Capacitance

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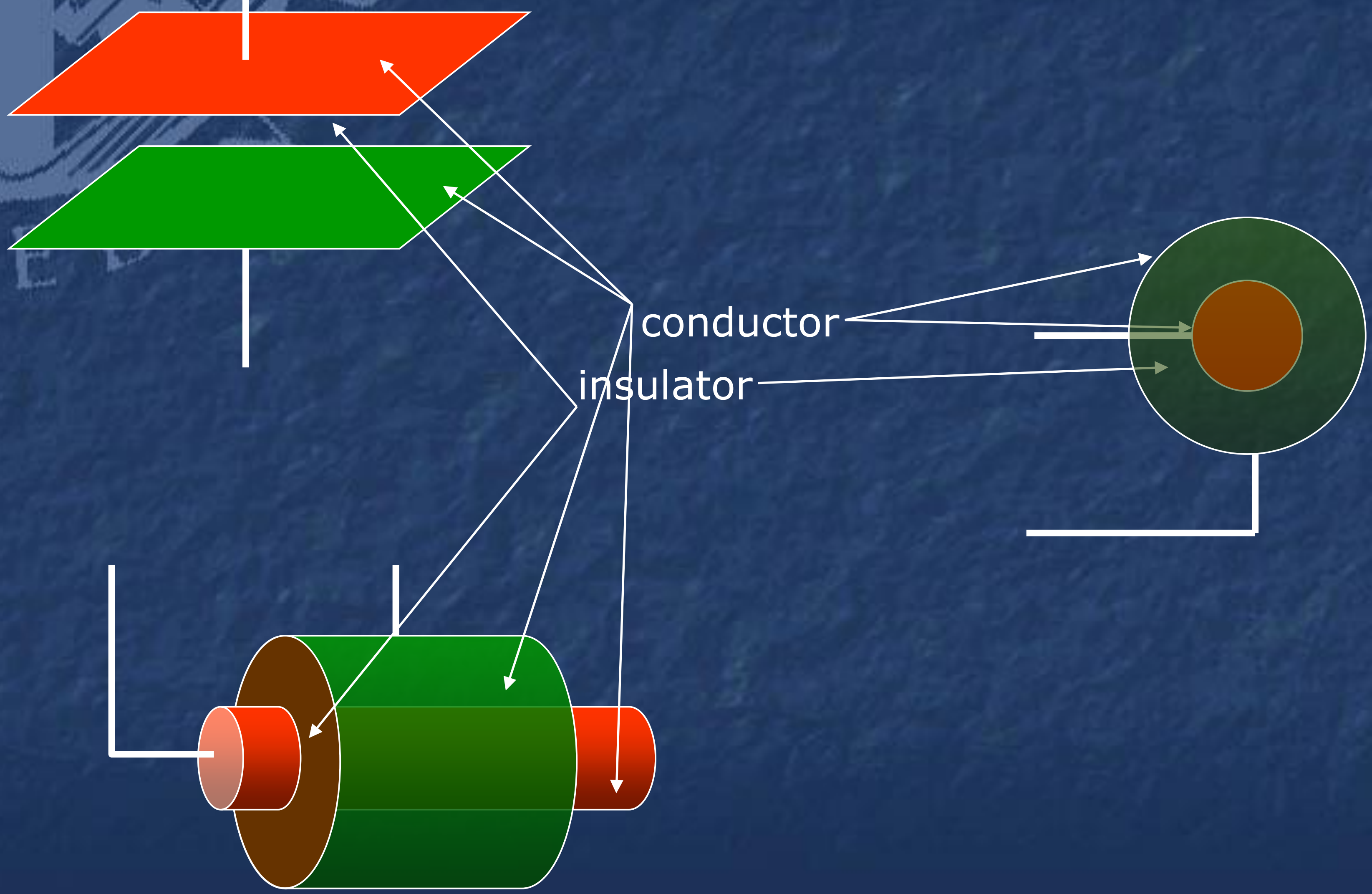
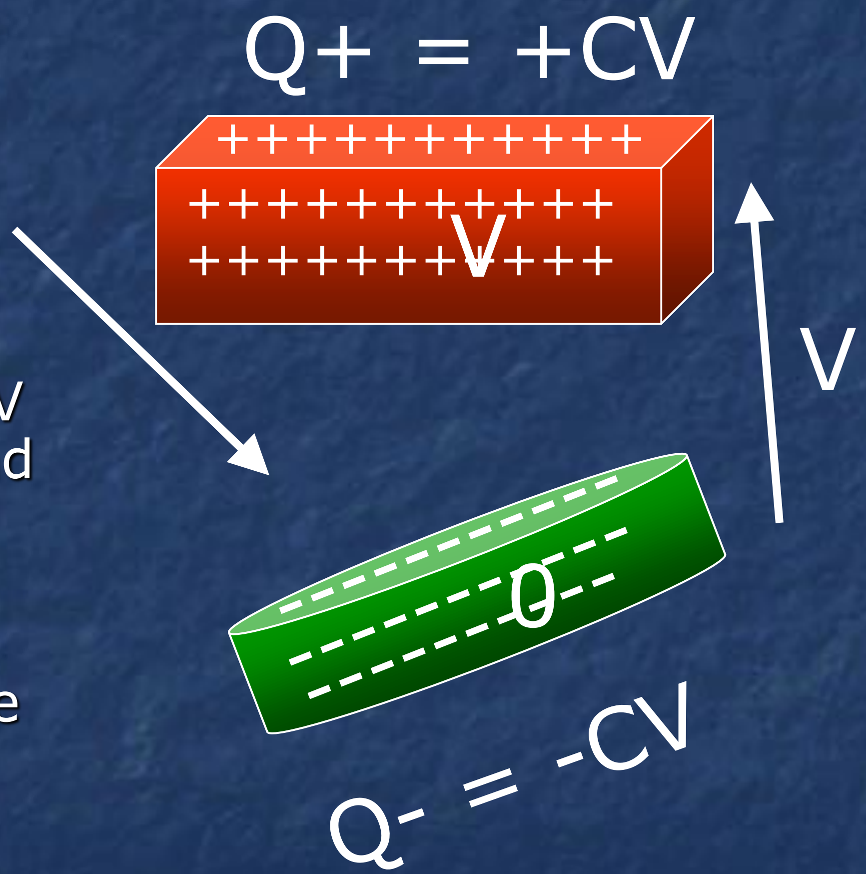
Some Capacitors



