The Spectrometer/Telescope for Imaging X-rays on-board Solar Orbiter: from photon to electron visibilities.

Università di Genova
DIMA | Dipartimento di Matematica

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SPHERE Workshop
June 22, 2023

Outline

1. From photon to electron visibilities & Visibility inversion algorithm
2. Photon maps → Electron maps → Regularized photon maps
3. Physics inferred from electron maps
4. Conclusions and future works

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1. **From photon to electron visibilities & Visibility inversion algorithm**
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From photon to electron visibilities

Photon visibilities:

$$V(u,v;\epsilon) = \mathcal{F}(I(x,y;\epsilon)) = \int\int I(x,y;\epsilon) e^{2\pi i(xu+yv)}\, dx\, dy \tag{1}$$

Electron visibilities:

$$W(u,v;E) = \frac{a}{4\pi R^2} \int\int N(x,y)\bar{F}(x,y;E) e^{2\pi i(xu+yv)}\, dx\, dy \tag{2}$$

Bremsstralhung equation for visibilities:

$$V(u,v;\epsilon) = \int_{\epsilon}^{\infty} W(u,v;E) Q(\epsilon,E)\, dE \tag{3}$$

Prato et al., *A Regularized Visibility-Based Approach to Astronomical Imaging Spectroscopy*, SIAM Journal on Imaging Sciences, (2009)

Piana et al., *Electron flux spectral imaging of solar flares through regularized analysis of hard x-ray source visibilities,* The Astrophysical Journal, (2007)

From photon to electron visibilities

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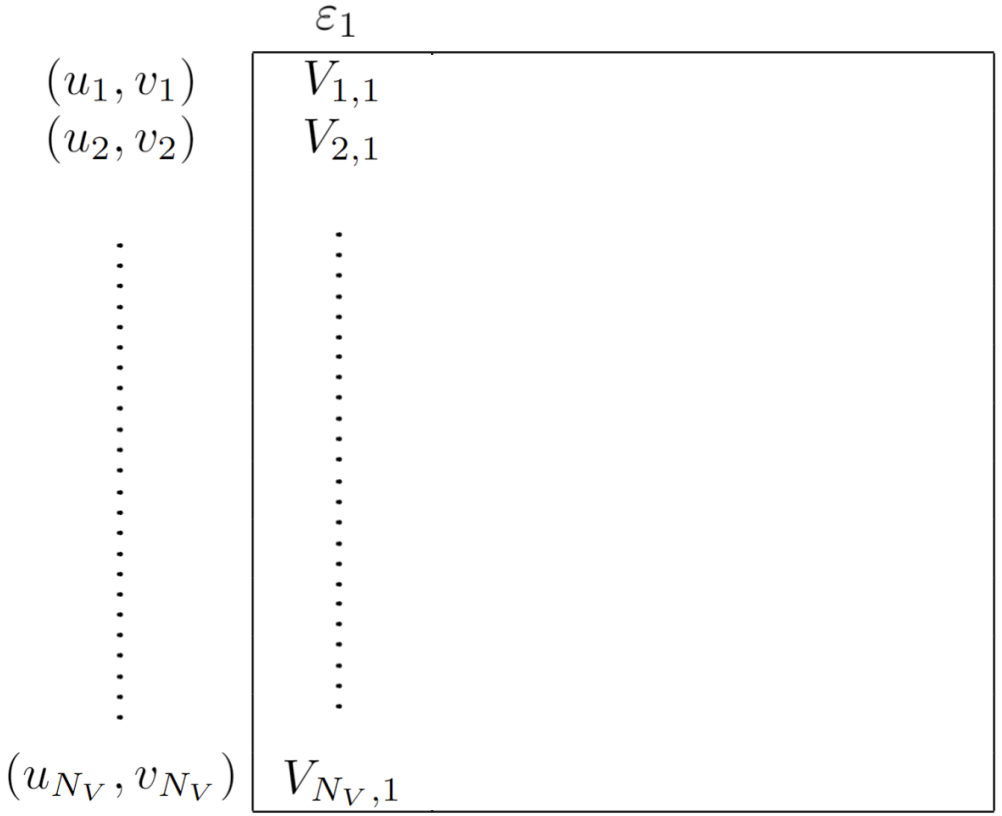
Visibility inversion algorithm - Photon visibilities

Photon visibilities



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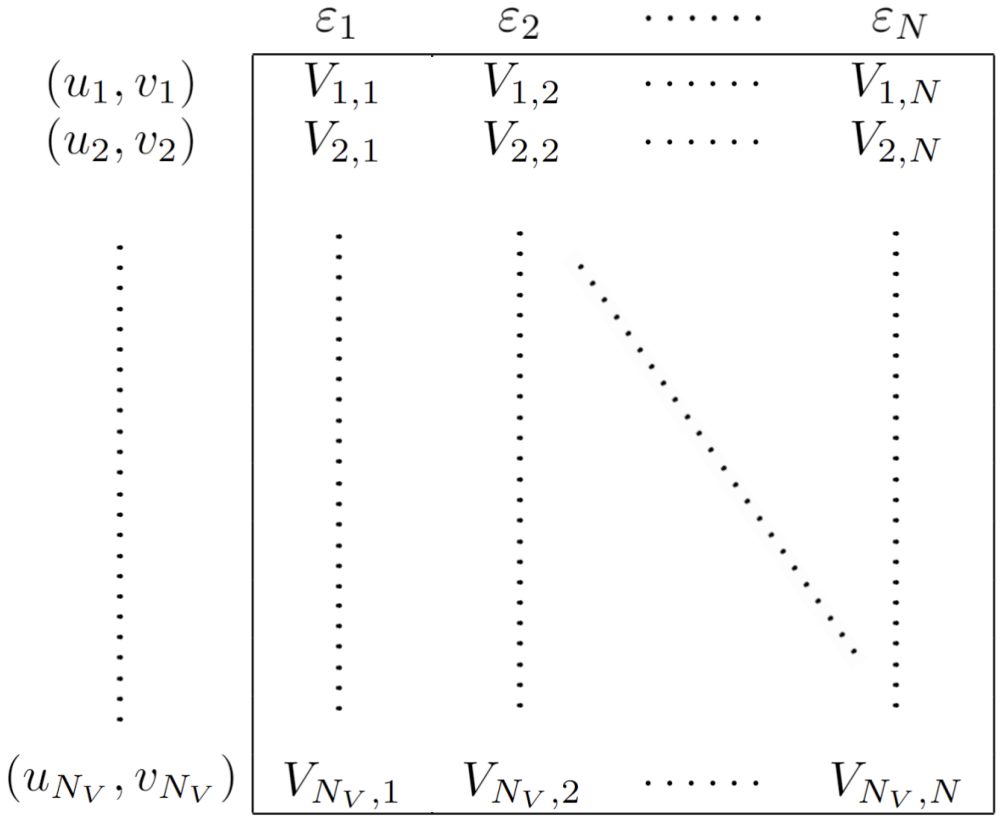

