CL exercise for
Tutorial 4

Introduction

Objectives

In this tutorial, you will:

- learn more about *sequents* and *combining predicates*;
- derive de Morgan's second law;
- do proofs in *sequent calculus*.

Tasks

Exercises 1 and 2 are mandatory. Exercises 3 is optional. Exercise 4 is for your own interest only.

Submit

a file called `cl-tutorial-4` with your answers (image or pdf).

Deadline

16:00 Tuesday 18 October

Reminder

Good Scholarly Practice

Please remember the good scholarly practice requirements of the University regarding work for credit.

You can find guidance at the School page

https://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct.

This also has links to the relevant University pages. Please do not publish solutions to these exercises on the internet or elsewhere, to avoid others copying your solutions.

Exercise 1 –mandatory—marked–

Read Chapter 14 (*Sequent Calculus*) of the textbook.

Derive the second of de Morgan's laws

$$\neg(a \wedge b) = \neg a \vee \neg b$$

using a similar argument to the one presented in the textbook for the first law on page 122.

Exercise 2 –mandatory—marked–

Write a proof which reduces the conclusion

$$(x \vee y) \wedge (x \vee z) \models x \vee (y \wedge z)$$

to premises that can't be reduced further.

Is it universally valid? If not, give a counterexample.

Exercise 3 –optional—marked–

Write a proof which reduces the conclusion

$$\models (x \wedge y) \vee (\neg(x \vee z) \vee (\neg y \vee z))$$

to premises that can't be reduced further.

Expressions φ like $(x \wedge y) \vee (\neg(x \vee z) \vee (\neg y \vee z))$ used in the antecedents and succedents of sequents are called:

- *tautologies* when $\models \varphi$ is valid (the antecedent is empty);
- *contradictions* when $\varphi \models$ is valid (the succedent is empty);

Is $(x \wedge y) \vee (\neg(x \vee z) \vee (\neg y \vee z))$ a tautology, a contradiction, or neither?

Exercise 4 –optional—not marked–

Do not submit a solution for this exercise. Discuss in tutorials if you wish!

Write proofs which reduce the conclusions

$$\neg a \wedge \neg b \models \neg(a \wedge b)$$

and

$$\neg(a \wedge b) \models \neg a \wedge \neg b$$

to premises that can't be reduced further.

Is one or both universally valid?

- If not, give a counterexample.
- If so, explain how that shows that $\neg a \wedge \neg b = \neg(a \wedge b)$.