

**Topic:** Center and radius of a circle**Question:** Find the center and radius of the circle.

$$4x^2 + 4y^2 + 4x - 12y + 1 = 0$$

**Answer choices:**

- |   |  |                         |
|---|--|-------------------------|
| A | Center at $\left(\frac{1}{2}, \frac{3}{2}\right)$  | Radius of $\frac{3}{2}$ |
| B | Center at $\left(-\frac{1}{2}, \frac{3}{2}\right)$ | Radius of $\frac{3}{2}$ |
| C | Center at $\left(\frac{1}{2}, \frac{3}{2}\right)$  | Radius of $\frac{1}{2}$ |
| D | Center at $\left(-\frac{1}{2}, \frac{3}{2}\right)$ | Radius of $\frac{1}{2}$ |



**Solution: B**

In order to find the center and radius, we need to convert the equation of the circle to standard form,  $(x - h)^2 + (y - k)^2 = r^2$ , where  $h$  and  $k$  are the coordinates of the center and  $r$  is the radius.

We'll begin by grouping the  $x$  terms separately from the  $y$  terms, and moving the constant term to the right side of the equation.

$$4x^2 + 4y^2 + 4x - 12y + 1 = 0$$

$$4x^2 + 4y^2 + 4x - 12y = -1$$

In standard form, the coefficients of the  $x^2$  term and the  $y^2$  term must be equal to 1. Since the coefficient of each of those terms is now 4, we'll factor out a 4 on the left side of the equation and then divide both sides by 4.

$$4(x^2 + y^2 + x - 3y) = -1$$

$$x^2 + y^2 + x - 3y = -\frac{1}{4}$$

In order to get this equation into standard form, we need to complete the square on both  $x$  and  $y$ .

$$(x^2 + x) + (y^2 - 3y) = -\frac{1}{4}$$

To complete the square on  $x$ , we need to find the number  $a$  that satisfies the equation

$$x^2 + x + a^2 = (x + a)^2$$



That is, we need to find the number  $a$  for which

$$x^2 + x + a^2 = x^2 + 2ax + a^2$$

This means that the coefficient of the  $x$  term of the expression inside the first set of parentheses must be equal to  $2a$ . That coefficient is 1, so we'll set  $2a$  to 1 and solve for  $a$ .

$$2a = 1 \quad \rightarrow \quad a = \frac{1}{2}$$

To keep our equation balanced, we need to add and subtract  $a^2$  ( $1/4$ ) inside that set of parentheses and then regroup.

$$(x^2 + x) + (y^2 - 3y) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + (y^2 - 3y) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + (y^2 - 3y) = -\frac{1}{4}$$

To complete the square on  $y$ , we need to find the number  $b$  that satisfies the equation

$$y^2 - 3y + b^2 = (y + b)^2$$

That is, we need to find the number  $b$  for which

$$y^2 - 3y + b^2 = y^2 + 2by + b^2$$



This means that the coefficient of the  $y$  term of the expression inside the second set of parentheses must be equal to  $2b$ . That coefficient is  $-3$ , so we'll set  $2b$  to  $-3$  and solve for  $b$ .

$$2b = -3 \rightarrow b = -\frac{3}{2}$$

To keep our equation balanced, we need to add and subtract  $b^2$  ( $9/4$ ) inside that set of parentheses and then regroup.

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + (y^2 - 3y) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + \left(y^2 - 3y + \frac{9}{4} - \frac{9}{4}\right) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + \left(y^2 - 3y + \frac{9}{4}\right) - \frac{9}{4} = -\frac{1}{4}$$

Moving the  $-1/4$  and  $-9/4$  to the right side, we have

$$\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 - 3y + \frac{9}{4}\right) = -\frac{1}{4} + \frac{1}{4} + \frac{9}{4}$$

Factoring the expressions in parentheses and simplifying the right side, we get

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

If we think of  $x + (1/2)$  and  $9/4$  as  $x - (-1/2)$  and  $(3/2)^2$ , respectively, we'll see that the center of the circle is at



$$(h, k) = \left(-\frac{1}{2}, \frac{3}{2}\right)$$

and its radius is

$$r = \frac{3}{2}$$



**Topic:** Center and radius of a circle

**Question:** Find the center and radius of the given circle.

$$x^2 + y^2 - 6y = 5$$

**Answer choices:**

- A Center is (0,3). Radius is  $\sqrt{14}$ .
- B Center is (0, - 3). Radius is  $\sqrt{14}$ .
- C Center is (0,3). Radius is 14.
- D Center is (0, - 3). Radius is 14.



