

# GERMANY DAX 30 index

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Table 1: Summary statistics of the returns

Statistics	Euro Stoxx 50	Standardized Residuals
Arithmetic Mean	0.0237	-0.0217
Median	0.0108	-0.0198
Maximum	10.7975	5.9586
Minimum	-13.7099	-14.5194
Stdev	1.2197	0.9996
Skewness	-0.3201 (0***)	-0.5228 (0***)
Excess Kurtosis	8.4292 (0***)	6.0751 (0***)
Jarque-Bera	43826.8388***	23304.9051***

Notes

<sup>1</sup> This table shows the descriptive statistics of the daily percentage returns of Dax30 over the period 1965-01-05 to 2021-06-04 (14719 observations). Including arithmetic mean, median, maximum, minimum, standard deviation. The skewness, excess kurtosis with p-value and significance and the Jarque-Bera test with significance.

<sup>2</sup> The standardized residual is derived from a maximum likelihood estimation (simple GARCH model) as follows:

$$R_t = \alpha_0 + \alpha_1 R_{t-1} + z_t \sigma_t$$

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2,$$

Where  $z$  is the standard residual (assumed to have a normal distribution).

<sup>3</sup> \*, \*\*, \*\*\* represent significance levels at the 5

Table 2: Maximum likelihood estimates of the ST-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
$\alpha_0$	0.052 (0.012)***	0.052 (0.013)***	0.014 (0.011)	0.019 (0.012)	0.055 (0.014)***	0.004 (0.012)	0.013 (0.013)	0.003 (0.013)
$\alpha_1$	-0.048 (0.013)***	-0.048 (0.013)***	-0.033 (0.012)***	-0.043 (0.012)***	-0.049 (0.014)***	-0.034 (0.012)***	-0.038 (0.011)***	-0.032 (0.012)**
$\beta_0$	0.017 (0.004)***	0.014 (0.004)***	0.004 (0.003)	0.021 (0.004)***	0	0.024 (0.003)***	0.023 (0.004)***	0.024 (0.002)***
$\beta_1$	0.095 (0.011)***	0.1 (0.011)***	-0.155 (0.011)***	0 (0.01)	0.072 (0.008)***	0.06 (0.013)***	0.078 (0.008)***	0.068 (0.004)***
$\beta_2$	0.899 (0.011)***	0.9	0.982 (0)***	0.902 (0.013)***	0.928	0.787 (0.022)***	0.922 (0.009)***	0.899 (0)***
$\xi$	0.918 (0.017)***	0.918 (0.017)***	0.89 (0.017)***	0.895 (0.016)***	0.915 (0.016)***	0.885 (0.017)***	0.891 (0.017)***	0.885 (0.017)***
$\eta$	3.301 (0.325)***	3.153 (0.273)***	3.95 (0.4355)***	3.8685 (0.422)***	3.624 (0.285)***	4.063 (0.4605)***	4.0335 (0.451)***	4.0635 (0.457)***
$\gamma$			0.107 (0.011)***	0.177 (0.02)***				
<i>shift</i>						1.567 (0.331)***		0.393 (0.014)***
<i>rot</i>							1 (0.069)***	1 (0.117)***
<i>LLH</i>	-8303.694	-8304.437	-8158.11	-8186.06	-8328.667	-8143.563	-8154.785	-8143.141

*Notes:*

This table shows the maximum likelihood estimates of various ST-GARCHmodels. The daily returns used on the DAX30 cover the period from 05 gennaio, 1965 to 04 giugno, 2021 (14719 observations).

The mean process is modeled as follows:  $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$  Where, in the 8 GARCH models estimated,  $\gamma$  is the asymmetry in volatility,  $\xi, \kappa$  and  $\eta$  are constant and robust standard errors based on the method of White (1982)) are displayed in parenthesis. *LLH* is the maximized log likelihood value.

