Algebra 1 Workbook Solutions

Systems of equations

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TWO-STEP PROBLEMS

1. Why can't we solve this two-step problem?

If $2(x-1)-3=9+x$, what is $y+2$?

Solution:

We can't solve the problem because there's no y variable in the first equation, so we can't get a value from the first equation to plug into $y+2$.

2. If $5-2x=17$, what is $x-1$?

Solution:

Solve the first equation for x.

$$5-2x=17$$

$$-2x=12$$

$$x=-6$$

Now plug $x=-6$ into the second expression.

$$x-1$$

$-6 - 1$

-7

3. If $3(2 - x) + 5 = -(4x - 2)$, what is $(x/2) + 1$?

Solution:

Solve the first equation for x.

$3(2 - x) + 5 = -(4x - 2)$

$6 - 3x + 5 = -4x + 2$

$11 - 3x = -4x + 2$

$9 = -x$

$x = -9$

Now plug $x = -9$ into the second expression.

$\frac{x}{2} + 1$

$\frac{-9}{2} + 1$

$-\frac{9}{2} + \frac{2}{2}$

$$-\frac{7}{2}$$

4. If $2(x+y)-6=3$, what is $x+y-1$?

Solution:

Solve the first equation for x.

$$2(x+y)-6=3$$

$$2(x+y)=9$$

$$x+y=\frac{9}{2}$$

Now substitute this value into the expression.

$$x+y-1$$

$$\frac{9}{2}-1$$

$$\frac{9}{2}-\frac{2}{2}$$

$$\frac{7}{2}$$

5. What went wrong in this solution?

If $2x + 3 = 7$, what is $x/3$?

$$2x + 3 = 7$$

$$2x = 4$$

$$\frac{x}{3} = \frac{4}{3}$$

Solution:

The equation wasn't solved for x. In the second step, it should be $2x = 4$ gives $x = 2$. Then $x = 2$ should get plugged into the expression $x/3$, $x/3 = 2/3$.

6. If $a + 2b = 6 - a$ and $b = 1$, what is $a/2$?

Solution:

First, plug $b = 1$ into $a + 2b = 6 - a$ and solve for a.

$$a + 2(1) = 6 - a$$

$$2a = 4$$

$$a = 2$$

Then plug $a = 2$ into $a/2$.

$$\frac{a}{2}$$

$$\frac{2}{2}$$

$$1$$

SOLVING SYSTEMS WITH SUBSTITUTION

1. Find the unique solution to the system of equations.

$$-x + 2y = 6$$

$$3x = y - 10$$

Solution:

Solve for x in the second equation.

$$3x = y - 10$$

$$x = \frac{y - 10}{3}$$

Plug this value for x into the first equation, then solve for y.

$$-x + 2y = 6$$

$$-\frac{y - 10}{3} + 2y = 6$$

$$-y + 10 + 6y = 18$$

$$5y = 8$$

$$y = \frac{8}{5}$$

Plug $y = 8/5$ back into the equation we found for x.

$$x = \frac{y - 10}{3}$$

$$x = \frac{\frac{8}{5} - 10}{3}$$

$$x = \frac{\frac{8}{5} - \frac{50}{5}}{3}$$

$$x = -\frac{42}{5} \cdot \frac{1}{3}$$

$$x = -\frac{14}{5}$$

The unique solution to the system is

$$\left(-\frac{14}{5}, \frac{8}{5}\right)$$

2. What is the easiest variable to get by itself? Set up but do not solve the substitution.

$$2y - x = 7$$

$$3x = 9 - 18y$$

Solution:

It’s easiest to solve for the x variable in the second equation by dividing both sides by 3 and then simplifying.

$$x = \frac{9 - 18y}{3}$$

$$x = \frac{9}{3} - \frac{18y}{3}$$

$$x = 3 - 6y$$

3. Find the unique solution to the system of equations.

$$-5x + y = 8$$

$$y = 3x - 8$$

Solution:

Taking the value for y given in the second equation as $y = 3x - 8$, we’ll substitute for y in the first equation.

$$-5x + y = 8$$

$$-5x + (3x - 8) = 8$$

$$-5x + 3x - 8 = 8$$

$$-2x = 16$$

$$x = -8$$

Now substitute $x = -8$ into the second equation to find a value for y.

$$y = 3x - 8$$

$$y = 3(-8) - 8$$

$$y = -32$$

The unique solution to the system is

$$(-8, -32)$$

4. Find the unique solution to the system of equations.

$$3 - y = 2x$$

$$-4x + 10 = 2y$$

Solution:

Solve the second equation for y.

$$-4x + 10 = 2y$$

$$-2x + 5 = y$$

Plug $y = -2x + 5$ into the first equation.

$$3 - y = 2x$$

$$3-(-2x+5)=2x$$

$$3+2x-5=2x$$

$$-2+2x=2x$$

$$-2=0$$

Since this is not true, there is no solution to the system.

5. What went wrong if a substitution was made in the system and the result was $2x-2-x=7$?

$$y=x-2$$

$$2y-x=7$$

Solution:

When substituting $y=x-2$ into the second equation, we get

$$2y-x=7$$

$$2(x-2)-x=7$$

$$2x-4-x=7$$

Therefore, in the substitution given, the 2 was not distributed to the -2.

6. Find the unique solution to the system of equations.

$$5y = 6 - 2x$$

$$6x + 15y = 18$$

Solution:

Solve for y in the first equation.

$$5y = 6 - 2x$$

$$y = \frac{6 - 2x}{5}$$

Plug this value for y into the second equation.

$$6x + 15y = 18$$

$$6x + 15\left(\frac{6 - 2x}{5}\right) = 18$$

$$6x + 3(6 - 2x) = 18$$

$$6x + 18 - 6x = 18$$

$$18 = 18$$

Since this equation is true, but we don't find a specific value for either variable, there are infinitely many solutions.

SOLVING SYSTEMS WITH ELIMINATION

1. What's the easiest way to set up the elimination method for the system of equations? Set up but do not solve the elimination.

$$6y - 3x = 8$$

$$x - 4y = 5$$

Solution:

The easiest way to solve the elimination is to multiply the second equation by 3 to get

$$x - 4y = 5$$

$$3x - 12y = 15$$

Then add the two equations together to eliminate x from the system.

$$6y - 3x + (3x - 12y) = 8 + (15)$$

$$6y - 12y = 8 + 15$$

$$-6y = 23$$

2. Find the unique solution to the system of equations.

$$2x - y = 5$$

$$-3x + y = 7$$

Solution:

If we add the two equations to eliminate y, we get

$$2x - y + (-3x + y) = 5 + (7)$$

$$2x - 3x = 12$$

$$-x = 12$$

$$x = -12$$

Plug $x = -12$ back into the second equation.

$$-3x + y = 7$$

$$-3(-12) + y = 7$$

$$y = -29$$

The solution to the system is

$$(-12, -29)$$

3. What went wrong if an elimination was done in the system and the result was $2y = 3$?

$$-4x + 3y = 7$$

$$-4x - y = 4$$

Solution:

When subtracting the two equations in order to eliminate x, $-y$ in the second equation was added, instead of subtracted. The elimination method should have given $4y = 3$.

■ 4. Find the unique solution to the system of equations.

$$x = 2y - 5$$

$$-3x + 6y = 15$$

Solution:

Multiplying the first equation by 3 gives

$$x = 2y - 5$$

$$3x = 6y - 15$$

Then adding $3x = 6y - 15$ to $-3x + 6y = 15$ gives

$$3x - 6y + (-3x + 6y) = -15 + (15)$$

$$3x - 6y - 3x + 6y = -15 + 15$$

$$-6y + 6y = -15 + 15$$

$$0 = 0$$

This is always true, so there are infinitely many solutions to the system of equations.

5. Find the unique solution to the system of equations.

$$4 - 2x = 6y$$

$$7 = x + 3y$$

Solution:

Rewrite the equations as

$$4 = 2x + 6y$$

$$7 = x + 3y$$

Multiplying the second equation by -2 gives

$$7 = x + 3y$$

$$-14 = -2x - 6y$$

Then adding the two equations gives

$$2x + 6y + (-2x - 6y) = 4 + (-14)$$

$$2x + 6y - 2x - 6y = 4 - 14$$

$$0 = -10$$

Since this is not true, there is no solution to the system of equations.

