

1

Data and methodology

1.1 Data

Here comes text...

1.1.1 Descriptives

Table of summary statistics

Here comes a table and description of the stats

Descriptive figures

As can be seen

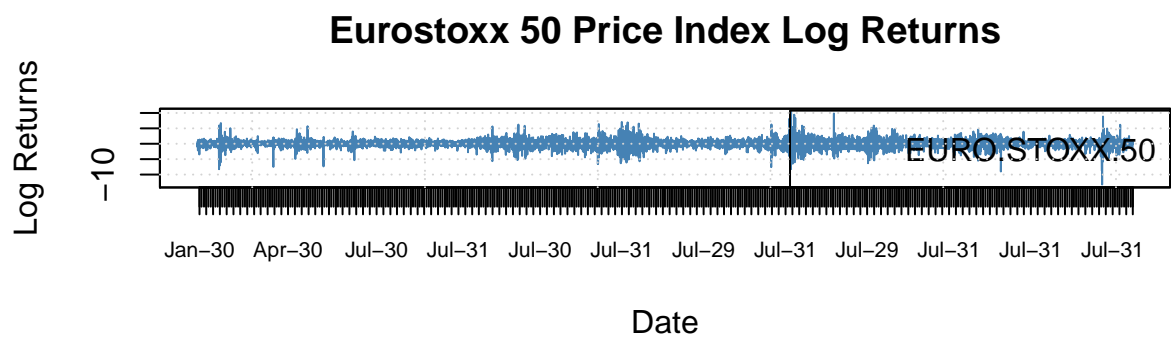
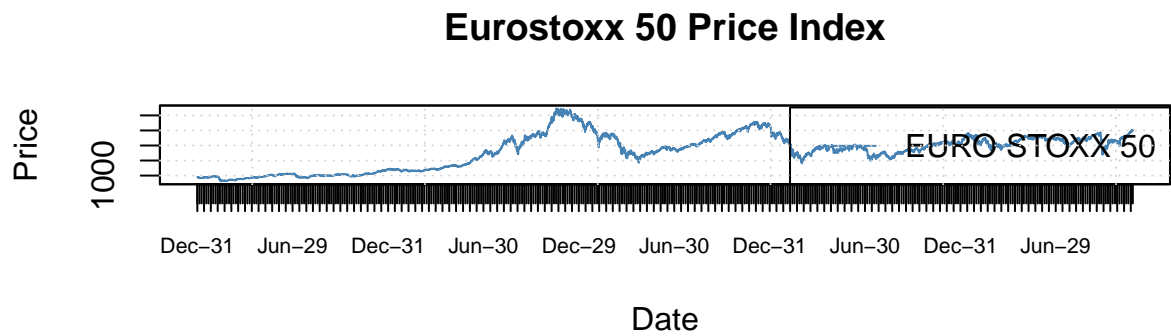


