

Topic: Systems of three equations

Question: Use any method to find the solution to the system of equations.

$$x + y - z = 4$$

$$x - y - z = 2$$

$$x + 2y + z = 1$$

Answer choices:

A $(x, y, z) = (-1, 1, 2)$

B $(x, y, z) = (1, -1, 2)$

C $(x, y, z) = (1, 1, -2)$

D $(x, y, z) = (1, 1, 2)$



Solution: C

So that we can stay organized, let's number the equations.

$$\text{[1]} \quad x + y - z = 4$$

$$\text{[2]} \quad x - y - z = 2$$

$$\text{[3]} \quad x + 2y + z = 1$$

Since the coefficients of x and z in [1] are equal to the coefficients of x and z in [2], we'll be able to eliminate the variables x and z (and then solve for y) if we subtract [2] from [1].

$$x + y - z - (x - y - z) = (4) - (2)$$

$$x + y - z - x + y + z = 4 - 2$$

$$2y = 2$$

$$y = 1$$

Let's substitute 1 for y in [2] and [3], which will give us two equations in the variables x and z only. So

$$\text{[2]} \quad x - y - z = 2$$

$$\text{[3]} \quad x + 2y + z = 1$$

become

$$x - 1 - z = 2$$

$$x + 2(1) + z = 1$$



and then

$$\text{[4]} \quad x - z = 3$$

$$\text{[5]} \quad x + z = -1$$

Next, we'll add **[4]** and **[5]** in order to eliminate z and solve for x .

$$(x - z) + (x + z) = (3) + (-1)$$

$$x - z + x + z = 3 - 1$$

$$2x = 2$$

$$x = 1$$

Now that we know that $x = 1$ and $y = 1$, we'll substitute 1 for both x and y in **[1]**, and then solve for z .

$$\text{[1]} \quad x + y - z = 4$$

$$1 + 1 - z = 4$$

$$2 - z = 4$$

$$-z = 2$$

$$z = -2$$

We've found a possible solution, $(1, 1, -2)$. Let's test it in the original system to make sure it satisfies the system.

$$\text{[1]} \quad x + y - z = 4$$



$$1 + 1 - (-2) = 4$$

$$1 + 1 + 2 = 4$$

$$4 = 4$$

and

$$\textbf{[2]} \quad x - y - z = 2$$

$$1 - 1 - (-2) = 2$$

$$1 - 1 + 2 = 2$$

$$2 = 2$$

and

$$\textbf{[3]} \quad x + 2y + z = 1$$

$$1 + 2(1) + (-2) = 1$$

$$1 + 2 - 2 = 1$$

$$1 = 1$$

We've shown that $(1, 1, -2)$ satisfies the system.



Topic: Systems of three equations

Question: Use any method to find the solution to the system of equations.

$$x - y + z = -6$$

$$3x - 4y - z = -4$$

$$-2x + 3y + 4z = 14$$

Answer choices:

A $(x, y, z) = (-60, -46, -8)$

B $(x, y, z) = (60, -46, 8)$

C $(x, y, z) = (-60, 46, 8)$

D $(x, y, z) = (-60, -46, 8)$



Solution: D

So that we can stay organized, let's number the equations.

$$\text{[1]} \quad x - y + z = -6$$

$$\text{[2]} \quad 3x - 4y - z = -4$$

$$\text{[3]} \quad -2x + 3y + 4z = 14$$

We'll add [1] and [2] to eliminate z .

$$(x - y + z) + (3x - 4y - z) = (-6) + (-4)$$

$$x - y + z + 3x - 4y - z = -6 - 4$$

$$\text{[4]} \quad 4x - 5y = -10$$

Now we'll multiply [2] by 4, so that we can add the result to [3] and eliminate z .

$$\text{[2]} \quad 3x - 4y - z = -4$$

$$4(3x - 4y - z) = 4(-4)$$

$$\text{[5]} \quad 12x - 16y - 4z = -16$$

Adding [3] and [5], we get

$$(-2x + 3y + 4z) + (12x - 16y - 4z) = (14) + (-16)$$

$$-2x + 3y + 4z + 12x - 16y - 4z = 14 - 16$$

$$\text{[6]} \quad 10x - 13y = -2$$



With **[4]** and **[6]**, we have a system of two equations in the variables x and y .

$$\mathbf{[4]} \quad 4x - 5y = -10$$

$$\mathbf{[6]} \quad 10x - 13y = -2$$

Let's solve **[4]** for y , and substitute the resulting expression for y into **[6]**, and then solve for x .

