Algebra 1 Workbook Solutions

Inequalities

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TRICHOTOMY

1. Solve the inequality.

$$2(x+1) \nleq -(8-x)$$

Solution:

By the Trichotomy Law, the inequality can be rewritten, and then simplified.

$$2(x+1) > -(8-x)$$

$$2x+2 > -8+x$$

$$2x > -10+x$$

$$x > -10$$

2. Give two ways to write the sentence in mathematical notation.

"x^2 is not greater than $4y$ and is also not equal to $4y$."

Solution:

The two ways to express the statement are

$x^2 \ngeq 4y$ and $x^2 < 4y$

3. Give the three possible relationships in the Law of Trichotomy.

Solution:

The three statements of the Trichotomy Law are

If $a \ngeq b$ then $a < b$.

If $a \nleq b$ then $a > b$.

If $a \ngtr b$ and $a \nless b$ then $a = b$.

4. Find a way to express the relationships as one equality or inequality.

$x^2 + x \nless 2$ and $x^2 + x \ngtr 2$

Solution:

By the Law of Trichotomy, we can rewrite the two statements as

$x^2 + x = 2$

5. Give two ways to write the statement in mathematical notation.

"$3(x+1)$ is not less than $-x-5$ and is also not equal to $-x-5$."

Solution:

The two ways to write the statement are

$3(x+1) \nleq -x-5$ and $3(x+1) > -x-5$

6. Solve the statement.

$-3(1-x) \ngtr 3(7-x)-2x$ and $-3(1-x) \nless 3(7-x)-2x$

Solution:

By the Law of Trichotomy, we can rewrite the statement as

$-3(1-x) = 3(7-x)-2x$

INEQUALITIES AND NEGATIVE NUMBERS

1. Solve the inequality.

$$-3x + 4 < 22$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -3.

$$-3x + 4 < 22$$

$$-3x < 18$$

$$x > -6$$

2. What went wrong in this set of steps?

$$-5x + 6 < 9 - 2x$$

$$-3x < 3$$

$$x < -1$$

Solution:

When the inequality was divided by -3, the inequality sign was not flipped. The solution should be $x > -1$.

3. Solve the inequality.

$$-(5-2x) \geq 3(x-3)+2x$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -3.

$$-(5-2x) \geq 3(x-3)+2x$$

$$-5+2x \geq 3x-9+2x$$

$$-5+2x \geq 5x-9$$

$$-5-3x \geq -9$$

$$-3x \geq -4$$

$$x \leq \frac{4}{3}$$

4. Solve the inequality.

$$-6x+7 > -3x+2$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -3.

$$-6x+7>-3x+2$$

$$-3x+7>2$$

$$-3x>-5$$

$$x<\frac{5}{3}$$

5. What went wrong in this set of steps?

$$-2(x+1)\geq 3(2+x)$$

$$-2x-2\geq 6+3x$$

$$-2x-3x-2\leq 6$$

Solution:

The inequality sign was flipped when $3x$ was subtracted from each side, but it should have remained the same and not been flipped.

6. Solve the inequality.

$$7(1 - x) \leq 2x$$

Solution:

Solve by isolating x using inverse operations, remembering to flip the inequality sign when dividing by -9.

$$7(1 - x) \leq 2x$$

$$7 - 7x \leq 2x$$

$$-7x \leq 2x - 7$$

$$-9x \leq -7$$

$$x \geq \frac{7}{9}$$

GRAPHING INEQUALITIES ON A NUMBER LINE

1. Give two inequalities that, when graphed on a number line, have open circles at $x = 3$.

Solution:

There are many correct answers. For example, $x < 3$ and $x > 3$ would both have open circles at $x = 3$.

2. Graph the inequality on a number line.

$$-2x < 4$$

Solution:

Rewrite the inequality, remembering to flip the sign when we divide by -2.

$$-2x < 4$$

$$x > -2$$

Then a graph of the inequality is

3. Graph the inequality on a number line.

$$x - 1 \geq 3$$

Solution:

Rewrite the inequality.

$$x - 1 \geq 3$$

$$x \geq 4$$

Then a graph of the inequality is

4. Graph the inequality on a number line.

$$5(-x + 3) < -3x + 7$$

Solution:

Rewrite the inequality, remembering to flip the sign when we divide by -2.

$$5(-x + 3) < -3x + 7$$

$$-5x + 15 < -3x + 7$$

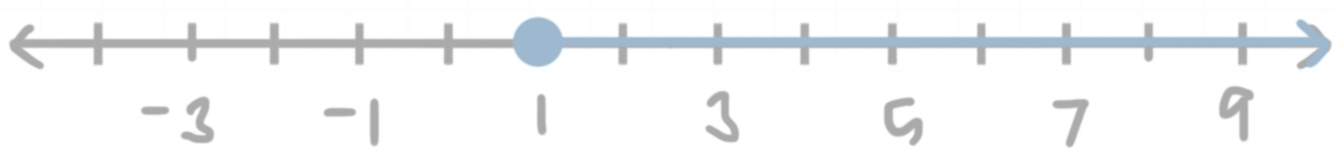
$$-5x < -3x - 8$$

$$-2x < -8$$

$$x > 4$$

Then a graph of the inequality is

■ 5. What's wrong with this graph of $x > 1$?



Solution:

There should be an open circle at 1, not a solid circle, since the inequality $x > 1$ does not include the value $x = 1$.

