Domain and range

Now that we understand when an equation is a function, we want to be able to define the domain and range of the function.

The domain

The **domain** of a function is all the values we can input into the function that don't cause it to be undefined. So the domain of a linear function (the equation of a line), is just all real numbers, because there are no values we can plug into a linear equation that make it undefined.

But if the function is a fraction, then any values that make the denominator 0 will be excluded from the domain, since the fraction is undefined wherever its denominator is 0. So these are some functions and their domains.

$$f(x) = \frac{1}{x} \qquad x \neq 0 \qquad\qquad g(x) = \frac{1}{x-3} \qquad x \neq 3$$

The function f has a domain that includes all numbers except 0 ($x \neq 0$ means x "not equal to" 0), and the function g has a domain that includes all numbers except 3 ($x \neq 3$ means x "not equal to" 3). The numbers excluded from the domain are the values that make the function undefined.

Let's also think about functions with roots. Any values that make the radicand (the value under the radical) negative will be excluded from the domain, since a radical is undefined wherever its radicand is negative. These are some more functions and their domains.

$$f(x) = \sqrt{x} \qquad x \geq 0 \qquad\qquad g(x) = \sqrt{x-3} \qquad x \geq 3$$

The notation $x \geq 0$ tells us that x has to be “greater than or equal to” 0, and the notation $x \geq 3$ tells us that x has to be “greater than or equal to” 3.

The range

Once we know the domain of the function, then we can identify its **range**, which is the entire set of output values that can result from all the inputs in the domain. So if the domain is all of the allowable x values, the range is all the possible y values.

For instance, these are the ranges of the fractional functions we’ve been looking at:

$$f(x) = \frac{1}{x} \qquad f(x) \neq 0 \qquad\qquad g(x) = \frac{1}{x-3} \qquad g(x) \neq 0$$

With both numerators equal to 1, there’s no way to make $f(x) = 0$ or $g(x) = 0$, so 0 is not part of either function’s range.

Similarly, for the radical functions, we’ve set the domains so that the radicands will never be 0, which means we’ll be taking the square root of a value that’s either 0 or positive. The square root of 0 is 0, and the square root of a positive number is a positive number, which means the range of each function is all values that aren’t 0.

$$f(x) = \sqrt{x} \quad f(x) \geq 0 \qquad\qquad g(x) = \sqrt{x-3} \quad g(x) \geq 0$$

The notation $f(x) \geq 0$ tells us that the function $f(x)$ has to be "greater than or equal to" 0.

Let's do an example where we find the domain of another function.

Example

What are the domain and range of the function?

$$f(x) = \frac{6}{x}$$

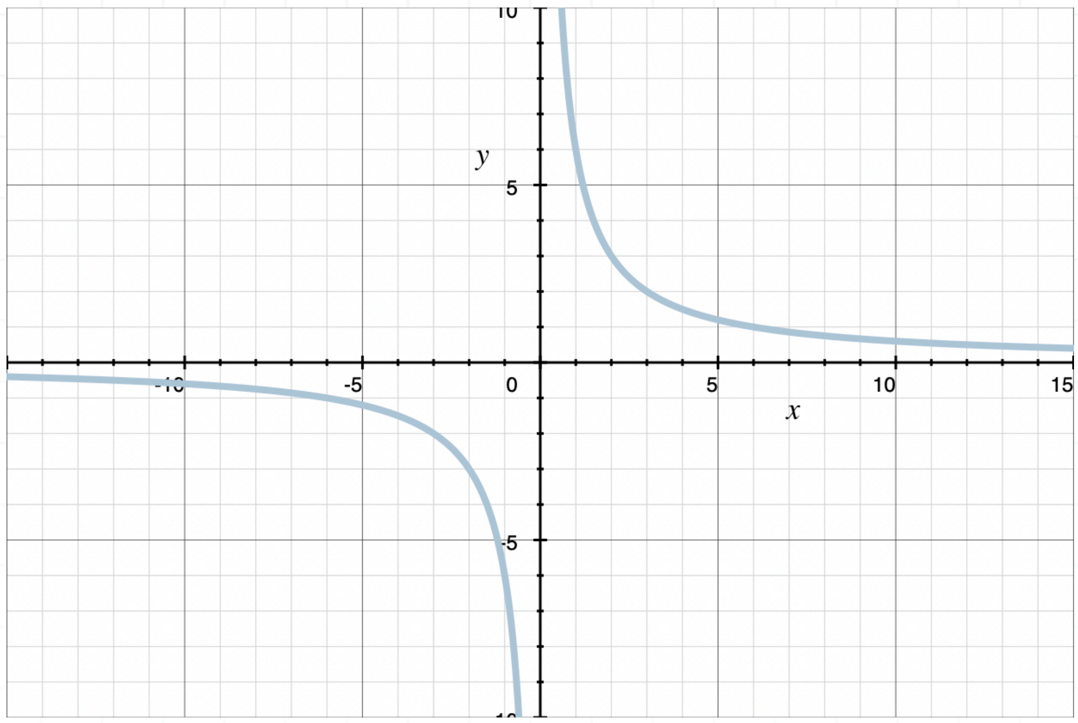
An input of $x = 0$ would make the denominator of the function 0, which would make the function undefined, so the domain includes all numbers except $x = 0$.

