1 Empirical Findings

1.1 Density of the returns

1.1.1 MLE distribution parameters

In table 1.1 we can see the estimated parameters of the unconditional distribution functions. Note that the Student-t and skewed Student-t distribution are usually noted with degrees of freedom as parameters. For consistency, we have parameterized them using limiting cases of the SGT-distribution. Note that to read the degrees of freedom for the two distributions, it is simply 2η. They are presented for the Skewed Generalized T-distribution (SGT) and limiting cases thereof previously discussed. Additionally, maximum likelihood score and the Akaike Information Criterion (AIC) is reported to compare goodness of fit of the different distributions but also taking into account simplicity of the models. We find that the SGT-distribution has the highest maximum likelihood score of all. All other distributions have relatively similar likelihood scores, though slightly lower and are therefore not the optimal distributions. However, when considering AIC it is a tie between SGT and SGED. This provides some indication that we have a valid case to test the suitability of different SGED-GARCH VaR models as an alternative for the SGT-GARCH VaR models. While sacrificing some goodness of fit, the SGED distribution has the advantage of requiring one parameter less, which could possibly result in less errors due to misspecification and easier implementation. For the SGT parameters the standard deviation and skewness are both significant at the 1% level. For the SGED parameters, the standard deviation and the skewness are both significant at respectively the 1% and 5% level. Both distributions are right-skewed. For both distributions the shape parameters are significant at the 1% level, though the q parameter was not estimated as it is by design set to infinity due to the SGED being a limiting case of SGT.[1]

Additionally, for every distribution fitted with MLE, plots are generated to compare the theoretical distribution with the observed returns. We see that except for the normal distribution which is quite off, the theoretical distributions are close to the actual data, except that they are too peaked. This problem is the least present for the SGT distribution.

[1]To check whether the relative ranking of distributions still holds in different periods, we have calculated the maximum likelihood score and AIC for three smaller periods: The period up to the dotcom collapse (1987-2001), up to the GFC (2002-2009) and up to the present Covid-crash (2009-2021). There is no qualitative difference in relative ranking with these subsamples. Results are reported in the appendix.

Table 1.1: Maximum likelihood estimates of unconditional distribution functions

dist	α	β	ξ	κ	η	*LLH*	AIC
SGT	0.014	1.441	-0.02	1.233	4.959	-8850.419	17710.84
	(0.025)	(0.031)***	(0.016)	(0.085)***	(1.368)***		
SGED	0.015	1.42	0.008	0.898	Inf	-8859.217	17710.84
	(0.003)***	(0.015)***	(0.002)***	(0.018)***			
GED	0	1.418	0	0.899	Inf	-8859.537	17725.07
	(0.002)	(0.023)***		(0.022)***			
ST	0.012	1.635	-0.043	2	2.817	-8873.843	17755.69
	(0.02)	(0.076)***	(0.016)***		(0.132)***		
T	0.045	1.64	0	2	1.403	-8877.165	17760.33
	(0.015)***	(0.078)***			(0.133)***		
Normal	0.01	1.439	0	2	Inf	-9477.660	18959.32
	(0.02)	(0.014)***					

Notes:
Table contains parameter estimates for SGT-distribution and some of its limiting cases. The underlying data is the daily return series of the Euro Stoxx 50 for the period between December 31. 1986 and April 27. 2021. Standard errors are reported between brackets. *LLH* is the maximum log-likelihood value. *, ** and *** point out significance at 10, 5 and 1 percent level.

1.2 Constant higher moments

To compare the goodness-of-fit of different combinations of GARCH models and distributions, we have portrayed the AICs in table 1.2. The smaller the criterium the better. As you can see in table 1.2 the AIC for the skewed Student-t distribution (ST) is the best for almost all the models. As also shown in appendix part ??. Only for the SGARCH and IGARCH the SGED distribution has a lower AIC. The best goodness-of-fit over all distributions seems to be the NAGARCH model.

Due to 1.2 being the result of an in-sample estimation, we will further examine the performance five different GARCH models (EGARCH, GJRGARCH, NAGARCH, AVGARCH and TGARCH).

Table 1.2: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
N	3.174	3.176	3.198	3.114	3.124	3.107	3.111	3.107
T	3.130	3.130	3.140	3.079	3.089	3.074	3.077	3.074
ST	3.127	3.127	3.135	3.072	3.083	3.067	3.071	3.067
GED	3.128	3.128	3.139	3.080	3.089	3.075	3.079	3.076
SGED	3.125	3.126	3.136	3.075	3.084	3.069	3.073	3.069

Notes
This table shows the AIC value for the respective model. With on the rows the distributions.