■ 6. Graph the inequality on a number line.

$$5(x + 7) - x \geq 3(x + 10) + 6$$

Solution:

Rewrite the inequality.

$$5(x+7)-x \geq 3(x+10)+6$$

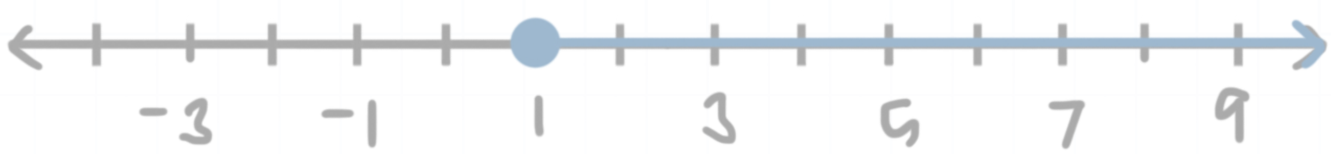
$$5x+35-x \geq 3x+30+6$$

$$4x+35 \geq 3x+36$$

$$4x \geq 3x+1$$

$$x \geq 1$$

Then a graph of the inequality is



GRAPHING DISJUNCTIONS ON A NUMBER LINE

■ 1. What's wrong with the graph of the disjunction?

$2x \leq 4$ or $x - 5 > 3$

Solution:

First, we'll solve and graph the two inequalities separately.

$2x \leq 4$	$x - 5 < 3$
$x \leq 2$	$x > 8$

The graph of the inequality $x \leq 2$ has a solid circle at 2, and the arrow goes to the left.

The graph of the inequality $x > 8$ has an open circle at 8, and the arrow goes to the right.

Therefore, the sketch of the disjunction is

2. Graph the disjunction.

$x + 2 \geq 2x + 3$ or $x - 5 \geq 0$

Solution:

First, we'll solve and graph the two inequalities separately.

$x + 2 \geq 2x + 3$	$x - 5 \geq 0$
$-x + 2 \geq 3$	$x \geq 5$
$-x \geq 1$	
$x \leq -1$	

The graph of the inequality $x \leq -1$ has a solid circle at -1, and the arrow goes to the left.

The graph of the inequality $x \geq 5$ has a solid circle at 5, and the arrow goes to the right.

Therefore, the sketch of the disjunction is

■ 3. Graph the disjunction of the inequalities.

$2(x-3)+x<2x+1$ or $2(x-1)-6>6$

Solution:

First, we'll solve and graph the two inequalities separately.

$$2(x-3)+x<2x+1$$

$$2x-6+x<2x+1$$

$$3x-6<2x+1$$

$$x-6<1$$

$$x<7$$

$$2(x-1)-6>6$$

$$2(x-1)>12$$

$$x-1>6$$

$$x>7$$

The graph of the inequality $x<7$ has an open circle at 7, and the arrow goes to the left.

The graph of the inequality $x > 7$ has an open circle at 7, and the arrow goes to the right.

Therefore, the sketch of the disjunction is

4. What's wrong with the graph of the disjunction?

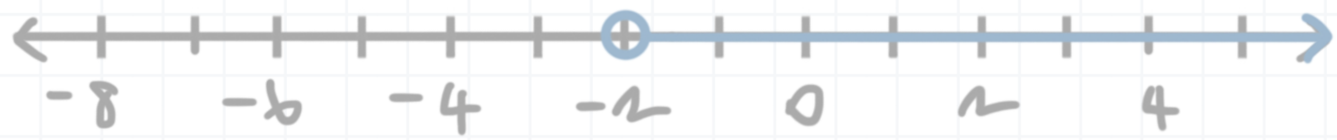
$-x + 3 < 5$ or $-2(x + 2) \geq 2$

Solution:

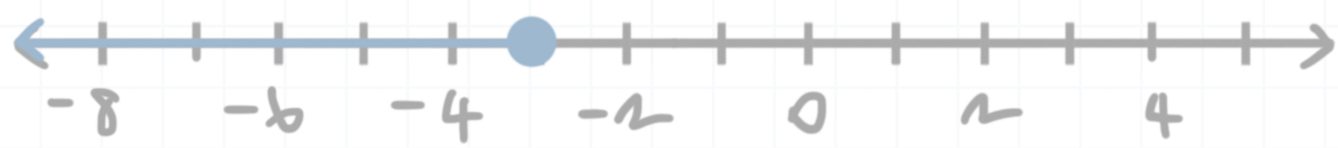
First, we'll solve and graph the two inequalities separately.

$-x + 3 < 5$	$-2(x + 2) \geq 2$
$-x < 2$	$x + 2 \leq -1$
$x > -2$	$x \leq -3$

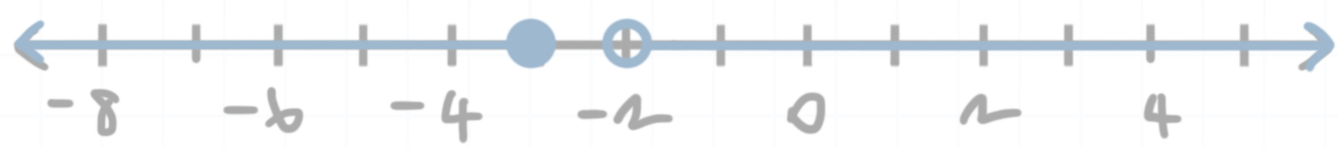
The graph of the inequality $x > -2$ has an open circle at -2, and the arrow goes to the right.



The graph of the inequality $x \le -3$ has a solid circle at -3, and the arrow goes to the left.



