



QUANTITATIVE APTITUDE FORMULA BANK



CONTENTS

Sr.No.	Topic	Page Number
1	Divisibility Rules and Numbers	1
2	HCF & LCM	3
3	Average	6
4	Percentage	7
5	Profit Loss	9
6	Simple Interest	11
7	Compound Interest	12
8	Speed Distance & Time	13
9	Problems on Trains	15
10	Boats and Streams	16
11	Calendar	17
12	Area and Volume	19
13	Alligation & Mixture	22
14	Permutation & Combination	23

1. DIVISIBILITY RULES AND NUMBERS

Divisible By	If (Rule)	Examples:
2	The last digit is even (0,2,4,6,8)	<ul style="list-style-type: none"> • 128 is • 129 is not
3	The sum of the digits is divisible by 3	<ul style="list-style-type: none"> • 381 ($3+8+1=12$, and $12 \div 3 = 4$) Yes 381 is divisible by 3 • 217 ($2+1+7=10$, and $10 \div 3 = 10/3$) No 217 is not divisible by 3
4	The last 2 digits are divisible by 4	<ul style="list-style-type: none"> • 1312 is ($12 \div 4 = 3$) divisible • 7019 is not divisible
5	The last digit is 0 or 5	<ul style="list-style-type: none"> • 175 is divisible by 5 • 809 is not divisible by 5
6	The number is divisible by both 2 and 3	<ul style="list-style-type: none"> • 114 (it is even hence divisible by 2 , and $1+1+4=6$ and $6 \div 3 = 2$, hence divisible by 3) So 114 is divisible by 6 • 308 (it is even hence divisible by 2 but $3+0+8=11$ and $11 \div 3 = 11/3$, hence not divisible 3) So 308 is not divisible by 6
7	If you double the last digit and subtract it from the rest of the number and the answer is: 0, or divisible by 7 (Note: you can apply this rule to that answer again if you want)	<ul style="list-style-type: none"> • 672 (last digit is 2 , Double of 2 is 4, $67-4=63$, and $63 \div 7 = 9$) Hence 672 is divisible by 7 • 905 (last digit is 5, Double of 5 is 10, $90-10=80$, and $80 \div 7 = 11 \frac{3}{7}$) Hence 905 is not divisible by 905.
8	The last three digits are divisible by 8	<ul style="list-style-type: none"> • 109816 ($816 \div 8 = 102$), hence 109816 is divisible by 8 • 216302 ($302 \div 8 = 151/4$) hence 216302 is not divisible by 8
9	The sum of the digits is divisible by 9 (Note: you can apply this rule to that answer again if you want)	<ul style="list-style-type: none"> • 1629 (sum=$1+6+2+9=18$, and $18/9$ is 2) hence 1629 is divisible by 9 • 2013 ($2+0+1+3=6$, 6 is not divisible by 9) hence 2013 is not divisible by 9
10	The number ends in 0	<ul style="list-style-type: none"> • 220 is divisible by 10 • 221 is not divisible by 10
11	Add and subtract digits in an alternating pattern (add first, subtract second, add third, etc). Then the answer must be: 0, or divisible by 11	<ul style="list-style-type: none"> • 1364 ($1-3+6-4 = 0$) , hence 1364 is divisible by 11 • 913 ($9-1+3 = 11$) • 3729 ($3-7+2-9 = -11$) <p>All above numbers are divisible by 11.</p>
12	The number is divisible by both 3 and 4	<ul style="list-style-type: none"> • 648: ($6+4+8=18$ and $18 \div 3 = 6$, hence 648 is divisible by 3) and, $48 \div 4 = 12$, hence 648 is divisible by 4, hence we can say 648 is divisible by 12.

Some Basic Formulae:

- i. $(a + b)(a - b) = (a^2 - b^2)$
- ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$
- iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$
- iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- vii. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- viii. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.
- ix. $(a + b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_{n-1} a b^{n-1} + b^n$

Law of Indices:

- x. $a^m \times a^n = a^{m+n}$
- xi. $a^m \div a^n = a^{m-n}$
- xii. $(a^m)^n = a^{mn}$
- xiii. $(ab)^n = a^n b^n$
- xiv. $a^0 = 1$

2.HCF & LCM

1. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.): The H.C.F. of two or more than two numbers is the greatest number that divided each of them exactly. There are two methods of finding the H.C.F. of a given set of numbers:

- **Factorization Method:** Express the each one of the given numbers as the product of prime factors. The product of common factors gives us the HCF.

