Algebra 2 Workbook Solutions

Advanced equations

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DIRECT VARIATION

■ 1. If $10k = 5$ and $kx = 3$, find x.

Solution:

Solve the first equation for k.

$$10k = 5$$

$$\frac{10k}{10} = \frac{5}{10}$$

$$k = \frac{1}{2}$$

Plug $k = 1/2$ into the second equation and solve for x.

$$kx = 3$$

$$\frac{1}{2}x = 3$$

$$\frac{2}{1} \cdot \frac{1}{2}x = 3 \cdot \frac{2}{1}$$

$$x = 6$$

■ 2. If x and y vary directly and the constant of variation, k, equals $1/3$, what is the value of y when $x = 54$, assuming the direct variation relationship between the variables is given by $y = kx$?

Solution:

The general form of direct variation is $y = kx$. Plug in the values for k and x.

$$y = \frac{1}{3}(54)$$

$$y = 18$$

■ 3. A restaurant takes in $\$15{,}000$ in a 5 hour period. Write a direct variation equation for the relationship between income and number of hours. Estimate how many hours it would take the restaurant to earn $\$35{,}750$.

Solution:

The general form of the direct variation equation is $y = kx$. If we let i represent income and h represent hours, then we can rewrite the direction variation equation as $i = kh$. Plug in the values for i and h to solve for the constant of variation k.

$$15{,}000 = k(5)$$

$$k = 3{,}000$$

Now that we have the constant of variation, plug $k = 3{,}000$ and $i = 35{,}750$ to solve for h.

$$35{,}750 = 3{,}000h$$

$$h = 11.9$$

So it takes slightly less than 12 hours for the restaurant to earn $\$35{,}750$.

■ 4. If x varies directly with y and $y = 4$ when $x = 20$, what is the value of the constant of variation, k, assuming the direct variation relationship between the variables is given by $y = kx$?

Solution:

The general form of direct variation is $y = kx$. Plug in the values for x and y.

$$4 = k(20)$$

$$\frac{4}{20} = \frac{20k}{20}$$

$$k = \frac{1}{5}$$

■ 5. If x varies directly with y and $y = 15$ when $x = 5$, what is the value of x when $y = 36$, assuming the direct variation relationship between the variables is given by $y = kx$?

Solution:

First, we need to find the constant of variation. The general form of direct variation is $y = kx$. Plug in the values for x and y to solve for k.

$$15 = k(5)$$

$$\frac{15}{5} = \frac{5k}{5}$$

$$k = 3$$

Now that we have the constant of variation, plug in $k = 3$ and $y = 36$ to solve for x.

$$36 = 3x$$

$$\frac{36}{3} = \frac{3x}{3}$$

$$x = 12$$

■ 6. If x varies directly with y and $y = 7$ when $x = 42$, what is the value of y when $x = 54$, assuming the direct variation relationship between the variables is given by $y = kx$?

Solution:

First, we need to find the constant of variation. The general form of direct variation is $y = kx$. Plug in the values for x and y to solve for k.

$$7 = k(42)$$

$$\frac{7}{42} = \frac{42k}{42}$$

$$k = \frac{1}{6}$$

Now that we have the constant of variation, plug in $k = 1/6$ and $x = 54$ to solve for y.

$$y = \frac{1}{6}(54)$$

$$y = 9$$

INVERSE VARIATION

■ 1. If $k/3 = 6$ and $k/x = 2$, find x.

Solution:

Solve the first equation for k.

$$\frac{k}{3} = 6$$

$$k = 18$$

Plug $k = 18$ into the second equation and solve for x.

$$\frac{k}{x} = 2$$

$$\frac{18}{x} = 2$$

$$2x = 18$$

$$x = 9$$

■ 2. The length of the base of a triangle with constant area varies inversely with the triangle's height. When the base is 5 cm long, the height is 6 cm. Find the length of the base when the height is 3 cm.

