Graphing log functions

Previously, we talked about the fact that exponential and logarithmic functions are inverses of each other. This is implied by the general log rule,

$$a^x = y \qquad \Longleftrightarrow \qquad \log_a(y) = x$$

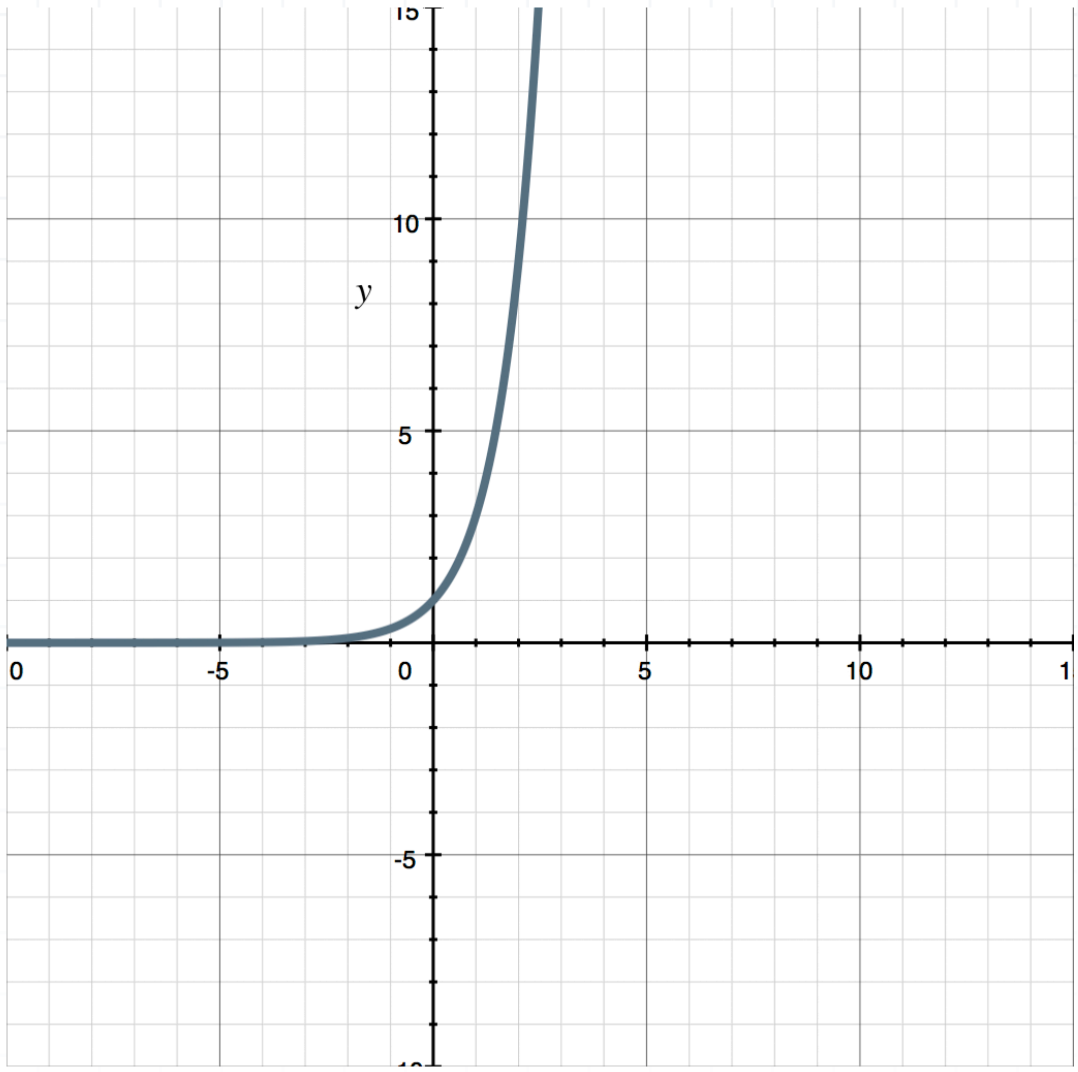
which allows us to convert back and forth between an exponential equation and the associated logarithmic equation. Remember that this is also true for natural logs, as

