

# 1

## Data and methodology

## Data We worked with daily returns on the EURO STOXX 50 Index denoted in EUR. It is the leading blue-chip index of the Eurozone and covers 50 stocks.

## 1.0.1 Descriptives

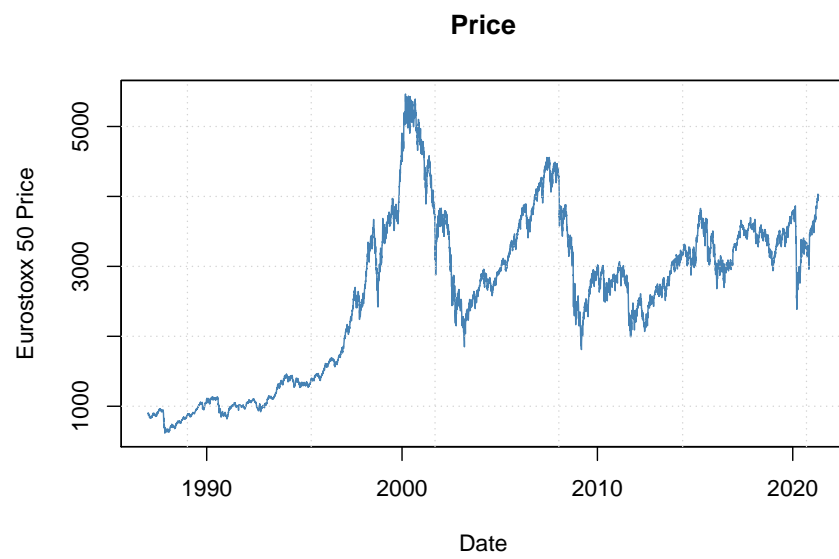
### Table of summary statistics

Here comes a table and description of the stats

**Table 1.1:** Summary statistics of the returns

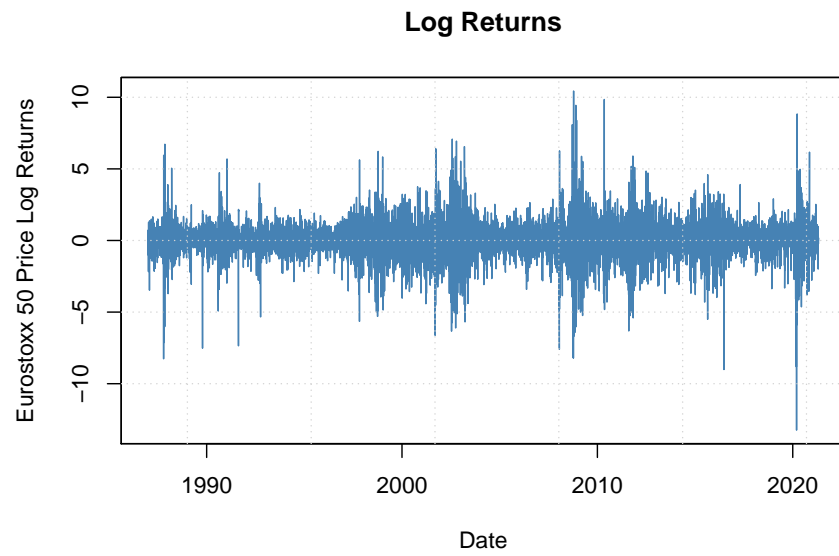
Statistics	Eurostoxx.50	Standardized.Residuals
Arithmetic Mean	-13.2404	-11.7732
Stddev	0.0357	-0.0193
Skewness	0.0167	-0.0409
NA	10.4376	5.7126
NA	1.307	0.9992
Skewness	-0.31 (0***)	-0.6327 (0***)
Excess Kurtosis	7.2083 (0***)	5.134 (0***)
Jarque-Bera	19528.6196***	10431.0514***

*Note:* This table shows the descriptive statistics of the returns of over the period 1987-01-01 to 2021-04-27. Including minimum, median, arithmetic average, maximum, standard deviation, skewness and excess kurtosis.

