_{mt} - \tilde{R}_{zt}\right) + \tilde{e}_{jt} \tag{6C}$$

$$\tilde{R}_{jt} - \tilde{R}_{zt} = B_j\frac{\left(\tilde{R}_{mt} - \tilde{R}_{zt}\right)}{V_{jt-1}} + \tilde{u}_{jt} \tag{6D}$$

4. This is the last month for which we had reliable data.

where R_{ft} = yield on Treasury Bills with 30 days to maturity.
$\tilde{R}_{jt}$ = observed rate of return on security j in month t.
$\tilde{R}_{mt}$ = observed rate of return on the market portfolio, as represented by Fisher's arithmetic index in month t.
V_{jt-1} = price per share, adjusted for splits, of security j at the end of month $t-1$.
$\tilde{R}_z$ = $\tilde{\gamma}_{0t}$; an estimate of the rate of return on a zero-beta portfolio, in month t.[5]

For future reference, let:

$$\tilde{r}_{jt} \equiv \tilde{R}_{jt} - R_{ft}, \qquad \tilde{r}_{mt} \equiv \tilde{R}_{mt} - R_{ft}$$

The four models will be referred to as models A, B, C and D.

Since the hypothesis here does not depend on a specific version of the CAPM, we have decided to use market models consistent with both the Sharpe-Lintner and the Black (1972) versions. Thus, we can test stationarity using A and B, or C and D.

It is important to note that models B and D differ from A and C only in the specification of the coefficient of $\tilde{R}_{mt}$, where V_{jt-1} is a parameter that changes over time, and may also change with some other variables in the model. The effect, if any, of V_{jt-1} on regression estimates will come about through the estimated residuals in the model. The problem here, in the time-series context, stems from serial-correlation in V_{jt}, where it can best be seen by rewriting $\tilde{u}_{jt}$ as $\tilde{v}_{jt}/V_{jt-1}$ (See Brenner and Smidt (1975), Appendix II, equation 10). Since successive values of V_{jt} are positively correlated, it is very probable that the serial correlation

$$\rho(\tilde{u}_{jt}, \tilde{u}_{jt-1}) = \rho\left(\frac{\tilde{v}_{jt}}{V_{jt-1}}, \frac{\tilde{v}_{jt-1}}{V_{jt-2}}\right) > 0.$$

Since the true residuals are unobservable, the seriousness of serial correlation is an empirical question. To test for serial correlation we used the conventional Durbin-Watson (DW) test and could not reject the hypothesis of no serial correlation.[6,7]

5. These estimates were supplied to one of the authors by E. F. Fama and J. MacBeth. The estimates are obtained from the model presented in Panel A, Table 3 of Fama and MacBeth (1973).

6. The problem with the DW test is that it is based on residuals estimated by OLS which will provide an underestimation of serial correlation if it does exist.

7. The DW statistic is computed by (Johnston [1972] p. 251)

$$d = \frac{\sum_{t=2}^{n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{n} \hat{u}_t^2} = \frac{\sum_{t=2}^{n} \hat{u}_t^2 + \sum_{t=2}^{n} \hat{u}_{t-1}^2 - 2\sum_{t=2}^{n} \hat{u}_t \hat{u}_{t-1}}{\sum_{t=1}^{n} \hat{u}_t^2}$$

It can be seen that the more the $\hat{u}_t$ are positively correlated the smaller and further from 2 d will be. Since, however, $\hat{u}_t$ is obtained by OLS, the computed $\sum_{t=2}^{n} \hat{u}_t \hat{u}_{t-1} < \sum_{t=2}^{n} u_t u_{t-1}$ and thus resulting in a value of d closer to 2. Only if $\sum_{t=2}^{n} \hat{u}_t \hat{u}_{t-1} < 0$ is $d > 2$. Since u_t, if serially correlated, is certainly positively correlated, (i.e., $\sum_{t=2}^{n} u_{t,t-1} > 0$) it seems unlikely, with our sample size, that $\sum_{t=2}^{n} \hat{u}_t \hat{u}_{t-1} < 0$, which is implied by our d value that is somewhat larger than 2. Our only conclusion is that serial correlation is not a serious problem in our context.

