JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS
Vol. XV, No. 1, March 1980

ON THE ESTIMATION AND STABILITY OF BETA

*Gordon J. Alexander and Norman L. Chervany**

I. Introduction

Beta coefficients were initially defined by Sharpe [11] as the slope term in the simple linear regression function where the rate of return on a market index was the independent variable and a security's rate of return was the dependent variable. As indicated by Brenner and Smidt [4], accurate estimation of beta coefficients is important for at least two reasons. First, they are important for understanding risk-return relationships in capital market theory. Second, they are important for use in making investment decisions. Some confusion has appeared, however, in recent research regarding both the optimal estimation interval and the intertemporal stability of beta coefficients. The purpose of this paper is to examine this confusion and present new evidence on the estimation and stability of beta.

In a recent article, Baesel [1] concluded that "the stability of beta is dependent upon both the estimation interval used and upon the extremity of the beta chosen."[1] Furthermore, he concludes that the longest of the estimation intervals evaluated, nine years, was optimal. However, Gonedes [6] has indicated that the optimal estimation interval is seven years. Part II of this paper will investigate the discrepancy between these two views and draw conclusions regarding the optimal estimation interval. In particular, the conclusions of Baesel will be shown to have been improperly drawn.

In another recent article, Porter and Ezzell [10] have questioned the stability of a portfolio's beta coefficient through time. In questioning the procedures used in past studies such as those of Blume [2] and Levy [8], Porter and Ezzell state:

> Evidence presented in this note suggests that the intertemporal stability of the beta coefficient... is sensitive to the procedure used to select portfolios. In particular, it was shown that if portfolios are randomly selected, the time-stability of beta is relatively slight and is totally unrelated to the number of securities in the portfolios.[2]

Both authors, University of Minnesota.

[1] Baesel [1, p. 1493].

[2] Porter and Ezzell [10, p. 369].

Part III of this paper will point out a possible reason for the differing opinions regarding the stability of portfolio beta coefficients. In particular, it will be shown that Porter and Ezzell's results are not inconsistent with those of Blume. Lastly, Part IV will draw conclusions and summarize the results previously presented.

II. Estimation of Security Betas

A. Use of Transition Matrices

Baesel calculated beta coefficients for 160 securities using different length estimation intervals over the period 1950 to 1967. After ranking the beta coefficients and placing the securities in pentiles for an estimation period t, he formed transition matrices by determining which pentile the securities were in during the following period t + 1. In interpreting the transition matrices, Baesel states that "the stability of the beta is dependent on the extremity of the beta chosen."[3] In particular, he noted that the probabilities at the extremes of the diagonal were greater than the interior diagonal probabilities, which implies that both the least risky and most risky securities have the most intertemporal stability.[4]

Baesel's interpretation of this finding is unfounded. Securities with betas placing them in either the highest or lowest pentile inherently have a greater probability of remaining in that pentile than those in the interior pentiles. Securities in the highest (lowest) pentile may move into a different pentile only if their betas decrease (increase) by a significant amount. If their betas rise (fall) significantly, they will remain in the same pentile. In contrast, securities in the interior pentiles may move to a different pentile if their betas either rise or fall. The existence of higher probabilities on the diagonal extremities of Baesel's transition matrices is a statistical artifact and is not sufficient evidence to warrant his conclusion.

B. Stability of Betas Contained in Extreme Pentiles

In order to determine if betas in the extreme pentiles are more stable than those in the interior pentiles, a sample of 160 common stocks listed on the New York Stock Exchange was taken. Rates of return were calculated for the sample monthly from 1950-1967 as well as for the Standard and Poor's 500 Composite Index as follows:

---

[3] Baesel [1, p. 1493].

[4] A similar observation is also apparent in transition matrices in Sharpe and Cooper [12].