Solution:

If we call the base of the triangle y, and call the height x, then we can substitute $y=5$ and $x=6$ into the inverse variation equation and solve for k.

$$y=\frac{k}{x}$$

$$5=\frac{k}{6}$$

$$k=30$$

Now plug $k=30$ and a height $x=3$ into the inverse variation equation.

$$y=\frac{30}{3}$$

$$y=10$$

■ 3. If x and y vary inversely and the constant of variation, k, equals $1/3$, what is the value of y when $x=8$?

Solution:

The general form of inverse variation is $y=k/x$. Plug in the values for k and x.

$$y = \frac{\frac{1}{3}}{8}$$

$$y = \frac{1}{3} \div \frac{8}{1}$$

$$y = \frac{1}{3} \cdot \frac{1}{8}$$

$$y = \frac{1}{24}$$

■ 4. If x varies inversely with y and $y = 5$ when $x = 6$, what is the value of the constant of variation, k?

Solution:

The general form of inverse variation is $y = k/x$. Plug in the values for x and y.

$$5 = \frac{k}{6}$$

$$k = 30$$

■ 5. If x varies inversely with y and $y = 4$ when $x = 2$, what is the value of x when $y = 1/2$?

Solution:

First, we need to find the constant of variation. The general form of inverse variation is $y = k/x$. Plug in the values for x and y to solve for k.

$$4 = \frac{k}{2}$$

$$k = 8$$

Now that we have the constant of variation, plug in $k = 8$ and $y = 1/2$ to solve for x.

$$\frac{1}{2} = \frac{8}{x}$$

$$1 \cdot x = 2 \cdot 8$$

$$x = 16$$

■ 6. If x varies inversely with y and $y = 3$ when $x = 9$, what is the value of y when $x = 1/4$?

Solution:

First, we need to find the constant of variation. The general form of inverse variation is $y = k/x$. Plug in the values for x and y to solve for k.

$$3 = \frac{k}{9}$$

$$k = 27$$

Now that we have the constant of variation, plug in $k = 27$ and $x = 1/4$ to solve for y.

$$y = \frac{27}{\frac{1}{4}}$$

$$y = 27 \div \frac{1}{4}$$

$$y = 27 \cdot 4$$

$$y = 108$$

DECIMAL EQUATIONS

■ 1. Solve the decimal equation.

$$0.34x - 0.62 = 1.25$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 100.

$$(0.34x - 0.62 = 1.25)100$$

$$0.34x(100) - 0.62(100) = 1.25(100)$$

$$34x - 62 = 125$$

$$34x = 187$$

$$x = 5.5$$

■ 2. Solve the decimal equation.

$$0.1(2.1a - 1.4a) + 3.57 = 2.8$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 100.

$$(0.1(2.1a - 1.4a) + 3.57)(100) = 2.8(100)$$

$$0.1(2.1a - 1.4a)(100) + 3.57(100) = 2.8(100)$$

$$10(2.1a - 1.4a) + 357 = 280$$

$$21a - 14a = 280 - 357$$

$$7a = -77$$

$$a = -11$$

■ 3. Solve the decimal equation.

$$4a + 6a = 1.7$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(4a + 6a = 1.7)10$$

$$4a(10) + 6a(10) = 1.7(10)$$

$$40a + 60a = 17$$

$$100a = 17$$

$$a = \frac{17}{100}$$

$$a = 0.17$$

■ 4. Solve the decimal equation.

$$0.12n + 3.6 = 4.8$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 100.

$$(0.12n + 3.6 = 4.8)100$$

$$0.12n(100) + 3.6(100) = 4.8(100)$$

$$12n + 360 = 480$$

$$12n = 120$$

$$n = 10$$

■ 5. Solve the decimal equation.

$$5n - 6.1 = -2.9$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(5n - 6.1 = -2.9)10$$

$$5n(10) - 6.1(10) = -2.9(10)$$

$$50n - 61 = -29$$

$$50n = 32$$

$$n = \frac{32}{50}$$

$$n = 0.64$$

■ 6. Solve the decimal equation.

$$3.2x + 2.6 = 1.8x - 4.4$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(3.2x + 2.6 = 1.8x - 4.4)10$$

$$3.2x(10) + 2.6(10) = 1.8x(10) - 4.4(10)$$

$$32x + 26 = 18x - 44$$

$$14x + 26 = -44$$

$$14x = -70$$

$x = -5$

FRACTIONAL EQUATIONS

■ 1. Solve for the variable.