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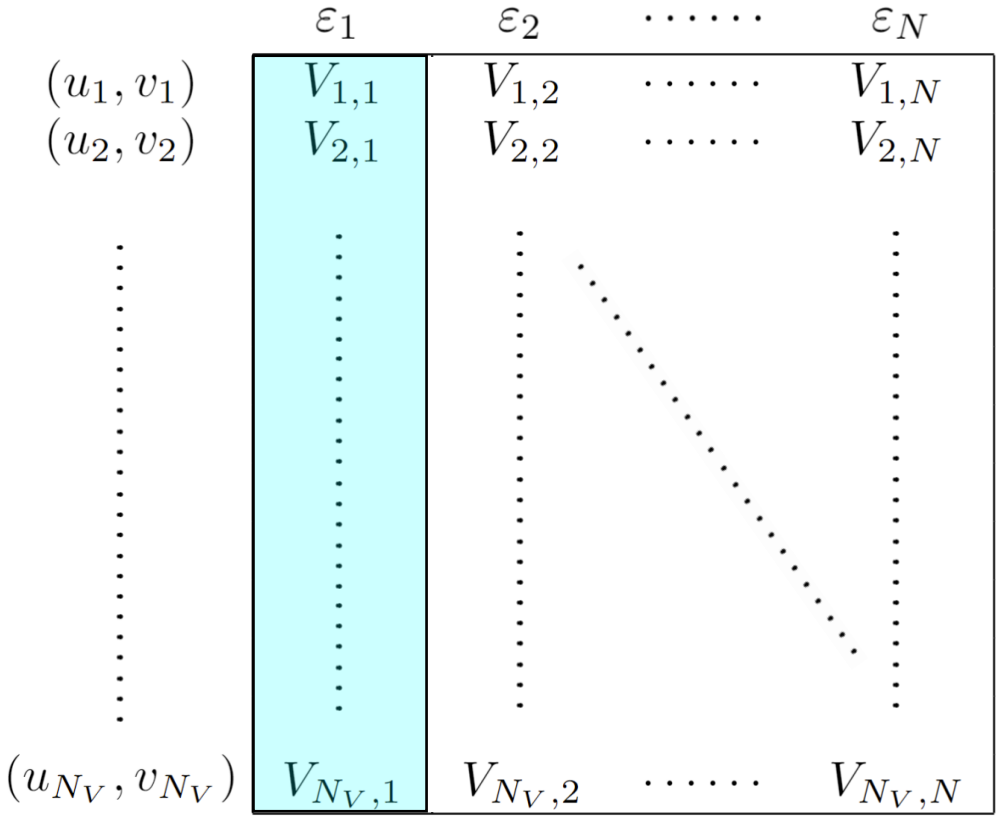

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Visibility inversion algorithm - Photon visibilities

Photon visibilities

$$
\begin{array}{c|cccc}
 & \varepsilon_1 & \varepsilon_2 & \cdots\cdots & \varepsilon_N \\
(u_1, v_1) & V_{1,1} & V_{1,2} & \cdots\cdots & V_{1,N} \\
(u_2, v_2) & V_{2,1} & V_{2,2} & \cdots\cdots & V_{2,N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
(u_{N_V}, v_{N_V}) & V_{N_V,1} & V_{N_V,2} & \cdots\cdots & V_{N_V,N}
\end{array}
$$

Solving image reconstruction problem via imaging methods.

May 7, 2021 – 18:51 UT

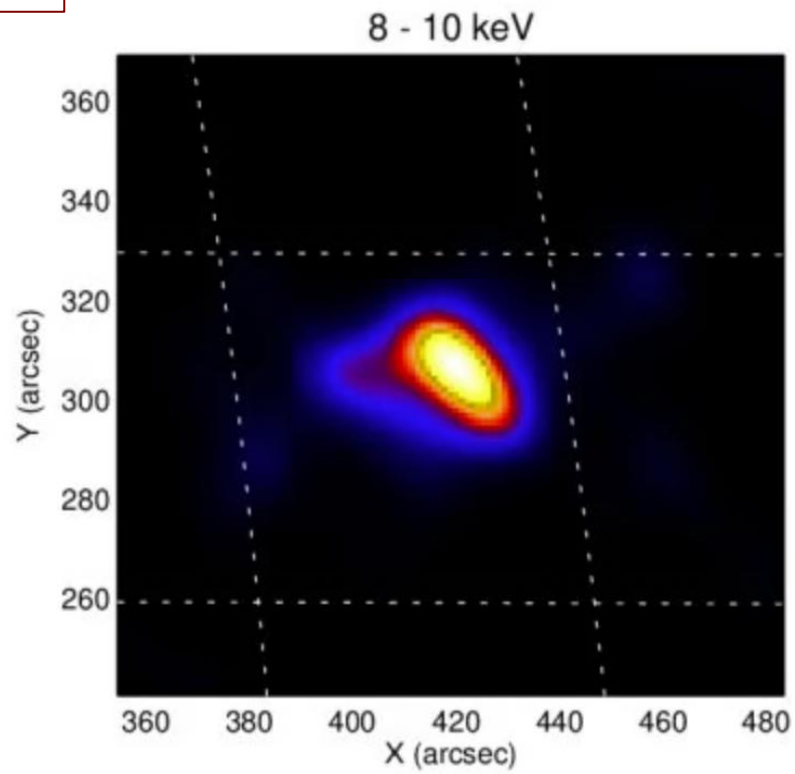

Figure: Reconstruction provided by MEM_GE, from photon visibilities.

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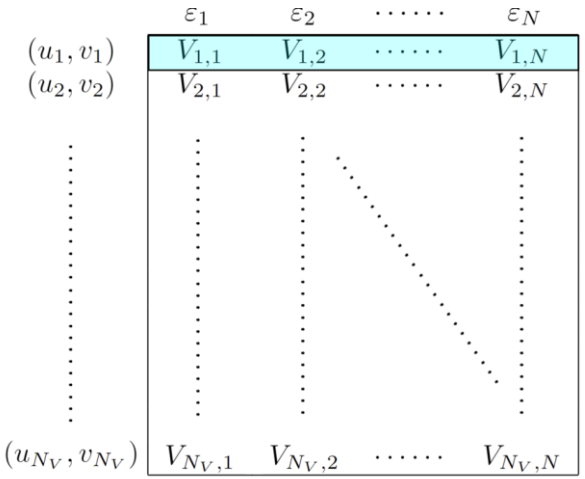

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Spectral inversion via regularized inversion technique (Tikhonov)

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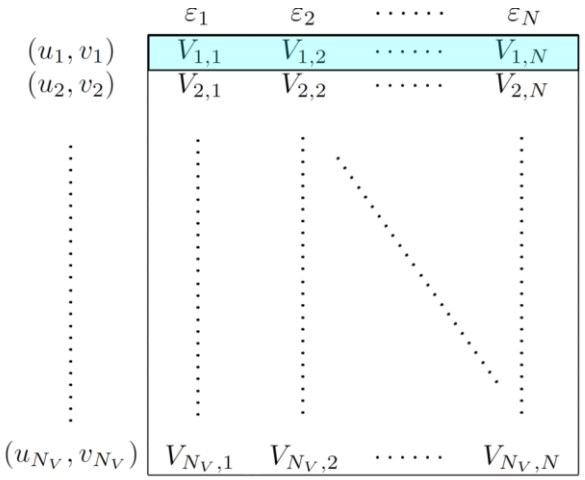
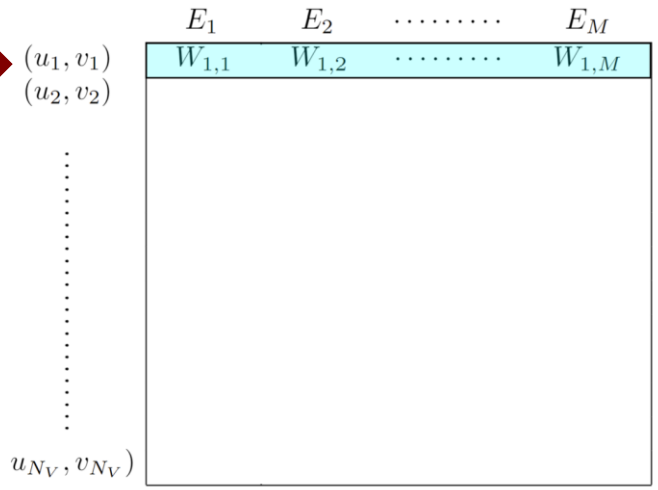


