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Image Acquisition and Analysis in Neuroscience

Assignment Sheet 6

Solution has to be uploaded by January 28, 2021, 8:00 a.m., via eCampus

If you have questions concerning the exercises, please use the forum on eCampus.

- Please work on this exercise in **small groups** of 3 students. Submit each solution only once, but clearly indicate who contributed to it by forming a team in eCampus. Remember that all team members have to be able to explain all answers.
- Please submit your answers in PDF format, and your scripts as *.py/*.ipynb files. If you are using Jupyter notebook, please also export your scripts and results as PDF.

Exercise 1 (Functional MRI, *4 Points*)

Indicate whether each of the following statements is true or false. Briefly justify your answers.

a) The functional MRI signal in a given voxel indicates how many action potentials have recently occurred within that voxel. (1P)

b) The "initial dip" in the hemodynamic response is often ignored when statistically modeling fMRI time courses. (1P)

c) When writing a t-test as a General Linear Model (GLM) and regressing out age as a variable of no interest, it is important to center ages to zero mean to avoid a statistical bias in the t-test. (1P)

d) Spatial Gaussian smoothing has a similar motivation in functional MRI analysis as in Voxel Based Morphometry. (1P)

Exercise 2 (Hemodynamic Response Function, *6 Points*)

a) Sample the "double gamma" canonical hemodynamic response function at 0.1 s temporal resolution and plot it. (2P)
Hint: You can make use of the gamma distribution available in `scipy`.

b) Using the result from a), predict the response to two box-shaped stimuli, which should have unit magnitude and 1 s or 2 s duration, respectively. Perform the required computations numerically at 0.1 s temporal resolution, and plot the results. (2P)
Hint: You can make use of the methods for convolution available in `numpy`.

c) Downsample your results from b) to the 2.5 s temporal resolution that is more typical of fMRI. Also perform the convolution at 2.5 s resolution and compare the results. (2P)

Exercise 3 (General Linear Model, *7 Points*)

a) You are given $2n$ regional gray matter volume measurements, which arise from measuring each of n subjects before and after learning a specific task. What is a suitable statistical test to check whether learning had an effect on gray matter volume? (1P) Please describe how you would prepare a measurement vector $\mathbf{y}$, a design matrix $\mathbf{X}$, and a contrast vector $\mathbf{c}$ to formalize this test as a General Linear Model. (3P)

b) You are now given regional gray matter volumes from a final follow-up scan of the same n subjects, two months after the second scan. Please describe how you would prepare a measurement vector $\mathbf{y}$, a design matrix $\mathbf{X}$, and a contrast matrix $\mathbf{C}$ to formalize an F-test of the null hypothesis that there is no significant difference between any of the three timepoints. (3P)

Exercise 4 (Normalized Spectral Clustering, *8 Points*)

In this exercise, you will derive normalized spectral clustering for clustering n data points into two separate clusters. The derivation will be similar to the one for unnormalized spectral clustering that was presented in the lecture.

a) The cluster indicator function $\mathbf{f} \in R^n$ of set $A \subset V$ is defined as

$$f_i = \begin{cases} \sqrt{\frac{vol(\bar{A})}{vol(A)}}, & \text{if } v_i \in A \\ -\sqrt{\frac{vol(A)}{vol(\bar{A})}}, & \text{if } v_i \in \bar{A} \end{cases}$$

Show that the quadratic form associated with the normalized graph Laplacian

$$\mathbf{f}^T\tilde{\mathbf{L}}\mathbf{f} = \frac{1}{2}\sum_{i,j=1}^{n} w_{i,j}\left(\frac{f_i}{\sqrt{d_i}} - \frac{f_j}{\sqrt{d_j}}\right)^2,$$

where $\tilde{\mathbf{L}} := \mathbf{D}^{-\frac{1}{2}}\mathbf{L}\mathbf{D}^{-\frac{1}{2}}$. (1P)

b) Show that the problem of minimizing the normalized cut is equivalent to the minimization problem

$$\min_{A} \quad \mathbf{f}^T\mathbf{L}\mathbf{f} \quad \text{subject to} \quad \mathbf{D}\mathbf{f}\perp\mathbb{1}, \quad \mathbf{f}^T\mathbf{D}\mathbf{f} = vol(V).$$

(2P)

c) Show that the relaxed problem takes the form below for $\mathbf{g} := \mathbf{D}^{\frac{1}{2}}\mathbf{f}$

$$\min_{R^n} \quad \mathbf{g}^T\tilde{\mathbf{L}}\mathbf{g} \quad \text{subject to} \quad \mathbf{g}\perp D^{\frac{1}{2}}\mathbb{1}, \quad \|\mathbf{g}\|^2 = vol(V).$$

(2P)

d) Show that the minimum of $\mathbf{g}^T\tilde{\mathbf{L}}\mathbf{g}$ is obtained when $\mathbf{g}$ is the eigenvector of $\tilde{\mathbf{L}}$ corresponding to its second smallest eigenvalue λ. (1P)

Hint: Apply the Rayleigh-Ritz theorem to the problem in (c).

e) Show that $\mathbf{f}$ is the solution of generalized eigenproblem $\mathbf{L}\mathbf{f} = \lambda\mathbf{D}\mathbf{f}$ where eigenvalue λ comes from d). (1P)

Hint: Use the fact that λ is an eigenvalue of $\tilde{\mathbf{L}}$ with eigenvector $\mathbf{D}^{\frac{1}{2}}\mathbf{u}$ if and only if λ and $\mathbf{u}$ solve the generalized eigenproblem $\mathbf{L}\mathbf{u} = \lambda\mathbf{D}\mathbf{u}$.

f) As a final step, we need to transform the real-valued solution vector $\mathbf{f}$ of the relaxed problem into a discrete indicator vector. Suggest a possible way to do so. (1P)

Good Luck!