$$2x - 5 = \frac{4x + 3}{5}$$

Solution:

Multiply both sides by the denominator from the right side.

$$(2x - 5) \cdot 5 = \frac{4x + 3}{5} \cdot 5$$

$$10x - 25 = 4x + 3$$

$$6x - 25 = 3$$

$$6x = 28$$

$$x = \frac{14}{3}$$

■ 2. Solve for the variable.

$$\frac{4}{3}x = 18$$

Solution:

Multiply both sides by the reciprocal of the fractional coefficient.

$$\frac{3}{4}\cdot\frac{4}{3}x = 18\cdot\frac{3}{4}$$

$$x = \frac{54}{4}$$

$$x = \frac{27}{2}$$

■ 3. Solve for the variable.

$$\frac{3}{4}x + \frac{5}{4} = \frac{7}{8}$$

Solution:

Multiply every term in the equation by the largest denominator.

$$\left(\frac{3}{4}x + \frac{5}{4} = \frac{7}{8}\right)8$$

$$\frac{3}{4}x\cdot 8 + \frac{5}{4}\cdot 8 = \frac{7}{8}\cdot 8$$

$$6x + 10 = 7$$

$$6x = -3$$

$$x = -\frac{1}{2}$$

■ 4. Solve for the variable.

$$\frac{4}{7}x + \frac{1}{7} = \frac{10}{7}$$

Solution:

Multiply every term in the equation by the largest denominator.

$$\left(\frac{4}{7}x + \frac{1}{7} = \frac{10}{7}\right)7$$

$$\frac{4}{7}x(7) + \frac{1}{7}(7) = \frac{10}{7}(7)$$

$$4x + 1 = 10$$

$$4x = 9$$

$$x = \frac{9}{4}$$

■ 5. Solve for the variable.

$$\frac{1}{2}a - \frac{5}{4}a = -\frac{10}{3} + \frac{5}{2}a$$

Solution:

The least common denominator (LCD) of the fractions is 12, because 12 is the least common multiple (LCM) of the denominators in the equation, 2, 3, and 4. So in order to remove the fractions, we'll multiply both sides of the equation by 12.

$$\left(\frac{1}{2}a-\frac{5}{4}a=-\frac{10}{3}+\frac{5}{2}a\right)12$$

$$\frac{1}{2}a\cdot 12-\frac{5}{4}a\cdot 12=-\frac{10}{3}\cdot 12+\frac{5}{2}a\cdot 12$$

$$6a-15a=-40+30a$$

$$-9a=-40+30a$$

$$-39a=-40$$

$$a=\frac{40}{39}$$

■ 6. Solve for the variable.

$$\frac{1}{2}\left(\frac{1}{2}x-\frac{1}{3}\right)=\frac{7}{3}+\frac{9}{2}$$

Solution:

The least common denominator (LCD) of the fractions is 6, because 6 is the least common multiple (LCM) of the denominators in the equation, 2 and 3. So in order to remove the fractions, we'll multiply both sides of the equation by 6.

$$6 \cdot \frac{1}{2}\left(\frac{1}{2}x - \frac{1}{3}\right) = 6 \cdot \frac{7}{3} + 6 \cdot \frac{9}{2}$$

$$3\left(\frac{1}{2}x - \frac{1}{3}\right) = 14 + 27$$

$$3\left(\frac{1}{2}x - \frac{1}{3}\right) = 41$$

$$\frac{1}{2}x - \frac{1}{3} = \frac{41}{3}$$

The least common denominator (LCD) of the fractions is 6, because 6 is the least common multiple (LCM) of the denominators in the equation, 2 and 3. So in order to remove the fractions, we'll multiply both sides of the equation by 6.

$$6 \cdot \frac{1}{2}x - 6 \cdot \frac{1}{3} = 6 \cdot \frac{41}{3}$$

$$3x - 2 = 82$$

$$3x = 84$$

$$x = 28$$

RATIONAL EQUATIONS

■ 1. Solve the equation.