Table 1.3 presents the maximum likelihood estimates for 8 GARCH models based on the ST distribution with constant skewness and kurtosis parameters (robust white errors are presented in parenthesis). The parameter α_0 are only statistically significant for the SGARCH, IGARCH and EWMA model with a value close to 0. The AR(1) coefficient, α_1, has parameters going from -0.049 to -0.032 with p values ranging from 0.011 to 0.014 suggesting significance, but indicating very small negative autocorrelation. The GARCH parameters in the conditional variance equations (β_0) are generally statistically significant except for the EGARCH model. The results of β_1 and β_2 show the presence of significant time-variation in the conditional volatility of the Euro Stoxx 50, in fact, the sum of β_1 and β_2 for the GARCH parameters is close to one (from 0.827 to 1), suggesting the presence of persistence in the volatility of the returns. The parameter ξ is highly significant for all the 8 models tested with values ranging from 0.885 to 0.918 confirming the presence of skewness in the returns. The shape parameter η, which, in our case, measures the number of degrees of freedom divided by two, determining the tail behavior, is significant for all the models and ranges between 3.153 and 4.0635. The parameter γ, which is present only for EGARCH and GJRGARCH is significant and with values around 0.14. The absolute value function in family GARCH models (NAGARCH, TGARCH and AVGARCH) is subject to the $shift$ and the rot parameters whose values are always positive and statistically significant. According to the log likelihood values (LLH), displayed in table 1.3, the model with the highest value is AVGARCH while, excluding the non-standard GARCH models from the analysis, the model that performs best is EGARCH.

Table 1.4 shows a very similar picture for the GARCH-SGED models as the GARCH-ST models.

Table 1.3: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.052	0.052	0.014	0.019	0.055	0.004	0.013	0.003
	(0.012)***	(0.013)***	(0.011)	(0.012)	(0.014)***	(0.012)	(0.013)	(0.013)
α_1	-0.048	-0.048	-0.033	-0.043	-0.049	-0.034	-0.038	-0.032
	(0.013)***	(0.013)***	(0.012)***	(0.012)***	(0.014)***	(0.012)***	(0.011)***	(0.012)**
β_0	0.017	0.014	0.004	0.021	0	0.024	0.023	0.024
	(0.004)***	(0.004)***	(0.003)	(0.004)***		(0.003)***	(0.004)***	(0.002)***
β_1	0.095	0.1	-0.155	0	0.072	0.06	0.078	0.068
	(0.011)***	(0.011)***	(0.011)***	(0.01)	(0.008)***	(0.013)***	(0.008)***	(0.004)***
β_2	0.899	0.9	0.982	0.902	0.928	0.787	0.922	0.899
	(0.011)***		(0)***	(0.013)***		(0.022)***	(0.009)***	(0)***
ξ	0.918	0.918	0.89	0.895	0.915	0.885	0.891	0.885
	(0.017)***	(0.017)***	(0.017)***	(0.016)***	(0.016)***	(0.017)***	(0.017)***	(0.017)***
η	3.301	3.153	3.95	3.868	3.624	4.062	4.0335	4.0635
	(0.325)***	(0.273)***	(0.4355)***	(0.422)***	(0.285)***	(0.46)***	(0.451)***	(0.457)***
γ			0.107	0.177				
			(0.011)***	(0.02)***				
$shift$						1.567		0.393
						(0.332)***		(0.014)***
rot							1	1
							(0.069)***	(0.117)***
LLH	-8303.694	-8304.437	-8158.11	-8186.06	-8328.667	-8143.563	-8154.785	-8143.141

Notes:
This table shows the maximum likelihood estimates of various ST-GARCH models. The daily returns used on the Euro Stoxx 50 cover the period from 03 gennaio, 2001 to 19 maggio, 2021 (5316 observations).
The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$ Where, in the 8 GARCH models estimated, γ is the asymmetry in volatility, ξ, κ and η are constant and robust standard errors based on the method of White (1982)) are displayed in parenthesis. LLH is the maximized log likelihood value.

