Thesis title

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For Yihui Xie

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Abstract

The greatest abstract all times

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List of Abbreviations

1-D, 2-D . . . One- or two-dimensional, referring in this thesis to spatial dimensions in an image.

Otter . . . . . One of the finest of water mammals.

Hedgehog . . . Quite a nice prickly friend.

Introduction

It has been shown that stock returns are not normally distributed and exhibit fat-tails and peakedness (. . .), these are called the stylized facts of returns. There are two viewpoints towards the predictability of stock prices. Firstly, some argue that stock prices are unpredictable or very difficult to predict by its past returns (i.e. have very little serial correlation) because they simply follow a Random Walk process (. . .). On the other hand, Lo & MacKinlay mention that “financial markets *are* predictable to some extent but far from being a symptom of inefficiency or irrationality, predictability is the oil that lubricates the gears of capitalism” . Furthermore, there is also no real robust evidence for the predictability of returns themselves, let alone be out-of-sample (Welch and Goyal 2008). This makes it difficult for corporations to manage market risk, i.e. the variability of stock prices.

Risk in general can be defined as the volatility of unexpected outcomes (Jorion 2007). The measure Value at Risk (VaR), developed in response to the financial disaster events of the early 1990s, has been very important in the financial world. Corporations have to manage their risks and thereby include a future risk measurement.

1

Literature review

1.1 Stylized facts of returns

1.2 Volatility

1.2.1 Rolling volatility

When volatility needs to be estimated on a specific trading day, the method used as a descriptive tool would be to use rolling standard deviations. Engle (2001) explains the calculation of rolling standard deviations, as the standard deviation over a fixed number of the most recent observations. For example, for the past month it would then be calculated as the equally weighted average of the squared deviations from the mean (i.e. residuals) from the last 22 observations (the average amount of trading or business days in a month). All these deviations are thus given an equal weight. Also, only a fixed number of past recent observations is examined. Engle regards this formulation as the first ARCH model.

1.2.2 ARCH model

Autoregressive Conditional Heteroscedasticity (ARCH) models, proposed by Engle (1982), was in the first case not used in financial markets but on inflation. Since then, it has been used as one of the workhorses of volatility modeling. To fully capture the logic behind GARCH models, the building blocks are examined in the first place. There are three building blocks of the ARCH model: returns. the innovation process and the variance process (or volatility function), written out in respectively equation (1.1), (1.2) and (1.3). Returns are written as a constant part (μ) and an unexpected part, called noise or the innovation process. The innovation process is the volatility (σ_t) times z_t, which is an independent identically distributed random variable with a mean of 0 (zero-mean) and a variance of 1 (unit-variance). The independent from iid, notes the fact that the z-values are not correlated, but completely independent of each other. The distribution is not yet assumed. The third component is the variance process or the expression for the volatility. The variance is given by a constant ω, plus the random part which depends on the return shock of the previous period squared (ε_{t-1}^2). In that sense when the uncertainty or surprise in the last period increases, then the variance becomes larger in the next period. The element σ_t^2 is thus known at time $t-1$, while it is a deterministic function of a random variable observed at time $t-1$ (i.e. ε_{t-1}^2).

$$R_t = \mu + \varepsilon_t \tag{1.1}$$

$$\varepsilon_t = \sigma_t * z_t,\ where\ z_t \overset{iid}{\sim} (0,1) \tag{1.2}$$

$$\sigma_t^2 = \omega + \alpha_1 * \varepsilon_{t-1}^2 \tag{1.3}$$

From these components we could look at the conditional moments (or expected returns and variance). We can plug in the component σ_t into the conditional mean innovation ε_t and use the conditional mean innovation to examine the conditional mean return. In equation (1.4) and (1.5) they are derived. Because the random variable z_t is distributed with a zero-mean, the conditional expectation is 0. As a consequence, the conditional mean return in equation (1.5) is equal to the unconditional mean in the most simple case. But variations are possible using ARMA (eg. AR(1)) processes.

$$\mathbb{E}_{t-1}(\varepsilon_t) = \mathbb{E}_{t-1}(\sqrt{\omega + \alpha_1 * \varepsilon_{t-1}^2} * z_t) = \sigma_t \mathbb{E}_{t-1}(z_t) = 0 \tag{1.4}$$

$$\mathbb{E}_{t-1}(R_t) = \mu + \mathbb{E}_{t-1}(\varepsilon_t) = \mu \tag{1.5}$$

For the conditional variance, knowing everything that happened until and including period $t-1$ the condiitional innovation variance is given by equation (1.6). This is equal to σ_t^2, while the variance of z_t is equal to 1. Then it is easy to derive the conditional variance of returns in equation (1.7), that is why equation (1.3) is called the variance equation.

