Algebra 1 Workbook Solutions

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MATH

VARIABLES

1. Which value can we identify as the variable in the expression?

$$3y^2 + ay - 6 = 1$$

Solution:

Variables are usually represented by letters at the end of the alphabet, so y is the variable in this equation. The letter a represents a constant coefficient.

2. Identify any constant(s) in the equation.

$$x^2 - 3x + 2 = 0$$

Solution:

The equation includes two constants, the 2 on the left and the 0 on the right. A constant is any real number that doesn't have a variable attached to it.

3. How many terms exist in the equation?

$$x^2 - 3x + 2 = 0$$

Solution:

A term is a single number or a variable, or numbers and variables multiplied together, so there are four terms in the equation, x^2, $3x$, 2, and 0.

4. Identify any coefficient(s) in the expression.

$$2x^2 + bx - c$$

Solution:

Coefficients are numbers attached to variables, or numbers that multiply variables. In this expression, 2 is a coefficient on x^2, and b is a coefficient on x. The constant c is not a coefficient, because it's not multiplying a variable.

5. Which value is the variable representing?

$$x - 7 = 2$$

Solution:

To determine the value of x, we need to figure out which number, when we take away 7, gives a result of 2, so $x = 9$. The value $x = 9$ is the value that makes the equation true.

$$x - 7 = 2$$

$$9 - 7 = 2$$

$$2 = 2$$

6. Which value is the variable representing?

$$y + 3 = 8$$

Solution:

To determine the value of y, we need to figure out which number can be added to 3 to get 8, so $y = 5$. The value $y = 5$ is the value that makes the equation true.

$$y + 3 = 8$$

$$5 + 3 = 8$$

$$8 = 8$$

IDENTIFYING MULTIPLICATION

1. Give three different examples of how we can write "a times b" mathematically.

Solution:

We can write the product of a and b as $a \times b$, $a \cdot b$, $(a)(b)$, or ab.

2. Simplify the expression.

$$5(2 \cdot 3) \times (1)(a)$$

Solution:

When all these values are multiplied together, the order in which we perform the multiplication won't affect the result.

$$5(6) \times (1)(a)$$

$$5(6) \times a$$

$$30 \times a$$

$$30a$$

3. Find the value of the expression.

$$4 \times 3(1)(2 \cdot 1)$$

Solution:

When all these values are multiplied together, the order in which we perform the multiplication won't affect the result.

$$4 \times 3(1)(2)$$

$$4 \times 3(2)$$

$$4 \times 6$$

$$24$$

4. Find the value of the expression.

$$2(4)(3 \cdot 4) \times (5)(2)$$

Solution:

When all these values are multiplied together, the order in which we perform the multiplication won't affect the result.

$$2(4)(12) \times (5)(2)$$

$$8(12) \times (5)(2)$$

$$96 \times (5)(2)$$

$$96 \times 10$$

$$960$$

5. Why do we have different ways to write multiplication?

Solution:

Because the “times” symbol, $\times$, can be confused with x.

6. Simplify the expression.

$$(-3)(2) \times 4 \cdot (-2)(2 \cdot 1)$$

Solution:

When all these values are multiplied together, the order in which we perform the multiplication won’t affect the result.

$$(-6) \times 4 \cdot (-2)(2 \cdot 1)$$

$(-6) \times 4 \cdot (-2)(2)$

$(-24) \cdot (-2)(2)$

$(-24) \cdot (-4)$

96

ASSOCIATIVE PROPERTY

1. Give an example of an expression that demonstrates the Associative Property of Multiplication.

Solution:

There are many correct answers. For example, the following all work:

$$2 \times (11 \times 3) = (2 \times 11) \times 3$$

$$2 \times (11 \times 3) \times 9 = (2 \times 11) \times 3 \times 9 = 2 \times 11 \times (3 \times 9)$$

However, this example does not work:

$$2 \times (11 + 3) = (2 \times 11) + 3$$

Parentheses can't be associated when the parentheses involve both addition and multiplication.

2. Using the Associative Property, rewrite and simplify $2 \times (3 \times 4)$.

Solution:

Move the parentheses to associate a different pair of constants.

$$(2 \times 3) \times 4$$

$$6 \times 4$$

$$24$$

3. According to the Associative Property, what number would make the most sense in the place of the variable?

$$42 + (31 + 17) = (42 + x) + 17$$

Solution:

The Associative Property tells us that we can associate different values in the expression and still keep the value the same.

On the left side of the equation, we're associating 31 and 17, but we can equivalently associate 42 and 31. So the missing value is $x = 31$.

4. Rearrange $(3 + 6) + 2$ using the Associative Property, then simplify.

Solution:

We'll use the Associative Property to associate $6 + 2$ instead of $3 + 6$, then we'll simplify.

$$3 + (6 + 2)$$

$$3 + 8$$

$$11$$

5. Give an example of an expression that demonstrates the Associative Property of Addition.

Solution:

There are many correct answers. For example, the following all work:

$$2 + (11 + 3) = (2 + 11) + 3$$

$$2 + (11 + 3) + 9 = (2 + 11) + 3 + 9 = 2 + 11 + (3 + 9)$$

However, this example does not work:

$$2 + (11 \times 3) = (2 + 11) \times 3$$

Parentheses can't be associated when the parentheses involve both addition and multiplication.

6. According to the Associative Property, what number would make the most sense in the place of the variable?

$$(4 \times 2) \times 9 = x \times (2 \times 9)$$

Solution:

The Associative Property tells us that we can associate different values in the expression and still keep the value the same.

On the left side of the equation, we're associating 4 and 2, but we can equivalently associate 4 and 9. So the missing value is $x = 4$.

COMMUTATIVE PROPERTY

■ 1. Using the Commutative Property, rewrite $6+19$ and then simplify.

Solution:

The Commutative Property lets us swap the order of terms being added.

$$19+6$$

$$25$$

■ 2. Give an example of an expression that demonstrates the Commutative Property of Multiplication.

Solution:

There are many correct answers. For example, the following all work:

$$11\times 3=3\times 11$$

$$2\times(11\times 3)=2\times(3\times 11)=(11\times 3)\times 2$$

However, these examples do not work:

$$3-4=4-3$$

$$3 \div 4 = 4 \div 3$$

The Commutative Property can't be used with subtraction and division.

3. According to the Commutative Property, what's the value of the variable in the equation?

$$11 + (23 + 6) = 11 + (6 + x)$$

Solution:

The Commutative Property tells us that we can change the order of the terms in a sum. In this equation, the order of the terms has changed, but the terms themselves need to stay the same, so the missing value must be $x = 23$.

4. Rearrange $(3 + 6) + 2$ using the Commutative Property and then the Associative Property.

Solution:

The Commutative Property lets us rearrange the order of the terms in the sum,

$$(6 + 3) + 2$$

and then the Associative Property lets us associate different terms in the sum.

$$6 + (3 + 2)$$

5. Give an example of an expression that demonstrates the Commutative Property of Addition.

Solution:

There are many correct answers. For example, the following all work:

$$11 + 3 = 3 + 11$$

$$2 + (11 + 3) = 2 + (3 + 11) = (11 + 3) + 2$$

However, these examples do not work:

$$3 - 4 = 4 - 3$$

$$3 \div 4 = 4 \div 3$$

The Commutative Property can't be used with subtraction and division.

6. According to the Commutative Property, what's the value of the variable in the equation?

$$(4 \times 2) \times 9 = (x \times 9) \times 4$$

Solution:

The Commutative Property tells us that we can change the order of the terms in a product. In this equation, the order of the terms has changed, but the terms themselves need to stay the same, so the missing value must be $x = 2$.

TRANSITIVE PROPERTY

1. If $AB = CD$ and $CD = EF$, what's another way to express EF?

Solution:

Because both AB and EF are equal to the same value, CD, we know they're also equal to each other.

$$EF = AB$$

2. According to the Transitive Property, if $x = 2y$ and $2y = 5z$, what's the value of x?

Solution:

Because both x and $5z$ are equal to the same value, $2y$, we know they're also equal to each other.

$$x = 5z$$

3. Give an example that demonstrates the Transitive Property.

Solution:

There are many correct answers. For example, the following all work:

If $x = 2$ and $y = x$ then $y = 2$.

If $z = x$ and $x = 2y$ then $z = 2y$.

However, this example does not work:

If $x = y$ and $z = 2$ then $x = 2$.

The Transitive Property can't be used when the equation statements aren't related to each another.

4. By the Transitive Property, what value would make the statement true?

If $2 + 3 = x$ and $4 + 1 = 5$, then $2 + 3 = 5$.

Solution:

If the conclusion is that $2 + 3 = 5$, then we need two other statements showing that both $2 + 3$ and 5 are equal to the same value. We're told that 5 is equal to $4 + 1$, which means we need to say that $2 + 3$ is also equal to $4 + 1$.

$$x = 4 + 1$$

5. Use the Transitive Property to write an equation that only includes x variables, without any y or z variables.

$$y = 2x + 3$$

$$y = z$$

$$z = 5x - 9$$

Solution:

We know that $y = 2x + 3$ and $y = z$, so we have two different values that are both equal to y, and we can use the Transitive Property to set those values equal to each other.

$$z = 2x + 3$$

Now we know that $z = 2x + 3$ and $z = 5x - 9$, so we have two different values that are both equal to z, and we can use the Transitive Property to set those values equal to each other.

$$2x + 3 = 5x - 9$$

6. According to the Transitive Property, what expression would make the most sense in the following statement?

If $x = 2y$ and $2y = ??$, then $x = 5z$.

Solution:

If the conclusion is that $x = 5z$, then we need two other statements showing that both x and $5z$ are equal to the same value. We're told that x is equal to $2y$, which means we need to say that $5z$ is also equal to $2y$.

$$?? = 5z$$

UNDERSTOOD 1

1. What happens when we multiply something by 1?

Solution:

Its value stays the same.

2. Simplify the expression.

$$\frac{1x^1}{1(1^1)}+\frac{1}{1(1x)}-1^1$$

Solution:

Remove all the "understood 1s" from the first term,

$$\frac{1x^1}{1(1^1)}+\frac{1}{1(1x)}-1^1$$

$$x+\frac{1}{1(1x)}-1^1$$

then from the second term,

$$x+\frac{1}{x}-1^1$$

and then from the third term.

$$x+\frac{1}{x}-1$$

■ 3. What value of x makes the equation true?

$$1(2^1)-\frac{1}{1(1)^1}+\frac{x^1}{1\times 1}=4$$

Solution:

Remove all the "understood 1s" from the first term,

$$2-\frac{1}{1(1)^1}+\frac{x^1}{1\times 1}=4$$

then from the second term,

$$2-1+\frac{x^1}{1\times 1}=4$$

and then from the third term.

$$2-1+x=4$$

$$1+x=4$$

The only value that x can take is $x=3$, because $1+3=4$.

4. Simplify the expression by removing any “understood 1s.”

$$\frac{x^1}{4x^3}+\frac{5x^4}{1x}$$

Solution:

Remove the “understood 1s.”

$$\frac{x}{4x^3}+\frac{5x^4}{x}$$

5. What happens when we divide something by 1?

Solution:

Its value stays the same.

6. Simplify the expression by removing any “understood 1s.”

$$\frac{x}{1^1}\cdot\frac{x^2+1(1)}{5x^2}$$

Solution:

Remove all the "understood 1s" from the first term,

$$x \cdot \frac{x^2 + 1(1)}{5x^2}$$

then from the second term.

$$x \cdot \frac{x^2 + 1}{5x^2}$$

ADDING AND SUBTRACTING LIKE TERMS

■ 1. Give an example of like terms that can added.

Solution:

There are many correct answers. Some examples include

$3x^2$ and $-5x^2$

$-x^3$ and $2x^3$

We can’t add terms unless they have the same base and same exponent. The following are pairs of terms that can’t be added.

x^2 and x^4

t^2 and y^2

■ 2. Simplify the expression.

$$-x + 6x - 8x + 3x$$

Solution:

All of these terms have the same exponent, so we add the coefficients.

$$(-1 + 6 - 8 + 3)x$$

$$0x$$

$$0$$

3. What stays the same when adding or subtracting like terms?

Solution:

The exponent and the base stay the same when adding and subtracting like terms, while the coefficient changes.

4. Simplify the expression.

$$x + 2x^2 - y - 5x^2 + 7y - 4x$$

Solution:

We start by grouping like terms,

$$(x - 4x) + (2x^2 - 5x^2) + (-y + 7y)$$

then we simplify the coefficients.

$$(1 - 4)x + (2 - 5)x^2 + (-1 + 7)y$$

$$-3x - 3x^2 + 6y$$

5. Simplify the expression.

$$\frac{1}{3}x - 5x^2 + \frac{1}{2}x^2 - x - y$$

Solution:

We start by grouping like terms, then we simplify the coefficients.

$$\left(\frac{1}{3}x - x\right) + \left(-5x^2 + \frac{1}{2}x^2\right) - y$$

$$\left(\frac{1}{3} - 1\right)x + \left(-5 + \frac{1}{2}\right)x^2 - y$$

$$-\frac{2}{3}x - \frac{9}{2}x^2 - y$$

6. Simplify the expression.

$$2a^2b - 5ab - 3ab^2 + a^2b + 4ab$$

Solution:

We start by grouping like terms, then we simplify the coefficients.

$$(2a^2b + a^2b) + (-5ab + 4ab) - 3ab^2$$

$$(2 + 1)a^2b + (-5 + 4)ab - 3ab^2$$

$$3a^2b - ab - 3ab^2$$

MULTIPLYING AND DIVIDING LIKE TERMS

1. Simplify the expression.

$$\frac{3x^2}{x^3}$$

Solution:

Cancel the x^2 in the numerator with the x^3 in the denominator.

$$\frac{3x^2}{x^2(x)}$$

$$\frac{3}{x}$$

2. Simplify the expression.

$$2a^2 \cdot 6b^3 \cdot ab^2$$

Solution:

Using rules of exponents, we can write the product as

$$2a^2 \cdot 6b^3 \cdot ab^2$$

$$12a^2b^3 \cdot ab^2$$

$$12a^{2+1}b^{3+2}$$

$$12a^3b^5$$

■ 3. Simplify the expression.

$$\frac{6x^a}{3x^b}$$

Solution:

Using rules of exponents, we can write the quotient as

$$\frac{6}{3}x^{a-b}$$

$$2x^{a-b}$$

■ 4. Simplify the expression.

$$3x^a \cdot 5x^b$$

Solution:

Using rules of exponents, we can write the product as

$$3 \cdot 5x^{a+b}$$

$$15x^{a+b}$$

■ 5. Simplify the expression.

$$\frac{5y^2 \cdot 4x^3 \cdot 2xy}{x^2y}$$

Solution:

Using rules of exponents, we can write the product in the numerator as

$$\frac{5y^2 \cdot 4x^3 \cdot 2xy}{x^2y}$$

$$\frac{20x^3y^2 \cdot 2xy}{x^2y}$$

$$\frac{40x^{3+1}y^{2+1}}{x^2y}$$

$$\frac{40x^4y^3}{x^2y}$$

Using rules of exponents, we can write the quotient as

$$40x^{4-2}y^{3-1}$$

$$40x^2y^2$$

6. Simplify the expression.

$$\frac{2y^2 \cdot 3x^3y \cdot x^2y^2}{x^4y^2}$$

Solution:

Using rules of exponents, we can write the product in the numerator as

$$\frac{2y^2 \cdot 3x^3y \cdot x^2y^2}{x^4y^2}$$

$$\frac{6x^3y^3 \cdot x^2y^2}{x^4y^2}$$

$$\frac{6x^5y^5}{x^4y^2}$$

Using rules of exponents, we can write the quotient as

$$6x^{5-4}y^{5-2}$$

$$6xy^3$$

DISTRIBUTIVE PROPERTY

1. Use the Distributive Property to simplify the expression.

$$5(x-2)+\frac{1}{2}(6-2x)$$

Solution:

Distribute the coefficients in front of the parentheses across the terms inside the parentheses.

$$5(x)-5(2)+\frac{1}{2}(6)-\frac{1}{2}(2x)$$

$$5x-10+3-x$$

$$4x-7$$

2. Use the Distributive Property to expand the expression.

$$-\frac{2}{5}(10-5x)$$

Solution:

Distribute the coefficient in front of the parentheses across the terms inside the parentheses.

$$-\frac{2}{5}(10)+\frac{2}{5}(5x)$$

$$-4+2x$$

■ 3. Give an example that demonstrates the Distributive Property with subtraction.

Solution:

There are many correct answers. For example, the following all work:

$$2(x-1)=2x-2$$

$$-\frac{1}{3}(9-2x)=-3+\frac{2}{3}x$$

However, this example does not work:

$$2(3-2x)=6-2x$$

The Distributive Property states that we have to multiply the coefficient outside of the parentheses by each term inside the parentheses.

■ 4. Which three main operations are used in the Distributive Property?

Solution:

Multiplication, addition, and subtraction.

■ 5. Use the Distributive Property to simplify the expression.

$$2(5-3x)-2(x-4)$$

Solution:

Distribute the coefficients in front of the parentheses across the terms inside the parentheses.

$$2(5)-2(3x)-2(x)-2(-4)$$

$$10-6x-2x+8$$

$$18-8x$$

■ 6. What value would make the following equation true?

$$2(x+3)=\ ??+6$$

Solution:

If we expand the left side, we get

$$2(x) + 2(3) = ?? + 6$$

$$2x + 6 = ?? + 6$$

We can see that the missing value has to be $2x$.

DISTRIBUTIVE PROPERTY WITH FRACTIONS

1. Use the Distributive Property to expand the expression.

$$-\frac{x^2z}{y^3}\left(\frac{y^2}{2}-\frac{xz^3}{z^2}\right)$$

Solution:

Distribute the coefficient fraction in front of the parentheses across the fractions inside the parentheses.

$$-\frac{x^2z}{y^3}\left(\frac{y^2}{2}\right)-\frac{x^2z}{y^3}\left(-\frac{xz^3}{z^2}\right)$$

$$-\frac{(x^2z)(y^2)}{(y^3)(2)}+\frac{(x^2z)(xz^3)}{(y^3)(z^2)}$$

$$-\frac{x^2y^2z}{2y^3}+\frac{x^3z^4}{y^3z^2}$$

Cancel common factors within each fraction.

$$-\frac{x^2z}{2y}+\frac{x^3z^2}{y^3}$$

2. Fill in the blanks.

"When we're distributing fractions, we multiply the numerator of the coefficient by the ___________ of the terms inside the parentheses, and we multiply the denominator of the coefficient by the ___________ of the terms inside the parentheses."

Solution:

numerators, denominators

3. Use the Distributive Property to expand the expression.

$$\frac{2}{3}\left(\frac{x}{2}-6\right)$$

Solution:

Distribute the coefficient fraction in front of the parentheses across the terms inside the parentheses.

$$\frac{2}{3}\left(\frac{x}{2}\right)-\frac{2}{3}(6)$$

$$\frac{2x}{6}-\frac{12}{3}$$

Cancel common factors from each fraction.

$$\frac{x}{3} - 4$$

■ 4. Explain why the two sides of the equation aren't equal to one another.

$$\frac{3}{2}\left(\frac{x}{5} - \frac{y}{2}\right) \neq \frac{3x}{10} - \frac{y}{2}$$

Solution:

The coefficient fraction was only distributed to the first fraction inside the parentheses, not to both, as it should have been. We should have seen

$$\frac{3}{2}\left(\frac{x}{5}\right) - \frac{3}{2}\left(\frac{y}{2}\right) = \frac{3x}{10} - \frac{3y}{4}$$

$$\frac{3x}{10} - \frac{3y}{4} \neq \frac{3x}{10} - \frac{y}{2}$$

■ 5. What missing value would make the equation true?

$$\frac{2ab}{c^2}\left(\frac{3ac}{b} + a^2c^2\right) = \frac{6a^2}{c} + ??$$

Solution:

Simplify the left side by distributing the $2ab/c^2$ across the terms inside the parentheses.

$$\frac{2ab}{c^2}\left(\frac{3ac}{b}\right)+\frac{2ab}{c^2}(a^2c^2)=\frac{6a^2}{c}+??$$

$$\frac{(2ab)(3ac)}{(c^2)(b)}+\frac{(2ab)(a^2c^2)}{c^2}=\frac{6a^2}{c}+??$$

$$\frac{6a^2bc}{bc^2}+\frac{2a^3bc^2}{c^2}=\frac{6a^2}{c}+??$$

Simplify within each fraction by dividing like terms.

$$\frac{6a^2}{c}+\frac{2a^3b}{1}=\frac{6a^2}{c}+??$$

$$\frac{6a^2}{c}+2a^3b=\frac{6a^2}{c}+??$$

Now that we have matching $6a^2/c$ terms on each side, we can see that the missing term is $2a^3b$.

6. Use the Distributive Property to show that the equation is true.

$$\frac{x^2}{3z}\left(\frac{2x}{z}+y^2\right)=\frac{2x^3}{3z^2}+\frac{x^2y^2}{3z}$$

Solution:

Simplify the left side by distributing the $x^2/3z$ across the terms inside the parentheses.

$$\frac{x^2}{3z}\left(\frac{2x}{z}+y^2\right)=\frac{x^2}{3z}\left(\frac{2x}{z}\right)+\frac{x^2}{3z}(y^2)$$

$$\frac{x^2}{3z}\left(\frac{2x}{z}+y^2\right)=\frac{(x^2)(2x)}{(3z)(z)}+\frac{(x^2)(y^2)}{(3z)(1)}$$

$$\frac{x^2}{3z}\left(\frac{2x}{z}+y^2\right)=\frac{2x^3}{3z^2}+\frac{x^2y^2}{3z}$$

This was the original equation we were asked to prove, and we've shown that the left and right sides are equivalent.

$$\frac{2x^3}{3z^2}+\frac{x^2y^2}{3z}=\frac{2x^3}{3z^2}+\frac{x^2y^2}{3z}$$

PEMDAS AND ORDER OF OPERATIONS

1. Simplify the expression.

$$\sqrt{2(5-3)}-|3[6-7]|$$

Solution:

Simplify the innermost parentheses first.

$$\sqrt{2(2)}-|3[-1]|$$

$$\sqrt{4}-|-3|$$

$$\sqrt{4}-3$$

Apply the root (which is like an exponent).

$$2-3$$

$$-1$$

2. Using PEMDAS, evaluate each expression separately to show that they are not equal.

$$4\times(3-1)-(4\div 2+2)$$

$$(4\times 3-1)-4\div(2+2)$$

Solution:

The first expression simplifies as

$$4 \times (3 - 1) - (4 \div 2 + 2)$$

$$4 \times 2 - (2 + 2)$$

$$4 \times 2 - 4$$

$$8 - 4$$

$$4$$

and the second expression simplifies as

$$(4 \times 3 - 1) - 4 \div (2 + 2)$$

$$(12 - 1) - 4 \div 4$$

$$11 - 4 \div 4$$

$$11 - 1$$

$$10$$

■ 3. Use order of operations to simplify the expression.

$$(10 - [(-1)^2 + 1 - 6 \div 6])^{1/2} + 4 \div 2$$

Solution:

Start by simplifying the innermost parentheses.

$$(10 - [(-1)^2 + 1 - 6 \div 6])^{1/2} + 4 \div 2$$

$$(10 - [1 + 1 - 6 \div 6])^{1/2} + 4 \div 2$$

$$(10 - [1 + 1 - 1])^{1/2} + 4 \div 2$$

$$(10 - 1)^{1/2} + 4 \div 2$$

$$9^{1/2} + 4 \div 2$$

Apply the exponent.

$$3 + 4 \div 2$$

Perform multiplication and division from left to right, then addition and subtraction from left to right.

$$3 + 2$$

$$5$$

■ 4. Use order of operations to simplify the expression.

$$3 - [(-2)^2x + (3 - 7)]$$

Solution:

Start by simplifying the innermost parentheses.

$$3 - [(-2)^2x + (-4)]$$

$$3 - [(-2)^2x - 4]$$

Apply the exponent.

$$3 - [4x - 4]$$

Distribute the -1 across the brackets, then perform addition and subtraction from left to right on like terms.

$$3 - 4x + 4$$

$$7 - 4x$$

5. Using order of operations, explain why $9 + 6 \div 3 \neq 5$.

Solution:

Order of operations tells us that we have to perform division before addition, so the expression would simplify as

$$9 + 6 \div 3$$

$$9 + 2$$

$$11$$

6. Use order of operations to simplify the expression.

$$\frac{-2+3-10\cdot 2\cdot[(5-4)+2]}{2}$$

Solution:

Start by simplifying the innermost parentheses.

$$\frac{-2+3-10\cdot 2\cdot[(5-4)+2]}{2}$$

$$\frac{-2+3-10\cdot 2\cdot[1+2]}{2}$$

$$\frac{-2+3-10\cdot 2\cdot 3}{2}$$

There are no exponents, so perform multiplication and division from left to right within the numerator.

$$\frac{-2+3-20\cdot 3}{2}$$

$$\frac{-2+3-60}{2}$$

Perform addition and subtraction from left to right within the numerator.

$$\frac{1-60}{2}$$

$$\frac{-59}{2}$$

$$-\frac{59}{2}$$

EVALUATING EXPRESSIONS

1. Explain what went wrong in the following statement?

If $x^2 - x + 1$ when $x = -2$, then $-2^2 - -2 + 1 = -4 + 2 + 1 = -1$.

Solution:

There were no parentheses used when plugging in $x = -2$, so the expression -2^2 was evaluated incorrectly. It should be $(-2)^2 - (-2) + 1 = 4 + 2 + 1 = 7$.

2. What does it mean to "evaluate an expression"?

Solution:

It means to replace (or plug in) a number for the given variable, and then simplify using PEMDAS until we've reached the simplest possible value.

3. Find the value of $y - 2z - 1$ when $y = 4$ and $z = -3$.

Solution:

Substitute $y = 4$ and $z = -3$ into the expression.

$$y - 2z - 1$$

$$4 - 2(-3) - 1$$

$$4 + 6 - 1$$

$$10 - 1$$

$$9$$

■ 4. Evaluate the expression when $a = 1$, $b = -3$, and $c = -4$.

$$\frac{\sqrt{b^2 - 4ac}}{2a}$$

Solution:

Substitute $a = 1$, $b = -3$, and $c = -4$ into the expression.

$$\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\frac{\sqrt{(-3)^2 - 4(1)(-4)}}{2(1)}$$

$$\frac{\sqrt{9 + 16}}{2}$$

$$\frac{5}{2}$$

5. Show that $x = -4$ by plugging it into the equation.

$$x^2 - 4 = -3x$$

Solution:

Substituting $x = -4$ into the equation gives

$$x^2 - 4 = -3x$$

$$(-4)^2 - 4 = -3(-4)$$

$$16 - 4 = 12$$

$$12 = 12$$

Because we find that both sides are equal, we know it must be true that $x = -4$.

6. Evaluate the expression when $a = -1$, $b = -2$, and $c = -3/2$.

$$\frac{5a + 1}{3 - 2b + 4c^2a}$$

Solution:

Substitute $a = -1$, $b = -2$, and $c = -3/2$ into the expression.

$$\frac{5a+1}{3-2b+4c^2a}$$

$$\frac{5(-1)+1}{3-2(-2)+4\left(-\frac{3}{2}\right)^2(-1)}$$

$$\frac{-5+1}{3+4-4\left(\frac{9}{4}\right)}$$

$$\frac{-4}{7-9}$$

$$\frac{-4}{-2}$$

$$2$$

INVERSE OPERATIONS

1. Use inverse operations to figure out what should replace the "?" in order to make the equation true.

$$5x \ ? = x$$

Solution:

Because 5 is being multiplied by x, we need to undo that operation by dividing by 5.

$$5x \div 5 = x$$

So the $?$ should be replaced with "$\div 5$".

2. What is the inverse operation of division?

Solution:

Multiplication

3. Using both division and multiplication, find two values that can replace the "?" in order to make the equation true.

$$\frac{1}{5}x \ ? = x$$

Solution:

We could say that x is being multiplied by $1/5$, so we could replace the $?$ with division by $1/5$, and the equation would be true.

$$\frac{1}{5}x \div \frac{1}{5} = x$$

But we could also say that x is being divided by 5, so we could replace the $?$ with multiplication by 5, and the equation would be true.

$$\frac{1}{5}x \cdot 5 = x$$

■ 4. What value of the missing exponent would make the equation true?

$$(x^3)^? = x$$

Solution:

Because x is being raised to the power of 3, we need to raise the result to $1/3$ in order to undo the exponent and get back to x. So the $?$ should be replaced with $1/3$.

5. Put an expression in place of the question mark that would make the equation true.

$$\frac{1}{7} ? = 1$$

Solution:

In this example, 1 is being divided by 7. To undo that operation, we need to multiply by 7.

$$\frac{1}{7} \cdot 7 = 1$$

$$1 = 1$$

6. Use inverse operations to find a value to replace the "?" that will make the equation true.

$$(\sqrt[4]{a+b})^{?} = a + b$$

Solution:

To undo a root, we need an exponent. Since we're taking the fourth root of $a + b$, we need to undo that operation by raising the result to the fourth power. Therefore, the $?$ should be replaced with the exponent 4.

SIMPLE EQUATIONS

■ 1. Solve the equation for x.

$$2x - 5 = 11$$

Solution:

Add 5 to both sides.

$$2x - 5 = 11$$

$$2x - 5 + 5 = 11 + 5$$

$$2x = 16$$

Divide both sides by 2 to get x by itself.

