BEL 20 index

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Table 1: Summary statistics of the returns

Statistics	Euro Stoxx 50	Standardized Residuals
Arithmetic Mean	0.0259	-0.0392
Median	0.0198	-0.0398
Maximum	9.3527	4.9795
Minimum	-15.3107	-12.3092
Stdev	1.1468	0.9992
Skewness	-0.3584	-0.447
	(0***)	(0***)
Excess Kurtosis	10.5055	4.4155
	(0***)	(0***)
Jarque-Bera	37818.9856***	6922.5233***

Notes

[1] This table shows the descriptive statistics of the daily percentage returns of Bel 20 over the period 1990-01-03 to 2021-05-19 (8186 observations). Including arithmetic mean, median, maximum, minimum, standard deviation. The skewness, excess kurtosis with p-value and signicance and the Jarque-Bera test with significance.

[2] The standardized residual is derived from a maximum likelihood estimation (simple GARCH model) as follows:
$R_t = \alpha_0 + \alpha_1 R_{t-1} + z_t \sigma_t$
$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2,$
Where z is the standard residual (assumed to have a normal distribution).

[3] *, **, *** represent significance levels at the 5

Table 2: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.064	0.064	0.035	0.037	0.066	0.026	0.033	0.025
	(0.01)***	(0.011)***	(0.016)**	(0.011)***	(0.012)***	(0.013)**	(0.011)***	(0.011)**
α_1	-0.01	-0.01	0	-0.003	-0.012	-0.001	0.002	-0.004
	(0.01)	(0.011)	(0.01)	(0.01)	(0.011)	(0.01)	(0.011)	(0.01)
β_0	0.019	0.014	0.004	0.024	0	0.025	0.023	0.025
	(0.004)***	(0.003)***	(0.002)**	(0.004)***		(0.008)***	(0.011)**	(0.002)***
β_1	0.086	0.093	-0.104	0.01	0.066	0.069	0.074	0.084
	(0.009)***	(0.009)***	(0.008)***	(0.006)	(0.006)***	(0.024)***	(0.026)***	(0.007)***
β_2	0.904	0.907	0.982	0.909	0.934	0.846	0.925	0.891
	(0.009)***		(0)***	(0.01)***		(0.016)***	(0.028)***	(0.001)***
ξ	0.938	0.938	0.927	0.929	0.936	0.921	0.928	0.921
	(0.014)***	(0.013)***	(0.015)***	(0.014)***	(0.012)***	(0.015)***	(0.015)***	(0.014)***
η	3.687	3.4105	3.913	3.954	3.933	4.006	3.9115	3.916
	(0.3235)***	(0.264)***	(0.363)***	(0.3755)***	(0.2735)***	(0.3775)***	(0.364)***	(0.37)***
γ			0.13	0.134				
			(0.011)***	(0.015)***				
$shift$						1.046		0.491
						(0.254)***		(0.039)***
rot							0.82	0.417
							(0.147)***	(0.077)***
LLH	-13416.297	-13420.311	-13292.268	-13315.393	-13457.9	-13269.273	-13284.978	-13272.231

Notes:
This table shows the maximum likelihood estimates of various ST-GARCHmodels. The daily returns used on the BEL 20 cover the period from 03 January, 1990 to 19 May, 2021 (8186 observations).
The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$ Where, in the 8 GARCH models estimated, γ is the asymmetry in volatility, ξ, κ and η are constant and robust standard errors based on the method of White (1982)) are displayed in parenthesis. LLH is the maximized log likelihood value.

Table 3: Maximum likelihood estimates of the SGED-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.053	0.053	0.027	0.028	0.052	0.018	0.026	0.017
	(0.01)***	(0.01)***	(0.008)***	(0.011)***	(0.011)***	(0.017)	(0.011)**	(0.008)**
α_1	-0.017	-0.017	-0.009	-0.011	-0.019	-0.01	-0.007	-0.013
	(0.01)*	(0.01)*	(0.01)	(0.011)	(0.01)*	(0.011)	(0.009)	(0.008)
β_0	0.023	0.015	0.005	0.027	0	0.028	0.026	0.027
	(0.005)***	(0.003)***	(0.002)***	(0.005)***		(0.019)	(0.015)*	(0.002)***
β_1	0.089	0.098	-0.102	0.011	0.065	0.069	0.073	0.088
	(0.01)***	(0.01)***	(0.008)***	(0.006)*	(0.006)***	(0.051)	(0.033)**	(0.002)***
β_2	0.898	0.902	0.98	0.907	0.935	0.849	0.924	0.883
	(0.01)***		(0)***	(0.01)***		(0.036)***	(0.037)***	(0)***
ξ	0.956	0.957	0.941	0.942	0.951	0.935	0.942	0.935
	(0.016)***	(0.015)***	(0.017)***	(0.017)***	(0.013)***	(0.018)***	(0.017)***	(0.015)***
η								
γ			0.129	0.131				
			(0.002)***	(0.015)***				
$shift$						1.017		0.592
						(0.552)*		(0.008)***
rot							0.804	0.312
							(0.188)***	(0.047)***
LLH	-13418.177	-13424.612	-13309.092	-13325.578	-13468.492	-13284.96	-13302.889	-13288.723

