Simplifying rational functions

In this lesson we'll look at how to simplify rational functions by cancelling factors. Remember that to cancel any factor, it must be a common factor in both the numerator and denominator.

A common factor in a rational function is a factor that's shared by all the terms of the numerator and all the terms of the denominator. For example, $3x$, $12x^2$, and $9x^3$ have a common factor of $3x$, because, $1 \cdot 3x = 3x$, $4x \cdot 3x = 12x^2$, and $3x^2 \cdot 3x = 9x^3$.

Let's look at a few examples.

Example

Simplify the rational function to lowest terms.

$$\frac{9x^3 - 12x^2 + 3x}{15x^2}$$

Let's look for a common factor. We can factor a $3x$ out of the numerator and the denominator, and then cancel it.

$$\frac{3x(3x^2 - 4x + 1)}{3x(5x)}$$

$$\frac{3x^2 - 4x + 1}{5x}$$

It would also work to take out each factor individually: either first the 3 and then the x, or first the x and then the 3.

Let's look at another example.

Example

Simplify each rational function in the difference.

$$\frac{12ab + 8a^2b^2}{10ab} - \frac{36a^3b^3 - 6a^2b^2}{6ab}$$

We want to factor everything that we can out of the numerator of each rational function in the difference. That means that for each numerator, we need to take out every constant, every factor of a, and every factor of b that's common to all the terms of that numerator; and that for each denominator, we need to take out every constant, every factor of a, and every factor of b that's common to all the terms of that denominator.

$$\frac{12ab + 8a^2b^2}{10ab} - \frac{36a^3b^3 - 6a^2b^2}{6ab}$$

$$\frac{2ab(6 + 4ab)}{2ab(5)} - \frac{6a^2b^2(6ab - 1)}{6ab}$$

$$\frac{2ab(6 + 4ab)}{2ab(5)} - \frac{6ab(ab)(6ab - 1)}{6ab}$$

Then for each rational function, we want to cancel what's common to its numerator and denominator.

$$\frac{6+4ab}{5} - \frac{(ab)(6ab-1)}{1}$$

$$\frac{6+4ab}{5} - ab(6ab-1)$$

Let's look at another example.

Example

Simplify the rational function to lowest terms.

$$\frac{x^2-x-20}{x^2-7x+10}$$

The quadratic expression in the numerator is factored as $(x+4)(x-5)$ and the quadratic expression in the denominator is factored as $(x-5)(x-2)$. Then we get

$$\frac{(x+4)(x-5)}{(x-2)(x-5)}$$

In this case, we can cancel the common factor $(x-5)$.

$$\frac{x+4}{x-2}$$