Capacitance : Definition

- Take two chunks of conductor
  - Separated by insulator
- Apply a potential V between them
- Charge will appear on the conductors, with $Q_+ = +CV$ on the higher-potential and $Q_- = -CV$ on the lower potential conductor
- C depends upon both the "geometry" and the nature of the material that is the insulator

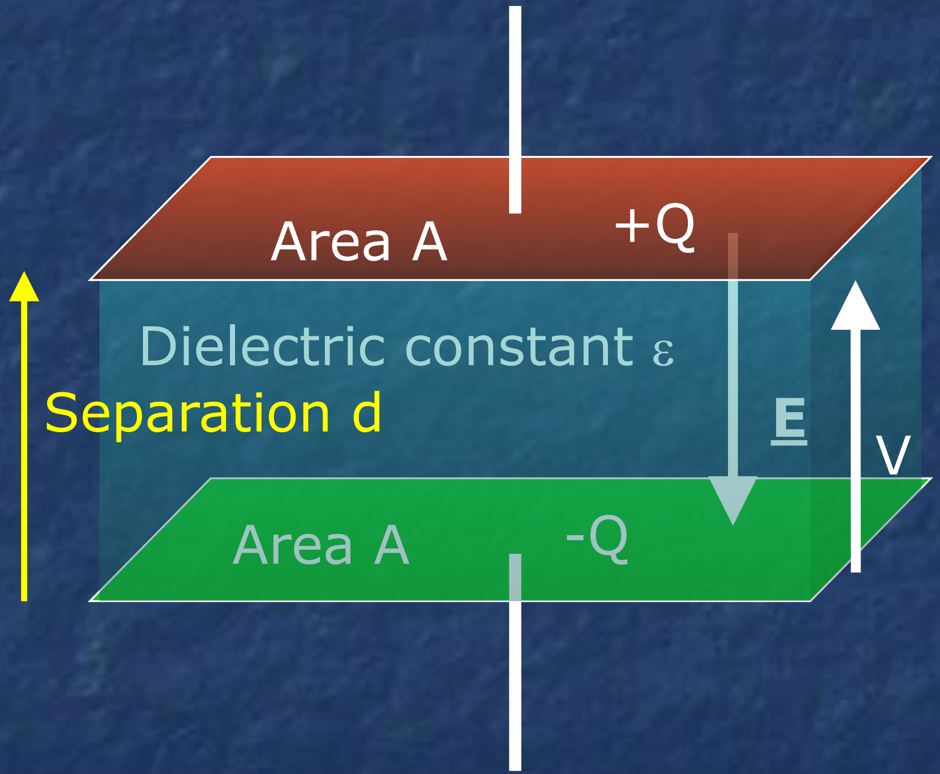


Calculating Capacitance?

- C = f(geometry, dielectric)
  - e.g. C = εArea/separation = $\varepsilon A/d$ for a parallel-plate capacitor
- With much symmetry, C can be calculated
  - And capacitors are often manufactured in simple geometries!
- Without such symmetry – approximation and estimation is necessary
  - Can be made arbitrarily accurate
  - Remember Laplace and field plotting?
- Tackle calculation, then estimation