$$\frac{x-3}{x+3}=\frac{4}{5}$$

Solution:

If $x+3=0$, then the equation is undefined, so this equation is true only if $x\neq -3$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $5(x+3)$. Multiply both sides of the equation by the LCD, $5(x+3)$.

$$\left(\frac{x-3}{x+3}=\frac{4}{5}\right)5(x+3)$$

$$\frac{x-3}{x+3}\cdot 5(x+3)=\frac{4}{5}\cdot 5(x+3)$$

$$5(x-3)=4(x+3)$$

$$5x-15=4x+12$$

$$x=27$$

■ 2. Solve the equation.

$$\frac{x}{6} - \frac{5}{3x} = \frac{1}{4}$$

Solution:

If $x = 0$, then the equation is undefined, so this equation is true only if $x \neq 0$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $12x$. Multiply both sides of the equation by the LCD, $12x$.

$$\left(\frac{x}{6} - \frac{5}{3x} = \frac{1}{4}\right) 12x$$

$$\frac{x}{6} \cdot 12x - \frac{5}{3x} \cdot 12x = \frac{1}{4} \cdot 12x$$

$$2x^2 - 20 = 3x$$

$$2x^2 - 3x - 20 = 0$$

$$(x-4)(2x+5) = 0$$

$x = -\dfrac{5}{2}$ and $x = 4$

■ 3. Solve the equation.

$$\frac{2}{x+5} = \frac{4}{x-1}$$

Solution:

If $x + 5 = 0$ or $x - 1 = 0$, then the equation is undefined, so this equation is true only if $x \neq -5$ and $x \neq 1$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $(x+5)(x-1)$. Multiply both sides of the equation by the LCD, $(x+5)(x-1)$.

$$\left(\frac{2}{x+5} = \frac{4}{x-1}\right)(x+5)(x-1)$$

$$\frac{2}{x+5} \cdot (x+5)(x-1) = \frac{4}{x-1} \cdot (x+5)(x-1)$$

$$2(x-1) = 4(x+5)$$

$$2x - 2 = 4x + 20$$

$$-2x = 22$$

$$x = -11$$

■ 4. Solve the equation.

$$2 + \frac{6}{x-7} = -\frac{8}{x+3}$$

Solution:

If $x-7=0$ or $x+3=0$, then the equation is undefined, so this equation is true only if $x\neq 7$ and $x\neq -3$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $(x-7)(x+3)$. Multiply both sides of the equation by the LCD, $(x-7)(x+3)$.

$$\left(2+\frac{6}{x-7}=-\frac{8}{x+3}\right)(x-7)(x+3)$$

$$2\cdot(x-7)(x+3)+\frac{6}{x-7}\cdot(x-7)(x+3)=-\frac{8}{x+3}\cdot(x-7)(x+3)$$

$$2\cdot(x^2-4x-21)+6(x+3)=-8(x-7)$$

$$2x^2-8x-42+6x+18=-8x+56$$

$$2x^2-2x-24=-8x+56$$

$$2x^2+6x-80=0$$

$$x^2+3x-40=0$$

$$(x+8)(x-5)=0$$

$x=-8$ and $x=5$

■ 5. Solve the equation.

$$\frac{5}{a-4}-\frac{3}{a+4}=-\frac{1}{a^2-16}$$

Solution:

Factor the denominator on the right side.

$$\frac{5}{a-4}-\frac{3}{a+4}=-\frac{1}{(a-4)(a+4)}$$

If $a-4=0$ or $a+4=0$, then the equation is undefined, so this equation is true only if $x\neq -4$ and $x\neq 4$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $(a-4)(a+4)$. Multiply both sides of the equation by the LCD, $(a-4)(a+4)$.

$$\left(\frac{5}{a-4}-\frac{3}{a+4}=-\frac{1}{(a-4)(a+4)}\right)(a-4)(a+4)$$

$$\frac{5}{a-4}\cdot(a-4)(a+4)-\frac{3}{a+4}\cdot(a-4)(a+4)=-\frac{1}{(a-4)(a+4)}\cdot(a-4)(a+4)$$

$$5(a+4)-3(a-4)=-1$$

$$5a+20-3a+12=-1$$

$$2a+32=-1$$

$$2a=-33$$

$$a = -\frac{33}{2}$$

■ 6. Solve the equation.