Electron visibilities



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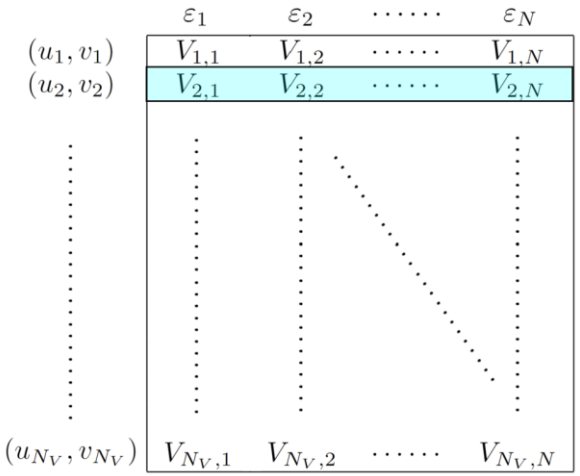

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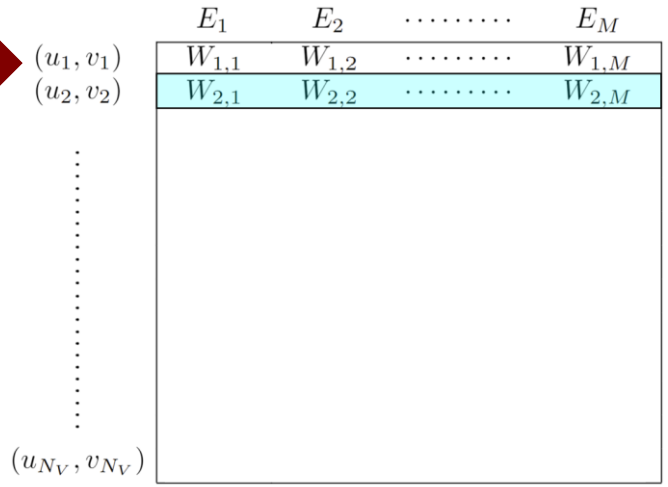

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$$\begin{array}{c|cccc} & \varepsilon_1 & \varepsilon_2 & \cdots\cdots & \varepsilon_N \\ (u_1,v_1) & V_{1,1} & V_{1,2} & \cdots\cdots & V_{1,N} \\ (u_2,v_2) & V_{2,1} & V_{2,2} & \cdots\cdots & V_{2,N} \\ \vdots & \vdots & \vdots & \iddots & \vdots \\ (u_{N_V},v_{N_V}) & V_{N_V,1} & V_{N_V,2} & \cdots\cdots & V_{N_V,N} \end{array}$$

Electron visibilities

$$\begin{array}{c|cccc} & E_1 & E_2 & \cdots\cdots\cdots & E_M \\ (u_1,v_1) & W_{1,1} & W_{1,2} & \cdots\cdots\cdots & W_{1,M} \\ (u_2,v_2) & W_{2,1} & W_{2,2} & \cdots\cdots\cdots & W_{2,M} \\ \vdots & \vdots & \vdots & \iddots & \vdots \\ (u_{N_V},v_{N_V}) & W_{N_V,1} & W_{N_V,2} & \cdots\cdots\cdots & W_{N_V,M} \end{array}$$

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May 7, 2021 – 18:51 UT

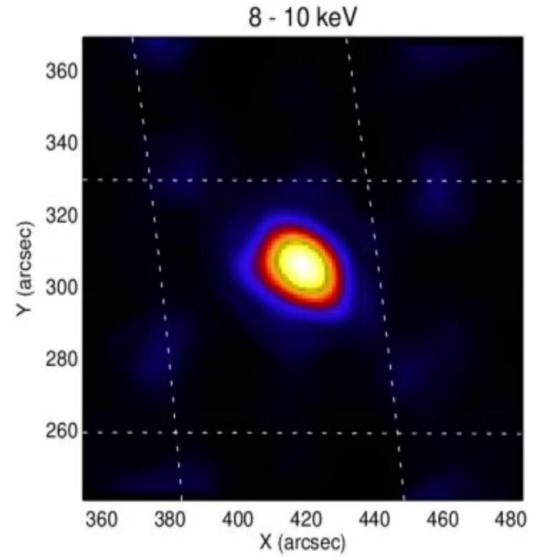

Figure: Reconstruction provided by MEM_GE, from electron visibilities.

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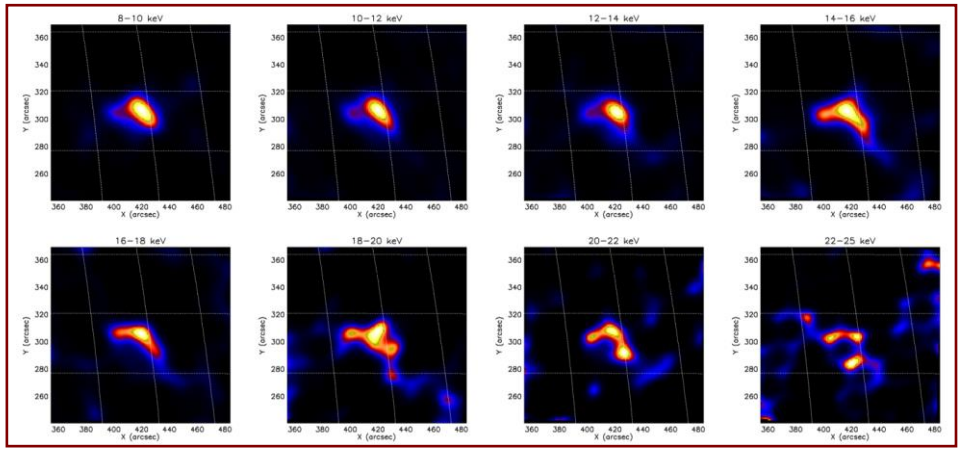
Photon images



