Algebra 1 Workbook Solutions

Equations

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MATH

EVALUATING EXPRESSIONS

■ 1. Explain what went wrong in the following statement?

If $x^2 - x + 1$ when $x = -2$, then $-2^2 - -2 + 1 = -4 + 2 + 1 = -1$.

Solution:

There were no parentheses used when plugging in $x = -2$, so the expression -2^2 was evaluated incorrectly. It should be $(-2)^2 - (-2) + 1 = 4 + 2 + 1 = 7$.

■ 2. What does it mean to “evaluate an expression”?

Solution:

It means to replace (or plug in) a number for the given variable, and then simplify using PEMDAS until we’ve reached the simplest possible value.

■ 3. Find the value of $y - 2z - 1$ when $y = 4$ and $z = -3$.

Solution:

Substitute $y = 4$ and $z = -3$ into the expression.

$$y - 2z - 1$$

$$4 - 2(-3) - 1$$

$$4 + 6 - 1$$

$$10 - 1$$

$$9$$

4. Evaluate the expression when $a = 1$, $b = -3$, and $c = -4$.

$$\frac{\sqrt{b^2 - 4ac}}{2a}$$

Solution:

Substitute $a = 1$, $b = -3$, and $c = -4$ into the expression.

$$\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\frac{\sqrt{(-3)^2 - 4(1)(-4)}}{2(1)}$$

$$\frac{\sqrt{9 + 16}}{2}$$

$$\frac{5}{2}$$

5. Show that $x = -4$ by plugging it into the equation.

$$x^2 - 4 = -3x$$

Solution:

Substituting $x = -4$ into the equation gives

$$x^2 - 4 = -3x$$

$$(-4)^2 - 4 = -3(-4)$$

$$16 - 4 = 12$$

$$12 = 12$$

Because we find that both sides are equal, we know it must be true that $x = -4$.

6. Evaluate the expression when $a = -1$, $b = -2$, and $c = -3/2$.

$$\frac{5a + 1}{3 - 2b + 4c^2a}$$

Solution:

Substitute $a=-1$, $b=-2$, and $c=-3/2$ into the expression.

$$\frac{5a+1}{3-2b+4c^2a}$$

$$\frac{5(-1)+1}{3-2(-2)+4\left(-\frac{3}{2}\right)^2(-1)}$$

$$\frac{-5+1}{3+4-4\left(\frac{9}{4}\right)}$$

$$\frac{-4}{7-9}$$

$$\frac{-4}{-2}$$

$$2$$

INVERSE OPERATIONS

1. Use inverse operations to figure out what should replace the "?" in order to make the equation true.

$$5x\ ? = x$$

Solution:

Because 5 is being multiplied by x, we need to undo that operation by dividing by 5.

$$5x \div 5 = x$$

So the $?$ should be replaced with "$\div 5$".

2. What is the inverse operation of division?

Solution:

Multiplication

3. Using both division and multiplication, find two values that can replace the "?" in order to make the equation true.

$$\frac{1}{5}x \ ? = x$$

Solution:

We could say that x is being multiplied by $1/5$, so we could replace the $?$ with division by $1/5$, and the equation would be true.

$$\frac{1}{5}x \div \frac{1}{5} = x$$

But we could also say that x is being divided by 5, so we could replace the $?$ with multiplication by 5, and the equation would be true.

$$\frac{1}{5}x \cdot 5 = x$$

■ 4. What value of the missing exponent would make the equation true?

$$(x^3)^? = x$$

Solution:

Because x is being raised to the power of 3, we need to raise the result to $1/3$ in order to undo the exponent and get back to x. So the $?$ should be replaced with $1/3$.

5. Put an expression in place of the question mark that would make the equation true.

$$\frac{1}{7}\ ? = 1$$

Solution:

In this example, 1 is being divided by 7. To undo that operation, we need to multiply by 7.

$$\frac{1}{7} \cdot 7 = 1$$

$$1 = 1$$

6. Use inverse operations to find a value to replace the "$?$" that will make the equation true.

$$(\sqrt[4]{a+b})^{?} = a + b$$

Solution:

To undo a root, we need an exponent. Since we're taking the fourth root of $a + b$, we need to undo that operation by raising the result to the fourth power. Therefore, the $?$ should be replaced with the exponent 4.

SIMPLE EQUATIONS

1. Solve the equation for x.

$$2x - 5 = 11$$

Solution:

Add 5 to both sides.

$$2x - 5 = 11$$

$$2x - 5 + 5 = 11 + 5$$

$$2x = 16$$

Divide both sides by 2 to get x by itself.

