Algebra 1 Formulas

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Operations

Variables

Variables: Symbols for numbers we don't know yet.

Constants: Numbers on their own.

Coefficients: A number attached to and in front of a variable.

Equations: Equations include an equals sign, and tell us that whatever we have on the left side of the equals sign is equivalent/equal to/has the same value as whatever we have on the right side of the equals sign.

Expressions: Expressions don't include an equals sign, but instead are just groups of terms, where a term is a single number or a variable, or numbers and variables multiplied together.

Identifying multiplication

Different ways to indicate multiplication:

Times	$a \times b = c$
Dot	$a \cdot b = c$
Parentheses	$(a)(b) = c$
Variables next to each other	$ab = c$

Associative Property

Associative Property:

Associative Property of Addition $(a+b)+c = a+(b+c)$

Associative Property of Multiplication $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Commutative Property

Commutative Property:

Commutative Property of Addition $a+b = b+a$

Commutative Property of Multiplication $a \cdot b = b \cdot a$

Transitive Property

Transitive Property: If $a = b$ and $b = c$, then $a = c$.

$$5 = \boxed{4+1}$$
$$\boxed{4+1} = 2+3$$

Understood 1

Understood 1: Every mathematical value has an implied coefficient of 1, an implied denominator of 1, and an implied exponent of 1.

$$x = \frac{1x^1}{1}$$

Adding and subtracting like terms

Like terms: When we're adding and subtracting, like terms are terms with equivalent bases and equivalent exponents.

$$1x^2 + 3x^2 = (1 + 3)x^2 = 4x^2$$

Multiplying and dividing like terms

Like terms: When we're multiplying and dividing, like terms are terms with equivalent bases.

$$4x^2 \cdot 3x^5 = (4 \cdot 3)x^{2+5} = 12x^7$$

Distributive Property

Distributive Property:

$$a(b + c) = ab + ac \qquad (a + b)c = ac + bc$$

$a(b-c) = ab - ac$

$(a-b)c = ac - bc$

Distributive Property with fractions

Distributing a fraction: When we multiply two fractions, we multiply the numerators to get the new numerator, and we multiply the denominators to get the new denominator.

$$\frac{a}{b}\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{ac}{bd}+\frac{ae}{bf}$$

Tips for distributing fractions:

- If we're multiplying a fraction by a term that isn't a fraction, we can rewrite any value with a denominator of 1. So given any term that isn't a fraction, we can turn it into a fraction by giving it a denominator of 1.
- Once we've applied the Distributive Property, we need to cancel any common factors from the numerator and denominator of any of the fractions in the result.

PEMDAS and order of operations

Grouping symbols:

Parentheses	( )
Brackets (square brackets)	[ ]

Braces (curly braces)	{ }
Absolute Value	| |

The fraction line that separates the numerator and denominator

PEMDAS, Order of operations:

Parentheses	(all grouping symbols)
Exponents	(powers and roots)
Multiplication/**D**ivision	(From left to right and top to bottom, performing each multiplication/division as we come to it)
Addition/**S**ubtraction	(From left to right and top to bottom, performing each addition/subtraction as we come to it)

Equations

Inverse operations

Inverse operations: Opposite operations that undo each other. Addition undoes subtraction and vice versa, and division undoes multiplication and vice versa. Exponents undo roots and vice versa.

Simple equations

Solving equations: Solving simple equations is really just undoing everything that's happening to the variable in order to get the variable by itself. We'll solve equations by working the order of operations in reverse.

Balancing equations

The equation scale: An equation is a two-sided scale that we always have to keep in balance. What we do to one side of an equation we have to do to the other, otherwise the scale won't stay balanced.

Equations with subscripts

Subscript: A small number that comes just after, and at a lower level than the variable.

Subscripted variable: A variable that has a subscript attached to it. In mathematics, we'll often use subscripted variables to represent the same kind of value for different subjects.

