LAB-16 EX-3

Given,

$N_1, N_2, N_3, N_4 \dots\dots N_k$ are observed data

while we know λ is a parameter we will estimate λ using Maximum Likelihood Estimation.

$$L = P\{N_1 = n_1, N_2 = n_2, \dots N_k = n_k \mid \lambda\}$$

$= f_{N_1, N_2, \dots N_k}(n_1, n_2, n_3 \dots n_k)$ using λ as parameter.

Let n_i be Random Variables for the process N_i (x_i - be count of an event)

then,

$$f(n_i \mid \lambda) = \frac{e^{-\lambda} \lambda^{n_i}}{n_i!}$$

Then,

$$\mathcal{L}(\lambda \mid n_1 \dots n_k) = \prod_{i=1}^{k} f(n_i \mid \lambda)$$

$$\mathcal{L}(\lambda \mid n_1 \dots n_k) = \prod_{i=1}^{k} \frac{e^{-\lambda} \lambda^{n_i}}{n_i!}$$

$$l(\lambda|n_1 \dots n_k) = \log(L(\lambda|n_1 \dots n_k))$$

$$l(\lambda|n_1 \dots n_k) = \sum_{i=1}^{k} \log\left[\frac{e^{-\lambda}\lambda^{n_i}}{n_i!}\right]$$

$$l(\lambda|n_1 \dots n_k) = \sum_{i=1}^{k}\left[\log e^{-\lambda} + n_i \log\lambda - \log n_i!\right]$$

$$l(\lambda|n_1 \dots n_k) \propto \sum_{i=1}^{k}\left[\log e^{-\lambda} + n_i \log\lambda\right]$$

$$l(\lambda|n_1 \dots n_k) \propto \sum_{i=1}^{k} -\lambda + n_i \log\lambda$$

Thus, as λ is the one which has maximum likelihood.

$$l'(\lambda|n_1 \dots n_k) = 0$$

$$-k + \frac{\sum_{i=1}^{k} n_i}{\lambda} = 0$$

$$\frac{\sum_{i=1}^{k} n_i}{\lambda} = k$$

$$\boxed{\lambda = \frac{\sum_{i=1}^{k} n_i}{k}}$$

Hence,

By Maximum Likelihood Estimation we get,

$$\lambda = \frac{\sum_{i=1}^{K} n_i}{K}$$

which is indeed the mean of n_i.