Graphing linear equations

A **linear equation** is the equation of a line, so when we graph linear equations, it means that we're graphing lines. To sketch the graph of a line, we want to start by putting the equation into point-slope form or slope-intercept form.

Point-slope form	$y - y_1 = m(x - x_1)$
Slope-intercept form	$y = mx + b$

Remember that, in these equations, m is the slope of the line and b is the y-intercept (the y-coordinate of the point where the line crosses the y-axis).

When we're graphing linear equations, we can also use the **intercepts** of the line to help us, which are the points where the line crosses the major axes. We already know that b is the y-intercept, and we can find the x-intercept by setting $y = 0$.

Let's do an example with a line in slope-intercept form.

Example

What is the y-intercept of the line?

$$y = -\frac{2}{3}x$$

This equation is in slope-intercept form, but the y-intercept is missing. However, we could actually rewrite the equation of the line as

$$y = -\frac{2}{3}x + 0$$

Written this way, we haven't changed the value of either side of the equation at all, but we can see that the y-intercept is 0, which means the line passes through the origin.

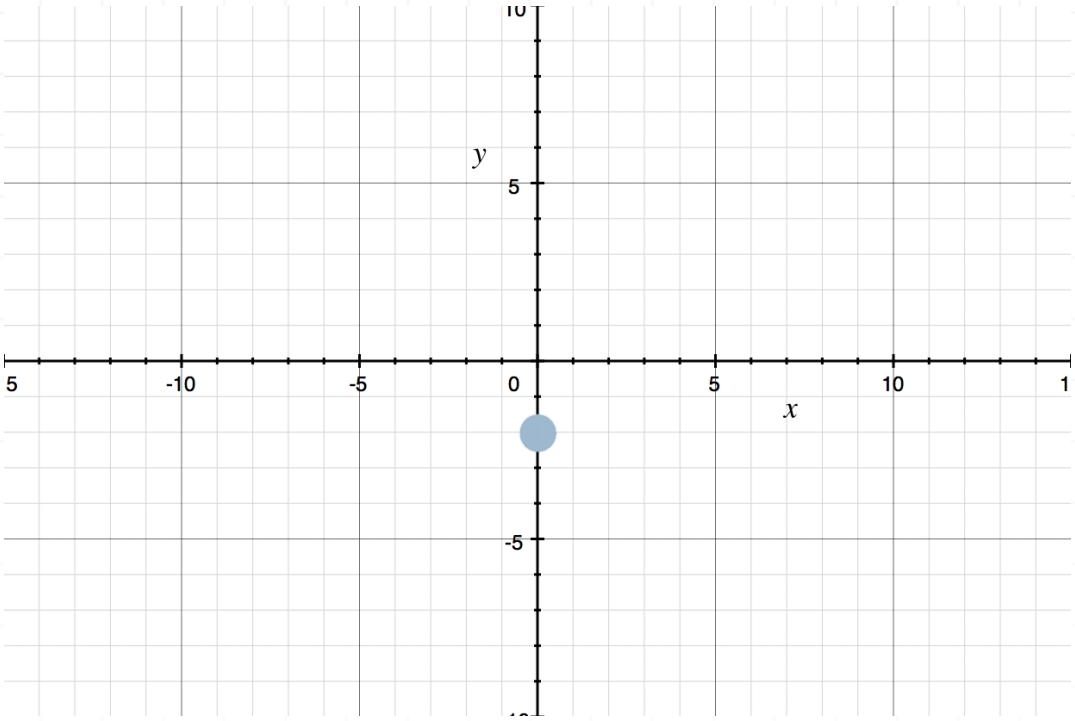
Let's look at an example where we graph a line from slope-intercept form.

