Algebra 2
Workbook Solutions

Exponential and logarithmic functions

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MATH

WHAT IS A LOGARITHM?

■ 1. How would you read the logarithmic equation out loud?

$$\log_7 57 = y$$

Solution:

This is read "log base 7 of 57 is equal to y."

■ 2. Rewrite the equation in logarithmic form.

$$64^{\frac{1}{2}} = 8$$

Solution:

For an exponential equation given as $a^x = y$, the associated log is $\log_a y = x$. For $64^{\frac{1}{2}} = 8$, $a = 64$, $x = 1/2$, and $y = 8$, so the equivalent logarithmic equation is

$$\log_{64} 8 = \frac{1}{2}$$

■ 3. Rewrite the equation in exponential form.

$$\log_u \frac{19}{20} = v$$

Solution:

For a logarithmic equation given as $\log_a y = x$, the associated log is $a^x = y$. For the given logarithmic equation, $a = u$, $y = 19/20$, and $x = v$, so the equivalent exponential equation is

$$u^v = \frac{19}{20}$$

■ 4. Use the general log rule to solve for x.

$$\log_5(125) = x$$

Solution:

The log equation is asking “How many times do you have to multiply 5 in order to get 125? Using the general log rule, we can rewrite the equation as

$$5^x = 125$$

$$5^x = 5^3$$

$$x = 3$$

■ 5. Use the general log rule to solve the logarithm.

$$\log_8 64 = x$$

Solution:

Using the general log rule, we can rewrite the log as

$$8^x = 64$$

$$x = 2$$

Which means that the value of the logarithm is 2.

$$\log_8 64 = 2$$

■ 6. Write the logarithm that answers this question: To what power do you have to raise 289 in order to get 17?

Solution:

A logarithm always makes the statement:

> "The base must be raised to the exponent in order to get the argument."

In this case, the exponent is unknown. If we wrote this as an exponential equation, we'd write:

$$289^x = 17$$

This means 289 is the base, x is the exponent, and 17 is the argument. So as a logarithm, we'd get

$$\log_{289} 17 = x$$

COMMON BASES AND RESTRICTED VALUES

■ 1. Is there something wrong with the logarithm? If so, what is it?

$$\log_{-5}(8)$$

Solution:

Yes, there's an error. The value of the base in the logarithm is negative ($a = -5$). The base of the logarithm can only be a positive number not equal to 1.

■ 2. Is there something wrong with the logarithm? If so, what is it?

$$\log_5(-8)$$

Solution:

Yes, there's an error. The value of the base in the logarithm is a positive number not equal to 1 ($a = 5$). The argument of -8 is the problem. Because the base of a logarithm can't be negative, the argument in a logarithm can't be negative either.

Let's look at what happens when we convert the logarithm to its exponential form.

$$\log_5(-8) = x$$

$$5^x = -8$$

But there's no value we can raise 5 to that will give us any negative number, so it's impossible for the result to be -8.

■ 3. Is the following statement true or false? Why?

$$\log 5 = \log_{10} 5$$

Solution:

True: This statement is true because base 10 is the common logarithm. If a logarithm is written without a base, it implies that the logarithm is base 10.

■ 4. How else can you write $\log_e 7$?

Solution:

$\log_e$ is the natural logarithm, which we can also indicate as $\ln$, so

$$\log_e 7 = \ln 7$$

■ 5. Solve the logarithm for x.

$$\log(10{,}000)=x$$

Solution:

When a logarithm is written without a base, it implies that the logarithm is the "common logarithm," which is base 10. Rewrite the logarithm as

$$\log_{10}(10{,}000)=x$$

Use the general log rule to write the exponential equivalent.

$$10^x=10{,}000$$

$$10^x=10^4$$

$$x=4$$

So the logarithm has a value of 4.

$$\log(10{,}000)=4$$

■ 6. Solve the logarithm for x.

$$\ln(e^5)=x$$

Solution:

$\ln$ is the natural log, so we can rewrite the equation as

$$\log_e(e^5) = x$$

Use the general log rule to convert this to its exponential form.

$$e^x = e^5$$

$$x = 5$$

EVALUATING LOGS

■ 1. Find the exact value of the logarithm.

$$\log_{\frac{1}{3}} \frac{1}{27}$$

Solution:

Rewrite the logarithm $\log_{\frac{1}{3}}(1/27) = x$ in its exponential form.

$$\left(\frac{1}{3}\right)^x = \frac{1}{27}$$

$$\left(\frac{1}{3}\right)^x = \frac{1}{3^3}$$

$$(3^{-1})^x = 3^{-3}$$

$$3^{-x} = 3^{-3}$$

$$-x = -3$$

$$x = 3$$

So the value of the logarithm is 3.

■ 2. Find the exact value of the logarithm.

$$\log_9 \frac{1}{81}$$

Solution:

Rewrite the logarithm $\log_9(1/81) = x$ in its exponential form.

$$9^x = \frac{1}{81}$$

$$9^x = \frac{1}{9^2}$$

$$9^x = 9^{-2}$$

$$x = -2$$

So the value of the logarithm is -2.

$$\log_9 \frac{1}{81} = -2$$

■ 3. Find the value given by the log.

$$\log_{7,776} 6$$

Solution:

Rewrite the logarithm $\log_{7,776} 6 = x$ as an exponential equation.

$$7{,}776^x = 6$$

$$(6^5)^x = 6$$

$$6^{5x} = 6^1$$

$$5x = 1$$

$$x = \frac{1}{5}$$

So the value of the logarithm is $1/5$.

$$\log_{7{,}776} 6 = \frac{1}{5}$$

■ 4. Find the exact value of the logarithm.

$$\log_{343} 7$$

Solution:

Rewrite the logarithm $\log_{343} 7 = x$ as an exponential function.

$$343^x = 7$$

$$(7^3)^x = 7$$

$$7^{3x} = 7^1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

So the value of the logarithm is $1/3$.

$$\log_{343} 7 = \frac{1}{3}$$

■ 5. Find the exact value of the logarithm.

$$\log_{\frac{1}{4}}(1{,}024)$$

Solution:

Rewrite the logarithm $\log_{\frac{1}{4}}(1{,}024) = x$ as an exponential equation.

$$\left(\frac{1}{4}\right)^x = 1{,}024$$

$$\left(\frac{1}{4}\right)^x = 4^5$$

$$(4^{-1})^x = 4^5$$

$$4^{-x} = 4^5$$

$$-x = 5$$

$$x = -5$$

■ 6. Find the exact value of the logarithm.

$$\log_{64} \frac{1}{4}$$

Solution:

Rewrite the logarithm $\log_{64}(1/4) = x$ as an exponential equation.

$$64^x = \frac{1}{4}$$

$$(4^3)^x = \frac{1}{4^1}$$

$$4^{3x} = 4^{-1}$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

So the value of the log is $-1/3$.

$$\log_{64} \frac{1}{4} = -\frac{1}{3}$$

LAWS OF LOGARITHMS

■ 1. Write the expression as a single logarithm. Solve if possible.

$$\log_2 2 + \log_2 4$$

Solution:

Use the product rule

$$\log_a x + \log_a y = \log_a(xy)$$

to rewrite the expression as a single logarithm.

$$\log_2 2 + \log_2 4$$

$$\log_2 8$$

Simplify further by converting the logarithm into an exponential expression using the rule, if $\log_a x = y$ then $a^y = x$.

$$\log_2 8 = y$$

$$2^y = 8$$

$$y = 3$$

So the value of the logarithm is 3.

$$\log_2 2 + \log_2 4 = 3$$

■ 2. Write the expression as a single logarithm. Solve if possible.

$$\log_3 216 - \log_3 24$$

Solution:

Use the quotient rule

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

to rewrite the expression as a single logarithm.

$$\log_3 216 - \log_3 24$$

$$\log_3 \frac{216}{24}$$

$$\log_3 9$$

Simplify further by converting the logarithm into an exponential expression using the rule, if $\log_a x = y$ then $a^y = x$.

$$\log_3 9 = y$$

$$3^y = 9$$

$$y = 2$$

So the value of the logarithm is 2.

$$\log_3 216 - \log_3 24 = 2$$

■ 3. Write the expression as a single logarithm. Solve if possible.

$$\log_4 10 - 3\log_4 2$$

Solution:

Use the power rule

$$n\log_a x = \log_a x^n$$

to rewrite the expression.

$$\log_4 10 - 3\log_4 2$$

$$\log_4 10 - \log_4 2^3$$

$$\log_4 10 - \log_4 8$$

Use the quotient rule

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

to rewrite the expression as a single logarithm.

$$\log_4 10 - \log_4 8$$

$$\log_4 \frac{10}{8}$$

$$\log_4 \frac{5}{4}$$

■ 4. Write the expression as a single logarithm. Solve if possible.

$$2\log_7 4 + 3\log_7 5$$

Solution:

Use the power rule

$$n\log_a x = \log_a x^n$$

to rewrite the expression.

$$2\log_7 4 + 3\log_7 5$$

$$\log_7 4^2 + \log_7 5^3$$

$$\log_7 16 + \log_7 125$$

Use the product rule

$$\log_a x + \log_a y = \log_a(xy)$$

to rewrite the expression as a single logarithm.

$$\log_7 16 + \log_7 125$$

$$\log_7 2{,}000$$

■ 5. Simplify the logarithmic expression.

$$\log_2\left(\frac{x^4y^2}{z^3}\right)$$

Solution:

Use the quotient rule,

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

to rewrite the expression as

$$\log_2\left(\frac{x^4y^2}{z^3}\right) = \log_2 x^4y^2 - \log_2 z^3$$

Then use the product rule,

$$\log_a(xy) = \log_a x + \log_a y$$

to rewrite $\log_2 x^4y^2$ as $\log_2 x^4y^2 = \log_2 x^4 + \log_2 y^2$ and find

$$\log_2\left(\frac{x^4y^2}{z^3}\right) = \log_2 x^4 + \log_2 y^2 - \log_2 z^3$$

Use the power rule,

$$\log_a x^n = n \log_a x$$

to rewrite the expression.

$$\log_2 \left(\frac{x^4 y^2}{z^3} \right) = 4 \log_2 x + 2 \log_2 y - 3 \log_2 z$$

■ 6. Simplify the logarithmic expression.

$$\log_9 \left(\frac{9x^3}{2y^4 z} \right)$$

Solution:

Use the quotient rule,

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

to rewrite the expression as

$$\log_9 \left(\frac{9x^3}{2y^4 z} \right) = \log_9 9x^3 - \log_9 2y^4 z$$

Then use the product rule,

$$\log_a(xy) = \log_a x + \log_a y$$

to rewrite $\log_9 2y^4z$ as

$$\log_9 2y^4z = \log_9 2 + \log_9 y^4 + \log_9 z$$

and $\log_9 9x^3$ as

$$\log_9 9x^3 = \log_9 9 + \log_9 x^3$$

Then

$$\log_9\left(\frac{9x^3}{2y^4z}\right) = \log_9 9 + \log_9 x^3 - (\log_9 2 + \log_9 y^4 + \log_9 z)$$

$$\log_9\left(\frac{9x^3}{2y^4z}\right) = \log_9 9 + \log_9 x^3 - \log_9 2 - \log_9 y^4 - \log_9 z$$

$$\log_9\left(\frac{9x^3}{2y^4z}\right) = 1 + \log_9 x^3 - \log_9 2 - \log_9 y^4 - \log_9 z$$

Use the power rule,

$$\log_a x^n = n\log_a x$$

to rewrite the expression.

$$\log_9\left(\frac{9x^3}{2y^4z}\right) = 1 + 3\log_9 x - \log_9 2 - 4\log_9 y - \log_9 z$$

LAWS OF NATURAL LOGS

■ 1. Condense the expression into a single logarithm.

$$2\ln 2 - 3\ln 4$$

Solution:

Apply the power rule

$$n\ln x = \ln(x^n)$$

to the expression.

$$2\ln 2 - 3\ln 4$$

$$\ln 2^2 - \ln 4^3$$

$$\ln 4 - \ln 64$$

Use the quotient rule for natural logs.

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

to get

$$\ln\frac{4}{64}$$

$$\ln \frac{1}{16}$$

■ 2. Condense the expression into a single logarithm.

$$\frac{5 \ln 2}{4}$$

Solution:

Apply the power rule

$$n \ln x = \ln(x^n)$$

to the expression.

$$\frac{5 \ln 2}{4}$$

$$\frac{5}{4} \ln 2$$

$$\ln (2)^{\frac{5}{4}}$$

Which can also be written as

$$\ln(\sqrt[4]{2})^5$$

■ 3. Condense the expression into a single logarithm.

$$3(\ln 2x - \ln 5) + (\ln 4 - \ln 3y)$$

Solution:

Distribute the 3 into the first set of parenthesis.

$$3\ln 2x - 3\ln 5 + (\ln 4 - \ln 3y)$$

Apply the power rule

$$n\ln x = \ln(x^n)$$

to rewrite the expression as

$$\ln(2x)^3 - \ln(5)^3 + (\ln 4 - \ln 3y)$$

$$(\ln 8x^3 - \ln 125) + (\ln 4 - \ln 3y)$$

Use the quotient rule for natural logs,

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

to get

$$\ln\frac{8x^3}{125} + \ln\frac{4}{3y}$$

Now use the product rule

$$\ln(xy) = \ln x + \ln y$$

to consolidate into one logarithm.

$$\ln\left(\frac{8x^3}{125} \cdot \frac{4}{3y}\right)$$

$$\ln\left(\frac{32x^3}{375y}\right)$$

■ 4. Expand the logarithm.

$$\ln(3 \cdot 5)^4$$

Solution:

Apply the power rule for natural logs

$$n \ln x = \ln(x^n)$$

to the expression to get

$$\ln(3 \cdot 5)^4$$

$$4 \ln(3 \cdot 5)$$

Now apply the product rule

$$\ln(xy) = \ln x + \ln y$$

to get

$$4(\ln 3 + \ln 5)$$

$$4\ln 3 + 4\ln 5$$

■ 5. Expand the logarithm.

$$\ln\left(\frac{4}{5}\right)^2$$

Solution:

Apply the power rule for natural logs

$$n\ln x = \ln(x^n)$$

to the expression to get

$$\ln\left(\frac{4}{5}\right)^2$$

$$2\ln\frac{4}{5}$$

Now apply the quotient rule

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

to get

$$2(\ln 4 - \ln 5)$$

$$2 \ln 4 - 2 \ln 5$$

■ 6. Expand the logarithm.

$$\ln(3xy^2)$$

Solution:

Use the product rule

$$\ln(xy) = \ln x + \ln y$$

to get

$$\ln(3xy^2)$$

$$\ln 3 + \ln x + \ln y^2$$

Use the power rule

$$n \ln x = \ln(x^n)$$

to get

$$\ln 3 + \ln x + 2 \ln y$$

SOLVING LOGARITHMIC EQUATIONS

■ 1. Solve the equation.

$$2\log_b x = \log_b 49$$

Solution:

Use the power rule,

$$n\log_a x = \log_a x^n$$

to rewrite the left side of the equation.

$$2\log_b x = \log_b 49$$

$$\log_b x^2 = \log_b 49$$

Since the logarithms are equal and have the same base, we know the arguments must be equivalent.

$$x^2 = 49$$

$$x = 7$$

■ 2. Solve the equation.

$$\log_{12} x = \frac{3}{2} \log_{12} 16$$

Solution:

Use the power rule,

$$n \log_a x = \log_a x^n$$

to rewrite the right side of the equation.

$$\log_{12} x = \frac{3}{2} \log_{12} 16$$

$$\log_{12} x = \log_{12} 16^{\frac{3}{2}}$$

$$\log_{12} x = \log_{12} 4^3$$

Since the logarithms are equal and have the same base, we know the arguments must be equivalent.

$$x = 4^3$$

$$x = 64$$

■ 3. Solve the equation.

$$\log_5(x-1) + \log_5(x+5) = \log_5 7$$

Solution:

Use the product rule,

$$\log_a x + \log_a y = \log_a(xy)$$

to rewrite the left side of the equation.

$$\log_5(x-1) + \log_5(x+5) = \log_5 7$$

$$\log_5(x-1)(x+5) = \log_5 7$$

$$\log_5(x^2+4x-5) = \log_5 7$$

Since the logarithms are equal and have the same base, we know the arguments must be equivalent.

$$x^2+4x-5=7$$

$$x^2+4x-12=0$$

$$(x+6)(x-2)=0$$

$x=-6$ and $x=2$

If we try to verify these solutions in the original equation, we see that $x=-6$ makes the first log on the left side of the equation

$$\log_5(-6-1)$$

$$\log_5(-7)$$

We can't evaluate the logarithm at a negative value because negative values are outside the domain of the logarithm, so $x=-6$ can't be a solution, and the only solution is $x=2$.

■ 4. Solve the equation.

$$\log_9(-4x)-\log_9 12=\log_9 5$$

Solution:

Use the quotient rule,

$$\log_a x-\log_a y=\log_a\left(\frac{x}{y}\right)$$

to rewrite the left side of the equation.

$$\log_9(-4x)-\log_9 12=\log_9 5$$

$$\log_9\left(\frac{-4x}{12}\right)=\log_9 5$$

$$\log_9\left(-\frac{x}{3}\right)=\log_9 5$$

Since the logarithms are equal and have the same base, we know the arguments must be equivalent.

$$-\frac{x}{3}=5$$

$$-x = 15$$

$$x = -15$$

■ 5. Solve the equation.

$$\log_a 2 + \log_a 4 = \log_a(x + 2)$$

Solution:

Use the product rule

$$\log_a x + \log_a y = \log_a(xy)$$

to rewrite the left side of the expression.

$$\log_a 2 + \log_a 4 = \log_a(x + 2)$$

$$\log_a 8 = \log_a(x + 2)$$

Since the logarithms are equal and have the same base, we know the arguments must be equivalent.

$$x + 2 = 8$$

$$x = 6$$

■ 6. Solve the equation.

$$\log_4(x+5) - \log_4(x-2) = \log_4 3$$

Solution:

Use the quotient rule

$$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$$

to rewrite the left side of the expression.

$$\log_4(x+5) - \log_4(x-2) = \log_4 3$$

$$\log_4\left(\frac{x+5}{x-2}\right) = \log_4 3$$

Since the logarithms are equal and have the same base, we know the arguments must be equivalent.

$$\frac{x+5}{x-2} = 3$$

$$x+5 = 3(x-2)$$

$$x+5 = 3x-6$$

$$11 = 2x$$

$$x = \frac{11}{2}$$

CHANGE OF BASE

■ 1. Find the value of the expression to four decimal places.

$$\log_3 6$$

Solution:

Use the change of base formula

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to change the base into a common log of base 10.

$$\log_3 6 = \frac{\log_{10} 6}{\log_{10} 3} \approx 1.6309$$

■ 2. Find the value of the expression to four decimal places.

$$\frac{\log 25}{\log 5} - \frac{\log 25}{\log 125}$$

Solution:

Use the change of base formula,

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to rewrite both fractions.

$$\frac{\log 25}{\log 5} - \frac{\log 25}{\log 125}$$

$$\log_5 25 - \log_{125} 25$$

Next, let $x = \log_5 25$, and use the general log rule to convert this to exponential form.

$$5^x = 25$$

$$5^x = 5^2$$

$$x = 2$$

Now let $x = \log_{125} 25$, and use the general log rule to convert this to exponential form.

$$125^x = 25$$

$$(5^3)^x = 5^2$$

$$5^{3x} = 5^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

Finally, substitute these values into the expression.

$$\log_5 25 - \log_{125} 25$$

$$2 - \frac{2}{3}$$

$$2\left(\frac{3}{3}\right) - \frac{2}{3}$$

$$\frac{6}{3} - \frac{2}{3}$$

$$\frac{4}{3}$$

■ 3. Find the approximate value to the expression to the nearest ten thousandth.

$$\frac{\log 18}{\log 2} - \frac{\log 9}{\log 2}$$

Solution:

Use the change of base formula,

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to rewrite both fractions.

$$\frac{\log 18}{\log 2} - \frac{\log 9}{\log 2}$$

$$\log_2 18 - \log_2 9$$

Then use the quotient rule,

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

to rewrite the expression as a single logarithm.

$$\log_2 \frac{18}{9}$$

$$\log_2 2$$

$$1$$

■ 4. Write the log expression in terms of natural logs.

$$\log_4 6.7$$

Solution:

We can use the change of base formula

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to change the base into a natural log of base e.

$$\log_4 6.7 = \frac{\log_e 6.7}{\log_e 4} = \frac{\ln 6.7}{\ln 4}$$

■ 5. Find the exact value of the logarithmic expression.

$$\frac{\ln(16{,}807)}{\ln(7)}$$

Solution:

Use the change of base formula

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to rewrite the expression as one log.

$$\frac{\ln(16807)}{\ln(7)} = \frac{\log_e(16{,}807)}{\log_e(7)} = \log_7(16{,}807)$$

Now use the general log rule to convert this to exponential form to solve the logarithm.

$$\log_7(16{,}807) = x$$

$$7^x = 16{,}807$$

$$7^x = 7^5$$

$$x = 5$$

So the value of the logarithmic expression is 5.

$$\frac{\ln(16{,}807)}{\ln(7)} = 5$$

■ 6. Use logarithms to solve the equation to the nearest ten thousandth.

$$8 \cdot 6^{3x} = 4{,}104$$

Solution:

Here we want to solve for the variable x, so we want to try to get the variable by itself. Divide both sides by 8.

$$8 \cdot 6^{3x} = 4{,}104$$

$$6^{3x} = 513$$

Now we can use the general log rule to change this into a logarithmic equation.

$$\log_6 513 = 3x$$

Use the change of base formula to convert this to common logarithms.

$$3x = \frac{\log 513}{\log 6}$$

$$x = \frac{\log 513}{3 \log 6}$$

$x \approx 1.1609$

GRAPHING EXPONENTIAL FUNCTIONS

■ 1. Will the function have a vertical or horizontal asymptote? Where is it located? What is the end behavior of the function?

$$y = \left(\frac{1}{3}\right)^{x-2} + 3$$

Solution:

Because the function is an exponential function that's solved for y in terms of x, it means the function will have a horizontal asymptote. To find it, we can plug both $x = 100$ and $x = -100$ into the equation.

For $x = 100$,

$$y = \left(\frac{1}{3}\right)^{100-2} + 3$$

$$y = \left(\frac{1}{3}\right)^{98} + 3$$

$$y = \frac{1^{98}}{3^{98}} + 3$$

$$y = \frac{1}{\text{a very large number}} + 3$$

$$y = 0 + 3$$

$$y = 3$$

For $x = -100$,

$$y = \left(\frac{1}{3}\right)^{-100-2} + 3$$

$$y = \left(\frac{1}{3}\right)^{-102} + 3$$

$$y = \frac{1^{-102}}{3^{-102}} + 3$$

$$y = \frac{3^{102}}{1^{102}} + 3$$

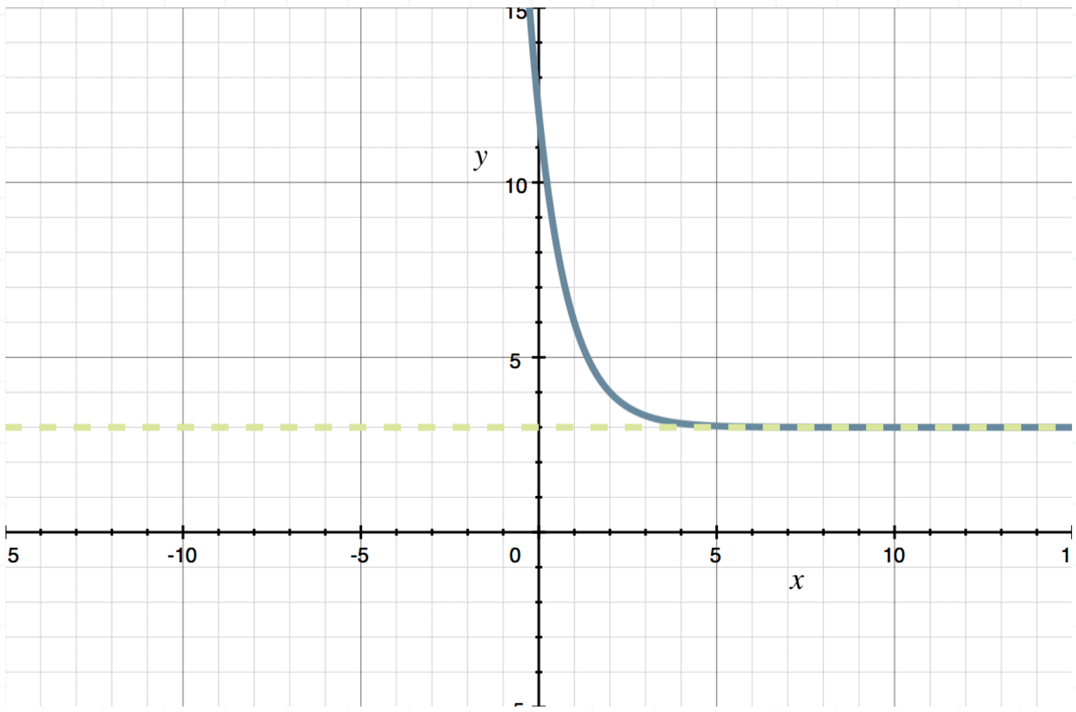
$$y = \frac{\text{a very large number}}{1} + 3$$

$$y = \infty + 3$$

$$y = \infty$$

Plugging in $x = 100$ and $x = -100$ gives us a picture of the end behavior of the graph of the function. The results tell us that the function has a horizontal asymptote at $y = 3$ and will tend toward ∞ as $x \to -\infty$.

