Equations with subscripts

Sometimes we'll encounter subscripted variables. A subscripted variable is just a variable that has a subscript attached to it, and a **subscript** is a small number that comes just after, and at a lower level than the variable.

When to use subscripts

It's really common to see subscripts in the sciences, like chemistry or physics. Think about the chemical formula for water, H_2O. The 2 that we see on the H is a subscript.

In mathematics, we'll often use subscripts to represent the same kind of value for different subjects.

For instance, let's say we're working a problem where two people, Abigail and Phoebe, are covering some distance at a particular rate of speed. We essentially have four variables in this situation: Abigail's distance, Phoebe's distance, Abigail's rate, and Phoebe's rate. Instead of using four variables like v, x, y, and z, we can use more descriptive variables, along with subscripts, to represent the four unknowns. Specifically, A_D, P_D, A_R, and P_R would make a lot of sense.

Solving equations with subscripts

Luckily, solving equations with subscripted variables isn't any different than solving equations with normal variables. In other words, we need to think about a subscripted variable just like we would a normal variable.

Let's do an example to get some practice with these.

Example

The pressure and volume of a gas are related according to the equation $P_1V_1 - P_2V_2 = 0$, where P_1 and V_1 are the original pressure and volume, and P_2 and V_2 are the new pressure and volume. If the original pressure is 1.4 the original volume is 210, and the new pressure is 28, what is the new volume?

We know that $P_1 = 1.4$, $V_1 = 210$, and $P_2 = 28$, so we can plug these values into the equation relating these variables, and we get

$$P_1V_1 - P_2V_2 = 0$$

$$(1.4)(210) - (28)V_2 = 0$$

Solve for new volume, V_2, by simplifying the left side and then using inverse operations to isolate V_2.

$$294 - 28V_2 = 0$$

$$294 - 28V_2 + 28V_2 = 0 + 28V_2$$

$$294 = 28V_2$$

$$\frac{294}{28} = \frac{28V_2}{28}$$

$$10.5 = V_2$$

$$V_2 = 10.5$$

Let's try another example with subscripted variables.

Example

A car travels at a 50 mph for 125 miles, then speeds up and travels at a new constant speed for another 153 miles. If the total time for the trip is 4.75 hours, how fast does the car go during the second part of the trip?

We'll use an equation that relates distance, rate, and time for an object in motion. The equation is

$$\text{distance} = \text{rate} \times \text{time}$$

and tells us that multiplying how fast something is moving and the amount of time it's been moving is equal to the distance that it's moved. We can manipulate this equation to solve for any of the three values. For example, we can divide both sides by rate to get an equation for time.

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

The total time for the trip (which we'll call t) is the sum of the times for the two parts. Let d_1 and r_1 be the distance and rate on the first part of the trip, and let d_2 and r_2 be the distance and rate on the second part. If we divide d_1 by r_1, we get the time for the first part; similarly if we divide d_2 by r_2, we get the time for the second part. Therefore,

$$\frac{d_1}{r_1} + \frac{d_2}{r_2} = t$$

Start by plugging in what we know, which is $d_1 = 125$, $r_1 = 50$, $d_2 = 153$, and $t = 4.75$.

$$\frac{125}{50} + \frac{153}{r_2} = 4.75$$

$$2.5 + \frac{153}{r_2} = 4.75$$

Use inverse operations to isolate r_2.

$$2.5 - 2.5 + \frac{153}{r_2} = 4.75 - 2.5$$

$$\frac{153}{r_2} = 2.25$$

$$\frac{153}{r_2} \cdot r_2 = 2.25 \cdot r_2$$

$$153 = 2.25r_2$$

$$\frac{153}{2.25} = \frac{2.25r_2}{2.25}$$

$$68 = r_2$$

$$r_2 = 68$$

The car travels at 68 mph for the second part of the trip.