Notes:
This table shows the maximum likelihood estimates of various SGED-
GARCH models. The daily returns used on the BEL 20 cover
the period from 03 January, 1990 to 19 May, 2021 (8186 observations).

Table 4: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
N	3.126	3.130	3.150	3.101	3.102	3.092	3.099	3.094
T	3.085	3.085	3.094	3.057	3.062	3.052	3.055	3.053
ST	3.083	3.083	3.092	3.054	3.060	3.049	3.053	3.050
GED	3.084	3.085	3.096	3.060	3.064	3.055	3.059	3.056
SGED	3.083	3.084	3.094	3.058	3.062	3.053	3.057	3.054

Notes

This table shows the AIC value for the respective model. With on the rows the distributions.

Table 5: VaR and CVaR test statistics

	EGARCH	GJRGARCH	TGARCH	NAGARCH	AVGARCH
Panel A: SGED					
AE VaR	1.192	1.176	1.192	1.16	1.16
AE CVaR	1.194**	1.177*	1.194*	1.161	1.161*
UC	2.174	1.833	2.174	1.52	1.52
CC	5.407*	1.856	5.407*	1.551	1.551
DQ	21.895***	12.51	27.427***	13.621*	22.355***
Panel B: GED					
AE VaR	1.288	1.353	1.305	1.288	1.321
AE CVaR	1.29***	1.355**	1.306**	1.29**	1.323***
UC	4.783**	7.033***	5.308**	4.783**	5.859**
CC	7.353**	7.051**	7.778**	4.784*	8.23**
DQ	29.151***	25.695***	28.348***	16.013**	27.735***
Panel C: ST					
AE CVaR	1.224	1.224	1.208	1.16	1.176
AE CVaR	1.226	1.226	1.21	1.161	1.177
UC	2.938*	2.938*	2.543	1.52	1.833
CC	5.939*	2.943	5.658*	1.551	1.856
DQ	21.917***	13.939*	28.637***	14.388*	21.782***
Panel D: T					
AE VaR	1.482	1.401	1.353	1.369	1.353
AE CVaR	1.484	1.403	1.355*	1.371	1.355
UC	12.675***	8.975***	7.033***	7.657***	7.033***
CC	14.194***	9.018**	9.215***	7.682**	9.215***
DQ	0.01	27.464***	29.179***	20.205***	30.607***
Panel E: N					
AE VaR	1.772	1.756	1.739	1.675	1.868
AE CVaR	1.774***	1.758***	1.742***	1.677***	1.871***
UC	30.371***	29.221***	28.09***	23.754***	37.655***
CC	32.1***	29.225***	28.679***	23.792***	38.952***
DQ	60.884***	52.57***	54.786***	45.838***	67.905***

Notes:

Table contains the ratio of actual to expected exceedances for VaR and Conditional VaR, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. *, ** and *** point out significance at 10, 5 and 1 percent level.

Table 6: Maximum likelihood estimates of the ST-ACD model with constant skewness and kurtosis parameters

	ACD GARCH
α_0	0.069 (0.011)***
α_1	-0.003 (0.013)
β_0	0.021
β_1	0.091 (0.001)***
β_2	0.897 (0.002)***
χ_0	-1.207 (0.116)***
χ_1	0.051 (0.019)***
χ_2	0.048 (0.027)*
ξ_1	0.658 (0.033)***
κ_0	0.016 (0.116)
κ_1	0.002 (0.05)
κ_2	1 (0.136)***
ψ_1	0.796 (0.041)***

Notes:
This table shows the maximum likelihood estimates of various SGED-GARCH models. The daily returns used on the BEL 20 cover the period from 03 January, 1990 to 19 May, 2021 (8186 observations). The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$. Where, in the model estimated, γ is the asymmetry in volatility, (calculated using robust standard errors based on the method of White (1982)) are displayed in parenthesis.

Table 7: VaR and ES test statistics (ACD-ST vs GARCH-ST)

	ACD	GARCH
AE VaR	2.5	2.5
AE CVaR	2.5	2.5
UC	7.288***	7.288***
CC	7.96	7.96
DQ	61.463***	61.577***

Notes:
Table contains the ratio of actual to expected exceedances for VaR and Expected Shortfall, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. *, ** and *** point out significance at 10, 5 and 1 percent level.