$$\mathbf{[4]} \quad 4x - 5y = -10$$

$$-5y = -4x - 10$$

$$5y = 4x + 10$$

$$\mathbf{[7]} \quad y = \frac{4}{5}x + 2$$

Now we'll plug this expression for y into **[6]**.

$$\mathbf{[6]} \quad 10x - 13y = -2$$

$$10x - 13\left(\frac{4}{5}x + 2\right) = -2$$

$$10x - \frac{52}{5}x - 26 = -2$$

$$5\left(10x - \frac{52}{5}x - 26\right) = 5(-2)$$

$$5(10x) + 5\left(-\frac{52}{5}x\right) + 5(-26) = 5(-2)$$



$$50x - 52x - 130 = -10$$

$$-2x = 120$$

$$x = -60$$

Next, we'll substitute -60 for x in **[7]**, and then compute the value of y .

$$\textbf{[7]} \quad y = \frac{4}{5}x + 2$$

$$y = \frac{4}{5}(-60) + 2$$

$$y = 4(-12) + 2$$

$$y = -48 + 2$$

$$y = -46$$

Now that we know that $x = -60$ and $y = -46$, we'll substitute -60 for x and -46 for y in **[1]**, and then solve for z .

$$\textbf{[1]} \quad x - y + z = -6$$

$$-60 - (-46) + z = -6$$

$$-60 + 46 + z = -6$$

$$-14 + z = -6$$

$$z = 8$$

We've found a possible solution, $(-60, -46, 8)$. Let's test it in the original system to make sure it satisfies the system.



$$\text{[1]} \quad x - y + z = -6$$

$$-60 - (-46) + 8 = -6$$

$$-60 + 46 + 8 = -6$$

$$-14 + 8 = -6$$

$$-6 = -6$$

and

$$\text{[2]} \quad 3x - 4y - z = -4$$

$$3(-60) - 4(-46) - 8 = -4$$

$$-180 + 184 - 8 = -4$$

$$4 - 8 = -4$$

$$-4 = -4$$

and

$$\text{[3]} \quad -2x + 3y + 4z = 14$$

$$-2(-60) + 3(-46) + 4(8) = 14$$

$$120 - 138 + 32 = 14$$

$$-18 + 32 = 14$$

$$14 = 14$$

We've shown that $(-60, -46, 8)$ satisfies the system.



Topic: Systems of three equations**Question:** Solve this system of equations for x , y , and z .

$$x + 2z = 3$$

$$3x - 2y + z = -11$$

$$2x + y + 3z = 9$$

Answer choices:

A $(x, y, z) = (3, -1, 5)$

B $(x, y, z) = (-1, 3, 2)$

C $(x, y, z) = (2, 5, -1)$

D $(x, y, z) = (-1, 5, 2)$



Solution: D

So that we can stay organized, let's number the equations.

$$\text{[1]} \quad x + 2z = 3$$

$$\text{[2]} \quad 3x - 2y + z = -11$$

$$\text{[3]} \quad 2x + y + 3z = 9$$

Solve [1] for x .

$$x + 2z = 3$$

$$x = 3 - 2z$$

Substitute this expression for x into [2] and [3], and then simplify.

$$\text{[2]} \quad 3x - 2y + z = -11$$

$$3(3 - 2z) - 2y + z = -11$$

$$9 - 6z - 2y + z = -11$$

$$-2y - 5z = -20$$

and

$$\text{[3]} \quad 2x + y + 3z = 9$$

$$2(3 - 2z) + y + 3z = 9$$

$$6 - 4z + y + 3z = 9$$

$$y - z = 3$$



Now we can solve the resulting equations as a system of two equations in the variables y and z .

$$-2y - 5z = -20$$

$$y - z = 3$$

We'll multiply the second equation by 2.

$$y - z = 3$$

$$2(y - z) = 2(3)$$

$$2y - 2z = 6$$

To eliminate y , we can add this equation to the equation $-2y - 5z = -20$ that we found earlier.

$$(-2y - 5z) + (2y - 2z) = (-20) + (6)$$

$$-2y - 5z + 2y - 2z = -20 + 6$$

$$-7z = -14$$

$$z = 2$$

Now we'll substitute 2 for z in the equation $y - z = 3$ that we found earlier, and then solve for y .

$$y - z = 3$$

$$y - 2 = 3$$

$$y = 5$$



Finally, we'll substitute 2 for z in the equation $x = 3 - 2z$ that we found much earlier, and then compute the value of x .

$$x = 3 - 2z$$

$$x = 3 - 2(2)$$

$$x = 3 - 4$$

$$x = -1$$

The solution is $(-1, 5, 2)$. If we plug the coordinates of this point into all three of the original equations, we'll see that it satisfies all of them.