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

2. If $x = 16$, what value of the "$??$" would make the equation true?

$$x - ?? = 11$$

Solution:

Substitute $x = 16$ into the equation.

$$x - ?? = 11$$

$$16 - ?? = 11$$

Subtract 16 from both sides.

$$16 - ?? - 16 = 11 - 16$$

Divide both sides by -1.

$$-?? = -5$$

$$\frac{-??}{-1} = \frac{-5}{-1}$$

$$?? = 5$$

3. Solve the equation for x.

$$\frac{x+1}{3} = 7$$

Solution:

Multiply both sides of the equation by 3.

$$\frac{x+1}{3}=7$$

$$\frac{x+1}{3}\cdot 3=7\cdot 3$$

$$x+1=21$$

Subtract 1 from both sides.

$$x+1-1=21-1$$

$$x=20$$

4. What went wrong in this set of steps?

$$2x-11=-3$$

$$2x=8$$

$$x=16$$

Solution:

The 2 was multiplied by both sides instead of divided. The steps should have been

$$2x-11=-3$$

$$2x-11+11=-3+11$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

5. What went wrong in this set of steps?

$$2 - \frac{1}{3}x = 1$$

$$-\frac{1}{3}x = 3$$

$$x = -9$$

Solution:

The 2 was subtracted from the left side and added to the right side, instead of subtracted from both sides. The inverse operation of addition is subtraction, so the steps should have been

$$2 - \frac{1}{3}x = 1$$

$$2 - \frac{1}{3}x - 2 = 1 - 2$$

$$-\frac{1}{3}x = -1$$

$$-\frac{1}{3}x(-3) = -1(-3)$$

$$x = 3$$

6. Solve the equation for x.

$$\frac{1}{4}x + 3 = 5$$

Solution:

Subtract 3 from both sides,

$$\frac{1}{4}x + 3 = 5$$

$$\frac{1}{4}x + 3 - 3 = 5 - 3$$

$$\frac{1}{4}x = 2$$

then multiply both sides by 4.

$$\frac{1}{4}x(4) = 2(4)$$

$$x = 8$$

BALANCING EQUATIONS

1. Solve the equation for x.

$$2(-3x+5)-1=-3(1-5x)$$

Solution:

Distribute the 2 and the -3 across the parentheses.

$$2(-3x+5)-1=-3(1-5x)$$

$$-6x+10-1=15x-3$$

$$-6x+9=15x-3$$

Add $6x$ to both sides,

$$9=21x-3$$

then add 3 to both sides,

$$12=21x$$

and divide both sides by 21.

$$\frac{12}{21}=x$$

2. Solve the equation for x.

$$x - 2(1 - x) + 5 = 3(2x + 4) - 6$$

Solution:

Distribute the -2 and the 3 across the parentheses.

$$x - 2(1 - x) + 5 = 3(2x + 4) - 6$$

$$x - 2 + 2x + 5 = 6x + 12 - 6$$

$$3x + 3 = 6x + 6$$

Subtract $3x$ from both sides,

$$3 = 3x + 6$$

subtract 6 from both sides,

$$-3 = 3x$$

then divide both sides by 3.

$$-1 = x$$

3. If $x = -2$, solve for y.

$$3x + 2y - 7 = 1 - 5x - y$$

Solution:

Substitute $x = -2$ into the equation.

$$3x + 2y - 7 = 1 - 5x - y$$

$$3(-2) + 2y - 7 = 1 - 5(-2) - y$$

$$-6 + 2y - 7 = 1 + 10 - y$$

$$-13 + 2y = 11 - y$$

Add y to both sides,

$$-13 + 3y = 11$$

add 13 to both sides,

$$3y = 24$$

then divide both sides by 3.

$$y = 8$$

■ 4. Solve for a.

$$7(4a - 3) = -(6a - 5) + 8$$

Solution:

First, we'll use the Distributive Property.

$$7(4a - 3) = -(6a - 5) + 8$$

$$7(4a) + 7(-3) = -(6a) - (-5) + 8$$

$$28a - 21 = -6a + 5 + 8$$

$$28a - 21 = -6a + 13$$

Add $6a$ to both sides,

$$28a + 6a - 21 = -6a + 6a + 13$$

$$34a - 21 = 13$$

add 21 to both sides,

$$34a - 21 + 21 = 13 + 21$$

$$34a = 34$$

then divide both sides by 34.