Table 1.4: Maximum likelihood estimates of the SGED-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
α_0	0.046	0.045	0.008	0.014	0.046	0	0.007	-0.002
	(0.011)***	(0.012)***	(0.008)	(0.012)	(0.015)***	(0.008)	(0.012)	(0.012)
α_1	-0.055	-0.055	-0.041	-0.05	-0.056	-0.043	-0.047	-0.041
	(0.013)***	(0.014)***	(0.012)***	(0.013)***	(0.016)***	(0.012)***	(0.011)***	(0.012)***
β_0	0.02	0.015	0.005	0.023	0	0.025	0.024	0.025
	(0.005)***	(0.004)***	(0.002)**	(0.004)***		(0.003)***	(0.005)***	(0.002)***
β_1	0.096	0.104	-0.151	0	0.07	0.059	0.077	0.067
	(0.012)***	(0.012)***	(0.01)***	(0.01)	(0.008)***	(0.007)***	(0.01)***	(0.003)***
β_2	0.895	0.896	0.981	0.901	0.93	0.793	0.922	0.897
	(0.012)***		(0)***	(0.014)***		(0.009)***	(0.01)***	(0)***
ξ	0.935	0.936	0.901	0.906	0.93	0.896	0.902	0.896
	(0.021)***	(0.023)***	(0.019)***	(0.02)***	(0.02)***	(0.017)***	(0.019)***	(0.018)***
η								
γ			0.107	0.175				
			(0.01)***	(0.021)***				
$shift$						1.547		0.435
						(0.147)***		(0.012)***
rot							1	0.959
							(0.076)***	(0.101)***
LLH	-8300.515	-8302.804	-8164.576	-8189.218	-8330.811	-8149.596	-8160.239	-8149.518

Notes:
This table shows the maximum likelihood estimates of various SGED-GARCH models. The daily returns used on the Euro Stoxx 50 Price index cover the period from 03 gennaio, 2001 to 19 maggio, 2021 (5316 observations). The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$ Where, in the 8 GARCH models estimated, γ is the asymmetry in volatility, ξ, κ and η are constant and robust standard errors based on the method of White (1982)) are displayed in parenthesis. LLH is the maximized log likelihood value.

1.2.1 Value-at-risk

As already mentioned 2 candidate models seem to be very appropriate. This includes the EGARCH and the NAGARCH. To check if they perform well out-of-sample we conduct a backtest by using a rolling forecasting technique. A simple graph is shown in figure 1.1 for the EGARCH-ST model. It seems that the VaR model for $\alpha = 0.05$ underestimates the downside events, while the VaR model for $\alpha = 0.01$ captures more of the downside events.

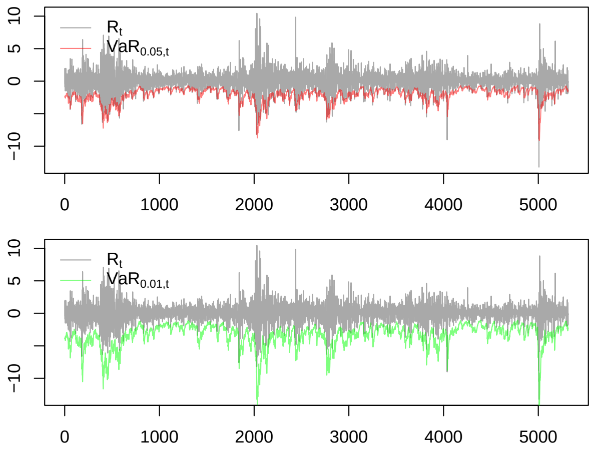

Figure 1.1: Value-at-Risk (in-sample) for the EGARCH-ST model

Let us examine this further using a rolling window approach whilst forecasting 1-day ahead results with re-estimating parameters every year.

Figure 1.2 shows that choosing an appropriate forecast period is important (with here the Eurobond crisis, the Brexit and Covid-crisis), so in order to avoid a look-ahead bias this rolling window approach was used instead of a static forecast method.

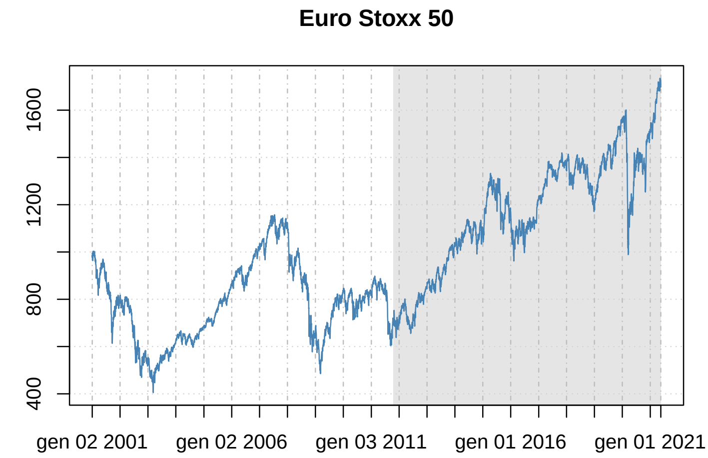

Figure 1.2: Selected period to start forecast from

If we look at the results of the rolling window, we can for example compare as in figure 1.3 the EGARCH-ST (with skewed Student-t distribution) with the EGARCH-N (with normal distribution). The EGARCH-N seems to capture the extreme events a bit less compared EGARCH-ST. But let us formally test this.

