**Topic**: Domains of composite functions

**Question**: What is the domain of $f \circ g$?

$$f(x) = x^2 - 5$$

$$g(x) = \sqrt{x + 4}$$

**Answer choices**:

A $x \leq -4$

B $x > -4$

C $x \geq -4$

D $x < -4$

**Solution**: C

First, find the domain of $g(x)$. The expression $\sqrt{x+4}$ is undefined if the radicand is negative. For example, if $x = -5$, then $x+4$ is $-1$. In general, if $x$ is any number less than $-4$, then $x+4$ is negative. However, $-4$ itself is okay, because $\sqrt{-4+4=0}$.

Therefore, the domain of $g(x)$ is all reals $x$ such that $x \geq -4$.

The algebraic expression for the composite function is

$$f(g(x)) = \left(\sqrt{x+4}\right)^2 - 5$$

$$f(g(x)) = (x+4) - 5$$

$$f(g(x)) = x - 1$$

For this simple binomial ($x-1$), no real numbers are excluded, so its domain is all reals. But because the domain of $g(x)$ excludes all $x < -4$ , those values of $x$ also have to be excluded from the domain of the composite function $f(g(x))$.

That means the domain of $f(g(x))$ is $x \geq -4$.

**Topic**: Domains of composite functions

**Question**: What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x+3}$$

$$g(x) = \frac{x}{x-2}$$

**Answer choices**:

A $\quad x \neq 2, \ -3$

B $\quad x \neq \frac{3}{2}, 2$

C $\quad x \neq -2, 3$

D $\quad x \neq -\frac{3}{2}, 2$

**Solution**: B

First, find the domain of $g(x)$. The expression $x/(x-2)$ is undefined if the denominator is $0$. That means $x=2$ isn't in the domain of $g(x)$. Therefore, the domain of $g(x)$ is all reals $x$ such that $x \neq 2$.

The algebraic expression for the composite function is

$$f(g(x)) = \frac{1}{\left(\frac{x}{x-2}\right) + 3}$$

$$f(g(x)) = \frac{1}{\left(\frac{x}{x-2}\right) + 3\left(\frac{x-2}{x-2}\right)}$$

$$f(g(x)) = \frac{1}{\left(\frac{x+3x-6}{x-2}\right)}$$

$$f(g(x)) = \frac{1}{\left(\frac{4x-6}{x-2}\right)}$$

$$f(g(x)) = \frac{x-2}{4x-6}$$

$$f(g(x)) = \frac{x-2}{2(2x-3)}$$

For this rational function ($(x-2)/[2(2x-3)]$), any numbers that make the denominator $0$ are excluded from the domain.

$$2(2x-3) = 0 \quad \rightarrow \quad 2x - 3 = 0 \quad \rightarrow \quad 2x = 3 \quad \rightarrow \quad x = \frac{3}{2}$$

Putting both exclusions together, the domain of the composite is all real numbers except $3/2$ and $2$, so

$$f(g(x)) = \frac{x-2}{2(2x-3)}, x \neq \frac{3}{2}, 2$$

**Topic**: Domains of composite functions

**Question**: What is the domain of $f \circ g$?

$$f(x) = \sqrt{x - 1}$$

$$g(x) = \frac{1}{x - 1}$$

**Answer choices**:

A $1 < x \leq 2$

B $1 \leq x \leq 2$

C $1 < x < 2$

D $1 \leq x < 2$

**Solution**: A

First, find the domain of $g(x)$. The expression $1/(x-1)$ is undefined if the denominator is $0$. That means $x=1$ isn't in the domain of $g(x)$. Therefore, the domain of $g(x)$ is all reals $x$ such that $x \neq 1$.

The algebraic expression for the composite function is

$$f(g(x)) = \sqrt{\frac{1}{x-1} - 1}$$

$$f(g(x)) = \sqrt{\frac{1-(x-1)}{x-1}}$$

$$f(g(x)) = \sqrt{\frac{2-x}{x-1}}$$

For the rational function under the radical sign ($(2-x)/(x-1)$), any numbers that make the denominator $0$ are excluded from the domain.

$$x-1=0 \quad \to \quad x=1$$

And any time that rational function is negative, the values of $x$ that make it negative will be excluded from the domain. A rational function is negative when either the numerator is negative and the denominator is positive, or vice versa. The numerator is negative when $2-x<0$.

$$2-x<0 \quad \to \quad -x<-2 \quad \to \quad x>2$$

The denominator is positive when $x-1>0$.

$$x-1>0 \quad \to \quad x>1$$

The values of $x$ where the numerator is negative and the denominator is positive are the values of $x$ such that $x > 2$ and $x > 1$. Notice that $x > 2$ and $x > 1$ if and only if $x > 2$.

The denominator is negative when $x - 1 < 0$.

$$x - 1 < 0 \quad \to \quad x < 1$$

The numerator is positive when $2 - x > 0$.

$$2 - x > 0 \quad \to \quad -x > -2 \quad \to \quad x < 2$$

The values of $x$ where the denominator is negative and the numerator is positive are the values of $x$ such that $x < 1$ and $x < 2$. Notice that $x < 1$ and $x < 2$ if and only if $x < 1$.

Therefore, the radicand is negative on the intervals $x > 2$ and $x < 1$, so the real numbers $x$ in those intervals are excluded from the domain of this composite function.

We found earlier that $x = 1$ is excluded from the domain of this composite function, so its domain is the set of all real numbers $x$ such that

$$1 < x \leq 2$$