Figure 1.1: Eurostoxx 50 prices and returns

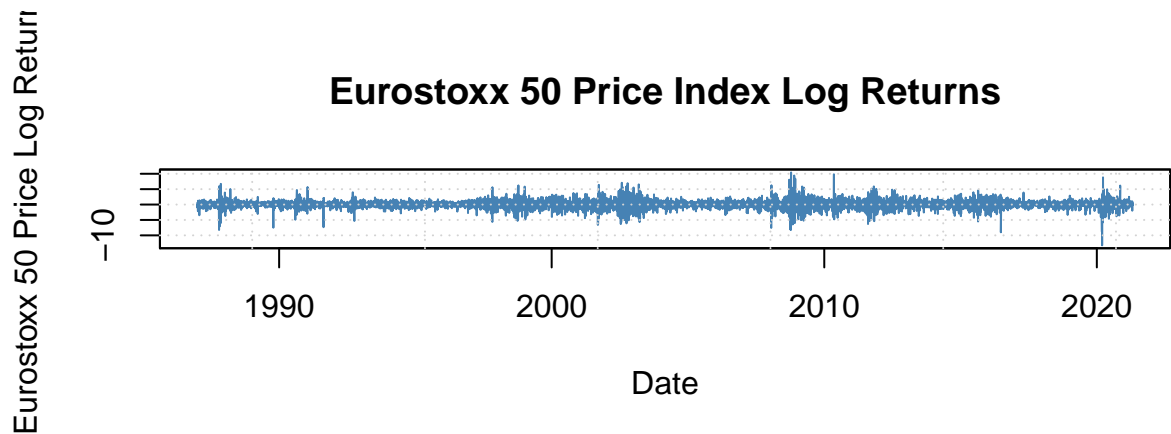
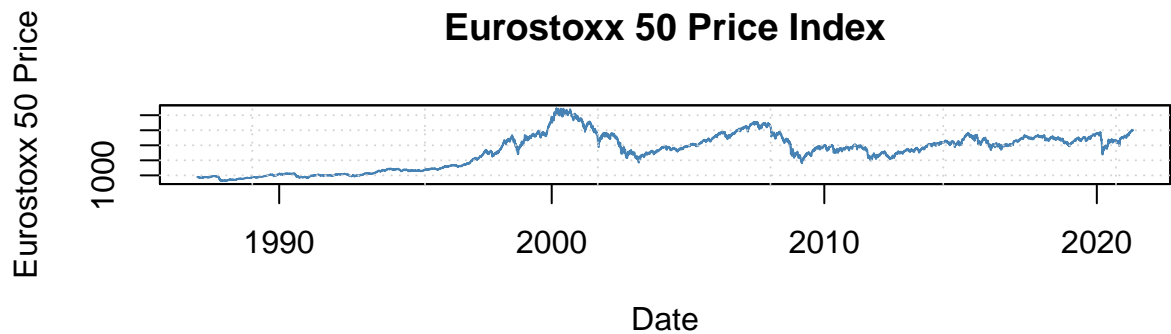
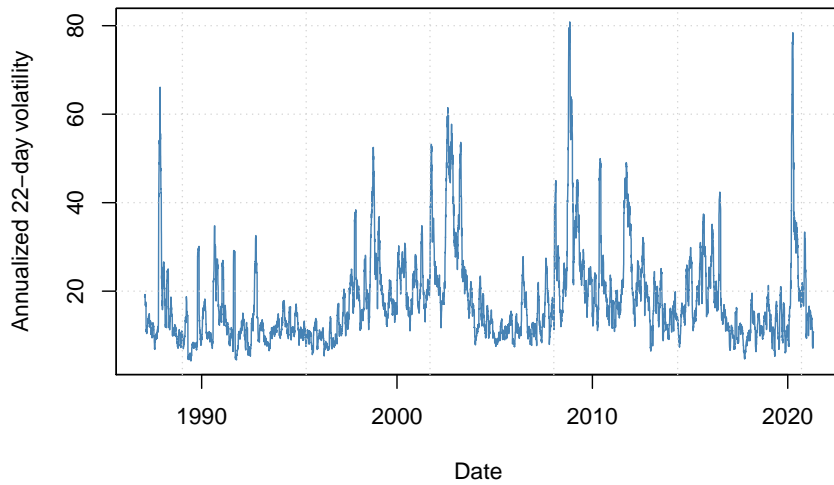


Figure 1.2: Eurostoxx 50 prices and returns

Table 1.1: Summary statistics of the returns

Statistics	Eurostoxx.50	Standardized.Residuals
Arithmetic Mean	-13.2404	-11.7732
Stdev	0.0357	-0.0193
Skewness	0.0167	-0.0409
NA	10.4376	5.7126
NA	1.307	0.9992
Skewness	-0.31 (0***)	-0.6327 (0***)
Excess Kurtosis	7.2083 (0***)	5.134 (0***)
Jarque-Bera	19528.6196***	10431.0514***

Note: This table shows the descriptive statistics of the returns of over the period 1987-01-01 to 2021-04-27. Including minimum, median, arithmetic average, maximum, standard deviation, skewness and excess kurtosis.