**Solution: A**

$x^2$  is already a perfect square, so we'll complete the square on  $y$ .

$$x^2 + (y^2 - 6y) = 5$$

To do that, we need to find the number  $a$  that satisfies the equation

$$y^2 - 6y + a^2 = (y + a)^2$$

That is, we need to find the number  $a$  for which

$$y^2 - 6y + a^2 = y^2 + 2ay + a^2$$

This means that the coefficient of the  $y$  term of the expression inside the parentheses must be equal to  $2a$ . That coefficient is  $-6$ , so we'll set  $2a$  equal to  $-6$  and solve for  $a$ .

$$2a = -6 \quad \rightarrow \quad a = -3$$

To keep our equation balanced, we need to add and subtract  $a^2$  (9) inside the parentheses and then regroup.

$$x^2 + (y^2 - 6y) = 5$$

$$x^2 + (y^2 - 6y + 9 - 9) = 5$$

$$x^2 + (y^2 - 6y + 9) - 9 = 5$$

Moving the  $-9$  to the right side, we have

$$x^2 + (y^2 - 6y + 9) = 5 + 9$$



Factoring the expression in parentheses and simplifying the right side, we get

$$x^2 + (y - 3)^2 = 14$$

If we think of  $x$  and 14 as  $x - 0$  and  $(\sqrt{14})^2$ , respectively, we'll see that the center of the circle is at  $(h, k) = (0, 3)$  and the radius is  $\sqrt{14}$ .





**Topic:** Center and radius of a circle**Question:** Find the center and radius of the given circle.

$$x^2 + y^2 + 10x - 4y + 13 = 0$$

**Answer choices:**

- A Center is  $(-5, 2)$ . Radius is 16.
- B Center is  $(5, -2)$ . Radius is 4.
- C Center is  $(-5, 2)$ . Radius is 4.
- D Center is  $(5, -2)$ . Radius is 16.



**Solution: C**

Starting with

$$x^2 + y^2 + 10x - 4y + 13 = 0$$

we'll group the terms in  $x$  separately from the terms in  $y$ , and subtract 13 from both sides.

$$x^2 + 10x + y^2 - 4y = -13$$

We need to complete the square on both  $x$  and  $y$ .

$$(x^2 + 10x) + (y^2 - 4y) = -13$$

To complete the square on  $x$ , we need to find the number  $a$  that satisfies the equation

$$x^2 + 10x + a^2 = (x + a)^2$$

That is, we need to find the number  $a$  for which

$$x^2 + 10x + a^2 = x^2 + 2ax + a^2$$

This means that the coefficient of the  $x$  term of the expression inside the first set of parentheses must be equal to  $2a$ . That coefficient is 10, so we'll set  $2a$  equal to 10 and solve for  $a$ .

$$2a = 10 \quad \rightarrow \quad a = 5$$

To keep our equation balanced, we need to add and subtract  $a^2$  (25) inside that set of parentheses and then regroup.



$$(x^2 + 10x) + (y^2 - 4y) = -13$$

$$(x^2 + 10x + 25 - 25) + (y^2 - 4y) = -13$$

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y) = -13$$

To complete the square on  $y$ , we need to find the number  $b$  that satisfies the equation

$$y^2 - 4y + b^2 = (y + b)^2$$

That is, we need to find the number  $b$  for which

$$y^2 - 4y + b^2 = y^2 + 2by + b^2$$

This means that the coefficient of the  $y$  term of the expression inside the second set of parentheses must be equal to  $2b$ . That coefficient is  $-4$ , so we'll set  $2b$  equal to  $-4$  and solve for  $b$ .

$$2b = -4 \quad \rightarrow \quad b = -2$$

To keep our equation balanced, we need to add and subtract  $b^2$  (4) inside that set of parentheses and then regroup.

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y) = -13$$

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y + 4 - 4) = -13$$

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y + 4) - 4 = -13$$

Moving the  $-25$  and  $-4$  to the right side, we have

$$(x^2 + 10x + 25) + (y^2 - 4y + 4) = -13 + 25 + 4$$



Factoring the expressions in parentheses and simplifying the right side, we get

$$(x + 5)^2 + (y - 2)^2 = 16$$

If we think of  $x + 5$  and 16 as  $x - (-5)$  and  $4^2$ , respectively, we'll see that the center of the circle is at  $(h, k) = (-5, 2)$  and the radius is 4.

