Powers of fractions

This lesson will cover how to find the power of a fraction as well as introduce how to work with fractional exponents.

Powers of fractions

Say we have something like

$$\left(\frac{a}{b}\right)^c$$

where a, b, and c are integers. This is like saying that we're doing a multiplication in which a/b appears as a factor c times (and there are no other factors). This turns the power problem into a fraction multiplication problem, where we multiply the numerators and the denominators separately. In this example, a is the numerator in each factor, and b is the denominator in each factor.

Example

Simplify the expression.

$$\left(\frac{3}{4}\right)^2$$



This is an example of a power of a fraction. The way the problem is written, it's like saying that we're multiplying 3/4 by itself, since the base is 3/4 and the exponent is 2. So the problem becomes

$$\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)$$

Now we've got a fraction multiplication problem. When we multiply fractions, we multiply the numerators and the denominators separately.

$$\frac{3\cdot 3}{4\cdot 4} = \frac{9}{16}$$

Let's look at an example with variables.

Example

Simplify the expression.

$$\left(\frac{x}{y^3}\right)^4$$

This is an example of a power of a fraction. The way the problem is written, it's like saying that we're doing a multiplication in which x/y^3 appears as a factor four times (and there are no other factors), since the base is x/y^3 and the exponent is 4. So the problem becomes



$$\left(\frac{x}{y^3}\right)\left(\frac{x}{y^3}\right)\left(\frac{x}{y^3}\right)\left(\frac{x}{y^3}\right)$$

Now we've got a fraction multiplication problem. Remember, when we multiply fractions, we multiply the numerators and the denominators separately.

$$\frac{x \cdot x \cdot x \cdot x}{y^3 \cdot y^3 \cdot y^3 \cdot y^3}$$

Now we have a like base of x in the numerator and a like base of y in the denominator.

In the numerator we can write x^4 because x appears as a factor four times.

Remember that, when we have like bases, we can add the exponents. We'll need to do this for the denominator. Let's look at the calculation for the denominator:

$$y^3 \cdot y^3 \cdot y^3 \cdot y^3 = y^{3+3+3+3} = y^{12}$$

So the simplified expression is

$$\frac{x^4}{y^{12}}$$

We can also simplify $(a/b)^c$ by rewriting it as a fraction in which the numerator and the denominator are separately raised to the power c.

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

