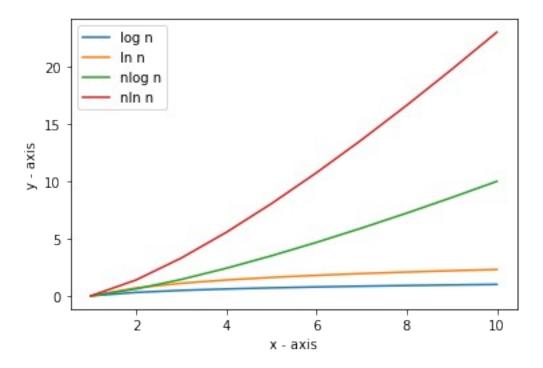
Graphs of functions

```
import math
x1=[]
y1=[]
for i in range(1,11):
    x1.append(i)
    a=math.log10(i)
    y1.append(a)
import matplotlib.pyplot as plt
plt.plot(x1, y1, label = "log n")
x2=[]
y2=[]
for i in range(1,11):
    x2.append(i)
    a=math.log(i)
    y2.append(a)
plt.plot(x2, y2, label = "ln n")
x3 = [1]
y3=[]
for i in range(1,11):
    x3.append(i)
    a=math.log10(i)
    b=i*a
    y3.append(b)
plt.plot(x3, y3, label = "nlog n")
x4=[]
y4=[]
for i in range(1,11):
    x4.append(i)
    a=math.log(i)
    b=i*a
    y4.append(b)
plt.plot(x4, y4, label = "nln n")
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.legend()
```

print("nlogn and nlnn are are continuously increasing functions. logn
and lnn are increasing functions")
print("The shapes of the graphs of log n and ln n are similar and the
shapes of the graphs of nlogn and nlnn are similar")

nlogn and nlnn are are continuously increasing functions. logn and lnn are increasing functions

The shapes of the graphs of log n and ln n are similar and the shapes of the graphs of nlogn and nlnn are similar



```
x5=[]
y5=[]
for i in range(1,21):
    x5.append(i)
    a=math.log10(i)
    b=a/i
    y5.append(b)

plt.plot(x5, y5, label = "log n/n")
x6=[]
y6=[]
for i in range(1,21):
    x6.append(i)
    a=math.log(i)
    b=a/i
    y6.append(b)
```

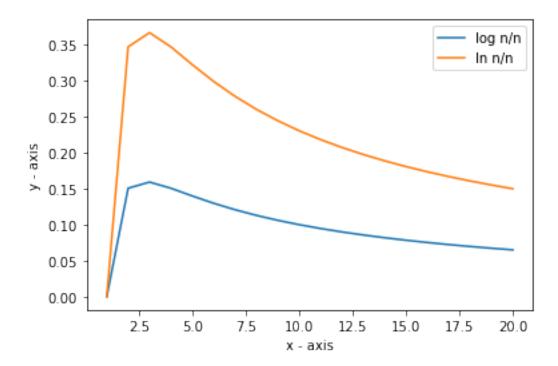
```
plt.plot(x6, y6, label = "ln n/n")

plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.legend()

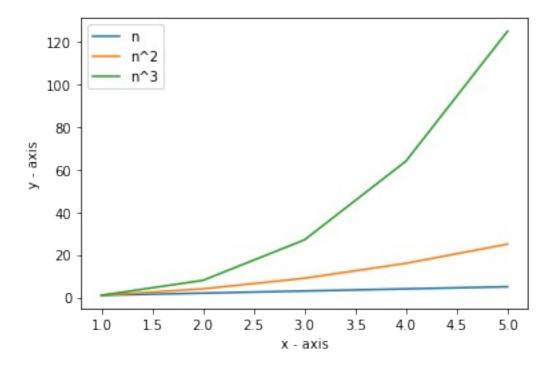
print(" Both the functions (logn)/n and (ln n)/ n first increase,
reach a point of maxima and then decrease")
print("The shapes of the graphs of (logn)/n and (ln n)/ n are
similar")
```

Both the functions $(\log n)/n$ and $(\ln n)/n$ first increase, reach a point of maxima and then decrease

The shapes of the graphs of $(\log n)/n$ and $(\ln n)/n$ are similar



```
y.append(i*i)
plt.plot(x, y, label = "n^2")
x=[1]
v=[]
for i in range(1,6):
    x.append(i)
    y.append(i*i*i)
plt.plot(x, y, label = "n^3")
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.legend()
print("the functions n,n^2 and n^3 are continuously functions")
print("The graph of n is a straight line. The graphs of n^2 and n^3
have a similar shape for all positive integers. For all integers >=2
n<n^2<n^3 . They are all equal at n=1")
the functions n,n^2 and n^3 are continuously functions
The graph of n is a straight line. The graphs of n^2 and n^3 have a
similar shape for all positive integers. For all integers >=2
n<n^2<n^3 . They are all equal at n=1
```

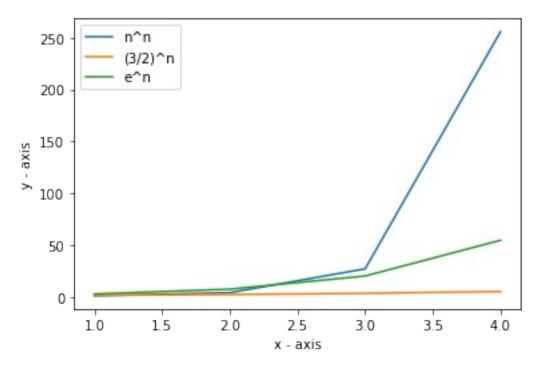


```
x=[]
y=[]
for i in range(1,21):
    x.append(i)
    y.append(i**0.5)
plt.plot(x, y, label = "n^1/2")
x=[]
y=[]
for i in range(1,21):
    x.append(i)
    y.append(i**(1/3))
plt.plot(x, y, label = "n^1/3")
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.legend()
print("The functions <math>n^{(1/2)} and n^{1/3} are increasing functions. Their
graphs have a similar shape")
```

