Multi-scale Clean

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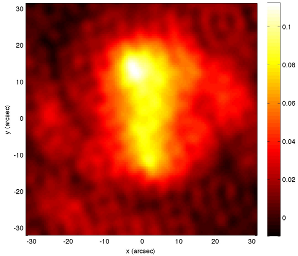
and the MIDA group

STIX Co-location, Windisch – November 15th, 2022

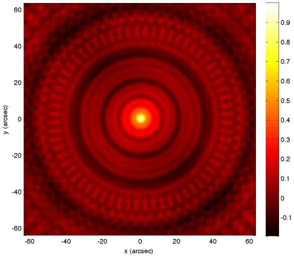
multi-scale CLEAN

Clean algorithm

Dirty map I^D



Dirty Beam K (PSF)



$$I^D(x,y) = (K * I)(x,y) := \int\int I(x',y')K(x-x',y-y')dx'dy'$$

Clean algorithm

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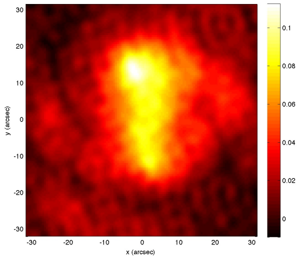

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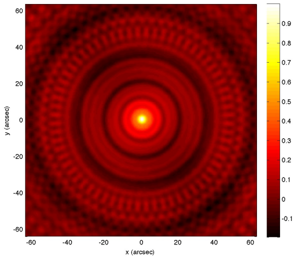
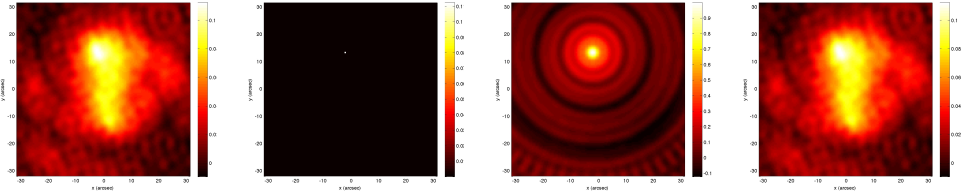


$$I^D(x,y) = (K * I)(x,y) := \int\int I(x',y')K(x-x',y-y')dx'dy'$$

CLEAN models the unknown source flux $I(x,y)$ as the sum of Q point sources (δ-Dirac distributions) plus background, i.e.:

$$I(x,y) = \sum_{q=1}^{Q} I_q \delta(x-x_q, y-y_q) + B(x,y) \quad \Longrightarrow \quad I^D(x,y) = \sum_{q=1}^{Q} I_q K(x-x_q, y-y_q) + (K*B)(x,y)$$

Clean algorithm



1. **Maximum Identification.**

$$(x_{max}^{(t)}, y_{max}^{(t)}) = \arg\max_{(x,y)} I^{(t-1)}(x,y) \qquad I_{max}^{(t)} = I^{(t-1)}(x_{max}^{(t)}, y_{max}^{(t)})$$

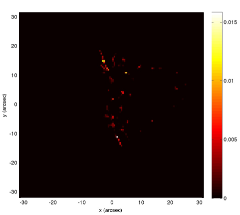
2. **Clean Components Update.**

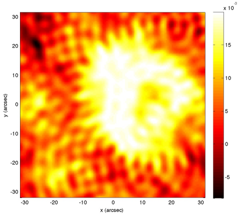
$$CC^{(t)}(x,y) = CC^{(t-1)}(x,y) + \frac{\gamma I_{max}^{(t)}}{\max_{(x,y)} |K(x,y)|} \delta(x - x_{max}^{(t)}, y - y_{max}^{(t)})$$

3. **Dirty Map Update.**

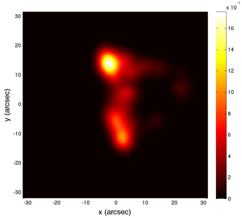
$$I^{(t)}(x,y) = I^{(t-1)}(x,y) - \frac{\gamma I_{max}^{(t)}}{\max_{(x,y)} |K(x,y)|} K(x - x_{max}^{(t)}, y - y_{max}^{(t)})$$

