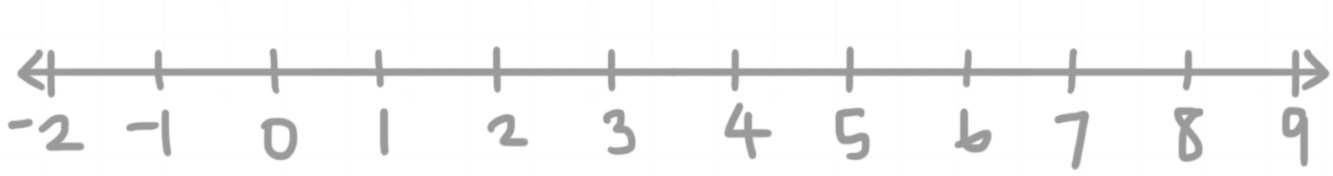
Graphing inequalities on a number line

In previous lessons we looked at how to graph points and lines in the Cartesian coordinate system, which was defined by the horizontal x-axis and the vertical y-axis.

Number lines

In this lesson we're talking about graphing inequalities, which will be done on a number line. We can think of the number line as simply the horizontal x-axis from the Cartesian coordinate system, with the vertical y-axis stripped away. So a number line might look like this,



and we can see that it's one-dimensional, where the Cartesian coordinate system was two-dimensional.

In other words, the number line is simply a horizontal line, extending infinitely in both directions. We'll see negative numbers to the left of 0 and positive numbers to the right of 0, and we'll be graphing inequalities on these kinds of number lines.

Solving inequalities

Before we can graph an inequality, we have to solve it. To make the inequality easier to graph, it's helpful to always write the solution where the variable is by itself on the left side of the inequality statement.

In other words, we'd like to get the solution into a form resembling one of these:

$x > a$ $\quad$ $x \neq a$ $\quad$ $x \geq a$

$x < a$ $\quad$ $x \leq a$

If we solve an inequality and the result has the variable on the right side instead of the left side, we just need to swap the sides and reverse the sign of the inequality. For example, if we solve an inequality and get $6 > x$, we can turn it around and write it as $x < 6$.

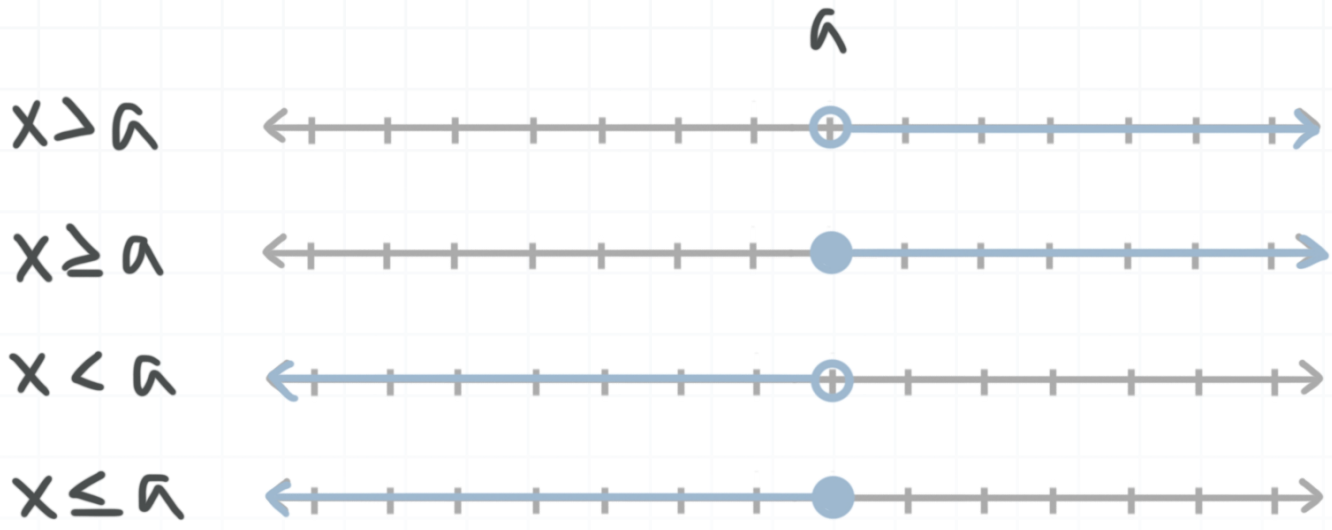
Graphing inequalities

Once we've solve the inequality and put the variable on the left side, we can graph it on a number line.