Therefore, the sketch of the disjunction is



5. Graph the disjunction.

$2x + 3 \ge 3$ or $2x + 5 < x$

Solution:

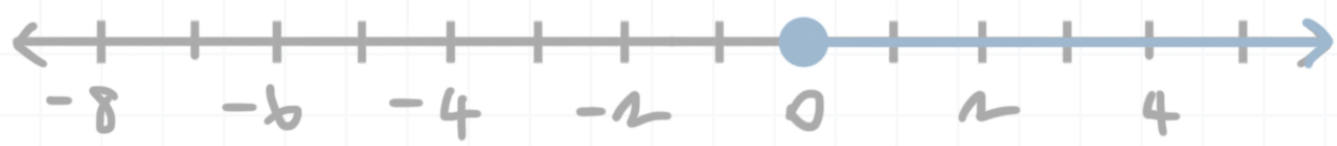
First, we'll solve and graph the two inequalities separately.

$2x + 3 \ge 3$ $\quad$ $2x + 5 < x$

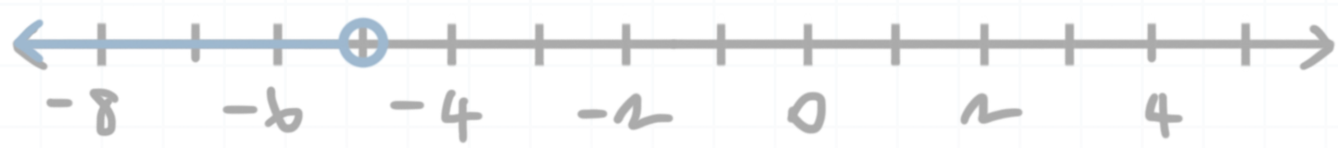
$2x \ge 0$ $\quad$ $x + 5 < 0$

$x \ge 0$ $\quad$ $x < -5$

The graph of the inequality $x \geq 0$ has a solid circle at 0, and the arrow goes to the right.



The graph of the inequality $x < -5$ has an open circle at -5, and the arrow goes to the left.



Therefore, the sketch of the disjunction is

■ 6. Graph the disjunction.

$$-2x + 5 \geq -1 \text{ or } x - 6 > -2$$

Solution:

The disjunction is the combination of

$$-2x + 5 \geq -1$$

$$-2x \geq -6$$

$x \leq 3$

and

$x - 6 > -2$

$x > 4$

So the disjunction "$x \leq 3$ or $x > 4$" is graphed as

GRAPHING CONJUNCTIONS ON A NUMBER LINE

1. Graph the conjunction of the inequalities $3(x-4) < x-2$ and $-2(x-6)+3 \geq 5$.

Solution:

First, we'll solve and graph the two inequalities separately.

$$3(x-4) < x-2 \qquad -2(x-6)+3 \geq 5$$

$$3x-12 < x-2 \qquad -2(x-6) \geq 2$$

$$2x-12 < -2 \qquad x-6 \leq -1$$

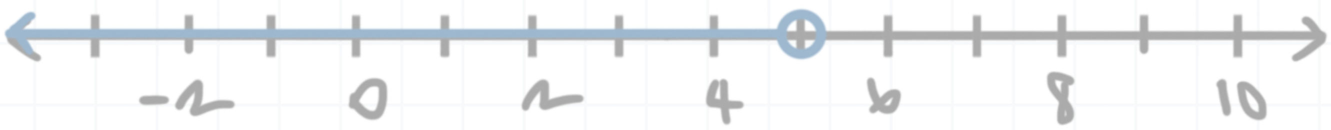
$$2x < 10 \qquad x \leq 5$$

$$x < 5$$

The graph of the inequality $x < 5$ has an open circle at 5, and the arrow goes to the left.

The graph of the inequality $x \leq 5$ has a solid circle at 5, and the arrow goes to the left.

Therefore, the sketch of the conjunction is



■ 2. Graph the conjunction.

$$-8 \leq -2x < 10$$

Solution:

Simplify the conjunction.

$$-8 \leq -2x < 10$$

$$4 \geq x > -5$$

$$-5 < x \leq 4$$

Then we can graph the conjunction on a number line.

■ 3. What’s wrong with the graph of the conjunction?

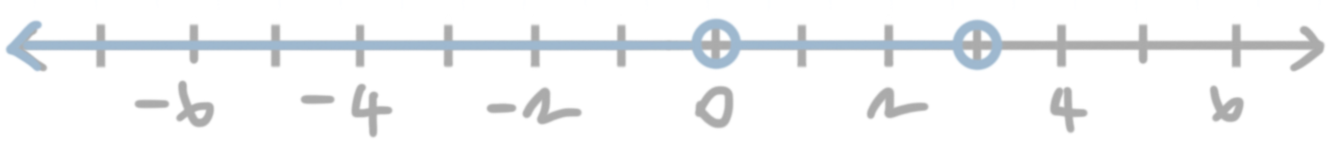
$x \leq 3$ and $x > -4$

Solution:

There should be an open circle at -4 since the inequality $x > -4$ does not include $x = -4$, and there should be a solid circle at 3 since the inequality $x \leq 3$ includes $x = 3$.

■ 4. What's wrong with the graph of the conjunction?

$x \leq 3$ and $x \neq 0$

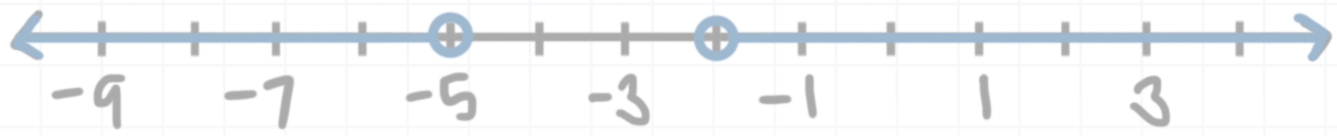


Solution:

There should be a closed circle at 3 since the inequality $x \leq 3$ includes the value $x = 3$.

