1
Data and methodology

1.1 Data

We worked with daily returns on the EURO STOXX 50 Index denoted in EUR. It is the leading blue-chip index of the Eurozone and covers 50 stocks.

1.1.1 Descriptives

Table of summary statistics

Here comes a table and description of the stats

Table 1.1: Summary statistics of the returns

Statistics	Eurostoxx.50	Standardized.Residuals
Arithmetic Mean	-13.2404	-11.7732
Stdev	0.0357	-0.0193
Skewness	0.0167	-0.0409
NA	10.4376	5.7126
NA	1.307	0.9992
Skewness	-0.31	-0.6327
	(0***)	(0***)
Excess Kurtosis	7.2083	5.134
	(0***)	(0***)
Jarque-Bera	19528.6196***	10431.0514***

Note: This table shows the descriptive statistics of the returns of over the period 1987-01-01 to 2021-04-27. Including minimum, median, arithmetic average, maximum, standard deviation, skewness and excess kurtosis.

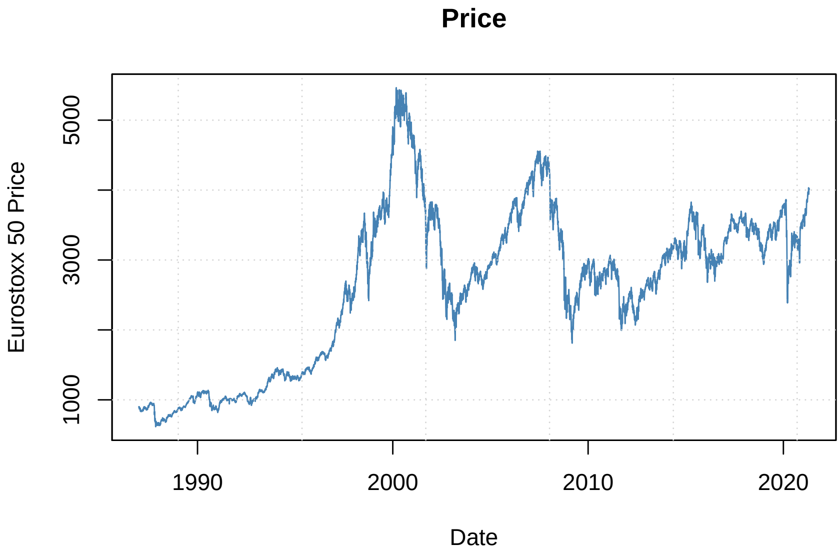


Figure 1.1: Eurostoxx 50 Price Index prices

Descriptive figures

As can be seen

Figure 1.2: Eurostoxx 50 Price Index log returns

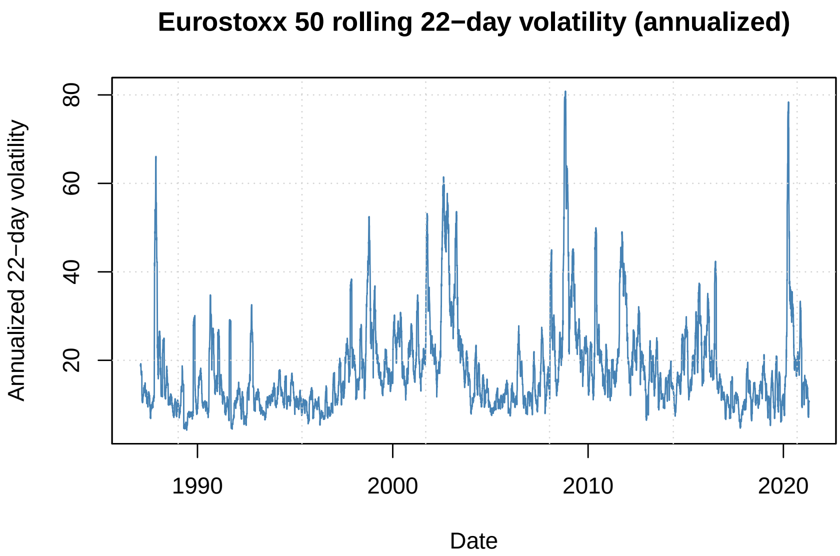


Figure 1.3: Eurostoxx 50 rolling volatility (22 days, calculated over 252 days)

