Graphing conjunctions on a number line

If disjunctions are "or statements," then conjunctions are "and statements."

Given a disjunction like "$x > 5$ or $x < 3$," values of x satisfy the disjunction when they either satisfies $x > 5$ or when it satisfies $x < 3$. But given a conjunction like "$1 \leq x$ and $x \leq 8$," values of x satisfy the conjunction only when they satisfy both $1 \leq x$ and $x \leq 8$.

Conjunctions as one inequality statement

The four "simple" forms for a conjunction of two inequalities are

$$a \leq x \leq b$$

$$a \leq x < b$$

$$a < x \leq b$$

$$a < x < b$$

Each of these may look like just a single inequality, but each of them is actually two inequalities. If we break each of these apart into an "and statement," we'd rewrite them this way:

$a \leq x \leq b$	$a \leq x$ and $x \leq b$
$a \leq x < b$	$a \leq x$ and $x < b$

$a < x \leq b$ $\qquad$ $a < x$ and $x \leq b$

$a < x < b$ $\qquad$ $a < x$ and $x < b$

As an example, $1 \leq x \leq 8$ is actually the pair of inequalities $1 \leq x$ and $x \leq 8$.

Graphing conjunctions

To graph a conjunction of two inequalities, we can first graph the inequalities separately and see where they overlap. Then we graph their conjunction (the overlap) on a separate number line, by including all the points that are on the graphs of both (not just one) of the inequalities (and no other points).

Let's look at an example of how to graph conjunctions by splitting the inequality into two separate inequalities, graphing each one, and then graphing the overlap.

Example

Graph the conjunction.

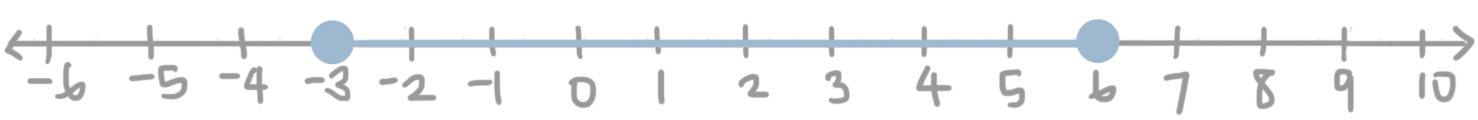
$$-3 \leq x \leq 6$$

We need to graph the conjunction of the inequalities $-3 \leq x$ and $x \leq 6$, but first let's see how they graph separately.

The inequality $-3 \leq x$ can be turned around and written as $x \geq -3$. So its graph has a solid circle at -3 (because we have a "greater than or equal to" sign), and because of the greater than part of the inequality, the arrow goes to the right.

The graph of the inequality $x \leq 6$ has a solid circle at 6 (because we have a "less than or equal to" sign), and because of the less than part of the inequality, the arrow goes to the left.

To graph the conjunction of the two inequalities, we'll graph only the overlap: a solid circle at -3 (because -3 is on the graphs of both of the inequalities), a solid circle at 6 (because 6 is on the graphs of both of the inequalities), and everything between -3 and 6.



Sometimes we'll have two separate inequalities and will need to graph their conjunction. To do that, we may need to solve one or both of the inequalities before we can write their conjunction in one of the four "simple" forms.

Example

Graph the conjunction of the inequalities $3x + 1 > -5$ and $2x - 4 \leq 6$.

First, we'll solve and graph the two inequalities separately. To begin solving $3x + 1 > -5$, subtract 1 from both sides.

$$3x + 1 > -5$$

$$3x + 1 - 1 > -5 - 1$$

$$3x > -6$$

Now divide both sides by 3.

$$\frac{3x}{3} > \frac{-6}{3}$$

$$x > -2$$

The graph of the inequality $x > -2$ has an open circle at -2 (because we have just a "greater than" sign), and because it's a greater than inequality, the arrow goes to the right.

To begin solving the inequality $2x - 4 \leq 6$, add 4 to both sides.

$$2x - 4 \leq 6$$

$$2x - 4 + 4 \leq 6 + 4$$

$$2x \leq 10$$

Now divide both sides by 2.

$$\frac{2x}{2} \leq \frac{10}{2}$$

$$x \leq 5$$

The graph of the inequality $x \leq 5$ has a solid circle at 5 (because we have a "less than or equal to" sign), and because of the less than part of the inequality, the arrow goes to the left.

We can combine the solutions we found ($x > -2$ and $x \leq 5$) and write their conjunction in one of the four "simple" forms. If we take the first inequality ($x > -2$) and turn it around, we get $-2 < x$.

So the conjunction of the inequalities $x > -2$ and $x \leq 5$ can be written as

$$-2 < x \leq 5$$

The graph of the conjunction will be the overlap of the graphs of the two separate inequalities: an open circle at -2 (because -2 isn't on the graph of $-2 < x$), a solid circle at 5 (because 5 is on the graphs of both of the separate inequalities), and everything between -2 and 5.

Let's try another example of graphing conjunctions.

Example

Graph the values of x that satisfy the following inequalities.

$x \geq 5$ and $x \neq 7$

We'll draw a solid circle at 5, because 5 is part of the solution, then we'll sketch the solution extending out to the right from that point, since the solution includes all the numbers greater than 5.

But we need to exclude $x = 7$, which means the arrow will go to the right, but it'll be interrupted by an open circle at $x = 7$.

One thing we should be aware of is that there are pairs of inequalities whose solutions have nothing in common.

For example, suppose we were given two inequalities, $x > 8$ and $x < -1$. In that case, the solution of their conjunction consists of all the numbers that are (simultaneously) greater than 8, and at the same time less than -1. Well, no such number exists (no number that's greater than 8 is also less than -1), so the graph of the conjunction of the inequalities $x > 8$ and $x < -1$ is just an empty number line, with nothing drawn on it.