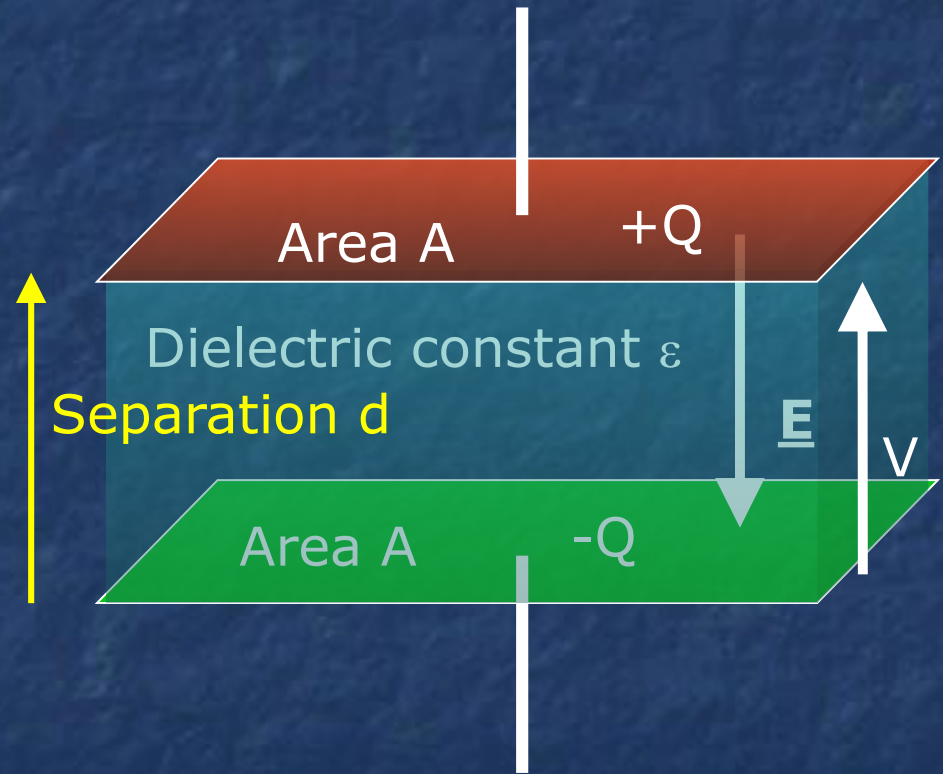


Example 1 : Parallel-Plate Capacitor

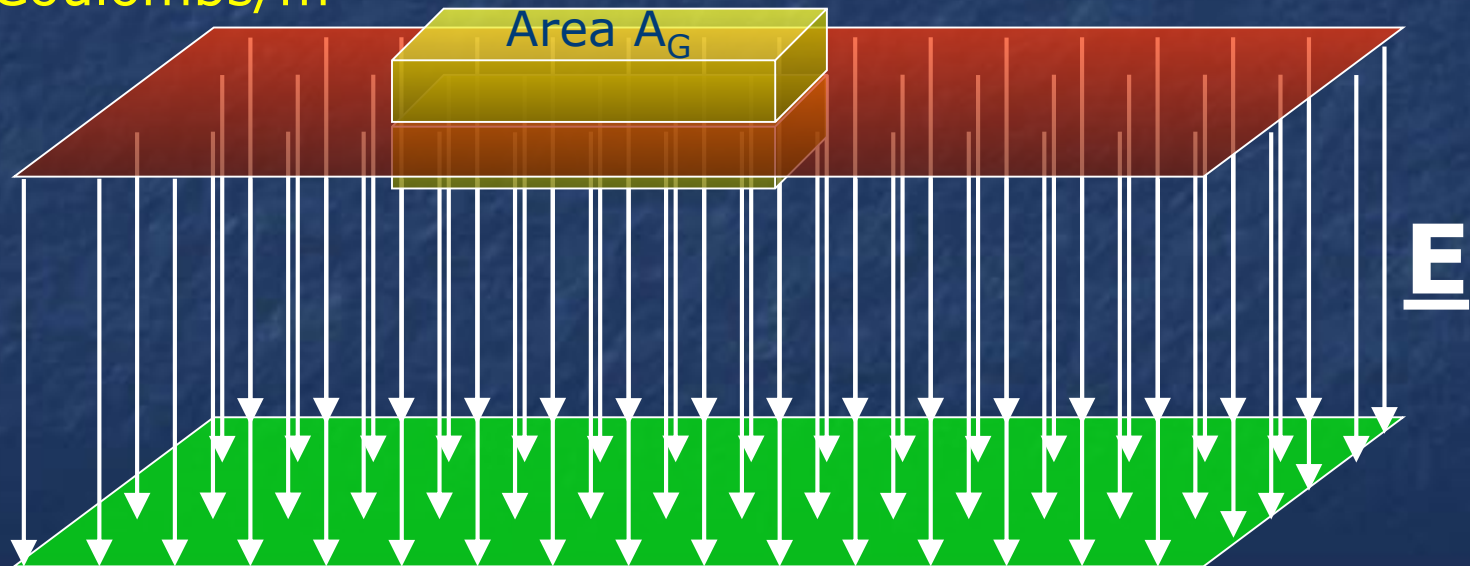
1. Calculate field strength \underline{E} as a function of charge $\pm Q$ on the plates
2. Integrate field to calculate potential V between the plates
3. $Q=CV$, $C = Q/V$