E.g. Find the H.C.F of 804 and 420

2	804
2	402
3	201
67	67
	1

Factors: $2 \times 2 \times 3 \times 67$

2	420
2	210
3	105
5	35
7	7
	1

Factors: $2 \times 2 \times 3 \times 5 \times 7$

Common Prime factors are 2, 2, and 3

$$\text{H.C.F.} = 2 \times 2 \times 3 = 12$$

- **Division Method:** Suppose we have to find the H.C.F. of two given numbers, divide the larger by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is required H.C.F.

E.g. Find the H.C.F of 804 and 210

$$804 / 210 \text{ ----- } 210 \times 3 = 630 \text{ (Remainder is 174)}$$

$$210 / 174 \text{ ---- } 174 \times 1 = 174 \text{ (Remainder is 36)}$$

$$174 / 36 \text{ ---- } 36 \times 4 = 144 \text{ (Remainder is 30)}$$

$$36 / 30 \text{ ---- } 30 \times 1 = 30 \text{ (Remainder is 6)}$$

$$30 / 6 \text{ ---- } 6 \times 5 = 30 \text{ (Remainder is 0)}$$

Hence HCF is 6

- **Finding the H.C.F. of more than two numbers:** Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number. Similarly, the H.C.F. of more than three numbers may be obtained.

E.g. Find the HCF of 20, 36 and 100

Then first we can find HCF of 20 and 36,

HCF of 20 and 36 will be 5

Now we can find HCF of 4 and 100

HCF of 100 and 4 is 4

Hence HCF of 20, 36 and 100 is 4

2. Least Common Multiple (L.C.M.): The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

- **Factorization Method:** Start prime factorization of the numbers with common multiples. Then, L.C.M. is the product of all the factors

Eg. Find the LCM of 24, 30, 16

Factors	Numbers		
2	24	30	16
2	12	15	8
2	6	15	4
3	3	15	2
2	1	5	2
5	1	5	1
	1	1	1

LCM: $2 \times 2 \times 2 \times 3 \times 2 \times 5 = 240$

Note: If the greatest number of the given number is a multiple of all remaining numbers then greatest number is LCM. E.g. Find the LCM of 24, 4 and 8

Here greatest number is 24 and it is a multiple of 4 and 8 hence LCM = 24

3. **Product of two numbers = Product of their H.C.F. and L.C.M.**

4. **Co-prime Numbers:** Two numbers are said to be co-prime, if their H.C.F. is 1. & $\text{LCM} = \text{Number}(1) \times \text{Number}(2)$

5. If in between 2 numbers, one number is multiple of other then their **HCF is smaller No and LCM is Larger No.**

6. **H.C.F. and L.C.M. of Decimal Fractions:**

$$\text{HCF of Fraction} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

$$\text{LCM of Fraction} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Remainder Based Problem

- If X is the remainder in each case when N is divided by x, y, z then the number is

$$N = K * [\text{LCM}(x, y, z)] + X, \text{ here } K \text{ is a natural number}$$

- If A, B, C are the remainders when N is divided by x, y, z and $x - A = y - B = z - C = P$ then the number is
$$N = K * [\text{LCM}(x, y, z)] - P$$

3. AVERAGES

$$\text{Average} = \frac{\text{Sum of Observations}}{\text{Number of Observations}}$$

Group 1

Average = A_1

Sum of Obs = S_1

Number of Obs. = N_1

Group 2

Average = A_2

Sum of Obs = S_2

Number of Obs. = N_2

If Group 1 and Group 2 come together then the combined average of the group will be $A = \frac{S_1 + S_2}{N_1 + N_2}$

Problem of Replacement (Most Common Type of Problem in Averages) :

If Average of a group of N people increases/decreases by P kg when one or more persons weighing W kg is replaced by a new person. Then, **the weight of new person is: $W + P \times N$ (if average increases) $W - P \times N$ (if average decreases)**

Eg.

