Direct variation

In this lesson we'll look at solving equations that express direct variation relationships, which are relationships in the form $kx = y$, where k is a constant. In a direct variation equation, we have two variables, usually x and y, and a constant, usually k.

The main idea in direct variation is that, as one variable increases, the other variable will also increase. That means if x increases y increases, and if y increases x increases. The number k is a constant, so it's the same for all pairs of numbers (x, y) that satisfy the equation.

In a direct variation problem, x and y are said to vary directly, and k is called the constant of proportionality.

This lesson will help us find the value of a variable in a direct variation equation, given other information (such as the corresponding value of the other variable and the constant k, or the corresponding value of the other variable and one pair (x, y) that satisfies the equation).

Let's look at an example.

Example

Two variables x and y vary directly. If the constant of proportionality, k, is 20, what is the value of y when x equals 15?

Remember that the general form of a direction variation equation is $y = kx$. In this example, we know that $k = 20$ and $x = 15$, and that we're looking for y. So

$$y = 20(15)$$

$$y = 300$$

Let's try another one.

Example

In a certain direct variation equation, the constant of proportionality, k, satisfies $5k = 50$. If $y = 85$, what is the value of x?

Remember that the general form of a direct variation equation is $y = kx$. In this example,

$$5k = 50$$

$$\frac{5k}{5} = \frac{50}{5}$$

$$k = 10$$

and

$$y = 85$$

So $85 = 10x$. Now let's solve for x.

$$\frac{85}{10} = \frac{10x}{10}$$

$$8.5 = x$$

Sometimes we'll be given a pair of equations that involve the same direct variation equation (they have the same constant of proportionality) and we'll need to solve for one of the variables. We call this type of problem a two-step problem.

Example

If $2k = 14$ and $kx = 56$, what is the value of x?

We'll solve the first equation for k.

$$2k = 14$$

$$\frac{2k}{2} = \frac{14}{2}$$

$$k = 7$$

Now we'll take the value we found for k and plug it into the second equation to solve for x.

$$kx = 56$$

$$7x = 56$$

$$\frac{7x}{7} = \frac{56}{7}$$

$$x = 8$$