Example 1 : Parallel-Plate Capacitor

- Gauss's Law – \underline{D} , $\underline{E} \neq 0$ only on bottom face
- Charge enclosed = $A_G \times Q/A$

+Q/A Coulombs/m²



-Q/A Coulombs/m²

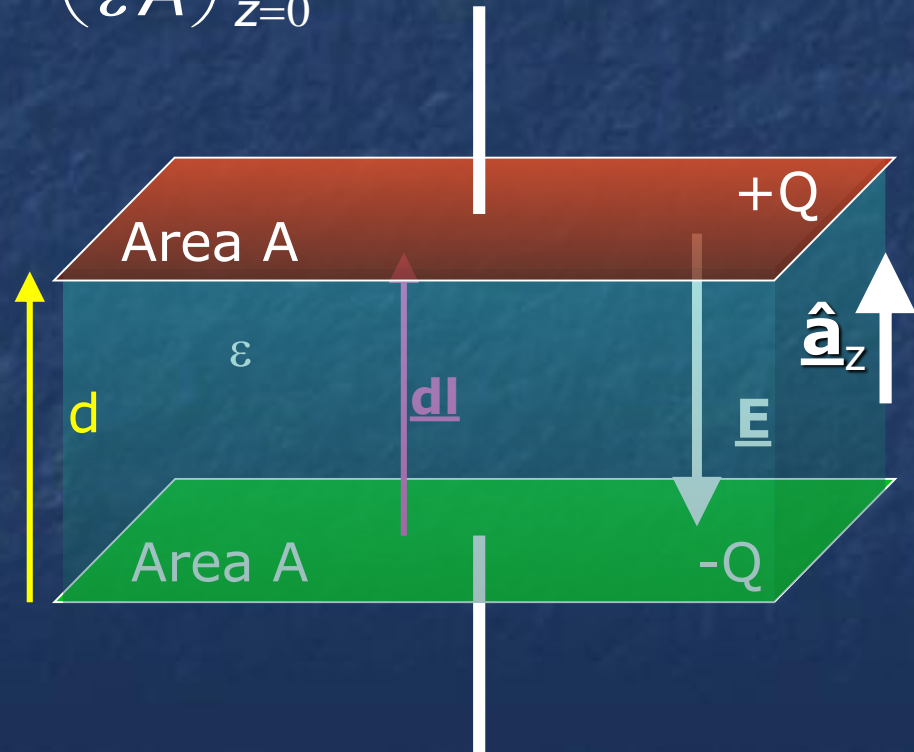
Example 1 : Parallel-Plate Capacitor

$$\underline{E} = -\frac{Q\underline{a}_z}{\varepsilon A} \text{ from Gauss's Law}$$

$$V = -\int_{z=0}^{z=d} \underline{E} \cdot d\underline{l} = -\int_{z=0}^{z=d} -\left(\frac{Q\underline{a}_z}{\varepsilon A}\right) \cdot d\underline{l} = \left(\frac{Q}{\varepsilon A}\right) \int_{z=0}^{z=d} \underline{a}_z \cdot d\underline{l}$$

