Negative exponents and product rule

This lesson will cover how to use the product rule for negative exponents to find the value of a positive number raised to a negative power.

Negative exponents in the numerator and denominator

If we have two positive real numbers a and b, then

$$a^{-b} = \frac{1}{a^b}$$

In order to change the exponent in a^{-b} from $-b$ to b, we move the a^{-b} from the numerator to the denominator to get $1/a^b$.

Alternatively, if we have two positive real numbers a and b, then

$$\frac{1}{a^{-b}} = a^b$$

In order to change the exponent in a^{-b} from $-b$ to b, we move the a^{-b} from the denominator to the numerator to get $1 \cdot a^b$, or just a^b.

In other words, if we have a negative exponent in the numerator, we can make it positive by moving that term to the denominator. Or if we have a negative exponent in the denominator, we can make it positive by moving that term to the numerator.

Also, we may have to deal with expressions like $(a/b)^{-n}$, in which case we need to swap the numerator and the denominator, and raise each of them to the nth power.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$$

Reciprocals

By the way, a^b and a^{-b} are reciprocals. As we may remember, two numbers whose product is equal to 1 are reciprocals (of each other). Sometimes we'll hear or read about negative exponents and their relationship to reciprocals, and that relationship follows from the product rule for negative exponents. If we multiply a^b by a^{-b}, we can see that the product is 1, which means a^b and a^{-b} are reciprocals of each other.

$$(a^b) \cdot (a^{-b})$$

$$a^{b+(-b)}$$

$$a^{b-b}$$

$$a^0$$

$$1$$

So these two pairs are reciprocals of one another:

a^b and $\dfrac{1}{a^b}$

$$a^{-b} \text{ and } \frac{1}{a^{-b}}$$

Let's look at a few examples.

Example

Rewrite the expression with no negative exponents.

$$2^{-1}$$

In order to get rid of the negative exponent, we change the exponent in 2^{-1} from -1 to 1 and move the resulting expression from the numerator to the denominator, so we get

$$\frac{1}{2^1}$$

Since $2^1 = 2$, we can write this as

$$\frac{1}{2}$$

Let's look at an example with a variable.

Example

Write the expression with no negative exponents.

$$x^{-5}$$

In order to get rid of the negative exponent, we change the exponent in x^{-5} from -5 to 5 and move the resulting expression from the numerator to the denominator. We get

$$\frac{1}{x^5}$$

Let's look at another example.

Example

Write this expression with no negative exponents.

$$\frac{1}{b^{-7}}$$

In order to get rid of the negative exponent, we change the exponent in b^{-7} from -7 to 7 and move the resulting expression from the denominator to the numerator. We get $1 \cdot b^7$, which is equal to b^7.

Let's look at a final example, this time with a number other than 1 in the numerator.

Example

Write the expression with no negative exponents.

$$\frac{3}{x^{-5}}$$

In order to get rid of the negative exponent, we change the exponent in x^{-5} from -5 to 5 and move the resulting expression from the denominator to the numerator. We get

$$3x^5$$