1
Empirical Findings

1.1 Density of the returns

1.1.1 MLE distribution parameters

In table ?? we can see the estimated parameters of the unconditional distribution functions. They are presented for the Skewed Generalized T-distribution (SGT) and limiting cases thereof previously discussed. Additionally, maximum likelihood score and the Aikake Information Criterion (AIC) is reported to compare goodness of fit of the different distributions. We find that the SGT-distribution has the highest maximum likelihood score of all. All other distributions have relatively similar likelihood scores, though slightly lower and are therefore not the optimal distributions. However, when considering AIC it is a tie between SGT and SGED. This provides some indication that we have a valid case to test the suitability of different SGED-GARCH VaR models as an alternative for the SGT-GARCH VaR models. While sacrificing some goodness of fit, the SGED distribution has the advantage of requiring one less parameter, which could possibly result in less errors due to misspecification and easier implementation. For the SGT parameters the standard deviation and skewness are both significant at the 1% level. For the SGED parameters, the standard deviation and the skewness are both significant at respectively the 1% and 5% level. Both distributions are right-skewed. For both distributions the shape parameters are significant at the 1% level, though the q parameter was not estimated as it is by design set to infinity due to the SGED being a limiting case of SGT.[1]

Additionally, for every distribution fitted with MLE, plots are generated to compare the theoretical distribution with the observed returns. We see that except for the normal distribution which is quite off, the theoretical distributions are close to the actual data, except that they are too peaked. This problem is the least present for the SGT distribution.

1.2 Constant higher moments

[1]To check whether the relative ranking of distributions still holds in different periods, we have calculated the maximum likelihood score and AIC for three smaller periods: The period up to the dotcom collapse (1987-2001), up to the GFC (2002-2009) and up to the present Covid-crash (2009-2021). There is no qualitative difference in relative ranking with these subsamples. Results are reported in the appendix.

Table 1.1: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.049	0.049	0.026	0.028	0.053	0.02	0.023	0.019
	(5.278)	(5.192)	(2.747)	(3.022)	(5.852)	(2.148)	(2.404)	(2.03)
α_1	-0.018	-0.018	-0.008	-0.008	-0.02	-0.005	-0.005	-0.006
	(-1.64)	(-1.635)	(-0.795)	(-0.768)	(-1.885)	(-0.485)	(-0.47)	(-0.611)
β_0	0.016	0.013	0.001	0.021	0	0.022	0.02	0.021
	(5.776)	(5.842)	(0.77)	(7.28)		(9.811)	(6.219)	(25.122)
β_1	0.094	0.101	-0.098	0.017	0.069	0.08	0.083	0.087
	(12.146)	(13.088)	(-15.524)	(3.021)	(15.02)	(6.286)	(9.717)	(30.759)
β_2	0.898	0.899	0.983	0.897	0.931	0.845	0.919	0.904
	(115.678)		(1557.507)	(115.012)		(86.237)	(107.22)	(365.502)
ξ	0.917	0.917	0.905	0.906	0.917	0.903	0.904	0.902
	(68.351)	(67.44)	(67.131)	(67.765)	(73.31)	(67.757)	(67.28)	(67.834)
κ								
η	6.339	5.997	6.897	6.819	7.036	6.974	6.928	6.944
	(15.442)	(16.925)	(14.582)	(14.635)	(18.325)	(14.536)	(14.568)	(14.514)
γ			0.144	0.143				
			(15.566)	(10.728)				
$shift$						0.904		0.214
						(10.355)		(9.66)
rot							0.723	0.552
							(12.112)	(9.638)
LLH	-13066.436	-13068.628	-12951.877	-12973.456	-13114.375	-12936.278	-12934.286	-12930.492

Notes

[1] This table shows the maximum likelihood estimates of various ST-GARCH models. The daily returns used on the Euro Stoxx 50 Price index cover the periodfrom 01 January, 1987 to 27 April, 2021 (8954 observations).
[2] The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$
Where, in the 8 GARCH models estimated, γ is the asymmetry in volatility , ξ, κ and η are constant and t statistics are displayed in parenthesis.
LLH is the maximized log likelihood value.

As you can see in table 1.2 the AIC for the skewed student's t-distribution (ST) is the best from all the models. As also shown in appendix part **??**. The best in all distributions of the GARCH models seems to be the NAGARCH model, but we do not want to overfit our models because of an in-sample estimation. That is why a parsimonous model like the EGARCH (which has the highest maximum likelihood for the standard GARCH models that are considered), but also the model AVGARCH will be examined using the ST distribution while it has the second-best (lowest) AIC.