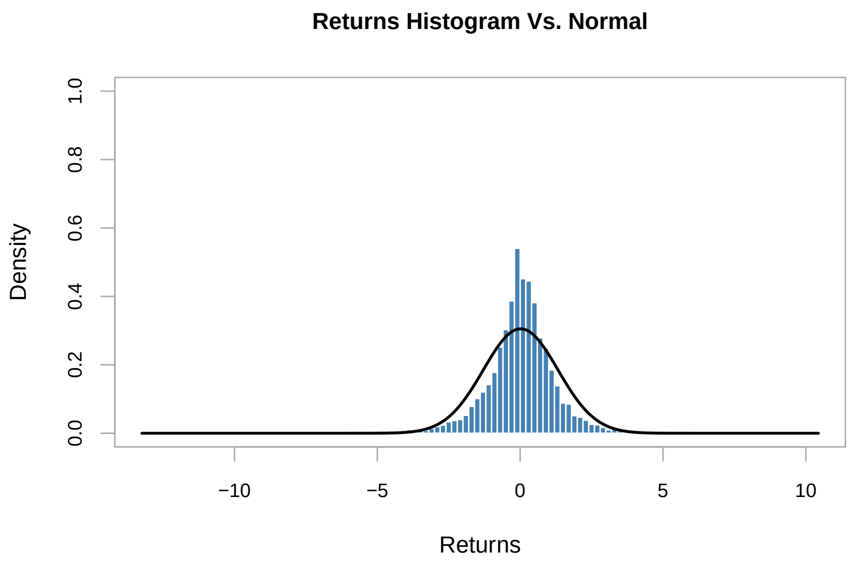


Figure 1.4: Density vs. Normal Eurostoxx 50 log returns)

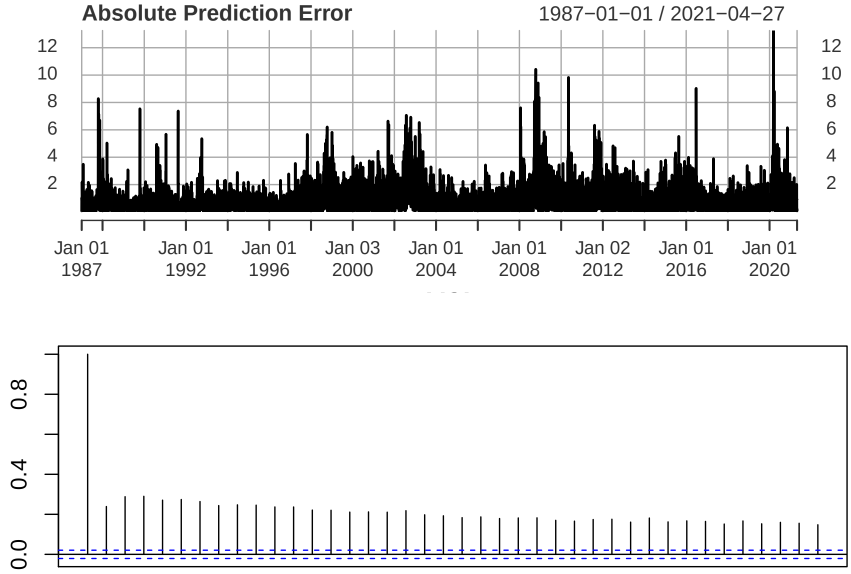


Figure 1.5: Absolute prediction errors

1.2 Methodology

1.2.1 Garch models

As already mentioned in ..., GARCH models GARCH, EGARCH, IGARCH, GJR-GARCH, NGARCH, TGARCH and NAGARCH (or TSGARCH) will be estimated. Additionally the distributions will be examined as well, including the normal, student-t distribution, skewed student-t distribution, generalized error distribution, skewed generalized error distribution and the skewed generalized t distribution.

They will be estimated using maximum likelihood. As already mentioned, fortunately, Alexios Ghalanos [1] has made it easy for us to implement this methodology in the R language (version 3.6.1) with the package "rugarch" version 1.4-4 (R univariate garch), which gives us a bit more time to focus on the results and the interpretation. Additionally

Maximum likelihood estimation is a method to find the distribution parameters that best fit the observed data, through maximization of the likelihood function, or the computationally more efficient log-likelihood function (by taking the natural logarithm). It is assumed that the return data is i.i.d. and that there is some underlying parametrized density function f with one or more parameters that generate the data, defined as a vector θ ((1.2)). These functions are based on the joint probability distribution of the observed data (equation (1.4)). Subsequently, the (log)likelihood function is maximized using an optimization algorithm (equation (1.6)).

$$y_1, y_2, ..., y_N \sim i.i.d \tag{1.1}$$

$$y_i \sim f(y|\theta) \tag{1.2}$$

$$L(\theta) = \prod_{i=1}^{N} f(y_i|\theta) \tag{1.3}$$

$$\log(L(\theta)) = \sum_{i=1}^{N} \log f(y_i|\theta) \tag{1.4}$$

$$\theta^* = arg\max_{\theta}[L] \tag{1.5}$$

$$\theta^* = arg\max_{\theta}[\log(L)] \tag{1.6}$$

1.2.2 ACD models

Following Ghalanos [2], arguments of ACD models are specified as in Hansen [3]. The density function $f(y|\alpha)$ has parameters $\alpha_t = (\mu_t, \sigma_t, \nu_t)$, with equation (1.7), the conditional mean equation. Equation (1.8) as the conditional variance. And $\nu_t = \nu(\theta, x_t)$ the remaining parameters of the distribution like the skewness and kurtosis (shape) parameters.

$$\mu_t = \mu\left(\theta, x_t\right) = E\left(y_t \mid x_t\right) \tag{1.7}$$

$$\sigma_t^2 = \sigma^2\left(\theta, x_t\right) = E\left(\left(y_t - \mu_t^2\right) \mid x_t\right) \tag{1.8}$$

To further explain the difference between GARCH and ACD. The scaled innovations are given by equation (1.9). The conditional density is given by equation (1.10) and related to the density function $f(y|\alpha)$ as in equation (1.2.2).