A sketch of the function shows the end behavior.



■ 2. Will the function have a vertical or horizontal asymptote? Where is it located? What is the end behavior of the function?

$$x = -4^{y+3} - 2$$

Solution:

Because the function is an exponential function that's solved for x in terms of y, it means the function will have a vertical asymptote. To find it, we can plug both $y = 100$ and $y = -100$ into the equation.

For $y = 100$,

$$x = -4^{100+3} - 2$$

$$x = -4^{103} - 2$$

$x = -$ (a very large number) $- 2$

$$x = -\infty - 2$$

$$x = -\infty$$

For $y = -100$,

$$x = -4^{-100+3} - 2$$

$$x = -4^{-97} - 2$$

$$x = -\frac{1}{4^{97}} - 2$$

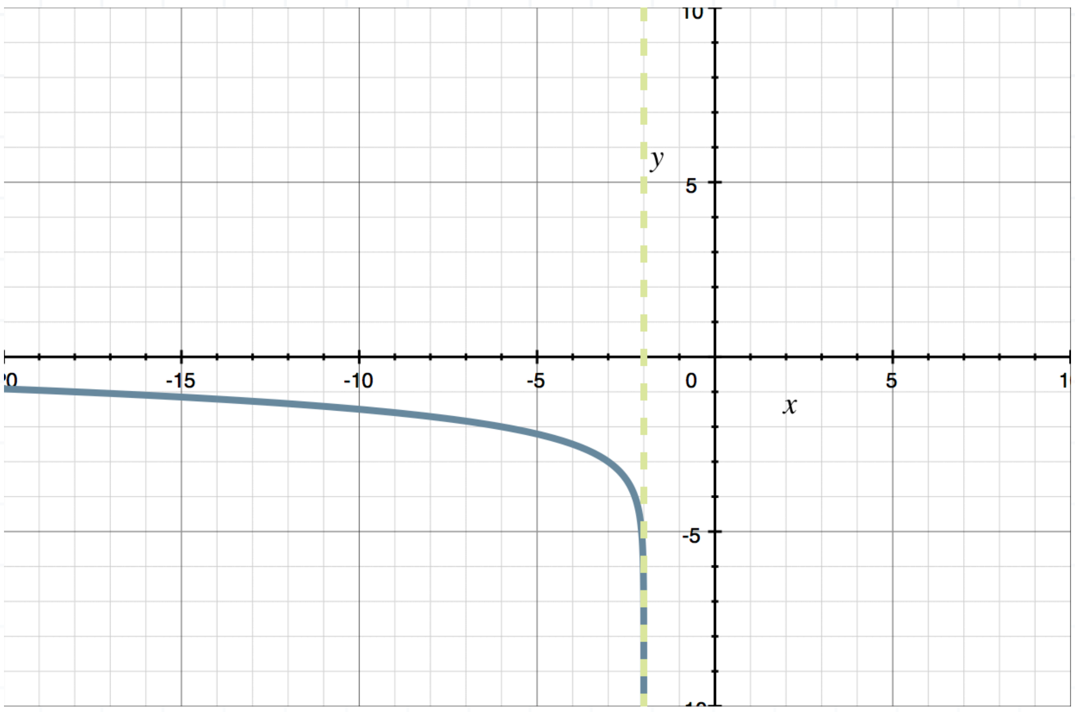
$$x = -\frac{1}{\text{a very large number}} - 2$$

$$x = 0 - 2$$

$$x = -2$$

Plugging in $y = 100$ and $y = -100$ gives us a picture of the end behavior of the graph of the function. The results tell us that the function has a vertical asymptote at $x = -2$ and will tend toward $-\infty$ as $y \to \infty$.

A sketch of the function shows the end behavior.



■ 3. Graph the exponential function.

$$f(x) = \left(\frac{4}{5}\right)^x$$

Solution:

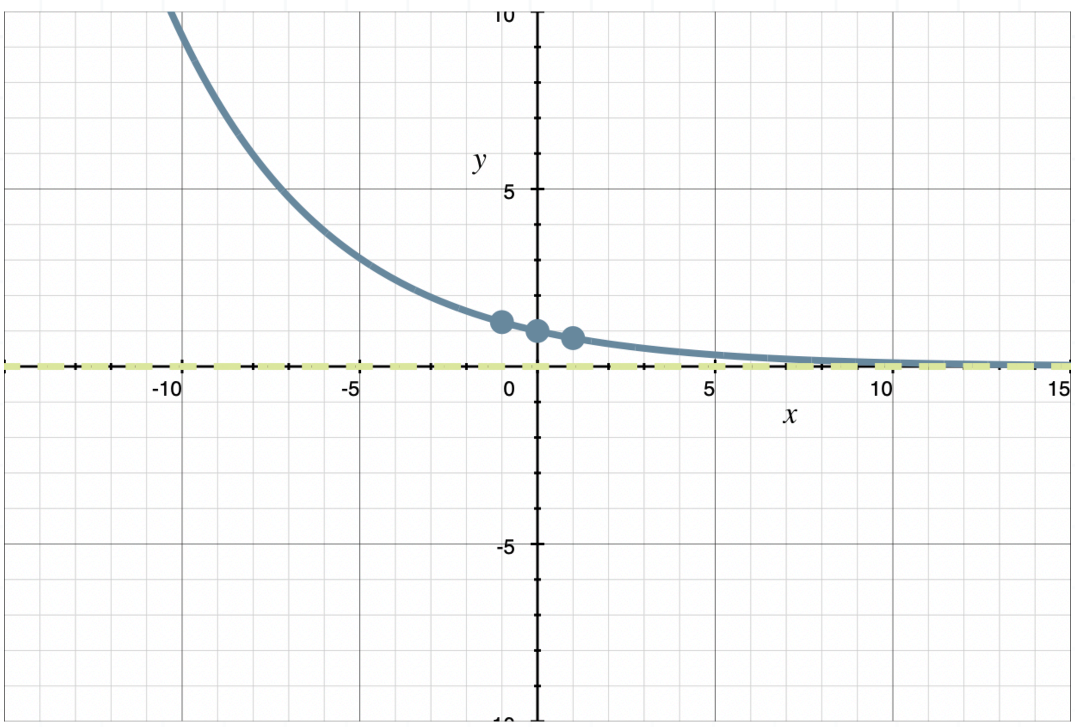
Since the base $0 < b < 1$, we know the function is decreasing. The horizontal asymptote is $y = 0$. We'll plug in a few values of x for which the value of $f(x)$ will be easy to calculate.

For $x = 0$: $f(0) = (4/5)^0 = 1$

For $x = -1$: $f(-1) = (4/5)^{-1} = 5/4$

For $x = 1$: $f(1) = (4/5)^1 = 4/5$

Now we have three points on the the graph of f: $(0,1)$, $(-1,5/4)$, and $(1,4/5)$. If we plot the points and connect them with a smooth curve that accounts for the horizontal asymptote $y=0$, we get



■ 4. What is the x-intercept of the function?

$$x=\left(\frac{11}{8}\right)^{y}-4$$

Solution:

The x-intercept of a function is where the value of y is equal to 0. We can find this by plugging 0 into the equation.

$$x=\left(\frac{11}{8}\right)^{0}-4$$

$$x = 1 - 4$$

$$x = -3$$

This means the coordinate of the x-intercept is the point $(-3,0)$.

■ 5. Sketch the graph of the function by finding the y-intercept and figuring out the function's end behavior.

$$y = -2^{x+1} - 5$$

Solution:

The y-intercept of a function is where the value of x is equal to 0. We can find this by plugging $x = 0$ into the equation.

$$y = -2^{0+1} - 5$$

$$y = -2^{1} - 5$$

$$y = -2 - 5$$

$$y = -7$$

This means the coordinate of the y-intercept is the point $(0, -7)$.

Because the function is an exponential function that's solved for y in terms of x, it means the function will have a horizontal asymptote. To find it, we can plug both $x = 100$ and $x = -100$ into the equation.

For $x = 100$,

$$y = -2^{100+1} - 5$$

$$y = -2^{101} - 5$$

$$y = -(\text{a very large number}) - 5$$

$$y = -\infty - 5$$

$$y = -\infty$$

For $x = -100$,

$$y = -2^{-100+1} - 5$$

$$y = -2^{-99} - 5$$

$$y = -\frac{1}{2^{99}} - 5$$

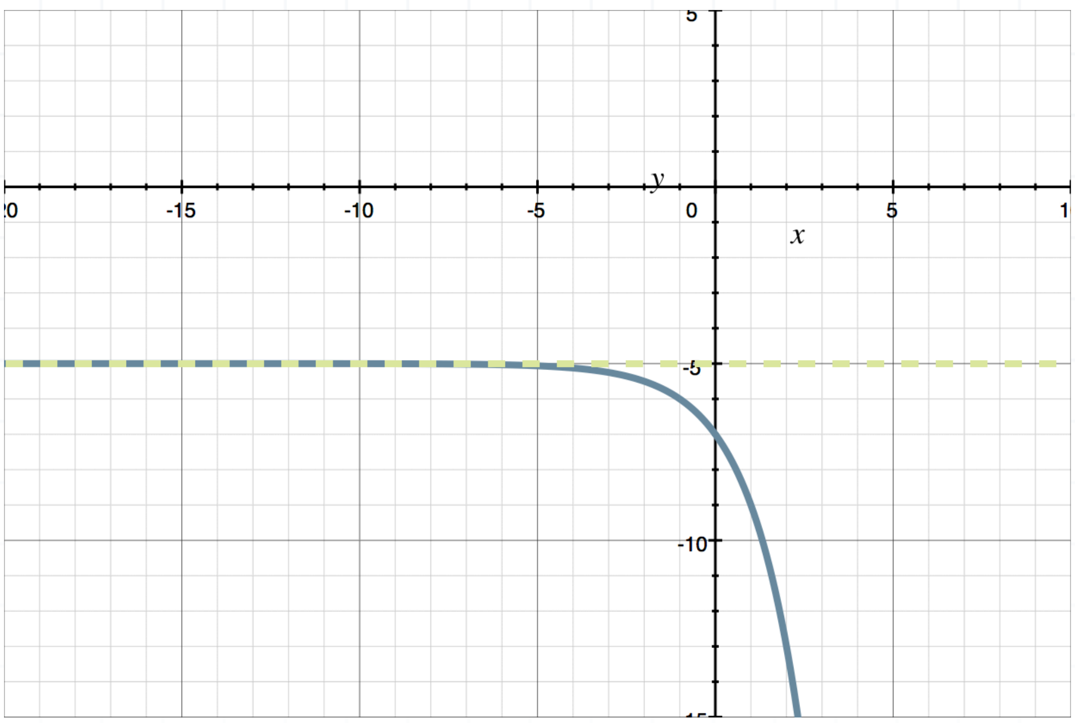
$$y = -\frac{1}{\text{a very large number}} - 5$$

$$y = -\frac{1}{\infty} - 5$$

$$y = -0 - 5$$

$$y = -5$$

Plugging in $x = 100$ and $x = -100$ gives us a picture of the end behavior of the graph of the function. The results tell us that the function will tend toward negative infinity as $x \to \infty$ and have a horizontal asymptote at $y = -5$ when $x \to -\infty$.



6. Sketch the graph of the function using the x-intercept and the end behavior.

$$x = \left(\frac{1}{4}\right)^{y+2} - 2$$

Solution:

The x-intercept of a function is where the value of y is equal to 0. We can find this by plugging $y = 0$ into the equation.

$$x = \left(\frac{1}{4}\right)^{0+2} - 2$$

$$x = \left(\frac{1}{4}\right)^2 - 2$$

$$x = \frac{1}{16} - 2$$

$$x = \frac{1}{16} - \frac{32}{16}$$

$$x = -\frac{31}{16}$$

This means the coordinate of the x-intercept is the point $(-31/16,0)$.

Because the function is an exponential function that's solved for x in terms of y, it means the function will have a vertical asymptote. To find it, we can plug both $y = 100$ and $y = -100$ into the equation.

For $y = 100$,

$$x = \left(\frac{1}{4}\right)^{y+2} - 2$$

$$x = \left(\frac{1}{4}\right)^{100+2} - 2$$

$$x = \left(\frac{1}{4}\right)^{102} - 2$$

$$x = \frac{1^{102}}{4^{102}} - 2$$

$$x = \frac{1}{\text{a very large number}} - 2$$

$$x = 0 - 2$$

$$x = -2$$

For $y = -100$,

$$x = \left(\frac{1}{4}\right)^{-100+2} - 2$$

$$x = \left(\frac{1}{4}\right)^{-98} - 2$$

$$x = \frac{1^{-98}}{4^{-98}} - 2$$

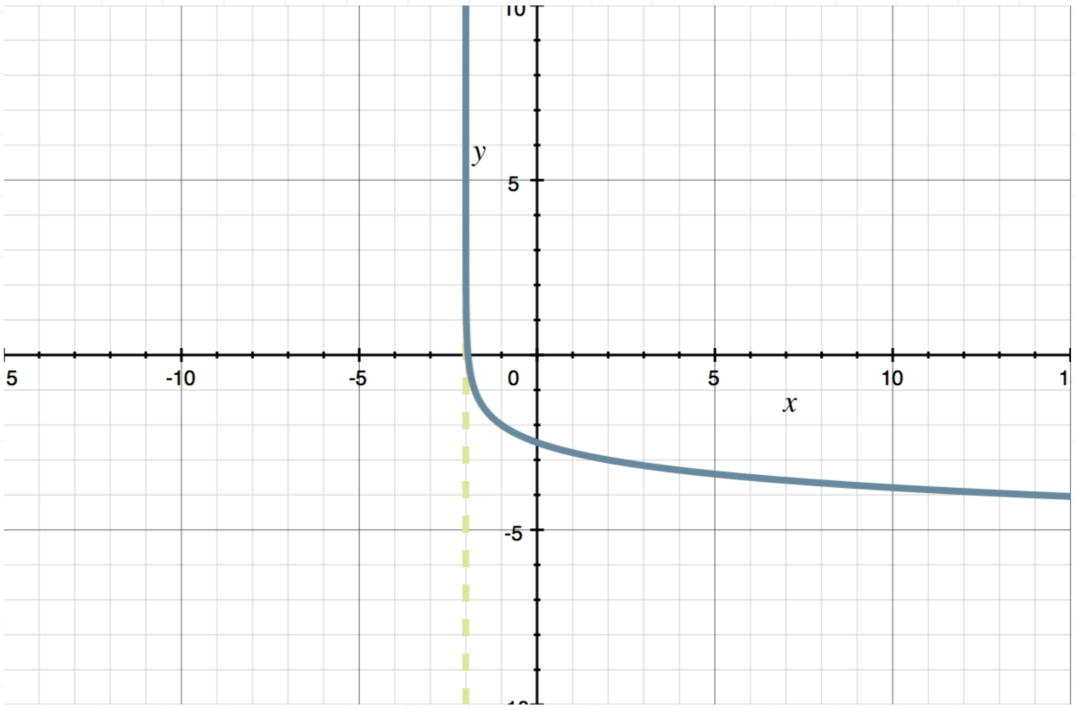
$$x = \frac{4^{98}}{1^{98}} - 2$$

$$x = \frac{\text{a very large number}}{1} - 2$$

$$x = \infty - 2$$

$$x = \infty$$

Plugging in $y = 100$ gives us a picture of the end behavior of the graph of the function. The results tell us that the function will tend toward infinity as $x \to -2$ and therefore have a vertical asymptote at $x = -2$ as $y \to \infty$.


10
y
5
5
-10
-5
0
5
10
1
x
-5

GRAPHING TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS

■ 1. Sketch the graph of the function.

$$y = 3^{x-2} - 3$$

Solution:

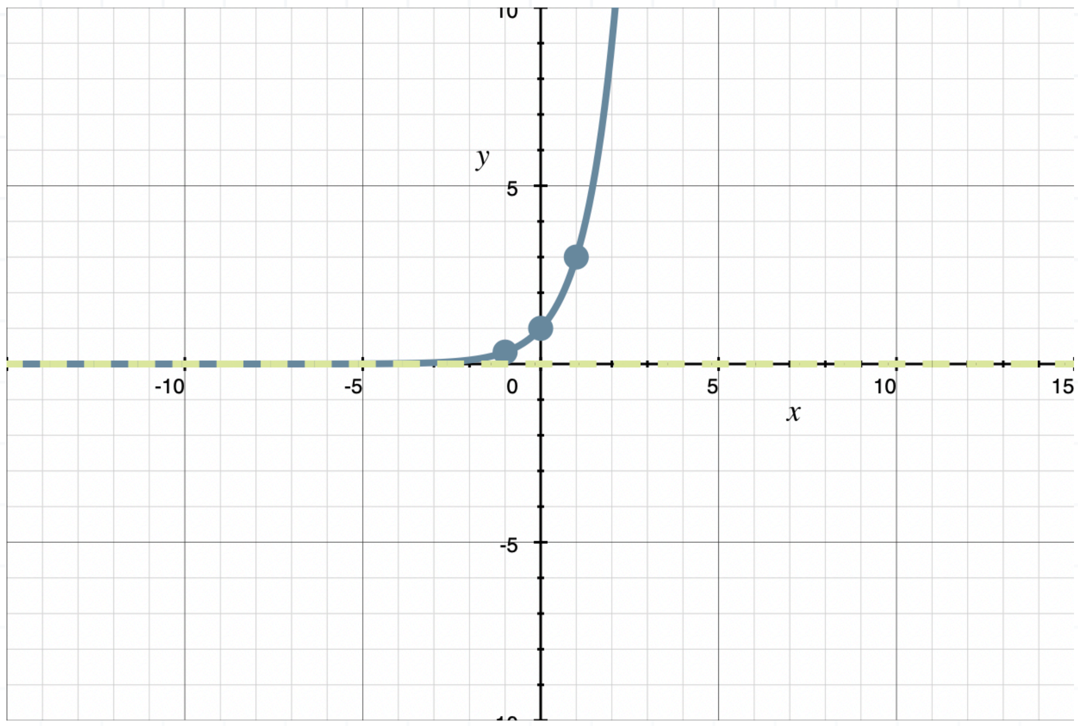
If we identify the parent function $y = 3^x$, we can plug in a few values of x for which the value of $f(x)$ will be easy to calculate.

For $x = 0$: $f(0) = 3^0 = 1$

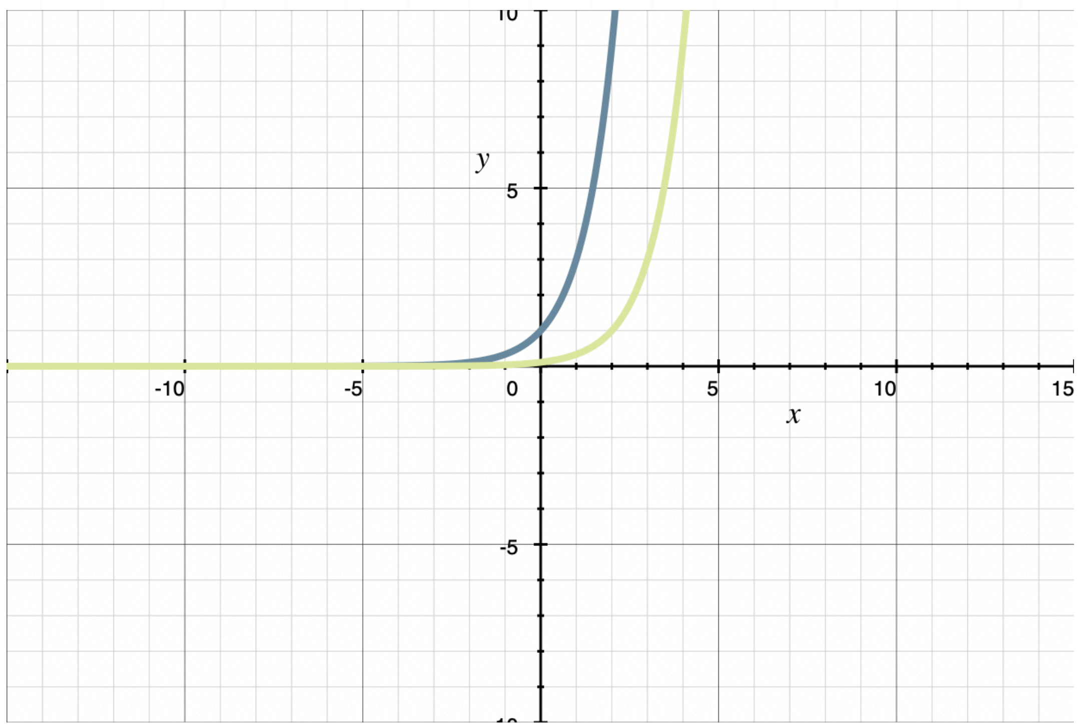
For $x = -1$: $f(-1) = 3^{-1} = 1/3$

For $x = 1$: $f(1) = 3^1 = 3$

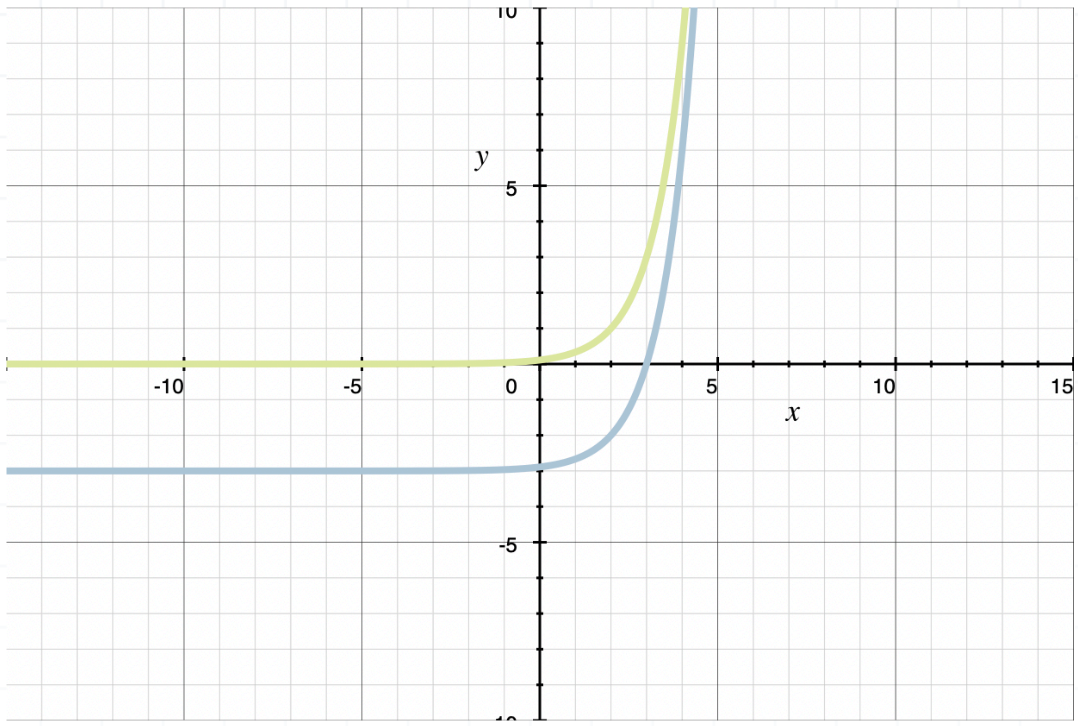
Now we have three points on the graph of f: $(0,1)$, $(-1,1/3)$, and $(1,3)$. If we plot them and draw the horizontal asymptote $y = 0$, we get



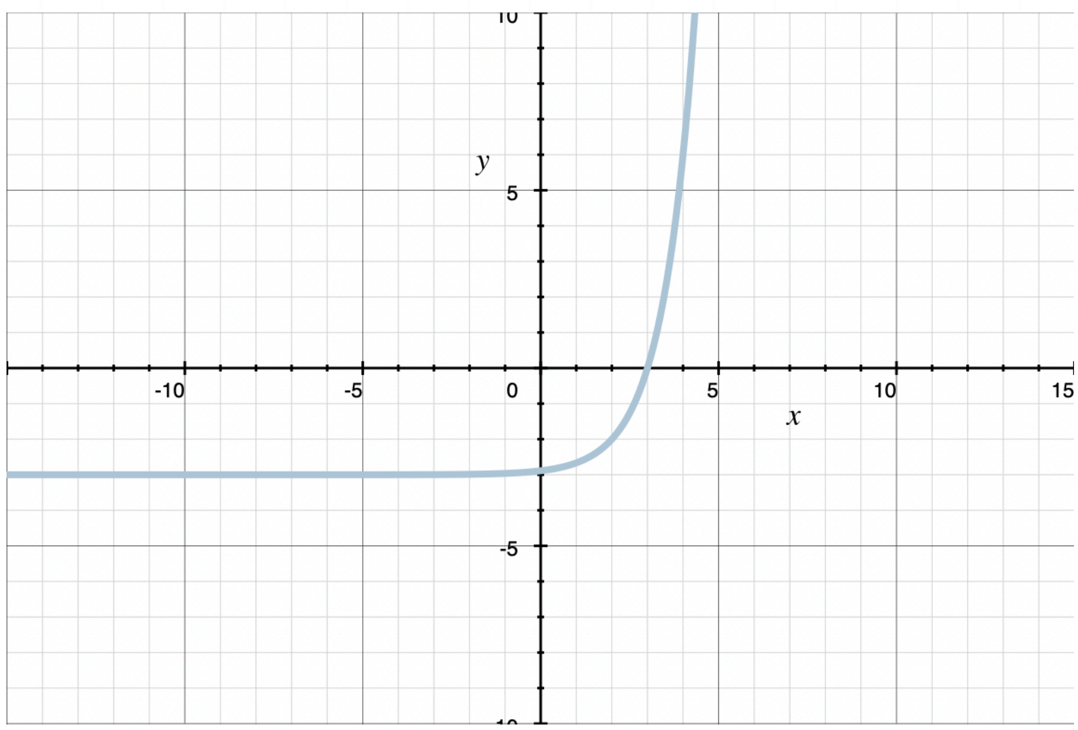
To go from $y = 3^x$ to $y = 3^{x-2}$, we shift the graph 2 units to the right,



and to go from $y = 3^{x-2}$ to $y = 3^{x-2} - 3$, we shift the graph 3 units down.