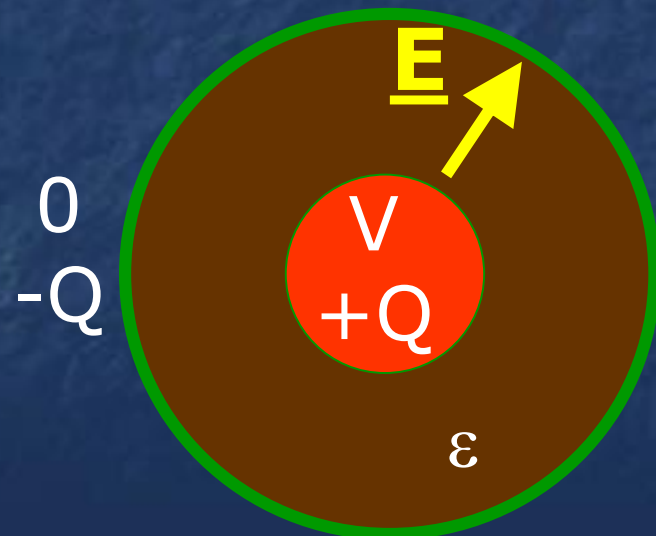
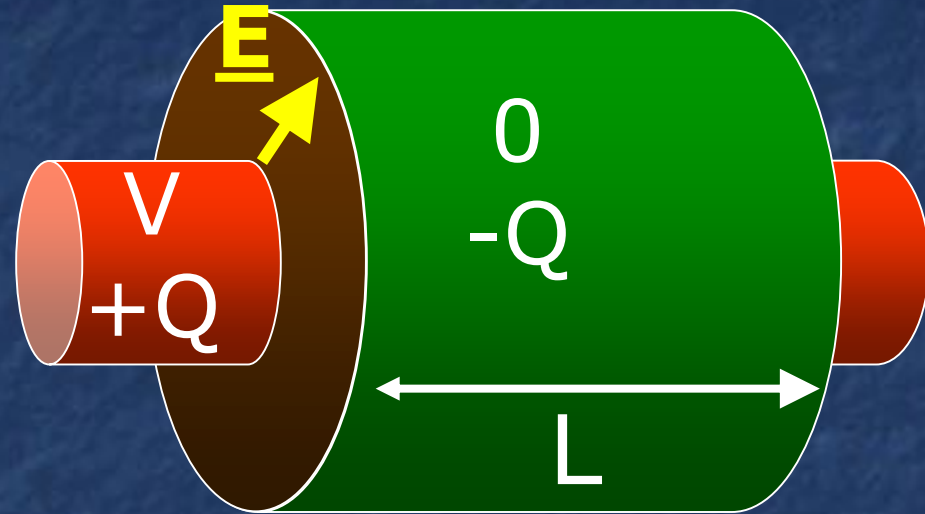
$$V = \left(\frac{Qd}{\varepsilon A}\right), Q = \left(\frac{\varepsilon A}{d}\right) V$$

$$C = \frac{\varepsilon A}{d}$$



Example 2 : Cylindrical Capacitor

- Two concentric cylindrical conductors, overlap length L
 - e.g. co-axial TV lead cable
- Separated by a dielectric (insulator)



Example 2 : Cylindrical Capacitor

$$\underline{E} = \frac{\rho_l \underline{a}_r}{2\pi\epsilon r} = \left(\frac{Q}{L}\right) \frac{\underline{a}_r}{2\pi\epsilon r} \quad \text{from Gauss's Law}$$

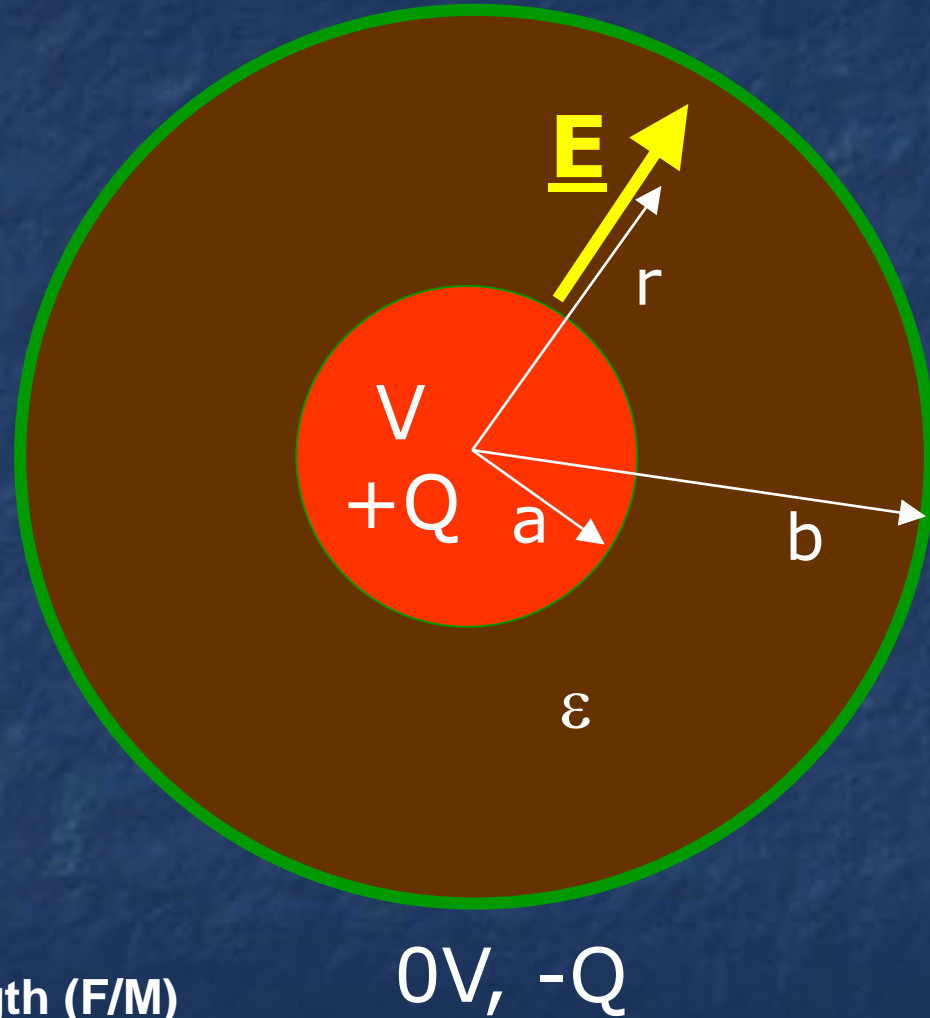
$$V = - \int_{l=a}^{l=b} \underline{E} \cdot d\underline{l} = - \int_{l=a}^{l=b} \frac{\rho_l \underline{a}_r \cdot d\underline{l}}{2\pi\epsilon r}$$

