Map Coloring

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1 Question

Given a map consisting of n circles, what is an upper bound for the number of colors needed to color in the map such that every space has it's own color and each adjacent space has a different color from it. Spaces touching at a finite number of points do not count, they must share an edge (thus infinite points). More specifically find a better upper bound then n^2.

2 Ideas

The solution to this that we are going to try and show that the upper bound is 2. One may get an intuitive sense as to why this is true by letting the outside space be color A and notice that only spaces which may be written in the form $\bigcap_{i \in (0,n]-\{k\}} ({S_i}^{\mathsf{c}}) \cap S_k$ so that it is the intersection of 1 circle with the compliment of all the others. Then those will only be touching areas that are the intersection of all the copmliments and areas that are the interesction of 2 circles and all the other copliments. This extends out no mater how the cirlces ae laid out. Though as I do not see how to prove this it is more of an intuitive idea then anything concrete.

Now let $\mathscr{U}$ is the set of all circles, then let $\mathcal{I} \subseteq \mathscr{U}$ and let $\mathcal{O} = \mathscr{U} \setminus \mathcal{I}$. Now if we are able to show that for any $\mathcal{I}$ we can have the space defined as $\left[\bigcap_{A \in \mathcal{I}} (A)\right] \cap \left[\bigcap_{A \in \mathcal{O}} (A^{\mathsf{c}})\right]$

2.1 construction

One can construct an algorithim to color any map of n circles with only 2 colors. This was found by Proffesor Guetter, although I will lay it out here. Start with a map only having 0 circles, and color the entire universe with color A. Now start to add circles, each time you do invert[1] the colors wherever that circle is. Continue adding circles like this until you have all n. This will result in 2 working colors, again a proof is beyond me.

[1]if the color is A then make it B and if the color is B then make it A