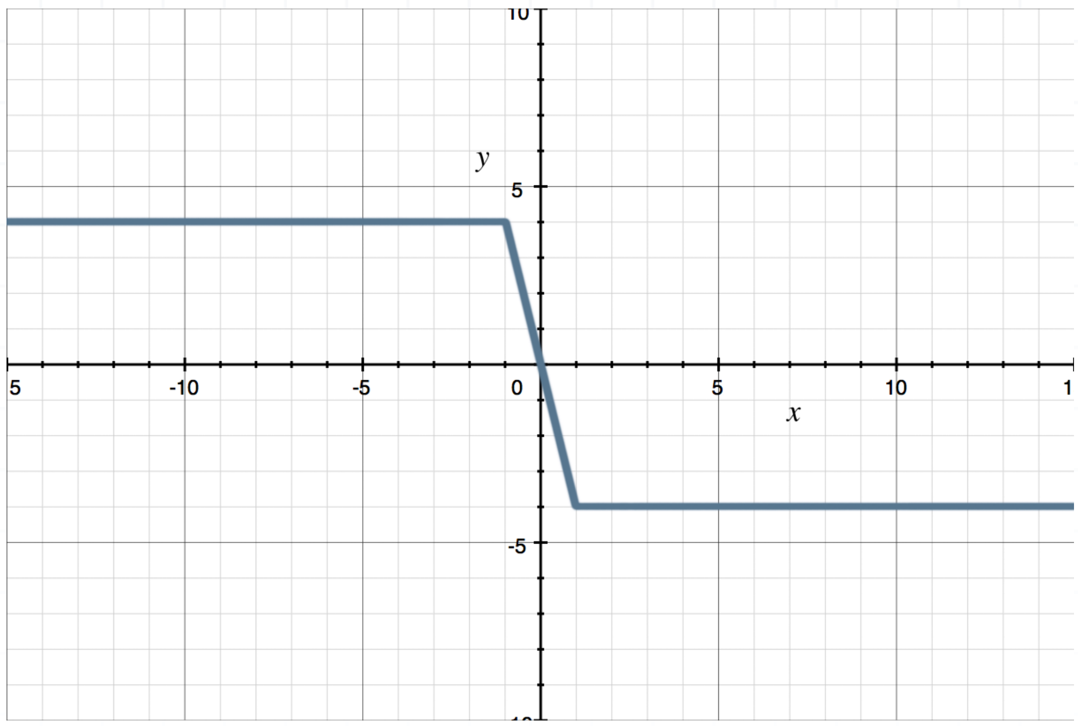
Topic: Modeling a piecewise-defined function

Question: What is the definition of the piecewise function shown in the graph?



Answer choices:

A $$f(x) = \begin{cases} -4 & \text{if} \quad x \leq -1 \\ -4x & \text{if} \quad -1 < x \leq 1 \\ 4 & \text{if} \quad x > 1 \end{cases}$$

B $$f(x) = \begin{cases} 4 & \text{if} \quad x \leq -1 \\ 4x & \text{if} \quad 1 < x \leq -1 \\ -4 & \text{if} \quad x > 1 \end{cases}$$

C $$f(x) = \begin{cases} 4 & \text{if} \quad x \leq -1 \\ -4x & \text{if} \quad -1 < x \leq 1 \\ -4 & \text{if} \quad x > 1 \end{cases}$$

D $$f(x) = \begin{cases} 4 & \text{if} \quad x < -1 \\ -4x & \text{if} \quad -1 < x \leq 1 \\ -4 & \text{if} \quad x > 1 \end{cases}$$

Solution: C

Going from left to right, the first part of the graph is part of the line $y = 4$ and it goes from the left, to $x = -1$. For this piece, we write 4 for the function and $x \le -1$ for its domain.

$$f(x) = \begin{cases} 4 & \text{if} \quad x \le -1 \\ & \text{if} \quad -1 < x \le 1 \\ & \text{if} \quad x > 1 \end{cases}$$