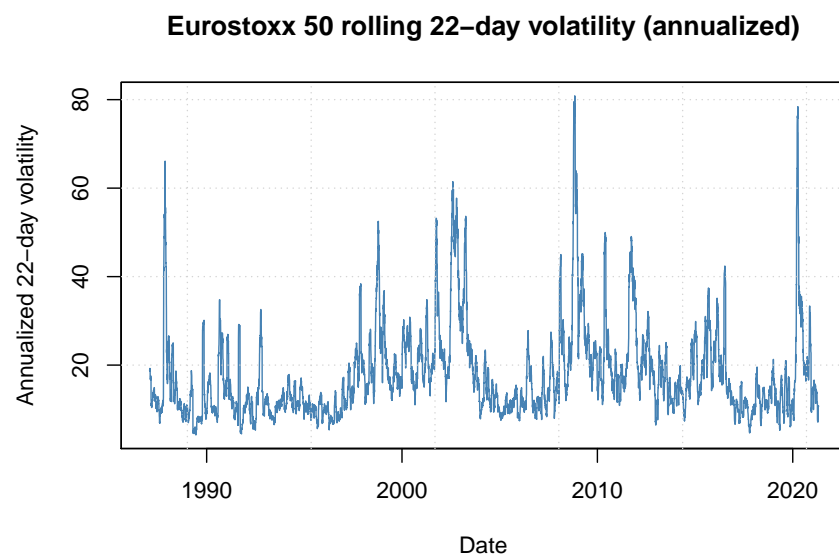


**Figure 1.1:** Eurostoxx 50 Price Index prices

Descriptive figures

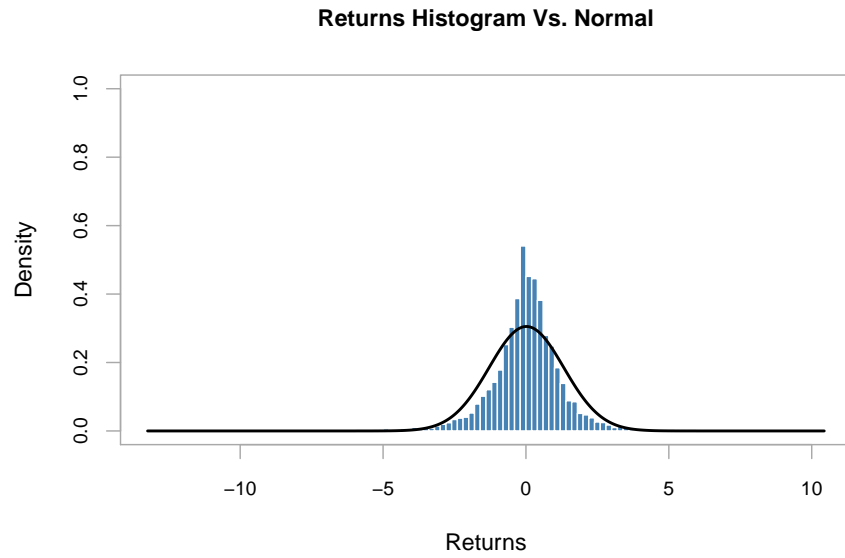


**Figure 1.2:** Eurostoxx 50 Price Index log returns

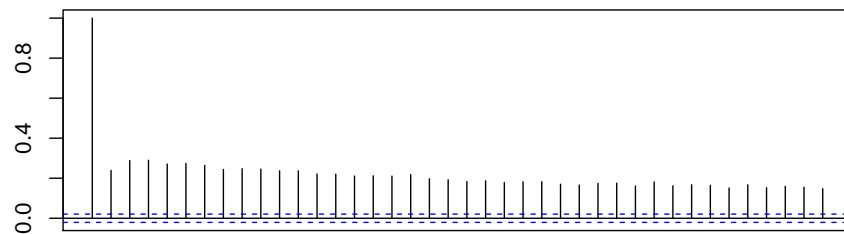
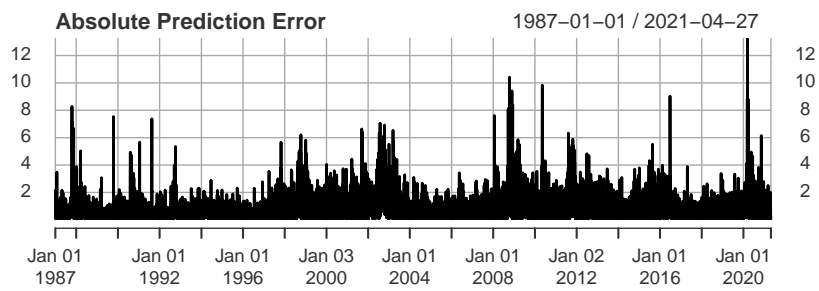


