# Zero Theorem

So far, we've been learning to factor quadratic expressions, but now we want to switch to solving quadratic equations. In other words, instead of factoring just $x^2 - x - 20$, we want to learn to solve $x^2 - x - 20 = 0$.

## The Zero Theorem

We already know that $x^2 - x - 20$ will factor as $(x + 4)(x - 5)$. Now that the quadratic is part of an equation, the factoring is no different. We can still factor the **quadratic equation** to rewrite it as

$$x^2 - x - 20 = 0$$

$$(x + 4)(x - 5) = 0$$

Once we've factored the quadratic equation, let's imagine that $A = x + 4$ and $B = x - 5$. Then this quadratic equation can be written as $AB = 0$.

What we can say about the values of $A$ and $B$ in $AB = 0$? Well, the only way to make the product $AB$ equal to $0$ is either for $A$ to be $0$, for $B$ to be $0$. This is the **Zero Theorem**, which tells us that, given $AB = 0$, we know

$$A = 0 \text{ or } B = 0$$

Which means that, using the example $(x + 4)(x - 5) = 0$, it must be true that $x + 4 = 0$ or $x - 5 = 0$, which means that $x = -4$ or $x = 5$. These are the values of $x$ that make the equation true, and we were able to find them by applying the Zero Theorem.

When we solve for the solutions of a quadratic, we can call them the "**solutions**," the "**roots**," or the "**zeros**" of the quadratic.

Keep in mind that this Theorem only works when one side of the equation is $0$. So given something like $(x+4)(x-5)=3$, we can't break that down as $x+4=3$ or $x-5=3$.

Let's look at another example where we have to factor the quadratic and then apply the Zero Theorem to find the roots.

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**Example**

Find the roots of the equation.

$$x^2-13x+36=0$$

The roots of this equation are the values of $x$ at which the polynomial on the left side is equal to $0$. If we factor the left side,

$$x^2-13x+36=0$$

$$(x-4)(x-9)=0$$

then the Zero Theorem tells us that

$$x-4=0 \quad \rightarrow \quad x=4$$

or

$$x-9=0 \quad \rightarrow \quad x=9$$

So the roots are $x = 4$ and $x = 9$.

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Let's do another example.

**Example**

Find the zeros of the equation.

$$x^2 - 8x + 7 = 0$$

Factor the left side.

$$x^2 - 8x + 7 = 0$$

$$(x - 7)(x - 1) = 0$$

The Zero Theorem tells us that

$$x - 7 = 0 \quad \rightarrow \quad x = 7$$

or

$$x - 1 = 0 \quad \rightarrow \quad x = 1$$

The zeros are $x = 1$ and $x = 7$.

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