Function notation

We already know that the math expression $y = 10 - 2x$ is an equation, because the equals sign tells us that the two expressions y and $10 - 2x$ are equivalent.

What we want to say now is that some, but not all, equations are functions. Specifically, a **function** is an equation that only gives one output of the dependent variable for each input of the independent variable.

In an equation defined by x and y, we typically say that the **independent variable** is x and that the **dependent variable** is y, because the output value we get for y "depends on" the input value we choose for x.

So we could say that $y = 10 - 2x$ is "a function y of x" if we never get multiple output values of y for any input value of x. The table below shows some input-output pairs for the equation $y = 10 - 2x$.

x Input	-2	-1	0	1	2
y Output	14	12	10	8	6

We substitute the value of x in the equation to find the associated value of y. For instance, substituting the input $x = 0$ gives an output $y = 10 - 2(0) = 10$.

Because we get a single output y for every input x, and because this will always be the case for any input we choose, we can say that $y = 10 - 2x$ is a function.

Equations that aren't functions

Any equation that gives multiple outputs for a single input is not a function. For example, $y^2 = x$ is not a function, because when we choose an input, like $x = 4$, we get $y = \pm 2$ as a pair of outputs, because both $(-2)^2 = 4$ and $2^2 = 4$.

Keep in mind that it's possible to find some inputs that only give one output, and other inputs that give multiple outputs. In $y^2 = x$, the input $x = 4$ gave two outputs $y = \pm 2$, but the input $x = 0$ gives only one output $y = 0$.

It doesn't matter if there are some inputs that only give one output. An equation is only a function if *all* inputs each only give one output. So if an equation gives one output at every input for all inputs, except for one single input that gives multiple outputs, that single input value is enough to disqualify the entire equation, and that equation is not a function.

The equations we've been dealing with so far in this course have all been equations of lines, quadratics, or other higher-degree polynomials. Every polynomial is always a function, with the exception of perfectly vertical lines. So any vertical line in the form $x = a$ is not a function, but all other lines and polynomials are always functions.

How to express functions

When the equation *is* a function, we have the option to write it in function notation. To express $y = 10 - 2x$ as a function, we'd usually write it as $y(x) = 10 - 2x$ or as $f(x) = 10 - 2x$. The notation $y(x)$ we read as "y of x," and it tells us that we're identifying the function with y, and the function y is defined in terms of the variable x. So when we see $y(x)$, the parentheses don't indicate multiplication; they tell us that we have a function for y in terms of x.

It's actually most common to see functions named with f, where the f stands for "function," but technically we can use any letter that we want as the name of the function. So the function $f(x)$ is the function f defined in terms of the variable x. We can also call x the **argument** of the function.

Input-output pairs satisfy the function

We showed earlier how to find input-output pairs for the function $f(x) = 10 - 2x$, and we listed some of those pairs in a table.

x Input	-2	-1	0	1	2
y Output	14	12	10	8	6

In function notation, we replace x with the desired input, which, for the input-output pairs in this table, looks like

$$f(-2) = 10 - 2(-2) = 10 + 4 = 14$$

$$f(-1) = 10 - 2(-1) = 10 + 2 = 12$$

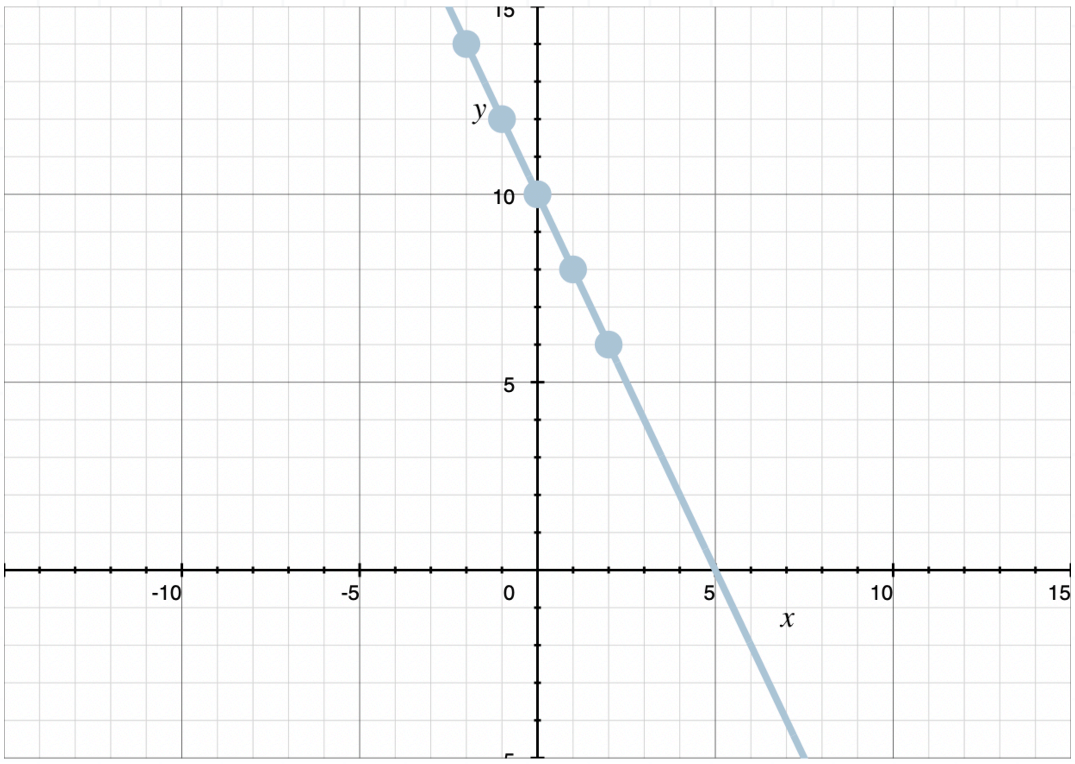
$$f(0) = 10 - 2(0) = 10 - 0 = 10$$

$$f(1) = 10 - 2(1) = 10 - 2 = 8$$

$$f(2) = 10 - 2(2) = 10 - 4 = 6$$

And keep in mind that these input-output pairs, because they satisfy the function, will exist as points on the graph of the function.

So if we sketch $f(x) = 10 - 2x$ in the Cartesian coordinate system, we can see that the graph passes through $(-2,14)$, $(-1,12)$, $(0,10)$, $(1,8)$, and $(2,6)$.



Let's do an example where we evaluate the function at a specific input.

Example

If $f(x) = 10 - 2x$, find $f(-6)$.

Here, -6 is a specific input (a specific value of the variable x), so we'll substitute -6 for the x in the function $f(x)$ (for the x in $10 - 2x$). Remember to use parentheses when plugging in numbers.

$$f(-6) = 10 - 2(-6)$$

$$f(-6) = 10 + 12$$

$$f(-6) = 22$$

Let's try another example with function notation, this time with a quadratic polynomial.

Example

If $f(x) = x^2 - 7x + 12$, find $f(4)$.

Plug in 4 for every x in the expression for $f(x)$ (for every x in $x^2 - 7x + 12$).

$$f(4) = 4^2 - 7(4) + 12$$

$$f(4) = 16 - 28 + 12$$

$$f(4) = -12 + 12$$

$$f(4) = 0$$