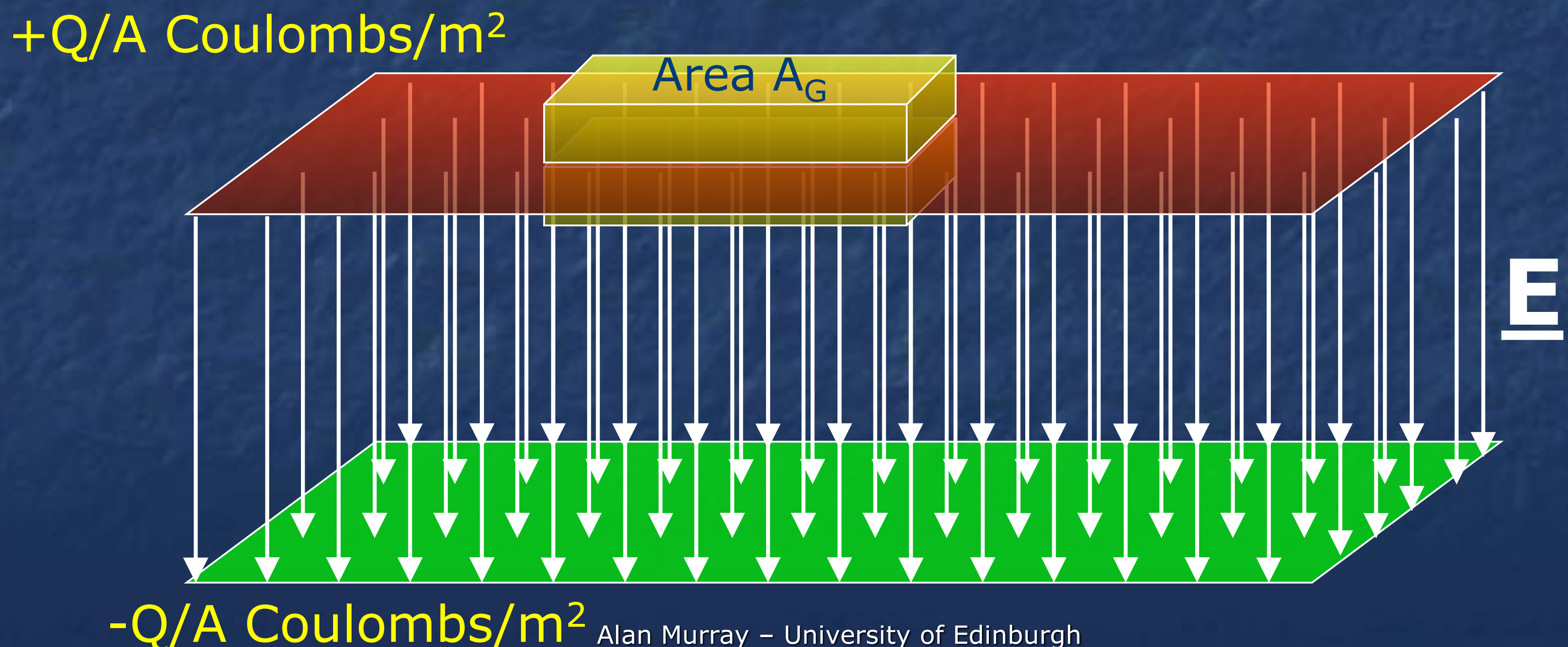
Example 1 : Parallel-Plate Capacitor

1. Calculate field strength **E** as a function of charge ±Q on the plates
2. Integrate field to calculate potential V between the plates
3. Q=CV, C = Q/V



Example 1 : Parallel-Plate Capacitor

- Gauss's Law – **D**, **E** $\neq 0$ only on bottom face
- Charge enclosed = $A_G \times Q/A$



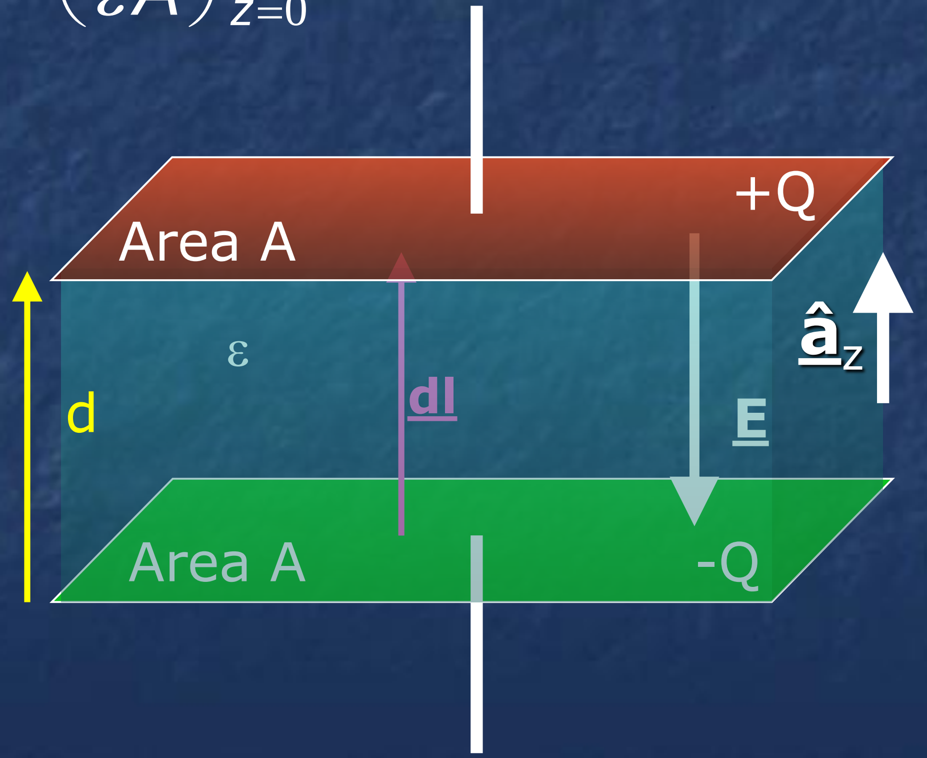
Example 1 : Parallel-Plate Capacitor

$$\underline{\mathbf{E}} = -\frac{Q\underline{\mathbf{a}}_z}{\varepsilon A}$$ **from Gauss's Law**

$$V = -\int_{z=0}^{z=d} \underline{\mathbf{E}}.\underline{\mathbf{dl}} = -\int_{z=0}^{z=d} -\left(\frac{Q\underline{\mathbf{a}}_z}{\varepsilon A}\right).\underline{\mathbf{dl}} = \left(\frac{Q}{\varepsilon A}\right)\int_{z=0}^{z=d} \underline{\mathbf{a}}_z.\underline{\mathbf{dl}}$$

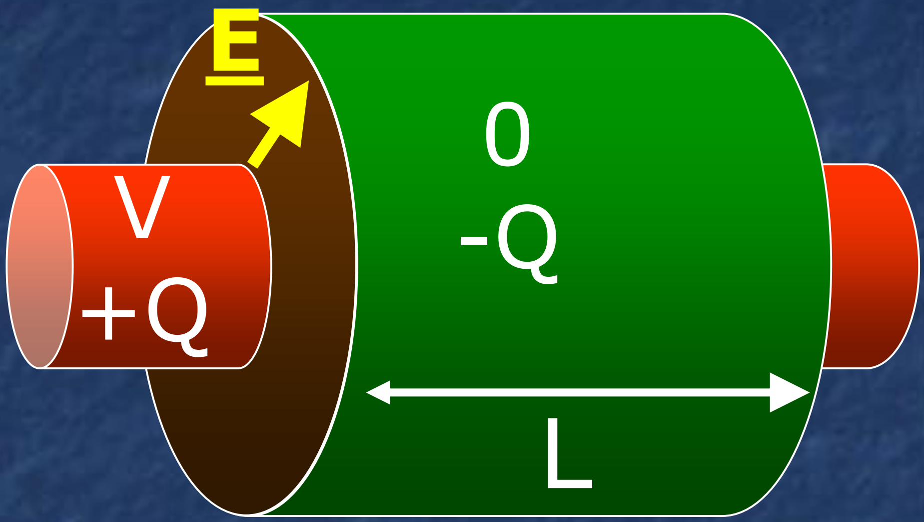
$$V = \left(\frac{Qd}{\varepsilon A}\right), Q = \left(\frac{\varepsilon A}{d}\right)V$$