Then the range will include all values except 0, because there's no value we can input for x that will cause the value of $f(x)$ to be 0.

Domain $\quad x \neq 0$

Range $\quad f(x) \neq 0$

If we graph this function, we see that it's not defined at $x = 0$ (along the vertical axis), and not defined at $f(x) = y = 0$ (along the horizontal axis).



We can also define a function by a set of (x, y) coordinate points. When a function is defined this way, the domain is the set of x-values in the coordinate points, and the range is the set of y-values in the coordinate points.

Let's look at an example where we find the domain and range of a function that's defined by a set of coordinate points.

Example

What are the domain and range of the function defined by the set of points?

$$(-2,4),\ (1,3),\ (2,5),\ (4,3)$$

If we collect all the x-values from the points, they make up the domain. And if we collect all the y-values from the points, they make up the range.

Domain: $-2, 1, 2, 4$

Range: $4, 3, 5, 3$

We don't need to list the same number more than once, and we'd prefer to arrange the values in ascending order, so we can actually give the domain and range as

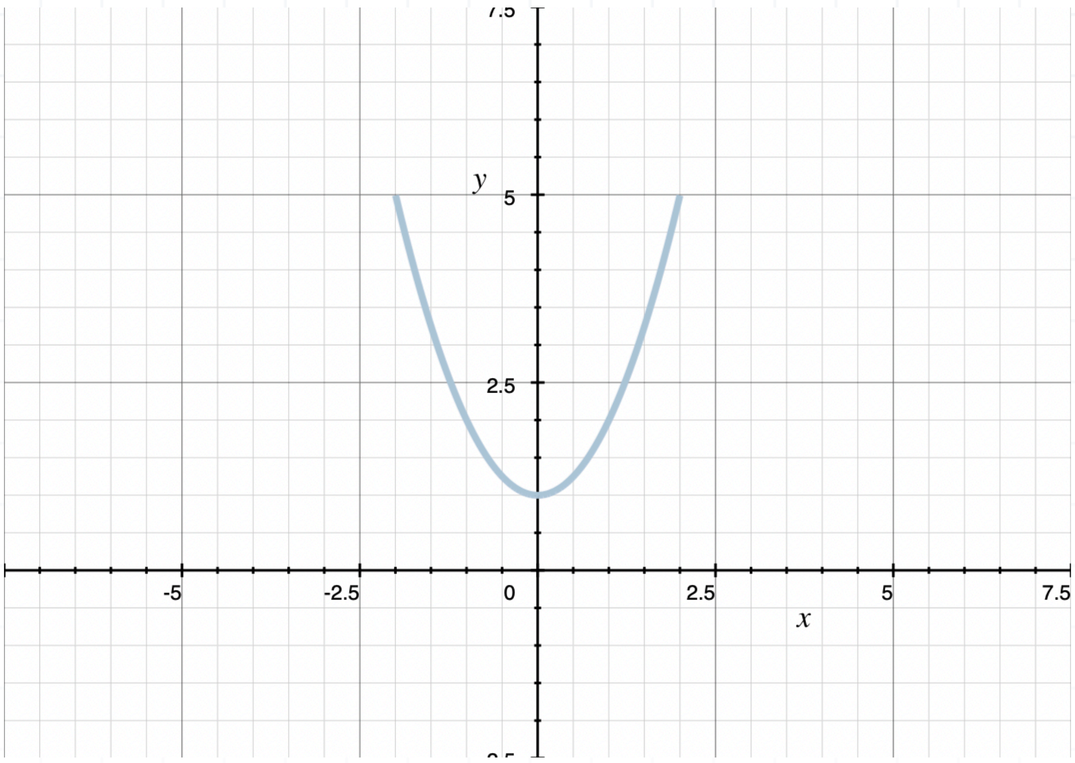
Domain: $-2, 1, 2, 4$

Range: $3, 4, 5$

We can also find the domain and range of a graph, just by looking at its sketch.