■ 6. Find the unique solution to the system of equations.

$$x = 2y - 8$$

$$3y = x + 5$$

Solution:

Rewrite the equations as

$$x - 2y = -8$$

$$-x + 3y = 5$$

Adding the two equations gives

$$x - 2y + (-x + 3y) = -8 + 5$$

$$x - 2y - x + 3y = -8 + 5$$

$$y = -3$$

Substitute $y = -3$ into the first equation to solve for x.

$$x = 2y - 8$$

$$x = 2(-3) - 8$$

$$x = -6 - 8$$

$$x = -14$$

Therefore, the solution to the system of equations is

$$(-14, -3)$$

SOLVING SYSTEMS THREE WAYS

1. Explain why using the graphing method would make the system easy to solve.

$$y = 3x - 4$$

$$y - 3 = 2(x + 1)$$

Solution:

The first equation is easy to graph because it's in slope-intercept form, $y = mx + b$. And the second equation is easy to graph because it's in point-slope form, $y - y_1 = m(x - x_1)$.

2. Find the unique solution to the system of equations using the elimination method.

$$2y = x + 5$$

$$3x - 2y = 11$$

Solution:

Rewrite the system as

$$-x + 2y = 5$$

$$3x - 2y = 11$$

Adding the two equations and solving for x gives

$$-x + 2y + (3x - 2y) = 5 + (11)$$

$$-x + 2y + 3x - 2y = 5 + 11$$

$$2x = 16$$

$$x = 8$$

Substitute $x = 8$ into the first equation.

$$2y = x + 5$$

$$2y = 8 + 5$$

$$y = \frac{13}{2}$$

The unique solution to the system of equations is

$$\left(8, \frac{13}{2}\right)$$

3. In words, describe the graphical solution to a system of equations.

Solution:

The solution to a system of equations on a graph is the intersection point of the two graphs.

4. Find the unique solution to the system of equations using the substitution method.

$$5y + x = 4$$

$$3y - 3x = 6$$

Solution:

Solve the first equation for x.

$$5y + x = 4$$

$$x = 4 - 5y$$

Substitute this into the second equation.

$$3y - 3x = 6$$

$$3y - 3(4 - 5y) = 6$$

$$3y - 12 + 15y = 6$$

$$18y = 18$$

$$y = 1$$

Plug $y = 1$ into the equation for x.

$$x = 4 - 5y$$

$$x = 4 - 5(1)$$

$$x = -1$$

The solution to the system of equations is

$$(-1,1)$$

5. Explain why the elimination method is a good way to solve this particular system.

$$3y - 2x = 7$$

$$2x = 4 - 6y$$

Solution:

If we add the two equations, the x terms immediately cancel, making it a very easy elimination method problem.

6. Find the unique solution to the system of equations using the graphing method.

$$y - 2 = -(x + 1)$$

$y = x + 1$

Solution:

In order to graph these equations, let's put both of them into slope-intercept form. We get

$y - 2 = -(x + 1)$

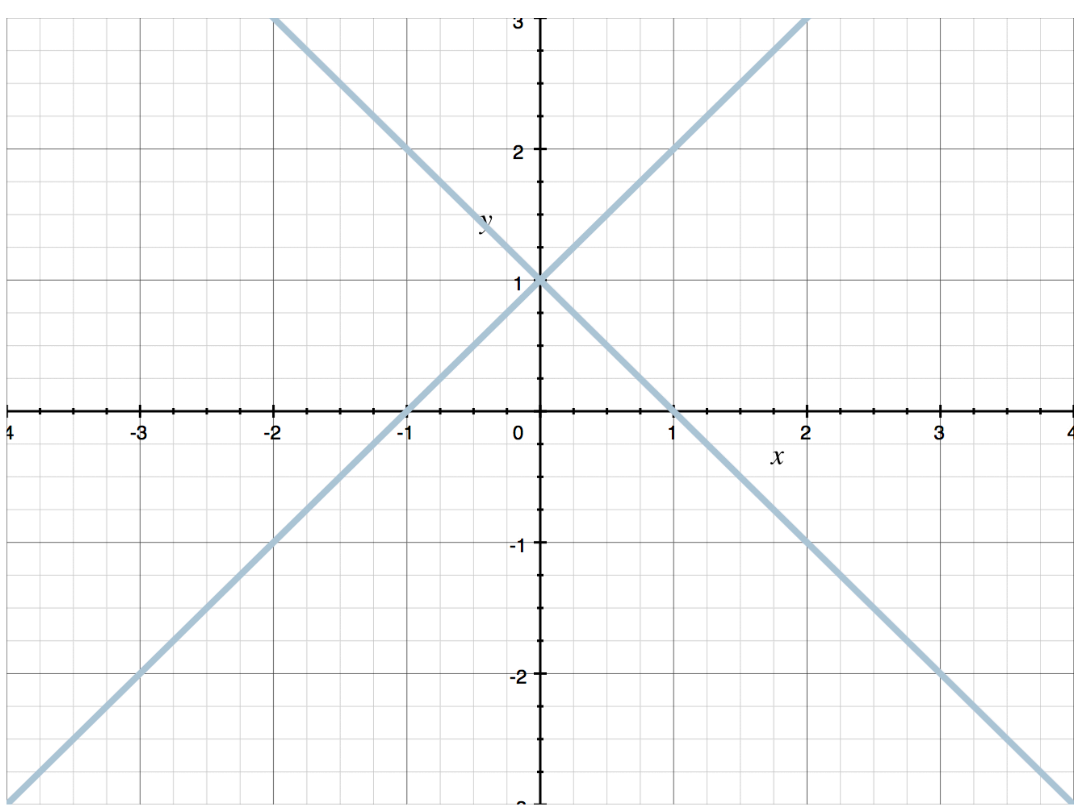
$y - 2 = -x - 1$

$y = -x + 1$

and

$y = x + 1$

The line $y = -x + 1$ intersects the y-axis at 1, and has a slope of -1. The line $y = x + 1$ intersects the y-axis at 1, and has a slope of 1.



From the sketch of the two lines, we can see that the intersection point is $(0,1)$ along the vertical axis, which means $(0,1)$ is the solution to the system.

SYSTEMS OF LINEAR INEQUALITIES

1. Graph the solution to the system of linear inequalities.

$$y > x + 1$$

$$y \leq 5 - x$$

Solution:

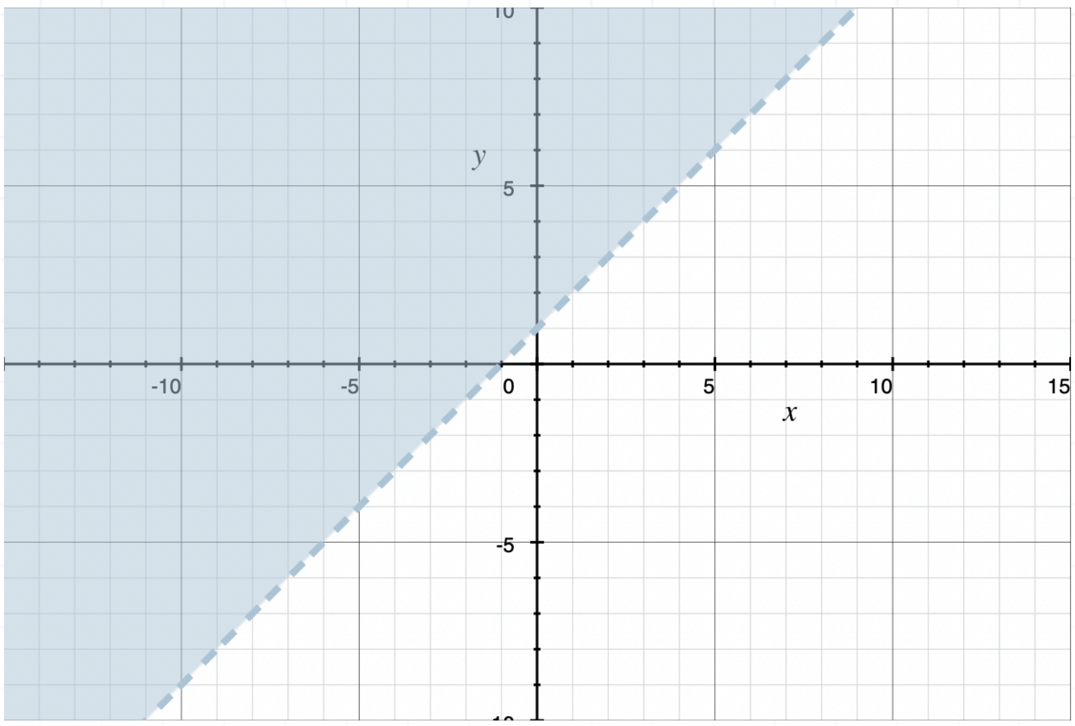
For $y = x + 1$, the slope is $m = 1$ and the y-intercept is $(0,1)$. The $>$ in $y > x + 1$ indicates the need for a dashed boundary line. Now let's test the origin to determine where to shade.

$$y > x + 1$$

$$0 > 0 + 1$$

$$0 > 1$$

Because this is a false statement, we shade away the origin .



