

Adding and subtracting polynomials

We've already defined expressions and equations, so we know that an expression is a collection of terms.

Now we want to define a **polynomial** as an expression that's the sum and/or difference of a finite number of terms, where the terms include only constants, variables, and positive integer exponents. A polynomial can't include division by a variable (we can't have a variable in any denominator).

The word "polynomial" is the combination of "poly" meaning "many" and "nomial" meaning "term." So we can think broadly about polynomials as "many terms." That being said, polynomials can still include only one term; they don't necessarily have to include multiple terms.

These are all examples of polynomials:

$$3x^2 + 6x - 8$$

$$2x^3 + 5x^2 - 7x + 1$$

$$-12x^2y^2 + y$$

$$3x$$

$$-7$$

$$x^3 + 3xy - 6x^2 - 4$$

$$3x^2 - \frac{1}{2}x + 1$$

But these are not polynomials,

$$3x^{-2} - y^2$$

because of the negative exponent

$$x^{\frac{1}{2}}$$

because of the fractional exponent



$$-\frac{1}{2x} + 1 - y^3 + y^2 \quad \text{because of the variable in the denominator}$$

It's most common to write the terms of a polynomial in descending order of their exponents, so the first term of the polynomial will usually be the term with the largest exponent.

The largest exponent is the **degree** of the polynomial. So, for example, if the largest exponent in the polynomial is 3, then the polynomial is a third-degree polynomial.

The sum or difference of polynomials

In this lesson we want to learn to add and subtract polynomials, which turns out is really just a matter of adding and subtracting like terms, something we already know how to do.

Remember that for the purposes of addition and subtraction, like terms are terms that have the same base and the same exponent. We combine like terms by adding or subtracting the coefficients while keeping the base and the exponent the same.

The sum or difference of polynomials will always itself be a polynomial. Let's do an example where we add two polynomials.

Example

Simplify the expression.



$$(3x^2 + 6x - 8) + (-12x^2 + 1)$$

First, remove the parentheses.

$$3x^2 + 6x - 8 - 12x^2 + 1$$

Group like terms together in descending order of their exponents.

$$(3x^2 - 12x^2) + 6x + (-8 + 1)$$

Combine like terms by performing the addition and/or subtraction of their coefficients.

$$(3 - 12)x^2 + 6x + (-8 + 1)$$

$$-9x^2 + 6x - 7$$

Let's try another example, this time we'll subtract one polynomial from the other.

Example

Simplify the expression.

$$(2x^3 + 5x^2 - 7x + 1) - (x^3 + 3x - 6x^2 - 4)$$



First, remove the parentheses. Because we're subtracting the second polynomial, we need to distribute the negative sign across each term in the second polynomial when we remove the parentheses.

$$2x^3 + 5x^2 - 7x + 1 - x^3 - 3x + 6x^2 + 4$$

Group like terms together in descending order of their exponents.

$$(2x^3 - x^3) + (5x^2 + 6x^2) + (-7x - 3x) + (1 + 4)$$

$$(2 - 1)x^3 + (5 + 6)x^2 + (-7 - 3)x + (1 + 4)$$

$$x^3 + 11x^2 - 10x + 5$$