Eurostoxx 50 rolling 22-day volatility (annualized)**Figure 1.3:** Eurostoxx 50 rolling volatility (22 days, calculated over 252 days)

1.1.2 Methodology

1.1.3 Garch models

As already mentioned in . . . , GARCH models GARCH, EGARCH, IGARCH, GJR-GARCH, NGARCH, TGARCH and NAGARCH (or TSGARCH) will be estimated. Additionally the distributions will be examined as well, including the normal, student-t distribution, skewed student-t distribution, generalized error distribution,

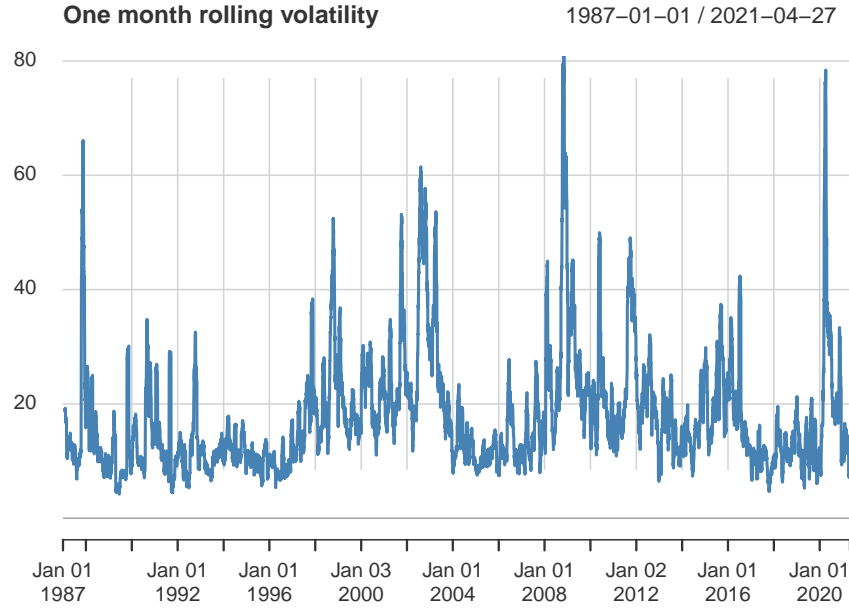


Figure 1.4: Eurostoxx 50 rolling volatility (22 days, calculated over 252 days)

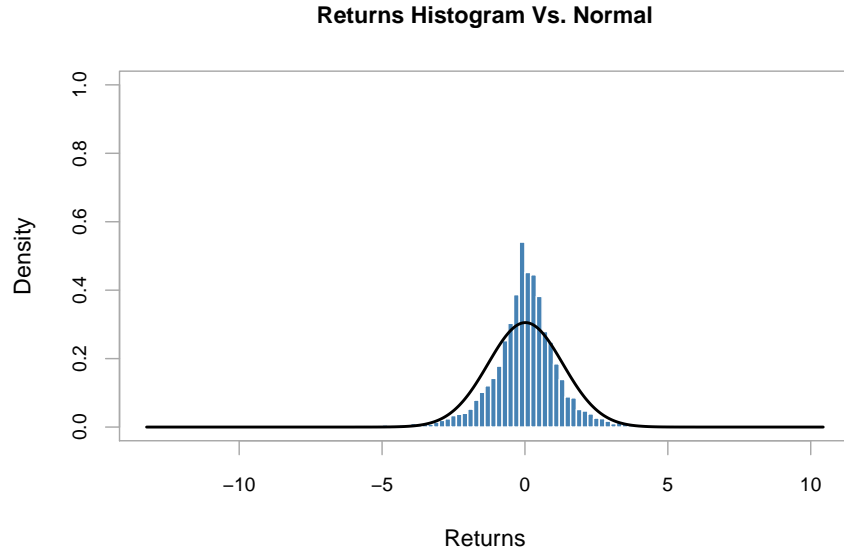


Figure 1.5: Density vs. Normal Eurostoxx 50 log returns)

skewed generalized error distribution and the skewed generalized t distribution.

