CNN operations cheatsheet

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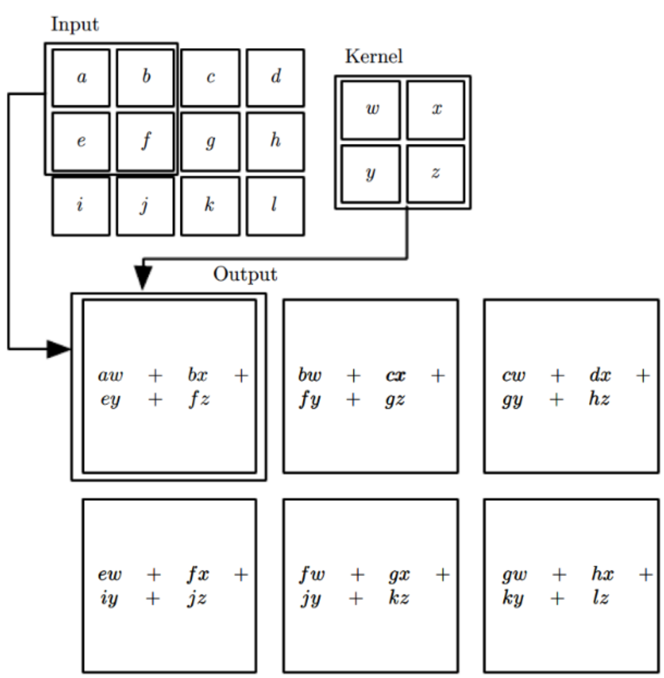
1 Basic Idea



2 Notation Summary

The following table provides a summary of the variables used in the example described in 3.

Explanation	Symbol
P	padding layers
X	input image
H	kernel
Y	output

Table 1: Summary of symbols used

To understand the formulation it is really important to clearify a couple of important things:

- The following formula, given the plain kernel, it flips it and applies it to the image, performing the so called **convolution** (if you don't flip it, it's just a correlation);
- For the kernel matrix, we use indexes in such a way that the central element has coordinates [0,0]. This means that we will end up with some negative indices.
- The input matrix needs to be considered with **zero padding**. The first row and column of the resulting matrix will have index -P;
- The output matrix has standard coordinates, so starting from [0,0];

3 Example

We are going to use the following formula, Input and Kernel:

$$Y[m,n] = \sum_{i=-P+m} \sum_{j=-P+n} X[i,j] \quad H[m-i,n-j] \tag{1}$$

i,j	-1	0	1	2	3
-1	0	0	0	0	0
0	0	1	2	3	0
1	0	4	5	6	0
2	0	7	8	9	0
3	0	0	0	0	0

Table 2: Input X

i,j	-1	0	1
-1	-1	-2	-3
-0	0	0	0
1	1	2	3

Table 3: Kernel H

As you can see padding is alredy applied in the input matrix, applying 1 layer of zeroes means that we are going to start counting indexes at -1.

$$
\begin{aligned}
Y[0,1] \Rightarrow m = 0, n = 1, P = -1 \\
& i = -1 \\
& X[-1,0] \cdot H[1,1] = 0 \cdot 3 + \\
& X[-1,1] \cdot H[1,0] = 0 \cdot 2 + \\
& X[-1,2] \cdot H[1,-1] = 0 \cdot 1 + \\
& i = 0 \\
& X[0,0] \cdot H[0,1] = 1 \cdot 0 + \\
& X[0,1] \cdot H[0,0] = 2 \cdot 0 + \\
& X[0,2] \cdot H[0,-1] = 3 \cdot 0 + \\
& i = 1 \\
& X[1,0] \cdot H[-1,1] = 4 \cdot -3 + \\
& X[1,1] \cdot H[-1,0] = 5 \cdot -2 + \\
& X[1,2] \cdot H[-1,-1] = 6 \cdot -1 + \\
& = -28
\end{aligned} \tag{2}
$$

As you can see in table 4, we flipped the kernel on both axis and overlapped it with the correct portion of the input.