(1) $$R_{it} = (P_{it}-P_{it-1}+D_{it})/P_{it-1} .$$

Here P_{it} reflects the price of security i at time t and D_{it} reflects any dividend payments on security i from time t-1 to time t. Beta coefficients β_i were then calculated using Sharpe's single index model as follows:

(2) $$R_{it} = \alpha_i+\beta_i R_{mt}+\varepsilon_{it} .$$

Here R_{mt} denotes the rate of return on the SP500 and α_i and β_i are the regression parameters to be estimated. For the 18-year sample time period, beta coefficients were calculated for consecutive one-year estimation intervals, resulting in 18 betas per security. Then nine betas were calculated for consecutive two-year estimation intervals per security. The procedure was repeated to produce four betas for four-year estimation intervals, three betas for six-year estimation intervals and two betas for nine-year estimation intervals. Thus for each security 36 beta coefficients were calculated.[5]

After calculation of the beta coefficients, transition matrices were formed for the varying estimation periods. Thus 17 comparisons of β_t with β_{t+1} were performed for each security using one-year estimation periods, eight comparisons for two-year estimation periods and so on, ending with one comparison for the nine-year estimation periods. The diagonal entries of the five transition matrices are displayed in Table 1. The bulging of probabilities for pentiles 1 and 5 are again apparent for the longer estimation intervals, although not as extreme as those present in Baesel's sample.[6]

In addition to the probabilities of remaining in a pentile, the mean absolute deviation (MAD) of changes in the betas were calculated. The MAD statistics displayed in Table 1 suggest that larger changes in beta occurred in the extreme pentiles than in the interior pentiles for all but one case, the fifth pentile using six-year estimation periods.[7] In order to determine if the

---

[5] Similar results to those reported here were also found using the time period 1958-75. The time period 1950-67 results are reported here as it coincides with the period analyzed by Baesel [1]. The basic procedure followed also coincides with that described by Baesel.

[6] Pentile 1 corresponds to the 20 percent smallest betas while pentile 5 corresponds to the 20 percent largest betas.

[7] The mean absolute deviation, $|\beta_t-\beta_{t+1}|$, is used here as a measure of stability of beta. Mood, Graybill, and Boes [9, p. 297] suggest using MAD as a possible loss function in parameter estimation.

TABLE 1

STABILITY OF BETA: PROBABILITIES AND MAD

Length of Estimation Period	β_t Pentile					
	1	2	3	4	5	H Test
1 Year:						
Probabilities	.18	.22	.22	.18	.24	
MAD	2.50	1.26	1.14	1.40	2.60	453.35*
2 Years:						
Probabilities	.21	.23	.21	.21	.29	
MAD	1.49	.75	.75	.92	1.32	157.42*
4 Years:						
Probabilities	.21	.23	.18	.23	.29	
MAD	.80	.48	.51	.57	.83	41.22*
6 Years:						
Probabilities	.30	.25	.16	.13	.23	
MAD	.87	.57	.54	.60	.53	8.27**
9 Years:						
Probabilities	.28	.25	.25	.25	.44	
MAD	.67	.43	.30	.41	.46	14.56*
Baesel Probabilities:						
1 Year	.33	.23	.23	.21	.34	
9 Years	.35	.41	.13	.21	.55	

*Significant at the .05 level.

**Significant at the .10 level.

differences in the MAD distributions were due to sampling error, a Kruskal-Wallis H test was conducted.[8] As indicated in Table 1, the null hypothesis of identical distributions of absolute deviations in each pentile can be rejected for all the estimation periods. It appears that Baesel's conclusion regarding beta stability in extreme pentiles is contradicted by the evidence presented in Table 1. Betas in extreme pentiles are less stable than those in interior pentiles as measured by the MAD of each pentile.

