Topic: Solving logarithmic equations

Question: Solve the logarithmic equation.

$$\log_3(4x - 2) = \log_3(x + 7)$$

Answer choices:

A $\quad x = 3$

B $\quad x = \frac{9}{5}$

C $\quad x = 1$

D $\quad x = \frac{5}{3}$

Solution: A

Because we have single, isolated logarithms on each side of the equation, and because the logarithms have equivalent bases (they both have base 3), we can set the arguments equal to each other.

$$4x - 2 = x + 7$$

$$3x = 9$$

$$x = 3$$

Topic: Solving logarithmic equations

Question: Solve the logarithmic equation.

$$\log(2x-5)+\log(x-2)=\log(x^2-2x+4)$$

Answer choices:

A $\quad x=\dfrac{11}{2}$

B $\quad x=6$

C $\quad x=1,\ 6$

D $\quad x=\dfrac{6}{7}$

Solution: C

To solve the logarithmic equation, our goal is to simplify both sides of the equation until we have a single, isolated logarithm on each side.

If we apply the product rule,

$$\log_a x + \log_a y = \log_a(xy)$$

to the left side of the logarithmic equation, we can rewrite it as

$$\log(2x - 5) + \log(x - 2) = \log(x^2 - 2x + 4)$$

$$\log((2x - 5)(x - 2)) = \log(x^2 - 2x + 4)$$

$$\log(2x^2 - 9x + 10) = \log(x^2 - 2x + 4)$$

Since the logarithms on each side have the same base, the equation can only be true if the arguments are equivalent, so we'll set the arguments each to each other.

$$2x^2 - 9x + 10 = x^2 - 2x + 4$$

$$x^2 - 7x + 6 = 0$$

$$(x - 1)(x - 6) = 0$$

$$x = 1,\ 6$$

Topic: Solving logarithmic equations

Question: Solve the logarithmic equation.

$$\ln(2x+1)=\ln 3+\ln(5x-4)$$

Answer choices:

A $x=-1$

B $x=1$

C $x=\frac{2}{3}$

D $x=0$

Solution: B

If we apply the product rule,

$$\log_a x + \log_a y = \log_a(xy)$$

to rewrite the right side of the equation, we get

$$\ln(2x + 1) = \ln 3 + \ln(5x - 4)$$

$$\ln(2x + 1) = \ln(3(5x - 4))$$

$$\ln(2x + 1) = \ln(15x - 12)$$

Since we've isolated the natural log on each side, we know the arguments must be equal.

$$2x + 1 = 15x - 12$$

$$13x = 13$$

$$x = 1$$