Algebra 2 Workbook Solutions

Manipulating functions

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COMBINATIONS OF FUNCTIONS

■ 1. Find $(f+g)(x)$.

$$f(x) = 2x^2 - x + 5$$

$$g(x) = x^2 + 4x - 7$$

Solution:

The combination $(f+g)(x)$ is the same as $f(x) + g(x)$, so we need to add the equations together.

$$(f+g)(x) = f(x) + g(x)$$

$$(f+g)(x) = 2x^2 - x + 5 + x^2 + 4x - 7$$

Combine like terms.

$$(f+g)(x) = 2x^2 + x^2 - x + 4x + 5 - 7$$

$$(f+g)(x) = 3x^2 + 3x - 2$$

■ 2. Find $(f-g)(2)$.

$$f(x) = 4x^2 - 2$$

$$g(x) = 3x^2 - 5x$$

Solution:

The combination $(f-g)(x)$ is the same as $f(x)-g(x)$, so we need to find the difference.

$$(f-g)(x)=f(x)-g(x)$$

$$(f-g)(x)=4x^2-2-(3x^2-5x)$$

$$(f-g)(x)=4x^2-2-3x^2+5x$$

Combine like terms.

$$(f-g)(x)=4x^2-3x^2+5x-2$$

$$(f-g)(x)=x^2+5x-2$$

Now substitute $x=2$ to get

$$(f-g)(2)=2^2+5(2)-2$$

$$(f-g)(2)=4+10-2$$

$$(f-g)(2)=12$$

■ 3. Find $(f-g)(x)$.

$$f(x)=x^2-3x+1$$

$$g(x)=2x-3$$

Solution:

The combination $(f-g)(x)$ is the same as $f(x)-g(x)$, so we need to find the difference.

$$(f-g)(x)=f(x)-g(x)$$

$$(f-g)(x)=x^2-3x+1-(2x-3)$$

$$(f-g)(x)=x^2-3x+1-2x+3$$

Group like terms together and combine.

$$(f-g)(x)=x^2-3x-2x+1+3$$

$$(f-g)(x)=x^2-5x+4$$

■ 4. Find $(f\cdot g)(x)$.

$$f(x)=2x-3$$

$$g(x)=3x^2+2$$

Solution:

The combination $(f\cdot g)(x)$ is the same as $f(x)\cdot g(x)$, so we need to find the product.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$(f \cdot g)(x) = (2x - 3)(3x^2 + 2)$$

Multiply using the FOIL method.

$$(f \cdot g)(x) = 6x^3 + 4x - 9x^2 - 6$$

$$(f \cdot g)(x) = 6x^3 - 9x^2 + 4x - 6$$

■ 5. Find $(f \div g)(x)$.

$$f(x) = x^2 + 6x$$

$$g(x) = x$$

Solution:

The combination $(f \div g)(x)$ is the same as $f(x)/g(x)$, so we need to find the quotient.

$$(f \div g)(x) = \frac{x^2 + 6x}{x}$$

$$(f \div g)(x) = \frac{x(x + 6)}{x}$$

$$(f \div g)(x) = x + 6, \; x \neq 0$$

6. Find $(g \div f)(x)$.

$$f(x) = x^2 + 6x$$

$$g(x) = x$$

Solution:

The combination $(g \div f)(x)$ is the same as $g(x)/f(x)$, so we need to find the quotient.

$$(g \div f)(x) = \frac{x}{x^2 + 6x}$$

$$(g \div f)(x) = \frac{x}{x(x+6)}$$

$$(g \div f)(x) = \frac{1}{x+6},\ x \neq 0$$

COMPOSITE FUNCTIONS

■ 1. Find the composite function $(g \circ f)(x)$.

$$f(x) = \sqrt{2x - 1}$$

$$g(x) = 3x^2$$

Solution:

When we take the composite $(g \circ f)(x)$, we plug $f(x)$ into $g(x)$.

$$(g \circ f)(x) = g(f(x)) = 3(\sqrt{2x - 1})^2$$

$$(g \circ f)(x) = g(f(x)) = 3(2x - 1)$$

$$(g \circ f)(x) = g(f(x)) = 6x - 3$$

■ 2. Find $f(g(x)) - g(f(x))$.

$$f(x) = x^2 - 4x + 3$$

$$g(x) = 2x + 1$$

Solution:

When we take the composite $f(g(x))$, we plug $g(x)$ into $f(x)$.

$$f(g(x)) = (2x+1)^2 - 4(2x+1) + 3$$

$$f(g(x)) = (2x+1)(2x+1) - 8x - 4 + 3$$

$$f(g(x)) = 4x^2 + 2x + 2x + 1 - 8x - 1$$

Combine like terms.

$$f(g(x)) = 4x^2 + 2x + 2x - 8x + 1 - 1$$

$$f(g(x)) = 4x^2 + 4x - 8x$$

$$f(g(x)) = 4x^2 - 4x$$

When we take the composite $g(f(x))$, we plug $f(x)$ into $g(x)$.

$$g(f(x)) = 2(x^2 - 4x + 3) + 1$$

$$g(f(x)) = 2x^2 - 8x + 6 + 1$$

$$g(f(x)) = 2x^2 - 8x + 7$$

Then $f(g(x)) - g(f(x))$ is

$$f(g(x)) - g(f(x)) = 4x^2 - 4x - (2x^2 - 8x + 7)$$

$$f(g(x)) - g(f(x)) = 4x^2 - 4x - 2x^2 + 8x - 7$$

$$f(g(x)) - g(f(x)) = 2x^2 + 4x - 7$$

■ 3. Find the composite function $(g \circ h)(x) - (h \circ h)(x)$.

$$g(x) = \frac{8}{x^3}$$

$$h(x) = \sqrt[3]{x+4}$$

Solution:

When we take the composite $(g \circ h)(x)$, we plug $h(x)$ into $g(x)$.

$$(g \circ h)(x) = g(h(x)) = \frac{8}{(\sqrt[3]{x+4})^3}$$

$$(g \circ h)(x) = g(h(x)) = \frac{8}{x+4}$$

When we take the composite $(h \circ h)(x)$, we plug $h(x)$ into $h(x)$.

$$(h \circ h)(x) = h(h(x)) = \sqrt[3]{\sqrt[3]{x+4}+4}$$

Then $(g \circ h)(x) - (h \circ h)(x)$ is

$$(g \circ h)(x) - (h \circ h)(x) = \frac{8}{x+4} - \sqrt[3]{\sqrt[3]{x+4}+4}$$

■ 4. Find the composite function $(h \circ g)(x)$.