$$a = 1$$

5. Solve for a.

$$-2(1 - a) + 3(a + 7) = -2$$

Solution:

Apply the Distributive Property,

$$-2(1 - a) + 3(a + 7) = -2$$

$$-2 + 2a + 3a + 21 = -2$$

$$5a + 19 = -2$$

subtract 19 from both sides,

$$5a = -21$$

then divide both sides by 5.

$$a = -\frac{21}{5}$$

■ 6. What missing number should replace the "$??$" in order to make the equation true?

$$-3(x - 5) = 2x - (3 - x)$$

$$??x + 15 = 3x - 3$$

Solution:

Apply the Distributive Property,

$$-3(x - 5) = 2x - (3 - x)$$

$$-3x + 15 = 2x - 3 + x$$

$$-3x + 15 = 3x - 3$$

This matches the form of the equation with the missing value. Comparing the two equations, we can see that $??$ should be replaced by -3.

EQUATIONS WITH SUBSCRIPTS

1. It takes Peter 6 hours to paint a room and Laura 8 hours to paint that same room. Use the equation below to determine how long it would take for Peter and Laura to paint the room together, where R_1 is the number of hours it takes Peter, R_2 is the number of hours it takes Laura, and T is the number of hours it takes them together.

$$\frac{R_1R_2}{R_1+R_2}=T$$

Solution:

Substituting the values we know into the formula, the time it takes them to paint the room together is

$$\frac{(6)(8)}{6+8}=T$$

$$\frac{48}{14}=T$$

$T=3.43$ hrs

2. Solve the equation for P_2.

$$P_1R+\frac{P_2}{V}=d$$

Solution:

Subtract P_1R from both sides of the equation.

$$P_1R + \frac{P_2}{V} = d$$

$$\frac{P_2}{V} = d - P_1R$$

Multiply both sides by V to get P_2 by itself.

$$P_2 = V(d - P_1R)$$

3. The profit function for a company is given by $P = Rx - C_1 - C_2x$, where P is the profit, R is the selling price of their product, C_1 is the company's fixed cost, C_2 is their variable cost, and x is the total number of products sold. What is the selling price R when $P = 114$, $C_1 = 550$, $C_2 = 3.50$, and $x = 16$?

Solution:

Plugging all the values we've been given into the profit function, we get

$$P = Rx - C_1 - C_2x$$

$$114 = R(16) - 550 - 3.50(16)$$

$$114 = 16R - 550 - 56$$

$$114 = 16R - 606$$

$$720 = 16R$$

$$R = 45$$

4. Solve the equation for x_1.

$$\frac{3V}{x_1} = td_0 + 2x_2d_1$$

Solution:

Multiply both sides of the equation by x_1,

$$\frac{3V}{x_1} = td_0 + 2x_2d_1$$

$$3V = x_1(td_0 + 2x_2d_1)$$

then divide through by $td_0 + 2x_2d_1$ to get x_1 by itself.

$$\frac{3V}{td_0 + 2x_2d_1} = x_1$$

5. Solve the equation for Y_2 when $t_1 = 2$, $t_2 = 11$, $D = 1/3$, and $Y_1 = 25$.

$$3t_1 + \frac{15t_2D}{Y_2} = Y_1 - 5$$

Solution:

Plugging all the values we've been given into the equation, we get

$$3t_1 + \frac{15t_2D}{Y_2} = Y_1 - 5$$

$$3(2) + \frac{15(11)(1/3)}{Y_2} = 25 - 5$$

$$6 + \frac{5(11)}{Y_2} = 20$$

$$6 + \frac{55}{Y_2} = 20$$

$$\frac{55}{Y_2} = 14$$

Now multiply both sides by Y_2 and divide by 14 in order to solve for Y_2.

$$55 = 14Y_2$$

$$Y_2 = \frac{55}{14}$$

6. The volume of the medium size box at the post office is given by $V = d_1 \times d_2 \times d_3$, where d_1, d_2, and d_3 are the length, width, and height, respectively. Given $d_1 = 4$ and $d_2 = 5$, find the relationship between volume and height.

Solution:

Plugging everything we've been given into the volume equation, we get

$$V = d_1 \times d_2 \times d_3$$

$$V = 4 \times 5 \times d_3$$

$$V = 20d_3$$

This new equation gives the relationship between volume V and height d_3, and tells us that the volume of the medium size box will always be equivalent to twenty times the height of the box.