They will be estimated using maximum likelihood. As already mentioned, fortunately, Alexios Ghalanos [1] has made it easy for us to implement this methodology in the R language (version 3.6.1) with the package “rugarch” version 1.4-4 (R univariate garch), which gives us a bit more time to focus on the results and the interpretation. Additionally

Maximum likelihood estimation is a method to find the distribution parameters

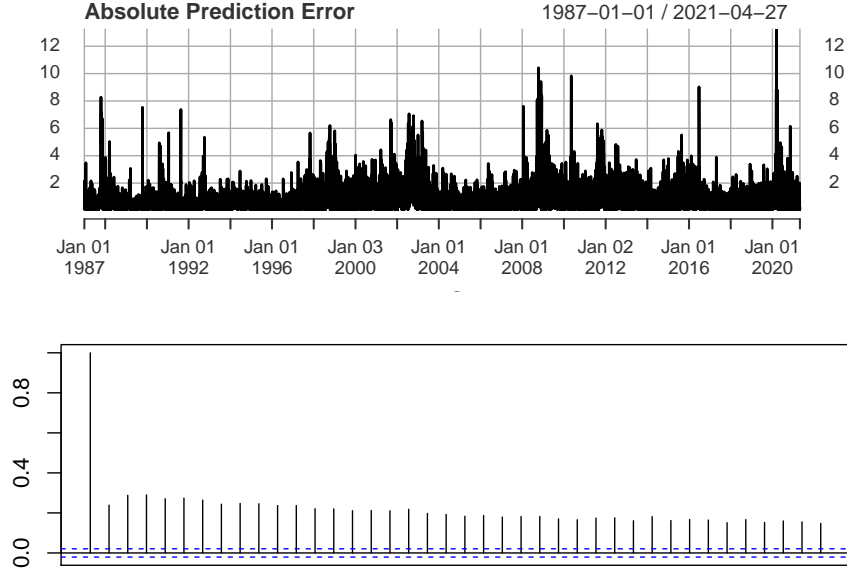


Figure 1.6: Absolute prediction errors

that best fit the observed data, through maximization of the likelihood function, or the computationally more efficient log-likelihood function (by taking the natural logarithm). It is assumed that the return data is i.i.d. and that there is some underlying parametrized density function f with one or more parameters θ that generates the data ((1.2)). These functions are based on the joint probability distribution of the observed data (equation (1.4)). Subsequently, the (log)likelihood function is maximized using an optimization algorithm (equation (1.6)).

