# Inverse variation

In this lesson we'll look at solving equations that express inverse variation relationships, which are relationships of the form

$$y = \frac{k}{x}$$

In an inverse variation relationship we have two variables, usually $x$ and $y$, and a constant, usually $k$.

The main idea in inverse variation is that as one variable increases the other variable decreases. That means that if $x$ is increasing $y$ is decreasing, and if $x$ is decreasing $y$ is increasing. The number $k$ is a constant, so it's the same for all pairs of numbers $(x, y)$ that satisfy the equation.

Inverse variation can also be called "inverse proportion," because the variables $x$ and $y$ can be said to be inversely proportional to each other. The constant of proportionality is the number $k$. (Similarly, direct variation can be called "direct proportion." In that case, the variables can be said to be directly proportional to each other.)

An inverse variation can be written in any of the following three forms: $xy = k$, $y = k/x$, and $x = k/y$.

Let's look at an example.

**Example**

If we know that $y$ varies inversely with $x$, and $x = 10$ when $y = 4$, what is the constant of proportionality? And if we express $y$ as a function of $x$, what is that function?

The formula for an inverse variation is $xy = k$.

We know that $x = 10$ and $y = 4$, so

$$10 \cdot 4 = k$$

$$40 = k$$

The constant of proportionality is $k = 40$. If we express $y$ as a function of $x$, we get

$$y = \frac{k}{x}$$

$$y = \frac{40}{x}$$

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Let's look at another example.

**Example**

The product of two numbers is always $100$. Describe the relationship between the two numbers as a function.

The equation $xy = k$ expresses an inverse relationship. It means that for all pairs $(x, y)$ that satisfy this equation, the product of $x$ and $y$ is always the same (namely, $k$).

In this example $xy = 100$, so $k = 100$. If we express $y$ as a function of $x$, we get

$$y = \frac{100}{x}$$

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Sometimes we'll be given a pair of equations that involve the same inverse variation equation (they have the same constant of proportionality) and we'll need to solve for one of the variables. Just as with direction variation, we call this type of problem a **two-step problem**.

**Example**

Solve the two-step problem, given that $x$ and $y$ vary inversely.

If $k/5 = 3$ and $k/x = y$, find the value of $x$ when $y = 30$.

We'll solve the first equation for $k$.

$$\frac{k}{5} = 3$$

$$\frac{k}{5} \cdot 5 = 3 \cdot 5$$

$$k = 15$$

Now we'll take the value we found for $k$, and we'll plug that and the value of $y$ (30) into the second equation to solve for $x$.

$$\frac{k}{x} = y$$

$$\frac{15}{x} = 30$$

$$\frac{15}{x}(x) = 30(x)$$

$$15 = 30x$$

$$\frac{15}{30} = \frac{30x}{30}$$

$$\frac{1}{2} = x$$