

RESPOSTAS DOS PROBLEMAS PROPOSTOS

Capítulo 6

seção 6.16, página 162

1. $a = \frac{-1 \pm \sqrt{13}}{2}$
2. $\vec{x} = (-17, -13, -15)$
3. $\vec{v} = (1, 1, 1)$
4. $(\frac{7}{9}, \frac{4}{9}, \frac{4}{9})$
5. \vec{u} não é e \vec{v} é
6. $\pm \frac{\sqrt{5}}{5}$
7. $m = -4$ ou $m = -5$
8. $m = 3$ ou $m = -\frac{13}{5}$
9. $m = -1$ ou $m = -3$
10. $2\sqrt{11} + \sqrt{12}$ u.c. $= 2(\sqrt{11} + \sqrt{3})$ u.c.
11. $P = (1, 0, 0)$
12. $\theta = 45^\circ$
13. 50
14. $\hat{A} = \arccos \frac{5\sqrt{7}}{21}$ $\hat{B} = \arccos \frac{2\sqrt{6}}{9}$ $\hat{C} = \arccos -\frac{\sqrt{42}}{21}$
15. $m = -4$

16. $n = \sqrt{15}$
17. $\alpha = -6$ ou $\alpha = 3$
18. $\vec{v} = (-3, 3, -6)$
19. qualquer vetor colinear a \vec{w} é ortogonal a \vec{u} ; isto é, $t(3, -2, 1), \forall t \in \mathbb{R}$
20. $\vec{v} = (2, -1, -3)$
21. Mostre que $\overrightarrow{BA} \bullet \overrightarrow{BC} = 0$
22. $\alpha = -3$ ou $\alpha = 2$
23. sim, $\hat{A} = 90^\circ$
24. $\vec{v} = (\pm 4, 3, 0)$
25. um deles é $(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
26. $(\frac{5\sqrt{6}}{6}, -\frac{5\sqrt{6}}{6}, \frac{10\sqrt{6}}{6})$
27. $\vec{v} = (2, 7, 1)$
28. $\vec{v} = (-1, 4, 0)$
29. $\text{proj}_{\vec{v}} \vec{u} = \frac{10}{9} (2, 1, -2)$
30. $\sqrt{37}$ e $\sqrt{13}$
31. $26 + 15\sqrt{2}$
32. $\vec{v} = (12, -6, 4)$
33. (a) $(2, 2, -1)$ (b) $(-1, -1, 0)$ (c) $(-2, -2, 2)$ (d) $(6, 6, -6)$
(e) 3 (f) -1 e -1 (g) $(4, -1, 3)$ e $(1, -4, -6)$ (h) 1
34. (a) $(-2, 4, -6)$ (b) $(4, -8, 12)$
35. $(112, -8, 12)$
36. $t(3, 7, 1), t \in \mathbb{R}$
38. $m = -5$
39. $x = -15b$ e $y = \frac{3}{2}c$
40. $(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3})$ ou $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$ e $(\frac{5\sqrt{3}}{3}, -\frac{5\sqrt{3}}{3}, -\frac{5\sqrt{3}}{3})$ ou $(-\frac{5\sqrt{3}}{3}, \frac{5\sqrt{3}}{3}, \frac{5\sqrt{3}}{3})$

41. 3

42. 2

43. $\sqrt{\frac{3}{10}} (2, 1, -5)$

44. $\sqrt{117}$ u.a.

45. $\sqrt{89}$ u.a.

46. $6\sqrt{5}$ u.a.

47. (a) $\sqrt{6}$ u.a. (b) $\frac{7}{2}$ u.a. (c) $9\frac{\sqrt{2}}{2}$ u.a. (d) $2\sqrt{6}$ u.a.

48. $\sqrt{74}$ u.a.