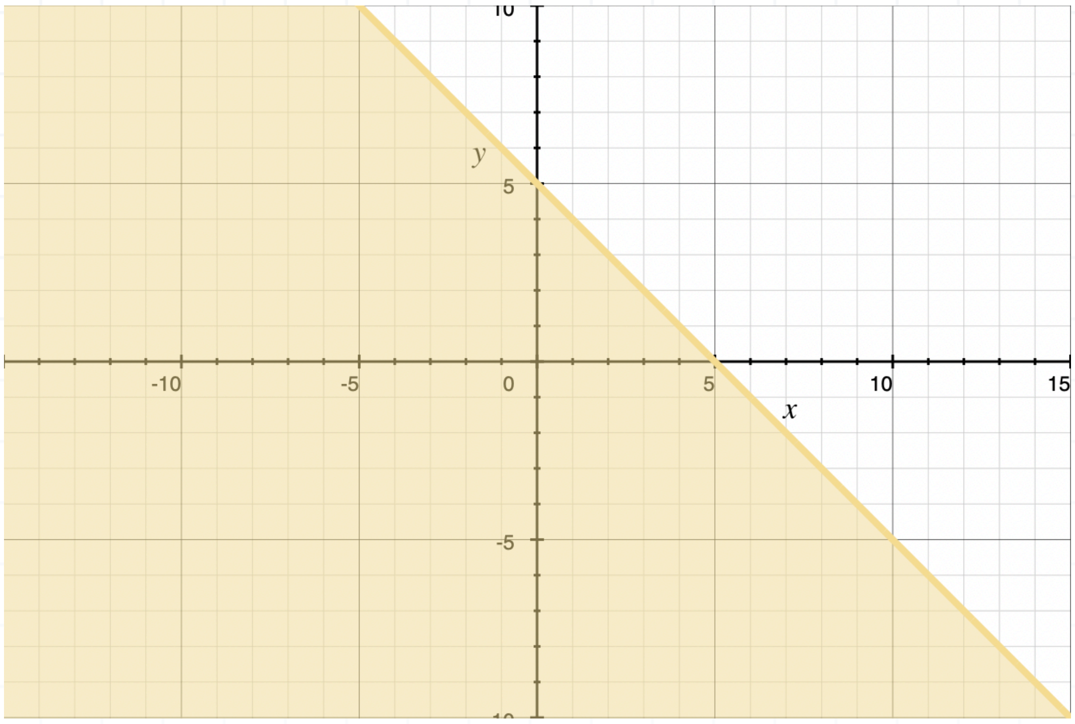
For $y = 5 - x$, the slope is $m = -1$ and the y-intercept is $(0,5)$. The $\leq$ in $y \leq 5 - x$ indicates the need for a solid boundary line. Now let's test the origin again.

$$y \leq 5 - x$$

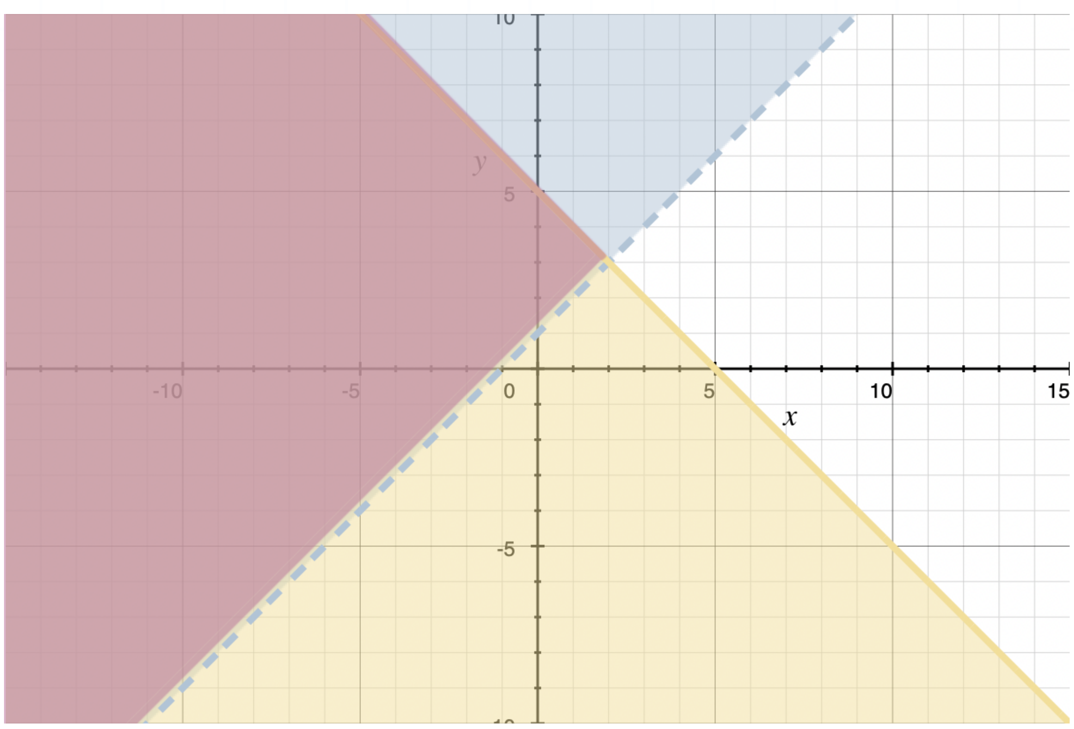
$$0 \leq 5 - 0$$

$$0 \leq 5$$

Because this is a true statement, we shade toward from the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



■ 2. Graph the solution to the system of linear inequalities.

$$2x + 2y \geq 4$$

$$y > -1$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$2x + 2y \geq 4$$

$$2y \geq -2x + 4$$

$$y \geq -x + 2$$

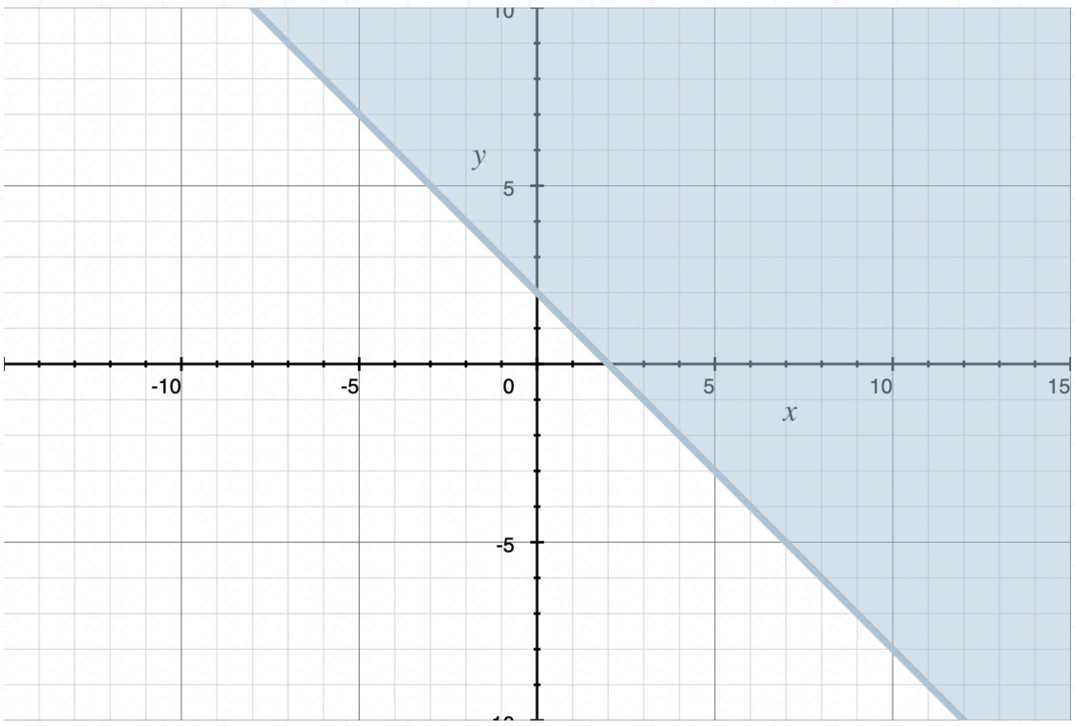
For $y = -x + 2$, the slope is $m = -1$ and the y-intercept is $(0,2)$. The $\geq$ in $y \geq -x + 2$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \geq -x + 2$$