**Figure 1.3:** Eurostoxx 50 rolling volatility (22 days, calculated over 252 days)

As can be seen



**Figure 1.4:** Density vs. Normal Eurostoxx 50 log returns)



**Figure 1.5:** Absolute prediction errors

## 1.1 Methodology

### 1.1.1 Garch models

As already mentioned in . . . , GARCH models GARCH, EGARCH, IGARCH, GJR-GARCH, NGARCH, TGARCH and NAGARCH (or TSGARCH) will be estimated. Additionally the distributions will be examined as well, including the normal, student-t distribution, skewed student-t distribution, generalized error distribution, skewed generalized error distribution and the skewed generalized t distribution.

They will be estimated using maximum likelihood. As already mentioned, fortunately, Alexios Ghalanos [1] has made it easy for us to implement this methodology in the R language (version 3.6.1) with the package “rugarch” version 1.4-4 (R univariate garch), which gives us a bit more time to focus on the results and the interpretation. Additionally

Maximum likelihood estimation is a method to find the distribution parameters that best fit the observed data, through maximization of the likelihood function, or the computationally more efficient log-likelihood function (by taking the natural logarithm). It is assumed that the return data is i.i.d. and that there is some underlying parametrized density function  $f$  with one or more parameters that generate the data, defined as a vector  $\theta$  ((1.2)). These functions are based on the joint probability distribution of the observed data (equation (1.4)). Subsequently, the (log)likelihood function is maximized using an optimization algorithm (equation (1.6)).

$$y_1, y_2, \dots, y_N \sim i.i.d \quad (1.1)$$

$$y_i \sim f(y|\theta) \quad (1.2)$$

$$L(\theta) = \prod_{i=1}^N f(y_i|\theta) \quad (1.3)$$

$$\log(L(\theta)) = \sum_{i=1}^N \log f(y_i|\theta) \quad (1.4)$$

$$\theta^* = \arg \max_{\theta} [L] \quad (1.5)$$

$$\theta^* = \arg \max_{\theta} [\log(L)] \quad (1.6)$$

### 1.1.2 ACD models

Following Ghalanos [2], arguments of ACD models are specified as in Hansen [3]. The density function  $f(y|\alpha)$  has parameters  $\alpha_t = (\mu_t, \sigma_t, \nu_t)$ , with equation (1.7), the conditional mean equation. Equation (1.8) as the conditional variance. And  $\nu_t = \nu(\theta, x_t)$  the remaining parameters of the distribution like the skewness and kurtosis (shape) parameters.

$$\mu_t = \mu(\theta, x_t) = E(y_t | x_t) \quad (1.7)$$

$$\sigma_t^2 = \sigma^2(\theta, x_t) = E((y_t - \mu_t)^2 | x_t) \quad (1.8)$$

To further explain the difference between GARCH and ACD. The scaled innovations are given by equation (1.9). The conditional density is given by equation (1.10) and related to the density function  $f(y|\alpha)$  as in equation (1.1.2).

