Number word problems

The purpose of number word problems is to give us practice in taking the words (about numbers) in problem statements, translating them to mathematical notation (variables, expressions, equations), using mathematics to solve the problems, and answering the specific questions that were asked in the problem statements.

For some people these are fun number games, but they do appear on tests and in math classes from time to time, so it's good to be comfortable with them.

In a number word problem, we're given information about a pair or group of numbers, and we usually need to translate the information into equations to solve for the numbers.

Some helpful vocabulary:

> Consecutive integers are integers that are in ascending order and have no integer between them, such as 4 and 5.
>
> Consecutive even numbers are even numbers that are in ascending order and have no even number between them, such as 4 and 6.
>
> Consecutive odd numbers are odd numbers that are ascending order and have no odd number between them, such as 5 and 7.

And of course "sum" means the result of addition, "difference" means the result of subtraction, and "product" means the result of a multiplication. Place value can also be used in these problems.

Let's do an example.

Example

The sum of two consecutive integers is 25. Find the numbers.

In solving any word problem, we should start by defining the variable(s). In this case, let's let X be the smaller integer. Then the larger integer is $X + 1$.

Let's write an equation for what we know about their sum.

$$X + (X + 1) = 25$$

Now let's solve for X.

$$X + X + 1 = 25$$

$$2X + 1 = 25$$

$$2X + 1 - 1 = 25 - 1$$

$$2X = 24$$

$$\frac{2X}{2} = \frac{24}{2}$$

$$X = 12$$

The smaller integer is 12, so the larger one is $12 + 1 = 13$. Therefore, the integers are 12 and 13. We can double check our answer and see that $12 + 13 = 25$.

Let's look at another style of problem.

Example

The sum of the digits of a certain two-digit number is 17. Reversing the digits gives a number which is 9 less than the original number. What is the original number?

Let T be the tens digit of the original number, and let U be its unit digit.

The value of the original number is

$$10T + U$$

Reversing the digits gives us a number whose value is

$$10U + T$$

The second number is 9 less than the original number, so we can write

$$\text{Original number} - 9 = \text{Second number}$$

In mathematical notation, that translates to the following equation:

$$(10T + U) - 9 = (10U + T)$$

$$10T + U - 9 = 10U + T$$

Move the $10U$ and the T to the left-hand side.

$$10T - T + U - 10U - 9 = 0$$

$$9T - 9U - 9 = 0$$

Dividing through by 9 gives

$$T - U - 1 = 0$$

$$T = U + 1$$

We know that the sum of the digits is 17, so we'll substitute the expression we just found for T into the equation $T + U = 17$, and then solve for U.

$$T + U = 17$$

$$(U + 1) + U = 17$$

$$2U + 1 = 17$$

$$2U = 16$$

$$U = 8$$

Substitute 8 for U in the equation $T + U = 17$, and then solve for T.

$$T + U = 17$$

$$T + 8 = 17$$

$$T = 9$$

The original number is 98. Also, the second number is 89, which is indeed 9 less than the original number: $89 = 98 - 9$.