Graphing exponential functions

We want to be able to graph **exponential functions**, which are functions in which the variable is in the exponent. These are all exponential functions:

$$f(x) = 3^x \qquad f(x) = 2\left(\frac{1}{3}\right)^{-x+2} - 4$$

$$f(x) = -3^{x-1} - 2 \qquad f(x) = 6 \cdot 2^{-x} + 1$$

Characteristics of the graph

To learn how to graph exponential functions, let's start by graphing $f(x) = ab^x$, where a is a nonzero number and b is a positive real number not equal to 1.

The coefficient a is the **initial value**, because $f = a$ when $x = 0$, which means the y-intercept of an exponential function $f(x) = ab^x$ is the point $(x, y) = (0,a)$.

The base b is the **growth factor**.

- When $b > 1$, the function grows at rate proportional to its size.
- When $0 < b < 1$, the function decays at a rate proportional to its size.

The base has to be positive in order to ensure that the function will have a real-number output. For instance, if $b = -16$ and $x = 1/2$, then $f(x) = ab^x$ becomes

$$f\left(\frac{1}{2}\right) = (-16)^{\frac{1}{2}} = \sqrt{-16}$$

and we can't take the square root of a negative number. The base also can't be equal to 1 because $f(x) = a(1^x)$ gives $f(x) = a$ for all values of x, and then the function is no longer an exponential, it's a constant function.

The domain of the exponential function is $(-\infty, \infty)$, and the range is all positive real numbers $(0,\infty)$ if $a > 0$ and all negative real numbers $(-\infty,0)$ if $a < 0$. The exponential function always has a horizontal asymptote at $y = 0$.

Let's do an example so that we can see how to graph a simple exponential function.

Example

Graph the exponential function.

$$f(x) = 2^x$$

If we rewrite the function as $f(x) = 1(2^x)$, then we can identify the initial value $a = 1$ and the growth factor $b = 2$.

The initial value tells us that the function passes through $(0,1)$. And since $b > 1$, we know the function is increasing above the horizontal asymptote at $y = 0$.

We'll plug in a couple more values of x for which the value of $f(x)$ will be easy to calculate.

For $x = -1$, $f(-1) = 2^{-1} = 1/2$

For $x = 1$, $f(1) = 2^1 = 2$

Now we have three points on the graph of f: the y-intercept $(0,1)$, and $(-1,1/2)$ and $(1,2)$. If we plot these points and connect them with a smooth curve, we get