Д/з по алгебре N10

N1

$$(i+3i)(8-i) = 8 - i + 24i - 3i^2 =$$

$$= 8 + 23i + 3 = 11 + 23i$$

$$(2+i)^2 = 3 + 4i$$

$$\frac{11+23i}{3+4i} = \frac{(11+23i)(3-4i)}{(3+4i)(3-4i)} = \frac{33-44i+69i+92}{9+16} =$$

$$= \frac{125+25i}{25} = 5+i$$

N2

$$(2+i)x + (1+2i)y = 2x + ix + y + 2iy =$$

$$= 2x+y + i(x+2y) = 1-4i$$

$$\begin{cases} 2x+y = 1 & (1) \\ x+2y = -4 & (2) \end{cases}$$

(1): $\underset{(3)}{y = 1-2x} \rightarrow (2): x + 2 - 4x = -4$

$-3x = -6; \quad x = 2 \rightarrow (3): \ y = 1-4 = -3$

$$\boxed{x=2; \ y=-3}$$

N3

г) $-3i = 0 - 3i. \Rightarrow \begin{cases} x=0 \\ y=-3 \end{cases}$

$r = \sqrt{0+(-3)^2} = 3$

$\begin{cases} 0 = 3\cdot\cos\varphi \\ -3 = 3\sin\varphi \end{cases} \Rightarrow \begin{cases} \cos\varphi = 0 \\ \sin\varphi = -1 \end{cases} \Rightarrow \varphi = -\frac{\pi}{2} + 2\pi k;$
$k \in Z$

$-3i = 3\left(\cos\left(-\frac{\pi}{2} + 2\pi k\right) + i\cdot\sin\left(-\frac{\pi}{2} + 2\pi k\right)\right); k \in Z$

д) $1+i \Rightarrow \begin{cases} x=1 \\ y=1 \end{cases};\quad r = \sqrt{2}$

$\begin{cases} 1 = r\cos\varphi \\ 1 = r\sin\varphi \end{cases} \Rightarrow \begin{cases} \cos\varphi = \frac{1}{\sqrt{2}} \\ \sin\varphi = \frac{1}{\sqrt{2}} \end{cases} \Rightarrow \varphi = \frac{\pi}{4} + 2\pi k;\ k \in Z$

$1+i = \sqrt{2}\left(\cos\left(\frac{\pi}{4} + 2\pi k\right) + i\sin\left(\frac{\pi}{4} + 2\pi k\right)\right);$
$k \in Z$

е) $1-i \Rightarrow \begin{cases} x=1 \\ y=-1 \end{cases};\quad r = \sqrt{2}$

$\begin{cases} 1 = r\cos\varphi \\ -1 = r\sin\varphi \end{cases} \Rightarrow \begin{cases} \cos\varphi = \frac{1}{\sqrt{2}} \\ \sin\varphi = -\frac{1}{\sqrt{2}} \end{cases} \Rightarrow \varphi = -\frac{\pi}{4} + 2\pi k;$
$k \in Z$

$1-i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4} + 2\pi k\right) + i\sin\left(-\frac{\pi}{4} + 2\pi k\right)\right)$
$k \in Z$

N4

ж) $1+i\sqrt{3}$; $r=2$

$$\begin{cases} \cos\varphi = \frac{1}{2} \\ \sin\varphi = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \varphi = \frac{\pi}{3} + 2\pi k;\ k \in Z$$

$$1+i\sqrt{3} = 2\left(\cos\left(\frac{\pi}{3}+2\pi k\right) + i\cdot\sin\left(\frac{\pi}{3}+2\pi k\right)\right)$$
$k \in Z$

и) $-1-i\sqrt{3}$; $r=2$

$$\begin{cases} \cos\varphi = -\frac{1}{2} \\ \sin\varphi = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \varphi = \frac{4\pi}{3} + 2\pi k;\ k \in Z$$

$$-1-i\sqrt{3} = 2\left(\cos\left(\frac{4\pi}{3}+2\pi k\right) + i\cdot\sin\left(\frac{4\pi}{3}+2\pi k\right)\right)$$
$k \in Z$