$$0 \geq -0 + 2$$

$$0 \geq 2$$

Because this is a false statement, we shade away from the origin.

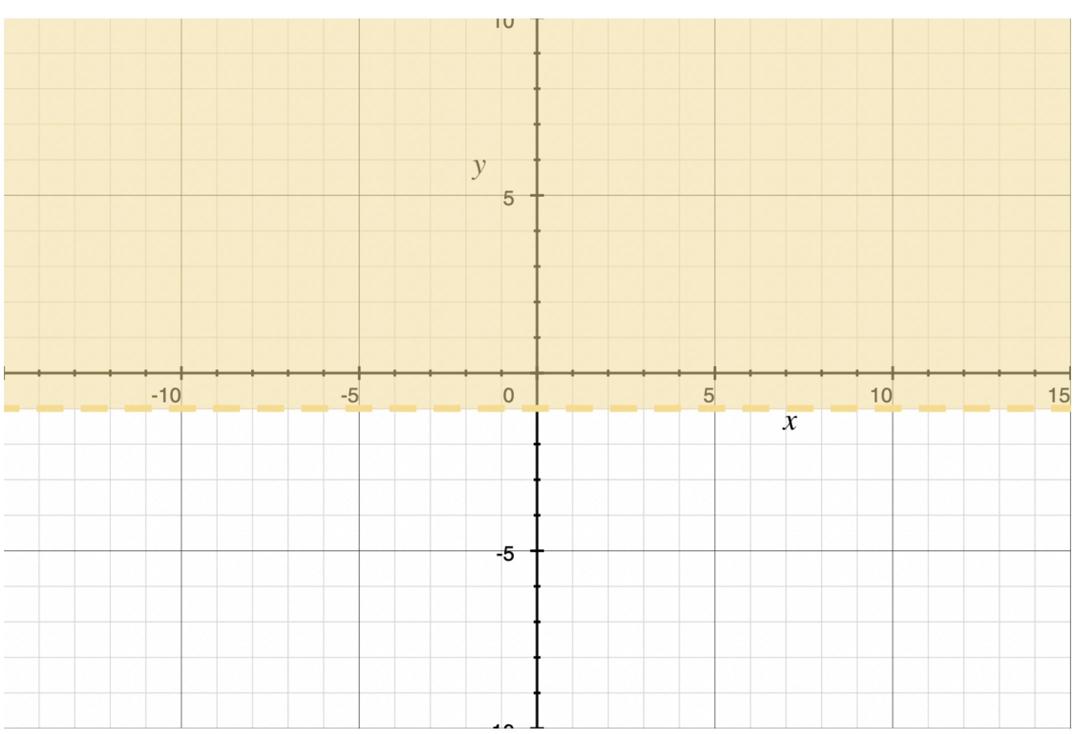


For $y = -1$, the slope is $m = 0$ and the y-intercept is $(0, -1)$; it's a perfectly horizontal line. The $>$ in $y > -1$ indicates the need for a dashed boundary line. Now let's test the origin again.

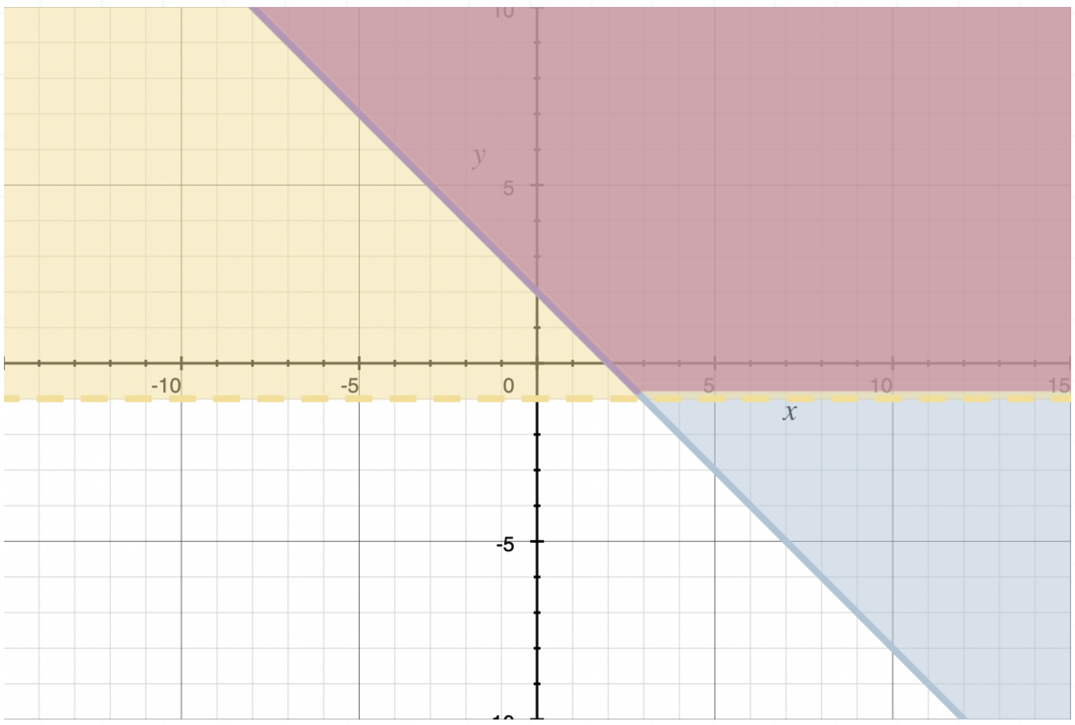
$$y > -1$$

$$0 > -1$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



3. Graph the solution to the system of linear inequalities.

$$x + 3y + 3 \geq 0$$

$$3x + y + 1 \geq 0$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$x + 3y + 3 \geq 0$$