Electron flux images

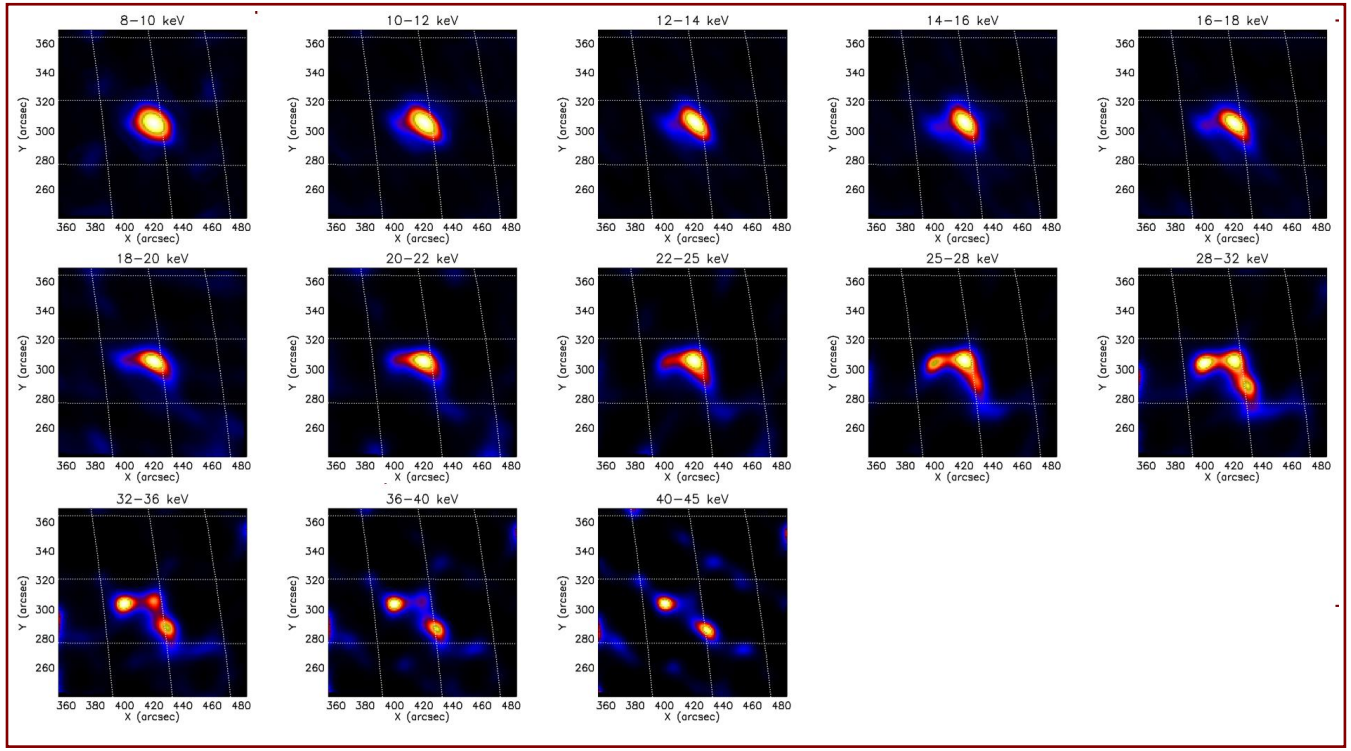


Figure: Photon images (*left panels*) for the energy intervals shown, compared with the electron flux images corresponding to the regularized electron visibilities (*right panels*) in the same energy range. The maps are produced using the MEM-GE algorithm.

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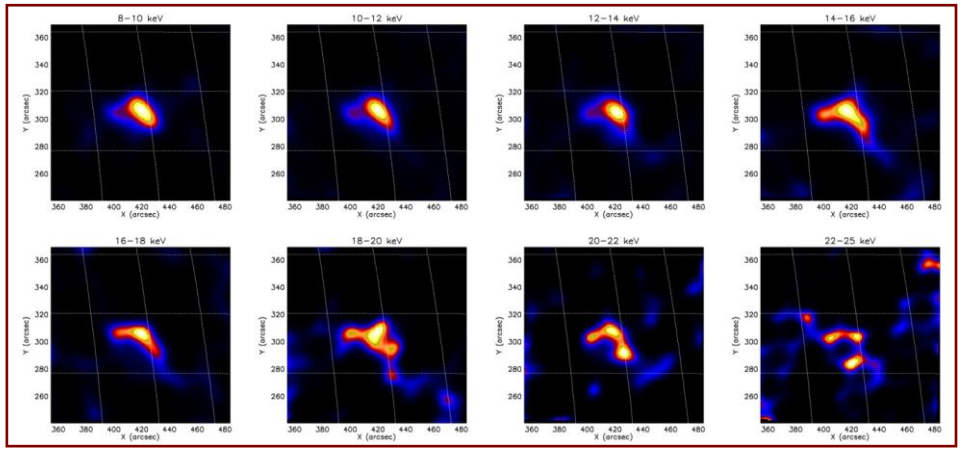
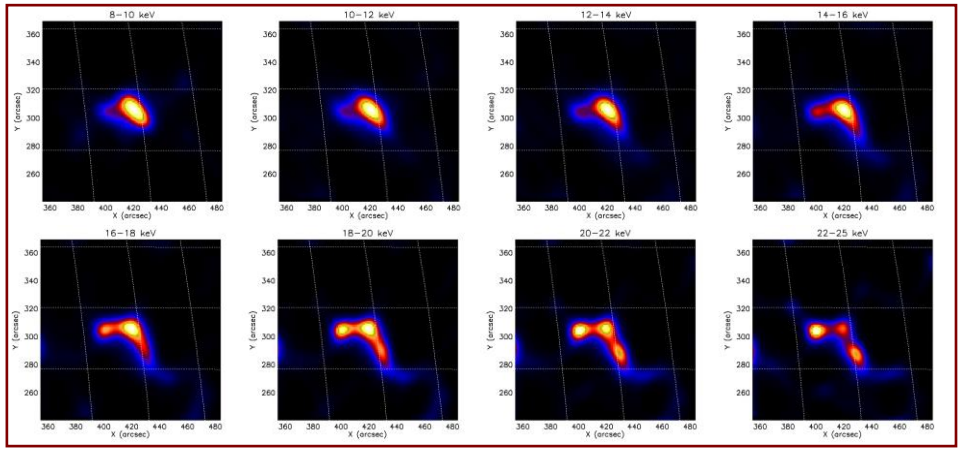
Photon images



Regularized photon images



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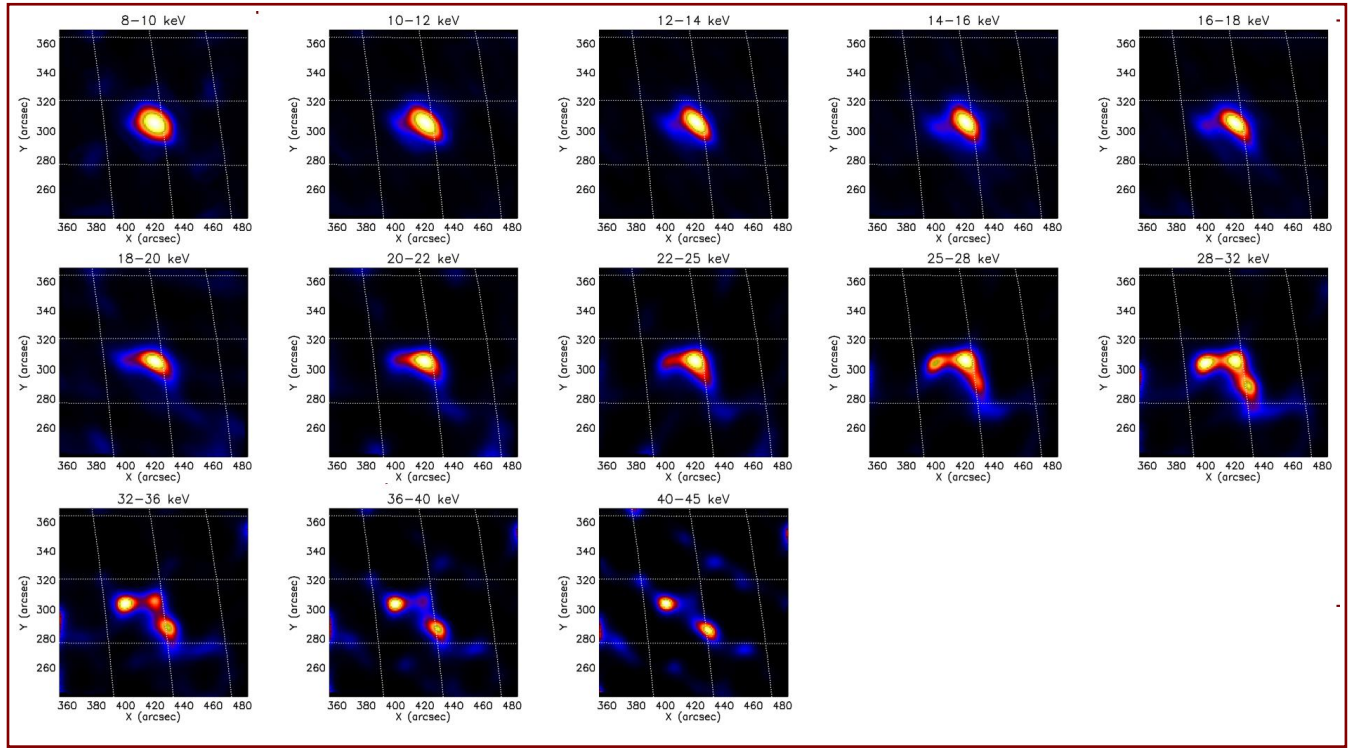

Figure: Photon images (*left top panels*) for the energy intervals shown, compared with the electron flux images (*right panels*) and regularized photon maps (*left bottom panels*) in the same energy range. The maps are produced using the MEM-GE algorithm.