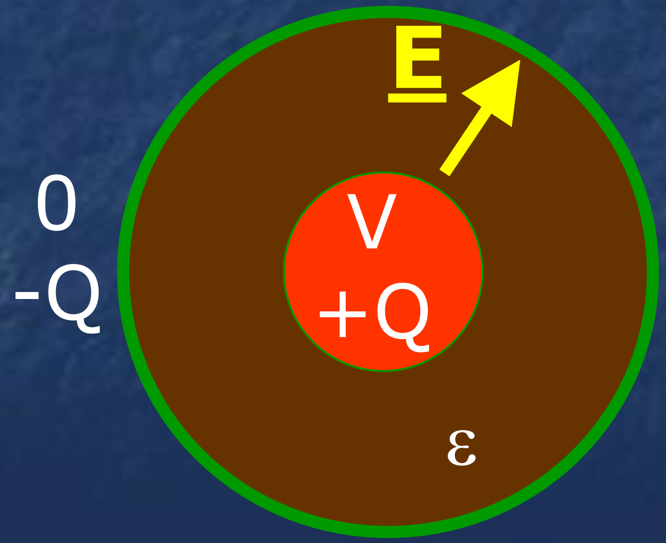
$$C = \frac{\varepsilon A}{d}$$



Example 2 : Cylindrical Capacitor

- Two concentric cylindrical conductors, overlap length L
  - e.g. co-axial TV lead cable
- Separated by a dielectric (insulator)





Example 2 : Cylindrical Capacitor

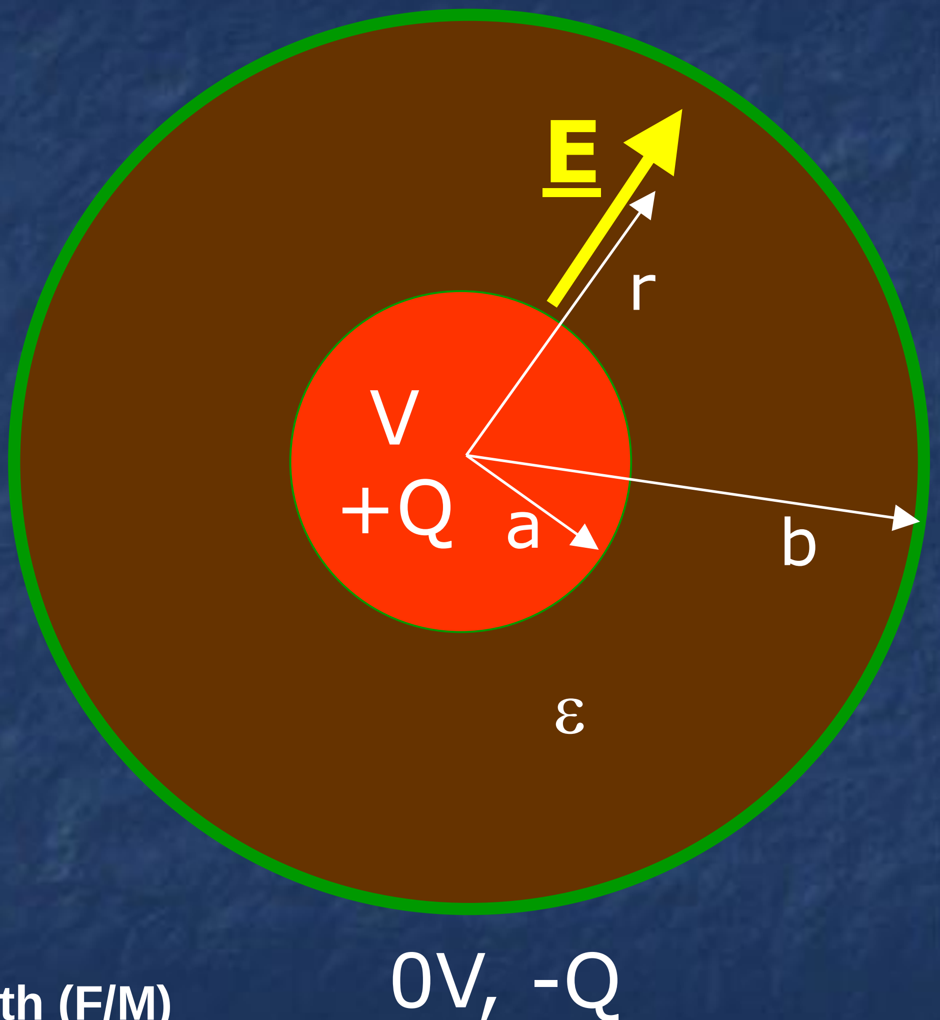
$$\underline{\mathbf{E}} = \frac{\rho_l \underline{\mathbf{a}}_r}{2\pi\varepsilon r} = \frac{\left(\frac{Q}{L}\right)\underline{\mathbf{a}}_r}{2\pi\varepsilon r}$$ **from Gauss's Law**

$$V = -\int_{l=a}^{l=b} \underline{\mathbf{E}}.\underline{\mathbf{dl}} = -\int_{l=a}^{l=b} \frac{\rho_l \underline{\mathbf{a}}_r.\underline{\mathbf{dl}}}{2\pi\varepsilon r}$$

$$V = -\frac{\rho_l}{2\pi\varepsilon}\int_{r=a}^{r=b}\frac{dr}{r} = -\frac{\rho_l}{2\pi\varepsilon}\left[\ln(r)\right]_a^b$$

$$V = -\frac{\rho_l}{2\pi\varepsilon}\left[\ln(b) - \ln(a)\right] = -\frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

