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# Kolmogorov axioms and Normal distribution in R

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# Contents

# Chapter 1

# Kolmogorov axioms

## 1.1 Historical background

Around 1900 the axiomatic approach to mathematics had spread well beyond its classical setting of Euclidean geometry, and the particular question of how to axiomatize Probability was highlighted as part of Hilbert's sixth problem:

Mathematical Treatment of the Axioms of Physics. The investigations on the foundations of geometry suggest the problem: To treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.

While Probability certainly involves some conceptually extra idea (relative to the rest of Mathematics), the issue was whether Probability required some new technical ingredient to be added to the rest of Mathematics. Kolmogorov's achievement was the realization that it didn't. Measure theory had been recently developed to resolve the technical conflict between the intuitive idea "every region in the plane has some area" and the axioms of set theory dealing with every subset of an uncountable set. This conflict has no conceptual connection with Probability, but Kolmogorov realized that the technical machinery (involved in its resolution) of measures, measurable sets, measurable functions could be reused as an axiomatic setting for Probability. In retrospect, because one special model within Probability is "pick a uniform random point from the unit square", it is clear that any general theory of Probability has to include measure theory, but (to reiterate) Kolmogorov's achievement was the realization that at the technical level it didn't require anything more.

With agreed axioms, mathematicians happily moved on with systematic development of theorem-proof Probability. The firm connection to the rest of theorem-proof Mathematics enabled researchers to use tools from other fields of Mathematics, particularly in the context of limit theorems. More prosaically, it is helpful to have coherent notation covering both discrete and continuous probability distributions and random variables.

## 1.2 The Axioms of Probability

### 1.2.1 Axiom 1: All probabilities are nonnegative

$$P(A) \geq 0 \text{ for all events A.}$$

### 1.2.2 Axiom 2: The probability of the whole sample space is 1

$$P(S) = 1.$$

### 1.2.3 Axiom 3 (Addition Rule): If two events A and B are disjoint (have no outcomes in common)

$$P(A \cup B) = P(A) + P(B)$$

The assumption that **P** is defined on a field guarantees that these axioms are non-vacuously instantiated, as are the various theorems that follow from them. The non-negativity and normalization axioms are largely matters of convention, although it is non-trivial that probability functions take at least the two values 0 and 1, and that they have a maximal value (unlike various other measures, such as length, volume, and so on, which are unbounded). We will return to finite additivity at a number of points below.

# Chapter 2

# Normal distribution in R

## 2.1 Normal distribution

The normal distribution is the most important distribution in statistics, since it arises naturally in numerous applications. The key reason is that large sums of (small) random variables often turn out to be normally distributed. A random variable $X$ is said to have the normal distribution with parameters $\mu$ and $\sigma$ if its density function is given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)\right\}$$

### 2.1.1 Generation of Random Normal Distribution Seed

```
size=1000
mean = 1
SD =0.6
x = rnorm(size,mean,SD)
plot(z,x)
hist(x)
plot(density(x))
```

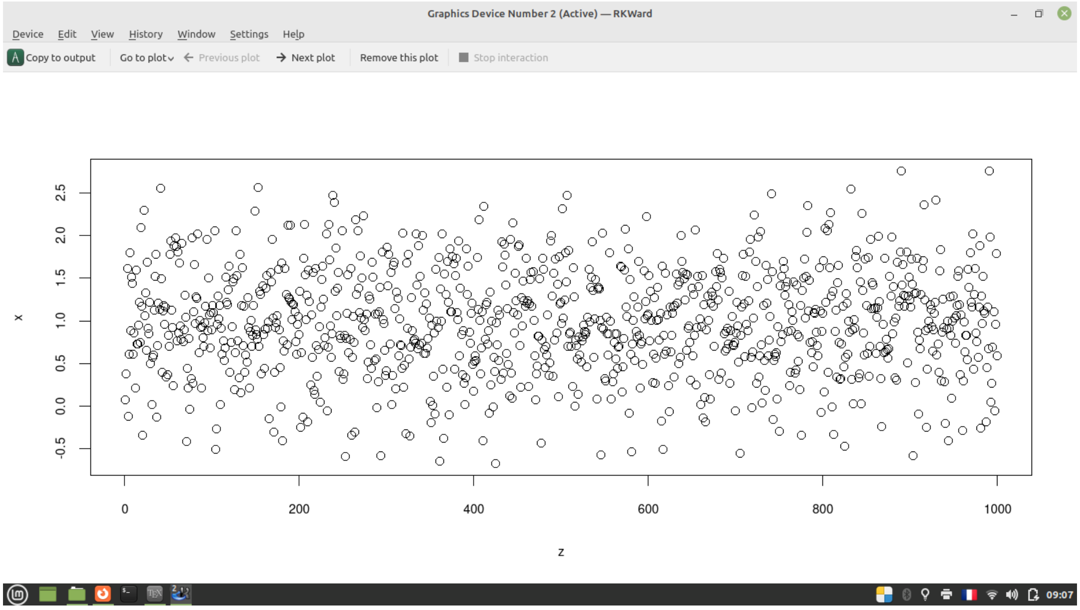


FIGURE 2.1: Seed plot.


FIGURE 2.2: Seed Histogram.

