1
Conclusion

This paper tested whether including higher-order moments in VaR and cVaR would make for more accurate risk measurement. The starting point for this was that there is a strong body of literature that suggests market returns are leptokurtotic and right-skewed. We chose to focus on daily data and forecast one period ahead, as this is the most relevant for short-term trading desks and the requirements of regulators.

Inspired by Bali, we selected 5 different distributions and 8 different GARCH models, for a total of 40 combinations, to estimate VaR and cVaR. We used daily data from the Eurostoxx 50 log return index between 2001 and 2021, a period which includes both major market crises from the 21th century: the Global Financial Crisis and the Covid-19 pandemic.

Our studied distributions are all limiting cases of the SGT-distribution studied by Bali, namely the SGED, GED, ST, T and Normal distribution. To provide some intuition on which distributions most closely match the observed log returns, we performed maximum likelihood estimation and compared the Aikaike Information Criterions to penalize complexity. We found that the SGED distribution is as good as the SGT (due to having one less parameter to estimate, even though it fits the data slightly worse). The normal distribution fitted the observed returns the worst, with both the maximum likelihood score and the AIC underperforming the other distributions. This affirmed previous literature that the normal distribution is inadequate to model returns.

As a second step we estimated the 8 GARCH models (SGARCH, IGARCH, EGARCH, EWMA, GJRGARCH, NAGARCH, TGARCH and AVGARCH) with the 5 underlying distributions in-sample and again reported the AIC. We found that the best model to forecast 1 day returns was the AVGARCH-ST. The ST distribution is generally the most performant over all GARCH models, with SGED being slightly better only for the SGARCH and IGARCH.

In a third step, we reported on the parameters of the different GARCH models for these two distributions, and find that the skewness and shape parameters are highly significant, suggesting that indeed higher order moments are relevant for estimating VaR and cVaR.

To avoid look-ahead bias as introduced by in-sample estimation, we applied a rolling window technique to 25 combinations (selecting the 5 best GARCH models on the basis of AIC). Here we refitted every 22 trading days and forecast 1 trading day. Subsequently we computed the VaR and cVaR and compared results for 4 performance tests: the ESTest (**Mcneil?**), Unconditional Coverage test, Conditional Coverage test and Dynamic Quantile test. We found that the smallest difference between predicted and actual exceedances over the $VaR_{0.01}$ was for the NAGARCH-ST. For most other models the SGED distribution has the smallest difference. For the $cVaR_{0.01}$ and associated ESTest, the results are that the SGED distribution gives the smallest difference between predicted and actual exceedances for the TGARCH and the AVGARCH and the same as the ST distribution for EGARCH and GJRGARCH. For both distributions, the uc, cc and dq are insignificant. The other distributions have significant values for the cc and uc test and for the most part for the dq test and are thus not as suitable for (c)VaR estimation.

To our knowledge, there has been little research on including time-varying moments beyond the variance in (c)VaR estimation. We allow for skewness and shape parameters to vary using an ACD model as proposed by (**Ghalanos?**). We compared the results of the SGARCH model with the ACD model and concluded that allowing higher moments to vary over time gives better goodness-of-fit in terms of maximum log-likelihood and AIC and more parameters (especially the shape parameter) significant. However it did not improve the (c)VaR estimation for any of the 4 performance tests. Future research might study ACD models for different GARCH models, than the standard one, better (c)VaR tests.

We performed 4 different robustness checks on the in-sample GARCH estimations to test if the residuals are normally distributed. First, we performed the Ljung-Box Test on the residuals and reject the null-hypothesis that there is no serial correlation of variance. Second, we applied the ARCH LM test on absolute, and squared residuals of our GARCH models and came to the same conclusion as with the Ljung-Box test (these are similar tests, except that the former tests the ARMA process and the latter the ARCH process). Third, we performed the GMM test to see if the residuals have moments equal to those of of the normal distribution. We find that this hypothesis can be rejected with high significance. Last, we performed the Jarque-Bera test to test if the kurtosis and skewness matches the normal distribution. We reject this hypothesis as well. There is thus ample evidence that the residuals are non-normal.

To study the external validity of our research, we also applied our methodology to 3 further datasets: the Eurostoxx 50 Price index (which goes back to 1987), CAC40 return index, BEL20 return index and FTSE100 return index, to estimate if the combination of models and distribution we found to be the most efficient models to calculate (c)VaR over a wide range of European markets stays the same. We found that XXX.

Different scenarios of ARMA order variation suggest that this does not affect results as much as the distribution or GARCH model choice.

To conclude by answering the research question "Do higher moments increase accuracy in the estimation of VaR and cVaR?" we can answer in the positive, though caution that more model complexity might not always mean better VaR and cVaR estimation.