Distributive Property

Let's imagine that we're shopping for fruit at the grocery store, and we decide to buy 3 apples and 8 oranges. We notice that the price for apples and oranges today is actually equivalent; they're all $\$0.75$ per piece of fruit.

To calculate the total cost of the fruit, we want to realize that, because the cost of both apples and oranges is equivalent, we can compute our total cost two ways.

1. Calculate the total cost of the apples, calculate the total cost of the oranges, and add those totals together to get the grand total.

$$\$0.75(A) + \$0.75(O)$$

$$\$0.75(3) + \$0.75(8)$$

$$\$2.25 + \$6.00$$

$$\$8.25$$

2. Find the total pieces of fruit, then calculate the cost of the total to get the grand total.

$$\$0.75(A + O)$$

$$\$0.75(3 + 8)$$

$$\$0.75(11)$$

$$\$8.25$$

The Distributive Property

The **Distributive Property** is the rule that tells us we're able to make this calculation both ways. Put simply it tells us that $a(b+c) = ab + ac$.

In other words, if we start at this point in our apples and oranges example,

$$\$0.75(3+8)$$

we can "distribute" the $\$0.75$ across the terms inside the parentheses to rewrite the expression as

$$\$0.75(3) + \$0.75(8)$$

Mathematically, we're saying that we can take the coefficient that's sitting in front of the parentheses, and multiply it across all the terms inside the parentheses. And the Distributive Property doesn't apply just to sums, we could also use the property to say

$$a(b+c) = ab + ac \qquad (a+b)c = ac + bc$$

$$a(b-c) = ab - ac \qquad (a-b)c = ac - bc$$

In other words, distributing removes the parentheses from the expression. When we're first learning to distribute, it's useful to write out every multiplication that results from the distribution, and then simplify them in a separate step.

Let's do an example where we introduce a variable into the expression.

Example

Use the Distributive Property to expand the expression.

$$4(3 - x)$$

Multiply the coefficient outside the parentheses, 4, by each term inside the parentheses.

$$4(3) - 4(x)$$

$$12 - 4x$$

Let's do another example that's a little more complicated.

Example

Use the Distributive Property to expand the expression.

$$-x(x^2 - 2x + 1)$$

It doesn't matter that there are now more than two terms inside the parentheses. We'll still just multiply the coefficient, $-x$, by each term inside the parentheses.

We'll pay special attention to the second term, $-x(-2x)$. If we ignore the negative signs, we know that $x(2x)$ is $2x^2$. The two negatives will cancel with one another to become a positive.

$$-x(x^2) - x(-2x) - x(1)$$

$$-x^3 + 2x^2 - x$$