$$V = - \frac{\rho_l}{2\pi\epsilon} \int_{r=a}^{r=b} \frac{dr}{r} = - \frac{\rho_l}{2\pi\epsilon} [\ln(r)]_a^b$$

$$V = - \frac{\rho_l}{2\pi\epsilon} [\ln(b) - \ln(a)] = - \frac{\rho_l}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)$$

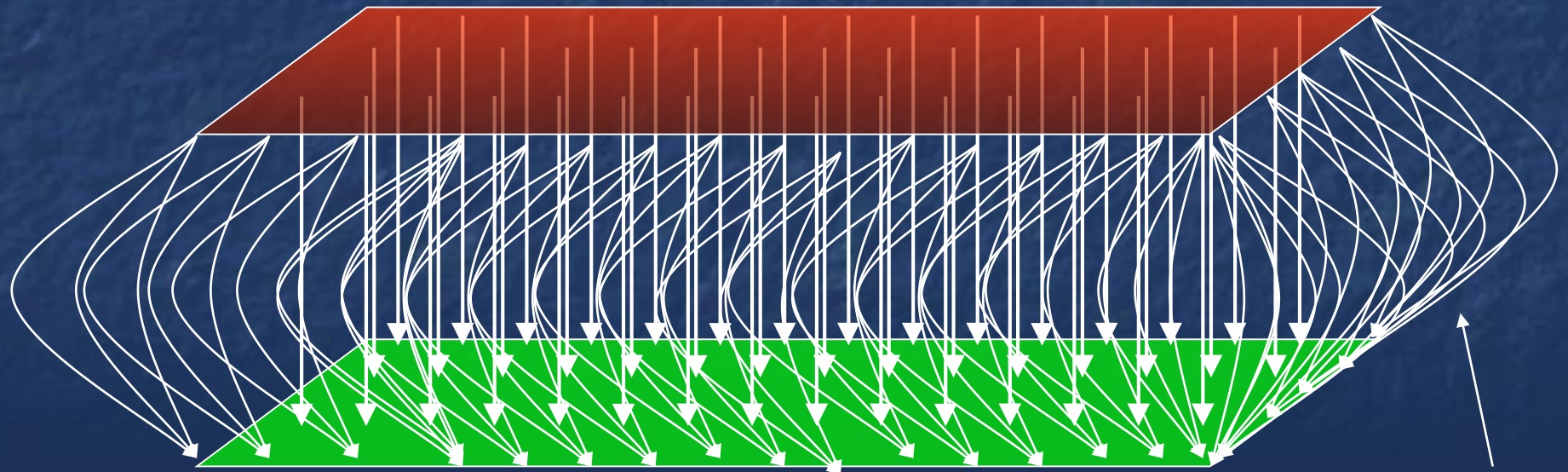
$$V = - \frac{\ln\left(\frac{b}{a}\right)}{2\pi\epsilon} \rho_l = - \frac{\ln\left(\frac{b}{a}\right)}{2\pi\epsilon} \frac{Q}{L}, \text{ so } C_L = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}$$

$$C_L = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)} \text{ is the Capacitance/unit length (F/M)}$$



Estimating Capacitance ...