Table 1.2: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
norm	2.995	2.998	3.034	2.962	2.967	2.955	2.957	2.954
std	2.924	2.924	2.935	2.900	2.904	2.897	2.896	2.896
sstd	2.920	2.920	2.930	2.895	2.900	2.891	2.891	2.890
ged	2.930	2.930	2.944	2.907	2.911	2.903	7.705	7.702
sged	2.927	2.927	2.940	2.902	2.906	2.898	7.675	7.672

Notes
[1] This table shows the AIC value for the respective model

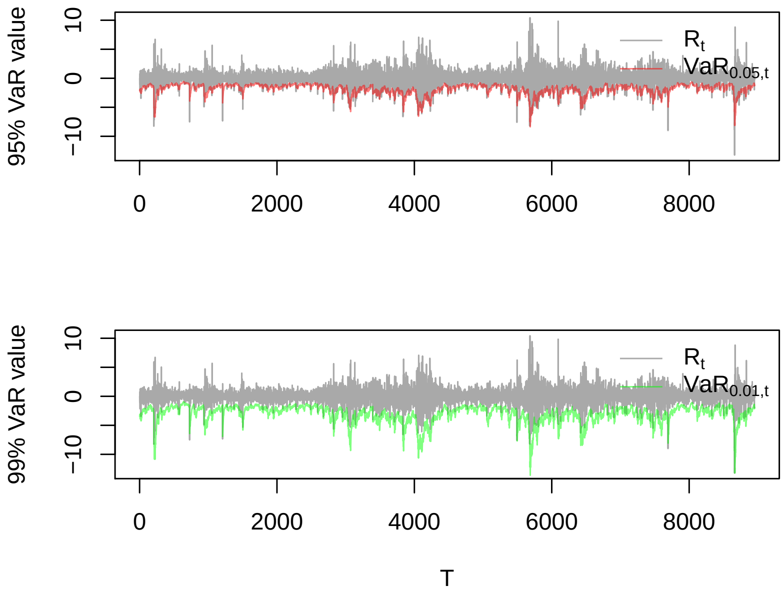
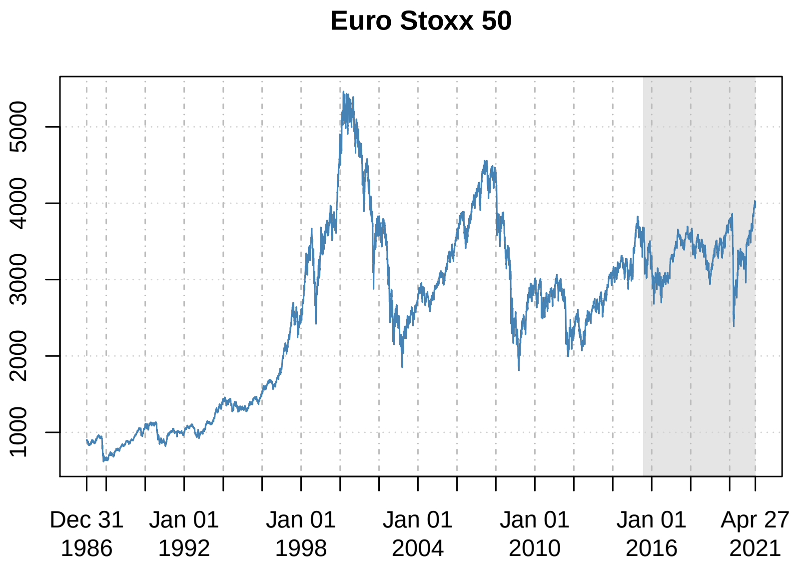


Figure 1.1: Value-at-Risk (in-sample) for the EGARCH-ST model

1.2.1 Value-at-risk

As already mentioned 3 candidate models seem to be candidates to check if they perform well using a forecasting technique and backtest. This includes the EGARCH, the NAGARCH and AVGARCH. A simple graph is shown in figure 1.1 for the EGARCH-ST model. It seems that the VaR model for $\alpha = 0.05$ underestimates the downside events, while the VaR model for $\alpha = 0.01$ overestimates a lot of the downside events.

Let us examine this further using a moving window approach whilst forecasting 1-day ahead results with a window size of 1500. Figure **??** shows that choosing an appropriate forecast period is important, while it includes the decline in 2016 with among which Brexit and the recent COVID-crisis.



Euro Stoxx 50
1000
2000
3000
4000
5000
Dec 31
1986
Jan 01
1992
Jan 01
1998
Jan 01
2004
Jan 01
2010
Jan 01
2016
Apr 27
2021

As you can see in figure @ref(fig.)

1.3 Time-varying higher moments