Ex 3 $\quad f(t) = e^{-|t|}$

$$\mathcal{F}(f)(\xi) = \int_{-\infty}^{\infty} f(t)\, e^{-2\pi i \xi t}\, dt$$

$$= \int_{-\infty}^{0} e^{t} \left(\cos(2\pi\xi t) - \sin(2\pi\xi t)\, i \right) dt$$

$$+ \int_{0}^{\infty} e^{-t} \left(\cos(2\pi\xi t) - \sin(2\pi\xi t)\, i \right) dt$$

$\cdot(-1)$ Trafo

$$= \int_{0}^{\infty} e^{-t}\left(\cos(2\pi\xi t) + \cancel{\sin(2\pi\xi t)\, i}\right) + e^{-t}\left(\cos(2\pi\xi t) - \cancel{\sin(2\pi\xi t)\, i}\right) dt$$

$$= 2 \int_{0}^{\infty} e^{-t} \cos(2\pi\xi t)\, dt =: (*)$$

p.I.

$$= 2 \left(\underbrace{\Big[(-e^{-t}) \cos(2\pi\xi t)\Big]_{0}^{\infty}}_{=1} - \int_{0}^{\infty} (-e^{-t}) \left(-\sin(2\pi\xi t)\, 2\pi\xi\right) dt \right)$$

$$= 2 \left(1 - \underbrace{\Big[(-e^{-t}) \sin(2\pi\xi t)\, 2\pi\xi\Big]_{0}^{\infty}}_{=0} - (-1) \cdot \int_{0}^{\infty} (-e^{-t}) \cos(2\pi\xi t) (2\pi\xi)^2\, dt \right)$$

$$\Leftrightarrow \quad (*) = 2 - (2\pi\xi)^2 (*)$$

$$\Rightarrow \quad (*) = \frac{2}{1 + (2\pi\xi)^2} = \mathcal{F}(f)(\xi)$$