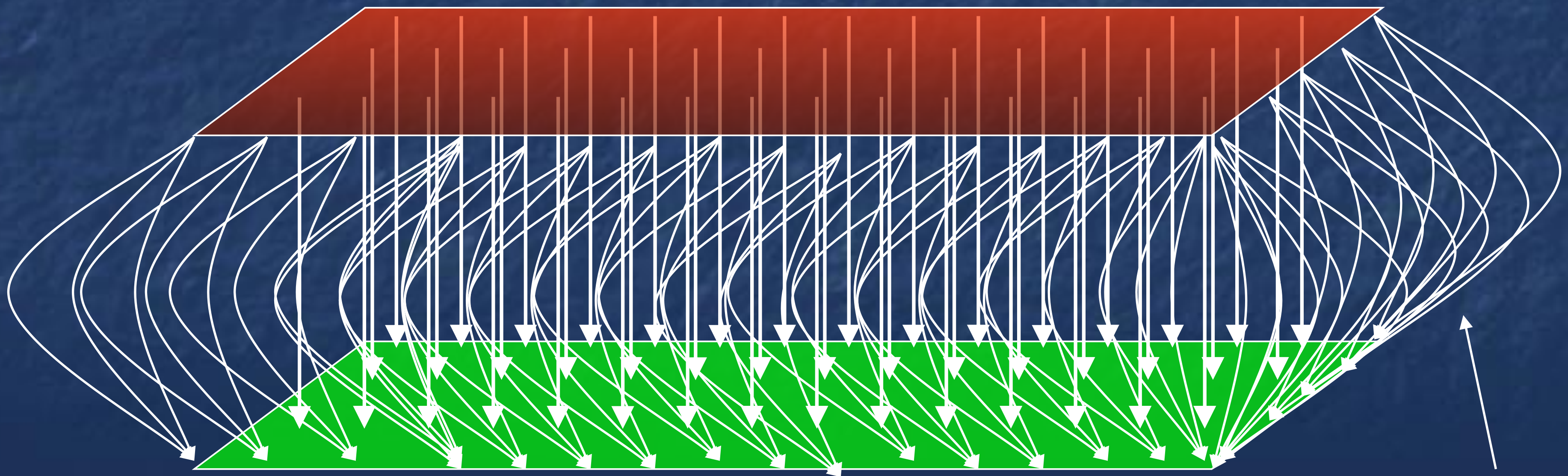
$$V = -\frac{\ln\left(\frac{b}{a}\right)}{2\pi\varepsilon}\rho_l = -\frac{\ln\left(\frac{b}{a}\right)}{2\pi\varepsilon}\frac{Q}{L},$$ **so** $C_L = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}$



$C_L = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}$ **is the Capacitance/unit length (F/M)**

Estimating Capacitance …

- When the electrodes are not as symmetrical as these examples
- Also – our "ideal" parallel-plate capacitor should really look thus:-

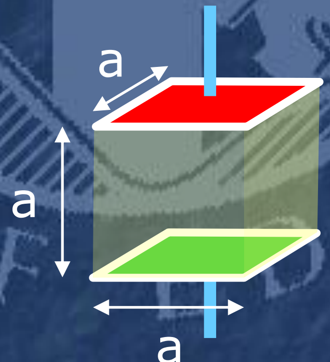


Fringing fields

Estimating Capacitance : Principle

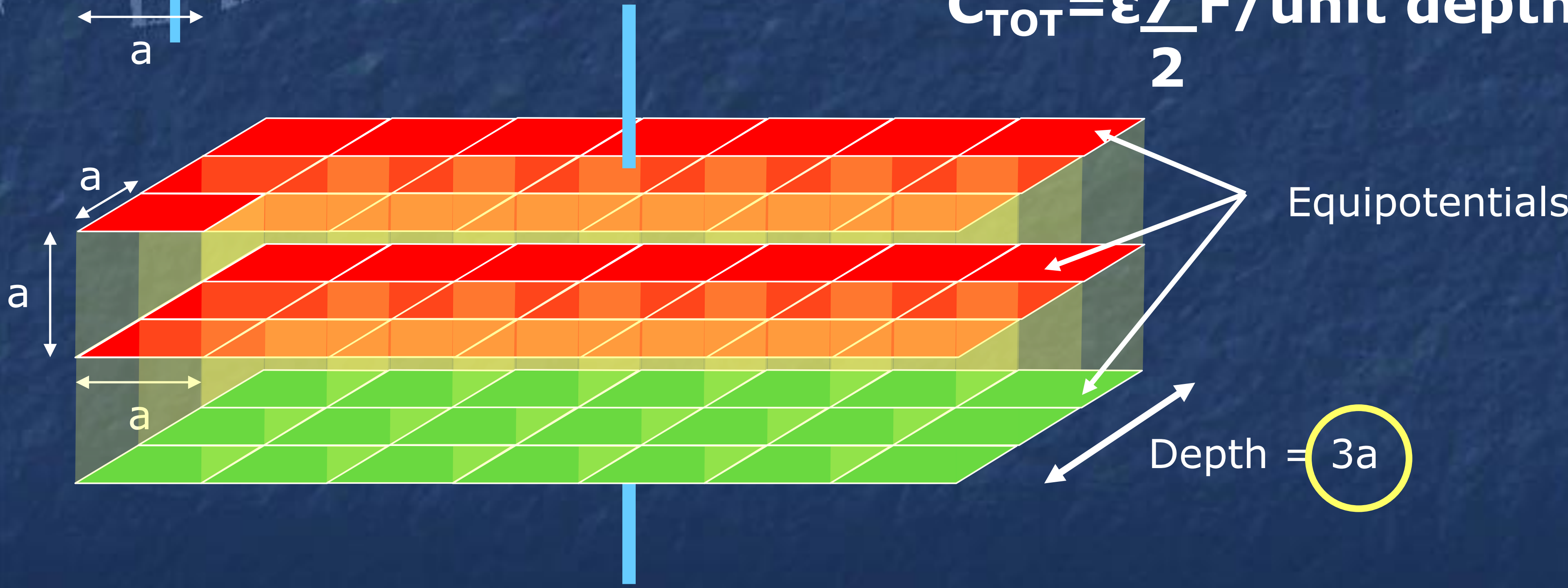
- Sketch equipotentials and field lines using field plotting
- Can be arbitrarily accurate
- More accuracy means more
  $V_0 = \frac{1}{4}(V_1+V_2+V_3+V_4)$
- Use a computer!