Table 3: Maximum likelihood estimates of the SGED-GARCH models with constant skewness and kurtosis parameters

	SGARCH	IGARCH	EGARCH	GJRGARCH	EWMA	NAGARCH	TGARCH	AVGARCH
$\alpha_0$	0.046 (0.011)***	0.045 (0.012)***	0.008 (0.008)	0.014 (0.012)	0.046 (0.015)***	0 (0.01)	0.007 (0.012)	-0.002 (0.013)
$\alpha_1$	-0.055 (0.013)***	-0.055 (0.014)***	-0.041 (0.012)***	-0.05 (0.013)***	-0.056 (0.016)***	-0.043 (0.012)***	-0.047 (0.011)***	-0.042 (0.012)***
$\beta_0$	0.02 (0.005)***	0.015 (0.004)***	0.005 (0.002)**	0.023 (0.004)***	0	0.025 (0.003)***	0.024 (0.005)***	0.025 (0.002)***
$\beta_1$	0.096 (0.012)***	0.104 (0.012)***	-0.151 (0.01)***	0 (0.01)	0.07 (0.008)***	0.059 (0.007)***	0.077 (0.01)***	0.067 (0.003)***
$\beta_2$	0.895 (0.012)***	0.896	0.981 (0)***	0.901 (0.014)***	0.93	0.793 (0.009)***	0.922 (0.01)***	0.9 (0)***
$\xi$	0.935 (0.021)***	0.936 (0.023)***	0.901 (0.019)***	0.906 (0.02)***	0.93 (0.02)***	0.896 (0.017)***	0.902 (0.019)***	0.896 (0.018)***
$\eta$								
$\gamma$			0.107 (0.01)***	0.175 (0.021)***				
<i>shift</i>						1.547 (0.147)***		0.391 (0.011)***
<i>rot</i>							1 (0.076)***	1 (0.114)***
<i>LLH</i>	-8300.515	-8302.804	-8164.576	-8189.218	-8330.811	-8149.596	-8160.239	-8149.442

*Notes:*

This table shows the maximum likelihood estimates of various SGED-GARCH models. The daily returns used on the DAX30 cover the period from 05 gennaio, 1965 to 04 giugno, 2021 (14719 observations).

Table 4: Model selection according to AIC

	SGARCH	IGARCH	EWMA	EGARCH	GJRGARCH	NAGARCH	TGARCH	AVGARCH
N	3.174	3.176	3.198	3.114	3.124	3.107	3.111	3.107
T	3.130	3.130	3.140	3.079	3.089	3.074	3.077	3.074
ST	3.127	3.127	3.135	3.072	3.083	3.067	3.071	3.067
GED	3.128	3.128	3.139	3.080	3.089	3.075	3.079	3.076
SGED	3.125	3.126	3.136	3.075	3.084	3.069	3.073	3.069

*Notes*

This table shows the AIC value for the respective model. With on the rows the distributions.

Table 5: VaR and CVaR test statistics

	EGARCH	GJRGARCH	TGARCH	NAGARCH	AVGARCH
<b>Panel A: SGED</b>					
AE VaR	1.21	1.24	1.21	1.21	1.21
AE ES	1.21**	1.25**	1.21**	1.21*	1.21**
UC	1.15	1.56	1.15	1.15	1.15
CC	1.98	2.44	1.98	1.98	1.98
DQ	7.76	13.29	12.58	5.22	5.84
<b>Panel B: GED</b>					
AE VaR	1.46	1.46	1.49	1.46	1.38
AE ES	1.46***	1.46**	1.5***	1.46**	1.39***
UC	5.18**	5.18**	5.97**	5.18**	3.76*
CC	6.4**	6.4**	7.24**	6.4**	4.86*
DQ	20.23***	16.56**	19.58**	11.69	10.59
<b>Panel C: ST</b>					
AE ES	1.21	1.24	1.28	1.17	1.24
AE ES	1.21	1.25	1.29	1.18	1.25
UC	1.15	1.56	2.03	0.8	1.56
CC	1.98	2.44	2.96	1.58	2.44
DQ	15.06*	13	13.94*	4.55	6.81
<b>Panel D: T</b>					
AE VaR	1.78	1.56	1.56	1.49	1.63
AE ES	1.79	1.57	1.57*	1.5	1.64
UC	13.9***	7.68***	7.68***	5.97**	9.58***
CC	15.71***	9.08**	9.08**	7.24**	11.11***
DQ	0.01	18.99**	22.76***	17.38**	35.87***
<b>Panel E: N</b>					
AE VaR	1.95	1.88	1.95	1.74	1.78
AE ES	1.96***	1.89***	1.96***	1.75***	1.79***
UC	20.22***	17.58***	20.22***	12.76***	13.9***
CC	22.41***	19.61***	22.41***	14.5***	15.71***
DQ	36.5***	32.87***	39.47***	27.1***	37.53***