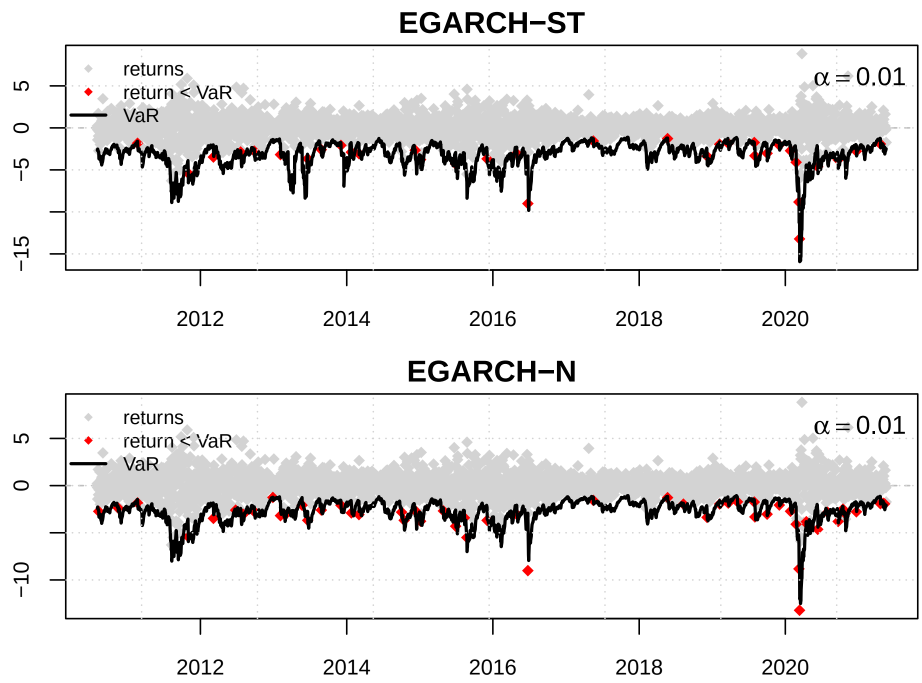

Figure 1.3: Comparison between VaR-EGARCH-ST and VaR-NAGARCH-N

With the SGED as underlying distribution, the ratio of actual to expected VaR exceedances is highest for the GJRGARCH model, with an excess exceedance rate of 24%. The other GARCH models still have a positive but slightly lower excess exceedance rate of 21%. The ratio of actual to expected CVaR exceedances gives a similar picture, with the highest excess exceedance rate being the GJRGARCH with 25% and for the other models 21%. These results are significant at the 5% level, except for the NAGARCH, which is significant only at the 10% level. This points that the difference between actual and expected exceedances is significantly different from 0. For all models under the SGED, the unconditional and conditional cover test and the dynamic quantile test statistics are insignificant. This suggests that the SGED is a good distribution for calculating VaR and CVaR.

With the ST as underlying distribution, the ratio of actual to expected VaR exceedances is highest for the TGARCH model, with an excess exceedance rate of 28%. The NAGARCH model has the lowest VaR excess exceedance rate of 17%. For the CVaR excess exceedance rate, there is a similar picture, with the TGARCH having an excess exceedance rate of 29% and the NAGARCH of 18%. These results are insignificant, thus it cannot be concluded that the excess exceedance for VaR and CVaR are significantly different from 0. Both the unconditional and conditional cover tests are insignificant. However, the dynamic quantile test is significant at the 10% level for the EGARCH and TGARCH. This suggests that the ST distribution is a good distribution for calculating VaR and CVaR, though not as good as the SGED.

The GED, T and normal distribution have significant unconditional and conditional and dynamic quantile test statistics for most models, which suggest they are less appropriate for modeling VaR and CVaR.