Example

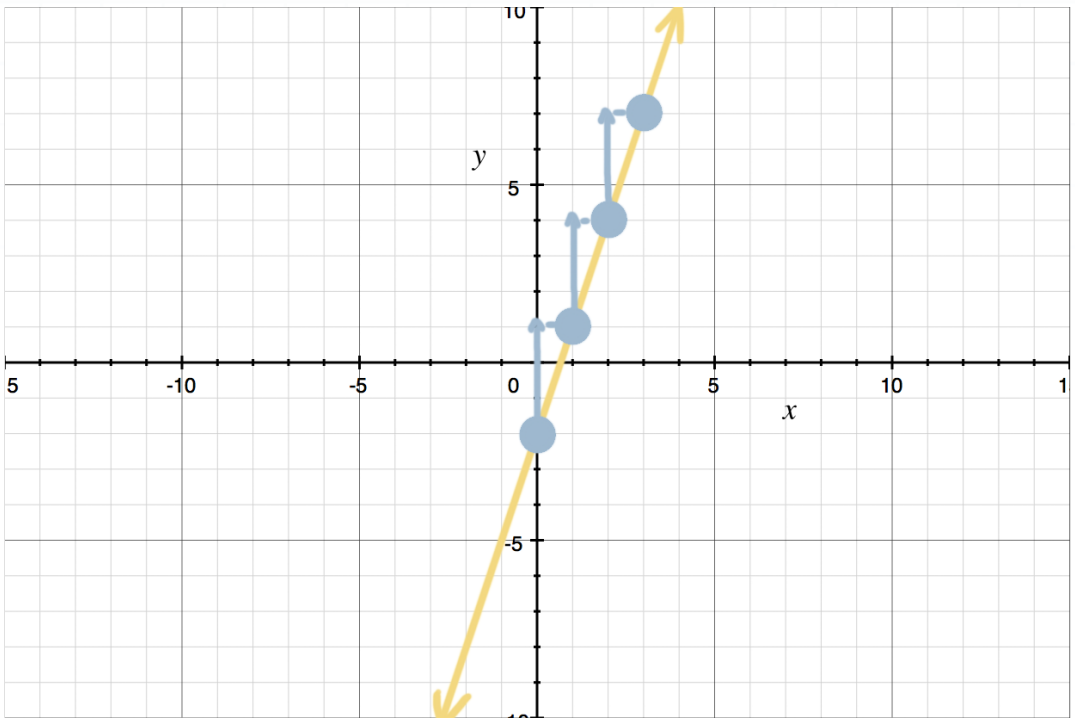
Graph the line.

$$y = 3x - 2$$

This equation is in slope-intercept form, so it's ready to be graphed. We'll plot the y-intercept $b = -2$ by placing a point at $(0, -2)$, or two units down from the origin on the y-axis.



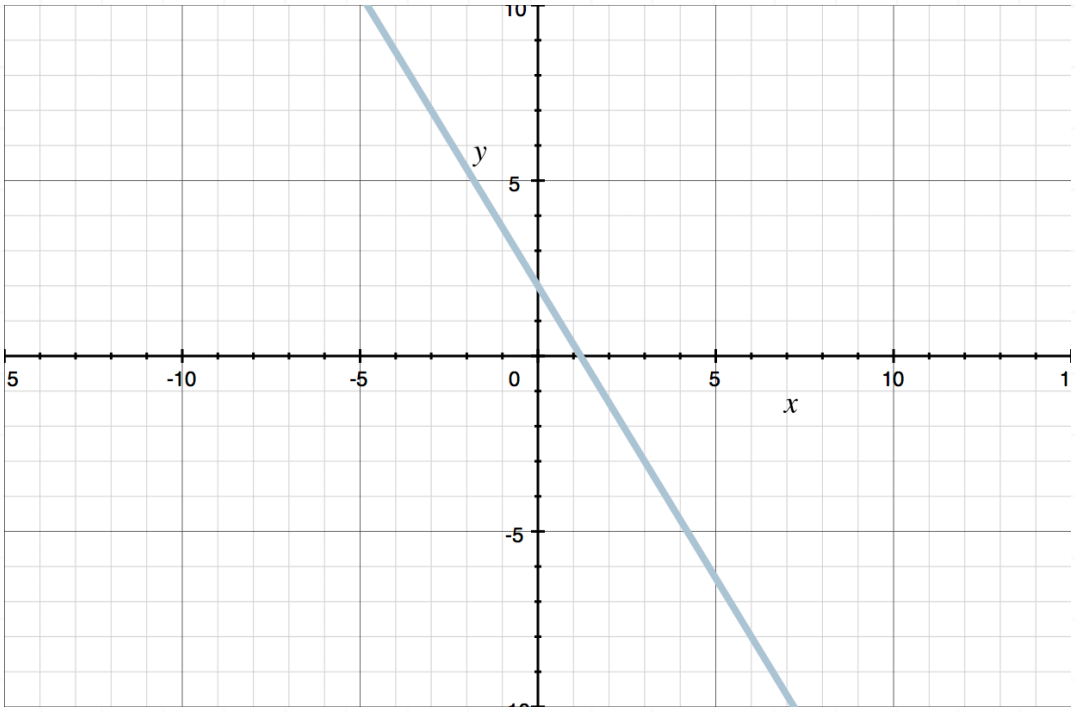
Since the slope of this line is $m = 3$, or $m = 3/1$, we'll move up 3 units and right 1 unit, and then plot a new point. So a sketch of the line is