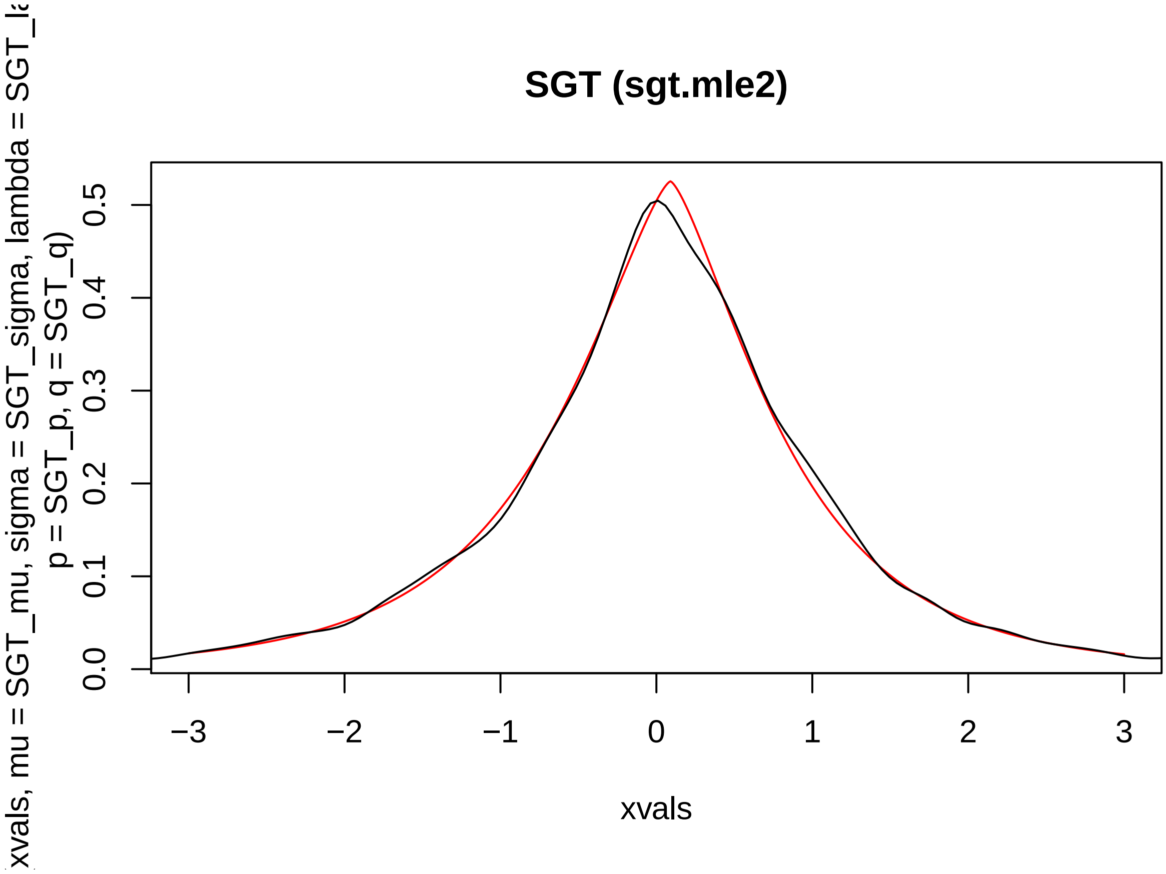
$$z_t(\theta) = \frac{y_t - \mu\left(\theta, x_t\right)}{\sigma\left(\theta, x_t\right)} \tag{1.9}$$

