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A BRIEF INTRODUCTION TO COQ

The Coq Proof Assistant

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PROOF ASSISTANTS

THE COQ PROOF ASSISTANT

usage and applicability of the Coq-proof-assistant

- → developed since 1983
- → community in the industry and research
  https://coq.inria.fr/community
  https://github.com/coq-community
- → basic concepts of logic
- → functional programming

Figure: The Rooster. Source:[2]

OTHER PROOF ASSISTANTS

There are numerous proof assistants.

- → Isabelle
  https://isabelle.in.tum.de
- → HOL
  https://hol-theorem-prover.org

LOGIC

''As as matter of fact, logic has turned out to be significantly more effective in computer science then it has been in mathematics." [1]

LOGIC

Reliability in software is amplified by the costs of bugs by insecurity up to multiple levels.

- → basic tools from logic
- → precise claims about programs
- → functional programming methods of programming and logical reasoning about programs

FUNCTIONAL PROGRAMMING IN COQ

SYSTEM REQUIREMENTS

Coq runs on Linux, Mac OS and Windows.
There are a lot of development environments available.

→ **Proof General**
  an Emacs based IDE
→ **CoqIDE**
  is a simple stand-alone IDE
→ **coquille**
  a vim plug-in
→ and others

SYSTEM REQUIREMENTS

Coq runs on Linux, Mac OS and Windows.
There are a lot of development environments.

→ **Proof General**
  an Emacs based IDE
  generic interface for proof assistants developed by multiple universities

→ **CoqIDE**
  is a simple stand-alone IDE
  user-friendly, included in the official Coq-installation

→ **coquille**
  a vim plug-in
  an open-source project

PROOF GENERAL

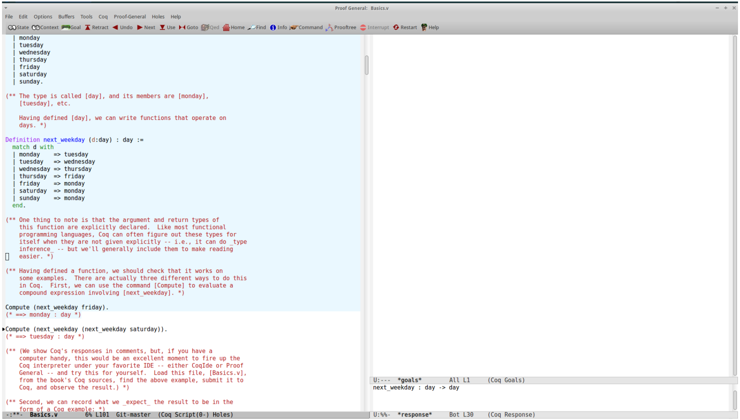

Figure: Coq interface in proof general.

COQIDE

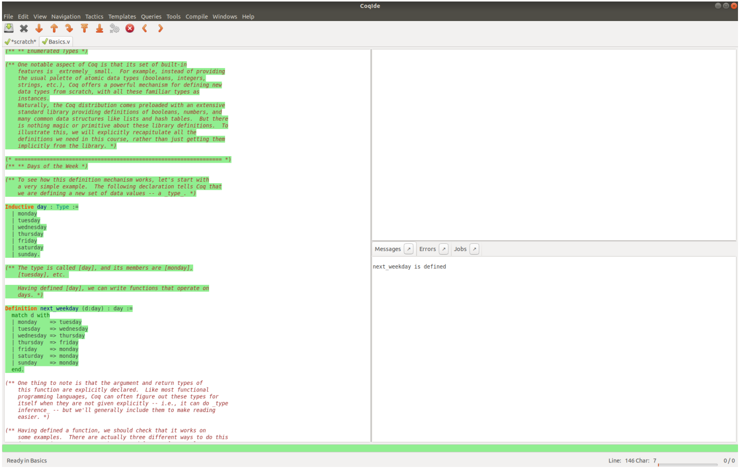

Figure: Coq interface in CoqIDE.

COQUILLE

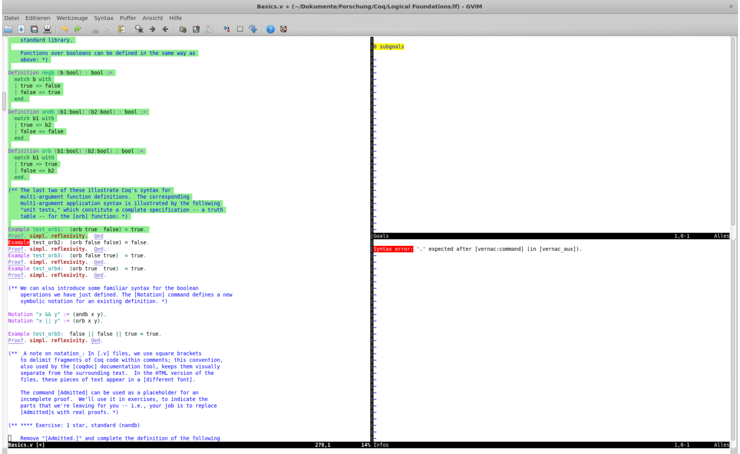

Figure: Coq interface in gvim using coquille.

