Common bases and restricted values

In the last section, we looked at logarithmic equations written as

$$\log_8(64) = 2$$

Remember that, in this case, the number 8 is called the "base." There are some bases that we use much more often than all others, so we need to give them some special attention.

Base 10

Sometimes we'll see logs written with no base at all, like

$$\log(100) = 2$$

When there's no subscript on the "log" (to indicate the base), it means that we're dealing with the **common logarithm**, which always has a base of 10. Common logs are used so much in the real world that we've decided to save ourselves some time and simplify $\log_{10}$ to just $\log$, and understand that base 10 is implied. This means that we can rewrite the equation $\log(100) = 2$ in either of the following ways:

$$\log_{10}(100) = 2$$

$$10^2 = 100$$

Base e

Perhaps the most basic logarithms are those that have a base called "e." e is known as Euler's number, and it's equal to about 2.72. Here are some more digits of e.

$$e \approx 2.71828182845904523536028747135 27...$$

Like π, e is an irrational number, so it has an infinite number of digits to the right of the decimal point and they don't repeat. Logarithms to base e are called **natural logarithms**, and we write them with $\ln$ (note the "n" for "natural") instead of with log. In other words,

$$\log_e(x) = \ln(x)$$

Because e is the base, whenever we have a natural log, we're asking "How many times does e need to appear as a factor in order to get a certain result?". For instance,

$\ln(54.598) = \log_e(54.598) \approx 4$, because $e^4 \approx 2.71828^4 \approx 54.598$

Example

Solve the equation for x.

$$\log(1{,}000) = x$$

We can use the general rule to rewrite the equation.

$$\log(1{,}000) = x$$

$$10^x = 1{,}000$$

$$x = 3$$

Restricted values

For logarithms to any base, there are two rules we always have to follow.

Let's remember the general rule that relates an exponential equation to the associated logarithmic equation:

Given the exponential equation $a^x = y$,

the associated logarithmic equation is $\log_a(y) = x$,

and vice versa.

In a logarithmic expression $\log_a(y)$,

the base a must be positive (and not equal to 1), and

the argument y must also be positive.

If we don't follow these rules, we can run into trouble and end up with equations that aren't true.