i, j	-1	0	1	2	3
-1	0	$0^{(3)}$	$0^{(2)}$	$0^{(1)}$	0
0	0	$1^{(0)}$	$2^{(0)}$	$3^{(0)}$	0
1	0	$4^{(-3)}$	$5^{(-2)}$	$6^{(-1)}$	0
2	0	7	8	9	0
3	0	0	0	0	0

Table 4: Convolution to compute element $Y[0,1]$ of output matrix

Let's write also the calculations made to compute $Y[1,2]$

$$
\begin{aligned}
Y[1,2] \Rightarrow\ & m = 1, n = 2, P = -1 \\
& i = 0 \\
& X[0,1] \cdot H[1,1] = 2 \cdot 3 + \\
& X[0,2] \cdot H[1,0] = 3 \cdot 2 + \\
& X[0,3] \cdot H[1,-1] = 0 \cdot 1 + \\
& i = 1 \\
& X[1,1] \cdot H[0,1] = 5 \cdot 0 + \\
& X[1,2] \cdot H[0,0] = 6 \cdot 0 + \\
& X[1,3] \cdot H[0,-1] = 0 \cdot 0 + \\
& i = 2 \\
& X[2,1] \cdot H[-1,1] = 8 \cdot -3 + \\
& X[2,2] \cdot H[-1,0] = 9 \cdot -2 + \\
& X[2,3] \cdot H[-1,-1] = 0 \cdot -1 + \\
& = -30
\end{aligned} \tag{3}
$$

i, j	-1	0	1	2	3
-1	0	0	0	0	0
0	0	1	$2^{(3)}$	$3^{(2)}$	$0^{(1)}$
1	0	4	$5^{(0)}$	$6^{(0)}$	$0^{(0)}$
2	0	7	$8^{(-3)}$	$9^{(-2)}$	$0^{(-1)}$
3	0	0	0	0	0

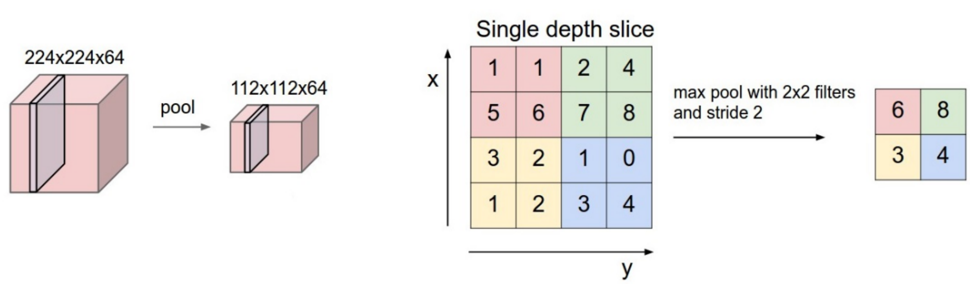
Table 5: Convolution to compute element $Y[1,2]$ of output matrix

4 Convolution Layers

In most cases CNNs are used with RGB images. As the name says, such images are made of 3 **channels** (Red, Green and Blue). You can imagine them as $h \times w \times 3$ cubes, or as 3 $h \times w$ matrixes stacked. This means that we have to imagine our kernels as $k \times k \times 3$ dimensional cubes as well! Now it's easy to imagine why for each convolutional layer there are $k \times k \times c \times N + N$ learnable parameters, where c represents the number of channels of the input data (3 in case of RGB images), and N is the number of filters for that layer. It's really important to remember that the number of channels for the next layer becomes N: if in the first layer we use 10 kernels to process our plain image, the input that needs to be processed by the next layer will have 10 channels!

5 Pooling Layers

Pooling layers are used as noise reduction layers or to perform dimensionality reduction. The most common types of pooling are *max pooling* and *average pooling*, which either compute the maximum value among the overlapped elements or the average value. The figure below should be clear enough.



The only 2 hyperparameters that need to be defined are

- Pool size k, in this case equal to 2;
- Pool stride s, in this case equal to 2 as well;

6 Output volume size