The average weight of 5 persons is decreased by 2kg when one of the men whose weight is 50kg is replaced by a new man. The weight of the new man is

In this problem average decrease hence weight of new person is: $50 - 5 \times 2 = 40$ kg

4. Percentage

- Expressing a value out of 100 means Percentage.
- Expressing Percentage to Fraction: 20% divide it by 100 ----- $20/100 = 1/5$
- Expressing Fraction to Percentage: $2/5$ multiply it by 100 ----- $2/5 \times 100 = 40\%$

Some Important Percentages and its Fraction	
Percentage	Fraction
100 %	1
50%	1/2
33.33%	1/3
25%	1/4
20%	1/5
16.66%	1/6
14.28%	1/7
12.5%	1/8
11.11%	1/9
10%	1/10
9.09%	1/11
8.33%	1/12
30%	3/10
40%	2/5
45%	9/20
60%	3/5
80%	4/5
75%	3/4
35%	7/20
15%	3/20

$$\text{Percentage Change} = \frac{\text{Change in the value (New value–Old value)}}{\text{Old Value}} \times 100$$

If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is:

$$\left[\frac{R}{(100 + R)} \times 100 \right] \%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

$$\left[\frac{R}{(100 - R)} \times 100 \right] \%$$

Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

- Population after n years = $P \left(1 + \frac{R}{100} \right)^n$

- Population n years ago = $\frac{P}{\left(1 + \frac{R}{100} \right)^n}$

5. Profit & Loss

Cost Price (C.P): The price, at which an article is purchased, is called its **cost price**

Selling Price (S.P): The price, at which an article is sold, is called its **selling prices**

Profit or Gain: If S.P. is greater than C.P., the seller is said to have a **profit** or **gain**.

Loss: If S.P. is less than C.P., the seller is said to have incurred a **loss**.

Marked Price (M.P): The price, at which the article is marked. If article is sold at marked price then selling price is equal to marked price. If some discount is given by shopkeeper then $S.P = M.P - \text{Discount}$

IMPORTANT FORMULA:

1. $\text{Gain} = (S.P.) - (C.P.)$

2. $\text{Loss} = (C.P.) - (S.P.)$

3. Loss or gain is always calculated on C.P.

4. Gain Percentage: (Gain %) :

$$\text{Gain \%} = \left(\frac{\text{Gain}}{C.P} \right) \times 100$$

5. Loss Percentage: (Loss %) :

$$\text{Loss \%} = \left(\frac{\text{Loss}}{C.P} \right) \times 100$$

6. Selling Price: (S.P.) :

$$SP = \left[\left(\frac{100 + \text{Gain}\%}{100} \right) \times CP \right]$$

7. Selling Price: (S.P.) :

$$SP = \left[\left(\frac{100 - \text{Loss}\%}{100} \right) \times CP \right]$$

8. Cost Price: (C.P.) :

$$CP = \left[\left(\frac{100}{100 + \text{Gain}\%} \right) \times SP \right]$$

9. Cost Price: (C.P.) :

$$CP = \left[\left(\frac{100}{100 - \text{Loss}\%} \right) \times SP \right]$$

10. If an article is sold at a gain of say 35%, then S.P. = 135% of C.P.

11. If an article is sold at a loss of say, 35% then S.P. = 65% of C.P.

12. When a person sells two similar items, one at a gain of say $x\%$, and the other at a loss of $x\%$, then the seller always incurs a loss given by:

$$\text{Loss \%} = \left(\frac{\text{Common Loss and Gain \%}}{10} \right)^2 = \left(\frac{x}{10} \right)^2.$$

13. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain \%} = \left(\frac{\text{Error}}{\text{True Value} - \text{Error}} \right) \times 100$$

6. SIMPLE INTEREST

1. Principal:

The money borrowed or lent out for a certain period is called the **principal** or the **sum**.

2. Interest:

Extra money paid for using other's money is called **interest**.

3. Simple Interest (S.I.):

If the interest on a sum borrowed for certain period is reckoned uniformly, then it is called **simple interest**.