Estimating capacitance

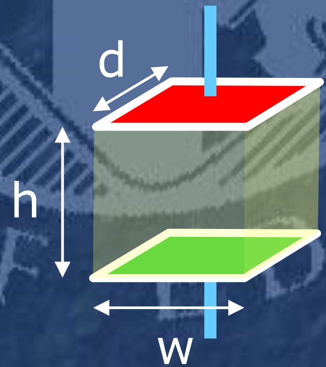


$$C=\varepsilon\frac{A}{d}=\varepsilon\frac{a \times a}{a}=\varepsilon a \rightarrow 7\varepsilon a \rightarrow \varepsilon 7 \times 3a \rightarrow \varepsilon\frac{7 \times 3a}{2}$$

$$\mathbf{C_{TOT}=\varepsilon\frac{7}{2}\ F/unit\ depth}$$



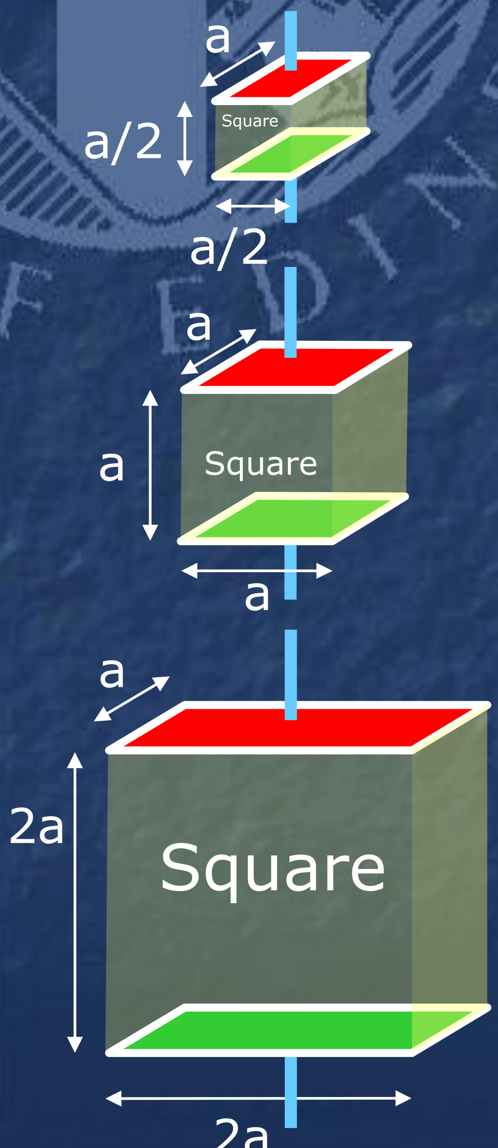
Estimating capacitance



$$C=\varepsilon\frac{A}{d}=\varepsilon\frac{w \times d}{h}=\varepsilon a \rightarrow 7\varepsilon a \rightarrow \varepsilon 7 \times 3a \rightarrow \varepsilon\frac{7 \times 3a}{2}$$

$$\mathbf{C_{TOT}=\varepsilon\frac{7}{2}\ F/unit\ depth}$$

Estimating capacitance



Capacitance? | Capacitance/unit depth (÷a)?

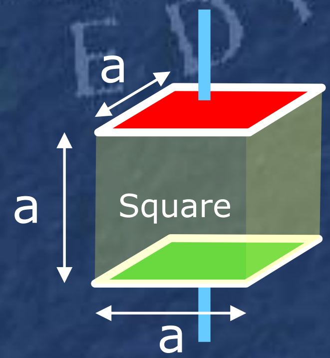
$$C=\varepsilon\frac{A}{d}=\varepsilon\frac{a\times 0.5a}{0.5a}=\varepsilon a \qquad =\varepsilon$$

$$C=\varepsilon\frac{A}{d}=\varepsilon\frac{a\times a}{a}=\varepsilon a \qquad =\varepsilon$$

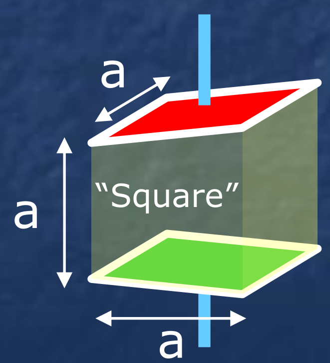
$$C=\varepsilon\frac{A}{d}=\varepsilon\frac{a\times 2a}{2a}=\varepsilon a \qquad =\varepsilon$$

Estimating capacitance

Capacitance? | Capacitance/unit depth (÷a)?