Clean algorithm



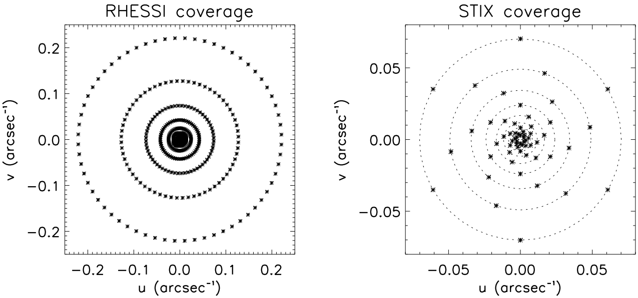


As the final step of CLEAN deconvolution, one constructs the CLEANed map as the convolution between the Clean Component map and an idealized version of the instrumental experimental PSF (**clean beam**) + background.



multi-scale Clean

We exploit the fact that hard X-ray telescopes are characterized by a PSF that can be written, under some assumptions, as the sum of a finite number of PSF components each one filtering a specific portion of the (u, v) plane.



If natural weighting is considered, then

$$K(x, y) = \sum_{j=1}^{N} K_j(x, y)$$

where the j-th dirty beam is computed by using just N_j visibilities as

$$K_j(x, y) = \sum_{l=1}^{N_j} \exp[-2\pi i(x u_l^j + y v_l^j)]\delta u \delta v$$

multi-scale Clean

Using the same approach, we can define a set of N dirty maps

$$I_j^D(x,y) = \sum_{l=1}^{N_j} V(u_l^j, v_l^j) \exp[-2\pi i(xu_l^j + yv_l^j)]\delta u \delta v \qquad j = 1, \ldots, N$$

such that

$$I^D(x,y) = \sum_{j=1}^{N} I_j^D(x,y).$$

multi-scale Clean: source model

Multi-scale Clean models the source image $I(x, y)$ as the superposition of basis functions:

$$I(x, y) = \sum_{i=1}^{N} \sum_{q_i=1}^{Q_i} I_{q_i} m_i(x - x_{q_i}, y - y_{q_i}) + B(x, y)$$

which means that at scale i, for $i = 1, \ldots, N$, there are Q_i sources, each one placed at (x_{q_i}, y_{q_i}) and with peak intensity I_{q_i} for $q_i = 1, \ldots, Q_i$

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By using this model, the convolution equation leads to

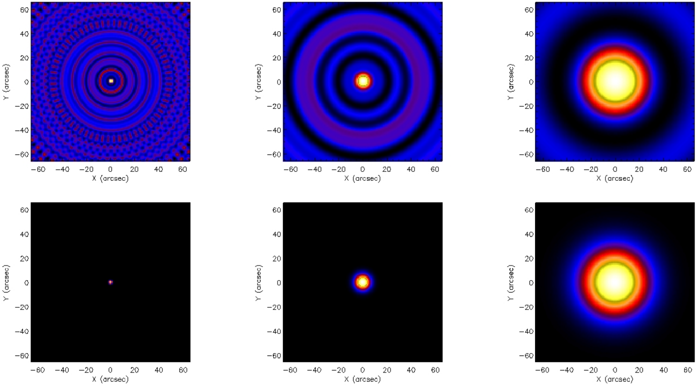
$$I^D(x, y) = \sum_{i=1}^{N} \sum_{q_i=1}^{Q_i} I_{q_i}(m_i * K)(x - x_{q_i}, y - y_{q_i}) + (K * B)(x, y)$$

$$\sum_{j}^{N} I_j^D(x, y) = \sum_{j=1}^{N} \left(\sum_{i=1}^{N} \sum_{q_i=1}^{Q_i} I_{q_i}(m_i * K_j)(x - x_{q_i}, y - y_{q_i}) + (K_j * B)(x, y) \right)$$

$$I_j^D(x, y) = \sum_{i=1}^{N} \sum_{q_i=1}^{Q_i} I_{q_i}(m_i * K_j)(x - x_{q_i}, y - y_{q_i}) + (K_j * B)(x, y)$$

multi-scale Clean: basis functions and PSFs

RHESSI	Det 1	Det 2	Det 3	Det 4	Det 5	Det 6	Det 7	Det 8	Det 9
Scale 1	x	x							
Scale 2			x	x	x				
Scale 3						x	x	x	x