к) $1-i\sqrt{3}$; $r=2$

$$\begin{cases} \cos\varphi = \frac{1}{2} \\ \sin\varphi = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \varphi = -\frac{\pi}{3} + 2\pi k;\ k \in Z$$

$$1-i\sqrt{3} = 2\left(\cos\left(-\frac{\pi}{3}+2\pi k\right) + i\sin\cdot\left(-\frac{\pi}{3}+2\pi k\right)\right);\ k \in Z$$

$-\sqrt{3}+i$; $r=2$

$$\begin{cases}\cos\varphi = -\frac{\sqrt{3}}{2} \\ \sin\varphi = \frac{1}{2}\end{cases} \Rightarrow \varphi = \frac{5\pi}{6} + 2\pi k;\ k\in Z$$

$$-\sqrt{3}+i = 2\left(\cos\left(\frac{5\pi}{6}+2\pi k\right) + i\cdot\sin\left(\frac{5\pi}{6}+2\pi k\right)\right)$$

$k\in Z$

$1+i\frac{\sqrt{3}}{3}$; $r=\sqrt{1+\frac{3}{9}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$

$$\begin{cases}\cos\varphi = \frac{\sqrt{3}}{2} \\ \sin\varphi = \frac{1}{2}\end{cases} \Rightarrow \varphi = \frac{\pi}{6} + 2\pi k;\ k\in Z$$

$$1+i\frac{\sqrt{3}}{3} = \frac{2}{\sqrt{3}}\left(\cos\left(\frac{\pi}{6}+2\pi k\right) + i\sin\left(\frac{\pi}{6}+2\pi k\right)\right);$$

$k\in Z$

N5

$\sin\alpha + i\cos\alpha$. $r=\sqrt{\cos^2\alpha+\sin^2\alpha} = 1$

$$\begin{cases}\cos\varphi = \sin\alpha \\ \sin\varphi = \cos\alpha\end{cases} \Leftrightarrow \begin{cases}\cos\varphi = \cos\left(\frac{\pi}{2}-\alpha\right) \\ \sin\varphi = \sin\left(\frac{\pi}{2}-\alpha\right)\end{cases} \Rightarrow$$

$$\Rightarrow \varphi = \frac{\pi}{2} - \alpha + 2\pi k;\ k\in Z$$

$$\sin\alpha + i\cos\alpha = \cos\left(\frac{\pi}{2}-\alpha+2\pi k\right) +$$

$$+ i\sin\left(\frac{\pi}{2}-\alpha+2\pi k\right);\ k\in Z$$

N6

$1+\cos\varphi + i\sin\varphi$

$x = 1+\cos\varphi$; $y = \sin\varphi$

$$r = \sqrt{1+2\cos\varphi+\cos^2\varphi+\sin^2\varphi} =$$

$$= \sqrt{2+2\cos\varphi}$$

Обозначим угол в аргументе триг. записи как w:

$$\begin{cases} \cos w = \dfrac{1+\cos\varphi}{\sqrt{2+2\cos\varphi}} \\ \sin w = \dfrac{\sin\varphi}{\sqrt{2+2\cos\varphi}} \end{cases}$$

$$\Downarrow$$

$$\cos w \cdot \sin w = \frac{\sin\varphi(1+\cos\varphi)}{2+2\cos\varphi}$$

$$\cos w \cdot \sin w = \frac{\sin\varphi}{2}$$

$$2\cos w\cdot\sin w = \sin\varphi$$

$$\sin 2w = \sin\varphi$$

$$\sin 2w - \sin\varphi = 0$$

$$2\cdot\sin\left(\frac{2w-\varphi}{2}\right)\cdot\cos\left(\frac{2w+\varphi}{2}\right) = 0$$

$$\Downarrow$$

$$\left[\begin{array}{l} \sin\left(\frac{2w-\varphi}{2}\right) = 0 \ (1) \\ \cos\left(\frac{2w+\varphi}{2}\right) = 0 \ (2) \end{array}\right.$$

(1): $\frac{2w - \varphi}{2} = \pi n ; \; n \in \mathbb{Z}$

$w = \pi n + \frac{\varphi}{2} ; \; n \in \mathbb{Z}$

(2): $\frac{2w + \varphi}{2} = \frac{\pi}{2} + \pi k ; \; k \in \mathbb{Z}$

$w = \frac{\pi - \varphi}{2} + \pi k ; \; k \in \mathbb{Z}$