1
Empirical Findings

1.1 Density of the returns

1.1.1 MLE distribution parameters

In table ?? we can see the estimated parameters of the unconditional distribution functions. They are presented for the Skewed Generalized T-distribution (SGT) and limiting cases thereof previously discussed. Additionally, maximum likelihood score and the Aikake Information Criterion (AIC) is reported to compare goodness of fit of the different distributions. We find that the SGT-distribution has the highest maximum likelihood score of all. All other distributions have relatively similar likelihood scores, though slightly lower and are therefore not the optimal distributions. However, when considering AIC it is a tie between SGT and SGED. This provides some indication that we have a valid case to test the suitability of different SGED-GARCH VaR models as an alternative for the SGT-GARCH VaR models. While sacrificing some goodness of fit, the SGED distribution has the advantage of requiring one less parameter, which could possibly result in less errors due to misspecification and easier implementation. For the SGT parameters the standard deviation and skewness are both significant at the 1% level. For the SGED parameters, the standard deviation and the skewness are both significant at respectively the 1% and 5% level. Both distributions are right-skewed. For both distributions the shape parameters are significant at the 1% level, though the q parameter was not estimated as it is by design set to infinity due to the SGED being a limiting case of SGT.[1]

Additionally, for every distribution fitted with MLE, plots are generated to compare the theoretical distribution with the observed returns. We see that except for the normal distribution which is quite off, the theoretical distributions are close to the actual data, except that they are too peaked. This problem is the least present for the SGT distribution.

1.2 Constant higher moments

1.1 presents the maximum likelihood estimates for 8 symmetric and asymmetric GARCH models based on the ST distribution with constant skewness and kurtosis parameters (t values are presented in parenthesis). The parameters in the conditional mean equations (α_0) are all statistically significant with t values from 6 to 11. The AR(1) coefficient, α_1, has parameters going from 2 to 2 with t values ranging from 4 to 5 not suggesting a high significance and indicating slight negative autocorrelation. The GARCH parameters in the conditional variance equations (β_0) are generally statistically significant with t values ranging from 1 to 11. The results of β_1 and β_2 show the presence of significant time-variation in the conditional volatility of the Euro Stoxx 50 Price Index daily returns, in fact, the sum of β_1 and β_2 for the GARCH parameters is close to one (from 20 to 34), suggesting the presence of persistence in the volatility of the returns. The parameter ξ is highly significant for all the 8 models tested with values ranging from 12 to 18 confirming the presence of Skewness in the returns. The shape parameter η, which, in our case, measures the number of degrees of freedom, determining the tail behavior, is significant for all the

[1] To check whether the relative ranking of distributions still holds in different periods, we have calculated the maximum likelihood score and AIC for three smaller periods: The period up to the dotcom collapse (1987-2001), up to the GFC (2002-2009) and up to the present Covid-crash (2009-2021). There is no qualitative difference in relative ranking with these subsamples. Results are reported in the appendix.

Table 1.1: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.049	0.049	0.026	0.028	0.053	0.02	0.023	0.019
	(5.278)	(5.192)	(2.747)	(3.022)	(5.852)	(2.148)	(2.404)	(2.03)
α_1	-0.018	-0.018	-0.008	-0.008	-0.02	-0.005	-0.005	-0.006
	(-1.64)	(-1.635)	(-0.795)	(-0.768)	(-1.885)	(-0.485)	(-0.47)	(-0.611)
β_0	0.016	0.013	0.001	0.021	0	0.022	0.02	0.021
	(5.776)	(5.842)	(0.77)	(7.28)		(9.811)	(6.219)	(25.122)
β_1	0.094	0.101	-0.098	0.017	0.069	0.08	0.083	0.087
	(12.146)	(13.088)	(-15.524)	(3.021)	(15.02)	(6.286)	(9.717)	(30.759)
β_2	0.898	0.899	0.983	0.897	0.931	0.845	0.919	0.904
	(115.678)		(1557.507)	(115.012)		(86.237)	(107.22)	(365.502)
ξ	0.917	0.917	0.905	0.906	0.917	0.903	0.904	0.902
	(68.351)	(67.44)	(67.131)	(67.765)	(73.31)	(67.757)	(67.28)	(67.834)
η	6.339	5.997	6.897	6.819	7.036	6.974	6.928	6.944
	(15.442)	(16.925)	(14.582)	(14.635)	(18.325)	(14.536)	(14.568)	(14.514)
γ			0.144	0.143				
			(15.566)	(10.728)				
$shift$						0.904		0.214
						(10.355)		(9.66)
rot							0.723	0.552
							(12.112)	(9.638)
LLH	-13066.436	-13068.628	-12951.877	-12973.456	-13114.375	-12936.278	-12934.286	-12930.492

