# A Stable Walking Strategy of Quadruped Robot Based on Foot Trajectory Planning

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Abstract— For a quadruped robot to implement steady moving on horizontal planes, avoiding the impact against the ground, a foot trajectory planning method underling low contact impact was presented. At first, kinematic model of the quadruped robot though D-H algorithm have been established. Then, the foot trajectories, which are continuous not only in displacements and velocities but also in accelerations, was scheduled based on the walking gait. Lastly, the steering and straight line movements of the robot were simulated, simulation experiments show that the quadruped robot motion stably on horizontal planes without impact and sliding.

Keywords-quadruped robot; kinematic; gait planning; foot trajectory; steering and straight line movement

## I. Introduction

As an integral part of the mobile robot, legged robot is gradually becoming the hotspot. With its discontinuous support motion and good performance in capacity, mobility and versatility, quadruped robot, such as BigDog [1] and HyQ [2] has great potential for applications in a complex environment. According to the nature of the stability, the gait of quadruped robots are classified into static gaits and dynamical gaits. Static gaits focuses on stability, whereas dynamical gaits focuses on fast and efficient [3]. Recently, a large number of control method, such as floating-base inverse dynamics control and predictive force control introduced in [4], have been proposed to control quadruped robots walking over non-perceived obstacles and rocky terrain. For steady moving on horizontal planes, the idea of planning smooth trajectories for robot motion has been proposed by several researchers [5-7]. The trajectory of quadruped robot like Cubic Trajectory, Cubic mixed with the linear Trajectory, Sinusoidal Trajectory and quintic polynomial Trajectory has been widely used. In this paper, quintic polynomial mixed with the linear interpolation algorithm based on static gaits were presented to make the robot do steering and straight line motion.

This paper is organized as follows. Kinematic model of the quadruped robot though D-H algorithm have been established in Section II. According to different forms of movement, gaits planning method underling low contact impact is presented. And then the foot trajectory underling low contact impact was scheduled based on the walking gait in Section III. In Section IV, the steering and straight line Qifeng Yang and Chong Tang

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movements of the robot were simulated, the simulation experiments show that the quadruped robot motion stably on horizontal planes without impact and sliding. Section V gives the conclusions of the paper.

### II. KINEMATIC MODEL OF THE QUADRUPED ROBOT

The quadruped robot of this paper is about 0.6m long, 0.4m wide, 0.5m tall, and weighs approximately 30kg. On the premise of not increasing the redundancy, the performance of the quadruped robot is satisfied, each leg of the quadruped robot has 3 degrees of freedom, actuated by motors with large gear ratios, two at each hip and one at each knee. For convenience of control and design, front and rear legs have same configuration.



Figure 1. Virtual prototype of the quadruped robot

According to the structure of the Fig. 1, single-leg coordinate system based on D-H algorithm of the quadruped robot has been established in Fig. 2, where L1, L2, and L3 denoted the length of hip, thigh and crus, respectively.

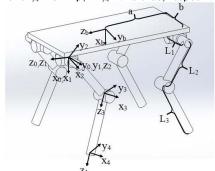


Figure 2. Illustration of the left front leg coordinate system



TABLE I. D-H PARAMETERS

i	$\alpha_{i-1}$	$\mathfrak{a}_{\scriptscriptstyle i\text{-}1}$	di	фі
1	0	0	0	$\boldsymbol{\Theta}_1$
2	<b>-</b> π/2	0	0	$\theta_2$
3	0	$L_2$	L <sub>1</sub>	$\theta_3$
4	0	$L_3$	0	0

Based on the homogeneous transformation matrix, we get the pose matrix of the foot

$${}_{4}^{0}T = {}_{1}^{0}T \cdot {}_{2}^{1}T \cdot {}_{2}^{2}T \cdot {}_{3}^{2}T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$$
 (1)

Here we are only concerned about the position of the foot.

