# Maximal Frequent Set Algorithm

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## 1 Pseudocode

Let $P(S)$ be read as the statement "$S$ is a special set", and let $\mathcal{U}$ be any finite superset of $\{x : P(\{x\})\}$.

**Algorithm 1** depth-first searth

```
function FindMaximalSpecialSet()
    return GetSpecialSuperset(𝒰, ∅, ∅)
end function

function GetSpecialSuperset(A, B, 𝒪)
    \\Step 1: thin out array
    for all x ∈ A do
        if ¬ P(B ∪ {x}) then
            A ← A − {x}
        end if
    end for

    \\Step 2: add to output, or call recursive
    if A = ∅ then

        \\If B is not a subset of any set in 𝒪 add it to 𝒪 and return.
        for all O ∈ 𝒪 do
            if B ⊆ O then
                return 𝒪                         ▷ Ends the function if the return statement is hit.
            end if
        end for
        return 𝒪 ∪ {B}
    else

        \\Make recursive calls, moving elements from A into B.
        for all a ∈ A do
            A ← A − {a}
            𝒪 ← GetSpecialSuperset(A, B ∪ {a}, 𝒪)        ▷ 𝒪 ⊆ GetSpecialSuperset(A, B, 𝒪)
        end for
        return 𝒪
    end if
end function
```

# 2 Explanation

## 2.1 details

We are going to tackle this problem with recursion, so we are going to try and show that our function "GetSpecialSuperset" will return the set of maximal special supersets of $B$, if $B$ is a special set and $A$ is a finite superset of the set of all elements which are special with $B$.[1]

We start by limiting $A$ to only being the set of elemetns with are special with $B$. If there are none, then $A$ will become the empty set, and we would know that $B$ is the only maximal special superset of $B$, and thus return the set $\{B\}$.

If $A$ is not the empty set (there are elements that are special with $B$), then for each element $x \in A$ we get the set, $\mathcal{R}$, of maximal special supersets of $B \cup \{x\}$. $\mathcal{R}$ is found by applying this function again with the new $A$ set as our current $A$, just without $x$, and the new $B$ set as $B \cup \{x\}$. We can see this is true as if some $k$ is speical with $B \cup \{x\}$ then it must also be special with $B$, $A$ is the set of all things special with $B$ and $A - \{x\}$ is the set of all things special with $B$ except $x$, which we know is in $B \cup \{x\}$, thus $A \supseteq \{k | k \text{ is special with } B \cup \{x\}\}$, which is our requirement for what we need $A$ to be. We also know that our new $B$ is special by defintion of $x$ being special with $B$. To take things one step further, we can remove $x$ from our current $A$ as if we have all maximal supersets $B \cup \{x\}$ then we never need to check $B$ with $x$ again as it would have already been found.[2] Once we have found $\mathcal{R}$ we add all sets in $\mathcal{R}$ to Output.

Once we have iterated through all $x \in A$ we return Output as it now contains all supersets of $B$ which are special.

## 2.2 example

Let $\mathcal{U} = \{0, 1, 2, 3\}$ and our set of maximal special sets, the expected output we will call $\mathcal{T}$ and we will let $\mathcal{T} = \{\{0, 2\}, \{1\}\}$, thus $P(S) = \exists_{E \in \mathcal{T}} (S \subseteq E)$.

$A : \{0, 1, 2, 3\}$
$B : \{\}$
$\mathcal{O} : \{\}$
$x : 0$
$P(B \cup \{x\})$
$x : 1$
$P(B \cup \{x\})$
$x : 2$
$P(B \cup \{x\})$
$x : 3$
$\neg P(B \cup \{x\})$
$A : \{0, 1, 2\}$
$A \neq \varnothing$
$a : 0$
$A : \{1, 2\}$
$\rightarrow$ $A : \{1, 2\}$
  $B : \{0\}$
  $\mathcal{O} : \{\}$
  $x : 1$
  $\neg P(B \cup \{x\})$
  $A : \{2\}$
  $x : 2$
  $P\{0, 2\}$
  $A \neq \varnothing$
  $a : 2$

[1] $x$ is special with $S$ iff $S \cup \{x\}$ is special and $x \notin S$.
[2] This justifies line 18.

$A : \{\}$
$\rightarrow \quad A : \{\}$
$B : \{0, 2\}$
$\mathcal{O} : \{\}$
$A = \varnothing$
$\mathcal{O} : \{\{0, 2\}\}$
$\mathcal{O} : \{\{0, 2\}\}$
$a : 1$
$A : \{2\}$
$\rightarrow \quad A : \{2\}$
$B : \{1\}$
$\mathcal{O} : \{\{0, 2\}\}$
$x : 2$
$\neg P(B \cup \{x\})$
$A : \{\}$
$A = \varnothing$
$O : \{0, 2\}$
$B \nsubseteq O$
$\mathcal{O} : \{\{0, 2\}, \{1\}\}$
$a : 2$
$A : \{\}$
$\rightarrow \quad A : \{\}$
$B : \{2\}$
$\mathcal{O} : \{\{0, 2\}, \{1\}\}$
$A = \varnothing$
$O : \{0, 2\}$
$B \subseteq O$
$\mathcal{O} : \{\{0, 2\}, \{1\}\}$