FUNCTIONAL PROGRAMMING

Coq's functional programming language *Gallina's* sub-types

1. vernacular language (top-level interaction):
   e.g.: **`Theorem`**, **`Proof`**, **`Qed`**.
2. tactics language:
   e.g.: **`intros`**, **`exact`**
3. the *language of Coq-terms*:
   e.g.: **`for all A: Prop, A → A.`**

AN EXAMPLE

AN EXAMPLE

```
Inductive day : Type :=
  | monday
  | tuesday
  | wednesday
  | thursday
  | friday
  | saturday
  | sunday.
```

Listing 1: definition of a datatype called `day`

AN EXAMPLE

```
Definition next_weekday (d:day) : day :=
    match d with
    | monday    ⇒ tuesday
    | tuesday   ⇒ wednesday
    | wednesday ⇒ thursday
    | thursday  ⇒ friday
    | friday    ⇒ monday
    | saturday  ⇒ monday
    | sunday    ⇒ monday
    end.
```

Listing 2: declaration of a function on `day`

AN EXAMPLE

```
Compute (next_weekday friday).
```

Listing 3: call `Compute`

```
= monday: day
```

Listing 4: output

APPLICATIONS

APPLICATIONS

A platform for *modeling programming languages* and an environment for *formally certifying software and hardware*

- → PROSA a felxible open-source foundation for formally proven schedulability analysis. PROSA uses the Coq proof assistant and the SSREFLECT extension library [1]
- → Jasmin: High-Assurance and High-Speed Cryptography [1]

PROSA

PROSA - A COQ LIBRARY

Figure: RT-proofs-logo. Source: [3].

1. Formal Proofs for real-time systems (RT-Proofs)
2. project running between multiple research faculties (DFG project between INRIA, MPI-SWS, Onera, TU Braunschweig and Verimag, running from 2018 until 2020)
3. The PROSA library is where this development takes place.

PROSA - A COQ LIBRARY

to reproduce the PROSA - ECRTS'16 Artifact Evaluation by mechanically proofing the case study the following framework is provided online

utility	source
`Ubuntu 15.10, wily`	VM-Image
`Coq Version 8.5pl`	VM-Image
`Proof General 4.3pre131011`	VM-Image
`mathcomp-1.6`, `ssreflect`-extension	VM-Image
`PROSA v.01`	former version of the Coq spec and proof development of the RT-PROOFS project

PROSA - A COQ LIBRARY

to reproduce the PROSA - ECRTS'16 Artifact Evaluation by mechanically proofing the case study the following framework is provided online

my current working directory:
https://gitlab.cs.hs-rm.de/almeroth/prosa_working_dir
members: Steffen Reith
due to documentation including all listings

PROSA - FORMALLY PROVEN SCHEDUBILITY ANALYSIS

- »We conclude that the develop foundations are sufficiently felixble and powerful to support a large fraction of existing literature and real-time scheduling, without compromising readability.« [1]
- »Mechanized schedulability proofs are feasible, to the point that non-trivial multiprocessor schedulability analyses can be formalized in a reasonable time frame.«[1]
- **case studies**

PROSA - FORMALLY PROVEN SCHEDUBILITY ANALYSIS

a case study provided: Ciriniei's RTA for EDF [2, p.160, Figure 17.3]
definition and proofs of termination and correctness of Bertonga and Cicerei's RTA for EDF

Theorem (Main Theorem)

```
Theorem edf_analysis_yields_response_time_bounds :
     tsk R,
    (tsk, R) \\In edf_claimed_bounds ts →
    response_time_bounded_by tsk R.
```

Listing 5: `theorem_edf_analysis` [1]

PROSA - FORMALLY PROVEN SCHEDUBILITY ANALYSIS

practical specification: correctness of the proof

mechanized proofs available

- → correctness of the work-loaded-based interference bound for work-conserving schedulers
- → EDF-specific interference bound definition and proof of termination and correctness of Bertongna and Cicerei's RTA for FP scheduling
- → definition and proof of termination and correctness of Bertongna and Cicerei's RTA for EDF scheduling ($\approx 1'320$ LOC)

