Topic: Completing the square

Question: Complete the square to solve the quadratic equation.

$$x^2 + 4x + 2 = 0$$

Answer choices:

A $x = -2 \pm \sqrt{2}$

B $x = 2 \pm \sqrt{2}$

C $x = -2 \pm \sqrt{3}$

D $x = 2 \pm \sqrt{3}$

Solution: A

The quadratic is given in standard form $ax^2 + bx + c$ with $a = 1$, so we can complete the square. We'll start by finding $(b/2)^2$,

$$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$$

then we'll add $(b/2)^2$ to both sides of the equation.

$$x^2 + 4x + 4 + 2 = 0 + 4$$

$$(x^2 + 4x + 4) + 2 = 4$$

$$x^2 + 4x + 4 = 2$$

The quadratic on the left factors as the perfect square $(x + 2)^2$.

$$(x + 2)(x + 2) = 2$$

$$(x + 2)^2 = 2$$

$$x + 2 = \pm\sqrt{2}$$

$$x = -2 \pm \sqrt{2}$$

The roots of the equation are therefore $x = -2 - \sqrt{2}$ and $x = -2 + \sqrt{2}$.

Topic: Completing the square

Question: Complete the square to find the roots of the quadratic.

$$u^2 - 4u + 3 = 0$$

Answer choices:

A $u = -1, \ -3$

B $u = 1, \ -3$

C $u = 1, 3$

D $u = -1, 3$

Solution: C

The quadratic is given in standard form $au^2 + bu + c$ with $a = 1$, so we can complete the square. We'll start by finding $(b/2)^2$,

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

then we'll add $(b/2)^2$ to both sides of the equation.

$$u^2 - 4u + 4 + 3 = 0 + 4$$

$$(u^2 - 4u + 4) + 3 = 4$$

$$u^2 - 4u + 4 = 1$$

The quadratic on the left factors as the perfect square $(u - 2)^2$.

$$(u - 2)(u - 2) = 1$$

$$(u - 2)^2 = 1$$

$$u - 2 = \pm\sqrt{1}$$

$$u = 2 \pm \sqrt{1}$$

$$u = 2 \pm 1$$

The roots of the equation are therefore $u = 2 - 1 = 1$ and $u = 2 + 1 = 3$.

Topic: Completing the square

Question: Solve the quadratic equation by completing the square.

$$x^2 - 2x + 9 = 0$$

Answer choices:

A $x = 1 \pm \sqrt{2}i$

B $x = 1 \pm 2\sqrt{2}i$

C $x = 2 \pm \sqrt{2}i$

D $x = 2 \pm 2\sqrt{2}i$

Solution: B

The quadratic is given in standard form $ax^2 + bx + c$ with $a = 1$, so we can complete the square. We'll start by finding $(b/2)^2$,

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

then we'll add $(b/2)^2$ to both sides of the equation.

$$x^2 - 2x + 1 + 9 = 0 + 1$$

$$(x^2 - 2x + 1) + 9 = 1$$

$$x^2 - 2x + 1 = -8$$

The quadratic on the left factors as the perfect square $(x - 1)^2$.

$$(x - 1)(x - 1) = -8$$

$$(x - 1)^2 = -8$$

$$x - 1 = \pm\sqrt{-8}$$

$$x = 1 \pm \sqrt{-8}$$

Use the imaginary number to rewrite the solutions.

$$x = 1 \pm \sqrt{8(-1)}$$

$$x = 1 \pm \sqrt{8}\sqrt{-1}$$

$$x = 1 \pm 2\sqrt{2}\sqrt{-1}$$

$$x = 1 \pm 2\sqrt{2}i$$

The roots of the equation are therefore $x = 1 - 2\sqrt{2}i$ and $x = 1 + 2\sqrt{2}i$.