Quadratic polynomials

Remember that the degree of a polynomial is given by its largest exponent. So, for example, if the largest exponent in the polynomial is 3, then the polynomial is a third-degree polynomial.

Quadratic polynomials, or **quadratics**, are second-degree polynomials. Our goal now is to learn to factor quadratics. In the same way that we said factoring out the greatest common factor was the inverse process to the Distributive Property, factoring a quadratic is the inverse process of "FOILing" two binomials.

In other words, we'll be starting with the quadratic and reversing the FOIL process to find the two binomial factors that were originally multiplied to get to the quadratic.

To factor a quadratic given in standard form, $ax^2 + bx + c$, where $a = 1$ and $c \neq 0$, we need to look for a pair of factors of c that multiply to c and sum to b.

Let's look at an example.

Example

Factor the quadratic polynomial.

$$x^2 - x - 20$$

This quadratic is given in standard form ax^2+bx+c, with $a=1$ and $c=-20\neq 0$. Which means we can try to factor it.

We'll start by listing all pairs of factors of the constant term, -20, and their sums. We're looking for the pair of factors that have a sum of $b=-1$.

Factors of -20	Sum
-1 and 20	19
1 and -20	-19
-2 and 10	8
2 and -10	-8
-4 and 5	1
4 and -5	-1

Since 4 and -5 are the only factor pair with sum of -1, they must be the factors we need. If the factors are x_1 and x_2, then the quadratic will always factor as $(x+x_1)(x+x_2)$. So the quadratic x^2-x-20 factor as

$$(x+4)(x-5)$$

To check that we factored the quadratic correctly, we'll use the Distributive Property to multiply the binomials $(x+4)(x-5)$ to get

$$(x)(x)+(x)(-5)+(4)(x)+(4)(-5)$$

$$x^2-5x+4x-20$$

$$x^2-x-20$$

Let's try another example of factoring a quadratic polynomial.

Example

Factor the quadratic as the product of two binomials.

$$x^2 - 8x + 15$$

This quadratic is given in standard form $ax^2 + bx + c$, with $a = 1$ and $c = 15 \neq 0$. Which means we can try to factor it.

We'll start by listing all pairs of factors of the constant term, 15, and their sums. We're looking for the pair of factors that have a sum of $b = -8$.

Factors of 15	Sum
1 and 15	16
-1 and -15	-16
3 and 5	8
-3 and -5	-8

The factors -3 and -5 have a sum of -8, so they're the correct factors, and the quadratic must factor as

$$(x-3)(x-5)$$

To check the factoring answer, we'll FOIL the product of the binomials.

$$(x)(x) + (x)(-5) + (-3)(x) + (-3)(-5)$$

$$x^2 - 5x - 3x + 15$$

$$x^2 - 8x + 15$$

If the coefficient a on the x^2 term in a quadratic is either -1 or the greatest common factor of the polynomial, we can first factor that out and then factor the remaining quadratic.

Example

Factor the quadratic polynomial.

$$4x^2 - 20x + 24$$

The greatest common factor of this polynomial is 4, so we first factor out a 4.

$$4(x^2 - 5x + 6)$$

The remaining quadratic is $x^2 - 5x + 6$, which means we're looking for a pair of factors of 6 that sum to -5.

Since $(-3)(-2) = 6$ and $(-3) + (-2) = -5$, we see that $x^2 - 5x + 6$ can be factored as $(x - 3)(x - 2)$, which means the given quadratic can be factored as

$$4(x - 3)(x - 2)$$

In later lessons, we'll learn how to factor more complicated quadratic polynomials, including quadratics in which

- the coefficient a on the x^2 term is neither 1 nor -1,
- the coefficient a on the x^2 term isn't the greatest common factor of the quadratic, and when
- the constant b is zero.