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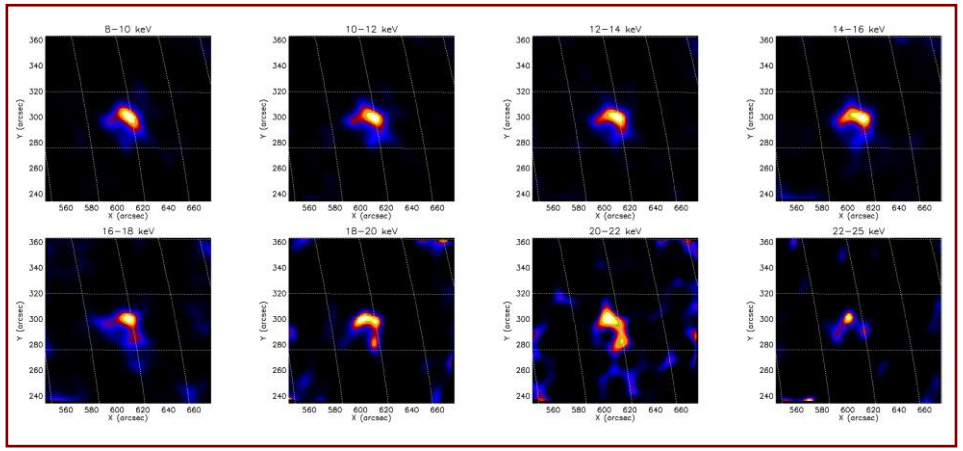

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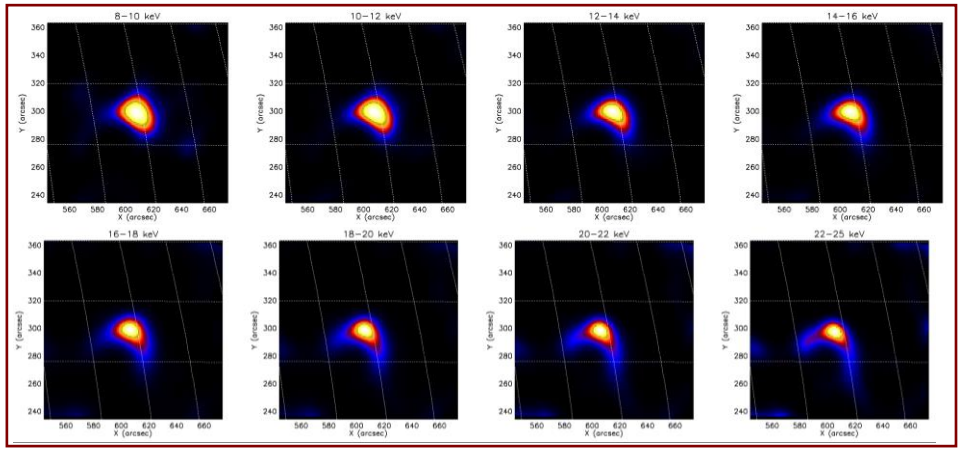

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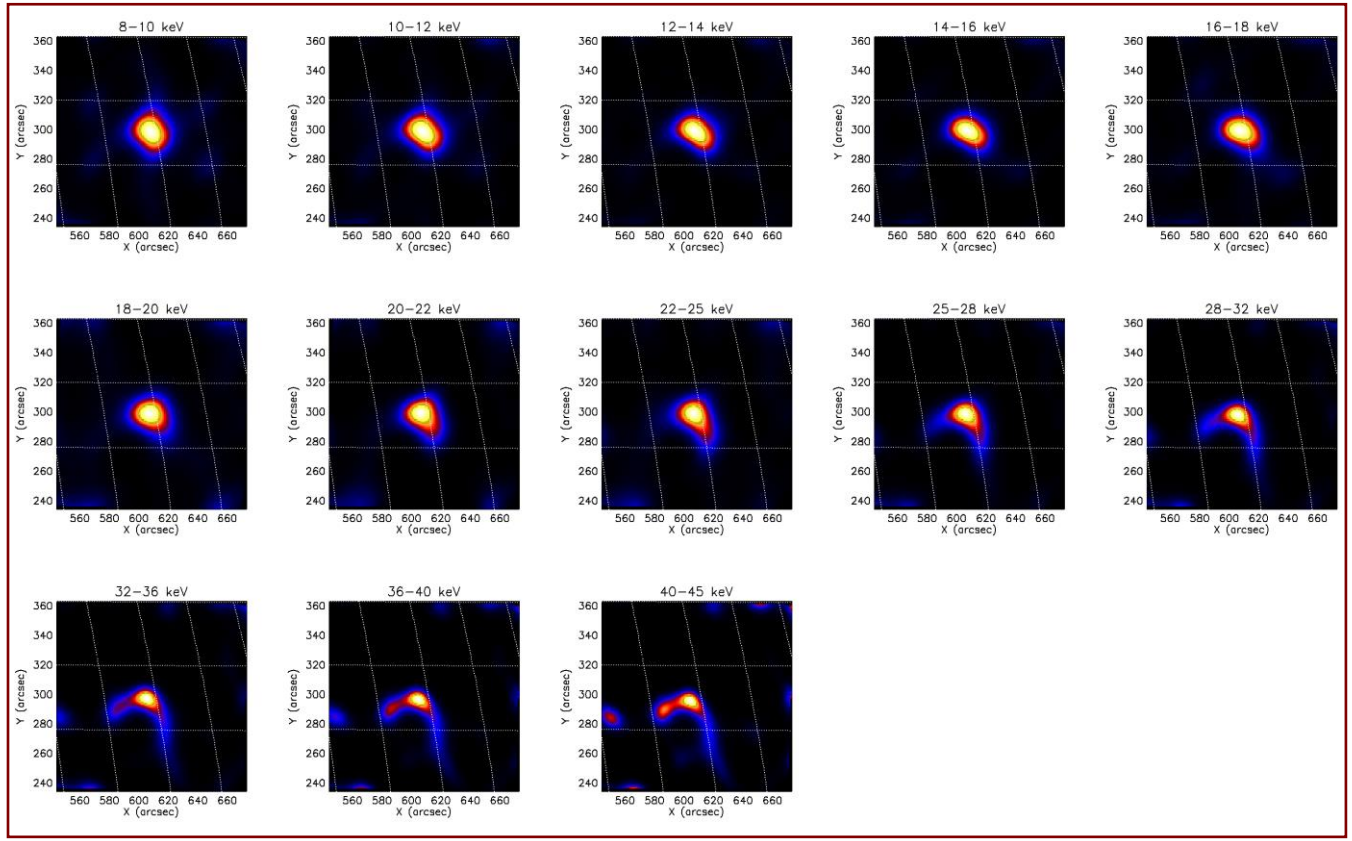