So the final sketch of $y = 3^{x-2} - 3$ is



■ 2. Sketch the graph of the function.

$$y = -2(2^{x+3}) + 1$$

Solution:

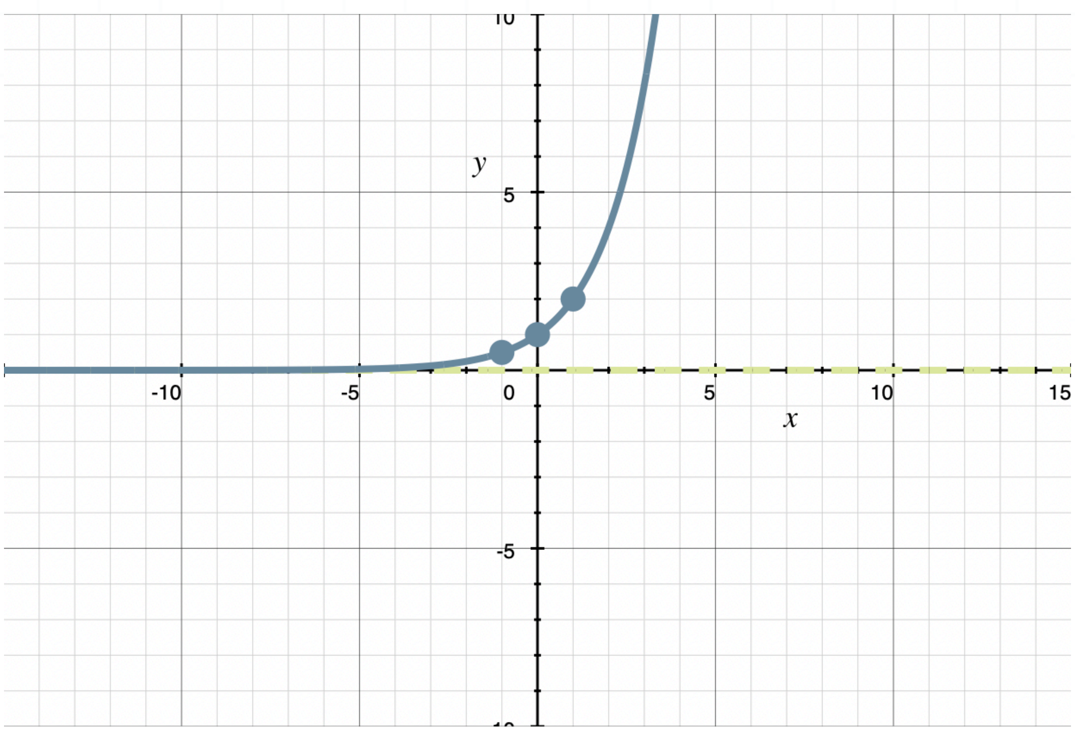
If we identify the parent function $y = 2^x$, we can plug in a few values of x for which the value of $f(x)$ will be easy to calculate.

For $x = 0$: $f(0) = 2^0 = 1$

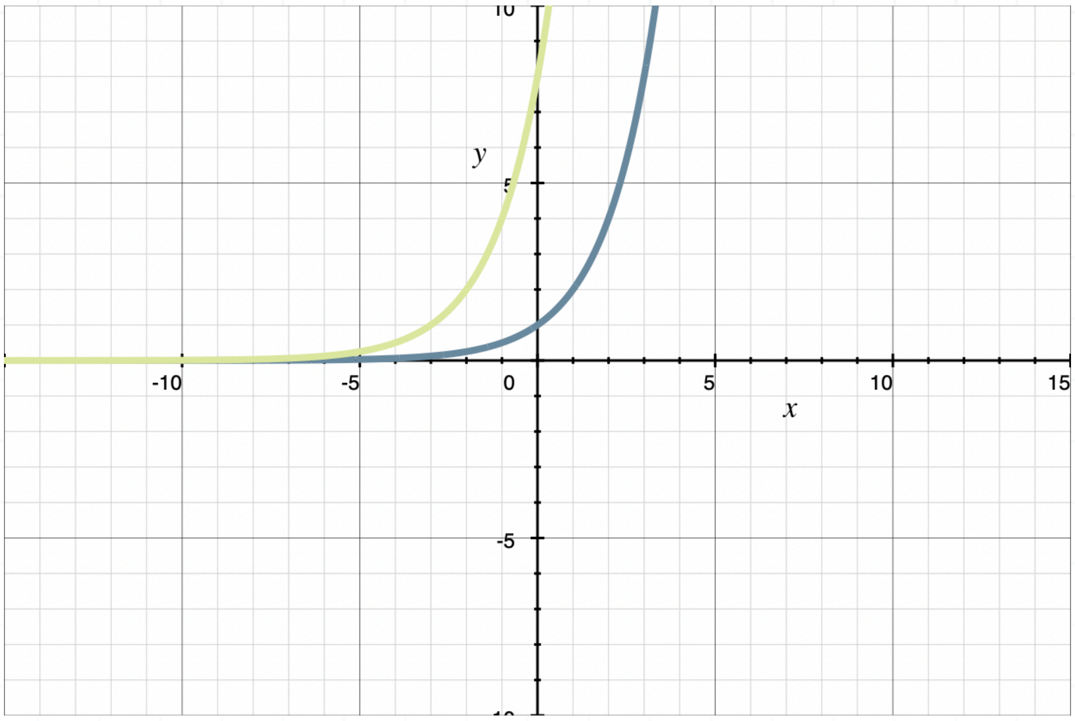
For $x = -1$: $f(-1) = 2^{-1} = 1/2$

For $x = 1$: $f(1) = 2^1 = 2$

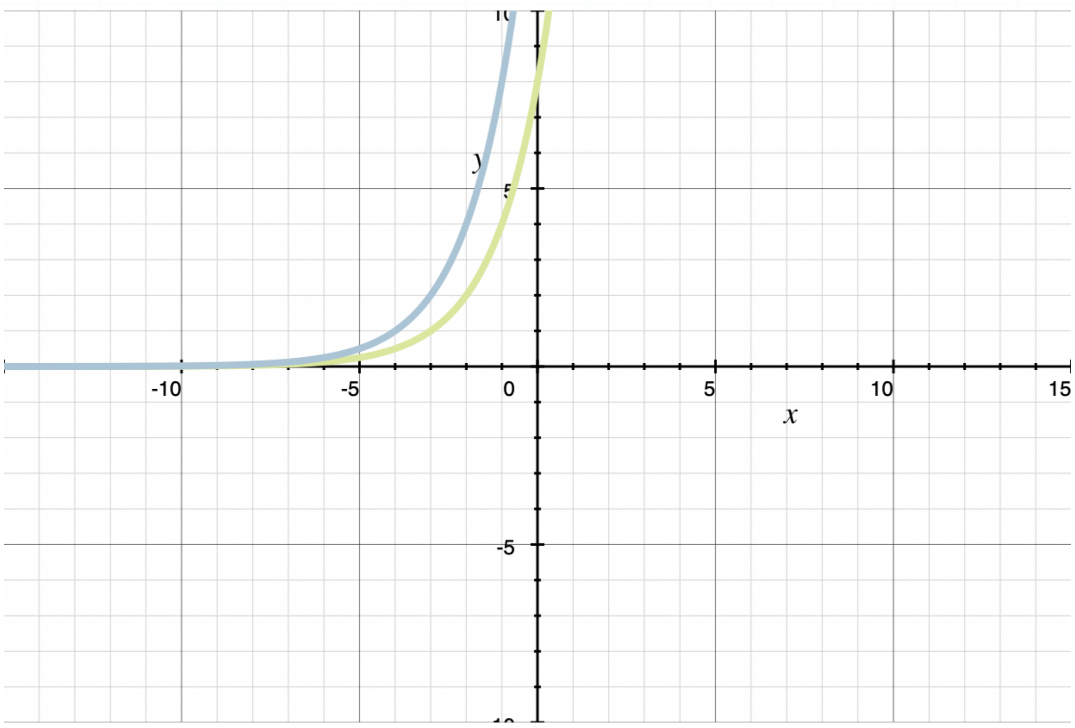
Now we have three points on the graph of f: $(0,1)$, $(-1,1/2)$, and $(1,2)$. If we plot them and draw the horizontal asymptote $y = 0$, we get



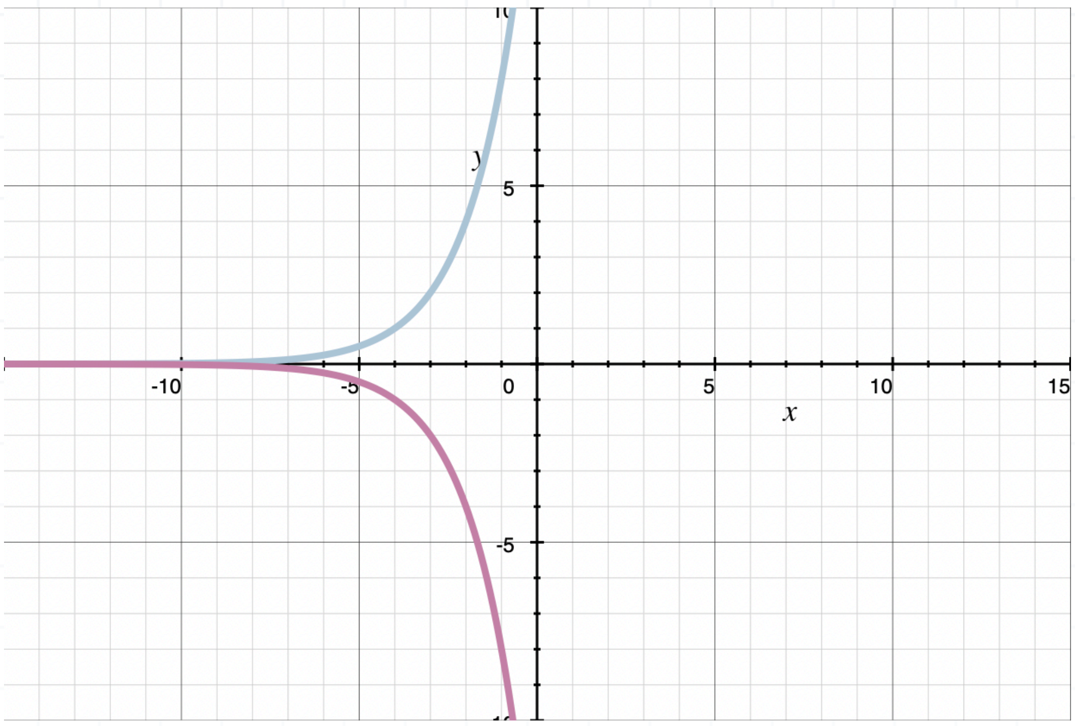
To go from $y = 2^x$ to $y = 2^{x+3}$, we shift the graph 3 units to the left,



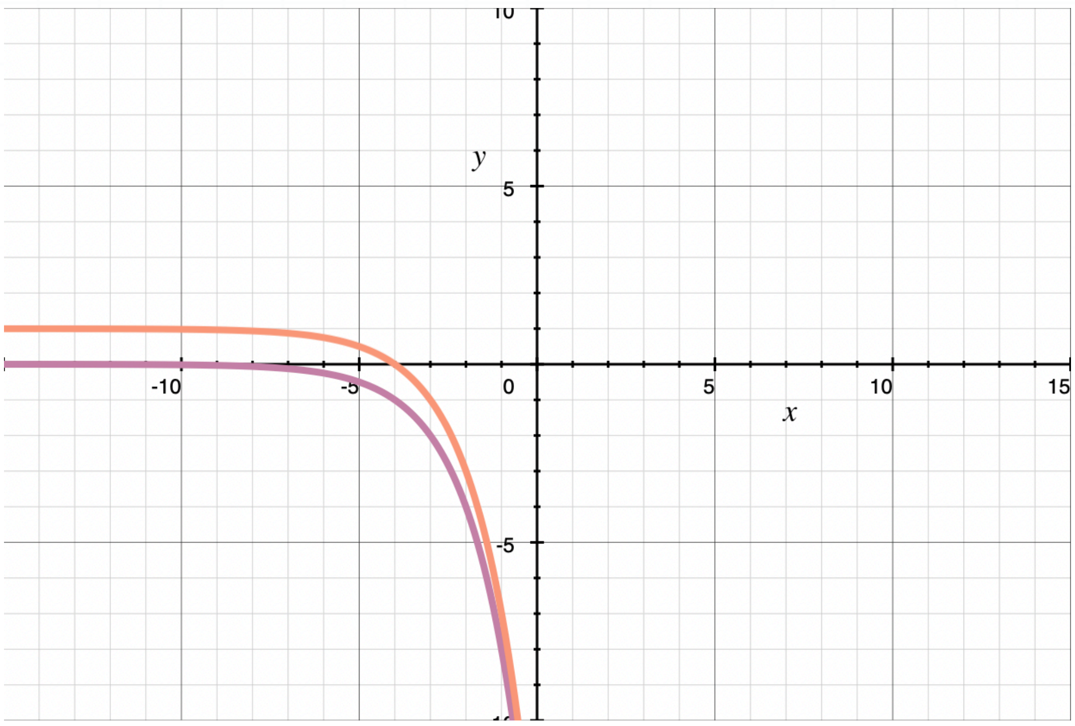
and to go from $y = 2^{x+3}$ to $y = 2(2^{x+3})$, we stretch the graph vertically by a factor of 2.



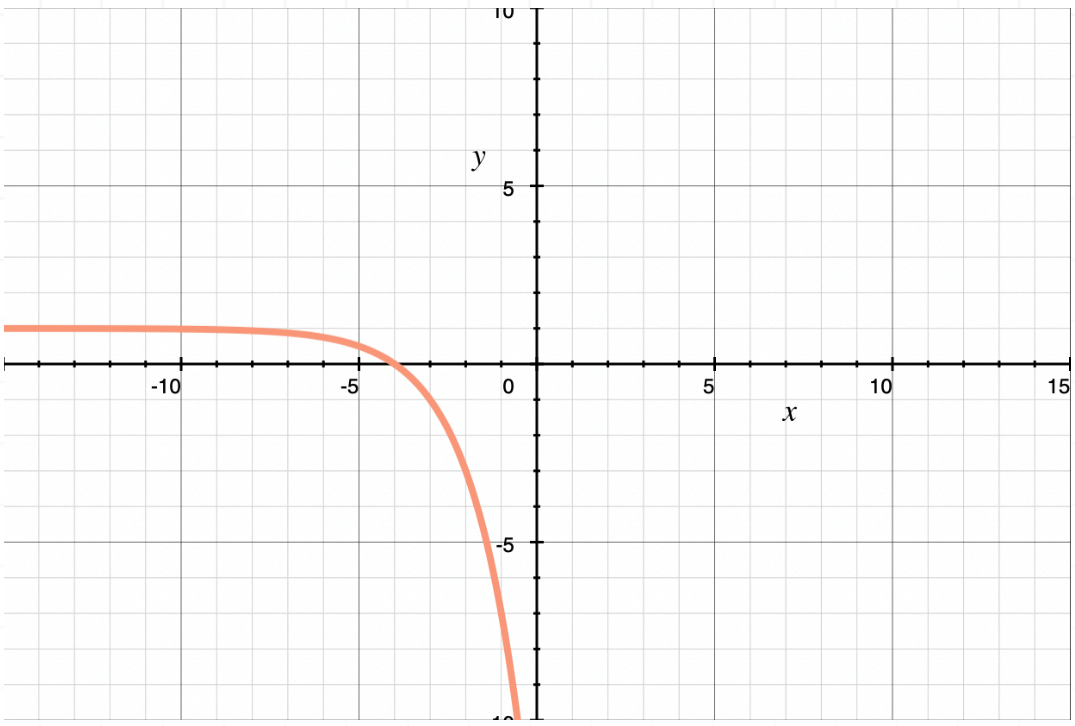
To go from $y = 2(2^{x+3})$ to $y = -2(2^{x+3})$, we reflect the graph across the x-axis,



and to go from $y = -2(2^{x+3})$ to $y = -2(2^{x+3}) + 1$, we move the graph up 1 unit.



So the final sketch of $y = -2(2^{x+3}) + 1$ is



■ 3. Graph the exponential function.

$$f(x) = \left(\frac{1}{2}\right)^{-x-2} - 5$$

Solution:

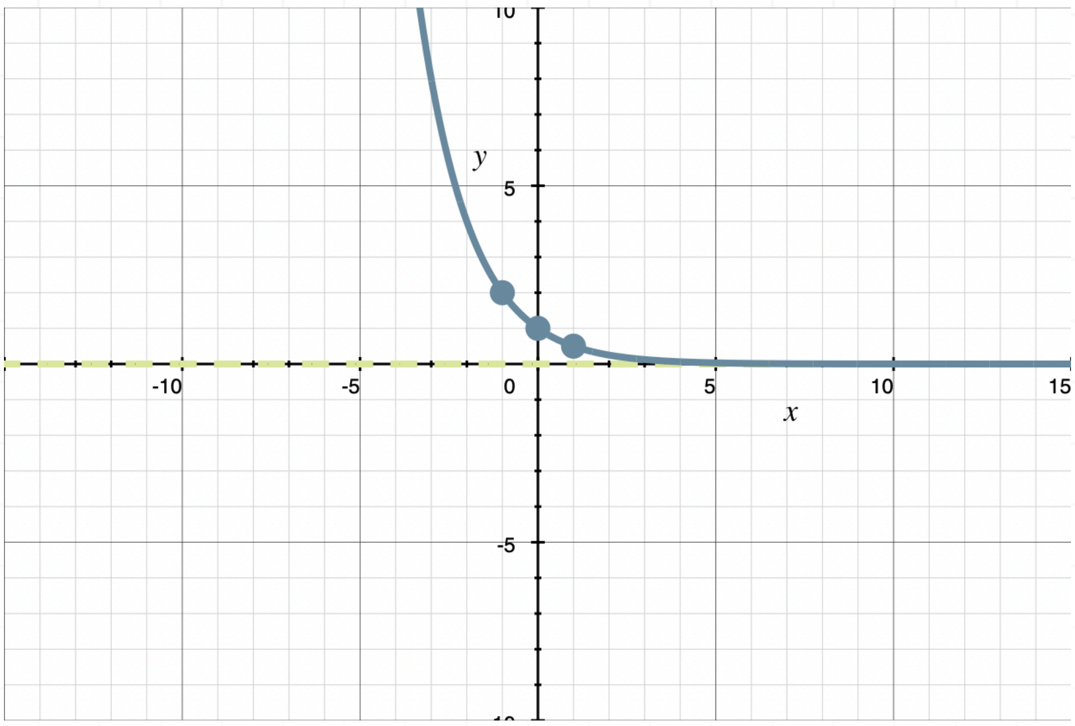
If we identify the parent function $y = (1/2)^x$, we can plug in a few values of x for which the value of $f(x)$ will be easy to calculate.

For $x = 0$: $f(0) = (1/2)^0 = 1$

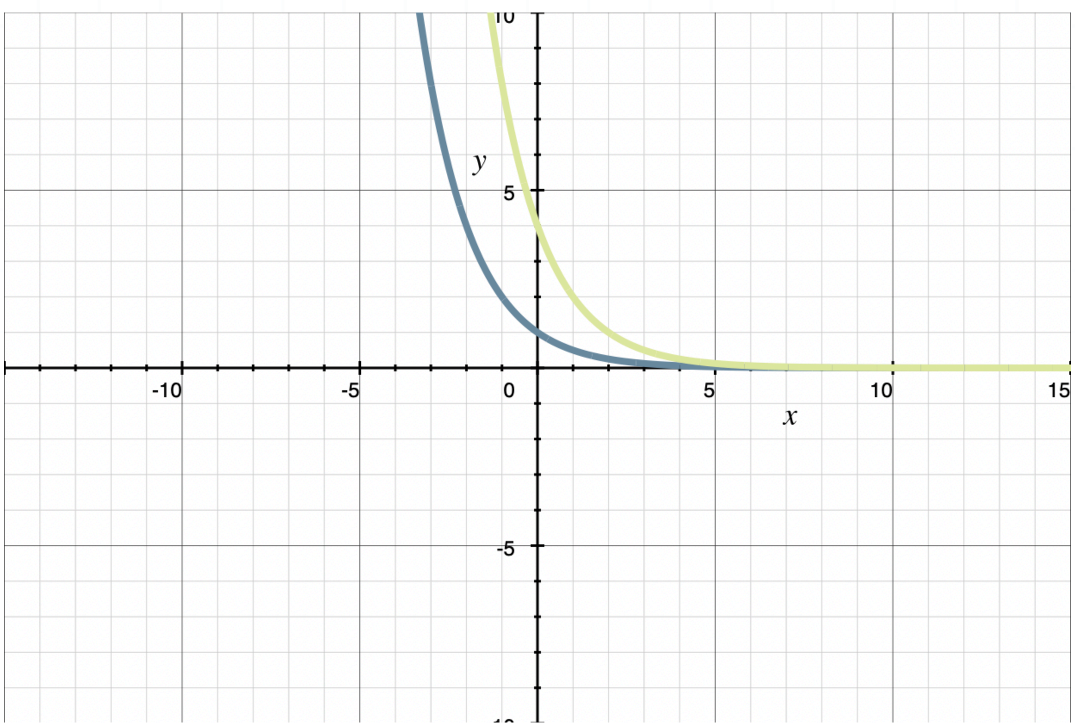
For $x = -1$: $f(-1) = (1/2)^{-1} = 2$

For $x = 1$: $f(1) = (1/2)^1 = 1/2$

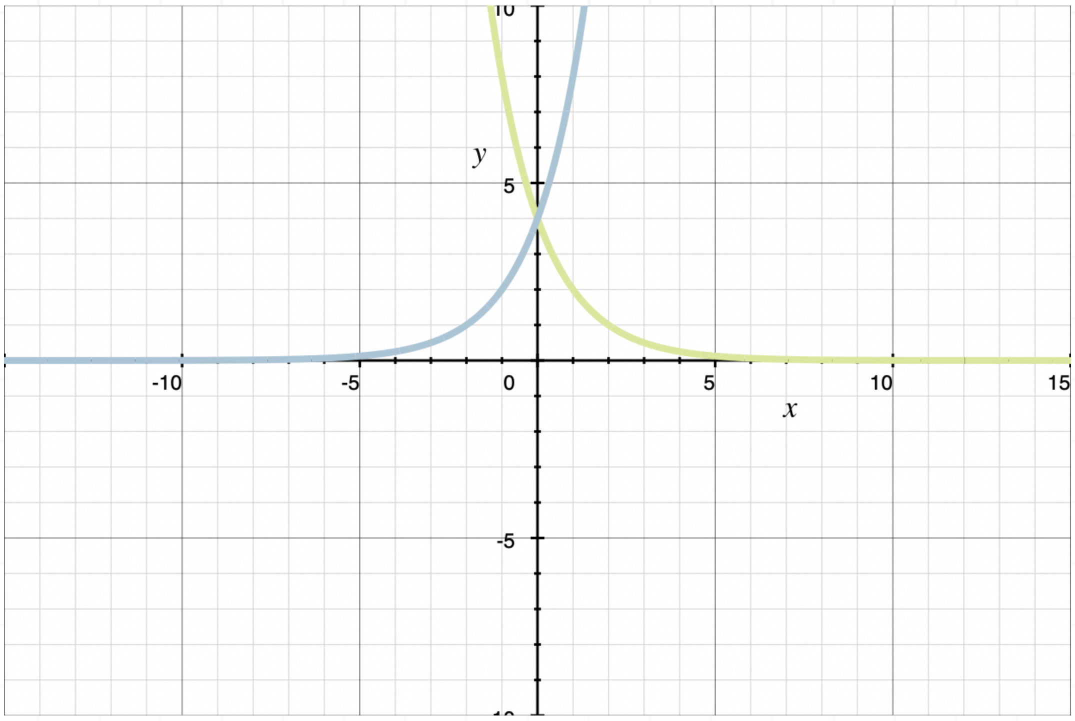
Now we have three points on the graph of f: $(0,1)$, $(-1,2)$, and $(1,1/2)$. If we plot them and draw the horizontal asymptote $y = 0$, we get