$$z_t(\theta) = \frac{y_t - \mu(\theta, x_t)}{\sigma(\theta, x_t)} \quad (1.9)$$

$$g(z | \eta_t) = \frac{d}{dz} P(z_t < z | \eta_t) \quad (1.10)$$

$$f(y_t | \mu_t, \sigma_t^2, \eta_t) = \frac{1}{\sigma_t} g(z_t | \eta_t) \quad (1.11)$$

```

##          mean          sd
## 0.01668214 1.30689172
##          mean          sd
## 0.01381119 0.00976596
## [1] -15101.73
##          df          ncp
## 4.31096001 0.03168827
##          df          ncp
## 0.14857777 0.01100453
## [1] -14149.5
##          mean          sd          nu
## 0.03160393 1.27550013 0.91274249
##          mean          sd          nu
## 0.008555584 0.015772159 0.016622605
## [1] -14009.53
##          mean          sd          nu          xi
## 0.01946361 1.27515748 0.91513166 0.98174821
##          mean          sd          nu          xi
## 0.013176090 0.015786515 0.016652983 0.009638209
## [1] -14008.63
## Skewed Generalized T MLE Fit
## Best Result with BFGS Maximization
## Convergence Code 0: Successful Convergence
## Iterations: NA, Log-Likelihood: -13973.01
##
##          Est. Std. Err.          z  P>|z|
## mu          0.0204      0.0131  1.5574 0.1194
## sigma      1.3214      0.0261 50.5971 0.0000 ***
## lambda -0.0397      0.0126 -3.1583 0.0016 **

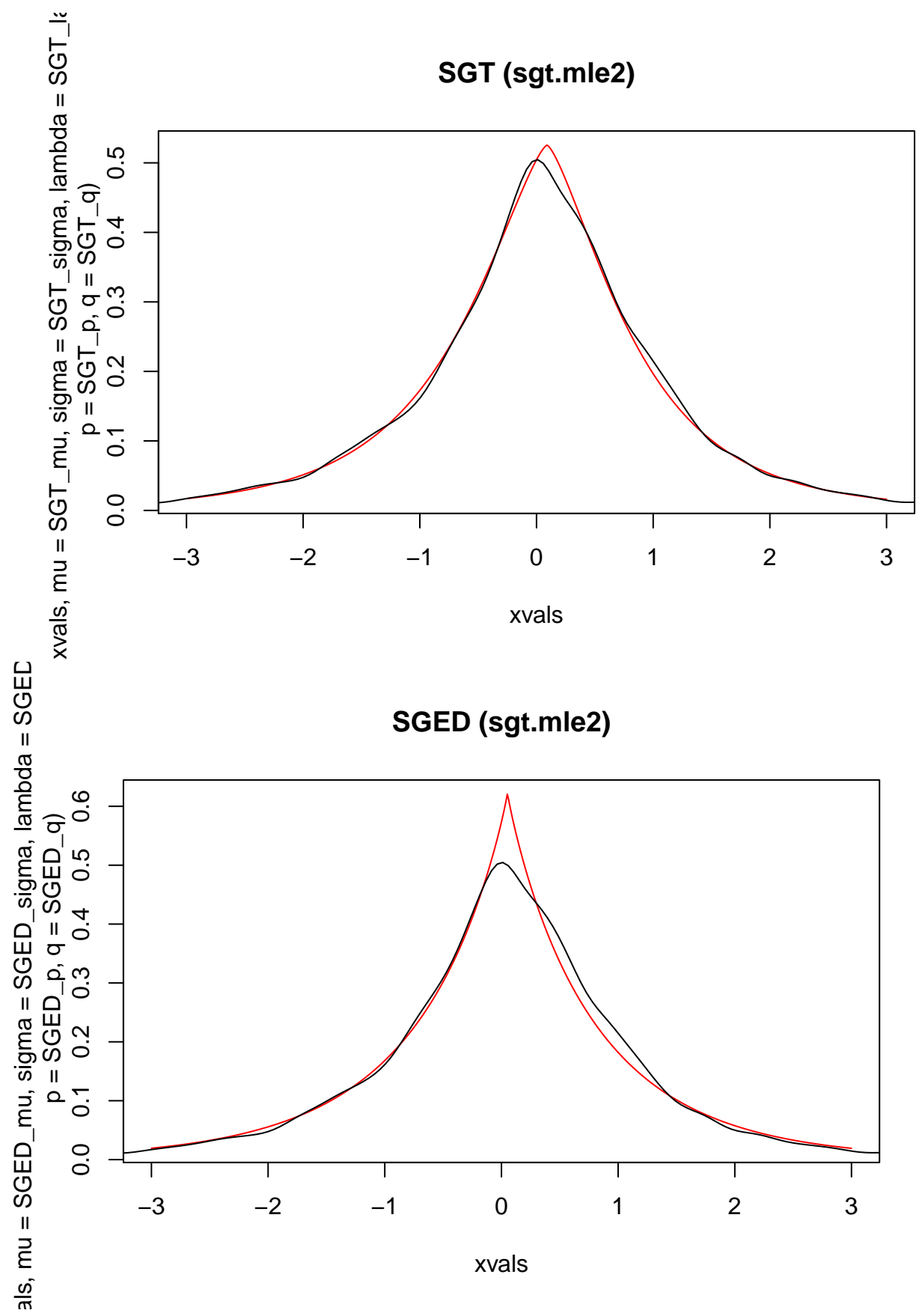
```