Table 1.5: VaR and CVaR test statistics

	EGARCH	GJRGARCH	TGARCH	NAGARCH	AVGARCH
Panel A: SGED					
AE VaR	1.207	1.243	1.207	1.207	1.207
AE CVaR	1.214**	1.25**	1.214**	1.214*	1.214**
UC	1.147	1.558	1.147	1.147	1.147
CC	1.979	2.439	1.979	1.979	1.979
DQ	7.757	13.286	12.578	5.223	5.846
Panel B: GED					
AE VaR	1.456	1.456	1.491	1.456	1.385
AE CVaR	1.464***	1.464**	1.5***	1.464**	1.393***
UC	5.184**	5.184**	5.969**	5.184**	3.764*
CC	6.396**	6.396**	7.242**	6.396**	4.859*
DQ	20.235***	16.564**	19.584**	11.688	10.508
Panel C: ST					
AE CVaR	1.207	1.243	1.278	1.172	1.243
AE CVaR	1.214	1.25	1.286	1.179	1.25
UC	1.147	1.558	2.026	0.796	1.558
CC	1.979	2.439	2.959	1.579	2.439
DQ	15.071*	13.002	13.935*	4.552	6.841
Panel D: T					
AE VaR	1.527	1.562	1.562	1.491	1.598
AE CVaR	1.536	1.571	1.571*	1.5	1.607
UC	6.803***	7.683***	7.683***	5.969**	8.61***
CC	8.137**	9.081**	9.081**	7.242**	10.072***
DQ	0.012	18.986**	22.759***	17.376**	25.561***
Panel E: N					
AE VaR	1.953	1.882	1.953	1.74	1.776
AE CVaR	1.964***	1.893***	1.964***	1.75***	1.786***
UC	20.217***	17.575***	20.217***	12.76***	13.904***
CC	22.409***	19.609***	22.409***	14.496***	15.712***
DQ	36.498***	32.87***	39.472***	27.102***	37.614***

Notes:
Table contains the ratio of actual to expected exceedances for VaR and Conditional VaR, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. *, ** and *** point out significance at 10, 5 and 1 percent level.

1.3 Time-varying higher moments

As we already pointed out in part **??**, it might also be interesting to look at time-varying moments and check if there is an improvement for estimating the VaR and CVaR as Bali, Mo, and Tang [1] did for example.

Table 1.6: Maximum likelihood estimates of the ST-ACD model with constant skewness and kurtosis parameters

	ACD GARCH
α_0	0.054 (0.111)
α_1	-0.044 (0.216)
β_0	0.02
β_1	0.098 (0.002)***
β_2	0.892 (0.009)***
χ_0	-1.548 (13.208)
χ_1	0.046 (0.401)
χ_2	0.032 (0.488)
ξ_1	0.563 (3.778)
κ_0	0.127 (0.196)
κ_1	0 (0.175)
κ_2	1 (0.249)***
ψ_1	0.788 (0.101)***

Notes:
This table shows the maximum likelihood estimates of various SGED-GARCH models. The daily returns used on the Euro Stoxx 50 Price index cover the period from 03 gennaio, 2001 to 19 maggio, 2021 (5316 observations).
The mean process is modeled as follows: $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$ Where, in the model estimated, γ is the asymmetry in volatility, (calculated using robust standard errors based on the method of White (1982)) are displayed in parenthesis.

The following figure 1.4 plots the conditional mean, the conditional volatility. The implied conditional time varying skewness and excess kurtosis for the Euro Stoxx 50 series.