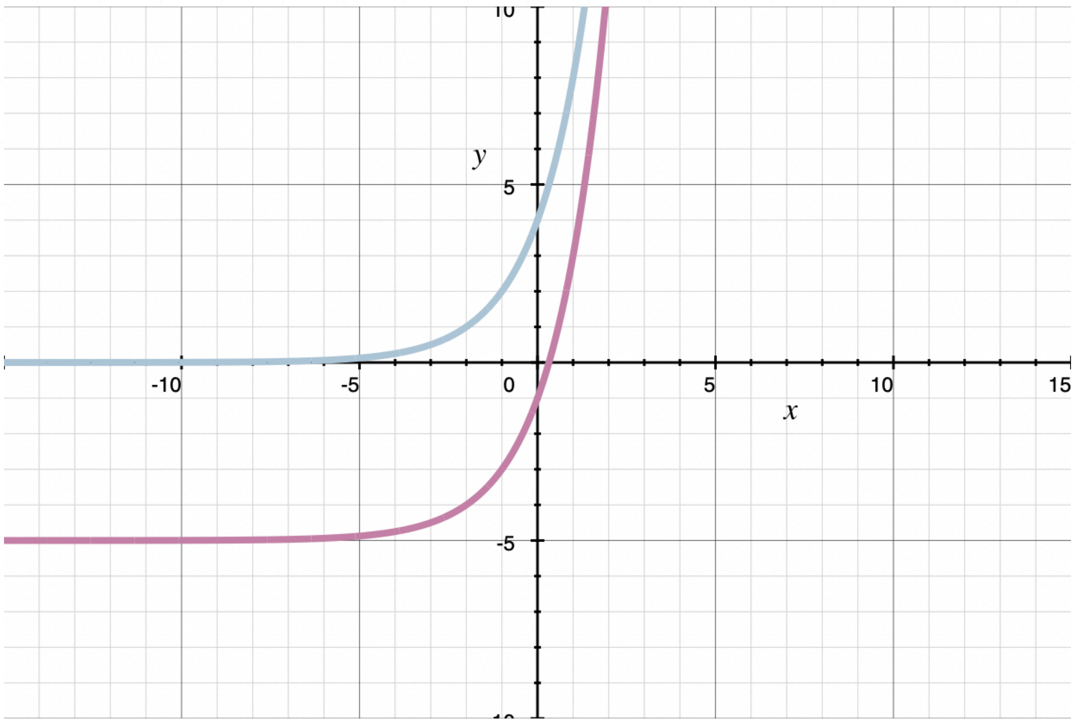
To go from $y = (1/2)^x$ to $y = (1/2)^{x-2}$, we shift the graph 2 units to the right,



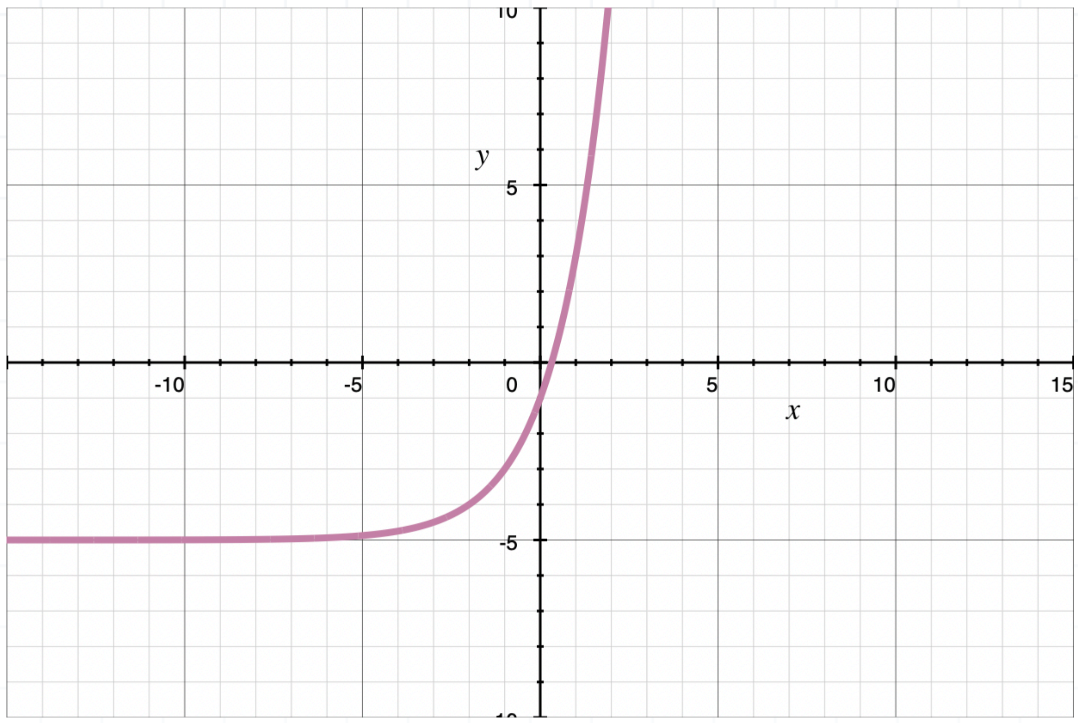
then to go from $y = (1/2)^{x-2}$ to $y = (1/2)^{-x-2}$, we reflect the graph across the y-axis.



To go from $y = (1/2)^{-x-2}$ to $y = (1/2)^{-x-2} - 5$, we shift the graph down 5 units.



So the final sketch of $f(x) = (1/2)^{-x-2} - 5$ is



4. Write the equation for the function that results from applying the list of transformations below to the parent function $y = 6^x$. The list of transformations is not necessarily given in the order in which they should be applied.

- Vertically stretch by a factor of 2
- Vertical shift 4 units up and horizontal shift 3 units to the right
- Reflection across the x-axis
- Reflection across the y-axis

Solution:

The transformations should be applied in the following order:

1. Horizontal shift: The function $y = 6^x$ changes to $y = 6^{x-3}$

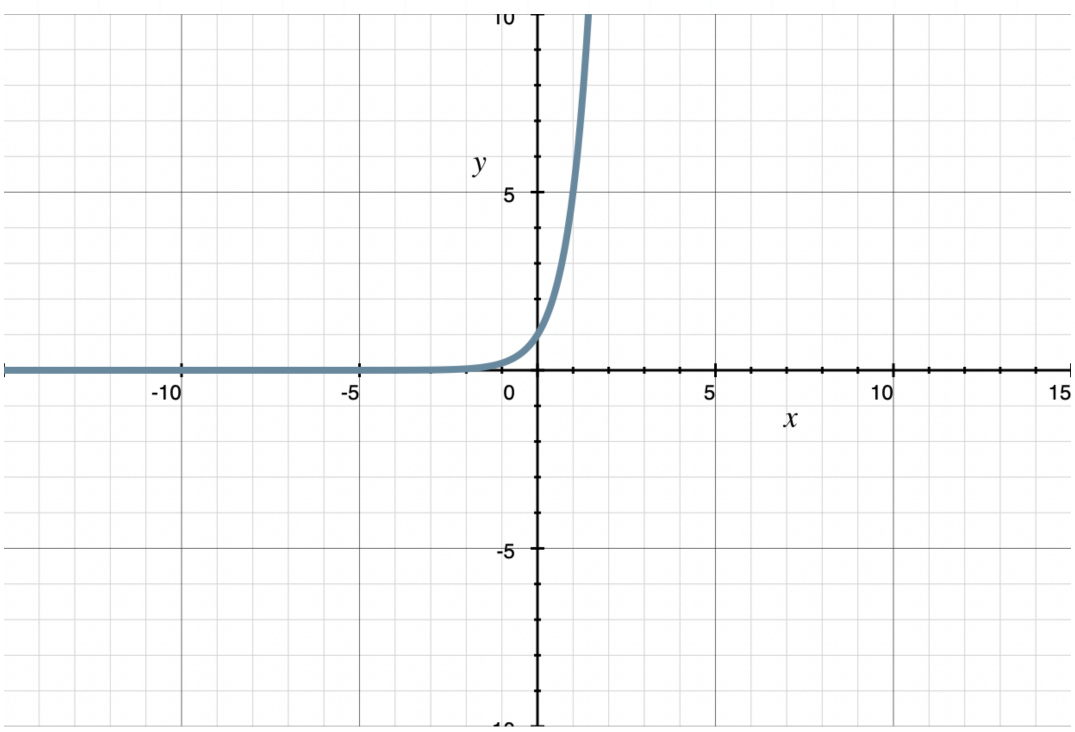
2. Horizontal reflection: The function $y = 6^{x-3}$ changes to $y = 6^{-x-3}$

3. Vertical stretch: The function $y = 6^{-x-3}$ changes to $y = 2(6^{-x-3})$

4. Vertical reflection: The function $y = 2(6^{-x-3})$ changes to $y = -2(6^{-x-3})$

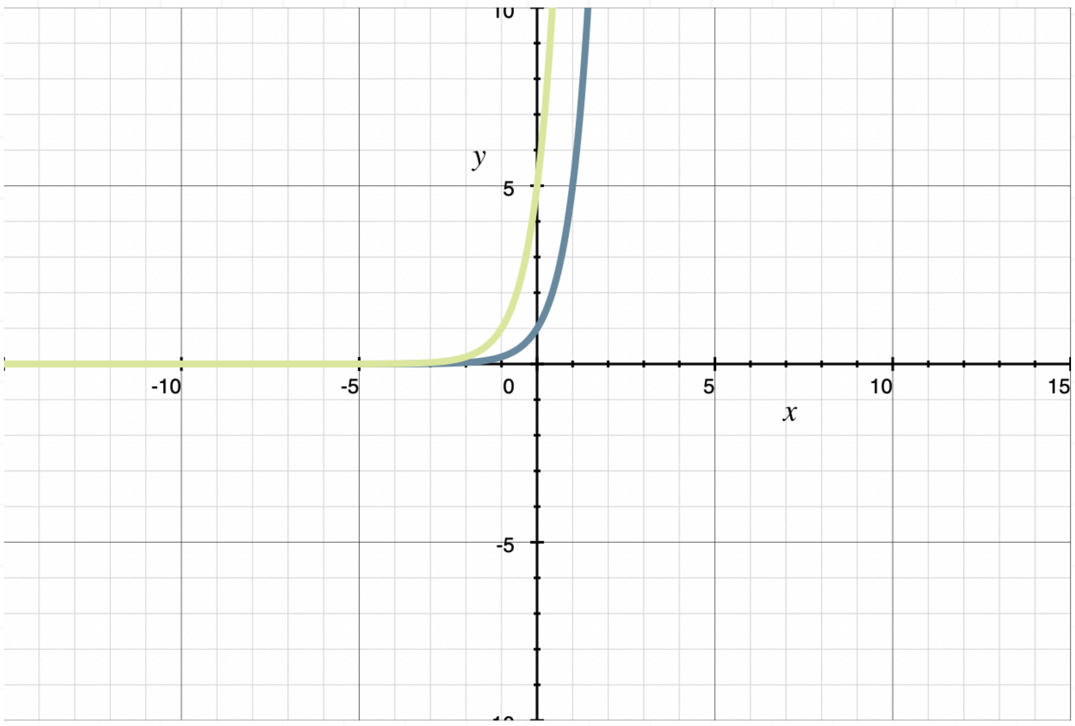
5. Vertical shift: The function $y = -2(6^{-x-3})$ changes to $y = -2(6^{-x-3}) + 4$

■ 5. Given the graph of $y = 5^x$, use transformations to graph $y = -5^{x+1} - 2$.

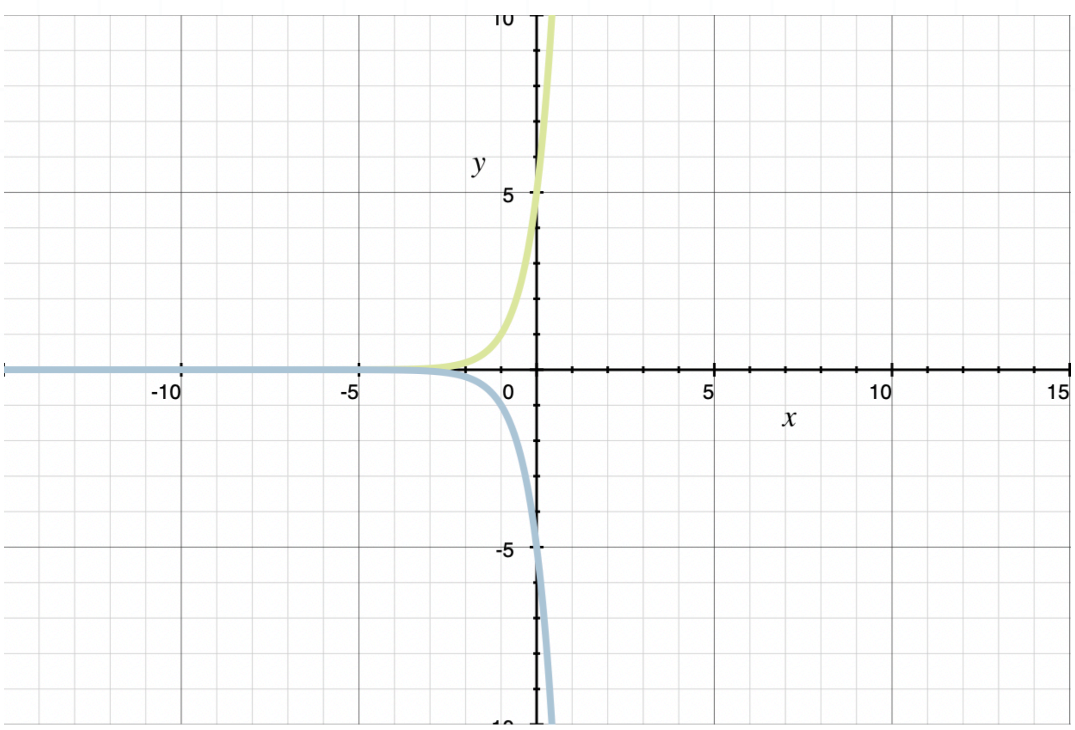


Solution:

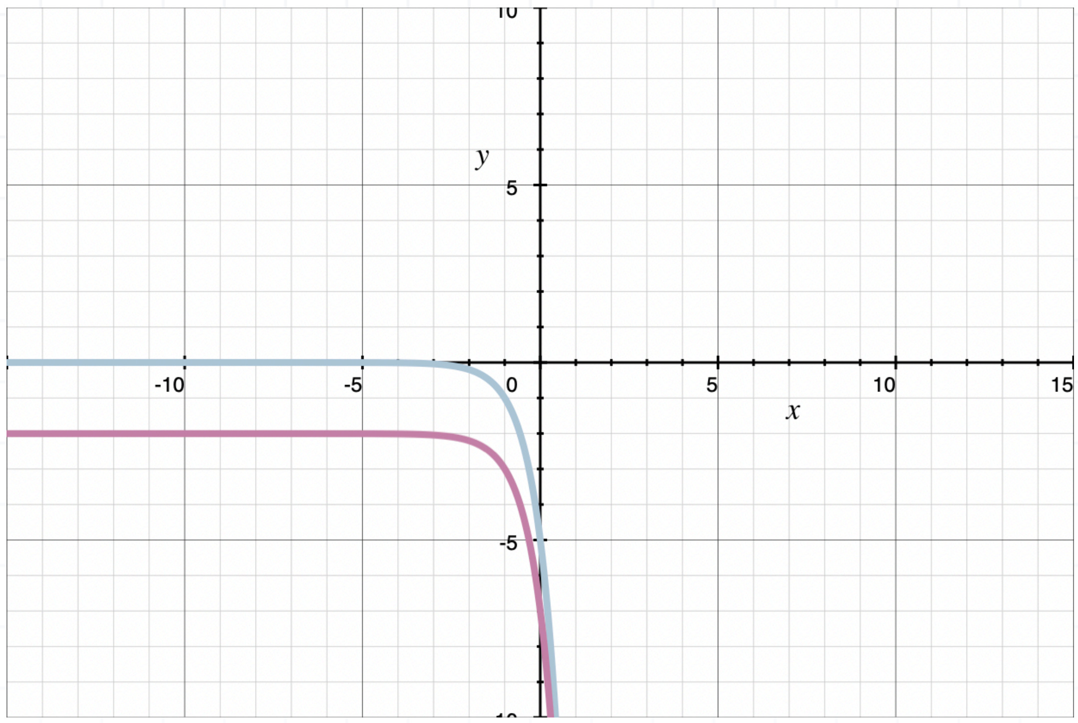
To change $y = 5^x$ to $y = 5^{x+1}$, we shift the graph 1 unit to the left.



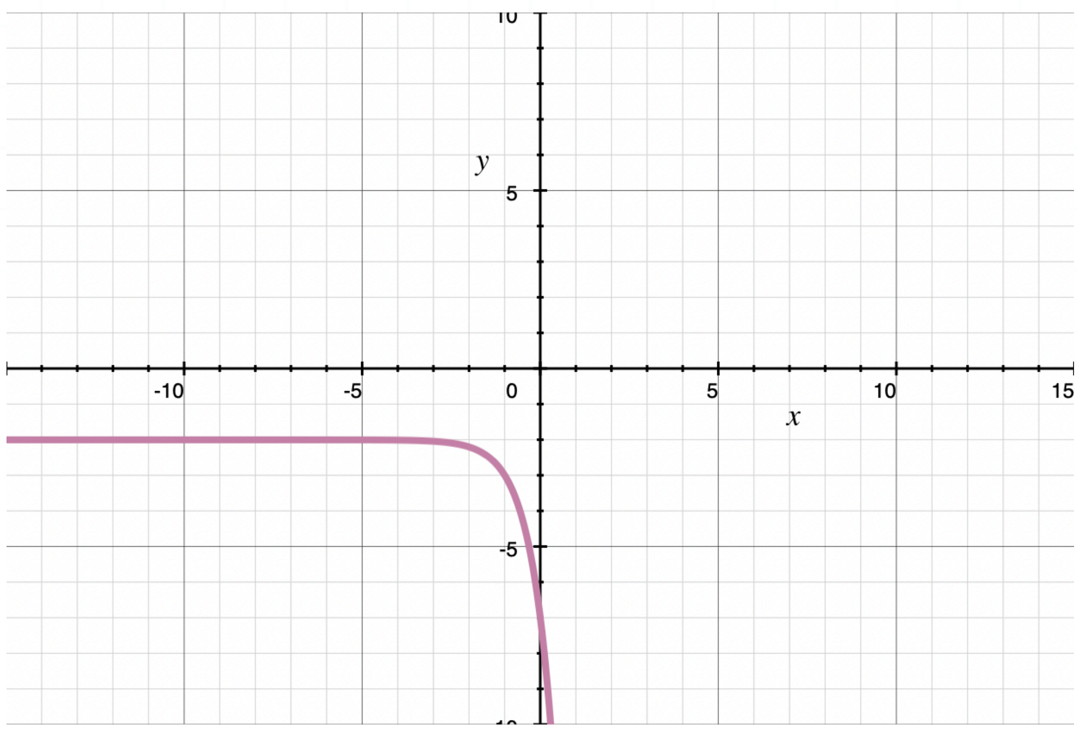
To change $y = 5^{x+1}$ to $y = -5^{x+1}$, we reflect the graph across the x-axis.



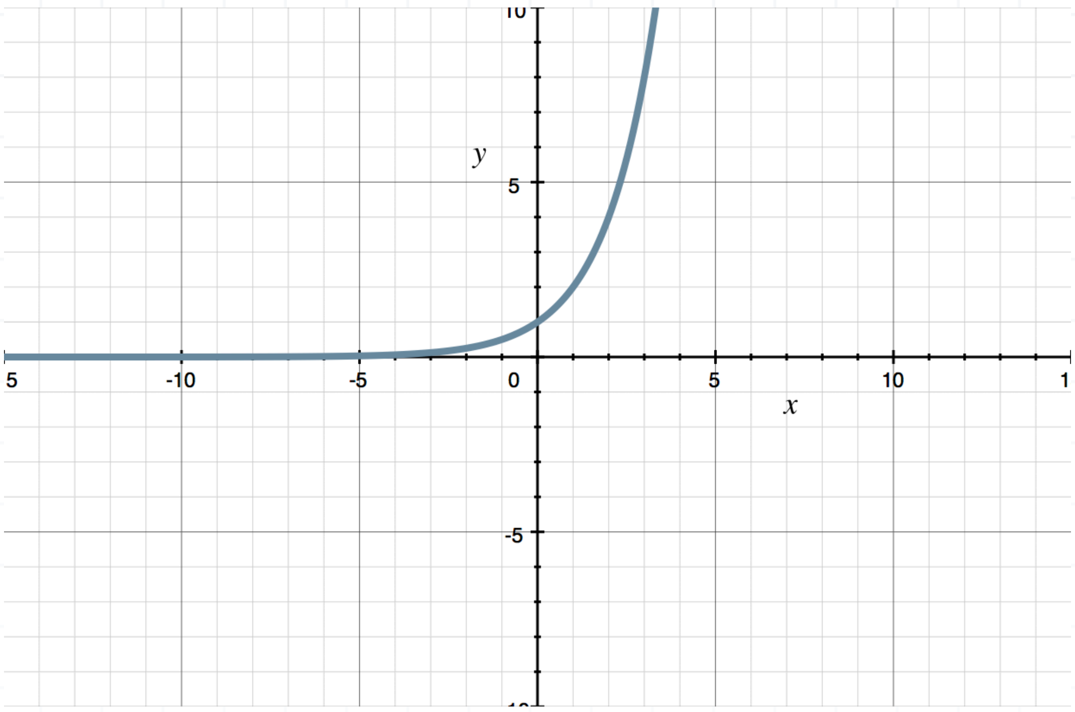
To change $y = -5^{x+1}$ to $y = -5^{x+1} - 2$, we shift the graph 2 units down.



So the final sketch of $y = -5^{x+1} - 2$ is



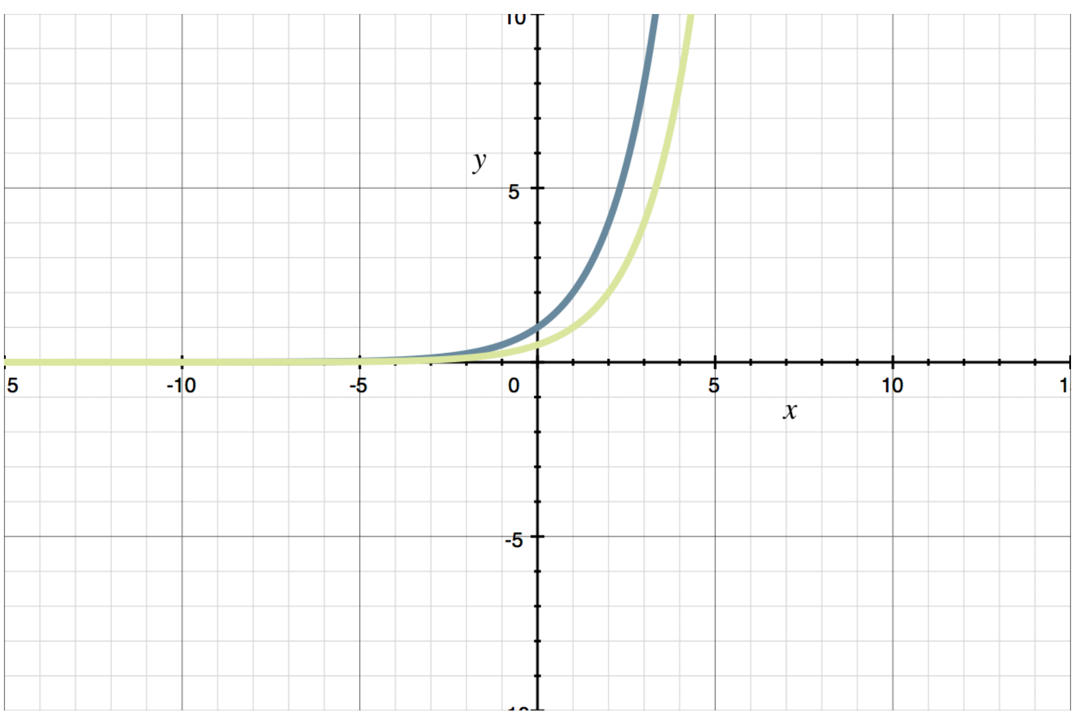
■ 6. Given the graph of $y = 2^x$, use transformations to graph $y = 2^{x-1} + 4$.



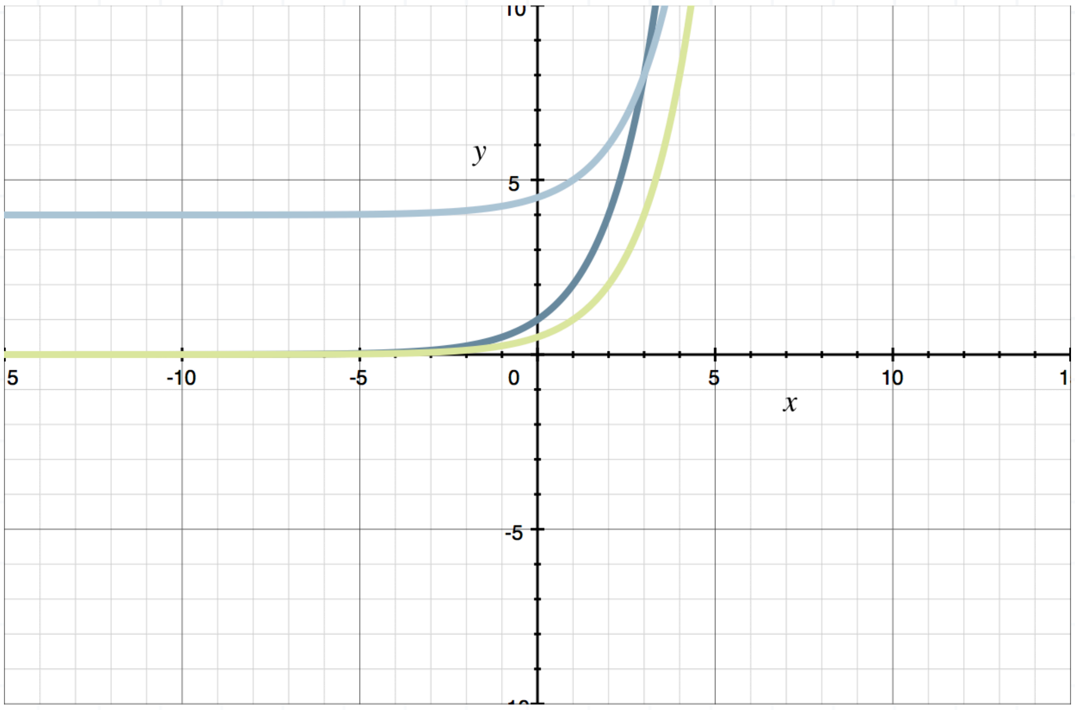
Solution:

We need to figure out which transformations get us from $y = 2^x$ to $y = 2^{x-1} + 4$.