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then

$$(i). \text{ Simple Interest} = \left(\frac{P \times R \times T}{100} \right)$$

$$(ii). P = \left(\frac{100 \times \text{S.I.}}{R \times T} \right) ; R = \left(\frac{100 \times \text{S.I.}}{P \times T} \right) \text{ and } T = \left(\frac{100 \times \text{S.I.}}{P \times R} \right).$$

7.COMPOUND INTEREST

1. Let Principal = P, Rate = R% per annum, Time = n years.

2. **When interest is compound Annually: (Considering a year)**

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

3. **When interest is compounded Half-yearly:**

$$\text{Amount} = P \left[1 + \frac{(R/2)}{100} \right]^{2n}$$

4. **When interest is compounded Quarterly:**

$$\text{Amount} = P \left[1 + \frac{(R/4)}{100} \right]^{4n}$$

5. **When interest is compounded annually but time is in fraction, say $3\frac{2}{5}$ years.**

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100} \right)$$

6. **When Rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd and 3rd year respectively.**

$$\text{Then, Amount} = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right).$$

7. **Present worth of Rs. x due n years hence is given by:**

$$\text{Present Worth} = \frac{x}{\left(1 + \frac{R}{100} \right)^n}.$$

8.SPEED DISTANCE AND TIME

- i. **Distance = Speed \times Time**
- ii. **Speed = Distance/Time**
- iii. **Time = Distance/speed**
- iv. **1(km/hr) = 5/18(m/s)**
- v. **1(m/s) = 18/5 (km/hr)**
- vi. If the ratio of the speeds of A and B is a:b then the ratio of times taken by them to cover the same distance is $1/a : 1/b$ or $b:a$.
- vii. **Average Speed= Total Distance/ Total Time**

E.g.

A man goes First 30 km of his journey at the speed of 15km/hr , next 40km of this journey with 10km/hr and last 30 km of his journey with 30km/hr. Then calculate the average speed of the man.

Here **Total distance is = 30km+40km+30km = 100km**

We need to calculate Total time. Total Time= Time for 1st Part + Time of 2nd Part +Time of 3rd Part.

$$\text{Time of 1}^{\text{st}} \text{ part} = \frac{30\text{km}}{15\text{km/hr}} = 2 \text{ hrs}$$

$$\text{Time of 2}^{\text{nd}} \text{ part} = \frac{40\text{km}}{10\text{km/hr}} = 4 \text{ hrs}$$

$$\text{Time of 3}^{\text{rd}} \text{ part} = \frac{30\text{km}}{30\text{km/hr}} = 1 \text{ hr}$$

Hence Total Time – 2+4+1 = 7 hours

$$\text{Hence Average speed} = \frac{100}{7} \text{ km/hr}$$

viii. If 2 objects are moving at the speed of Speed(1) and Speed(2) respectively for a certain distance then:

If the Distance is constant

If time is constant

$$\text{Average speed} = \frac{2(\text{Speed}(1) + \text{Speed}(2))}{\text{Speed}(1) + \text{Speed}(2)}$$

$$\text{Average speed} = \frac{\text{Speed}(1) + \text{Speed}(2)}{2}$$

Relative Speed:

- If Two bodies are moving in opposite directions at speed S_1 & S_2 respectively.

The relative speed is defined as $S_r = S_1 + S_2$

- Two bodies are moving in same directions at speed S_1 & S_2 respectively.

The relative speed is defined as $S_r = |S_1 - S_2|$

9. PROBLEMS ON TRAINS

1. Time taken by a train of length X meters to pass a pole or standing man or a signal post is = **the time taken by the train to cover X meters.**
2. Time taken by a train of length X meters to pass a stationary object of length b meters is = **the time taken by the train to cover $(X + b)$ meters.**
3. Suppose two trains or two objects bodies are moving in the **same direction** at u m/s and v m/s, where $u > v$, **then their relative speed is = $(u - v)$ m/s.**
4. Suppose two trains or two objects bodies are moving in **opposite directions** at u m/s and v m/s, then **their relative speed is = $(u + v)$ m/s.**
5. If two trains of length a meters and b meters are moving in opposite directions at u m/s and v m/s, then:

$$\text{The time taken by the trains to cross each other} = \frac{(a + b)}{(u + v)} \text{ sec.}$$

6. If two trains of length a meters and b meters are moving in the same direction at u m/s and v m/s, then:

$$\text{The time taken by the faster train to cross the slower train} = \frac{(a + b)}{(u - v)} \text{ sec.}$$

7. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then:

$$(\text{A's speed}) : (\text{B's speed}) = (\sqrt{b} : \sqrt{a})$$

10. BOATS AND STREAMS

- **Downstream / Upstream :**

In Water, the direction along the stream is called **Downstream**.