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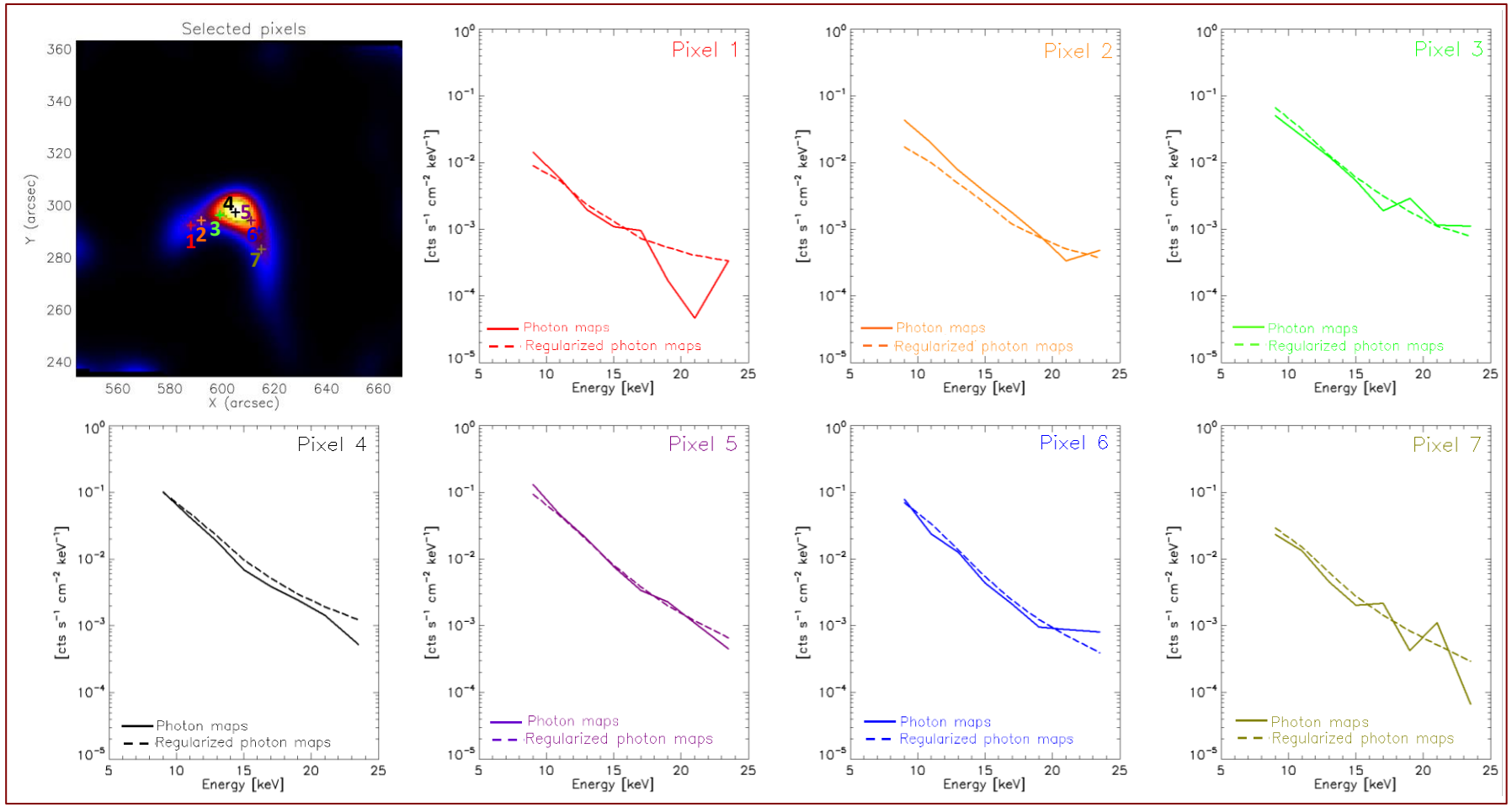

Figure: Pixel-wise spectrum obtained from photon maps and regularized photon maps. Top left panel: selected pixels are indicated with colored crosses. The other panels show the pixel-wise spectrum obtained from photon maps (*solid line*) and regularized photon maps (*dotted line*). The pixels selected in the top left panel and their respective spectra are indicated with the same colour. Plots are logarithmic scaled on the y-axis.

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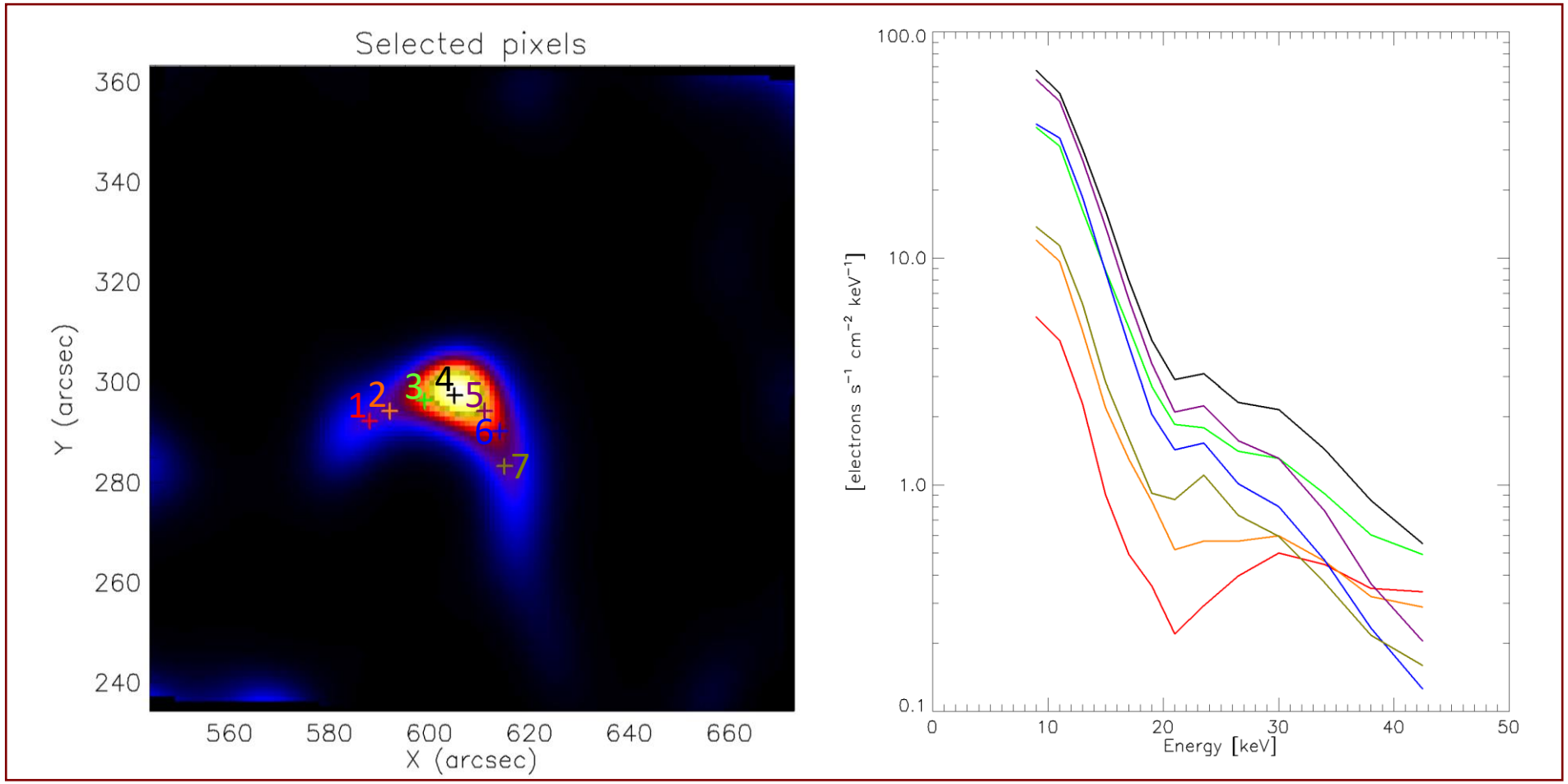

Figure: Pixel-wise spectrum obtained from regularized electron maps. Left panel: selected pixels are indicated with colored crosses. Right panel shows the pixel-wise spectrum obtained from regularized electron maps. The pixels selected in the top left panel and their respective spectra are indicated with the same colour. Plots are logarithmic scaled on the y-axis.