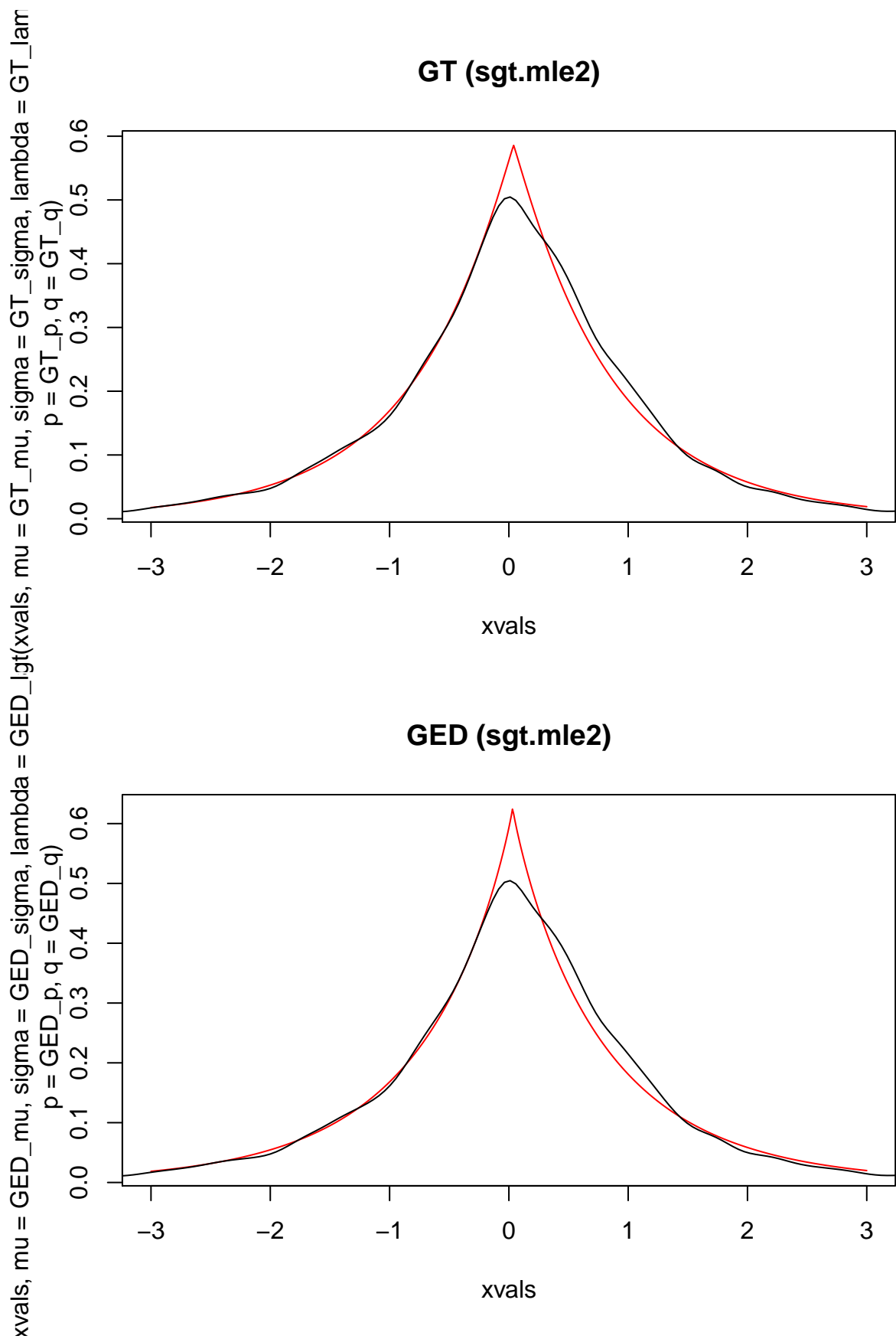


```

## p      1.3818      0.0708 19.5077 0.0000 ***
## q      3.3093      0.5333  6.2058 0.0000 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Fitting of the distribution ' sgt ' by maximum likelihood
## Parameters :
##           estimate Std. Error
## mu      0.01974156 0.01263035
## sigma    1.27919321 0.01674109
## lambda  -0.03189521 0.01159236
## p        1.09667765          NaN
## q        9.37999498          NaN
## Loglikelihood: -13984.5   AIC:  27978.99   BIC:  28014.49
## Correlation matrix:
##           mu      sigma      lambda      p      q
## mu      1.00000000 -0.04998713 0.70347249 NaN NaN
## sigma  -0.04998713  1.00000000 0.04648083 NaN NaN
## lambda  0.70347249  0.04648083 1.00000000 NaN NaN
## p        NaN          NaN          NaN    1 NaN
## q        NaN          NaN          NaN NaN    1

