Dynamic Programming for Resource Allocation in Cloud Computing

Abstract- Dynamic Programming (DP) offers a robust solution for optimizing resource allocation in cloud computing environments. This paper explores the efficacy of both top-down and bottom-up DP strategies, providing a structured approach to minimize costs associated with resource distribution across virtual machines (VMs). We analyze the performance of these strategies against the traditional naive recursive approach, both analytically and empirically. Our findings indicate that DP not only enhances computational efficiency but also ensures adherence to the constraints of resource demand and availability. The implementations of top-down and bottom-up algorithms demonstrate a significant reduction in computational overhead, making them viable for large-scale cloud infrastructures where resource allocation decisions critically impact operational costs and system performance.

1 Introduction

The challenge of resource allocation in cloud computing entails efficiently distributing computing resources among virtual machines (VMs) to optimize performance and minimize costs. While resource allocation problems in multidimensional settings are solely depend on load balancing and system architecture, requiring heuristic or approximate solutions, this project proposes a dynamic programming (DP) approach for an optimal solution in specific cloud computing environments.

This approach stands as a significant contribution to cloud computing resource management, providing a foundation for further exploration and refinement.

1.1 Motivation

The increasing demand for cloud computing services highlights the critical need to explore efficient resource allocation strategies to strike a balance between cost, performance, and energy efficiency. Cloud service providers and users seek to minimize costs while judiciously allocating resources to meet fluctuating workloads, enhancing economic viability. Furthermore, optimizing performance is crucial to meet user expectations for seamless and prompt access to cloud resources.

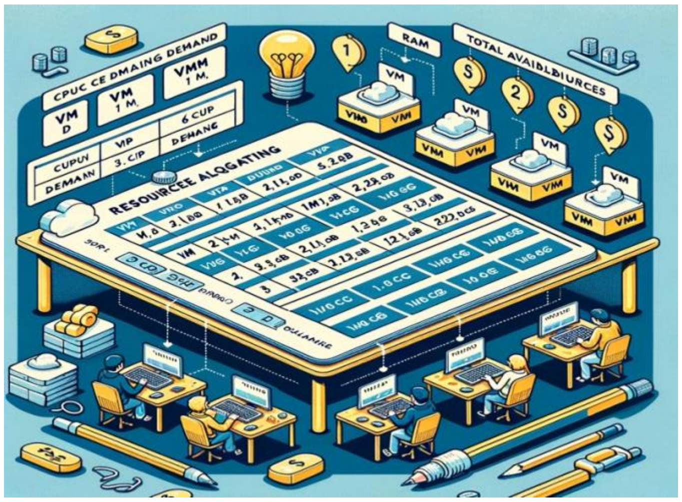


Fig. 1. scenario of resource allocation of machines in the server

1.2 Formulation

The model involves binary decision variables that indicate whether a specific resource is allocated to a VM, and cost variables representing the expense of such allocations. The objective is to minimize the total allocation cost while adhering to constraints that ensure each VM's demand is met without exceeding the available resources.

The formulation employs a recursive function that calculates the minimum cost for allocating resources to

VMs, accounting for the demands and the remaining resources at each step. This recursive strategy ensures an efficient allocation that balances demand satisfaction with cost minimization.

Variables:

x_{ij}: Binary variable indicating whether resource j is allocated to VM_{ij}:

C_{ij}: Cost of allocating resource j to VM_i

R_j**:** Total amount of resource j available.

D_{ij}**:** Demand of VM_i for resources.

i traverses the set of virtual machines requiring resources, while j iterates through the available types of resources such as CPU and memory for allocation.

Objective Function:

Minimize the total cost of resource allocation:

$$min\sum_{i=1}^{n}\sum_{j=1}^{m}(Cij \cdot xij)$$

Constraints:

Each VM's resource demand must be met:

$$\sum_{j=1}^{m} x_{ij} \geq D_i \text{ , } \forall\ i$$

Resources cannot be oversubscribed:

$$\sum_{i=1}^{n} x_{ij} \geq R_j \text{ , } \forall\ j$$

DP Formulation:

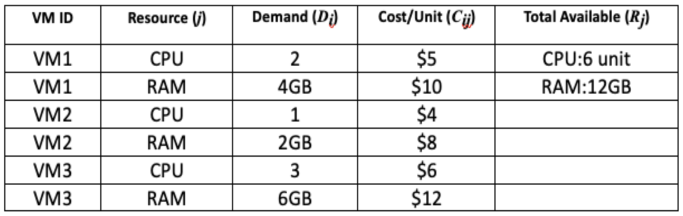
Let $F(i,r)$ be the minimum cost to allocate resources to VMs 1 through i with r resources remaining.