If the solution is either $x \geq a$ or $x \leq a$, then a is included in the solution and we draw a solid circle at $x = a$ on the number line, and then a ray that extends from a to the right or left.

If the solution is either $x > a$ or $x < a$, then a is excluded from the solution and we draw an open circle at $x = a$ on the number line, and then a ray that extends from a to the right or left.

To summarize, given $>$ or $\geq$, the arrow will go to the right, but if we have $<$ or $\leq$, the arrow will go to the left. If we have $>$ or $<$, the circle will be open, and if we have $\geq$ or $\leq$, the circle will be solid.



If the inequality is $x \neq a$ ("x not equal to a"), the solution consists of all the numbers other than a, so we draw an open circle at a and then sketch two rays, one that extends out to the left from a, and the other that extends out to the right from a.

Let's do an example with a "less than" inequality.

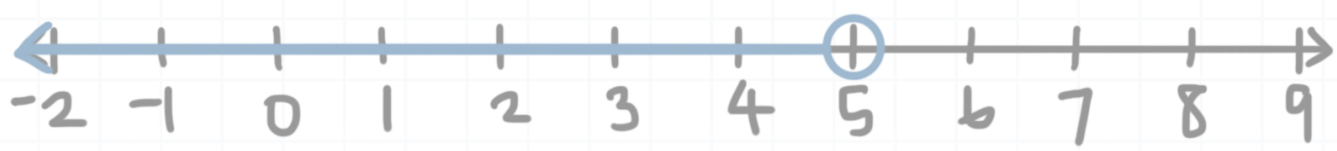
Example

Graph the inequality on a number line.

$$x < 5$$

First, we'll draw a number line that includes 5.

Next, we'll draw an open circle at 5 because 5 isn't part of the solution. Finally, we'll sketch a ray that points left from 5, since the solution consists of all the numbers less than 5.



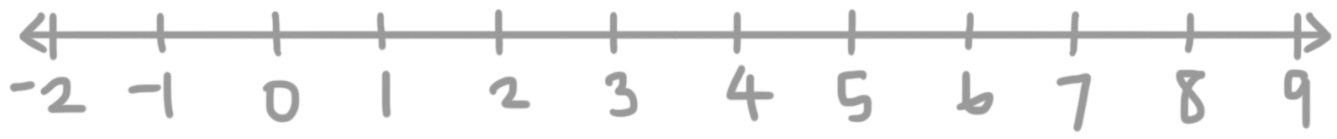
Let's try another example, this time with a "greater than or equal to" inequality.

Example

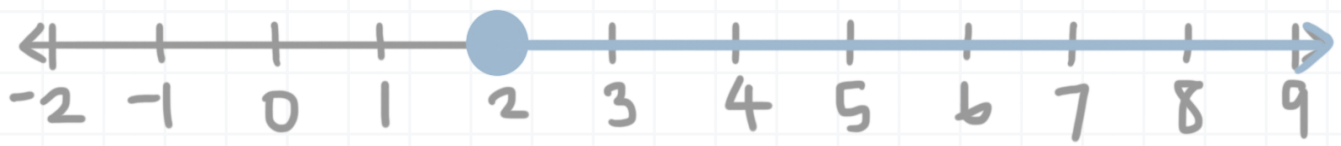
Graph the inequality on a number line.

$$x \geq 2$$

First, we'll draw a number line that includes 2.



Next, we'll draw a solid circle at 2 because 2 is part of the solution. Finally, we'll sketch a ray that points right from 2, since the solution consists of all the numbers greater than or equal to 2.



Let's try another example of graphing inequalities, this time where we have to simplify the inequality first.

Example

Graph $3(x+2)+1 \leq x-5$ on a number line.

Use the Distributive Property to simplify the parentheses.

$$3(x+2)+1 \leq x-5$$

$$3x+6+1 \leq x-5$$

$$3x+7 \leq x-5$$

Subtract 7 from both sides.

$$3x+7-7 \leq x-5-7$$

$$3x \leq x-12$$

Subtract x from both sides.

$$3x-x \leq x-x-12$$

$$2x \leq -12$$

Divide both sides by 2. We don't have to flip the direction of the inequality because we're dividing by a positive number, not a negative number.

$$x \leq -6$$

Because the solution is a "less than or equal to" inequality, we'll draw a solid circle at $x = -6$, and then a ray that extends out to the left from that point.