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

■ 2. If $x = 16$, what value of the "$??$" would make the equation true?

$$x - ?? = 11$$

Solution:

Substitute $x = 16$ into the equation.

$$x-?? = 11$$

$$16-?? = 11$$

Subtract 16 from both sides.

$$16-?? - 16 = 11 - 16$$

Divide both sides by -1.

$$-?? = -5$$

$$\frac{-??}{-1} = \frac{-5}{-1}$$

$$?? = 5$$

■ 3. Solve the equation for x.

$$\frac{x+1}{3} = 7$$

Solution:

Multiply both sides of the equation by 3.

$$\frac{x+1}{3} = 7$$

$$\frac{x+1}{3} \cdot 3 = 7 \cdot 3$$

$$x + 1 = 21$$

Subtract 1 from both sides.

$$x + 1 - 1 = 21 - 1$$

$$x = 20$$

■ 4. What went wrong in this set of steps?

$$2x - 11 = -3$$

$$2x = 8$$

$$x = 16$$

Solution:

The 2 was multiplied by both sides instead of divided. The steps should have been

$$2x - 11 = -3$$

$$2x - 11 + 11 = -3 + 11$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

5. What went wrong in this set of steps?

$$2 - \frac{1}{3}x = 1$$

$$-\frac{1}{3}x = 3$$

$$x = -9$$

Solution:

The 2 was subtracted from the left side and added to the right side, instead of subtracted from both sides. The inverse operation of addition is subtraction, so the steps should have been

$$2 - \frac{1}{3}x = 1$$

$$2 - \frac{1}{3}x - 2 = 1 - 2$$

$$-\frac{1}{3}x = -1$$

$$-\frac{1}{3}x(-3) = -1(-3)$$

$$x = 3$$

6. Solve the equation for x.

$$\frac{1}{4}x + 3 = 5$$

Solution:

Subtract 3 from both sides,

$$\frac{1}{4}x + 3 = 5$$

$$\frac{1}{4}x + 3 - 3 = 5 - 3$$

$$\frac{1}{4}x = 2$$

then multiply both sides by 4.

$$\frac{1}{4}x(4) = 2(4)$$

$$x = 8$$

BALANCING EQUATIONS

1. Solve the equation for x.

$$2(-3x+5)-1=-3(1-5x)$$

Solution:

Distribute the 2 and the -3 across the parentheses.

$$2(-3x+5)-1=-3(1-5x)$$

$$-6x+10-1=15x-3$$

$$-6x+9=15x-3$$

Add $6x$ to both sides,

$$9=21x-3$$

then add 3 to both sides,

$$12=21x$$

and divide both sides by 21.

$$\frac{12}{21}=x$$

2. Solve the equation for x.

$$x - 2(1 - x) + 5 = 3(2x + 4) - 6$$

Solution:

Distribute the -2 and the 3 across the parentheses.

$$x - 2(1 - x) + 5 = 3(2x + 4) - 6$$

$$x - 2 + 2x + 5 = 6x + 12 - 6$$

$$3x + 3 = 6x + 6$$

Subtract $3x$ from both sides,

$$3 = 3x + 6$$

subtract 6 from both sides,

$$-3 = 3x$$

then divide both sides by 3.

$$-1 = x$$

3. If $x = -2$, solve for y.

$$3x + 2y - 7 = 1 - 5x - y$$

Solution:

Substitute $x = -2$ into the equation.

$$3x + 2y - 7 = 1 - 5x - y$$

$$3(-2) + 2y - 7 = 1 - 5(-2) - y$$

$$-6 + 2y - 7 = 1 + 10 - y$$

$$-13 + 2y = 11 - y$$

Add y to both sides,

$$-13 + 3y = 11$$

add 13 to both sides,

$$3y = 24$$

then divide both sides by 3.

$$y = 8$$

■ 4. Solve for a.

$$7(4a - 3) = -(6a - 5) + 8$$

Solution:

First, we'll use the Distributive Property.

$$7(4a-3)=-(6a-5)+8$$

$$7(4a)+7(-3)=-(6a)-(-5)+8$$

$$28a-21=-6a+5+8$$

$$28a-21=-6a+13$$

Add $6a$ to both sides,

$$28a+6a-21=-6a+6a+13$$

$$34a-21=13$$

add 21 to both sides,

$$34a-21+21=13+21$$

$$34a=34$$

then divide both sides by 34.

$$a=1$$

5. Solve for a.

$$-2(1-a)+3(a+7)=-2$$

Solution:

Apply the Distributive Property,

$$-2(1-a)+3(a+7)=-2$$

$$-2+2a+3a+21=-2$$

$$5a+19=-2$$

subtract 19 from both sides,

$$5a=-21$$

then divide both sides by 5.

$$a=-\frac{21}{5}$$

6. What missing number should replace the "??" in order to make the equation true?

$$-3(x-5)=2x-(3-x)$$

$$??x+15=3x-3$$

Solution:

Apply the Distributive Property,

$$-3(x-5)=2x-(3-x)$$

$$-3x+15=2x-3+x$$

$$-3x+15=3x-3$$

This matches the form of the equation with the missing value. Comparing the two equations, we can see that $??$ should be replaced by -3.

EQUATIONS WITH SUBSCRIPTS

1. It takes Peter 6 hours to paint a room and Laura 8 hours to paint that same room. Use the equation below to determine how long it would take for Peter and Laura to paint the room together, where R_1 is the number of hours it takes Peter, R_2 is the number of hours it takes Laura, and T is the number of hours it takes them together.

$$\frac{R_1R_2}{R_1+R_2}=T$$

Solution:

Substituting the values we know into the formula, the time it takes them to paint the room together is

$$\frac{(6)(8)}{6+8}=T$$

$$\frac{48}{14}=T$$

$T=3.43$ hrs

2. Solve the equation for P_2.

$$P_1R+\frac{P_2}{V}=d$$

Solution:

Subtract P_1R from both sides of the equation.

$$P_1R + \frac{P_2}{V} = d$$

$$\frac{P_2}{V} = d - P_1R$$

Multiply both sides by V to get P_2 by itself.

$$P_2 = V(d - P_1R)$$

■ 3. The profit function for a company is given by $P = Rx - C_1 - C_2x$, where P is the profit, R is the selling price of their product, C_1 is the company's fixed cost, C_2 is their variable cost, and x is the total number of products sold. What is the selling price R when $P = 114$, $C_1 = 550$, $C_2 = 3.50$, and $x = 16$?

Solution:

Plugging all the values we've been given into the profit function, we get

$$P = Rx - C_1 - C_2x$$

$$114 = R(16) - 550 - 3.50(16)$$

$$114 = 16R - 550 - 56$$

$$114 = 16R - 606$$

$$720 = 16R$$

$$R = 45$$

■ 4. Solve the equation for x_1.

$$\frac{3V}{x_1} = td_0 + 2x_2d_1$$

Solution:

Multiply both sides of the equation by x_1,

$$\frac{3V}{x_1} = td_0 + 2x_2d_1$$

$$3V = x_1(td_0 + 2x_2d_1)$$

then divide through by $td_0 + 2x_2d_1$ to get x_1 by itself.

$$\frac{3V}{td_0 + 2x_2d_1} = x_1$$

■ 5. Solve the equation for Y_2 when $t_1 = 2$, $t_2 = 11$, $D = 1/3$, and $Y_1 = 25$.

$$3t_1 + \frac{15t_2D}{Y_2} = Y_1 - 5$$

Solution:

Plugging all the values we've been given into the equation, we get

$$3t_1 + \frac{15t_2D}{Y_2} = Y_1 - 5$$

$$3(2) + \frac{15(11)(1/3)}{Y_2} = 25 - 5$$

$$6 + \frac{5(11)}{Y_2} = 20$$

$$6 + \frac{55}{Y_2} = 20$$

$$\frac{55}{Y_2} = 14$$

Now multiply both sides by Y_2 and divide by 14 in order to solve for Y_2.

$$55 = 14Y_2$$

$$Y_2 = \frac{55}{14}$$

6. The volume of the medium size box at the post office is given by $V = d_1 \times d_2 \times d_3$, where d_1, d_2, and d_3 are the length, width, and height, respectively. Given $d_1 = 4$ and $d_2 = 5$, find the relationship between volume and height.

Solution:

Plugging everything we've been given into the volume equation, we get

$$V = d_1 \times d_2 \times d_3$$

$$V = 4 \times 5 \times d_3$$

$$V = 20d_3$$

This new equation gives the relationship between volume V and height d_3, and tells us that the volume of the medium size box will always be equivalent to twenty times the height of the box.

WORD PROBLEMS INTO EQUATIONS

■ 1. Write the phrase as an algebraic expression.

Six more than three times a number

Solution:

"Three times a number" can be written as $3x$, so "six more than" that value must be $3x + 6$.

■ 2. Find the value of the expression.

The quotient of 150 and 5

Solution:

The quotient of two values is the division of the first number by the second number, so the quotient is

$$\frac{150}{5}$$

$$30$$

3. Write the phrase as an algebraic expression.

Half of five times a number

Solution:

"Five times a number" can be written as $5x$, so half of that value must be

$$\frac{1}{2}(5x)$$

$$\frac{5x}{2}$$

4. Find the value of the expression.

3 less than the product of 2 and 7

Solution:

The "product of 2 and 7" is 2×7, and three less than that value is

$$(2 \times 7) - 3$$

$$14 - 3$$

$$11$$

5. Find the value of the expression.

$$\frac{1}{3} \text{ of } 2 \text{ more than } 7$$

Solution:

"2 more than 7" is the sum $7+2$, and $1/3$ of that value is

$$\frac{1}{3}(7+2)$$

$$\frac{1}{3}(9)$$

$$\frac{9}{3}$$

6. David's age is five more than twice Jane's age. If Jane is 6, how old is David?

Solution:

We have two quantities: Jane's age and David's age. So we'll say that Jane's age is J and that David's age is D. "Twice Jane's age" is $2J$, and "five more than twice Jane's age" is therefore $2J+5$. This is David's age, so

$$D = 2J + 5$$

Jane is 6, so David's age must be

$$D = 2(6) + 5$$

$$D = 12 + 5$$

$$D = 17$$

CONSECUTIVE INTEGERS

1. Write the next five consecutive integers following -4.

Solution:

The five consecutive integers following -4 are

$$-3,\ -2,\ -1,\ 0,\ 1$$

2. Give an example of three consecutive negative integers.

Solution:

There are many correct answers. Some examples include

$$-11,\ -10,\ -9$$

$$-23,\ -22,\ -21$$

$$-3,\ -2,\ -1$$

But $-5,\ -3,\ -1$ is not an example, because those integers are not one unit apart from each other.

3. Write the four consecutive integers that precede -3.

Solution:

The four consecutive integers preceding -3 are

$$-7, \ -6, \ -5, \ -4$$

4. Find three consecutive integers that sum to 60.

Solution:

Since the sum of the three consecutive integers is 60, we can write

$$n + (n + 1) + (n + 2) = 60$$

$$n + n + 1 + n + 2 = 60$$

$$3n + 3 = 60$$

$$3n = 57$$

$$n = 19$$

Because the integers are n, $n + 1$, and $n + 2$, and the first integer is $n = 19$, the other two consecutive integers are

$$n + 1 = 19 + 1 = 20$$

$$n + 2 = 19 + 2 = 21$$

and therefore the three consecutive integers that sum to 60 are 19, 20, and 21.

5. Find three consecutive odd integers that sum to 21.

Solution:

We can model three consecutive odd integers as n, $n + 2$, and $n = 4$. Since the sum of the three consecutive odd integers is 21, we have

$$n + (n + 2) + (n + 4) = 21$$

$$n + n + 2 + n + 4 = 21$$

$$3n + 6 = 21$$

$$3n = 15$$

$$n = 5$$

Because the integers are n, $n + 2$, and $n + 4$, and the first integer is $n = 5$, the other two consecutive odd integers are

$$n + 2 = 5 + 2 = 7$$

$$n + 4 = 5 + 4 = 9$$

and therefore the three consecutive odd integers that sum to 21 are 5, 7, and 9.

6. If, given three consecutive integers, the third integer is 10 more than the sum of the first two integers, what is the third integer?

Solution:

Since the three integers here are consecutive, we can identify them as x, $x+1$, and $x+2$. The sum of the first two integers is

$$x+(x+1)$$

$$2x+1$$

The third integer is 10 more than the sum of the first two, which we can express as

$$2x+1+10$$

$$2x+11$$

And the third integer we identified as $x+2$, so we now have two ways of expressing the value of the third integer, and we can set them equal to each other.

$$2x+11=x+2$$

$$x+11=2$$

$$x = -9$$

The second and third integers are therefore

$$x + 1 = -9 + 1 = -8$$

$$x + 2 = -9 + 2 = -7$$

So those integers are -9, -8, -7, and the value of the third integer is -7.

ADDING AND SUBTRACTING POLYNOMIALS

■ 1. Which part(s) of the terms stay the same when we add or subtract like terms?

Solution:

Both the base and the exponent stay the same when we add or subtract like terms. Only the coefficient changes.

■ 2. Simplify the expression.

$$(2x^3 - 5x^2 + x - 3) - (x^2 - 2x + 7)$$

Solution:

Distribute the subtraction across the second set of parentheses.

$$(2x^3 - 5x^2 + x - 3) - (x^2 - 2x + 7)$$

$$2x^3 - 5x^2 + x - 3 - x^2 + 2x - 7$$

Combine like terms.

$$2x^3 - 6x^2 + 3x - 10$$

3. What went wrong in this set of steps?

$$6x^3 + 7 + x^2$$

$$7x^3 + 7$$

Solution:

The terms $6x^3$ and x^2 were added together but they aren't like terms. The exponents aren't the same, so they can't be added together.

4. Simplify the expression.

$$(10a^2b + 3ab^2 - ab) + (2ab^2 - a^2b + ab)$$

Solution:

Simplifying the expression by combining like terms.

$$(10a^2b + 3ab^2 - ab) + (2ab^2 - a^2b + ab)$$

$$10a^2b + 3ab^2 - ab + 2ab^2 - a^2b + ab$$

$$9a^2b + 5ab^2$$

5. Simplify the expression.

$$(x^4 - 5y^3 + z - xy) - (2y^4 + 6xy - z + x^4)$$

Solution:

Distribute the subtraction across the second set of parentheses.

$$(x^4 - 5y^3 + z - xy) - (2y^4 + 6xy - z + x^4)$$

$$x^4 - 5y^3 + z - xy - 2y^4 - 6xy + z - x^4$$

Combine like terms.

$$-5y^3 + 2z - 7xy - 2y^4$$

$$-2y^4 - 5y^3 - 7xy + 2z$$

6. What went wrong in this set of steps?

$$9 - x^3 + 3 + 4x^3$$

$$12 + 3x^6$$

Solution:

The terms $-x^3$ and $4x^3$ were added together. They're like terms, so we do want to add them, but when the terms were added, the exponents were added as well. The sum should be $3x^3$, not $3x^6$.

MULTIPLYING POLYNOMIALS

1. Use the Distributive Property to expand the expression.

$$\frac{1}{2}(6x+4)(x-1)$$

Solution:

Distribute the $1/2$ across the $(6x+4)$.

$$\frac{1}{2}(6x+4)(x-1)$$

$$\left(\frac{1}{2}(6x)+\frac{1}{2}(4)\right)(x-1)$$

$$(3x+2)(x-1)$$

Use FOIL to multiply the binomials.

$$(3x)(x)+(3x)(-1)+(2)(x)+(2)(-1)$$

$$3x^2-3x+2x-2$$

$$3x^2-x-2$$

2. What should we put in place of the "??" to make the expression true?

$$(2x+1)(5-x) = ?? + 10x - x + 5$$

Solution:

Use FOIL to expand the left side of the equation.

$$(2x+1)(5-x)$$

$$(2x)(5) + (2x)(-x) + (1)(5) + (1)(-x)$$

$$10x - 2x^2 + 5 - x$$

Matching this expanded left side to the form of the right side, $?? + 10x - x + 5$, we can see that the missing value is $-2x^2$.

■ 3. What went wrong in this set of steps?

$$(a-2)^2$$

$$a^2 - 4$$

Solution:

The expression was not interpreted correctly, because the exponent was distributed to both terms directly, when it should have been expanded as

$$(a-2)^2$$

$$(a-2)(a-2)$$

Then FOIL should have been used to expand it.

$$a^2-2a-2a+4$$

$$a^2-4a+4$$

■ 4. Use the Distributive Property to expand the expression.

$$4(2-x)(3+2x)$$

Solution:

Use FOIL to multiply the binomials,

$$4(2-x)(3+2x)$$

$$4(6+4x-3x-2x^2)$$

$$4(6+x-2x^2)$$

then use the Distributive Property to distribute the 4 across the parentheses.

$$24+4x-8x^2$$

■ 5. Fill in the blank.

$$(3 - a)(5 + a) = 15 + \_\_\_ - a^2$$

Solution:

If we FOIL the product on the left, we get

$$(3 - a)(5 + a)$$

$$15 + 3a - 5a - a^2$$

$$15 - 2a - a^2$$

Comparing this to the right side of the original equation, the value that goes in the blank must be $-2a$.

6. Expand the expression.

$$(x^2 - 3)(2 - x)$$

Solution:

The expression is expanded and simplified as

$$(x^2 - 3)(2 - x)$$

$$2x^2 - x^3 - 6 + 3x$$

$$-x^3 + 2x^2 + 3x - 6$$

DIVIDING POLYNOMIALS

■ 1. Simplify the expression using polynomial long division.

$$(3x^3 - x^2 + 5) \div (x + 2)$$

Solution:

Using polynomial long division, we get

$$
\begin{array}{r|l}
 & 3x^2 - 7x + 14 \\
\hline
x+2 & 3x^3 - x^2 + 0x + 5 \\
 & \underline{-(3x^3 + 6x^2)} \\
 & \quad -7x^2 + 0x \\
 & \quad \underline{-(-7x^2 - 14x)} \\
 & \qquad 14x + 5 \\
 & \qquad \underline{-(14x + 28)} \\
 & \qquad\qquad -23
\end{array}
$$

Therefore, the solution is

$$3x^2 - 7x + 14 - \frac{23}{x+2}$$

■ 2. What went wrong in setting up the long division problem?

$(5x^4 - 3x^2 + x - 2) \div (x^2 + 1)$

$$5x^4 - 3x^2 + x - 2 \,\overline{\big)\; x^2 + 1}$$

Solution:

The dividend and divisor were placed incorrectly. We should have set up the division problem as

$$x^2 + 1 \,\overline{\big)\; 5x^4 - 3x^2 + x - 2}$$

3. Express the full solution of the polynomial long division.

$$
\begin{array}{r}
3x \quad -1 \\
x^2 - 3 \,\overline{\big)\; 3x^3 - x^2 + x - 5} \\
-(3x^3 + 0x^2 - 9x) \quad \\
\hline
-x^2 + 10x - 5 \\
-(-x^2 + 0x + 3) \\
\hline
10x - 8
\end{array}
$$

Solution:

The solution should be written as

$$\text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

$$3x - 1 + \frac{10x - 8}{x^2 - 3}$$

4. Simplify the expression using polynomial long division.

$$(2x^5 - 3x^3 + x^2 + 4x - 1) \div (x^2 + 2)$$

Solution:

Using polynomial long division, we get

$$
\begin{array}{r l}
 & 2x^3 \quad - 7x \quad + 1 \\
\cline{2-2}
x^2 + 2 \big) & 2x^5 - 3x^3 + x^2 + 4x - 1 \\
 & -(2x^5 + 4x^3) \\
\cline{2-2}
 & -7x^3 + x^2 + 4x \\
 & -(-7x^3 + 0x^2 - 14x) \\
\cline{2-2}
 & x^2 + 18x - 1 \\
 & -(x^2 + 0x + 2) \\
\cline{2-2}
 & 18x - 3
\end{array}
$$

Therefore, the solution is

$$2x^3 - 7x + 1 + \frac{18x - 3}{x^2 + 2}$$

5. Simplify the expression using polynomial long division.

$$\frac{x^5 - x^3 + 4x^2 - x + 6}{2x^3 - 5}$$

Solution:

Using polynomial long division, we get

$$
\begin{array}{r l}
 & \frac{1}{2}x^2 - \frac{1}{2} \\
2x^3 - 5 & \overline{\big) x^5 - x^3 + 4x^2 - x + 6} \\
 & -(x^5 + 0x^3 - \frac{5}{2}x^2) \\
\hline
 & -x^3 + \frac{13}{2}x^2 - x + 6 \\
 & -(-x^3 + 0x^2 + 0x + \frac{5}{2}) \\
\hline
 & \frac{13}{2}x^2 - x + \frac{7}{2}
\end{array}
$$

Therefore, the solution is

$$\frac{1}{2}x^2 - \frac{1}{2} + \frac{\frac{13}{2}x^2 - x + \frac{7}{2}}{2x^3 - 5}$$

6. Simplify the expression using polynomial long division.

$$(3x^2 + 2x + 5) \div (3x + 5)$$

Solution:

Using polynomial long division, we get

$$\begin{array}{r|l}
 & x \quad -1 \\
\hline
3x+5 & 3x^2+2x+5 \\
 & -(3x^2+5x) \\
\hline
 & \qquad -3x+5 \\
 & \quad -(-3x-5) \\
\hline
 & \qquad\qquad 10
\end{array}$$

Therefore, the solution is

$$x - 1 + \frac{10}{3x + 5}$$

MULTIPLYING MULTIVARIABLE POLYNOMIALS

1. Simplify the expression.

$$(a-3y)(2a+y)$$

Solution:

Use FOIL to expand the product of the binomials.

$$(a-3y)(2a+y)$$

$$(a)(2a)+(a)(y)+(-3y)(2a)+(-3y)(y)$$

$$2a^2+ay-6ay-3y^2$$

$$2a^2-5ay-3y^2$$

2. Simplify the expression.

$$(x-2y)(x+y)+(3x-y)(4x+4y)$$

Solution:

Use FOIL to expand each pair of binomials.

$$(x-2y)(x+y)+(3x-y)(4x+4y)$$

$$(x)(x) + (x)(y) + (-2y)(x) + (-2y)(y)$$

$$+(3x)(4x) + (3x)(4y) + (-y)(4x) + (-y)(4y)$$

$$x^2 + xy - 2xy - 2y^2 + 12x^2 + 12xy - 4xy - 4y^2$$

$$x^2 - xy - 2y^2 + 12x^2 + 8xy - 4y^2$$

$$13x^2 + 7xy - 6y^2$$

■ 3. Fill in the blanks with the correct terms.

$$(5a - b)(7b - 3a)$$

$$35ab - 15a^2 + \_\_\_ + 3ab$$

$$\_\_\_ - 15a^2 + \_\_\_$$

Solution:

Expanding and simplifying the binomial expression gives

$$(5a - b)(7b - 3a)$$

$$35ab - 15a^2 - 7b^2 + 3ab$$

$$38ab - 15a^2 - 7b^2$$

Therefore, the first blank should be filled with $-7b^2$, the second blank with $38ab$, and the last blank with $-7b^2$.

4. What went wrong in this set of steps?

$$(a^2 + 6b)(-a - b^2)$$

$$-a^3 - a^2b^2 - 6ab - b^3$$

$$-a^3 - 7ab - b^3$$

Solution:

In the first step, the terms $6b$ and $-b^2$ were multiplied incorrectly. Their product was shown as $-b^3$, but it should have been $-6b^3$. In the second step, the terms $-a^2b^2$ and $-6ab$ were added, but they shouldn't have been added because they're not like terms.

5. Fill in the the multiplication chart with the correct terms, given the following product of binomials.

$$(4a + 3b)(-a + 2b^2)$$

		3b
-a		-3ab

Solution:

The chart should be filled in as

	4a	3b
-a	$-4a^2$	$-3ab$
$2b^2$	$8ab^2$	$6b^3$

6. Simplify the expression.

$$(5ax - 3by)(a + y) - (a - y)(2ax + 4by)$$

Solution:

We'll use FOIL to expand both pairs of binomials.

$$(5ax - 3by)(a + y) - (a - y)(2ax + 4by)$$

$$(5a^2x + 5axy - 3aby - 3by^2) - (2a^2x + 4aby - 2axy - 4by^2)$$

Distribute the subtraction across the second set of parentheses,

$$5a^2x + 5axy - 3aby - 3by^2 - 2a^2x - 4aby + 2axy + 4by^2$$

then combine like terms.

$$3a^2x + 7axy - 7aby + by^2$$

DIVIDING MULTIVARIABLE POLYNOMIALS

1. Find the quotient.

$$\frac{3x^2 + 6xy - 2y^2}{x - 2y}$$

Solution:

Using polynomial long division,

$$\begin{array}{r|l}
 & 3x \quad +12y \\
\cline{2-2}
x-2y & 3x^2+6xy-2y^2 \\
 & \underline{-(3x^2-6xy)} \\
 & \qquad\quad 12xy-2y^2 \\
 & \qquad \underline{-(12xy-24y^2)} \\
 & \qquad\qquad\qquad 22y^2
\end{array}$$

we can see that the solution is

$$3x + 12y + \frac{22y^2}{x - 2y}$$

2. Identify the quotient, remainder, and divisor.

$$
\begin{array}{r l}
 & x^2 \quad -xy \qquad +y^2 \\
\cline{2-2}
x+y \!\!\! & \big) \, x^3+0x^2y+0xy^2+y^3 \\
 & -(x^3+x^2y) \\
\cline{2-2}
 & \qquad -x^2y+0xy^2 \\
 & \quad -(-x^2y-xy^2) \\
\cline{2-2}
 & \qquad\qquad\qquad xy^2+y^3 \\
 & \qquad\qquad\quad -(xy^2+y^3) \\
\cline{2-2}
 & \qquad\qquad\qquad\qquad\quad 0
\end{array}
$$

Solution:

The quotient is $x^2 - xy + y^2$, the remainder is 0, and the divisor is $x + y$.

■ 3. How should we rewrite the expression before starting the long division?

$$\frac{2y^3 - xy^2 + x^3}{x - y}$$

Solution:

Because the leading term int the divisor is x, we want to reorder the terms in the dividend by descending power of x, which means we should rewrite the quotient as

$$\frac{x^3 - xy^2 + 2y^3}{x - y}$$

4. Find the quotient.

$$\frac{6x^2 - xy + 2y^2}{2x - y}$$

Solution:

Using polynomial long division,

$$
\begin{array}{r|rrr}
 & 3x & +y & \\ \hline
2x-y & 6x^2 & -xy & +2y^2 \\
 & -(6x^2 & -3xy) & \\ \cline{2-3}
 & & 2xy & +2y^2 \\
 & & -(2xy & -y^2) \\ \cline{3-4}
 & & & 3y^2
\end{array}
$$

we can see that the solution is

$$3x + y + \frac{3y^2}{2x - y}$$

5. In words, what's the first question we should ask when solving this long division problem?

$$2x+3y\overline{\big)\,6x^4-x^2y+xy^2+4y^4}$$

Solution:

To begin the long division, the first question we need to ask is "What do we need to multiply by $2x$ in order to get $6x^4$?" The answer to that question will be the first term in the quotient.

6. Find the quotient.

$$(y^2+xy-3x^2)\div(y+x)$$

Solution:

Using polynomial long division,

$$
\begin{array}{r|l}
 & -3x + 4y \\
\hline
x+y & -3x^2 + xy + y^2 \\
 & \underline{-(-3x^2 - 3xy)} \\
 & \qquad 4xy + y^2 \\
 & \qquad \underline{-(4xy + 4y^2)} \\
 & \qquad\qquad -3y^2
\end{array}
$$

we can see that the solution is

$$-3x + 4y - \frac{3y^2}{x + y}$$

GREATEST COMMON FACTOR

1. Factor out the greatest common factor.

$$3x^2y^3 + 12x^3y^2 - 9x^4y^4$$

Solution:

The greatest common factor is $3x^2y^2$, so the expression is factored as

$$3x^2y^2(y + 4x - 3x^2y^2)$$

2. Factor the polynomial in the numerator and simplify the resulting expression. Fill in the blank with the correct term.

$$\frac{3x^3 - 12x}{3x} = x^2 - \underline{\qquad}$$

Solution:

The blank should be filled in with 4.

$$\frac{3x^3 - 12x}{3x}$$

$$\frac{3x(x^2 - 4)}{3x}$$

$$x^2 - 4$$

3. Factor the expression.

$$9s^3t^2 + 15s^2t^5 - 24s^5t + 6s^4t^2$$

Solution:

The greatest common factor is $3s^2t$. When we factor out the $3s^2t$, we have to divide each term by $3s^2t$.

$$3s^2t(3st + 5t^4 - 8s^3 + 2s^2t)$$

4. What went wrong when the polynomial was factored?

$$10x^3y^4 - 5x^4y^2 - 20x^6y^3$$

$$x^3y^2(10y^2 - 5x - 20x^3y)$$

Solution:

There's a factor of 5 in each term that was not factored out. The factoring should have been

$$5x^3y^2(2y^2 - x - 4x^3y)$$

5. Factor the polynomial in the numerator and simplify the resulting expression.

$$\frac{4x^4 - 8x^3 - 32x^2}{4x^2}$$

Solution:

Factor the greatest common factor out of the numerator,

$$\frac{4x^4 - 8x^3 - 32x^2}{4x^2}$$

$$\frac{4x^2(x^2 - 2x - 8)}{4x^2}$$

then cancel like terms from the numerator and denominator.

$$x^2 - 2x - 8$$

6. Fill in the blank with the correct term.

$$4a^3b - 10ab^2 + \_\_\_\_ = 2ab(2a^2 - 5b + 3a^2b^2)$$

Solution:

The blank should be filled in with $6a^3b^3$. We can see this by distributing the $2ab$ across the parentheses.

$$2ab(2a^2 - 5b + 3a^2b^2)$$

$$4a^3b - 10ab^2 + 6a^3b^3$$

QUADRATIC POLYNOMIALS

1. Factor the quadratic expression.

$$2x^2+2x-12$$

Solution:

The greatest common factor is 2, so we first factor out a 2.

$$2(x^2+x-6)$$

Since $(3)(-2)=-6$ and $(3)+(-2)=1$, we see that x^2+x-6 factors as

$$(x+3)(x-2)$$

So the quadratic polynomial can be factored as

$$2(x+3)(x-2)$$

2. What went wrong when the polynomial was factored?

$$x^2-4x+3$$

$$(x-3)(x+1)$$

Solution:

The second factor should have been $(x - 1)$, instead of $(x + 1)$. If we expand the expression $(x - 3)(x + 1)$, we get

$$(x - 3)(x + 1)$$

$$x^2 + x - 3x - 3$$

$$x^2 - 2x - 3$$

But if we instead factor $x^2 - 4x + 3$ as $(x - 3)(x - 1)$, then we get back to the correct expression.

$$(x - 3)(x - 1)$$

$$x^2 - x - 3x + 3$$

$$x^2 - 4x + 3$$

3. Factor the quadratic expression.

$$x^2 + 3x - 28$$

Solution:

Since $(-4)(7) = -28$ and $(-4) + (7) = 3$, we see that $x^2 + 3x - 28$ factors as

$$(x - 4)(x + 7)$$

4. Factor the quadratic expression.

$$x^2 - 9x + 18$$

Solution:

Since $(-3)(-6) = 18$ and $(-3) + (-6) = -9$, we see that $x^2 - 9x + 18$ factors as

$$(x - 3)(x - 6)$$

5. Fill in the blank with the correct term.

$$5x^2 - 40x + 60 = \underline{\quad}(x - 2)(x - \underline{\quad})$$

Solution:

The greatest common factor of the polynomial on the left is 5, so we first factor out a 5.

$$5(x^2 - 8x + 12)$$

Since $(-6)(-2) = 12$ and $(-6) + (-2) = -8$, we see that $x^2 - 8x + 12$ factors as

$$(x - 6)(x - 2)$$

So the quadratic polynomial can be factored as

$$5(x - 6)(x - 2)$$

6. Factor the quadratic expression.

$$x^2 - x - 2$$

Solution:

Since $(-2)(1) = -2$ and $(-2) + 1 = -1$, we see that $x^2 - x - 2$ factors as

$$(x - 2)(x + 1)$$

DIFFERENCE OF SQUARES

1. Factor the expression.

$$4y^2 - 36$$

Solution:

The expression can be rewritten as

$$4y^2 - 36$$

$$(2y)^2 - (6)^2$$

and factored as

$$(2y - 6)(2y + 6)$$

2. What went wrong when the polynomial was factored?

$$9a^4 - 25b^2$$

$$(9a^2 - 25b)(9a^2 + 25b)$$

Solution:

The coefficients were not taken into consideration when factoring the expression. It should be first written as

$$9a^4 - 25b^2$$

$$(3a^2)^2 - (5b)^2$$

and then factored as the difference of squares.

$$(3a^2 - 5b)(3a^2 + 5b)$$

■ 3. Factor the expression.

$$49x^6y^2 - 36z^4$$

Solution:

The expression can be rewritten as

$$49x^6y^2 - 36z^4$$

$$(7x^3y)^2 - (6z^2)^2$$

and factored as

$$(7x^3y - 6z^2)(7x^3y + 6z^2)$$

■ 4. Fill in the blank with the correct term.

$$\_\_\_\_ - 25y^2 = (2xz^2 - 5y)(2xz^2 + 5y)$$

Solution:

The blank should be filled in with $4x^2z^4$.