Recurrence Relation for optimal solution:

$$F(i,r) = min_{d \leq r}\ (C_{id} + F(i+1, r-d)),$$

where d is the decision to allocate d units of resource to VM_i. Using the above variables, functions and relationships, dynamic programming can be applied to find the allocation of resources to VMs that minimizes the total cost while satisfying all demands and not exceeding available resources.

1.3 Sample Input



VM ID	Resource (j)	Demand (D_i)	Cost/Unit (C_{ij})	Total Available (R_j)
VM1	CPU	2	$5	CPU:6 unit
VM1	RAM	4GB	$10	RAM:12GB
VM2	CPU	1	$4	
VM2	RAM	2GB	$8	
VM3	CPU	3	$6	
VM3	RAM	6GB	$12	

Fig. 2. Visualization of sample input for resource allocation

The decision variable x_{ij} plays a pivotal role, indicating the allocation status of resource j to virtual machine i, where $x_{ij} = 1$ signifies that the resource is allocated, and $x_{ij} = 0$ otherwise. Figure 1 considers a scenario with three VMs, each with specified demands: VM1 requires 2 CPU units and 4GB RAM, VM2 requires 1 CPU unit and 2GB RAM, and VM3 needs 3 CPU units and 6GB RAM. The cost of allocating these resources is denoted by C_{ij} with CPU costs per unit at \$5, \$4, and \$6 for VM1, VM2, and VM3 respectively, and RAM costs per GB at \$10, \$8, and \$12 respectively. The total available resources are 6 units and 12GB RAM, denoted by R_j.

The objective is to allocate resources across VMs in a way that minimizes total costs while ensuring the fulfilment of all VM demands without exceeding the available resource pool.

2 Cloud Resource Allocation

A traditional naive approach to the cloud resource allocation problem would enumerate all possible combinations of resource assignments to virtual machines (VMs). For each VM_i, it would consider every permutation of resource j allocations. The complexity would be $\boldsymbol{O(2^{nm})}$, where n is the number of VMs and m is the number of resources. This is because each VM can have a binary decision (allocate or not) for each resource type.

The algorithm would iterate through every possible allocation x_{ij}, where i ranges over VMs and j over resources, calculating the total cost $\sum_{i=1}^{n}\sum_{j=1}^{m}(Cij \cdot xij)$. After computing the cost for an allocation, it would check whether the allocation meets all VM demands D_i and does not exceed the available resources R_j. The allocation with the lowest cost that satisfies the constraints is selected as the optimal solution.

```
NAIVE-ALLOCATION-APPROACH(VMs, Resources, R, D)
1. BestCost = ∞
2. BestAllocation = [ ]
3. Generate all possible combinations of resource allocations
4. for each combination do
5.     AllocationCost = CalculateCost(combination, C)
6.     if AllocationCost < BestCost and IsValidAllocation(combination, R, D) then
7.         BestCost = AllocationCost
8.         BestAllocation = combination
9.     end if
10. end for
11. return BestAllocation, BestCost
```

```
CALCULATE-COST(combination, C)
1. TotalCost = 0
2. for i = 1 to length(VMs) do
3.     for j = 1 to length(Resources) do
4.         if combination[i][j] == 1 then
5.             TotalCost += C[i][j]
6.         end if
7.     end for
8. end for
9. return TotalCost
```

```
IS-VALID-ALLOCATION(combination, R, D)
1. for j = 1 to length(Resources) do
2.     ResourceSum = 0
3.     for i = 1 to length(VMs) do
4.         ResourceSum += combination[i][j]
5.     end for
6.     if ResourceSum > R[j] then
7.         return False
8.     end if
9. end for
10. for i = 1 to length(VMs) do
11.     if ∑(combination[i]) < D[i] then
12.         return False
13.     end if
14. end for
15. return True
```

2.1 Dynamic Programming Approach

While the classical naive approach is straightforward, it becomes intractable for large n and m, as the number of possible allocations grows exponentially. The DP approach avoids redundant calculations by solving each subproblem only once and storing its solution. The complexity is significantly reduced to $\boldsymbol{O(n \cdot m)}$, where n is the number of VMs, m is the number of Resources.

