hw3

October 20, 2024

1 Problem 4

[117]:
```
from sklearn.datasets import load_digits
from sklearn.metrics import log_loss, accuracy_score
from sklearn.ensemble import GradientBoostingClassifier
from sklearn.model_selection import GridSearchCV

import matplotlib.pyplot  as plt
```

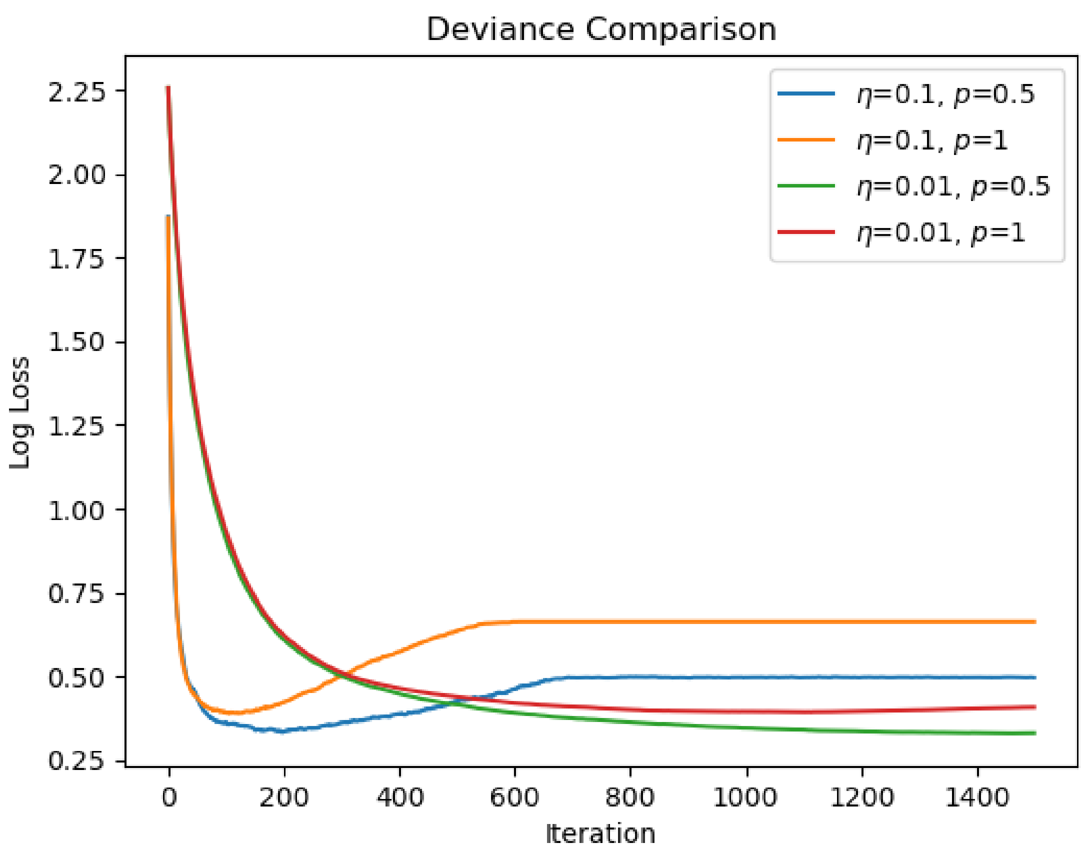
[85]:
```
digits = load_digits()
X = digits.data
y = digits.target

X_train, X_test = X[:1000], X[1000:]
y_train, y_test = y[:1000], y[1000:]
```

[ ]:
```
deviance_dct = {
    (0.1,0.5): None,
    (0.1, 1) : None,
    (0.01, 0.5): None,
    (0.01, 1): None
}

for lr in [0.1, 0.01]:
    for p in [0.5, 1]:
        gb_clf = GradientBoostingClassifier(n_estimators=1500, # T = 1500
                                            learning_rate=lr, # 0.1 and 0.01
                                            max_leaf_nodes=4,
                                            min_samples_split=5,
                                            subsample=p, # 0.5 and 1
                                            random_state=42)



        gb_clf.fit(X_train, y_train)

        y_pred_proba_stages = gb_clf.staged_predict_proba(X_test)
```

```
        deviances = []
        for y_pred_proba in y_pred_proba_stages:
            deviance = log_loss(y_test, y_pred_proba)
            deviances.append(deviance)

        deviance_dct[(lr, p)] = deviances
```

