Topic: Quadratic inequalities

Question: Solve $x^2 + 2x - 3 < 0$ graphically.

Answer choices:

A $-3 < x < 1$

B $-1 < x < 3$

C $-\infty < x < -3$ and $1 < x < \infty$

D $-\infty < x < -1$ and $3 < x < \infty$

Solution: A

The quadratic is in standard form, so we can identify $a = 1 > 0$, which means the parabola opens upward.

The vertex is at

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

$$\left(\frac{-2}{2(1)}, f\left(\frac{-2}{2(1)}\right)\right)$$

$$(-1, f(-1))$$

$$(-1, (-1)^2 + 2(-1) - 3)$$

$$(-1, -4)$$

To find the x-intercepts, we can factor the quadratic and use the Zero Theorem to solve for the roots.

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x = -3, 1$$

Then the x-intercepts are $(-3,0)$ and $(1,0)$. So if the vertex is below the horizontal axis and the parabola opens up, that means the parabola is above the horizontal axis to the left of $(-3,0)$, below the horizontal axis between $(-3,0)$ and $(1,0)$, and above the horizontal axis to the right of $(1,0)$.

Which means the inequality $x^2 + 2x - 3 < 0$ is satisfied when x is between $x = -3$ and $x = 1$.

Topic: Quadratic inequalities

Question: Solve $x^2 + 10x + 21 > 0$ algebraically.

Answer choices:

A $-7 < x < 3$

B $-\infty < x < 3$ and $7 < x < \infty$

C $-\infty < x < -3$ and $7 < x < \infty$

D $-\infty < x < -7$ and $-3 < x < \infty$

Solution: D

Factor the quadratic to find the zeros.

$$x^2 + 10x + 21 = 0$$

$$(x + 7)(x + 3) = 0$$

$$x = -7, \ -3$$

The zeros divide the horizontal axis into three intervals: the interval to the left of $x = -7$, the interval between $x = -7$ and $x = -3$, and the interval to the right of $x = -3$.

We need to find the sign of the quadratic inequality on each interval. Let's choose three test points, one from each interval, and substitute them into the quadratic expression.

For $x = -8$: $(-8)^2 + 10(-8) + 21 = 5 > 0$

For $x = -4$: $(-4)^2 + 10(-4) + 21 = -3 < 0$

For $x = 0$: $(0)^2 + 10(0) + 21 = 21 > 0$

Because the inequality asks for the values of x where the quadratic is above the horizontal axis (positive), we can see that the inequality is only satisfied to the left of $x = -7$ and to the right of $x = -3$.

Topic: Quadratic inequalities

Question: Solve the quadratic inequality $-x^2 + x + 12 \geq 0$.

Answer choices:

A $-4 \leq x \leq 3$

B $-3 \leq x \leq 4$

C $-\infty < x \leq -3$ and $4 \leq x < \infty$

D $-4 < x < 3$

Solution: B

Factor the quadratic to find the zeros.

$$-x^2 + x + 12 = 0$$

$$-(x^2 - x - 12) = 0$$

$$-(x - 4)(x + 3) = 0$$

$$x = -3, 4$$

The zeros divide the horizontal axis into three intervals: the interval to the left of $x = -3$, the interval between $x = -3$ and $x = 4$, and the interval to the right of $x = 4$.

We need to find the sign of the quadratic expression on each interval.

Let's choose three test points, one from each interval, and substitute them into the quadratic inequality.

For $x = -4$: $-(-4)^2 + (-4) + 12 = -8 < 0$

For $x = 0$: $-(0)^2 + 0 + 12 = 12 > 0$

For $x = 6$: $-(6)^2 + 6 + 12 = -18 < 0$

Because the inequality asks for the values of x where the quadratic is above the horizontal axis (positive) or on the horizontal axis (equal to zero), we can see that the inequality is satisfied when x is between $x = -3$ and $x = 4$, at $x = 3$ itself, and at $x = 4$ itself.