WORD PROBLEMS INTO EQUATIONS

1. Write the phrase as an algebraic expression.

Six more than three times a number

Solution:

"Three times a number" can be written as $3x$, so "six more than" that value must be $3x + 6$.

2. Find the value of the expression.

The quotient of 150 and 5

Solution:

The quotient of two values is the division of the first number by the second number, so the quotient is

$$\frac{150}{5}$$

$$30$$

3. Write the phrase as an algebraic expression.

Half of five times a number

Solution:

"Five times a number" can be written as $5x$, so half of that value must be

$$\frac{1}{2}(5x)$$

$$\frac{5x}{2}$$

4. Find the value of the expression.

3 less than the product of 2 and 7

Solution:

The "product of 2 and 7" is 2×7, and three less than that value is

$$(2 \times 7) - 3$$

$$14 - 3$$

$$11$$

5. Find the value of the expression.

$$\frac{1}{3} \text{ of } 2 \text{ more than } 7$$

Solution:

"2 more than 7" is the sum $7+2$, and $1/3$ of that value is

$$\frac{1}{3}(7+2)$$

$$\frac{1}{3}(9)$$

$$\frac{9}{3}$$

6. David's age is five more than twice Jane's age. If Jane is 6, how old is David?

Solution:

We have two quantities: Jane's age and David's age. So we'll say that Jane's age is J and that David's age is D. "Twice Jane's age" is $2J$, and "five more than twice Jane's age" is therefore $2J+5$. This is David's age, so

$$D = 2J + 5$$

Jane is 6, so David's age must be

$$D = 2(6) + 5$$

$$D = 12 + 5$$

$$D = 17$$

CONSECUTIVE INTEGERS

1. Write the next five consecutive integers following -4.

Solution:

The five consecutive integers following -4 are

$$-3,\ -2,\ -1,\ 0,\ 1$$

2. Give an example of three consecutive negative integers.

Solution:

There are many correct answers. Some examples include

$$-11,\ -10,\ -9$$

$$-23,\ -22,\ -21$$

$$-3,\ -2,\ -1$$

But $-5,\ -3,\ -1$ is not an example, because those integers are not one unit apart from each other.

3. Write the four consecutive integers that precede -3.

Solution:

The four consecutive integers preceding -3 are

$$-7,\ -6,\ -5,\ -4$$

4. Find three consecutive integers that sum to 60.

Solution:

Since the sum of the three consecutive integers is 60, we can write

$$n+(n+1)+(n+2)=60$$

$$n+n+1+n+2=60$$

$$3n+3=60$$

$$3n=57$$

$$n=19$$

Because the integers are n, $n+1$, and $n+2$, and the first integer is $n=19$, the other two consecutive integers are

$$n+1=19+1=20$$

$$n + 2 = 19 + 2 = 21$$

and therefore the three consecutive integers that sum to 60 are 19, 20, and 21.

5. Find three consecutive odd integers that sum to 21.

Solution:

We can model three consecutive odd integers as n, $n + 2$, and $n = 4$. Since the sum of the three consecutive odd integers is 21, we have

$$n + (n + 2) + (n + 4) = 21$$

$$n + n + 2 + n + 4 = 21$$

$$3n + 6 = 21$$

$$3n = 15$$

$$n = 5$$

Because the integers are n, $n + 2$, and $n + 4$, and the first integer is $n = 5$, the other two consecutive odd integers are

$$n + 2 = 5 + 2 = 7$$

$$n + 4 = 5 + 4 = 9$$

and therefore the three consecutive odd integers that sum to 21 are 5, 7, and 9.

6. If, given three consecutive integers, the third integer is 10 more than the sum of the first two integers, what is the third integer?

Solution:

Since the three integers here are consecutive, we can identify them as x, $x+1$, and $x+2$. The sum of the first two integers is

$$x+(x+1)$$

$$2x+1$$

The third integer is 10 more than the sum of the first two, which we can express as

$$2x+1+10$$

$$2x+11$$

And the third integer we identified as $x+2$, so we now have two ways of expressing the value of the third integer, and we can set them equal to each other.

$$2x+11=x+2$$

$$x+11=2$$

$$x = -9$$

The second and third integers are therefore

$$x + 1 = -9 + 1 = -8$$

$$x + 2 = -9 + 2 = -7$$

So those integers are -9, -8, -7, and the value of the third integer is -7.

W W W . K R I S T A K I N G M A T H . C O M