$$\frac{1}{2x^2} + \frac{3}{4x} = \frac{x+7}{x^2}$$

Solution:

If $x = 0$, then the equation is undefined, so this equation is true only if $x \neq 0$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $4x^2$. Multiply both sides of the equation by the LCD, $4x^2$.

$$\left(\frac{1}{2x^2} + \frac{3}{4x} = \frac{x+7}{x^2}\right) 4x^2$$

$$\frac{1}{2x^2} \cdot 4x^2 + \frac{3}{4x} \cdot 4x^2 = \frac{x+7}{x^2} \cdot 4x^2$$

$$2 + 3x = 4(x+7)$$

$$2 + 3x = 4x + 28$$

$$2 = x + 28$$

$$x = -26$$

RADICAL EQUATIONS

■ 1. Solve the radical equation for the variable.

$$\sqrt[3]{2x-1}+5=7$$

Solution:

To solve the radical equation, first isolate the radical by subtracting 5 from both sides of the equation.

$$\sqrt[3]{2x-1}=2$$

$$(\sqrt[3]{2x-1})^3=2^3$$

$$2x-1=8$$

$$2x=9$$

$$x=\frac{9}{2}$$

Plug the solution back into the original equation to make sure it's an actual solution.

$$\sqrt[3]{2\cdot\frac{9}{2}-1}+5=7$$

$$\sqrt[3]{9-1}+5=7$$

$$\sqrt[3]{8} + 5 = 7$$

$$2 + 5 = 7$$

$$7 = 7$$

So $x = 9/2$ is a solution to the radical equation.

■ 2. Solve the radical equation for the variable.

$$2\sqrt{x} = 14$$

Solution:

To solve the radical equation, first isolate the radical by dividing both sides of the equation by 2.

$$2\sqrt{x} = 14$$

$$\sqrt{x} = 7$$

$$(\sqrt{x})^2 = 7^2$$

$$x = 49$$

Plug the solution back into the original equation to make sure it satisfies it.

$$2\sqrt{49} = 14$$

$$2(7) = 14$$

$$14 = 14$$

So $x = 49$ is a solution to the radical equation.

■ 3. Solve the radical equation for the variable.

$$\sqrt{x+1} - 3 = 2$$

Solution:

To solve the radical equation, first isolate the radical by adding 3 to both sides of the equation.

$$\sqrt{x+1} - 3 = 2$$

$$\sqrt{x+1} = 5$$

$$(\sqrt{x+1})^2 = 5^2$$

$$x + 1 = 25$$

$$x = 24$$

Plug the solution back into the original equation to make sure it's an actual solution.

$$\sqrt{24+1} - 3 = 2$$

$$\sqrt{25} - 3 = 2$$

$$5 - 3 = 2$$

$$2 = 2$$

So $x = 24$ is a solution to the radical equation.

■ 4. Solve the radical equation for the variable.

$$\sqrt{x-3} + 2 = \sqrt{2x+1}$$

Solution:

Square both sides of the equation. Remember to FOIL the right side of the equation.

$$(\sqrt{x-3} + 2)^2 = (\sqrt{2x+1})^2$$

$$(x-3) + 4\sqrt{x-3} + 4 = 2x + 1$$

$$x + 1 + 4\sqrt{x-3} = 2x + 1$$

Isolate the radical.

$$4\sqrt{x-3} = 2x + 1 - x - 1$$

$$4\sqrt{x-3} = x$$

$$\sqrt{x-3} = \frac{x}{4}$$

Square both sides.

$$(\sqrt{x-3})^2 = \left(\frac{x}{4}\right)^2$$

$$x - 3 = \frac{x^2}{16}$$

$$\frac{x^2}{16} - x + 3 = 0$$

$$x^2 - 16x + 48 = 0$$

$$(x-4)(x-12) = 0$$

Then the solutions are

$$x - 4 = 0$$

$$x = 4$$

and

$$x - 12 = 0$$

$$x = 12$$

Plug both solutions back into the original equation to make sure they're actual solutions. We get

$$\sqrt{4-3} + 2 = \sqrt{2(4)+1}$$

$$\sqrt{1} + 2 = \sqrt{8+1}$$

$$1+2=\sqrt{9}$$

$$3=3$$

and

$$\sqrt{12-3}+2=\sqrt{2(12)+1}$$

$$\sqrt{9}+2=\sqrt{24+1}$$

$$3+2=\sqrt{25}$$

$$5=5$$

So $x=4$ and $x=12$ are solutions to the radical equation.