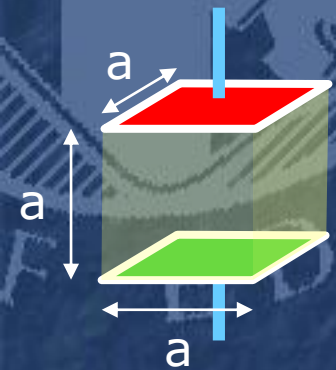
- When the electrodes are not as symmetrical as these examples
- Also – our “ideal” parallel-plate capacitor should really look thus:-



Estimating Capacitance : Principle

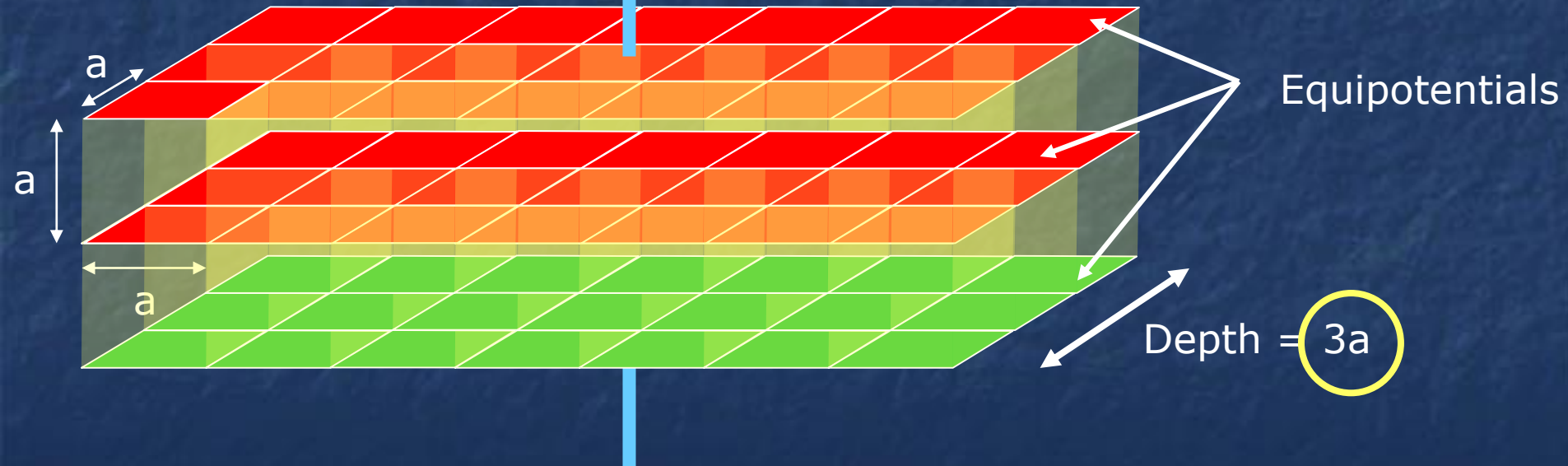
- Sketch equipotentials and field lines using field plotting
- Can be arbitrarily accurate
- More accuracy means more
$$V_0 = \frac{1}{4} (V_1 + V_2 + V_3 + V_4)$$
- Use a computer!

Estimating capacitance

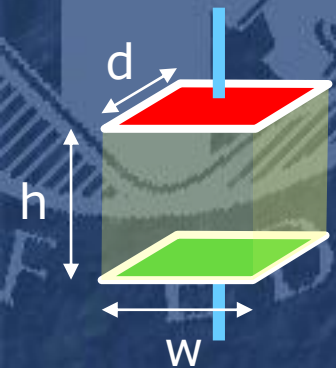


$$C = \epsilon \frac{A}{d} = \epsilon \frac{a \times a}{a} = \epsilon a \rightarrow 7\epsilon a \rightarrow \epsilon 7 \times 3a \rightarrow \frac{\epsilon 7 \times 3a}{2}$$

$$C_{TOT} = \frac{\epsilon 7}{2} F / \text{unit depth}$$



Estimating capacitance

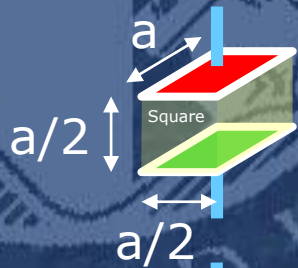


$$C = \epsilon \frac{A}{d} = \epsilon \frac{w \times d}{h} = \epsilon a \rightarrow 7\epsilon a \rightarrow \epsilon 7 \times 3a \rightarrow \frac{\epsilon 7 \times 3a}{2}$$

$$C_{TOT} = \frac{\epsilon 7}{2} \text{ F/unit depth}$$



Estimating capacitance

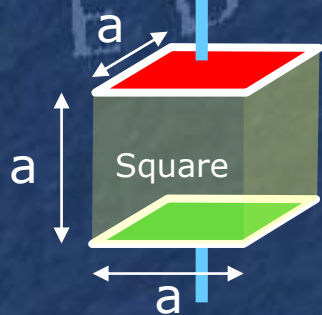


Capacitance?