Example

What are the domain and range of the graph?



The domain is all of the x-values defined by the sketch, so we need to look at where the function is defined horizontally in the sketch, from its left-most point to its right-most point.

The range is all of the y-values defined by the sketch, so we need to look at where the function is defined vertically in the sketch, from its lowest point to its highest point.

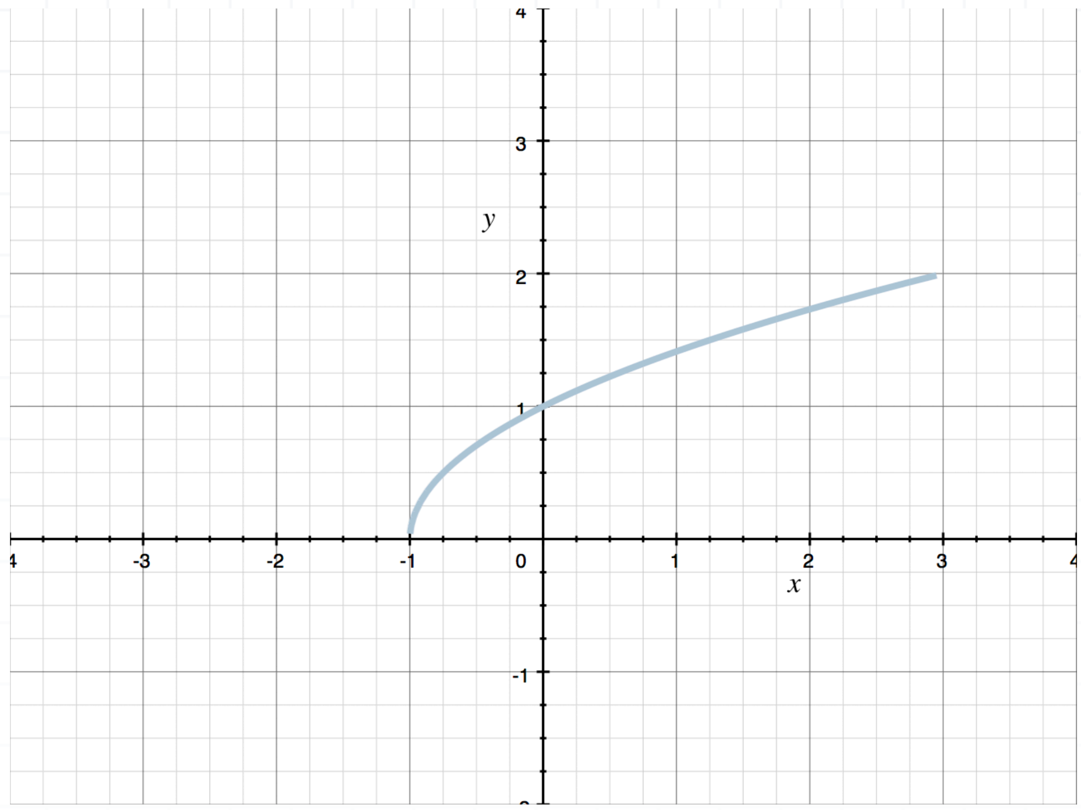
Domain: $-2 \leq x \leq 2$

Range: $1 \leq y \leq 5$

Let's try another example of finding domain and range from a graph.

Example

What are the domain and range of the function?



The domain is all of the x-values defined by the sketch, so we need to look at where the function is defined horizontally in the sketch, from its left-most point to its right-most point.

The range is all of the y-values defined by the sketch, so we need to look at where the function is defined vertically in the sketch, from its lowest point to its highest point.

Domain: $-1 \leq x \leq 3$

Range: $0 \leq y \leq 2$
