Rationalizing the denominator

What is rationalizing the denominator? When we rationalize the denominator, it means we're taking all of the radicals out of the denominator of a fraction and moving them to the numerator. We do this by multiplying the numerator and denominator by the product of all the radicals in the original denominator.

Remember two facts:

Fact 1: We can multiply any number by 1 without changing its value.

$$\frac{1}{\sqrt{2}} \cdot 1 = \frac{1}{\sqrt{2}}$$

Fact 2: We can divide any nonzero number by itself, and it'll equal 1.

$$\frac{\sqrt{2}}{\sqrt{2}} = 1$$

Let's look at a simple example so we can see how rationalizing the denominator works.

Example

Rationalize the denominator.

$$\frac{1}{\sqrt{2}}$$

We begin by multiplying the numerator and denominator by the radical in the denominator.

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{1\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2}$$

This means

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Notice that we changed only how the number looks; we didn't change its value.

When we have a multistep problem, we might need to perform other operations as well.

Here are some helpful rules to remember about radicals:

Rule 1: $$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Rule 2: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Rule 3: $a \cdot \sqrt{b} = a\sqrt{b}$

Rule 4: $\sqrt{a} \cdot \sqrt{a} = a$

Example

Simplify the expression, making sure to rationalize the denominator.

$$\sqrt{\frac{8}{49}} + \sqrt{\frac{14}{28}}$$

First simplify the fraction in the second radical to lowest terms.

$$\frac{14}{28}$$

$$\frac{1}{2}$$

Now we have

$$\sqrt{\frac{8}{49}} + \sqrt{\frac{1}{2}}$$

When we take the square root of a fraction, we can take the square roots of the numerator and denominator separately. Therefore, we can rewrite the expression as

$$\frac{\sqrt{8}}{\sqrt{49}}+\frac{\sqrt{1}}{\sqrt{2}}$$

Rewrite this by taking the square roots of any perfect squares.

$$\frac{\sqrt{8}}{7}+\frac{1}{\sqrt{2}}$$

Now we need to find a common denominator. Since we have only two terms, we can do this by multiplying the numerator and denominator of each fraction by the denominator of the other fraction.

$$\frac{\sqrt{8}}{7}\cdot\frac{\sqrt{2}}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cdot\frac{7}{7}$$

$$\frac{\sqrt{8}\sqrt{2}}{7\cdot\sqrt{2}}+\frac{1\cdot 7}{\sqrt{2}\cdot 7}$$

$$\frac{\sqrt{8}\sqrt{2}}{7\cdot\sqrt{2}}+\frac{1\cdot 7}{7\cdot\sqrt{2}}$$

$$\frac{\sqrt{16}}{7\sqrt{2}}+\frac{7}{7\sqrt{2}}$$

Now that we have a common denominator, add the fractions.

$$\frac{\sqrt{16}+7}{7\sqrt{2}}$$

$$\frac{4+7}{7\sqrt{2}}$$

$$\frac{11}{7\sqrt{2}}$$

Now we need to rationalize the denominator.

$$\frac{11}{7\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{11\sqrt{2}}{7\sqrt{2}\sqrt{2}}$$

$$\frac{11\sqrt{2}}{7 \cdot 2}$$

$$\frac{11\sqrt{2}}{14}$$

Let's look at one more example.

Example

Simplify the expression, making sure to rationalize the denominator.

$$4\sqrt{\frac{2}{3}} - 5\sqrt{\frac{3}{2}} + \sqrt{150}$$

We know that when we take the square root of a fraction, we can take the square roots of the numerator and denominator separately.

$$4\frac{\sqrt{2}}{\sqrt{3}}-5\frac{\sqrt{3}}{\sqrt{2}}+\sqrt{150}$$

Now we'll factor the 150 (under the radical sign in the last term) as $25 \cdot 6$, which will ultimately help us to simplify things because 25 is a perfect square.

$$4\frac{\sqrt{2}}{\sqrt{3}}-5\frac{\sqrt{3}}{\sqrt{2}}+\sqrt{25 \cdot 6}$$

$$4\frac{\sqrt{2}}{\sqrt{3}}-5\frac{\sqrt{3}}{\sqrt{2}}+\sqrt{25}\sqrt{6}$$

$$\frac{4\sqrt{2}}{\sqrt{3}}-\frac{5\sqrt{3}}{\sqrt{2}}+5\sqrt{6}$$

We can divide the $5\sqrt{6}$ in the last term by 1, which won't change its value.

$$\frac{4\sqrt{2}}{\sqrt{3}}-\frac{5\sqrt{3}}{\sqrt{2}}+\frac{5\sqrt{6}}{1}$$

Now we need to find a common denominator so that we can combine the fractions. We can use the product of the three denominators ($\sqrt{3}$, $\sqrt{2}$, and 1) as our common denominator:

$$\sqrt{3} \cdot \sqrt{2} \cdot 1$$

$$\sqrt{6}$$

We'll multiply the numerator and denominator of each fraction by whatever gets us $\sqrt{6}$ in the denominator.

$$\frac{4\sqrt{2}}{\sqrt{3}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)-\frac{5\sqrt{3}}{\sqrt{2}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)+\frac{5\sqrt{6}}{1}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$\frac{4\sqrt{2}\sqrt{2}}{\sqrt{3}\sqrt{2}}-\frac{5\sqrt{3}\sqrt{3}}{\sqrt{2}\sqrt{3}}+\frac{5\sqrt{6}\sqrt{6}}{1\sqrt{6}}$$

$$\frac{4\cdot 2}{\sqrt{6}}-\frac{5\cdot 3}{\sqrt{6}}+\frac{5\cdot 6}{\sqrt{6}}$$

$$\frac{8}{\sqrt{6}}-\frac{15}{\sqrt{6}}+\frac{30}{\sqrt{6}}$$

$$\frac{23}{\sqrt{6}}$$

Now we need to rationalize the denominator.

$$\frac{23}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{23\sqrt{6}}{6}$$