$$g(x) = \frac{8}{x^3}$$

$$h(x) = \sqrt[3]{x+4}$$

Solution:

When we take the composite $(h \circ g)(x)$, we plug $g(x)$ into $h(x)$.

$$(h \circ g)(x) = h(g(x)) = \sqrt[3]{\frac{8}{x^3} + 4}$$

$$(h \circ g)(x) = h(g(x)) = \sqrt[3]{\frac{8}{x^3} + 4\frac{x^3}{x^3}}$$

$$(h \circ g)(x) = h(g(x)) = \sqrt[3]{\frac{8 + 4x^3}{x^3}}$$

Take the root of the numerator and denominator separately.

$$(h \circ g)(x) = h(g(x)) = \frac{\sqrt[3]{8 + 4x^3}}{\sqrt[3]{x^3}}$$

$$(h \circ g)(x) = h(g(x)) = \frac{\sqrt[3]{8 + 4x^3}}{x}$$

■ 5. Find the composite function $g(g(x))$.

$$g(x) = \frac{1}{x}$$

$$h(x) = 3x^2 - x$$

Solution:

When we take the composite $g(g(x))$, we plug $g(x)$ into $g(x)$.

$$g(g(x)) = \frac{1}{\frac{1}{x}}$$

$$g(g(x)) = \frac{x}{1}$$

$$g(g(x)) = x$$

■ 6. Find the composite functions $h(g(2))$ and $g(h(2))$.

$$g(x) = \frac{1}{x}$$

$$h(x) = 3x^2 - x$$

Solution:

When we take the composite $h(g(x))$, we plug $g(x)$ into $h(x)$.

$$h(g(x)) = 3\left(\frac{1}{x}\right)^2 - \frac{1}{x}$$

$$h(g(x)) = \frac{3}{x^2} - \frac{1}{x}$$

Find a common denominator to combine the fractions.

$$h(g(x)) = \frac{3}{x^2} - \frac{1}{x}\left(\frac{x}{x}\right)$$

$$h(g(x)) = \frac{3}{x^2} - \frac{x}{x^2}$$

$$h(g(x)) = \frac{3 - x}{x^2}$$

Evaluating the composite at $x = 2$ gives

$$h(g(2)) = \frac{3 - 2}{2^2}$$

$$h(g(2)) = \frac{1}{4}$$

When we take the composite $g(h(x))$, we plug $h(x)$ into $g(x)$.

$$g(h(x)) = \frac{1}{3x^2 - x}$$

Evaluating the composite at $x = 2$ gives

$$g(h(2)) = \frac{1}{3 \cdot 2^2 - 2}$$

$$g(h(2)) = \frac{1}{12 - 2}$$

$$g(h(2)) = \frac{1}{10}$$

So $h(g(2)) = 1/4$ and $g(h(2)) = 1/10$.

DOMAINS OF COMPOSITE FUNCTIONS

■ 1. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x}$$

$$g(x) = x + 5$$

Solution:

Find the domain of $g(x)$. In this case, $g(x)$ is a simple binomial with no domain restrictions.

Now find $f \circ g$.

$$f \circ g = \frac{1}{x+5}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$x + 5 \neq 0$$

$$x \neq -5$$

There were no restrictions on the domain of $g(x)$, so the domain of $f(g(x))$ is just $x \neq -5$.

2. What is the domain of $f \circ g$?

$$f(x) = \frac{2}{x-1}$$

$$g(x) = \sqrt{x-4}$$

Solution:

Find the domain of $g(x)$. Remember that you can't take the square root of negative numbers, since negative roots can't be defined by real numbers, so the expression inside the root must be positive or equal to 0.

$$x - 4 \geq 0$$

$$x \geq 4$$

Now find $f \circ g$.

$$f \circ g = \frac{2}{\sqrt{x-4} - 1}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$\sqrt{x-4} - 1 \neq 0$$

$$\sqrt{x-4} \neq 1$$

$$x - 4 \neq 1$$

$$x \neq 5$$

The domain of the composite has to account for domain restrictions on $g(x)$ and the composite function $f \circ g$ itself, so the domain of $f(g(x))$ is $x \geq 4$, $x \neq 5$.

■ 3. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x} + 4$$

$$g(x) = \frac{3}{2x - 7}$$

Solution:

Find the domain of $g(x)$. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$2x - 7 \neq 0$$

$$2x \neq 7$$

$$x \neq \frac{7}{2}$$

Now find $f \circ g$.

$$f \circ g = \frac{1}{\frac{3}{2x-7}} + 4$$

$$f \circ g = \frac{2x-7}{3} + 4$$

$$f \circ g = \frac{2x-7}{3} + \frac{12}{3}$$

$$f \circ g = \frac{2x-7+12}{3}$$

$$f \circ g = \frac{2x+5}{3}$$

We need to consider the domain of this composite function. But there are no domain restrictions on $f \circ g$, so the only restriction we need to consider is the one on $g(x)$. The domain of $f(g(x))$ is $x \neq 7/2$.

■ 4. What is the domain of $f \circ g$?

$$f(x) = \frac{2}{x-3}$$

$$g(x) = \frac{4}{x+2}$$

Solution:

Find the domain of $g(x)$. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$x+2\neq 0$$

$$x\neq -2$$

Now find $f\circ g$.

$$f\circ g=\frac{2}{\frac{4}{x+2}-3}$$

$$f\circ g=\frac{2}{\frac{4}{x+2}-3\left(\frac{x+2}{x+2}\right)}$$

$$f\circ g=\frac{2}{\frac{4}{x+2}-\frac{3x+6}{x+2}}$$

Combine fractions within the denominator.

$$f\circ g=\frac{2}{\frac{4-3x-6}{x+2}}$$

$$f\circ g=\frac{2}{\frac{-3x-2}{x+2}}$$

$$f\circ g=\frac{2(x+2)}{-3x-2}$$

$$f\circ g=\frac{2x+4}{-3x-2}$$

$$f \circ g = -\frac{2x+4}{3x+2}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$3x + 2 \neq 0$$

$$3x \neq -2$$

$$x \neq -\frac{2}{3}$$

Putting this restriction together with the one we found for $g(x)$, we can say that the domain of $f(g(x))$ is $x \neq -2, \ -2/3$.