49. 3 ou $\frac{1}{5}$

50. $\frac{3\sqrt{35}}{7}$ uc

51. $\vec{v} = (1, 0, 1)$

52. $\vec{x} = (-2, 2, -4)$

53. $\vec{x} = (2b - 9, b, 3)$, $b \in \mathbb{R}$

54. $(1, -2, 1)$ e $\frac{1}{7}(-10, -13, -19)$

55. $\vec{x} = \frac{\vec{u}}{\|\vec{u}\|^2}$

56. $\vec{x} = \frac{\vec{u} \wedge \vec{v}}{\|\vec{u}\|^2} + \frac{m\vec{u}}{\|\vec{u}\|^2}$

Capítulo 7

seção 7.5, página 172

1. P sim e Q não

2. $P = (4, 1, 5)$

3. $m = -2$ e $n = -5$

4. (a) $(5, -2, -2)$ (b) $(-7, 4, 10)$ (c) $(2, -\frac{1}{2}, 1)$

5. $P = (2, 1, 9)$

7. $m = -5$

8. infinitas respostas

9. (a) $X = (1, -2, 4) + t(1, 0, 0), t \in \mathbb{R}$

(b) $X = (4, -1, 2) + t(1, -1, 0), t \in \mathbb{R}$

(c) $X = (2, -3, 4) + t(0, 2, -1), t \in \mathbb{R}$

Capítulo 8

seção 8.3, página 178

1. (a) $\Pi: X = (1, 1, 0) + \lambda(0, 2, 1) + \mu(2, 1, 0)$ $\Pi: \begin{cases} x = 1 + 2\mu \\ y = 1 + 2\lambda + \mu \\ z = \lambda \end{cases}, \lambda, \mu \in \mathbb{R}$

(b) $\Pi: X = (1, 0, 1) + \lambda(1, -1, 2) + \mu(1, 1, 1)$ $\Pi: \begin{cases} x = 1 + \lambda + \mu \\ y = -\lambda + \mu \\ z = 1 + 2\lambda + \mu \end{cases}, \lambda, \mu \in \mathbb{R}$

(c) $\Pi: X = (1, 0, 1) + \lambda(1, 1, -2) + \mu(1, 2, -1)$ $\Pi: \begin{cases} x = 1 + \lambda + \mu \\ y = \lambda + 2\mu \\ z = 1 - 2\lambda - \mu \end{cases}, \lambda, \mu \in \mathbb{R}$

(d) os pontos A, B e C determinam infinitos planos

2. (a) sim (b) sim (c) não (d) sim

3. $(11, 7, 4) + (-10, -5, 0)$

4. $A = (-2, -4, -1), B = (4, 5, 2), X = (4, 5, 2) + \lambda(2, 3, 1), \lambda \in \mathbb{R}$

5. $xOy: \begin{cases} x = \lambda \\ y = \mu \\ z = 0 \end{cases} \quad xOz: \begin{cases} x = \lambda \\ y = 0 \\ z = \mu \end{cases} \quad yOz: \begin{cases} x = 0 \\ y = \lambda \\ z = \mu \end{cases}, \lambda, \mu \in \mathbb{R}$

6. $\begin{cases} x = 1 + \lambda + 2\mu \\ y = 1 + 2\lambda + \mu \\ z = 2 - \lambda \end{cases}, \lambda, \mu \in \mathbb{R}$

seção 8.6, página 183

1. (a) $x - 2y + 4z + 1 = 0$ (c) $3x - y + z - 4 = 0$
(b) $3x - y - 2z - 1 = 0$ (d) plano não determinado
2. (a) $x - y - 1 = 0$ (b) $8x - 4y - z + 4 = 0$
3. (a) $2x - y - 3z + 7 = 0$ (b) $y - 2 = 0$
4. (a) $\Pi_1 : x + y + z - 1 = 0$ $\Pi_2 : x - y - z = 0$ $\Pi_3 : x + 2y - z - 2 = 0$
(b) $P = (\frac{1}{2}, \frac{2}{3}, -\frac{1}{6})$
5. (b) $\Pi : 8x + 6y - z - 39 = 0$
6. (a) sim (b) não (c) não
7. (a) Sim, $P = (-2, 2, -7)$, $\Pi : 17x - 7y - 6z + 6 = 0$
(b) Sim, $P = (-2, 6, -6)$, $\Pi : 4x - y - 3z - 4 = 0$
8. (a) $(4, 3, -2)$ (b) $(1, 9, 2)$ (c) $k = -2$ (d) $(0, -2, -1)$
9. (a) $2x - 3y - z + d = 0$ (e) $z + 3 = 0$
(b) $3x + 2y - 6 = 0$ (f) $4x + 5y + 3z - 6 = 0$
(c) $y - 2z + 4 = 0$ (g) $x - 2y = 0$
(d) $x + 2z - 2 = 0$ (h) $z - 3 = 0$
10. $\alpha = -3$
11. (a) $5x - 4y - 3z - 6 = 0$ (c) $2x + 2y + z + 2 = 0$
(b) $5x - 2y + 4z - 21 = 0$ (d) $2x + y - 2z + 3 = 0$
12. (a) $7x + 11y + 2z - 28 = 0$ (d) $x + y = 0$
(b) $2x + 3y + z + 1 = 0$ (e) $2y + z = 0$
(c) $6x - 2y + z - 3 = 0$
13. $\begin{cases} x = 2 + \mu \\ y = \lambda \\ z = 2\lambda + 3\mu \end{cases}, \lambda, \mu \in \mathbb{R}$