$$e^x = y \qquad \Longleftrightarrow \qquad \log_e(y) = x$$

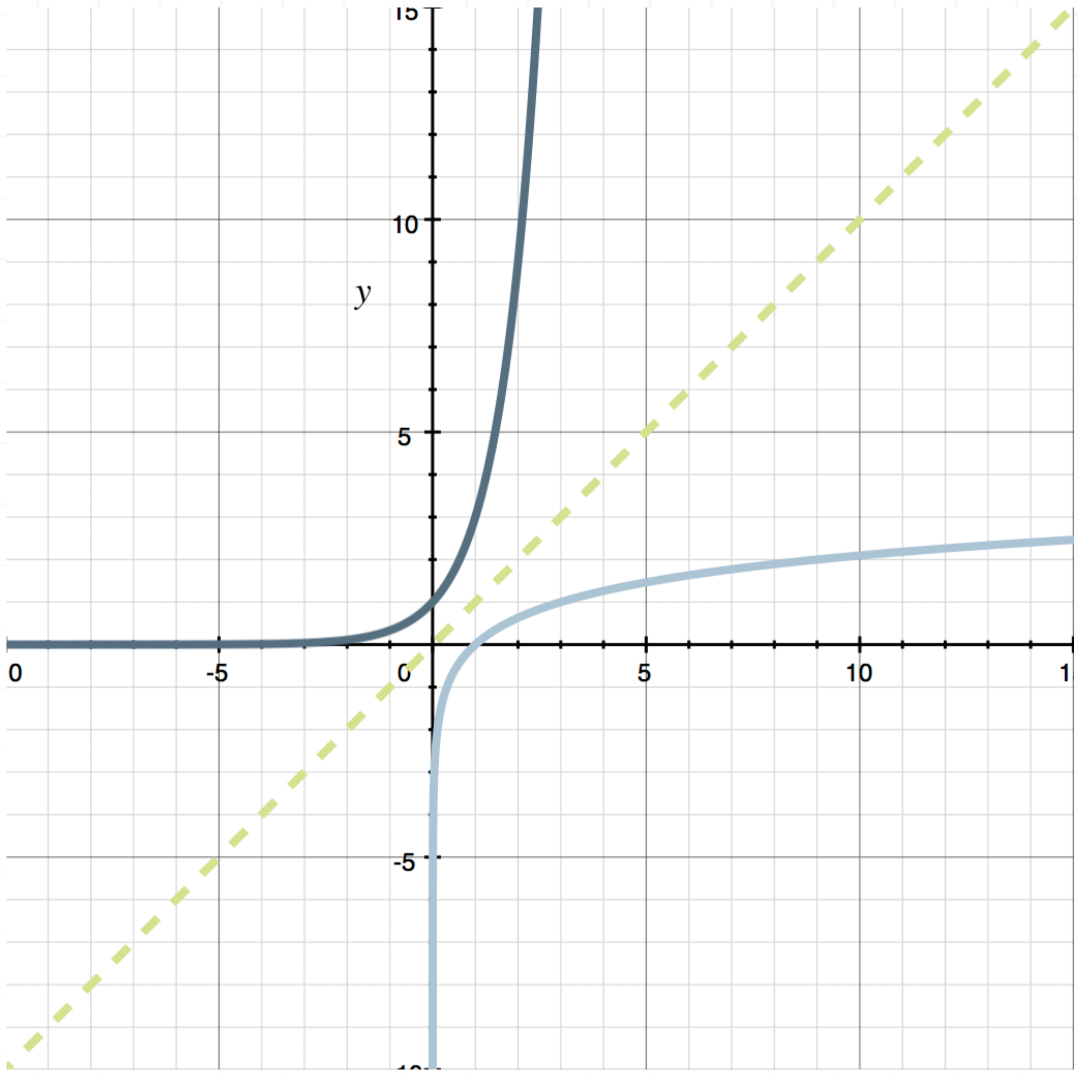
In earlier lessons, we talked about the graphs of exponential functions, including how to graph transformations of exponential functions.

