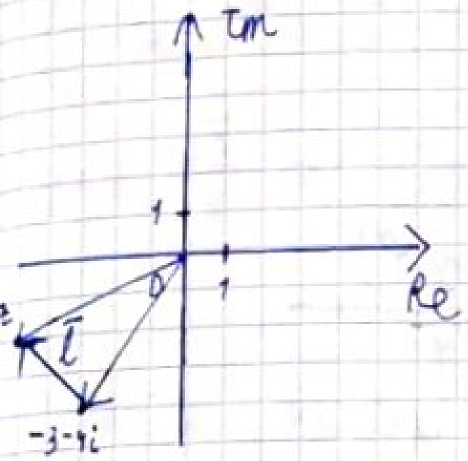
Д/з по алгебре N 11

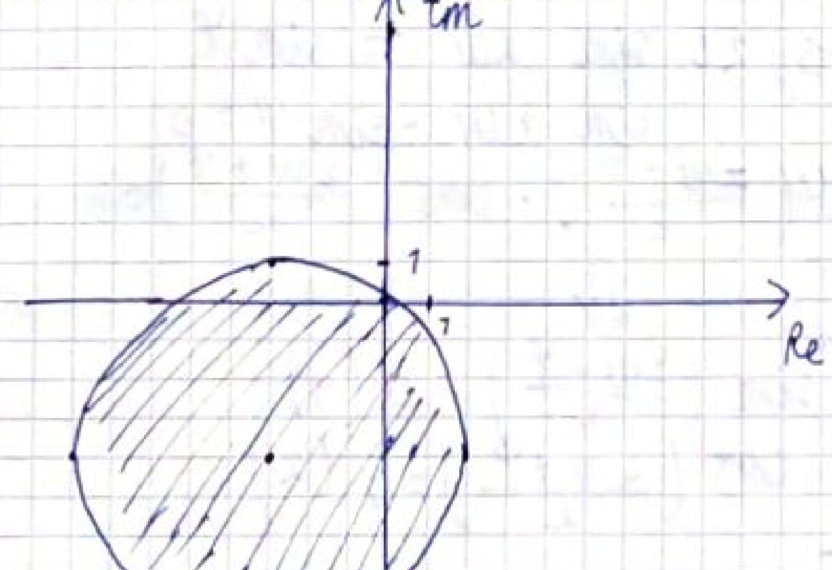
N 1

$|z+3+4i| \le 5$

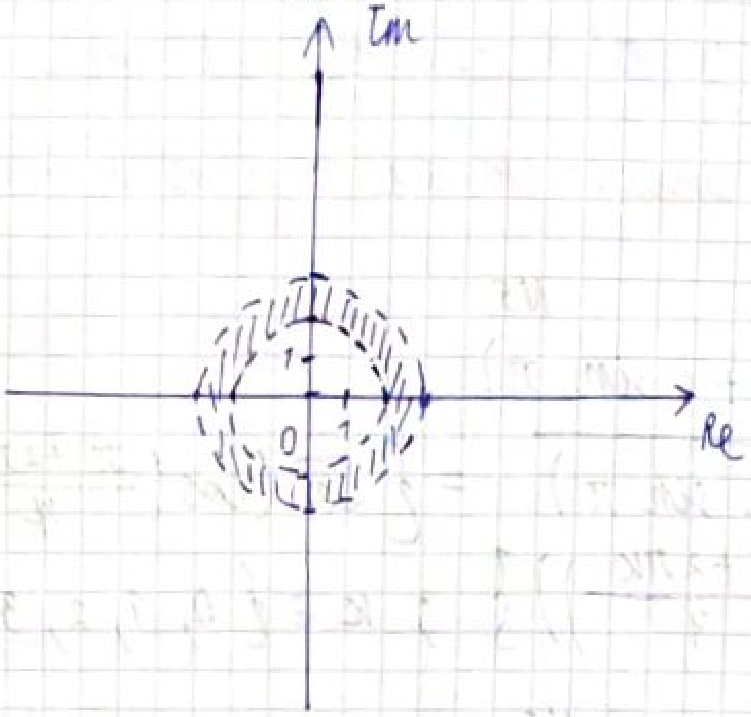
$|z-(-3-4i)| \le 5$



$|\overline{l}| \le 5$. Изобразим множество этих точек на комплексной плоскости:



N2



N3

$$\frac{1-i\sqrt{3}}{2} = 1\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$

$$\left(\frac{1-i\sqrt{3}}{2}\right)^n = 1^n\left(\cos\frac{5\pi n}{3} + i\sin\frac{5\pi n}{3}\right) =$$

$$= \cos\frac{5\pi n}{3} + i\sin\frac{5\pi}{3}$$

N 4

$$(\cos x + i\sin x)^5 = \cos 5x + i\sin 5x$$

$$(a+b)^5 = C_5^k\, a^{5-k}\cdot b^k = a^5 + 5\cdot a^4\cdot b + 10a^3b^2 +$$
$$+ 10a^2b^3 + 5ab^4 + b^5$$