C. Optimal Size of Estimation Interval[9]

Using the estimation equations for a simple regression model, the estimated value of a security's beta coefficient $\hat{\beta}_1$ is calculated as follows:

$$\hat{\beta}_i = [\sum_{t=1}^{n} (R_{mt}-\bar{R}_{mt})(R_{it}-\bar{R}_{it})] / \sum_{t=1}^{n} (R_{mt}-\bar{R}_{mt})^2. \tag{3}$$

Here n refers to the number of sampling observations and the terms with bars refer to average values of the particular random variables. The estimated variance of $\hat{\beta}_i$ can be calculated as follows:

$$\hat{\sigma}_{\beta}^2 = \hat{\sigma}_{\varepsilon_i}^2 / \sum_{t=1}^{n} (R_{mt}-\bar{R}_{mt})^2 \tag{4}$$

where

$$\hat{\sigma}_{\varepsilon_i}^2 = \sum_{t=1}^{n} \hat{\varepsilon}_{it}^2/(n-2). \tag{5}$$

Gonedes [6] noted that increasing the sample size n will reduce the sampling error for $\hat{\beta}_i$, $\hat{\sigma}_{\beta}^2$, as calculated in (4). This implies that larger estimation intervals will result in more precise estimations of beta. However, Gonedes also considers the effect of including, as a result of increasing the estimation interval, returns that were generated under different structural conditions.

---

[8] The Kruskal-Wallis H test, as described by Winkler and Hays [14, pp. 862-864], is a nonparametric test similar to the analysis of variance F tests. The parametric F test assumes normality and equal variance among the treatments. Since the data indicated possible nonnormality and unequal variances among the pentiles, the nonparametric H test of equal distributions was employed.

[9] While the estimation of beta coefficients described in this section is based on classical statistics, recent developments by Blume [3], Eubank and Zumwalt [5], Klemkosky and Martin [7], and Vasicek [13] have suggested the possible use of Bayesian estimation techniques. The Bayesian approach requires a prior estimate of beta. Regression estimates are one important source of these prior estimates. As such, an understanding of the appropriate estimation interval is necessary.

Assume the beta of security i has a value β_{1i} over the first half of the estimation interval and the value β_{2i} over the second half of the estimation interval, where $\beta_{1i} \neq \beta_{2i}$ due to a structural change in the company. If $\hat{\beta}_i$ denotes the empirical estimate of beta over the entire estimation interval, then $E(\hat{\beta}_i) \neq \beta_{1i}$ and $E(\hat{\beta}_i) \neq \beta_{2i}$. This implies that $\hat{\beta}_i$ is no longer an unbiased estimate of either β_{1i} or β_{2i}. In summary, while increasing the size of the estimation interval n will result in less sampling error, it will potentially increase the bias of the parameter estimates due to possible structural changes. An important issue, then, involves the empirical question of what is the optimal size of the estimation interval.

Baesel concludes that "the forecaster will be better off using a longer estimation interval."[10] This statement, however, is not based on any statistical test, but rather upon a cursory examination of the transition matrices. By converting Baesel's transition matrices from probabilities to observations and noting the proportions appearing on the diagonals of the respective matrices, difference-in-proportions tests, as described by Winkler and Hays [14, p. 448], can be conducted. A one-tailed test, based on the null hypothesis that beta stability does not increase with longer estimation periods, suggests the improvement is significant at the .05 level (i.e., the null hypothesis would be rejected). If, however, the extremities of the diagonal are excluded from consideration, the null hypothesis cannot be rejected; a Z score of .06 indicates an insignificant improvement even at the .25 level. Excluding the extreme pentiles can be justified by the previous analysis because they inaccurately reflect the stability of high or low betas.

Table 2 presents a breakdown, by pentile and overall for varying estimation intervals, of the average value of $\beta_t - \beta_{t+1}$ and $|\beta_t - \beta_{t+1}|$. In constructing this table, prediction intervals of one year were used for all estimation intervals.[11] As Eubank and Zumwalt [5] note, empirical studies on the examination of beta stability typically use a prediction interval of length identical to the estimation interval. Changes in the length of the estimation interval have then been matched by equivalent changes in the length of the prediction interval.[12]

---

[10] Baesel [1, p. 1493].