$$P = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} -L_3 L_1 c_1 c_2 s_1 + L_2 c_1 c_2 \\ L_3 L_1 s_1 c_2 s_2 + L_2 s_1 c_2 \\ -L_3 s_2 s_3 - L_2 s_2 \end{bmatrix}$$
(2)

Vice versa, each joint angle of the leg can be obtained with the assistance of the inverse kinematics as long as give the desired location of foot in task space.

$$\theta_{1} = \arctan(\frac{p_{y}}{p_{x}}) - \arctan(\frac{L_{1}}{\sqrt{(p_{x}^{2} + p_{y}^{2} - L_{1}^{2})}})$$
 (3)

$$\theta_3 = \arccos(\frac{p_x^2 + p_y^2 + p_z^2 - L_1^2 - L_2^2 - L_3^2}{2L_1L_2})$$
 (4)

$$\theta_2 = \arctan\left(\frac{L_2 s_3 (p_x c_1 + p_y s_1) - (L_3 + L_2 c_3) p_z}{L_1 p_z s_3 - (L_3 + L_2 c_3) (p_z p_x c_1 + p_y s_1)}\right) - \theta_3$$
 (5)

## III. GAIT TRAJECTORY PLANNING

## A. Gait Planning

Different gaits, such as walk, pace, trot and gallop, are widely used in quadruped animals to adapt to rough terrain environment and speeds. In this paper, we use the quasistatic walking gait that the projection of the center of gravity (COG) of the quadruped robot is always within the polygon which is composed of the supporting legs [8].

One of six swing possible leg sequences satisfies the requirement that the COG of the quadruped robot always within the polygon as the quadruped go straight. The gait cycle as follows: Front-Left to Back-Right, Back-Right to Front-Right, Front-Right to Back-Left. An overview of this cyclic gait is sketched in Fig. 3. The sequence diagram of walk is shown in Fig. 4.

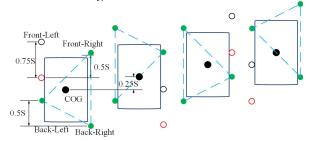


Figure 3. The linear walk gait locomotion strategy

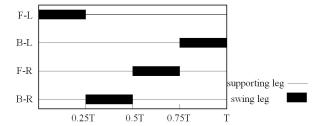


Figure 4. The sequence diagram of linear walk

Fig. 5 and Fig. 6 show the gait cycle and sequence diagram of the steering walk gait. The gait cycle as follows: Front-Left to Back- Left, Back- Left to Back-Right, Back-Right to Front-Right, Front-Right to Front-Left.

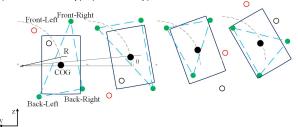


Figure 5. The steering walk gait locomotion strategy

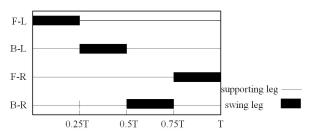


Figure 6. The sequence diagram of steering walk

### B. Zero Impact Trajectory Planning

In order to prevent the foot contact with the ground to generate a larger sliding and impact, [5] mentioned that the swing leg fall back before the foot contact with the ground and the body move along the Z axis at a constant speed during the swing phase. However, the trajectories of foot in [5] still have problem, such as acceleration jump at began and end point, which may enlarge the contact forces between ground and the foot. [6] presented the algorithm of trajectory planning that the velocity and accelerations along the X axis of the swing leg is zero at movement of touchdown and liftoff in (6).

$$g(H,T,t) = \begin{cases} 2H \left[ \frac{8t}{T} - \frac{1}{4\pi} \sin(4\pi \frac{8t}{T}) \right] & 0 \le t < \frac{T}{8} \\ 2H \left[ 1 - (\frac{8t}{T} - \frac{1}{4\pi} \sin(4\pi \frac{8t}{T})) \right] \frac{T}{8} \le t < \frac{T}{4} \end{cases}$$
(6)

Here are some important design aspects for the gait planning:

- The body move along the forward direction at a constant speed during the swing phase.
- The trajectory of the foot is like an animal.
- The planned trajectories of foot are continuous not only in displacements and velocities but also in accelerations.
- Avoid unnecessary movements

In this paper, quintic polynomial interpolation algorithm mixed with the linear has been used to solve the problem of jump acceleration.

$$f(L,T,t) = \begin{cases} L \times \left(-\frac{16384t^5}{T^5} + \frac{10240t^4}{T^4} - \frac{5120t^3}{3T^3} + \frac{8t}{3T} + 1\right) & 0 \le t < \frac{T}{4} \\ L \times \left(\frac{8t}{3T} - \frac{5}{3}\right) & \frac{T}{4} \le t < T \end{cases}$$
(7)

Looking at Fig. 5, it is apparent that the unit of the trajectories the planned trajectories are continuous not only in displacements and velocities but also in accelerations.

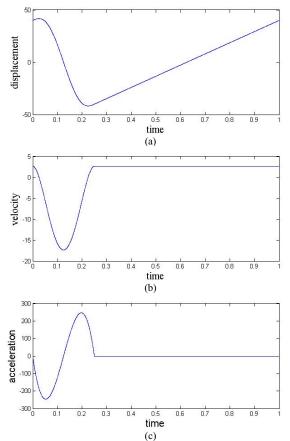


Figure 7. (a), (b), (c) show the displacement, velocity and acceleration of the trajectories

When walking in straight a line, Trajectory planning of quadruped robot are

$$P_{x,FL} = g(H,T,t) \tag{8}$$

$$P_{v,FL} = 0 (9)$$

$$P_{z.FL} = f(L, T, t) \tag{10}$$

When walking in circular, Trajectory planning of quadruped robot are

$$P_{x,FL} = g(H,T,t) \tag{11}$$

$$P_{y,FL} = (R - a)\sin(f(\beta, T, t)) + b\cos(f(\beta, T, t)) - (R - a)\sin(f(0, T, t)) - b\cos(f(0, T, t))$$
(12)

$$P_{z,FL} = (R+a)\cos(f(\beta,T,t)) + b\sin(f(\beta,T,t)) - (R+a)\cos(f(0,T,t)) - b\sin(f(0,T,t))$$
(13)

Where H represents the maximum height of leg raise, L represents stride length, R represents radius of rotation, T represents cycle time,  $\beta$  represents the rotation angle. a and b are geometry parameters of the torso specified in Fig. 2.

### IV. THE SIMULATION EXPERIMENT

In order to verify the accuracy of the calculation of the kinematics and the validity of gait trajectory planning algorithm, simulation has been carried out in this paper. To improve simulation efficiency further, the virtual prototype model of the quadruped robot closed to actual situation was built after reasonable simplification. The body weighs 30kg, stride length is 120mm, the maximum height of leg raise is 40mm, cycle time is 0.5s.

After planning the foot trajectory walking in a straight line based on the walking gait, the joint angles, which calculated by inverse kinematics are transferred to drive the motor. The results of simulation are shown in Fig. 8 to Fig. 10.

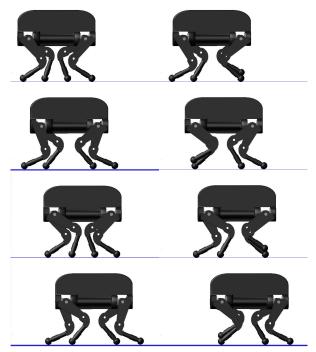


Figure 8. Snapshots of the line walking simulation

The torque of hip and knee have been shown in Fig. 9, which indicate that there is no impact during walking.

What's more, the simulation results provide important references to complete the DC motor selection.