Table 8: Diagnostic Tests for Heteroscedasticity

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Panel A: Ljung Box Test on the standardized squared values of the residuals								
Norm	31.559*	21.198	25.233	20.735	21.579	21.748	35.642**	45.131***
T	34.485**	19.397	22.137	19.541	21.018	19.401	35.801**	43.348***
ST	34.823**	19.27	22.897	19.478	21.131	19.516	36.112**	43.205***
GED	33.233*	19.524	23.033	19.835	21.17	19.796	35.666**	44.217***
SGED	33.523*	19.377	23.65	19.812	21.296	19.896	35.915**	43.787***
Panel B: ARCH LM Test on the standardized squared values of the residuals								
Norm	32.433*	21.812	25.523	21.126	21.921	21.774	36.828**	44.203***
T	35.081**	19.871	22.41	19.895	21.519	19.471	36.355**	42.589***
ST	35.35**	19.71	23.106	19.796	21.593	19.505	36.558**	42.424***
GED	33.954**	20.028	23.329	20.194	21.609	19.864	36.426**	43.356***
SGED	34.191**	19.841	23.891	20.133	21.707	19.878	36.591**	42.923***

Notes

Table displays the Ljung box statistics and the ARCH LM Test for the standardized squared residuals of the models analyzed. The underlying data is the daily return series of the BEL 20 for the period between 1990-01-03 and 2021-05-19.

*, ** and *** point out respectively significance at 10, 5 and 1 percent level.

The null hypothesis of the test in both panels are described as follows:

H_0: $Corr(Z_t^2, Z_{t-1}^2) = Corr(Z_t^2, Z_{t-2}^2) = ... = Corr(Z_t^2, Z_{t-22}^2) = 0$

Table 9: GMM Tests

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Panel A: SGED								
Mean	-0.022*	0.002	0.012	0.01	0.002	0.004	-0.021*	-0.022*
Variance	0.01*	0.011	0.007	0.008	0.009	0.009	-0.026*	0.112*
Skewness	-0.482*	-0.469	-0.4	-0.409	-0.446	-0.457	-0.484*	-0.654*
Excess Kurtosis	2.659*	3.74	3.024	3.209	3.287	3.743	2.488*	4.89*
Panel B: GED								
Mean	-0.032***	-0.012	-0.005	-0.006	-0.012	-0.01	-0.032***	-0.032***
Variance	0.003***	0.006	0.004	0.005	0.004	0.005	-0.031***	0.111***
Skewness	-0.507***	-0.508	-0.451	-0.454	-0.485	-0.494	-0.508***	-0.684***
Excess Kurtosis	2.598***	3.673	3.081	3.2	3.277	3.678	2.436***	4.926***
Panel C: ST								
Mean	-0.032***	-0.006	0.003	0.002	-0.008	-0.004	-0.032***	-0.037***
Variance	0.01***	0.013	0.008	0.01	0.011	0.012	-0.024***	0.114***
Skewness	-0.522***	-0.508	-0.438	-0.445	-0.49	-0.494	-0.517***	-0.703***
Excess Kurtosis	2.766***	3.942	3.229	3.379	3.52	3.966	2.563***	4.977***
Panel D: T								
Mean	-0.044***	-0.021*	-0.012	-0.013	-0.022*	-0.018	-0.044***	-0.046***
Variance	0.003***	0.011*	0.008	0.01	0.008*	0.01	-0.03***	0.114***
Skewness	-0.55***	-0.548*	-0.488	-0.491	-0.531*	-0.534	-0.547***	-0.734***
Excess Kurtosis	2.707***	3.916*	3.334	3.417	3.553*	3.944	2.516***	5.017***
Panel E: N								
Mean	-0.033***	-0.007	0.008	0.006	-0.002	-0.005	-0.032***	-0.027**
Variance	0***	0	-0.001	0	0	-0.001	-0.042***	0.11**
Skewness	-0.487***	-0.467	-0.392	-0.396	-0.427	-0.452	-0.489***	-0.661**
Excess Kurtosis	2.447***	3.344	2.78	2.894	2.915	3.319	2.249***	4.892**

Notes Table displays the GMM test statistics for the standardized residuals. The underlying data is the daily return series of the BEL 20 for the period between 1990-01-03 and 2021-05-19. The null hypothesis of the test for each variable are described as follows: $H_0 : E[z_t] = 0$ for the mean, $H_0 : E[z_t^2 - 1] = 0$ for the variance. $H_0 : E[z_t^3] = 0$ for the skewness and $H_0 : E[z_t^4 - 3] = 0$ for the excess kurtosis.