$$C = \epsilon \frac{A}{d} = \epsilon \frac{a \times 0.5a}{0.5a} = \epsilon a$$

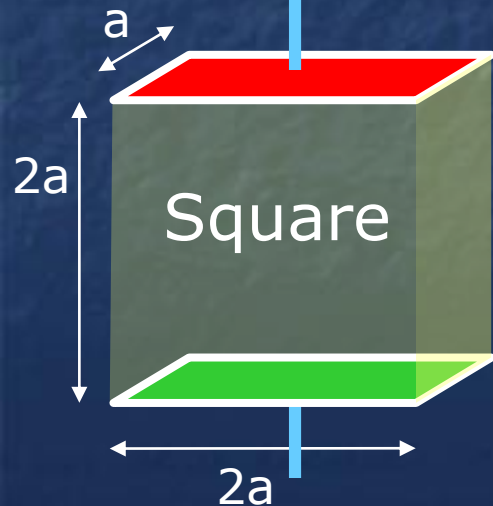
Capacitance/unit depth ($\div a$)?

$$= \epsilon$$



$$C = \epsilon \frac{A}{d} = \epsilon \frac{a \times a}{a} = \epsilon a$$

$$= \epsilon$$



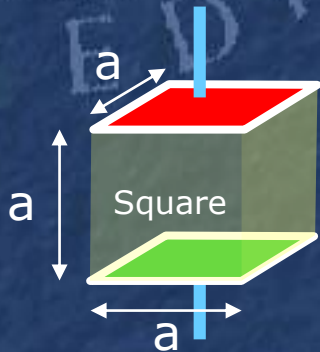
$$C = \epsilon \frac{A}{d} = \epsilon \frac{a \times 2a}{2a} = \epsilon a$$

$$= \epsilon$$

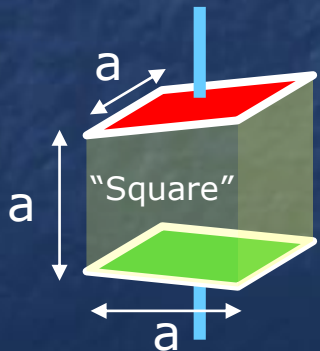
Estimating capacitance

Capacitance?

Capacitance/unit depth ($\div a$)?

