Ex 4

a) $w_{k+1} := w_k - \frac{1}{\lambda} Df(w_k)^t$

with Taylor 1st order: $f(x) \approx f(x_0) + Df(x_0)(x - x_0)$:

$$f(w_{k+1}) \approx f(w_k) + Df(w_k)\left(-\frac{1}{\lambda} Df(w_k)^t\right)$$

$$\Rightarrow \text{progress } P_{f, w_k}(\lambda) \approx -\frac{1}{\lambda} Df(w_k) Df(w_k)^t \qquad (*)$$

if ~~fails~~ progress small: Start line search (1D) along the line $w_k + \mathbb{R} \cdot Df(w_k)^t$

$D(f\circ\alpha id)(\alpha^{-1}x) = \alpha Df(x)$

b) from $(*)$ get: $P_{f\circ\alpha\cdot id,\, \alpha^{-1}\cdot w_k}(\lambda) = \frac{1}{\lambda}\alpha^2 \cdot Df(w_k) Df(w_k)^t$

$$= \alpha^2 P_{f, w_k}(\lambda)$$

c) $D^2(f\circ\alpha id)(\alpha^{-1}x) = \alpha^2 D^2 f(\alpha\cdot\alpha^{-1}\cdot x) = \alpha^2 D^2 f(x)$

$$\Rightarrow P_{f, w_k}(\lambda \cdot D^2 f(\cdot)) = P_{(f\circ\alpha id), (\alpha^{-1} w_k)}(\lambda \cdot D^2(f\circ\alpha id)(\cdot))$$

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also have: $\left.\frac{(Dh^2(\cdot))^t\, Dh^2(\cdot)}{\tilde\lambda\,(Dh(\cdot))^2}\right|_{\cdot = x} = \frac{\alpha^2}{\alpha^2} \cdot \left(\frac{\cdots}{\cdots}\right)$

$\cdot\, h'(x)$ works as well

$$= \left.\frac{(\alpha Dh^2(\cdot))^t(\alpha Dh^2(\cdot))}{\tilde\lambda(\alpha Dh(\cdot))^2}\right|_{\cdot = x} = \left.\frac{([D(h\circ\alpha)^2](\alpha^{-1}\cdot))^t\, [D(h\circ\alpha)^2](\alpha^{-1}\cdot)}{\tilde\lambda\left([D(h\circ\alpha)](\alpha^{-1}\cdot)\right)^2}\right|_{\cdot = x}$$

$\square$