

# AEX index

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Table 1: Summary statistics of the returns

Statistics	Euro Stoxx 50	Standardized Residuals
Arithmetic Mean	0.0326	-0.0403
Median	0.0552	-0.0193
Maximum	11.1751	3.926
Minimum	-12.7843	-9.9798
Stdev	1.3113	0.9993
Skewness	-0.3547 (0***)	-0.5781 (0***)
Excess Kurtosis	9.2462 (0***)	3.2452 (0***)
Jarque-Bera	32140.8785***	4435.7757***

Notes

<sup>1</sup> This table shows the descriptive statistics of the daily percentage returns of Aex over the period 1987-01-01 to 2021-05-19 (8970 observations). Including arithmetic mean, median, maximum, minimum, standard deviation. The skewness, excess kurtosis with p-value and significance and the Jarque-Bera test with significance.

<sup>2</sup> The standardized residual is derived from a maximum likelihood estimation (simple GARCH model) as follows:

$$R_t = \alpha_0 + \alpha_1 R_{t-1} + z_t \sigma_t$$

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2,$$

Where  $z$  is the standard residual (assumed to have a normal distribution).

<sup>3</sup> \*, \*\*, \*\*\* represent significance levels at the 5

Table 2: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
$\alpha_0$	0.066 (0.009)***	0.065 (0.009)***	0.045 (0.009)***	0.046 (0.01)***	0.069 (0.01)***	0.039 (0.01)***	0.044 (0.011)***	0.037 (0.01)***
$\alpha_1$	0.003 (0.01)	0.003 (0.011)	0.012 (0.01)	0.013 (0.01)	0.001 (0.011)	0.016 (0.01)	0.014 (0.018)	0.009 (0.009)
$\beta_0$	0.015 (0.003)***	0.012 (0.002)***	0.001 (0.002)	0.017 (0.003)***	0	0.021 (0.003)***	0.02 (0.005)***	0.022 (0.006)***
$\beta_1$	0.095 (0.009)***	0.102 (0.009)***	-0.093 (0.008)***	0.021 (0.008)***	0.071 (0.006)***	0.085 (0.01)***	0.085 (0.015)***	0.098 (0.01)***
$\beta_2$	0.896 (0.009)***	0.898	0.984 (0)***	0.906 (0.011)***	0.929	0.854 (0.012)***	0.917 (0.016)***	0.889 (0.012)***
$\xi$	0.907 (0.013)***	0.907 (0.013)***	0.904 (0.013)***	0.904 (0.013)***	0.903 (0.012)***	0.903 (0.013)***	0.905 (0.014)***	0.9 (0.013)***
$\eta$	3.6185 (0.2975)***	3.389 (0.255)***	3.818 (0.321)***	3.889 (0.3355)***	3.9065 (0.268)***	3.9695 (0.3495)***	3.836 (0.324)***	3.9215 (0.339)***
$\gamma$			0.152 (0.011)***	0.119 (0.014)***				
<i>shift</i>						0.772 (0.068)***		0.356 (0.046)***
<i>rot</i>							0.646 (0.078)***	0.344 (0.061)***
<i>LLH</i>	-12811.378	-12814.376	-12723.208	-12728.713	-12859.197	-12698.715	-12715.397	-12701.513

*Notes:*

This table shows the maximum likelihood estimates of various ST-GARCH models. The daily returns used on the AEX cover the period from 01 gennaio, 1987 to 19 maggio, 2021 (8970 observations).

The mean process is modeled as follows:  $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$  Where, in the 8 GARCH models estimated,  $\gamma$  is the asymmetry in volatility,  $\xi, \kappa$  and  $\eta$  are constant and robust standard errors based on the method of White (1982)) are displayed in parenthesis. *LLH* is the maximized log likelihood value.