[101]:
```
for lr in [0.1, 0.01]:
    for p in [0.5, 1]:

        plt.plot(deviance_dct[(lr,p)], label=f'$\eta$={lr}, $p$={p}')

plt.title('Deviance Comparison')
plt.legend()
plt.xlabel('Iteration')
plt.ylabel('Log Loss')
```

[101]:
```
Text(0, 0.5, 'Log Loss')
```



[103]:
```python
param_grid = {
    'n_estimators': [20, 30, 40, 60, 80, 100, 200, 500]
}

gb_clf = GradientBoostingClassifier(learning_rate=0.1,  # Example learning rate
                                    max_leaf_nodes=4,
                                    min_samples_split=5,
                                    subsample=0.5,      # Example subsample
                                    random_state=42)

grid_search = GridSearchCV(estimator=gb_clf,
                           param_grid=param_grid,
                           cv=10,
                           scoring='neg_log_loss',
                           n_jobs=-1)

grid_search.fit(X_train, y_train)

best_n_estimators = grid_search.best_params_['n_estimators']
```

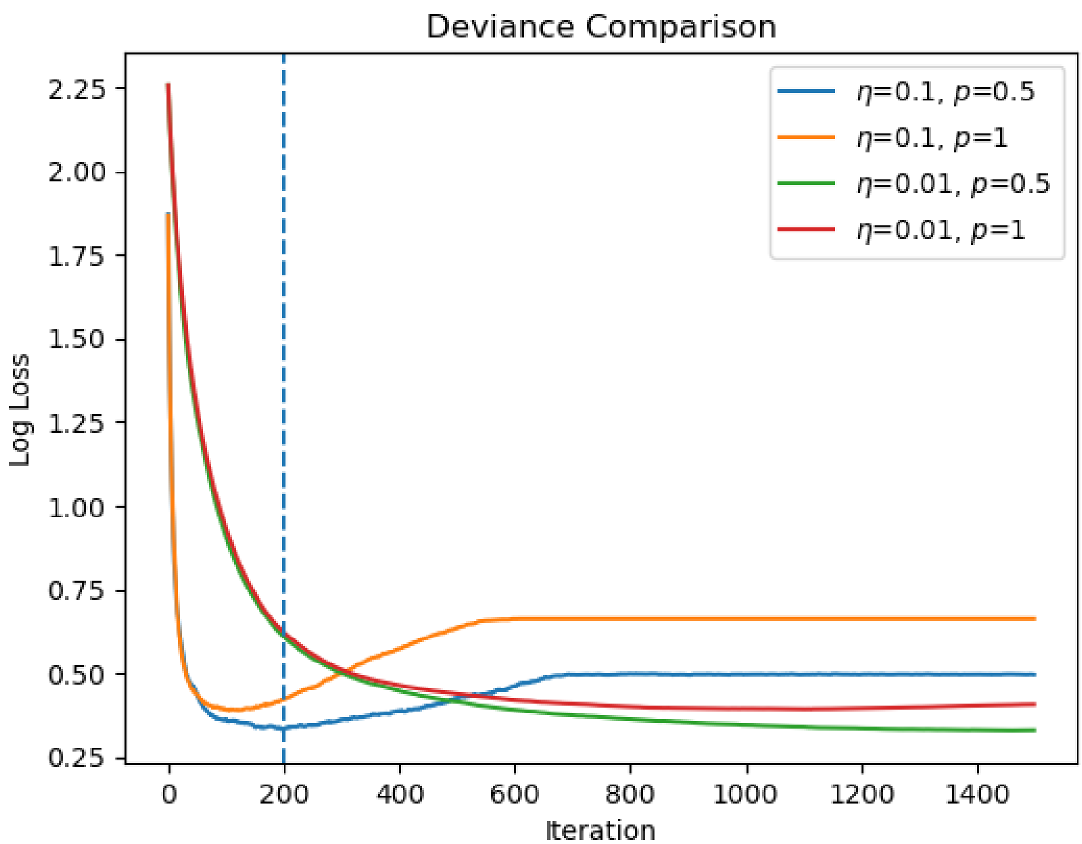
[108]:
```python
import pandas as pd
pd.DataFrame(grid_search.cv_results_)
```

