

# Ratios and proportions

In this lesson we'll learn how to set up and solve ratio and proportion word problems.

For word problems, the best thing to do is to look at a few examples, but first let's review a few vocabulary terms.

**Ratio:** A ratio is a comparison of two numbers (or other mathematical expressions), and it is often written as a fraction. For example, if there are 2 boys and 5 girls in a group, then the ratio can be written as  $2 : 5$  or  $2/5$ .

**Proportion:** A proportion is an equality between two ratios.

One process that will come in handy in dealing with ratio and proportion is **cross multiplication**. When we cross multiply, we start with a proportion like

$$\frac{a}{b} = \frac{c}{d}$$

Then we multiply the numerator on the left side by the denominator on the right side (we multiply  $a$  by  $d$ ), and we multiply the numerator on the right side by the denominator on the left side (we multiply  $c$  by  $b$ ). And finally, we equate those two products:

$$a \cdot d = c \cdot b$$



To see why this works (and is mathematically legal!), notice that if we start with the equation  $a/b = c/d$ , we can multiply both sides by both denominators ( $b$  and  $d$ ):

$$\left(\frac{a}{b}\right) \cdot b \cdot d = \left(\frac{c}{d}\right) \cdot b \cdot d$$

On the left side, we can cancel the  $b$ 's, and on the right side we can cancel the  $d$ 's, so we get

$$a \cdot d = c \cdot b$$

This is exactly the same equation we get when we start with the proportion  $a/b = c/d$  and cross multiply.

Now let's do some problems.

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### Example

A pet store has 12 dogs, 14 fish, and 32 cats. What is the ratio of cats to total animals?

We know there are 32 cats, and we're looking for the ratio

$$\frac{\text{cats}}{\text{total}}$$

We can find the total number of animals by adding the numbers of animals in the three groups.

$$12 + 14 + 32 = 58$$



So we get the ratio

$$\frac{32}{58}$$

We need to simplify the ratio to lowest terms.

$$\frac{16(2)}{29(2)}$$

$$\frac{16}{29}$$

There are 16 cats for every 29 animals.

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Let's look at another type of word problem.

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### Example

Two numbers have a ratio of 1 to 9, and a sum of 80. What are the two numbers?

Let's call the two unknown numbers  $x$  (for the first one) and  $y$  (for the second one). Then we'll set up a proportion, by equating the ratio  $x/y$  to the ratio  $1/9$ , and solve for one of the variables in terms of the other.

$$\frac{x}{y} = \frac{1}{9}$$

Cross multiply.



$$9x = 1y$$

$$y = 9x$$

Next, set up an equation for  $x$  and  $y$  using what we know about their sum.

$$x + y = 80$$

Let's solve this for  $y$  in terms of  $x$ , since we already know that  $y = 9x$ .

$$y = 80 - x$$

Now let's use our system of equations and the substitution method. We know:

$$y = 80 - x$$

$$y = 9x$$

Since we have two expressions equal to  $y$ , we can set those expressions equal to each other and solve for  $x$ .

$$80 - x = 9x$$

$$80 = 9x + x$$

$$80 = 10x$$

$$\frac{80}{10} = \frac{10x}{10}$$

$$8 = x$$

$$x = 8$$



Now we can use  $y = 9x$  and the fact that  $x = 8$  to solve for  $y$ .

$$y = 9(8)$$

$$y = 72$$

Let's check our work.

It's true that

$$\frac{8}{72} = \frac{1}{9}$$

and that

$$8 + 72 = 80$$

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Let's do one more like that one.

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### Example

Two numbers have a ratio of 7 to 13, and a sum of 300. What are the two numbers?

Let's call the two unknown numbers  $x$  and  $y$ . Set up a proportion, by equation the ratio  $x/y$  to the ratio  $7/13$ , and solve for one of the variables in terms of the other.

$$\frac{x}{y} = \frac{7}{13}$$



Cross multiply.

$$13x = 7y$$

Solve for  $x$ .

$$\frac{13x}{13} = \frac{7y}{13}$$

$$x = \frac{7}{13}y$$

Next, set up an equation for  $x$  and  $y$  using what we know about their sum.

$$x + y = 300$$

Let's solve for  $x$  in terms of  $y$ , since we already know that  $x = (7/13)y$ .

$$x = 300 - y$$

Now let's use our system of equations and the substitution method. We know:

$$x = 300 - y$$

$$x = \frac{7}{13}y$$

Since we have two expressions equal to  $x$ , we can set those expressions equal to each other and solve for  $y$ .

$$\frac{7}{13}y = 300 - y$$



$$\frac{7}{13}y + y = 300$$

$$\frac{7}{13}y + \frac{13}{13}y = 300$$

$$\frac{20}{13}y = 300$$

$$\frac{13}{20} \cdot \frac{20}{13}y = \frac{13}{20} \cdot 300$$

$$y = 13 \cdot 15$$

$$y = 195$$

Now solve for  $x$ .

$$x = 300 - 195$$

$$x = 105$$

We can use the original equations to check double-check that

$$\frac{105}{195} = \frac{7}{13}$$

and that

$$105 + 195 = 300$$

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