BM6140
Theoretical and Computational Neuroscience

Simulation of action potential from HH equations

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We know that the four hodgkin-huxley equations are :

$$I_{inj} = c_m dv/dt + I_{ion}(v,t)$$

$$\frac{dn}{dt} = (n_\infty(t) - n)/\tau_n$$

$$\frac{dm}{dt} = (m_\infty(t) - m)/\tau_m$$

$$\frac{dh}{dt} = (h_\infty(t) - h)/\tau_h$$

With the help of these four equations we can simulate an action potential.
Given an initial voltage V, we first calculate $n_\infty(V)$ and $\tau_n(V)$, $m_\infty(V)$ and $\tau_m(V)$ and $h_\infty(V)$ and $\tau_h(V)$. For a state variable x, we can write that

$$x(t) = x(t - 1) + \frac{dx}{dt} * \delta t$$

Here x can be n, m or h and δt is the small timestep. Now the ionic current in the cell can be given by the expression

$$I_{ion} = I_{Na} + I_k + I_L$$

Here I_{Na} is the ionic current due to the flow of sodium ions, I_k is the ionic current due to the flow of potassium ions and I_L is the leaky current. We also know that the ionic currents are of the form

$$I_{Na} = \overline{G}_{Na} m^3 h (V - E_{Na})$$

$$I_K = \overline{G}_K n^4 (V - E_K)$$

$$I_L = \overline{G}_L (V - E_K)$$

By adding all the three currents we get the total current injected. With the help of the total current we can calculate the new potential at this time step as potential is nothing but current times conductance. This process is to be repeated by substituting in this new value of the potential and repeating the entire process for the complete cycle.

The values of different constants are :

- g_K = 36.0 (mS/cm^2)
- g_{Na} = 120.0 mS/cm^2)

- g_L = 0.3 (mS/cm^2)
- C_m = 0.9 (uF/cm^2)
- V_K = -90.0 (mV)
- V_{Na} = 65.0 (mV)
- V_l = 10.613 (mV)

The rate equations are of the form:

$$\alpha_n = 0.01\,(V - 10)\,/\,(1 - \exp[(10 - V)\,/\,10]),$$

$$\beta_n = 0.125\,\exp[-V\,/\,80],$$

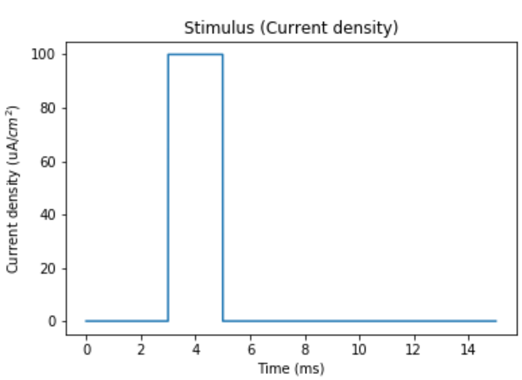
$$\alpha_m = 0.1\,(V - 25)\,/\,(1 - \exp[25 - V\,/\,10]),$$

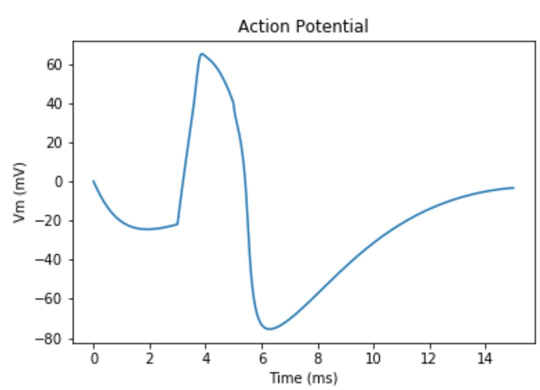
$$\beta_m = 4\,\exp[-V\,/\,18],$$

$$\alpha_h = 0.7\,\exp[-V\,/\,20],$$

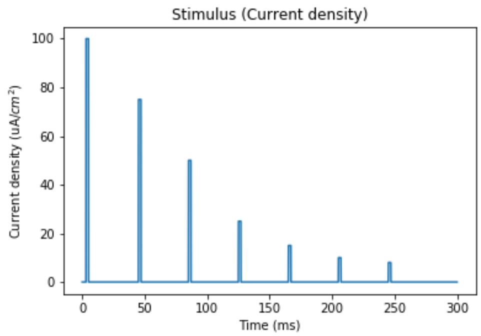
$$\beta_h = 1\,/\,(1 + \exp[30 - V\,/\,10]).$$

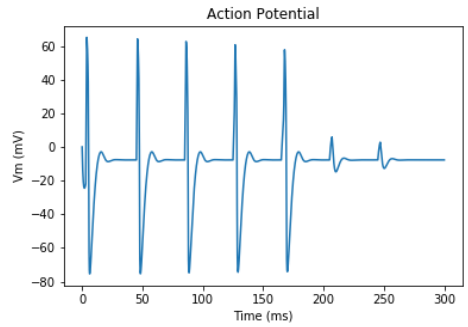
A current was injected for a small duration of 2 ms to simulate an action potential. The result of the simulation was





The simulation was then again repeated by giving multiple current injections but by changing the current densities.





In the above simulation, we can clearly see that the magnitude of the current density doesn't affect the magnitude of the action potential once it crosses a particular threshold. We can see in the figure above that for current densities equal to 100, 75, 50, 25 and 15 the action potentials remain fairly similar but when the current density is changed from 15 to 10, the action potential changes significantly.

We can even see how the state variables i.e. n,m and vary based on the rate equations seen above.