```

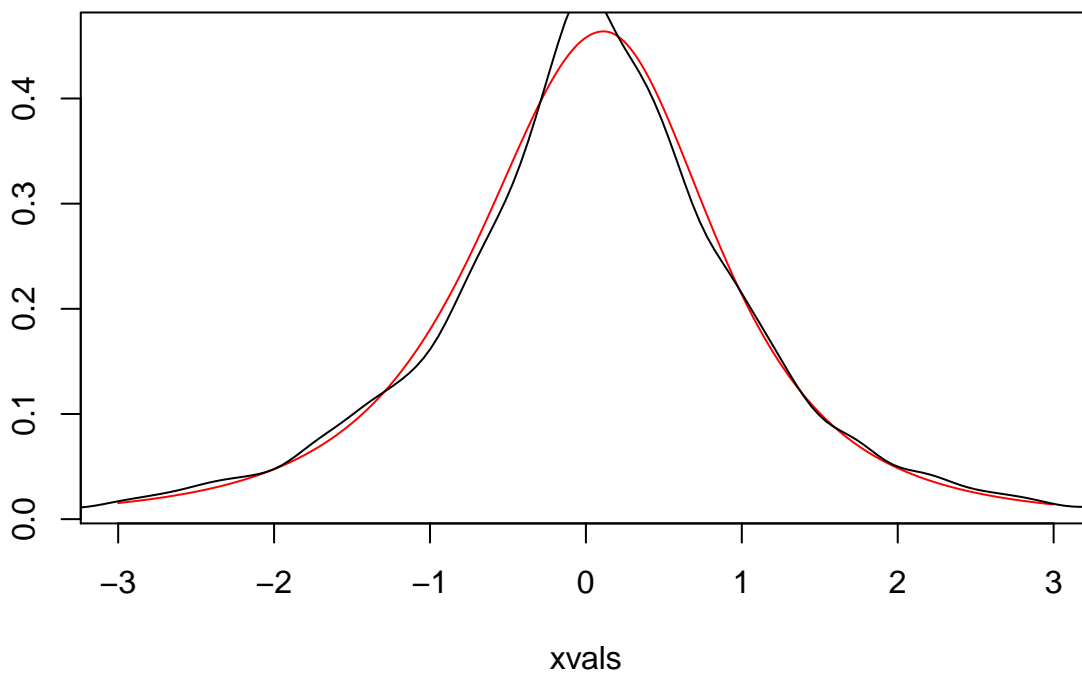




```
## [1] 28002.7
```

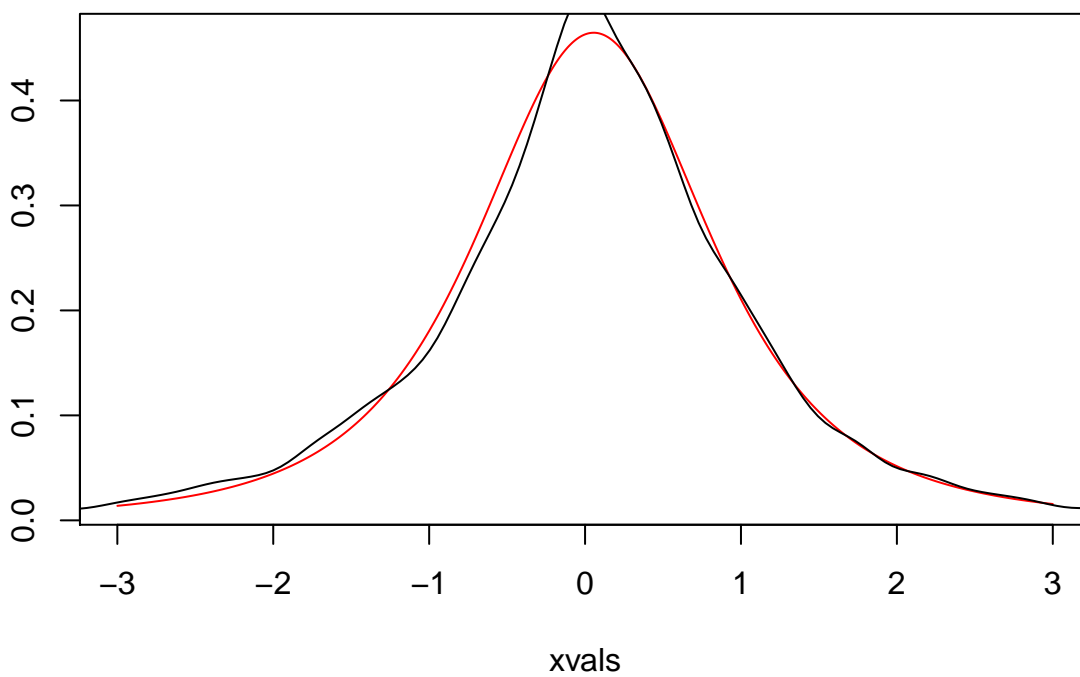
```
std(xvals, mean = ST_mean, sd = ST_sd, nu = ST_nu, xi = S
```