$$3y \geq -x - 3$$

$$y \geq -\frac{1}{3}x - 1$$

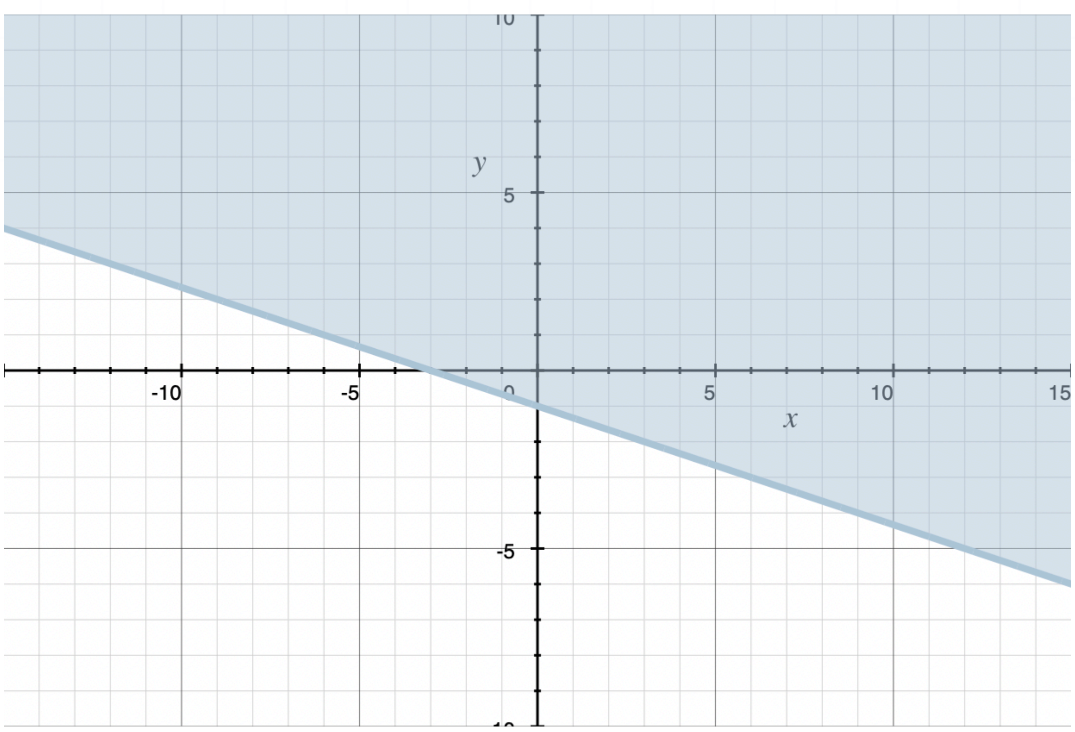
For $y = (-1/3)x - 1$, the slope is $m = -1/3$ and the y-intercept is $(0, -1)$. The $\geq$ in $y \geq (-1/3)x - 1$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \geq -\frac{1}{3}x - 1$$

$$0 \geq -\frac{1}{3}(0) - 1$$

$$0 \geq -1$$

Because this is a true statement, we shade toward the origin.



Rewrite the second inequality so that it's in slope-intercept form.

$$3x + y + 1 \geq 0$$

$$y \geq -3x - 1$$

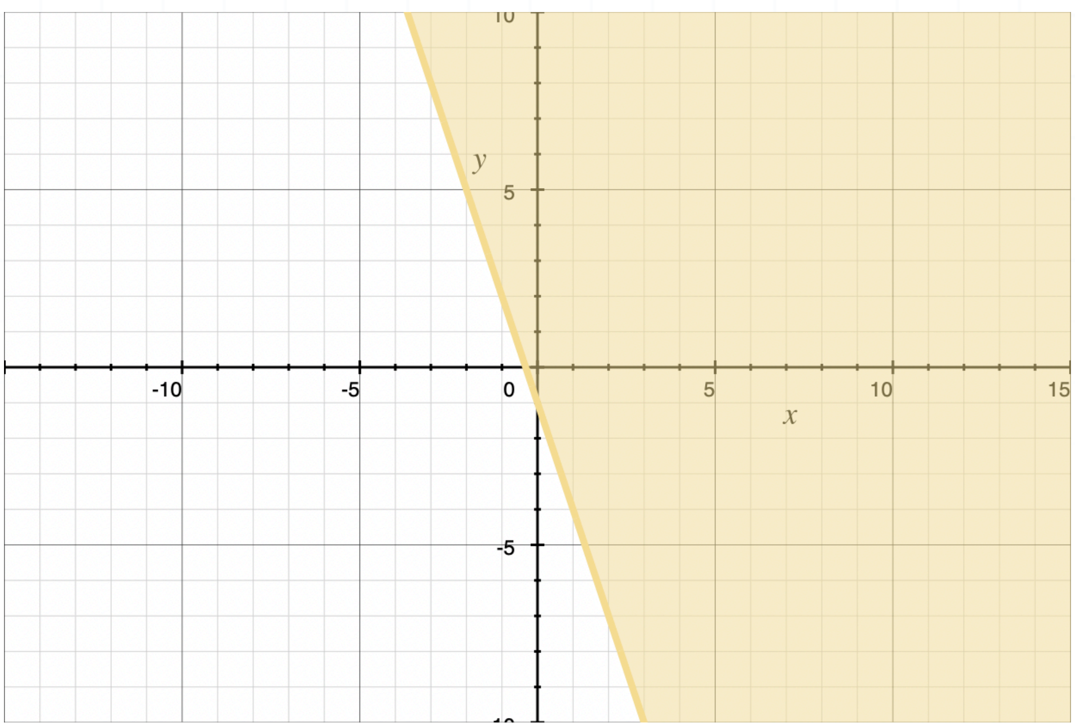
For $y = -3x - 1$, the slope is $m = -3$ and the y-intercept is $(0, -1)$. The $\geq$ in $y \geq -3x - 1$ indicates the need for a solid boundary line. Now let's test the origin again.

$$y \geq -3x - 1$$

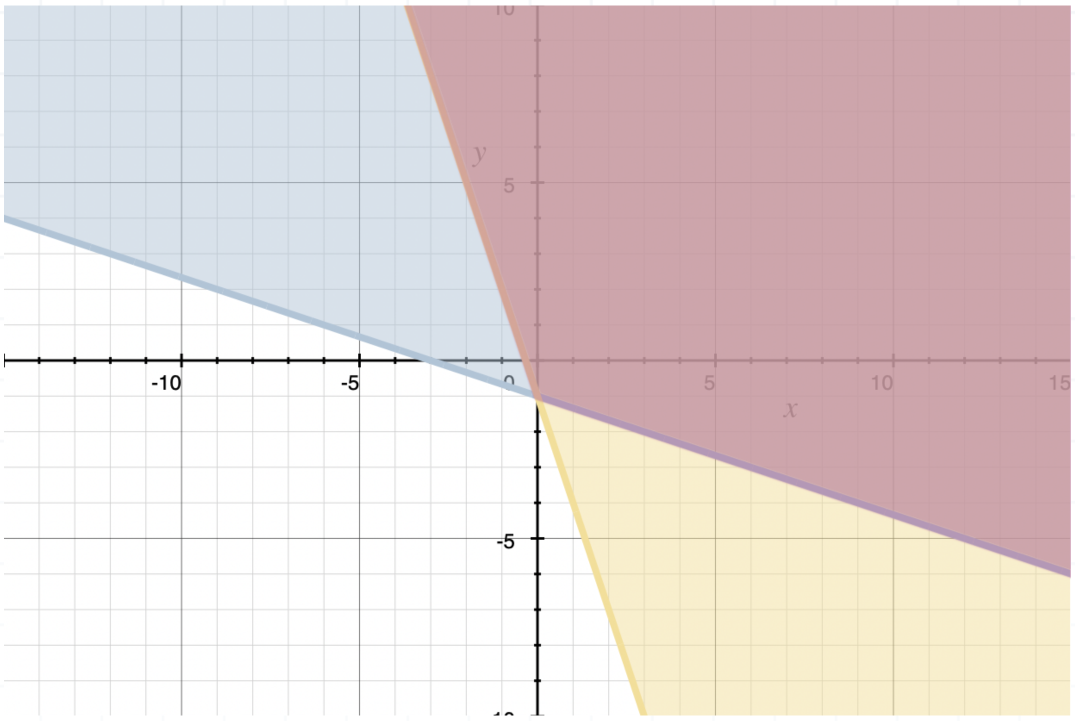
$$0 \geq -3(0) - 1$$

$$0 \geq -1$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



4. Graph the solution to the system of linear inequalities.

$y > 2x$

$x > 2y$

Solution:

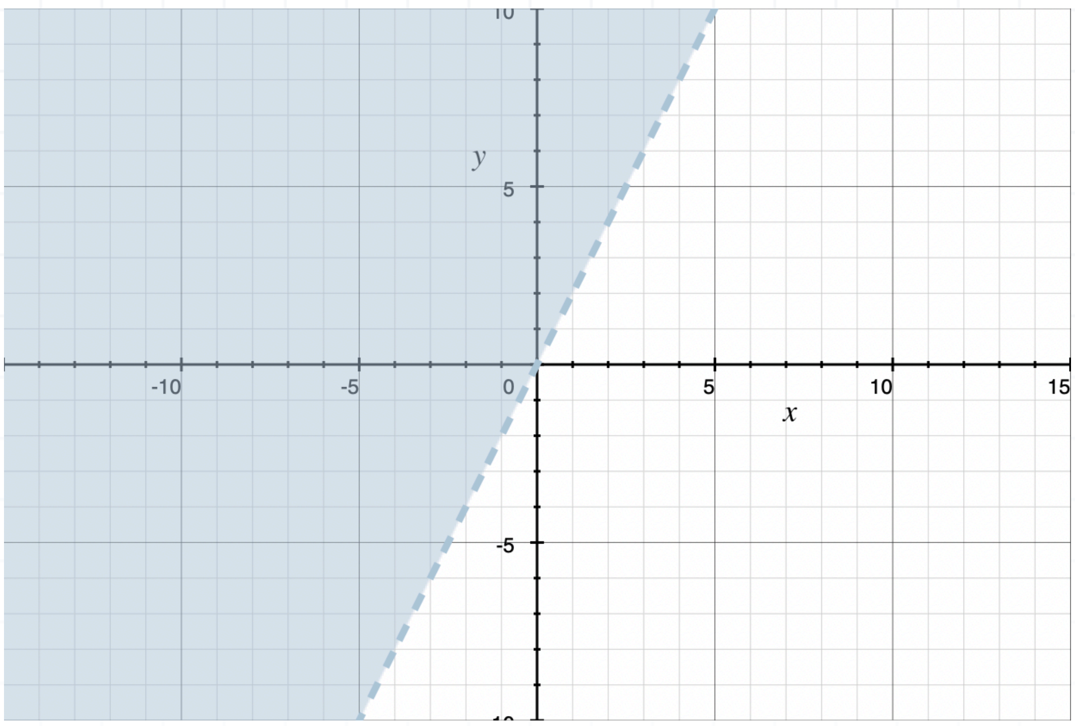
For $y = 2x$, the slope is $m = 2$ and the y-intercept is $(0,0)$. The $>$ in $y > 2x$ indicates the need for a dashed boundary line. Now let's test the point $(1,0)$ to determine where to shade.

$y > 2x$

$0 > 2(1)$

$0 > 2$

Because this is a false statement, we shade away from $(1,0)$.



Rewrite the second inequality so that it's in slope-intercept form.

$$y < \frac{1}{2}x$$

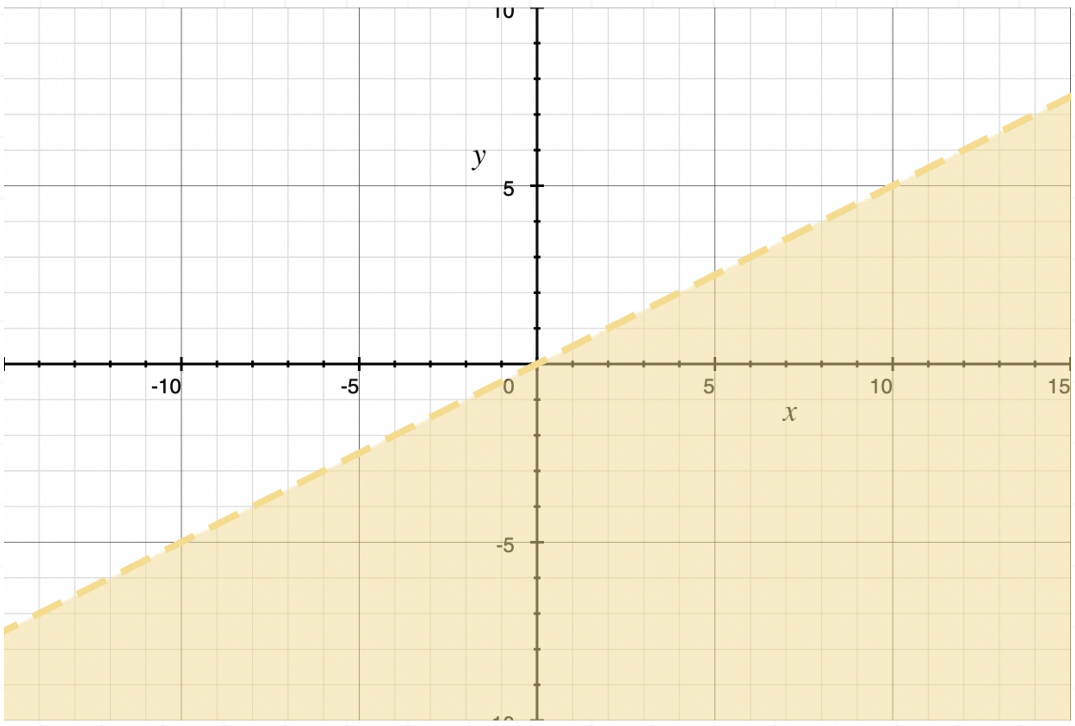
For $y = (1/2)x$, the slope is $m = 1/2$ and the y-intercept is $(0,0)$. The $<$ in $y < (1/2)x$ indicates the need for a dashed boundary line. Now let's test $(1,0)$ again.

$$y < \frac{1}{2}x$$

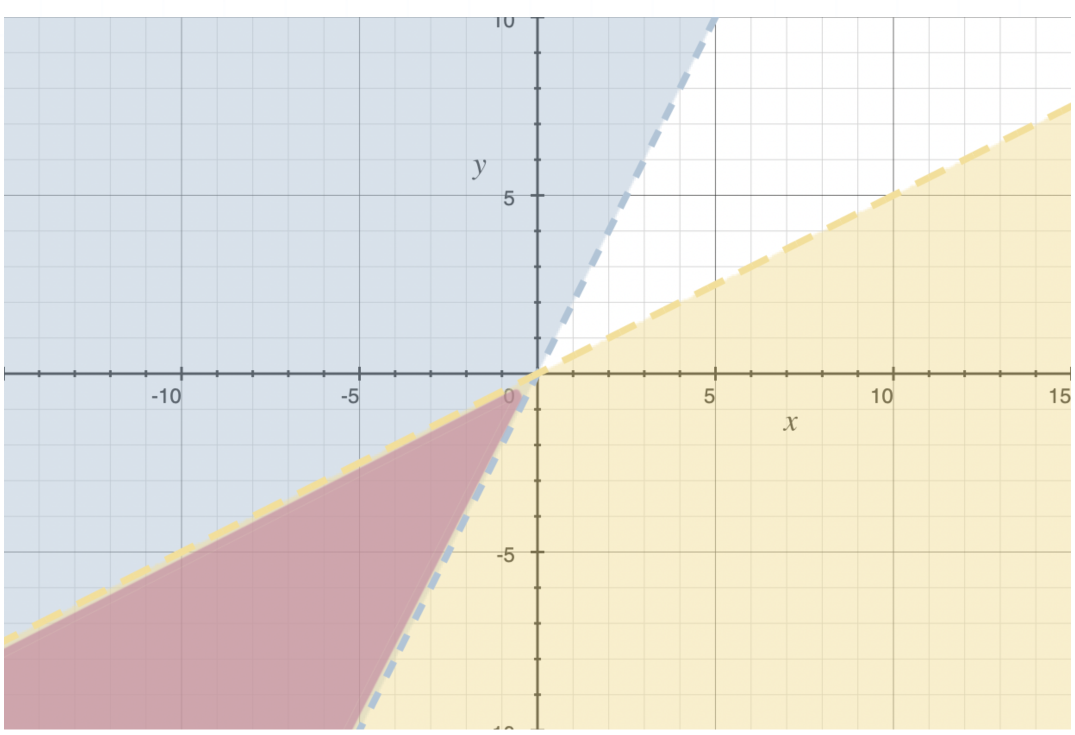
$$0 < \frac{1}{2}(1)$$

$$0 < \frac{1}{2}$$

Because this is a true statement, we shade toward $(1,0)$.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