[11] As described by Eubank and Zumwalt [5, p. 26], "the 'estimation period' is the time period used to calculate ex post betas for the estimation of ex ante betas. The 'prediction period' is the time period used to calculate realized or predicted betas for comparison with the estimated betas."

[12] Studies cited by Eubank and Zumwalt which have followed this procedure include [1], [2], [7], [8], and [12].

TABLE 2

AVERAGE VALUE OF $\beta_t - \beta_{t+1}$ AND $|\beta_t - \beta_{t+1}|$

Pentile		Estimation Interval				
		1 YR	2 YR	4 YR	6 YR	9 YR
1:	Mean	-2.2891	-1.7272	-1.2204	- .4833	- .3740
	MAD	2.4978	2.1435	1.7078	.7805	1.1480
2:	Mean	- .6746	- .2531	- .2869	- .3599	- .0235
	MAD	1.2616	1.0734	1.2523	.7221	1.1764
3:	Mean	.0147	- .1088	- .1982	- .0732	.4374
	MAD	1.1390	1.1751	1.4893	.6650	1.5144
4:	Mean	.6190	.5123	.1920	.2487	.1622
	MAD	1.4024	1.4212	1.6673	.8802	1.6028
5:	Mean	2.1760	1.0354	.5002	.8189	1.7987
	MAD	2.5991	2.0068	1.7991	1.0149	2.3389
Overall:						
	Mean	- .0308	- .1083	- .2027	- .0302	.4002
	MAD	1.7800	1.5640	1.5831	.8125	1.5561
H Tests:						
	Mean	1135.22*	295.21*	63.84*	67.19*	29.86*
	MAD	453.35*	108.84*	17.61*	14.20*	24.07*

* Significant at the .05 level.

Since the focus here is on the optimal estimation interval, the prediction interval was kept constant at one year for all estimation intervals.

Several interesting features are to be noted in Table 2. First, the mean change in beta is negative for the first and second pentiles and positive for the fourth and fifth pentiles, regardless of the estimation interval used. This is consistent with Blume's [2] results, where the tendency of betas to drift toward a value of one was noted. Also, the mean change in beta generally tends towards zero the closer the pentile is to the middle. This observation suggests a greater likelihood of betas moving either up or down (but of smaller magnitude) for betas closer to one. The Kruskal-Wallis H test results suggest that the null hypothesis of equal distributions of changes in beta between the pentiles can be rejected for any estimation interval tested.

Second, a measure of the magnitude of the change in beta, mean absolute deviation, suggests that generally the change in beta will be of smaller magnitude the closer it is to the middle pentile, regardless of the estimation interval. Again, Kruskal-Wallis H test results indicate that the null hypothesis of identical distributions of absolute deviations between the pentiles can be rejected for any estimation interval tested.

Third, for each pentile and overall the mean absolute deviation is smallest for the six-year estimation interval. This is consistent with the findings of Gonedes, as previously mentioned, but is inconsistent with the conclusions of Baesel. A possible reconciliation of these contradictory findings, however, could be that the difference in beta stability between six- and nine-year estimation intervals is not statistically significant.[13] In order to address this issue, the sample of 160 securities was split into two subsamples of 80 securities each. Then, the sampling distribution of $|\beta_t - \beta_{t+1}|$ was derived from one subsample for one, two, four, and nine-year estimation intervals, and for six years for the other subsample. Then pairwise comparisons were made overall and with each pentile between the six-year estimation interval changes and the other intervals.[14] The results are reported in Table 3. Based on the Mann-Whitney U

---

[13]While Gonedes was concerned with the estimation of the entire market model and not just beta, neither he nor Baesel conducted statistical tests of significance in determining the optimal estimation interval.