$$y_1, y_2, \dots, y_N \sim i.i.d \quad (1.1)$$

$$y_i \sim f(y|\theta) \quad (1.2)$$

$$L(\theta) = \prod_{i=1}^N f(y_i|\theta) \quad (1.3)$$

$$\log(L(\theta)) = \sum_{i=1}^N \log f(y_i|\theta) \quad (1.4)$$

$$\theta^* = \arg \max_{\theta} [L] \quad (1.5)$$

$$\theta^* = \arg \max_{\theta} [\log(L)] \quad (1.6)$$

```

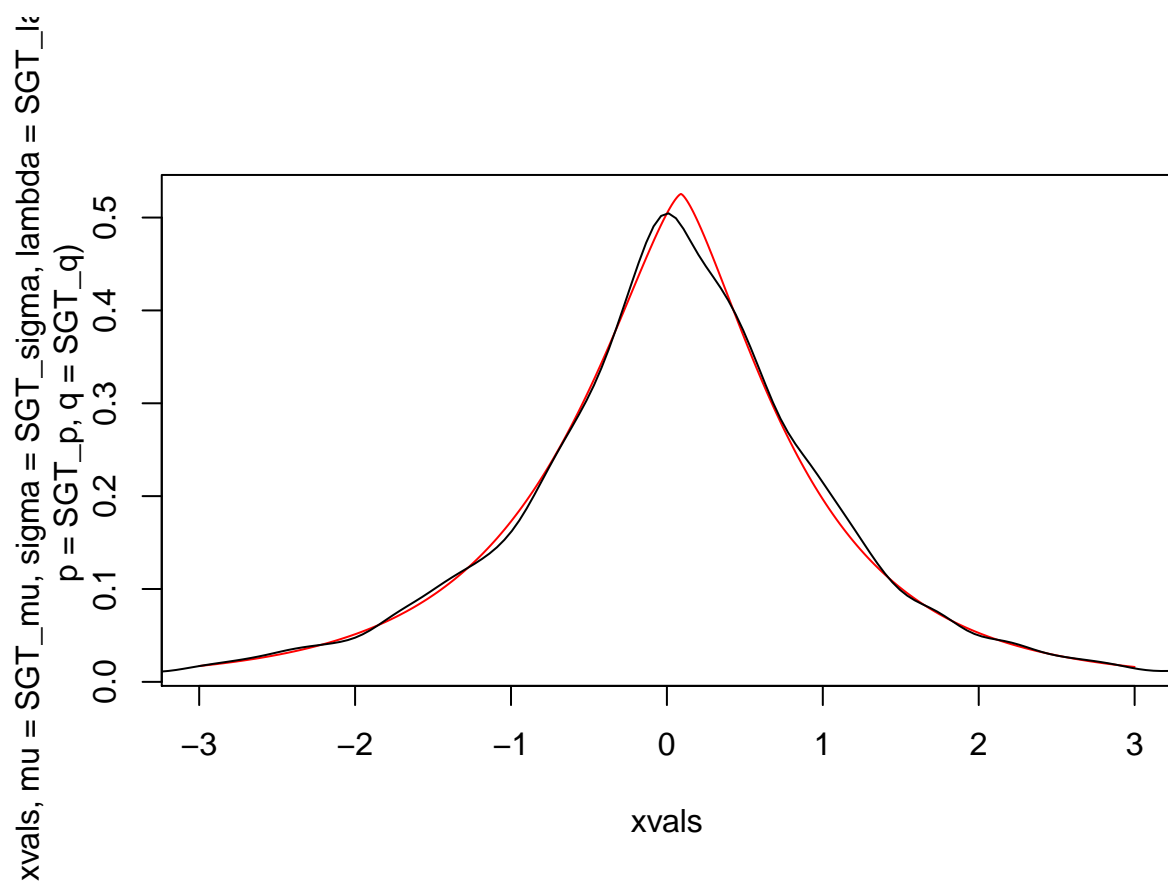
##          mean          sd
## 0.01668214 1.30689172
##          mean          sd
## 0.01381119 0.00976596
## [1] -15101.73
##          df          ncp
## 4.31096001 0.03168827
##          df          ncp
## 0.14857777 0.01100453
## [1] -14149.5
##          mean          sd          nu
## 0.03160393 1.27550013 0.91274249
##          mean          sd          nu
## 0.008555584 0.015772159 0.016622605
## [1] -14009.53
##          mean          sd          nu          xi
## 0.01946361 1.27515748 0.91513166 0.98174821
##          mean          sd          nu          xi
## 0.013176090 0.015786515 0.016652983 0.009638209
## [1] -14008.63
##          mean          sd          nu          xi
## 0.0187729 1.4868913 2.7847974 0.9485825
##          mean          sd          nu          xi
## 0.01375064 0.05550991 0.09972285 0.01270650
## [1] -13997.35
## Skewed Generalized T MLE Fit
## Best Result with BFGS Maximization
## Convergence Code 0: Successful Convergence
## Iterations: NA, Log-Likelihood: -13973.01