■ 5. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x^2 - 3}$$

$$g(x) = \sqrt{x-1}$$

Solution:

Find the domain of $g(x)$. Remember that you can't take the square root of negative numbers, since negative roots can't be defined by real numbers, so the expression inside the root must be positive or equal to 0.

$$x - 1 \geq 0$$

$$x \geq 1$$

Now find $f \circ g$.

$$f \circ g = \frac{1}{(\sqrt{x-1})^2 - 3}$$

$$f \circ g = \frac{1}{x - 1 - 3}$$

$$f \circ g = \frac{1}{x - 4}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$x - 4 \neq 0$$

$$x \neq 4$$

Putting this restriction together with the one we found for $g(x)$, we can say that the domain of $f(g(x))$ is $x \geq 1$, $x \neq 4$.

■ 6. What is the domain of $f \circ g$?

$$f(x) = 2x^2 - x + 1$$

$$g(x) = x - 3$$

Solution:

Find the domain of $g(x)$. In this case $g(x)$ is a simple binomial with no domain restrictions.

Now find $f \circ g$.

$$f \circ g = 2(x-3)^2 - (x-3) + 1$$

$$f \circ g = 2(x^2 - 6x + 9) - x + 3 + 1$$

$$f \circ g = 2x^2 - 12x + 18 - x + 4$$

$$f \circ g = 2x^2 - 13x + 22$$

We need to consider the domain of this composite function. But there are no domain restrictions on $f \circ g$. And, since there are also no domain restrictions for $g(x)$, the domain of $f(g(x))$ is all real numbers.

DECOMPOSING COMPOSITE FUNCTIONS

■ 1. Write $f(x)$ as the composition of two functions.

$$f(x)=\frac{4}{(2x^2-5x)^3}$$

Solution:

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x)=h(g(x))$. We might notice that the expression $2x^2-5x$ is inside the parentheses in the denominator, so we could decompose the function as

$$g(x)=2x^2-5x \text{ and } h(x)=\frac{4}{x^3}$$

We can check our answer by recomposing the function.

$$h(g(x))=h(2x^2-5x)=\frac{4}{(2x^2-5x)^3}$$

■ 2. Write $f(x)=\ln(\ln x)$ as the composition of two functions.

Solution:

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x) = h(g(x))$. We might notice that the expression $\ln x$ is inside the other log function, so we could decompose the function as

$$g(x) = \ln x \text{ and } h(x) = \ln x$$

We can check our answer by recomposing the function.

$$h(g(x)) = h(\ln x) = \ln(\ln x)$$

■ 3. Write $f(x) = 5(2\sqrt[3]{x})^2 - 8$ as the composition of two functions.

Solution:

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x) = h(g(x))$. We might notice that the expression $2\sqrt[3]{x}$ is inside the parentheses, so we could decompose the function as

$$g(x) = 2\sqrt[3]{x} \text{ and } h(x) = 5x^2 - 8$$

We can check our answer by recomposing the function.

$$h(g(x)) = h(2\sqrt[3]{x}) = 5(2\sqrt[3]{x})^2 - 8$$

■ 4. Write $f(x)$ as the composition of two functions.

$$f(x) = \frac{\frac{1}{x+1}}{\frac{1}{x+1} - 1}$$

Solution:

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x) = h(g(x))$. We might notice that the expression $1/(x+1)$ shows up twice inside the fraction, so we could decompose the function as

$$g(x) = \frac{1}{x+1} \text{ and } h(x) = \frac{x}{x-1}$$

We can check our answer by recomposing the function.

$$h(g(x)) = h\left(\frac{1}{x+1}\right) = \frac{\frac{1}{x+1}}{\frac{1}{x+1} - 1}$$

■ 5. Write $f(x) = 5(2x+3)^4 + 3(2x+3)^2 - 7$ as the composition of two functions.

Solution:

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x) = h(g(x))$. We might notice that the expression $2x+3$ shows up twice inside the function, so we could decompose the function as

$$g(x) = 2x + 3 \text{ and } h(x) = 5x^4 + 3x^2 - 7$$

We can check our answer by recomposing the function.

$$h(g(x)) = h(2x + 3) = 5(2x + 3)^4 + 3(2x + 3)^2 - 7$$

■ 6. Write $f(x)$ as the composition of two functions.

$$f(x) = \frac{4}{x^2 - 7}$$

Solution:

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x) = h(g(x))$. We might notice that the expression $x^2 - 7$ is the denominator of the fraction, so we could decompose the function as

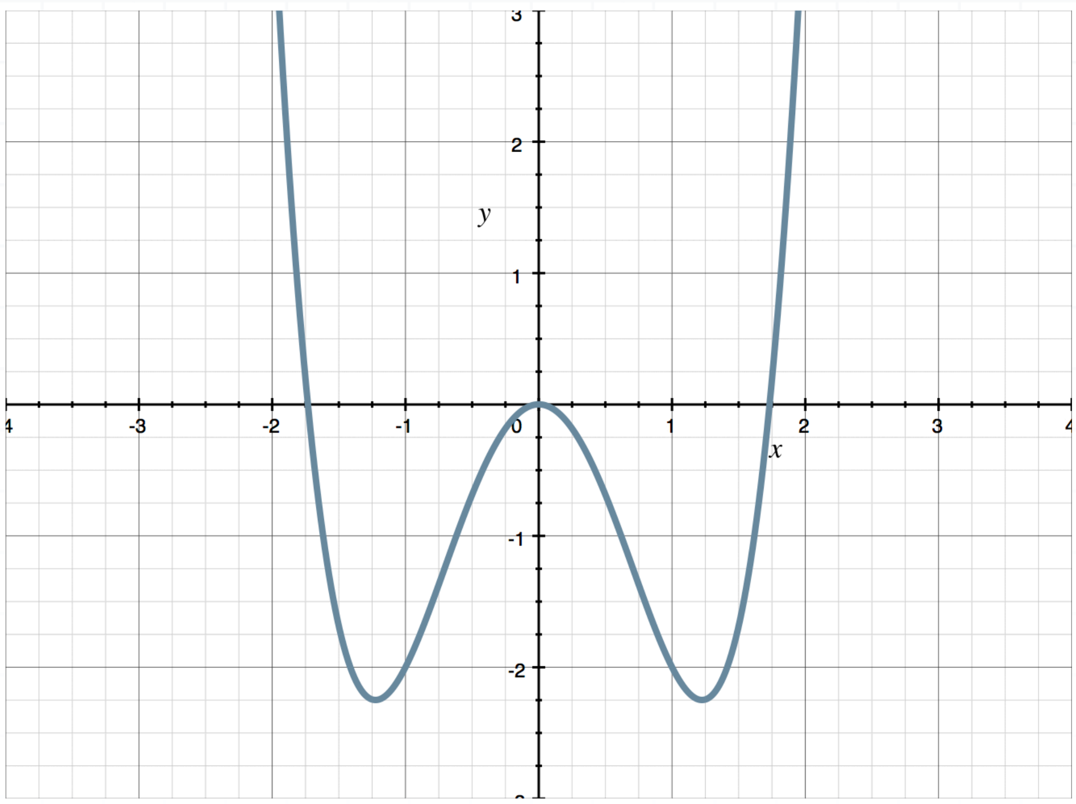
$$g(x) = x^2 - 7 \text{ and } h(x) = \frac{4}{x}$$