Another problem that should be considered is the possible heteroscedasticity in models (6A) or (6B). Heteroscedasticity occurs when the residual variance, $\sigma^2(e)$, is dependent on the exogenous variable, denoted X, or on t. Test statistics that are based on the residual variance and assume a constant $\sigma^2(e)$ (e.g., R^2, Chow test) will be biased. To test whether $\sigma^2(e)$ is a function of X or t we chose a random sample of 200 stocks with data for the 120 months ending June 1968 and applied a parametric test suggested by Glejser (1969). For each company for both models, (6A) and (6B), we employed the following regressions

$$|\hat{e}_t| = a_0 + a_l X_t + w_t$$

$$|\hat{e}_t| = b_0 + b_l t + \epsilon_t$$

where X_t is r_{mt} in (6A) or r_{mt}/V_{t-1} in (6B) and $|\hat{e}_t|$ are the absolute values of the residuals obtained from (6A) or (6B). Using the first regression and model (6A) (i.e., $X_t = r_{mt}$) we could not reject, at a 5% significance level, the hypothesis of homoscedasticity for 165 out of 200 companies. For model (6B) we could not reject the hypothesis for 168 out of 200 companies (about 85%). Additional evidence that support the assumption of homoscedasticity was provided in an article by Martin and Klemkosky (1976) that employed several tests of heteroscedasticity on a model like (6A)[8] and conclude "...that heteroscedasticity was not a serious problem in the market model..."

The first regression tests for a relation between the error and the independent variable are not necessarily testing for the constancy of $\sigma^2(e)$ through time. However, results that show no relation of $|\hat{e}|$ with t are consistent with the hypothesis that the residual variance was constant over time. The results from using t in the regression with $|\hat{e}_t|$ were similar to the other results; for model (6A) we could not reject the hypothesis of constant residual variance in 164 out of 200 companies while for (6B) we could not reject the hypothesis for 156 of the companies (where about 2/3 of the rejected cases showed a positive relation and $\frac{1}{3}$ a negative one). Even for companies that showed a significant relation the estimated correlations between the error term and X_t or t were rather low, around 20–25%, and therefore may produce only a trivial asymptotic bias.[9] We conclude, therefore, that over the period of 120 months used in this study heteroscedasticity in models (6A) and (6B) is not a serious problem and thus we can use the tests that assume homoscedasticity.

In testing the models we have used seven time intervals: the entire 120 month period ending June 1968; two successive 60 month periods ending on this date; and four successive 30 month periods ending on this date.[10] The tests presented in this paper are conducted in the following way. First, each of the two competing models is tested separately. That is, for each model we assume that the basic properties of the model hold (e.g., regression assumptions) and the null hypothesis of stationarity

8. Their tests indicated that "less than 15 percent of the companies provided any evidence of heteroscedasticity" (p. 85).

9. See Malinvaud (1970), pp. 304–308.

10. Since some of the different time intervals share at least part of their data, the test results are not always independent. Nevertheless, this facilitates some of our comparisons that are concerned with the length of the interval.

is tested against a composite hypothesis of non-stationarity. Even though each model is tested separately. The mere existence of another specific model makes our test more powerful. Our interests lie, of course, in the second stage when we compare the separate stationarity tests to see which model does better. At this stage it was not necessary to employ significance tests since the size of our sample is so large that there is no need to do the calculations to see when two statistics differ.

In many cases, the reader will observe that the statistics for the two models differ significantly. Even though the difference appears to be *statistically* significant, it may not be important for practical purposes (e.g., How important is an average R^2 of .26 vs. an average R^2 of .24?). Moreover, in studies like this that rely on a large data base [the CRSP tapes] that have been studied extensively, the conventionally computed levels of significance for standard statistical tests grossly overstate the true level of significance. This caution applies particularly to our results, where a sequence of observations have been made (i.e., test statistics computed) on a particular subset of the data; but it could be valid even if we had computed only one test statistic from this data. For that reason, although we follow the custom of reporting our results in a "test of hypothesis" framework, we believe it is at least equally useful to interpret them as a description or estimation of the characteristics