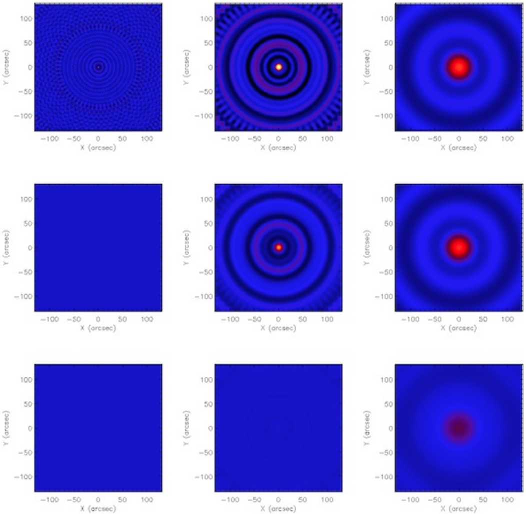


multi-scale Clean: cross-convolutions

$$I_j^D(x, y) = \sum_{i=1}^{N} \sum_{q_i=1}^{Q_i} I_{q_i} (m_i * K_j)(x - x_{q_i}, y - y_{q_i}) + (K_j * B)(x, y)$$

multi-scale Clean: cross-convolutions

Row i–column j $(i, j = 1, 2, 3)$ contains cross-convolution $m_i * K_j$



multi-scale Clean algorithm

The multi-scale CLEAN algorithm can be summarized with the following loop:

1. **Rescaling.** At each iteration $t \geq 1$, all dirty maps are re-scaled in such a way that small scales will be favored at the first iterations. *This rescaling is just a technical fact and re-scaled maps are utilized just in the maximum search process*

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2. **Maximum Identification.** We determine the $\bar{j}$-th re-scaled dirty map that presents the maximum intensity value and select the corresponding basis function $m_{\bar{j}}(x, y)$

$$(x_{max}^{(t)}, y_{max}^{(t)}) = \arg\max_{(x,y)} I_{\bar{j}}^{(t-1)}(x, y) \qquad I_{max}^{(t)} = I_{\bar{j}}^{(t-1)}(x_{max}^{(t)}, y_{max}^{(t)})$$

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3. **CLEAN Components Update.**

$$CC^{(t)}(x, y) = CC^{(t-1)}(x, y) + \frac{\gamma I_{max}^{(t)}}{\max_{(x,y)} |(m_{\bar{j}} * K_{\bar{j}})(x, y)|} m_{\bar{j}}(x - x_{max}^{(t)}, y - y_{max}^{(t)})$$

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4. **Dirty Map Update.** At each scale $j = 1, \ldots, N$, the dirty map component is updated in such a way that
$$I_j^{(t)}(x, y) = I_j^{(t-1)}(x, y) - \frac{\gamma I_{max}^{(t)}}{\max_{(x,y)} |(m_{\bar{j}} * K_{\bar{j}})(x, y)|} (m_{\bar{j}} * K_j)(x - x_{max}^{(t)}, y - y_{max}^{(t)}).$$

Rescaling

At each iteration, all dirty maps are re-scaled in such a way that the re-scaled dirty map components $\mathcal{I}_j^{(t)}$, $j = 1, \ldots, N$ become

$$\mathcal{I}_j^{(t)}(x, y) = \eta_j \frac{I_j^{(t-1)}(x, y)}{\max_{(x,y)} I_j^{(t-1)}(x, y)} \tag{1}$$

where η_j, $j = 1, \ldots, N$ are **scale bias** factors decreasing as j increases.