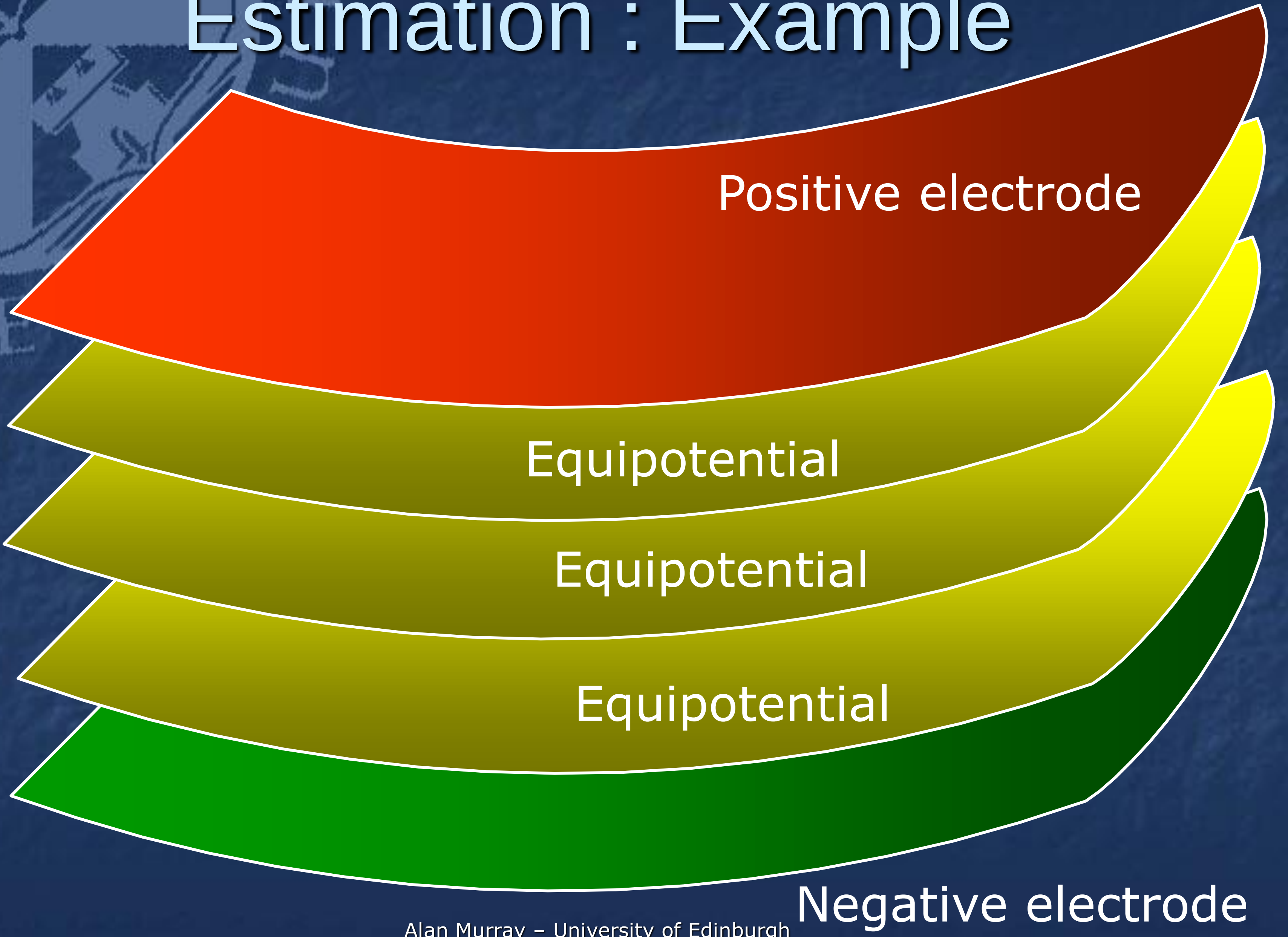


$$C=\varepsilon\frac{A}{d}=\varepsilon\frac{axa}{a}=\varepsilon a \qquad =\varepsilon$$