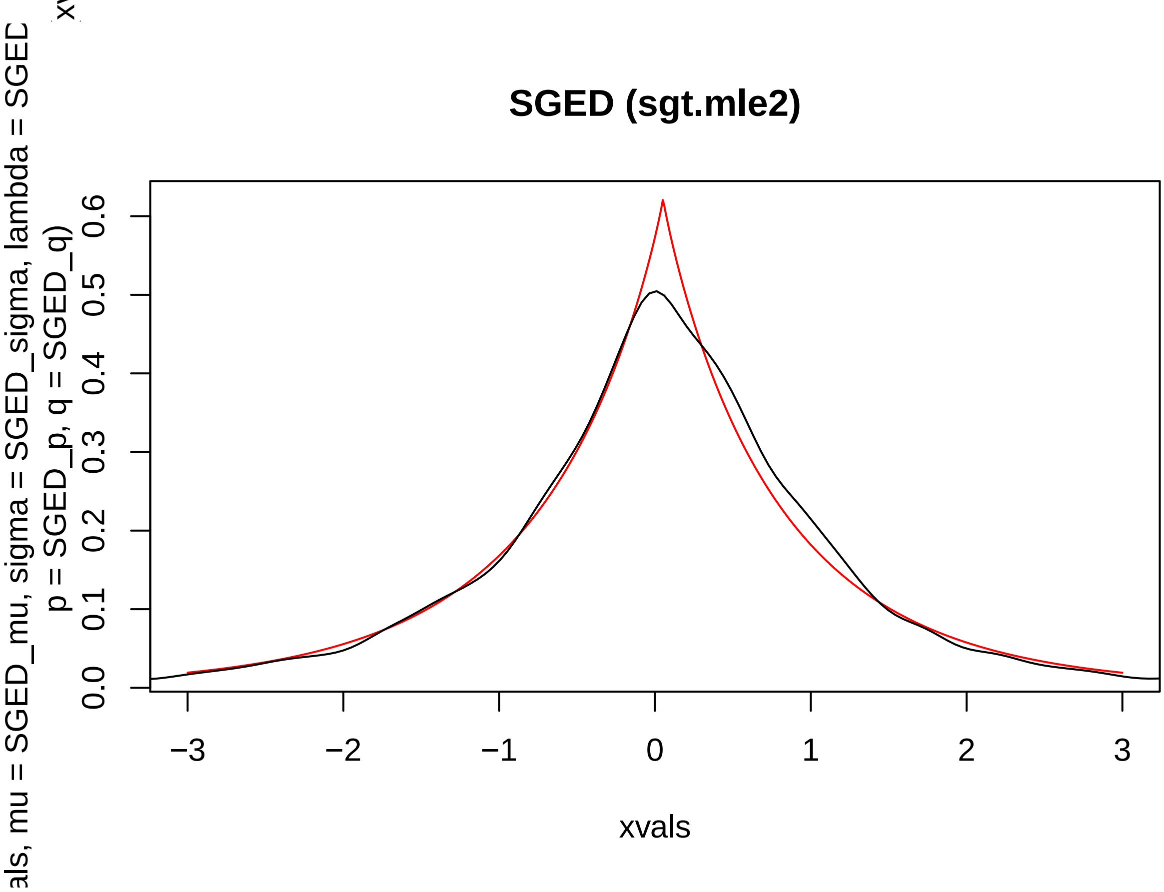
$$g\left(z \mid \eta_t\right) = \frac{d}{dz} P\left(z_t < z \mid \eta_t\right) \tag{1.10}$$

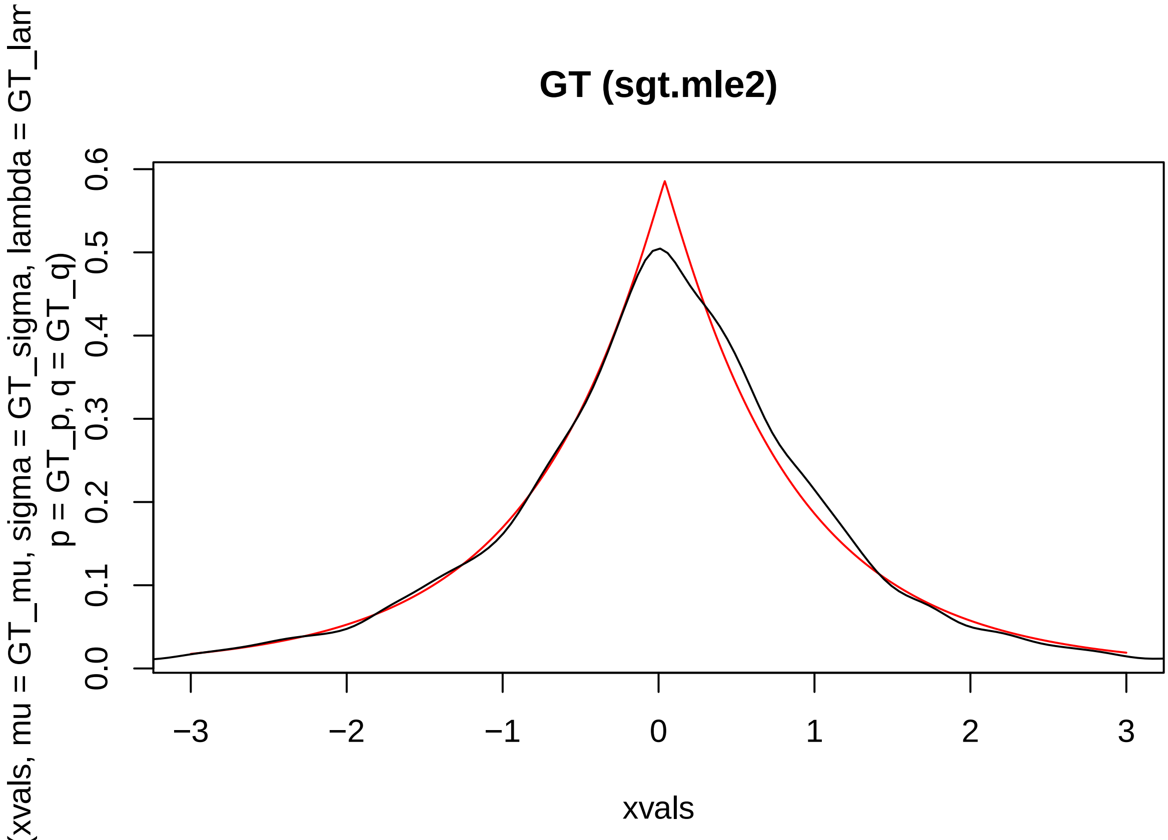
$$f\left(y_t \mid \mu_t, \sigma_t^2, \eta_t\right) = \frac{1}{\sigma_t} g\left(z_t \mid \eta_t\right) \tag{1.11}$$