Scale bias

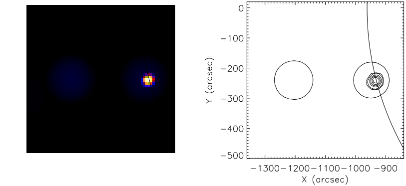
The definition of the scale bias η_j is crucial in order to emphasize smaller scales at first iterations. **We use a relation reflecting the geometric progression of the radii of the circles sampling the** (u, v) **plane**:

$$\eta_j := \left(\frac{1}{r}\right)^{\alpha(j)} \qquad j = 1, \ldots, N. \tag{2}$$

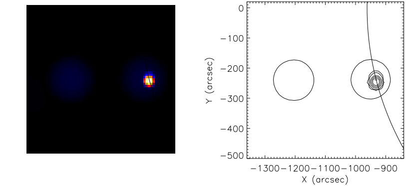
where r is the ratio of the geometric progression ($r = \sqrt{3}$ for RHESSI and r t.b.d. for STIX)

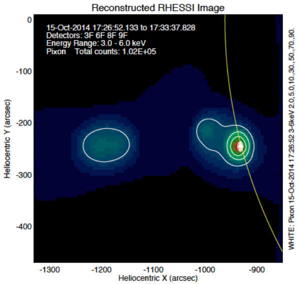
October 15, 2014 event

3 scales: det 3 - det 6 - det 8,9



2 scales: det 3,6 - det 8,9



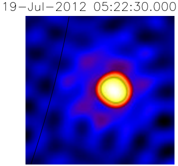


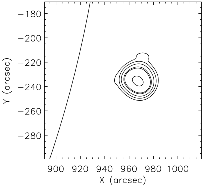
July 19, 2012 event (Krucker & Battaglia, 2014)

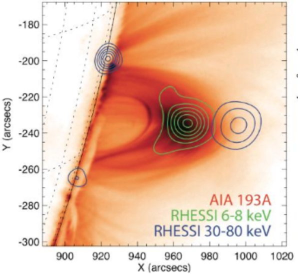
05:21:00 - 05:24:00 UT

Energy range: 6-8 keV

3 scales: det 3,4 - det 5,6 - det 7,8,9



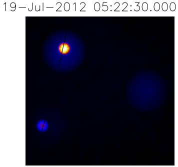


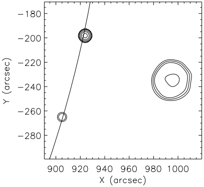


green contours at 20%, 35%, 50%, 65%, 70%, 95%
blue contours (footpoints) at 30%, 50%, 70%, 90%
blue contours (coronal source) at 2%, 2.5%, 3%

Energy range: 30-80 keV

3 scales: det 3,4 - det 5,6,7 - det 8,9





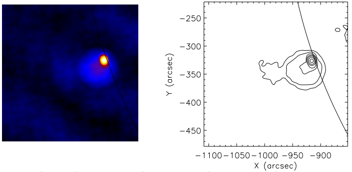
$\Longleftarrow$ contours (footpoints) at **15%, 20%**, 30%, 50%, 70%, 90%
contours (coronal source) at 2%, 2.5%, 3%, **5%**

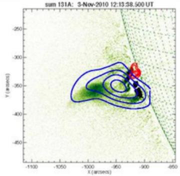
November 3, 2010 event (Glesener et al., 2012)

18-40 keV - 30 sec integration time

3 scales: det 3 - det 4,5 - det 6,7,8,9

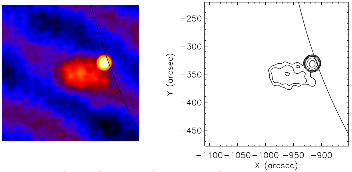
3−Nov−2010 12:13:38.000 UT

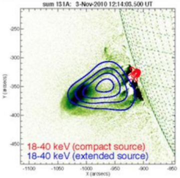




4 scales: det 3,4 - det 5,6 - det 7,8 - det 9

3−Nov−2010 12:14:03.000 UT





3 scales: det 3 - det 4,5 - det 6,7,8,9

3−Nov−2010 12:14:39.000 UT