Understood 1

We talked earlier about x as a variable, and about how, in a term like $3x$, the 3 is a constant coefficient on x that multiplies it. So $3x$ is "3 multiplied by x." What we want to say now is that, while x by itself looks like it has no coefficient, it actually has a coefficient of 1.

The "Understood 1"

In other words, whenever anything has a coefficient of 1, we just don't write it, because we agree that it's implied. So when we see x, we know automatically that it means $1x$. So think about $3x + x$ as $3x + 1x$, which simplifies to $4x$. We'll talk more later about how to add like terms like $3x + x$.

The same goes for subtraction. We should know that $3x - x$ is means $3x - 1x$, which simplifies to $2x$.

And if we think about it more, we realize that essentially everything has an understood coefficient of 1. To pick a random example, even the number 7 we could think of as $1(7)$. There's can always be an implied coefficient of 1, because we know that multiplying something by 1 doesn't change its value (because 1 is the identity number for multiplication, which we learned about in Pre-Algebra).

This idea of the "understood 1" also extends to denominators. Every value has an implied denominator of 1, since dividing something by 1 doesn't change its value. So x is understood to be $x/1$, in the same way that 7 is understood to be $7/1$.

And finally, we can extend this idea to exponents, so x is understood to be x^1, and 7 is understood to be 7^1, since raising any value to the power of 1 doesn't change its value.

It's a little absurd, but technically we could combine this "understood 1" rule for coefficients, denominators, and exponents, and rewrite x and 7 as

$$x = \frac{1x^1}{1} \qquad 7 = \frac{1(7)^1}{1}$$

Why does this matter?

Let's be honest, it might seem a little odd to make this weird point that $x = 1x^1/1$. So why are we even bothering?

We've already seen that this concept helps us simplify something like $3x + x$ or $3x - x$, but let's look at some more examples to see when this "understood 1" comes into play.

Example

Rewrite the expression as one fraction.

$$\frac{2}{3} + x$$

If we want to rewrite this sum as only one fraction, our first step is to realize that x by itself is actually a fraction, too. If we rewrite x as $x/1$, now we have the sum of two fractions. Start by simplifying the expression in parentheses, using the fact that $x = 1x$.

$$\frac{2}{3} + \frac{x}{1}$$

Remember from Pre-Algebra that, in order to add fractions, we need a common denominator, meaning that the denominators have to the be same. Right now, one denominator is 3 and the other is 1. If we multiply the second fraction by $3/3$ (which is like multiplying by 1, and therefore doesn't change the value of the original fraction), then we'll make the denominators the same.

$$\frac{2}{3} + \frac{x \cdot 3}{1 \cdot 3}$$

$$\frac{2}{3} + \frac{3x}{3}$$

Now that the denominators are equivalent, we can add the fractions. When we do, the denominator stays the same, and the numerators get added.

$$\frac{2 + 3x}{3}$$

This time let's go in the opposite direction, by simplifying an expression that contains some "understood 1s" that we can eliminate.

Example

Simplify the expression.

$$\frac{1}{1(1x^1)}$$

We know that x^1 is the same as x by itself, so we can remove the exponent without changing the value of the expression.

$$\frac{1}{1(1x)}$$

We know that $1x$ is the same as x by itself, so we remove the coefficient without changing the value of the expression.

$$\frac{1}{1(x)}$$

Within the denominator, the parentheses indicate multiplication, so we have 1 multiplied by x. We can simplify the denominator to $1x$, and then remove the "understood 1" coefficient, and the expression becomes just

$$\frac{1}{x}$$

This is as far as we can simplify. The "understood 1" concept allows us to simplify $x/1$ to just x, but we can't simplify $1/x$ to just x. Dividing something by 1 doesn't change its value, which is why $x/1$ simplifies to x. But dividing 1 by x is the opposite operation and doesn't simplify the same way.