Let's try another example, this time where we work backwards from the graph of the line to find the equation.

Example

Write the equation of the line shown in the graph.



First, identify the y-intercept. In this case the graph of the line crosses the y-axis at $b = 2$. Next, we'll find the slope by identifying another clear point on the graph, like $(3, -3)$. To get from the y-intercept to the point $(3, -3)$, we'll go 5 units down and then 3 units to the right, so the slope is $m = -5/3$ and the equation of the line is

$$y = mx + b$$

$$y = -\frac{5}{3}x + 2$$

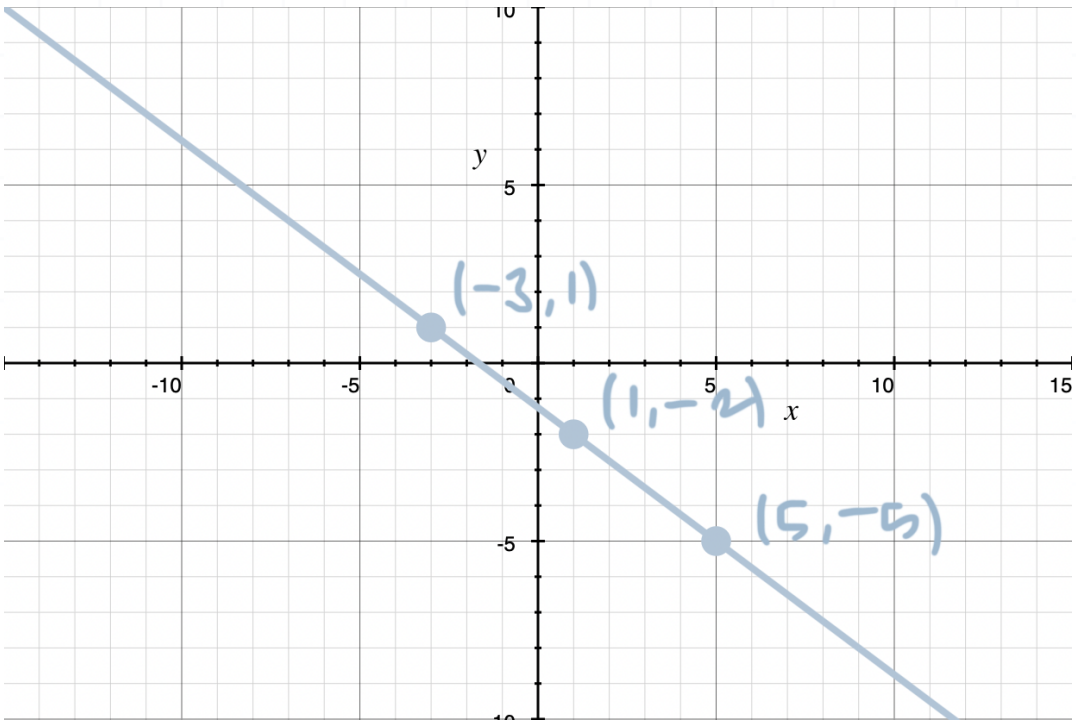
Let's try one more example where we have to interpret the slope of the line.

Example

How can we use the slope to find another point on the graph if the slope is $m = -3/4$ and the line passes through $(x_1, y_1) = (1, -2)$?

Starting at the point $(1, -2)$, we can find a second point in two ways.

We can either move up 3 and to the left 4 to plot the second point at $(1-4, -2+3) = (-3,1)$, or we can move down 3 and to the right 4 to plot the second point at $(1+4, -2-3) = (5, -5)$.



And we could keep going, moving up and left or down and right, plotting more points along the line.