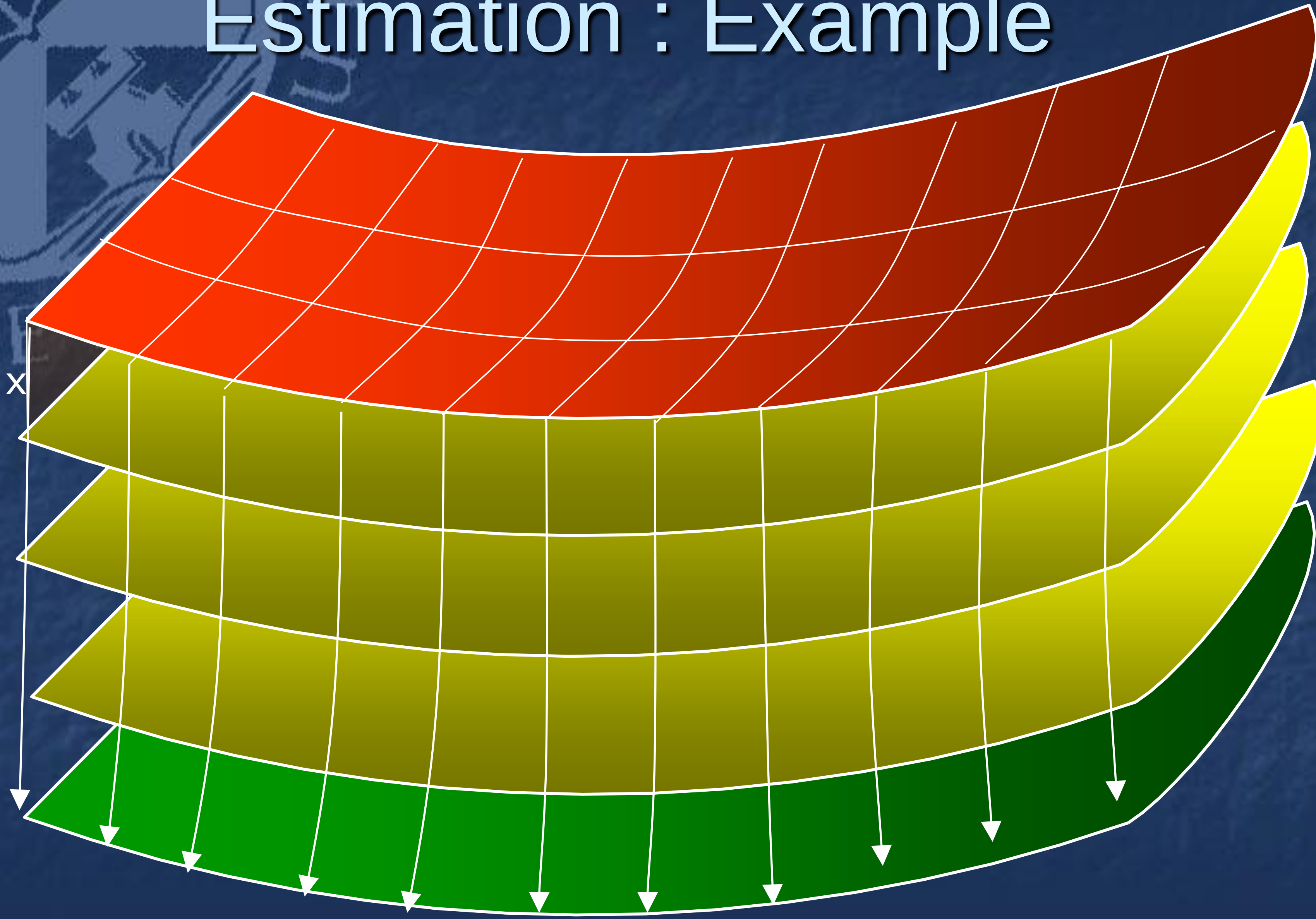


$$C\approx\varepsilon\frac{A}{d}=\varepsilon\frac{axa}{a}=\varepsilon a \qquad \approx\varepsilon$$

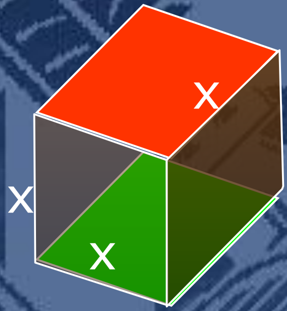
Estimation : Example



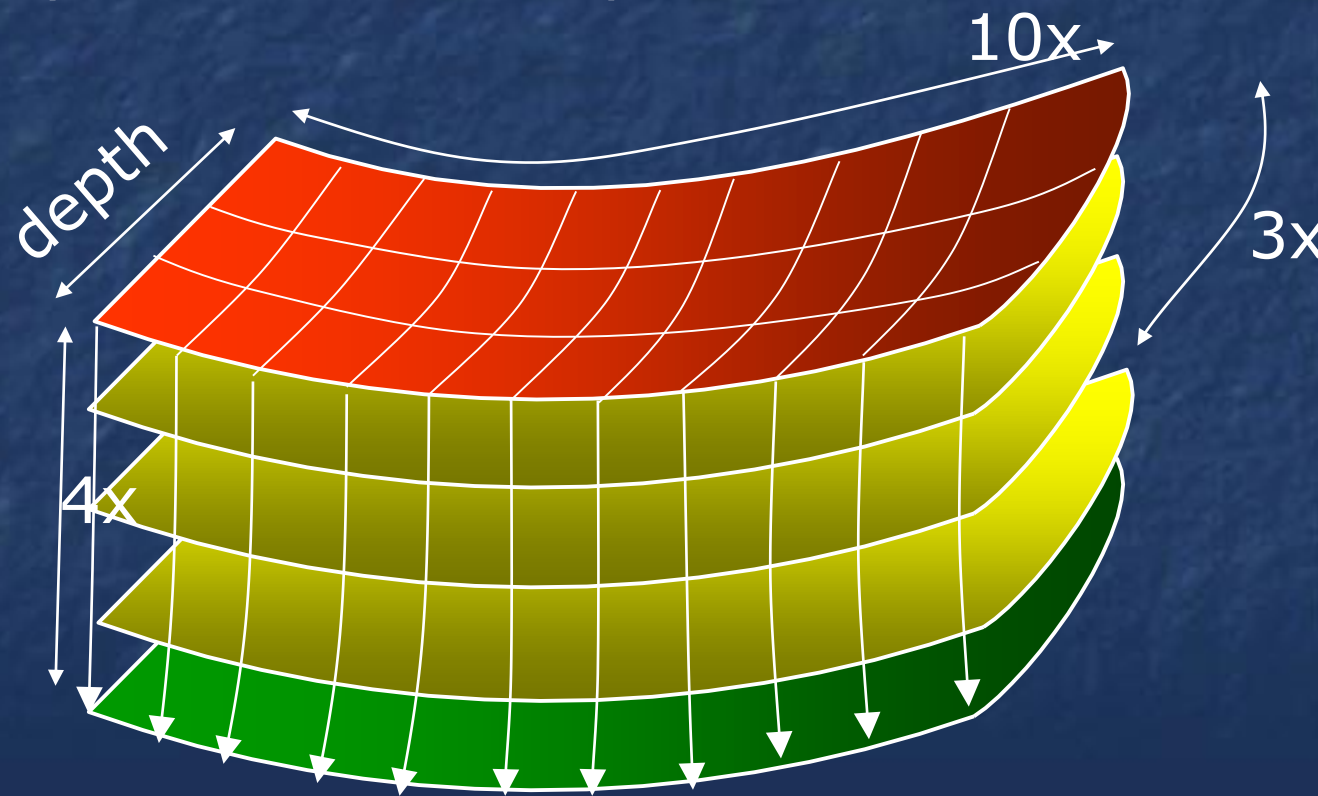
Estimation : Example



Estimation : Example



Each of these is $\varepsilon A/d = \varepsilon \times x \times x/x$
4 in series, 30 in parallel
Capacitance = $30x\varepsilon/4$
Or capacitance/unit depth = $10\varepsilon/4$



Estimating Capacitance : Recipe

- Draw equipotentials as accurately as you have time for
  - Using field mapping in reality
- Draw field lines to make square “cells” (cubes in 3D)
  - Field line and equipotentials cross at 90°
  - Make cells as square as possible
- Count series and parallel – each is a capacitance of εx (ε per unit depth when using a 2D diagram)