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Empirical Findings

1.1 Density of the returns

1.1.1 MLE distribution parameters

In table ?? we can see the estimated parameters of the unconditional distribution functions. They are presented for the Skewed Generalized T-distribution (SGT) and limiting cases thereof previously discussed. Additionally, maximum likelihood score and the Aikake Information Criterion (AIC) is reported to compare goodness of fit of the different distributions. We find that the SGT-distribution has the highest maximum likelihood score of all. All other distributions have relatively similar likelihood scores, though slightly lower and are therefore not the optimal distributions. However, when considering AIC it is a tie between SGT and SGED. This provides some indication that we have a valid case to test the suitability of different SGED-GARCH VaR models as an alternative for the SGT-GARCH VaR models. While sacrificing some goodness of fit, the SGED distribution has the advantage of requiring one less parameter, which could possibly result in less errors due to misspecification and easier implementation. For the SGT parameters the standard deviation and skewness are both significant at the 1% level. For the SGED parameters, the standard deviation and the skewness are both significant at respectively the 1% and 5% level. Both distributions are right-skewed. For both distributions the shape parameters are significant at the 1% level, though the q parameter was not estimated as it is by design set to infinity due to the SGED being a limiting case of SGT.[1]

Additionally, for every distribution fitted with MLE, plots are generated to compare the theoretical distribution with the observed returns. We see that except for the normal distribution which is quite off, the theoretical distributions are close to the actual data, except that they are too peaked. This problem is the least present for the SGT distribution.

[1] To check whether the relative ranking of distributions still holds in different periods, we have calculated the maximum likelihood score and AIC for three smaller periods: The period up to the dotcom collapse (1987-2001), up to the GFC (2002-2009) and up to the present Covid-crash (2009-2021). There is no qualitative difference in relative ranking with these subsamples. Results are reported in the appendix.

1.2 Constant higher moments

1.1 presents the maximum likelihood estimates for 8 symmetric and asymmetric GARCH models based on the ST distribution with constant skewness and kurtosis parameters (t values are presented in parenthesis). The parameters in the conditional mean equations (α_0) are all statistically significant with t values from 6 to 11. The AR(1) coefficient, α_1, has parameters going from 2 to 2 with t values ranging from 4 to 5 not suggesting a high significance and indicating slight negative autocorrelation. The GARCH parameters in the conditional variance equations (β_0) are generally statistically significant with t values ranging from 1 to 11. The results of β_1 and β_2 show the presence of significant time-variation in the conditional volatility of the Euro Stoxx 50 Price Index daily returns, in fact, the sum of β_1 and β_2 for the GARCH parameters is close to one (from 20 to 33), suggesting the presence of persistence in the volatility of the returns. The parameter ξ is highly significant for all the 8 models tested with values ranging from 12 to 18 confirming the presence of Skewness in the returns. The shape parameter η, which, in our case, measures the number of degrees of freedom, determining the tail behavior, is significant for all the models and ranges between 14 and 19. The parameter γ, which is present only for eGARCH and gjrGARCH is significant and with values around 4.5. The absolute value function in fGARCH models (NAGARCH, TGARCH and AVGARCH) is subject to the $shift$ and the rot parameters whose values are always positive and statistically significant. According to the log likelihood values (LLH), displayed in 1.1, the model with the highest value is eGARCH while, excluding the non-standard GARCH models from the analysis, the model that performs best is eGARCH.