Because exponential and logarithmic functions are inverses of each other, if we have the graph of an exponential function, we can get the graph of the corresponding log function in two ways: by reflecting the graph of the exponential function with respect to the line $y = x$, or by switching the x- and y-coordinates of all the points.

For instance, we already know that the graph of the exponential function $y = 3^x$ is



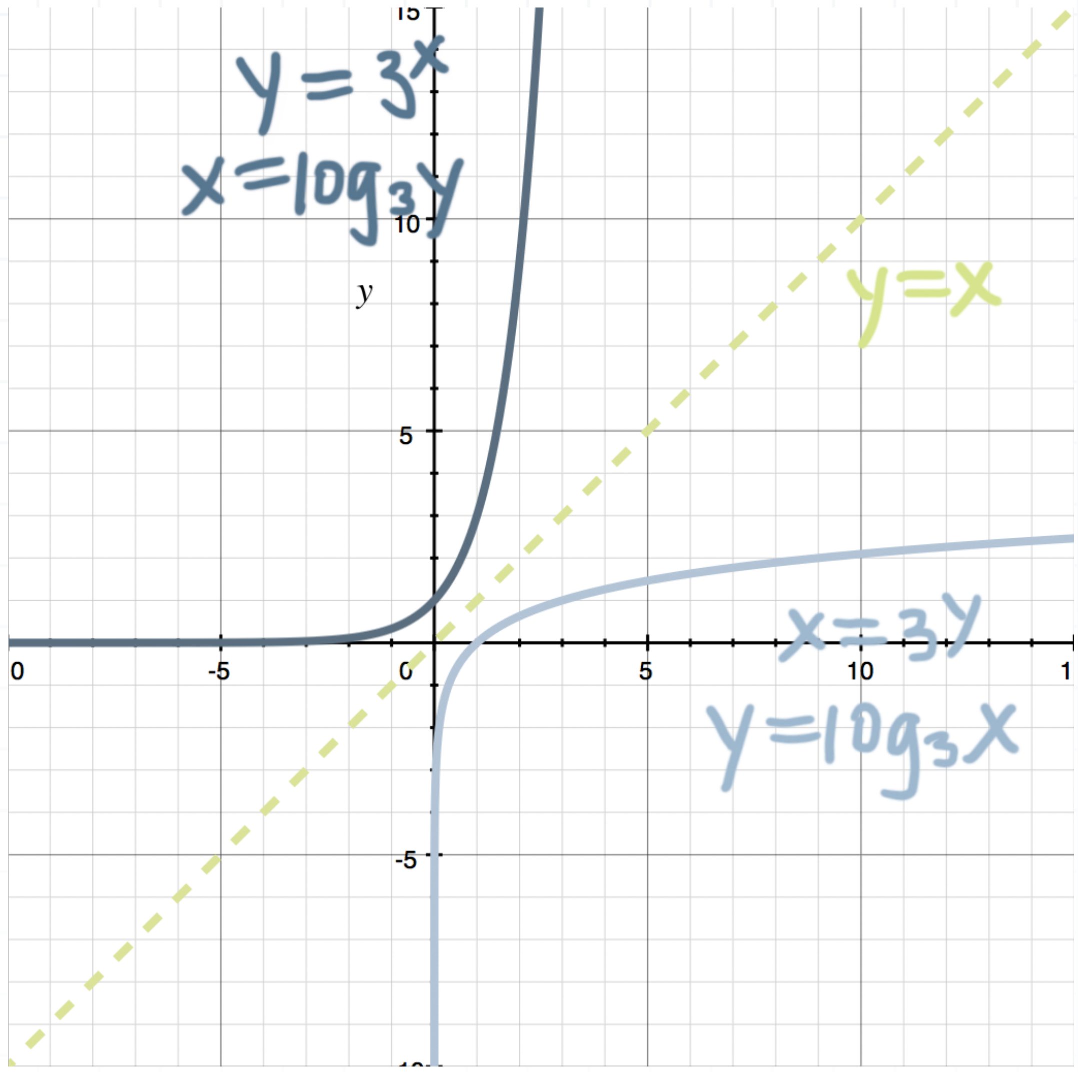
Let’s now find the inverse of this exponential function. Functions which are inverses of each other simply have their x- and y-values flipped, which means that the inverse of $y = 3^x$ can simply be given by $x = 3^y$ (the same equation, just with x and y swapped). If we sketch $x = 3^y$, we see that we get the mirror image of $y = 3^x$, reflected over the line $y = x$.



