Algebra 2 Workbook Solutions

Exponents and radicals

krista king MATH

POWERS OF NEGATIVE BASES

■ 1. Simplify the expression.

$$-2^2$$

Solution:

PEMDAS tells us that we need to first simplify the exponent, and then multiply by the negative sign.

$$-2^2 = -(2 \cdot 2) = -4$$

■ 2. Simplify the expression.

$$(-3)^4$$

Solution:

Since the negative sign is inside the parentheses, it means we we'll multiply -3 by itself four times.

$$(-3)^4 = (-3)(-3)(-3)(-3) = 81$$

or, since 4 is even, then $(-3)^4 = 3^4 = 81$.

■ 3. Simplify the expression.

$$(-5)^3$$

Solution:

Since the negative sign is inside the parentheses, we'll multiply -5 by itself three times.

$$(-5)^3 = (-5)(-5)(-5) = 25(-5) = -125$$

■ 4. Simplify the expression.

$$-3^3 + (-1)^5 \cdot 9^2$$

Solution:

PEMDAS tells us that we need to first simplify the exponents.

$$-3^3 = -(3 \cdot 3 \cdot 3) = -(27) = -27$$

$$(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$$

$$9^2 = 81$$

Substituting these values into the original expression, we get

$$-3^3 + (-1)^5 \cdot 9^2$$

$$-27 + (-1) \cdot 81$$

$$-27 - 81$$

$$-108$$

■ 5. Simplify the expression.

$$-4^2 \cdot (-2)^3 + 0^4$$

Solution:

PEMDAS tells us that we need to first simplify the exponents.

$$-4^2 = -(4 \cdot 4) = -(16) = -16$$

$$(-2)^3 = (-2)(-2)(-2) = -8$$

$$0^4 = 0 \cdot 0 \cdot 0 \cdot 0 = 0$$

Substituting these values into the original expression, we get

$$-4^2 \cdot (-2)^3 + 0^4$$

$$(-16)(-8) + 0$$

$$128$$

■ 6. Simplify the expression.

$$-9^2 + (-1)^6 - 3^2 \cdot (-5)^3$$

Solution:

PEMDAS tells us that we need to first simplify the exponents.

$$-9^2 = -(9 \cdot 9) = -(81) = -81$$

$$(-1)^6 = (-1)(-1)(-1)(-1)(-1)(-1) = 1$$

$$-3^2 = -(3 \cdot 3) = -(9) = -9$$

$$(-5)^3 = (-5)(-5)(-5) = -125$$

Substituting these values into the original expression, we get

$$-9^2 + (-1)^6 - 3^2 \cdot (-5)^3$$

$$-81 + 1 - 9(-125)$$

$$-80 + 1{,}125$$

$$1{,}045$$

POWERS OF FRACTIONS

■ 1. Simplify the expression.

$$\left(\frac{5}{6}\right)^2$$

Solution:

Apply the exponents separately to the numerator and denominator.

$$\frac{5^2}{6^2}=\frac{25}{36}$$

■ 2. Simplify the expression.

$$\left(\frac{1}{2}\right)^3\cdot\left(\frac{2}{3}\right)^2$$

Solution:

Apply the exponents separately to the numerator and denominator for each fraction. We get

$$\frac{1^3}{2^3}=\frac{1}{8}$$

and

$$\frac{2^2}{3^2} = \frac{4}{9}$$

Then the expression simplifies to

$$\left(\frac{1}{2}\right)^3 \cdot \left(\frac{2}{3}\right)^2 = \frac{1}{8} \cdot \frac{4}{9} = \frac{1}{18}$$

■ 3. Simplify the expression.

$$\left(\frac{x^3}{y^5}\right)^2 \cdot \left(\frac{xy}{z^2}\right)^4$$

Solution:

Apply the exponents separately to the numerator and denominator for each fraction. We get

$$\left(\frac{x^3}{y^5}\right)^2 = \frac{x^{3\cdot 2}}{y^{5\cdot 2}} = \frac{x^6}{y^{10}}$$

and

$$\left(\frac{xy}{z^2}\right)^4 = \frac{x^{1\cdot 4} \cdot y^{1\cdot 4}}{z^{2\cdot 4}} = \frac{x^4y^4}{z^8}$$

Then the expression simplifies to

$$\left(\frac{x^3}{y^5}\right)^2 \cdot \left(\frac{xy}{z^2}\right)^4 = \frac{x^6}{y^{10}} \cdot \frac{x^4y^4}{z^8} = \frac{x^{6+4}}{y^{10-4}z^8} = \frac{x^{10}}{y^6z^8}$$

■ 4. Simplify the expression.

$$\left(\frac{2}{3}\right)^4$$

Solution:

Apply the exponent to the entire fraction and multiply it by itself four times.

$$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \left(\frac{4}{9}\right)\left(\frac{4}{9}\right) = \frac{16}{81}$$

■ 5. Simplify the expression.

$$\left(\frac{x^3}{y^2}\right)^5$$

Solution:

Apply the exponent to the entire fraction and multiply it by itself five times.

$$\left(\frac{x^3}{y^2}\right)\left(\frac{x^3}{y^2}\right)\left(\frac{x^3}{y^2}\right)\left(\frac{x^3}{y^2}\right)\left(\frac{x^3}{y^2}\right)$$

$$\frac{x^3 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x^3}{y^2 \cdot y^2 \cdot y^2 \cdot y^2 \cdot y^2}$$

$$\frac{x^{3+3+3+3+3}}{y^{2+2+2+2+2}}$$

$$\frac{x^{15}}{y^{10}}$$

■ 6. Simplify the expression.

$$\left(\frac{a^2}{b}\right)^4$$

Solution:

Apply the exponent to the entire fraction and multiply it by itself four times.

$$\left(\frac{a^2}{b}\right)\left(\frac{a^2}{b}\right)\left(\frac{a^2}{b}\right)\left(\frac{a^2}{b}\right)$$

$$\frac{a^2 \cdot a^2 \cdot a^2 \cdot a^2}{b \cdot b \cdot b \cdot b}$$

$$\frac{a^{2+2+2+2}}{b^{1+1+1+1}}$$

$$\frac{a^8}{b^4}$$

ZERO AS AN EXPONENT

■ 1. Simplify the expression.

$$\frac{4^0 \cdot 9^2}{(-2)^0 + 2^1}$$

Solution:

Any nonzero real number raised to the power of 0 is 1.

$$\frac{4^0 \cdot 9^2}{(-2)^0 + 2^1} = \frac{1 \cdot 81}{1 + 2} = \frac{81}{3} = 27$$

■ 2. Simplify the expression.

$$1{,}042^0$$

Solution:

Any nonzero real number raised to the power of 0 is 1.

■ 3. Simplify the expression.

$$(10^2 + 3^3)^0$$

Solution:

Any nonzero real number raised to the power of 0 is 1.

$$(10^2 + 3^3)^0 = 1$$

■ 4. Simplify the expression.

$$(-1)^0$$

Solution:

Any nonzero real number raised to the power of 0 is 1.

■ 5. Simplify the expression.

$$(2ac - 4x)^0$$

Solution:

Any nonzero real number raised to the power of 0 is 1. Therefore, the answer is 1 if we assume that $2ac - 4x \neq 0$.

■ 6. Simplify the expression.

$$(-100b)^0$$

Solution:

Any nonzero real number raised to the power of 0 is 1. Therefore, the answer is 1 if we assume that $b \neq 0$.