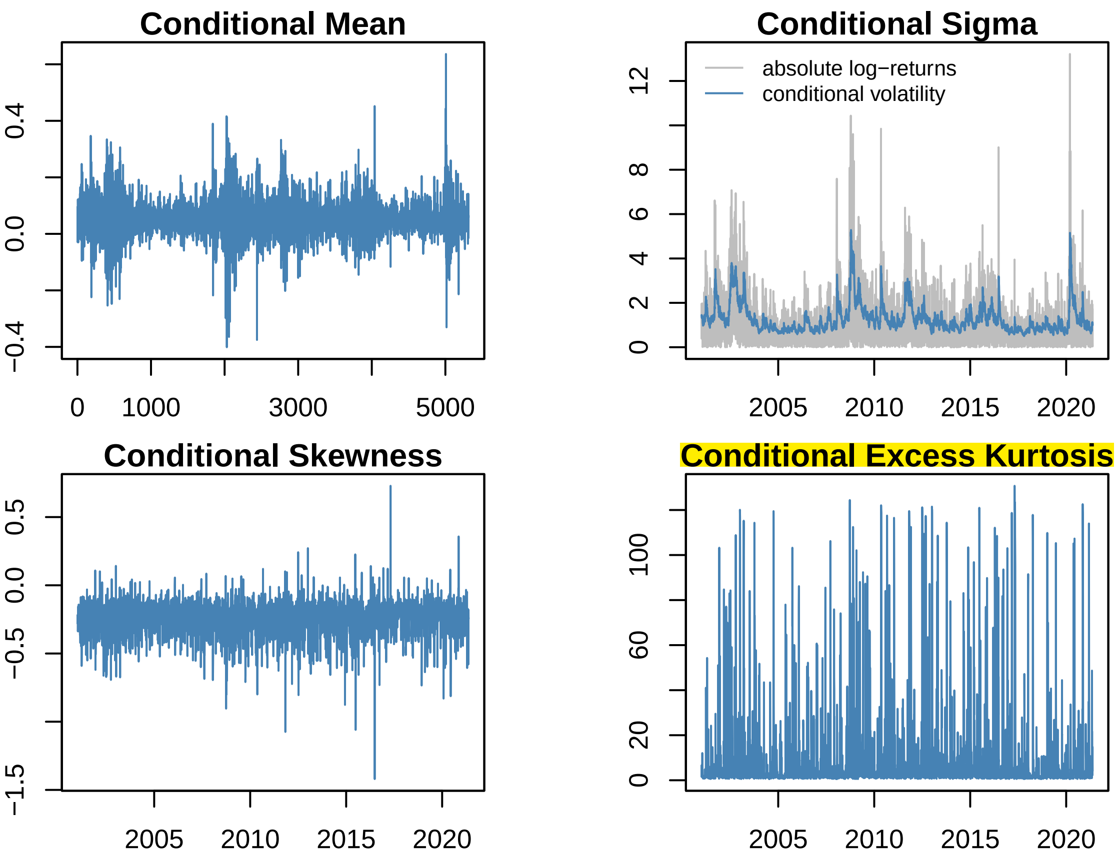

Figure 1.4: Dynamics of the ACD model

After performing a backtest[2], we observe that the ACD model is slightly worse than the GARCH equivalent in estimating the VaR. Although there are much less models tested here because of the difficulty of the procedure of ACD models, we find less convincing evidence of the merit of using conditional higher moments in VaR and CVaR estimation. So contrary to Bali, Mo, and Tang [1], we find that the ACD models seem to underperform to the GARCH model. With a rejection of unconditional coverage test statistic at 10% significance level. It is not surprising that the GARCH model performs bad as well looking at the rejection of the dynamic quantile test and the CVaR test.

[2]This backtest contains the following features: recursive, window of 2500, refitted every 250 trading days (approximately one year).

Table 1.7: VaR and ES test statistics (ACD-ST vs GARCH-ST)

	ACD	GARCH
AE VaR	1.357	1.286
AE CVaR	1.357**	1.286**
UC	3.131*	2.026
CC	4.171	2.959
DQ	16.118**	14.587*

Notes:
Table contains the ratio of actual to expected exceedances for VaR and Expected Shortfall, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. *, ** and *** point out significance at 10, 5 and 1 percent level.

2
Robustness checks

In order to check if the models are specified correctly, some diagnostic checks have to be performed. The specification checks have to be done on the standardized residuals of the estimated GARCH model given by the following equation:

$$\hat{Z}_t = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_t} = \frac{R_t - \hat{\mu}}{\hat{\sigma}_t}$$

.

Table 2.1 Panel A displays the Ljung-box test on the squared standardized residuals of the GARCH models. Panel B displays the ARCH LM test on the squared standardized residuals. The ARCH LM test checks if the ARCH process is adequately fitted.

The description of table

Table 2.1: Diagnostic Tests for Heteroscedasticity

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Panel A: Ljung Box Test on the standardized squared values of the residuals								
Norm	31.157*	25.321	22.263	23.03	32.186*	26.208	32.727*	44.066***
T	33.907**	24.81	21.321	21.862	34.34**	26.658	34.183**	41.765***
ST	33.961**	25.024	21.412	22.051	34.607**	26.811	34.187**	40.605***
GED	32.493*	24.826	21.63	22.191	33.365*	26.142	33.361*	42.627***
SGED	32.569*	25.065	21.57	22.341	33.747*	26.351	33.342*	41.333***
Panel B: ARCH LM Test on the standardized squared values of the residuals								
Norm	32.322*	26.461	23.081	23.474	34.475**	26.991	33.857*	42.773***
T	34.687**	25.958	22.063	22.218	37.875**	27.634	34.912**	40.719***
ST	34.605**	26.138	22.129	22.36	38.174**	27.718	34.756**	39.559**
GED	33.431*	25.973	22.433	22.598	36.379**	27.023	34.228**	41.433***
SGED	33.393*	26.173	22.319	22.698	36.859**	27.167	34.071**	40.155***

Notes

Table displays the Ljung box statistics and the ARCH LM Test for the standardized squared residuals of the models analyzed. The underlying data is the daily return series of the Euro Stoxx 50 for the period between 2001-01-03 and 2021-05-19.