[14]The comparisons made were designed so that an equal number of observations of $|\beta_t - \beta_{t+1}|$ were in each of the two estimation intervals being compared. Furthermore, attempts were made so that the one-year prediction period and preceding estimation period were from the same time period. For example, in comparing the change in beta using one-year and six-year estimation intervals, six-year estimation intervals from 1950-1955 and 1956-1961 were used for one 80 security subsample, resulting in 160 observations. Then, for the one-year estimation intervals, the periods 1955 and 1961 were used for the other 80 security subsample, resulting in 160 observations.

test, it appears that the six-year estimation interval, overall and for certain pentiles, is superior to the one, two and nine-year estimation intervals.[15] However, the difference between the four and six-year estimation intervals was not significant. It appears reasonable, then, to conclude that the optimal estimation interval includes both four and six-year periods.

III. Stability of Portfolio Betas

As previously noted, Porter and Ezzell [10] (hereafter denoted PE) mentioned that the major methodological difference between Blume's and their study was in the method of portfolio formation. Blume first ranked individual securities in ascending order of their beta coefficients, and then sequentially formed n security portfolios. PE, in contrast, formed their portfolios of n securities randomly. For both studies a seven-year period was used to determine the securities' beta coefficients. Then the portfolio beta coefficients were calculated as the average of the betas of the securities in each portfolio. After determining the portfolios' beta coefficients for the succeeding seven-year period, product moment and rank order correlations were calculated by comparing the two sets of portfolio beta coefficients. The process was repeated over a total of six consecutive seven-year periods, beginning in July 1926 and ending in June 1968. While Blume noted that the correlations increased monotonically with n (the number of securities in each portfolio), PE observed that the correlations did not show any discernible trend. This led PE to conclude that the stability of portfolio beta coefficients for randomly selected portfolios was slight and unrelated to n. PE have not, however, indicated the magnitude of the instability of portfolio beta coefficients.[16]

It is possible for the correlations of portfolio beta coefficients to remain relatively low and essentially unchanged for increasing size portfolios while the magnitude of the change in portfolio beta coefficients simultaneously decreases. As n increases, portfolio beta coefficients will be more tightly distributed around one when the portfolios are formed randomly, as the Central Limit Theorem [14, p. 314] is applicable. Smaller changes in beta can thus occur as n increases without any discernable change in the correlation coefficient.

---

[15] The Mann-Whitney U test, as described by Winkler and Hays [14, pp. 852-854], is a nonparametric test of the equality of two population distributions.

[16] PE have argued that the relative rankings of portfolios by beta coefficients are important measures of the intertemporal stability of the beta coefficients. It is suggested here, however, that a better measure is the mean absolute deviation of the change in the beta coefficients.

TABLE 3

ESTIMATION INTERVAL TESTS

	1-Year Comparison			2-Year Comparison			4-Year Comparison			9-Year Comparison		
	MAD			MAD			MAD			MAD		
Pentile:	1 Year	6 Year	U Test	2 Year	6 Year	U Test	4 Year	6 Year	U Test	9 Year	6 Year	U Test
1	2.2564	.7960	32.26*	1.5672	.7746	13.14*	1.0198	.7746	1.04	1.0512	.7988	.89
2	1.1738	.8293	5.03*	.9202	.6487	2.30	.6963	.6487	.23	1.1890	1.0342	.07
3	.7087	.6828	.29	.7090	.5429	2.82**	.6205	.5429	.94	1.6212	.7084	6.76*
4	.9516	.7855	.61	.7176	1.0400	2.32	.9522	1.0400	.51	1.3568	.5024	8.64*
5	1.6177	.9134	7.95*	1.1774	1.1121	.35	1.0703	1.1121	.05	2.3365	.9489	12.29*
Overall:	1.3417	.8014	28.70*	1.0183	.8237	6.60*	.8718	.8237	.54	1.5109	.7986	20.48*

* Significant at the .05 level.
**Significant at the .10 level.