Word problems into equations

Translating words into mathematical notation:

	Words	Phrases	Expressions
Addition	sum, total, more than, added, increased, plus	3 more than a number, the sum of 5 and a number	3+x 5+n
Subtraction	less, minus, decreased by, difference, less than	12 decreased by a number, the difference of 7 and a number	12-n 7-x
Multiplication	product, times, multiplied, of	the product of a number and 2, 2/3 of a number	2x (2/3)n
Division	quotient, divided by, divided into	15 divided by a number, the quotient of a number and 4	15/n x/4

Consecutive integers

Integers: "Whole numbers" that are either positive, negative, or 0.

Consecutive integers: Integers that are one unit apart from each other.

Polynomials

Adding and subtracting polynomials

Polynomial: An expression that's the sum and/or difference of a finite number of terms, where the terms include only constants, variables, and positive integer exponents.

Degree of a polynomial: Given by the largest exponent in the polynomial.

Adding and subtracting polynomials: For the purposes of addition and subtraction, like terms are terms that have the same base and the same exponent. We combine like terms by adding or subtracting the coefficients while keeping the base and the exponent the same.

Multiplying polynomials

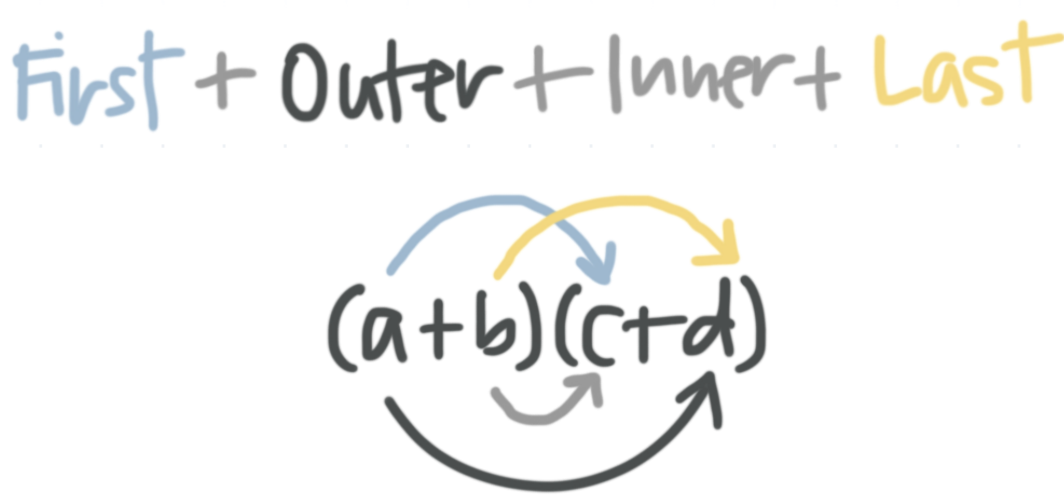
Monomial: A single term

Binomial: Two terms

Trinomial: Three terms

Polynomial: Many terms

FOIL two binomials:



$$(a+b)(c+d) = ac + ad + bc + bd$$

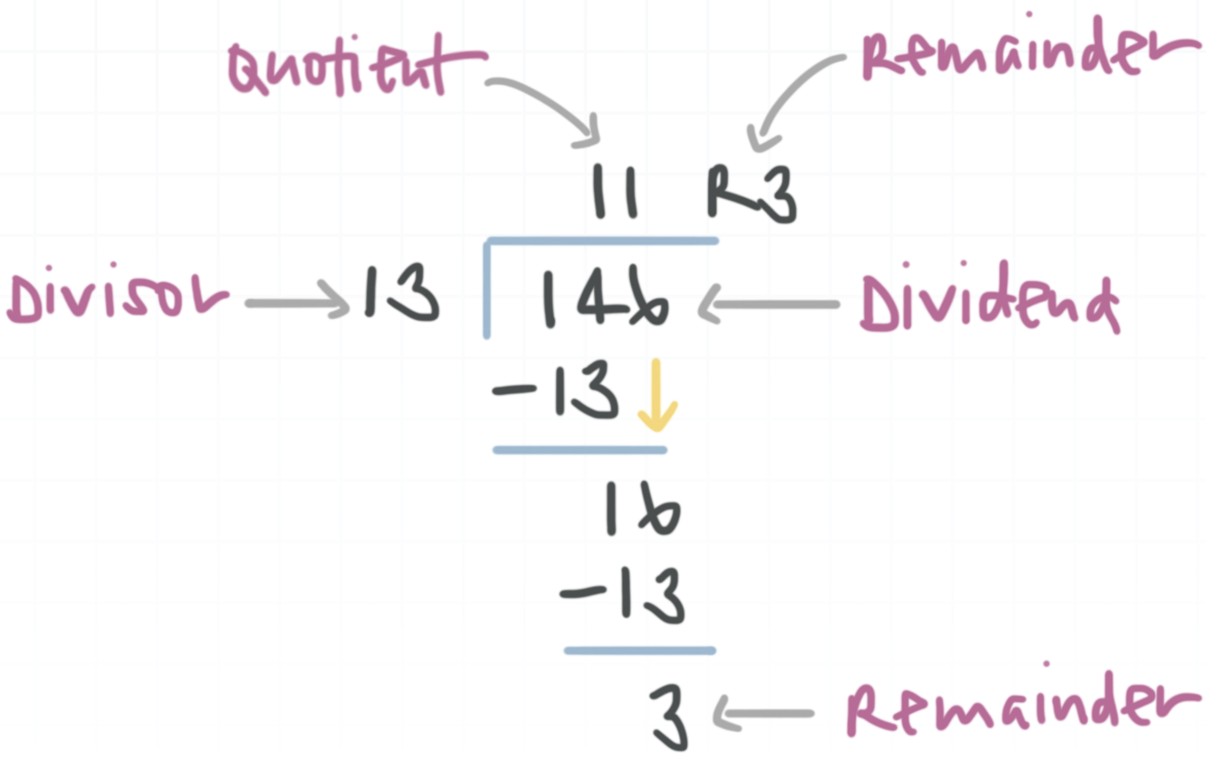
$$(a+b)(c-d) = ac - ad + bc - bd$$

$$(a-b)(c+d) = ac + ad - bc - bd$$

$$(a-b)(c-d) = ac - ad - bc + bd$$

Dividing polynomials

Dividend, divisor, quotient, and remainder:



Ordering the dividend's terms: Order the terms in the dividend by descending power of the dividend's variable.

Multiplying multivariable polynomials

Multivariable polynomial: A polynomial that includes two or more variables.

Multiplying multivariable polynomials: If we multiply terms with like bases, then the base stays the same and we add the exponents.

Dividing multivariable polynomials

Ordering the dividend's terms: Order the terms in the dividend by descending power of the leading variable in the divisor.

Factoring

Greatest common factor

Factoring: "Un-distributing"

Common factor: Any factor that's shared by all the terms.

Greatest common factor (GCF): The factor that consists of everything that's shared by all the terms.

Quadratic polynomials

Quadratic polynomials, or quadratics: Second-degree polynomials.

Factoring quadratics: To factor a quadratic in standard form, $ax^2 + bx + c$, where $a = 1$ and $c \neq 0$, we need to look for a pair of factors of c that multiply to c and sum to b.

Difference of squares

Factoring the difference of squares:

$$ax^2 - c = (\sqrt{a}x + \sqrt{c})(\sqrt{a}x - \sqrt{c})$$

Zero Theorem

Quadratic equation: $ax^2 + bx + c = 0$. The values that satisfy the quadratic equation are its solutions, roots, or zeros.

Zero Theorem: Given $AB = 0$, we know $A = 0$ or $B = 0$.

Completing the square

Completing the square: A method we can use when we can't find the roots of a quadratic by factoring.