The functions $n^{(1/2)}$ and $n^{1/3}$ are increasing functions. Their graphs have a similar shape

```
4.5
                 n^1/2
                 n^1/3
   4.0
   3.5
3.0
- A
2.5
   2.0
   1.5
   1.0
                                 7.5
               2.5
                        5.0
                                         10.0
                                                  12.5
                                                          15.0
                                                                   17.5
                                                                            20.0
                                         x - axis
```

```
x=[]
y=[]
for i in range(1,5):
    x.append(i)
    y.append(i**i)
plt.plot(x, y, label = "n^n")
x=[]
y=[]
for i in range(1,5):
    x.append(i)
    y.append(1.5**i)
plt.plot(x, y, label = "(3/2)^n")
x=[]
y=[]
for i in range(1,5):
    x.append(i)
    y.append(math.exp(i))
plt.plot(x, y, label = "e^n")
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.legend()
```



```
x=[]
y=[]
for i in range(2,11):
    x.append(i)
    y.append(math.log10(math.log10(i)))
plt.plot(x, y, label = "log logn")
x=[]
y=[]
for i in range(2,11):
    x.append(i)
    y.append(math.log(math.log(i)))
plt.plot(x, y, label = "ln lnn")
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.legend()
print("The functions log logn and ln lnn are increasing functions and
their graphs have a similiar shape")
```

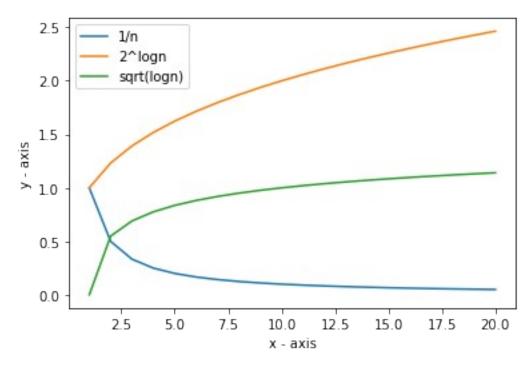
The functions log logn and ln lnn are increasing functions and their graphs have a similiar shape

```
log logn
     0.8
                   In Inn
     0.6
     0.4
y - axis
     0.2
     0.0
   -0.2
   -0.4
             2
                      3
                                       5
                                                         7
                                                                 8
                               4
                                                6
                                                                          9
                                                                                  10
                                            x - axis
```

```
x=[]
y=[]
for i in range(1,21):
    x.append(i)
    y.append(1/i)
plt.plot(x, y, label = "1/n")
x=[]
y=[]
for i in range(1,21):
    x.append(i)
    y.append(2**math.log10(i))
plt.plot(x, y, label = "2^logn")
x=[]
y=[]
for i in range(1,21):
    x.append(i)
    y.append(math.log10(i)**0.5)
plt.plot(x, y, label = "sqrt(logn)")
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.legend()
```

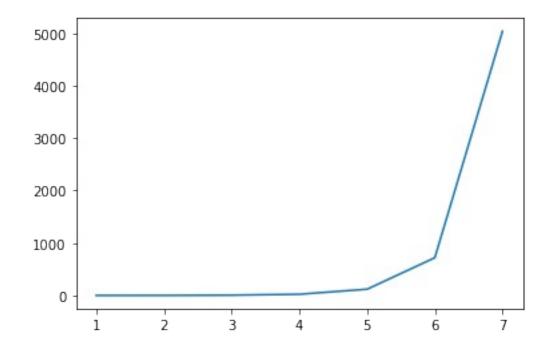
```
print("1/n is a decreasing function")
print("sqrt(logn) and 2^logn are increasing functions")
```

<matplotlib.legend.Legend at 0x214eb573400>



```
x=[]
y=[]
for i in range(1,8):
        x.append(i)
        y.append(math.factorial(i))

plt.plot(x, y, label = "n!")
print("n! is a strictly increasing function")
n! is a strictly increasing function
```



Binary search

```
def binarySearch(a,low,high,x):
    if high >= low:
        mid = (high + low) // 2
        if a[mid] == x:
            return mid

        elif a[mid] > x:
            return binarySearch(a, low, mid - 1, x)

        else:
            return binarySearch(a, mid + 1, high, x)
        else:
            return -1

Case 1: Object is present

a=[1,2,3,4,5,6,7,8,9,10]
x=8
index=binarySearch(a,0,len(a)-1,x)
```

```
if index!=-1:
    print("The object is present at index ",index)

else:
    print("The object is not present")

The object is present at index 7

Case 2: Object is not present

x=15
index=binarySearch(a,0,len(a)-1,x)
if index!=-1:
    print("The object is present at index ",index)

else:
    print("The object is not present")
```

The object is not present

Initilisation: The program is initialised assuming the array is sorted and the object is present in the loop. Maintainence: x must be found between a[low] and a[m--1]. The n for the recursive call is $n = m - 1 - low = \lfloor (low + high)/2 \rfloor - 1 - low$. (Note that $\lfloor x \rfloor$ is the floor of x, which rounds it down toward negative infinity.) If low+high is odd, then n = (low + high - 1)/2 - 1 - low = (high - low)/2 - 1, which is definitely smaller than high-low. If low+high is even then n = (low + high)/2 - 1 - low = (high-low)/2, which is also smaller than k + 1 = high-low because high-low = k + 1 > 0. Termination: The loop is terminated if element is the mid of any sub array in the recursive steps