$$14. \begin{cases} x = 1 + \lambda + 3\mu \\ y = 1 + 3\mu \\ z = 3\lambda - \mu \end{cases}, \quad \lambda, \mu \in \mathbb{R}$$

seção 8.9, página 189

1. (a) $\vec{n} = (1, 0, 0)$ (b) $\vec{n} = (1, 4, 2)$ (c) $\vec{n} = (1, -2, 4)$
2. $\Pi : x - y + 2z - 4 = 0$
3. $\Pi : x - 2z = 0$
4. $\Pi : x + 2y - z = 0$
5. $\vec{w}_1 = (-3, 0, 5)$ e $\vec{w}_2 = (0, 4, 0)$
6. $r : X = (1, 2, 3) + \lambda (2, 1, -1), \lambda \in \mathbb{R}$
7. $r : \begin{cases} x = 3 + 3\lambda \\ y = -2 \\ z = 5 + 2\lambda \end{cases}, \quad \lambda, \mu \in \mathbb{R}$
8. $r : \begin{cases} x = \lambda \\ y = \lambda \\ z = 0 \end{cases}, \quad \lambda, \mu \in \mathbb{R}$

Capítulo 9**seção 9.3, página 193**

1. (a) paralelas distintas (e) concorrentes em $P = (-2, 6, -6)$
 (b) concorrentes em $P = (1, -1, 0)$ (f) concorrentes em $P = (-2, 2, -7)$
 (c) reversas (g) reversas
 (d) $r = s$ (h) reversas
2. (a) $m = 1$ (b) $m = 1$ (c) $\forall m$ (d) $\nexists m$ (e) $m \neq 0$ e $m \neq 1$
3. (a) $3x - 4y - 10z + 3 = 0$
 (b) $x - z - 1 = 0$

- (e) $4x - y - 3z - 4 = 0$
(f) $17x - 7y - 6z + 6 = 0$
4. (c) $\Pi : 4x - 2y - z + 3 = 0$
(g) $\Pi : 7x - 11y + 3z + 7 = 0$
(h) $\Pi : 5x - 4y + z + 20 = 0$
5. para $m = \frac{2}{3}$ concorrentes no ponto $(-9, -5, -13)$ e determinam o plano $\Pi : 2x - y - z = 0$

seção 9.6, página 197

1. (a) transversais, $P = (1, 0, -1)$ (d) $r // \Pi$
(b) $r \parallel \Pi$ (e) r transversal a Π , $P = (-\frac{1}{9}, -\frac{4}{9}, -\frac{1}{9})$
(c) $r \subset \Pi$ (f) $r // \Pi$
2. $m = 2$
3. $m = 1$ e $n = 7$
4. $\forall m \neq 0$

seção 9.9, página 200

1. (a) $\Pi_1 = \Pi_2$ (b) paralelos distintos
(c) transversais, $r = \Pi_1 \cap \Pi_2 : X = (0, 0, 1) + \lambda(-6, 7, 10)$
2. (a) $\nexists m$, pois $(1, 1, 0) \in \Pi_1 \cap \Pi_2, \forall m$
(b) $m = -\frac{5}{2}$
4. (a) $t: X = (1, 1, 1) + \lambda(1, -1, -1), \lambda \in \mathbb{R}$
(b) $t: X = (-2, 2, 4) + \lambda(0, 1, 0), \lambda \in \mathbb{R}$
(c) $\nexists t$, pois $P \in r \cap t$
(d) $t: X = (1, -2, -1) + \lambda(1, 2, 1), \lambda \in \mathbb{R}$
(e) $t: X = (1, 0, 3) + \lambda(6, -2, 7), \lambda \in \mathbb{R}$