$$var_{t-1}(\varepsilon_t) = \mathbb{E}_{t-1}(\varepsilon_t^2) = \mathbb{E}_{t-1}(\sigma_t^2 * z_t^2) = \sigma_t^2 \mathbb{E}_{t-1}(z_t^2) = \sigma_t^2 \tag{1.6}$$

$$var_{t-1}(R_t) = var_{t-1}(\varepsilon_t) = \sigma_t^2 \tag{1.7}$$

The unconditional variance is also interesting to derive, while this is the long-run variance, which will be derived in (1.11). After deriving this using the law of iterated expectations and assuming stationarity for the variance process, one would get (1.8) for the unconditional variance, equal to the constant c and divided by $1 - \alpha_1$, the slope of the variance equation.

$$\sigma^2 = \frac{\omega}{1 - \alpha_1} \tag{1.8}$$

This leads to the properties of ARCH models.

- Stationarity condition for variance: $\omega > 0$ and $0 \leq \alpha_1 < 1$.
- Zero-mean innovations
- Uncorrelated innovations

Thus a weak white noise process ε_t

The unconditional 4th moment, kurtosis $\mathbb{E}(\varepsilon_t^4)/\sigma^4$ of an ARCH model is given by equation (1.9). This term is larger than 3, which implicates that the fat-tails (a stylised fact of returns).

$$3\frac{1-\alpha_1^2}{1-3\alpha_1^2} \tag{1.9}$$

Another property of ARCH models is that it takes into account volatility clustering. Because we know that $var(\varepsilon_t) = \mathbb{E}(\varepsilon_t^2) = \sigma^2 = \omega/(1-\alpha_1)$, we can plug in ω for the conditional variance $var_t(\varepsilon_{t+1}) = \mathbb{E}(\varepsilon_{t+1}^2) = \sigma_{t+1}^2 = c + \alpha_1 * \varepsilon_t^2$. Thus it follows that equation (1.10) displays volatility clustering. If we examine the RHS, as $\alpha_1 > 0$ (condition for stationarity), when shock ε_t^2 is larger than what you expect it to be on average σ^2 the LHS will also be positive. Then the conditional variance will be larger than the unconditional variance. Briefly, large shocks will be followed by more large shocks.

$$\sigma_{t+1}^2 - \sigma^2 = \alpha_1 * (\varepsilon_t^2 - \sigma^2) \tag{1.10}$$

Excess kurtosis can be modeled, even when the conditional distribution is assumed to be normally distributed. The third moment, skewness, can be introduced using a skewed conditional distribution as we will see in part 1.2.4. The serial correlation for squared innovations is positive if fourth moment exists (equation (1.9), this is volatility clustering once again.

The estimation of ARCH model and in a next step GARCH models will be explained in the methodology. However how will then the variance be forecasted?

Well, the conditional variance for the k-periods ahead , denoted as period $T+k$, is given by equation (1.11). This can already be simplified, while we know that $\sigma^2_{T+1} = \omega + \alpha_1 * \varepsilon^2_T$ from equation (1.3).

$$\begin{aligned} \mathbb{E}_T(\varepsilon^2_{T+k}) &= \omega * (1 + \alpha_1 + ... + \alpha^{k-2}) + \alpha^{k-1} * \sigma^2_{T+1} \\ &= \omega * (1 + \alpha_1 + ... + \alpha^{k-1}) + \alpha^{k} * \sigma^2_{T} \end{aligned} \tag{1.11}$$

It can be shown that then the conditional variance in period $T+k$ is equal to equation (1.12). The LHS is the predicted conditional variance k-periods ahead above its unconditional variance, σ^2. The RHS is the difference current last-observed return residual ε^2_T above the unconditional average multiplied by α_1^k, a decreasing function of k (given that $0 \leq \alpha_1 < 1$). The further ahead predicting the variance, the closer α_1^k comes to zero, the closer to the unconditional variance, i.e. the long-run variance.

$$\mathbb{E}_T(\varepsilon^2_{T+k}) - \sigma^2 = \alpha_1^k * (\varepsilon^2_T - \sigma^2) \tag{1.12}$$

1.2.3 Univariate GARCH models

Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models come in because of the fact that rolling period standard deviations give an equal weight to the deviations, by such not taking into account volatility clustering, which can be identified as positive autocorrelation in the absolute returns. All these GARCH models are estimated using the package rugarch by Alexios Ghalanos (2020b). We use specifications similar to Ghalanos (2020a).

sGARCH model

The standard GARCH model (Bollerslev 1986) is written consistent with Alexios Ghalanos (2020a) as in equation (1.13) without external regressors.

