Distributive Property with fractions

Remember that the Distributive Property is a method we can use to simplify an expression by multiplying the coefficient outside the parentheses by each term inside the parentheses.

$$a(b+c)=ab+ac$$

All we want to say now is that the Distributive Property applies to fractions in exactly the same way. Introducing fractions just means we have to remember rules for fraction multiplication.

When we multiply two fractions, we multiply the numerators to get the new numerator, and we multiply the denominators to get the new denominator.

$$\frac{a}{b}\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{ac}{bd}+\frac{ae}{bf}$$

Understood 1 and cancelling common factors

There are two other important points to keep in mind when we're applying the Distributive Property and there are fractions involved.

First, if we're multiplying a fraction by a term that isn't a fraction, remember that the idea of the "understood 1" tells us that we can rewrite any value with a denominator of 1. So given any term that isn't a fraction, we can turn it into a fraction by giving it a denominator of 1.

Second, once we've applied the Distributive Property, we need to decide whether we can cancel any common factors from the numerator and denominator of any of the fractions in the result.

When we first learned about fractions in Pre-Algebra, we talked about how to cancel common factors to simplify something like $2/4$ to $1/2$, or to simplify $10/30$ to $1/3$. Now that we have variables involved, we'll need to be able to cancel common factors to turn something like $3x/x^2$ into $3/x$, or something like y^4/y into $y^3/1 = y^3$.

Let's do some examples.

Example

Use the Distributive Property to rewrite the expression.

$$-\frac{1}{3}(3y + 15)$$

Multiply the coefficient outside the parentheses, $-1/3$, by each term inside the parentheses.

$$-\frac{1}{3}(3y) - \frac{1}{3}(15)$$

When we multiply a fraction by a non-fraction, the non-fraction just multiplies the numerator,

$$-\frac{1(3y)}{3} - \frac{1(15)}{3}$$

$$-\frac{3y}{3}-\frac{15}{3}$$

but we could also change the non-fractions into fractions by giving them a denominator of 1, and we'll arrive at the same place.

$$-\frac{1}{3}\left(\frac{3y}{1}\right)-\frac{1}{3}\left(\frac{15}{1}\right)$$

$$-\frac{1(3y)}{3(1)}-\frac{1(15)}{3(1)}$$

$$-\frac{3y}{3}-\frac{15}{3}$$

Either way, now that we've applied the Distributive Property, we need to consider both fractions to see if we can cancel any common factors. In the first fraction, we see a common factor of 3 in the numerator and denominator, so we can cancel it. Remember that this is because we're cancelling out the $3/3$, and $3/3=1$, and cancelling 1 out of the fraction won't change the value of the fraction.

$$-y-\frac{15}{3}$$

In the second fraction, we see a common factor of 3 in the numerator and denominator, so we can cancel it.

$$-y-\frac{3(5)}{3}$$

$$-y-5$$

Let's do another example that's a little more complicated.

Example

Use the Distributive Property to expand the expression.

$$\frac{3a}{b^2}\left(\frac{4c}{5b}+\frac{a^3}{3b^2}\right)$$

Multiply the coefficient $3a/b^2$ by each term inside the parentheses.

$$\frac{3a}{b^2}\left(\frac{4c}{5b}\right)+\frac{3a}{b^2}\left(\frac{a^3}{3b^2}\right)$$

In each term, we'll use fraction multiplication, multiplying the numerators together to get the new numerator, and multiplying the denominators together to get the new denominator.

$$\frac{3a(4c)}{b^2(5b)}+\frac{3a(a^3)}{b^2(3b^2)}$$

$$\frac{12ac}{5b^3}+\frac{3a^4}{3b^4}$$

We need to consider whether we can cancel any common factors within each of the remaining fractions. Within the first fraction, there's no common factors between 12 and 5, so we can't simplify the coefficients.

The numerator includes a and c only, and the denominator includes b only, so there are no common factors that we can see among those constants.

Within the second fraction, the only common factor is a common factor of 3 in the numerator and denominator, so we'll cancel that, and the resulting expression will be

$$\frac{12ac}{5b^3}+\frac{a^4}{b^4}$$

Let's try one more example of the Distributive Property with fractions.

Example

Use the distributive property to expand the expression.

$$\left(xz^2-\frac{x^2y^3}{z^2}\right)\frac{xy^2}{z}$$

Multiply each term inside the parentheses by the coefficient outside the parentheses, xy^2/z.

$$xz^2\left(\frac{xy^2}{z}\right)-\frac{x^2y^3}{z^2}\left(\frac{xy^2}{z}\right)$$

Multiply the numerators and the denominators separately (remember that if there's no denominator, then the denominator is 1).

$$\frac{xz^2(xy^2)}{1(z)}-\frac{x^2y^3(xy^2)}{z^2(z)}$$

$$\frac{x^2y^2z^2}{z}-\frac{x^3y^5}{z^3}$$

Within the first fraction, we can cancel one factor of z.

$$\frac{x^2y^2z(z)}{z(1)}-\frac{x^3y^5}{z^3}$$

$$\frac{x^2y^2z}{1}-\frac{x^3y^5}{z^3}$$

$$x^2y^2z-\frac{x^3y^5}{z^3}$$

There are no common factors in the second fraction, since the numerator contains only x and y, while the denominator contains only z.