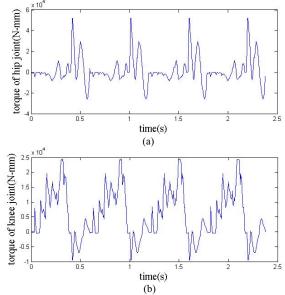


Figure 9. The torque of hip and knee

Motion trajectory of FL leg show in Fig. 10 show that the trajectories of foot based on the linear walk gait locomotion strategy are continuous and smooth.

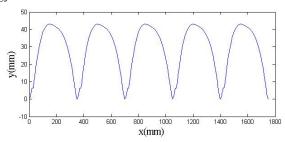
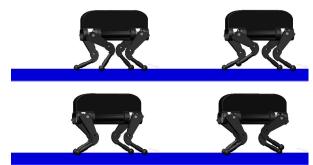


Figure 10. motion trajectory of FL leg in straight line

When the quadruped robot do steering motion, rotation angle is 20 degree, the maximum height of leg raise is 40mm, cycle time is 2s, complete 360 degree arc motion need 18 cycles. The results of simulation are shown in Fig. 11 to Fig. 13. Trajectory of COG is a continuous arc curve, which indicate that the body do a continuous, smooth steering motion without unnecessary movements.



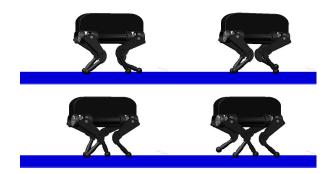


Figure 11. Snapshots of the circular walking simulation

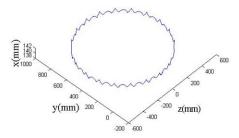


Figure 12. COG trajectory in circular

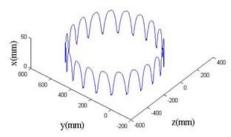


Figure 13. motion trajectory of FL leg in circular

## V. SUMMARY

In this paper, a gait planning method underling low contact impact is presented to make the quadruped robot have stable gaits on horizontal planes. The results of simulation show that the trajectories of foot based on walk gait are continuous and smooth, the quadruped robot can stably motion on horizontal planes without impact and sliding.

#### REFERENCES

- [1] M. Raibert, K. Blankspoor, G. Nelson, R. Playter, Bigdog, the rough-terrain quadruped robot [C]// Proceedings of 17thWorld Congress, The International Federation of Automatic Control. Seoul, Korea. USA: IFAC, 2008:10822-10825.
- I. Havoutis, J. Ortiz, S. Bazeille, V. Barasuol, C. Semini, Onboard perception-based trotting and walking with the hydraulic quadruped robot (HyQ). The IEEE international conference on intelligent robots and systems, 2013, pp.6052-6057.
- [3] M. Kalakrishnan, J. Buchli, P. Pastor, M. Mistry, and S. Schaal, Fast, Robust Quadruped Locomotion over Challenging Terrain, International Conference on Robotics and Automation, 2010, pp. 2665–2670.

- [4] J. Buchli, M. Kalakrishnan, M. Mistry, P. Pastor, and S. Schaal, Compliant Quadruped Locomotion Over Rough Terrain, The IEEE international conference on intelligent robots and systems, 2009, pp.814–820.
- [5] J. Meng, Y. B. Li, B. Li, Control method and its implementation of quadruped robot in omni-directional trotting gait, Robot, vol. 37, pp. 74–84, Jan. 2015.
- [6] L. P. Wang, J. Z. Wang, S. K. Wang, Y. D. He, Strategy of foot trajectory generation for hydraulic quadruped robots gait planning, Journal of Mechanical Engineering, vol. 49, pp. 39–44, Jan. 2013.
- [7] N. Hu, S. Y. Li, D. Huang, F. Gao, Gait planning and control of quadruped robot with high payload, Journal of System Simulation, vol. 27, pp. 529533, Mar. 2015.
- [8] M Kalakrishnan, J. Buchli, P. Pastor, M. Mistry and S. Schaal, Learning, planning, and control for quadruped locomotion over challenging terrain, The International Journal of Robotics Research, vol. 30, pp. 236-258, Nov. 2010.