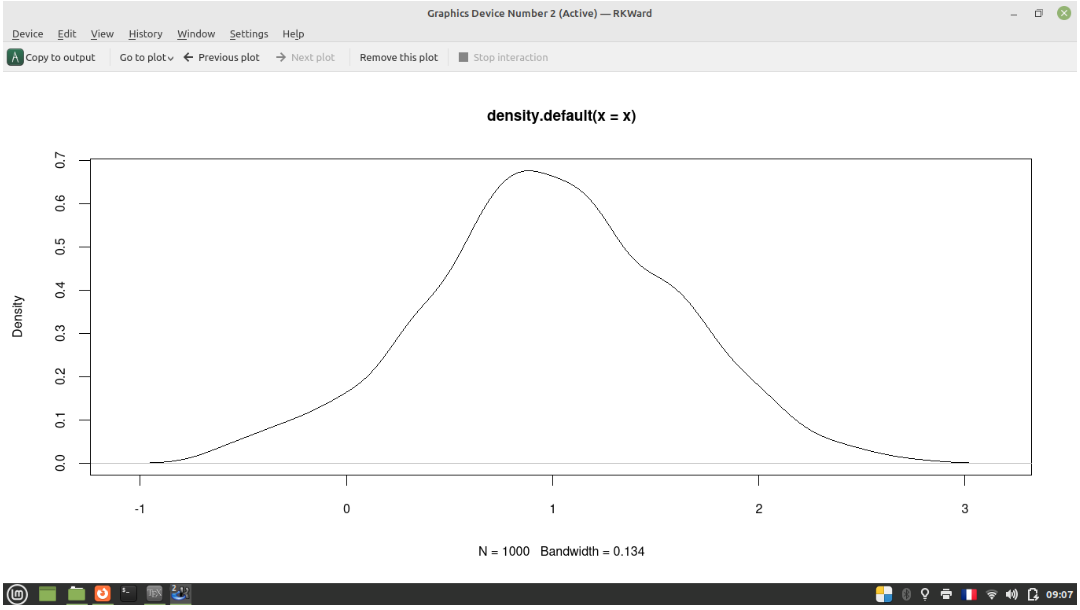


FIGURE 2.3: Seed Density.

### 2.1.2 Standard Deviation from Expected Values

```
# Expectation of Squares
for(i in 1:1000)
y[i]=sum((x[1:i])^2)/i

# Squares of Expectations
for(i in 1:1000)
h[i]=sum(x[1:i]/i)^2

# Finding Standard Deviation
for(i in 1:1000)
d=sqrt(y-h)
d
plot(z,d)
```

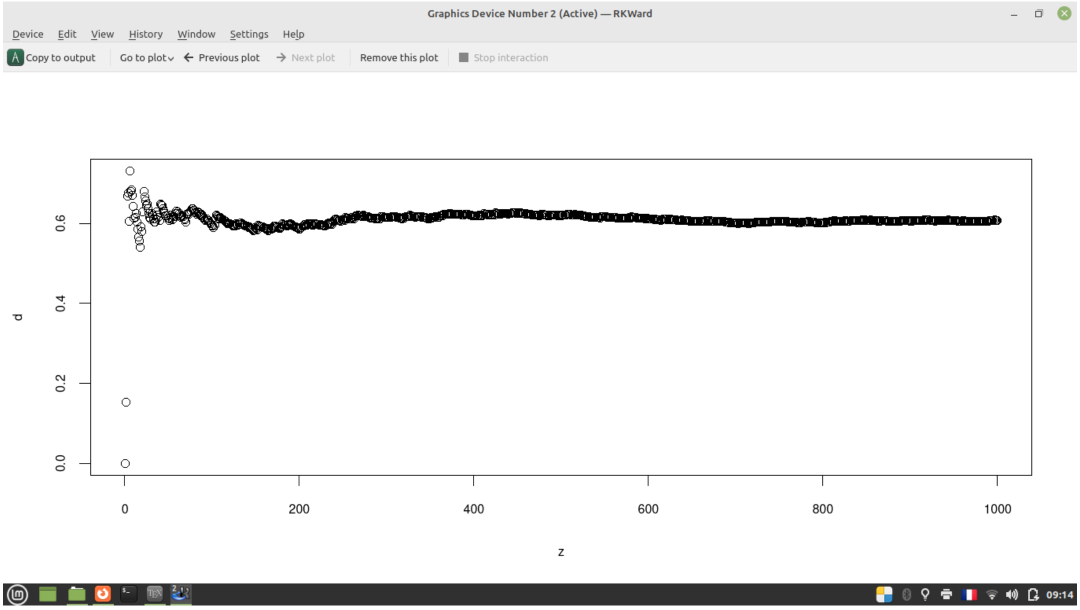


FIGURE 2.4: Standard Deviation plot.

### 2.1.3 Conclusion

As we can see w started our randomly generated seed of normal deattributed population of 1000 with a mean of 1 and standard deviation of 0.6 with the help of **rnorm** function in R. then we calculated the square of the sum of our expected value, and the expected values of the sum of squares. then we subtracted them and took the square root of the of the result which is a formula for finding the standard deviation, but when applied to our population we get a curve that starts a bit random then slowly converges to the standard deviation of 0.6.

# Appendix A

# Appendix A

## A.1 R code

```
# Declaration of variables
y=0
h=0
d=1
z=1:1000

# Generation of Random Normal Distribution Seed
size=1000
mean = 1
SD =0.6
x = rnorm(size,mean,SD)
plot(z,x)
hist(x)
plot(density(x))

# Expectation of Squares
for(i in 1:1000)
y[i]=sum((x[1:i])^2)/i

# Squares of Expectations
for(i in 1:1000)
h[i]=sum(x[1:i]/i)^2

# Finding Standard Deviation
for(i in 1:1000)
d=sqrt(y-h)
d
plot(z,d)
```

# Bibliography

[1] Alan Hájek. "Interpretations of Probability". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Fall 2019. Metaphysics Research Lab, Stanford University, 2019. URL: https://plato.stanford.edu/archives/fall2019/entries/probability-interpret/ (visited on 04/07/2022).

[2] *What is the significance of the Kolmogorov axioms?* URL: https://www.stat.berkeley.edu/~aldous/Real_World/kolmogorov.html (visited on 04/07/2022).