The second part of the graph is part of the line that has a slope of -4 and a y-intercept of 0, so the equation of this line is $y = -4x$. Remember: the slope-intercept of the equation of a line is $y = mx + b$.) To see how to get the slope, notice that $(-1,4)$ and $(0,0)$ are points on this line, so its slope is

$$m = \frac{0-4}{0-(-1)} = \frac{-4}{1} = -4$$

This part of the graph goes from $x = -1$ to $x = 1$. So for the second piece, we write $-4x$ for the function and $-1 < x \le 1$ for its domain. We can't include $x = -1$ in the domain of this piece, because we included $x = -1$ in the domain of the first piece.

$$f(x) = \begin{cases} 4 & \text{if} \quad x \le -1 \\ -4x & \text{if} \quad -1 < x \le 1 \\ & \text{if} \quad x > 1 \end{cases}$$

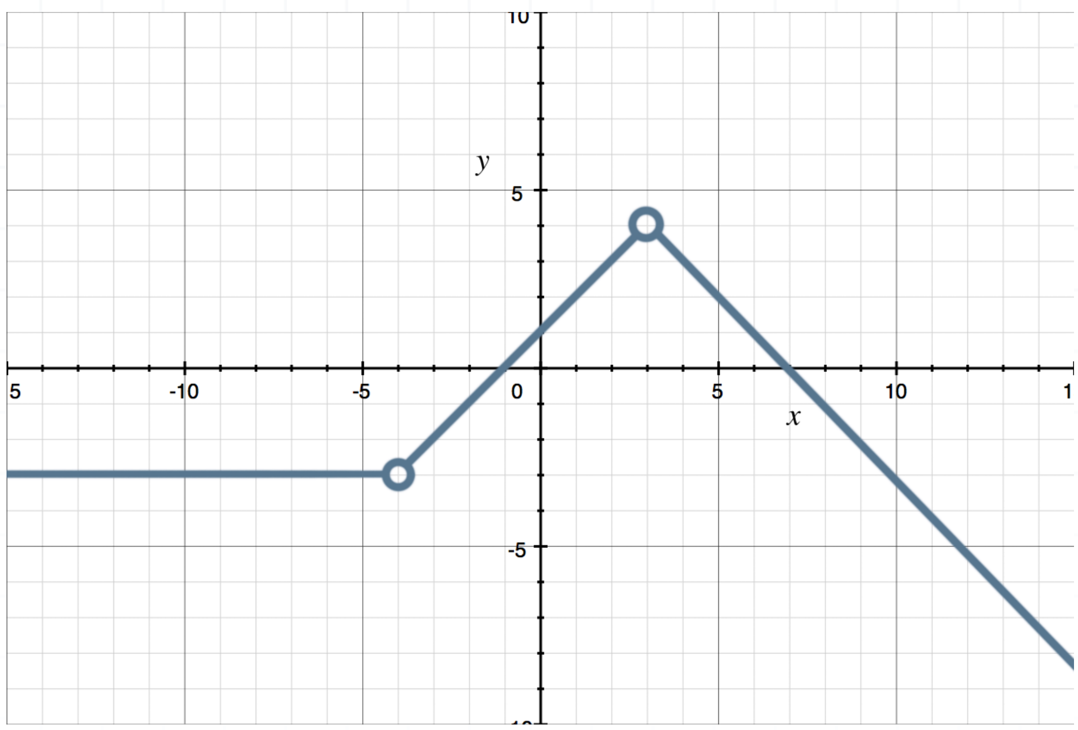
The third part of the graph is part of the line $y = -4$, and it goes from $x = 1$ to the right. For this piece, we write -4 for the function and $x > 1$ for its domain. We can't include $x = 1$ in the domain of this piece, because we included $x = 1$ in the domain of the second piece.

When we put the pieces together, we get

$$f(x) = \begin{cases} 4 & \text{if} \quad x \le -1 \\ -4x & \text{if} \quad -1 < x \le 1 \\ -4 & \text{if} \quad x > 1 \end{cases}$$

Topic: Modeling a piecewise-defined function

Question: What is the definition of the piecewise function shown in the graph?