We can check our answer by recomposing the function.

$$h(g(x)) = h(x^2 - 7) = \frac{4}{x^2 - 7}$$

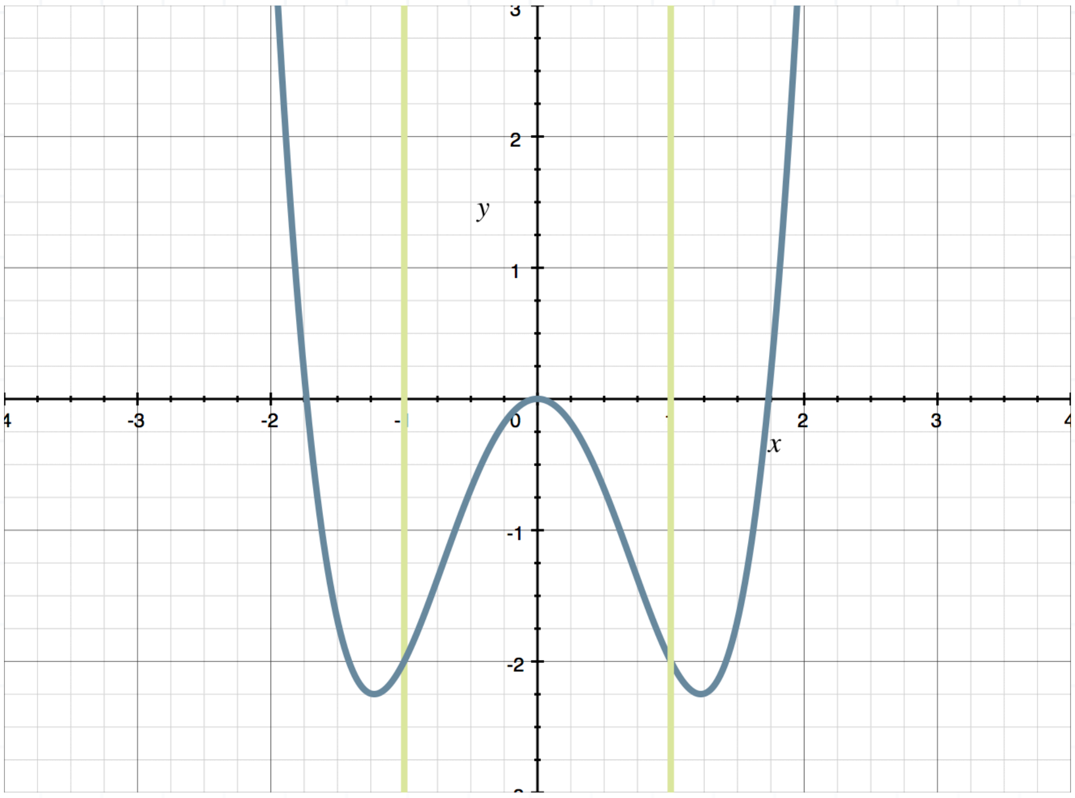
ONE-TO-ONE FUNCTIONS AND THE HORIZONTAL LINE TEST

■ 1. Does the graph represent a one-to-one function?

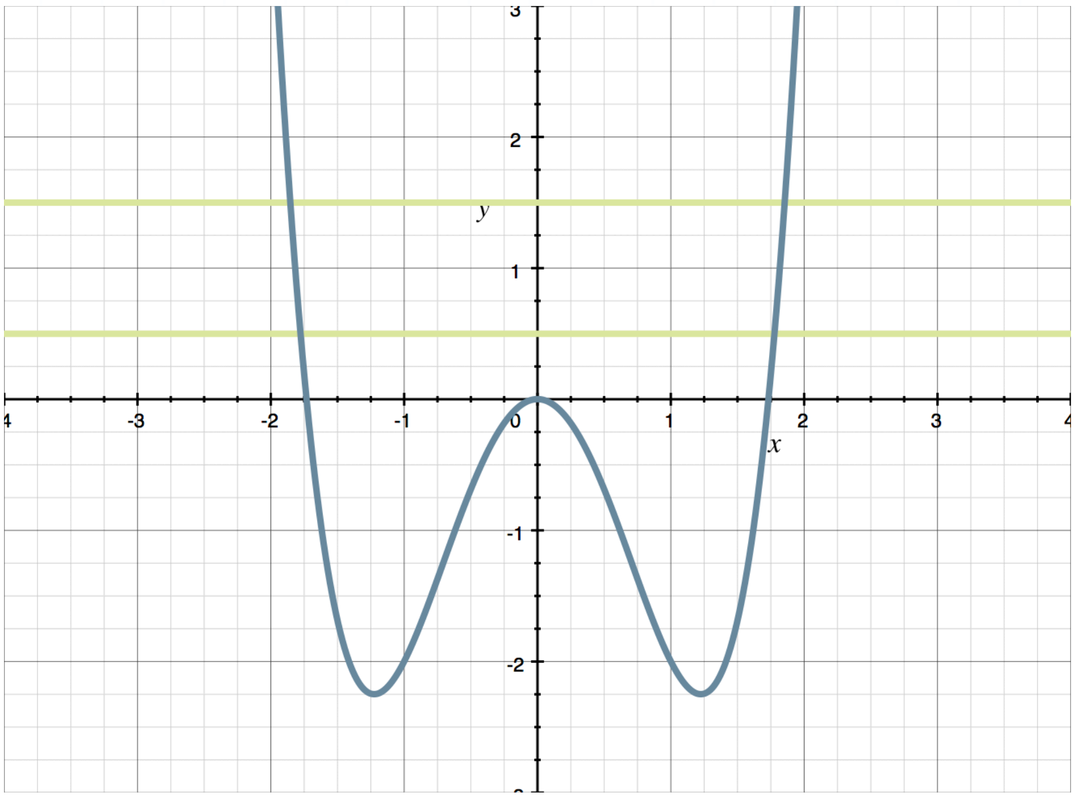


Solution:

First, see if the graph passes the Vertical Line Test. If any vertical line passes through the graph at two or more points, it will fail the Vertical Line Test and isn't a function.

