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Image Acquisition and Analysis in Neuroscience

Assignment Sheet 2

Solution has to be uploaded by November 26, 2020, 8:00 a.m., via eCampus

If you have questions concerning the exercises, please use the forum on eCampus.

- Please work on this exercise in **small groups** of 3 students. Submit each solution only once, but clearly indicate who contributed to it by forming a team in eCampus. Remember that all team members have to be able to explain all answers.
- Please submit your answers in PDF format, and your scripts as *.py/*.ipynb files. If you are using Jupyter notebook, please also export your scripts and results as PDF.

Exercise 1 (Images in k-Space, *17+5 Bonus Points*)

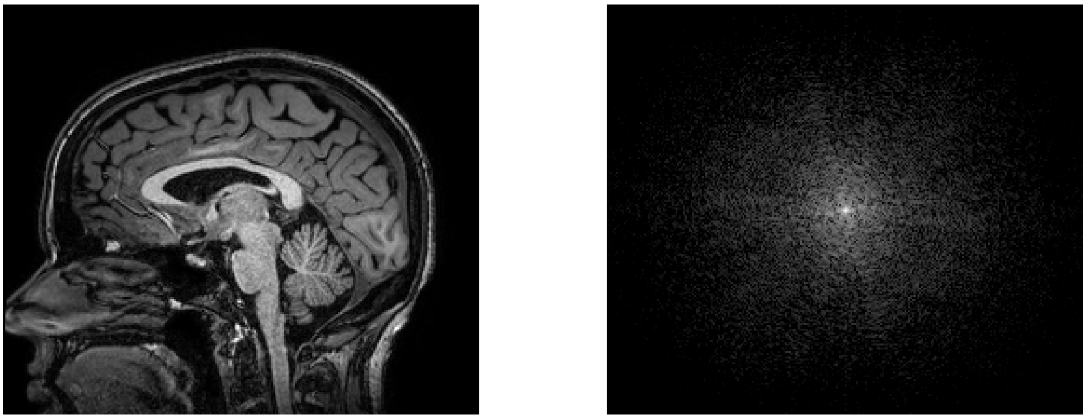

Figure 1: Logarithmic power spectrum of a brain image.

a) Write a Python script to display the logarithmic power spectrum of arbitrary grayscale images. In particular, your script should read an image (whose name is given as a command-line argument), transform it into k-space, compute the squared magnitude of the complex number at each location, take the logarithm, and display the result as an image. Please make proper use of the so-called FFT shift to ensure that the center of k-space (zero spatial frequency) is displayed at the center of the final image, as shown in Fig. 1. (5P)

   Hints: You can use `scipy.fftpack` to perform the Fourier transformation and FFT shift. You will have to be careful with very small values when taking the logarithm; it is acceptable to clamp them to a reasonable ϵ.

b) Some MR protocols only fill a rectangle at the center of k-space with measured values, and leave the rest of k-space empty (i.e., at zero values). Argue based on the theory of Fourier analysis how the image that results from such "zero-padding" compares to one for which only the measured part of k-space has been used for reconstruction (3P).

Illustrate your reasoning by writing a script that computes the k-space representation of any input image, copies the result into the center of a larger matrix (padded with zeroes), transforms it back, and displays the resulting image (4P).

In a), we computed the power spectrum of the input image. Briefly explain why we cannot reconstruct an image from the power spectrum alone. What information is missing? (1P)

c) The file `sepi.npy` along with this sheet on eCampus contains a vector with 4096 complex numbers, providing the raw MR measurements from an EPI experiment. Reconstruct the image that corresponds to these raw measurements (Fig. 2), correctly accounting for the zig-zag fashion in which EPI traverses k-space (illustrated in Fig. 3 left). (5P)

d) **Bonus task:** The image reconstructed in c) suffers from severe artifacts. They are due to a slight time shift between even and odd echoes, which stems from eddy currents and is illustrated in Fig. 3 right. The file `spc.npy` contains 128 complex numbers, which are the raw measurements from two echoes with positive and negative polarity, respectively, and no phase encoding. You can determine the time shift by simply plotting these two signals and comparing the location of their maxima. You can earn five bonus points by implementing a 1D interpolation technique that performs the required k-space shift before reconstruction, which will greatly reduce the artifact.

