Algebra 2 Workbook Solutions

Factoring

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COEFFICIENTS IN QUADRATICS

1. Factor the quadratic.

$$6x^2 + 11x - 10$$

Solution:

The only pairs of factors of 6 are $(6,1)$ and $(3,2)$, so we'll have one of these:

$(6x \quad)(x \quad)$ or $(3x \quad)(2x \quad)$

The only pairs of factors of 10 are $(10,1)$ and $(5,2)$, which means we'll have one of these possibilities:

$(6x \quad 10)(x \quad 1)$ $\qquad$ $(3x \quad 10)(2x \quad 1)$

$(6x \quad 1)(x \quad 10)$ $\qquad$ $(3x \quad 1)(2x \quad 10)$

$(6x \quad 5)(x \quad 2)$ $\qquad$ $(3x \quad 5)(2x \quad 2)$

$(6x \quad 2)(x \quad 5)$ $\qquad$ $(3x \quad 2)(2x \quad 5)$

If we do the factoring as $(3x \quad 2)(2x \quad 5)$, we'll need to combine $15x$ and $4x$ to get $11x$, which we can do by making $15x$ positive and $4x$ negative. Therefore, we have to use 5 as the constant term in the second factor in parentheses (because $15x = 3x \cdot 5$), and -2 as the constant term in the first factor in parentheses (because $-4x = -2 \cdot 2x$), so we get

$$(3x - 2)(2x + 5)$$

2. Factor the quadratic.

$$20x^2 - 23x + 6$$

Solution:

The only pairs of factors of 20 are $(20,1)$, $(10,2)$, and $(5,4)$, so we'll have one of these:

$$(20x \qquad)(x \qquad)$$

$$(10x \qquad)(2x \qquad)$$

$$(5x \qquad)(4x \qquad)$$

The only pairs of factors of 6 are $(6,1)$ and $(3,2)$, which means we'll have one of these possibilities:

$$(20x \quad 6)(x \quad 1) \qquad (20x \quad 1)(x \quad 6)$$

$$(20x \quad 3)(x \quad 2) \qquad (20x \quad 2)(x \quad 3)$$

$$(10x \quad 6)(2x \quad 1) \qquad (10x \quad 1)(2x \quad 6)$$

$$(10x \quad 3)(2x \quad 2) \qquad (10x \quad 2)(2x \quad 3)$$

$$(5x \quad 6)(4x \quad 1) \qquad (5x \quad 1)(4x \quad 6)$$

$$(5x \quad\quad 3)(4x \quad\quad 2) \qquad\qquad (5x \quad\quad 2)(4x \quad\quad 3)$$

If we do the factoring as $(5x \quad\quad 2)(4x \quad\quad 3)$, we'll need to combine $15x$ and $8x$ to get $-23x$, which we can do by making $15x$ negative and $8x$ negative. Therefore, we have to use -3 as the constant term in the second factor in parentheses (because $-15x = -3 \cdot 5x$), and -2 as the constant term in the first factor in parentheses (because $-8x = -2 \cdot 4x$), so we get

$$(5x - 2)(4x - 3)$$

■ 3. Factor the quadratic.

$$4x^2 + 26x + 36$$

Solution:

Divide through by 2.

$$2(2x^2 + 13x + 18)$$

The only factors of 2 are 2 and 1, so we know we'll have

$$(2x \quad\quad)(x \quad\quad)$$

The only pairs of factors of 18 are $(18,1)$, $(9,2)$, and $(6,3)$, which means we'll have one of these possibilities:

$$(2x \quad 18)(x \quad 1) \qquad\qquad (2x \quad 1)(x \quad 18)$$

$(2x \quad 9)(x \quad 2)$ $(2x \quad 2)(x \quad 9)$

$(2x \quad 6)(x \quad 3)$ $(2x \quad 3)(x \quad 6)$

If we do the factoring as $(2x \quad 9)(x \quad 2)$, we'll need to combine $4x$ and $9x$ to get $13x$, which we can do by making $4x$ positive and $9x$ positive. Therefore, we have to use 2 as the constant term in the second factor in parentheses (because $4x = 2 \cdot 2x$), and 9 as the constant term in the first factor in parentheses (because $9x = 9 \cdot x$), so we get

$$2(2x + 9)(x + 2)$$

■ 4. Factor the quadratic.

$$14x^2 + 15x + 4$$

Solution:

The only pairs of factors of 14 are $(14,1)$ and $(7,2)$, so we'll have one of these:

$(14x \quad)(x \quad)$ or $(7x \quad)(2x \quad)$

The only pairs of factors of 4 are $(4,1)$ and $(2,2)$, which means we'll have one of these possibilities:

$(14x \quad 4)(x \quad 1)$ $(14x \quad 1)(x \quad 4)$

$(14x \quad 2)(x \quad 2)$ $(7x \quad 2)(2x \quad 2)$

$(7x \quad 4)(2x \quad 1)$ $\qquad$ $(7x \quad 1)(2x \quad 4)$

If we do the factoring as $(7x \quad 4)(2x \quad 1)$, we'll need to combine $7x$ and $8x$ to get $15x$, which we can do by making $7x$ positive and $8x$ positive. Therefore, we have to use 1 as the constant term in the second factor in parentheses (because $7x = 1 \cdot 7x$), and 4 as the constant term in the first factor in parentheses (because $8x = 4 \cdot 2x$), so we get

$$(7x + 4)(2x + 1)$$

■ 5. Factor the quadratic.

$$12x^2 + 4x - 1$$

Solution:

The only pairs of factors of 12 are $(12,1)$, $(6,2)$, and $(4,3)$, so we'll have one of these:

$$(12x \quad)(x \quad)$$

$$(6x \quad)(2x \quad)$$

$$(4x \quad)(3x \quad)$$

The only factors of 1 are 1 and 1, which means we'll have one of these possibilities:

$$(12x \quad 1)(x \quad 1)$$

$$(6x \quad 1)(2x \quad 1)$$

$$(4x \quad 1)(3x \quad 1)$$

If we do the factoring as $(6x \quad 1)(2x \quad 1)$, we'll need to combine $6x$ and $2x$ to get $4x$, which we can do by making $6x$ positive and $2x$ negative. Therefore, we have to use 1 as the constant term in the second factor in parentheses (because $6x = 1 \cdot 6x$), and -1 as the constant term in the first factor in parentheses (because $-2x = -1 \cdot 2x$), so we get

$$(6x - 1)(2x + 1)$$

■ 6. Factor the quadratic.

$$8x^2 - 10x - 63$$

Solution:

The only pairs of factors of 8 are $(8,1)$ and $(4,2)$, so we'll have one of these:

$(8x \quad)(x \quad)$ or $(4x \quad)(2x \quad)$

The only factors of 63 are 9 and 7, which means we'll have one of these possibilities:

$(8x \quad 9)(x \quad 7)$ $(8x \quad 7)(x \quad 9)$

$(4x \quad 9)(2x \quad 7)$ $(4x \quad 7)(2x \quad 9)$

If we do the factoring as $(4x \quad 9)(2x \quad 7)$, we'll need to combine $28x$ and $18x$ to get $-10x$, which we can do by making $28x$ negative and $18x$ positive. Therefore, we have to use -7 as the constant term in the second factor in parentheses (because $-28x = -7 \cdot 4x$), and 9 as the constant term in the first factor in parentheses (because $18x = 9 \cdot 2x$), so we get

$$(4x + 9)(2x - 7)$$

GROUPING

■ 1. Factor the expression by grouping.

$$2x - 3y - 4ax + 6ay$$

Solution:

Find terms that have factors in common, and group those terms.

$$(2x - 4ax) + (-3y + 6ay)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2x(1 - 2a) + (-3y + 6ay)$$

$$2x(1 - 2a) - 3y(1 - 2a)$$

Now because both terms happen to have $(1 - 2a)$ in common, we're able to factor $(1 - 2a)$ out of each term, leaving only $2x$ from the first term, and $3y$ from the second term.

$$(1 - 2a)(2x - 3y)$$

■ 2. Factor the quadratic by grouping.

$$4x^2 + 2xy + 10x + 5y$$

Solution:

Find terms that have factors in common, and group those terms.

$$(4x^2 + 2xy) + (10x + 5y)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2x(2x + y) + (10x + 5y)$$

$$2x(2x + y) + 5(2x + y)$$

Now because both terms happen to have $(2x + y)$ in common, we're able to factor $(2x + y)$ out of each term, leaving only $2x$ from the first term, and 5 from the second term.

$$(2x + y)(2x + 5)$$

■ 3. Factor the expression by grouping.

$$8ab + 2b - 4a - 1$$

Solution:

Find terms that have factors in common, and group those terms.

$$(8ab + 2b) + (-4a - 1)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2b(4a+1)+(-4a-1)$$

$$2b(4a+1)-1(4a+1)$$

Now because both terms happen to have $(4a+1)$ in common, we're able to factor $(4a+1)$ out of each term, leaving only $2b$ from the first term, and -1 from the second term.

$$(4a+1)(2b-1)$$

■ 4. Factor the expression by grouping.

$$9z+9qr+5ayz+5ayqr$$

Solution:

Find terms that have factors in common, and group those terms.

