

List of Symbols

Symbol	Interpretation
α_0	The constant in the conditional mean equation. In other papers sometimes referred to as μ .
α_1	The slope in the conditional mean equation.
β_0	The constant in the variance equation and (G)ARCH models. In other papers sometimes referred to as ω .
β_1	The parameter estimate of the innovation process in the variance equation and (G)ARCH models. In other papers sometimes referred to as α . This is also known as the ARCH component (p).
β_2	The parameter estimate of the autoregressive component in the variance equation and GARCH models. In other papers sometimes referred to as β . This is also known as the GARCH component (q).
γ	The leverage term in the asymmetric GARCH models: GJR-GARCH and EGARCH.
ε	The innovation term or “noise” term. The random part or return shock of the previous period in the variance equation in GARCH models.
θ	The vector of parameters to estimate by the log likelihood functions in the maximum likelihood estimation of GARCH and distribution parameters.
η	This is the tail-thickness parameter of the unconditional and conditional distribution. Following the notation of Bali, Mo, and Tang [1], the parameter rises in the SGT distribution. For the T distribution this parameter is equal to the $\nu/2$.
κ	The peakedness parameter which is only present in the SGT distribution and equal to 2 for the other distributions used in this paper.
ν	The degree of freedom parameter from the T distribution.
ξ	The skewness parameter in the skewed distributions (SGT, SGED, ST).

Introduction

A general assumption in finance is that stock returns are normally distributed. However, various authors have shown that this assumption does not hold in practice: stock returns are not normally distributed [Among which 2–4]. For example, Theodossiou [2] mentions that “empirical distributions of log-returns of several financial assets exhibit strong higher-order moment dependencies which exist mainly in daily and weekly log-returns and prevent monthly, bimonthly and quarterly log-returns from obeying the normality law implied by the central limit theorem. As a consequence, price changes do not follow the geometric Brownian motion.” So in reality, stock returns exhibit fat-tails and peakedness [5], these are some of the so-called stylized facts of returns.

Additionally, a point of interest is the predictability of stock prices. Fama [6] explains that the question in academic and business circles is: “To what extent can the past history of a common stock’s price be used to make meaningful predictions concerning the future price of the stock?”. There are two viewpoints towards the predictability of stock prices. Firstly, some argue that stock prices are unpredictable or very difficult to predict by their past returns (i.e. have very little serial correlation) because they simply follow a Random Walk process [7]. On the other hand, Lo & MacKinlay mention that “financial markets *are* predictable to some extent but far from being a symptom of inefficiency or irrationality, predictability is the oil that lubricates the gears of capitalism”. Furthermore, there is also no real robust evidence for the predictability of returns themselves, let alone be out-of-sample [8]. This makes it difficult for corporations to manage market risk, i.e. the variability of stock prices.

Risk, in general, can be defined as the volatility of unexpected outcomes [9]. The measure Value at Risk (VaR), developed in response to the financial disaster events of the early 1990s, has been very important in the financial world. Corporations have to manage their risks and thereby include a future risk measurement. The tool of VaR has now become a standard measure of risk for many financial institutions going from banks, that use VaR to calculate the adequacy of their capital structure, to other financial services companies to assess the exposure of their positions and portfolios. The 5% VaR can be informally defined as the maximum loss of a portfolio, during a time horizon, excluding all the negative events with a combined probability lower than 5% while the Conditional Value at Risk (CVaR) can be informally defined as the average of the events that are lower than the VaR. Bali, Mo, and Tang [1] explains that many implementations of the CVaR have the assumption that asset and portfolio's returns are normally distributed but that it is an inconsistency with the evidence empirically available which outlines a more skewed distribution with fatter tails than the normal. This lead to the conclusion that the assumption of normality, which simplifies the computation of VaR, can bring to incorrect numbers, underestimating the probability of extreme events happening.

This paper has the aim to replicate and update the research made by Bali, Mo, and Tang [1] on US indexes, analyzing the dynamics proposed with a European outlook. The main contribution of the research is to provide the industry with a new approach to calculating VaR with a flexible tool for modeling the empirical distribution of returns with higher accuracy and characterization of the tails.

The paper is organized as follows. Chapter ?? discusses at first the alternative distribution than the normal that we are going to evaluate during the analysis (Student's t-distribution, Generalized Error Distribution, Skewed t-distribution, Skewed Generalized Error Distribution, Skewed Generalized t-distribution), then the discrete time GARCH models used (GARCH, IGARCH, EGARCH, GJRARCH, NAGARCH, TGARCH, TSGARCH or AVGARCH and EWMA) are presented as

extensions of the Engle [10] 's ARCH model. Chapter ?? describes the dataset used and the methodology followed in modeling the volatility with the GARCH model by Bollerslev [11] and with its refinements using Maximum likelihood estimation to find the distribution parameters. Then a description is given of how are performed the control tests (un- and conditional coverage test, dynamic quantile test) used in the paper to evaluate the performances of the different GARCH models and underlying distributions. In chapter ??, findings are presented and discussed, in chapter ?? the results of the performed control tests are shown and interpreted and in chapter ?? the investigation and the results are summarized.

References

- [1] Turan G. Bali, Hengyong Mo, and Yi Tang. “The role of autoregressive conditional skewness and kurtosis in the estimation of conditional VaR”. In: *Journal of Banking and Finance* 32.2 (Feb. 2008). Publisher: North-Holland, pp. 269–282.
- [2] Peter Theodossiou. “Skewed Generalized Error Distribution of Financial Assets and Option Pricing”. In: *SSRN Electronic Journal* (2000). URL: <http://www.ssrn.com/abstract=219679>.
- [3] M.T. Subbotin. “On the Law of Frequency of Error.” In: *Matematicheskii Sbornik* 31 (1923), pp. 296–301.
- [4] Panayiotis Theodossiou. “Skewed Generalized Error Distribution of Financial Assets and Option Pricing”. In: *Multinational Finance Journal* 19.4 (2015), pp. 223–266.
- [5] R. R. Officer. *The Distribution of Stock Returns*. Tech. rep. 340. 1972, pp. 807–812.
- [6] Eugene F. Fama. “The Behavior of Stock-Market Prices”. In: *The Journal of Business* 38.1 (1965), pp. 34–105. URL: <http://www.jstor.org/stable/2350752>.
- [7] Eugene Fama. *Efficient Capital Markets: A Review of Theory and Empirical Work*. Tech. rep. 2. 1970, pp. 383–417.
- [8] Ivo Welch and Amit Goyal. “A Comprehensive Look at The Empirical Performance of Equity Premium Prediction”. In: *Review of Financial Studies* 21.4 (July 2008), pp. 1455–1508. URL: <https://academic.oup.com/rfs/article-lookup/doi/10.1093/rfs/hhm014>.
- [9] Philippe Jorion. *Value at Risk: The New Benchmark For Managing Financial Risk*. 3rd ed. McGraw-Hill, 2007.
- [10] R. F. Engle. “Autoregressive Conditional Heteroscedacity with Estimates of variance of United Kingdom Inflation,journal of Econometrica, Volume 50, Issue 4 (Jul., 1982),987-1008.” In: *Econometrica* 50.4 (1982), pp. 987–1008.
- [11] Tim Bollerslev. “Generalized Autoregressive Conditional Heteroskedasticity”. In: *Journal of Econometrics* 31 (1986), pp. 307–327.