```

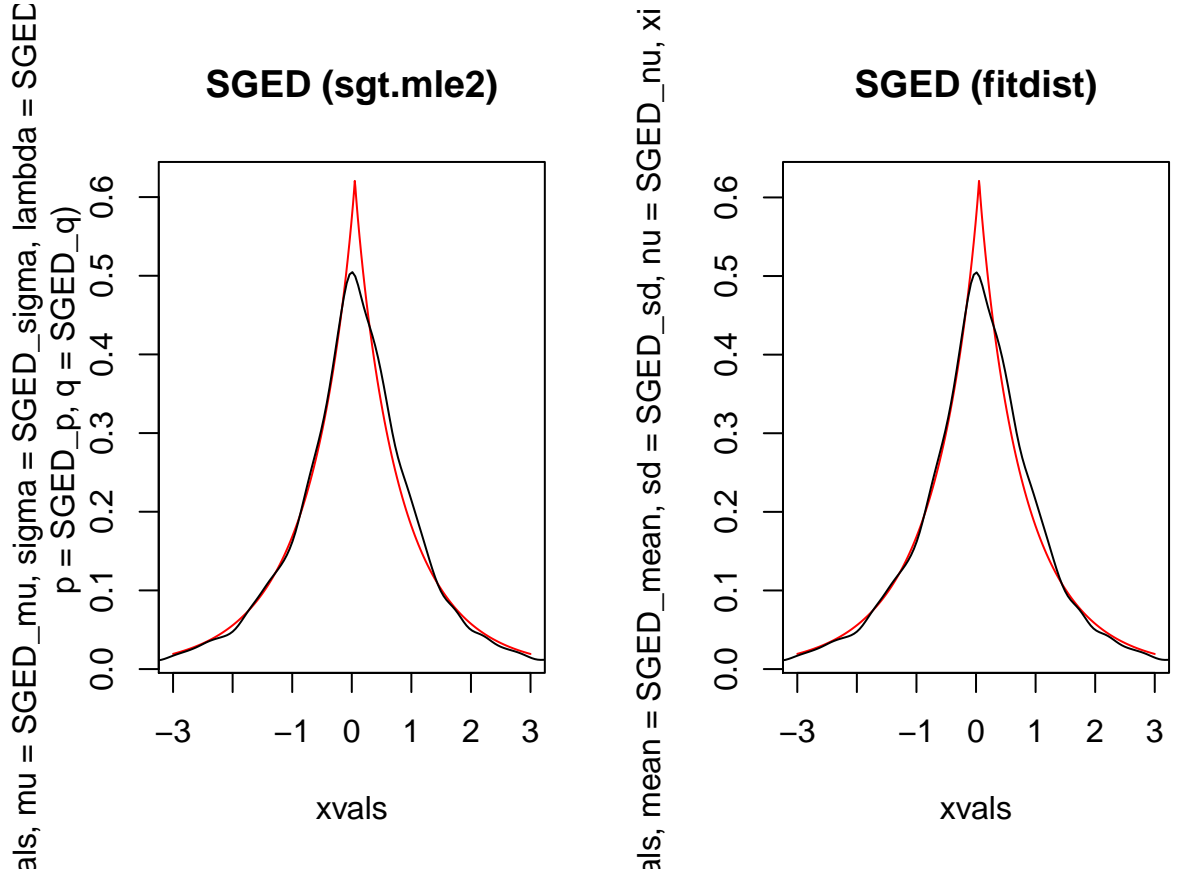
```

##
##          Est. Std. Err.      z    P>|z|
## mu      0.0204      0.0131  1.5574 0.1194
## sigma   1.3214      0.0261 50.5971 0.0000 ***
## lambda -0.0397      0.0126 -3.1583 0.0016 **
## p       1.3818      0.0708 19.5077 0.0000 ***
## q       3.3093      0.5333  6.2058 0.0000 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Fitting of the distribution ' sgt ' by maximum likelihood
## Parameters :
##          estimate Std. Error
## mu      0.01974156 0.01263035
## sigma   1.27919321 0.01674109
## lambda -0.03189521 0.01159236
## p       1.09667765      NaN
## q       9.37999498      NaN
## Loglikelihood: -13984.5   AIC:  27978.99   BIC:  28014.49
## Correlation matrix:
##          mu      sigma      lambda    p    q
## mu      1.00000000 -0.04998713 0.70347249 NaN NaN
## sigma  -0.04998713  1.00000000 0.04648083 NaN NaN
## lambda  0.70347249  0.04648083 1.00000000 NaN NaN
## p              NaN      NaN      NaN    1 NaN
## q              NaN      NaN      NaN NaN    1