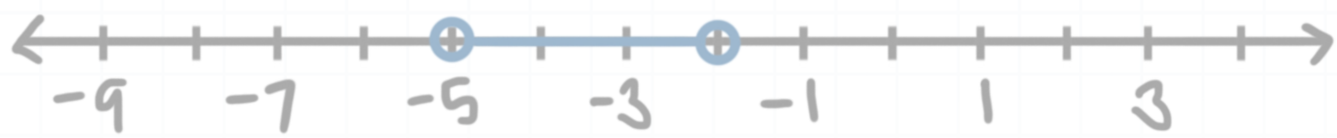
■ 5. What's wrong with the graph of the conjunction?

$x < -2$ and $x > -5$



Solution:

The graph is showing the disjunction "$x < -5$ or $x > -2$" instead of the conjunction "$x < -2$ and $x > -5$." The graph should be



6. Graph the conjunction.

$2x - 1 \geq 3$ and $-x \geq -9$

Solution:

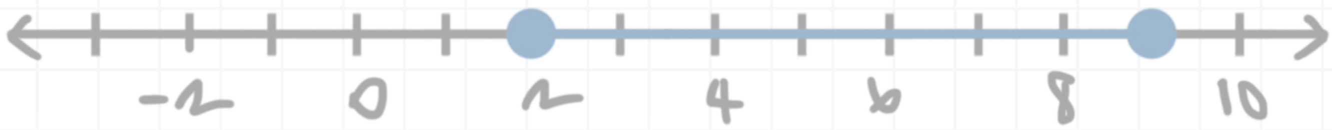
The individual inequalities simplify to

$$2x - 1 \geq 3 \qquad -x \geq -9$$

$$2x \geq 4 \qquad x \leq 9$$

$$x \geq 2$$

So $x \geq 2$ and $x \leq 9$ form a conjunction that is graphed as


-2
0
2
4
6
8
10

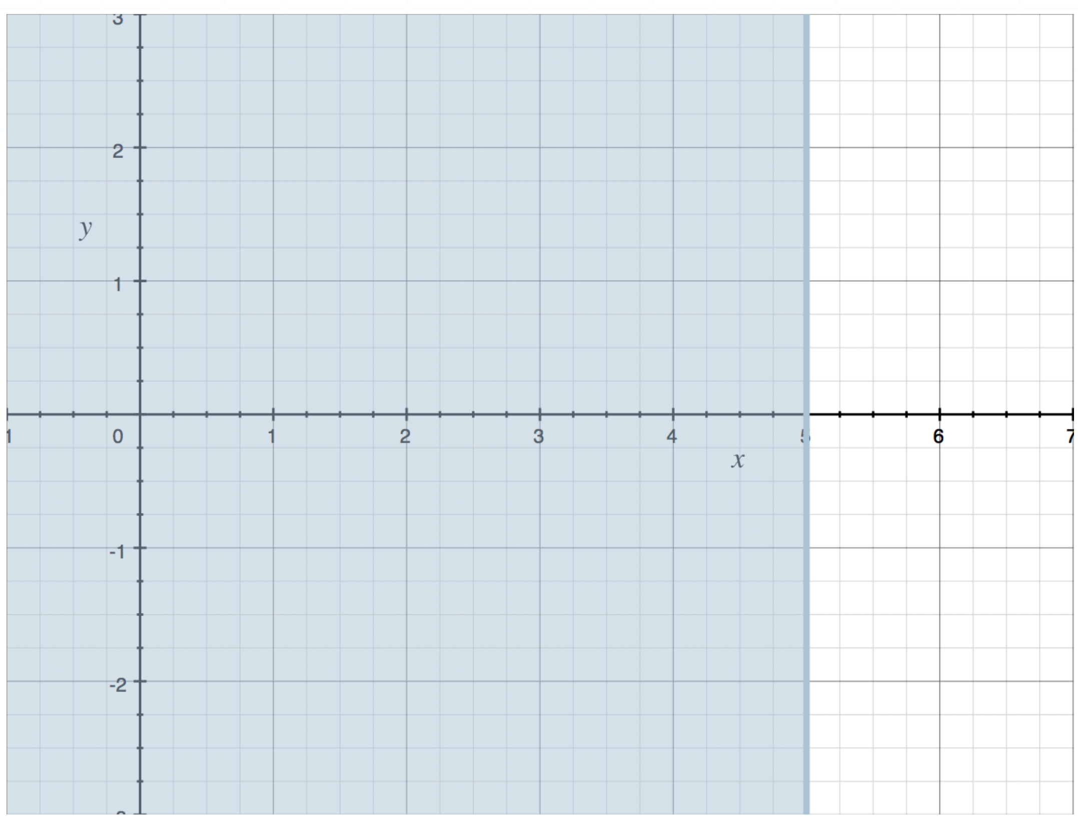
GRAPHING INEQUALITIES IN THE PLANE

1. Graph the inequality in the Cartesian coordinate plane.

$$x \leq 5$$

Solution:

Start by graphing the vertical line $x = 5$. Make it a solid line since the inequality is "less than or equal to." Since the inequality is "less than," we'll shade to the left of the vertical line.