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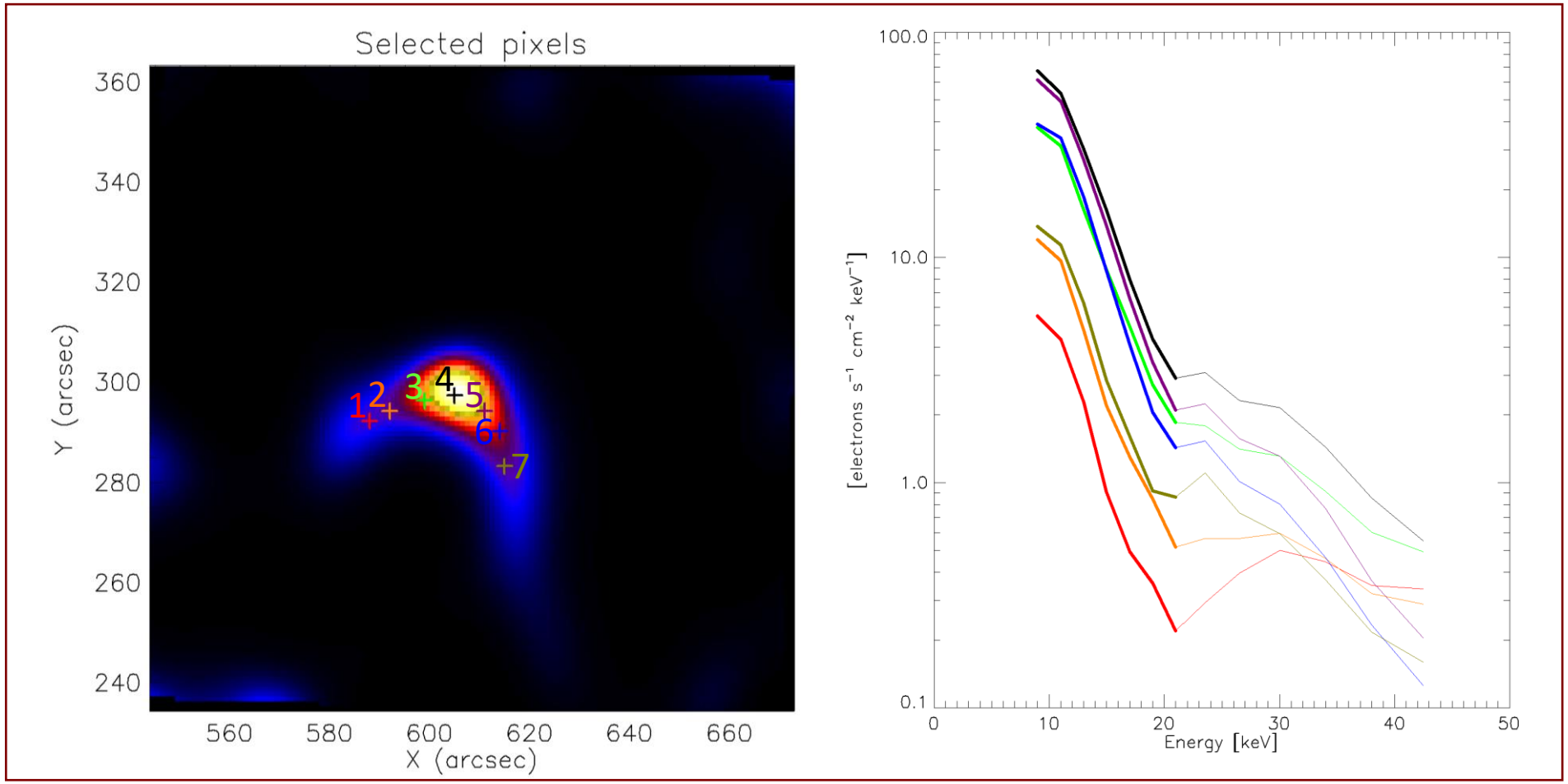

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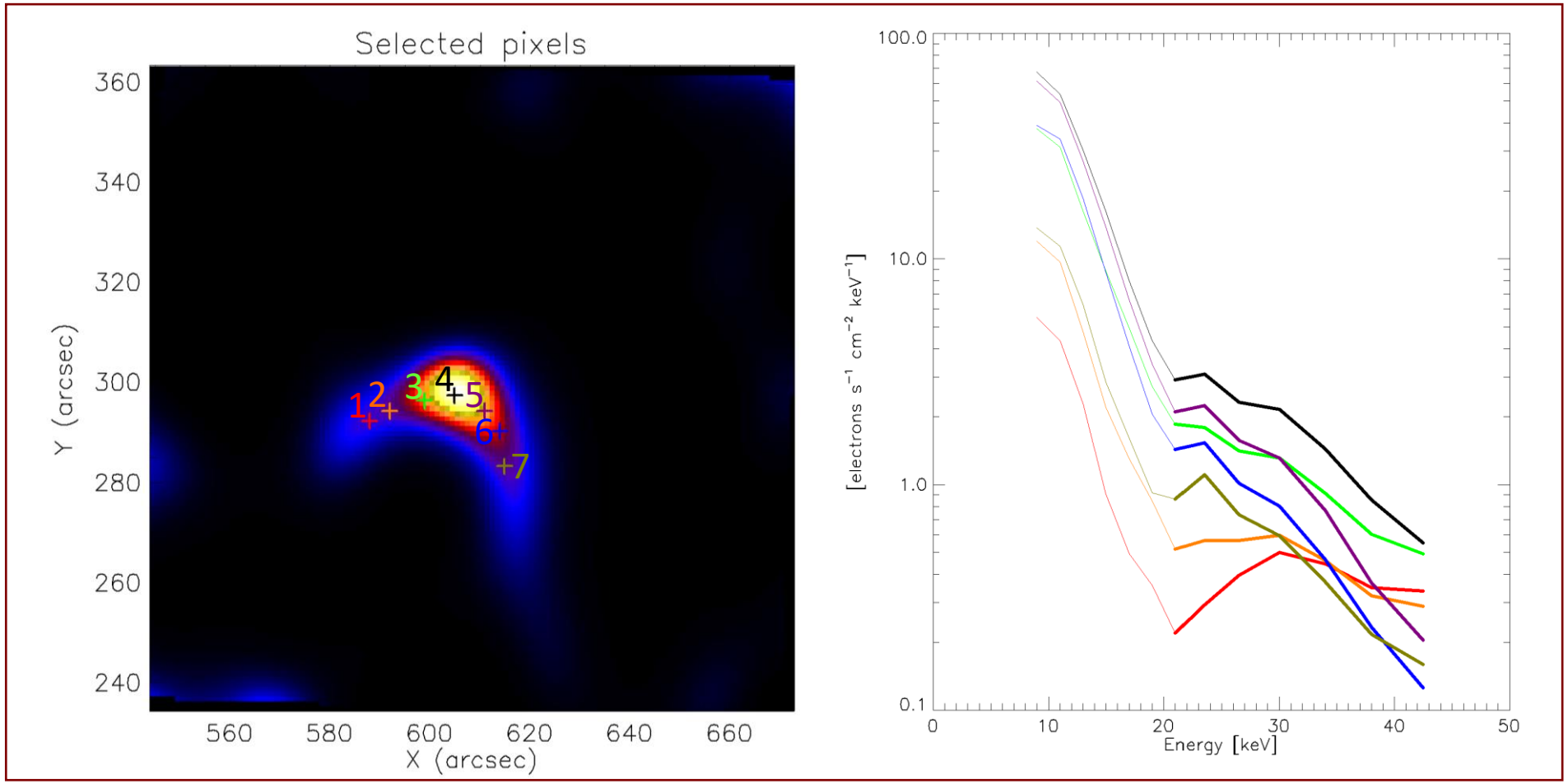

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The solution obtaining by inverting Equation (2)

$$W(u,v;E) = \frac{a}{4\pi R^2} \int \int N(x,y)\bar{F}(x,y;E)e^{2\pi i(xu+yv)}\,dx\,dy \qquad (2)$$

provides the quantity:

$$REM_s(x,y;E) := N(x,y)\bar{F}(x,y;E)$$

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From electron flux maps we have: $REM_s(x, y; E) \sim \bar{n}(x, y)\, \ell(x, y) \bar{F}(x, y; E)$ (4)

