Fractional equations

In this lesson we'll look at how to solve equations that include fractions as coefficients and standalone terms. There are a couple of things we need to remember about multiplying fractions.

1. Multiplying a fraction by its reciprocal will always give us a value of 1.

   For example, $4/5$ has a reciprocal of $5/4$ because

$$\frac{4}{5} \cdot \frac{5}{4} = 1$$

2. To clear a fraction from an equation, multiply both sides of the equation by the fraction's denominator.

   For example, to clear the 2 from the fraction in $5x + 1/2 = 12$, multiply both sides of the equation by 2.

$$2\left(5x + \frac{1}{2}\right) = 2(12)$$

$$2(5x) + 2\left(\frac{1}{2}\right) = 2(12)$$

$$10x + 1 = 24$$

In general, we'll use the same set of steps to solve fractional equations.

1. Find the lowest common denominator (LCD).

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2. Multiply both sides of the equation by the LCD to remove the fractions.

3. Solve the equation and check the solution.

Let's do a few examples where we solve an equation with a fraction. First let's look at an equation that has a fractional coefficient.

Example

Solve for the variable.

$$\frac{4}{5}n = 20$$

To get rid of a fractional coefficient, we'll multiply both sides of the equation by the fraction's reciprocal, because that'll change the coefficient to 1.

$$\frac{4}{5}n = 20$$

$$\frac{5}{4} \cdot \frac{4}{5}n = \frac{5}{4} \cdot 20$$

$$\frac{20}{20}n = \frac{100}{4}$$

$$1n = 25$$

$$n = 25$$

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If we have a fractional coefficient and another term on the same side of the equation, we can isolate the term with the variable and then multiply both sides by the reciprocal of the fractional coefficient.

Example

Solve for the variable.

$$\frac{4}{7}x + 14 = 22$$

First isolate the fractional term.

$$\frac{4}{7}x + 14 - 14 = 22 - 14$$

$$\frac{4}{7}x = 8$$

Now get rid of the fractional coefficient by multiplying both sides of the equation by the reciprocal of $4/7$.

$$\frac{7}{4} \cdot \frac{4}{7}x = \frac{7}{4} \cdot 8$$

$$\frac{28}{28}x = \frac{56}{4}$$

$$1x = 14$$

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$$x = 14$$

We could also solve this problem by first clearing the fraction. In order to get rid of the fraction, we have to multiply both sides of the equation by the fraction's denominator.

$$7\left(\frac{4}{7}x + 14\right) = 7(22)$$

$$7 \cdot \frac{4}{7}x + 7 \cdot 14 = 7 \cdot 22$$

$$4x + 98 = 154$$

Now we can solve for the variable using inverse operations.

$$4x + 98 - 98 = 154 - 98$$

$$4x = 56$$

$$x = 14$$

Let's do one more example.

Example

Solve for the variable.

$$\frac{2x}{5} + \frac{x}{3} = \frac{3}{5}$$

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The lowest common denominator (LCD) of the three fractions is 15, because 15 is the least common multiple (LCM) of the denominators in the equation, 5 and 3. So in order to remove the fractions but keep the equation balanced, we'll multiply both sides of the equation by 15.

$$15\left(\frac{2x}{5}+\frac{x}{3}\right)=15\left(\frac{3}{5}\right)$$

$$15\left(\frac{2x}{5}\right)+15\left(\frac{x}{3}\right)=15\left(\frac{3}{5}\right)$$

$$6x+5x=9$$

$$11x=9$$

$$x=\frac{9}{11}$$

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