2. Graph the inequality in the Cartesian coordinate plane.

$$y < -2x + 4$$

Solution:

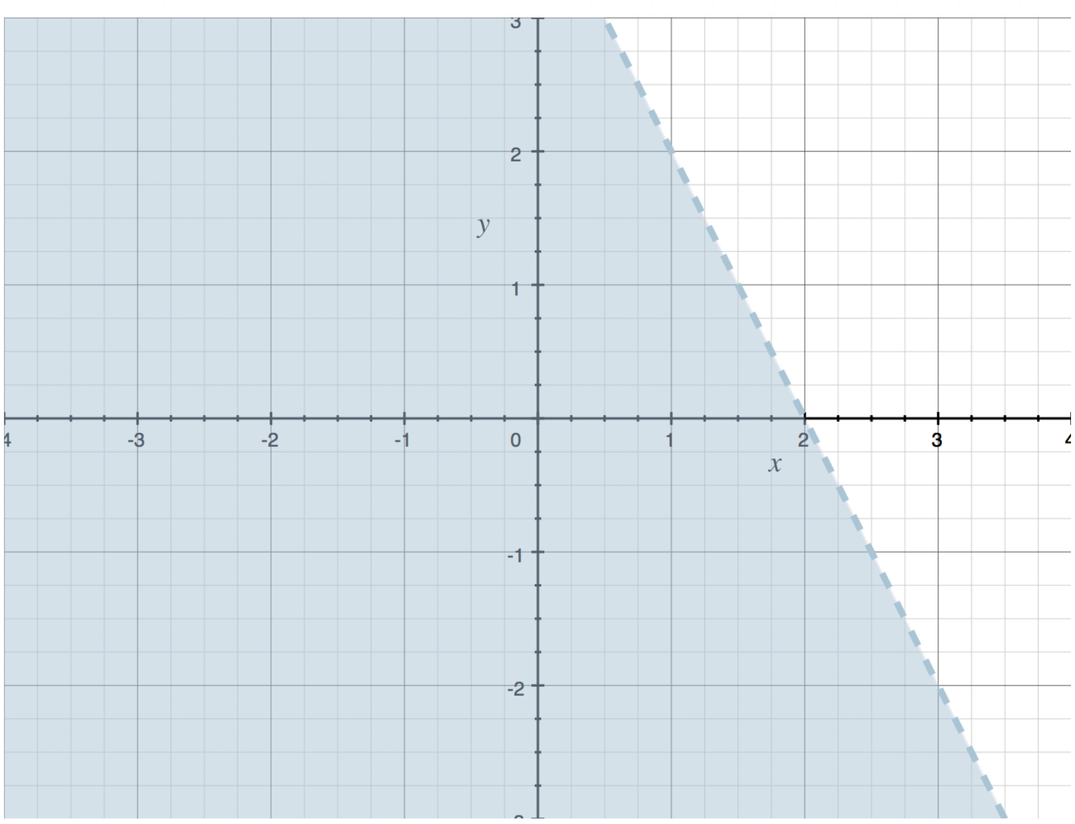
Start by graphing the line $y = -2x + 4$. Make it a dashed line since the inequality is strictly "less than." To determine where to shade, let's test $(0,0)$ by substituting it into the inequality.

$$y < -2x + 4$$

$$0 < -2(0) + 4$$

$$0 < 4$$

Since this inequality is true, we shade on the side of the dashed line that contains the test point $(0,0)$.

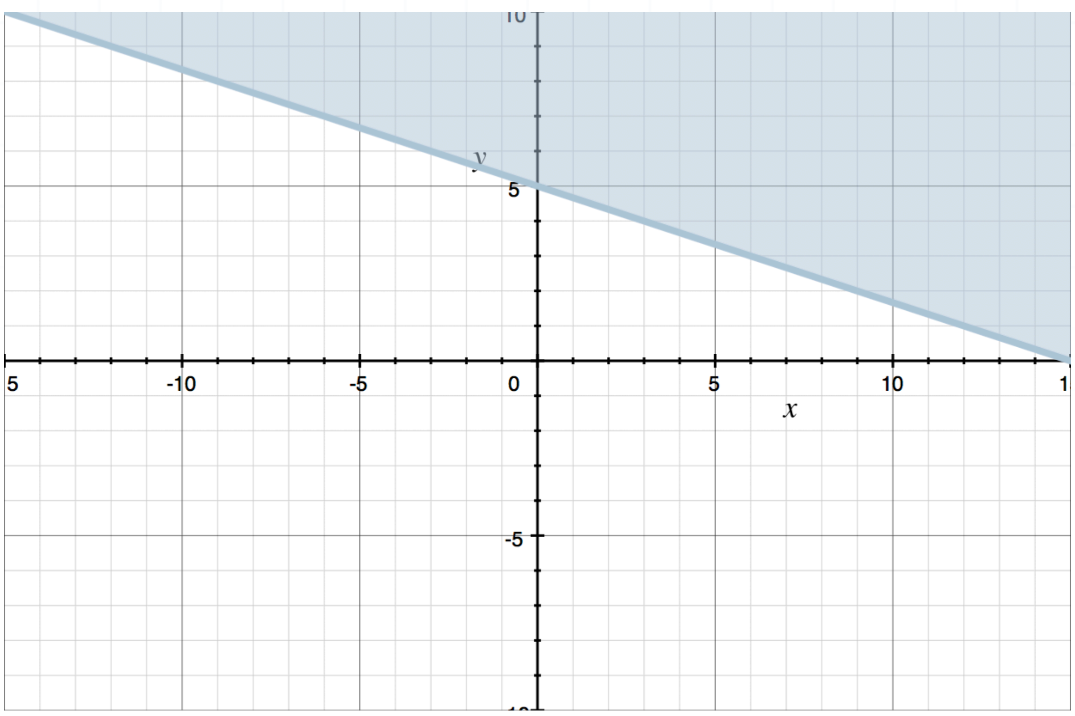


3. Graph the inequality in the Cartesian coordinate plane.

$$y \geq -\frac{1}{3}x + 5$$

Solution:

Start by graphing the line $y = -(1/3)x + 5$. Make it a solid line since the inequality is "greater than or equal to." Since the inequality is "greater than," we'll shade above the line.