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From:

1. the solution of (5)

$$\frac{dE}{ds} = -\frac{K\,n}{E} \qquad K = 2\pi e^4 \Lambda \qquad (5)$$

2. the electron continuity equation in (6):

$$F(E)\;dE = F(E_o)\,dE_o \qquad (6)$$

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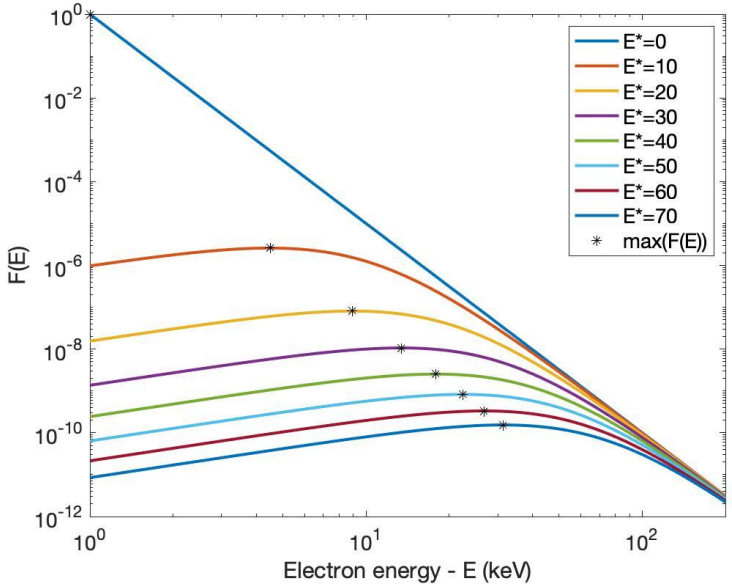
$$REM_s(x,y;E) = A(x,y)E(E^2 + E_*^2(x,y))^{-\frac{\delta+1}{2}} \qquad (7)$$

where $E_*(x,y) = \sqrt{2K\tilde{N}(x,y)}$

Figure: The position of the maximum of $F(E) = E\left(E^2 + E_*^2\right)^{\frac{\delta+1}{2}}$ (in the plot $\delta = 5$) is linked to the value of E_* by:

$$E_{MAX} = \frac{E_*}{\sqrt{\delta}}$$



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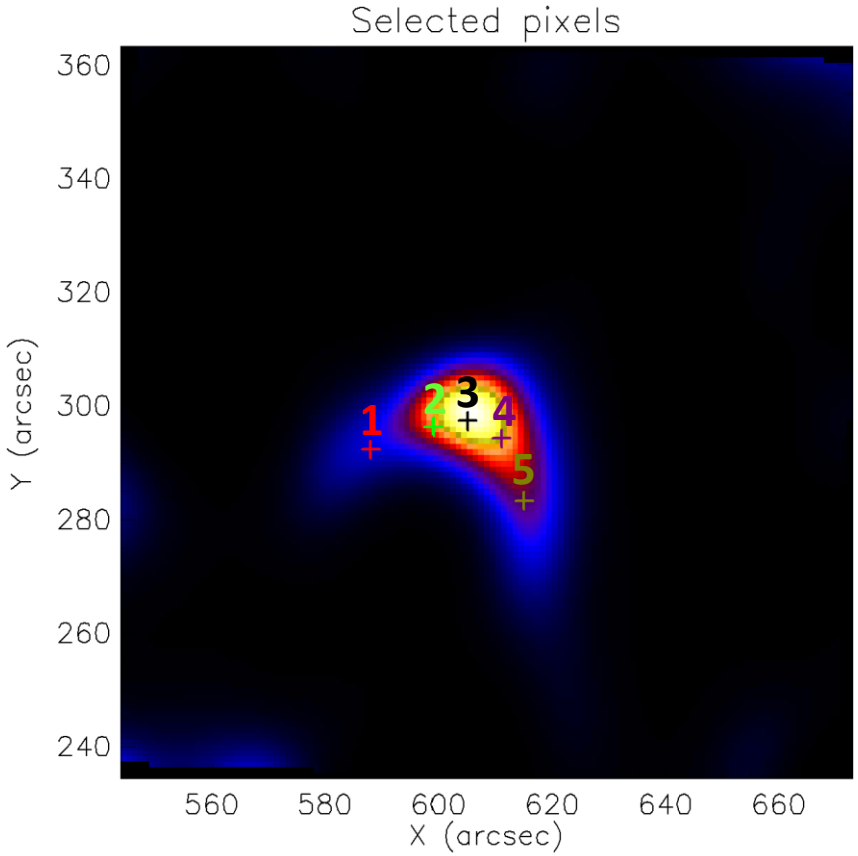


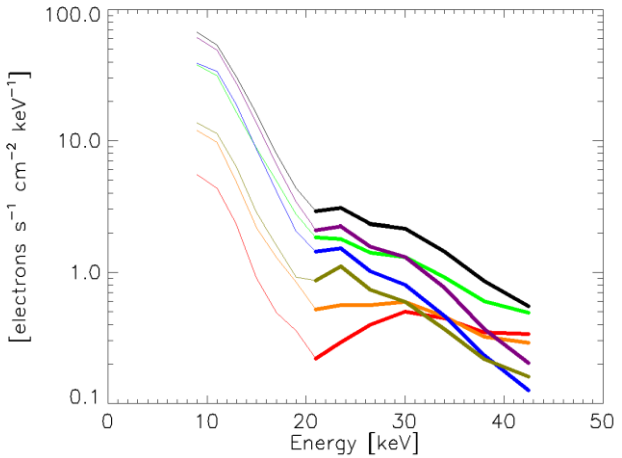
Figure: Top panel shows the selected pixels.

Table: E_* values for the selected pixels in the top panel. The further you move away from pixel 3, the more the value of E_* increases.

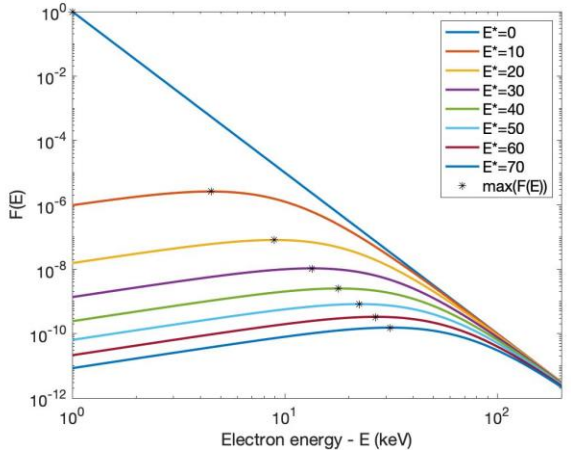
$$REM_s(x, y; E) = A(x, y)E(E^2 + E_*^2(x, y))^{-\frac{\delta+1}{2}} \quad (7)$$

	Pixel 1	Pixel 2	Pixel 3	Pixel 4	Pixel 5
E_*	67.5	53.8	47.5	51.4	60.1

Observed spectrum



Theoretical spectrum



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Conclusions and future works

- ☑ We have described a new approach to solar hard X-ray imaging spectroscopy:
  - ☑ two-dimensional Fourier transforms of the image in the photon domain are transformed into Fourier transforms of the electron flux maps.
  - ☑ This tool also provides regularized photon visibilities corresponding to the regularized electron visibilities.

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$$REM_s(x,y;E) = A(x,y)E(E^2 + E_*^2(x,y))^{-\frac{\delta+1}{2}} \longrightarrow \widetilde{N}(x,y) = \frac{E_*^2(x,y)}{2K}$$

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Conclusions and future works

- ☑ We have described a new approach to solar hard X-ray imaging spectroscopy:
  - ☑ two-dimensional Fourier transforms of the image in the photon domain are transformed into Fourier transforms of the electron flux maps.
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