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# A BRIEF INTRODUCTION TO COQ

## The Coq Proof Assistant and Schedulability Analysis

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RECAP

## LAST TIME

- proof assistants
- functional programming in Coq
- applications of Coq
- PROSA and RT-Proofs

# FORMER OUTLOOK

- lecture notes for Steffen Reith, incorporate review
- incorporate linear temporal logic
- Steffen Reith's review
- incorporate Steffen Reith's review on the PROSA working directory
- tutorial on `Gallina` and `SSreflect` (4 person days)
- formally proof the Kaiser's EDF scheduler in PROSA hopefully less then ( $\approx 1'320$  LOC)
- see at what is the best way to approach the complete kernel

FORMALLY PROVE KAISER'S  
EDF-SCHEDULER

# PROSA FORMAL SCHEDULABILITY ANALYSIS

- basic tools from logic
- precise claims about programs
- functional programming methods of programming and logical reasoning about programs

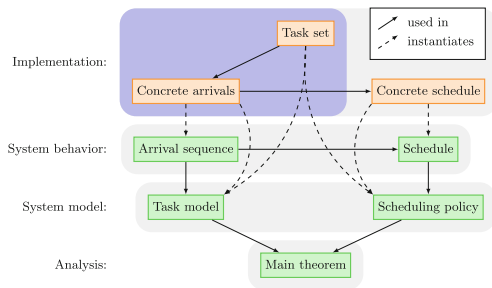


Figure: PROSA layers. Source:[1]

# MAIN THEOREM: PROSA ECRTS16 ARTIFACT EVALUTAION

practical specification: correctness of the proof

- EDF-specific interference bound definition and proof of termination and correctness of Bertongna and Cicerei's RTA for FP scheduling
- **definition and proof of termination and correctness of Bertongna and Cicerei's RTA for EDF scheduling**  
( $\approx 1'320$  LOC)

my current working directory:

**[https://gitlab.cs.hs-rm.de/almeroth/prosa\\_working\\_dir](https://gitlab.cs.hs-rm.de/almeroth/prosa_working_dir)**

due to documentation including all listings ( $\approx 13'070$  LOC)



# LTL AND MULTIPROCESSOR SCHEDULING

# LTL AND MULTIPROCESSOR SCHEDULING

## Taxonomy of Multiprocessor Scheduling

- fixed task priority
- fixed job priority
- dynamic priority

Idea: apply linear temporal logic to verify static and dynamic priority scheduling methods.

Suggestion: Predicate logic-based approaches fail for dynamic priority scheduling methods.

# WHAT IS LTL

- Linear temporal logic is linear like a linear tree (graph)
- apply this linearity approach to a discrete time in a state-space system
- LTL's origins are in the natural language descriptions
- natural numbers  $\mathbb{N}$  represent a moment in time

Example: typical temporal operators

- $\bigcirc\phi$  means  $\phi$  is true in the next moment
- $\Box\phi$  means  $\phi$  is true in all future moments
- $\Diamond\phi$  means  $\phi$  is true in some future moment
- $\phi\mathcal{U}\psi$  means  $\phi$  is true until  $\psi$  is true

Goal: verification of dynamic priority scheduling methods

# MUTIPROCESSOR SCHEDULING

# PROBLEM

In *hard real-time systems* we say a process or task is defined as an sequential execution of a program or a job on a processor. The execution ends after a *finite number of steps*.

→»One big train of commands has to pass in a finite amount of time.«

## Definition

Multiprocessor scheduling can be viewed as attempting to solve two problems.

- (i) The *allocation problem*, or on which processor the task should execute.
- (ii) The *priority problem*, or in what order with respect to jobs of tasks each job should be executed.

# PRIORITY PROBLEM

Kasier's encapsulated EDF-scheduler [3]

- real-time as in [1]
- hierarchical (local and global distinction)
- multi-core (in the real-time-sense)
- EDF (earlines deadline first on a local level)
- sporadic
- disruptive

Main theorem: correctness of analysis although it is a deductive derivation

# PROBLEM: DEFINE A JOB OR TASK

## Definition

Let

$\delta_p \in \mathbb{N}$  be a periodic disruptive process and

$\delta_s \in \mathbb{N}$  be an asporadic disruptive process.

Let  $P := \{1, \dots, n\} \subset \mathbb{N}^n$

be a job, a set of disruptable processes

with fixed priority order ascading.

Let  $\delta p_i \in \mathbb{N}$ , for  $i = 1, \dots, n$  denote the period and

$\delta e_i \in \mathbb{N}$  for  $i = 1, \dots, n$  denote the execution time of a proces.

# MULTIPROCESSOR SCHEDULING ALGORITHM

## Definition

We say a multiprocessor scheduling algorithm  $\sigma$  is given by

$$\sigma : \mathbb{N}^n \times \mathbb{N}^n \longrightarrow \mathbb{N} \times \{1, 2, \dots, m\} \quad (1)$$

$$\forall i, \delta e_j \in \mathbb{N} \quad (i, \delta e_j) \mapsto (t, k) \in \mathbb{N} \times \{1, 2, \dots, m\}. \quad (2)$$

map time  $i$  and execution time  $\delta e_i$  to time-sequence  $t$  and processor  $k$



# MULTIPROCESSOR SCHEDULING ALGORITHM

## Definition

A scheduler satisfying the EDF-regulation

$$U(P) = \sum_{i=1}^n \frac{\delta e_i}{\delta p_i} \leq \frac{\Delta e_{sv}}{\Delta p_{sv}} \quad (3)$$

is called feasible.

## Theorem

*A hard-real-time, hierarchical, multi-core, EDF, sporadic and disruptive time scheduler is feasible iff*

$$\sum_{i=1}^n \lfloor \frac{t}{\Delta p_i} \rfloor \Delta e_i \leq \lfloor \frac{t}{\Delta p_{sv}} \rfloor \Delta e_{sv} + \max(0, t \bmod \Delta p_{sv} - \Delta e_s - \Delta e_p). \quad (4)$$

# OUTLOOK

apply the theorem of Akra -Bazzi

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