[108]:
```
   mean_fit_time  std_fit_time  mean_score_time  std_score_time  \
0       1.113934      0.024499         0.006466        0.004081
1       1.633280      0.044832         0.003623        0.000530
2       2.151802      0.046762         0.003758        0.000719
3       3.099959      0.082132         0.004153        0.000775
4       4.124060      0.086896         0.004514        0.000750
5       5.278415      0.084862         0.005714        0.002679
6      10.710508      0.210531         0.006764        0.000723
7      21.778956      0.112896         0.009756        0.000930

  param_n_estimators                  params  split0_test_score  \
0                 20   {'n_estimators': 20}          -0.634718
1                 30   {'n_estimators': 30}          -0.517642
2                 40   {'n_estimators': 40}          -0.462130
3                 60   {'n_estimators': 60}          -0.414317
4                 80   {'n_estimators': 80}          -0.384714
5                100  {'n_estimators': 100}          -0.352984
6                200  {'n_estimators': 200}          -0.326890
7                500  {'n_estimators': 500}          -0.484325

   split1_test_score  split2_test_score  split3_test_score  split4_test_score  \
0          -0.410750          -0.357796          -0.463038          -0.542924
1          -0.306412          -0.261578          -0.343196          -0.406640
2          -0.256677          -0.211052          -0.306566          -0.337621
```

```
3          -0.200579          -0.160666          -0.253328          -0.285509
4          -0.179854          -0.145694          -0.217918          -0.251432
5          -0.166531          -0.121922          -0.189873          -0.226205
6          -0.137073          -0.098897          -0.120130          -0.223812
7          -0.171181          -0.106960          -0.103007          -0.292377

   split5_test_score  split6_test_score  split7_test_score  split8_test_score  \
0          -0.585417          -0.483604          -0.511252          -0.511213
1          -0.407291          -0.317340          -0.397742          -0.420775
2          -0.341101          -0.251725          -0.328141          -0.387675
3          -0.274873          -0.167223          -0.262561          -0.316265
4          -0.235531          -0.128219          -0.246509          -0.282631
5          -0.204619          -0.102296          -0.232413          -0.245616
6          -0.159400          -0.048569          -0.227744          -0.165468
7          -0.157914          -0.016387          -0.301402          -0.141732

   split9_test_score  mean_test_score  std_test_score  rank_test_score
0          -0.462232        -0.496294        0.076584                8
1          -0.341912        -0.372053        0.069221                7
2          -0.290532        -0.317322        0.068645                6
3          -0.212774        -0.254810        0.072020                5
4          -0.179247        -0.225175        0.070668                4
5          -0.159207        -0.200167        0.067655                3
6          -0.117145        -0.162513        0.075221                1
7          -0.160283        -0.193557        0.125934                2
```

[115]:
```python
for lr in [0.1, 0.01]:
    for p in [0.5, 1]:

        plt.plot(deviance_dct[(lr,p)], label=f'$\eta$={lr}, $p$={p}')

plt.axvline(best_n_estimators, linestyle='--')

plt.title('Deviance Comparison')
plt.legend()
plt.xlabel('Iteration')
plt.ylabel('Log Loss')
```

[115]:
```
Text(0, 0.5, 'Log Loss')
```



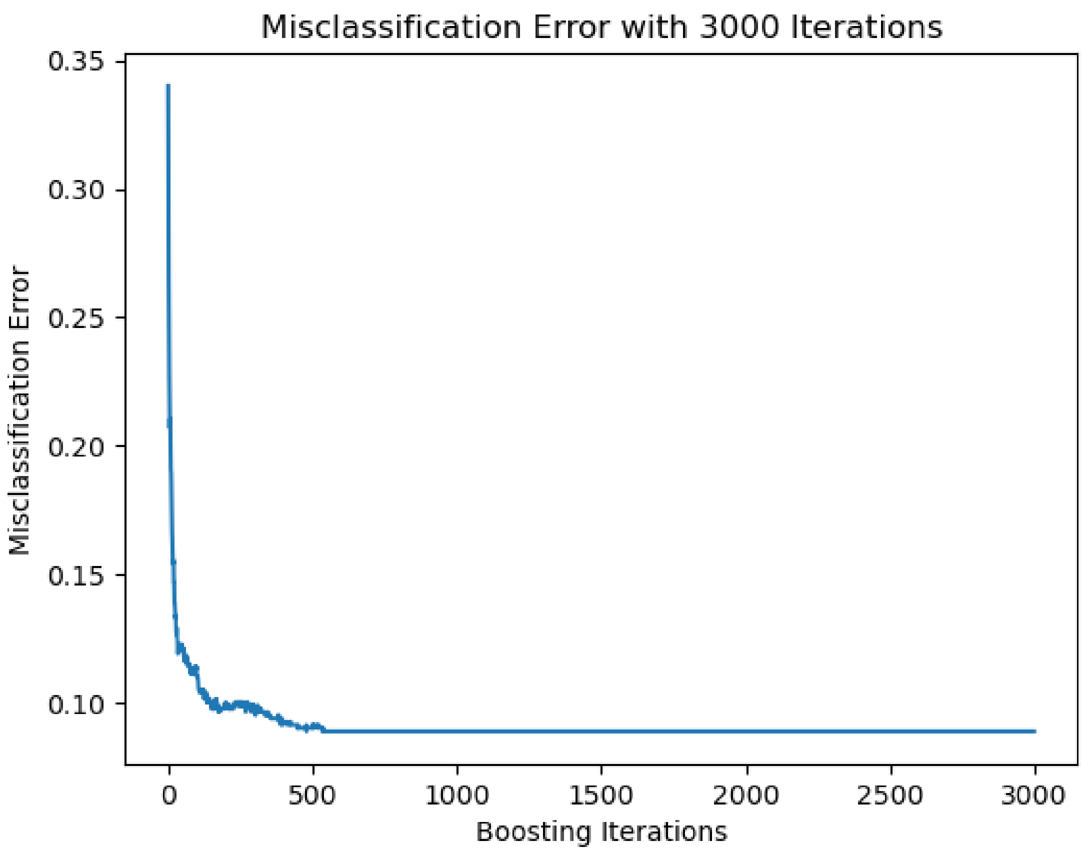
[116]:
```
gb_clf = GradientBoostingClassifier(n_estimators=3000,  # T = 1500
                                     learning_rate=0.1,  # 0.1 and 0.01
                                     max_leaf_nodes=4,
                                     min_samples_split=5,
                                     subsample=1,      # 0.5 and 1
                                     random_state=42)



gb_clf.fit(X_train, y_train)
```