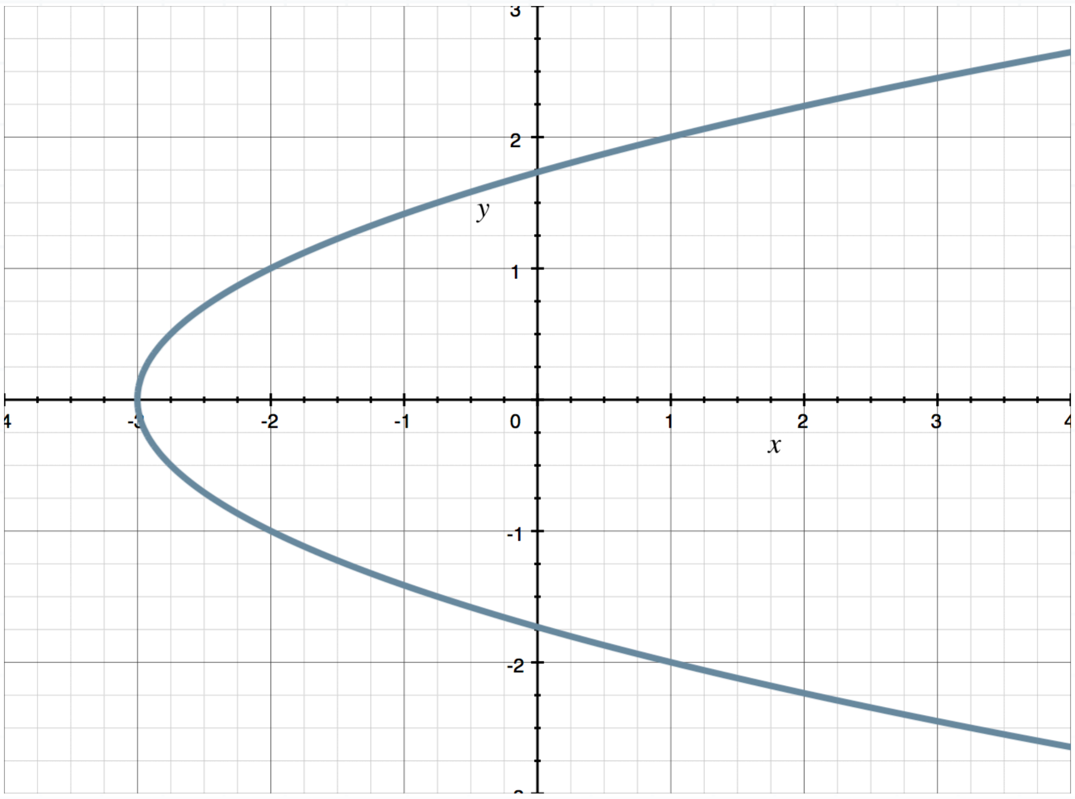


The graph passes the Vertical Line Test, which means that it represents a function. Next, see if the graph passes the Horizontal Line Test. If any horizontal line passes through the graph at two or more points, it will fail the Horizontal Line Test and isn't a one-to-one function.



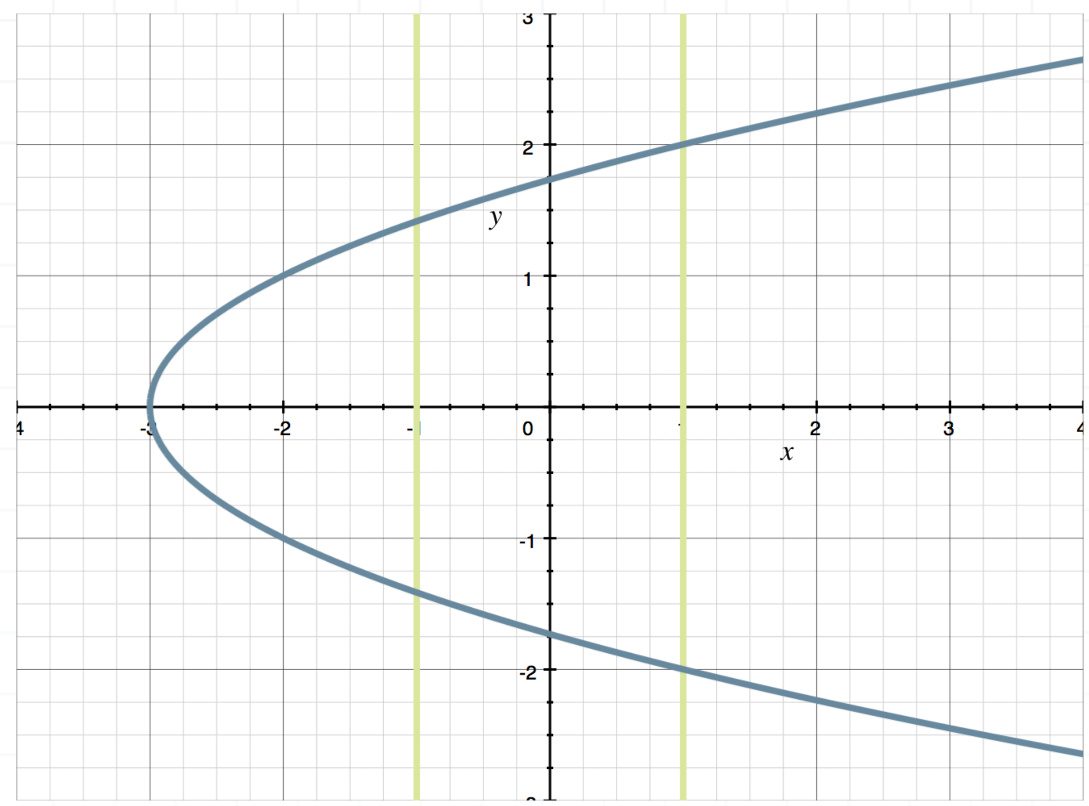
The graph does not pass the Horizontal Line Test, so it's still a function, just not a one-to-one function.

■ 2. Does the graph represent a one-to-one function?