TABLE 1

CROSS-SECTIONAL MEANS AND STANDARD DEVIATIONS* OF REGRESSION STATISTICS

		Time Interval (in months)						
Statistic	Model	120	60(1)	60(2)	30(1)	30(2)	30(3)	30(4)
R^2	*A*	.256	.293	.228	.184	.373	.164	.275
		(.099)	(.120)	(.123)	(.128)	(.160)	(.126)	(.157)
	B	.240	.291	.225	.184	.370	.168	.268
		(.099)	(.120)	(.120)	(.126)	(.162)	(.130)	(.154)
	C	.301	.214	.364	.148	.259	.389	.351
		(.115)	(.131)	(1.31)	(.128)	(.179)	(.156)	(.165)
	D	.273	.209	.348	.146	.255	.391	.340
		(.116)	(.130)	(.133)	(.127)	(.175)	(.161)	(.164)
t-values	*A*	6.66	5.16	4.39	2.75	4.35	2.67	3.44
		(1.74)	(1.51)	(1.60)	(1.16)	(1.54)	(1.32)	(1.43)
	B	6.39	5.13	4.35	2.76	4.33	2.71	3.38
		(1.69)	(1.50)	(1.53)	(1.14)	(1.54)	(1.32)	(1.38)
	C	7.39	4.03	6.29	2.36	3.30	5.16	4.11
		(2.04)	(1.70)	(1.72)	(1.23)	(1.67)	(1.62)	(1.57)
	D	6.91	3.96	6.09	2.35	3.25	5.18	4.00
		(2.00)	(1.69)	(1.69)	(.123)	(1.62)	(1.65)	(1.52)
$\rho(t,t-1)$	*A*	−.063	−.072	−.058	−.045	−.048	−.044	−.043
		(.102)	(.133)	(.136)	(.180)	(.181)	(.182)	(.176)
	B	−.054	−.063	−.032	−.044	−.044	−.041	−.028
		(.104)	(.134)	(.137)	(.180)	(.184)	(.184)	(.178)
	C	−.076	−.081	−.062	−.066	−.047	−.056	−.050
		(.107)	(.137)	(.139)	(.183)	(.184)	(.179)	(.182)
	D	−.064	−.076	−.054	−.075	.039	−.047	−.043
		(.110)	(.137)	(.140)	(.177)	(.187)	(.187)	(.184)

* Standard Deviations are in parenthesis.

of the data. The stationarity of β is important mainly for two purposes; 1. understanding the factors affecting differential rates of return (i.e., studies of capital market theory). 2. Forming good prediction of future β's for use in investment decisions. We have attempted to interpret our results in the light of these objectives, and we hope that the reader will also.

B. *First Test*: *Goodness-of-fit*

For each of the 762 stocks in the sample, regression equations were fitted to each of the four models described in (6) over each of the seven time intervals previously described. A t-value, R^2 and serial correlation coefficient (ρ) were obtained from each regression. These data are summarized in Table 1 which reports the cross-sectional means and standard deviations of each regression statistic, from each model and time period.

If the absolute amount of systematic risk (B_j) tended to remain constant, then the regression of model B (or D) should tend to explain month-by-month returns (r_{jt}), better than the corresponding regressions of model A (or C) and vice versa. Based on R^2 (or equivalently, on t-values) model A does somewhat better than model B in all except one period. (Similarly model C does better than model D.) The differences, however, are quite small. Considering the drastic differences in the independent variables, the most striking result is the similarity of the two models in their explanatory power (R^2). In addition, the small difference in favor of model A (or C) declines and even reverses, as the time interval gets shorter. If anything, this may indicate that our hypothesis may provide better estimates for shorter time periods.

In comparing the serial correlation coefficient (ρ), we find no important differences between A and B (or between C and D).

C. *Second Test*; *The Chow Test*.

The stationarity of β_j or B_j coefficients can be tested by one form of the so called Chow Test that tests for the equality of the coefficients in two time periods.[11] If we suspect that the relationship between the variables is not the same in both periods, the observations in those two periods will be used to form a test that will tell us whether the relationship might have changed.[12]

Formally, the test statistic is given by the F ratio:

$$F(k, n_1+n_2-2k) = \frac{Q_3/k}{Q_2/(n_1+n_2-2k)} \tag{7}$$

where k = number of parameters

n_1 = number of observations in first period

n_2 = number of observations in second period

Q_1 = Sum of squared residuals (SSR) in a regression over n_1+n_2

Q_2 = SSR in n_1 + SSR in n_2

$Q_3 = Q_1 - Q_2$

11. See Johnston (1972), p. 207.

12. The test statistic is distributed as F under the null hypothesis of no change in β and a constant $\sigma^2(e)$. As explained above (p. 7), in general, the data supports the assumption of a constant residual variance.

If $F > F_\alpha$, reject the hypothesis $\beta_1 = \beta_2 = \beta$ (or $B_1 = B_2 = B$) where α pertains to the chosen level of significance.

The F ratio was computed for all 4 models, for 762 companies, for three sets of observations:

1. The entire 120 months with two subsets of 60 months each;
2. The first 60 months with two subsets of 30 months each;
3. The second 60 months with two subsets of 30 months each.