```



```
##      mean      sd      nu      xi
## 0.01946361 1.27515748 0.91513166 0.98174821
```

```
## [1] -14008.18
```

```
## [1] -14008.63
```

```
Aθ + B = 0 ##### Control Tests
```

Unconditional coverage test of Kupiec [2] A number of tests are computed to see if the value-at-risk estimations capture the actual losses well. A first one is the unconditional coverage test by Kupiec [2]. The unconditional coverage or proportion of failures method tests if the actual value-at-risk exceedances are consistent with the expected exceedances (a chosen percentile, e.g. 1% percentile) of the VaR model. Following Kupiec [2] and Ghalanos [3], the number of exceedence follow a binomial distribution (with thus probability equal to the significance level or expected proportion) under the null hypothesis of a correct VaR model. The test is conducted as a likelihood ratio test with statistic like in equation (1.7), with p the probability of an exceedence for a confidence level, N the sample size and X the number of exceedence. The null hypothesis states that the test statistic

LR^{uc} is χ^2 -distributed with one degree of freedom or that the probability of failure \hat{p} is equal to the chosen percentile α .

$$LR^{uc} = -2 \ln \left(\frac{(1-p)^{N-X} p^X}{\left(1 - \frac{X}{N}\right)^{N-X} \left(\frac{X}{N}\right)^X} \right) \quad (1.7)$$

Conditional coverage test of Christoffersen, Hahn, and Inoue [4] Christoffersen, Hahn, and Inoue [4] proposed the conditional coverage test. It is tests for unconditional coverage and serial independence. The serial independence is important while the LR^{uc} can give a false picture while at any point in time it classifies inaccurate VaR estimates as “acceptably accurate” [5]. For a certain VaR estimate an indicator variable, $I_t(\alpha)$, is computed as equation (1.8).

$$I_t(\alpha) = \begin{cases} 1 & \text{if exceedence occurs} \\ 0 & \text{if no exceedence occurs} \end{cases} \quad (1.8)$$

It involves a likelihood ratio test’s null hypothesis is that the statistic is χ^2 -distributed with two degrees of freedom or that the probability of violation \hat{p} (unconditional coverage) as well as the conditional coverage (independence) is equal to the chosen percentile α .

Dynamic quantile test Engle2004 with the aim to provide completeness to the conditional coverage test of Christoffersen, Hahn, and Inoue [4] developed the Dynamic quantile test. It consists in testing some restriction in a

References

- [1] Alexios Ghalanos. *rugarch: Univariate GARCH models*. R package version 1.4-4. 2020.
- [2] P.H. Kupiec. “Techniques for Verifying the Accuracy of Risk Measurement Models”. In: *Journal of Derivatives* 3.2 (1995), pp. 73–84.
- [3] Alexios Ghalanos. *Introduction to the rugarch package. (Version 1.4-3)*. Tech. rep. 2020. URL: <http://cran.r-project.org/web/packages/>.
- [4] Peter Christoffersen, Jinyong Hahn, and Atsushi Inoue. “Testing and comparing Value-at-Risk measures”. In: *Journal of Empirical Finance* 8.3 (July 2001), pp. 325–342. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0927539801000251>.
- [5] Turan G. Bali and Panayiotis Theodossiou. “A conditional-SGT-VaR approach with alternative GARCH models”. In: *Annals of Operations Research* 151.1 (Feb. 22, 2007), pp. 241–267. URL: <http://link.springer.com/10.1007/s10479-006-0118-4>.