[116]:
```
GradientBoostingClassifier(max_leaf_nodes=4, min_samples_split=5,
                           n_estimators=3000, random_state=42, subsample=1)
```

[124]:
```
errors = []
for y_pred in gb_clf.staged_predict(X_test):
    errors.append(
        1 - accuracy_score(y_pred, y_test)
    )
```

[132]:
```
plt.plot(errors)
# plt.yscale('log')
# plt.ylim(0,0.1)
plt.xlabel("Boosting Iterations")
plt.ylabel("Misclassification Error")
plt.title("Misclassification Error with 3000 Iterations")
```

[132]:
```
Text(0.5, 1.0, 'Misclassification Error with 3000 Iterations')
```



2 Problem 5

[206]:
```
from sklearn . neighbors import RadiusNeighborsClassifier
from collections import Counter
import random
import numpy as np
```

[142]:
```
d=2
mu0 = [
    [-2,-3.5],
```

```
    [0,0],[2,-3.5]
]

Id = [
    [25,0],
    [0,25]
]

def Data_Generate(n):
    outX = []
    outY = np.random.choice(
        list(range(3)), n, replace=True, p=[0.5,0.25,0.25]
    )

    for i in range(n):
        mu = mu0[outY[i]]
        X = np.random.multivariate_normal(mu,Id,1)[0].tolist()
        outX.append(X)

    return np.array(outX), np.array(outY)
```

[183]:
```
n = 500
testX, testY = Data_Generate(n)
```

[169]:
```
def q_quantile_interpoint_distance(data, q):
    distances = np.linalg.norm(data[:, np.newaxis, :] - data[np.newaxis, :, :],
 ↪axis=2)
    # print(distances)
    # print(distances.shape)

    interpoint_distances = distances[np.triu_indices_from(distances, k=1)]
    # print(interpoint_distances)
    # print(interpoint_distances.shape)
    return np.quantile(interpoint_distances, q)
```