And the direction against the stream is called **Upstream**.

- If the speed of boat in still water is u km/hr & the speed of stream is v km/hr, then:

Speed Downstream: $(u + v)$ km/hr

Speed Upstream: $(u - v)$ km/hr

- If the Speed Downstream is a km/hr & the Speed Upstream is b km/hr, then:

Speed in still water $= (a + b) / 2$ km/hr

Rate of stream $= (a - b) / 2$ km/hr

11.CALENDER

1. **Odd Days:** Number of days more than the complete weeks are called odd days in a given period.

2. **Leap Year:** A leap year has 366 days.

In a leap year, the month of February has 29 days

a) Every year divisible by 4 is a leap year, if it is not a century.

Examples: 1952, 2008, 1680 etc. are leap years but 1991, 2003 etc. are not leap years

b) Every 4th century is a leap year and no other century is a leap year.

Examples:

400, 800, 1200 etc. are leap years.

100, 200, 1900 etc. are not leap years

3. **Ordinary Year**

The year which is not a leap year is an ordinary year.

An ordinary year has 365 days.

4. **Counting odd days and Calculating the day of any particular date:**

I. 1 ordinary year \equiv 365 days \equiv (52 weeks + 1 day)
Hence number of odd days in 1 ordinary year = 1.

II. 1 leap year \equiv 366 days \equiv (52 weeks + 2 days)
Hence number of odd days in 1 leap year = 2.

III. 100 years \equiv (76 ordinary years + 24 leap years)

\equiv (76 x 1 + 24 x 2) odd days

\equiv 124 odd days.

\equiv (17 weeks + 5 days)

\equiv 5 odd days.

Hence number of odd days in 100 years = 5.

IV. Number of odd days in 200 years = (5 x 2) = 10 \equiv 3 odd days

V. Number of odd days in 300 years = (5 x 3) = 15 \equiv 1 odd days

VI. Number of odd days in 400 years = (5 x 4 + 1) = 21 \equiv 0 odd days

VII. Similarly, the number of odd days in all 4th centuries (400, 800, 1200 etc.) = 0

VIII. Mapping of the number of odd day to the day of the week

Number of Odd Days	:	0	1	2	3	4	5	6
Day of the week	:	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

Last day of a century cannot be Tuesday or Thursday or Saturday.

For the calendars of two different years to be the same, the following conditions must be satisfied.

a) Both years must be of the same type. i.e., both years must be ordinary years or both years must be leap years.

b) 1st January of both the years must be the same day of the week.

One solved example: What day of the week does Nov 16 1991 fall on ?

- **Odd days in Century: 3 centuries completed after 1600 years**
Final odd days of 3 centuries = 1 odd day
- **Odd days in Year: Completed years are 90.**
In 90 years, $(90/4) \rightarrow 22$ leap years, remaining 68 ordinary years.
 $22 \text{ leap years} = 22 * 2 = 44 \text{ odd days}$
 $68 \text{ ordinary years} = 68 * 1 = 68 \text{ odd days}$
 $\text{Total odd days} = 44 + 68 = 112$
Final odd days = $(112/7) = 0 \text{ odd day}$
- **Odd days of Month: Completed months are up to October.**
 $3(\text{Jan}) + 0(\text{Feb}) + 3(\text{Mar}) + 2(\text{Apr}) + 3(\text{May}) + 2(\text{June}) + 3(\text{July}) + 3(\text{Aug}) + 2(\text{Sep}) + 3(\text{Oct}) = 24 \text{ Odd days}$
Final odd days = $(24/7) = 3 \text{ odd days}$
- **Odd days of date: 16**
Final odd days of date = $(16/7) = 2 \text{ odd day}$
Final Odd day = Century + Year + Month + Date = $1 + 0 + 3 + 2 = 6 \text{ odd day}$
Odd day = 6 = Saturday