We create a subproblem in DP, defined by a partial allocation of resources to a subset of VMs. This is represented by $F(i,r)$, defined before which is the minimum cost to allocate resources to VMs 1 through i with r resources remaining.

The relation $F(i,r) = min_{d \leq r}\ (C_{id} + F(i+1, r-d))$ is used, where represents the decision to allocate d units of resource to VM_i. We construct an optimal allocation by tracing back the decisions from $F(1,r)$, where r is the requested resources.

DP scales well with the problem size and is capable of handling larger problems(n and m>>100) that are typical in cloud computing scenarios. It uses memoization to store the results of subproblems, avoiding the exponential computation for large n due to its exponential time complexity, ensuring optimality with much lower time complexity.

```
DYNAMIC-ALLOCATION(VMs, Resources, R, D)
1. Initialize DP table, DP[0...length(VMs)][0...R] with ∞
2. DP[0][0] = 0  // Base case: no cost for allocating nothing
3. for i = 1 to length(VMs) do
4.     for r = 0 to R do
5.         for d = 0 to min(r, D[i]) do
6.             for j = 1 to length(Resources) do
7.                 if r >= d then
8.                         DP[i][r] = min(DP[i][r], DP[i-1][r-d] + C[i][j] * d)
9.                 end if
10.            end for
11.        end for
12.    end for
13. end for
14. Construct the solution from DP table
15. return DP[length(VMs)][R], RECONSTRUCTION(DP, length(VMs), R)
```

```
RECONSTRUCTION(DP, n, R)
1. Allocation = Array of length(n) filled with 0
2. remainingR = R
3. for i = n down to 1 do
4.     for d = remainingR down to 0 do
5.         if DP[i][remainingR] == DP[i-1][remainingR-d] + C[i][j] * d then
6.             Allocation[i] = d
7.             remainingR - = d
8.             break
9.         end if
10.    end for
11. end for
12. return Allocation
```

2.1.1 *Optimal Substructure*

The optimal substructure in the context of the dynamic programming approach to cloud resource allocation relies on the recurrence relation $F(i,r) = min_{d \leq r} \ (C_{id} + F(i+1, r-d))$. This relation defines the cost of allocating resources to a set of VMs as the minimum of either allocating a certain number of resources d to the current VM_i, plus the cost of optimally allocating the remaining resources r-d to the subsequent VMs $i+1$ through n, or not allocating those resources and proceeding to find the optimal allocation for the remaining VMs and resources. C_{id} represents the cost associated with allocating d units of resources to VM_i, $F(i+1,r-d)$ represents the minimum cost of allocating the remaining resources after VM_i has been considered, and r is the total remaining resources. Essentially, if you allocate resources optimally to the first i VMs with r resources, this solution incorporates the optimal allocation to the first $i-1$ VMs with r' resources, where r' is the remaining resources after allocating to the i^{th} VM. This recursive relationship allows for building up the solution piece by piece, ensuring each step is based on the best possible decisions made in the previous steps. Each subproblem solution is optimal in its own right and contributes to the overall optimal solution.

2.1.2 *Overlapping Subproblems*

Specifically, in resource allocation, an overlapping subproblem occurs when we need to calculate the optimal allocation of a set number of resources to a set number of VMs more than once.
For example, let's consider the following scenario:

- $VM_1, VM_2, VM_3 \ldots .VM_n$: Virtual Machines that need resources.
- Resources are quantified in units (e.g., CPU hours, GB of RAM).
- We want to allocate a total of R units of resources optimally across these VMs.

In a naive recursive solution, the subproblem of allocating R units of resources to VMs 1 through i might be solved independently many times as part of different larger problems (like allocating resources to VMs 1 through $i+1$, $i+2$, etc.). Each time we consider a different VM, we may need to reconsider how to allocate resources to all the previous VMs, leading to recalculating the same allocations.

Dynamic Programming solves each subproblem once and stores the result, so the next time that particular allocation needs to be considered, it simply looks up the stored solution instead of recalculating it. This significantly reduces the computational complexity from exponential to polynomial time. The tree structure of these computations would reveal the same nodes (subproblems) appearing at different parts of the tree, representing the overlap.

2.2 Top-Down/Memoized Algorithm

To negate the exponential time complexity and achieve the time complexity mentioned using DP, we come up with the memoized algorithm keeping the recurrence relation that we derived from the optimal substructure and overlapping subproblem.

