Balancing equations

Remember that equations have an equals sign and expressions do not. We balance or solve equations, but simplify or evaluate expressions. And we've seen now how to solve simple equations by using inverse operations to isolate the variable.

Now we'll look in more detail about how to solve equations by keeping them balanced.

The equation scale

Think of an equation as a two-sided scale that we always have to keep in balance.

What we do to one side of an equation we have to do to the other, otherwise the scale won't stay balanced. For example, if we add something to one side, we have to add the same value to the other side.

Or if we take something away from one side, we also have to take it away from the other side.

But don't confuse changing the amount of weight on the scale with moving weight around on the same side of the scale. We can still rewrite one side of the equation without rewriting the other side, we just can't change the value of one side of the equation without changing the value of the other side.

So given an equation like,

$$3x(x+2) = 6x - 2$$

if we just want to distribute the $3x$ across the $(x+2)$ on the left, then we can rewrite the equation as

$$3x(x) + 3x(2) = 6x - 2$$

$$3x^2 + 6x = 6x - 2$$

All we did here was rewrite the left side; we didn't actually change its value. In other words, it's like we changed the left side from $3+1$ to $2+2$. It looks different, but the value isn't different.

In contrast, if we want to add 4 to one side of the original equation, then we have to add 4 to both sides,

$$3x(x+2) + 4 = 6x - 2 + 4$$

because adding 4 actually does affect the value. So if we want to make any change to one side of the equation that'll affect the "weight" on that side of the "scale," then we need to make sure we equally change the

weight on the other side of the scale (equally change the amount on the other side of the equation).

In general, to solve equations, we'll

1. Simplify both sides of the equation as much as possible using the order of operations.
2. If the variable we're trying to solve for appears on both sides of the equation, combine those terms on one side using inverse operations.
3. Move all constant values to the other side of the equation (opposite the variable) using inverse operations.

We'll remember to always keep the equation balanced as we go. Let's do an example.

Example

Solve for the variable by keeping the equation balanced.

$$-2(3x+1) = 3(-5x+11) + 1$$

Simplify both sides of the equation by distributing and then combining like terms.

$$-2(3x) - 2(1) = 3(-5x) + 3(11) + 1$$

$$-6x - 2 = -15x + 33 + 1$$

$$-6x - 2 = -15x + 34$$

Use inverse operations to move all the x terms to one side.

$$-6x + 15x - 2 = -15x + 15x + 34$$

$$9x - 2 = 34$$

Use inverse operations to move all the constants to the other side of the equation.

$$9x - 2 + 2 = 34 + 2$$

$$9x = 36$$

Use inverse operations to solve for x.

$$\frac{9x}{9} = \frac{36}{9}$$

$$x = 4$$

Let's try another example of balancing equations.

Example

Solve for the variable.

$$5(6a - 3) = -(1 - 9a) + 7$$

Simplify both sides of the equation by distributing and then combining like terms.

$$5(6a) + 5(-3) = -(1) - (-9a) + 7$$

$$30a - 15 = -1 + 9a + 7$$

$$30a - 15 = 9a + 6$$

Use inverse operations to move all the a terms to one side.

$$30a - 9a - 15 = 9a - 9a + 6$$

$$21a - 15 = 6$$

Use inverse operations to move all the constants to the other side of the equation.

$$21a - 15 + 15 = 6 + 15$$

$$21a = 21$$

Use inverse operations to solve for a.

$$\frac{21a}{21} = \frac{21}{21}$$

$$a = 1$$

Let's look at another example.

Example

Solve for the variable.

$$2x - 3 = 3x + 1$$

Both sides are already as simplified as they can be, so we'll start by moving all the x terms to one side.

$$2x - 2x - 3 = 3x - 2x + 1$$

$$-3 = x + 1$$

Use inverse operations to move all the constants to the other side.

$$-3 - 1 = x + 1 - 1$$

$$-4 = x$$

$$x = -4$$

Let's try another example of solving equations with variables on both sides.

Example

Solve for the variable.

$$10x - 13 = 4x + x - 6$$

Start by combining like terms to simplify the right side of the equation.

$$10x - 13 = 5x - 6$$

Move all the x terms to one side.

$$10x - 5x - 13 = 5x - 5x - 6$$

$$5x - 13 = -6$$

Move all the constants to the other side.

$$5x - 13 + 13 = -6 + 13$$

$$5x = 7$$

Solve for x.

$$\frac{5x}{5} = \frac{7}{5}$$

$$x = \frac{7}{5}$$