How to complete the square: If the coefficient a in $ax^2 + bx + c = 0$ is $a \neq 1$, then divide through by a to make the coefficient 1. Then with the quadratic in the form $x^2 + bx + c = 0$,

1. Calculate $b/2$, then square the result to get $(b/2)^2$.
2. Add $(b/2)^2$ to both sides of the equation to get

$$x^2 + bx + \left(\frac{b}{2}\right)^2 + c = \left(\frac{b}{2}\right)^2$$

3. Subtract c from both sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$$

4. Factor the left side. It will always factor as a perfect square.

$$\left(x+\frac{b}{2}\right)\left(x+\frac{b}{2}\right)=\left(\frac{b}{2}\right)^2-c$$

$$\left(x+\frac{b}{2}\right)^2=\left(\frac{b}{2}\right)^2-c$$

5. Take the square root of both sides, remembering to include a $\pm$ sign on the right side, then subtract $b/2$ from both sides.

$$x+\frac{b}{2}=\pm\sqrt{\left(\frac{b}{2}\right)^2-c}$$

$$x=-\frac{b}{2}\pm\sqrt{\left(\frac{b}{2}\right)^2-c}$$

Number of solutions to the equation:

- If $\left(\frac{b}{2}\right)^2-c<0$, then the quadratic has zero real roots (the roots are complex)
- If $\left(\frac{b}{2}\right)^2-c=0$, then the quadratic has one root, $x=-\frac{b}{2}$
- If $\left(\frac{b}{2}\right)^2-c>0$, then the quadratic has two roots,

$$x=-\frac{b}{2}\pm\sqrt{\left(\frac{b}{2}\right)^2-c}$$

Complex numbers: Numbers that include both real and imaginary numbers.

Imaginary number: Any number that includes the imaginary number i, where $i = \sqrt{-1}$.

Quadratic formula

The discriminant of a quadratic equation, $ax^2 + bx + c = 0$: $b^2 - 4ac$

Number of solutions based on the value of the discriminant:

- When $b^2 - 4ac = 0$, the solution is one real number
- When $b^2 - 4ac > 0$, the solutions are two real numbers
- When $b^2 - 4ac < 0$, the solutions are two real complex numbers

Functions and graphing

Cartesian coordinate system

Cartesian coordinate system: The two-dimensional plane in which we graph points and equations.

Coordinate axes: A pair of perpendicular number lines, one horizontal and one vertical. The horizontal axis is the x-axis and the vertical axis is the y-axis, and they meet at the origin, which is the point $(0,0)$.

Coordinates: We represent every point in the plane by its coordinates (x, y), where x (the horizontal coordinate or the x-coordinate) is the horizontal (left-right) location of the point, and y (the vertical coordinate or the y-coordinate) is the vertical (up-down) location of the point.

Quadrants: Quadrant I is where x and y are both positive. The other three quadrants are named in counterclockwise order.