Table 3: Maximum likelihood estimates of the SGED-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
$\alpha_0$	0.059 (0.009)***	0.058 (0.009)***	0.038 (0.011)***	0.039 (0.009)***	0.06 (0.011)***	0.033 (0.01)***	0.038 (0.01)***	0.03 (0.01)***
$\alpha_1$	-0.002 (0.01)	-0.002 (0.011)	0.006 (0.01)	0.007 (0.009)	-0.004 (0.012)	0.01 (0.01)	0.007 (0.01)	0.003 (0.01)
$\beta_0$	0.017 (0.003)***	0.013 (0.003)***	0.001 (0.002)	0.019 (0.004)***	0	0.023 (0.004)***	0.021 (0.005)***	0.024 (0.006)***
$\beta_1$	0.098 (0.01)***	0.107 (0.01)***	-0.091 (0.008)***	0.022 (0.008)***	0.071 (0.006)***	0.085 (0.009)***	0.085 (0.014)***	0.098 (0.011)***
$\beta_2$	0.891 (0.009)***	0.893	0.983 (0)***	0.903 (0.011)***	0.929	0.853 (0.013)***	0.916 (0.015)***	0.887 (0.012)***
$\xi$	0.917 (0.014)***	0.918 (0.014)***	0.91 (0.014)***	0.909 (0.014)***	0.913 (0.013)***	0.908 (0.014)***	0.91 (0.014)***	0.905 (0.013)***
$\eta$								
$\gamma$			0.151 (0.012)***	0.119 (0.015)***				
<i>shift</i>						0.764 (0.073)***		0.37 (0.061)***
<i>rot</i>							0.637 (0.076)***	0.329 (0.075)***
<i>LLH</i>	-12823.616	-12828.523	-12737.961	-12742.501	-12880.49	-12712.937	-12730.417	-12715.259

*Notes:*

This table shows the maximum likelihood estimates of various SGED-GARCH models. The daily returns used on the AEX cover the period from 01 gennaio, 1987 to 19 maggio, 2021 (8970 observations).

Table 4: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
N	2.913	2.915	2.942	2.888	2.889	2.879	2.885	2.881
T	2.863	2.863	2.874	2.844	2.845	2.838	2.842	2.840
ST	2.858	2.859	2.868	2.839	2.840	2.833	2.837	2.834
GED	2.865	2.866	2.879	2.847	2.848	2.842	2.845	2.843
SGED	2.861	2.862	2.873	2.842	2.843	2.836	2.840	2.837

*Notes*

This table shows the AIC value for the respective model. With on the rows the distributions.

Table 5: VaR and CVaR test statistics

	EGARCH	GJRGARCH	TGARCH	NAGARCH	AVGARCH
<b>Panel A: SGED</b>					
AE VaR	1.19	1.314	1.206	1.175	1.144
AE CVaR	1.203**	1.328	1.219**	1.188	1.156**
UC	2.226	5.856**	2.591	1.888	1.29
CC	4.082	8.119**	4.495	3.695	3.003
DQ	20.427***	31.251***	19.896**	20.281***	19.726**
<b>Panel B: GED</b>					
AE VaR	1.515	1.607	1.546	1.468	1.484
AE CVaR	1.531***	1.625*	1.562**	1.484*	1.5**
UC	14.954***	20.365***	16.677***	12.526***	13.314***
CC	15.138***	23.763***	16.903***	15.357***	13.459***
DQ	28.256***	42.34***	36.034***	28.808***	25.72***
<b>Panel C: ST</b>					
AE CVaR	1.221	1.345	1.236	1.19	1.128
AE CVaR	1.234	1.359	1.25	1.203	1.141
UC	2.982*	7.007***	3.399*	2.226	1.033
CC	4.936*	9.379***	5.403*	4.082	2.699
DQ	20.241***	26.194***	20.21***	18.892**	20.353***
<b>Panel D: T</b>					
AE VaR	1.669	1.607	1.577	1.53	1.53
AE CVaR	1.688	1.625	1.594	1.547	1.547
UC	24.365***	20.365***	18.481***	15.805***	15.805***
CC	24.805***	23.763***	18.754***	18.882***	16.01***
DQ	0.011	39.714***	40.245***	31.991***	28.662***
<b>Panel E: N</b>					
AE VaR	1.994	1.855	1.932	1.886	1.963
AE CVaR	2.016***	1.875***	1.953***	1.906***	1.984***
UC	50.081***	38.134***	44.608***	40.674***	47.312***
CC	50.224***	39.011***	44.686***	41.637***	48.507***
DQ	79.218***	60.905***	73.891***	63.622***	82.198***

*Notes:*

Table contains the ratio of actual to expected exceedances for VaR and Conditional VaR, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. \*, \*\* and \*\*\* point out significance at 10, 5 and 1 percent level.

Table 6: VaR and ES test statistics (ACD-ST vs GARCH-ST)

	ACD	GARCH
AE VaR	1.22	1.237
AE CVaR	1.22	1.237
UC	2.402	2.795*
CC	4.16	4.602
DQ	26.733***	32.084***

*Notes:*

Table contains the ratio of actual to expected exceedances for VaR and Expected Shortfall, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. \*, \*\* and \*\*\* point out significance at 10, 5 and 1 percent level.