■ 5. Solve the radical equation for the variable.

$$\sqrt{1-x}-x=5$$

Solution:

To solve the radical equation, first isolate the radical by adding x from both sides of the equation.

$$\sqrt{1-x}-x=5$$

$$\sqrt{1-x}=x+5$$

Square both sides of the equation. Remember to FOIL the right side of the equation.

$$(\sqrt{1-x})^2 = (x+5)^2$$

$$1 - x = x^2 + 10x + 25$$

Move all terms to one side of the equation.

$$x^2 + 11x + 24 = 0$$

$$(x+8)(x+3) = 0$$

Then the solutions are

$$x + 8 = 0$$

$$x = -8$$

and

$$x + 3 = 0$$

$$x = -3$$

Plug both solutions back into the original equation to make sure they're actual solutions.

$$\sqrt{1-(-8)} - (-8) = 5$$

$$\sqrt{1+8} + 8 = 5$$

$$\sqrt{9} + 8 = 5$$

$$3 + 8 = 5$$

$$11 = 5$$

and

$$\sqrt{1 - (-3)} - (-3) = 5$$

$$\sqrt{1 + 3} + 3 = 5$$

$$\sqrt{4} + 3 = 5$$

$$2 + 3 = 5$$

$$5 = 5$$

So $x = -3$ is the only solution.

■ 6. Solve the radical equation for the variable.

$$\sqrt{x^2 - 2x + 4} + 4 = x$$

Solution:

To solve the radical equation, first isolate the radical by subtracting 4 from both sides of the equation.

$$\sqrt{x^2 - 2x + 4} + 4 = x$$

$$\sqrt{x^2 - 2x + 4} = x - 4$$

Square both sides of the equation. Remember to FOIL the right side of the equation.

$$(\sqrt{x^2 - 2x + 4})^2 = (x - 4)^2$$

$$x^2 - 2x + 4 = x^2 - 8x + 16$$

Move all terms to one side of the equation.

$$x^2 - x^2 - 2x + 8x + 4 - 16 = x^2 - x^2 - 8x + 8x + 16 - 16$$

$$6x - 12 = 0$$

Solve for x.

$$6x - 12 + 12 = 0 + 12$$

$$6x = 12$$

$$x = 2$$

Plug the solution back into the original equation to make sure it satisfies it.

$$\sqrt{2^2 - 2(2) + 4} + 4 = 2$$

$$\sqrt{4 - 4 + 4} + 4 = 2$$

$$\sqrt{4} + 4 = 2$$

$$2 + 4 = 2$$

$$6 = 2$$

So there's no solution to the radical equation.

MULTIVARIABLE EQUATIONS

■ 1. Solve for x if $y = z/x$.

Solution:

Multiply both sides by x.

$$y = \frac{z}{x}$$

$$x \cdot y = \frac{z}{x} \cdot x$$

$$xy = z$$

Divide both sides by y.

$$\frac{xy}{y} = \frac{z}{y}$$

$$x = \frac{z}{y}$$

■ 2. Solve for t if $4s - 3t + u = 5$.

Solution:

Get $-3t$ by itself by subtracting $4s$ and u from both sides of the equation.

$$4s - 3t + u = 5$$

$$4s - 4s - 3t + u = 5 - 4s$$

$$-3t + u = 5 - 4s$$

$$-3t + u - u = 5 - 4s - u$$

$$-3t = 5 - 4s - u$$

Divide both sides by -3.

$$\frac{-3t}{-3} = \frac{5 - 4s - u}{-3}$$

$$t = -\frac{5 - 4s - u}{3}$$

$$t = \frac{4s + u - 5}{3}$$

■ 3. Solve for y if $z - x + 4y = 3x + z$.

