$$L = P\{N_1 = n_1, N_2 = n_2, \ldots, N_k = n_k \mid \lambda\}$$

$$= f_{N_1, N_2, N_k}\{n_1, n_2, \ldots, n_k\}$$ using λ as parameter

$$f(n_i \mid \lambda) = \frac{e^{-\lambda} \lambda^{n_i}}{n_i!}$$

$$L(\lambda / n_1 \ldots n_k) = \prod_{i=1}^{k} f(n_i / \lambda)$$

$$L(\lambda \mid n_1 \ldots n_k) = \prod_{i=1}^{k} \frac{e^{-\lambda} \lambda^{n_i}}{n_i!}$$

$$l(\lambda / n_1 \ldots n_k) = \log(L(\lambda / n_1 \ldots n_k))$$

$$l(\lambda \mid n_1 \ldots n_k) = \sum_{i=1}^{k} \log\left(\frac{e^{-\lambda} \lambda^{n_i}}{n_i!}\right)$$

$$l(\lambda \mid n_1 \ldots n_k) = \sum_{i=1}^{k} \left(\log e^{-\lambda} + n_i \log \lambda - \log n_i!\right)$$

$$l(\lambda / n_1 \ldots n_k) \propto \sum_{i=1}^{k} -\lambda + n_i \log \lambda$$

Thus, as λ is the on which has maximum likelyhood.

$$l'(\lambda \mid n_1 \ldots n_k) = 0$$

$$\Rightarrow -k + \frac{\sum_{i=1}^{k} n_i}{\lambda} = 0$$

$$\Rightarrow k = \frac{\sum_{i=1}^{k} n_i}{\lambda}.$$

$$\boxed{\Rightarrow \lambda = \frac{\sum_{i=1}^{k} n_i}{k}}$$