Negative exponents

This lesson will cover how to find the value of a positive number raised to a negative power.

Remember that any number can be written as itself divided by 1. For example, 3 is equal to $3/1$. Also, remember that any number can be written as itself multiplied by 1, and that any nonzero number divided by itself is equal to 1.

The rule for negative exponents

If we have two positive real numbers a and b, then

$$a^{-b} = \frac{1}{a^b}$$

First we need to realize that a^{-b} is equal to

$$\frac{a^{-b}}{1}$$

We'll change the exponent in a^{-b} from $-b$ to b, and move the resulting expression from the numerator to the denominator, by performing a series of algebraic operations that we're already familiar with. First, we'll multiply $(a^{-b})/1$ by 1, which doesn't change its value.

$$\frac{a^{-b}}{1} \cdot 1$$

Next, we'll write that new factor of 1 as $(a^b)/(a^b)$.

$$\left(\frac{a^{-b}}{1}\right)\left(\frac{a^b}{a^b}\right)$$

Now we'll multiply the fractions, remembering to multiply their numerators and denominators separately.

$$\frac{(a^{-b})(a^b)}{(1)(a^b)}$$

Using the fact that $(1)(a^b) = a^b$, we get

$$\frac{(a^{-b})(a^b)}{a^b}$$

Remember that when we multiply numbers that have like bases (as in our numerator, where the base of each factor is a), we keep the base and add the exponents.

$$\frac{a^{(-b+b)}}{a^b}$$

By addition of the exponents ($-b$ and b), we get

$$\frac{a^0}{a^b}$$

Since a is nonzero, we have $a^0 = 1$.

$$\frac{1}{a^b}$$

Let's look at a few examples.

Example

Simplify the expression.

$$4^{-2}$$

Remember that 4^{-2} is equal to

$$\frac{4^{-2}}{1}$$

We'll change the exponent in 4^{-2} from -2 to 2 and move the resulting expression from the numerator to the denominator, which gives,

$$\frac{1}{4^2}$$

Now we'll perform the calculation in the denominator.

$$\frac{1}{4^2} = \frac{1}{4 \cdot 4} = \frac{1}{16}$$

Let's look at an example with a negative sign in front of the base.

Example

Simplify the expression.

$$-5^{-3}$$

Remember, we can rewrite -5^{-3} as

$$\frac{-5^{-3}}{1}$$

because they are of equal value.

We'll change the exponent in 5^{-3} from -3 to 3 and move the resulting expression (including the negative sign out in front) from the numerator to the denominator.

$$\frac{1}{-5^{3}}$$

We have to apply the exponent before we apply the negative sign, so the expression becomes

$$\frac{1}{-125}$$

$$-\frac{1}{125}$$

Let's also take a look at an example with a variable as the base.

Example

Write the expression with with no negative exponents.

$$x^{-3}$$

First, we need to realize that the expression x^{-3} is equal to

$$\frac{x^{-3}}{1}$$

We'll change the exponent in x^{-3} from -3 to 3 and move the resulting expression from the numerator to the denominator.

$$\frac{1}{x^3}$$

Of course, this is defined only if $x \neq 0$.