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{1.13}$$

where σ_t^2 denotes the conditional variance, ω the intercept and ε_t^2 the residuals from the used mean process. The GARCH order is defined by (q, p) (ARCH, GARCH). As Ghalanos (2020a) describes: "one of the key features of the observed behavior of financial data which GARCH models capture is volatility clustering which may be quantified in the persistence parameter $\hat{P}$" specified as in equation (1.14).

$$\hat{P} = \sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j. \tag{1.14}$$

The unconditional variance of the standard GARCH model of Bollerslev is very similar to the ARCH model, but with the Garch parameters (β's) included as in equation (1.15).

$$\begin{aligned} \hat{\sigma}^2 &= \frac{\hat{\omega}}{1-\hat{P}} \\ &= \frac{\hat{\omega}}{1-\alpha-\beta} \end{aligned} \tag{1.15}$$

iGARCH model

Following Alexios Ghalanos (2020a), the integrated GARCH model (Bollerslev 1986) can also be estimated. This model assumes the persistence $\hat{P} = 1$. This is done by Ghalanos, by setting the sum of the ARCH and GARCH parameters to 1. Because of this unit-persistence, the unconditional variance cannot be calculated.

eGARCH model

The eGARCH model or exponential GARCH model (Nelson 1991) is defined as in equation (1.16),

$$\log_e(\sigma_t^2) = \omega + \sum_{j=1}^{q}(\alpha_j z_{t-j} + \gamma_j(|z_{t-j}| - E|z_{t-j}|)) + \sum_{j=1}^{p}\beta_j \log_e(\sigma_{t-j}^2) \tag{1.16}$$

where α_j captures the sign effect and γ_j the size effect.

gjrGARCH model

The gjrGARCH model (Glosten et al. 1993) models both positive as negative shocks on the conditional variance asymmetrically by using an indicator variable I, it is specified as in equation (1.17).

$$\sigma_t^2 = \omega + \sum_{j=1}^{q}(\alpha_j \varepsilon_{t-j}^2 + \gamma_j I_{t-j}\varepsilon_{t-j}^2) + \sum_{j=1}^{p}\beta_j \sigma_{t-j}^2 \tag{1.17}$$

where γ_j represents the *leverage* term. The indicator function I takes on value 1 for $\varepsilon \leq 0$, 0 otherwise. Because of the indicator function, persistence of the model now crucially depends on the asymmetry of the conditional distribution used according to Ghalanos (2020a).

naGARCH model (Engle & Ng)

tGARCH model (Zakoian)

avGARCH model (in our paper: TS-paper to Taylor and Schwert)

1.2.4 Conditional distributions

1.3 Value at Risk

1.4 Conditional Value at risk

2

Data and methodology

2.1 Data

Here comes text. . .

2.1.1 Descriptives

Table of summary statistics

Here comes a table and description of the stats

Correlation

Here comes a table and description of the correlations

Visualizations (eye-balling)

Figure 2.1: EuroStoxx 50 price and price log return evolution

Notes: This figure plots the price and price log return respectively for the EuroStoxx 50 index

As can be seen

2.1.2 Methodology

Here comes text…

As already mentioned in …, GARCH models sGARCH, eGARCH, iGARCH, gjrGARCH, nGARCH, tGARCH and tsGARCH will be estimated. Additionally the distributions will be examined as well, including the normal, student-t distribution, skewed student-t distribution, generalised error distribution, skewed generalised error distribution and the skewed generalised Theodossiou distribution.

They will be estimated using maximum likelyhood. As already mentioned, fortunately, Alexios Ghalanos (2020b) has made it easy for us to implement this methodology in R (version 3.6.1) with the package rugarch version 1.4-4 (R univariate garch), which gives us a bit more time to focus on the results and the interpretation.

Let's add an image:

```r
# knitr::include_graphics("figures/sample-content/captain.jpeg")
```

3
Empirical Findings

3.1 Main analysis title

Here comes our main part

4

Robustness Analysis

4.1 Specification checks

In order to check if the models are specified correctly, some specification checks have to be performed. The specification checks have to be done on the standardized residuals of the estimated GARCH model given by the following equation:

$$\hat{Z}_t = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_t} = \frac{R_t - \hat{\mu}}{\hat{\sigma}_t}$$

Residual heteroscedasticity Ljung-Box test on the squared or absolute standardized residuals.

4.1.1 Eye-balling econometrics

Autocorrelation function of the standardized residuals and autocorrelation function of the squared standardized residuals.

Then the density can be examined standardized residuals and compared with the normal distribution.

Also the QQ-plot can be examined.

4.1.2 GMM test

zero-mean unit-variance not skewed no excess kurtosis no serial correlation in the squares no serial correlation in the cubes no serial correlation in the squares

Conclusion

Appendices

A
Appendix

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