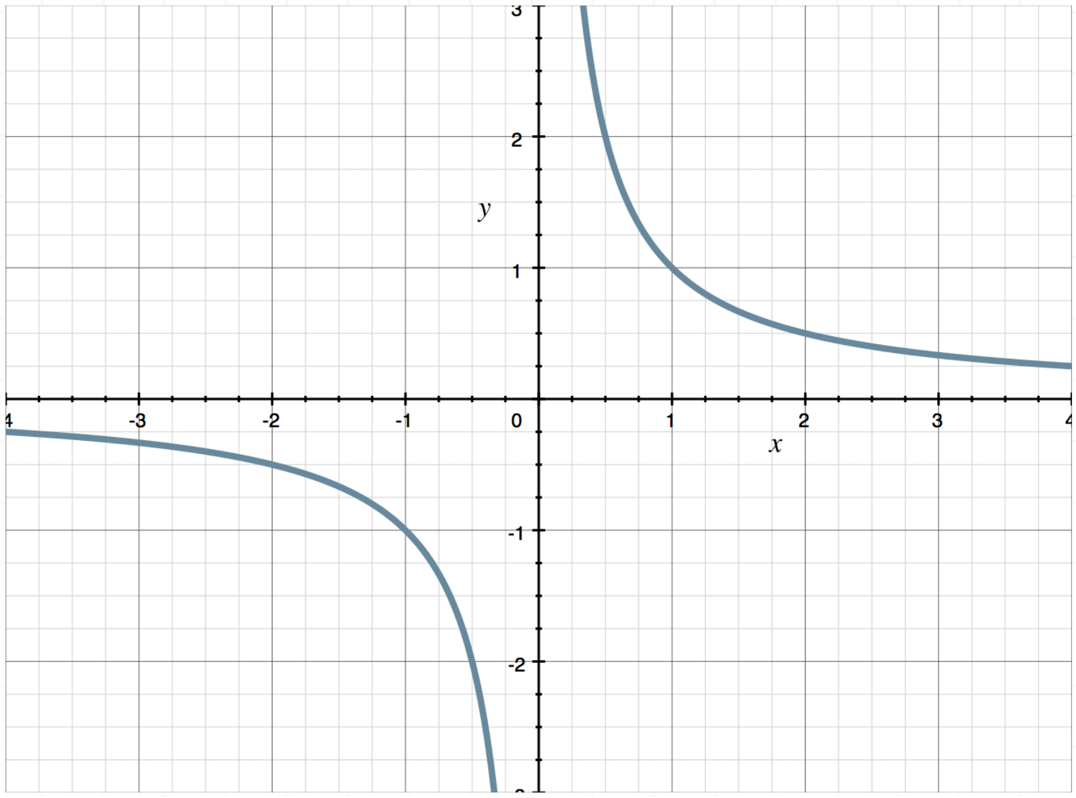
Solution:

First, see if the graph passes the Vertical Line Test. If any vertical line passes through the graph at two or more points, it will fail the Vertical Line Test and isn't a function.



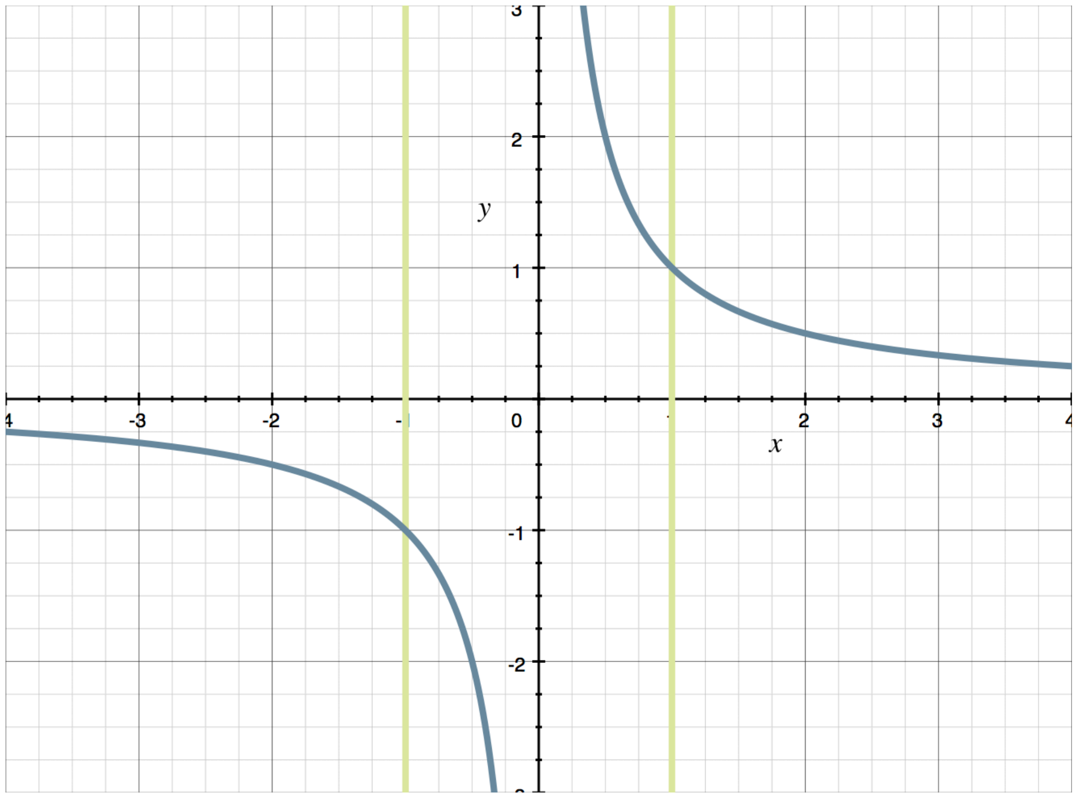
Clearly, a vertical line can cross the graph at two or more points. This means that the graph fails the Vertical Line Test and does not represent a function. Since the graph isn't a function, it can't be a one-to-one function, even if it passes the Horizontal Line Test.

■ 3. Does the graph represent a one-to-one function?

