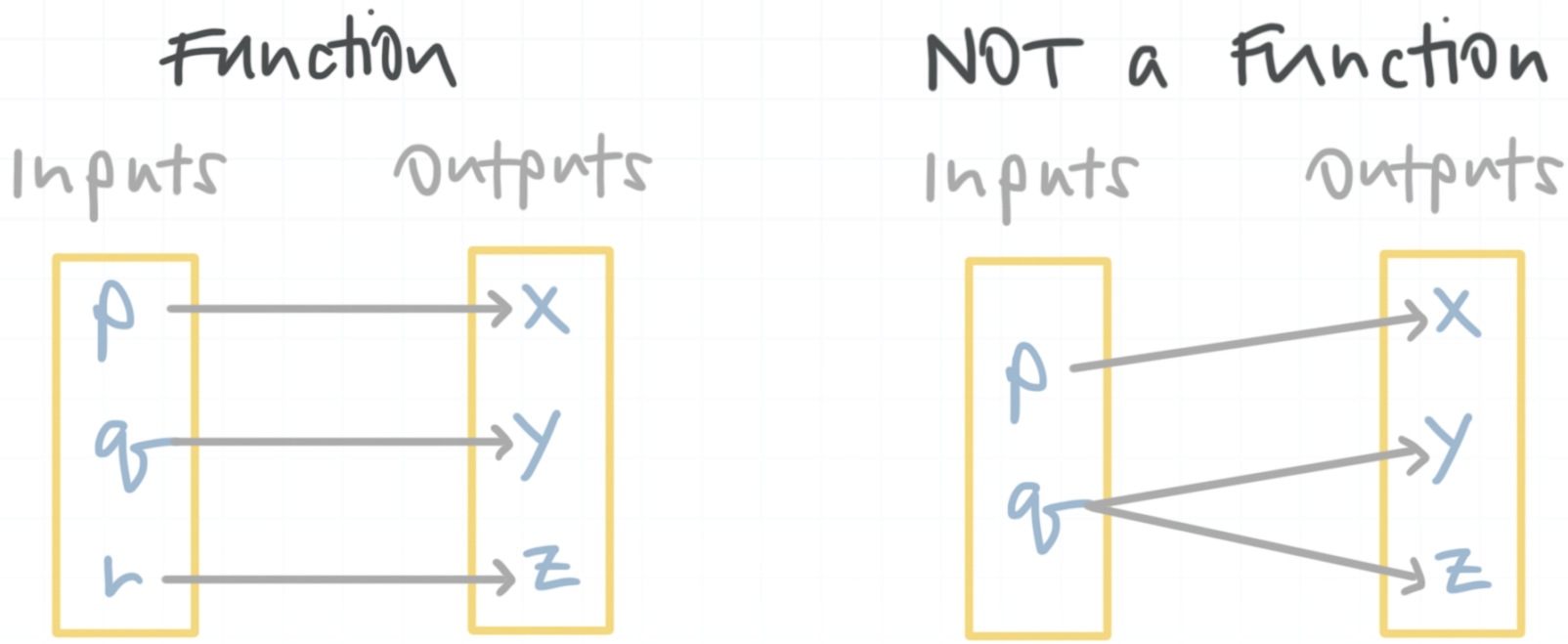
Testing for functions

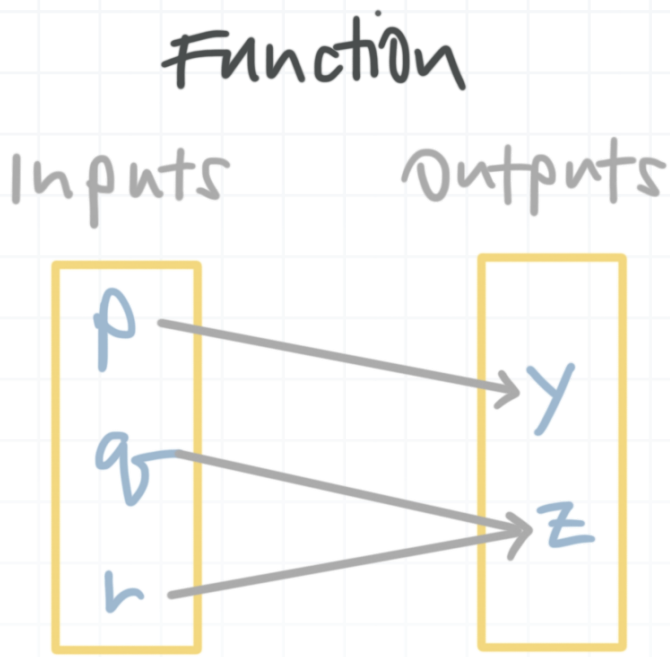
We already know that an equation is a function if every input is associated with only one output.

If we create a visual representation of this idea, it might look like this:



The image on the left could define a function because each of the inputs connects to only one output. In contrast, the image on the right is definitely not a function because there's an input that produces more than one output.

Keep in mind that there's a third scenario, which we can illustrate as



This scenario is still a function, because each input is still only associated with one output. The fact that two different inputs happen to have the same output value doesn't matter, because the one-input-one-output relationship is still preserved, so the image still represents s function.

That being said, if we're only given a set of coordinate points that represent the relation, we can still test that point set to determine whether or not it represents s function. We just need to make sure that there's only one y-value for every x-value.

Let's do an example.

Example

Which point set, A or B, represents a function?

A: $(1,2)$, $(2,4)$, $(2,3)$

B: $(1,-3)$, $(2,-4)$, $(3,-5)$

Set B represents a function because each input has only one output. Set A doesn't represent a function because the input 2 has both an output of 3 and an output of 4, which means there are multiple outputs associated with the single input.

We can also try to determine algebraically whether an equation represents a function. Let's do an example with an equation.

Example

Determine algebraically whether the equation represents a function.

$$x^2 + y^2 = 1$$

If we can show that there are multiple outputs y associated with a single input x, then we'll prove that the equation doesn't represent a function.

Let's see what's happening with the equation at $x = 0$ by plugging in $x = 0$.

$$(0)^2 + y^2 = 1$$

$$y^2 = 1$$

$$y = \pm 1$$

From this result, we can conclude that, at $x = 0$, y can be both 1 and -1. Since a function can have only one output y for any input x, this equation can't represent a function.