$$(9z+9qr)+(5ayz+5ayqr)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$9(z+qr)+(5ayz+5ayqr)$$

$$9(z+qr)+5ay(z+qr)$$

Now because both terms happen to have $(z + qr)$ in common, we're able to factor $(z + qr)$ out of each term, leaving only 9 from the first term, and $5ay$ from the second term.

$$(z + qr)(9 + 5ay)$$

■ 5. Factor the quadratic by grouping.

$$3k^2 + 7k - 6$$

Solution:

First find the factors of $a \cdot c$ that combine to equal b. Start with the fact that $a = 3$, $b = 7$, and $c = -6$.

$$a \cdot c = 3(-6) = -18$$

The factors of -18 that combine to equal 7 are 9 and -2. Rewrite the quadratic by replacing $7k$ with $9k - 2k$.

$$3k^2 + 9k - 2k - 6$$

Find terms that have factors in common, and group those terms.

$$(3k^2 + 9k) + (-2k - 6)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$3k(k+3)+(-2k-6)$$

$$3k(k+3)-2(k+3)$$

Now because both terms happen to have $(k+3)$ in common, we're able to factor $(k+3)$ out of each term, leaving only $3k$ from the first term, and -2 from the second term.

$$(k+3)(3k-2)$$

■ 6. Factor the quadratic by grouping.

$$6x^2+13x-5$$

Solution:

First find the factors of $a \cdot c$ that combine to equal b. Start with the fact that $a=6$, $b=13$, and $c=-5$.

$$a \cdot c = 6(-5) = -30$$

The factors of -30 that combine to equal 13 are 15 and -2. Rewrite the quadratic by replacing $13x$ with $15x-2x$.

$$6x^2+15x-2x-5$$

Find terms that have factors in common, and group those terms.

$$(6x^2-2x)+(15x-5)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2x(3x-1)+(15x-5)$$

$$2x(3x-1)+5(3x-1)$$

Now because both terms happen to have $(3x-1)$ in common, we're able to factor $(3x-1)$ out of each term, leaving only $2x$ from the first term, and 5 from the second term.

$$(3x-1)(2x+5)$$

DIFFERENCE OF CUBES

■ 1. Factor the polynomial.

$$x^3 - 27y^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{x^3} = x$$

$$\sqrt[3]{27y^9} = 3y^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = x$ and $b = 3y^3$. Therefore, we get

$$(x - 3y^3)(x^2 + x(3y^3) + (3y^3)^2)$$

$$(x - 3y^3)(x^2 + 3xy^3 + 9y^6)$$

■ 2. Factor the polynomial.

$$8x^3y^6 - 64z^{21}$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{8x^3y^6} = 2xy^2$$

$$\sqrt[3]{64z^{21}} = 4z^7$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 2xy^2$ and $b = 4z^7$. Therefore, we get

$$(2xy^2 - 4z^7)((2xy^2)^2 + (2xy^2)(4z^7) + (4z^7)^2)$$

$$(2xy^2 - 4z^7)(4x^2y^4 + 8xy^2z^7 + 16z^{14})$$

■ 3. Factor the polynomial.

$$a^3b^{12} - 125c^6$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{a^3b^{12}} = ab^4$$

$$\sqrt[3]{125c^6} = 5c^2$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = ab^4$ and $b = 5c^2$. Therefore, we get

$$(ab^4 - 5c^2)((ab^4)^2 + (ab^4)(5c^2) + (5c^2)^2)$$

$$(ab^4 - 5c^2)(a^2b^8 + 5ab^4c^2 + 25c^4)$$

4. Factor the polynomial.

$$27y^6z^3 - 216x^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{27y^6z^3} = 3y^2z$$

$$\sqrt[3]{216x^9} = 6x^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 3y^2z$ and $b = 6x^3$. Therefore, we get

$$(3y^2z - 6x^3)((3y^2z)^2 + (3y^2z)(6x^3) + (6x^3)^2)$$

$$(3y^2z - 6x^3)(9y^4z^2 + 18x^3y^2z + 36x^6)$$

■ 5. Factor the polynomial.

$$8x^{15} - 27y^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{8x^{15}} = 2x^5$$

$$\sqrt[3]{27y^9} = 3y^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 2x^5$ and $b = 3y^3$. Therefore, we get

$$(2x^5 - 3y^3)((2x^5)^2 + (2x^5)(3y^3) + (3y^3)^2)$$

$$(2x^5 - 3y^3)(4x^{10} + 6x^5y^3 + 9y^6)$$

■ 6. Factor the polynomial.

$$216a^3b^6 - 125c^{24}d^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{216a^3b^6} = 6ab^2$$

$$\sqrt[3]{125c^{24}d^3} = 5c^8d$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 6ab^2$ and $b = 5c^8d$. Therefore, we get

$$(6ab^2 - 5c^8d)((6ab^2)^2 + (6ab^2)(5c^8d) + (5c^8d)^2)$$

$$(6ab^2 - 5c^8d)(36a^2b^4 + 30ab^2c^8d + 25c^{16}d^2)$$

SUM OF CUBES

■ 1. If $x^2 - 2xy^2 + 4y^4 = 5$ and $x + 2y^2 = 8$, what is the value of $8x^3 + 64y^6$?