4. Graph the inequality in the Cartesian coordinate plane.

$$y \leq x - 1$$

Solution:

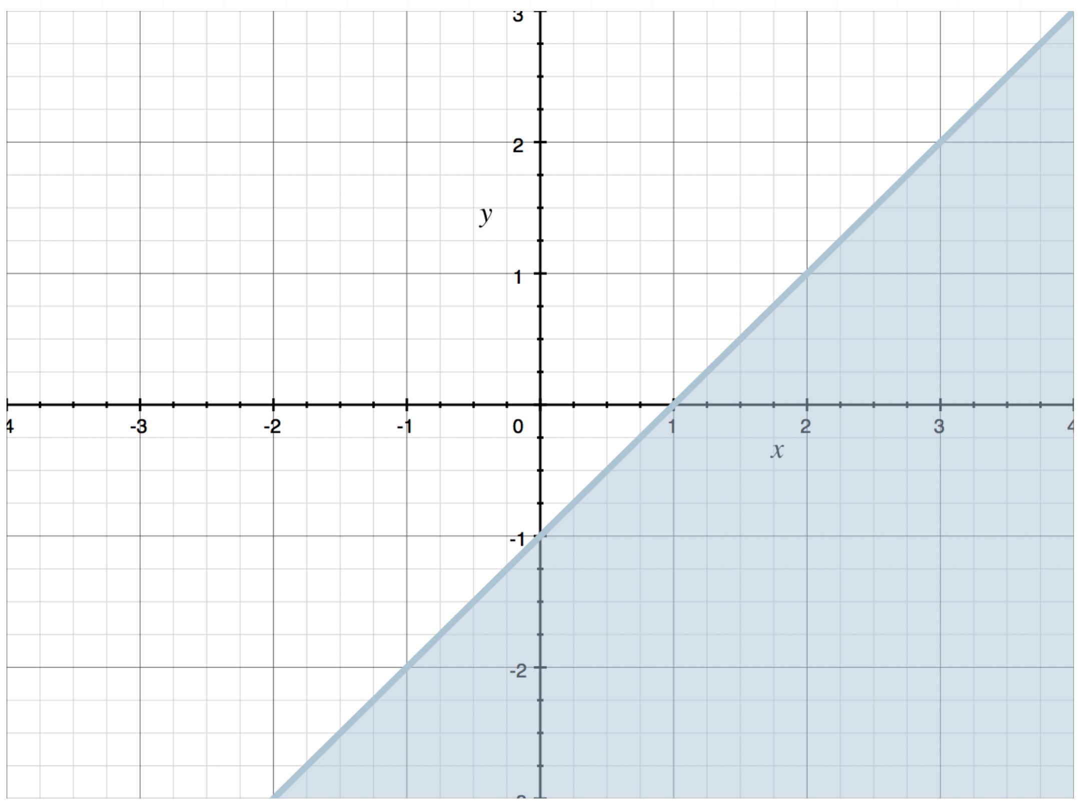
Start by graphing the line $y = x - 1$. Make it a solid line since the inequality is "less than or equal to." To determine where to shade, let's test $(0,0)$ by substituting it into the inequality.

$$y \leq x - 1$$

$$0 \leq 0 - 1$$

$$0 \leq -1$$

Since this inequality is false, we shade on the side of the solid line that does not contain the test point $(0,0)$.

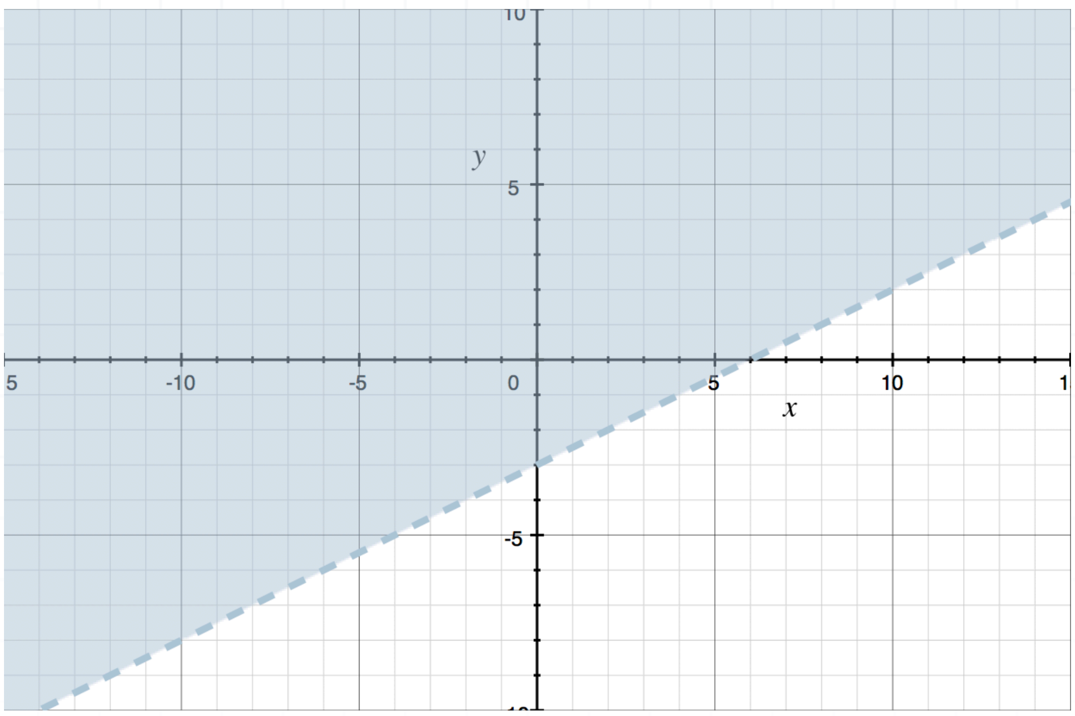


5. Graph the inequality in the Cartesian coordinate plane.

$$y > \frac{1}{2}x - 3$$

Solution:

Start by graphing the line $y = (1/2)x - 3$. Make it a dashed line since the inequality is strictly "greater than." Since the inequality is "greater than," we'll shade above the line.



