Solving systems with elimination

We know how to solve systems using substitution, but now we want to look at a second method: elimination.

Solving with elimination

As the name suggests, the goal of the elimination method is to eliminate one of the variables from the system by adding or subtracting the equations. With one variable eliminated, we'll be left with just one equation in terms of one variable.

We can solve that equation for the value of that variable, and then use the value we found to go back and find the value of the other variable.

Here are the steps we'll follow when we use the **elimination method** to solve a system of two linear equations:

1. If necessary, rearrange both equations so that the x-terms are first, followed by the y-terms, the equals sign, and the constant term (in that order). If an equation appears to have no constant term, that means that the constant term is 0.

2. Multiply one (or both) equations by a constant that will allow either the x-terms or the y-terms to cancel when the equations are added or subtracted.

3. Add or subtract the equations to eliminate one of the variables.

4. Solve for the remaining variable.

5. Plug the result of step 4 into one of the original equations, then solve for the other variable.

Let's look at an example so that we can see how these steps work.

Example

Find the solution to the system of equations.

$$3x + 4y = 12$$

$$-3x + 2y = 18$$

Steps 1 and 2 are done, since the individual parts of each equation are in the correct places, and the x-terms ($3x$ in the first equation and $-3x$ in the second equation) will cancel when we add the equations. So we'll skip to step 3 and add the equations.

$$3x + 4y = 12 \qquad + \qquad -3x + 2y = 18$$

$$3x + 4y + (-3x + 2y) = 12 + (18)$$

$$3x + 4y - 3x + 2y = 12 + 18$$

$$3x - 3x + 4y + 2y = 30$$

$$0 + 6y = 30$$

$$6y = 30$$

$$y = 5$$

Now, we'll substitute $y = 5$ into the first equation and solve for x.

$$3x + 4y = 12$$

$$3x + 4(5) = 12$$

$$3x + 20 = 12$$

Subtract 20 from both sides.

$$3x + 20 - 20 = 12 - 20$$

$$3x = -8$$

$$x = -\frac{8}{3}$$

The solution to the system is $(-8/3, 5)$.

Let's try another example of solving with elimination.

Example

Find the unique solution to the system of equations.

$$y = 3x - 4$$

$$-x + 2y = 12$$

First, we'll rearrange the first equation so that its individual parts are in the correct places for elimination. Subtract $3x$ from both sides.

$$y = 3x - 4$$

$$-3x + y = 3x - 3x - 4$$

$$-3x + y = -4$$

Now our system is

$$-3x + y = -4$$

$$-x + 2y = 12$$

Next, multiply this new first equation by 2,

$$2(-3x + y) = 2(-4)$$

$$-6x + 2y = -8$$

such that the new system becomes

$$-6x + 2y = -8$$

$$-x + 2y = 12$$

We multiplied the first equation by 2 so that the y-terms will cancel when we subtract the equations.

$$-6x + 2y = -8 \qquad - \qquad -x + 2y = 12$$

$$-6x + 2y - (-x + 2y) = -8 - (12)$$

$$-6x + 2y + x - 2y = -8 - 12$$

$$-6x + x + 2y - 2y = -20$$

$$-5x + 0 = -20$$

$$-5x = -20$$

$$x = 4$$

To solve for y, we'll substitute $x = 4$ into the original first equation.

$$y = 3x - 4$$

$$y = 3(4) - 4$$

$$y = 12 - 4$$

$$y = 8$$

The solution to the system is $(4,8)$.