Ratios and proportions with complex fractions

When we have complex fractions in a proportion that includes a variable, we can solve for the variable in one of two ways. We can either use cross multiplication, or we can multiply by the reciprocals of the fractions in the denominators.

Remember that cross multiplication tells us that $a/b = c/d$ can be rewritten as $ad = bc$.

Let's look at a few examples.

Example

Solve for the variable.

$$\frac{\left(\frac{1}{3}\right)}{x} = \frac{\left(\frac{1}{6}\right)}{\left(\frac{1}{7}\right)}$$

We'll cross multiply.

$$\frac{1}{3} \cdot \frac{1}{7} = x \cdot \frac{1}{6}$$

Now we can simplify by multiplying the fractions.

$$\frac{1 \cdot 1}{3 \cdot 7} = \frac{x}{6}$$

$$\frac{1}{21} = \frac{x}{6}$$

Multiply both sides of this equation by 6 to solve for x.

$$\frac{6}{21} = x$$

$$x = \frac{2}{7}$$

Let's look at one more example.

Example

Solve for the variable.

$$\frac{\left(\frac{x}{4}\right)}{\left(\frac{8}{3}\right)} = \frac{\left(\frac{4}{3}\right)}{\left(\frac{5}{4}\right)}$$

Instead of dividing by the fractions in the denominators, we can multiply by their reciprocals.

$$\frac{x}{4} \cdot \frac{3}{8} = \frac{4}{3} \cdot \frac{4}{5}$$

After multiplying we get

$$\frac{3x}{32} = \frac{16}{15}$$

Multiply both sides by 32.

$$3x = 32 \cdot \frac{16}{15}$$

Divide both sides by 3 to solve for x. Then multiply fractions to simplify.

$$x = \frac{32}{3} \cdot \frac{16}{15}$$

$$x = \frac{512}{45}$$