Since the models presented in 6.A-6.D do not involve an intercept term we have supressed the intercept in all regressions. Thus, only the stationarity of the slope coefficient is tested.

Thus, we have a vector of 762 F ratios for each model for each of the above set of observations. Using a significance level of $\alpha = .05$, Table 2 gives the proportion of companies for which $F < F_\alpha$ (i.e., the proportion of companies for which the hypothesis $\beta_1 = \beta_2 = \beta$(or $B_1 = B_2 = B$) cannot be rejected). The results in Table 2 are, generally, consistent with the results in Table 1. That is, in the shorter period model B performs about as well as the conventional model (A) while in the long period the conventional model performs better. This apparent superiority is more pronounced in the case of zero beta models, C and D. The percentage of companies with stable coefficients in the long period (120 months) is 67% for model A vs. 53% for model B, while the corresponding percentages for models C and D are 80% and 57% respectively.

Since the models do not differ dramatically in terms of percentages of companies with stationary coefficients, it is interesting to examine to what extent companies tend to have stationary coefficients under both models, or under neither model, or under only one of the models. This data is summarized for the five percent significance level in Table 3. If we compare adjacent 60 month observation periods we observe that only 14–20% of the companies appear to have non-stationary coefficients under both models while 36–39% have stationary coefficients under one of the models, not under both.

It is apparent from Table 2 that under either model, over half of the companies would be classified as having stationary coefficients. Thus even if how a company was classified by one model was statistically independent of how it was classified by the other, a large percentage of the companies might be expected to be classified as stationary by both models. A second possibility is that companies classified as having stationary coefficients by one model are more likely to be classified as having non-stationary coefficients by the other model. In that case the empirical investigator may wish to determine which model is appropriate for a given

TABLE 2

PROPORTION OF COMPANIES WITH $F < F_\alpha$ ($\alpha = 5\%$)

Model	120(60, 60)	60(1)(30, 30)	60(2)(30, 30)
A	.668	.656	.672
B	.533	.639	.685
C	.796	.643	.595
D	.570	.630	.535

TABLE 3

PERCENTAGES OF COMPANIES WITH STATIONARY COEFFICIENTS UNDER BOTH MODELS, NEITHER MODEL, OR ONLY ONE MODEL AT THE 5% SIGNIFICANCE LEVEL.

	Percentage of Companies With Stationary Coefficients			
Models and Period	Under Both Models	Under Neither Model	Under Only One Model	Total
120 (60, 60)				
A and *B*	41	20	39	100
C and *D*	50	14	36	100
60 (1) (30, 30)				
A and *B*	38	44	17	100
C and *D*	56	29	15	100
60 (2) (30, 30)				
A and *B*	61	25	14	100
C and *D*	45	32	24	100

company. A third possibility is that companies will tend to receive the same classification regardless of whether we assume constant absolute risk or constant relative risk.

To examine these possibilities we compare in Table 4, the percentage of companies with stationary coefficients in both models (from Table 3) with the percentage that would be expected under statistical independence. For example using adjacent 60 month observation periods we note from Table 2 that under model *A* 67% and under model *B* 53% of the companies were classified as having stationary coefficients. If these classifications were statistically independent we would expect to observe 36% [=(.67)(.53)(100)] of the companies being classified as having stationary coefficients under both models. In fact, we observe (from Table 3) that 41% of the companies fall in this category. Thus there is a tendency for companies to be classified the same way by both models. This tendency is observed in nearly every comparison, although the strength of the tendency varies somewhat.

This suggests that some factor (or factors) contributing to unstable beta coefficients are not properly accounted for by either model.

TABLE 4

PERCENTAGES OF COMPANIES CLASSIFIED AS HAVING STATIONARY COEFFICIENTS UNDER BOTH MODELS VERSUS PERCENTAGE EXPECTED ASSUMING INDEPENDENCE
5% SIGNIFICANCE LEVEL

Model and Period	Percent Observed	Percent Expected	Observed-Expected
120(60, 60)			
A and *B*	41	36	5
C and *D*	50	45	5
60 (1) (30, 30)			
A and *B*	38	42	−4
C and *D*	56	41	15
60 (2) (30, 30)			
A and *B*	61	46	15
C and *D*	45	32	13

D. *Third Test; A Test Based on Mean Square Error*

Unpredictable instability in the risk coefficient of the CAPM is important mainly because it reduces the predictive accuracy of the model. Thus it seems appropriate to evaluate possible instability in the coefficients by studying the predictive accuracy of the model.