Solution:

Get $4y$ by itself by subtracting z and adding x to both sides of the equation.

$$z - x + 4y = 3x + z$$

$$z - z - x + 4y = 3x + z - z$$

$$-x + 4y = 3x$$

$$-x + x + 4y = 3x + x$$

$$4y = 4x$$

Divide both sides by 4.

$$\frac{4y}{4} = \frac{4x}{4}$$

$$y = x$$

■ 4. Solve for c if $2a - b + 3c = 2b - 4a + c$.

Solution:

Get $3c$ by itself by subtracting $2a$ and adding b to both sides of the equation.

$$2a - b + 3c = 2b - 4a + c$$

$$2a - 2a - b + 3c = 2b - 4a - 2a + c$$

$$-b + 3c = 2b - 6a + c$$

$$-b + b + 3c = 2b + b - 6a + c$$

$$3c = 3b - 6a + c$$

Get all c-terms to the same side of the equation by subtracting c from both sides.

$$3c - c = 3b - 6a + c - c$$

$$2c = 3b - 6a$$

Divide both sides by 2.

$$\frac{2c}{2} = \frac{3b - 6a}{2}$$

$$c = \frac{3b - 6a}{2}$$

■ 5. Solve for y if $2x - y + z = 3x$.

Solution:

Get $-y$ by itself by subtracting $2x$ and z from both sides of the equation.

$$2x - y + z = 3x$$

$$2x - 2x - y + z = 3x - 2x$$

$$-y + z = x$$

$$-y + z - z = x - z$$

$$-y = x - z$$

Multiply both sides by -1.

$$-1 \cdot -y = -1(x - z)$$

$$y = -x + z$$

$$y = z - x$$

6. Solve for a if $x + y = 3ab + c$.

Solution:

Get $3ab$ by itself by subtracting c from both sides of the equation.

$$x + y = 3ab + c$$

$$x + y - c = 3ab + c - c$$

$$x + y - c = 3ab$$

Divide both sides by $3b$.

$$\frac{x + y - c}{3b} = \frac{3ab}{3b}$$

$$\frac{x + y - c}{3b} = a$$

$$a = \frac{x + y - c}{3b}$$

MULTIVARIABLE RATIONAL EQUATIONS

■ 1. Solve the abstract equation for x, if $x \neq 0$.

$$\frac{1}{x} - z = y$$

Solution:

Multiply every term of the equation by the denominator of the fraction.

$$\left(\frac{1}{x} - z = y\right)x$$

$$\frac{1}{x}(x) - z(x) = y(x)$$

$$1 - xz = xy$$

Move all the terms containing x to the left side of the equation, and move all the terms that don't contain x to the right side of the equation.

$$1 - 1 - xz - xy = xy - xy - 1$$

$$-xz - xy = -1$$

Factor out -1 from the left side of the equation.

$$-1(xz + xy) = -1$$

$$xz + xy = 1$$

Factor out x from the left side of the equation.

$$x(z+y)=1$$

Divide both sides by $(z+y)$.

$$\frac{x(z+y)}{(z+y)}=\frac{1}{(z+y)}$$

$$x=\frac{1}{z+y}$$

■ 2. Solve the abstract equation for y, if $x \neq 0$.

$$\frac{y}{x}+3x=2z$$

Solution:

Multiply every term of the equation by the denominator of the fraction.

$$\left(\frac{y}{x}+3x=2z\right)x$$

$$\frac{y}{x}(x)+3x(x)=2z(x)$$

$$y+3x^2=2xz$$

Move all the terms containing y to the left side of the equation, and move all the terms that don't contain y to the right side of the equation.

$$y + 3x^2 - 3x^2 = 2xz - 3x^2$$

$$y = 2xz - 3x^2$$

■ 3. Solve the abstract equation for a, if $a \neq 0$ and $b \neq 0$.

$$\frac{bc}{a} - cxy = \frac{z}{b}$$

Solution:

Multiply every term of the equation by the denominator of both fractions.