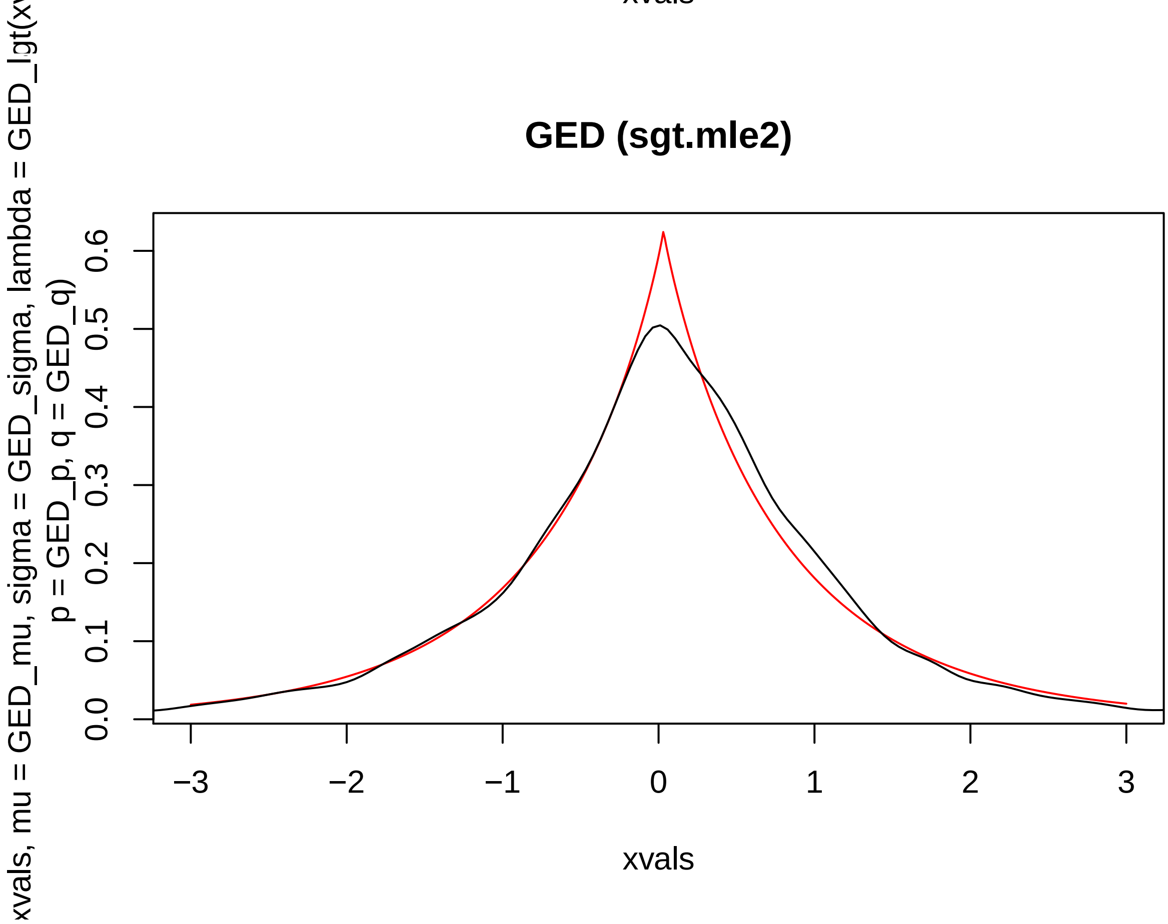
```
##       mean         sd
## 0.01668214 1.30689172
##       mean         sd
## 0.01381119 0.00976596
## [1] -15101.73
##         df        ncp
## 4.31096001 0.03168827
##         df        ncp
## 0.14857777 0.01100453
## [1] -14149.5
##       mean         sd         nu
## 0.03160393 1.27550013 0.91274249
##        mean          sd          nu
## 0.008555584 0.015772159 0.016622605
## [1] -14009.53
##       mean         sd         nu         xi
## 0.01946361 1.27515748 0.91513166 0.98174821
##        mean          sd          nu          xi
## 0.013176090 0.015786515 0.016652983 0.009638209
## [1] -14008.63
## Skewed Generalized T MLE Fit
## Best Result with BFGS Maximization
## Convergence Code 0: Successful Convergence
## Iterations: NA, Log-Likelihood: -13973.01
##
##           Est. Std. Err.       z  P>|z|
## mu      0.0204    0.0131  1.5574 0.1194
## sigma   1.3214    0.0261 50.5971 0.0000 ***
## lambda -0.0397    0.0126 -3.1583 0.0016  **
```

```
## p       1.3818    0.0708 19.5077 0.0000 ***
## q       3.3093    0.5333  6.2058 0.0000 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Fitting of the distribution ' sgt ' by maximum likelihood
## Parameters :
##           estimate Std. Error
## mu      0.01974156 0.01263035
## sigma   1.27919321 0.01674109
## lambda -0.03189521 0.01159236
## p       1.09667765        NaN
## q       9.37999498        NaN
## Loglikelihood:  -13984.5   AIC:  27978.99   BIC:  28014.49
## Correlation matrix:
##                 mu       sigma     lambda   p   q
## mu      1.00000000 -0.04998713 0.70347249 NaN NaN
## sigma  -0.04998713  1.00000000 0.04648083 NaN NaN
## lambda  0.70347249  0.04648083 1.00000000 NaN NaN
## p              NaN         NaN        NaN   1 NaN
## q              NaN         NaN        NaN NaN   1
```