Quadrant I: both x and y are positive $(x, y) = (+, +)$

Quadrant II: x is negative and y is positive $(x, y) = (-, +)$

Quadrant III: both x and y are negative $(x, y) = (-, -)$

Quadrant IV: x is positive and y is negative $(x, y) = (+, -)$

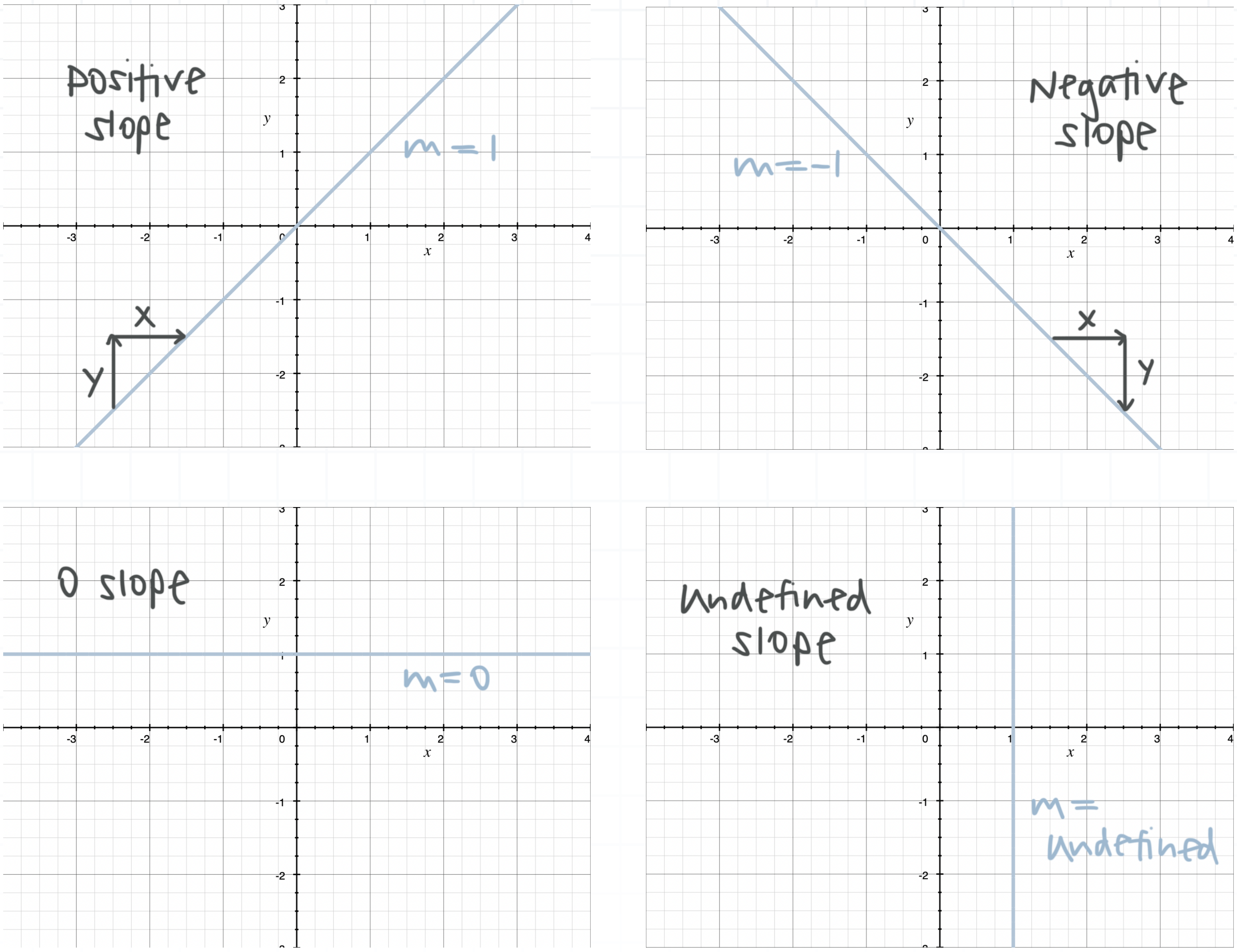
Slope

Slope: The "steepness" of the line, or the rate of change of the y-coordinates of the points on the graph as we move horizontally from left to right. If we think about the vertical change as "rise" and the horizontal change as "run," then we can write the slope formula as

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Sign of the slope:



Point-slope and slope-intercept forms of a line

Two ways to write the equation of a line:

Point-slope form $y - y_1 = m(x - x_1)$

Slope-intercept form $y = mx + b$

Information needed to find the equation of a line: We must know at least two of the following pieces of information about the line, in order to find its equation:

1. One point, (x_1, y_1)

2. A second point, (x_2, y_2)

3. The slope, m

4. The y-intercept, b (the y-value where the line crosses the y-axis)

Graphing linear equations

Linear equation: The equation of a line.

Intercepts: The points where the line crosses the major axes.

Function notation

Function: An equation that only gives one output of the dependent variable for each input of the independent variable. An equation is only a function if *all* inputs each only give one output.