*Notes:*

Table contains the ratio of actual to expected exceedances for VaR and Conditional VaR, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. \*, \*\* and \*\*\* point out significance at 10, 5 and 1 percent level.

Table 6: Maximum likelihood estimates of the ST-ACD model with constant skewness and kurtosis parameters

	ACD GARCH
$\alpha_0$	0.055 (0.107)
$\alpha_1$	-0.044 (0.096)
$\beta_0$	0.02
$\beta_1$	0.098 (0.002)***
$\beta_2$	0.892 (0.008)***
$\chi_0$	-1.483 (3.984)
$\chi_1$	0.045 (0.121)
$\chi_2$	0.032 (0.169)
$\xi_1$	0.582 (1.134)
$\kappa_0$	0.128 (0.167)
$\kappa_1$	0 (0.111)
$\kappa_2$	1 (0.355)***
$\psi_1$	0.788 (0.098)***

*Notes:*

This table shows the maximum likelihood estimates of various SGED-GARCH models. The daily returns used on the DAX30 cover the period from 05 gennaio, 1965 to 04 giugno, 2021 (14719 observations). The mean process is modeled as follows:  $R_t = \alpha_0 + \alpha_1 \times R_{t-1} + \varepsilon_t$ . Where, in the model estimated,  $\gamma$  is the asymmetry in volatility, (calculated using robust standard errors based on the method of White (1982)) are displayed in parenthesis.

Table 7: VaR and ES test statistics (ACD-ST vs GARCH-ST)

	ACD	GARCH
AE VaR	1.59	1.508
AE CVaR	1.59	1.508
UC	17.973***	13.573***
CC	18.194	13.694
DQ	38.252***	31.168***

*Notes:*

Table contains the ratio of actual to expected exceedances for VaR and Expected Shortfall, the unconditional and conditional coverage test statistic and the dynamic quantile test statistic for VaR. Significance levels for the VaR ratio not reported. \*, \*\* and \*\*\* point out significance at 10, 5 and 1 percent level.

Table 8: Diagnostic Tests for Heteroscedasticity

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
<b>Panel A: Ljung Box Test on the standardized squared values of the residuals</b>								
Norm	31.157*	25.321	22.263	23.03	32.186*	26.208	32.727*	44.066***
T	33.907**	24.81	21.321	21.863	34.34**	26.658	34.183**	41.765***
ST	33.961**	25.024	21.412	22.051	34.607**	26.811	34.187**	40.605***
GED	32.493*	24.826	21.63	22.192	33.364*	26.142	33.361*	42.627***
SGED	32.569*	25.065	21.711	22.341	33.747*	26.351	33.342*	41.333***
<b>Panel B: ARCH LM Test on the standardized squared values of the residuals</b>								
Norm	32.322*	26.461	23.081	23.474	34.475**	26.991	33.857*	42.773***
T	34.687**	25.958	22.063	22.218	37.875**	27.634	34.912**	40.719***
ST	34.605**	26.138	22.129	22.36	38.173**	27.718	34.756**	39.559**
GED	33.431*	25.973	22.433	22.599	36.378**	27.023	34.228**	41.433***
SGED	33.393*	26.173	22.48	22.698	36.859**	27.167	34.071**	40.155***

*Notes*

Table displays the Ljung box statistics and the ARCH LM Test for the standardized squared residuals of the models analyzed. The underlying data is the daily return series of the GERMANY DAX 30 for the period between 1965-01-05 and 2021-06-04.

