let $f_x(a)$ be the pdf and $F_x(a)$ be the cdf of both x and y. And x, y are both are independent variables

1) let, $Q = \min(x, y)$

CDF of Q

$$P(Q \le a) = P(x \le a) \cup P(y \le a)$$
$$= P(x \le a) + P(y \le a) - P(x \le a) \cdot P(y \le a)$$
$$= F_x(a) + F_x(a) - (F_x(a))^2$$

$$\text{CDF} = P(\min(x,y) \le a)$$
$$\cancel{= F_x(a) - (F_x(a))^2}$$
$$= 2F_x(a) - (F_x(a))^2$$

PDF = derivative of CDF

$$= 2f_x(a) - 2(F_x(a))(f_x(a))$$

$$= 2f_x(a)(1 - F_x(a))$$

$$\boxed{\Rightarrow \text{PdF} = 2f_x(a) \int_a^{\infty} f_x(x)\,dx}$$

Date
Page

(2) Let $Q = \max(X, Y)$

$CDF - P(Q \leq a) = P(x \leq a) \cap P(Y \leq a)$

$= P(X \leq a) \cdot P(Y \leq a)$

$CDF = (F_X(a))^2$

$\Rightarrow PDF = 2(F_X(a))(f_X(a))$

$\Rightarrow PDF = 2 f_X(a) \int_{-\infty}^{a} F_X(x)\, dx.$

Hence for Min (X, Y)

$CDF - 2F_X(a) - (F_X(a))^2$

$PDF = 2f_X(a) \cdot \int_{a}^{\infty} f_X(x)\, dx.$

Hence for max (X, Y)

$CDF = (F_X(a))^2$

$PDF = 2f_X(a) \cdot \int_{-\infty}^{a} f_X(x)\, dx$

X ———— X