5. (a) não existe solução

(b) $t: X = (-\frac{1}{5}, \frac{2}{5}, -\frac{2}{5}) + \lambda (1, 0, 1), \lambda \in \mathbb{R}$

(c) $t: X = (6, 10, 0) + \lambda (3, 2, -1), \lambda \in \mathbb{R}$

6. (a) $t: X = (1, 1, 0) + \lambda (1, -3, -1)$

(b) infinitas soluções

(c) $r // t, \nexists t$

7. \nexists solução, pois $s // \Pi, s \not\subset \Pi$ e $t \subset \Pi$

8. $t: X = (1, 0, 2) + \lambda (0, -1, 2)$

9. $\Pi: x - 2y + 1 = 0$

10. $\Pi: x + y + z - 8 = 0$

11. $Q = (-1, -4, -2)$

Capítulo 10

seção 10.3, página 205

1. (a) sim (b) não (c) sim (d) sim (e) não

2. (a) $t: \frac{x-2}{1} = \frac{y-0}{1} = \frac{z-0}{-2}$

(b) $t: X = (\frac{7}{2}, \frac{9}{2}, \frac{1}{2}) + \lambda (0, 1, -1), \lambda \in \mathbb{R}$

seção 10.6, página 206

1. (a) não (b) sim (c) sim (d) não (e) sim

2. (a) $r: \begin{cases} x = 1 - \lambda \\ y = -1 \\ z = \lambda \end{cases}, \lambda \in \mathbb{R}$

(b) $r: \begin{cases} x = 1 + 2\lambda \\ y = 3 - \lambda \\ z = 7 + \lambda \end{cases}, \lambda \in \mathbb{R}$

3. (a) $\Pi: x - y + z + 2 = 0$

(b) $\Pi: x + y + z - 2 = 0$

(c) $\Pi: x - y + z = 0$

4. (a) $Q = (2, -1, -2)$ (b) $Q = (-\frac{1}{11}, \frac{7}{11}, \frac{29}{11})$ (c) $Q = (-\frac{8}{19}, \frac{18}{19}, -\frac{7}{19})$
5. (a) $Q = (3, 2, 4)$ (b) $Q = (1, -1, 2)$
6. (a) $(\frac{58}{25}, \frac{56}{25}, 1)$
(b) $X = (-\frac{3}{2}, -\frac{3}{2}, 0) + \lambda (8, 10, 1)$
7. $\Pi: x + y - z - 1 = 0$
8. $B = (\frac{1}{3}, \frac{4}{3}, \frac{4}{3})$

seção 10.9, página 208

1. (a) não (b) sim (c) sim (d) sim
2. $\Pi: x - 2y - z = 0$
3. $x + y - z - 1 = 0$

Capítulo 11

seção 11.7, página 213

1. (a) $\cos \theta = \frac{20}{21}$
(b) $\cos \theta = \frac{1}{2}, \theta = 60^\circ$
(c) $\cos \theta = \frac{\sqrt{2}}{2}, \theta = 45^\circ$
(d) $\cos \theta = 0, \theta = 90^\circ$
2. (a) $\theta = \frac{\pi}{4}$ rd (b) $\sin \theta = \frac{\sqrt{3}}{3}$ (c) $\sin \theta = \frac{\sqrt{2}}{10}$ (d) $\sin \theta = \frac{2\sqrt{2}}{3}$ (e) $\theta = \frac{\pi}{6}$ rd
3. (a) $\cos \theta = \frac{\sqrt{66}}{33}$ (b) $\cos \theta = \frac{\sqrt{3}}{3}$ (c) $\cos \theta = \frac{\sqrt{2}}{2}$
4. $r: X = (\frac{5}{2}, 2, -\frac{3}{2}) + \lambda (1, 1, 1), \lambda \in \mathbb{R}$
ou $r: X = (-3, -\frac{5}{3}, 4) + \lambda (1, 1, -1), \lambda \in \mathbb{R}$
ou $r: X = (\frac{7}{5}, \frac{19}{5}, -\frac{2}{5}) + \lambda (-1, 1, 1), \lambda \in \mathbb{R}$
ou $r: X = (\frac{1}{7}, \frac{3}{7}, \frac{6}{7}) + \lambda (-1, 1, -1), \lambda \in \mathbb{R}$