Independent and dependent variables: In an equation defined by x and y, we typically say that the independent variable is x and that the dependent variable is y, because the output value we get for y "depends on" the input value we choose for x.

Argument: The function $f(x)$ is the function f defined in terms of the argument x.

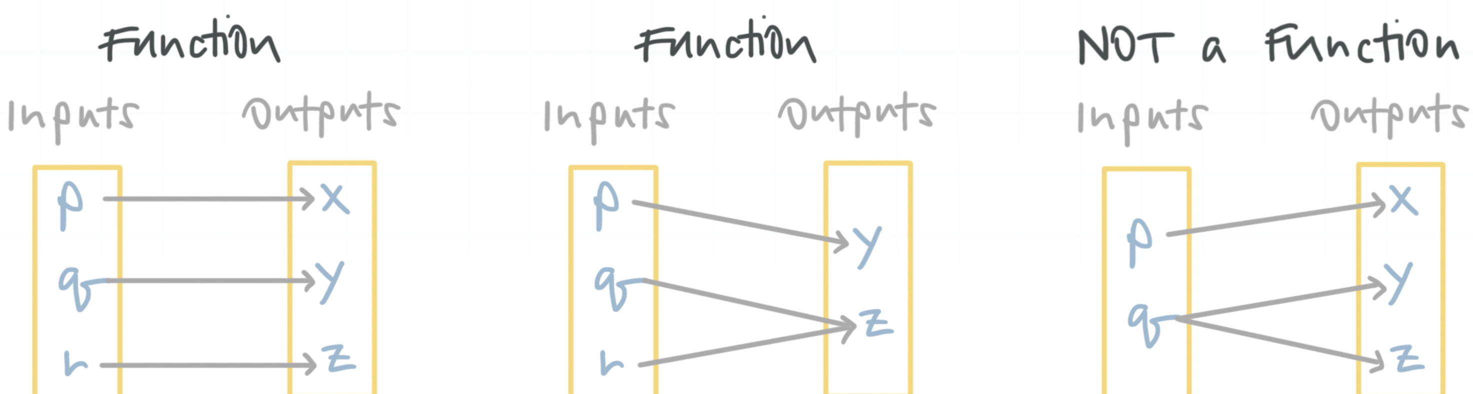
Domain and range

Domain: The domain of a function is all the values we can input into the function that don't cause it to be undefined.

Range: The entire set of output values that can result from all the inputs in the domain. So if the domain is all of the allowable x values, the range is all the possible y values.

Testing for functions

Visual representation of a function:



Vertical Line Test

Vertical Line Test (VLT): A graph represents a function if no perfectly vertical line crosses the graph more than once.

Sum of functions

Sum of functions: $(f+g)(x) = f(x) + g(x)$

Product of functions

Product of functions: $(fg)(x) = f(x) \cdot g(x)$

Even, odd, or neither

Even functions: Functions that are even are symmetric with respect to the y-axis. Opposite values of x will have equivalent values of y.

$$f(-x) = f(x)$$

Odd functions: Functions that are odd are symmetric with respect to the origin. Opposite values of x will have opposite values of y.

$$f(-x) = -f(x)$$

Inequalities

Trichotomy

The Law of Trichotomy: Two numbers (or expressions) can have exactly one of three possible relationships:

- The first number is smaller than the second number, $a < b$

- The first number is greater than the second number, $a > b$
- The first number is equal to the second number, $a = b$

Three corollaries to the Law of Trichotomy:

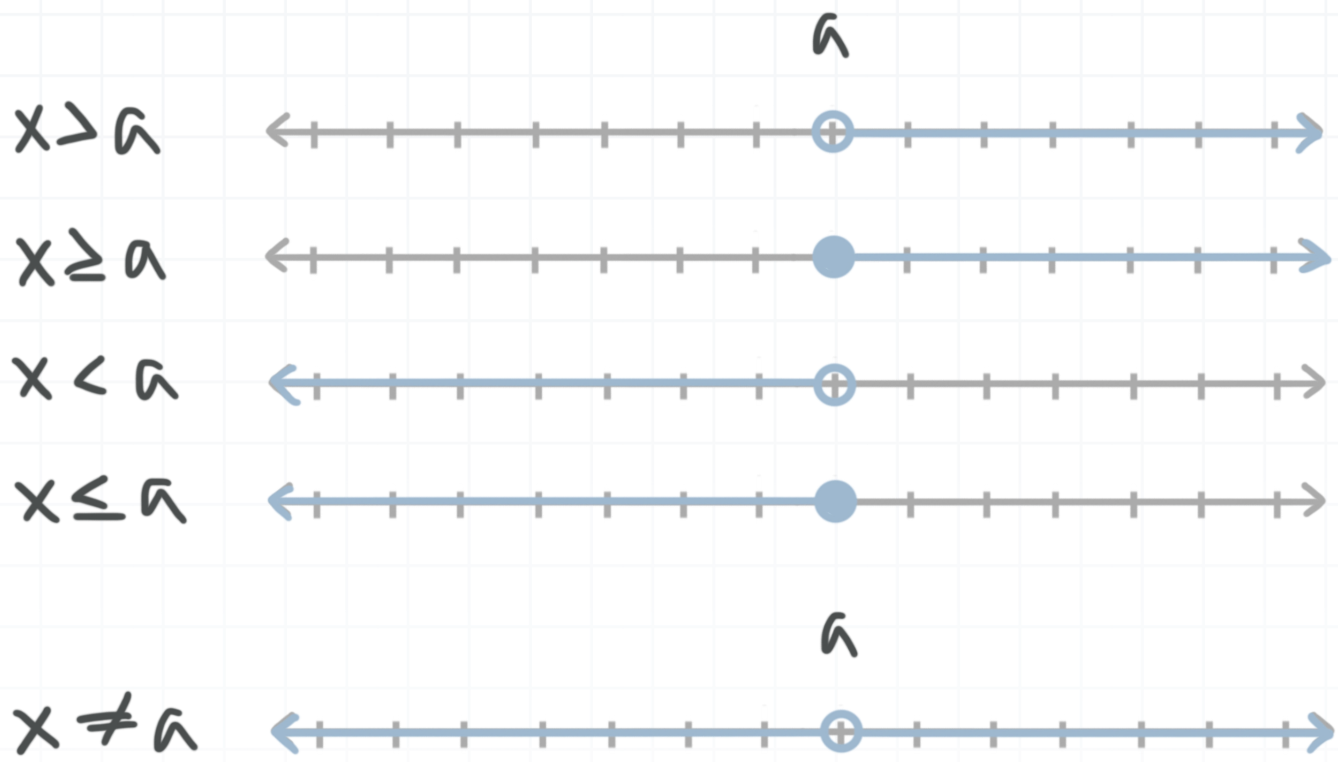
- If a is not greater than b and also not equal to b, then a must be less than b. If $a \ngeq b$, then $a < b$.
- If a is not less than b and also not equal to b, then a must be greater than b. If $a \nleq b$, then $a > b$.
- If a is not greater than b and also not less than b, then a must be equal to b. If $a \nless b$ and $a \ngtr b$, then $a = b$.

Inequalities and negative numbers

Rule for negative numbers: When we multiply or divide both sides of an inequality by a negative value, we have to reverse the direction of the inequality.

Graphing inequalities on a number line

The sketches of inequalities solved for x:



Graphing disjunctions on a number line

Disjunction: “Or statement” where the solutions are any values that satisfy either one inequality in the statement *or* the other inequality in the statement. Values not included in the solution set are values that don’t satisfy either inequality in the statement.

Graphing conjunctions on a number line

Conjunction: “And statement” where the solutions are any values that satisfy both one inequality in the statement *and* the other inequality in the statement. Values not included in the solution set are any values that don’t satisfy both inequalities in the statement.

