Topic: Change of base

Question: Write the expression in terms of base-10 logs.

$$\log_3 12$$

Answer choices:

A $\log 12 - \log 3$

B $3^x = 12$

C 4

D $\dfrac{\log 12}{\log 3}$

Solution: D

To rewrite the given expression in terms of base-10 logs, use the change of base formula.

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Starting with $\log_3 12$, we get

$$\frac{\log_{10} 12}{\log_{10} 3}$$

The common logarithm function $\log_{10}$ can be written as just $\log$ without the base.

$$\frac{\log 12}{\log 3}$$

Topic: Change of base

Question: Find the exact value of the expression.

$$\frac{\log 729}{\log 9}$$

Answer choices:

A 2

B 3

C 8

D 16

Solution: B

Use the change of base formula,

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to rewrite the given expression as one log.

$$\frac{\log 729}{\log 9}$$

$$\log_9 729$$

Let $x = \log_9 729$, and use the general log rule to convert this to exponential form.

$$9^x = 729$$

$$9^x = 9^3$$

$$x = 3$$

Topic: Change of base

Question: Find the exact value of the expression.

$$\frac{\log 4}{\log 16} - \frac{\log 2}{\log 64}$$

Answer choices:

A $\frac{1}{3}$

B $\frac{1}{4}$

C $\frac{1}{6}$

D $\frac{1}{8}$

Solution: A

Use the change of base formula,

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to rewrite both fractions.

$$\frac{\log 4}{\log 16} - \frac{\log 2}{\log 64}$$

$$\log_{16} 4 - \log_{64} 2$$

Next, let $x = \log_{16} 4$, and use the general log rule to convert this to exponential form.

$$16^x = 4$$

$$(4^2)^x = 4^1$$

$$4^{2x} = 4^1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Now let $x = \log_{64} 2$, and use the general log rule to convert this to exponential form.

$$64^x = 2$$

$$(2^6)^x = 2^1$$

$$2^{6x} = 2^1$$

$$6x = 1$$

$$x = \frac{1}{6}$$

Finally, substitute the values of $\log_{64} 2$, and use the general log rule to convert this to exponential form.

$$\log_{16} 4 - \log_{64} 2$$

$$\frac{1}{2} - \frac{1}{6}$$

$$\frac{1}{2}\left(\frac{3}{3}\right) - \frac{1}{6}$$

$$\frac{3}{6} - \frac{1}{6}$$

$$\frac{2}{6}$$

$$\frac{1}{3}$$