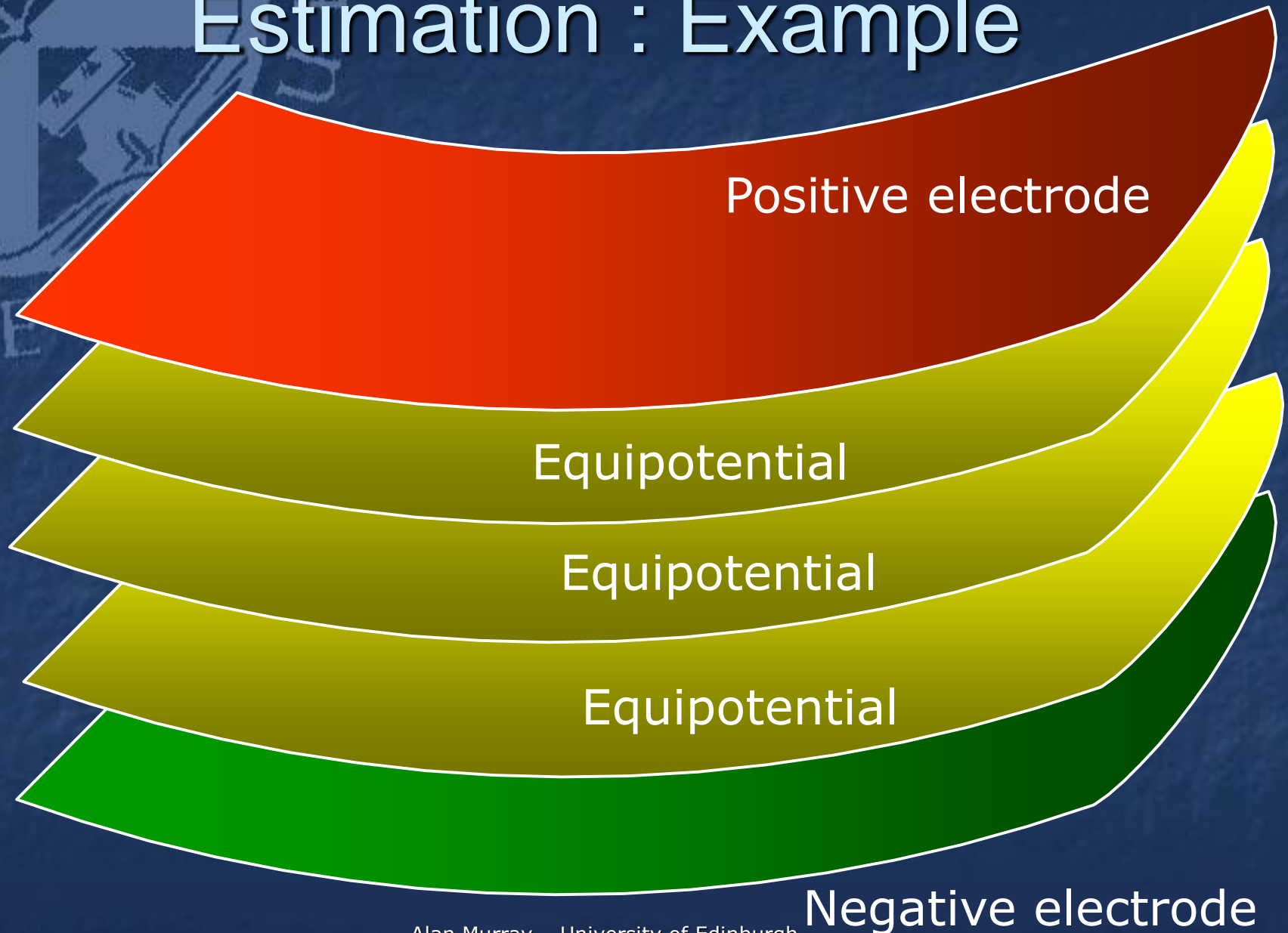


$$C = \epsilon \frac{A}{d} = \epsilon \frac{a \times a}{a} = \epsilon a = \epsilon$$

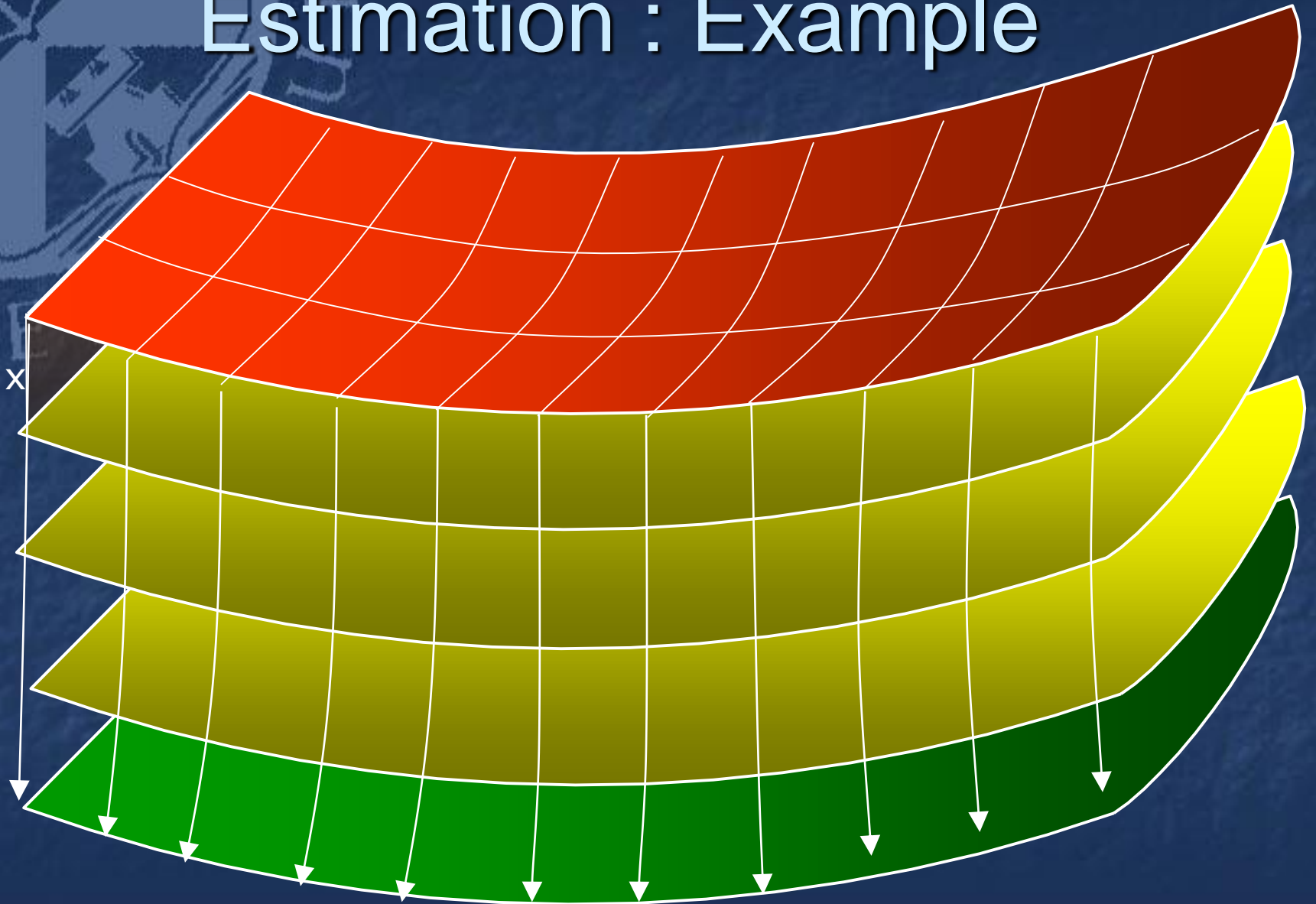


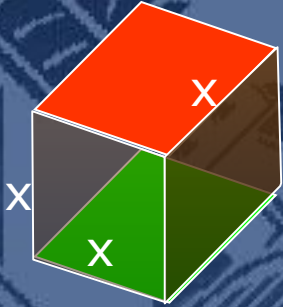
$$C \approx \epsilon \frac{A}{d} = \epsilon \frac{a \times a}{a} = \epsilon a \approx \epsilon$$

