Topic: Systems with non-linear equations

Question: Use any method to find the solution(s) to the system of equations.

$$x^2 + y^2 = 25$$

$$3x - y = 5$$

Answer choices:

A	$(0,5)$	and	$(-3, -4)$
B	$(0, -5)$	and	$(-3, -4)$
C	$(0,5)$	and	$(3,4)$
D	$(0, -5)$	and	$(3,4)$

Solution: D

We'll solve the second equation for y, and then substitute the resulting expression for y into the first equation.

$$3x - y = 5$$

$$-y = 5 - 3x$$

$$y = 3x - 5$$

Plug this expression for y into the first equation, and then solve for x.

$$x^2 + y^2 = 25$$

$$x^2 + (3x - 5)^2 = 25$$

$$x^2 + (3x - 5)(3x - 5) = 25$$

$$x^2 + 9x^2 - 15x - 15x + 25 = 25$$

$$10x^2 - 30x + 25 = 25$$

$$10x^2 - 30x = 0$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0, 3$$

Now we'll plug these values of x into the expression we found for y, to get the corresponding values of y.

For $x = 0$:

$$y = 3x - 5$$

$$y = 3(0) - 5$$

$$y = 0 - 5$$

$$y = -5$$

For $x = 3$:

$$y = 3x - 5$$

$$y = 3(3) - 5$$

$$y = 9 - 5$$

$$y = 4$$

The points $(0, -5)$ and $(3,4)$ satisfy the linear equation in this system. We have to make sure they also satisfy the non-linear equation.

For $(0, -5)$:

$$x^2 + y^2 = 25$$

$$(0)^2 + (-5)^2 = 25$$

$$0 + 25 = 25$$

$$25 = 25$$

For $(3,4)$:

$$x^2 + y^2 = 25$$

$$(3)^2 + (4)^2 = 25$$

$$9 + 16 = 25$$

$$25 = 25$$

Both $(0, -5)$ and $(3,4)$ are solutions to the system of equations.

Topic: Systems with non-linear equations

Question: Solve the system for x and y.

$$(x-2)^2+y^2=4$$

$$\frac{x}{2}-y=-1$$

Answer choices:

A $(2,2)$ and $\left(\frac{6}{5},\frac{2}{5}\right)$

B $\left(\frac{6}{5},2\right)$ and $\left(\frac{2}{5},2\right)$

C $\left(2,\frac{2}{5}\right)$ and $\left(2,\frac{6}{5}\right)$

D $(2,2)$ and $\left(\frac{2}{5},\frac{6}{5}\right)$

Solution: D

Solve the second equation for y.

$$\frac{x}{2} - y = -1$$

$$-y = -\frac{x}{2} - 1$$

$$y = \frac{x}{2} + 1$$

Substitute this expression for y into the first equation.

$$(x-2)^2 + y^2 = 4$$

$$(x-2)^2 + \left(\frac{x}{2} + 1\right)^2 = 4$$

Expand the squares.

$$(x-2)(x-2) + \left(\frac{x}{2} + 1\right)\left(\frac{x}{2} + 1\right) = 4$$

$$x^2 - 4x + 4 + \frac{x^2}{4} + x + 1 = 4$$

$$x^2 + \frac{x^2}{4} - 4x + x + 4 + 1 = 4$$

$$x^2 + \frac{x^2}{4} - 3x + 1 = 0$$

Clear the fraction by multiplying both sides by 4.

$$4x^2 + x^2 - 12x + 4 = 0$$

$$5x^2 - 12x + 4 = 0$$

Factor, and then solve for x.

$$(5x - 2)(x - 2) = 0$$

$5x - 2 = 0$ gives $x = 2/5$

$x - 2 = 0$ gives $x = 2$

Plug these values of x into the expression we found for y, to get the corresponding values of y.

For $x = 2/5$:

$$y = \frac{x}{2} + 1$$

$$y = \frac{1}{2}\left(\frac{2}{5}\right) + 1$$

$$y = \frac{1}{5} + 1$$

$$y = \frac{6}{5}$$

For $x = 2$:

$$y = \frac{x}{2} + 1$$

$$y = \frac{1}{2}(2) + 1$$

$$y = \frac{2}{2} + 1$$

$$y = 1 + 1$$

$$y = 2$$

This tells us that the points $(2,2)$ and $(2/5,6/5)$ satisfy the linear equation in this system. If we plug the coordinates of these points into the non-linear equation, we'll find that they satisfy that one as well.

Topic: Systems with non-linear equations

Question: Solve the system for x and y.

$$2x^2 - 12x - y + 19 = 0$$

$$2x + y = 11$$

Answer choices:

A $(1,3)$ and $(4,9)$

B $(4,3)$ and $(1,9)$

C $(9,3)$ and $(1,4)$

D $(3,1)$ and $(4,9)$

Solution: B

Solve the second equation for y.

$$2x + y = 11$$

$$y = -2x + 11$$

Substitute this expression for y into the first equation, and then simplify.

$$2x^2 - 12x - y + 19 = 0$$

$$2x^2 - 12x - (-2x + 11) + 19 = 0$$

$$2x^2 - 12x + 2x - 11 + 19 = 0$$

$$2x^2 - 10x + 8 = 0$$

Divide both sides by 2.

$$x^2 - 5x + 4 = 0$$

Factor, and then solve for x.

$$(x - 4)(x - 1) = 0$$

$x - 4 = 0$ gives $x = 4$

$x - 1 = 0$ gives $x = 1$

Plug these values of x into the expression we found for y, to get the corresponding values of y.

For $x = 4$:

$$y = -2x + 11$$

$$y = -2(4) + 11$$

$$y = -8 + 11$$

$$y = 3$$

For $x = 1$:

$$y = -2x + 11$$

$$y = -2(1) + 11$$

$$y = -2 + 11$$

$$y = 9$$

So the points $(4,3)$ and $(1,9)$ satisfy the linear equation in this system.

If we plug the coordinates of these points into the non-linear equation, we'll find that they satisfy that one as well.