5. Graph the solution to the system of linear inequalities.

$$2y + 3x \geq -4$$

$$x > y - 1$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$2y + 3x \geq -4$$

$$2y \geq -3x - 4$$

$$y \geq -\frac{3}{2}x - 2$$

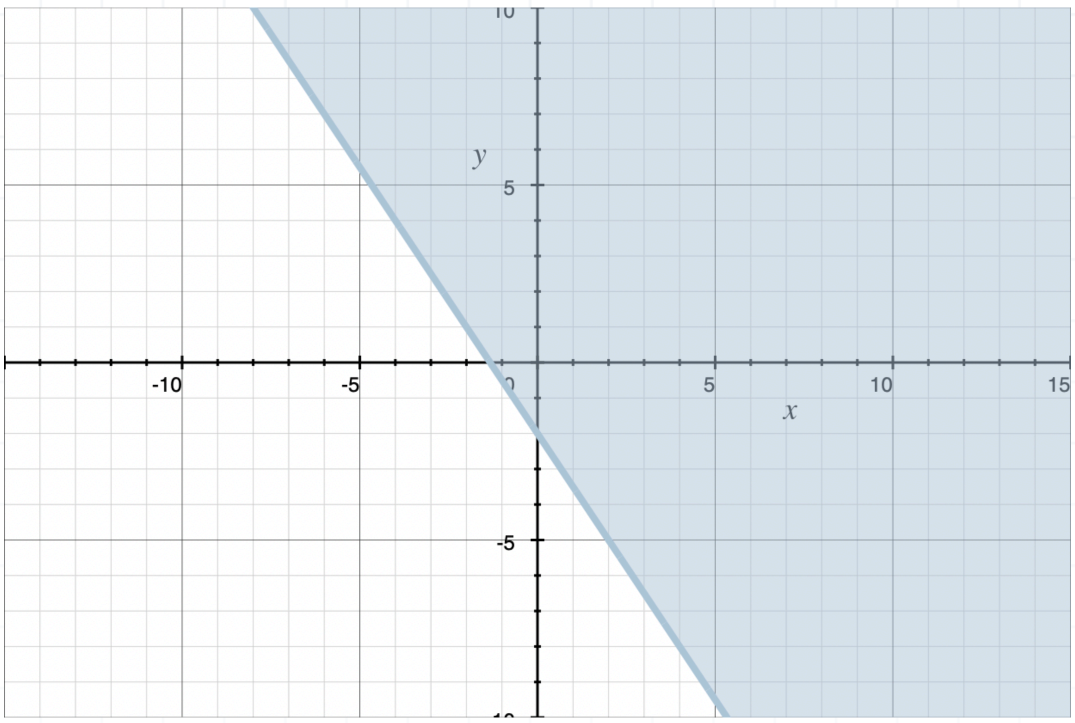
For $y = (-3/2)x - 2$, the slope is $m = -3/2$ and the y-intercept is $(0, -2)$. The $\geq$ in $y \geq (-3/2)x - 2$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \geq -\frac{3}{2}x - 2$$

$$0 \geq -\frac{3}{2}(0) - 2$$

$$0 \geq -2$$

Because this is a true statement, we shade toward the origin .



Rewrite the second inequality so that it's in slope-intercept form.

$$x > y - 1$$

$$x + 1 > y$$

$$y < x + 1$$

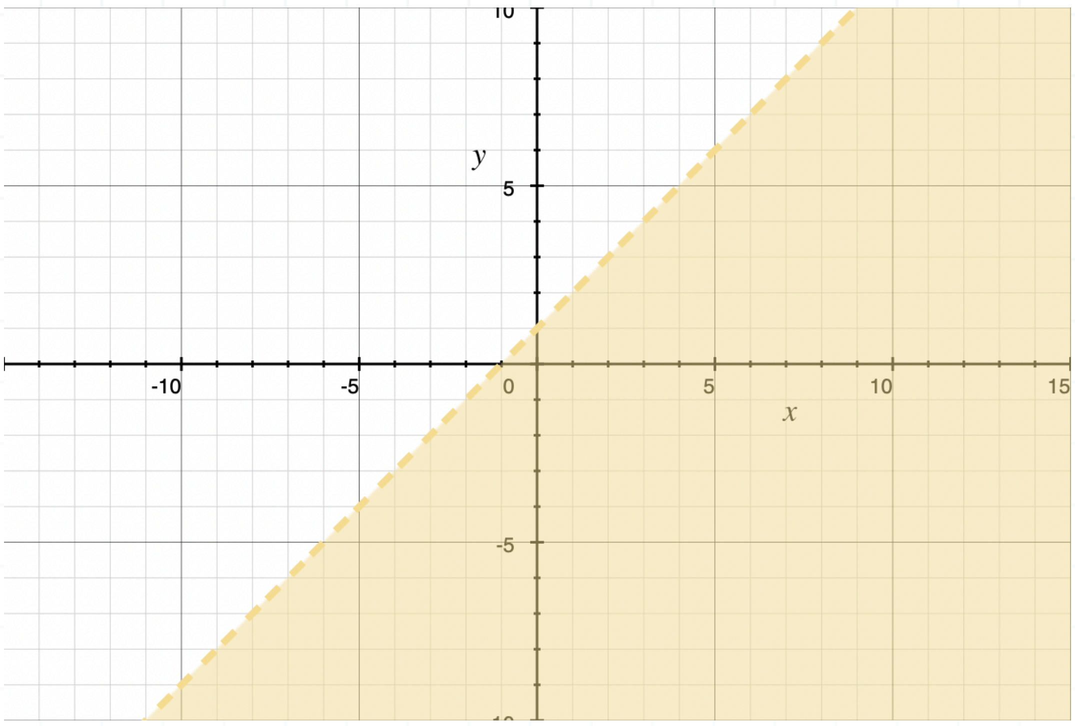
For $y = x + 1$, the slope is $m = 1$ and the y-intercept is $(0,1)$. The $<$ in $y < x + 1$ indicates the need for a dashed boundary line. Now let's test the origin again.

$$y < x + 1$$

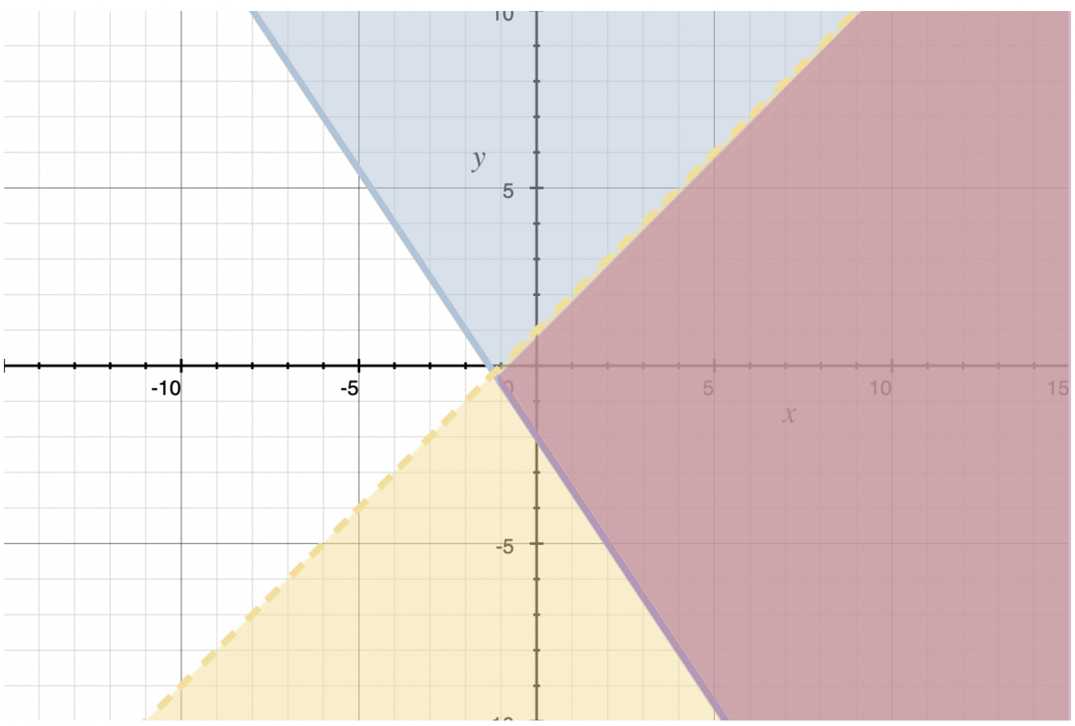
$$0 < 0 + 1$$

$$0 < 1$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



6. Graph the solution to the system of linear inequalities.

$$4x - 2y - 4 \geq 0$$

$$y \geq 2x - 2$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$4x - 2y - 4 \geq 0$$