6. Graph the inequality in the Cartesian coordinate plane.

$$y \geq 3x - 2$$

Solution:

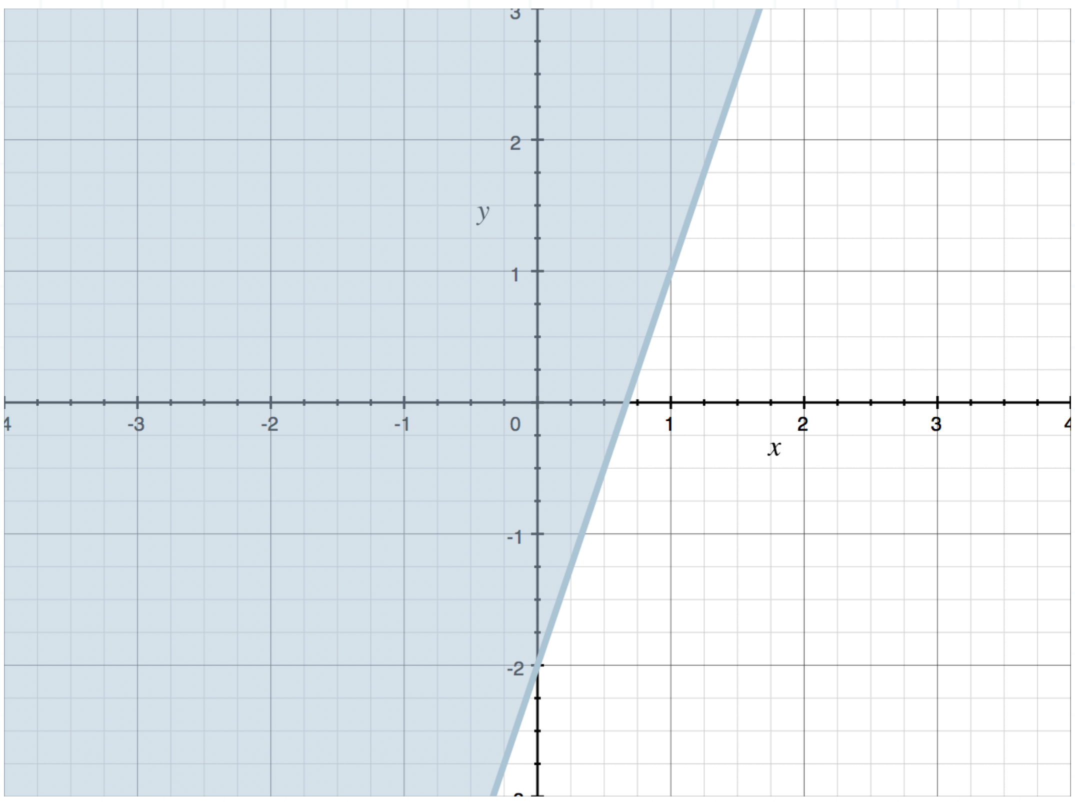
Start by graphing the line $y = 3x - 2$. Make it a solid line since the inequality is "greater than or equal to." To determine where to shade, let's test $(0,0)$ by substituting it into the inequality.

$$y \geq 3x - 2$$

$$0 \geq 3(0) - 2$$

$$0 \geq -2$$

Since this inequality is true, we shade on the side of the solid line that contains the test point $(0,0)$.



ABSOLUTE VALUE EQUATIONS

1. Solve $|3 - x| = 1$.

Solution:

Solve the two related equations.

$3 - x = 1$ $\qquad$ $3 - x = -1$

$x = 2$ $\qquad$ $x = 4$

Check each solution by substituting them into the original absolute value equation.

Check $x = 2$:

$|3 - x| = 1$

$|3 - 2| = 1$

$|1| = 1$

$1 = 1$

Check $x = 4$:

$|3 - x| = 1$

$|3 - 4| = 1$

$|-1| = 1$

$1 = 1$

Both equations are true, so $x = 2$ and $x = 4$ are the solutions to the absolute value equation.

2. Solve $|4x-8|=3x-6$.

Solution:

Solve the two related equations.

$$4x-8=3x-6$$

$$x=2$$

$$4x-8=-(3x-6)$$

$$7x=14$$

$$x=2$$

Check whether $x=2$ is a solution by substituting it into the original absolute value equation.

$$|4x-8|=3x-6$$

$$|4(2)-8|=3(2)-6$$

$$|0|=0$$

$$0=0$$

The equation is true, so $x=2$ is the solution to the absolute value equation.

3. Solve $|2x-2|=x-6$.

Solution:

Solve the two related equations.

$$2x - 2 = x - 6 \qquad 2x - 2 = -(x - 6)$$

$$x = -4 \qquad 3x = 8$$

$$x = \frac{8}{3}$$

Check each solution by substituting them into the original absolute value equation.

Check $x = -4$:

$$|2x - 2| = x - 6$$

$$|2(-4) - 2| = -4 - 6$$

$$|-10| = -10$$

$$10 = -10$$

Check $x = 8/3$:

$$|2x - 2| = x - 6$$

$$\left|2 \cdot \frac{8}{3} - 2\right| = \frac{8}{3} - 6$$

$$\left|\frac{10}{3}\right| = -\frac{10}{3}$$

$$\frac{10}{3} = -\frac{10}{3}$$

Both equations are false, so there are no solutions to the absolute value equations.