5. Factor the expression.

$$2x^2 - 288$$

Solution:

The greatest common factor of this polynomial is 2, so we first factor out a 2.

$$2(x^2 - 144)$$

Since x^2 and 144 are both perfect squares (the squares of x and 12, respectively), $x^2 - 144$ is factored as $(x - 12)(x + 12)$, so the polynomial factors as

$$2(x - 12)(x + 12)$$

6. Factor the expression.

$$5a^3 - 20ab^2$$

Solution:

The greatest common factor of this polynomial is $5a$, so we first factor out a $5a$.

$$5a(a^2 - 4b^2)$$

Since a^2 and $4b^2$ are both perfect squares (the squares of a and $2b$, respectively), $a^2 - 4b^2$ is factored as $(a - 2b)(a + 2b)$, so the polynomial factors as

$$5a(a - 2b)(a + 2b)$$

ZERO THEOREM

1. Find the zeros of the function.

$$y = x^2 - 5x + 6$$

Solution:

The zeros are the x-values when $y = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 2 = 0$$

$$x = 2$$

and

$$x - 3 = 0$$

$$x = 3$$

The roots are $x = 2$ and $x = 3$.

2. Find the zeros of the function.

$$y = x^2 - 4x - 5$$

Solution:

The zeros are the x-values when $y = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 5 = 0$$

$$x = 5$$

and

$$x + 1 = 0$$

$$x = -1$$

The roots are $x = 5$ and $x = -1$.

3. Find the x-intercepts.

$$f(x) = x^2 + 10x + 24$$

Solution:

The x-intercepts are the x-values when $f(x) = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 + 10x + 24 = 0$$

$$(x + 6)(x + 4) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x + 6 = 0$$

$$x = -6$$

and

$$x + 4 = 0$$

$$x = -4$$

The solutions are $x = -6$ and $x = -4$.

4. Find the x-intercepts.

$$f(x) = x^2 - 7x + 6$$

Solution:

The x-intercepts are the x-values when $f(x) = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 7x + 6 = 0$$

$$(x - 6)(x - 1) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 6 = 0$$

$$x = 6$$

and

$$x - 1 = 0$$

$$x = 1$$

The solutions are $x = 6$ and $x = 1$.

5. Use the Zero Theorem to find the solutions to the quadratic equation.

$$4x^2 - 16 = 0$$

Solution:

Factor the left side as the difference of squares.

$$4x^2 - 16 = 0$$

$$(2x - 4)(2x + 4) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$2x - 4 = 0$$

$$2x = 4$$

$$x = 2$$

and

$$2x + 4 = 0$$

$$2x = -4$$

$$x = -2$$

The solutions are $x = 2$ and $x = -2$.

6. Use the Zero Theorem to find the solutions to the quadratic equation.

$$25 - 9x^2 = 0$$

Solution:

Factor the left side as the difference of squares.

$$25 - 9x^2 = 0$$

$$(5 - 3x)(5 + 3x) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$5 - 3x = 0$$

$$3x = 5$$

$$x = \frac{5}{3}$$

and

$$5 + 3x = 0$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

The solutions are $x = 5/3$ and $x = -5/3$.

COMPLETING THE SQUARE

1. Solve for x by completing the square.

$$x^2 - 6x + 5 = 0$$

Solution:

Completing the square gives

$$x^2 - 6x = -5$$

$$x^2 - 6x + 9 = -5 + 9$$

$$(x - 3)^2 = 4$$

$$x - 3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 1, 5$$

2. Fill in the blank with the correct term.

$$x^2 - \_\_\_\_ + \frac{9}{4} = -2 + \frac{9}{4}$$

Solution:

We have the equation in the form

$$x^2+bx+\left(\frac{b}{2}\right)^2=-c+\left(\frac{b}{2}\right)^2$$

Therefore,

$$\left(\frac{b}{2}\right)^2=\frac{9}{4}$$

$$\frac{b}{2}=\frac{3}{2}$$

$$b=3$$

The blank should be the term $3x$.

3. Complete the square but don’t solve for the roots.

$$y^2-4y+1=0$$

Solution:

To complete the square, we first write the expression as

$$y^2-4y=-1$$

Now complete the square as

$$y^2 - 4y + 4 = -1 + 4$$

$$(y-2)^2 = 3$$

4. Solve for y by completing the square.

$$y^2 + 3y = 1$$

Solution:

Completing the square gives

$$y^2 + 3y + \frac{9}{4} = 1 + \frac{9}{4}$$

$$\left(y + \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$y = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

$$y = -\frac{3 \pm \sqrt{13}}{2}$$

5. Solve for x by completing the square.

$$x^2 + 6x + 11 = 0$$

Solution:

Completing the square gives

$$x^2 + 6x = -11$$

$$x^2 + 6x + 9 = -11 + 9$$

$$(x + 3)^2 = -2$$

$$x + 3 = \pm\sqrt{-2}$$

$$x = -3 \pm \sqrt{2}i$$

6. Solve for x by completing the square.

$$2x^2 + 8x + 35 = 0$$

Solution:

Completing the square gives

$$2x^2 + 8x = -35$$

$$x^2 + 4x = -\frac{35}{2}$$

$$x^2 + 4x + 4 = -\frac{35}{2} + 4$$

$$(x+2)^2 = -\frac{27}{2}$$

$$x + 2 = \pm\sqrt{-\frac{27}{2}}$$

$$x = -2 \pm \sqrt{\frac{27}{2}}i$$

$$x = -2 \pm 3\sqrt{\frac{3}{2}}i$$

QUADRATIC FORMULA

1. Write the quadratic formula for the following quadratic equation.

$$x^2 - 5x - 24 = 0$$

Solution:

In this problem $a = 1$, $b = -5$, and $c = -24$. The quadratic formula for the expression is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-24)}}{2(1)}$$

We could continue to simplify to solve for the roots.

$$x = \frac{5 \pm \sqrt{25 + 96}}{2}$$

$$x = \frac{5 \pm \sqrt{121}}{2}$$

$$x = \frac{5 \pm 11}{2}$$

$$x = -3,\ 8$$

2. What went wrong in the way the quadratic formula was applied?

$$3x^2 - 5x + 10 = 0$$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(10)}}{2(3)}$$

Solution:

The $-b$ at the beginning of the quadratic formula is written as -5, but $b = -5$. Which means it should be written as $-(-5)$.

3. Solve for z using the quadratic formula.

$$z^2 = z + 3$$

Solution:

Rewrite the expression as

$$z^2 = z + 3$$

$$z^2 - z - 3 = 0$$

In this problem $a = 1$, $b = -1$, and $c = -3$. Then the quadratic formula gives

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$$z = \frac{1 \pm \sqrt{13}}{2}$$

4. Fill in the blank with the correct term if the quadratic formula below was built from the quadratic equation.

$$\_\_\_\_ x^2 + 3x - 5 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(-2)(-5)}}{2(-2)}$$

Solution:

The blank should be filled in with -2.

5. What went wrong if the quadratic formula below was built from the quadratic equation?

$$x^2 + 2x = 7$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(7)}}{2(1)}$$

Solution:

The expression was not written in the correct form before using the quadratic formula. It should be written as $x^2 + 2x - 7 = 0$, for which the quadratic formula would then be

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-7)}}{2(1)}$$

■ 6. Solve for t using the quadratic formula.

$$4t^2 - 1 = -8t$$

Solution:

Rewrite the expression as

$$4t^2 - 1 = -8t$$

$$4t^2 + 8t - 1 = 0$$

In this problem $a = 4$, $b = 8$, and $c = -1$. Then the quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-8 \pm \sqrt{8^2 - 4(4)(-1)}}{2(4)}$$

$$t = \frac{-8 \pm \sqrt{64 + 16}}{8}$$

$$t = \frac{-8 \pm 4\sqrt{5}}{8}$$

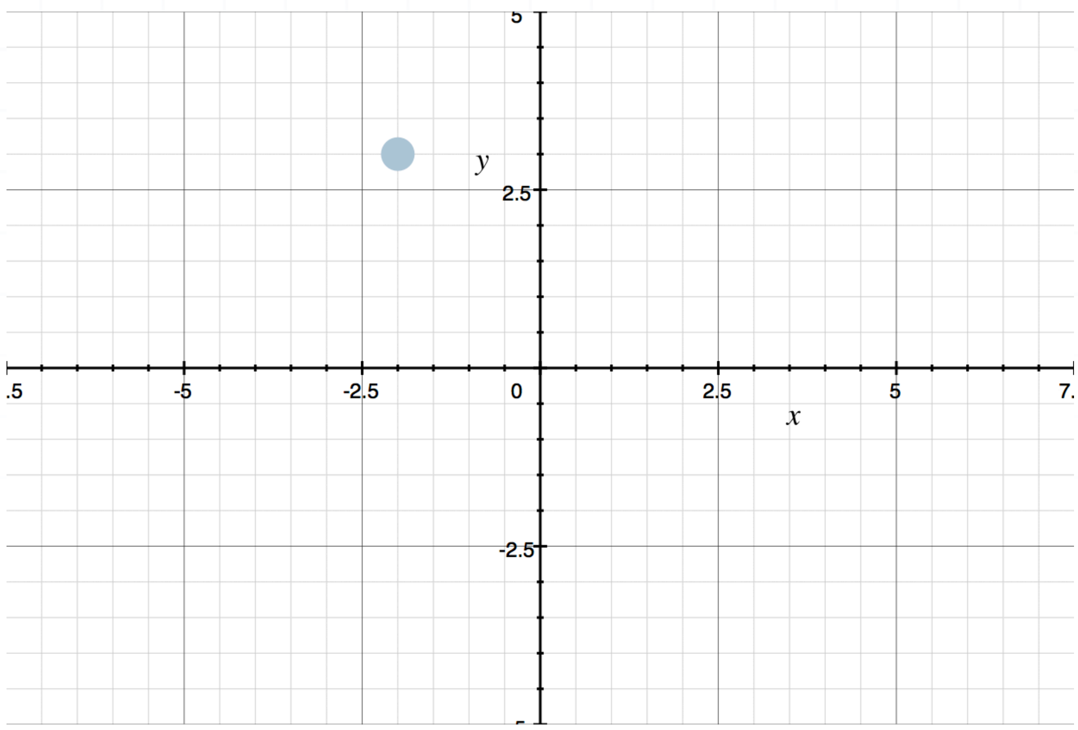
$$t = \frac{-2 \pm \sqrt{5}}{2}$$

CARTESIAN COORDINATE SYSTEM

1. Graph the point $(-2,3)$ in the Cartesian plane.

Solution:

The graph of the point is



2. In which quadrant should we plot the point $(1,6)$?

Solution:

Since both the x- and the y-coordinates are positive, this point is graphed in Quadrant I.

3. What is the y-coordinate of any point that lies on the x-axis? Give an example of a coordinate point that lies on the x-axis.

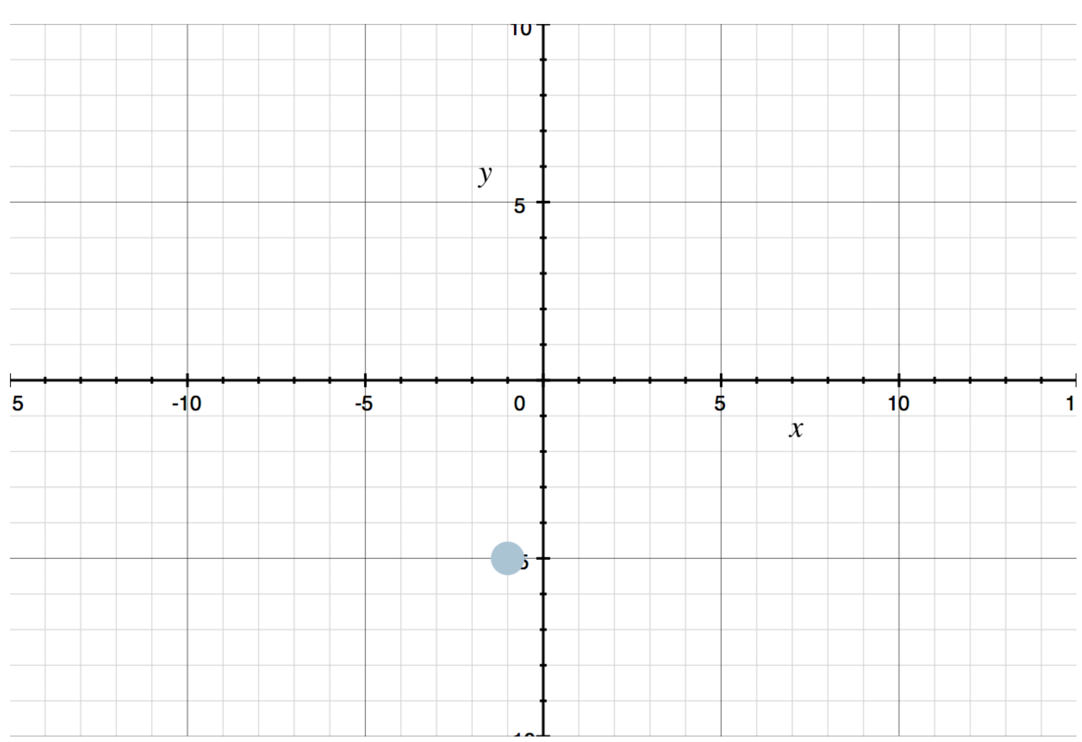
Solution:

The y-coordinate of any point on the x-axis is always $y = 0$. For example, $(3,0)$ is a point on the x-axis.

4. Graph the point $(-1, -5)$ in the Cartesian plane.

Solution:

The graph of the point is



5. In which quadrant should we plot $(3, -7)$?

Solution:

Since the x-coordinate is positive and the y-coordinate is negative, this point is graphed in Quadrant IV.

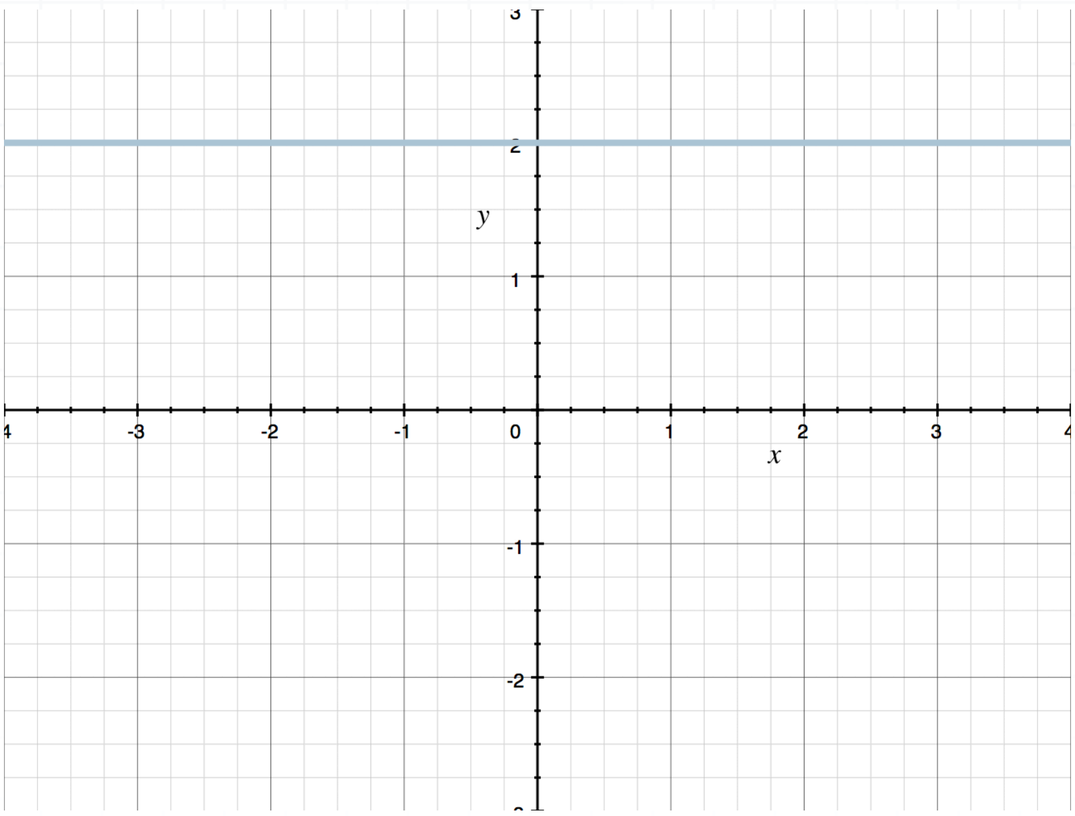
6. What is the x-coordinate of any point that lies on the y-axis? Give an example of a coordinate point that lies on the y-axis.

Solution:

The x-coordinate of any point on the y-axis is always $x = 0$. For example, $(0, -7)$ is a point on the y-axis.

SLOPE

1. What is the slope of the line?



Solution:

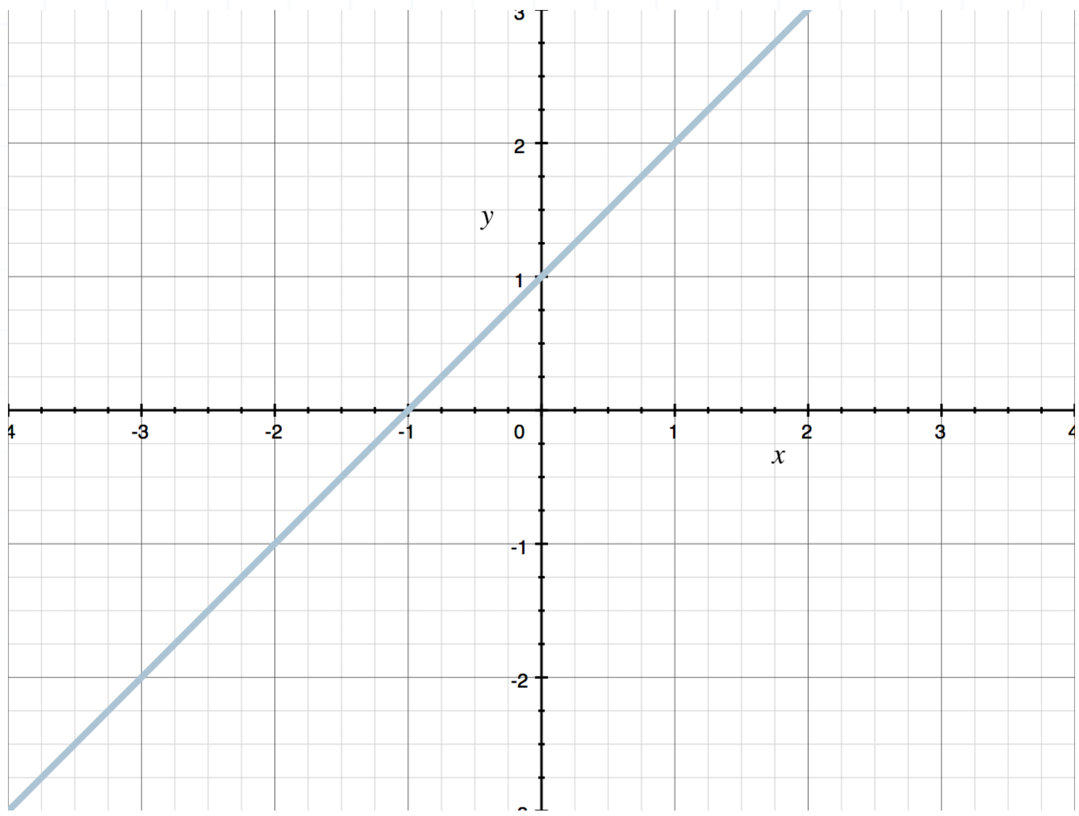
Since the line is a horizontal line, the slope is 0.

2. What direction is an undefined slope: horizontal or vertical? Use the formula for the slope to explain why.

Solution:

The direction of an undefined slope is vertical. It is because the change in x of a vertical line is 0, so the slope has a 0 in the denominator and is therefore undefined.

3. What is the slope of the line?



Solution:

Notice that the graph passes through the points $(-1,0)$ and $(0,1)$, which means the slope can be defined as

$$m = \frac{1-0}{0-(-1)}$$

$$m = \frac{1}{1}$$

$$m = 1$$

4. What is the slope of the line that passes through the points $(-1,3)$ and $(4,-7)$?

Solution:

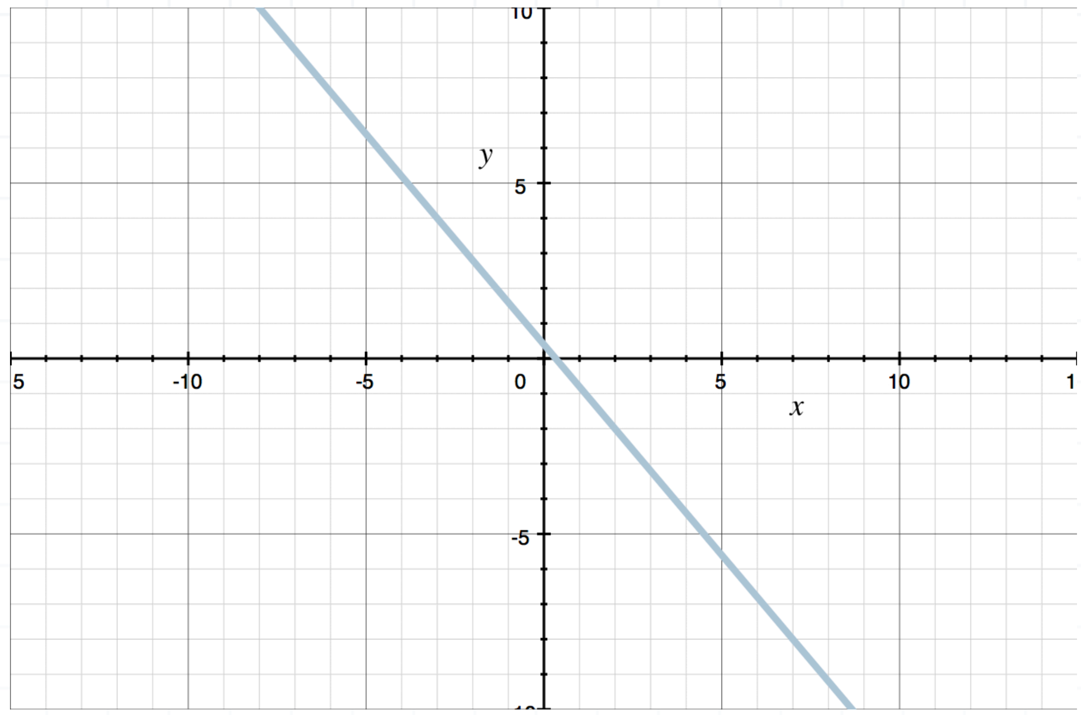
The graph passes through the points $(-1,3)$ and $(4,-7)$, so the slope is defined as

$$m = \frac{-7-3}{4-(-1)}$$

$$m = \frac{-10}{5}$$

$$m = -2$$

5. What is the slope of the line?



Solution:

Notice that the graph passes through the points $(-3,4)$ and $(2,-2)$, so the slope is

$$m=\frac{-2-4}{2-(-3)}$$

$$m=\frac{-6}{5}$$

$$m=-\frac{6}{5}$$

6. Find the slope of the line that passes through $(3,5)$ and $(-1,5)$.

Solution:

The graph passes through the points $(3,5)$ and $(-1,5)$, so the slope is defined as

$$m = \frac{5-5}{-1-3}$$

$$m = \frac{0}{-4}$$

$$m = 0$$

POINT-SLOPE AND SLOPE-INTERCEPT FORMS OF A LINE

■ 1. Find the equation of the line that passes through $(3,0)$ with slope -2.

Solution:

Using point-slope form, the equation of the line is

$$y - 0 = -2(x - 3)$$

■ 2. Find the equation of the line that passes through the points $(-2,3)$ and $(2,-4)$.

Solution:

We first need to calculate the slope of the line as follows

$$m = \frac{-4 - 3}{2 - (-2)}$$

$$m = \frac{-7}{4}$$

$$m = -\frac{7}{4}$$

Using point-slope form, the equation of the line is either of the following:

$$y - 3 = -\frac{7}{4}(x + 2)$$

$$y + 4 = -\frac{7}{4}(x - 2)$$

■ 3. Find the equation of the line that passes through the points $(5, -4)$ and $(6,0)$.

Solution:

We first need to calculate the slope of the line as

$$m = \frac{0 - (-4)}{6 - 5}$$

$$m = \frac{4}{1}$$

$$m = 4$$

Using point-slope form, the equation of the line is then either of the following:

$$y + 4 = 4(x - 5)$$

$$y = 4(x - 6)$$

■ 4. Identify the y-intercept and slope m defining the line.

$$y = -\frac{1}{4}(x + 12)$$

Solution:

We need to rewrite the equation in slope-intercept form, $y = mx + b$.

$$y = -\frac{1}{4}(x + 12)$$

$$y = -\frac{1}{4}x - 3$$

Notice that the slope of the line given is $-1/4$ and the y-intercept (when $x = 0$) is $(0, -3)$.

5. Convert the point-slope equation into a slope-intercept equation.

$$y - 3 = \frac{1}{3}(x - 6)$$

Solution:

Converting to slope-intercept form means that we need to solve for y, and simplify as much as we can.

$$y - 3 = \frac{1}{3}(x - 6)$$

$$y - 3 = \frac{1}{3}x - 2$$

$$y = \frac{1}{3}x - 2 + 3$$

$$y = \frac{1}{3}x + 1$$

6. Find the equation of a line that passes through the points $(1, -1)$ and $(0,3)$. Write the solution in slope-intercept form.

Solution:

We first need to calculate the slope of the line as

$$m = \frac{3 - (-1)}{0 - 1}$$

$$m = \frac{4}{-1}$$

$$m = -4$$

Using slope-intercept form, noting that the y-intercept is 3, the equation of the line is

$$y = -4x + 3$$

GRAPHING LINEAR EQUATIONS

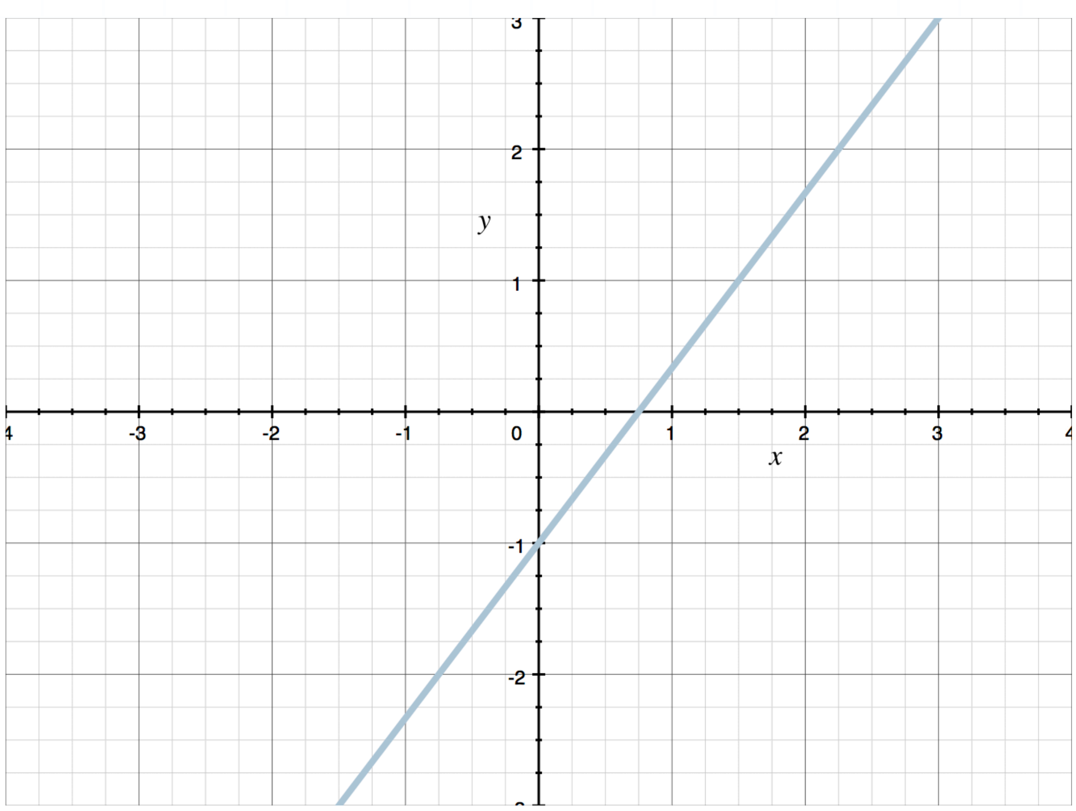
1. Graph the line.

$$y = \frac{4}{3}x - 1$$

Solution:

The linear equation is already in slope-intercept form, so we can see that the slope is $m = 4/3$ and the y-intercept is $b = -1$.

Since the slope is positive, we know that the line will lean to the right. The graph of the line is



■ 2. Describe how we would use the slope to find another point on the line if the slope is $m = 2/3$ and the line passes through $(x_1, y_1) = (-1,2)$.

Solution:

Starting at the point $(-1,2)$, move up 2 and to the right 3 to get the point $(2,4)$, or move down 2 and to the left 3 to get the point $(-4,0)$.

■ 3. Graph the line.

$$y + 2 = -3x + 1$$

Solution:

The linear equation isn't already in slope-intercept form, so we'll subtract 2 from both sides in order to solve for y.

