Difference of squares

Factoring the difference of squares is a special case of factoring a quadratic.

We know we have a difference of squares when the quadratic is in standard form $ax^2 + bx + c$, when $b = 0$, and when ax^2 and c are both perfect squares, such that the difference of squares is given as

$$ax^2 - c$$

Whenever we have a quadratic that we can define as the difference of squares, it will always factor as $(\sqrt{a}x + \sqrt{c})(\sqrt{a}x - \sqrt{c})$.

All the roots make this binomial look complicated, but all we need to do to factor the difference of squares is to take the square root of each term, then one binomial is the sum of the roots, and the other binomial is the difference of the roots.

Let's look at an example so that we can see how to factor the difference of squares.

Example

Factor the difference of squares.

$$9x^2 - 16$$

We can see that both $9x^2$ and 16 are perfect squares. The term $9x^2$ is the perfect square of $3x$ because $(3x)^2 = 9x^2$, and the term 16 is the perfect square of 4 because $4^2 = 16$. In other words, we could rewrite the difference of squares as

$$9x^2 - 16$$

$$(3x)^2 - 4^2$$

To factor this difference of squares, we'll split the quadratic into the product of two binomials, which will be the sum and difference of $3x$ and 4.

$$(3x + 4)(3x - 4)$$

We can double check our factoring by multiplying out the binomials.

$$(3x)(3x) + (3x)(-4) + (4)(3x) + (4)(-4)$$

$$9x^2 - 12x + 12x - 16$$

$$9x^2 - 16$$

Let's do another example.

Example

Factor the difference of squares.

$$x^2 - 25$$

Since x^2 and 25 are both perfect squares (the squares of x and 5, respectively), $x^2 - 25$ is factored as

$$(x+5)(x-5)$$

Let's try another example, this time where we factor a multivariable difference of squares.

Example

Factor the binomial.

$$64x^4y^2 - 9z^6$$

Notice that $64x^4y^2 = (8x^2y)^2$, and that $9z^6 = (3z^3)^2$. Therefore, $64x^4y^2 - 9z^6$ a difference of squares that we can factor as

$$(8x^2y + 3z^3)(8x^2y - 3z^3)$$