■ 4. Solve $|3x + 1| + x = 1$.

Solution:

First, isolate the absolute value on the left side of the equation.

$$|3x + 1| + x = 1$$

$$|3x + 1| = 1 - x$$

Solve the two related equations.

$$3x + 1 = 1 - x$$

$$4x = 0$$

$$x = 0$$

$$3x + 1 = -(1 - x)$$

$$2x = -2$$

$$x = -1$$

Check each solution by substituting them into the original absolute value equation.

Check $x = 0$:

$$|3x + 1| + x = 1$$

$$|3(0) + 1| + 0 = 1$$

$$|1| = 1$$

$$1 = 1$$

Check $x = -1$:

$$|3x + 1| + x = 1$$

$$|3(-1) + 1| - 1 = 1$$

$$|-2| = 2$$

$$2 = 2$$

Both equations are true, so $x = 0$ and $x = -1$ are the solutions to the absolute value equation.

5. Solve $|2x+5| = 3x+6$.

Solution:

Solve the two related equations.

$$2x+5 = 3x+6$$

$$-x = 1$$

$$x = -1$$

$$2x+5 = -(3x+6)$$

$$5x = -11$$

$$x = -\frac{11}{5}$$

Check each solution by substituting them into the original absolute value equation.

Check $x = -1$:

$$|2x+5| = 3x+6$$

$$|2(-1)+5| = 3(-1)+6$$

$$|3| = 3$$

$$3 = 3$$

Check $x = -11/5$:

$$|2x+5| = 3x+6$$

$$\left|2\left(-\frac{11}{5}\right)+5\right| = 3\left(-\frac{11}{5}\right)+6$$

$$\left|\frac{3}{5}\right| = -\frac{3}{5}$$

$$\frac{3}{5} = -\frac{3}{5}$$

The equation is only true for $x = -1$, so the only solution to the absolute value equation is $x = -1$.

6. Solve $|3x+2| = |3x+4|$.

Solution:

Solve the two related equations.

$3x+2=3x+4$	$3x+2=-(3x+4)$
$0=2$	$6x=-6$
No solutions	$x=-1$

Check whether $x=-1$ is a solution by substituting it into the original absolute value equation.

$$|3x+2| = |3x+4|$$

$$|3(-1)+2| = |3(-1)+4|$$

$$|-1| = |1|$$

$$1=1$$

The equation is true, so $x=-1$ is the solution to the absolute value equation.

ABSOLUTE VALUE INEQUALITIES

1. Rewrite the inequality by taking away the absolute value.

$$|3x-7| \geq 2$$

Solution:

We can take away the absolute value by rewriting the inequality as the disjunction

$3x-7 \geq 2$ or $3x-7 \leq -2$

2. Graph the inequality.

$$5|1-x|-7<3$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$5|1-x|-7<3$$

$$5|1-x|<10$$

$$|1-x|<2$$

Since $2 > 0$, we can rewrite the inequality as the conjunction

$$-2 < 1 - x < 2$$

$$-3 < -x < 1$$

$$3 > x > -1$$

$$-1 < x < 3$$

A sketch of the inequality is therefore

3. Graph the inequality.

$$2(|x-4|-1)+6 \le 4$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$2(|x-4|-1)+6 \le 4$$

$$2(|x-4|-1) \le -2$$

$$|x-4|-1 \le -1$$

$$|x-4| \le 0$$

The absolute value is always positive, so this expression is telling us

“positive or zero” $\leq$ zero

We can’t have “positive $\leq$ zero,” so the only equation that’s possible is “zero $\leq$ zero”, so we get

$$x - 4 = 0$$

$$x = 4$$

A sketch of the inequality is therefore

4. Graph the inequality.

$$-2|x+2| - 3 \geq 1$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$-2|x+2| - 3 \geq 1$$

$$-2|x+2| \geq 4$$

$$|x+2| \leq -2$$

The absolute value is always positive, so this expression is telling us

positive $\leq$ negative

There's no value that can make this inequality true, so there's no solution, and a sketch of the inequality is therefore an empty number line.

5. Graph the inequality.

$$2(3+|x-5|)-4 \geq 10$$

Solution:

Isolate the absolute value on the left side of the inequality.

$$2(3+|x-5|)-4 \geq 10$$

$$2(3+|x-5|) \geq 14$$

$$3+|x-5| \geq 7$$

$$|x-5| \geq 4$$

Since $4>0$, we can rewrite the inequality as the disjunction

$4 \leq x-5$ or $x-5 \leq -4$

$9 \le x$ or $x \le 1$

A sketch of the inequality is therefore

6. Graph the inequality.

$$|6 - 2x| \le 4$$

Solution:

Since $4 > 0$, we can rewrite the inequality as the conjunction

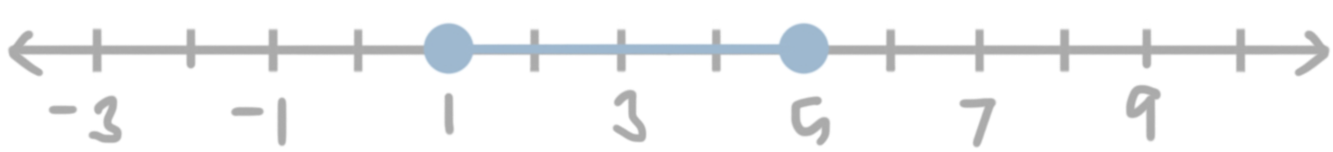
$$-4 \le 6 - 2x \le 4$$

$$-10 \le -2x \le -2$$

$$5 \ge x \ge 1$$

$$1 \le x \le 5$$

A sketch of the inequality is therefore



W W W . K R I S T A K I N G M A T H . C O M