The interesting thing is that, by the general log rule applied to the function $y = \log_3 x$, we get

$$x = 3^y \qquad \Longleftrightarrow \qquad y = \log_3 x$$

Therefore, we can actually say that $y = 3^x$ and $y = \log_3 x$ are inverses of one another. Similarly, $x = 3^y$ and $x = \log_3 y$ are inverses of one another. Let's show both the exponential and logarithmic expression of both functions, and how they are inverses of each other, reflected over $y = x$.



Let's walk through a couple of examples of graphing logarithmic functions, keeping in mind that we can always use the general log rule to convert logarithmic equations to their exponential form, and then graph the resulting exponential equations using the steps we used in the previous lesson.

Example

Graph the logarithmic function.

$$y = \log_3 x$$

There are several ways to go about this. First, we could use the general rule for logs to convert the logarithmic equation $y = \log_3 x$ to its exponential form, $x = 3^y$. Then we can follow the steps from the previous lesson, but this time by plugging in values of y to get values of x, starting with $y = 100$ and $y = -100$.

For $y = 100$:

$$x = 3^{100}$$

$$x = \text{very large positive number}$$

$$x = \infty$$

For $y = -100$:

$$x = 3^{-100}$$

$$x = \frac{1}{3^{100}}$$

$$x = \frac{1}{\text{very large positive number}}$$

$$x = \text{very small positive number}$$

$$x = 0$$

This basically allowed us to evaluate end behavior, and we've learned that the function has a vertical asymptote at $x = 0$, and heads up toward ∞ as y gets very large.

We'll plug in a few simple-to-calculate values for y.

For $y = -1$:

$$x = 3^{-1}$$

$$x = \frac{1}{3}$$

For $y = 0$:

$$x = 3^0$$

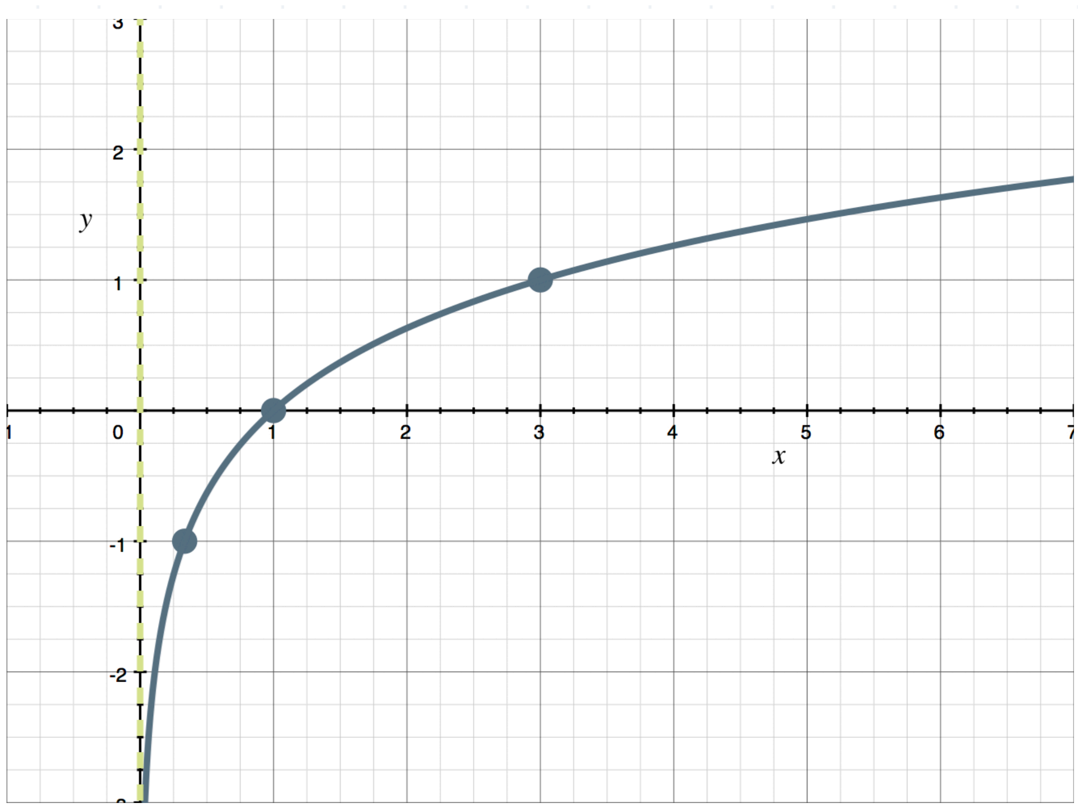
$$x = 1$$

For $y = 1$:

$$x = 3^1$$

$$x = 3$$

If we plot these points, along with the vertical asymptote $x = 0$, and then connect the points, we get the graph of $x = 3^y$.



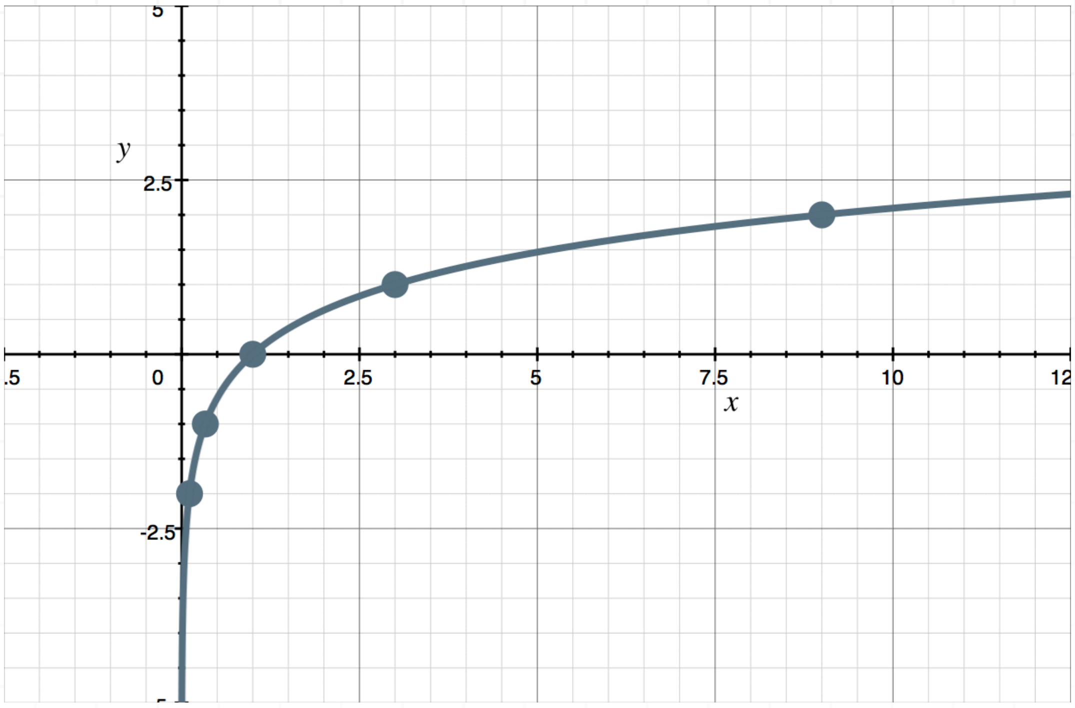
We could also graph the log function using a table of points. Since the logarithmic equation $y = \log_3 x$ corresponds to the exponential equation $x = 3^y$, we can find the coordinates of some points that satisfy this exponential equation. It'll be easier for us to plug in values of y, and see which x-values come out of the equation. For instance, if we plug $y = 0$ into $x = 3^y$, we get $x = 1$.