Notes
This table shows the maximum likelihood estimates of various ST-GARCH models. The daily returns used on the Euro Stoxx 50 Price index cover the periodfrom 01 January, 1987 to 27 April, 2021 (8954 observations).
The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$
Where, in the 8 GARCH models estimated, γ is the asymmetry in volatility , ξ, κ and η are constant and t statistics are displayed in parenthesis.
LLH is the maximized log likelihood value.

models and ranges between 14 and 20. The parameter γ, which is present only for eGARCH and gjrGARCH is significant and with values around 4.5. The absolute value function in fGARCH models (NAGARCH, TGARCH and AVGARCH) is subject to the $shift$ and the rot parameters whose values are always positive and statistically significant. According to the log likelihood values (LLH), displayed in 1.1, the model with the highest value is eGARCH while, excluding the non-standard GARCH models from the analysis, the model that performs best is eGARCH.

As you can see in table 1.2 the AIC for the skewed student's t-distribution (ST) is the best from all the models. As also shown in appendix part **??**. The best in all distributions of the GARCH models seems to be the NAGARCH model, but we do not want to overfit our models because of an in-sample estimation. That is why a parsimonious model like the EGARCH (which has the highest maximum likelihood for the standard GARCH models that are considered), but also the model AVGARCH will be examined using the ST distribution while it has the second-best (lowest) AIC.

Table 1.2: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
norm	2.995	2.998	3.034	2.962	2.967	2.955	2.957	2.954
std	2.924	2.924	2.935	2.900	2.904	2.897	2.896	2.896
sstd	2.920	2.920	2.930	2.895	2.900	2.891	2.891	2.890
ged	2.930	2.930	2.944	2.907	2.911	2.903	7.705	7.702
sged	2.927	2.927	2.940	2.902	2.906	2.898	7.675	7.672

Notes

[1] This table shows the AIC value for the respective model

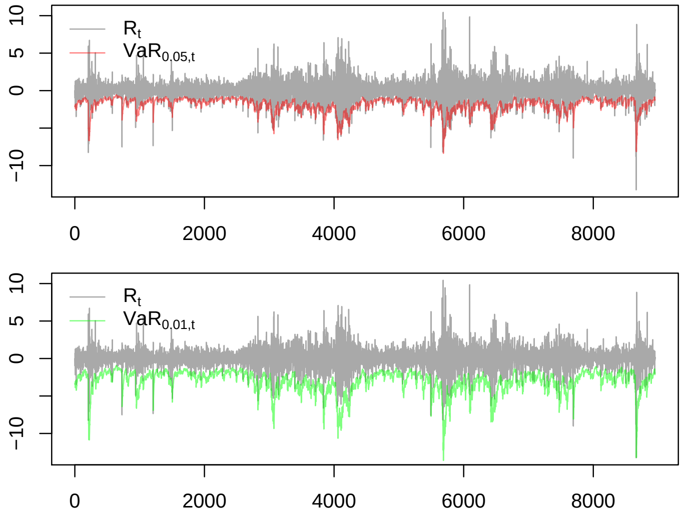

Figure 1.1: Value-at-Risk (in-sample) for the EGARCH-ST model

1.2.1 Value-at-risk

As already mentioned 2 candidate models seem to be very appropriate. This includes the EGARCH and the NAGARCH So to check if they perform well out-of-sample we conduct a backtest by using a rolling forecasting technique. A simple graph is shown in figure 1.1 for the EGARCH-ST model.

Let us examine this further using a rolling window approach whilst forecasting 1-day ahead results with re-estimating note for prof. Annaert: choices: n.start = 1500 days before the end of the series, refit.every = 100, solver = hybrid using a cluster = 10 to run on 10 cores to speed up the process of estimation of the roll object (took 5-10 minutes per backtest with some solvers. . .) the parameters every 100 days. Figure 1.2 shows that choosing an appropriate forecast period is important, while it includes the decline in 2016 with among which Brexit and the recent Covid-crisis.

Figure 1.2: Selected period to start forecast from

As you can see in figure @ref(fig.)

1.2.2 Expected shortfall

1.3 Time-varying higher moments