Composite functions

In a **composite function**, one function is used as a variable in the other function. For instance, the composite $f(g(x))$ treats $g(x)$ as the variable in $f(x)$, and we can also write $f(g(x))$ as $(f \circ g)(x)$.

We also know that the composites $f(g(x))$ and $g(f(x))$ are different functions. It's possible that $f(g(x)) = g(f(x))$, but that's usually not the case, and we should always treat them as different functions.

Remember, the composite of two functions is not the same as the product of two functions.

Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Composite	$(f \circ g)(x) = f(g(x))$

Let's look at a few examples where we find the composite function.

Example

Find the composite function $(g \circ f)(x)$.

$$g(x) = \frac{2}{x^4}$$

$$f(x) = \sqrt[4]{x-3}$$

To find the composite function $(g \circ f)(x)$, we plug $f(x)$ into $g(x)$, which means that we take the algebraic expression for $f(x)$ and substitute it for x into the algebraic expression for $g(x)$.

$$(g \circ f)(x) = g(f(x)) = \frac{2}{(\sqrt[4]{x-3})^4}$$

$$(g \circ f)(x) = g(f(x)) = \frac{2}{x-3}$$

Here's another example.

Example

Find $h(g(x))$ if $h(x) = 3x^2 - 2$ and $g(x) = x - 4$.

To find the composite function $h(g(x))$, we plug $g(x)$ into $h(x)$, which means that we take the algebraic expression for $g(x)$ and substitute it for x into the algebraic expression for $h(x)$.

$$h(g(x)) = 3(x-4)^2 - 2$$

$$h(g(x)) = 3(x^2 - 8x + 16) - 2$$

$$h(g(x)) = 3x^2 - 24x + 48 - 2$$

$$h(g(x)) = 3x^2 - 24x + 46$$

Evaluating composites

To evaluate the composite function, we start from the inside. For instance, to evaluate $f(g(x))$ at $x = a$, we start on the inside and first find $g(a)$. Then we plug $g(a)$ into f to find $f(g(a))$. In general, we'll use these steps:

1. Evaluate the inside function using the given input value or variable.

2. Use the output obtained in the first step as the input into the outside function.

Let's work through an example.

Example

Find $h(g(1))$ if $h(x) = x^2 - 2x$ and $g(x) = 4x - 3$.

To find the composite function $h(g(x))$, we plug $g(x)$ into $h(x)$, which means that we take the algebraic expression for $g(x)$ and substitute it for x into the algebraic expression for $h(x)$.

$$h(g(x)) = (4x - 3)^2 - 2(4x - 3)$$

$$h(g(x)) = (4x - 3)(4x - 3) - 8x + 6$$

$$h(g(x)) = 16x^2 - 12x - 12x + 9 - 8x + 6$$

$$h(g(x)) = 16x^2 - 32x + 15$$

Now substitute $x = 1$ into composite to find $h(g(1))$.

$$h(g(1)) = 16(1)^2 - 32(1) + 15$$

$$h(g(1)) = 16 - 32 + 15$$

$$h(g(1)) = -1$$

Alternatively, we could have started by finding $g(1)$,

$$g(1) = 4(1) - 3$$

$$g(1) = 1$$

and then plugged this result into $f(x)$ to find $h(g(1))$ as

$$h(g(1)) = 1^2 - 2(1)$$

$$h(g(1)) = 1 - 2$$

$$h(g(1)) = -1$$