Table 1.1: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.049	0.049	0.026	0.028	0.053	0.02	0.023	0.018
	(5.281)	(5.195)	(2.762)	(3.026)	(5.855)	(2.15)	(2.394)	(2.292)
α_1	-0.018	-0.018	-0.008	-0.008	-0.02	-0.005	-0.005	-0.007
	(-1.64)	(-1.634)	(-0.766)	(-0.769)	(-1.885)	(-0.485)	(-0.464)	(-0.755)
β_0	0.016	0.013	0.001	0.021	0	0.022	0.02	0.022
	(5.778)	(5.842)	(0.768)	(7.281)		(9.947)	(6.224)	(2.808)
β_1	0.094	0.101	-0.098	0.017	0.069	0.08	0.083	0.088
	(12.149)	(13.092)	(-15.506)	(3.023)	(15.022)	(6.335)	(9.728)	(4.962)
β_2	0.898	0.899	0.983	0.897	0.931	0.845	0.919	0.902
	(115.671)		(1557.528)	(115.021)		(86.838)	(107.318)	(49.085)
ξ	0.917	0.917	0.905	0.906	0.917	0.903	0.904	0.902
	(68.347)	(67.434)	(67.158)	(67.761)	(73.304)	(67.75)	(67.219)	(69.587)
η	6.342	6	6.899	6.823	7.037	6.975	6.932	6.95
	(15.441)	(16.919)	(14.583)	(14.632)	(18.327)	(14.539)	(14.564)	(14.526)
γ			0.144	0.143				
			(15.568)	(10.728)				
$shift$						0.904		0.248
						(10.462)		(3.067)
rot							0.723	0.523
							(12.112)	(8.67)
LLH	-13065.425	-13067.628	-12950.977	-12972.473	-13113.368	-12935.328	-12933.581	-12929.723

Notes
This table shows the maximum likelihood estimates of various ST-GARCH models. The daily returns used on the Euro Stoxx 50 Price index cover the periodfrom 02 January, 1987 to 27 April, 2021 (8953 observations).
The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$
Where, in the 8 GARCH models estimated, γ is the asymmetry in volatility , ξ, κ and η are constant and t statistics are displayed in parenthesis.
LLH is the maximized log likelihood value.

As you can see in table 1.2 the AIC for the skewed student's t-distribution (ST) is the best from all the models. As also shown in appendix part **??**. The best in all distributions of the GARCH models seems to be the NAGARCH model, but we do not want to overfit our models because of an in-sample estimation. That is why a parsimonous model like the EGARCH (which has the highest maximum likelihood for the standard GARCH models that are considered), but also the model AVGARCH will be examined using the ST distribution while it has the second-best (lowest) AIC.

Table 1.2: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
norm	2.995	2.998	3.034	2.962	2.967	2.955	2.957	2.955
std	2.924	2.924	2.935	2.900	2.905	2.897	2.896	2.896
sstd	2.920	2.921	2.930	2.895	2.900	2.891	2.891	2.890
ged	2.930	2.931	2.945	2.907	2.911	2.903	7.704	7.701
sged	2.927	2.928	2.940	2.902	2.907	2.898	7.675	7.672

Notes

[1] This table shows the AIC value for the respective model

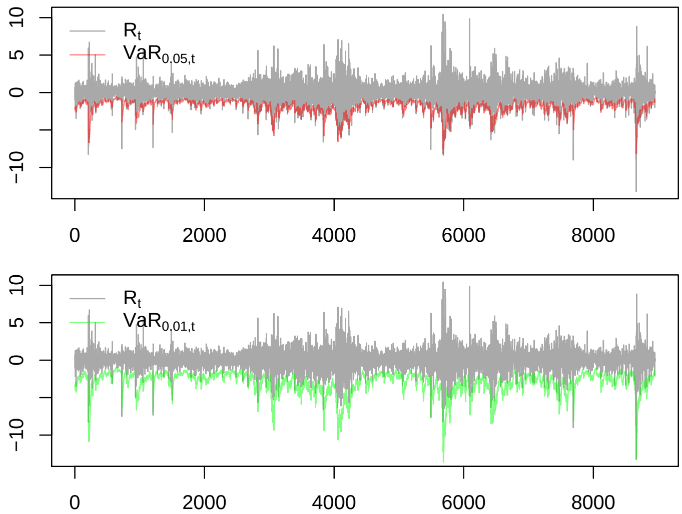

Figure 1.1: Value-at-Risk (in-sample) for the EGARCH-ST model

1.2.1 Value-at-risk

As already mentioned 2 candidate models seem to be very appropriate. This includes the EGARCH and the NAGARCH So to check if they perform well out-of-sample we conduct a backtest by using a rolling forecasting technique. A simple graph is shown in figure 1.1 for the EGARCH-ST model. It seems that the VaR model for $\alpha = 0.05$ underestimates the downside events, while the VaR model for $\alpha = 0.01$ captures more of the downside events.