\*, \*\* and \*\*\* point out respectively significance at 10, 5 and 1 percent level.

The null hypothesis of the test in both panels are described as follows:

$$H_0: Corr(Z_t^2, Z_{t-1}^2) = Corr(Z_t^2, Z_{t-2}^2) = \dots = Corr(Z_t^2, Z_{t-22}^2) = 0$$



Table 9: GMM Tests

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
<b>Panel A: SGED</b>								
Mean	-0.036**	-0.002	0.007	0.006	-0.006	-0.001	-0.036**	-0.039***
Variance	0.015**	0.009	0.007	0.007	0.01	0.008	-0.016**	0.121***
Skewness	-0.46**	-0.392	-0.372	-0.381	-0.406	-0.374	-0.443**	-0.566***
Excess Kurtosis	1.974**	1.782	1.8	1.804	1.693	1.696	1.755**	3.821***
<b>Panel B: GED</b>								
Mean	-0.051***	-0.024*	-0.015	-0.017	-0.026*	-0.022	-0.05***	-0.051***
Variance	0.004***	0.004*	0.004	0.004	0.003*	0.003	-0.024***	0.12***
Skewness	-0.49***	-0.452*	-0.438	-0.444	-0.459*	-0.435	-0.476***	-0.602***
Excess Kurtosis	1.858***	1.763*	1.815	1.808	1.663*	1.689	1.661***	3.824***
<b>Panel C: ST</b>								
Mean	-0.042***	-0.009	0.002	0.001	-0.012	-0.007	-0.042***	-0.049***
Variance	0.006***	0.005	0.003	0.003	0.004	0.005	-0.016***	0.124***
Skewness	-0.473***	-0.414	-0.388	-0.396	-0.421	-0.393	-0.46***	-0.596***
Excess Kurtosis	1.944***	1.79	1.809	1.808	1.671	1.702	1.777***	3.855***
<b>Panel D: T</b>								
Mean	-0.059***	-0.03**	-0.019	-0.02	-0.032**	-0.027*	-0.059***	-0.062***
Variance	-0.007***	0.001**	0.003	0.002	-0.001**	0.002*	-0.024***	0.124***
Skewness	-0.511***	-0.473**	-0.452	-0.458	-0.476**	-0.451*	-0.501***	-0.638***
Excess Kurtosis	1.812***	1.783**	1.848	1.84	1.658**	1.707*	1.674***	3.856***
<b>Panel E: N</b>								
Mean	-0.051***	-0.007	0.004	0.002	-0.01	-0.004	-0.051***	-0.047***
Variance	0.001***	0.001	0	0	0.001	0	-0.035***	0.118***
Skewness	-0.482***	-0.391	-0.371	-0.378	-0.405	-0.372	-0.465***	-0.583***
Excess Kurtosis	1.775***	1.684	1.736	1.721	1.605	1.616	1.516***	3.807***

*Notes* Table displays the GMM test statistics for the standardized residuals. The underlying data is the daily return series of the GERMANY DAX 30 for the period between 1965-01-05 and 2021-06-04. The null hypothesis of the test for each variable are described as follows:  $H_0 : E[z_t] = 0$  for the mean,  $H_0 : E[z_t^2 - 1] = 0$  for the variance.  $H_0 : E[z_t^3] = 0$  for the skewness and  $H_0 : E[z_t^4 - 3] = 0$  for the excess kurtosis.

Table 10: Jarque-Bera Test on standardized residuals

	SGARCH	EGARCH	AVGARCH	NAGARCH	GJRGARCH	TGARCH	IGARCH	EWMA
Norm	739.888***	735.991***	791.98***	784.479***	676.532***	679.569***	804.423***	1382.222***
T	822.929***	789.875***	839.862***	845.393***	703.17***	718.176***	851.465***	1350.929***
ST	845.468***	787.206***	833.066***	842.486***	700.449***	712.976***	884.557***	1360.366***
GED	785.374***	759.794***	810.002***	811.786***	685.789***	697.098***	841.477***	1367.133***
SGED	803.315***	758.539***	803.488***	811.318***	684.155***	691.427***	869.762***	1372.361***

*Notes* Table displays the Jarque-Bera statistic  $JB = \frac{n}{6}(S^2 + \frac{1}{4}(K - 3)^2)$  with  $n$  the sample size,  $K$  the kurtosis and  $S$  the skewness for the residuals of the models. The JB statistic is distributed  $\chi^2$  with  $\nu = 2$ . The underlying data is the daily return series of the GERMANY DAX 30 for the period between 1965-01-05 and 2021-06-04.