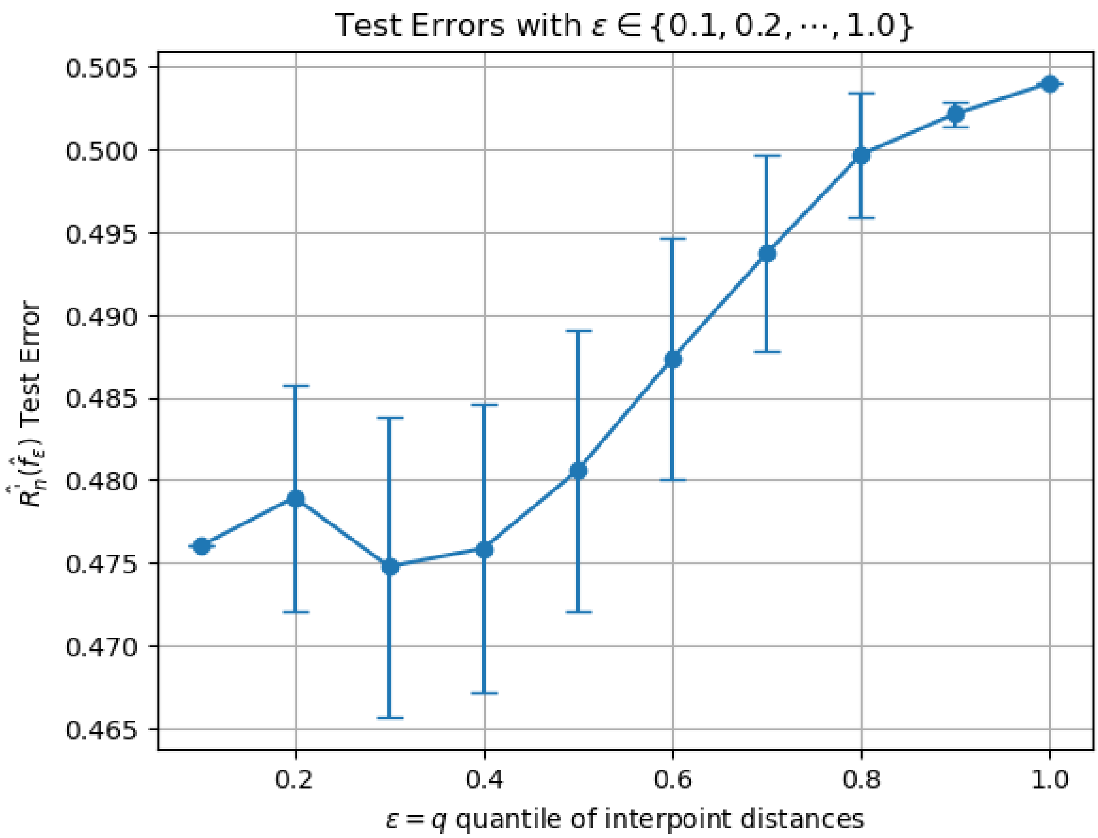
[ ]:
```
NUM_EXPERIMENT = 100

well_defined_proportions = [[] for _ in range(11)]
errors = [[] for _ in range(11)]
# repeat 1000 times

for experiment in range(NUM_EXPERIMENT):
    trainX, trainY = Data_Generate(n)

    for q in range(1, 11):
        dist = q_quantile_interpoint_distance(trainX, q/10)
        # print(q/10, dist)
```

```
        ENN = RadiusNeighborsClassifier(radius=dist)
        ENN.fit(trainX, trainY)

        neigh_indices = ENN.radius_neighbors(testX, return_distance=False)
        well_defined_count = 0
        for indices in (neigh_indices):
            if len(indices):
                well_defined_count += 1

        # print(experiment, q, well_defined_count, well_defined_count / n)
        well_defined_proportion = well_defined_count / n
        well_defined_proportions[q].append(well_defined_proportion)

        try:
            y_pred = ENN.predict(testX)

            test_error = 1 - accuracy_score(testY, y_pred)
            errors[q].append(test_error)

            # print(neigh_indices)
            # print(len(neigh_indices))
            # print(neigh_indices[0].shape, neigh_indices[39].shape,␣
↪neigh_indices[139].shape)
        except ValueError as e:
            # print(e)
            pass
```

plot error

[246]:
```
mus = []
stds = []

for q in range(1, 11):
    err = np.array(errors[q])
    mu = err.mean()
    std = err.std()

    mus.append(mu)
    stds.append(std)

x_values = np.arange(1, 11) / 10

plt.errorbar(x_values, mus, yerr=stds, fmt='o-', capsize=5)

# Add labels and title
plt.xlabel("$\epsilon = q$ quantile of interpoint distances")
```

```
plt.ylabel("$\hat{R_n^'} (\hat{f}_\epsilon)$ Test Error")
plt.title("Test Errors with $\epsilon \in \{0.1, 0.2, \cdots, 1.0 \}$")

# Show the plot
plt.grid(True)
plt.show()
```



[275]:
```
mus = []
stds = []

for q in range(1, 11):
    arr = np.array(well_defined_proportions[q])
    # print(arr)
    mu = arr.mean()
    std = arr.std()

    mus.append(mu)
    stds.append(std)

x_values = np.arange(1, 11) / 10
```

```
plt.errorbar(x_values, mus, yerr=stds, fmt='o-', capsize=5)

# Add labels and title
plt.xlabel("$\epsilon = q$ quantile of interpoint distances")
plt.ylabel("Ratio of well-defined $\hat{f}_\epsilon$")
plt.title("Ratio of well-defined $\hat{f}_\epsilon$ with $\epsilon \in \{0.1, 0.
↪2, \cdots, 1.0 \}$")

# Show the plot
plt.grid(True)
plt.show()
```