*, ** and *** point out respectively significance at 10, 5 and 1 percent level.

The null hypothesis of the test in both panels are described as follows:

H_0: $Corr(Z_t^2,Z_{t-1}^2)=Corr(Z_t^2,Z_{t-2}^2)= ... =Corr(Z_t^2,Z_{t-22}^2) = 0$

The Generalized Method of Moments (GMM) test by Hansen [2] is also a test that checks if the model is correctly specified or not. Here only the results of the t-tests for the four individual moments are examined from the GMM test[1]. The mean (first moment) should be equal to zero (or or $E\left[z_t\right] = 0$). The variance (second moment) should be equal to one (or $E\left[z_t^2 - 1\right] = 0$). The skewness should be equal to zero and the excess kurtosis (third and fourth moment) should be equal to zero (respectively $E\left[z_t^3\right] = 0$ and $E\left[z_t^4 - 3\right] = 0$).

[1] We already looked at the joint conditional moment: serial correlation in the squares or in the variances

Table 2.2: GMM Tests

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Panel A: SGED								
Mean	-0.036**	-0.002	0.007	0.006	-0.006	-0.001	-0.036**	-0.039***
Variance	0.015**	0.009	0.007	0.007	0.01	0.008	-0.016**	0.121***
Skewness	-0.46**	-0.392	-0.374	-0.381	-0.406	-0.374	-0.443**	-0.566***
Excess Kurtosis	1.974**	1.782	1.809	1.803	1.693	1.696	1.755**	3.821***
Panel B: GED								
Mean	-0.051***	-0.024*	-0.015	-0.017	-0.026*	-0.022	-0.05***	-0.051***
Variance	0.004***	0.004*	0.004	0.004	0.003*	0.003	-0.024***	0.12***
Skewness	-0.49***	-0.452*	-0.438	-0.444	-0.459*	-0.435	-0.476***	-0.602***
Excess Kurtosis	1.858***	1.763*	1.815	1.809	1.662*	1.689	1.661***	3.824***
Panel C: ST								
Mean	-0.042***	-0.009	0.002	0.001	-0.012	-0.007	-0.042***	-0.049***
Variance	0.006***	0.005	0.003	0.003	0.004	0.005	-0.016***	0.124***
Skewness	-0.473***	-0.414	-0.388	-0.396	-0.421	-0.393	-0.46***	-0.596***
Excess Kurtosis	1.944***	1.79	1.809	1.808	1.671	1.702	1.777***	3.855***
Panel D: T								
Mean	-0.059***	-0.03**	-0.019	-0.02	-0.032**	-0.027*	-0.059***	-0.062***
Variance	-0.007***	0.001**	0.003	0.002	-0.001**	0.002*	-0.024***	0.124***
Skewness	-0.511***	-0.473**	-0.452	-0.458	-0.476**	-0.451*	-0.501***	-0.638***
Excess Kurtosis	1.812***	1.783**	1.848	1.84	1.658**	1.707*	1.674***	3.856***
Panel E: N								
Mean	-0.051***	-0.007	0.004	0.002	-0.01	-0.004	-0.051***	-0.047***
Variance	0.001***	0.001	0	0	0.001	0	-0.035***	0.118***
Skewness	-0.482***	-0.391	-0.371	-0.378	-0.405	-0.372	-0.465***	-0.583***
Excess Kurtosis	1.775***	1.684	1.736	1.721	1.605	1.616	1.516***	3.807***

Notes Table displays the GMM test statistics for the standardized residuals. The underlying data is the daily return series of the Euro Stoxx 50 for the period between 2001-01-03 and 2021-05-19. The null hypothesis of the test for each variable are described as follows: $H_0 : E[z_t] = 0$ for the mean, $H_0 : E[z_t^2 - 1] = 0$ for the variance. $H_0 : E[z_t^3] = 0$ for the skewness and $H_0 : E[z_t^4 - 3] = 0$ for the excess kurtosis.

