

Multivariable equations

In this lesson we'll look at how to solve a multivariable equation for a certain variable in terms of the others.

When we solve an equation for a variable, we need to use inverse operations to isolate the variable we're solving for.

For example, if we have $y = z + w + x$ and we want to solve the equation for w in terms of x , y , and z , we need to move the terms other than w so that the w is by itself on one side of the equation.

$$y = z + w + x$$

Move the z and the x by subtracting them from both sides.

$$y - z - x = z - z + w + x - x$$

$$y - z - x = w$$

Now we've isolated the variable we were solving for (w). If we want to end up with that variable on the left-hand side, we can switch the two sides of this equation.

$$w = y - z - x$$

Let's do a few more examples so we can get the hang of it.

Example

Solve for t in terms of v , w , x , and z .



$$wx = -tvz$$

We want to get t by itself on one side of the equation.

We can do this by dividing both sides by -1 , then dividing both sides by vz .

$$wx = -tvz$$

$$\frac{wx}{-1} = \frac{-tvz}{-1}$$

$$-wx = tvz$$

$$\frac{-wx}{vz} = \frac{tvz}{vz}$$

$$\frac{-wx}{vz} = t$$

If we'd like, we can move the t to the left-hand side. Another thing we can do is write the negative sign in front of the fraction.

$$t = -\frac{wx}{vz}$$

Let's do one with terms that have coefficients other than 1.

Example

Isolate the variable s .



$$2s - 3t + 2u = 7$$

We need to get s by itself on one side of the equation. Let's move the $2u$ first.

$$2s - 3t + 2u = 7$$

$$2s - 3t + 2u - 2u = 7 - 2u$$

$$2s - 3t = 7 - 2u$$

Next, let's move the $-3t$.

$$2s - 3t + 3t = 7 - 2u + 3t$$

$$2s = 7 - 2u + 3t$$

Finally, we'll divide both sides by 2.

$$\frac{2s}{2} = \frac{7 - 2u + 3t}{2}$$

$$s = \frac{7 - 2u + 3t}{2}$$

Let's do one more, this time with terms that have coefficients other than 1 on both sides.

Example



Solve for b in terms of a and c .

$$3a - 4c - b = 5a - 3c$$

We need to get b by itself on one side of the equation. Let's move the $3a$ first.

$$3a - 4c - b = 5a - 3c$$

$$3a - 3a - 4c - b = 5a - 3a - 3c$$

$$-4c - b = 2a - 3c$$

Next, let's move the $-4c$.

$$-4c + 4c - b = 2a - 3c + 4c$$

$$-b = 2a + c$$

Finally, we'll multiply by -1 to isolate b .

$$-1(-b) = -1(2a + c)$$

$$b = -2a - c$$

