Word problems into equations

Word problems can be tricky at first, because it can be challenging to translate the words into actual mathematical equations that we can solve.

Translating word problems

Since this translation is usually the hardest part, we want to outline exactly how certain words and phrases are translated into expressions or equations. The table shows some common words and phrases.

	Words	Phrases	Expressions
Addition	sum, total, more than, added, increased, plus	3 more than a number, the sum of 5 and a number	3+x 5+n
Subtraction	less, minus, decreased by, difference, less than	12 decreased by a number, the difference of 7 and a number	12-n 7-x
Multiplication	product, times, multiplied, of	the product of a number and 2, 2/3 of a number	2x (2/3)n
Division	quotient, divided by, divided into	15 divided by a number, the quotient of a number and 4	15/n x/4

Let’s work through an example where we translate a simple phrase into a mathematical expression.

Example

Write the phrase as an algebraic expression.

"four less than twice x"

The phrase "twice x" means "2 times x," which we know means to multiply, and so we can write it as $2x$. Now we have

"four less than $2x$"

"Less" means subtraction, so we'll subtract 4 from $2x$.

$$2x - 4$$

We might be tempted to write the 4 first and express "four less than $2x$" as $4 - 2x$, but that would be incorrect, and we can use specific numbers to visualize this.

If we were asked what number is four less than 10, we'd know it's 6, and that we'd have to subtract 4 from 10, which is written as $10 - 4$. So "four less than $2x$" will be $2x - 4$.

Once we've translated a phrase into a math expression, we can work with the math in different ways. This next example asks us to translate a phase into math, but then simplify the mathematical expression.

Example

Find the value of the expression.

$$\frac{1}{4} \text{ of } 120$$

In math, the word "of" (immediately after a fraction) tells us to multiply. Therefore, the mathematical expression of the phrase will be

$$\frac{1}{4} \cdot 120$$

Because we were asked to actually find the value of the expression, we'll perform the multiplication to get the simplified value.

$$\frac{1(120)}{4}$$

$$30$$

Not only can we translate phrases into expressions, but we can write equations from some phrases as well.

Example

John's age is four less than twice Mary's age. If Mary is 18, how old is John?

The first step in solving a word problem like this is to define the variables, meaning that we want to state the particular quantity that each variable stands for.

In this problem, we have two quantities: Mary's age and John's age, so let's let M be Mary's age, and let J be John's age.

Now we need to translate each word or phrase into mathematical symbols. Here, "John's age" is translated as "$2M - 4$."

How about the word "is" (in "John's age is four less than twice Mary's age")? Well, "is" is translated as an equals sign. To see this, it may help to think of the word "is" as having the same meaning (in math) as "is equal to." Combining all of these, we get the equation

$$J = 2M - 4$$

We're given Mary's age as 18, so we substitute 18 for M and then solve for J.

$$J = 2(18) - 4$$

$$J = 36 - 4$$

$$J = 32$$

The final step is to answer the question that was asked. Here, we're asked for John's age. Since we defined J as John's age, the answer is 32.

If instead of the last example, we'd been asked,

> "Currently, John's age is four less than twice Mary's age. If Mary is now 18, how old will John be seven years from now?"

then it would be convenient to define M as Mary's age now, and J as John's age now, because we're given a relationship between Mary's current age and John's current age.

Then in the last step (answering the question that was asked), we'd have to evaluate $J + 7$ (to get John's age seven years from now), and our answer would be $32 + 7 = 39$.