Тогда, с другой стороны:

$$\mathrm{Re}\left((\cos x + i\sin x)^5\right) = \cos^5 x - 10\cos^3 x\sin^2 x$$
$$+ 5\cos x\sin^4 x \quad (*)$$

$$\left.\begin{aligned}\sin^2 x &= 1-\cos^2 x\\ \sin^4 x &= (1-\cos^2 x)^2 = 1 - 2\cos^2 x + \cos^4 x\end{aligned}\right\} \to (*).$$

$$\cos^5 x - 10\cos^3 x(1-\cos^2 x) + 5\cos x(1-2\cos^2 x +$$
$$+\cos^4 x) =$$

$= \cos^5 x - 10\cos^3 x + 10\cos^5 x + 5\cos x - 10\cos^3 x + 5\cos^5 x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

В итоге имеем: $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

N5

$-4 = 4(\cos\pi + i\sin\pi)$

$\sqrt[4]{4(\cos\pi + i\sin\pi)} = \left\{\sqrt{2}\left(\cos\left(\frac{\pi + 2\pi k}{4}\right) + i\sin\left(\frac{\pi+2\pi k}{4}\right)\right)\right\};\ k \in \{0, 1, 2, 3\}$

N6

$\sqrt[6]{64} = 64(\cos 0 + i\sin 0)$

$\sqrt[6]{64(\cos 0 + i\sin 0)} = \left\{2\left(\cos\frac{2\pi k}{6} + i\sin\frac{2\pi k}{6}\right)\right\}$

$= \left\{2\left(\cos\frac{\pi k}{3} + i\sin\frac{\pi k}{3}\right)\right\};\ k \in \{0, 1 \ldots 5\}$

N7

$2 - 2i = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$\sqrt[3]{2-2i} = \left\{8^{\frac{3}{2}}\left(\cos\frac{\frac{7\pi}{4} + 2\pi k}{3} + i\sin\frac{\frac{7\pi}{4}+2\pi k}{3}\right)\right\} =$

$= \left\{8^{\frac{3}{2}}\left(\cos\frac{7\pi + 8\pi k}{12} + i\sin\frac{7\pi + 8\pi k}{12}\right)\right\}$

$k \in \{0, 1, 2\}$

N8

$(z+1)^n - (z-1)^n = 0$

$\Updownarrow \longleftarrow z = 1$ не явл. корнем при $\forall n$

$\left(\frac{z+1}{z-1}\right)^n = 1$

$\Updownarrow$

$\frac{z+1}{z-1} = \sqrt[n]{1} \quad (*)$

$1 = \cos 0 + i \sin 0$

$$\sqrt[n]{1} = \left\{ \cos \frac{2\pi k}{n} + i \sin \frac{2\pi k}{n} \right\} = e^{\frac{2\pi k i}{n}}$$

$$k \in \{0, 1, 2 \dots n-1\}$$

$$(*): \quad \frac{z+1}{z-1} = e^{\frac{2\pi k i}{n}}$$

Пусть $\alpha = e^{\frac{2\pi k i}{n}}$. Заметим, что при $k=0$:

$$\frac{z+1}{z-1} = 1; \quad z+1 = z-1; \quad 1 = -1 \text{ — неверно}$$

$$\Rightarrow k \neq 0 \Leftrightarrow \alpha \neq 1$$

$$\frac{z+1}{z-1} = \alpha; \quad z+1 = \alpha z - \alpha$$

$$1+\alpha = z(\alpha - 1) \underset{\alpha \neq 1}{\Rightarrow} z = \frac{1+\alpha}{\alpha - 1}$$

$$z = \frac{e^{\frac{2\pi k i}{n}} + 1}{e^{\frac{2\pi k i}{n}} - 1}; \quad k \in \{1, 2 \dots n-1\}$$

№9

$256x^8 + 1 = 0$ (рассмотрим как многочлен)

$$x^8 = -\frac{1}{256}$$

$$x \in \sqrt[8]{-\frac{1}{256}} = \left\{ \frac{1}{2} \left(\cos \frac{\pi + 2\pi k}{8} + i \sin \frac{\pi + 2\pi k}{8} \right) \right.$$

$$k \in \{0, 1 \dots 7\}$$

$$256x^8 + 1 = \frac{1}{256} \left(x - \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right) \right) \cdot$$

$$\cdot \left(x - \left(\cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8} \right) \right) \cdot \ldots \cdot \left(x - \right.$$

$$\left. \left(\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8} \right) \right) \; (!)$$

Рассмотрим произведение вида:

$$(x-(\cos(2\pi-\alpha)+i\sin(2\pi-\alpha)))\cdot$$

$$\cdot(x-(\cos\alpha+i\sin\alpha))\ ;\ \alpha\in\left\{\frac{9\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}\right\}$$

$(*)$

перемножили компл. числа

$$(*)=x^2-x(\cos\alpha+i\sin\alpha)-$$

$$-x(\cos(2\pi-\alpha)+i\sin(2\pi-\alpha))-1=$$

$$=x^2-x(\cos\alpha+\cos\alpha+i\sin\alpha-i\sin\alpha)-1=$$

$$=x^2-2x\cos\alpha-1$$

Произведений такого вида в (!) 4 шт., значит:

$$256x^8+1=\left(x^2-2x\cos\frac{9\pi}{8}-1\right)\left(x^2-2x\cos\frac{11\pi}{8}-1\right)$$

$$\left(x^2-2x\cos\frac{13\pi}{8}-1\right)\left(x^2-2x\cos\frac{15\pi}{8}-1\right)$$