In order to reexamine PE's results, beta coefficients for 500 New York Stock Exchange listed stocks were calculated using equation (2) over two consecutive seven-year estimation intervals, 1962-1968 and 1969-1975. Monthly returns were used and the Standard and Poor's 500 Composite Index was used as the market index. Portfolios consisting of 1, 2, 4, 7, 10, 20, 35 and 50 securities (equally weighted) were formed both randomly and by a ranking procedure based on the securities' beta coefficients over the first seven-year estimation interval.[17] Table 4 presents a descriptive summary of these two sets of portfolios. Of importance is the observation that the range and standard deviation of the randomly formed portfolio beta coefficients become smaller as the number of securities in the portfolio rises. A similar observation is not as apparent for the portfolios formed by the ranking procedure.

Table 5 presents a summary of alternative measures of portfolio beta stability resulting from comparisons of the portfolios' beta coefficients between the two seven-year periods. Both the product moment (PM) and rank order correlation coefficients became larger as the portfolio size was increased for the ranked portfolios, consistent with Blume's results, suggesting greater beta stability for more diversified portfolios. The correlation coefficients remain at a level of about .5 as portfolio size increases for the randomly formed portfolios, consistent with PE's results, suggesting no discernible improvement in beta stability for more diversified portfolios. Examination of an alternative measure of beta stability, mean absolute deviation, suggests that PE and Blume's results are not inconsistent. This measure generally decreased as the size of the portfolios increased for both portfolio formation techniques. These improvements in beta stability were most noticeable as portfolio size increased to ten securities, with relatively little improvement thereafter.

IV. Summary and Conclusion

The conclusion presented by Baesel [1] suggesting that betas of securities belonging to extreme pentiles are more stable than those belonging to interior pentiles has been shown to be erroneous. Using mean absolute deviation as a measure of beta stability, evidence was presented supporting a diametrically opposite conclusion. In particular, extreme betas were shown to be less stable than interior betas.

Evidence was also presented contradicting Baesel's claim that longer estimation intervals produced more stable betas. Statistical tests, using mean absolute deviation as a measure of stability, indicated that the optimal

[17] The basic procedure followed coincides with that of Porter and Ezzell.

TABLE 4

DESCRIPTIVE SUMMARY OF BETA COEFFICIENTS

Estimation Interval	Number of Securities Per Portfolio	Number of Portfolios	Random Portfolios			Ranked Portfolios		
			Mean	Std. Dev.	Range	Mean	Std. Dev.	Range
1962-1968	1	500	1.09	.32	.28-2.30	1.09	.32	.28-2.30
	2	250	1.09	.23	.43-1.76	1.09	.32	.31-2.22
	4	125	1.09	.16	.71-1.66	1.09	.32	.35-2.12
	7	71	1.09	.12	.78-1.45	1.08	.31	.38-1.91
	10	50	1.09	.11	.92-1.44	1.09	.32	.41-1.98
	20	25	1.09	.07	1.00-1.34	1.09	.32	.47-1.88
	35	14	1.08	.07	.96-1.26	1.07	.30	.53-1.67
	50	10	1.09	.06	1.01-1.23	1.09	.32	.58-1.72
1969-1975	1	500	1.14	.32	.22-2.53	1.14	.32	.22-2.53
	2	250	1.14	.23	.57-1.84	1.14	.24	.54-2.38
	4	125	1.14	.16	.83-1.63	1.14	.20	.69-1.92
	7	71	1.14	.14	.88-1.52	1.14	.17	.77-1.61
	10	50	1.14	.12	.88-1.38	1.14	.17	.78-1.63
	20	25	1.14	.07	1.03-1.28	1.14	.15	.88-1.56
	35	14	1.14	.07	1.02-1.24	1.14	.14	.87-1.43
	50	10	1.14	.06	1.06-1.23	1.14	.14	.94-1.42

