Multiplying multivariable polynomials

We define a **multivariable polynomial** as a polynomial that includes two or more variables. For instance, $2x^3 - 3xy - y^2$ is a multivariable polynomial because it includes multiple variables, x and y.

We already know how to multiply like terms, and we already know how to multiply single variable polynomials, and we'll be able to multiply multivariable polynomials if we combine these two concepts.

Remember that, if we multiply terms with like bases, then the base stays the same and we add the exponents. So $3x^2(x^4) = 3x^{2+4} = 3x^6$. If we're multiplying multivariable terms, then we still add exponents for like bases. So $3x^2y(x^4y^3) = 3x^{2+4}y^{1+3} = 3x^6y^4$.

With multiple variables involved, and as the polynomials get longer, it can be especially useful here to use a chart to organize the polynomial multiplication so that we don't get lost as we're working through each term.

Let's look at an example where we multiply multivariable polynomials.

Example

Expand the expression.

$$(x - 2y)(2x^3 - 3xy - y^2)$$

Let's use a chart to make sure we distribute every term in the first polynomial by every term in the second polynomial.

	$2x^3$	$-3xy$	$-y^2$
x	$2x^4$	$-3x^2y$	$-xy^2$
-2y	$-4x^3y$	$6xy^2$	$2y^3$

If we add every term in the body of the chart, we get

$$2x^4 - 3x^2y - xy^2 - 4x^3y + 6xy^2 + 2y^3$$

Next, we'll rearrange the terms in descending order of powers of x. We'll list any terms that don't include x at the end, in ascending order of powers of y.

$$2x^4 - 4x^3y - 3x^2y - xy^2 + 6xy^2 + 2y^3$$

$$2x^4 - 4x^3y - 3x^2y + (-xy^2 + 6xy^2) + 2y^3$$

$$2x^4 - 4x^3y - 3x^2y + 5xy^2 + 2y^3$$

Let's try another example of multiplying multivariable polynomials.

Example

Expand the expression.

$$(2x + 3y)(x - y) + (x + y)(4x - 2y)$$

We'll multiply the first pair of binomials.

$$(2x + 3y)(x - y) + (x + y)(4x - 2y)$$

$$2x(x - y) + 3y(x - y) + (x + y)(4x - 2y)$$

$$2x(x) - 2x(y) + 3y(x) - 3y(y) + (x + y)(4x - 2y)$$

$$2x^2 - 2xy + 3xy - 3y^2 + (x + y)(4x - 2y)$$

$$2x^2 + xy - 3y^2 + (x + y)(4x - 2y)$$

Multiply the second pair of binomials.

$$2x^2 + xy - 3y^2 + x(4x - 2y) + y(4x - 2y)$$

$$2x^2 + xy - 3y^2 + x(4x) - x(2y) + y(4x) - y(2y)$$

$$2x^2 + xy - 3y^2 + 4x^2 - 2xy + 4xy - 2y^2$$

$$(2x^2 + 4x^2) + (xy - 2xy + 4xy) + (-3y^2 - 2y^2)$$

$$(2 + 4)x^2 + (1 - 2 + 4)xy + (-3 - 2)y^2$$

$$6x^2 + 3xy - 5y^2$$