$\approx 13'070$ LOC

PROSA - FORMALLY PROVEN SCHEDUBILITY ANALYSIS

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- → **definition and proof of termination and correctness of Bertongna and Cicerei's RTA for EDF scheduling** ($\approx 1'320$ LOC)

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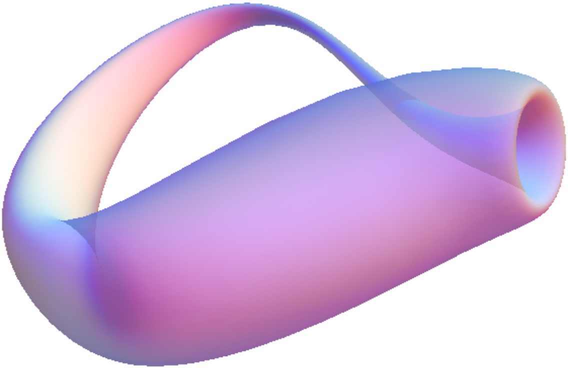
BREAK

BREAK - LET'S TALK ABOUT INSIDE AND OUTSIDE.

https://www.youtube.com/watch?v=SyD4p8_y8Kw



Klein Bottle with slight transparency, rendered with Mathematica 8 using the parameterisation provided by Robert Israel. Copyright by wikipedia, Creative Commons Attribution-Share Alike 3.0 Unported

PROSA - FORMALLY PROVEN SCHEDUBILITY ANALYSIS

Kasier's encapsulated EDF-scheduler [3]

- → real-time as in [1]
- → hierarchical (local and global distinction)
- → multi-core (in the real-time-sense)
- → EDF (earlines deadline first on a local level)
- → sporadic
- → disruptive

Main theorem: correctness of analysis although it is a deductive derivation

PROSA - FORMALLY PROVEN SCHEDUBILITY ANALYSIS

Definition

The *process* is defined as a sequential execution of a program on a processor. The execution ends after a finite number of steps. Therefore it corresponds to a finite execution of machine commands and is not separable.

Definition

A process is called *periodic* if it should be restated after a certain time called *the period*.
Otherwise a process is called *aperiodic* or *sporadic*. Furthermore, whenever as process is said to be *non-preemptive* the execution may not be interrupted between the beginning and ending of the process.
It is called *preemptive* if it may be interrupted after any instruction.

PROSA - FORMALLY PROVEN SCHEDUBILITY ANALYSIS

The **slotted prirority modell** from [1] with sporadic and periodic processes ([3]). Due to [1] the major requirement is said to be as in the following:

> *»If a system itself the execution of a real-time and non-real-time thread in alternate intervals the intervals in which real-time threads execute are scheduled to be in every l time unit, then it must be ensured that the interval begin at time t where $kl \leq t \leq kl + \epsilon \quad \forall \epsilon \geq 0$.«*

Moreover there is this requirement $\mathcal{B}$.

> *»For $L > \epsilon$ (for a suitable ϵ) for which the real-time thread schedule has asserted a real-time thread τ to be expected on the CPU, there must be a function of the method by which the minimum number of CPU cycles available to execute the instructions of τ can be determined.«*

PROSA-FORMALLY PROVEN SCHEDULABILITY ANALYSIS

Definition

$$\delta_p \in \mathbb{N} \quad \text{be a periodic disruptive process and} \tag{1}$$

$$\delta_s \in \mathbb{N} \quad \text{be an asporadic disruptive process.} \tag{2}$$

$$\text{Let} \quad P := \{1, \cdots, n\} \subset \mathbb{N}^n \quad \text{be an disruptable process.} \tag{3}$$

$$\text{Let} \quad \delta p_i, \quad \text{for} \quad i = 1, .., n \quad \text{denote the period and} \tag{4}$$

$$\delta e_i \quad \text{for} \quad i = 1, .., n \quad \text{denote the execution time.} \tag{5}$$

PROSA-FORMALLY PROVEN SCHEDULABILITY ANALYSIS

Moreover, we define the slotted priority model by Bollea with Kaiser's sporadic and periodic disruptive process with a discrete time in contrast to these authors. This chioce is due to the real-time model from [?] and the real-time kernel as in [1, chp. 5.3] and the exclusion model [1, p.12].

Definition

Time is $\mathbb{N}$.

```
...

Admitted.
```

OUTLOOK

- → lecture notes for Steffen Reith, incorporate review
- → incorporate linear temporal logic
- → Steffen Reith's review
- → incorporate Steffen Reith's review on the PROSA working directory
- → tutorial on `Gallina` and `SSreflect` (4 person days)
- → formally proof the Kaiser's EDF scheduler in PROSA hopefully less then ($\approx 1'320$ LOC)
- → see at what is the best way to approach the complete kernel

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