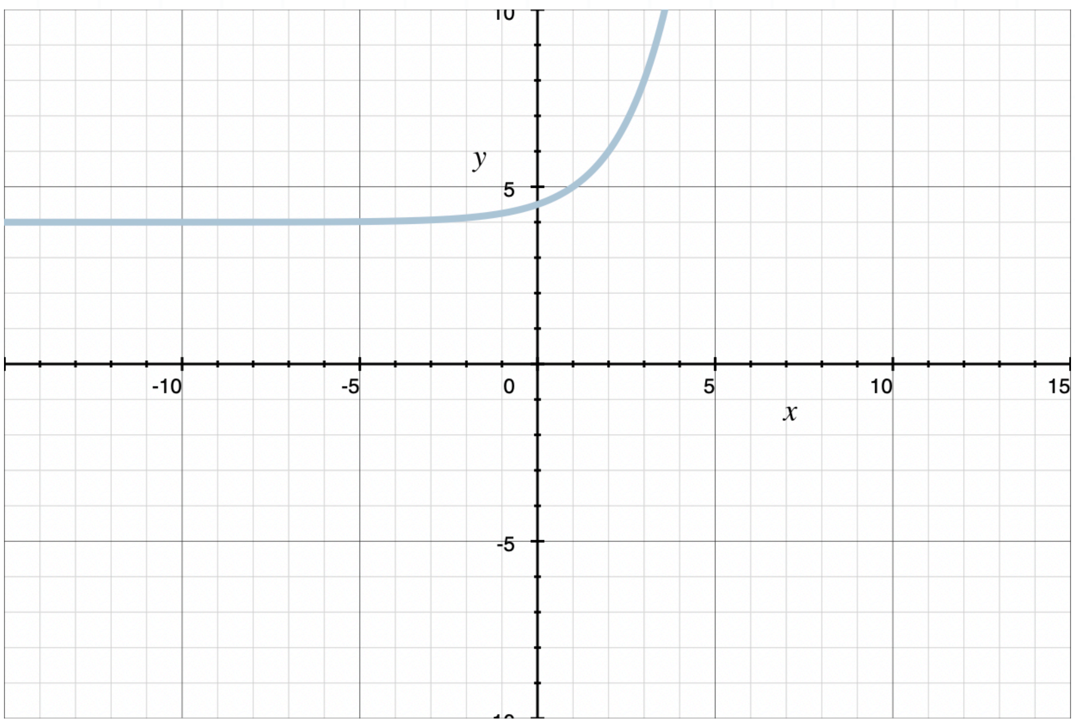
To change $y = 2^x$ to $y = 2^{x-1}$, we move the graph to the right 1 unit.



To change $y = 2^{x-1}$ to $y = 2^{x-1} + 4$, we move the graph up 4 units.



So the final sketch of $y = 2^{x-1} + 4$ is



THE GENERAL LOG RULE

■ 1. Write the inverse of the log function.

$$\log y = 2x$$

Solution:

When the logarithm is written without a base, it means it's the common logarithm, with base 10. So rewrite the logarithm with base 10.

$$\log_{10} y = 2x$$

The inverse function will switch the x- and y-values, so the inverse function is

$$\log_{10} x = 2y$$

$$\log x = 2y$$

We could leave the answer this way, or solve it for y.

$$y = \frac{1}{2} \log x$$

■ 2. Write the inverse of the log function.

$$\ln y = x$$

Solution:

When the logarithm is $\ln$, it's the natural logarithm, which can be rewritten as a log with base e.

$$\log_e y = x$$

The inverse function will switch the x- and y-values, so the inverse function is

$$\log_e x = y$$

$$y = \log_e x$$

$$y = \ln x$$

■ 3. The table shows points that satisfy an exponential function. Write a set of four points that will satisfy its inverse.

x	1	2	3	4
$y=a^x$	1.5	2.25	3.375	5.0625

Solution:

Inverse functions have their x- and y-values swapped. The point set given in the table is $(1,1.5)$, $(2,2.25)$, $(3,3.375)$, and $(4,5.0625)$.

If we want to find a set of points that's on the inverse function, the easiest way is to flip the x- and y-values in each point. So,

$(1,1.5)$ becomes $(1.5,1)$

$(2,2.25)$ becomes $(2.25,2)$

$(3,3.375)$ becomes $(3.375,3)$

$(4,5.0625)$ becomes $(5.0625,4)$

The point set that satisfies the inverse function is therefore

$(1.5,1)$, $(2.25,2)$, $(3.375,3)$, $(5.0625,4)$

■ 4. The table shows points that satisfy a logarithmic function. Write a set of four points that will satisfy its inverse.

x	10	100	1,000	10,000
y=log(x)	1	2	3	4

Solution:

Inverse functions have their x- and y-values swapped. The point set given in the table is $(10,1)$, $(100,2)$, $(1{,}000,3)$, and $(10{,}000,4)$.

If we want to find a set of points that's on the inverse function, the easiest way is to flip the x- and y-values in each point. So,

$(10,1)$ becomes $(1,10)$

$(100,2)$ becomes $(2,100)$

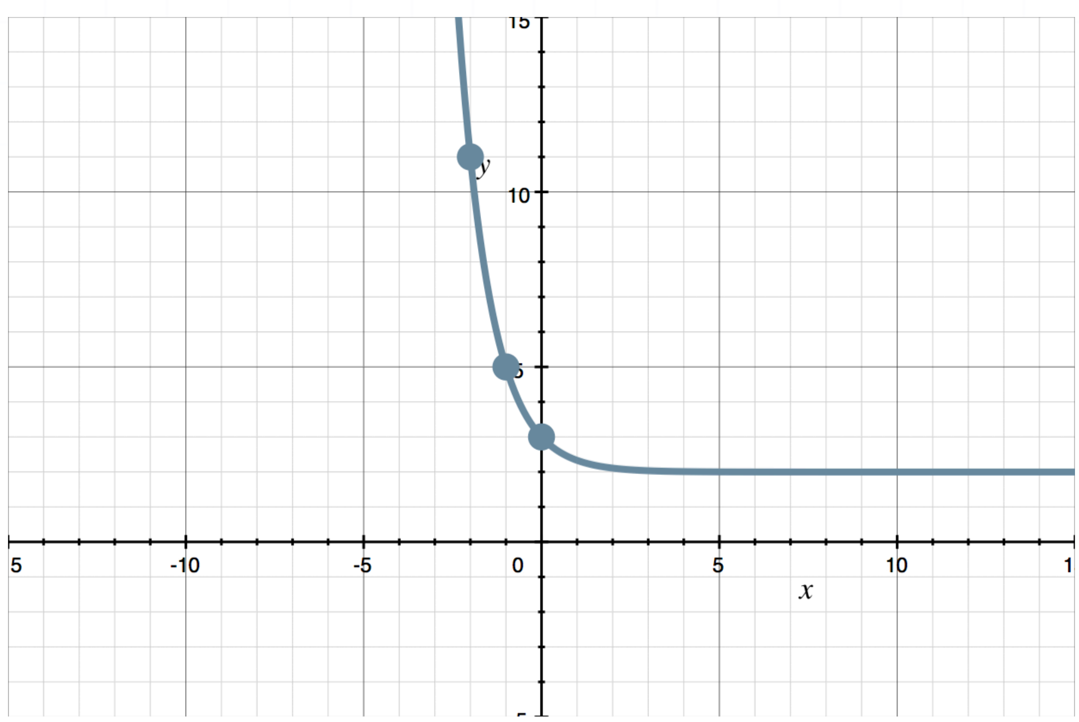
$(1{,}000,3)$ becomes $(3,1{,}000)$

$(10{,}000,4)$ becomes $(4,10{,}000)$

The point set that satisfies the inverse function is therefore

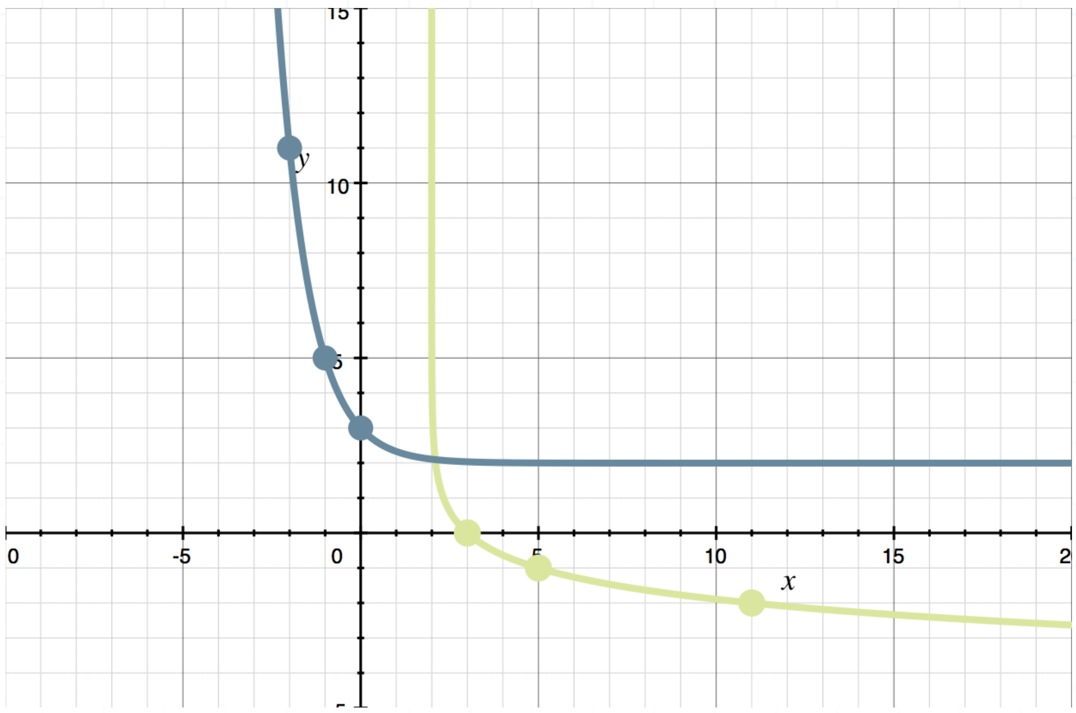
$(1,10)$, $(2,100)$, $(3,1{,}000)$, $(4,10{,}000)$

■ 5. The graph shown passes through $(-2,11)$, $(-1,5)$, and $(0,3)$. Sketch the graph of its inverse.

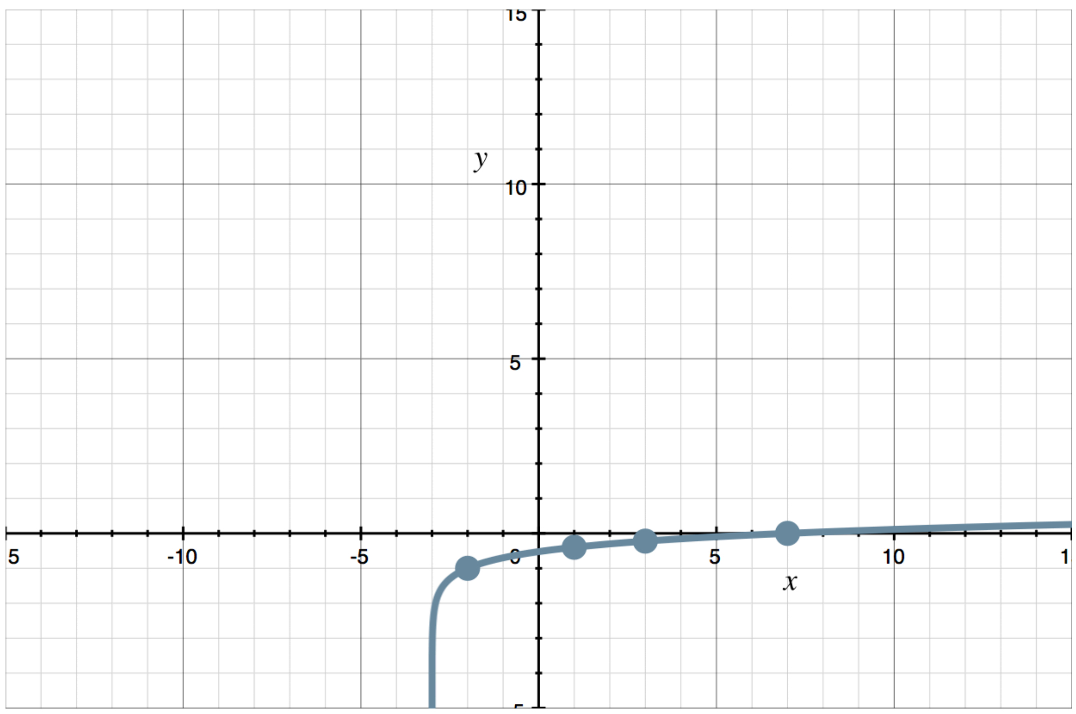


Solution:

Because the graph passes through $(-2,11)$, $(-1,5)$, and $(0,3)$, that means the inverse function must pass through $(11,-2)$, $(5,-1)$, and $(3,0)$. Plot these points on the same set of axes. Then connect the new point set with a curve that reflects the original graph over the line $y=x$.

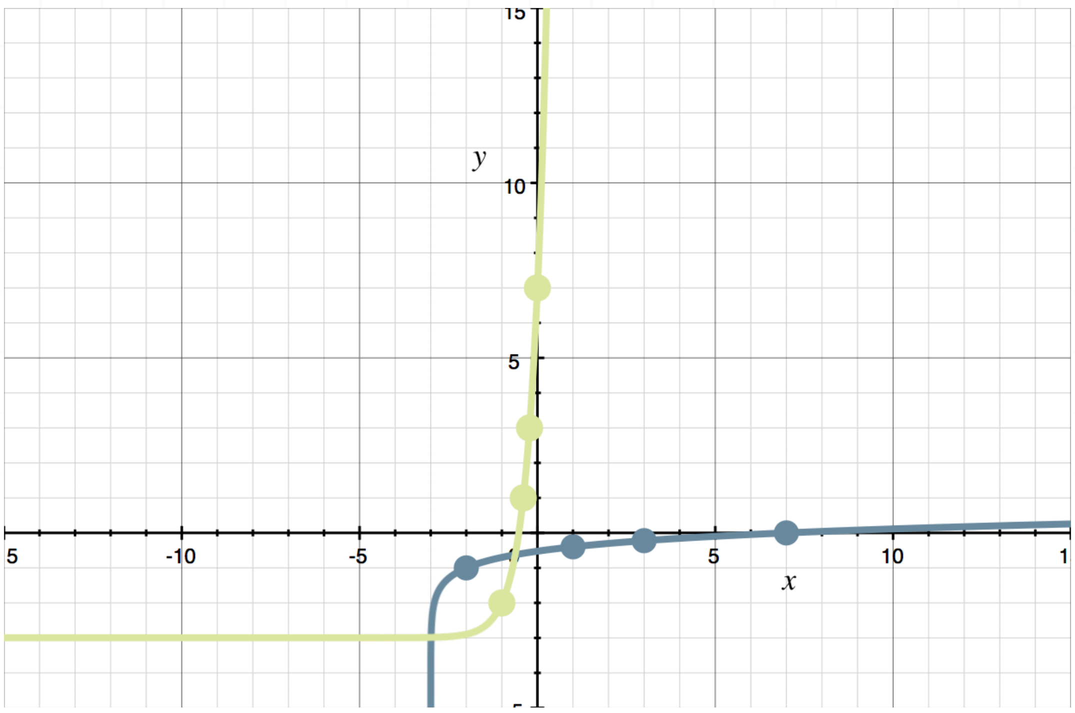


■ 6. The graph shown passes through $(-2,-1)$, $(1,-0.3979)$, $(3,-0.2218)$, and $(7,0)$. Sketch the graph of its inverse.



Solution:

Because the graph passes through $(-2,-1)$, $(1,-0.3979)$, $(3,-0.2218)$ and $(7,0)$, that means the inverse function must pass through $(-1,-2)$, $(-0.3979,1)$, $(-0.2218,3)$, and $(0,7)$. Plot these points on the same set of axes. Then connect the new point set with a curve that reflects the original graph over the line $y=x$.



GRAPHING LOG FUNCTIONS

■ 1. Will the function have a vertical or horizontal asymptote? Where is it located?

$$y = \log_2(x - 1)$$

Solution:

Convert the logarithmic equation to its exponential form.

$$2^y = x - 1$$

$$x = 2^y + 1$$

Now check what happens for large values of y.

For $y = 100$,

$$x = 2^{100} + 1$$

$x =$ a very large number $+ 1$

$$x = \infty$$

For $y = -100$,

$$x = 2^{-100} + 1$$

$$x = \frac{1}{2^{100}} + 1$$

$$x = \frac{1}{\text{a very large number}} + 1$$

$$x = 0 + 1$$

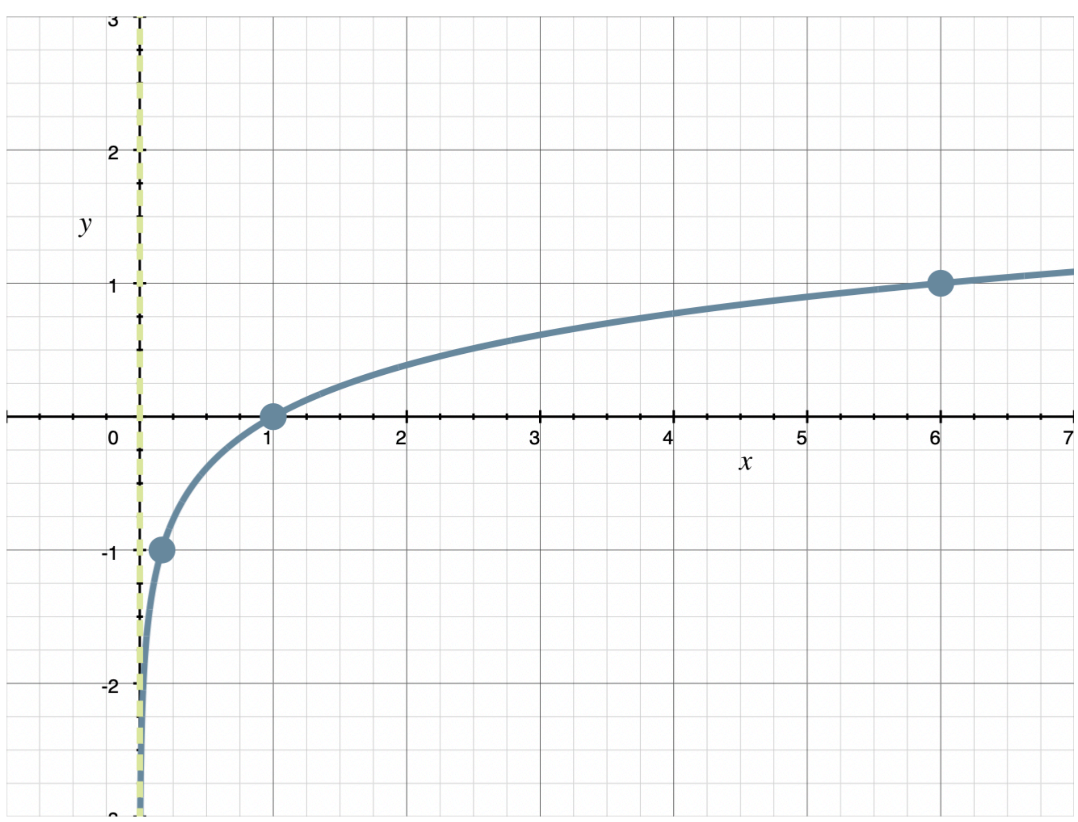
$$x = 1$$

This shows the graph will have a vertical asymptote at $x = 1$.

■ 2. Graph the function $y = \log_6(x)$.

Solution:

We'll sketch the vertical asymptote $x = 0$, and plot the x-intercept $(1,0)$ and the points $(b,1) = (6,1)$ and $(1/b, -1) = (1/6, -1)$, connecting them with a smooth curve to get the graph of $y = \log_6 x$.



■ 3. Sketch the graph of the function using intercepts and end behavior.

$$y = 5\log_2(x+4)$$

Solution:

First, we could use the general rule for logs to convert the logarithmic equation $y = 5\log_2(x+4)$ to its exponential form, $x = 2^{\frac{y}{5}} - 4$. Then we can plug in values of y to get values of x, starting with $y = 100$ and $y = -100$.

For $y = 100$:

$$x = 2^{\frac{100}{5}} - 4$$

$$x = \text{very large positive number} - 4$$

$$x = \infty$$

For $y = -100$:

$$x = 2^{\frac{-100}{5}} - 4$$

$$x = \frac{1}{2^{\frac{100}{5}}} - 4$$

$$x = \frac{1}{2^{\frac{100}{5}}} - 4$$

$$x = 0 - 4$$

$$x = -4$$

We've learned that the function has a vertical asymptote at $x = -4$, and heads up toward ∞ as y gets very large.

We'll plug in a few simple-to-calculate values for y.

For $y = -5$:

$$x = 2^{\frac{-5}{5}} - 4$$

$$x = 2^{-1} - 4$$

$$x = -\frac{7}{2}$$

For $y = 0$:

$$x = 2^{\frac{0}{5}} - 4$$

$$x = 1 - 4$$

$$x = -3$$

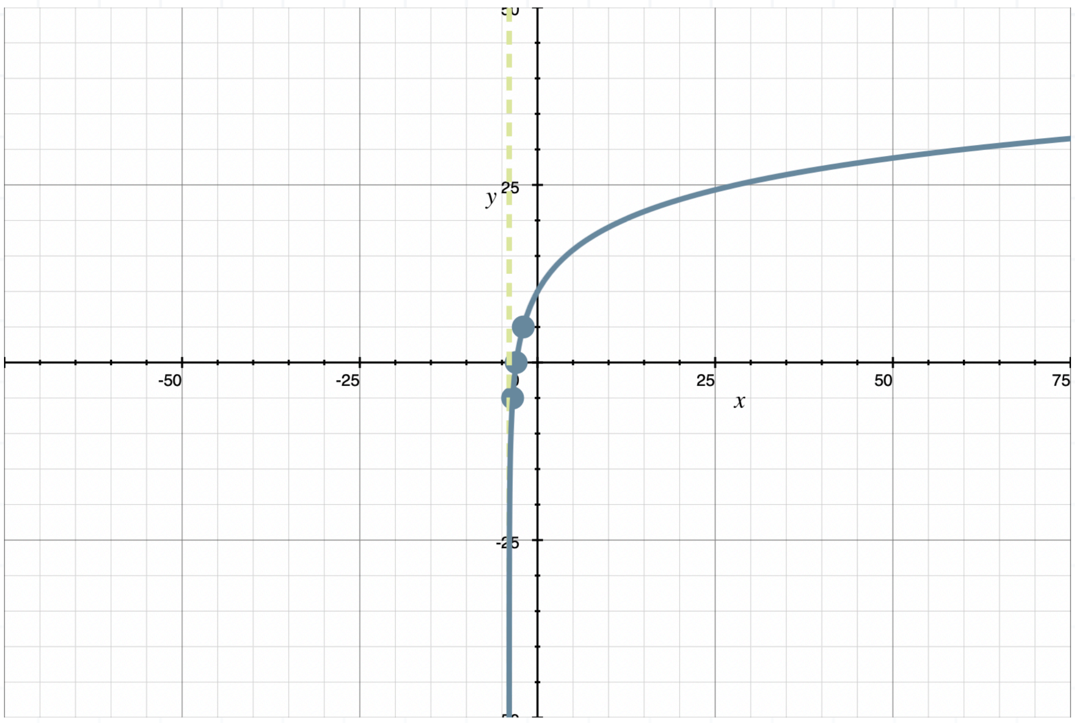
For $y = 5$:

$$x = 2^{\frac{5}{5}} - 4$$

$$x = 2 - 4$$

$$x = -2$$

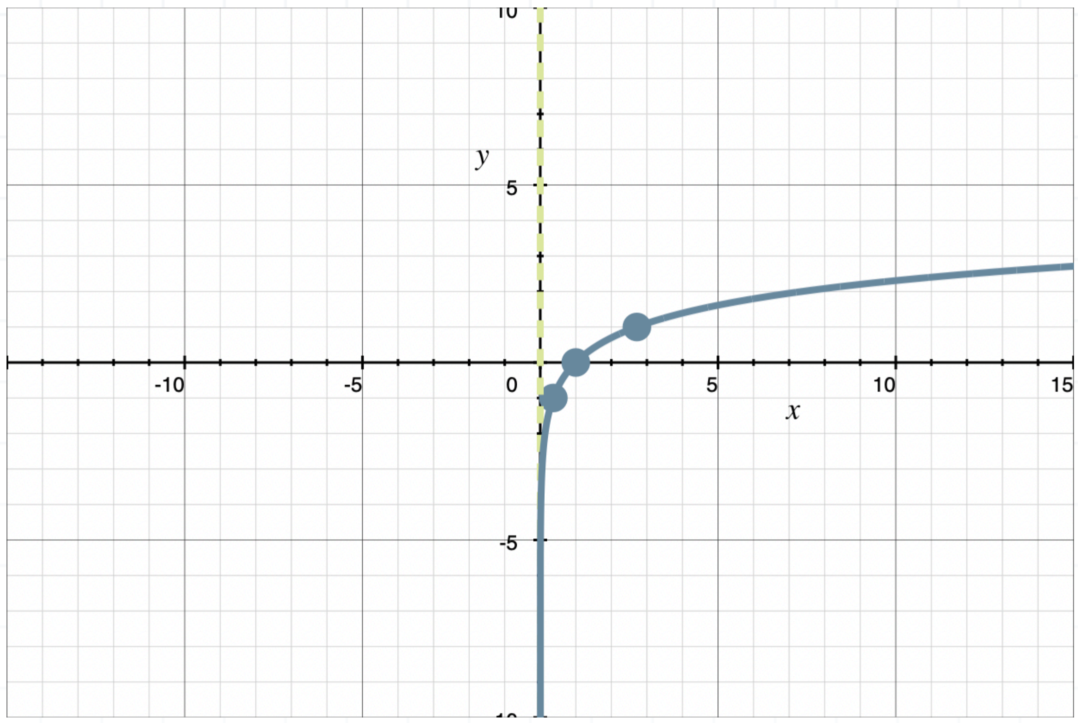
If we plot these points, along with the vertical asymptote $x = -4$, and then connect the points, we get the graph of $x = 2^{\frac{y}{5}} - 4$.



