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Empirical Findings

1.1 Density of the returns

1.1.1 MLE distribution parameters

In table **??** we can see the estimated parameters of the unconditional distribution functions. They are presented for the Skewed Generalized T-distribution (SGT) and limiting cases thereof previously discussed. Additionally, maximum likelihood score and the Aikake Information Criterion (AIC) is reported to compare goodness of fit of the different distributions. We find that the SGT-distribution has the highest maximum likelihood score of all. All other distributions have relatively similar likelihood scores, though slightly lower and are therefore not the optimal distributions. However, when considering AIC it is a tie between SGT and SGED. This provides some indication that we have a valid case to test the suitability of different SGED-GARCH VaR models as an alternative for the SGT-GARCH VaR models. While sacrificing some goodness of fit, the SGED distribution has the advantage of requiring one less parameter, which could possibly result in less errors due to misspecification and easier implementation. For the SGT parameters the standard deviation and skewness are both significant at the 1% level. For the SGED parameters, the standard deviation and the skewness are both significant

at respectively the 1% and 5% level. Both distributions are right-skewed. For both distributions the shape parameters are significant at the 1% level, though the q parameter was not estimated as it is by design set to infinity due to the SGED being a limiting case of SGT.[1]

Additionally, for every distribution fitted with MLE, plots are generated to compare the theoretical distribution with the observed returns. We see that except for the normal distribution which is quite off, the theoretical distributions are close to the actual data, except that they are too peaked. This problem is the least present for the SGT distribution.

1.2 Results of GARCH with constant higher moments

1.3 Results of GARCH with time-varying higher moments

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[1]To check whether the relative ranking of distributions still holds in different periods, we have calculated the maximum likelihood score and AIC for three smaller periods: The period up to the dotcom collapse (1987-2001), up to the GFC (2002-2009) and up to the present Covid-crash (2009-2021). There is no qualitative difference in relative ranking with these subsamples. Results are reported in the appendix.

Table 1.1: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.049	0.049	0.026	0.028	0.053	0.02	0.023	0.019
	(5.278)	(5.192)	(2.747)	(3.022)	(5.852)	(2.148)	(2.404)	(2.03)
α_1	-0.018	-0.018	-0.008	-0.008	-0.02	-0.005	-0.005	-0.006
	(-1.64)	(-1.635)	(-0.795)	(-0.768)	(-1.885)	(-0.485)	(-0.47)	(-0.611)
β_0	0.016	0.013	0.001	0.021	0	0.022	0.02	0.021
	(5.776)	(5.842)	(0.77)	(7.28)		(9.811)	(6.219)	(25.122)
β_1	0.094	0.101	-0.098	0.017	0.069	0.08	0.083	0.087
	(12.146)	(13.088)	(-15.524)	(3.021)	(15.02)	(6.286)	(9.717)	(30.759)
β_2	0.898	0.899	0.983	0.897	0.931	0.845	0.919	0.904
	(115.678)		(1557.507)	(115.012)		(86.237)	(107.22)	(365.502)
ξ	0.917	0.917	0.905	0.906	0.917	0.903	0.904	0.902
	(68.351)	(67.44)	(67.131)	(67.765)	(73.31)	(67.757)	(67.28)	(67.834)
κ								
η	6.339	5.997	6.897	6.819	7.036	6.974	6.928	6.944
	(15.442)	(16.925)	(14.582)	(14.635)	(18.325)	(14.536)	(14.568)	(14.514)
γ			0.144	0.143				
			(15.566)	(10.728)				
$shift$						0.904		0.214
						(10.355)		(9.66)
rot							0.723	0.552
							(12.112)	(9.638)
LLH	-13066.436	-13068.628	-12951.877	-12973.456	-13114.375	-12936.278	-12934.286	-12930.492

Notes

Notes

[1] This table shows the maximum likelyhood estimates of various AR1-ST-GARCH models. The daily returns used on the Euro Stoxx 50 Price index cover the period from 1987-01-01 to 2021-04-27(8954 observations).

[2] The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$

Where, in the 8 GARCH models estimated, γ is the asymmetry in volatility , ξ, κ and η are constant and t statistics are displayed in parenthesis. LLH is the maximized log likelihood value.

Table 1.2: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
norm	2.995	2.998	3.034	2.962	2.967	2.955	2.957	2.954
std	2.924	2.924	2.935	2.900	2.904	2.897	2.896	2.896
sstd	2.920	2.920	2.930	2.895	2.900	2.891	2.891	2.890
ged	2.930	2.930	2.944	2.907	2.911	2.903	7.705	7.702
sged	2.927	2.927	2.940	2.902	2.906	2.898	7.675	7.672

Notes

[1] This table shows the AIC value for the respective model