Table 2.3: Jarque-Bera Test on standardized residuals

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Norm	739.888***	735.992***	791.977***	784.475***	676.533***	679.569***	804.423***	1382.222***
T	822.929***	789.877***	839.862***	845.41***	703.17***	718.176***	851.465***	1350.929***
ST	845.468***	787.192***	833.066***	842.468***	700.469***	712.976***	884.557***	1360.366***
GED	785.374***	759.794***	810.002***	811.834***	685.775***	697.098***	841.477***	1367.132***
SGED	803.315***	758.538***	810.37***	811.343***	684.155***	691.427***	869.762***	1372.361***

Notes Table displays the Jarque-Bera statistic $JB = \frac{n}{6}(S^2 + \frac{1}{4}(K-3)^2)$ with n the sample size, K the kurtosis and S the skewness for the residuals of the models. The JB statistic is distributed χ^2 with $\nu = 2$. The underlying data is the daily return series of the Euro Stoxx 50 for the period between 2001-01-03 and 2021-05-19.
*, ** and *** point out respectively significance at 10, 5 and 1 percent level. The null hypothesis is that S and K are not significantly different than what would be found under normality (0 and 3).

2

DESCRIPTION GRAPHS

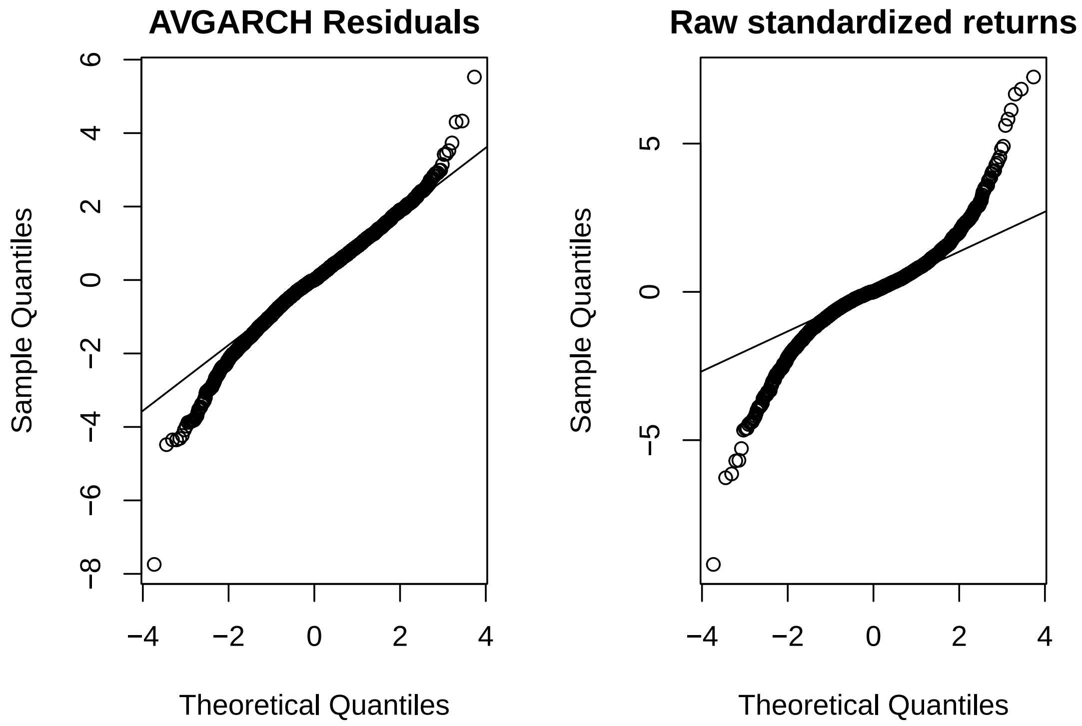

Figure 2.1: QQ plots of AVGARCH residuals versus the standardized returns of the series

[2] Note that using monthly returns instead of daily, it cannot be rejected that the residuals are normally distributed.

References

[1] Turan G. Bali, Hengyong Mo, and Yi Tang. “The role of autoregressive conditional skewness and kurtosis in the estimation of conditional VaR”. In: *Journal of Banking and Finance* 32.2 (Feb. 2008). Publisher: North-Holland, pp. 269–282.

[2] Lars Peter Hansen. “Large Sample Properties of Generalized Method of Moments Estimators”. In: *Econometrica* 50.4 (1982), pp. 1029–1054. URL: `https://www.jstor.org/stable/1912775`.