\*, \*\* and \*\*\* point out respectively significance at 10, 5 and 1 percent level. The null hypothesis is that  $S$  and  $K$  are not significantly different than what would be found under normality (0 and 3).

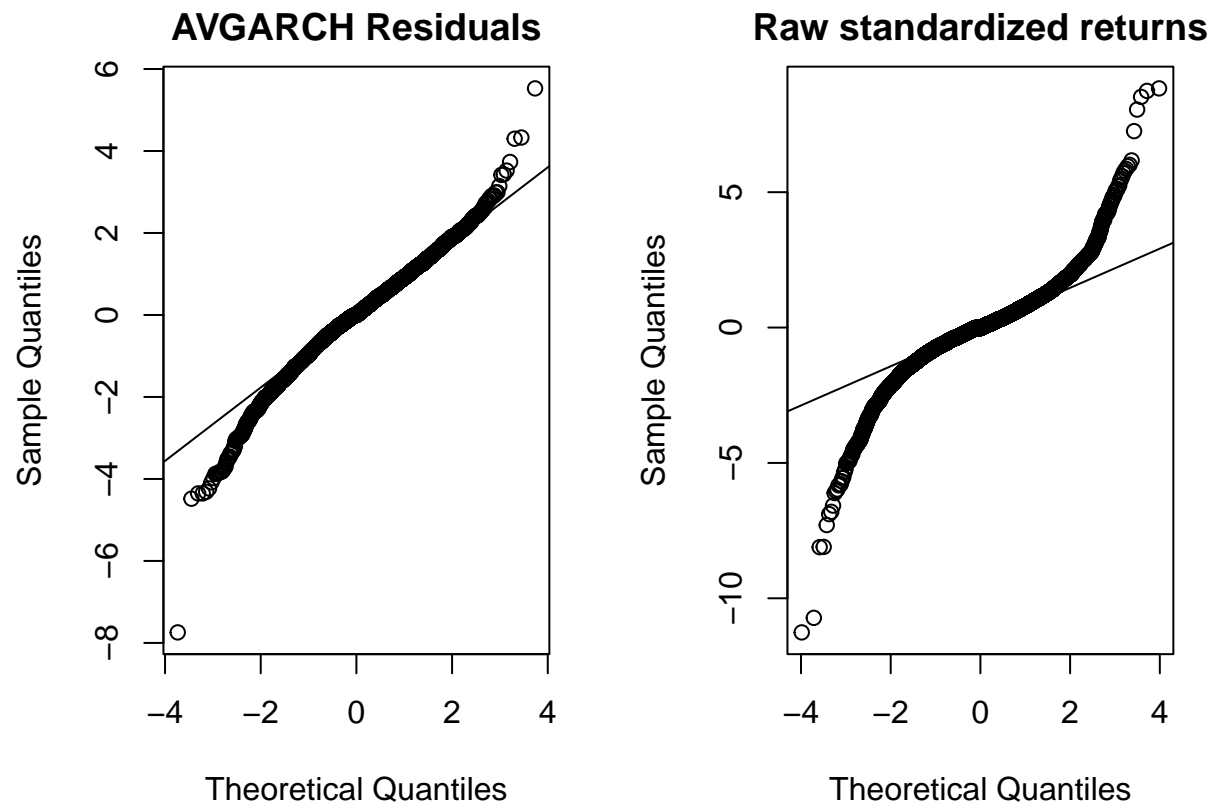


Figure 1: QQ plots of AVGARCH residuals versus the standardized returns of the series