Table 7: Diagnostic Tests for Heteroscedasticity

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
<b>Panel A: Ljung Box Test on the standardized squared values of the residuals</b>								
Norm	21.258	23.629	21.789	18.907	19.166	26.532	22.888	31.913*
T	24.188	23.421	21.196	19.123	20.31	25.784	24.569	31.434*
ST	24.938	24.327	22.023	19.445	20.368	27.274	25.101	31.546*
GED	22.896	23.148	20.922	18.813	19.759	25.537	23.826	31.793*
SGED	23.584	24.225	21.8	19.158	19.791	27.322	24.242	31.555*
<b>Panel B: ARCH LM Test on the standardized squared values of the residuals</b>								
Norm	21.748	24.327	22.862	19.432	19.704	27.726	23.552	29.784
T	24.246	23.85	21.966	19.607	20.887	26.748	24.621	29.34
ST	24.846	24.721	22.753	19.928	20.887	28.16	24.959	29.439
GED	23.151	23.687	21.792	19.306	20.335	26.607	24.129	29.655
SGED	23.714	24.727	22.622	19.659	20.324	28.295	24.36	29.456

*Notes*

Table displays the Ljung box statistics and the ARCH LM Test for the standardized squared residuals of the models analyzed. The underlying data is the daily return series of the AEX for the period between 1987-01-01 and 2021-05-19.

\*, \*\* and \*\*\* point out respectively significance at 10, 5 and 1 percent level.

The null hypothesis of the test in both panels are described as follows:

$$H_0: \text{Corr}(Z_t^2, Z_{t-1}^2) = \text{Corr}(Z_t^2, Z_{t-2}^2) = \dots = \text{Corr}(Z_t^2, Z_{t-22}^2) = 0$$



Table 8: GMM Tests

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
<b>Panel A: SGED</b>								
Mean	-0.024**	0	0.008	0.006	-0.001	0.001	-0.023**	-0.029**
Variance	0.019**	0.012	0.007	0.01	0.012	0.01	-0.014**	0.132**
Skewness	-0.694**	-0.531	-0.491	-0.505	-0.561	-0.511	-0.676**	-0.958**
Excess Kurtosis	3.723**	2.886	2.78	2.796	2.997	2.794	3.465**	6.781**
<b>Panel B: GED</b>								
Mean	-0.044***	-0.022**	-0.015	-0.016	-0.023**	-0.02*	-0.044***	-0.047***
Variance	0.007***	0.006**	0.005	0.005	0.006**	0.005*	-0.024***	0.133***
Skewness	-0.73***	-0.59**	-0.556	-0.563	-0.615**	-0.567*	-0.715***	-1.013***
Excess Kurtosis	3.539***	2.88**	2.819	2.8	2.955**	2.783*	3.299***	6.841***
<b>Panel C: ST</b>								
Mean	-0.032***	-0.008	0.001	-0.001	-0.009	-0.006	-0.031***	-0.039***
Variance	0.018***	0.011	0.007	0.009	0.012	0.009	-0.014***	0.132***
Skewness	-0.723***	-0.558	-0.518	-0.531	-0.591	-0.536	-0.702***	-0.99***
Excess Kurtosis	3.797***	2.956	2.865	2.88	3.076	2.857	3.523***	6.817***
<b>Panel D: T</b>								
Mean	-0.051***	-0.027**	-0.019*	-0.02*	-0.028**	-0.025**	-0.051***	-0.054***
Variance	0.006***	0.007**	0.007*	0.008*	0.008**	0.006**	-0.023***	0.134***
Skewness	-0.756***	-0.613**	-0.576*	-0.584*	-0.639**	-0.588**	-0.738***	-1.036***
Excess Kurtosis	3.621***	2.98**	2.923*	2.916*	3.06**	2.878**	3.358***	6.88***
<b>Panel E: N</b>								
Mean	-0.04***	-0.004	0.007	0.004	-0.005	-0.003	-0.039***	-0.035***
Variance	-0.001***	0	-0.001	0	0	-0.001	-0.037***	0.132***
Skewness	-0.695***	-0.522	-0.476	-0.491	-0.549	-0.505	-0.676***	-0.974***
Excess Kurtosis	3.332***	2.699	2.602	2.607	2.777	2.615	3.038***	6.82***

*Notes* Table displays the GMM test statistics for the standardized residuals. The underlying data is the daily return series of the AEX for the period between 1987-01-01 and 2021-05-19. The null hypothesis of the test for each variable are described as follows:  $H_0 : E[z_t] = 0$  for the mean,  $H_0 : E[z_t^2 - 1] = 0$  for the variance.  $H_0 : E[z_t^3] = 0$  for the skewness and  $H_0 : E[z_t^4 - 3] = 0$  for the excess kurtosis.