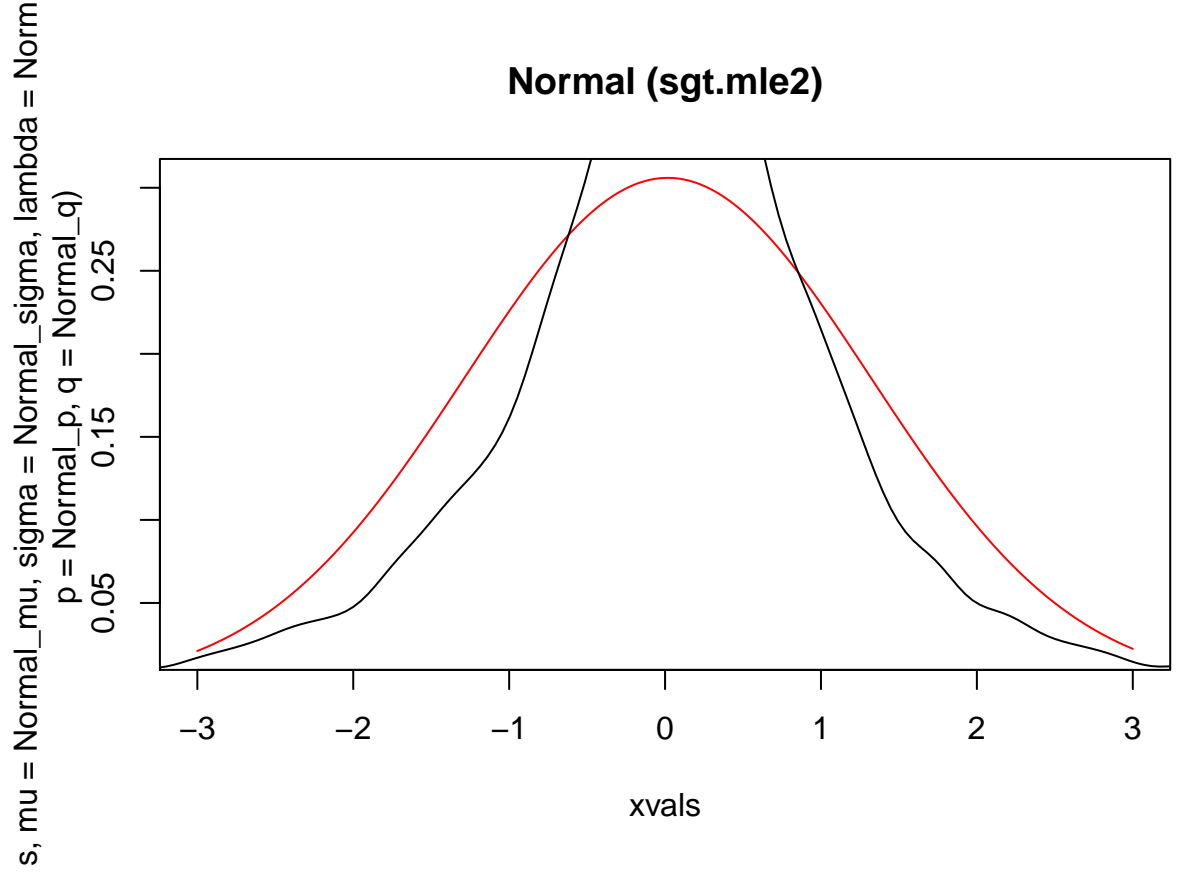
**ST (fitdist)**



```
dstd(xvals, mean = T_mean, sd = T_sd, nu = T_nu)
```