Answer choices:

A $f(x) = \begin{cases} -4 & \text{if} \quad x < -3 \\ x+1 & \text{if} \quad -4 < x < 3 \\ -x+7 & \text{if} \quad x > 3 \end{cases}$

B $f(x) = \begin{cases} -3 & \text{if} \quad x < -4 \\ x+1 & \text{if} \quad -4 < x < 3 \\ -x+7 & \text{if} \quad x > 3 \end{cases}$

C $f(x) = \begin{cases} -3 & \text{if} \quad x \le -4 \\ x+1 & \text{if} \quad -4 < x < 3 \\ -x+7 & \text{if} \quad x \ge 3 \end{cases}$

D $f(x) = \begin{cases} -3 & \text{if} \quad x < -4 \\ x-1 & \text{if} \quad -4 < x < 3 \\ x+7 & \text{if} \quad x > 3 \end{cases}$

Solution: B

Going from left to right, the first part of the graph is part of the line $y = -3$, and it includes all values of x in the interval $x < -4$ (but not $x = -4$, because there's an open circle on the graph at $x = -4$). For this piece, we write -3 for the function and $x < -4$ for its domain.

$$f(x) = \begin{cases} -3 & \text{if} \quad x < -4 \\ & \text{if} \quad -4 < x < 3 \\ & \text{if} \quad x > 3 \end{cases}$$

The second part of the graph is (part of) the line that has a slope of 1, so the equation of this line is $y = x + 1$. To see how to get the slope, notice that $(-4, -3)$ and $(0,1)$ are points on this line, so its slope is

$$m = \frac{1 - (-3)}{0 - (-4)} = \frac{4}{4} = 1$$

This piece goes from $x = -4$ to $x = 3$, but neither -4 nor 3 is in its domain (or in the domain of this entire piecewise function), because there's an open circle at each of those two values of x on the graph. So for this piece, we write $x + 1$ for the function and $-4 < x < 3$ for its domain.

$$f(x) = \begin{cases} -3 & \text{if} \quad x < -4 \\ x + 1 & \text{if} \quad -4 < x < 3 \\ & \text{if} \quad x > 3 \end{cases}$$

The graph of the third part is part of the line that has a slope of -1 and a y-intercept of 7, so the equation of this line is $y = -x + 7$. To see this, we'll first compute the slope from the points $(3,4)$ and $(5,2)$, both of which are on this line. Then we'll use the slope and the point $(3,4)$ to get the point-slope form of the equation of the line (and then use that to get the slope-intercept form). The slope is

$$m = \frac{2-4}{5-3} = \frac{-2}{2} = -1$$

Combining the three pieces, we get this function:

$$f(x) = \begin{cases} -3 & \text{if} \quad x < -4 \\ x+1 & \text{if} \quad -4 < x < 3 \\ -x+7 & \text{if} \quad x > 3 \end{cases}$$

Topic: Modeling a piecewise-defined function

Question: For the given function, evaluate $f(-4)+f(8)+f(3)$.

$$f(x)=\begin{cases} -\frac{1}{2}x-3 & \text{if} \quad -6 \le x \le 2 \\ -4 & \text{if} \quad 2 < x \le 7 \\ 3x-25 & \text{if} \quad 7 < x \le 9 \end{cases}$$

Answer choices:

A $\quad -10$

B $\quad -6$

C $\quad -2$

D $\quad 2$

Solution: B

First, evaluate $f(-4)$. Notice that -4 is in the interval $-6 \leq x \leq 2$, so we use the function for the first piece.

$$f(x) = -\frac{1}{2}x - 3$$

$$f(-4) = -\frac{1}{2}(-4) - 3 = 2 - 3 = -1$$

Next, evaluate $f(8)$. Notice that 8 is in the interval $7 < x \leq 9$, so we use the function for the third piece.

$$f(x) = 3x - 25$$

$$f(8) = 3(8) - 25 = 24 - 25 = -1$$

Now, evaluate $f(3)$. Notice that 3 is in the interval $2 < x \leq 7$, so we use the function for the second piece.

$$f(x) = -4$$

$$f(3) = -4$$

Finally, compute the sum of the three values.

$$f(-4) + f(8) + f(3) = -1 + (-1) + (-4) = -6$$