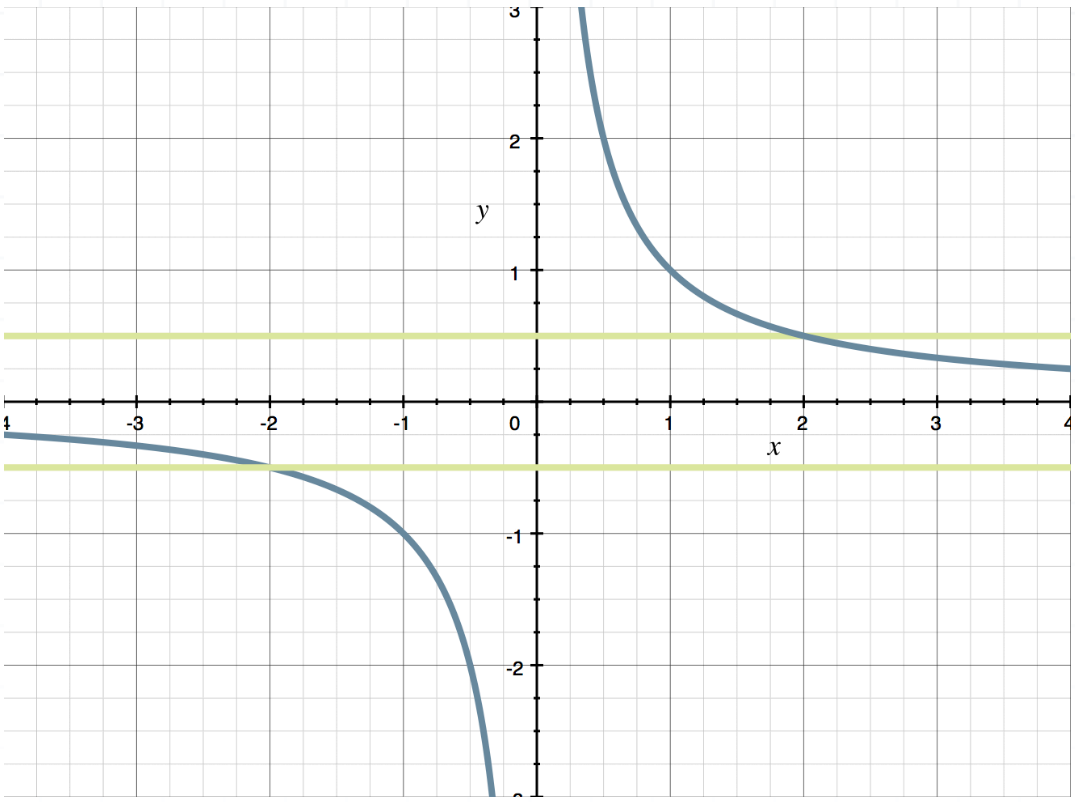


Solution:

First, see if the graph passes the Vertical Line Test. If any vertical line passes through the graph at two or more points, it will fail the Vertical Line Test and isn't a function.



The graph passes the Vertical Line Test, which means that it represents a function. Next, see if the graph passes the Horizontal Line Test. If any horizontal line passes through the graph at two or more points, it will fail the Horizontal Line Test and isn't a one-to-one function.



The graph passes the Horizontal Line Test, so it's a one-to-one function.

4. Show that the function is one-to-one by showing that $f(a) = f(b)$ leads to $a = b$.

$$f(x) = 3x - 4$$

Solution:

Start by replacing x with a and b.

$$3a-4$$

$$3b-4$$

Set these equal to one another, then simplify.

$$3a-4=3b-4$$

$$3a=3b$$

$$a=b$$

Since $a=b$, $f(x)$ is a one-to-one function.

■ 5. Show that the function is one-to-one by showing that $f(a)=f(b)$ leads to $a=b$.

$$f(x)=\frac{x+1}{x-5}$$

Solution:

Start by replacing x with a and b.

$$\frac{a+1}{a-5}$$

$$\frac{b+1}{b-5}$$

Set these equal to one another, then simplify.

$$\frac{a+1}{a-5} = \frac{b+1}{b-5}$$

$$(a+1)(b-5) = (b+1)(a-5)$$

$$ab - 5a + b - 5 = ab - 5b + a - 5$$

$$ab - 5a + b = ab - 5b + a$$

$$-5a + b = -5b + a$$

$$6b = 6a$$

$$b = a$$

Since $a = b$, $f(x)$ is a one-to-one function.

■ 6. Show that the function is not one-to-one by showing that $f(a) = f(b)$ does not lead to $a = b$.

$$f(x) = (x+3)(x-2)$$

Solution:

All we need is one counterexample to show that $f(a) = f(b)$ doesn't imply that $a = b$. Let's use $a = -3$ and $b = 2$.

$$f(a) = f(-3) = (-3+3)(-3-2) = 0(-5) = 0$$

$$f(b) = f(2) = (2+3)(2-2) = 5(0) = 0$$

Since we get the same value for both functions, $f(a)=f(b)$, but we used different values to get the same answer, $a \neq b$, the function is not one-to-one.

INVERSE FUNCTIONS

1. What is the inverse of the function?

$$f(x) = \frac{1}{2}x - 3$$

Solution:

Start by replacing $f(x)$ with y.

$$y = \frac{1}{2}x - 3$$

Switch x and y, then solve for y.

$$x = \frac{1}{2}y - 3$$

$$x + 3 = \frac{1}{2}y$$

$$2x + 6 = y$$

Replace y with $f^{-1}(x)$ to write the inverse function.

$$f^{-1}(x) = 2x + 6$$

2. What is the inverse of the function?

$$f(x) = -4x + 5$$

Solution:

Start by replacing $f(x)$ with y.

$$y = -4x + 5$$

Switch x and y, then solve for y.

$$x = -4y + 5$$

$$x - 5 = -4y$$

$$-\frac{1}{4}x + \frac{5}{4} = y$$

Replace y with $f^{-1}(x)$ to write the inverse function.

$$f^{-1}(x) = -\frac{1}{4}x + \frac{5}{4}$$

■ 3. What is the inverse of the function?

$$f(x) = \frac{2x}{x-5}$$

Solution:

Start by replacing $f(x)$ with y.

$$y = \frac{2x}{x-5}$$

Switch x and y, then solve for y.

$$x = \frac{2y}{y-5}$$

$$x(y-5) = 2y$$

$$xy - 5x = 2y$$

$$-5x = 2y - xy$$

$$-5x = y(2-x)$$

$$-\frac{5x}{2-x} = y$$

Replace y with $f^{-1}(x)$ to write the inverse function.

$$f^{-1}(x) = -\frac{5x}{2-x}$$

■ 4. What is the inverse of the function?

$$f(x) = \frac{1}{x} + 3$$

Solution:

Start by replacing $f(x)$ with y.

$$y = \frac{1}{x} + 3$$

Switch x and y, then solve for y.

$$x = \frac{1}{y} + 3$$

$$x - 3 = \frac{1}{y}$$

$$y(x - 3) = 1$$

$$y = \frac{1}{x - 3}$$

Replace y with $f^{-1}(x)$ to write the inverse function.

$$f^{-1}(x) = \frac{1}{x - 3}$$

■ 5. What is the inverse of the function?

$$f(x) = -\frac{3}{x - 2} - 4$$

Solution:

Start by replacing $f(x)$ with y.