12. AREA AND VOLUME

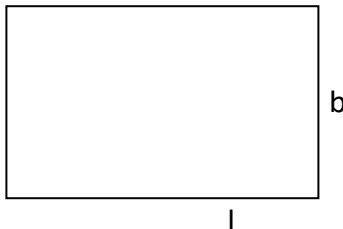
Square



Perimeter: $P = 4s$

Area: $A = s^2$

Rectangle

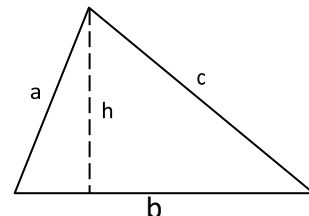


Perimeter: $P = 2l + 2b$

Area: $A = l \times b$

Diagonal of rectangle = $\sqrt{l^2 + b^2}$

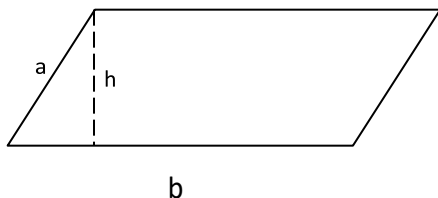
Triangle



Perimeter: $a + b + c$

Area = $\frac{1}{2} \times b \times h$

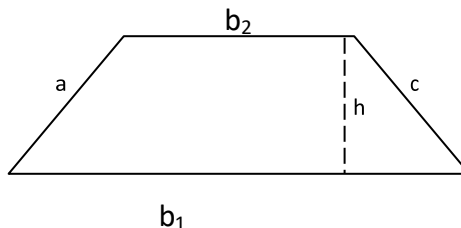
Parallelogram



Perimeter: $P = 2a + 2b$

Area: $A = b \times h$

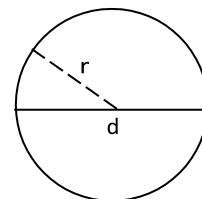
Trapezoid



Perimeter: $P = a + b_1 + c + b_2$

Area = $\frac{1}{2} \times (b_1 + b_2) \times h$

Circle



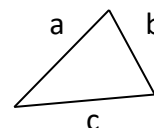
Circumference: $C = 2\pi r$

or $C = \pi d$

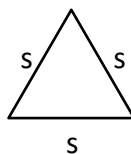
Area: $A = \pi r^2$

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where a, b, c are the sides of the triangle and $s = \frac{a+b+c}{2}$



Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (s)^2$

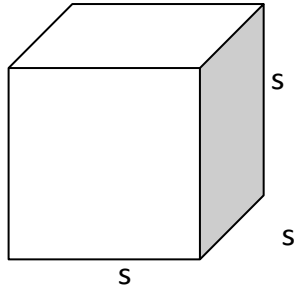


Circumference of a semi-circle = $\pi r + 2r$



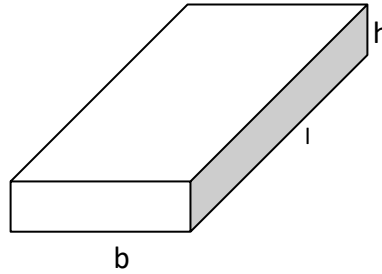
Area of Semi Circle: $\frac{\pi r^2}{2}$

Cube



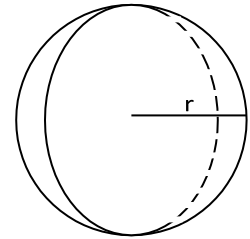
Volume: $V = s^3$
 Surface Area: $S = 6s^2$

Rectangular Prism
 (Box) (Cuboid)



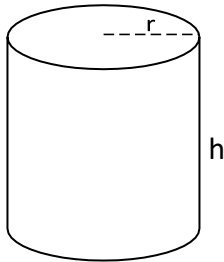
Volume: $V = l \times b \times h$
 Surface Area: $S = 2(lb + 2bh + 2lh)$

