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Image Acquisition and Analysis in Neuroscience

Assignment Sheet 4

Solution has to be uploaded by January 7, 2021, 8:00 a.m., via eCampus

If you have questions concerning the exercises, please use the forum on eCampus.

- Please work on this exercise in **small groups** of 3 students. Submit each solution only once, but clearly indicate who contributed to it by forming a team in eCampus. Remember that all team members have to be able to explain all answers.
- Please submit your answers in PDF format, and your scripts as *.py/*.ipynb files. If you are using Jupyter notebook, please also export your scripts and results as PDF.

Exercise 1 (Chan-Vese Model, *5 Points*)

a) Find the equation of the regularized Heaviside step function H_ϵ which corresponds to the equation for the regularized delta distribution δ_ϵ that was provided in Chapter 5, slide 34. (2P)
   Hint: We recommend using a table of integrals. Clearly state which rule from the table you applied, *do not* just provide the final result.
b) Extend the energy functional of the Chan-Vese model to regularize the total area of the foreground object in addition to the length of its boundary. Introduce a new regularization parameter λ that allows us to weight the contribution of this new term. (1P)
c) Use the Euler-Langrange formalism to derive the additional term that your extension of the energy function contributes to the update equation. (2P)

Exercise 2 (GMMs and EM Algorithm for Image Segmentation, *10 Points*)

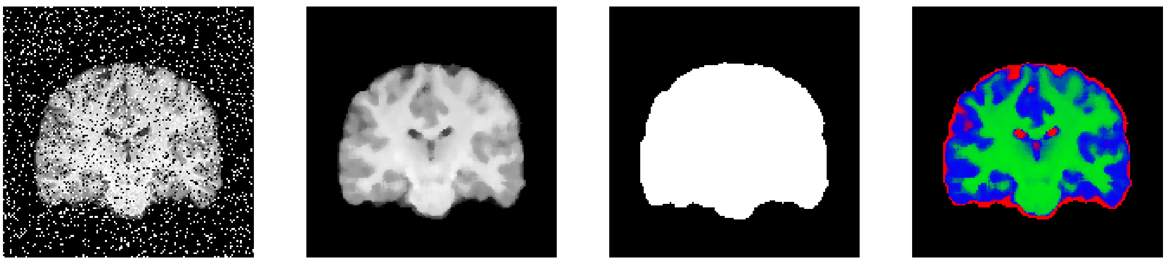

Figure 1: From left to right: The input image, after denoising, the binary mask, and the segmented brain image.

In this exercise, you will implement a Gaussian Mixture Model (GMM) to produce a probabilistic image segmentation, and find suitable parameters automatically by using the EM algorithm.

a) Read the grayscale image `brain-noisy.png`, which is provided along with this sheet on eCampus. Reduce the salt and pepper noise in the image using a median filter. Produce a binary mask that marks all pixels with an intensity greater than zero. In all further steps, only treat pixels within that mask. (2P)

Hint: You do not have to implement the median filter yourself, you may use a suitable Python package.

b) Plot a log-scaled histogram of the pixels within the mask. It should show how frequently different intensity values occur in the image. What do the peaks in this histogram represent? *Hint:* One way to find out is to create masks that highlight the pixels belonging to each peak. (2P)

c) Now, we will use a three-compartment Gaussian Mixture Model for image segmentation: Based on their gray level, pixels that fall within the mask from b) should be assigned to one of three Gaussians, capturing corticospinal fluid (dark), gray matter (medium), or white matter (bright). To start this process, initialize the parameters of a three-compartment GMM to reasonable values and use them to compute the responsibilities ρ_{ik} of cluster k for pixel i. (2P)

d) Visualize the responsibilities by mapping the probabilities of belonging to the CSF, gray matter, and white matter clusters to the red, blue, and green color channels, respectively. Please submit the resulting image. (1P)

e) Use the update rules provided in the lecture to re-compute the parameters μ_k, σ_k, and π_k. (1P)

f) Iterate the E and M steps of the algorithm until convergence. Please submit the final parameter values, a visualization of the final responsibilities, and your code. (2P)

Exercise 3 (Markov Random Fields, *10 Points*)

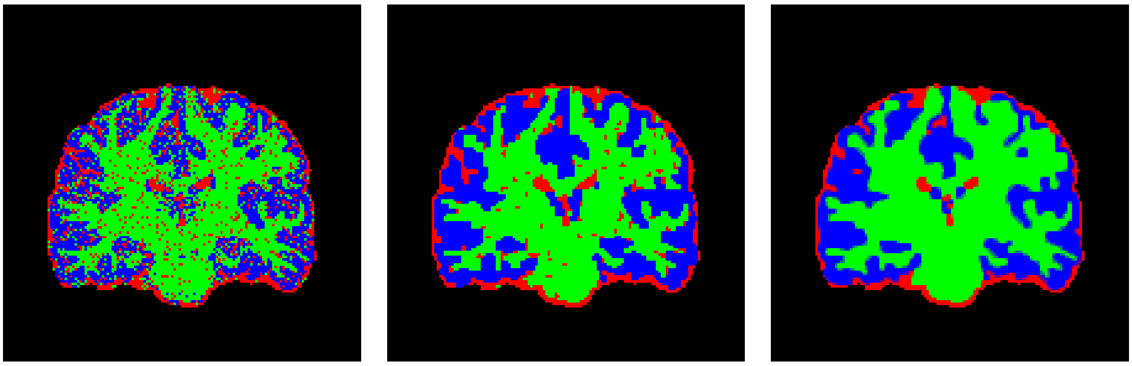

Figure 2: A hard labeling of the noisy image based on the algorithm from Exercise 2 (left), the result after repeated application of Iterated Conditional Modes with higher values of β (center), and the final probabilistic segmentation result with the MRF term (right).

a) Load the noisy brain image `brain-noisy.png` again and download the `mask.png` which is provided along with this sheet on eCampus. Based on your implementation of the EM algorithm from Exercise 2, but leaving out the median filtering, create a discrete (hard / non-probabilistic) label image that contains the most likely material for each pixel. Output it as an RGB image. For the segmentation use the `mask.png` in order to apply the algorithms only on the foreground pixels. (2P)

b) Implement one iteration of the Iterated Conditional Modes (ICM) algorithm for a Markov Random Field that uses the Potts model and $\beta = 0.5$. Use your EM parameters to initialize the unary potentials and use the labels of the neighbouring pixels to compute the pairwise potentials. Finally, for each pixel pick the label that minimizes the energy. Output the result as an RGB image. (3P)

c) Apply your ICM iteration five times overall. Output the number of pixels whose label changes in each iteration, and output the final labels as an RGB image. (2P)

d) Integrate your implementation of the ICM into the EM algorithm and run it until convergence. Output the final result as an RGB image. (2P)

e) Increase the β parameter and repeat task d). At which value of β is the final segmentation almost noise-free similar to Fig. 2? Output the final result. (1P)

Good Luck!