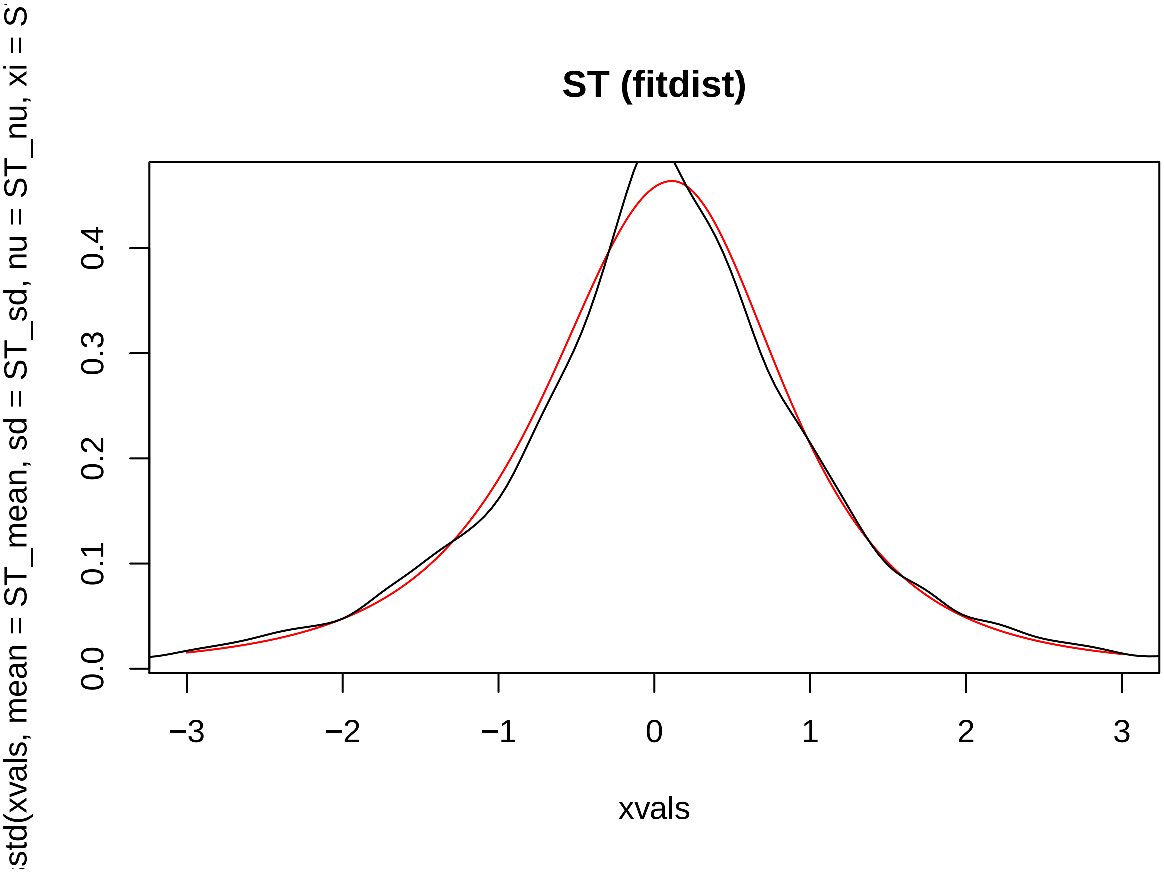

SGT (sgt.mle2)
xvals, mu = SGT_mu, sigma = SGT_sigma, lambda = SGT_la
p = SGT_p, q = SGT_q)
0.0
0.1
0.2
0.3
0.4
0.5
−3
−2
−1
0
1
2
3
xvals


SGED (sgt.mle2)
als, mu = SGED_mu, sigma = SGED_sigma, lambda = SGED
p = SGED_p, q = SGED_q)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−3
−2
−1
0
1
2
3
xvals