4. Graph the function $y = \ln(x)$.

Solution:

Sketch the vertical asymptote, $x = 0$, and plot the x-intercept $(1,0)$ and the points $(b,1) = (e,1)$ and $(1/b, -1) = (1/e, -1)$, connecting them with a smooth curve to get



■ 5. Sketch the graph of the function by making a table of values.

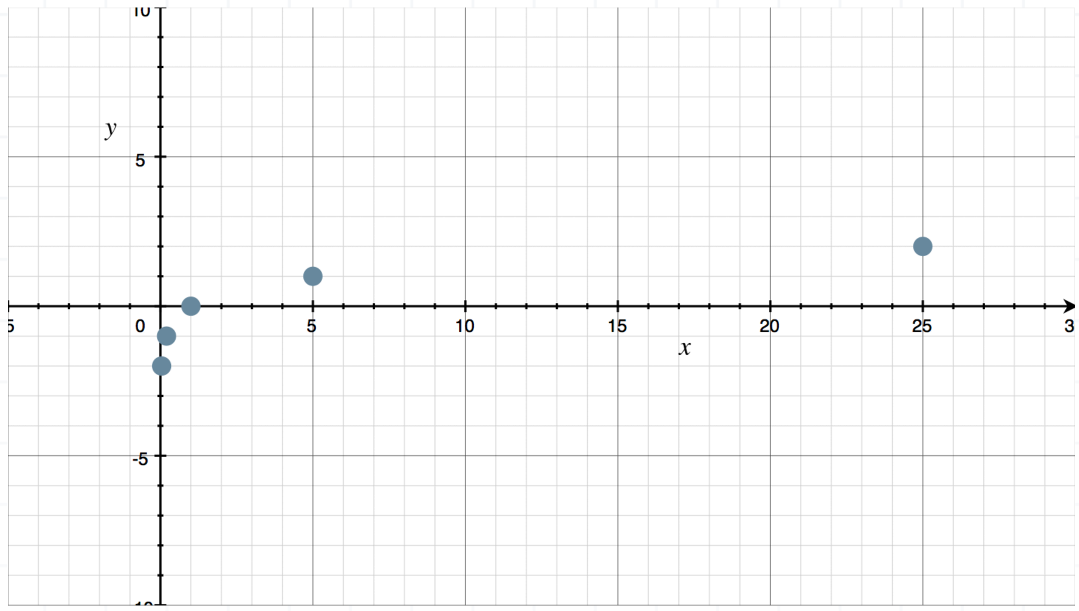
$$y = \log_5 x$$

Solution:

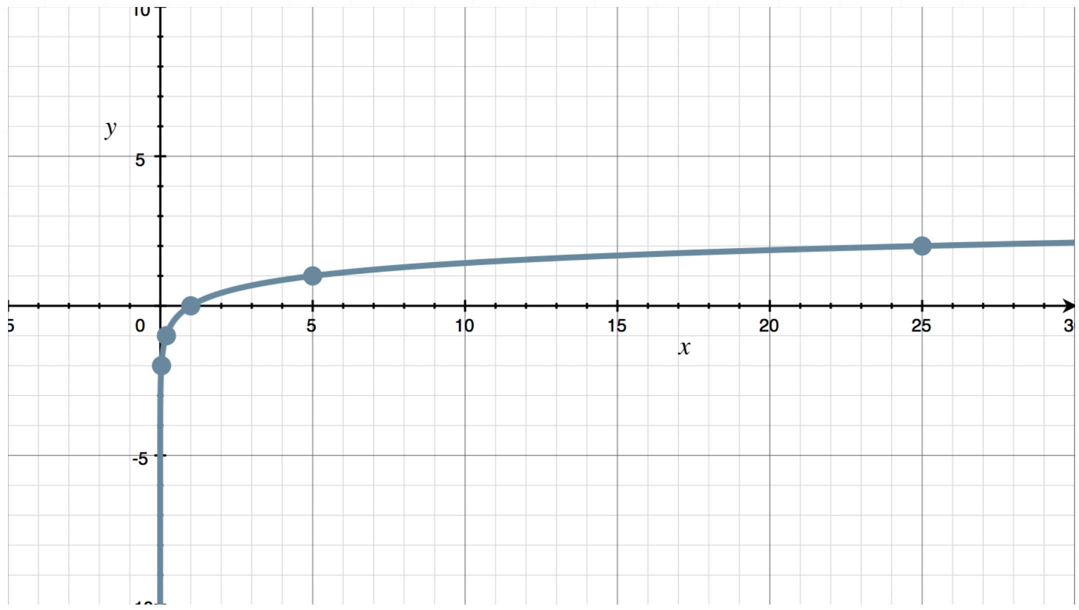
Since we can use the general log rule to rewrite $y = \log_5 x$ as $5^y = x$. We can create a quick table using the exponential function. It's easiest to plug in the y-values to solve for x. Choose any easy y-values for the table.

x	$5^{-2}=1/25$	$5^{-1}=1/5$	$5^0=1$	$5^1=5$	$5^2=25$
y	-2	-1	0	1	2

Now plot these points on the graph,



then connect them with a smooth curve.



■ 6. Sketch the graph of the function using intercepts and end behavior.

$$x = 2\log_3(y+1)$$

Solution:

Use algebra to move the coefficient off the log function.

$$x = 2\log_3(y+1)$$

$$\frac{x}{2} = \log_3(y+1)$$

Use the general log rule to convert the logarithmic function to its associated exponential form.

$$3^{\frac{x}{2}} = y + 1$$

$$y = 3^{\frac{x}{2}} - 1$$

Plug in $x = 100$ and $x = -100$ to see what the function is doing as x starts getting close to $-\infty$ or ∞.

For $x = 100$:

$$y = 3^{\frac{100}{2}} - 1$$

$$y = 3^{50} - 1$$

$y =$ a very large number $- 1$

$y =$ a very large number

$$y = \infty$$

For $x = -100$:

$$y = 3^{\frac{-100}{2}} - 1$$

$$y = 3^{-50} - 1$$

$$y = \frac{1}{3^{50}} - 1$$

$$y = \frac{1}{\text{a very large number}} - 1$$

$$y = 0 - 1$$

$$y = -1$$

This means $y = -1$ will be a horizontal asymptote, and as x tends toward ∞, the y-values will approach ∞.

We can calculate a few points to plot as well. Let's do $x = -2, 0, 2$.

For $x = -2$,

$$y = 3^{\frac{-2}{2}} - 1$$

$$y = 3^{-1} - 1$$

$$y = \frac{1}{3^1} - 1$$

$$y = \frac{1}{3} - \frac{3}{3}$$

$$y = -\frac{2}{3}$$

For $x = 0$,

$$y = 3^{\frac{0}{2}} - 1$$

$$y = 3^0 - 1$$

$$y = 1 - 1$$

$$y = 0$$

For $x = 2$,

$$y = 3^{\frac{2}{2}} - 1$$

$$y = 3^1 - 1$$

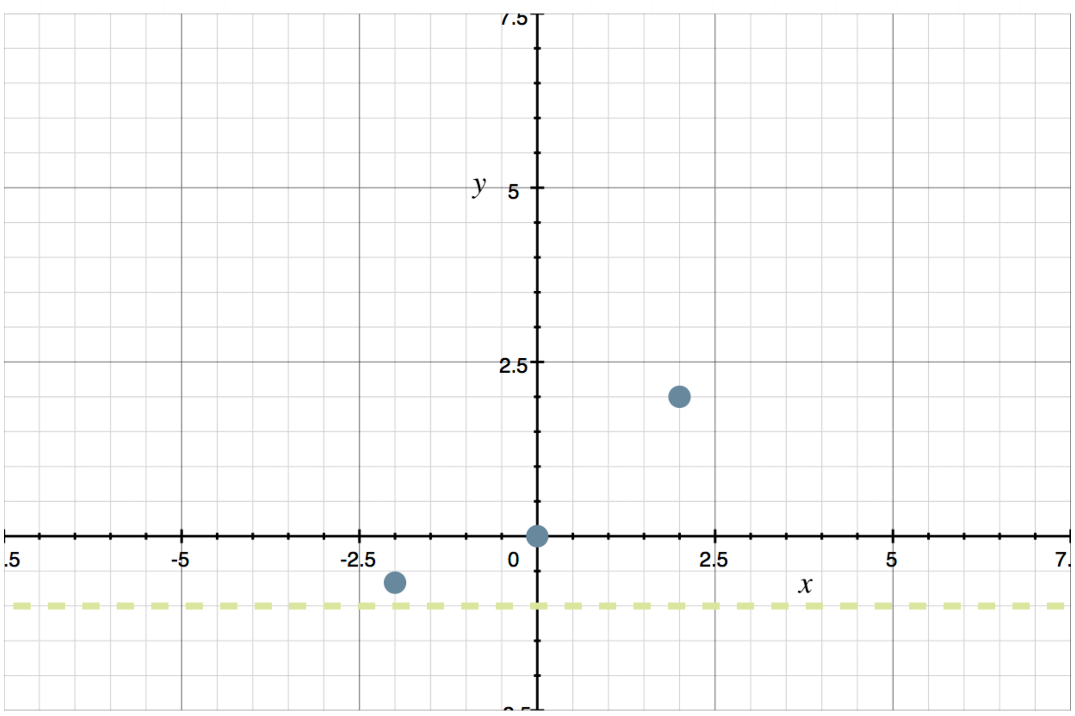
$$y = 3 - 1$$

$$y = 2$$

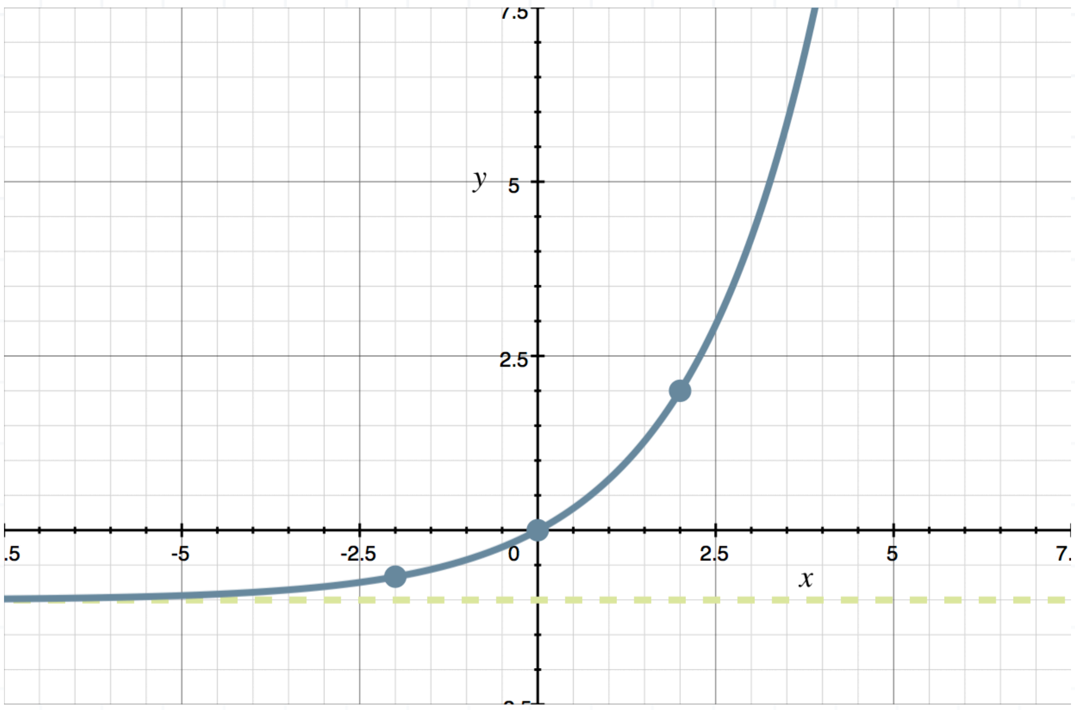
Now we have the three points:

$$\left(-2, -\frac{2}{3}\right), (0,0), (2,2)$$

If we plot these points and the horizontal asymptote $y = -1$, we get



We can see that the logarithmic function will go along the horizontal asymptote at $y = -1$, and then as $x \to \infty$, the function's value heads toward ∞. Connecting the points gives us the graph.

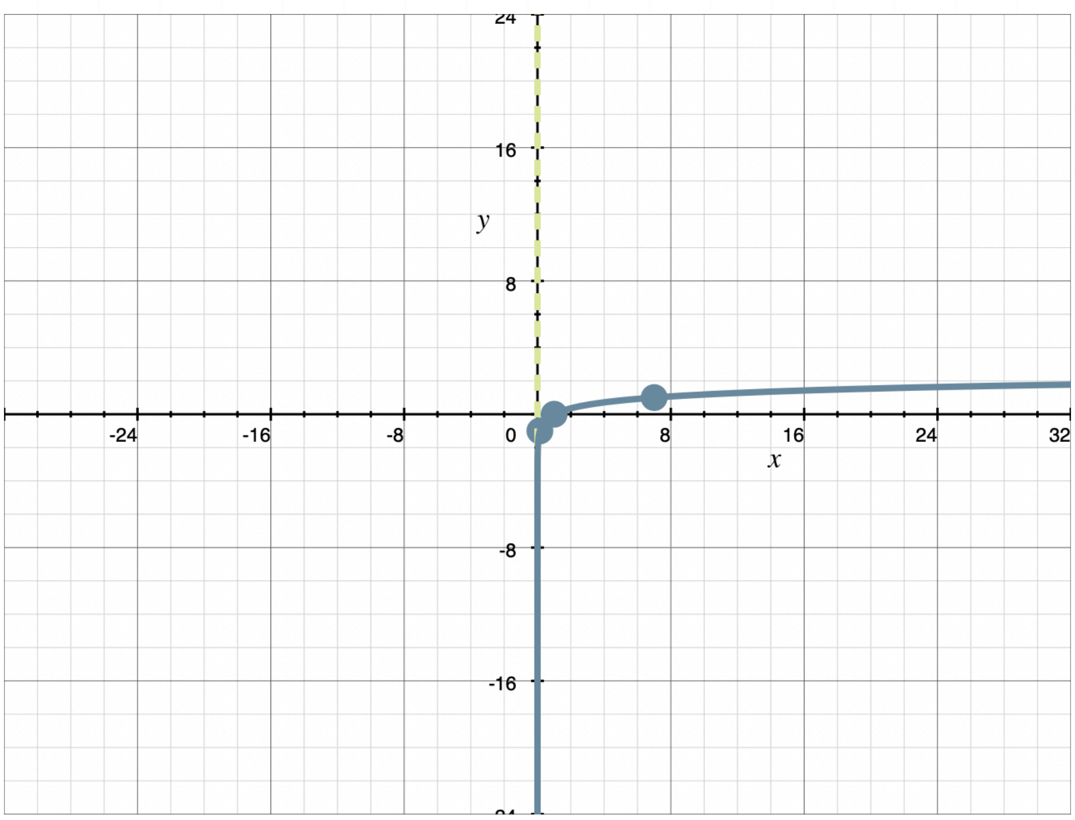

7.5
y
5
2.5
.5
-5
-2.5
0
2.5
5
7.
x

GRAPHING TRANSFORMATIONS OF LOG FUNCTIONS

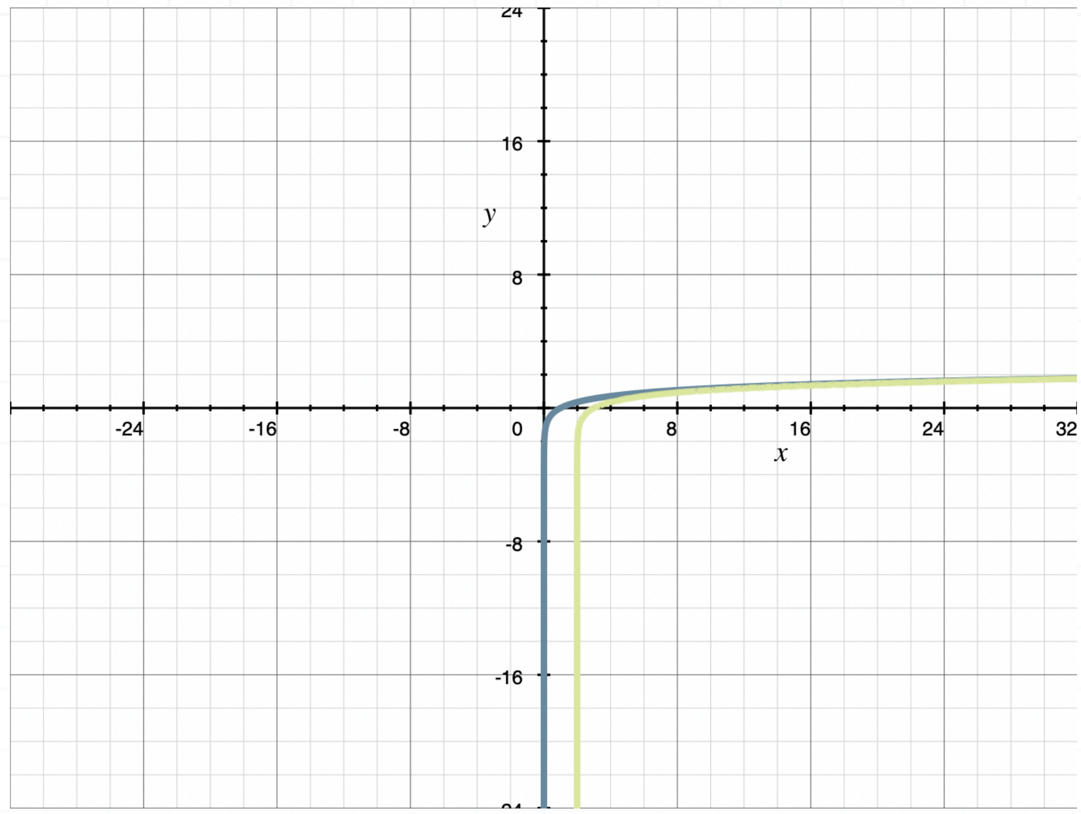
■ 1. Use transformations to sketch the graph of $y = -3\log_7(x-2)+4$.

Solution:

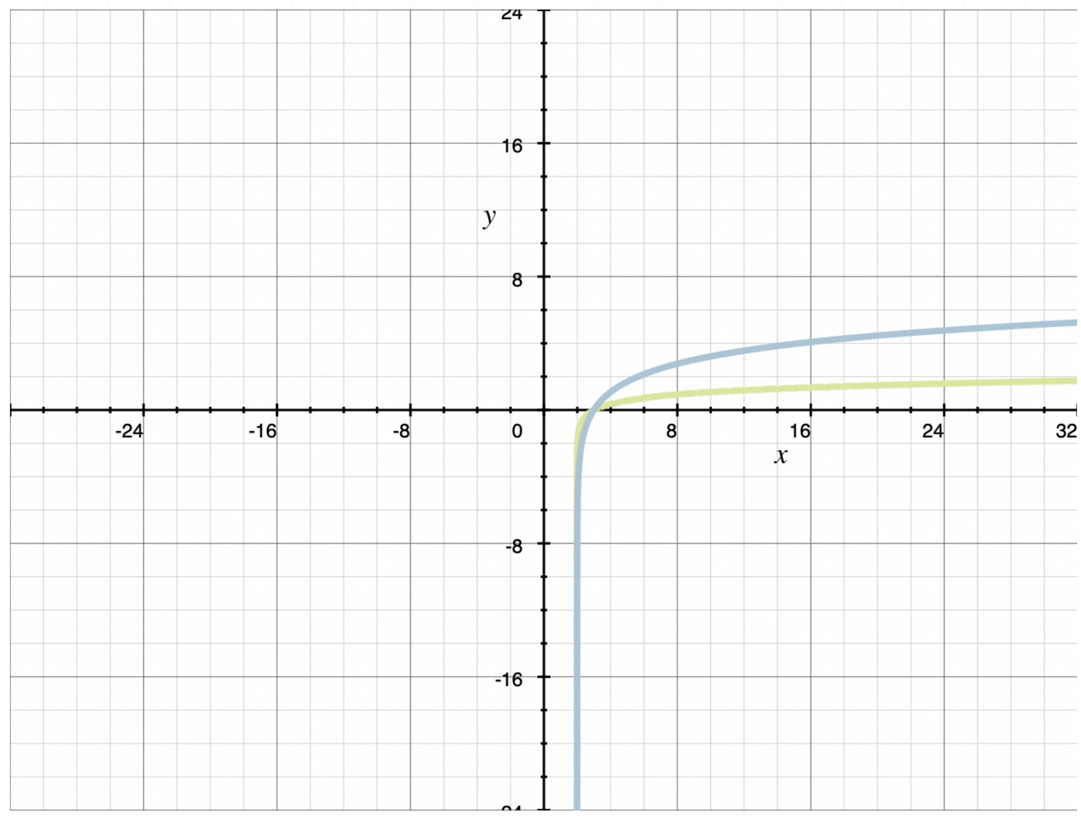
First we need to graph the parent function $y = \log_7 x$ by setting up the vertical asymptote, $x = 0$, and plotting the x-intercept $(1,0)$ and the points $(b,1) = (7,1)$ and $(1/b, -1) = (1/7, -1)$, and connecting them with a smooth curve.



