Trichotomy

Up to now we've been dealing almost exclusively with equations, which tell us that the expressions on the left and right sides of the equals sign are equivalent.

But there are other ways to describe the relationship between two numbers or two expressions. Instead of defining them as equal, we can say that one is greater than the other, or that one is less than the other.

The Law of Trichotomy

In fact, the **Law of Trichotomy** tells us that two numbers (or expressions) can have exactly one of three possible relationships:

- The first number is smaller than the second number, $a < b$

- The first number is greater than the second number, $a > b$

- The first number is equal to the second number, $a = b$

Given two numbers, we know that they must have exactly one of these three relationships. It's impossible for them to have more than one of these relationships at the same time, and it's also impossible that they aren't related in one of these three ways.

So, no matter which two numbers (or expressions) we choose, the Law of Trichotomy tells us that they'll take on exactly one of these three relationships.

Because of this fact, we can also make the following three statements:

- If a is not greater than b and also not equal to b, then a must be less than b. If $a \ngtr b$, then $a < b$.

- If a is not less than b and also not equal to b, then a must be greater than b. If $a \nless b$, then $a > b$.

- If a is not greater than b and also not less than b, then a must be equal to b. If $a \nless b$ and $a \ngtr b$, then $a = b$.

Let's do an example where we use the Law of Trichotomy to describe the relationship between two numbers.

Example

Describe the relationship between 2 and -7.

Using the Law of Trichotomy, we know we can describe this relationship in three says. First, we know that 2 and -7 are not equal to one another. But we can also say that -7 is less than 2, or that 2 is greater than -7.

$$2 \neq -7$$

$$2 > -7$$

$$-7 < 2$$

Let's do another example, but this time we'll have to make a slightly different conclusion.

Example

If $4x + 5 \not< 2x + 7$, how can we rewrite the inequality?

The inequality symbol tells us that $4x + 5$ is not less than $2x + 7$, and also that $4x + 5$ is not equal to $2x + 7$.

But the Law of Trichotomy tells us that two expressions must either be equal, or that one has to be greater than the other or less than the other.

So if $4x + 5$ is not less than $2x + 7$ and also not equal to $2x + 7$, then the only possibility is that $4x + 5$ is greater than $2x + 7$. So we can rewrite the inequality statement as

$$4x + 5 > 2x + 7$$