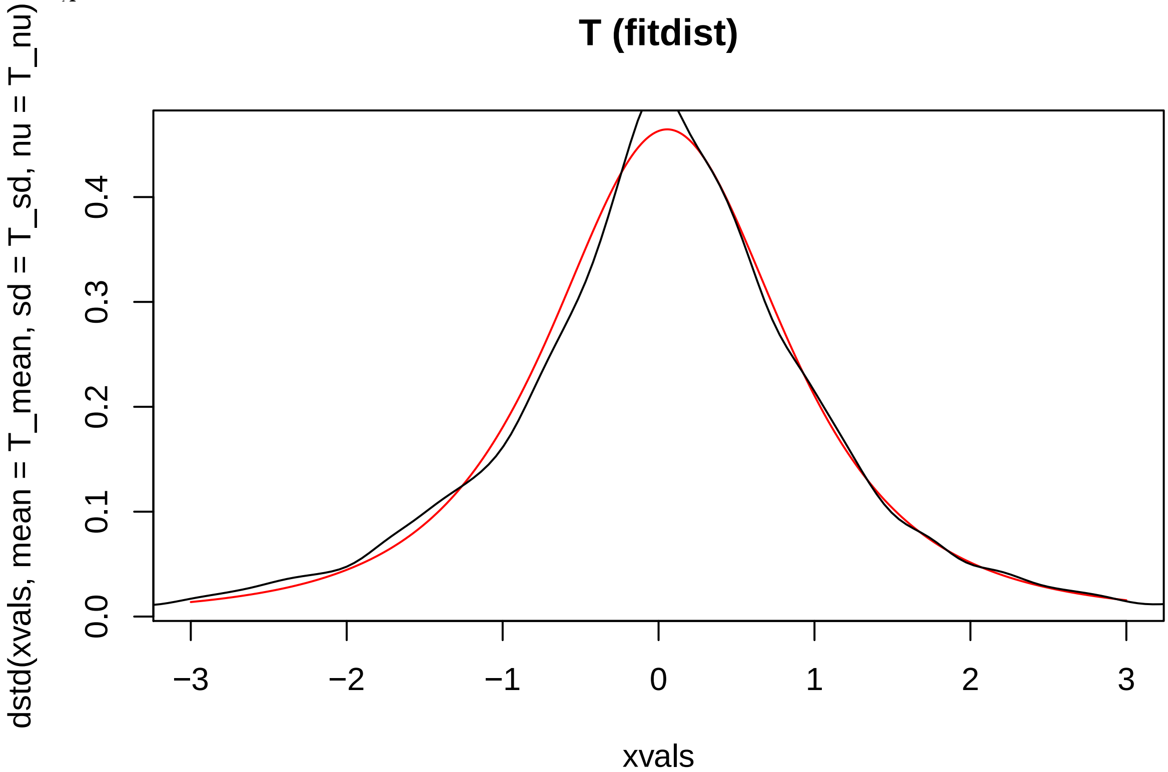

GT (sgt.mle2)
xvals, mu = GT_mu, sigma = GT_sigma, lambda = GT_lam
p = GT_p, q = GT_q)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−3
−2
−1
0
1
2
3
xvals


GED (sgt.mle2)
xvals, mu = GED_mu, sigma = GED_sigma, lambda = GED_lgt(xvals, mu = GED_mu, sigma = GED_sigma, lambda = GED_lambda
p = GED_p, q = GED_q)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−3
−2
−1
0
1
2
3
xvals