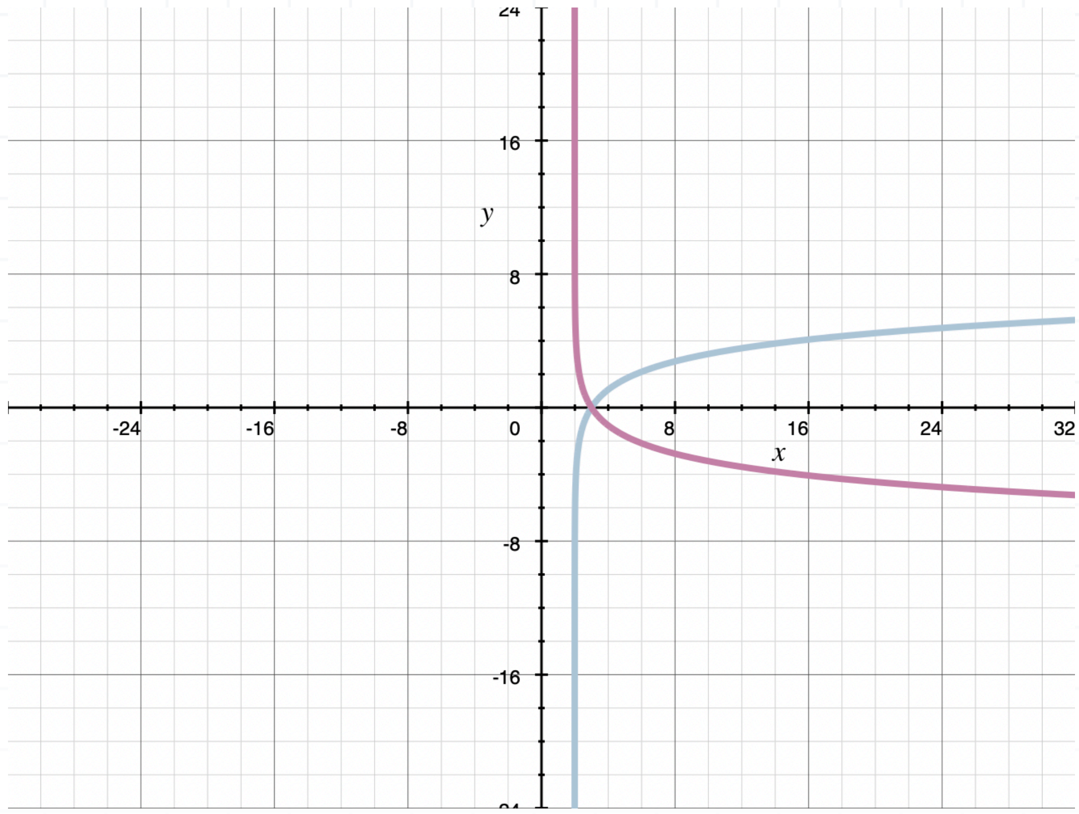
To change $y = \log_7 x$ to $y = \log_7(x-2)$, we shift the graph 2 units to the right,



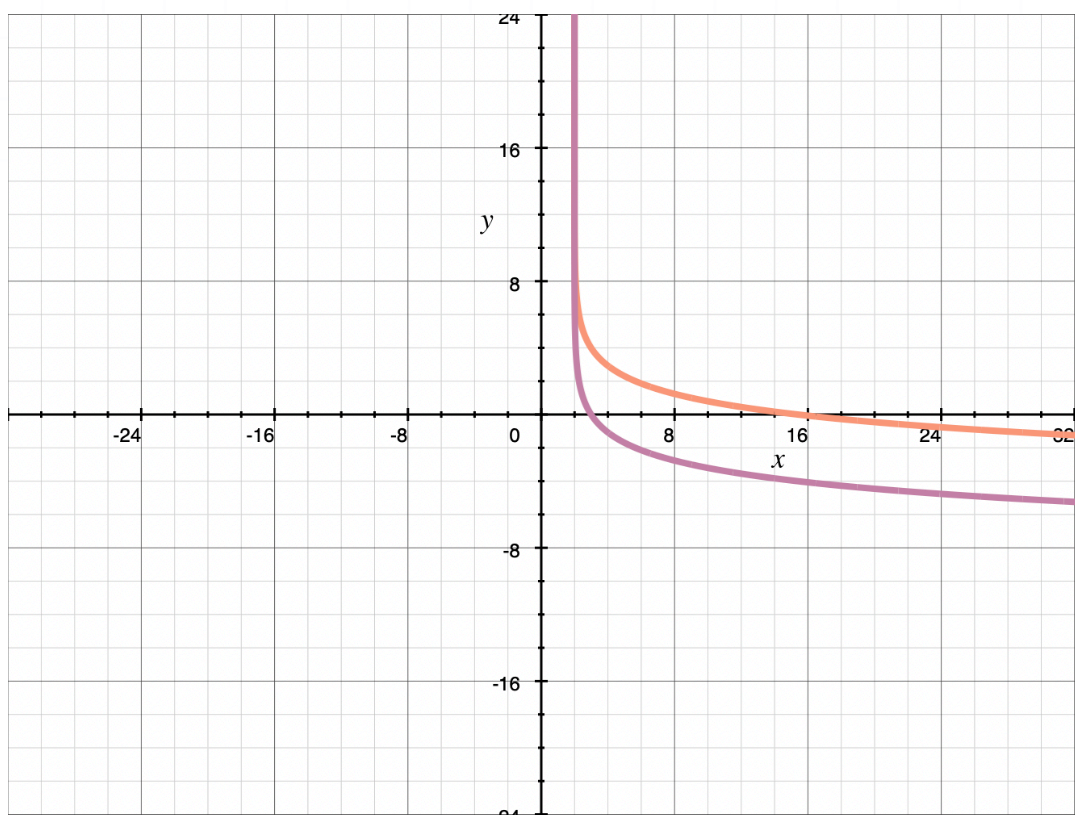
and then to change $y = \log_7(x - 2)$ to $y = 3\log_7(x - 2)$, we stretch the graph vertically by a factor of 3.



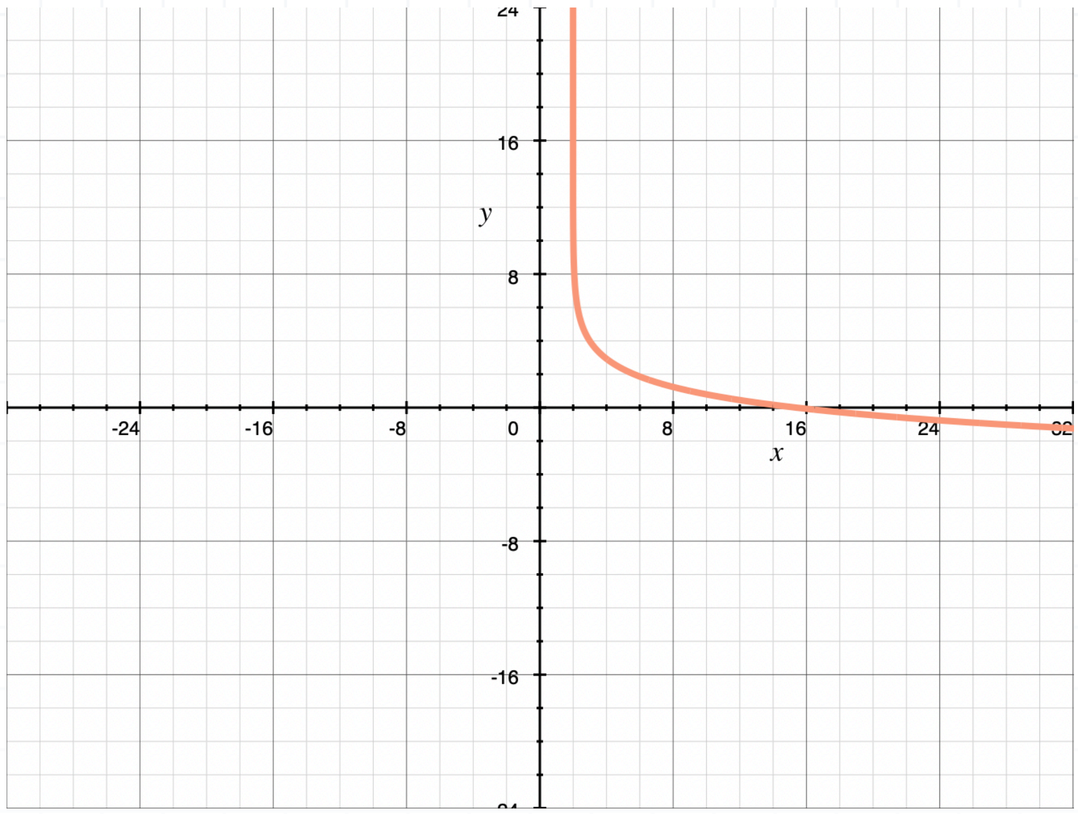
To change $y = 3\log_7(x - 2)$ to $y = -3\log_7(x - 2)$, we reflect the graph across the x-axis,



and to change $y = -3\log_7(x-2)$ to $y = -3\log_7(x-2) + 4$, we shift the graph up 4 units.



So the final sketch of $y = -3\log_7(x-2) + 4$ is



■ 2. Will the function have a vertical or horizontal asymptote? Where is it located?

$$y = \log_4(2x) + \log_4(3)$$

Solution:

Simplify the equation using the product rule for logarithms,

$$\log_a(xy) = \log_a x + \log_a y$$

We get

$$y = \log_4(2x) + \log_4(3)$$

$$y = \log_4(3 \cdot 2x)$$

$$y = \log_4(6x)$$

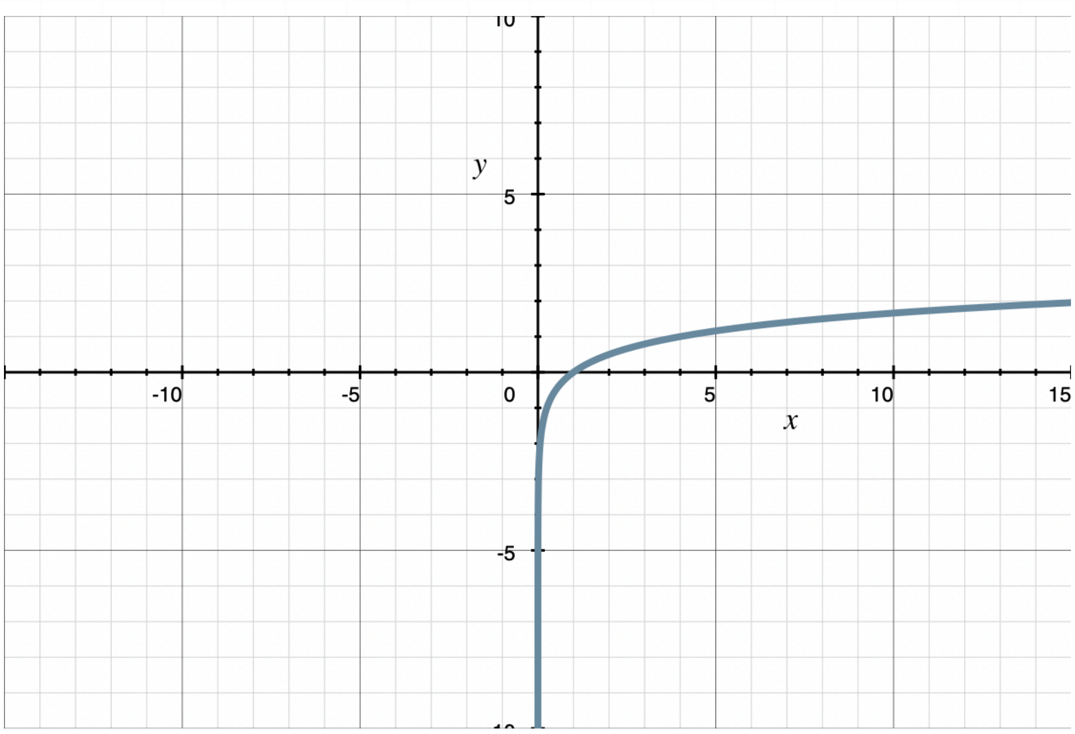
To find the vertical asymptote of the logarithmic function, we'll set its argument equal to zero,

$$6x = 0$$

$$x = 0$$

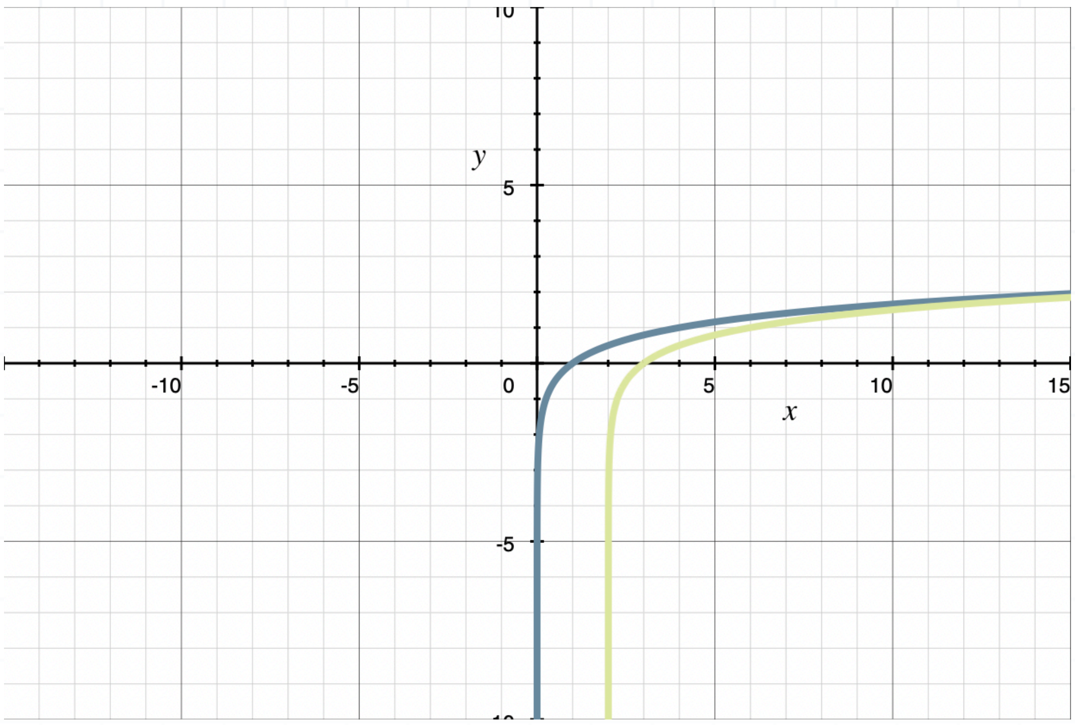
The vertical asymptote is $x = 0$.

■ 3. Given the graph of $y = \log_4(x)$, use transformations to sketch the graph of $y = \log_4(x - 2) - 3$.

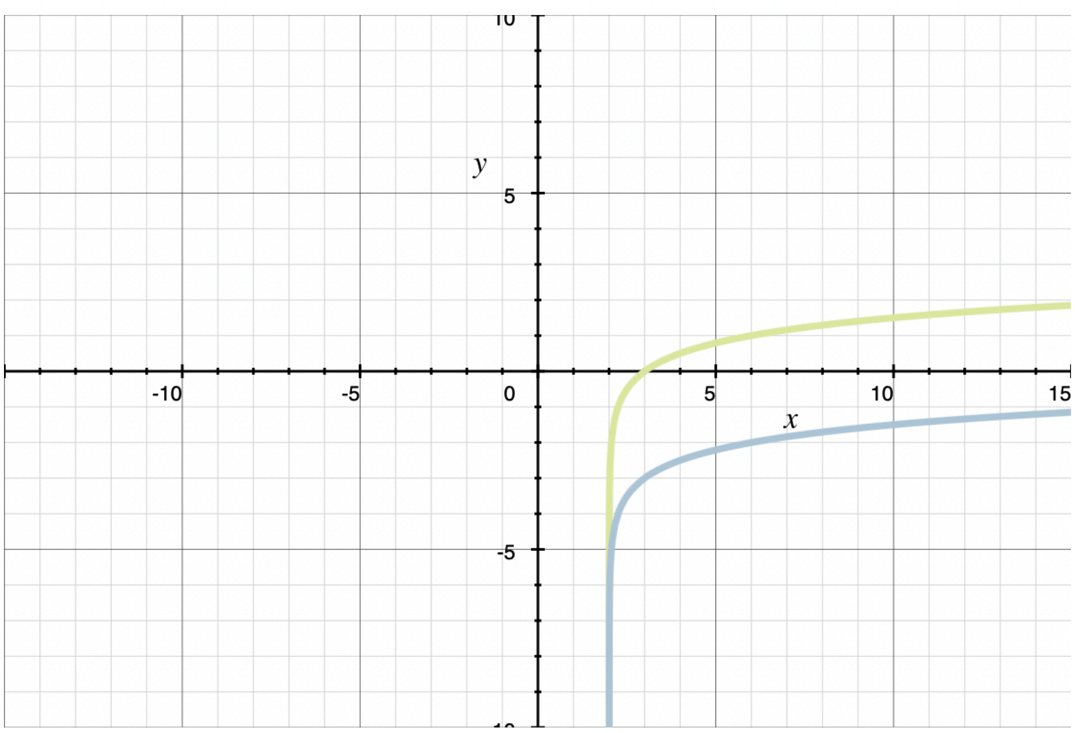


Solution:

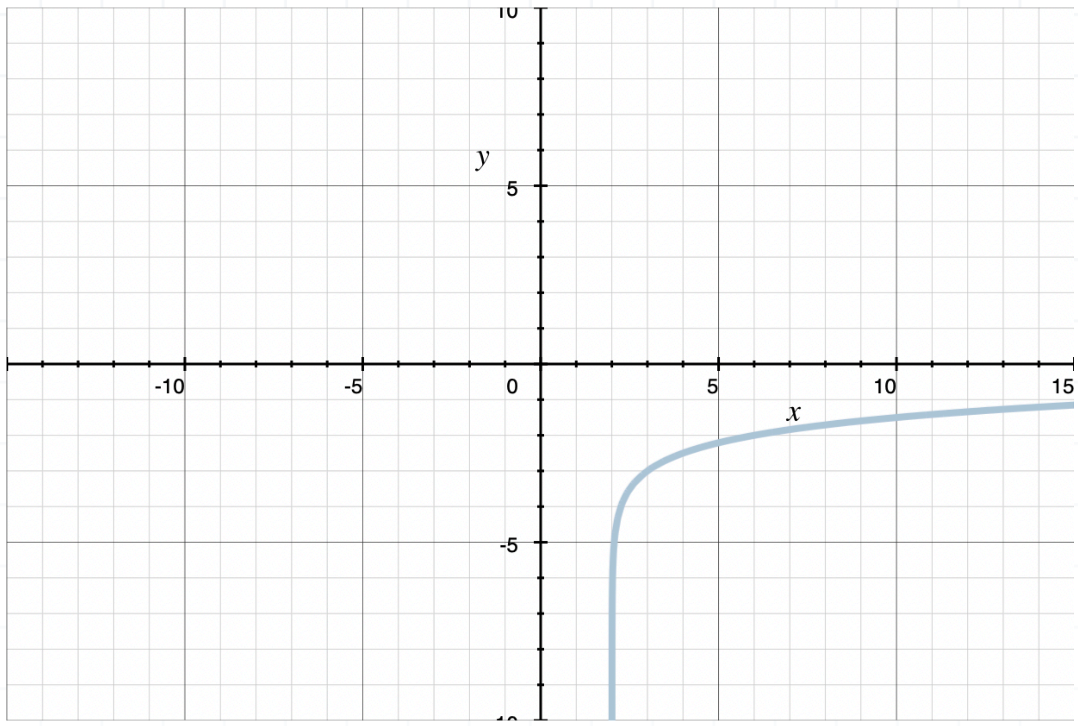
To change $y = \log_4(x)$ to $y = \log_4(x-2)$, we shift the graph 2 units to the right,



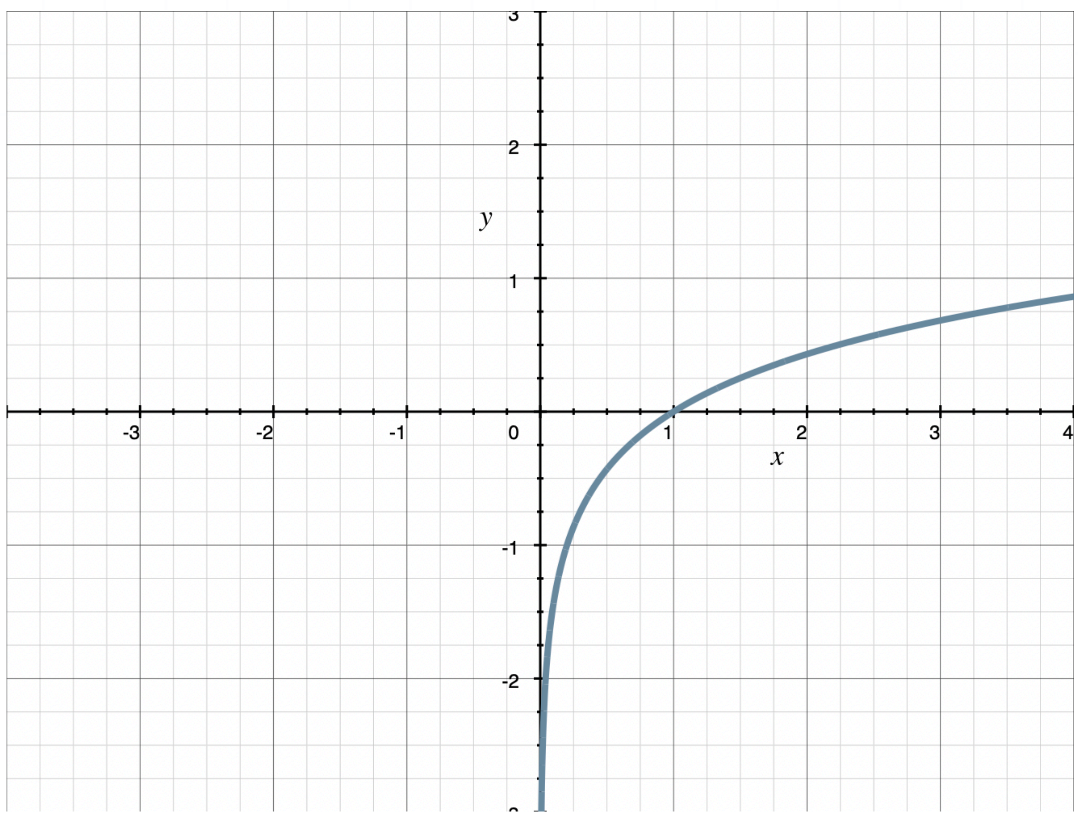
and then to change $y = \log_4(x-2)$ to $y = \log_4(x-2) - 3$, we shift the graph 3 units down.



So the final sketch of $y = \log_4(x-2) - 3$ is

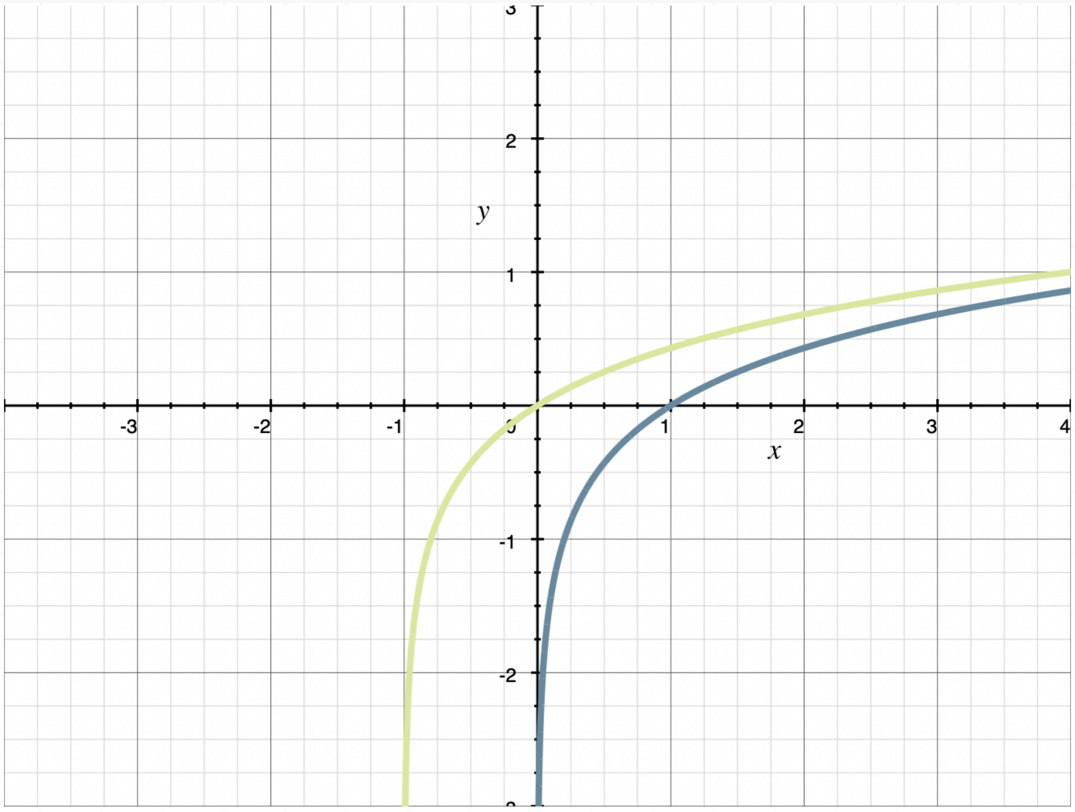


■ 4. Given the graph of $y = \log_5(x)$, use transformations to sketch the graph of $y = -\log_5(x+1) - 2$.

