72

## Consecutive integers

Remember that **integers** are "whole numbers" that are either positive, negative, or 0, which means we're not including fractions or decimals.

## Consecutive integer word problems

Consecutive integers are integers that are one unit apart from each other.

$$\dots$$
 -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5  $\dots$ 

So, for example, the three consecutive integers that immediately follow -2 are -1, 0, 1, and the three consecutive integers that immediately precede -1 are -4, -3, -2.

Solving for a set of consecutive integers is a word problem we commonly see in Algebra, so let's do some examples.

## **Example**

Find three consecutive integers that sum to 39.

First we'll define a variable, n, as the smallest of these three consecutive integers. Since consecutive integers are one unit apart from each other, the other two consecutive integers must be n + 1 and n + 1 + 1, or n + 2. Since the sum of the three consecutive integers is 39, we can say

$$n + (n + 1) + (n + 2) = 39$$

Removing the parentheses, and then grouping and combining like terms, we get

$$n + n + 1 + n + 2 = 39$$

$$(n+n+n) + (1+2) = 39$$

$$3n + 3 = 39$$

With both sides of the equation simplified, we'll use inverse operations to solve for n.

$$3n + 3 - 3 = 39 - 3$$

$$3n = 36$$

$$\frac{3n}{3} = \frac{36}{3}$$

$$n = 12$$

It's really important to remember here that n=12 isn't automatically the correct answer. We have to look back at the specific question that was asked, and give the answer to it. We were asked to find three consecutive integers that have a sum of 39.

We found n = 12 to be the smallest of the three consecutive integers, so the other two integers are n + 1 = 12 + 1 = 13 and n + 2 = 12 + 2 = 14. We can double-check that the three consecutive integers are 12, 13, and 14 by plugging them back into the sum equation.

| n + (n + 1) + (n + 2) |
|-----------------------|
| 12 + 13 + 14          |
| 25 + 14               |
| 39                    |

Let's look at another common integer problem. This time, we'll be looking for consecutive even integers.

## **Example**

Find three consecutive even integers that have a sum of 54.

First we'll define a variable, n, as the smallest of the three consecutive even integers. The next two consecutive even integers are n + 2 and n + 2 + 2, or n + 4. Since the sum of the three consecutive even integers is 54, we have

$$n + (n + 2) + (n + 4) = 54$$

Removing the parentheses, and then grouping and combining like terms, we get

$$n + n + 2 + n + 4 = 54$$

$$(n + n + n) + (2 + 4) = 54$$



$$3n + 6 = 54$$

With both sides of the equation simplified, we'll use inverse operations to solve for n.

$$3n + 6 - 6 = 54 - 6$$

$$3n = 48$$

$$\frac{3n}{3} = \frac{48}{3}$$

$$n = 16$$

Again, we have to look back at the question that was asked, and give the answer to it. We were asked to find three consecutive even integers with a sum of 54.

What we found is that the smallest of the three consecutive even integers is n = 16. The other two consecutive integers are n + 2 = 16 + 2 = 18 and n + 4 = 16 + 4 = 20. We can double-check that the three consecutive integers are 16, 18, and 20 by plugging them back into the sum equation.

$$n + (n+2) + (n+4)$$

$$16 + 18 + 20$$

$$34 + 20$$

54