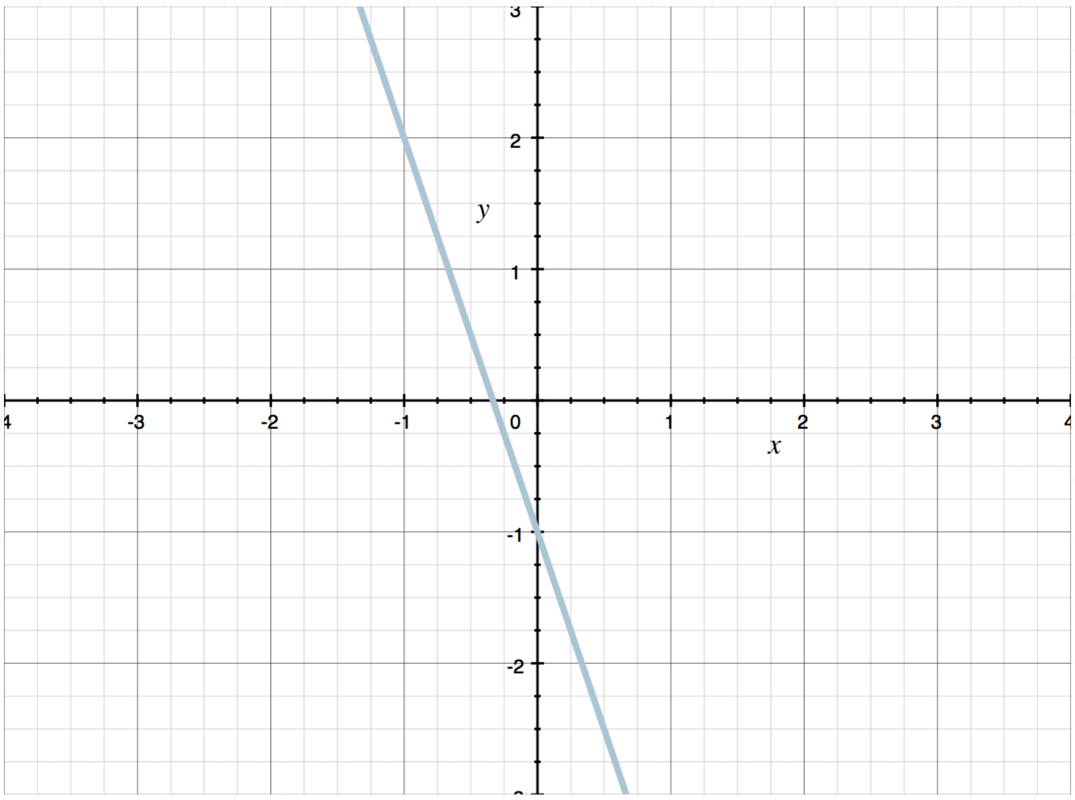
$$y + 2 = -3x + 1$$

$$y + 2 - 2 = -3x + 1 - 2$$

$$y = -3x - 1$$

With the equation in slope-intercept form, we can identify that the slope is $m = -3$ and the y-intercept is -1.

Since the slope is negative, we know that the line will lean to the left. The graph of the line is



4. Use the slope $m = 1/3$ to find two more points on the line passing through $(1,2)$. Move right to determine one point and left to determine another.

Solution:

Going right, we get the point $(4,3)$. Going left, we get the point $(-2,1)$.

5. Graph the line.

$$y = -2(3x + 1)$$

Solution:

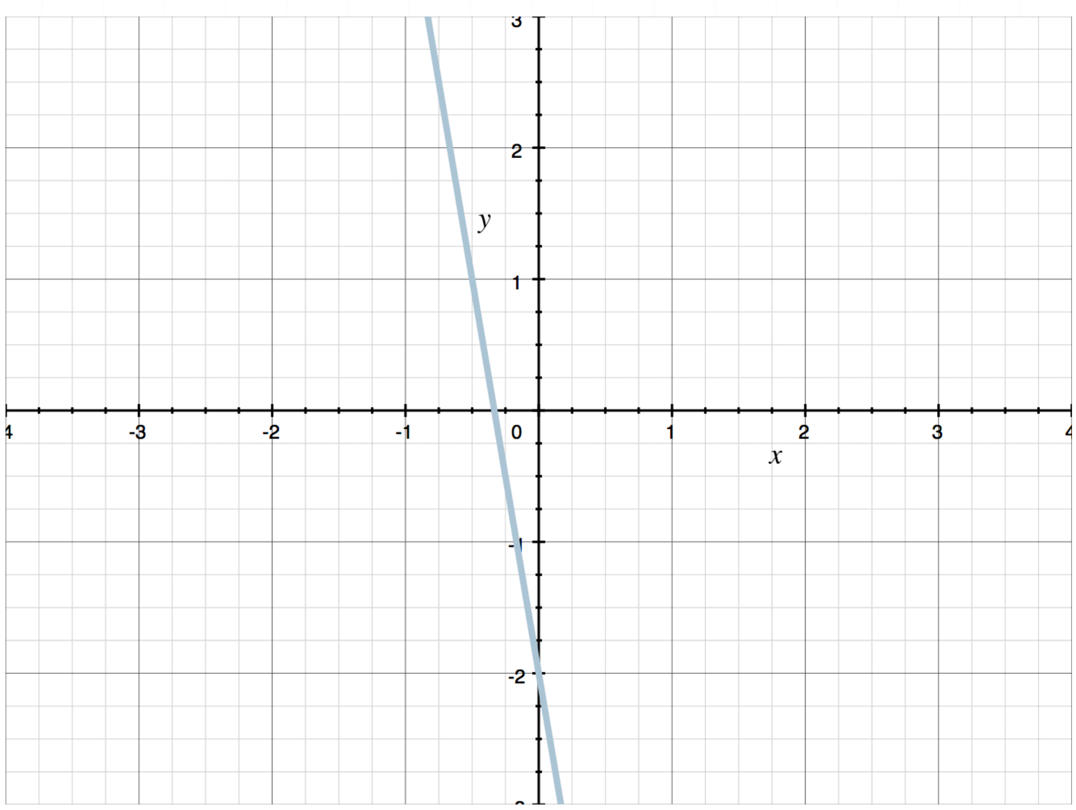
The linear equation isn't already in slope-intercept form, so we'll distribute the -2 across the parentheses.

$$y = -2(3x + 1)$$

$$y = -6x - 2$$

With the equation in slope-intercept form, we can identify that the slope is $m = -6$ and the y-intercept is -2.

Since the slope is negative, we know that the line will lean to the left. The graph of the line is



■ 6. Give two points that lie on the line, find the slope, and graph the line.

$$y + 3 = -\frac{1}{2}(4x + 10)$$

Solution:

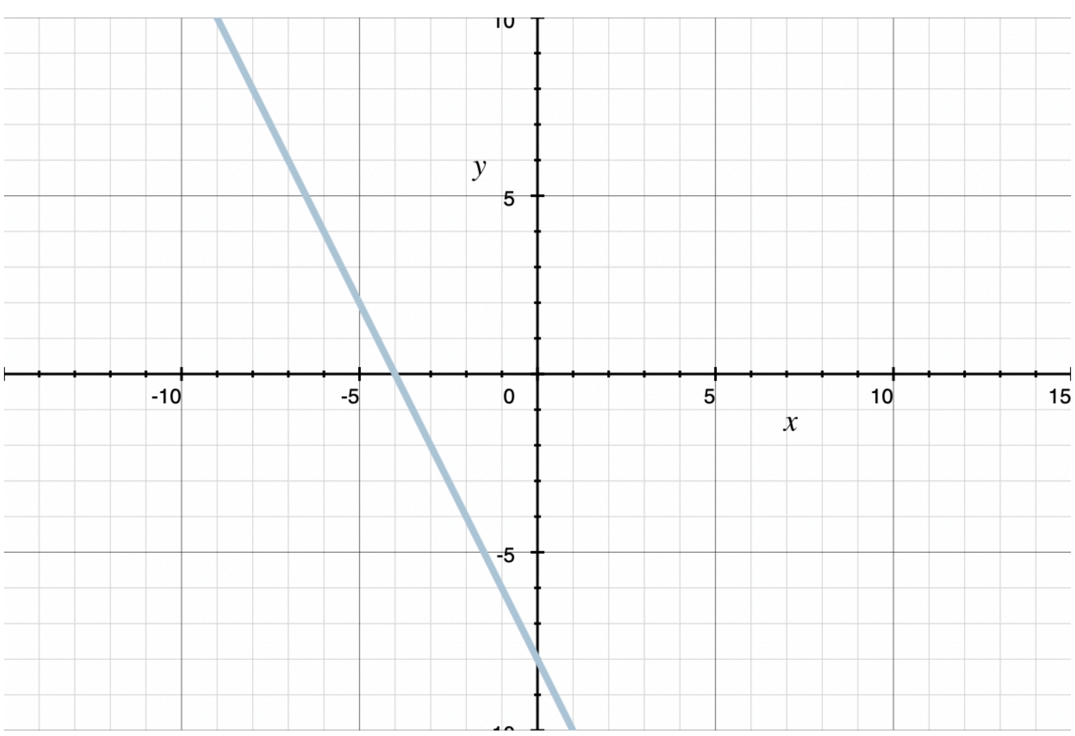
The linear equation isn't already in slope-intercept form, so we'll rewrite it.

$$y + 3 = -\frac{1}{2}(4x + 10)$$

$$y + 3 = -2x - 5$$

$$y = -2x - 8$$

With the equation in slope-intercept form, we can identify that the slope is $m = -2$ and the y-intercept is -8. There are an infinite number of points on the line, like $(0, -8)$ and $(-1, -6)$. The graph of the line is



FUNCTION NOTATION

1. Find and simplify $f(x+1)$ if $f(x)=4x-5$.

Solution:

To find $f(x+1)$, we plug $x+1$ into $f(x)$ in place of x.

$$f(x+1)=4(x+1)-5$$

$$f(x+1)=4x+4-5$$

$$f(x+1)=4x-1$$

2. What went wrong in this set of steps?

Evaluate $f(x)=x^2+1$ at $x=-2$.

$$f(-2)=-2^2+1$$

$$f(-2)=-4+1$$

$$f(-2)=-3$$

Solution:

To find $f(-2)$, we need to plug $x = -2$ into $f(x)$. But we needed to wrap the -2 in parentheses, to force the exponent to apply to the negative sign, not just to the 2. So the evaluation should have been

$$f(-2) = (-2)^2 + 1$$

$$f(-2) = 4 + 1$$

$$f(-2) = 5$$

3. Find and simplify $h(s^2)$ if $h(s) = -s^2 + 3s - 1$.

Solution:

To find $h(s^2)$, we plug s^2 into $h(s)$ in place of s.

$$h(s^2) = -(s^2)^2 + 3(s^2) - 1$$

$$h(s^2) = -s^4 + 3s^2 - 1$$

4. If $g(x) = x^3 - x + 1$, what do we need to plug into the function in order to get the following expression?

$$g(??) = (2x + 1)^3 - (2x + 1) + 1$$

Solution:

Notice that everywhere there's an x in $g(x)$, there's a $2x+1$ in the new function. Therefore, the value that got plugged in must have been $2x+1$.

5. Find the value of the expression if $f(x)=x^2+x-1$.

$$\frac{f(x+h)-f(x)}{h}$$

Solution:

To find $f(x+h)$, we plug $x+h$ into $f(x)$ in place of x.

$$f(x+h)=(x+h)^2+(x+h)-1$$

$$f(x+h)=x^2+2hx+h^2+x+h-1$$

Now substitute and simplify.

$$\frac{f(x+h)-f(x)}{h}=\frac{x^2+2hx+h^2+x+h-1-(x^2+x-1)}{h}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{x^2+2hx+h^2+x+h-1-x^2-x+1}{h}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{2hx+h^2+h}{h}$$

$$\frac{f(x+h)-f(x)}{h}=2x+h+1$$

6. What went wrong in this set of steps?

Find $f(1)$ if $f(x) = x^3 + 3x^2 - 5x + 2$.

$$f(1) = 1^3 + 3(1)^2 - 5(1) + 2$$

$$f(1) = 1 + 9 - 5 + 2$$

$$f(1) = 7$$

Solution:

When evaluating $f(1)$, notice that instead of evaluating $3(1)^2$ as $3(1)^2 = 3(1) = 3$, the steps show $3(1)^2 = 9$. However, the 3 is not being squared, so that's incorrect. The steps should have been

$$f(1) = 1^3 + 3(1)^2 - 5(1) + 2$$

$$f(1) = 1 + 3 - 5 + 2$$

$$f(1) = 1$$

DOMAIN AND RANGE

1. Find the domain of $f(x)$.

$$f(x)=\frac{3}{x(x+1)}+x^2$$

Solution:

In this function, the denominator can't be 0. The values of x that make the denominator 0 are $x=0$ and $x=-1$. So the domain of the function is all $x\neq 0,-1$.

2. Find the domain and range of the point set.

$$(-1,-3),\quad (0,5),\quad (-3,6),\quad (0,-3)$$

Solution:

The domain is all the x-values and the range is all the y-values. Therefore the domain and range are

Domain: $-3,\ -1,\ 0$

Range: $-3,\ 5,\ 6$

3. Find the domain and range of $g(x)$.

$$g(x) = \frac{\sqrt{x-2}}{3}$$

Solution:

In this function, the radicand (the expression under the square root) must be 0 or positive. So $x - 2 \geq 0$ is the domain. Since the square root function can't be negative, the numerator is guaranteed to be positive or zero, while the denominator is guaranteed to be positive. Since a positive divided by a positive is a positive, and zero divided by a positive is zero, the range is $g(x) \geq 0$.

4. Find the domain and range of the function.

$$f(x) = \frac{2}{x} + 1$$

Solution:

In this function, the denominator can't be 0, which means $x \neq 0$. Therefore the domain of the function is $x \neq 0$.

Since $2/x$ will never be 0, $f(x)$ can never be 1. Therefore the range of the function is $f(x) \neq 1$.

5. Find the domain and range of $g(x)$.

$$g(x) = -x^2 + 5$$

Solution:

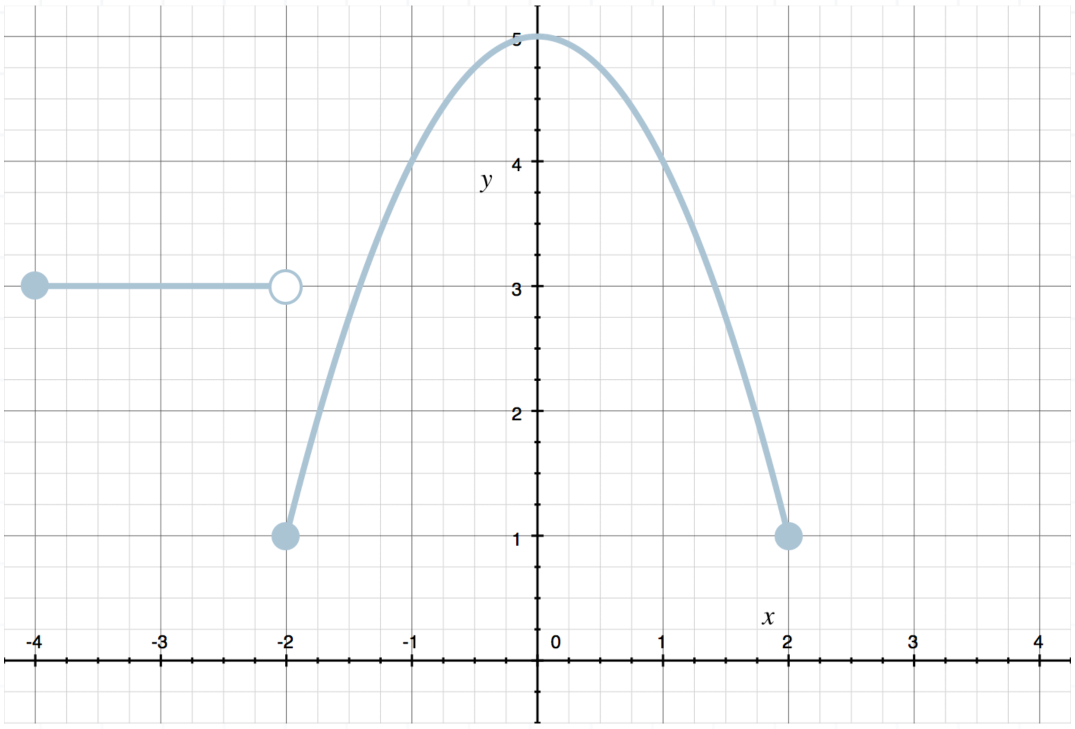
There is no real number that makes the expression undefined. So, the domain of the function is all real numbers.

The range is the set of all valid y values. If we switch the order of the terms to rewrite the function as

$$g(x) = 5 - x^2$$

we can see that we'll always be subtracting zero or a positive number from 5. Since x^2 will never be negative, it can only take on a zero value, in which case we'll get $f(x) = 5 - 0 = 5$, or a positive value, in which case we'll find some value less than 5. So the range is $g(x) \leq 5$.

6. What is the domain and range of the graph? Hint: An empty circle indicates that exact point *is not* included as part of the graph, while a solid circle indicates that exact point *is* included as part of the graph.



Solution:

The domain of the graph is determined by the x-values, which are defined between $x = -4$ and $x = 2$. The solid circles indicate that $x = -4$ and $x = 2$ are included in the graph. There's an empty circle at $x = -2$, but also a solid circle at the same value of x, which means $x = -2$ is also included in the domain. Therefore, the domain is $-4 \leq x \leq 2$.

The range is determined by the y-values, which are defined between $y = 1$ and $y = 5$. The solid circles indicate that $y = 1$ is included in the graph. Therefore, the range is $1 \leq y \leq 5$.

TESTING FOR FUNCTIONS

■ 1. Determine whether or not the point set represents a function.

$$(2,-1), (-1,0), (0,-1), (3,2)$$

Solution:

For every x-value, there is only one y-value, so the set of points represents a function. The x-values are -1, 0, 2, 3 and the y-values are -1, 0, 2. Even though both $x=2$ and $x=0$ are mapping to $y=-1$, the point set is still a function.

The point set would not be a function if we found two different y-values mapping to the same x-value.

■ 2. Fill in the blanks in the definition of a function.

For every ____________, there is only one unique ____________.

Solution:

x (or input), y (or output)

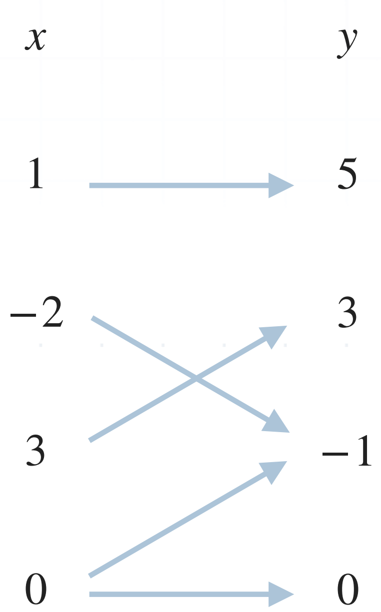
3. Determine whether or not the point set represents a function.

$$(1,2),\ (-1,5),\ (1,-3),\ (0,1)$$

Solution:

Notice that for $x=1$, there are two y-values, $y=2$ and $y=-3$. Since for one input there are two different outputs, the point set doesn't represent a function.

4. Determine whether the mapping represents a function.



Solution:

Notice that for $x=0$, there are two y-values: $y=0$ and $y=-1$. Since for one input there are two different outputs, this doesn't represent a function.

5. Determine algebraically whether or not the equation represents a function.

$$(x-1)^2 + y = 3$$

Solution:

Solve the equation for y.

$$(x-1)^2 + y = 3$$

$$y = 3 - (x-1)^2$$

Simplify the right side.

$$y = 3 - (x^2 - 2x + 1)$$

$$y = 3 - x^2 + 2x - 1$$

$$y = -x^2 + 2x + 2$$

Each value we plug in for x will give a unique value for y, so the equation represents a function.

6. Determine algebraically whether or not the equation represents a function.

$$y^2 = x + 1$$

Solution:

Solve the equation for y.

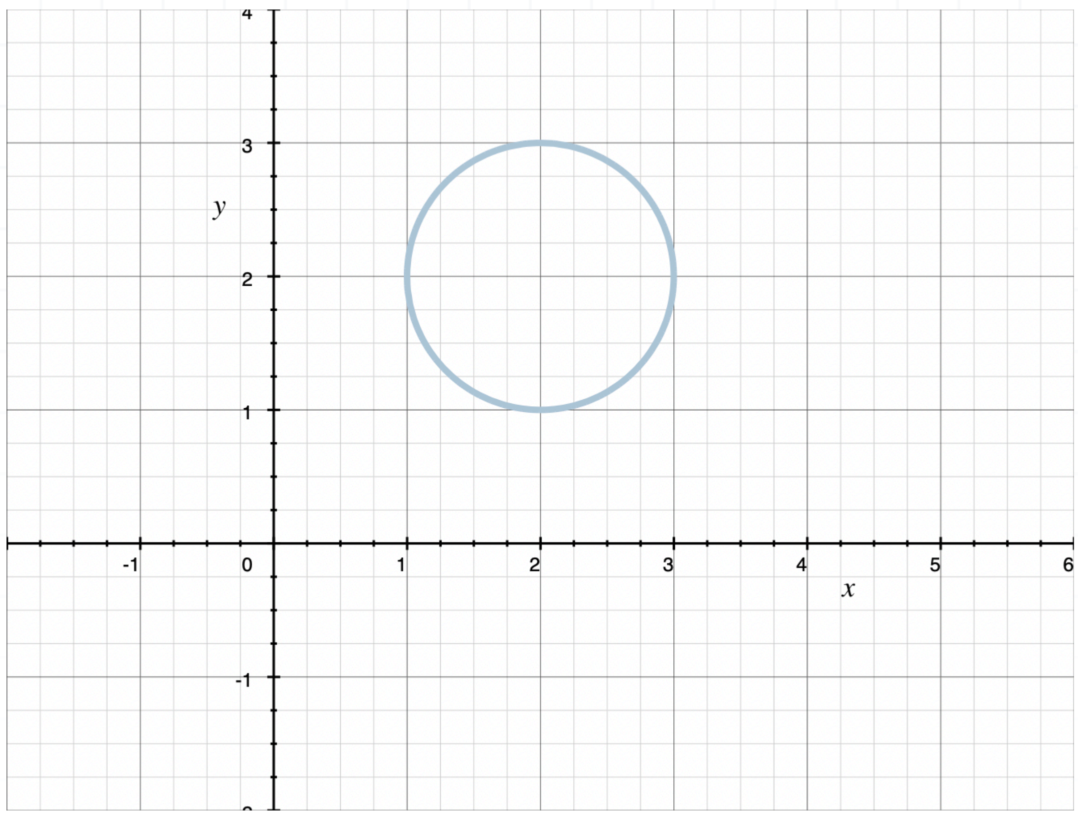
$$y^2 = x + 1$$

$$y = \pm\sqrt{x + 1}$$

Given this equation for y, there are values of x that will give multiple values for y. For instance, at $x = 0$ y takes on the values $y = -1$ and $y = 1$. So for one input there are two outputs and the equation doesn't represent a function.

VERTICAL LINE TEST

1. Use the Vertical Line Test to determine whether or not the graph is the graph of a function.

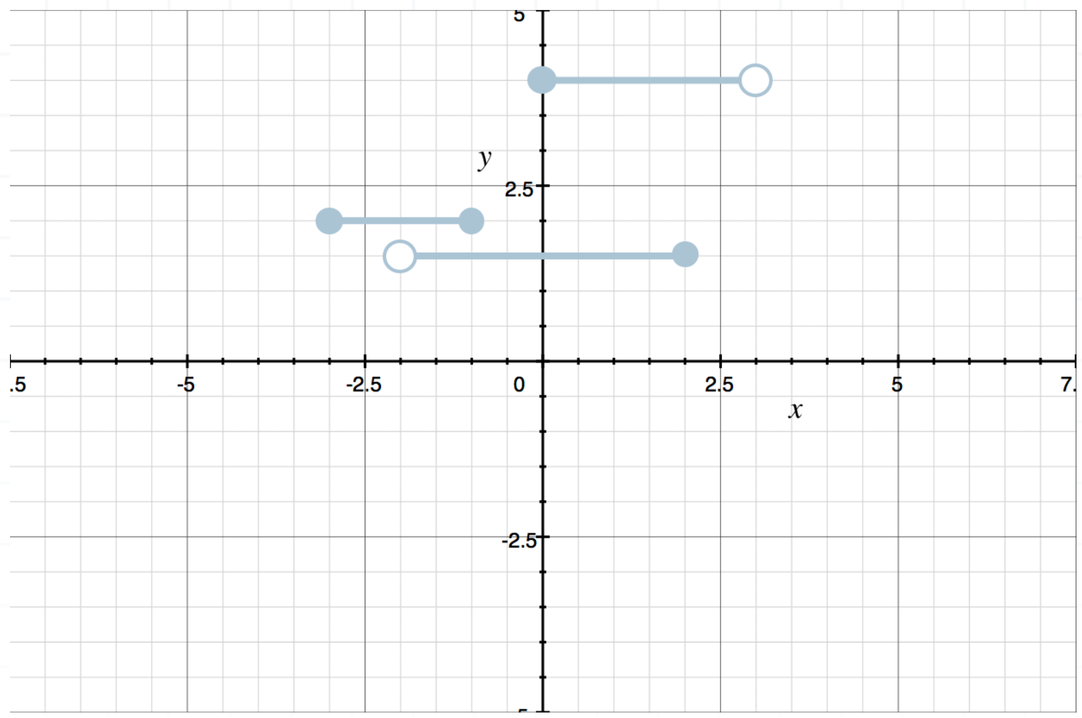


Solution:

The graph does not pass the Vertical Line Test, because any vertical line between the left edge of the circle and the right edge of the circle intersects the graph more than once. Therefore, the graph doesn't represent a function.

2. Use the Vertical Line Test to determine whether or not the graph represents a function. Hint: an empty circle indicates that exact point isn't

included in the graph, where a solid circle indicates that exact point is included in the graph.



Solution:

There are different vertical lines that intersect the graph more than once. An example would be $x = 0$, which intersects the graph at $y = 3/2$ and $y = 4$. So by the Vertical Line Test, the graph is not a graph of a function.

■ 3. Explain why the Vertical Line Test can determine whether or not a graph represents a function.

Solution:

There are many correct answers. But they should all more or less say something like:

> "The Vertical Line Test can show whether or not a graph represents a function, because if any perfectly vertical line crosses the graph more than once, it proves that there are two output values of y for the one input value of x."

■ 4. Fill in the blanks using the words "equations" and "functions."

Not all __________ are __________.

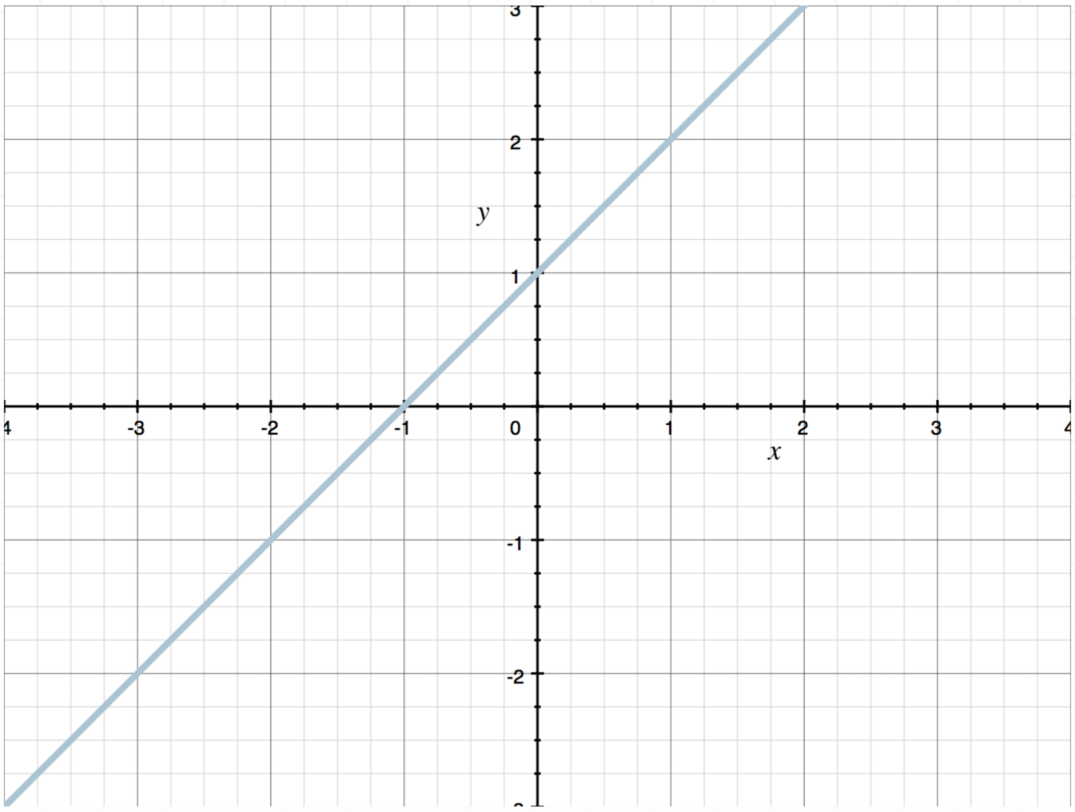
Solution:

The statement should read "Not all equations are functions."

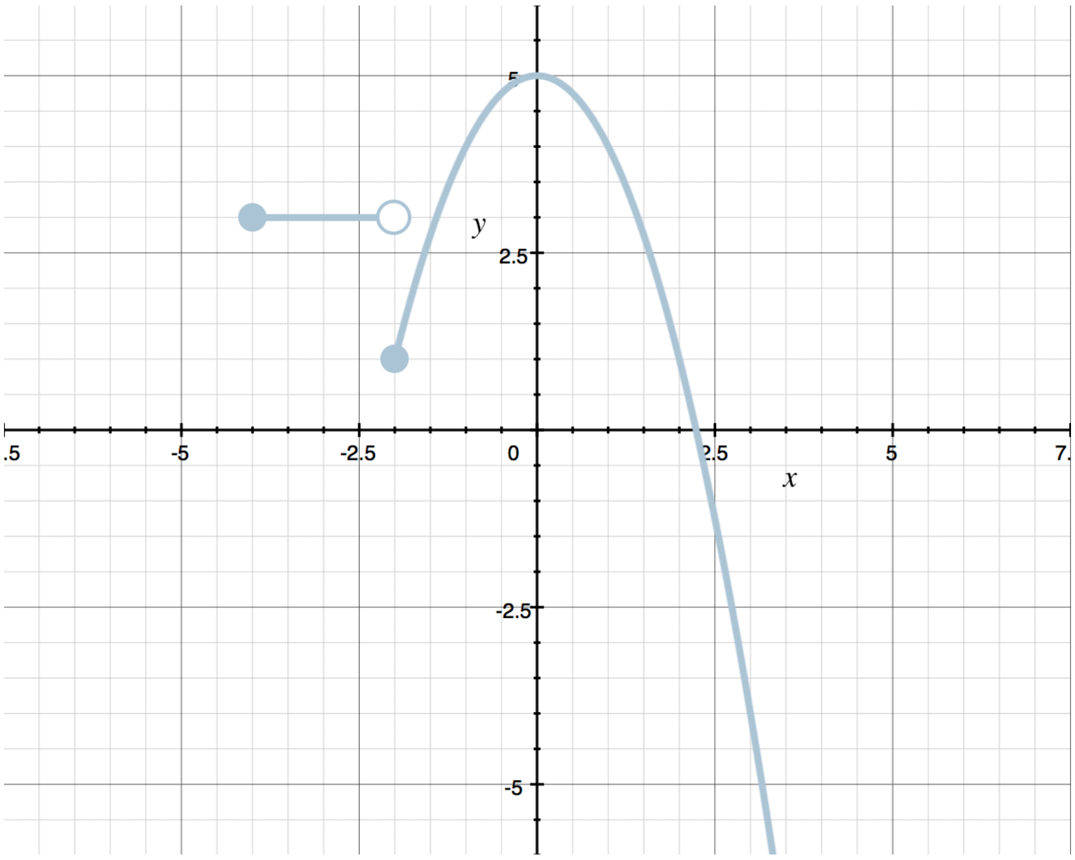
■ 5. Draw a graph that represents a function, and explain why it's a function.