**T (fitdist)**





### 1.1.3 Control Tests

#### Unconditional coverage test of Kupiec [4]

A number of tests are computed to see if the value-at-risk estimations capture the actual losses well. A first one is the unconditional coverage test by Kupiec [4]. The unconditional coverage or proportion of failures method tests if the actual value-at-risk exceedances are consistent with the expected exceedances (a chosen percentile, e.g. 1% percentile) of the VaR model. Following Kupiec [4] and Ghalanos [5], the number of exceedence follow a binomial distribution (with thus probability equal to the significance level or expected proportion) under the null hypothesis of a correct VaR model. The test is conducted as a likelihood ratio test with statistic like in equation (1.12), with  $p$  the probability of an exceedence for a confidence level,  $N$  the sample size and  $X$  the number of exceedence. The null hypothesis states that the test statistic  $LR^{uc}$  is  $\chi^2$ -distributed with one degree of freedom or that the probability of failure  $\hat{p}$  is equal to the chosen percentile  $\alpha$ .

$$LR^{uc} = -2 \ln \left( \frac{(1-p)^{N-X} p^X}{\left(1 - \frac{X}{N}\right)^{N-X} \left(\frac{X}{N}\right)^X} \right) \quad (1.12)$$

### **Conditional coverage test of Christoffersen, Hahn, and Inoue [6]**

Christoffersen, Hahn, and Inoue [6] proposed the conditional coverage test. It is tests for unconditional coverage and serial independence. The serial independence is important while the  $LR^{uc}$  can give a false picture while at any point in time it classifies inaccurate VaR estimates as “acceptably accurate” [7]. For a certain VaR estimate an indicator variable,  $I_t(\alpha)$ , is computed as equation (1.13).

$$I_t(\alpha) = \begin{cases} 1 & \text{if exceedence occurs} \\ 0 & \text{if no exceedence occurs} \end{cases} \quad (1.13)$$

It involves a likelihood ratio test’s null hypothesis is that the statistic is  $\chi^2$ -distributed with two degrees of freedom or that the probability of violation  $\hat{p}$  (unconditional coverage) as well as the conditional coverage (independence) is equal to the chosen percentile  $\alpha$ .

### **Dynamic quantile test**

Engle and Manganelli [8] with the aim to provide completeness to the conditional coverage test of Christoffersen, Hahn, and Inoue [6] developed the Dynamic quantile test. It consists in testing some restriction in a

## References

- [1] Alexios Ghalanos. *rugarch: Univariate GARCH models*. R package version 1.4-4. 2020.
- [2] Alexios Ghalanos. *racd: Autoregressive Conditional Density Models*. <http://www.unstarched.net>, <https://bitbucket.org/alexiosg/>. 2016.
- [3] Bruce E. Hansen. “Autoregressive Conditional Density Estimation”. In: *International Economic Review* 35.3 (1994), pp. 705–730.
- [4] P.H. Kupiec. “Techniques for Verifying the Accuracy of Risk Measurement Models”. In: *Journal of Derivatives* 3.2 (1995), pp. 73–84.
- [5] Alexios Ghalanos. *Introduction to the rugarch package. (Version 1.4-3)*. Tech. rep. 2020. URL: <http://cran.r-project.org/web/packages/>.
- [6] Peter Christoffersen, Jinyong Hahn, and Atsushi Inoue. “Testing and comparing Value-at-Risk measures”. In: *Journal of Empirical Finance* 8.3 (July 2001), pp. 325–342. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0927539801000251>.
- [7] Turan G. Bali and Panayiotis Theodossiou. “A conditional-SGT-VaR approach with alternative GARCH models”. In: *Annals of Operations Research* 151.1 (Feb. 22, 2007), pp. 241–267. URL: <http://link.springer.com/10.1007/s10479-006-0118-4>.
- [8] Robert F. Engle and S. Manganelli. *CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles*. Tech. rep. San Diego: UC San Diego, 1999. URL: <http://www.jstor.org/stable/1392044>.