$$y = -\frac{3}{x-2} - 4$$

Switch x and y, then solve for y.

$$x = -\frac{3}{y-2} - 4$$

$$x + 4 = -\frac{3}{y-2}$$

$$(y-2)(x+4) = -3$$

$$y - 2 = -\frac{3}{x+4}$$

$$y = -\frac{3}{x+4} + 2$$

Replace y with $f^{-1}(x)$ to write the inverse function.

$$f^{-1}(x) = -\frac{3}{x+4} + 2$$

■ 6. What is the inverse of the function?

$$f(x) = \frac{x-2}{x+3}$$

Solution:

Start by replacing $f(x)$ with y.

$$y = \frac{x-2}{x+3}$$

Switch x and y, then solve for y.

$$x = \frac{y-2}{y+3}$$

$$x(y+3) = y-2$$

$$xy + 3x = y - 2$$

$$xy - y = -3x - 2$$

$$y(x-1) = -3x - 2$$

$$y(x-1) = -(3x+2)$$

$$y = -\frac{3x+2}{x-1}$$

Replace y with $f^{-1}(x)$ to write the inverse function.

$$f^{-1}(x) = -\frac{3x+2}{x-1}$$

FINDING A FUNCTION FROM ITS INVERSE

■ 1. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(1) = -2$$

$$f^{-1}(-3) = -1$$

Solution:

$(1,-2)$ and $(-3,-1)$ are points on the inverse function $f^{-1}(x)$. Switch the x and y-values in those points in order to get points on $f(x)$.

$$(-2,1)$$

$$(-1,-3)$$

Find the slope between these points to find the slope of $f(x)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-1}{-1-(-2)} = \frac{-4}{1} = -4$$

Use point-slope form $y - y_1 = m(x - x_1)$ to find the equation of the line by plugging in this slope, and either of the points $(-2,1)$ and $(-1,-3)$. We'll use $(-2,1)$.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -4(x-(-2))$$

$$y - 1 = -4(x + 2)$$

$$y - 1 = -4x - 8$$

$$y = -4x - 7$$

So $f(x)$ is given by

$$f(x) = -4x - 7$$

■ 2. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(0) = 3$$

$$f^{-1}(-2) = 1$$

Solution:

$(0,3)$ and $(-2,1)$ are points on the inverse function $f^{-1}(x)$. Switch the x and y -values in those points in order to get points on $f(x)$.

$$(3,0)$$

$$(1, -2)$$

Find the slope between these points to find the slope of $f(x)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{1 - 3} = \frac{-2}{-2} = 1$$

Use point-slope form $y - y_1 = m(x - x_1)$ to find the equation of the line by plugging in this slope, and either of the points $(3,0)$ and $(1, -2)$. We'll use $(3,0)$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 1(x - 3)$$

$$y = x - 3$$

So $f(x)$ is given by

$$f(x) = x - 3$$

■ 3. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(2) = 5$$

$$f^{-1}(4) = 9$$

Solution:

$(2,5)$ and $(4,9)$ are points on the inverse function $f^{-1}(x)$. Switch the x and y -values in those points in order to get points on $f(x)$.

$$(5,2)$$

$$(9,4)$$

Find the slope between these points to find the slope of $f(x)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{9 - 5} = \frac{2}{4} = \frac{1}{2}$$

Use point-slope form $y - y_1 = m(x - x_1)$ to find the equation of the line by plugging in this slope, and either of the points $(5,2)$ and $(9,4)$. We'll use $(5,2)$.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{2}(x - 5)$$

$$y - 2 = \frac{1}{2}x - \frac{5}{2}$$

$$y = \frac{1}{2}x - \frac{5}{2} + \frac{4}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2}$$

So $f(x)$ is given by

$$f(x) = \frac{1}{2}x - \frac{1}{2}$$

■ 4. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(-4) = 7$$

$$f^{-1}(-1) = 14$$

Solution:

$(-4,7)$ and $(-1,14)$ are points on the inverse function $f^{-1}(x)$. Switch the x and y-values in those points in order to get points on $f(x)$.

$$(7,-4)$$

$$(14,-1)$$

Find the slope between these points to find the slope of $f(x)$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-1-(-4)}{14-7}=\frac{3}{7}$$

Use point-slope form $y-y_1=m(x-x_1)$ to find the equation of the line by plugging in this slope, and either of the points $(7,-4)$ and $(14,-1)$. We'll use $(14,-1)$.

$$y-y_1=m(x-x_1)$$

$$y-(-1)=\frac{3}{7}(x-14)$$

$$y+1=\frac{3}{7}x-6$$

$$y=\frac{3}{7}x-7$$

So $f(x)$ is given by

$$f(x)=\frac{3}{7}x-7$$

5. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(5) = -4$$

$$f^{-1}(10) = -12$$

Solution:

$(5, -4)$ and $(10, -12)$ are points on the inverse function $f^{-1}(x)$. Switch the x and y-values in those points in order to get points on $f(x)$.

$$(-4,5)$$

$$(-12,10)$$

Find the slope between these points to find the slope of $f(x)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 5}{-12 - (-4)} = \frac{5}{-8} = -\frac{5}{8}$$

Use point-slope form $y - y_1 = m(x - x_1)$ to find the equation of the line by plugging in this slope, and either of the points $(-4,5)$ and $(-12,10)$. We'll use $(-4,5)$.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{5}{8}(x - (-4))$$

$$y - 5 = -\frac{5}{8}(x + 4)$$

$$y - 5 = -\frac{5}{8}x - \frac{5}{2}$$

$$y = -\frac{5}{8}x - \frac{5}{2} + \frac{10}{2}$$

$$y = -\frac{5}{8}x + \frac{5}{2}$$

So $f(x)$ is given by

$$f(x) = -\frac{5}{8}x + \frac{5}{2}$$

■ 6. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(1) = 3$$

$$f^{-1}(2) = 6$$

Solution:

$(1,3)$ and $(2,6)$ are points on the inverse function $f^{-1}(x)$. Switch the x and y -values in those points in order to get points on $f(x)$.

$$(3,1)$$

$$(6,2)$$

Find the slope between these points to find the slope of $f(x)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{6-3} = \frac{1}{3}$$

Use point-slope form $y - y_1 = m(x - x_1)$ to find the equation of the line by plugging in this slope, and either of the points $(3,1)$ and $(6,2)$. We'll use $(3,1)$.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{3}(x - 3)$$

$$y - 1 = \frac{1}{3}x - 1$$

$$y = \frac{1}{3}x$$

So $f(x)$ is given by

$$f(x) = \frac{1}{3}x$$

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