Uniform motion problems

In this lesson we'll look at how to compare and solve for values of the variables in the distance equation,

Distance = Rate · Time

$$D = RT$$

when we have a case of uniform motion and a pair of related scenarios, including

- a pair of scenarios with the same distance but different speeds and times,
- a pair of scenarios with the same speed but different distances and times, or
- a pair of scenarios with the same time but different distances and speeds.

Let’s talk about the units of each of these values.

Distance has units of inches, feet, miles, etc., or of centimeters, meters, kilometers, etc.

Time has units of seconds, minutes, hours, etc.

Rate has units of distance/time, for example inches/second, miles/hour, or kilometers/hour.

Before we can use the formula $D = RT$, we need to make sure that the units for distance and time are the same as the units in of the rate. If they aren't, we'll need to change them so we're working with the same units.

Let's do an example of a standard distance, rate, and time problem.

Example

Heather ran 56 km in 5 hours. What was Heather's rate in km/hr?

We'll use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Let's write down what we know.

$$D = 56 \text{ km}$$

$$T = 5 \text{ hr}$$

If we plug these into the distance formula, we get

$$56 \text{ km} = R \cdot 5 \text{ hr}$$

Now solve for the rate.

$$\frac{56 \text{ km}}{5 \text{ hr}} = \frac{R \cdot 5 \text{ hr}}{5 \text{ hr}}$$

$$R = 11.2 \ \frac{\text{km}}{\text{hr}}$$

Let’s try one with two people.

Example

Susan and Benjamin were 60 miles apart on a straight trail. Susan started walking toward Benjamin at a rate of 5 mph at 7:30 a.m. Benjamin left three hours later, and they met on the trail at 3:30 p.m. How fast did Benjamin walk?

We’ve been given information about distance, rate, and time, so we’ll use the formula

Distance = Rate · Time

$$D = RT$$

We can use subscripts to create unique equations for Susan and Benjamin; we’ll use S for Susan, and B for Benjamin.

Susan $\qquad D_S = R_S T_S$

Benjamin $\qquad D_B = R_B T_B$

We know that in order to meet each other, they must have covered a distance of 60 miles between them. Therefore,

$$D_S + D_B = 60$$

Since we know that $D_S = R_S T_S$ and $D_B = R_B T_B$, we can rewrite this equation as

$$R_S T_S + R_B T_B = 60$$

and then substitute the known quantities (Susan's speed and time, and Benjamin's time) into the equation. The problem tells us that Susan walked at a speed of 5 mph, and that she walked for 8 hours, since she walked from 7:30 a.m. until 3:30 p.m. So

$$(5)(8) + R_B T_B = 60$$

$$40 + R_B T_B = 60$$

$$R_B T_B = 20$$

Benjamin left three hours after Susan, which means he started walking at 10:30 a.m., and he kept walking until they met at 3:30 p.m., which means he walked for 5 hours. So

$$R_B(5) = 20$$

$$R_B = 4$$

Which means that Benjamin walks at a speed of 4 mph.

Let's look at another uniform motion problem.

Example

One train leaves Station A at a constant speed and arrives at Station B in 8 hours. A second train leaves Station A at a constant rate of 40 mph and arrives at Station B in 10 hours. What was the speed of the first train?

Since each train is traveling at a uniform speed, we recognize this as a uniform motion problem. We can use subscripts to create a unique equation for each train. We'll call them Train 1 and Train 2.

Train 1: $D_1 = R_1T_1$

Train 2: $D_2 = R_2T_2$

Let's organize the information we know about each train.

Train 1:

$D_1 = ?$

$R_1 = ?$

$T_1 = 8$ hours

Train 2:

$D_2 = ?$

$R_2 = 40$ mph

$T_2 = 10$ hours

Now let's plug this information into the equations for Train 1 and Train 2.

$D_1 = R_1 T_1$

$D_1 = R_1(8 \text{ hrs})$

and

$D_2 = R_2 T_2$

$D_2 = (40 \text{ mph})(10 \text{ hrs})$

$D_2 = 400 \text{ miles}$

The two trains traveled the same distance ($D_1 = D_2$), so we can equate the value we just found for D_2 to the expression we found for D_1 (and then solve for R_1).

$D_2 = D_1$

$400 \text{ miles} = r_1(8 \text{ hrs})$

$50 \text{ mph} = r_1$

The first train traveled at a constant speed of 50 mph from Station A to Station B.

Let's do one more example.

Example

Cassie is driving at a constant rate of 30 mph on the highway. Four hours later, her friend Susan starts from the same point and drives at a constant rate of 60 mph and passes Cassie. How many hours had each woman been traveling at the time that Susan passed Cassie? And how far had each woman traveled at that time?

Since each woman is traveling at a uniform rate, we recognize this as a uniform motion problem, so we can use the equation $D = RT$, where D is the distance each of them traveled, R is the rate at which they traveled, and T is the time it took them to get to the place where Susan passed Cassie. We can use subscripts to set up a unique equation for each woman's travel.

Cassie: $D_c = R_c T_c$

Susan: $D_s = R_s T_s$

The problem tells us that Cassie traveled at a rate of 30 mph, that Susan traveled at a rate of 60 mph, and that it took Susan 4 hours less than Cassie to travel the same distance (because she left 4 hours later).

Let's set up what we know.

Cassie:

$D_c = ?$

$R_c = 30$ mph

$T_c = ?$

$D_c = (30 \text{ mph})T_c$

Susan:

$$D_s = ?$$

$$R_s = 60 \text{ mph}$$

$$T_s = T_c - 4$$

$$D_s = (60 \text{ mph})(T_c - 4)$$

Cassie and Susan traveled the same distance ($D_c = D_s$), so can equate the expression we found for D_c to the expression we found for D_s (and then solve for T_c).

$$(30 \text{ mph})T_c = (60 \text{ mph})(T_c - 4)$$

$$\frac{(30 \text{ mph})T_c}{60 \text{ mph}} = \frac{(60 \text{ mph})(T_c - 4)}{60 \text{ mph}}$$

$$\frac{1}{2}T_c = T_c - 4$$

$$\frac{1}{2}T_c - T_c = T_c - T_c - 4$$

$$-\frac{1}{2}T_c = -4$$

$$-2\left(-\frac{1}{2}T_c\right) = -2(-4)$$

$$T_c = 8 \text{ hours}$$

We now know that it took Cassie 8 hours to get to the point at which Susan passed her, so we can substitute 8 for T_c in the equation we found for T_s (and then compute the value of T_s).

$$T_s = T_c - 4$$

$$T_s = 8 - 4$$

$$T_s = 4 \text{ hours}$$

Now that we have a rate and a time for both Cassie and Susan, we can find the distance that each of them traveled (and verify that it was the same for both of them).

Cassie:

$$D_c = R_c T_c$$

$$D_c = 30(8)$$

$$D_c = 240 \text{ miles}$$

Susan:

$$D_s = R_s T_s$$

$$D_s = 60(4)$$

$$D_s = 240 \text{ miles}$$