List of Symbols

Symbol	Interpretation
α	Significance level, or the percentile chosen for the VaR.
α_0	The constant in the conditional mean equation. In other papers sometimes referred to as μ.
α_1	The slope in the conditional mean equation.
β_0	The constant in the variance equation and (G)ARCH models. In other papers sometimes referred to as ω.
β_1	The parameter estimate of the innovation process in the variance equation and (G)ARCH models. In other papers sometimes referred to as α. This is also known as the ARCH component (p).
β_2	The parameter estimate of the autoregressive component in the variance equation and GARCH models. In other papers sometimes referred to as β. This is also known as the GARCH componenet (q)
γ	The leverage term in the asymmetric GARCH models: GJRGARCH and EGARCH.
ε	The innovation term or "noise" term. The random part or return shock of the previous period in the variance equation in GARCH models.
ζ	The time-varying kurtosis parameter in the ACD specification following a piecewise linear dynamic.
θ	The vector of parameters to estimate by the log likelihood functions in the maximum likelihood estimation of GARCH and distribution parameters.
η	This is the tail-thickness parameter of the unconditional and conditional distribution. Following the notation of Bali, Mo, and Tang [1], the parameter rises in the SGT distribution. For the T distribution this parameter is equal to the $\nu/2$.
κ	The peakedness parameter which is only present in the SGT distribution and equal to 2 for the other distributions used in this paper.
ν	The degree of freedom parameter from the T distribution.
ξ	The skewness parameter in the skewed distributions (SGT, SGED, ST) and in the GARCH models. Sometimes referred to as λ.
ρ	The time-varying skewness parameter in the ACD specification following a piecewise linear dynamic.

Introduction

A general assumption in finance is that stock returns are normally distributed. However, various authors have shown that this assumption does not hold in practice: stock returns are not normally distributed [among which 2–4]. For example, Theodossiou [2] mentions that "empirical distributions of log-returns of several financial assets exhibit strong higher-order moment dependencies which exist mainly in daily and weekly log-returns and prevent monthly, bimonthly and quarterly log-returns from obeying the normality law implied by the central limit theorem. As a consequence, price changes do not follow the geometric Brownian motion." So in reality, stock returns exhibit fat-tails and peakedness [5], these are some of the so-called stylized facts of returns. Additionally, a point of interest is the predictability of stock prices [6–8]. This makes it difficult for corporations to manage market risk, i.e. the variability of stock prices.

Risk, in general, can be defined as the volatility of unexpected outcomes [9]. Corporations have to manage their risks both for their own sake and due to regulatory requirements. Consequentially, the Value at Risk metric (VaR), developed in response to the financial disaster events of the early 1990s, has been very influential in the financial world. The tool of VaR has become a standard measure of risk for many financial institutions going from banks, that use VaR to calculate the adequacy of their capital structure, to other financial services companies to assess the exposure of their positions and portfolios. The 1% VaR can be defined as the maximum loss of a portfolio, during a time horizon, excluding all the negative events with a combined probability lower than 1% while the Conditional Value at Risk (CVaR) can be defined as the average of the events that are lower than the VaR. Also Bali, Mo, and Tang [1] explains that many implementations of the CVaR have the assumption that asset and portfolio's returns are normally distributed but that this is inconsistent with the evidence. Assuming normality can lead to incorrect VaR and CVaR numbers, an underestimation of the probability of extreme events happening and ultimately a poor understanding of risk exposure.

This paper has *two goals*. The first goal is to *develop a deeper understanding of GARCH models*, their mechanics and the properties of these models in regard to risk management. Second, we aim to *replicate and update the research made by Bali, Mo, and Tang [1] on US indexes, analyzing the dynamics proposed with a European outlook*. Namely, we use GARCH models to predict the volatility of the Euro Stoxx 50 return index. After this estimation, we compute VaR and CVaR and conduct a backtest to see if the GARCH models predict the VaR and CVaR appropriately. This is summarized in our research question:

> **Do higher moments increase accuracy in the estimation of VaR and CVaR?**

The paper is organized as follows. First, chapter **??** discusses the stylized facts, conditional distributions and the GARCH models. Chapter **??** describes the data used and the methodology followed in modeling the volatility with GARCH models. Then a description is given of the control tests used to evaluate the performances of the different GARCH models and underlying distributions. In chapter **??**, the findings are presented and discussed. in chapter **??** the results of some diagnostic tests are shown and interpreted. Finally, chapter **??** summarizes the results and answers the research question.

References

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