Solution:

There are many correct answers. Below is an example of a function because, for every input x, there is only one output y. In fact, every line that's not a perfectly vertical line will always be a function.



6. Determine whether or not the graph represents a function. Hint: an empty circle indicates that exact point isn't included in the graph, where a solid circle indicates that exact point is included in the graph.



Solution:

For every x-value, there's a single unique y-value. So this graph represents a function. At $x = -2$, it looks like the graph might include two different y-values. But the empty circle at $y = 3$ indicates that point is not included in the graph, which means the input $x = -2$ only produces the one output at $y = 1$.

SUM OF FUNCTIONS

1. Find $(f+h)(-1)$ if $f(x)=x^2+1$ and $h(x)=2x-2$.

Solution:

Find $(f+h)(x)$.

$$(f+h)(x)=(x^2+1)+(2x-2)$$

$$(f+h)(x)=x^2+2x-1$$

To find $(f+h)(-1)$, we plug $x=-1$ into $(f+h)(x)$.

$$(f+h)(-1)=(-1)^2+2(-1)-1$$

$$(f+h)(-1)=1-2-1$$

$$(f+h)(-1)=-2$$

2. Find and simplify $(h+g)(x)$ if $g(x)=x^2+3x-1$ and $h(x)=-2x^2+4x-5$.

Solution:

To find $(h+g)(x)$, we'll start by finding $h(x)+g(x)$.

$$(h+g)(x)=h(x)+g(x)=(-2x^2+4x-5)+(x^2+3x-1)$$

which simplifies as

$$(h+g)(x) = -2x^2 + 4x - 5 + x^2 + 3x - 1$$

$$(h+g)(x) = -x^2 + 7x - 6$$

■ 3. If $f(-2) = 6$, $g(-2) = -3$, and $h(-2) = 4$, find $(f+g+h)(-2)$.

Solution:

By the definition of the sum of functions, we get

$$(f+g+h)(-2) = f(-2) + g(-2) + h(-2)$$

$$(f+g+h)(-2) = 6 + (-3) + 4$$

$$(f+g+h)(-2) = 7$$

■ 4. Find $f(x)$ and $g(x)$.

$$(f+g)(x) = (-x^2 + 3x + 2) + (x - 7)$$

Solution:

By the definition of the sum of functions, we can see that

$f(x) = -x^2 + 3x + 2$ and $g(x) = x - 7$

It could also be correct to say that

$$g(x) = -x^2 + 3x + 2 \text{ and } f(x) = x - 7$$

5. Let $a(x) = x^3 - x^2 + x - 1$ and $b(x) = -x^3 + x^2 + x - 1$. Determine the value of $(a + b)(-1)$.

Solution:

First, we can find the values for $a(-1)$ and $b(-1)$.

$$a(-1) = (-1)^3 - (-1)^2 + (-1) - 1$$

$$a(-1) = -1 - 1 - 1 - 1$$

$$a(-1) = -4$$

and

$$b(-1) = -(-1)^3 + (-1)^2 + (-1) - 1$$

$$b(-1) = 1 + 1 - 1 - 1$$

$$b(-1) = 0$$

Therefore,

$$(a + b)(-1) = a(-1) + b(-1)$$

$$(a + b)(-1) = -4 + 0$$

$$(a+b)(-1) = -4$$

6. If $f(0) = 3$ and $(f+g)(0) = 8$, find $g(0)$.

Solution:

By the definition of the sum of functions, we get

$$(f+g)(0) = f(0) + g(0)$$

$$(f+g)(0) = 3 + g(0)$$

Since $(f+g)(0) = 8$, we get

$$8 = 3 + g(0)$$

$$g(0) = 5$$

PRODUCT OF FUNCTIONS

1. Find and simplify $(ab)(x)$ if $a(x)=x+3$ and $b(x)=5x-4$.

Solution:

By the definition of the product of two functions, we have

$$(ab)(x)=a(x)b(x)$$

$$(ab)(x)=(x+3)(5x-4)$$

$$(ab)(x)=5x^2+15x-4x-12$$

$$(ab)(x)=5x^2+11x-12$$

2. Find $(fg)(-1)$ if $f(x)=x^2+3$ and $g(x)=x-5$.

Solution:

We'll find $f(-1)$ and $g(-1)$.

$$f(-1)=(-1)^2+3=1+3=4$$

$$g(-1)=(-1)-5=-6$$

Then the product of these functions is

$$(fg)(-1) = f(-1)g(-1)$$

$$(fg)(-1) = (4)(-6)$$

$$(fg)(-1) = -24$$

■ 3. If $g(0) = -2$ and $(gh)(0) = -14$, find $h(0)$.

Solution:

From the definition of the product of functions, we have

$$-14 = (gh)(0)$$

$$-14 = g(0)h(0)$$

$$-14 = (-2)h(0)$$

$$h(0) = 7$$

■ 4. Given the expanded expression, determine $f(x)$ and $g(x)$.

$$(gf)(x) = x^2(x-7) - x(x-7) + 5(x-7)$$

Solution:

Factor the $(x-7)$ out of the expression.

$$(gf)(x) = (x - 7)(x^2 - x + 5)$$

Then the two functions are $f(x) = (x - 7)$ and $g(x) = x^2 - x + 5$. We could also define the functions as $g(x) = (x - 7)$ and $f(x) = x^2 - x + 5$.

5. Find $(fh)(5)$ if $f(x) = -x^2 + 2x$ and $h(x) = 2x + 7$.

Solution:

By the definition of the product of functions, we have

$$(fh)(x) = f(x)h(x)$$

$$(fh)(x) = (-x^2 + 2x)(2x + 7)$$

$$(fh)(x) = -2x^3 + 4x^2 - 7x^2 + 14x$$

$$(fh)(x) = -2x^3 - 3x^2 + 14x$$

Evaluating the product at $x = 5$ gives

$$(fh)(5) = -2(5)^3 - 3(5)^2 + 14(5)$$

$$(fh)(5) = -250 - 75 + 70$$

$$(fh)(5) = -255$$

6. Find and simplify $(gh)(x)$ if $g(x) = x^2 + 1$ and $h(x) = 2x^2 + 3$.

Solution:

By the definition of the product of functions, we have

$$(gh)(x) = g(x)h(x)$$

$$(gh)(x) = (x^2 + 1)(2x^2 + 3)$$

$$(gh)(x) = 2x^4 + 3x^2 + 2x^2 + 3$$

$$(gh)(x) = 2x^4 + 5x^2 + 3$$

EVEN, ODD, OR NEITHER

1. Is the function even, odd, or neither?

$$f(x) = -x^5 + 2x^2 - 1$$

Solution:

Substitute $-x$ for x.

$$f(-x) = -(-x)^5 + 2(-x)^2 - 1$$

$$f(-x) = x^5 + 2x^2 - 1$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(-x^5 + 2x^2 - 1) = x^5 - 2x^2 + 1$$

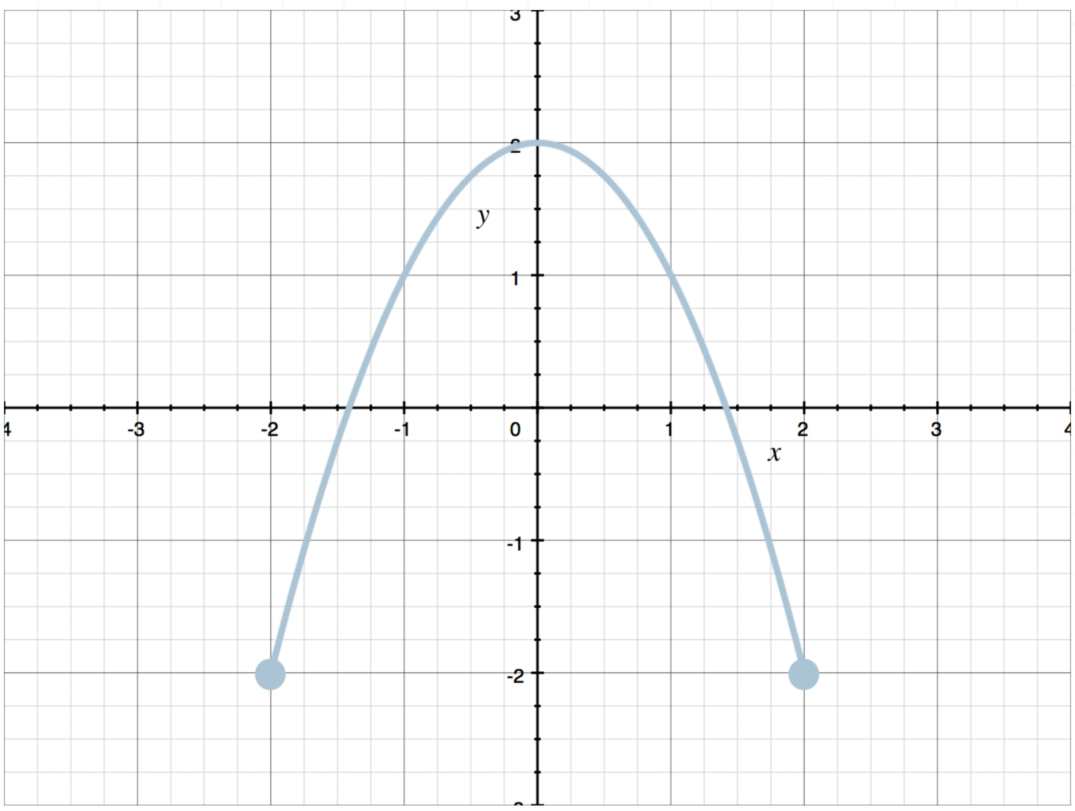
Because $f(-x) \neq -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

2. Describe the symmetry of an even function, and give an example of an even function.

Solution:

An even function is symmetric about the y-axis. There are many examples of even functions, one being $f(x) = x^2$.

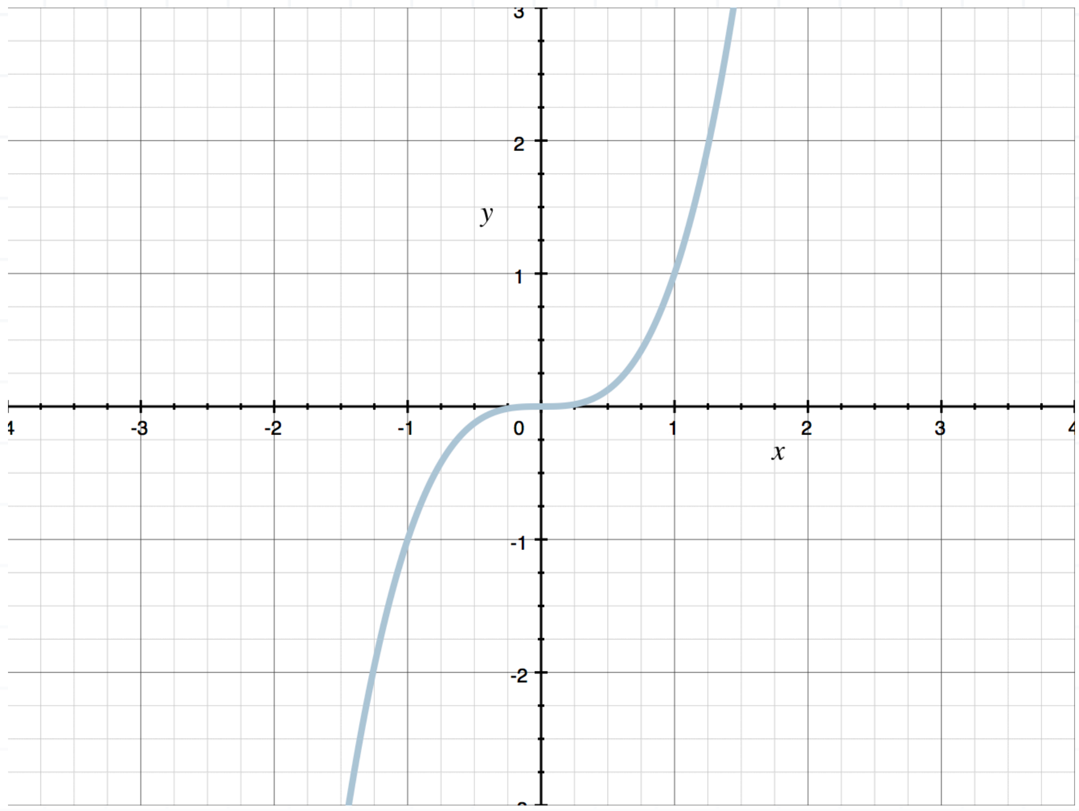
3. Determine whether the graph represents a function that's even, odd, or neither.



Solution:

Notice that the graph is symmetric about the y-axis and therefore the graph is the graph of an even function.

4. Determine whether the graph represents a function that's even, odd, or neither.



Solution:

Notice that the graph is symmetric about the origin, and therefore the graph is the graph of an odd function.

5. Is the function even, odd, or neither?

$$h(x) = x^3 - 3x$$

Solution:

Substitute $-x$ for x.

$$h(-x) = (-x)^3 - 3(-x)$$

$$h(-x) = -x^3 + 3x$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(x^3 - 3x)$$

$$-f(x) = -x^3 + 3x$$

Because $f(-x) = -f(x)$, the function is odd.

6. Is the function even, odd, or neither?

$$(-2,3),\ (-1,0),\ (0,-1),\ (1,0),\ (2,3)$$

Solution:

The y-values are symmetric across the vertical axis, so $f(-x) = f(x)$ and the function is even. For example, $f(-2) = f(2)$ and $f(-1) = f(1)$.

TRICHOTOMY

1. Solve the inequality.

$$2(x+1) \nleq -(8-x)$$

Solution:

By the Trichotomy Law, the inequality can be rewritten, and then simplified.

$$2(x+1) > -(8-x)$$

$$2x+2 > -8+x$$

$$2x > -10+x$$

$$x > -10$$

2. Give two ways to write the sentence in mathematical notation.

"x^2 is not greater than $4y$ and is also not equal to $4y$."

Solution:

The two ways to express the statement are

$x^2 \ngeq 4y$ and $x^2 < 4y$

■ 3. Give the three possible relationships in the Law of Trichotomy.

Solution:

The three statements of the Trichotomy Law are

If $a \ngeq b$ then $a < b$.

If $a \nleq b$ then $a > b$.

If $a \ngtr b$ and $a \nless b$ then $a = b$.

■ 4. Find a way to express the relationships as one equality or inequality.

$x^2 + x \nless 2$ and $x^2 + x \ngtr 2$

Solution:

By the Law of Trichotomy, we can rewrite the two statements as

$x^2 + x = 2$

■ 5. Give two ways to write the statement in mathematical notation.

"$3(x+1)$ is not less than $-x-5$ and is also not equal to $-x-5$."

Solution:

The two ways to write the statement are

$3(x+1) \nleq -x-5$ and $3(x+1) > -x-5$

6. Solve the statement.

$-3(1-x) \ngtr 3(7-x)-2x$ and $-3(1-x) \nless 3(7-x)-2x$

Solution:

By the Law of Trichotomy, we can rewrite the statement as

$-3(1-x) = 3(7-x)-2x$

INEQUALITIES AND NEGATIVE NUMBERS

■ 1. Solve the inequality.

$$-3x + 4 < 22$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -3.

$$-3x + 4 < 22$$

$$-3x < 18$$

$$x > -6$$

■ 2. What went wrong in this set of steps?

$$-5x + 6 < 9 - 2x$$

$$-3x < 3$$

$$x < -1$$

Solution:

When the inequality was divided by -3, the inequality sign was not flipped. The solution should be $x > -1$.

■ 3. Solve the inequality.

$$-(5-2x) \geq 3(x-3)+2x$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -3.

$$-(5-2x) \geq 3(x-3)+2x$$

$$-5+2x \geq 3x-9+2x$$

$$-5+2x \geq 5x-9$$

$$-5-3x \geq -9$$

$$-3x \geq -4$$

$$x \leq \frac{4}{3}$$

■ 4. Solve the inequality.

$$-6x+7 > -3x+2$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -3.

$$-6x + 7 > -3x + 2$$

$$-3x + 7 > 2$$

$$-3x > -5$$

$$x < \frac{5}{3}$$

■ 5. What went wrong in this set of steps?

$$-2(x + 1) \geq 3(2 + x)$$

$$-2x - 2 \geq 6 + 3x$$

$$-2x - 3x - 2 \leq 6$$

Solution:

The inequality sign was flipped when $3x$ was subtracted from each side, but it should have remained the same and not been flipped.

6. Solve the inequality.

$$7(1 - x) \leq 2x$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -9.

$$7(1 - x) \leq 2x$$

$$7 - 7x \leq 2x$$

$$-7x \leq 2x - 7$$

$$-9x \leq -7$$

$$x \geq \frac{7}{9}$$

GRAPHING INEQUALITIES ON A NUMBER LINE

1. Give two inequalities that, when graphed on a number line, have open circles at $x = 3$.

Solution:

There are many correct answers. For example, $x < 3$ and $x > 3$ would both have open circles at $x = 3$.

2. Graph the inequality on a number line.

$$-2x < 4$$

Solution:

Rewrite the inequality, remembering to flip the sign when we divide by -2.

$$-2x < 4$$

$$x > -2$$

Then a graph of the inequality is

3. Graph the inequality on a number line.

$$x - 1 \geq 3$$

Solution:

Rewrite the inequality.

$$x - 1 \geq 3$$

$$x \geq 4$$

Then a graph of the inequality is

4. Graph the inequality on a number line.

$$5(-x + 3) < -3x + 7$$

Solution:

Rewrite the inequality, remembering to flip the sign when we divide by -2.

$$5(-x + 3) < -3x + 7$$

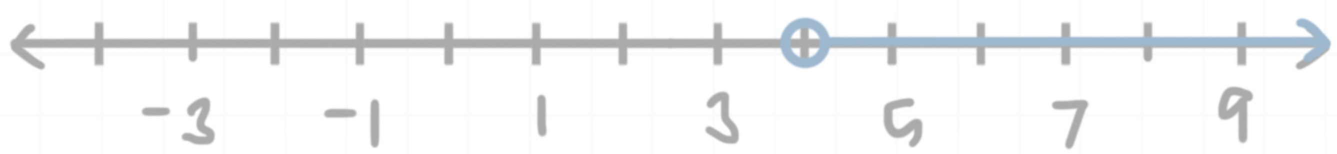
$$-5x + 15 < -3x + 7$$

$$-5x < -3x - 8$$

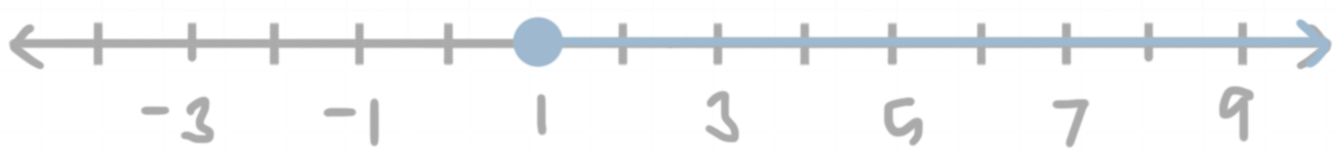
$$-2x < -8$$

$$x > 4$$

Then a graph of the inequality is



5. What's wrong with this graph of $x > 1$?



Solution:

There should be an open circle at 1, not a solid circle, since the inequality $x > 1$ does not include the value $x = 1$.

6. Graph the inequality on a number line.

$$5(x + 7) - x \geq 3(x + 10) + 6$$

Solution:

Rewrite the inequality.

$$5(x+7)-x \geq 3(x+10)+6$$

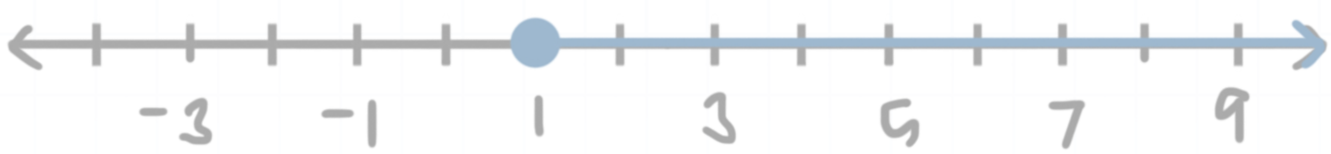
$$5x+35-x \geq 3x+30+6$$

$$4x+35 \geq 3x+36$$

$$4x \geq 3x+1$$

$$x \geq 1$$

Then a graph of the inequality is



GRAPHING DISJUNCTIONS ON A NUMBER LINE

1. What's wrong with the graph of the disjunction?

$2x \leq 4$ or $x - 5 > 3$

Solution:

First, we'll solve and graph the two inequalities separately.

$2x \leq 4$ $\qquad\qquad$ $x - 5 < 3$

$x \leq 2$ $\qquad\qquad$ $x > 8$

The graph of the inequality $x \leq 2$ has a solid circle at 2, and the arrow goes to the left.

The graph of the inequality $x > 8$ has an open circle at 8, and the arrow goes to the right.

Therefore, the sketch of the disjunction is

2. Graph the disjunction.

$x + 2 \geq 2x + 3$ or $x - 5 \geq 0$

Solution:

First, we'll solve and graph the two inequalities separately.

$x + 2 \geq 2x + 3$

$-x + 2 \geq 3$

$-x \geq 1$

$x \leq -1$

$x - 5 \geq 0$

$x \geq 5$

The graph of the inequality $x \leq -1$ has a solid circle at -1, and the arrow goes to the left.

The graph of the inequality $x \geq 5$ has a solid circle at 5, and the arrow goes to the right.

Therefore, the sketch of the disjunction is

3. Graph the disjunction of the inequalities.

$$2(x-3)+x<2x+1 \text{ or } 2(x-1)-6>6$$

Solution:

First, we'll solve and graph the two inequalities separately.

$$2(x-3)+x<2x+1$$

$$2x-6+x<2x+1$$

$$3x-6<2x+1$$

$$x-6<1$$

$$x<7$$

$$2(x-1)-6>6$$

$$2(x-1)>12$$

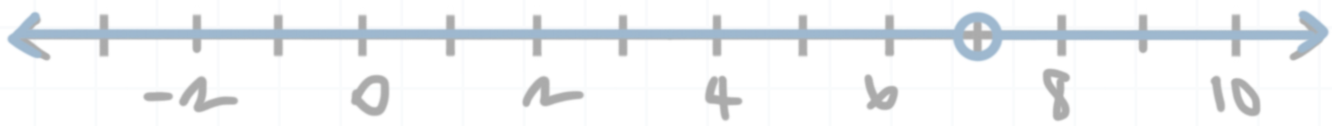
$$x-1>6$$

$$x>7$$

The graph of the inequality $x<7$ has an open circle at 7, and the arrow goes to the left.

The graph of the inequality $x > 7$ has an open circle at 7, and the arrow goes to the right.

Therefore, the sketch of the disjunction is



4. What's wrong with the graph of the disjunction?

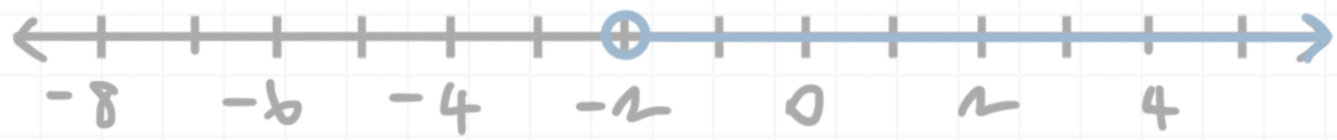
$-x+3<5$ or $-2(x+2) \geq 2$

Solution:

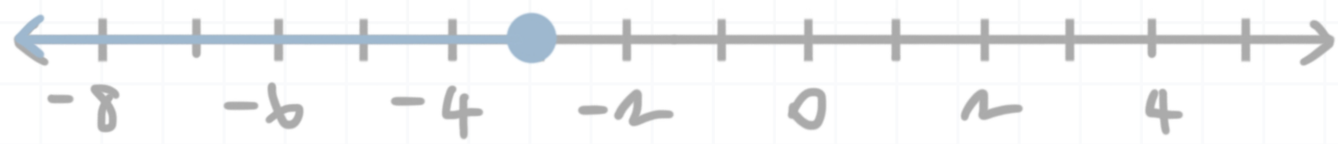
First, we'll solve and graph the two inequalities separately.

$-x+3<5$	$-2(x+2) \geq 2$
$-x<2$	$x+2 \leq -1$
$x>-2$	$x \leq -3$

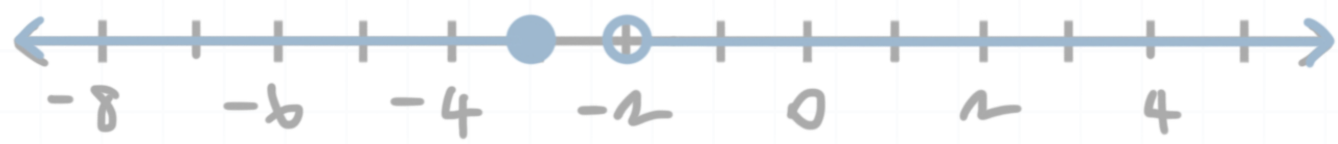
The graph of the inequality $x > -2$ has an open circle at -2, and the arrow goes to the right.



The graph of the inequality $x \leq -3$ has a solid circle at -3, and the arrow goes to the left.



Therefore, the sketch of the disjunction is



5. Graph the disjunction.

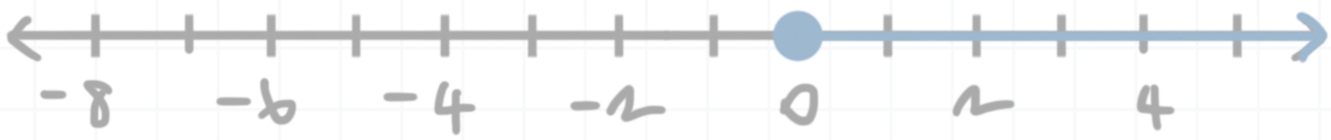
$2x + 3 \geq 3$ or $2x + 5 < x$

Solution:

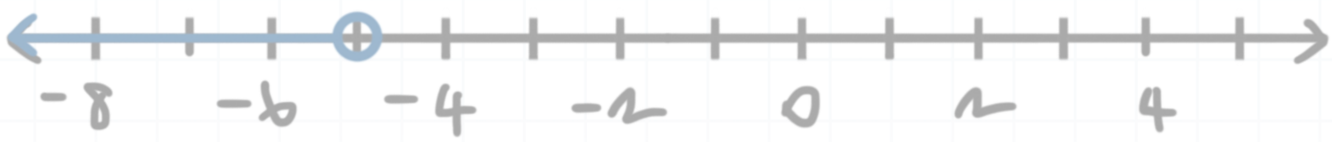
First, we'll solve and graph the two inequalities separately.

$2x + 3 \geq 3$	$2x + 5 < x$
$2x \geq 0$	$x + 5 < 0$
$x \geq 0$	$x < -5$

The graph of the inequality $x \geq 0$ has a solid circle at 0, and the arrow goes to the right.



The graph of the inequality $x < -5$ has an open circle at -5, and the arrow goes to the left.



Therefore, the sketch of the disjunction is

■ 6. Graph the disjunction.

$$-2x + 5 \geq -1 \text{ or } x - 6 > -2$$

Solution:

The disjunction is the combination of

$$-2x + 5 \geq -1$$

$$-2x \geq -6$$

$x \leq 3$

and

$x - 6 > -2$

$x > 4$

So the disjunction "$x \leq 3$ or $x > 4$" is graphed as

GRAPHING CONJUNCTIONS ON A NUMBER LINE

1. Graph the conjunction of the inequalities $3(x-4) < x-2$ and $-2(x-6)+3 \geq 5$.

Solution:

First, we'll solve and graph the two inequalities separately.

$$3(x-4) < x-2 \qquad -2(x-6)+3 \geq 5$$

$$3x-12 < x-2 \qquad -2(x-6) \geq 2$$

$$2x-12 < -2 \qquad x-6 \leq -1$$

$$2x < 10 \qquad x \leq 5$$

$$x < 5$$

The graph of the inequality $x < 5$ has an open circle at 5, and the arrow goes to the left.

The graph of the inequality $x \leq 5$ has a solid circle at 5, and the arrow goes to the left.

Therefore, the sketch of the conjunction is

2. Graph the conjunction.

$$-8 \leq -2x < 10$$

Solution:

Simplify the conjunction.

$$-8 \leq -2x < 10$$

$$4 \geq x > -5$$

$$-5 < x \leq 4$$

Then we can graph the conjunction on a number line.

3. What's wrong with the graph of the conjunction?

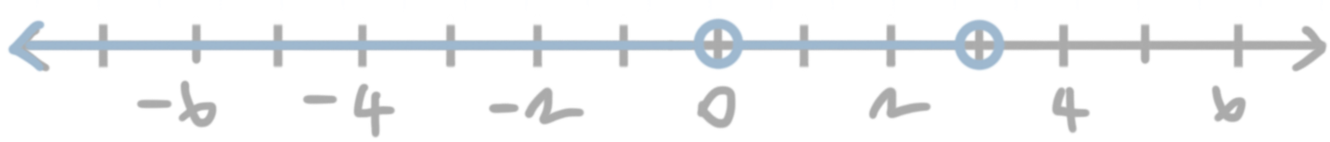
$x \leq 3$ and $x > -4$

Solution:

There should be an open circle at -4 since the inequality $x > -4$ does not include $x = -4$, and there should be a solid circle at 3 since the inequality $x \leq 3$ includes $x = 3$.

4. What's wrong with the graph of the conjunction?

$x \leq 3$ and $x \neq 0$

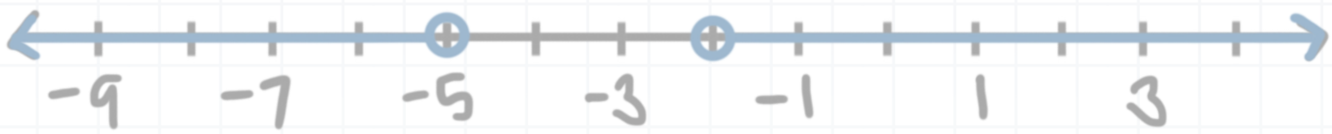


Solution:

There should be a closed circle at 3 since the inequality $x \leq 3$ includes the value $x = 3$.