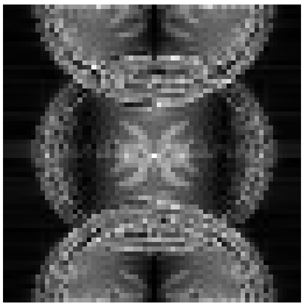

Figure 2: Reconstructed MR image with eddy current EPI artifact.

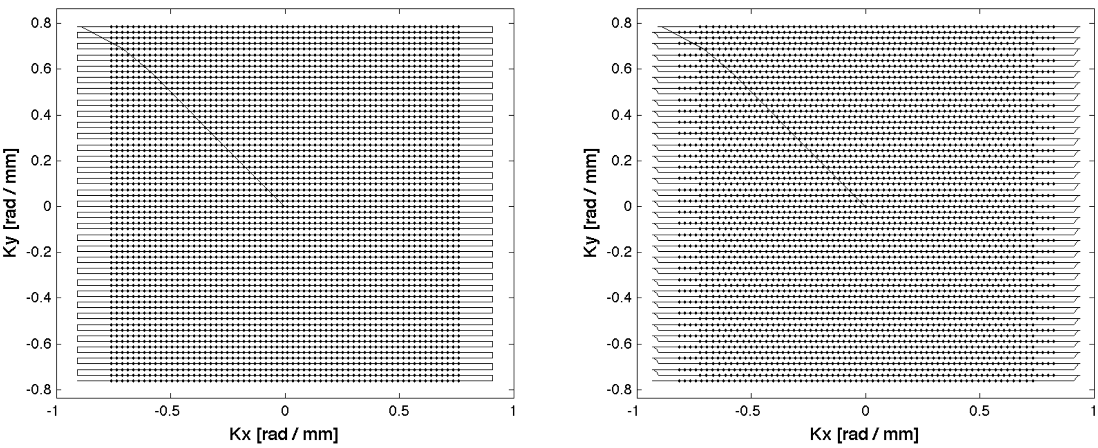

Figure 3: Left: Ideal EPI k-space traversal. Right: EPI k-space traversal with time shift.

Exercise 2 (Equilibrium Magnetization, *8+1 Bonus Points*)

Chapter 3, Slide 18 discusses how the equilibrium magnetization on which Magnetic Resonance Imaging is based arises from the alignment of nuclear magnetic moments with an external magnetic field $\mathbf{B}_0$, and from a tiny excess of spins in the lower energy state. It also claims that the magnitude M_0 of the resulting macroscopic magnetization is proportional to the strength B_0 of the external field. This exercise will help you better understand how these statements fit together.

a) Given the overall number n of hydrogen nuclei in a voxel, and the Boltzmann statistics $r = n_\uparrow / n_\downarrow$, compute the number of spins in the lower energy state that are *not* canceled by another spin in the higher energy state. Explain why this leads to the following equation for the strength of the resulting macroscopic magnetization:

$$M_0 = \frac{1-r}{1+r} \frac{n\hbar\gamma}{2} \tag{1}$$

Plug in values for a clinical scanner $B_0 = 1.5\,\mathrm{T}$ and body temperature $T \approx 310\,\mathrm{K}$ as well as an approximate value $n \approx 6.69 \times 10^{19}$ of protons in one cubic millimeter of water to obtain a numerical result, including physical units. (4P)

You can earn one bonus point for researching the order of magnitude of the magnetic moment of a compass needle for comparison (please cite your source).

b) The claimed proportionality between M_0 and B_0 holds approximately, for values of B_0 that are small enough to lead to a Boltzmann statistic that is very close to one. For notational simplicity, we could express its dependence on B_0 as

$$r = \frac{n_\downarrow}{n_\uparrow} = e^{\zeta B_0} \quad \text{with} \quad \zeta = -\frac{\hbar\gamma}{k_B T}. \tag{2}$$

Compute the derivative of Equation (1) with respect to B_0 (2P). Use it to specify a Taylor approximation of $M_0(B_0)$ around $B_0 = 0$ that is truncated after the linear part. (1P) Observe that it is linear in B_0 and confirm that, for clinical field strengths such as $B_0 = 1\,\mathrm{T}$ or $B_0 = 3\,\mathrm{T}$, it yields very similar numerical values as the exact Equation (1). (1P)

Good Luck!