TABLE 5

STABILITY OF BETA OVER TIME

Portfolio Formation Method	Number of Securities Per Portfolio	Correlation PM	Correlation Rank	Mean Absolute Deviation
Ranked	1	.4479	.4255	.2733
	2	.5869	.5689	.2157
	4	.7265	.7089	.1889
	7	.8197	.7964	.1694
	10	.8660	.8618	.1657
	20	.9529	.9631	.1538
	35	.9713	.9736	.1432
	50	.9789	.9879	.1510
Random	1	.4479	.4255	.2733
	2	.4957	.4705	.1863
	4	.4922	.5104	.1333
	7	.5060	.5079	.1218
	10	.4815	.4679	.1091
	20	.3964	.4954	.0825
	35	.6498	.6308	.0659
	50	.5285	.4788	.0633

estimation interval was generally four-six years. Such a finding is consistent with previous research by Gonedes [6], where the optimal estimation interval was suggested to be seven years. The tendency of betas to drift toward one, as cited by Blume [2,3], was also noted.

The conclusion of Porter and Ezzell [10] that beta coefficients of randomly selected portfolios are relatively unstable and unrelated to the number of securities in the portfolio was shown to be consistent with the conclusions of Blume [2] and Levy [8]. It is suggested here that the magnitude of intertemporal changes in portfolio beta coefficients decreases as the number of securities in the portfolio rises, regardless of how the portfolios are formed. Attempts to measure the stability of portfolio beta coefficients by the use of correlation coefficients masks this trend for portfolios formed by random selection. The time stability of portfolio beta coefficients continues to be directly related to the number of securities in the portfolio and is significantly stable for portfolios of ten or more securities, where stability is measured by mean absolute deviation.

REFERENCES

1 Baesel, Jerome B. "On the Assessment of Risk: Some Further Considerations." *Journal of Finance,* Vol. 29 (December 1974), pp. 1491-1494.

2 Blume, Marshall E. "On the Assessment of Risk." *Journal of Finance,* Vol. 26 (March 1971), pp. 1-10.

3 __________. "Betas and Their Regression Tendencies." *Journal of Finance,* Vol. 30 (June 1975), pp. 785-795.

4 Brenner, Menachem, and Seymour Smidt. "A Simple Model of Non-Stationarity of Systematic Risk." *Journal of Finance,* Vol. 32 (September 1977), pp. 1081-1092.

5 Eubank, Arthur A., Jr., and J. Kenton Zumwalt. "The Forecast Error Impact of Alternative Length Beta Estimation Periods, Adjustment Techniques, and Risk Classes." Paper presented at 1977 Western Finance Association Meeting.

6 Gonedes, Nicholas J. "Evidence on the Information Content of Accounting Numbers: Accounting-based and Market-based Estimates of Systematic Risk." *Journal of Financial and Quantitative Analysis,* Vol. 8 (June 1973), pp. 407-443.

7 Klemkosky, Robert C., and John D. Martin. "The Adjustment of Beta Forecasts." *Journal of Finance,* Vol. 30 (September 1975), pp. 1123-1128.

8 Levy, Robert A. "Stationarity of Beta Coefficients." *Financial Analysts Journal,* Vol. 27 (November-December 1971), pp. 55-62.

9 Mood, Alexander M.; Franklin A. Graybill; and Duane C. Boes. *Introduction to the Theory of Statistics,* 3rd ed. New York: McGraw-Hill (1974).

10 Porter, R. Burr, and John R. Ezzell. "A Note on the Predictive Ability of Beta Coefficients." *Journal of Business Research,* Vol. 3 (October 1975), pp. 367-372.

11 Sharpe, William F. "A Simplified Model for Portfolio Analysis." *Management Science,* Vol. 9 (January 1963), pp. 277-293.

12 Sharpe, William F., and Guy M. Cooper. "Risk-Return Classes of New York Stock Exchange Common Stocks." *Financial Analysts Journal,* Vol. 28 (March-April 1972), 46ff.

13 Vasicek, Oldrich A. "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas." *Journal of Finance,* Vol. 28 (December 1973), pp. 1233-1239.

14 Winkler, Robert L., and William L. Hays. *Statistics,* 2nd ed. New York: Holt, Rinehart and Winston (1975).