5. $h: X = (0, 2, 1) + \lambda (1, -1, -1), \quad \lambda \in \mathbb{R}$
ou $h: X = (0, 2, 1) + \lambda (7, -1, -1), \quad \lambda \in \mathbb{R}$
ou $h: X = (0, 2, 1) + \lambda (3, -1, 1), \quad \lambda \in \mathbb{R}$
ou $h: X = (0, 2, 1) + \lambda (3, 1, -1), \quad \lambda \in \mathbb{R}$
6. $r: \frac{x-1}{\sqrt{2}} = \frac{y+2}{1} = \frac{z-3}{1}$
ou $r: \frac{x-1}{\sqrt{2}} = \frac{y+2}{1} = \frac{z-3}{-1}$
ou $r: \frac{x-1}{\sqrt{2}} = \frac{y+2}{-1} = \frac{z-3}{1}$
ou $r: \frac{x-1}{\sqrt{2}} = \frac{y+2}{-1} = \frac{z-3}{-1}$
7. $t: X = (1, 1, 1) + \lambda (4, -1, -1), \quad \lambda \in \mathbb{R}$
ou $t: X = (1, 1, 1) + \lambda (0, 1, 1), \quad \lambda \in \mathbb{R}$
8. $(-2 + \sqrt{3}, 1, 1 - \sqrt{3})$ ou $(-2 - \sqrt{3}, 1, 1 + \sqrt{3})$
9. $\Pi: 2x - 3y + z - 5 = 0$ ou $\Pi: 3x - y - 2z - 4 = 0$
10. $\Pi: x + y - 3z - 1 = 0$ ou $\Pi: x - y - 3z + 3 = 0$

Capítulo 12

seção 12.11, página 222

1. (a) $\sqrt{5}$ u.c. (b) $\sqrt{173}$ u.c.
2. (a) $\sqrt{5}$ u.c. (b) $\frac{\sqrt{34}}{7}$ u.c. (c) $\sqrt{\frac{270}{29}}$ u.c. (d) $\sqrt{\frac{90}{7}}$ u.c.
3. (a) $\sqrt{\frac{41}{21}}$ u.c. (b) $5\sqrt{\frac{5}{6}}$ u.c.
4. (a) 2 u.c. (b) $\frac{51}{14}$ u.c. (c) $\frac{94}{13}$ u.c. (d) 1 u.c.
5. $\frac{1}{2}$ u.c. (b) $\frac{\sqrt{3}}{6}$ u.c. (c) $\frac{2\sqrt{3}}{3}$ u.c.
6. (a) $\frac{2\sqrt{46}}{23}$ u.c. (b) 13 u.c. (c) $\frac{7\sqrt{26}}{26}$ u.c.
7. $P = (2, 0, 2)$ ou $Q = (0, 2, -2)$

8. $P = (-1, -1, -1)$
9. $P = (2, 0, 2)$ ou $Q = (0, 2, -2)$
10. $P = (1, 0, 0)$ ou $Q = (\frac{19}{3}, \frac{8}{3}, \frac{16}{3})$
11. $r: X = (-1, 3, -3) + \lambda(1, 0, 2), \lambda \in \mathbb{R}$ ou $r: X = (-1, \frac{17}{9}, -\frac{7}{9}) + \lambda(1, 0, 2) \lambda \in \mathbb{R}$
12. $(1, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}), (1, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}), (2, 0, 0)$
13. $P = (-3, 5, -8)$ e $Q = (9, -7, 16)$
14. $P = (3, 1, 2)$ ou $Q = (-1, -1, -2)$
15. $\Pi: x + z - 2 = 0$
16. $\Pi: y - 1 = 0$ ou $\Pi: 6x - 2y - 3z - 7 = 0$
17. $\Pi: z - 1 = 0$ ou $\Pi: x + y + 2z - 4 = 0$

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