Sum of functions

We can actually do arithmetic operations with functions, like addition and multiplication, in the same way we might do operations with polynomials.

Let's start by looking at how to add add functions, and in the next lesson we'll look at function multiplication.

Adding functions

To add functions, we simply add the equations together, combining terms only when they're alike. So the sum of two functions f and g is

$$(f+g)(x) = f(x) + g(x)$$

We'll only be able to combine terms in f with terms in g if we have like terms. Once we've added the functions together, we can evaluate the sum at a specific value of x.

Alternatively, if we're ultimately trying to find the value of the sum at a particular value of x, we could evaluate each function individually at that value, and then add the results.

Let's do an example so that we can see both of these routes to the solution.

Example

Find $(f+g)(3)$ if $f(x) = x^2 - x + 4$ and $g(x) = x - 2$.

We need to find $(f+g)(3)$, which we could rewrite as $f(3)+g(3)$. So we can input $x=3$ into each function and then add the outputs.

First, let's find $f(3)$.

$$f(x)=x^2-x+4$$

$$f(3)=3^2-3+4$$

$$f(3)=9-3+4$$

$$f(3)=10$$

Now let's find $g(3)$.

$$g(x)=x-2$$

$$g(3)=3-2$$

$$g(3)=1$$

Now we can add the outputs to find the sum.

$$(f+g)(3)=f(3)+g(3)$$

$$(f+g)(3)=10+1$$

$$(f+g)(3)=11$$

We could also have added the functions first, and then plugged in $x=3$ to get the answer.

$$(f+g)(x) = (x^2 - x + 4) + (x - 2)$$

$$(f+g)(x) = x^2 - x + 4 + x - 2$$

$$(f+g)(x) = x^2 + 2$$

Evaluate the sum at $x = 3$.

$$(f+g)(3) = 3^2 + 2$$

$$(f+g)(3) = 9 + 2$$

$$(f+g)(3) = 11$$

Let's try another example of a sum of functions.

Example

Find $(g+h)(-2)$ if $g(x) = x^2 + 5x$ and $h(x) = 3 - x$.

We need to find $(g+h)(-2)$, which we could rewrite as $g(-2) + h(-2)$. So we can input $x = -2$ into the expression for each function and then add the outputs.

First, let's find $g(-2)$.

$$g(x) = x^2 + 5x$$

$$g(-2) = (-2)^2 + 5(-2)$$

$$g(-2) = 4 - 10$$

$$g(-2) = -6$$

Now let's find $h(-2)$.

$$h(x) = 3 - x$$

$$h(-2) = 3 - (-2)$$

$$h(-2) = 5$$

Now we'll find the sum of the functions at $x = -2$.

$$(g + h)(-2) = g(-2) + h(-2)$$

$$(g + h)(-2) = -6 + 5$$

$$(g + h)(-2) = -1$$

We could also have added the functions first, and then plugged in $x = -2$ to get the answer.

$$(g + h)(x) = (x^2 + 5x) + (3 - x)$$

$$(g + h)(x) = x^2 + 5x + 3 - x$$

$$(g + h)(x) = x^2 + 4x + 3$$

Evaluate the sum at $x = -2$.

$$(g + h)(-2) = (-2)^2 + 4(-2) + 3$$

$$(g + h)(-2) = 4 - 8 + 3$$

$$(g+h)(-2) = -4+3$$

$$(g+h)(-2) = -1$$

If the only information we have are pairs of inputs and outputs for each function, instead of their equations, there's only one way to add them, and that's to add the outputs. Suppose, for example, that we have functions f and g given as point sets.

$$f: (1,-9),\ (-2,8),\ (-5,16),\ (3,2)$$

$$g: (1,12),\ (-2,10),\ (-5,9),\ (3,-4)$$

Then the only way to find $(f+g)(-5)$ is to add the values of $f(-5)$ and $g(-5)$.

$$(f+g)(-5) = f(-5) + g(-5)$$

$$(f+g)(-5) = 16+9$$

$$(f+g)(-5) = 25$$