Estimation : Example



Estimation : Example



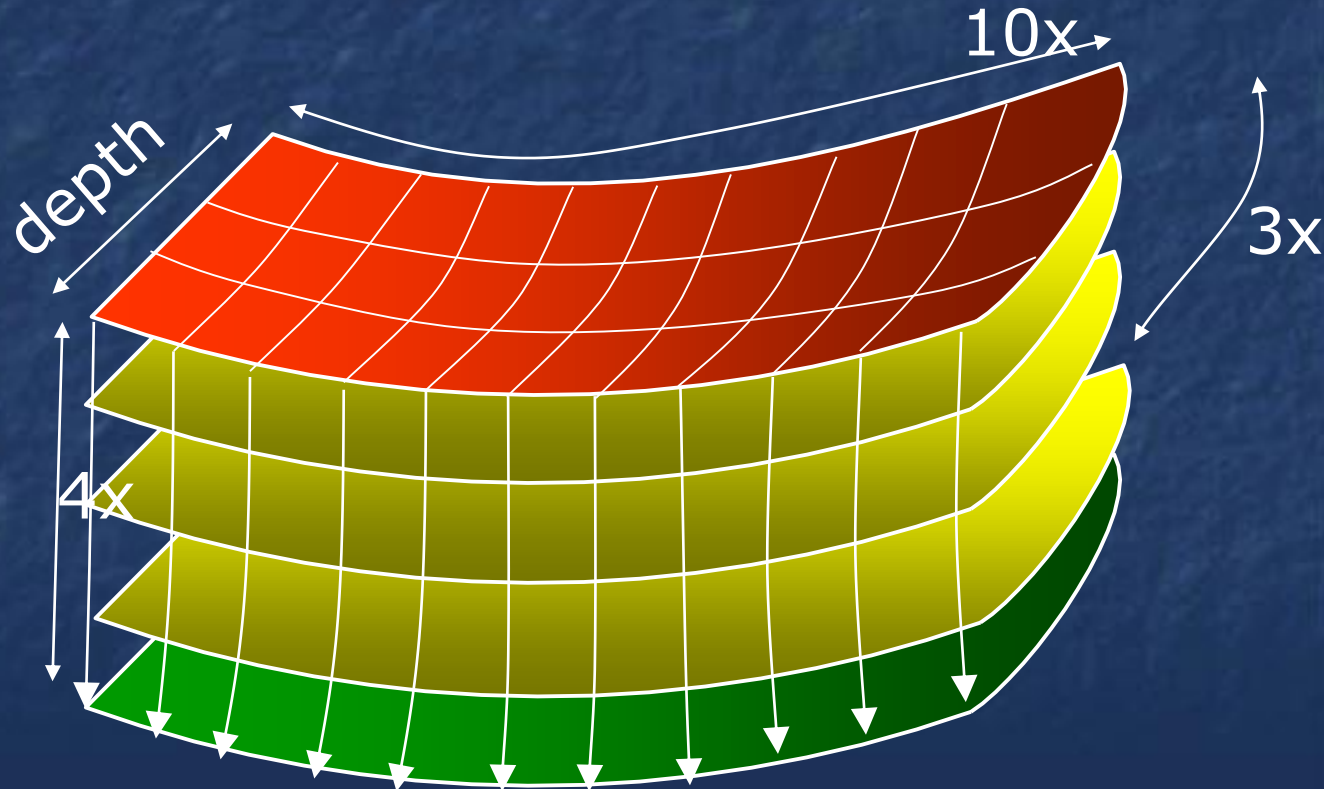


Estimation : Example

Each of these is $\epsilon A/d = \epsilon \times x \times x / x$
 4 in series, 30 in parallel

Capacitance = $30x\epsilon/4$

Or capacitance/unit depth = $10\epsilon/4$



Estimating Capacitance : Recipe

- Draw equipotentials as accurately as you have time for
 - Using field mapping in reality
- Draw field lines to make square “cells” (cubes in 3D)
 - Field line and equipotentials cross at 90°
 - Make cells as square as possible
- Count series and parallel – each is a capacitance of ϵx (ϵ per unit depth when using a 2D diagram)