$$\left(\frac{bc}{a} - cxy = \frac{z}{b}\right) ab$$

$$\frac{bc}{a}(ab) - cxy(ab) = \frac{z}{b}(ab)$$

$$b^2c - abcxy = az$$

Move all the terms containing a to the left side of the equation, and move all the terms that don't contain a to the right side of the equation.

$$b^2c - b^2c - abcxy - az = az - az - b^2c$$

$$-abcxy - az = -b^2c$$

Factor out -1 from the left side of the equation.

$$-1(abcxy + az) = -b^2c$$

$$abcxy + az = b^2c$$

Factor out a from the left side of the equation.

$$a(bcxy + z) = b^2c$$

Divide both sides by $(bcxy + z)$.

$$\frac{a(bcxy + z)}{(bcxy + z)} = \frac{b^2c}{(bcxy + z)}$$

$$a = \frac{b^2c}{bcxy + z}$$

■ 4. Solve the abstract equation for y, if $y \neq 0$, $b \neq 0$, and $n \neq 0$.

$$\frac{1}{y} + \frac{a}{b} = \frac{m}{n}$$

Solution:

Multiply every term of the equation by the denominator of each fraction.

$$\left(\frac{1}{y} + \frac{a}{b} = \frac{m}{n}\right) bny$$

$$\frac{1}{y}(bny) + \frac{a}{b}(bny) = \frac{m}{n}(bny)$$

$$bn + any = bmy$$

Move all the terms containing y to the left side of the equation, and move all the terms that don't contain y to the right side of the equation.

$$bn - bn + any - bmy = bmy - bmy - bn$$

$$any - bmy = -bn$$

Factor out y from the left side of the equation.

$$y(an - bm) = -bn$$

Divide both sides by $(an - bm)$.

$$\frac{y(an - bm)}{(an - bm)} = \frac{-bn}{(an - bm)}$$

$$y = -\frac{bn}{an - bm}$$

■ 5. Solve the abstract equation for x, if $z \neq 0$, $n \neq 0$, and $b \neq 0$.

$$\frac{2x + y}{z} - \frac{m}{n} = \frac{a}{b}$$

Solution:

Multiply every term of the equation by the denominator of each fraction.

$$\left(\frac{2x + y}{z} - \frac{m}{n} = \frac{a}{b}\right) bnz$$

$$\frac{2x+y}{z}(bnz) - \frac{m}{n}(bnz) = \frac{a}{b}(bnz)$$

$$bn(2x+y) - bmz = anz$$

Distribute bn through $(2x+y)$.

$$2bnx + bny - bmz = anz$$

Move all the terms containing x to the left side of the equation, and move all the terms that don't contain x to the right side of the equation.

$$2bnx + bny - bny - bmz + bmz = anz - bny + bmz$$

$$2bnx = anz - bny + bmz$$

Divide both sides by $2bn$.

$$\frac{2bnx}{2bn} = \frac{anz - bny + bmz}{2bn}$$

$$x = \frac{anz - bny + bmz}{2bn}$$

■ 6. Solve the abstract equation for x, if $x \neq 0$ and $y + z \neq 0$.

$$\frac{1}{x} + \frac{2}{y+z} = 3$$

Solution:

Multiply every term of the equation by the denominator of each fraction.

$$\left(\frac{1}{x}+\frac{2}{y+z}=3\right)\cdot x(y+z)$$

$$\frac{1}{x}\cdot x(y+z)+\frac{2}{y+z}\cdot x(y+z)=3\cdot x(y+z)$$

$$y+z+2x=3x(y+z)$$

Distribute $3x$ through $(y+z)$.

$$y+z+2x=3xy+3xz$$

Move all the terms containing x to the left side of the equation, and move all the terms that don't contain x to the right side of the equation.

$$y-y+z-z+2x-3xy-3xz=3xy-3xy+3xz-3xz-y-z$$

$$2x-3xy-3xz=-y-z$$

Factor out x on the left side of the equation.

$$x(2-3y-3z)=-y-z$$

Divide both sides by $(2-3y-3z)$.

$$x=-\frac{y+z}{2-3y-3z}$$