Table 10: Jarque-Bera Test on standardized residuals

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Norm	2296.009***	4279.11***	3068.995***	3284.894***	3305.843***	4240.493***	2827.44***	4401.869***
T	2701.486***	5334.03***	3987.609***	4140.842***	4484.032***	5439.063***	3075.452***	4490.214***
ST	2693.802***	5390.968***	3816.071***	4109.426***	4382.332***	5506.673***	3082.355***	4445.395***
GED	2506.353***	4901.291***	3552.002***	3792.61***	3967.108***	4939.562***	2972.597***	4416.122***
SGED	2507.29***	4985.231***	3409.815***	3782.004***	3915.489***	5031.173***	2981.969***	4370.963***

Notes   Table displays the Jarque-Bera statistic $JB = \frac{n}{6}(S^2 + \frac{1}{4}(K-3)^2)$ with n the sample size, K the kurtosis and S the skewness for the residuals of the models. The JB statistic is distributed χ^2 with $\nu = 2$. The underlying data is the daily return series of the BEL 20 for the period between 1990-01-03 and 2021-05-19.   *, ** and *** point out respectively significance at 10, 5 and 1 percent level.   The null hypothesis is that S and K are not significantly different than what would be found under normality (0 and 3).
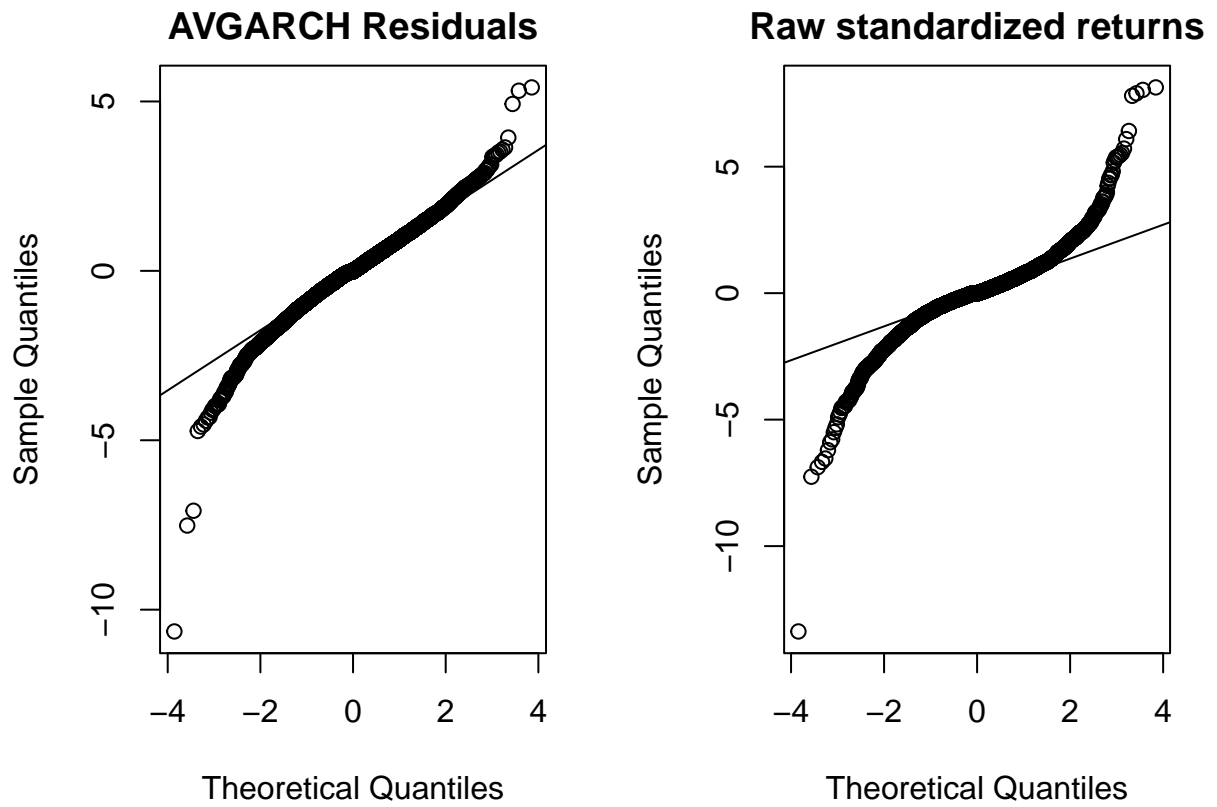
Figure 1: QQ plots of AVGARCH residuals versus the standardized returns of the series