Д/з по матану N5

N1

a) $$\lim_{n\to\infty} \frac{3n^2+2n-1}{3-n-4n^2} = \lim_{n\to\infty} \frac{3+\frac{2}{n}-\frac{1}{n^2}}{\frac{3}{n^2}-\frac{1}{n}-4} \overset{\text{а.п.}}{=}$$

$$= -\frac{3}{4}$$

в) $$\lim_{n\to\infty} n - \frac{3}{\frac{3}{n}-\frac{3}{n^2}+\frac{1}{n^3}} =$$

$$= \lim_{n\to\infty} n - \frac{3n^3}{3n^2-3n+1} = \lim_{n\to\infty} 3$$

$$= \lim_{n\to\infty} \frac{3n^3-3n^2+n-3n^3}{3n^2-3n+1} = \lim_{n\to\infty} \frac{n-3n^2}{3n^2-3n+1} =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}-3}{3-\frac{3}{n}+\frac{1}{n^2}} \overset{\text{а.п.}}{=} -1$$

c) $$\lim_{n\to\infty} \frac{2n-\sqrt{4n^2-1}}{\sqrt{n^2+3}-n} =$$

$$= \lim_{n\to\infty} \frac{(2n-\sqrt{4n^2-1})(2n+\sqrt{4n^2-1})}{(2n+\sqrt{4n^2-1})(\sqrt{n^2+3}-n)} =$$

$$= \lim_{n\to\infty} \frac{4n^2-4n^2+1}{(2n+\sqrt{4n^2-1})(\sqrt{n^2+3}-n)} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+3}+n}{(2n+\sqrt{4n^2-1})(n^2+3-n^2)}$$

$$= \frac{1}{3} \lim_{n\to\infty} \frac{\sqrt{n^2+3}+n}{2n+\sqrt{4n^2-1}} = \frac{1}{3} \lim_{n\to\infty} \frac{\sqrt{1+\frac{3}{n^2}}+1}{2+\sqrt{4-\frac{1}{n^2}}} \overset{\text{a.n}}{=}$$

$$= \frac{1}{3} \cdot \frac{2}{4} = \frac{1}{6}.$$

d) $$\lim_{n\to\infty} \sqrt[3]{n+1} - \sqrt[3]{n-1} =$$

$$= \lim_{n\to\infty} \frac{\left(\sqrt[3]{n+1} - \sqrt[3]{n-1}\right)\left((n+1)^{\frac{2}{3}} + (n^2-1)^{\frac{1}{3}} + (n-1)^{\frac{2}{3}}\right)}{(n+1)^{\frac{2}{3}} + (n^2-1)^{\frac{1}{3}} + (n-1)^{\frac{2}{3}}}$$

$$= \lim_{n\to\infty} \frac{2}{\sqrt[3]{n^2+2n+1} + \sqrt[3]{n^2-1} + \sqrt[3]{n^2-2n+1}} = L$$

$\lim \left(\sqrt[3]{n^2+2n+1} + \sqrt[3]{n^2-1} + \sqrt[3]{n^2-2n+1}\right) = +\infty$
(тк каждое из слагаемых в пределе уходит в $+\infty$)

$\Rightarrow \quad L = 0$

e) $$\lim_{n\to\infty} \frac{10^n + n!}{2^n + (n+1)!} = \lim_{n\to\infty} \frac{\frac{10^n}{n!} + 1}{\frac{2^n}{n!} + \frac{(n+1)!}{n!}} \overset{\text{a.n.}}{=}$$

$$= \lim_{n\to\infty} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{n!}{n! \cdot (n+1)} = \lim_{n\to\infty} \frac{1}{n+1} =$$

$= 0$

g) $\lim\limits_{n\to\infty} \sqrt[n]{n^3+3n}$

$$\sqrt[n]{n} \le \sqrt[n]{n^3+3n} \le \sqrt[n]{4n^3} \quad (*)$$

$$\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim\limits_{n\to\infty} \sqrt[n]{4n^3} = \lim\limits_{n\to\infty} \sqrt[n]{4}\cdot\sqrt[n]{n}\cdot\sqrt[n]{n}\cdot\sqrt[n]{n} \overset{\text{а.л.}}{=} 1$$

Но тогда, по теореме о зажатой последовательности, из $(*) \Rightarrow$

$$\Rightarrow \lim\limits_{n\to\infty} \sqrt{n^3+3n} = 1$$

h) $\lim\limits_{n\to\infty} \left(\dfrac{2n-1}{5n+1}\right)^{n^2}$

$$0 \le \left(\frac{2n-1}{5n+1}\right)^{n^2} \le \left(\frac{3n}{4n}\right)^{n^2} \quad (*)$$

$\lim\limits_{n\to\infty} \left(\dfrac{3n}{4n}\right)^{n^2} = 0 \quad \lim\limits_{n\to\infty} \left(\dfrac{3}{4}\right)^{n^2}$ (при n^2 стремление к нулю ещё быстрее, чем при (n)).

Тогда, по теореме о зажатой последовательности, из $(*) \Rightarrow \lim\limits_{n\to\infty} \left(\dfrac{2n-1}{5n+1}\right)^{n^2} = 0$

f) $\dfrac{\sqrt[n]{8}-1}{\sqrt[n]{16}-1} = \dfrac{(\sqrt[n]{2})^3-1}{\left((\sqrt[n]{2})^2\right)^2-1} =$

$$= \frac{(\sqrt[n]{2}-1)(\sqrt[n]{4}+\sqrt[n]{2}+1)}{\left((\sqrt[n]{2})^2-1\right)(\sqrt[n]{4}+1)} =$$

$$= \frac{(\sqrt[n]{2}-1)(\sqrt[n]{4}+\sqrt[n]{2}+1)}{(\sqrt[n]{2}-1)(\sqrt[n]{2}+1)(\sqrt[n]{4}+1)} =$$

$$= \frac{\sqrt[n]{4}+\sqrt[n]{2}+1}{(\sqrt[n]{2}+1)(\sqrt[n]{4}+1)}$$

Тогда: $\lim\limits_{n\to\infty} \dfrac{\sqrt[n]{8}-1}{\sqrt[n]{16}-1} = \lim\limits_{n\to\infty} \dfrac{\sqrt[n]{4}+\sqrt[n]{2}+1}{(\sqrt[n]{2}+1)(\sqrt[n]{4}+1)} =$

$$\overset{\text{a.r.}}{=} \frac{1+1+1}{(1+1)(1+1)} = \frac{3}{4}$$