

Inequalities and negative numbers

Now that we understand the Law of Trichotomy and the idea behind inequalities and inequality statements, let's turn toward learning to actually solve these inequalities.

Solving inequalities

We'll solve inequalities using all the same methods we used to solve equations. In other words, just like equations, we want to follow order of operations and keep the inequality balanced as we do. We can essentially treat the inequality sign as an equals sign, and solve it as if it were an equation.

The only difference between equations and inequalities is what we have to do with the multiplication or division of negative values. When we multiply or divide both sides of an inequality by a negative value, we have to reverse the direction of the inequality.

So if we started with a less than sign ($<$) and we multiply or divide through the inequality by negative number, we need to change the less than sign $<$ to a greater than sign $>$. Likewise, if we start with $>$ and multiply or divide through by a negative number, we need to flip the inequality to $<$.

If we have a greater than or equal to sign (\geq), or a less than or equal to sign (\leq), the "equals" part of the inequality sign is unaffected when we multiply or divide by a negative number, but the less than or greater than



part still flips. In other words, multiplying or dividing by a negative number changes \geq to \leq , and changes \leq to \geq .

Let's do an example so that we can solve an inequality, including flipping the sign.

Example

Solve the inequality.

$$-x + 3 > 12$$

First, subtract 3 from both sides, just like we would if this inequality was an equation instead.

$$-x + 3 - 3 > 12 - 3$$

$$-x > 9$$

Now, to solve for x , we have to multiply both sides by -1 , which means we have to change the direction of the inequality sign at the same time that we do the multiplication.

$$(-1)(-x) < 9(-1)$$

$$x < -9$$

Let's try another example of solving inequalities with negatives.



Example

Solve the inequality.

$$-2x + 4 \geq -6$$

First, subtract 4 from both sides, just like we would if this inequality was an equation instead.

$$-2x + 4 - 4 \geq -6 - 4$$

$$-2x \geq -10$$

Now we have to divide both sides by -2 , so we have to change the direction of the inequality sign at the same time that we do the division.

$$\frac{-2x}{-2} \leq \frac{-10}{-2}$$

$$x \leq 5$$