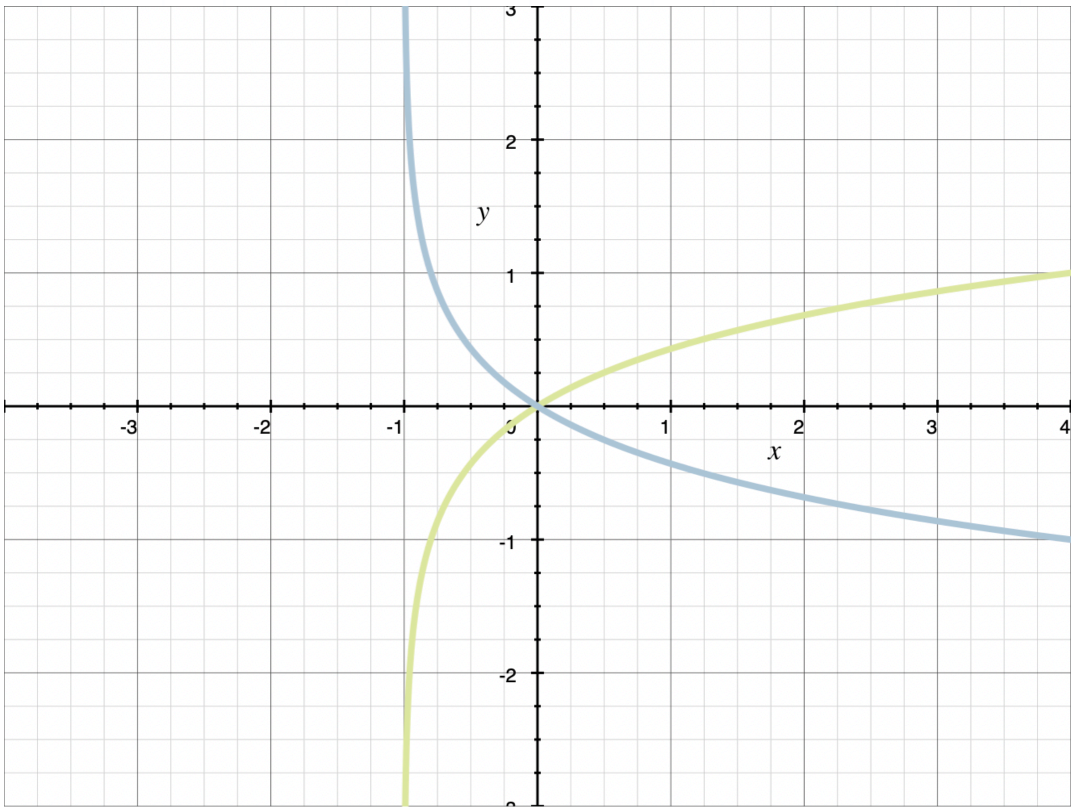


Solution:

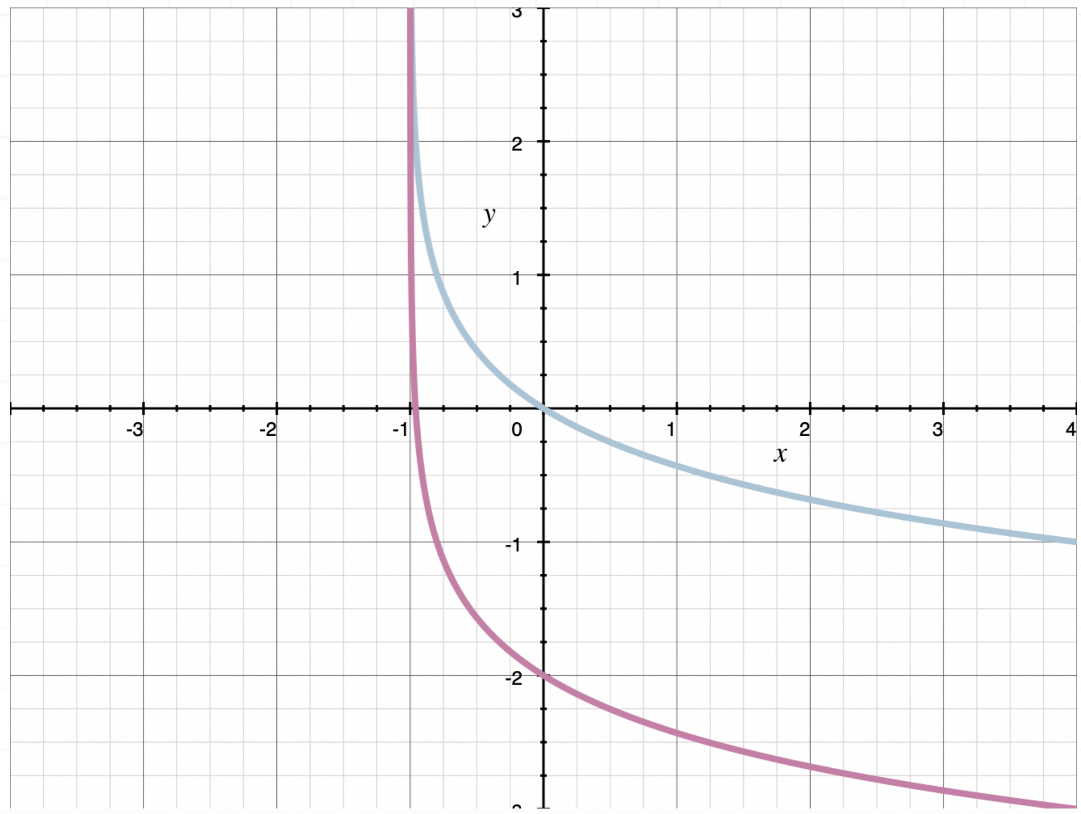
To change $y = \log_5(x)$ to $y = \log_5(x + 1)$, we shift the graph to the left 1 unit,



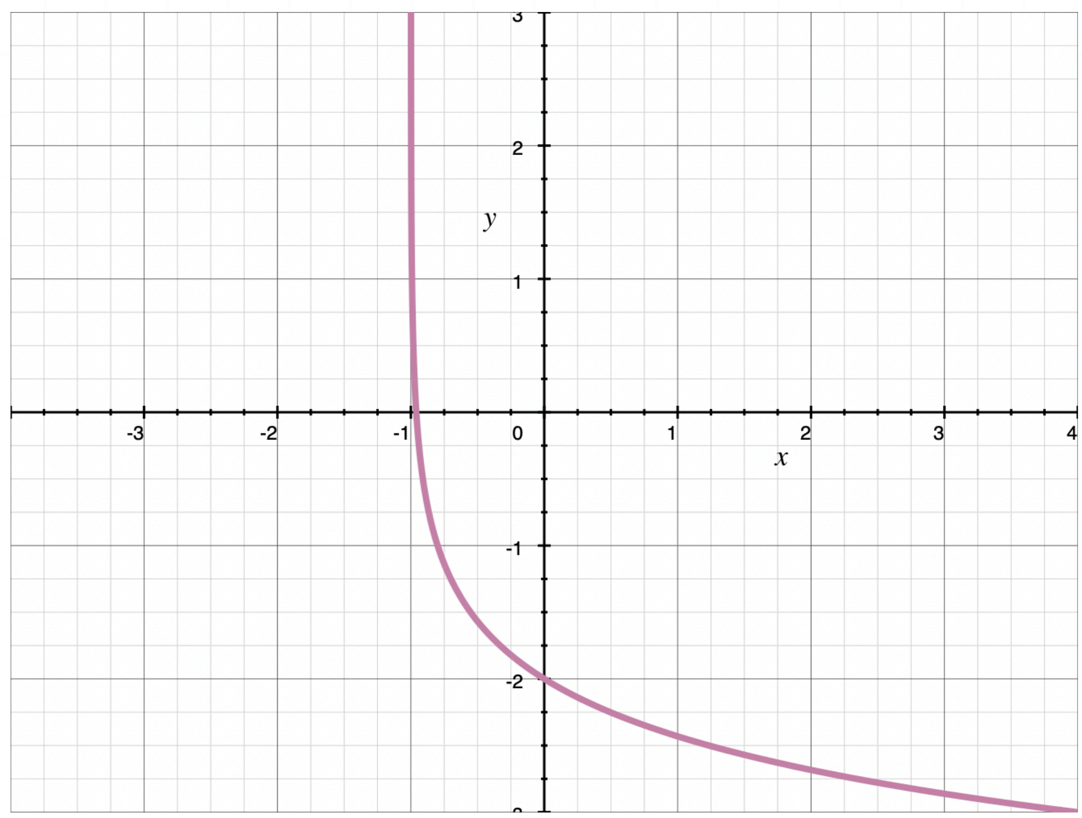
and to change $y = \log_5(x + 1)$ to $y = -\log_5(x + 1)$, we reflect the graph over the x-axis to get the light blue graph.



To change $y = -\log_5(x + 1)$ to $y = -\log_5(x + 1) - 2$, we shift the graph down 2 units.



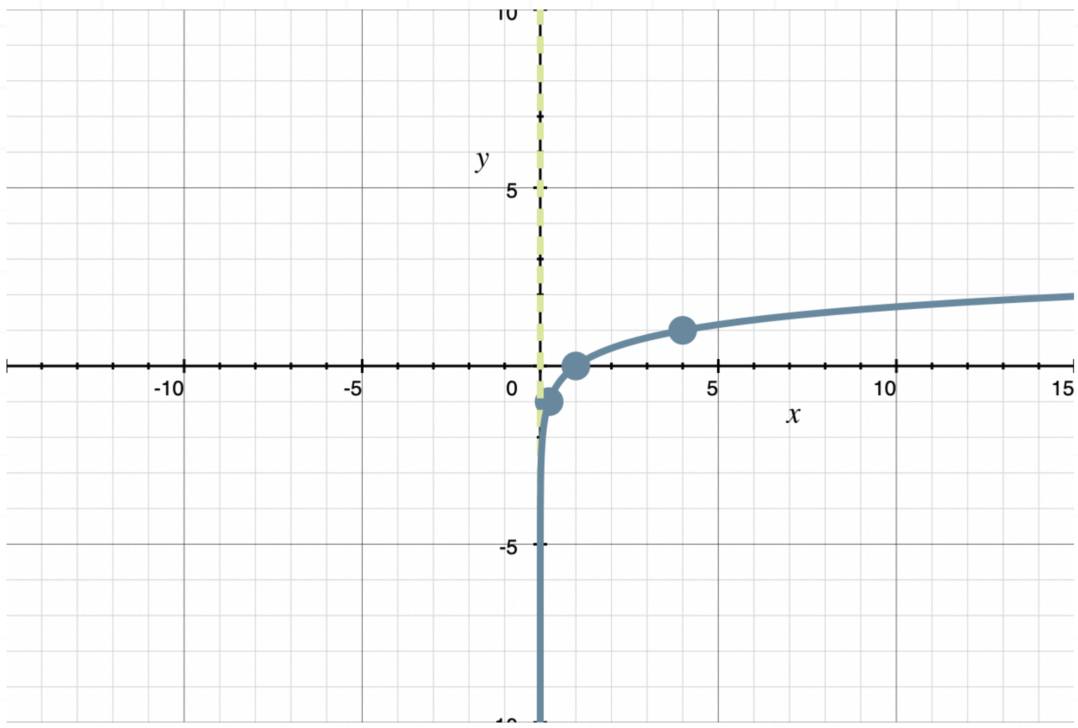
So the final sketch of $y = -\log_5(x+1) - 2$ is



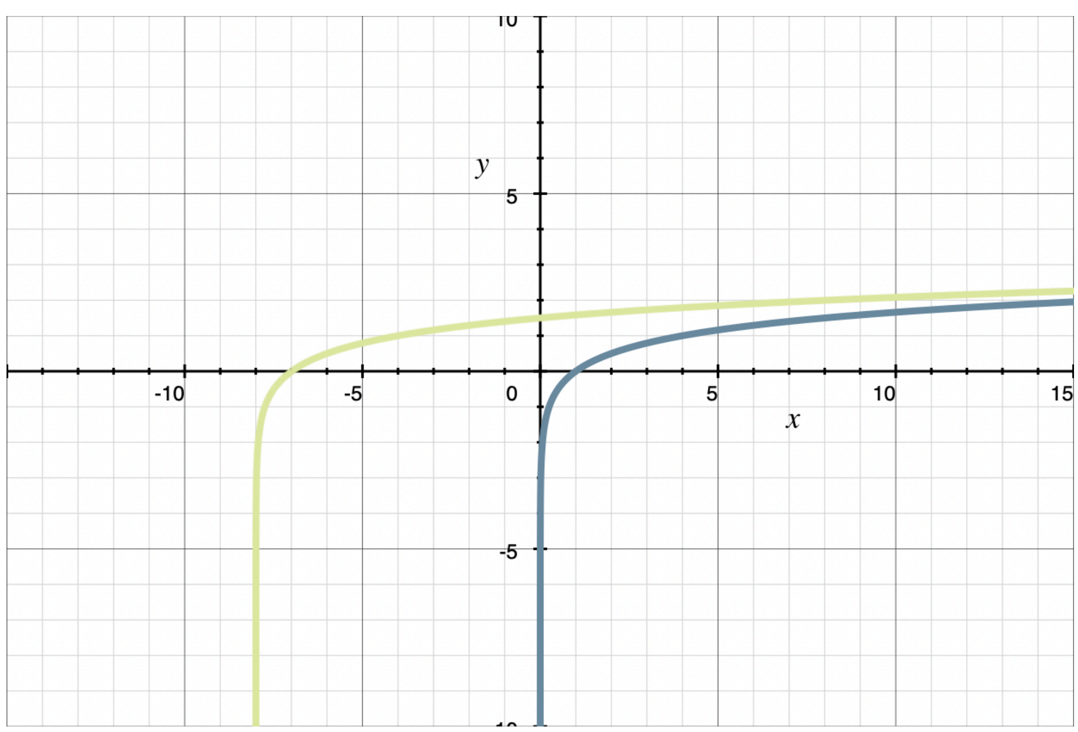
■ 5. Use transformations to sketch the graph of $y = \log_4(x+8)$.

Solution:

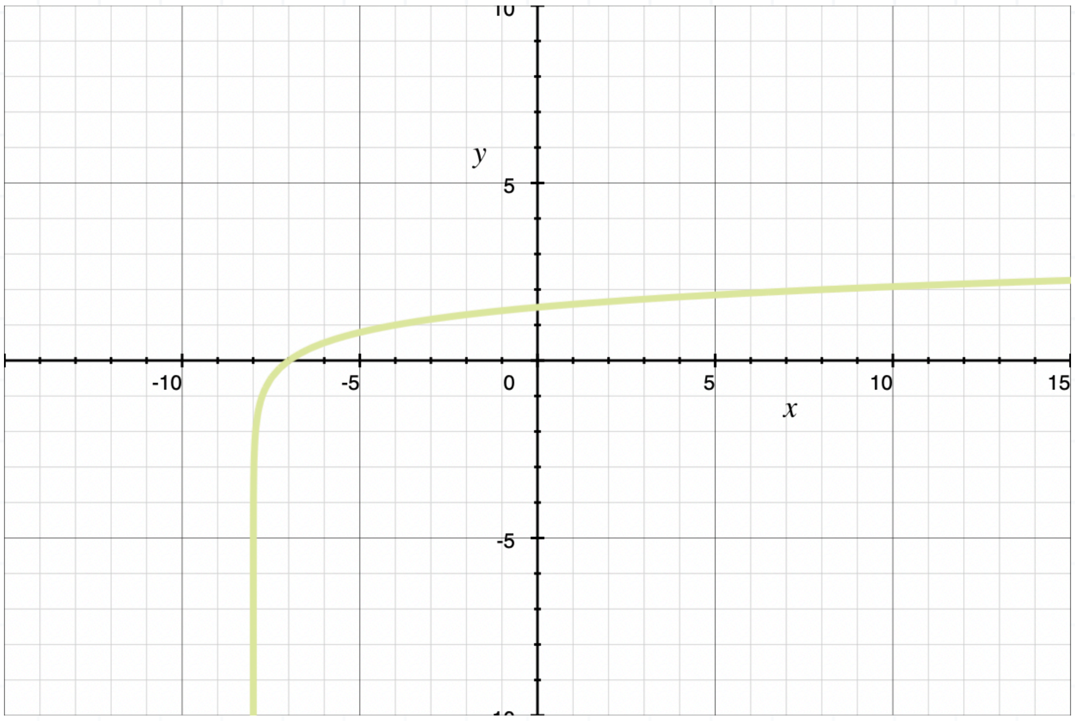
First we need to graph the parent function $y = \log_4 x$. We sketch the vertical asymptote, $x = 0$, and plot the x-intercept $(1,0)$ and the points $(b,1) = (4,1)$ and $(1/b, -1) = (1/4, -1)$, connecting them with a smooth curve to get



To change $y = \log_4 x$ to $y = \log_4(x + 8)$, we shift the graph left 8 units.



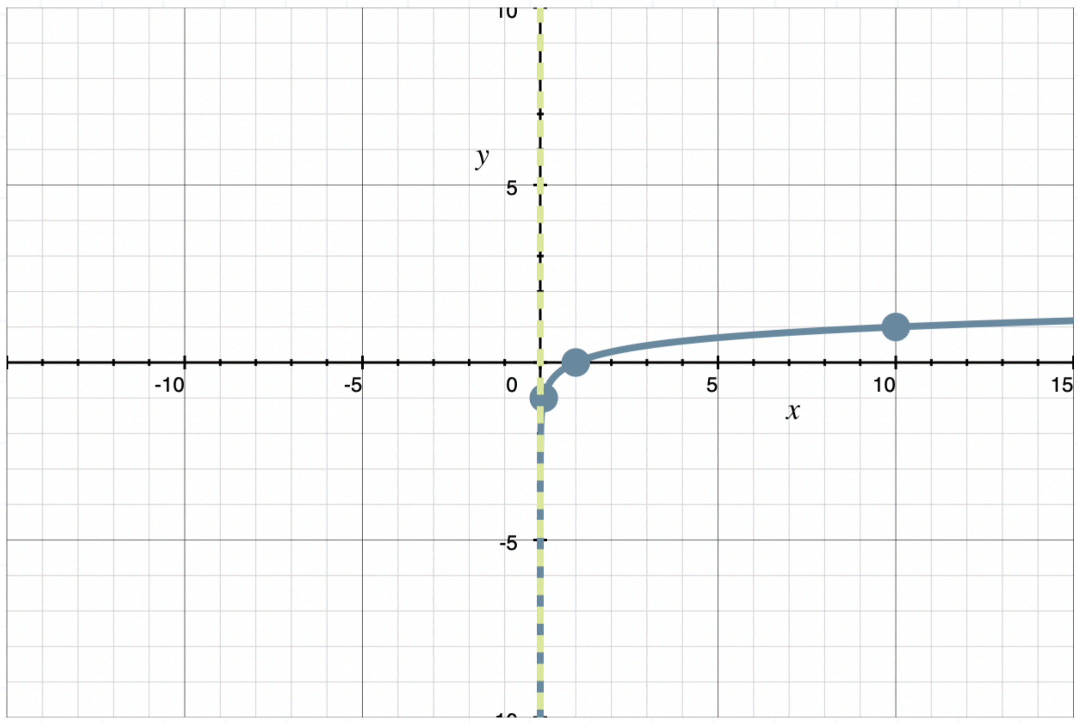
So the final sketch of $y = \log(5x) - 3$ is



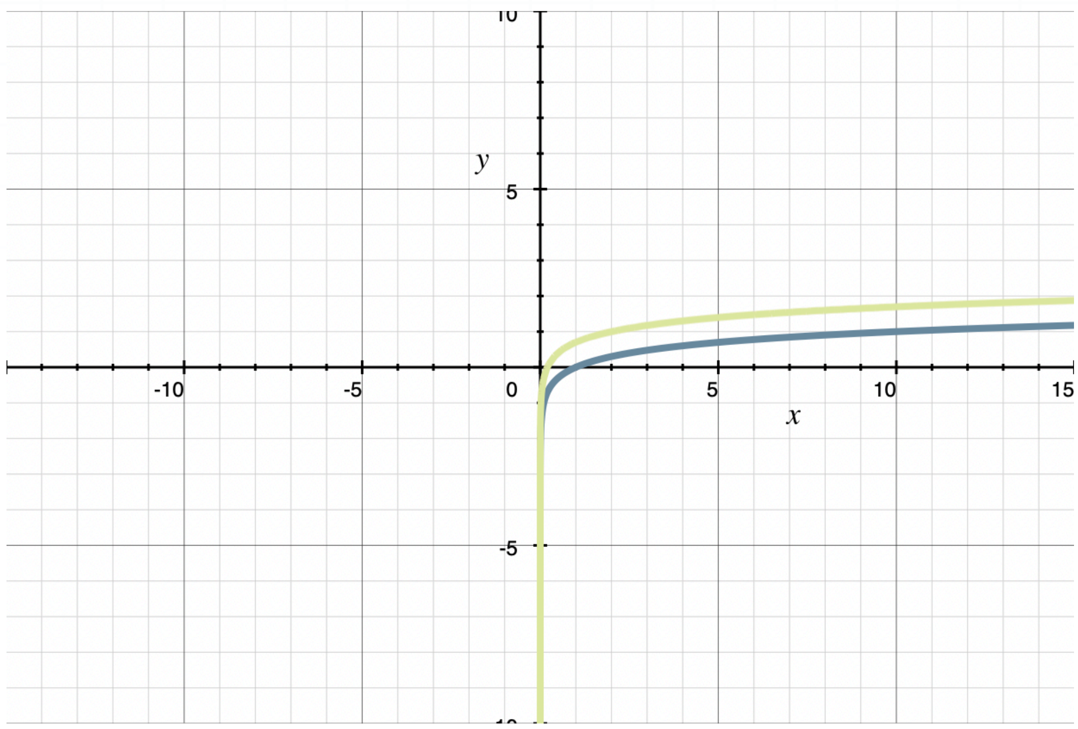
■ 6. Use transformations to sketch the graph of $y = \log(5x) - 3$.

Solution:

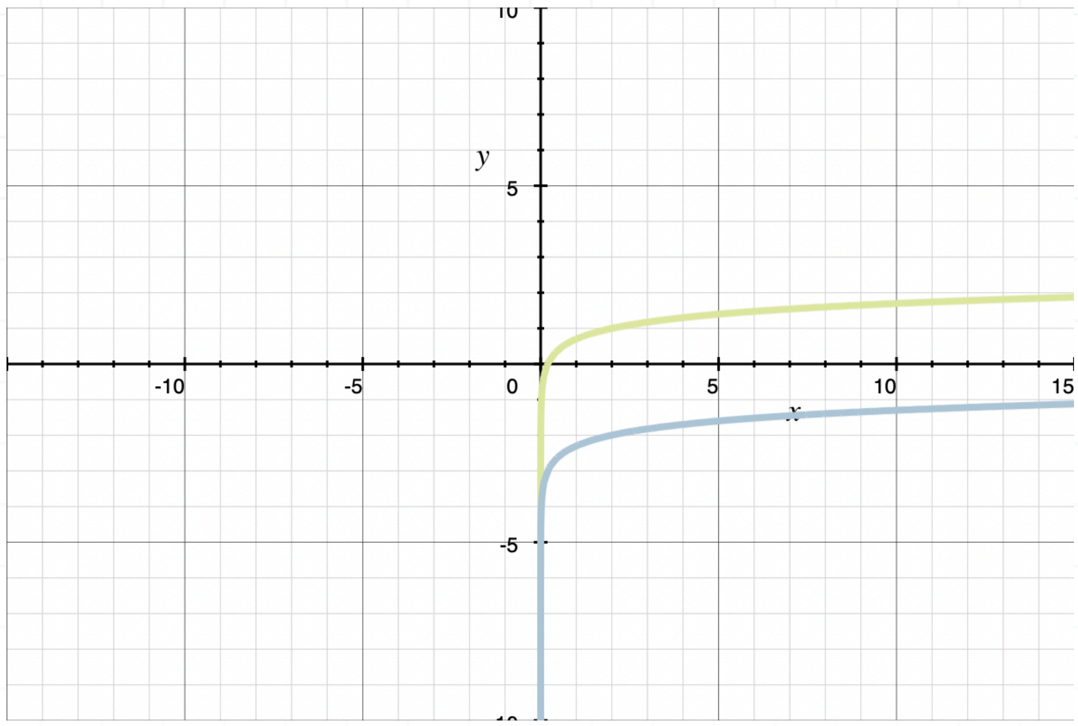
First we need to graph the parent function $y = \log x$. We sketch the vertical asymptote, $x = 0$, and plot the x-intercept $(1,0)$ and the points $(b,1) = (10,1)$ and $(1/b, -1) = (1/10, -1)$, connecting them with a smooth curve to get



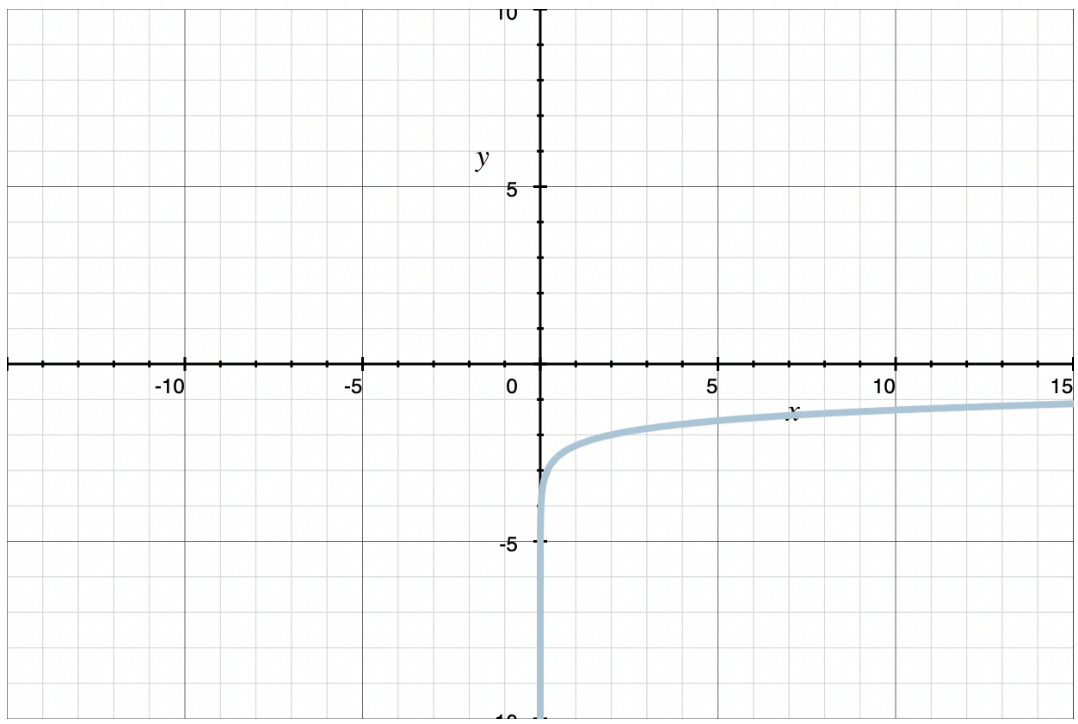
To change $y = \log x$ to $y = \log(5x)$, we compress the function horizontally by a factor of 5,



and then to change $y = \log(5x)$ to $y = \log(5x) - 3$, we shift the graph by down by 3 units.



So the final sketch of $y = \log(5x) - 3$ is



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