CL Tutorial 5:

Q1:

$$\frac{a \vdash \neg b \qquad c \vdash b}{\dfrac{a \vdash \neg b \qquad \neg b \vdash \neg c}{a \vdash \neg c}}$$

∴ It checks $a \vdash \neg c$

(II) a must exist under existential assumption. Thus the pure a area is not empty. But $a \cap c$ is empty as it's $\neg c$. ∴ c is not empty. ∴ The correct new diagram is with pure a not shaded, $a \cap c$ shaded, and pure c not shaded.

Q2.

barbara: $\dfrac{x \vdash y \qquad y \vdash z}{x \vdash z}$

$x = a$

$y = b$

$z = \neg c$

$$\frac{y \vdash z \qquad x \vdash y}{\neg z \vdash \neg x}$$ ↔ → Negation.

$$\frac{y \vdash z \qquad \neg z \# \neg x}{x \# y}$$ ↔ contraposition of rules.

$$\frac{b \vdash \neg c \qquad c \# \neg a}{a \# b}$$ substitution.

CL - Tutorial

Q 3

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\neg p \wedge \neg q \vdash \neg p \wedge \neg q}\ \text{Immediate}}{\neg p, \neg q \vdash \neg p \wedge \neg q}\ \wedge L}{\vdash (\neg p \wedge \neg q), p, q}\ \neg L}{\vdash (\neg p \wedge \neg q) \vee (p \vee q)}\ \vee R$$

Q 4

$$\dfrac{\Gamma, q \vdash \Delta \quad \Gamma \vdash p, \Delta}{\Gamma, p \to q \vdash \Delta}\ (\to L) \qquad \dfrac{\Gamma p \vdash q, \Delta}{\Gamma \vdash p \to q, \Delta}\ (\to R)$$

Workings:

$$\dfrac{\dfrac{q \vdash \quad \vdash p}{q \vdash \quad \neg q \vdash p}}{\neg q \vee \neg p \vdash}$$

$$\Gamma \quad p \to q \vdash \Delta$$

$$\dfrac{p \vdash q}{\vdash q, \neg p}$$

$$\vdash q \vee \neg p$$

$$\vdash p \to q$$