$\boxed{\text{JAAM}}$

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Sheet 1

Ex 2 : $A = (a_{\alpha,\beta})_{\alpha,\beta \in \{1,\dots,n\}}$

a) (1) $\Rightarrow$ $a_{\alpha,\beta} = e^{-2\pi i \alpha \frac{\beta}{N}}$

b) $\frac{1}{N} A^* \overset{a)}{=} \frac{1}{N}(e^{+2\pi i \alpha \frac{\beta}{N}})_{\alpha,\beta \in \{1,\dots,n\}} \overset{(2)}{\Leftrightarrow} \frac{1}{N} A^* = A^{-1} =: (\#)$

c) (#) $\Rightarrow$ $\frac{1}{\sqrt{N}} A^* = \sqrt{N} A^{-1} = \left(\frac{A}{\sqrt{N}}\right)^{-1}$

$\Rightarrow$ let $\tilde{A} := \frac{1}{\sqrt{N}} A$ $\rightsquigarrow$ fullfills $\tilde{A}^* = \tilde{A}^{-1}$

1.)

(a) $(1-i)\cdot(0+2i)$

$= (1\cdot 0) + (2i) - (i\cdot 0) - (i\cdot 2i)$

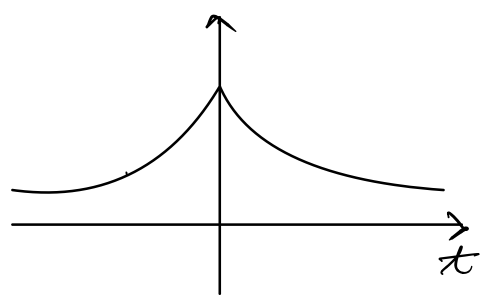
$= 0 + 2i - 0 - 2(-1) = 2i + 2$

$= \underline{\underline{2(1+i)}}$

(b) $\dfrac{(0+i)}{(1-\sqrt{3}i)} \quad \dfrac{(1+\sqrt{3}i)}{(1+\sqrt{3}i)} = \dfrac{\sqrt{3}i + i + \sqrt{3}(-1)}{(1)^2 - (\sqrt{3}i)^2}$

$= \dfrac{-\sqrt{3} + (1+\sqrt{3})i}{1+3} = \underline{\underline{\dfrac{-\sqrt{3} + (1+\sqrt{3})i}{4}}}$

3) $f(t) = \begin{cases} e^{-t} & t \geq 0 \\ e^{t} & t < 0 \end{cases}$



$f(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$

$= \int_{-\infty}^{0} e^{t} e^{-wit}\, dt + \int_{0}^{\infty} e^{-t} e^{-i\omega t}\, dt$

$= \int^{0} e^{t(1-i\omega)} + \int_{0}^{\infty} e^{-t(1+i\omega)}\, dt$

$$= \left.\frac{e^{t(1-i\omega)}}{1-i\omega}\right|_{-\infty}^{0} + \left.\frac{e^{-t(1+i\omega)}}{-(1+\omega i)}\right|_{0}^{\infty}$$

$$= \frac{1}{1-i\omega} - 0 + 0 - \left(-\frac{1}{(1+i\omega)}\right)$$

$$= \frac{1}{1-i\omega} + \frac{1}{1+i\omega} = \frac{(1+\cancel{i\omega}) + (1-\cancel{i\omega})}{(1)^2-(i\omega)^2}$$

$$= \frac{2}{1-(-1)\omega^2} = \frac{2}{1+\omega^2}$$

$$\underline{f(u) = \frac{2}{1+\omega^2}}$$