Let us examine this further using a rolling window approach whilst forecasting 1-day ahead results with re-estimating parameters every year note for prof. Annaert: choices: n.start = 1500 days before the end of the series, refit.every = 252 (trading days in a year), solver = hybrid using a cluster = 10 to run on 10 cores to speed up the process of estimation of the roll object (took 5-10 minutes per backtest with some

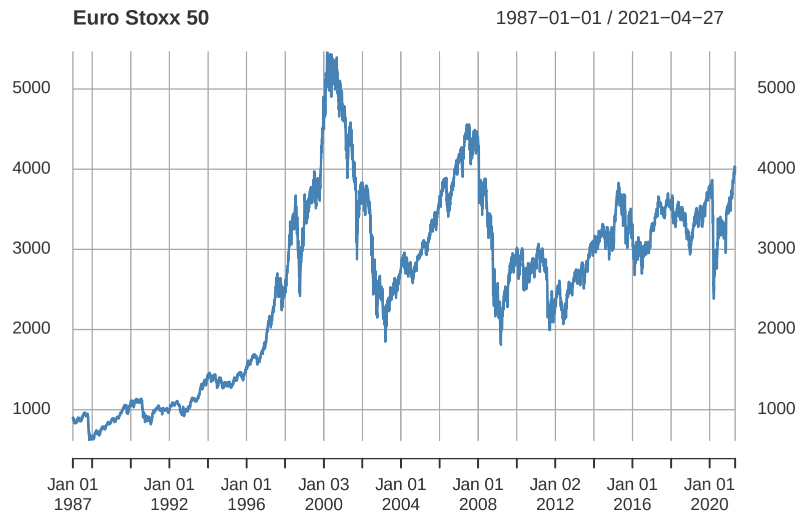

Figure 1.2: Selected period to start forecast from

solvers, now with parallel package...). Figure 1.2 shows that choosing an appropriate forecast period is important (with here the Eurobond crisis, the Brexit and Covid-crisis), so in order to avoid a look-ahead bias this rolling window approach was used.

As you can see in figure **??** the EGARCH with a normal distribution seems to capture the extreme events a bit less compared with the skewed t-distribution.

But let us formally test this.

	EGARCH	GJRGARCH	TGARCH	NAGARCH	AVGARCH
Panel A: SGED					
AP.ratio	1.193243	1.131257	4.029134	1.208740	4.029134
UC	2.292336	1.077299	339.749534	2.662682	339.749534
CC	2.299459	2.748130	377.424279	4.571735	380.220681
DQ	34.442542	24.936113	1783.621469	25.812320	1805.881981
Panel B: GED					
AP.ratio.1	1.410197	1.549667	4.215094	1.425693	4.215094
UC.1	9.728556	16.865292	374.509356	10.435424	374.509356
CC.1	9.798167	20.014037	407.453235	13.097152	410.034187
DQ.1	38.252121	45.476044	1802.464000	38.449617	1818.799317
Panel C: ST					
AP.ratio.2	1.193243	1.162250	1.177747	1.177747	1.162250
UC.2	2.292336	1.630851	1.948278	1.948278	1.630851
CC.2	2.299459	3.395044	1.960383	3.760115	1.649281
DQ.2	34.302619	25.005120	33.249369	19.102820	22.753461
Panel D: T					
AP.ratio.3	1.472184	1.642647	1.487680	1.456687	1.503177
UC.3	12.687425	22.547261	13.481127	11.915090	14.295977
CC.3	12.922959	26.088554	13.628718	14.694682	14.482276
DQ.3	43.912495	52.784288	41.642033	39.803194	54.968600
Panel E: N					
AP.ratio.4	1.983574	2.076554	1.983574	1.937083	1.828607
UC.4	49.027087	57.648354	49.027087	44.930069	35.947622
CC.4	49.109426	57.902257	49.109426	45.011116	36.252515
DQ.4	79.685528	90.173737	83.390986	75.628854	78.372757

1.2.2 Expected shortfall

1.3 Time-varying higher moments

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