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{8x^3} = 2x$$

$$\sqrt[3]{64y^6} = 4y^2$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = 2x$ and $b = 4y^2$. Therefore, we get

$$(2x + 4y^2)((2x)^2 - (2x)(4y^2) + (4y^2)^2)$$

$$(2x + 4y^2)(4x^2 - 8xy^2 + 16y^4)$$

$$2(x + 2y^2)(4x^2 - 8xy^2 + 16y^4)$$

$$2(x + 2y^2)4(x^2 - 2xy^2 + 4y^4)$$

$$8(x + 2y^2)(x^2 - 2xy^2 + 4y^4)$$

Then we can substitute $x^2 - 2xy^2 + 4y^4 = 5$ and $x + 2y^2 = 8$ into the equation to get

$$8(8)(5)$$

$$320$$

■ 2. Factor the polynomial.

$$216a^{21} + 64b^{15}c^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{216a^{21}} = 6a^7$$

$$\sqrt[3]{64b^{15}c^9} = 4b^5c^3$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

with $a = 6a^7$ and $b = 4b^5c^3$. Therefore, we get

$$(6a^7 + 4b^5c^3)((6a^7)^2 - (6a^7)(4b^5c^3) + (4b^5c^3)^2)$$

$$(6a^7 + 4b^5c^3)(36a^{14} - 24a^7b^5c^3 + 16b^{10}c^6)$$

$$2(3a^7 + 2b^5c^3)(36a^{14} - 24a^7b^5c^3 + 16b^{10}c^6)$$

$$2(3a^7 + 2b^5c^3)4(9a^{14} - 6a^7b^5c^3 + 4b^{10}c^6)$$

$$8(3a^7 + 2b^5c^3)(9a^{14} - 6a^7b^5c^3 + 4b^{10}c^6)$$

■ 3. Factor the polynomial.

$$512z^{24} + 125m^6r^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{512z^{24}} = 8z^8$$

$$\sqrt[3]{125m^6r^3} = 5m^2r$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

with $a = 8z^8$ and $b = 5m^2r$. Therefore, we get

$$(8z^8 + 5m^2r)((8z^8)^2 - (8z^8)(5m^2r) + (5m^2r)^2)$$

$$(8z^8 + 5m^2r)(64z^{16} - 40m^2rz^8 + 25m^4r^2)$$

■ 4. Factor the polynomial.

$$64j^3k^6 + 8r^{12}t^6$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{64j^3k^6} = 4jk^2$$

$$\sqrt[3]{8r^{12}t^6} = 2r^4t^2$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

with $a = 4jk^2$ and $b = 2r^4t^2$. Therefore, we get

$$(4jk^2 + 2r^4t^2)((4jk^2)^2 - (4jk^2)(2r^4t^2) + (2r^4t^2)^2)$$

$$(4jk^2 + 2r^4t^2)(16j^2k^4 - 8jk^2r^4t^2 + 4r^8t^4)$$

$$2(2jk^2 + r^4t^2)4(4j^2k^4 - 2jk^2r^4t^2 + r^8t^4)$$

$$8(2jk^2 + r^4t^2)(4j^2k^4 - 2jk^2r^4t^2 + r^8t^4)$$

■ 5. Factor the polynomial.

$$729x^{18} + 216y^6$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{729x^{18}} = 9x^6$$

$$\sqrt[3]{216y^6} = 6y^2$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

with $a = 9x^6$ and $b = 6y^2$. Therefore, we get

$$(9x^6 + 6y^2)((9x^6)^2 - (9x^6)(6y^2) + (6y^2)^2)$$

$$(9x^6 + 6y^2)(81x^{12} - 54x^6y^2 + 36y^4)$$

$$3(3x^6 + 2y^2)9(9x^{12} - 6x^6y^2 + 4y^4)$$

$$27(3x^6 + 2y^2)(9x^{12} - 6x^6y^2 + 4y^4)$$

■ 6. Factor the polynomial.

$$(x-5)^3 + 125$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{(x-5)^3} = x - 5$$

$$\sqrt[3]{125} = 5$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = x - 5$ and $b = 5$. Therefore, we get

$$(x - 5 + 5)((x - 5)^2 - (x - 5)(5) + 5^2)$$

$$x(x^2 - 10x + 25 - 5x + 25 + 25)$$

$$x(x^2 - 15x + 75)$$

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