Topic: Covariance

Question: Calculate the covariance of the sample.

X	2	3	4	5	6
Y	3	5	7	9	11

Answer choices:

A $s_{XY} = 8$

B $s_{XY} = 7$

C $s_{XY} = 5$

D $s_{XY} = 4$

Solution: C

Find the mean of X,

$$\bar{X} = \frac{2+3+4+5+6}{5}$$

$$\bar{X} = \frac{20}{5}$$

$$\bar{X} = 4$$

and then the mean of Y.

$$\bar{Y} = \frac{3+5+7+9+11}{5}$$

$$\bar{Y} = \frac{35}{5}$$

$$\bar{Y} = 7$$

Now use the means to find the sample covariance.

$$s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2-4)(3-7) + (3-4)(5-7)$$

$$+(4-4)(7-7) + (5-4)(9-7) + (6-4)(11-7)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = -2(-4) - 1(-2) + 0(0) + 1(2) + 2(4)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 8 + 2 + 2 + 8$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 20$$

$$s_{XY} = \frac{20}{5 - 1}$$

$$s_{XY} = 5$$

Topic: Covariance

Question: If X takes on the sample values $\{2, 4, 6, 8, 10, 15\}$, and Y takes on the sample values $\{12, 17, 23, 25, 33, 40\}$, find the covariance of X and Y.

Answer choices:

A 39.2

B 46

C 47

D 56

Solution: C

Find the mean of X,

$$\bar{X} = \frac{2+4+6+8+10+15}{6}$$

$$\bar{X} = \frac{45}{6}$$

$$\bar{X} = 7.5$$

and then the mean of Y.

$$\bar{Y} = \frac{12+17+23+25+33+40}{6}$$

$$\bar{Y} = \frac{150}{6}$$

$$\bar{Y} = 25$$

Now use the means to find the sample covariance.

$$s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2-7.5)(12-25) + (4-7.5)(17-25)$$

$$+(6-7.5)(23-25) + (8-7.5)(25-25)$$

$$+(10-7.5)(33-25) + (15-7.5)(40-25)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 235$$

$$s_{XY} = \frac{235}{6-1}$$

$$s_{XY} = 47$$

Topic: Covariance

Question: Two corporations record their stock returns between 2010 and 2014. From the sample, calculate the covariance of their stock returns.

	2010	2011	2012	2013	2014
X	2%	1%	-2%	4%	-1%
Y	3%	0%	1%	2%	1%

Answer choices:

A 0.95

B 1.08

C 1.35

D 1.49

Solution: C

Find the mean of X,

$$\bar{X} = \frac{2 + 1 + (-2) + 4 + (-1)}{5}$$

$$\bar{X} = \frac{4}{5}$$

$$\bar{X} = 0.8$$

and then the mean of Y.

$$\bar{Y} = \frac{3 + 0 + 1 + 2 + 1}{5}$$

$$\bar{Y} = \frac{7}{5}$$

$$\bar{Y} = 1.4$$

Now use the means to find the sample covariance.

$$s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2 - 0.8)(3 - 1.4) + (1 - 0.8)(0 - 1.4)$$

$$+(-2 - 0.8)(1 - 1.4) + (4 - 0.8)(2 - 1.4) + (-1 - 0.8)(1 - 1.4)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 5.4$$

$$s_{XY} = \frac{5.4}{5 - 1}$$

$s_{XY} = 1.35$