Sphere



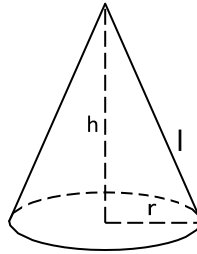
Volume: $V = \frac{4}{3}\pi r^3$
 Total Surface area = $4\pi r^2$

Cylinder

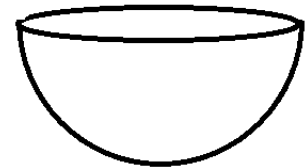


Volume = $\pi r^2 h$
 Curved Surface Area = $2\pi r h$
 Total Surface Area = $2\pi r h + 2\pi r^2$

Cone



Volume = $\frac{1}{3}\pi r^2 h$
 Curved Surface Area = $\pi r l$
 Total Surface Area = $\pi r l + 2\pi r^2$



Volume: $V = \frac{2}{3}\pi r^3$
 Curved Surface area = $2\pi r^2$
 Total Surface area = $3\pi r^2$

13. ALLIGATION AND MIXTURE

1. Alligation:

It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price.

2. Mean Price:

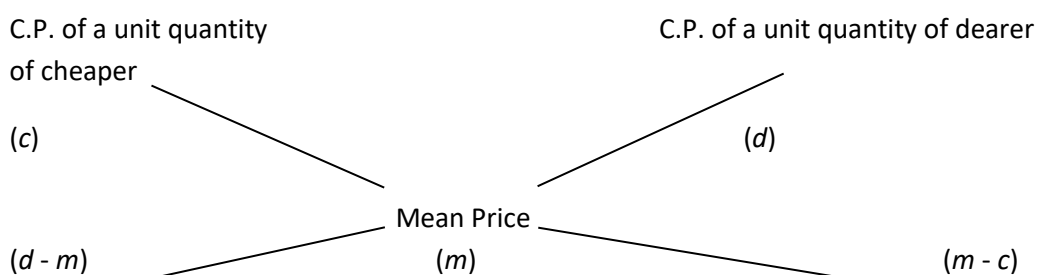
The cost of a unit quantity of the mixture is called the mean price.

3. Rule of Alligation:

If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} \right) = \left(\frac{\text{C.P. of dearer} - \text{Mean Price}}{\text{Mean price} - \text{C.P. of cheaper}} \right)$$

We present as under:



$$\therefore (\text{Cheaper quantity}) : (\text{Dearer quantity}) = (d - m) : (m - c).$$

4. Suppose a container contains x of liquid from which y units are taken out and replaced by water.

After n operations, the quantity of pure liquid = $x \left[1 - \frac{y}{x} \right]^n$

14. PERMUTATION AND COMBINATION

Factorial Notation:

Let n be a positive integer. Then, factorial n , denoted $n!$ is defined as:

$$n! = n(n-1)(n-2) \dots 3.2.1.$$

Examples:

- We define $0! = 1$.
- $4! = (4 \times 3 \times 2 \times 1) = 24$.
- $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

Permutations:

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Examples:

- All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb).
- All permutations made with the letters a, b, c taking all at a time are: ($abc, acb, bac, bca, cab, cba$)

Number of Permutations:

Number of all permutations of n things, taken r at a time, is given by:

$${}^n P_r = \frac{n!}{(n-r)!}$$

Examples:

- ${}^6 P_2 = (6 \times 5) = 30$.
- ${}^7 P_3 = (7 \times 6 \times 5) = 210$.

Combinations:

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a **combination**.

Examples:

- Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.
- Note: AB and BA represent the same selection.
- All the combinations formed by a, b, c taking ab, bc, ca .
- The only combination that can be formed of three letters a, b, c taken all at a time is abc .
- Various groups of 2 out of four persons A, B, C, D are:
 - AB, AC, AD, BC, BD, CD.
- Note that ab ba is two different permutations but they represent the same combination.

Number of Combinations:

The number of all combinations of n things, taken r at a time is:

$${}^nC_r = \frac{n!}{(r!)(n-r)!}$$

Note:

- ${}^nC_n = 1$ and ${}^nC_0 = 1$.
- ${}^nC_r = {}^nC_{(n-r)}$

Examples:

$${}^{11}C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330.$$

$${}^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = \frac{16 \times 15 \times 14}{6} = 560.$$