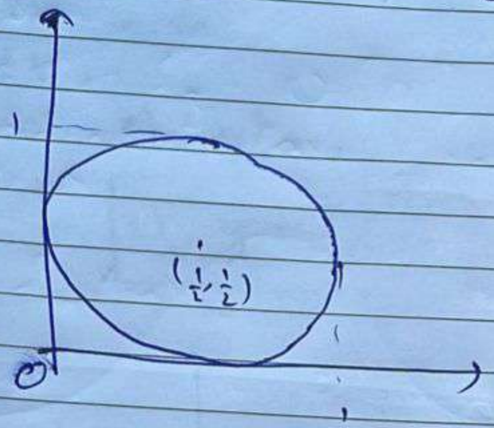
LAB 11

(1) let $x(\omega)$ & $y(\omega)$ are i.i.d $U[0,1]$

So, radius $= \frac{1}{2}$ & center is $(\frac{1}{2}, \frac{1}{2})$



As $x = r\cos\theta$

$y = r\sin\theta$

& $r = \frac{1}{2}$

$\Rightarrow x = \frac{\cos\theta}{2}, \ y = \frac{\sin\theta}{2}$

For $U[0,1]$

$$f_x(a) = \begin{cases} 1, & a \in [0,1] \\ 0, & \text{otherwise} \end{cases}$$

$$f_y(a) \begin{cases} 1, & a \in [0,1] \\ 0, & \text{otherwise.} \end{cases}$$

$x = \frac{\cos\theta}{2}.$

$\Rightarrow \theta = \cos^{-1}(2x)$

$F_\theta(a) = P(\theta \leq a) = P(\cos^{-1}(2x) \leq a)$

$\Rightarrow F_\Theta(a) = P\left(X \leq \frac{\cos a}{2}\right)$

$= F_X\left(\frac{\cos a}{2}\right)$

We know that,

$f_\Theta = \frac{d}{da} F_\Theta$

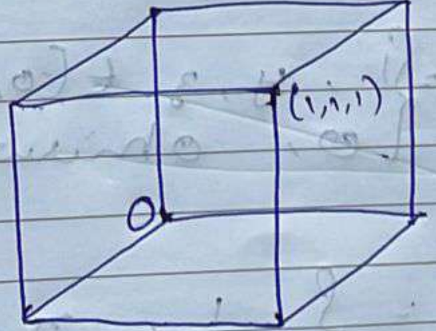
$\Rightarrow f_\Theta = \frac{d}{da} F_X\left(\frac{\cos a}{2}\right)$

$$\boxed{\Rightarrow f_\Theta = -\frac{\sin a}{2} \times f_X\left(\frac{\cos a}{2}\right)}$$

$$\Rightarrow f_\Theta = \begin{cases} -\frac{\sin a}{2}, & a \in \left[2n\pi - \frac{\pi}{2}, 2m\pi + \frac{\pi}{2}\right) \\ 0, & \text{otherwise} \end{cases}$$

x ——————— x

(2) Let X, Y, Z are three random variables.



(i) Let position of fly $= (X(\omega), Y(\omega), Z(\omega))$

Let f_{XYZ} is Joint PdF.

$f_x = f_y = f_z = U[0,1]$, since the coordinates are independent of each other.

i) $f_x(x) \begin{cases} 1, & x \in [0,1] \\ 0, & \text{otherwise} \end{cases}$

$f_y(x) \begin{cases} 1, & x \in [0,1] \\ 0, & \text{otherwis} \end{cases}$

$f_z(x) \begin{cases} 1, & x \in [0,1] \\ 0, & \text{otherwise} \end{cases}$

Since X, Y, Z are all i.i.d

$f_{XY} = f_X \cdot f_Y = \begin{cases} 1, & x, y \in [0,1] \\ 0, & \text{otherwise} \end{cases}$

Similarly

$f_{YZ} = f_Y \cdot f_Z = \begin{cases} 1, & y, z \in [0,1] \\ 0, & \text{otherwise} \end{cases}$

$f_{ZX} = f_Z \cdot f_X = \begin{cases} 1, & x, z \in [0,1] \\ 0, & \text{otherwise} \end{cases}$

$f_{XYZ} = f_X \cdot f_Y \cdot f_Z = \begin{cases} 1, & x, y, z \in [0,1] \\ 0, & \text{otherwise} \end{cases}$

Since X, Y, Z are all independent,

$$Cov(X,Y) = Cov(Y,Z) = Cov(Z,X) = 0.$$

(ii) Here, $I = [1, 1, 1]^T$

$$I - A \cdot [X, Y, Z]^T = \begin{bmatrix} 1-X \\ 1-Y \\ 1-Z \end{bmatrix} = \begin{bmatrix} U \\ V \\ W \end{bmatrix}^T.$$

If the animal is sitting at $(1,1,1)$, the position of the fly w.r.t the animal is $(1-\cancel{X}, 1-y, 1-z)$

(iii) $I + [x, y, z]^T = \begin{bmatrix} 1 + x \\ 1 + 2y \\ 1 + 3z \end{bmatrix}$

To the animal, the room appears a cuboid of the size $1 \times 2 \times 3$.

(iv) The PDF will look like a gaussian