The time complexity for the top-down approach $\boldsymbol{O}(\mathbf{n} \cdot \boldsymbol{R}^{\boldsymbol{m}})$, where n is the number of VMs, m is the number of Resources and R is the total amount of resources present. We start with a list of VMs and RAM and CPU cost demand for each VM to be allocate, the total CPU and RAM costs that is available. We are interested to find the minimum cost of allocating the demanded resources to each VM in an optimal manner. Using the recurrence relation to calculate the minimum cost for each subproblem, we build and store the results in a memo and call the Allocate function recursively by calculating the minimum cost at each level keeping the demands in check for each VM and then update the memo storage.

```
Algorithm CloudResourceAllocationTopDown
Input:  VMs, a list of Virtual Machines;
        CPU_Costs and RAM_Costs, cost of allocating CPU and RAM to each VM;
        CPU_Resources and RAM_Resources, total available CPU and RAM
Output: Minimum cost of allocating resources to all VMs

1. Initialize memoization table Memo with all entries as undefined
2. Define function Allocate(i, remainingCPU, remainingRAM) recursively:
    a. If (i, remainingCPU, remainingRAM) is in Memo, return Memo[(i, remainingCPU,
       remainingRAM)]
    b. If i > number of VMs, return 0 (base case)
    c. Initialize minCost as infinity
    d. For each possible allocation of CPU and RAM to VM i:
        i. Compute cost = CPU_Costs[i] * allocatedCPU + RAM_Costs[i] * allocatedRAM +
           Allocate(i+1, remainingCPU - allocatedCPU, remainingRAM - allocatedRAM)
        ii. Update minCost = min(minCost, cost), considering the demands and not
            exceeding remaining resources
    e. Update Memo[(i, remainingCPU, remainingRAM)] = minCost
    f. Return minCost
3. Call Allocate(1, TotalCPU_Resources, TotalRAM_Resources) to start the allocation
   process
4. Return the result of Allocate(1, TotalCPU_Resources, TotalRAM_Resources) as the
   minimum cost
```

Fig. 3. Top-down approach to optimise resource allocation

2.3 Bottom-Up Approach

The bottom-up approach iteratively builds the solution from the simplest subproblems (allocating resources to no VMs) towards the original problem (allocating resources across all VMs). The same recurrence relation guides the filling of a DP table but is applied in a non-recursive, iterative manner:

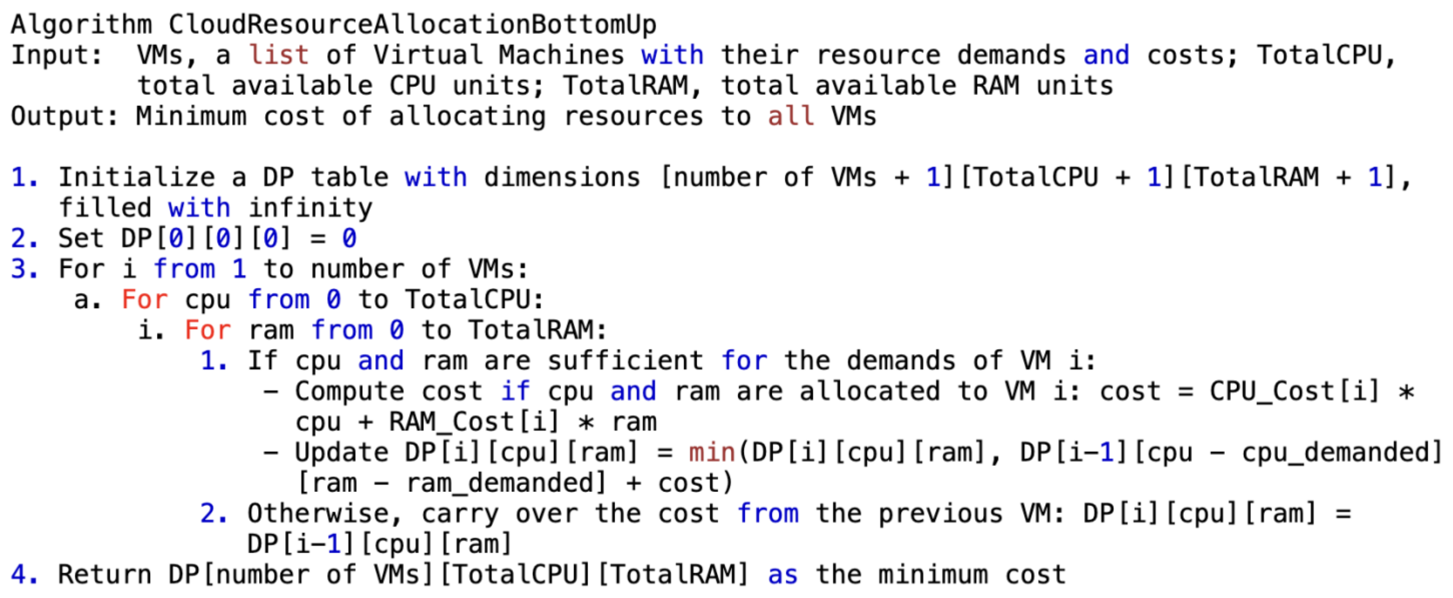
Algorithmic Steps:
Initialization: A three-dimensional dynamic programming (DP) table is initialized, where each entry DP[i][cpu][ram] holds the minimum cost of allocating cpu CPU units and ram RAM units to the first i VMs. Initially, all entries are set to infinity, representing an undefined state, indicating that no minimum cost has been calculated for these allocations.
`	*Base Case:* The base case is set at DP[0][0][0] = 0, indicating that no resources allocated to no VMs incur no cost. This provides a starting point for the algorithm to build upon.
Iterative Computation: The algorithm iterates over each VM_i and systematically considers every possible allocation of CPU and RAM resources within the constraints of the total available resources (TotalCPU and TotalRAM).