$$-2y \geq 4 - 4x$$

$$y \leq 2x - 2$$

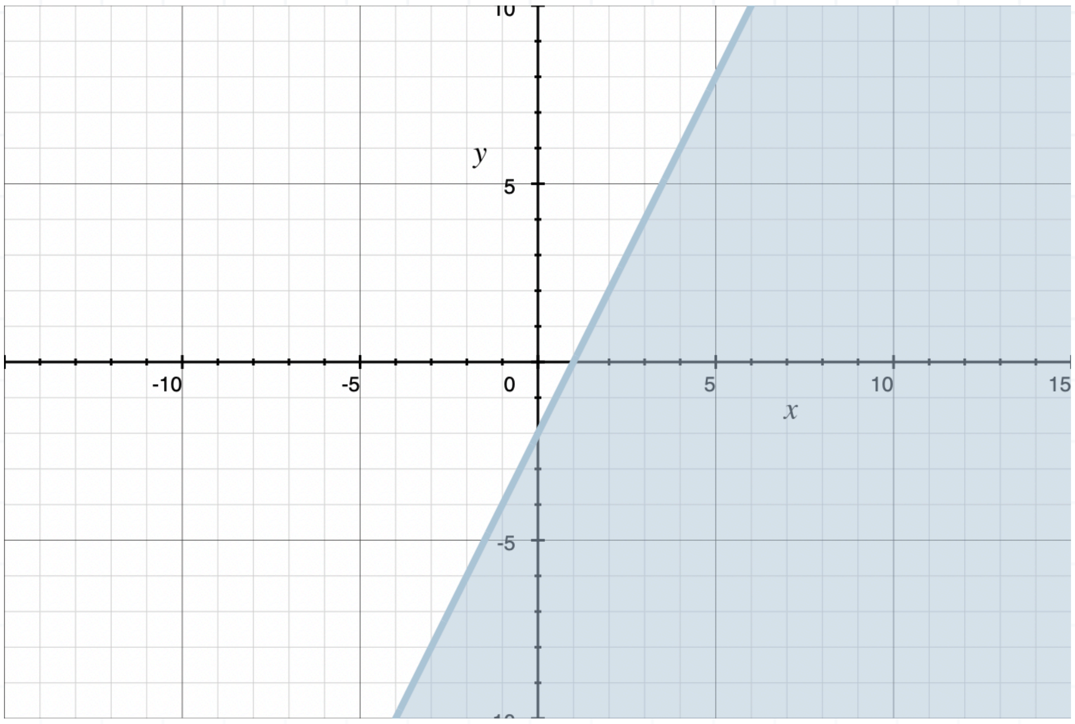
For $y = 2x - 2$, the slope is $m = 2$ and the y-intercept is $(0, -2)$. The $\leq$ in $y \leq 2x - 2$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \leq 2x - 2$$

$$0 \leq 2(0) - 2$$

$$0 \leq -2$$

Because this is a false statement, we shade away from the origin .



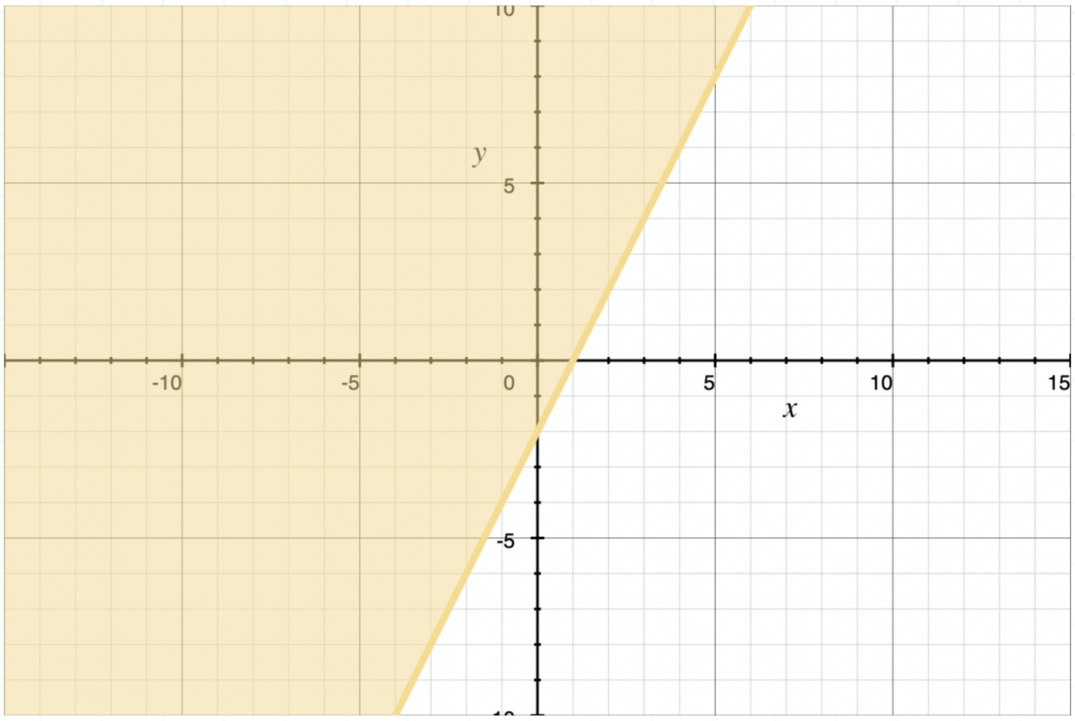
For $y = 2x - 2$, the slope is $m = 2$ and the y-intercept is $(0, -2)$. The $\geq$ in $y \geq 2x - 2$ indicates the need for a solid boundary line. Now let's test the origin again.

$$y \geq 2x - 2$$

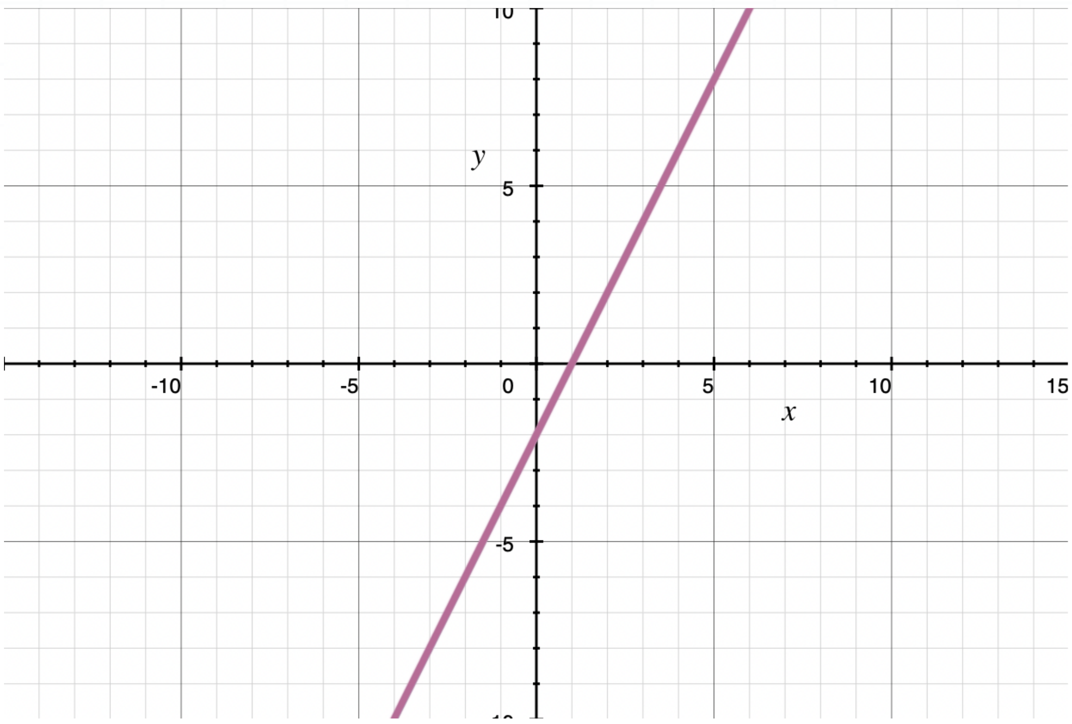
$$0 \geq 2(0) - 2$$

$$0 \geq -2$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify that the only overlapping portion is the line $y = 2x - 2$ itself. So the solution is



W W W . K R I S T A K I N G M A T H . C O M