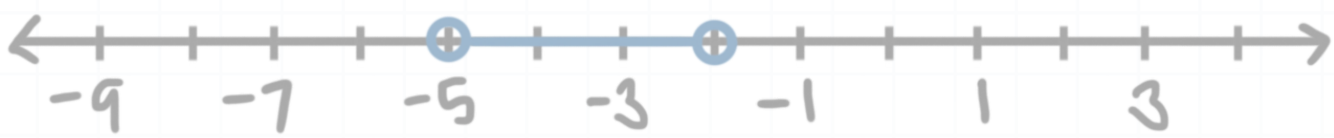
5. What's wrong with the graph of the conjunction?

$x < -2$ and $x > -5$



Solution:

The graph is showing the disjunction "$x < -5$ or $x > -2$" instead of the conjunction "$x < -2$ and $x > -5$." The graph should be



■ 6. Graph the conjunction.

$2x - 1 \geq 3$ and $-x \geq -9$

Solution:

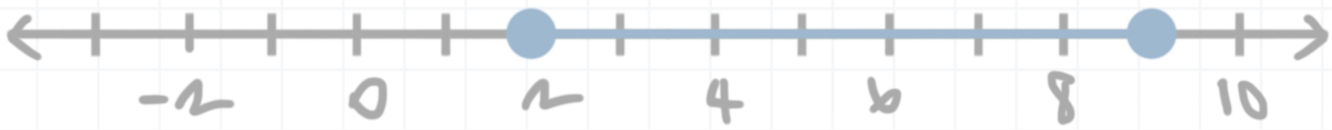
The individual inequalities simplify to

$2x - 1 \geq 3$ $\qquad\qquad$ $-x \geq -9$

$2x \geq 4$ $\qquad\qquad$ $x \leq 9$

$x \geq 2$

So $x \geq 2$ and $x \leq 9$ form a conjunction that is graphed as


-2
0
2
4
6
8
10

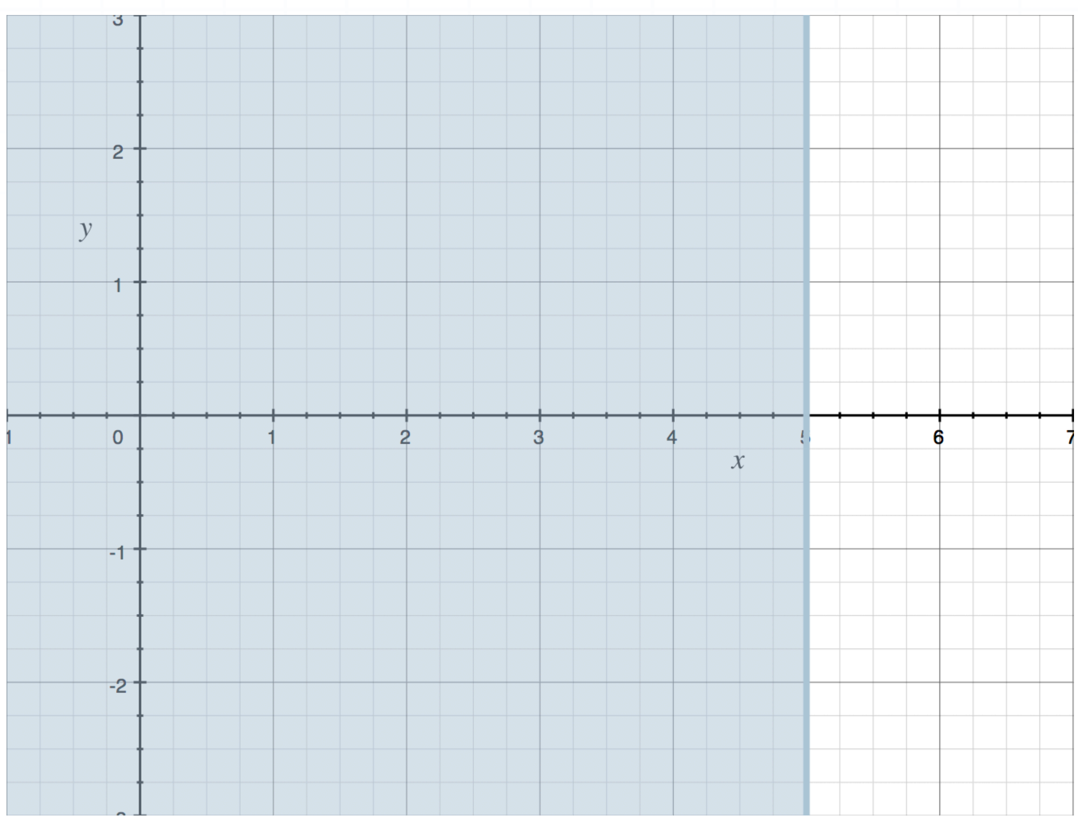
GRAPHING INEQUALITIES IN THE PLANE

1. Graph the inequality in the Cartesian coordinate plane.

$$x \leq 5$$

Solution:

Start by graphing the vertical line $x = 5$. Make it a solid line since the inequality is "less than or equal to." Since the inequality is "less than," we'll shade to the left of the vertical line.



2. Graph the inequality in the Cartesian coordinate plane.

$$y < -2x + 4$$

Solution:

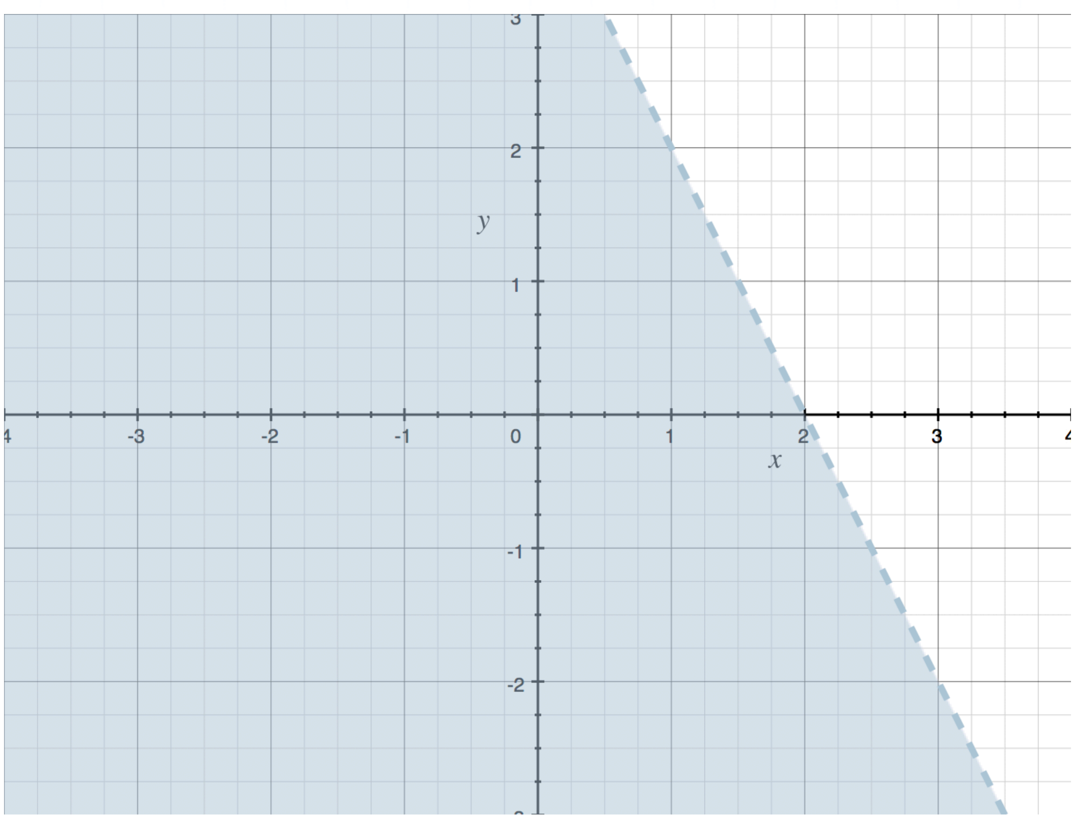
Start by graphing the line $y = -2x + 4$. Make it a dashed line since the inequality is strictly "less than." To determine where to shade, let's test $(0,0)$ by substituting it into the inequality.

$$y < -2x + 4$$

$$0 < -2(0) + 4$$

$$0 < 4$$

Since this inequality is true, we shade on the side of the dashed line that contains the test point $(0,0)$.

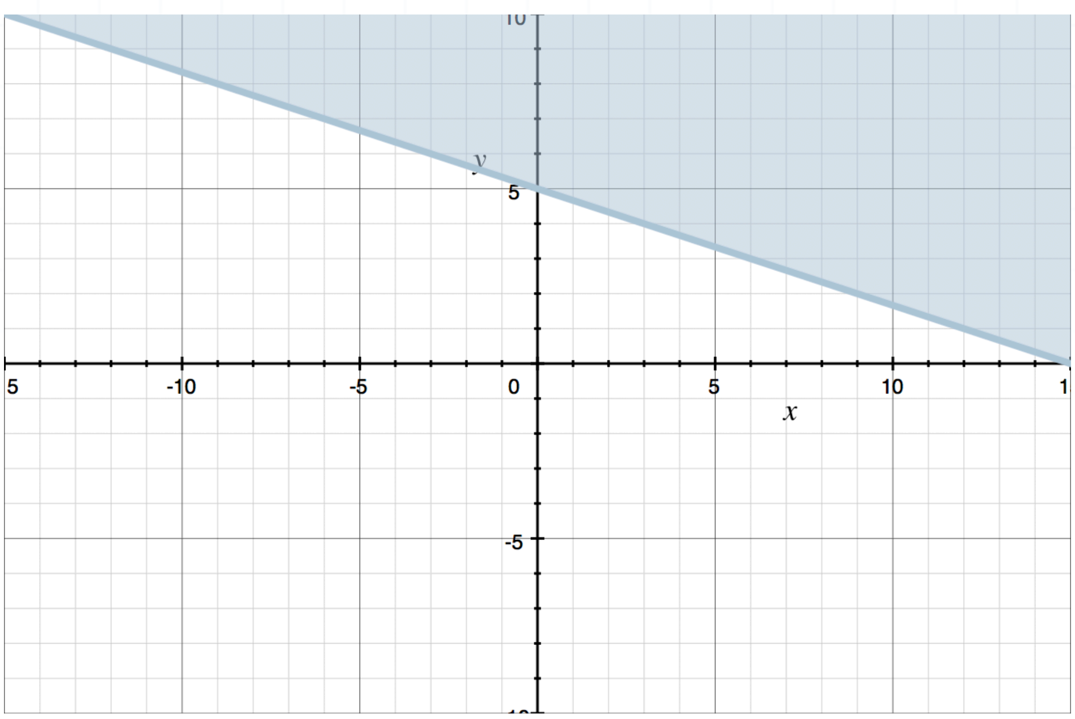


3. Graph the inequality in the Cartesian coordinate plane.

$$y \geq -\frac{1}{3}x + 5$$

Solution:

Start by graphing the line $y = -(1/3)x + 5$. Make it a solid line since the inequality is "greater than or equal to." Since the inequality is "greater than," we'll shade above the line.



4. Graph the inequality in the Cartesian coordinate plane.

$$y \leq x - 1$$

Solution:

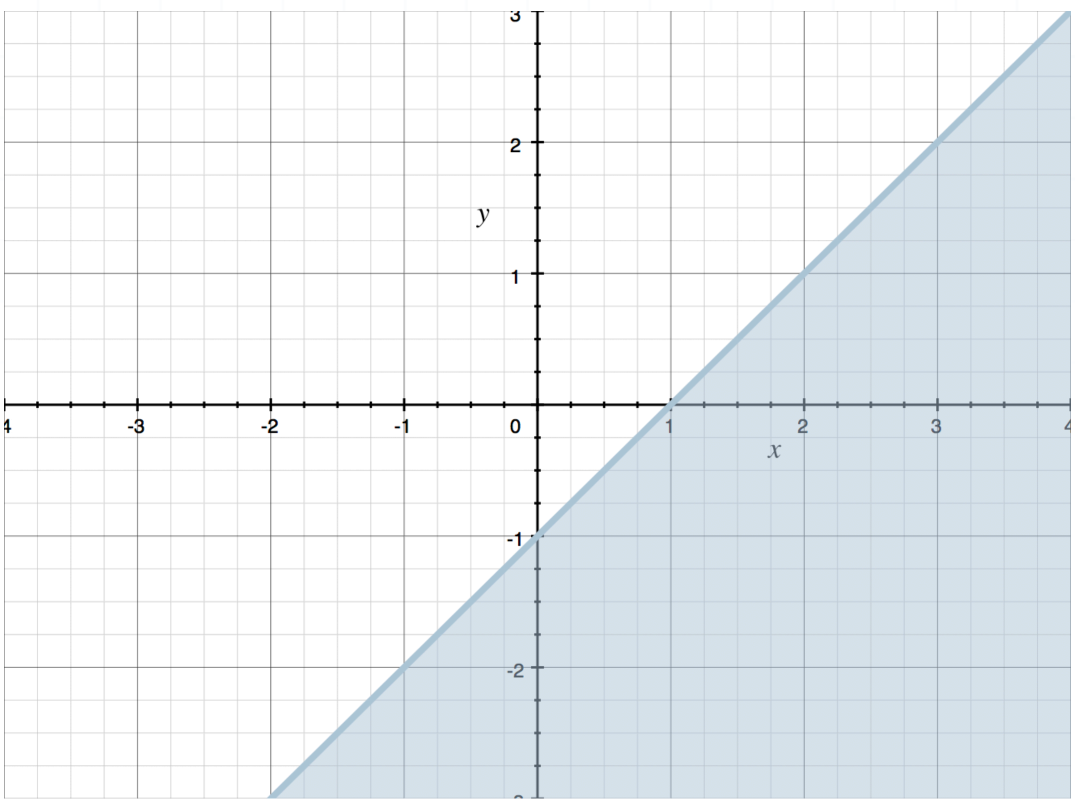
Start by graphing the line $y = x - 1$. Make it a solid line since the inequality is "less than or equal to." To determine where to shade, let's test $(0,0)$ by substituting it into the inequality.

$$y \le x - 1$$

$$0 \le 0 - 1$$

$$0 \le -1$$

Since this inequality is false, we shade on the side of the solid line that does not contain the test point $(0,0)$.

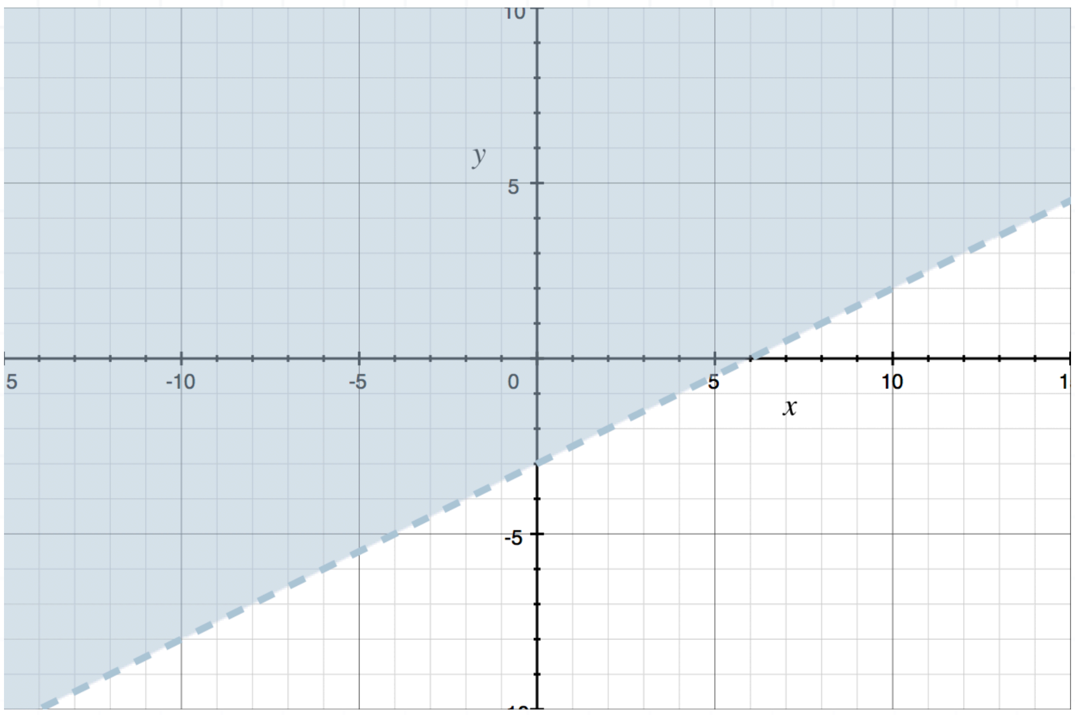


5. Graph the inequality in the Cartesian coordinate plane.

$$y > \frac{1}{2}x - 3$$

Solution:

Start by graphing the line $y = (1/2)x - 3$. Make it a dashed line since the inequality is strictly "greater than." Since the inequality is "greater than," we'll shade above the line.



6. Graph the inequality in the Cartesian coordinate plane.

$$y \geq 3x - 2$$

Solution:

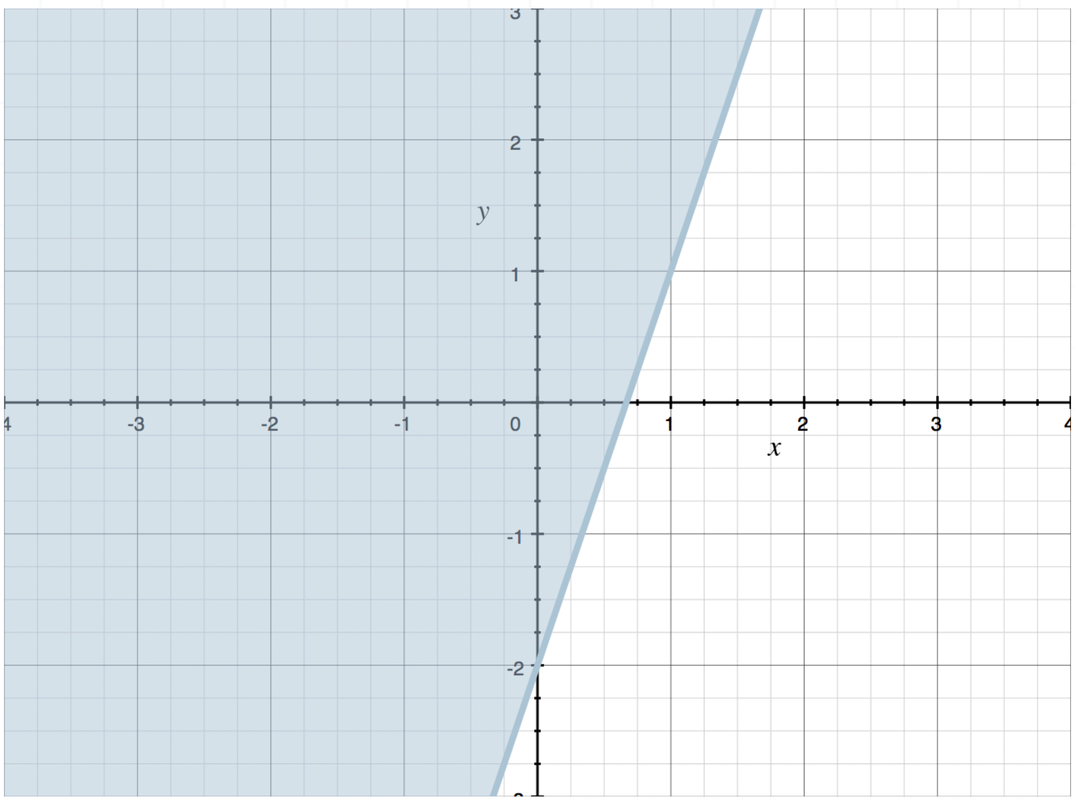
Start by graphing the line $y = 3x - 2$. Make it a solid line since the inequality is "greater than or equal to." To determine where to shade, let's test $(0,0)$ by substituting it into the inequality.

$$y \geq 3x - 2$$

$$0 \geq 3(0) - 2$$

$$0 \geq -2$$

Since this inequality is true, we shade on the side of the solid line that contains the test point $(0,0)$.



ABSOLUTE VALUE EQUATIONS

1. Solve $|3-x|=1$.

Solution:

Solve the two related equations.

$$3-x=1 \qquad\qquad 3-x=-1$$

$$x=2 \qquad\qquad x=4$$

Check each solution by substituting them into the original absolute value equation.

Check $x=2$:

$$|3-x|=1$$

$$|3-2|=1$$

$$|1|=1$$

$$1=1$$

Check $x=4$:

$$|3-x|=1$$

$$|3-4|=1$$

$$|-1|=1$$

$$1=1$$

Both equations are true, so $x=2$ and $x=4$ are the solutions to the absolute value equation.

2. Solve $|4x - 8| = 3x - 6$.

Solution:

Solve the two related equations.

$$4x - 8 = 3x - 6$$

$$x = 2$$

$$4x - 8 = -(3x - 6)$$

$$7x = 14$$

$$x = 2$$

Check whether $x = 2$ is a solution by substituting it into the original absolute value equation.

$$|4x - 8| = 3x - 6$$

$$|4(2) - 8| = 3(2) - 6$$

$$|0| = 0$$

$$0 = 0$$

The equation is true, so $x = 2$ is the solution to the absolute value equation.

3. Solve $|2x - 2| = x - 6$.

Solution:

Solve the two related equations.

$$2x - 2 = x - 6$$

$$x = -4$$

$$2x - 2 = -(x - 6)$$

$$3x = 8$$

$$x = \frac{8}{3}$$

Check each solution by substituting them into the original absolute value equation.

Check $x = -4$:

$$|2x - 2| = x - 6$$

$$|2(-4) - 2| = -4 - 6$$

$$|-10| = -10$$

$$10 = -10$$

Check $x = 8/3$:

$$|2x - 2| = x - 6$$

$$\left|2 \cdot \frac{8}{3} - 2\right| = \frac{8}{3} - 6$$

$$\left|\frac{10}{3}\right| = -\frac{10}{3}$$

$$\frac{10}{3} = -\frac{10}{3}$$

Both equations are false, so there are no solutions to the absolute value equations.

■ 4. Solve $|3x + 1| + x = 1$.

Solution:

First, isolate the absolute value on the left side of the equation.

$$|3x+1|+x=1$$

$$|3x+1|=1-x$$

Solve the two related equations.

$$3x+1=1-x$$

$$4x=0$$

$$x=0$$

$$3x+1=-(1-x)$$

$$2x=-2$$

$$x=-1$$

Check each solution by substituting them into the original absolute value equation.

Check $x=0$:

$$|3x+1|+x=1$$

$$|3(0)+1|+0=1$$

$$|1|=1$$

$$1=1$$

Check $x=-1$:

$$|3x+1|+x=1$$

$$|3(-1)+1|-1=1$$

$$|-2|=2$$

$$2=2$$

Both equations are true, so $x=0$ and $x=-1$ are the solutions to the absolute value equation.

5. Solve $|2x+5|=3x+6$.

Solution:

Solve the two related equations.

$$2x+5=3x+6$$

$$-x=1$$

$$x=-1$$

$$2x+5=-(3x+6)$$

$$5x=-11$$

$$x=-\frac{11}{5}$$

Check each solution by substituting them into the original absolute value equation.

Check $x=-1$:

$$|2x+5|=3x+6$$

$$|2(-1)+5|=3(-1)+6$$

$$|3|=3$$

$$3=3$$

Check $x=-11/5$:

$$|2x+5|=3x+6$$

$$\left|2\left(-\frac{11}{5}\right)+5\right|=3\left(-\frac{11}{5}\right)+6$$

$$\left|\frac{3}{5}\right|=-\frac{3}{5}$$

$$\frac{3}{5}=-\frac{3}{5}$$

The equation is only true for $x=-1$, so the only solution to the absolute value equation is $x=-1$.

6. Solve $|3x+2| = |3x+4|$.

Solution:

Solve the two related equations.

$3x+2=3x+4$	$3x+2=-(3x+4)$
$0=2$	$6x=-6$
No solutions	$x=-1$

Check whether $x=-1$ is a solution by substituting it into the original absolute value equation.

$$|3x+2| = |3x+4|$$

$$|3(-1)+2| = |3(-1)+4|$$

$$|-1| = |1|$$

$$1=1$$

The equation is true, so $x=-1$ is the solution to the absolute value equation.

ABSOLUTE VALUE INEQUALITIES

1. Rewrite the inequality by taking away the absolute value.

$$|3x-7| \geq 2$$

Solution:

We can take away the absolute value by rewriting the inequality as the disjunction

$$3x-7 \geq 2 \text{ or } 3x-7 \leq -2$$

2. Graph the inequality.

$$5|1-x|-7<3$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$5|1-x|-7<3$$

$$5|1-x|<10$$

$$|1-x|<2$$

Since $2 > 0$, we can rewrite the inequality as the conjunction

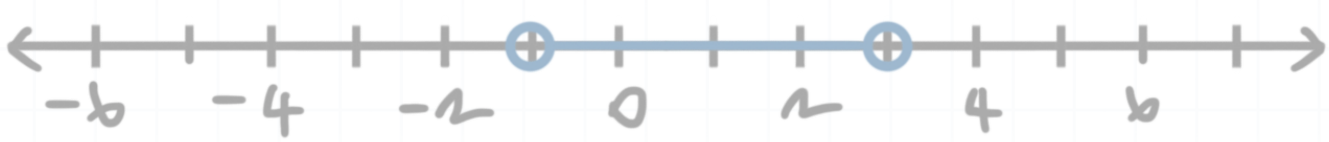
$$-2 < 1 - x < 2$$

$$-3 < -x < 1$$

$$3 > x > -1$$

$$-1 < x < 3$$

A sketch of the inequality is therefore



3. Graph the inequality.

$$2(|x-4|-1)+6 \le 4$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$2(|x-4|-1)+6 \le 4$$

$$2(|x-4|-1) \le -2$$

$$|x-4|-1 \le -1$$

$$|x-4| \le 0$$

The absolute value is always positive, so this expression is telling us

“positive or zero” $\leq$ zero

We can’t have “positive $\leq$ zero,” so the only equation that’s possible is “zero $\leq$ zero”, so we get

$$x - 4 = 0$$

$$x = 4$$

A sketch of the inequality is therefore

4. Graph the inequality.

$$-2|x + 2| - 3 \geq 1$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$-2|x + 2| - 3 \geq 1$$

$$-2|x + 2| \geq 4$$

$$|x + 2| \leq -2$$

The absolute value is always positive, so this expression is telling us

positive $\leq$ negative

There's no value that can make this inequality true, so there's no solution, and a sketch of the inequality is therefore an empty number line.

■ 5. Graph the inequality.

$$2(3+|x-5|)-4 \geq 10$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$2(3+|x-5|)-4 \geq 10$$

$$2(3+|x-5|) \geq 14$$

$$3+|x-5| \geq 7$$

$$|x-5| \geq 4$$

Since $4>0$, we can rewrite the inequality as the disjunction

$4 \leq x-5$ or $x-5 \leq -4$

$9 \leq x$ or $x \leq 1$

A sketch of the inequality is therefore

6. Graph the inequality.

$$|6 - 2x| \leq 4$$

Solution:

Since $4 > 0$, we can rewrite the inequality as the conjunction

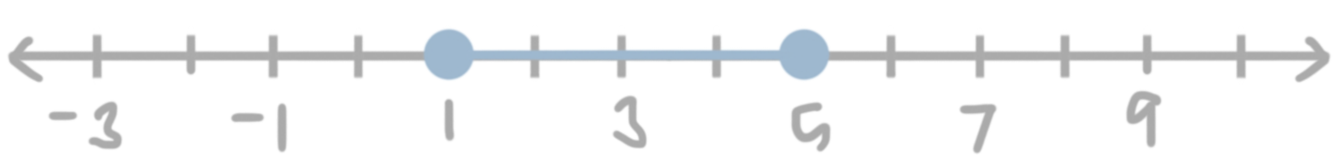
$$-4 \leq 6 - 2x \leq 4$$

$$-10 \leq -2x \leq -2$$

$$5 \geq x \geq 1$$

$$1 \leq x \leq 5$$

A sketch of the inequality is therefore



TWO-STEP PROBLEMS

1. Why can't we solve this two-step problem?

If $2(x-1)-3=9+x$, what is $y+2$?

Solution:

We can't solve the problem because there's no y variable in the first equation, so we can't get a value from the first equation to plug into $y+2$.

2. If $5-2x=17$, what is $x-1$?

Solution:

Solve the first equation for x.

$$5-2x=17$$

$$-2x=12$$

$$x=-6$$

Now plug $x=-6$ into the second expression.

$$x-1$$

$$-6-1$$

$$-7$$

3. If $3(2-x)+5=-(4x-2)$, what is $(x/2)+1$?

Solution:

Solve the first equation for x.

$$3(2-x)+5=-(4x-2)$$

$$6-3x+5=-4x+2$$

$$11-3x=-4x+2$$

$$9=-x$$

$$x=-9$$

Now plug $x=-9$ into the second expression.

$$\frac{x}{2}+1$$

$$\frac{-9}{2}+1$$

$$-\frac{9}{2}+\frac{2}{2}$$

$$-\frac{7}{2}$$

4. If $2(x+y)-6=3$, what is $x+y-1$?

Solution:

Solve the first equation for x.

$$2(x+y)-6=3$$

$$2(x+y)=9$$

$$x+y=\frac{9}{2}$$

Now substitute this value into the expression.

$$x+y-1$$

$$\frac{9}{2}-1$$

$$\frac{9}{2}-\frac{2}{2}$$

$$\frac{7}{2}$$

5. What went wrong in this solution?

If $2x + 3 = 7$, what is $x/3$?

$$2x + 3 = 7$$

$$2x = 4$$

$$\frac{x}{3} = \frac{4}{3}$$

Solution:

The equation wasn't solved for x. In the second step, it should be $2x = 4$ gives $x = 2$. Then $x = 2$ should get plugged into the expression $x/3$, $x/3 = 2/3$.