For each possible allocation of resources (cpu and ram), the algorithm calculates the cost of allocating these resources to the current VM i, considering both the CPU and RAM cost for that VM. If the allocation meets the demands of VM i (does not exceed the VM's required CPU and RAM), the algorithm updates the DP table with the minimum cost for that allocation.

If the current allocation does not meet the VM's demands, the algorithm carries over the cost from the previous VM to ensure continuity in resource allocation.

Final Solution: The final minimum cost for allocating resources to all VMs is obtained from the entry DP[number of VMs][TotalCPU][TotalRAM] in the DP table, which reflects the culmination of all optimal decisions made throughout the iterative process.

The bottom-up approach also has a time complexity of $\boldsymbol{O}(\mathbf{n}\cdot \boldsymbol{R}^{\boldsymbol{m}})$. It fills in a DP table iteratively, ensuring that each subproblem is calculated based on previously solved subproblems. It often has the same theoretical complexity as the top-down approach but can be more space-efficient and faster in practice because it avoids the overhead of recursive calls.



```
Algorithm CloudResourceAllocationBottomUp
Input:  VMs, a list of Virtual Machines with their resource demands and costs; TotalCPU,
        total available CPU units; TotalRAM, total available RAM units
Output: Minimum cost of allocating resources to all VMs

1. Initialize a DP table with dimensions [number of VMs + 1][TotalCPU + 1][TotalRAM + 1],
   filled with infinity
2. Set DP[0][0][0] = 0
3. For i from 1 to number of VMs:
    a. For cpu from 0 to TotalCPU:
        i. For ram from 0 to TotalRAM:
            1. If cpu and ram are sufficient for the demands of VM i:
                - Compute cost if cpu and ram are allocated to VM i: cost = CPU_Cost[i] *
                  cpu + RAM_Cost[i] * ram
                - Update DP[i][cpu][ram] = min(DP[i][cpu][ram], DP[i-1][cpu - cpu_demanded]
                  [ram - ram_demanded] + cost)
            2. Otherwise, carry over the cost from the previous VM: DP[i][cpu][ram] =
               DP[i-1][cpu][ram]
4. Return DP[number of VMs][TotalCPU][TotalRAM] as the minimum cost
```

Fig. 3. Top-down approach to optimise resource allocation

3. Conclusion

This study has presented a comprehensive examination of the allocation of computational resources in cloud environments using dynamic programming techniques. Through the implementation of both top-down with memoization and bottom-up approaches, we demonstrated efficient strategies for minimizing the cost associated with distributing CPU and RAM resources among virtual machines. The bottom-up dynamic programming method, in particular, offers an effective solution, systematically constructing the minimum-cost allocation from smaller subproblems and avoiding the redundancies inherent in naive recursive methods.

Our findings suggest that dynamic programming is not only theoretically robust but also practically applicable to real-world cloud resource management problems. While the naive recursive approach quickly becomes infeasible as the number of virtual machines increases, dynamic programming maintains a polynomial time complexity, rendering it a scalable solution even for large cloud infrastructures.

References

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