x			1		
y			0		

Let's fill out the rest of the chart with a few other easy-to-calculate values. It doesn't matter which values we pick, because we're just trying to get a few points that we can use to graph the logarithmic function.

x	1/9	1/3	1	3	9
y	-2	-1	0	1	2

With these five points, we should be able to pretty easily sketch the graph.



Properties of the graphs of log functions

For log functions in the form $y = \log_b(x)$ with $b > 1$,

- The x-intercept is $(1,0)$ because $0 = \log_b(1)$ means that $b^0 = 1$, which is true for any value of b.
- The point $(b,1)$ satisfies the function because $1 = \log_b(b)$ means that $b^1 = b$, which is true for any value of b.
- The point $(1/b, -1)$ satisfies the function because $-1 = \log_b(1/b)$ means that $b^{-1} = 1/b$, which is true for any value of b.
- The y-axis is a vertical asymptote.
- The domain is $(0,\infty)$ and the range is $(-\infty, \infty)$.

Additionally, we know that for log functions in the form $y = \log_b(x)$ with $b > 0$ and $b \neq 1$, the function is one-to-one, and the graph is decreasing for $0 < b < 1$ and increasing for $b > 1$.

So to graph the logarithmic function $f(x) = \log_b(x)$, we'll

1. Set up the vertical asymptote, $x = 0$, and plot the x-intercept $(1,0)$, as well as the points $(b,1)$ and $(1/b, -1)$.

2. Rewrite $f(x) = \log_b(x)$ in exponential form $b^y = x$ to find additional points if needed. Choose a few small values of y to find the corresponding x-values. Plot the points on the graph.

3. Connect the points with a smooth curve.

Just as we've built graphs of exponential functions in the previous lesson, we can use the general rule for logs to convert the logarithmic equation into its exponential form in order to graph the function. Let's do one more example.

Example

Graph the function $y = -\log_3(x-1)$.

First, we could use the general rule for logs to convert the logarithmic equation $y = -\log_3(x-1)$ to its exponential form,

$$x = 1 + \frac{1}{3^y}$$

Then we can follow the steps from the previous lesson, but this time by plugging in values of y to get values of x, starting with $y = 100$ and $y = -100$.

For $y = 100$:

$$x = 1 + \frac{1}{3^{100}}$$

$x = 1 +$ very small positive number

$$x = 1$$

For $y = -100$:

$$x = 1 + \frac{1}{3^{-100}}$$

$$x = 1 + 3^{100}$$

$x = 1 +$ very large positive number

$$x = \infty$$

Doing this allowed us to evaluate end behavior, and we've learned that the function has a vertical asymptote at $x = 1$, and heads up toward ∞ as y gets very small.

We'll plug in a few simple-to-calculate values for y.

For $y = -1$:

$$x = 1 + \frac{1}{3^{-1}}$$

$$x = 1 + 3$$

$$x = 4$$

For $y = 0$:

$$x = 1 + \frac{1}{3^0}$$

$$x = 1 + 1$$

$$x = 2$$

For $y = 1$:

$$x = 1 + \frac{1}{3^1}$$

$$x = \frac{4}{3}$$

If we plot these points, along with the vertical asymptote $x = 1$, and then connect the points, we get the graph of the function.