6. If $a + 2b = 6 - a$ and $b = 1$, what is $a/2$?

Solution:

First, plug $b = 1$ into $a + 2b = 6 - a$ and solve for a.

$$a + 2(1) = 6 - a$$

$$2a = 4$$

$$a = 2$$

Then plug $a = 2$ into $a/2$.

$$\frac{a}{2}$$

$$\frac{2}{2}$$

$$1$$

SOLVING SYSTEMS WITH SUBSTITUTION

1. Find the unique solution to the system of equations.

$$-x + 2y = 6$$

$$3x = y - 10$$

Solution:

Solve for x in the second equation.

$$3x = y - 10$$

$$x = \frac{y - 10}{3}$$

Plug this value for x into the first equation, then solve for y.

$$-x + 2y = 6$$

$$-\frac{y - 10}{3} + 2y = 6$$

$$-y + 10 + 6y = 18$$

$$5y = 8$$

$$y = \frac{8}{5}$$

Plug $y = 8/5$ back into the equation we found for x.

$$x = \frac{y - 10}{3}$$

$$x = \frac{\frac{8}{5} - 10}{3}$$

$$x = \frac{\frac{8}{5} - \frac{50}{5}}{3}$$

$$x = -\frac{42}{5} \cdot \frac{1}{3}$$

$$x = -\frac{14}{5}$$

The unique solution to the system is

$$\left(-\frac{14}{5}, \frac{8}{5}\right)$$

2. What is the easiest variable to get by itself? Set up but do not solve the substitution.

$$2y - x = 7$$

$$3x = 9 - 18y$$

Solution:

It's easiest to solve for the x variable in the second equation by dividing both sides by 3 and then simplifying.

$$x = \frac{9 - 18y}{3}$$

$$x = \frac{9}{3} - \frac{18y}{3}$$

$$x = 3 - 6y$$

3. Find the unique solution to the system of equations.

$$-5x + y = 8$$

$$y = 3x - 8$$

Solution:

Taking the value for y given in the second equation as $y = 3x - 8$, we'll substitute for y in the first equation.

$$-5x + y = 8$$

$$-5x + (3x - 8) = 8$$

$$-5x + 3x - 8 = 8$$

$$-2x = 16$$

$$x = -8$$

Now substitute $x = -8$ into the second equation to find a value for y.

$$y = 3x - 8$$

$$y = 3(-8) - 8$$

$$y = -32$$

The unique solution to the system is

$$(-8, -32)$$

4. Find the unique solution to the system of equations.

$$3 - y = 2x$$

$$-4x + 10 = 2y$$

Solution:

Solve the second equation for y.

$$-4x + 10 = 2y$$

$$-2x + 5 = y$$

Plug $y = -2x + 5$ into the first equation.

$$3 - y = 2x$$

$$3 - (-2x + 5) = 2x$$

$$3 + 2x - 5 = 2x$$

$$-2 + 2x = 2x$$

$$-2 = 0$$

Since this is not true, there is no solution to the system.

5. What went wrong if a substitution was made in the system and the result was $2x - 2 - x = 7$?

$$y = x - 2$$

$$2y - x = 7$$

Solution:

When substituting $y = x - 2$ into the second equation, we get

$$2y - x = 7$$

$$2(x - 2) - x = 7$$

$$2x - 4 - x = 7$$

Therefore, in the substitution given, the 2 was not distributed to the -2.

6. Find the unique solution to the system of equations.

$$5y = 6 - 2x$$

$$6x + 15y = 18$$

Solution:

Solve for y in the first equation.

$$5y = 6 - 2x$$

$$y = \frac{6 - 2x}{5}$$

Plug this value for y into the second equation.

$$6x + 15y = 18$$

$$6x + 15\left(\frac{6 - 2x}{5}\right) = 18$$

$$6x + 3(6 - 2x) = 18$$

$$6x + 18 - 6x = 18$$

$$18 = 18$$

Since this equation is true, but we don't find a specific value for either variable, there are infinitely many solutions.

SOLVING SYSTEMS WITH ELIMINATION

1. What's the easiest way to set up the elimination method for the system of equations? Set up but do not solve the elimination.

$$6y - 3x = 8$$

$$x - 4y = 5$$

Solution:

The easiest way to solve the elimination is to multiply the second equation by 3 to get

$$x - 4y = 5$$

$$3x - 12y = 15$$

Then add the two equations together to eliminate x from the system.

$$6y - 3x + (3x - 12y) = 8 + (15)$$

$$6y - 12y = 8 + 15$$

$$-6y = 23$$

2. Find the unique solution to the system of equations.

$$2x - y = 5$$

$$-3x + y = 7$$

Solution:

If we add the two equations to eliminate y, we get

$$2x - y + (-3x + y) = 5 + (7)$$

$$2x - 3x = 12$$

$$-x = 12$$

$$x = -12$$

Plug $x = -12$ back into the second equation.

$$-3x + y = 7$$

$$-3(-12) + y = 7$$

$$y = -29$$

The solution to the system is

$$(-12, -29)$$

■ 3. What went wrong if an elimination was done in the system and the result was $2y = 3$?

$$-4x + 3y = 7$$

$$-4x - y = 4$$

Solution:

When subtracting the two equations in order to eliminate x, $-y$ in the second equation was added, instead of subtracted. The elimination method should have given $4y = 3$.

■ 4. Find the unique solution to the system of equations.

$$x = 2y - 5$$

$$-3x + 6y = 15$$

Solution:

Multiplying the first equation by 3 gives

$$x = 2y - 5$$

$$3x = 6y - 15$$

Then adding $3x = 6y - 15$ to $-3x + 6y = 15$ gives

$$3x - 6y + (-3x + 6y) = -15 + (15)$$

$$3x - 6y - 3x + 6y = -15 + 15$$

$$-6y + 6y = -15 + 15$$

$$0 = 0$$

This is always true, so there are infinitely many solutions to the system of equations.

5. Find the unique solution to the system of equations.

$$4 - 2x = 6y$$

$$7 = x + 3y$$

Solution:

Rewrite the equations as

$$4 = 2x + 6y$$

$$7 = x + 3y$$

Multiplying the second equation by -2 gives

$$7 = x + 3y$$

$$-14 = -2x - 6y$$

Then adding the two equations gives

$$2x + 6y + (-2x - 6y) = 4 + (-14)$$

$$2x + 6y - 2x - 6y = 4 - 14$$

$$0 = -10$$

Since this is not true, there is no solution to the system of equations.

6. Find the unique solution to the system of equations.

$$x = 2y - 8$$

$$3y = x + 5$$

Solution:

Rewrite the equations as

$$x - 2y = -8$$

$$-x + 3y = 5$$

Adding the two equations gives

$$x - 2y + (-x + 3y) = -8 + 5$$

$$x - 2y - x + 3y = -8 + 5$$

$$y = -3$$

Substitute $y = -3$ into the first equation to solve for x.

$$x = 2y - 8$$

$$x = 2(-3) - 8$$

$$x = -6 - 8$$

$$x = -14$$

Therefore, the solution to the system of equations is

$$(-14, -3)$$

SOLVING SYSTEMS THREE WAYS

1. Explain why using the graphing method would make the system easy to solve.

$$y = 3x - 4$$

$$y - 3 = 2(x + 1)$$

Solution:

The first equation is easy to graph because it's in slope-intercept form, $y = mx + b$. And the second equation is easy to graph because it's in point-slope form, $y - y_1 = m(x - x_1)$.

2. Find the unique solution to the system of equations using the elimination method.

$$2y = x + 5$$

$$3x - 2y = 11$$

Solution:

Rewrite the system as

$$-x + 2y = 5$$

$$3x - 2y = 11$$

Adding the two equations and solving for x gives

$$-x + 2y + (3x - 2y) = 5 + (11)$$

$$-x + 2y + 3x - 2y = 5 + 11$$

$$2x = 16$$

$$x = 8$$

Substitute $x = 8$ into the first equation.

$$2y = x + 5$$

$$2y = 8 + 5$$

$$y = \frac{13}{2}$$

The unique solution to the system of equations is

$$\left(8, \frac{13}{2}\right)$$

3. In words, describe the graphical solution to a system of equations.

Solution:

The solution to a system of equations on a graph is the intersection point of the two graphs.

4. Find the unique solution to the system of equations using the substitution method.

$$5y + x = 4$$

$$3y - 3x = 6$$

Solution:

Solve the first equation for x.

$$5y + x = 4$$

$$x = 4 - 5y$$

Substitute this into the second equation.

$$3y - 3x = 6$$

$$3y - 3(4 - 5y) = 6$$

$$3y - 12 + 15y = 6$$

$$18y = 18$$

$$y = 1$$

Plug $y = 1$ into the equation for x.

$$x = 4 - 5y$$

$$x = 4 - 5(1)$$

$$x = -1$$

The solution to the system of equations is

$$(-1,1)$$

5. Explain why the elimination method is a good way to solve this particular system.

$$3y - 2x = 7$$

$$2x = 4 - 6y$$

Solution:

If we add the two equations, the x terms immediately cancel, making it a very easy elimination method problem.

6. Find the unique solution to the system of equations using the graphing method.

$$y - 2 = -(x + 1)$$

$$y = x + 1$$

Solution:

In order to graph these equations, let's put both of them into slope-intercept form. We get

$$y - 2 = -(x + 1)$$

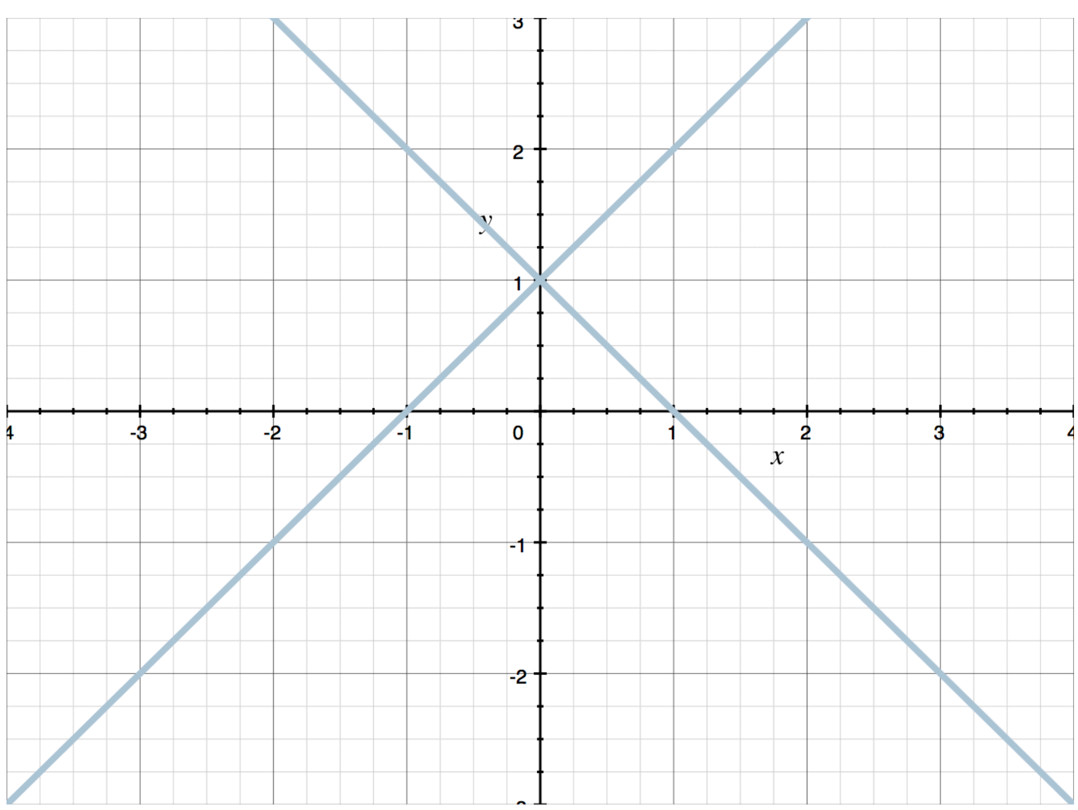
$$y - 2 = -x - 1$$

$$y = -x + 1$$

and

$$y = x + 1$$

The line $y = -x + 1$ intersects the y-axis at 1, and has a slope of -1. The line $y = x + 1$ intersects the y-axis at 1, and has a slope of 1.



From the sketch of the two lines, we can see that the intersection point is $(0,1)$ along the vertical axis, which means $(0,1)$ is the solution to the system.

SYSTEMS OF LINEAR INEQUALITIES

1. Graph the solution to the system of linear inequalities.

$$y > x + 1$$

$$y \leq 5 - x$$

Solution:

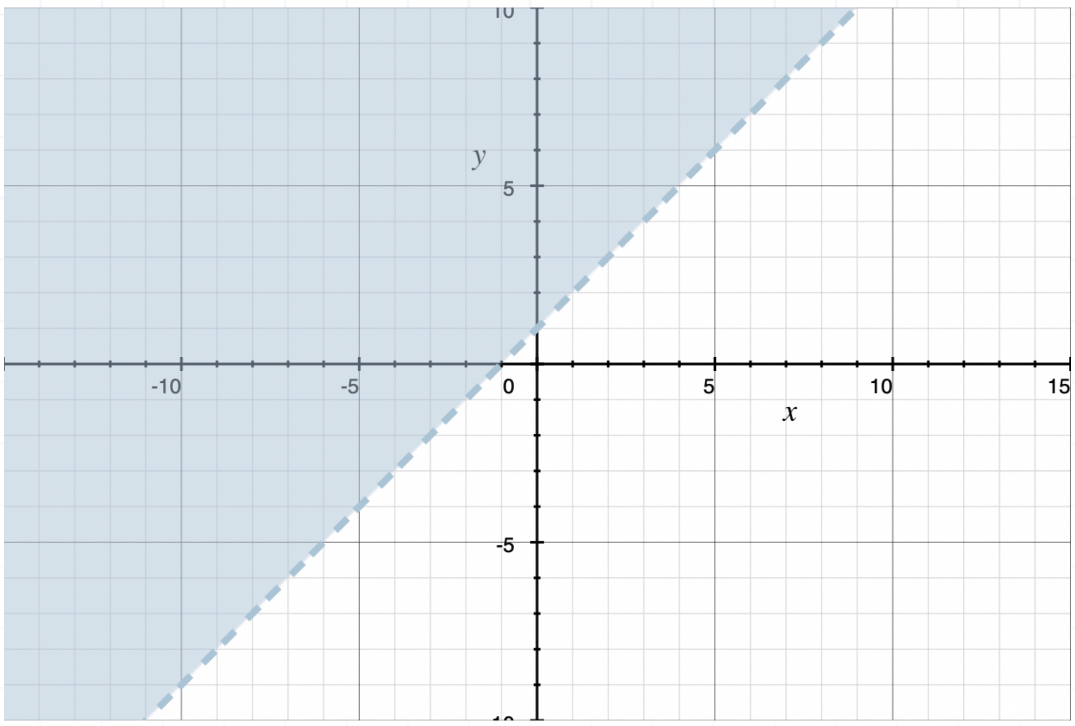
For $y = x + 1$, the slope is $m = 1$ and the y-intercept is $(0,1)$. The $>$ in $y > x + 1$ indicates the need for a dashed boundary line. Now let's test the origin to determine where to shade.

$$y > x + 1$$

$$0 > 0 + 1$$

$$0 > 1$$

Because this is a false statement, we shade away the origin .



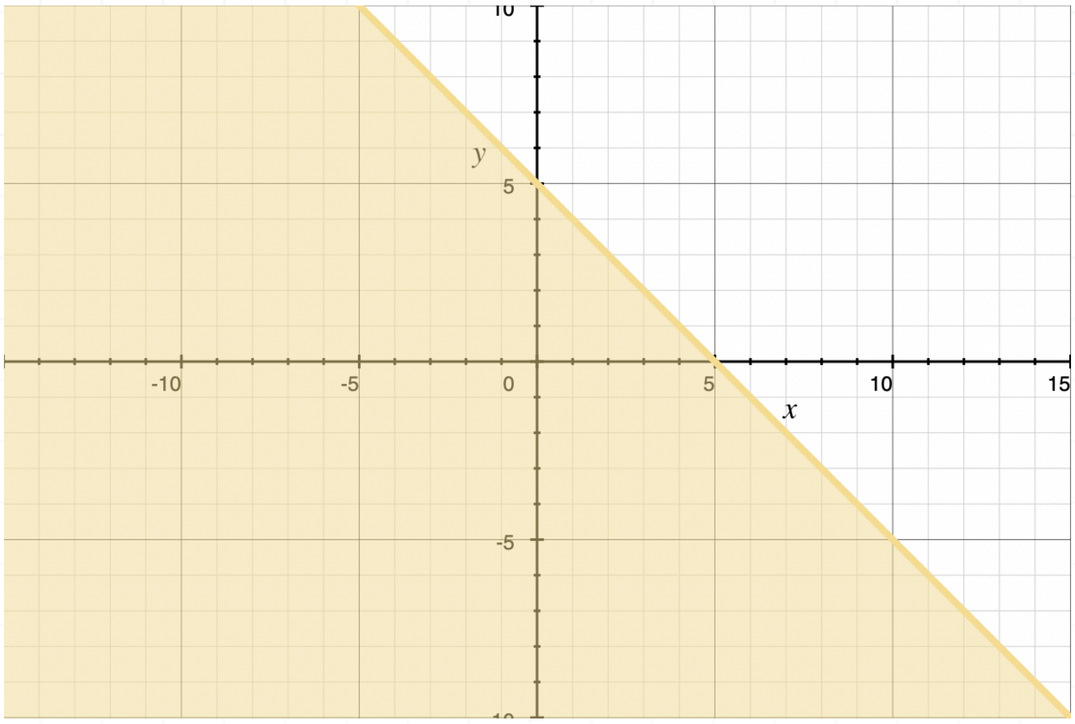
For $y = 5 - x$, the slope is $m = -1$ and the y-intercept is $(0,5)$. The $\leq$ in $y \leq 5 - x$ indicates the need for a solid boundary line. Now let's test the origin again.

$$y \leq 5 - x$$

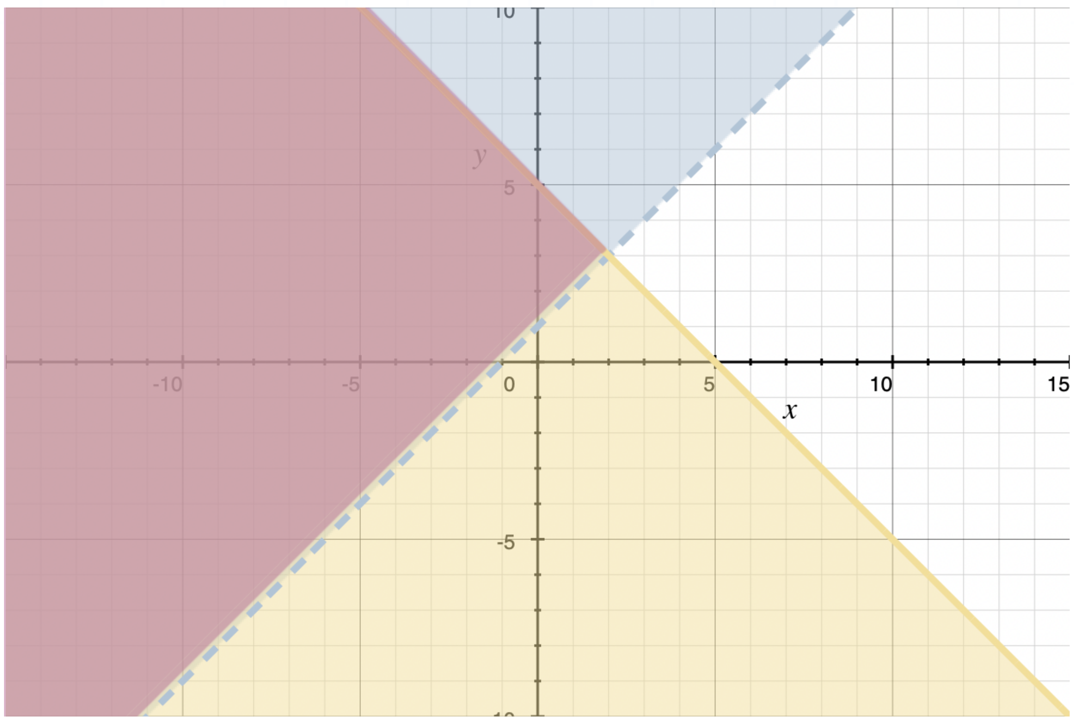
$$0 \leq 5 - 0$$

$$0 \leq 5$$

Because this is a true statement, we shade toward from the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



2. Graph the solution to the system of linear inequalities.

$$2x + 2y \geq 4$$

$$y > -1$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$2x + 2y \geq 4$$

$$2y \geq -2x + 4$$

$$y \geq -x + 2$$

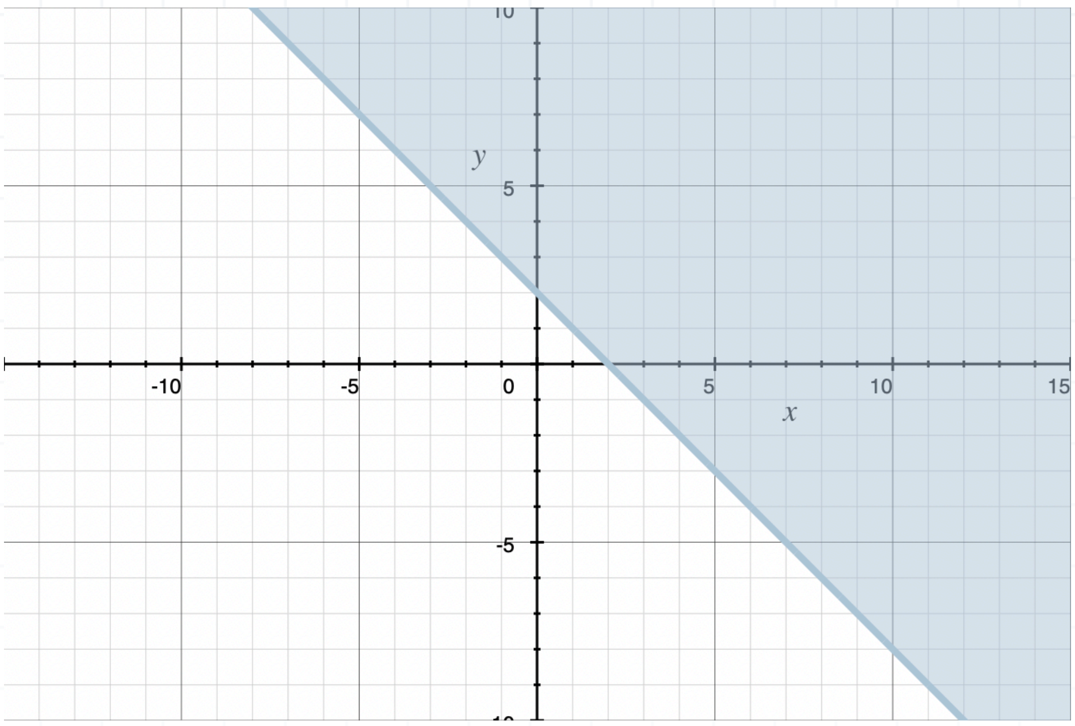
For $y = -x + 2$, the slope is $m = -1$ and the y-intercept is $(0,2)$. The $\geq$ in $y \geq -x + 2$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \geq -x + 2$$

$$0 \geq -0 + 2$$

$$0 \geq 2$$

Because this is a false statement, we shade away from the origin.

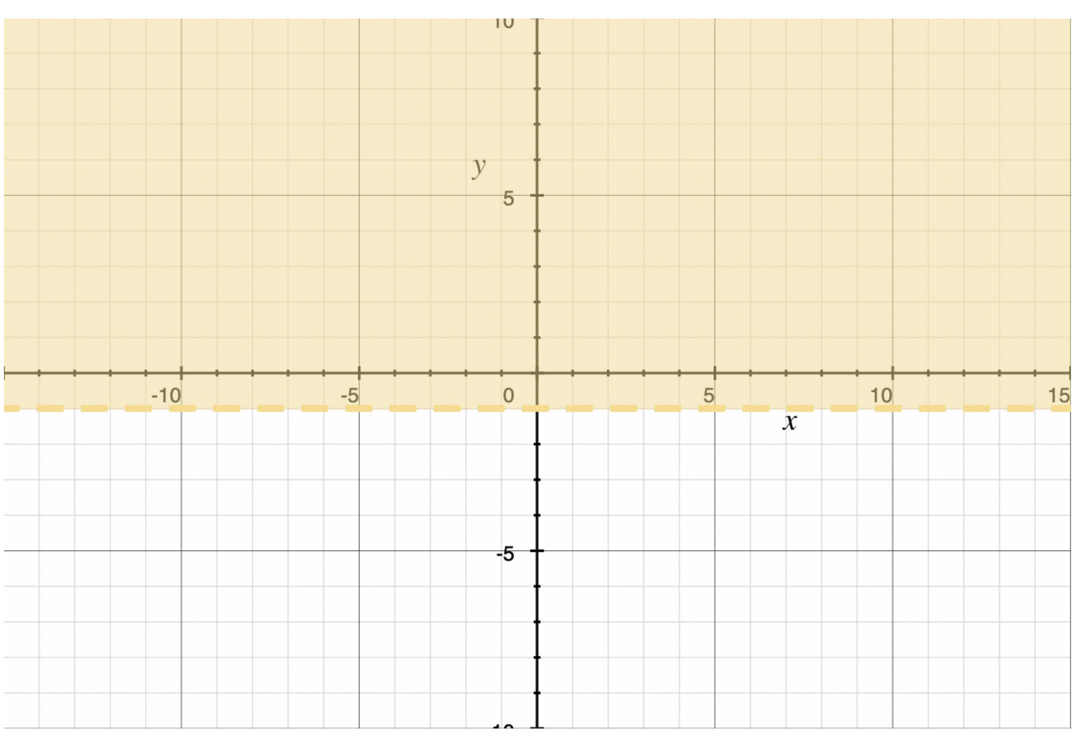


For $y = -1$, the slope is $m = 0$ and the y-intercept is $(0, -1)$; it's a perfectly horizontal line. The $>$ in $y > -1$ indicates the need for a dashed boundary line. Now let's test the origin again.

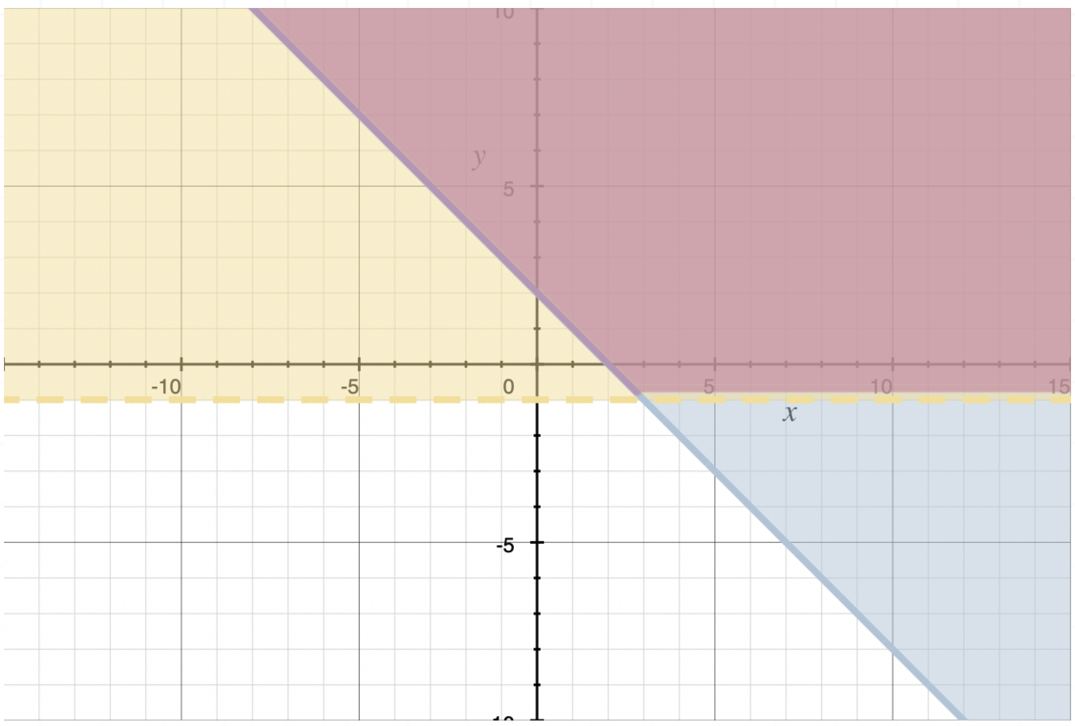
$$y > -1$$

$$0 > -1$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



3. Graph the solution to the system of linear inequalities.

$$x + 3y + 3 \geq 0$$

$$3x + y + 1 \geq 0$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$x + 3y + 3 \geq 0$$

$$3y \geq -x - 3$$

$$y \geq -\frac{1}{3}x - 1$$

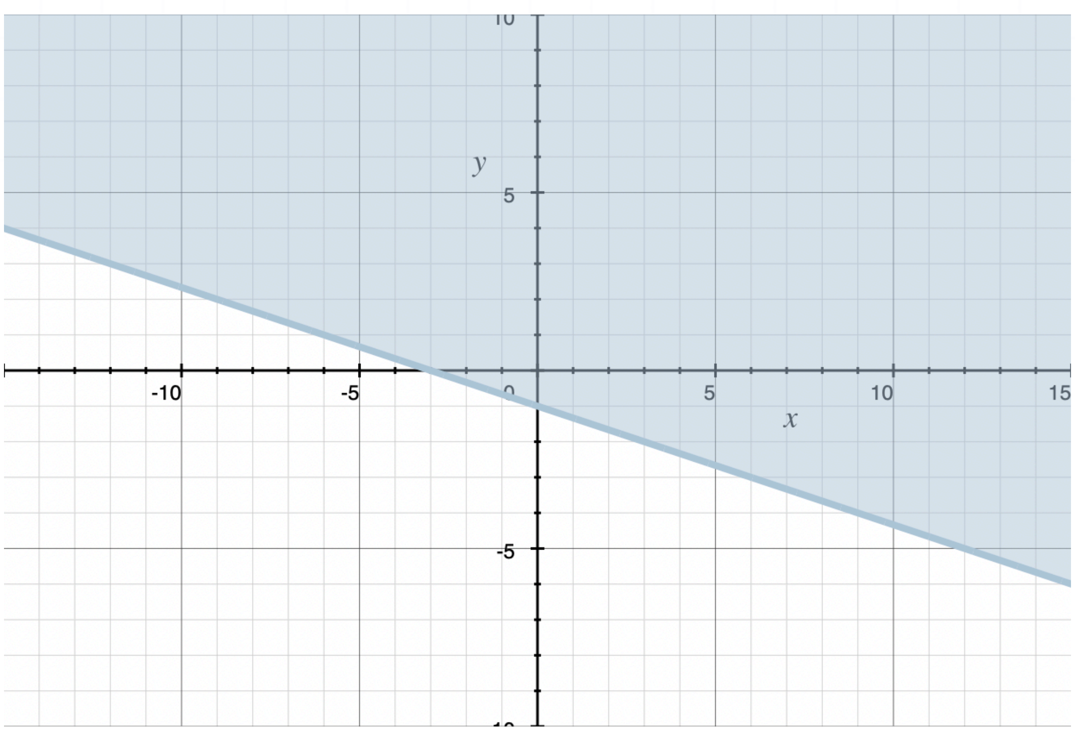
For $y = (-1/3)x - 1$, the slope is $m = -1/3$ and the y-intercept is $(0, -1)$. The $\geq$ in $y \geq (-1/3)x - 1$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \geq -\frac{1}{3}x - 1$$

$$0 \geq -\frac{1}{3}(0) - 1$$

$$0 \geq -1$$

Because this is a true statement, we shade toward the origin.