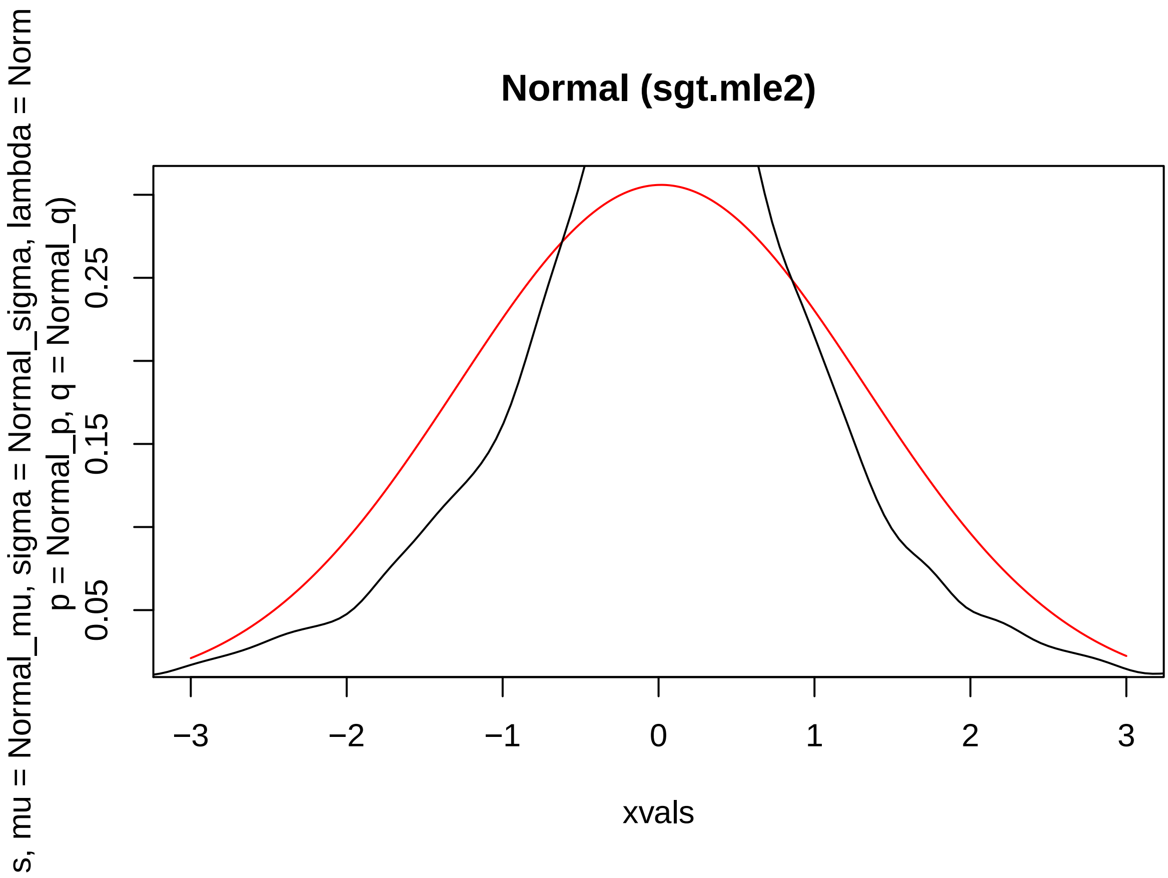
```
## [1] 28002.7
```



ST (fitdist)
dstd(xvals, mean = ST_mean, sd = ST_sd, nu = ST_nu, xi = S
0.0
0.1
0.2
0.3
0.4
−3
−2
−1
0
1
2
3
xvals



T (fitdist)
dstd(xvals, mean = T_mean, sd = T_sd, nu = T_nu)
0.0
0.1
0.2
0.3
0.4
−3
−2
−1
0
1
2
3
xvals



1.2.3 Control Tests

Unconditional coverage test of Kupiec [4]

A number of tests are computed to see if the value-at-risk estimations capture the actual losses well. A first one is the unconditional coverage test by Kupiec [4]. The unconditional coverage or proportion of failures method tests if the actual value-at-risk exceedances are consistent with the expected exceedances (a chosen percentile, e.g. 1% percentile) of the VaR model. Following Kupiec [4] and Ghalanos [5], the number of exceedence follow a binomial distribution (with thus probability equal to the significance level or expected proportion) under the null hypothesis of a correct VaR model. The test is conducted as a likelihood ratio test with statistic like in equation (1.12), with p the probability of an exceedence for a confidence level, N the sample size and X the number of exceedence. The null hypothesis states that the test statistic LR^{uc} is χ^2-distributed with one degree of freedom or that the probability of failure $\hat{p}$ is equal to the chosen percentile α.

$$LR^{uc} = -2\ln\left(\frac{(1-p)^{N-X}p^X}{\left(1-\frac{X}{N}\right)^{N-X}\left(\frac{X}{N}\right)^X}\right) \tag{1.12}$$

Conditional coverage test of Christoffersen, Hahn, and Inoue [6]

Christoffersen, Hahn, and Inoue [6] proposed the conditional coverage test. It is tests for unconditional covrage and serial independence. The serial independence is important while the LR^{uc} can give a false picture while at any point in time it classifies inaccurate VaR estimates as "acceptably accurate" [7]. For a certain VaR estimate an indicator variable, $I_t(\alpha)$, is computed as equation (1.13).

$$I_t(\alpha) = \begin{cases} 1 & \text{if exceedence occurs} \\ 0 & \text{if no exceedence occurs} \end{cases}. \tag{1.13}$$

It involves a likelihood ratio test's null hypothesis is that the statistic is χ^2-distributed with two degrees of freedom or that the probability of violation $\hat{p}$ (unconditional coverage) as well as the conditional coverage (independence) is equal to the chosen percentile α.

Dynamic quantile test

Engle and Manganelli [8] with the aim to provide completeness to the conditional coverage test of Christoffersen, Hahn, and Inoue [6] developed the Dynamic quantile test. It consists in testing some restriction in a

References

[1] Alexios Ghalanos. *rugarch: Univariate GARCH models.* R package version 1.4-4. 2020.

[2] Alexios Ghalanos. *racd: Autoregressive Conditional Density Models.* http://www.unstarched.net, https://bitbucket.org/alexiosg/. 2016.

[3] Bruce E. Hansen. “Autoregressive Conditional Density Estimation”. In: *International Economic Review* 35.3 (1994), pp. 705–730.

[4] P.H. Kupiec. “Techniques for Verifying the Accuracy of Risk Measurement Models”. In: *Journal of Derivatives* 3.2 (1995), pp. 73–84.

[5] Alexios Ghalanos. *Introduction to the rugarch package. (Version 1.4-3).* Tech. rep. 2020. URL: `http://cran.r-project.org/web/packages/`.

[6] Peter Christoffersen, Jinyong Hahn, and Atsushi Inoue. “Testing and comparing Value-at-Risk measures”. In: *Journal of Empirical Finance* 8.3 (July 2001), pp. 325–342. URL: `https://linkinghub.elsevier.com/retrieve/pii/S0927539801000251`.

[7] Turan G. Bali and Panayiotis Theodossiou. “A conditional-SGT-VaR approach with alternative GARCH models”. In: *Annals of Operations Research* 151.1 (Feb. 22, 2007), pp. 241–267. URL: `http://link.springer.com/10.1007/s10479-006-0118-4`.

[8] Robert F. Engle and S. Manganelli. *CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles.* Tech. rep. San Diego: UC San Diego, 1999. URL: `http://www.jstor.org/stable/1392044`.