Table 9: Jarque-Bera Test on standardized residuals

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Norm	4435.776***	3086.969***	2928.107***	2923.066***	3269.811***	2908.607***	4877.981***	8551.424***
T	4909.338***	3410.689***	3319.309***	3289.181***	3599.587***	3205.115***	5263.033***	8498.222***
ST	5085.053***	3361.225***	3270.155***	3262.915***	3622.283***	3173.472***	5492.51***	8486.025***
GED	4704.842***	3250.257***	3162.011***	3114.776***	3434.205***	3054.355***	5149.266***	8495.544***
SGED	4882.473***	3219.064***	3129.533***	3103.423***	3469.624***	3044.308***	5378.214***	8492.402***

*Notes* Table displays the Jarque-Bera statistic  $JB = \frac{n}{6}(S^2 + \frac{1}{4}(K - 3)^2)$  with  $n$  the sample size,  $K$  the kurtosis and  $S$  the skewness for the residuals of the models. The JB statistic is distributed  $\chi^2$  with  $\nu = 2$ . The underlying data is the daily return series of the AEX for the period between 1987-01-01 and 2021-05-19. \*, \*\* and \*\*\* point out respectively significance at 10, 5 and 1 percent level. The null hypothesis is that  $S$  and  $K$  are not significantly different than what would be found under normality (0 and 3).

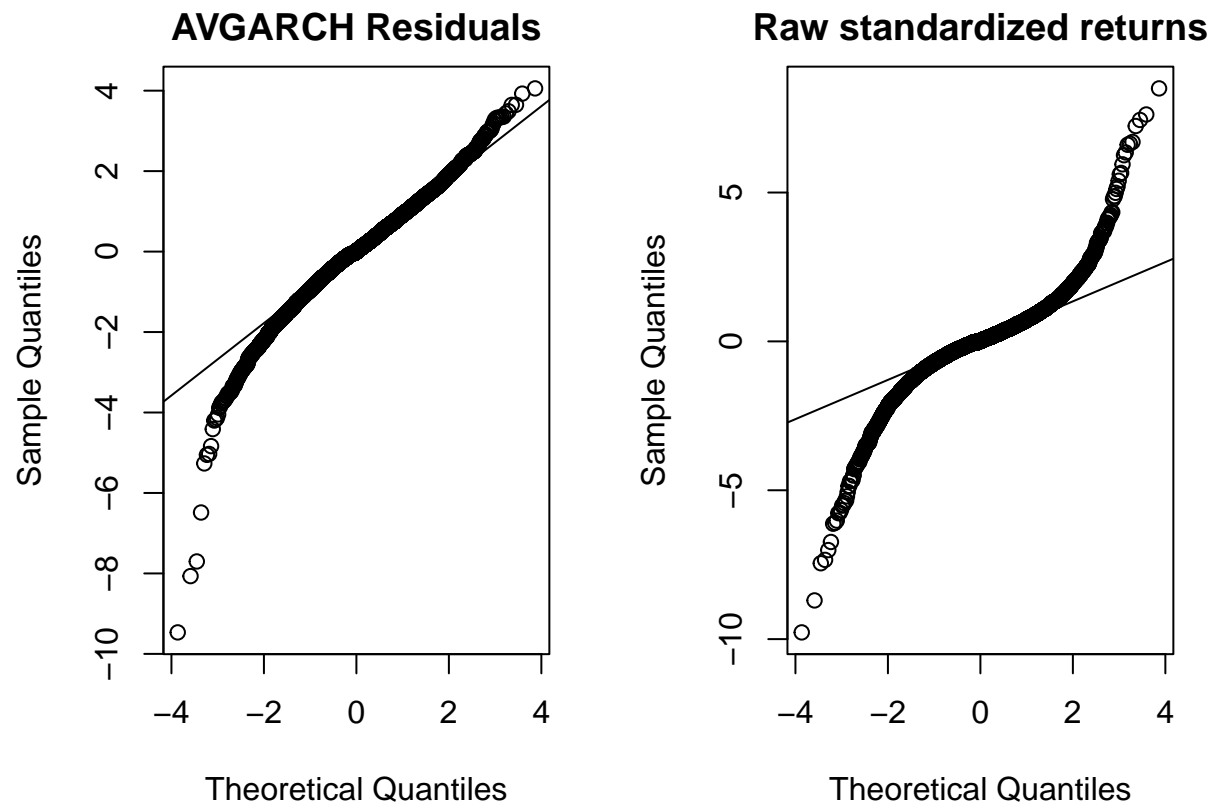


Figure 1: QQ plots of AVGARCH residuals versus the standardized returns of the series