Rewrite the second inequality so that it's in slope-intercept form.

$$3x + y + 1 \geq 0$$

$$y \geq -3x - 1$$

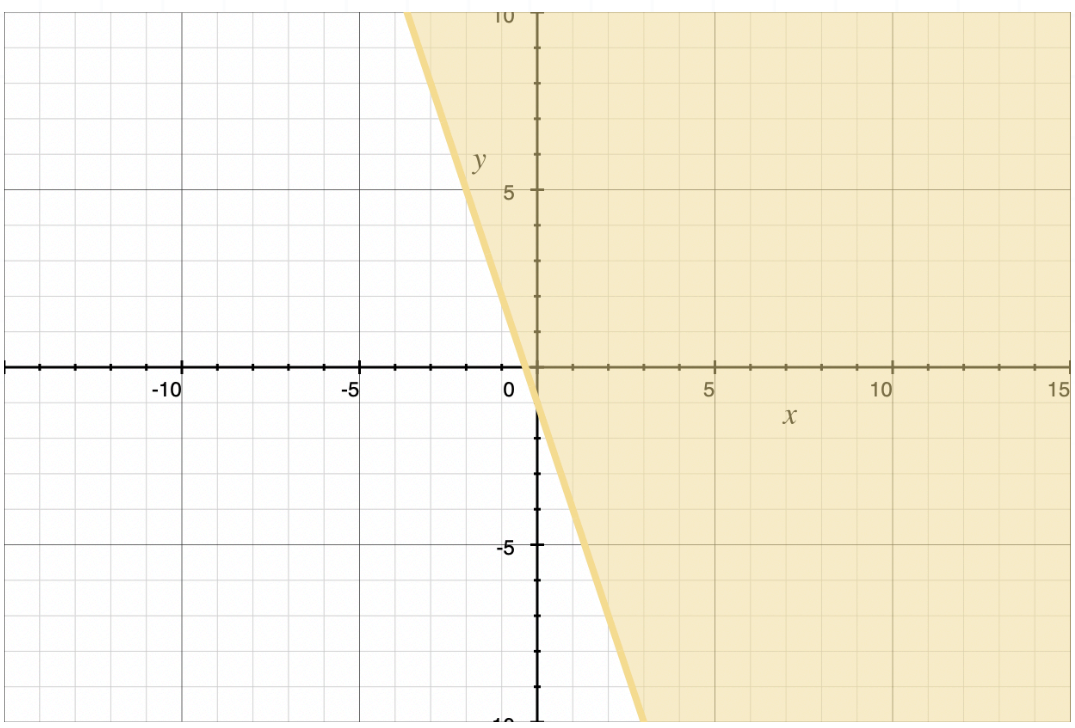
For $y = -3x - 1$, the slope is $m = -3$ and the y-intercept is $(0, -1)$. The $\geq$ in $y \geq -3x - 1$ indicates the need for a solid boundary line. Now let's test the origin again.

$$y \geq -3x - 1$$

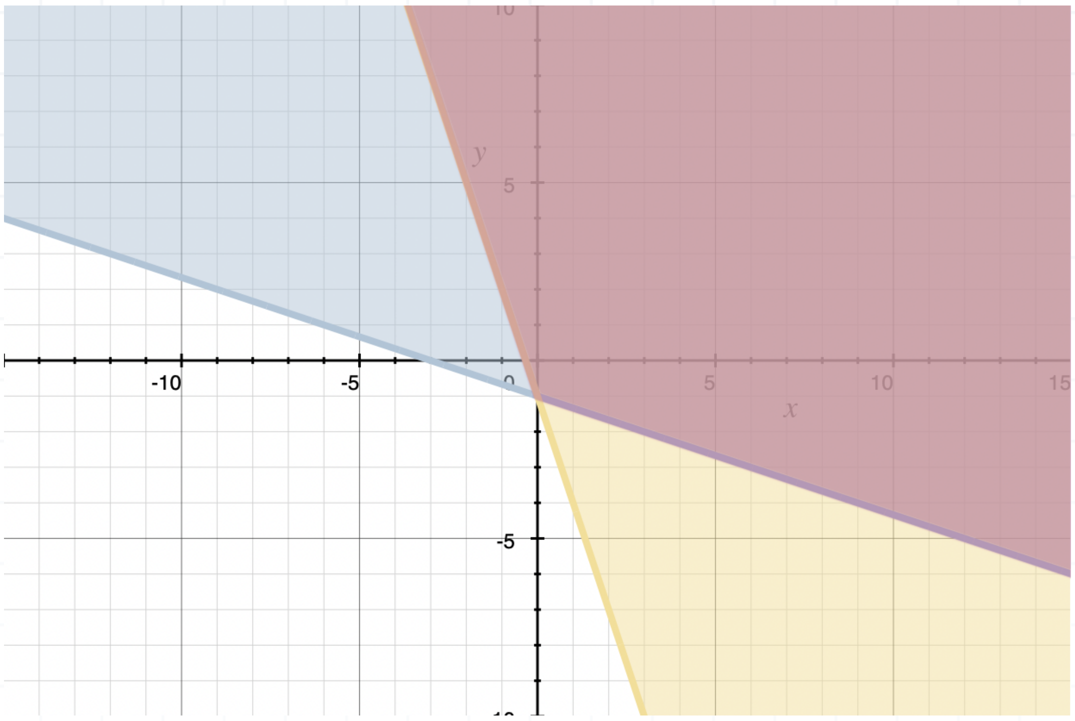
$$0 \geq -3(0) - 1$$

$$0 \geq -1$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



4. Graph the solution to the system of linear inequalities.

$$y > 2x$$

$$x > 2y$$

Solution:

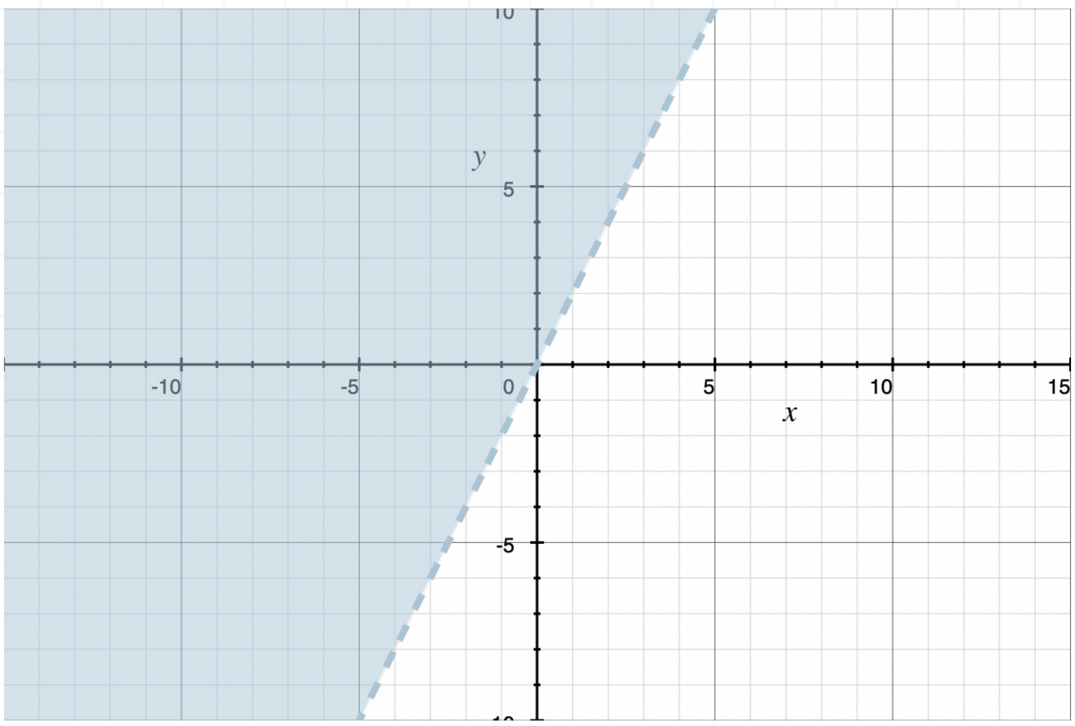
For $y = 2x$, the slope is $m = 2$ and the y-intercept is $(0,0)$. The $>$ in $y > 2x$ indicates the need for a dashed boundary line. Now let's test the point $(1,0)$ to determine where to shade.

$$y > 2x$$

$$0 > 2(1)$$

$$0 > 2$$

Because this is a false statement, we shade away from $(1,0)$.



Rewrite the second inequality so that it's in slope-intercept form.

$$y < \frac{1}{2}x$$

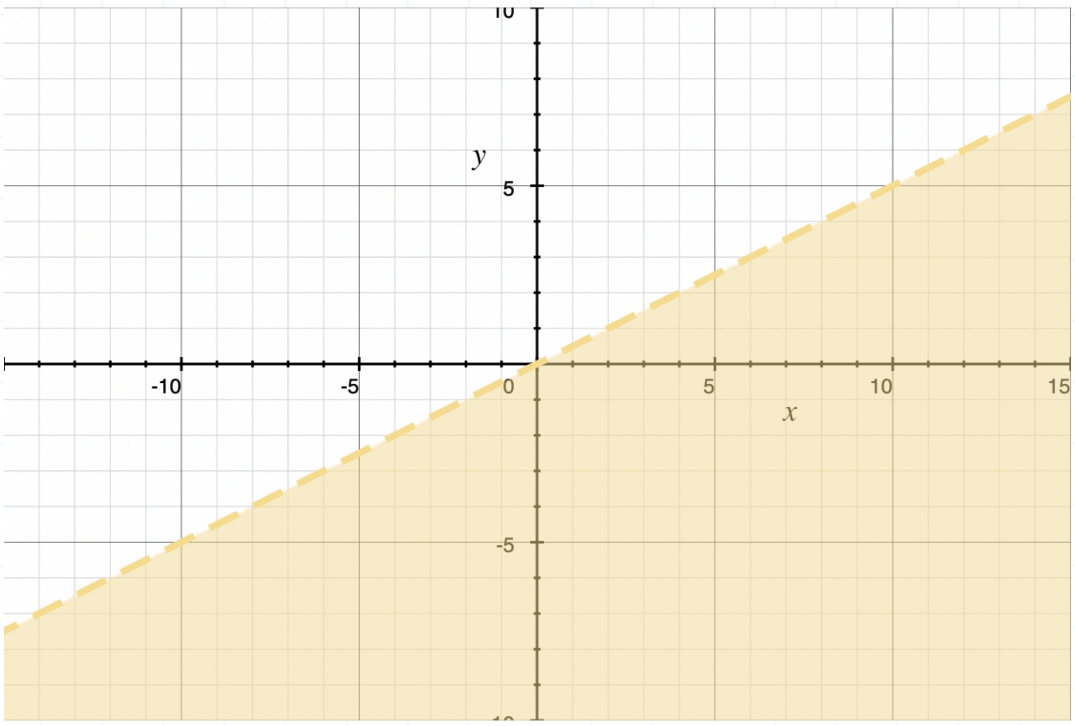
For $y = (1/2)x$, the slope is $m = 1/2$ and the y-intercept is $(0,0)$. The $<$ in $y < (1/2)x$ indicates the need for a dashed boundary line. Now let's test $(1,0)$ again.

$$y < \frac{1}{2}x$$

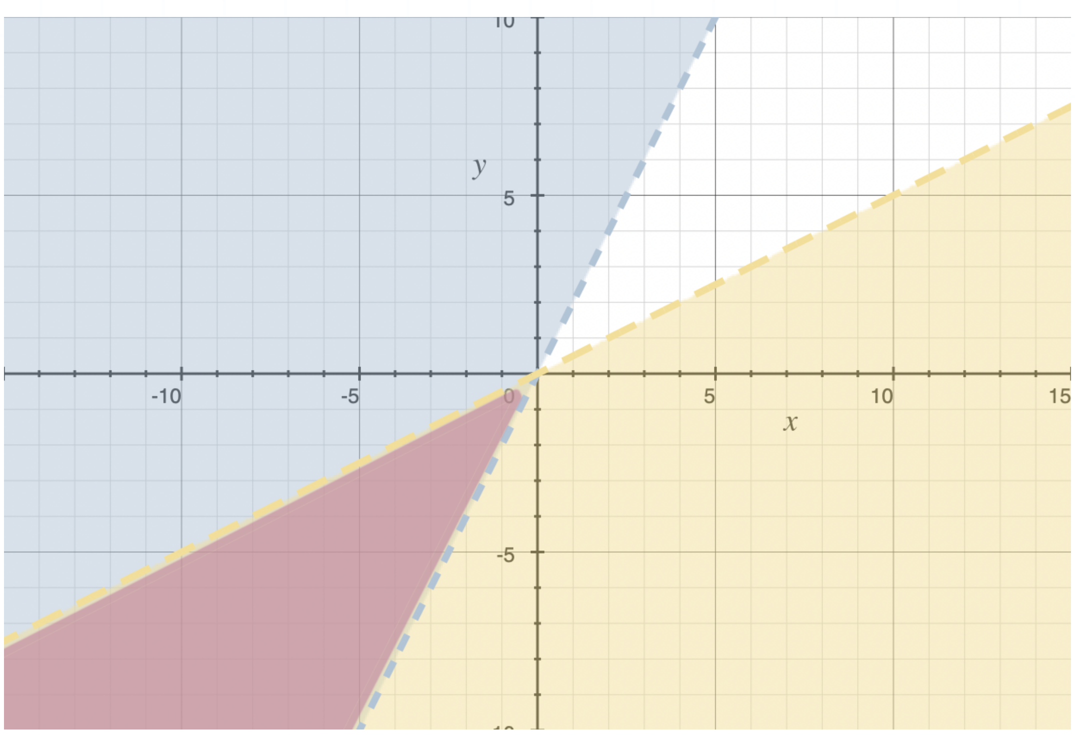
$$0 < \frac{1}{2}(1)$$

$$0 < \frac{1}{2}$$

Because this is a true statement, we shade toward $(1,0)$.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



5. Graph the solution to the system of linear inequalities.

$$2y + 3x \geq -4$$

$$x > y - 1$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$2y + 3x \geq -4$$

$$2y \geq -3x - 4$$

$$y \geq -\frac{3}{2}x - 2$$

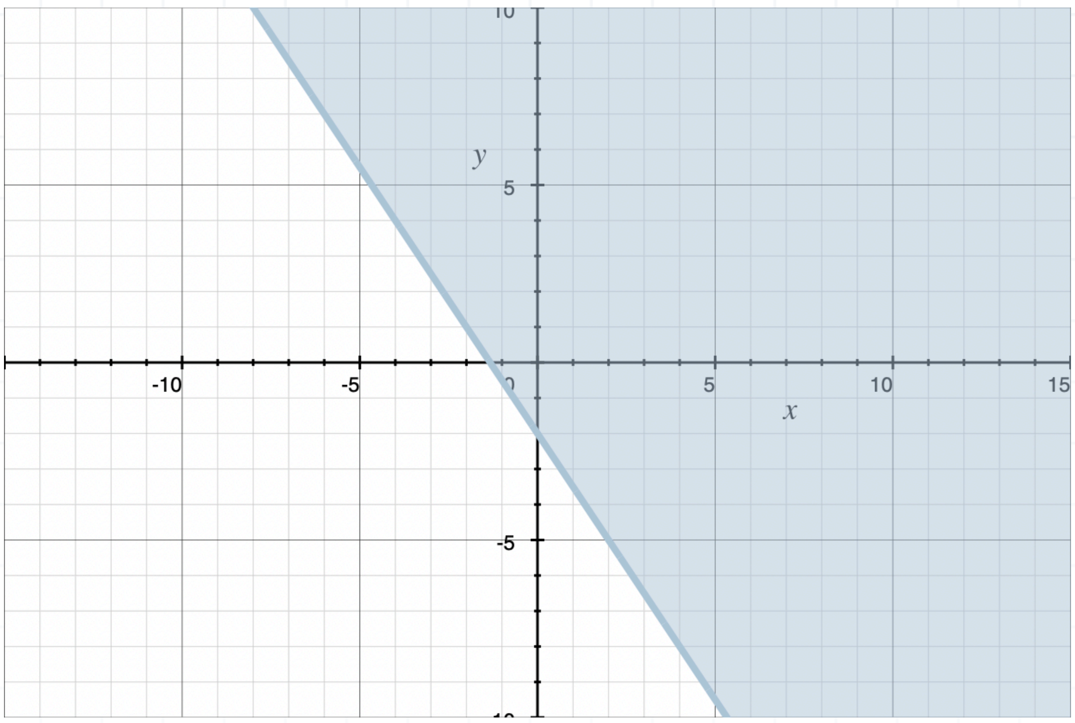
For $y = (-3/2)x - 2$, the slope is $m = -3/2$ and the y-intercept is $(0, -2)$. The $\geq$ in $y \geq (-3/2)x - 2$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \geq -\frac{3}{2}x - 2$$

$$0 \geq -\frac{3}{2}(0) - 2$$

$$0 \geq -2$$

Because this is a true statement, we shade toward the origin .



Rewrite the second inequality so that it's in slope-intercept form.

$$x > y - 1$$

$$x + 1 > y$$

$$y < x + 1$$

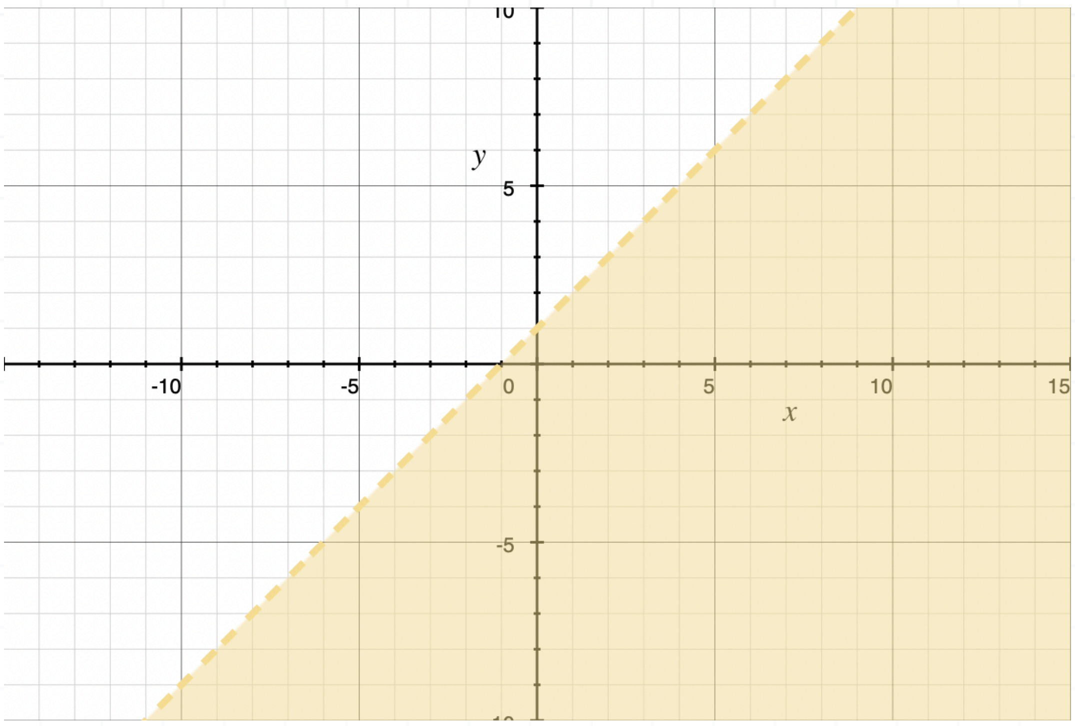
For $y = x + 1$, the slope is $m = 1$ and the y-intercept is $(0,1)$. The $<$ in $y < x + 1$ indicates the need for a dashed boundary line. Now let's test the origin again.

$$y < x + 1$$

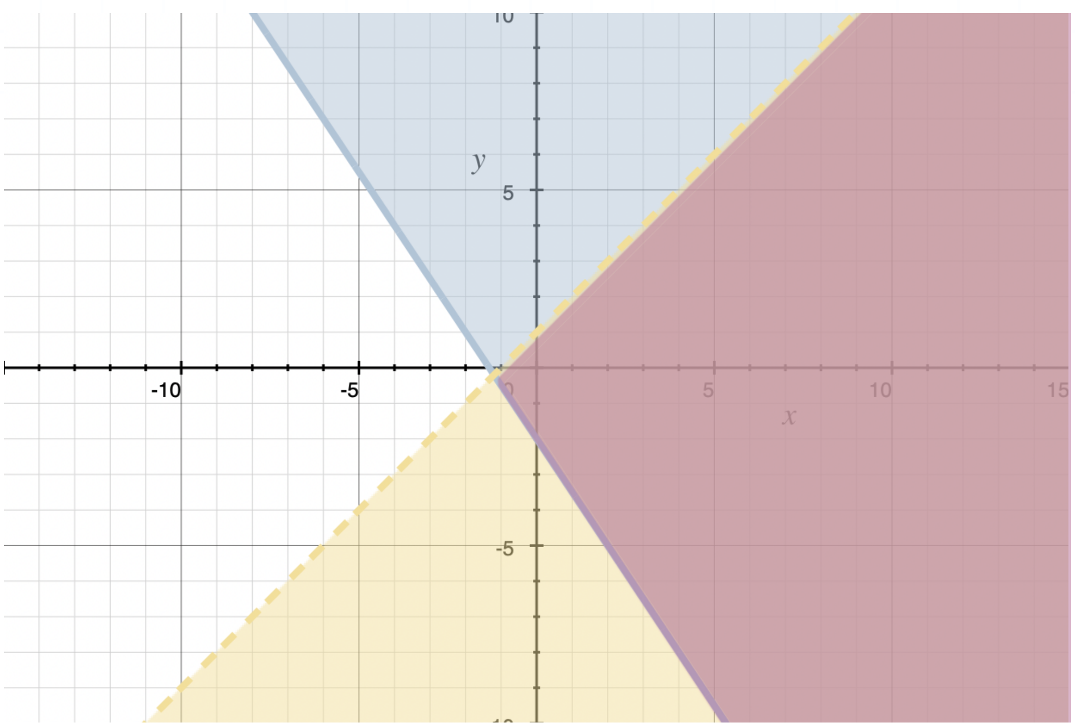
$$0 < 0 + 1$$

$$0 < 1$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify the overlapping portion as the solution to the system of inequalities. Remember that the solution includes any solid boundary of the solution region, and excludes any dashed boundary of the solution region.



6. Graph the solution to the system of linear inequalities.

$$4x - 2y - 4 \geq 0$$

$$y \geq 2x - 2$$

Solution:

Rewrite the first inequality so that it's in slope-intercept form.

$$4x - 2y - 4 \geq 0$$

$$-2y \geq 4 - 4x$$

$$y \leq 2x - 2$$

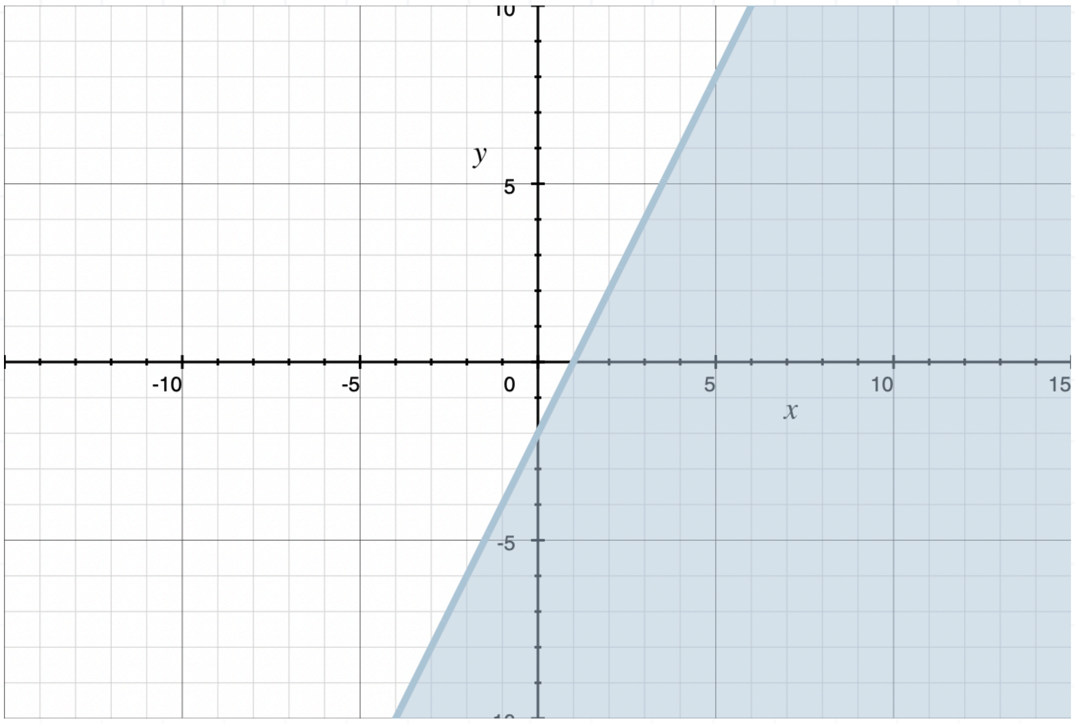
For $y = 2x - 2$, the slope is $m = 2$ and the y-intercept is $(0, -2)$. The $\leq$ in $y \leq 2x - 2$ indicates the need for a solid boundary line. Now let's test the origin to determine where to shade.

$$y \leq 2x - 2$$

$$0 \leq 2(0) - 2$$

$$0 \leq -2$$

Because this is a false statement, we shade away from the origin .



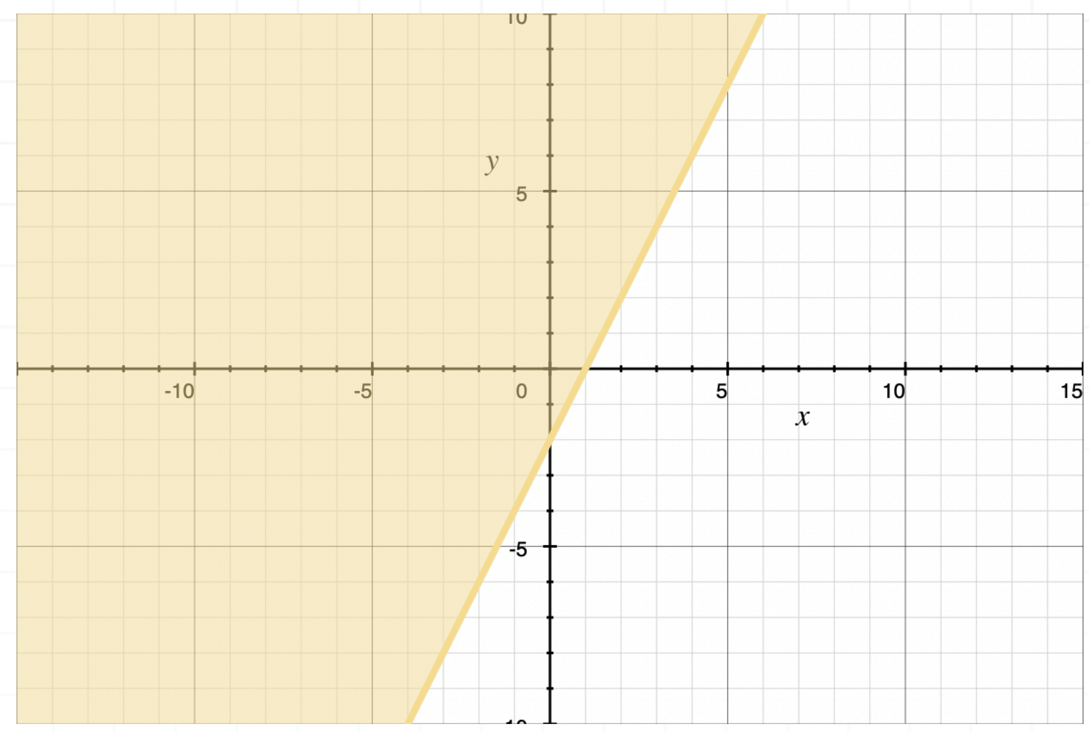
For $y = 2x - 2$, the slope is $m = 2$ and the y-intercept is $(0, -2)$. The $\geq$ in $y \geq 2x - 2$ indicates the need for a solid boundary line. Now let's test the origin again.

$$y \geq 2x - 2$$

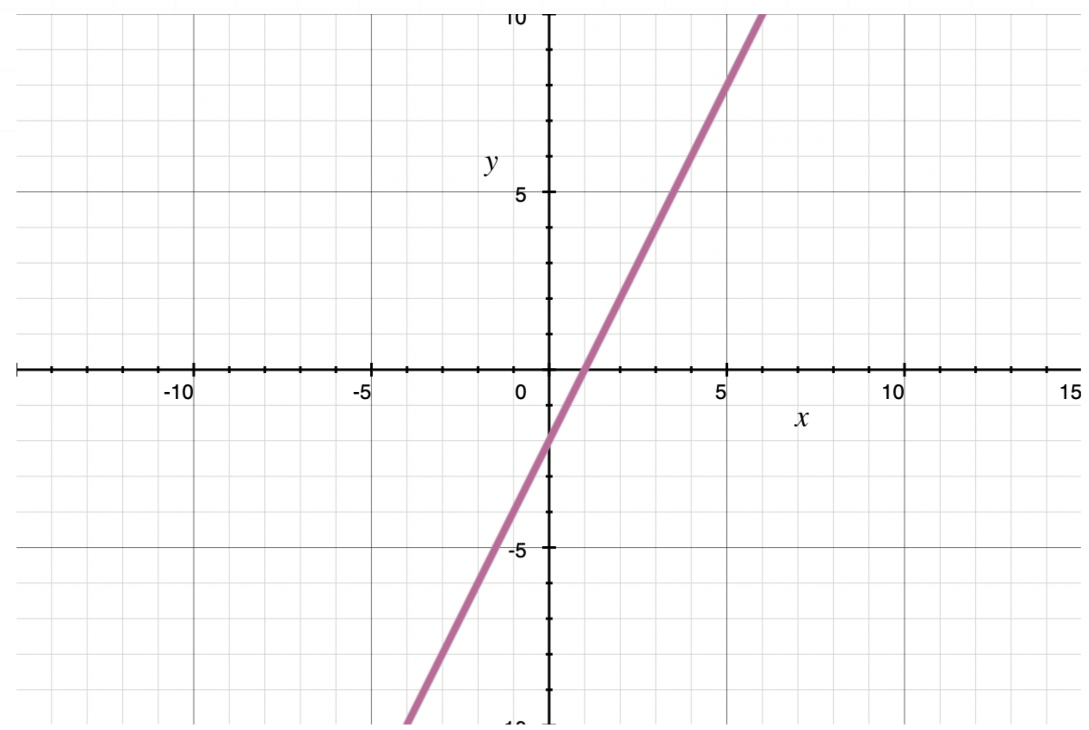
$$0 \geq 2(0) - 2$$

$$0 \geq -2$$

Because this is a true statement, we shade toward the origin.



Overlaying these two regions on the same graph, we can identify that the only overlapping portion is the line $y = 2x - 2$ itself. So the solution is



W W W . K R I S T A K I N G M A T H . C O M