$a \leq x \leq b$ means $a \leq x$ and $x \leq b$

$a \le x < b$	means	$a \le x$ and $x < b$
$a < x \le b$	means	$a < x$ and $x \le b$
$a < x < b$	means	$a < x$ and $x < b$

Graphing inequalities in the plane

Steps for sketching an inequality in the plane:

Start by drawing the boundary line.

- The boundary line will be dashed if the inequality is $<$ or $>$, which indicates that the boundary line isn't part of the graph of the inequality.

- The boundary line will be solid if the inequality is $\le$ or $\ge$, which indicates that the boundary line is part of the graph of the inequality.

After we draw the boundary line, we'll shade in the side of the line that satisfies the inequality.

- Shade above the line if we have a $>$ or $\ge$ inequality.

- Shade below the line if we have a $<$ or $\le$ inequality.

Using a test point to determine whether to shade on one side of the inequality or the other: Substitute any point that's not on the boundary line of the inequality (the origin $(0,0)$ is the easiest). If the resulting inequality is true, shade on the side of the line that includes the test point;

if the resulting inequality is false, shade on the side of the line that doesn't include the test point.

Absolute value equations

Absolute value: The distance from 0. Opposite values of x have the same absolute value because they're both equally distant from 0.

- If $a > 0$, then $|f(x)| = a$ has two solutions.
- If $a = 0$, then $|f(x)| = a$ has one solution.
- If $a < 0$, then $|f(x)| = a$ has no solution.

Solving equations with one absolute value:

1. Isolate the absolute value expression on one side of the equation.
2. Check the value of a. If $a > 0$, then set up and solve two equations, $f(x) = a$ and $f(x) = -a$. If $a = 0$, set up the equation $f(x) = 0$. And if $a < 0$, we know the equation has no solutions.
3. For any values we find in Step 2, verify that they satisfy the original absolute value equation.

Solving equations with two absolute values: If $|m| = |n|$, then $m = n$ or $m = -n$.

Absolute value inequalities

Solutions to an absolute value inequality:

$\lvert$absolute value$\rvert$ $<$ negative	No solution
$\lvert$absolute value$\rvert$ $>$ negative	Solution is all real numbers
$\lvert$absolute value$\rvert$ $<$ positive a	Conjunction
	$-a <$ absolute value $< a$
$\lvert$absolute value$\rvert$ $>$ positive a	Disjunction
	absolute value $< -a$
	or absolute value $> a$

Systems of equations

Solving systems with substitution

Substitution method for solving systems:

1. Get a variable by itself in one of the equations.
2. Substitute the expression from step 1 into the other equation.
3. Solve the equation in step 2 for the remaining variable.
4. Substitute the result from step 3 into the equation from step 1.

Number of solutions: There are three possible solutions to a system of equations.

- one solution (called the unique solution), or
- no solutions (parallel lines), or
- infinitely many solutions (identical lines).

Solving systems with elimination

Elimination method for solving systems:

1. If necessary, rearrange both equations so that the x-terms are first, followed by the y-terms, the equals sign, and the constant term (in that order). If an equation appears to have no constant term, that means that the constant term is 0.
2. Multiply one (or both) equations by a constant that will allow either the x-terms or the y-terms to cancel when the equations are added or subtracted.
3. Add or subtract the equations to eliminate one of the variables.
4. Solve for the remaining variable.
5. Plug the result of step 4 into one of the original equations, then solve for the other variable.

Solving systems three ways

Graphing method for solving systems:

1. Solve for y in each equation.

2. Graph both equations on the same Cartesian coordinate system.

3. Find the point of intersection of the lines (the point where the lines cross).

Systems of linear inequalities

Solution to a system of linear inequalities: A region in the plane.

Steps for finding the solution to the system:

1. Graph the boundary lines

2. Determine whether the boundary lines are dashed ($<$ or $<$) or solid ($\leq$ or $\geq$)

3. Determine which side of each boundary line to shade

4. Identify the overlapping shaded region as the solution, keeping only the overlap shaded

W W W . K R I S T A K I N G M A T H . C O M