A natural statistic for this purpose is the Mean Square Error (MSE), given by

$$\mathrm{MSE}=\frac{1}{N}\frac{1}{T}\sum_{j}\sum_{t}(\hat{r}_{jt}-r_{jt})^2 \tag{8}$$

where r_{jt} is the actual excess rate of return for company j in month t, and $\hat{r}_{jt}$ is the predicted rate of return *conditional* on the value of the independent variable. The MSE can also be decomposed into two contributing components; the square of the average bias and the variance around the prediction error.

$$\mathrm{MSE}=\bar{r}_m^2\frac{1}{N}\sum_{j}\left(\hat{\beta}_j-\beta_j\right)^2+\frac{1}{N}\sum\sigma^2(e_j) \tag{9}$$

In some contexts, when MSE is used to evaluate a predictor, the ideal would be an MSE of zero. But in the CAPM model context, an ideal predictor would have an expected MSE equal to the variance of the residual, $\sigma^2(e)$. Therefore the absolute value of MSE is not measure of predictive accuracy in a CAPM context. However, in comparing two models (which share a common $\sigma^2(e)$), the difference between two MSE's can be used as a test statistic. The difference reflects the extent to which one model is more biased than the other. In this comparative sense, the model with the smaller MSE is better.

The MSE was computed for all 4 models and for 4 subsets of data.[13] In comparing a pair of models using the same data, it is the difference in the MSE's that is relevant.

TABLE 5

MSE For Four Models and Four Time Periods

MSE	60	30	30	30
A	.00528	.00471	.00421	.00666
B	.00558	.00486	.00436	.00659
C	.00531	.00449	.00445	.00623
D	.00584	.00469	.00471	.00665

The differences between the models seem minute and would not lead us to strongly prefer one model over the other. The results in Table 5 are consistent with the earlier ones, and suggest that, most probably, the two different hypotheses characterize different time periods and/or different companies.

In summary, the empirical tests employed in this section were unable to distinguish between the two alternative hypotheses tested in this paper despite the

13. The prediction period was always separated from the estimation period. Beta was estimated from the first 60 monthly observations and the MSE was computed from the next 60 observations; then beta was estimated from 30 monthly observations and the MSE from the next 30 observations and so on.

fundamental difference in the theoretical models and in their empirical counterparts. The results clearly indicate, in addition to the theoretical argument, that stationarity of β_j is not a good assumption; but without any better specific alternative it may not be so bad.

V. Conclusions

This paper is concerned with the stationarity of beta coefficients. The hypothesis that beta coefficients are stationary is contrasted with an alternative specific hypothesis of non-stationarity and both are subjected to empirical testing.

The CAPM specifies a relationship between the expected return of a security and its beta coefficient. The assumptions that are necessary to derive the relationship are not sufficient to imply that beta coefficients are stationary. In a static equilibrium framework, there is no theoretical support for the proposition that beta coefficients are stable. A plausible alternative to the assumption of a constant amount of risk per dollar of market value (which is implied by a constant beta coefficient) is the assumption that the absolute amount of risk is constant (and therefore that the risk per dollar of market value varies inversely with the market value). Empirical applications of the CAPM require specifying a market model (a statistical generating process) for actual rate of return. In practice the market models used have arbitrarily assumed constant risk per dollar of market value. An alternative market model that is consistent with the CAPM and that assumes the absolute amount of risk is constant (beta coefficient values are inversely proportional to market values) is presented.[14]

In a series of tests we contrast and compare the two null hypothesis: constant relative risk and constant absolute risk, for a sample of 762 New York Stock Exchange listed securities using data for the 10 year period ending June 30, 1968. Two findings emerge from these tests: that there is very little difference between the two hypotheses; and that the slight difference that does exist tends to favor the hypothesis of constant beta coefficients. A plausible interpretation of these findings is that both market models are incorrect specifications of the true data generating process. If this interpretation is correct the observed instability of beta coefficient estimates for individual securities may be due less to measurement error than had previously been believed, and more to misspecification of market models.[15]

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14. Brenner and Smidt (1975) derive such a market model from some more fundamental assumptions about the characteristics of the cash flows that will be received by the security holder, and the process by which the security holder updates his expectations about the magnitudes of these cash flows.

15. Consider the following view "...a significant portion of the measured changes in estimated beta values may not be due to changes in the true values, but rather to measurement errors. This observation is particularly applicable to individual security betas where the standard errors tend to be large." [Modigliani and Pogue (1974), p. 77].

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