Topic: Correlation coefficient

Question: Given the sample covariance $s_{xy} = 0.456$ and the sample standard deviations $s_x = 2.53$ and $s_y = 0.25$, calculate the value of the correlation coefficient.

Answer choices:

A 0.0480

B 0.2132

C 0.2884

D 0.7209

Solution: D

We can substitute the sample covariance $s_{xy}=0.456$ and the sample standard deviations $s_x=2.53$ and $s_y=0.25$ into the formula for the correlation coefficient.

$$r_{xy}=\frac{s_{xy}}{s_xs_y}$$

$$r_{xy}=\frac{0.456}{2.53\cdot 0.25}$$

$$r_{xy}=0.7209$$

Topic: Correlation coefficient

Question: Two corporations record their stock returns between 2010 and 2014. Calculate the value of the correlation coefficient and interpret the result.

	2010	2011	2012	2013	2014
X	2%	1%	-2%	4%	-1%
Y	3%	0%	1%	2%	1%

Answer choices:

A $r_{XY} \approx 1.35$

B $r_{XY} \approx 0.4961$

C $r_{XY} \approx 0.8644$

D $r_{XY} \approx -0.8644$

Solution: B

Find the mean of X,

$$\bar{X} = \frac{2 + 1 + (-2) + 4 + (-1)}{5}$$

$$\bar{X} = \frac{4}{5}$$

$$\bar{X} = 0.8$$

and then the mean of Y.

$$\bar{Y} = \frac{3 + 0 + 1 + 2 + 1}{5}$$

$$\bar{Y} = \frac{7}{5}$$

$$\bar{Y} = 1.4$$

Now use the means to find the sample covariance.

$$s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2 - 0.8)(3 - 1.4) + (1 - 0.8)(0 - 1.4)$$

$$+(-2 - 0.8)(1 - 1.4) + (4 - 0.8)(2 - 1.4) + (-1 - 0.8)(1 - 1.4)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 5.4$$

$$s_{XY} = \frac{5.4}{5 - 1}$$

$$s_{XY} = 1.35$$

Next we'll need the standard deviation for corporation X,

$$s_X = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}}$$

$$\sum_{i=1}^{n} (X_i - \bar{X})^2 = (2 - 0.8)^2 + (1 - 0.8)^2 + (-2 - 0.8)^2$$

$$+(4 - 0.8)^2 + (-1 - 0.8)^2$$

$$\sum_{i=1}^{n} (X_i - \bar{X})^2 = 22.8$$

$$s_X = \sqrt{\frac{22.8}{5 - 1}}$$

$$s_X \approx 2.387$$

and the standard deviation for corporation Y.

$$s_Y = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}{n - 1}}$$

$$\sum_{i=1}^{n} (Y_i - \bar{Y})^2 = (3 - 1.4)^2 + (0 - 1.4)^2$$

$$+(1 - 1.4)^2 + (2 - 1.4)^2 + (1 - 1.4)^2$$

$$\sum_{i=1}^{n} (Y_i - \bar{Y})^2 = 5.2$$

$$s_Y = \sqrt{\frac{5.2}{5-1}}$$

$$s_Y \approx 1.140$$

Now we can plug the covariance and standard deviations into the formula for correlation.

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

$$r_{XY} \approx \frac{1.35}{2.387 \cdot 1.140}$$

$$r_{XY} \approx 0.4961$$

The correlation coefficient tells us that there's a moderate positive correlation between the annual stock return of the two corporations.

From what we know generally about the stock market, we might suspect that it's not necessarily the return of one stock that's causing the return of the other, but instead that broader market forces might be causing the returns of both stocks.

Topic: Correlation coefficient

Question: Given the sample covariance $s_{XY} = -0.12$ of the data set, calculate the value of the correlation coefficient.

X	12	5	8	18	6	5	11
Y	0.5	0.8	0.3	0.4	0.55	0.25	0.67

Answer choices:

A -0.3455

B -0.1282

C 0.1282

D 0.3455

Solution: B

Find the mean of X,

$$\bar{X}=\frac{12+5+8+18+6+5+11}{7}$$

$$\bar{X}\approx 9.286$$

and then the mean of Y.

$$\bar{Y}=\frac{0.5+0.8+0.3+0.4+0.55+0.25+0.67}{7}$$

$$\bar{Y}\approx 0.496$$

Next we'll need the standard deviation of X,

$$s_X=\sqrt{\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}$$

$$\sum_{i=1}^{n}(X_i-\bar{X})^2=(12-9.286)^2+(5-9.286)^2+(8-9.286)^2$$

$$+(18-9.286)^2+(6-9.286)^2+(5-9.286)^2+(11-9.286)^2$$

$$\sum_{i=1}^{n}(X_i-\bar{X})^2\approx 135.429$$

$$s_X=\sqrt{\frac{135.429}{7-1}}$$

$$s_X\approx 4.751$$

and the standard deviation of Y.

$$s_Y = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}{n-1}}$$

$$\sum_{i=1}^{n} (Y_i - \bar{Y})^2 = (0.5 - 0.496)^2 + (0.8 - 0.496)^2 + (0.3 - 0.496)^2$$

$$+(0.4 - 0.496)^2 + (0.55 - 0.496)^2$$

$$+(0.25 - 0.496)^2 + (0.67 - 0.496)^2$$

$$\sum_{i=1}^{n} (Y_i - \bar{Y})^2 \approx 0.234$$

$$s_Y = \sqrt{\frac{0.234}{7-1}}$$

$$s_Y \approx 0.197$$

Now we can plug the covariance and standard deviations into the formula for correlation.

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

$$r_{XY} \approx \frac{-0.12}{4.751 \cdot 0.197}$$

$$r_{XY} \approx -0.1282$$