NEGATIVE EXPONENTS

■ 1. Simplify the expression.

$$3 \cdot 5^{-2} \cdot 6^{-2}$$

Solution:

To make the exponents positive, move any terms with the negative exponent from the numerator to the denominator.

$$3 \cdot 5^{-2} \cdot 6^{-2}$$

$$\frac{3}{5^2 \cdot 6^2}$$

$$\frac{3}{25 \cdot 36}$$

$$\frac{1}{300}$$

■ 2. Simplify the expression.

$$4^{-3}$$

Solution:

Remember that 4^{-3} is the same as

$$\frac{4^{-3}}{1}$$

To make the exponent positive, move the term with the negative exponent from the numerator to the denominator.

$$\frac{1}{4^3} = \frac{1}{64}$$

■ 3. Simplify the expression.

$$-3^{-1}$$

Solution:

Remember that -3^{-1} is the same as

$$\frac{-3^{-1}}{1}$$

To make the exponent positive, move the term with the negative exponent from the numerator to the denominator.

$$\frac{1}{-3^1}$$

PEMDAS tells us that we need to first simplify the exponent, and then multiply by the negative sign.

$$-\frac{1}{3}$$

■ 4. Simplify the expression.

$$-2^{-2} \cdot (-2)^{-2}$$

Solution:

To make the exponents positive move the entire value from the numerator to the denominator.

$$\frac{1}{-2^2} \cdot \frac{1}{(-2)^2}$$

PEMDAS tells us that we need to first simplify the exponents, and then multiply by the negative sign.

$$\frac{1}{-4} \cdot \frac{1}{4}$$

$$-\frac{1}{16}$$

■ 5. Write the expression with only positive exponents.

$$a^{-5}$$

Solution:

Remember that a^{-5} is the same as

$$\frac{a^{-5}}{1}$$

To make the exponent positive, move the term with the negative exponent from the numerator to the denominator.

$$\frac{1}{a^5}$$

■ 6. Write the expression with only positive exponents.

$$\frac{x^{-3}y^2}{x^4y^7}$$

Solution:

To make the exponent positive, move that factor from the numerator to the denominator.

$$\frac{y^2}{x^4x^3y^7}$$

$$\frac{1}{x^{4+3}y^{7-2}}$$

$$\frac{1}{x^7y^5}$$

NEGATIVE EXPONENTS AND PRODUCT RULE

■ 1. Write the expression without any negative exponents.

$$\frac{(2ab)^{-2}a^2}{b^{-4}\cdot(ab)^0}$$

Solution:

To make the exponents positive, move the term with the negative exponent from the numerator to the denominator or move the term with the negative exponent from the denominator to the numerator.

$$\frac{(2ab)^{-2}a^2}{b^{-4}\cdot(ab)^0}$$

$$\frac{a^2b^4}{(2ab)^2\cdot(ab)^0}$$

$$\frac{a^2b^4}{4a^2b^2\cdot 1}$$

$$\frac{b^4}{4b^2}$$

$$\frac{b^2}{4}$$

■ 2.Write the expression without any negative exponents.

$$\frac{2x^0y^{-5}}{z^{-1}(xy^2)^{-3}}$$

Solution:

To make the exponents positive, move the term with the negative exponent from the numerator to the denominator or move the term with the negative exponent from the denominator to the numerator.

$$\frac{2x^0y^{-5}}{z^{-1}(xy^2)^{-3}}$$

$$\frac{2x^0 \cdot (xy^2)^3z^1}{y^5}$$

$$\frac{2(1)(x^3y^6)z}{y^5}$$

$$\frac{2x^3yz}{1}$$

$$2x^3yz$$

■ 3. Write the expression without any negative exponents.

$$\frac{1}{a^{-8}}$$

Solution:

To make the exponent positive, move the term with the negative exponent from the denominator to the numerator.

$$\frac{a^8}{1} = a^8$$

■ 4. Write the expression without any negative exponents.

$$\frac{8}{z^{-3}}$$

Solution:

To make the exponent positive, move the term with the negative exponent from the denominator to the numerator.

$$\frac{8z^3}{1} = 8z^3$$

■ 5. Write the expression without any negative exponents.

$$\frac{2y^{-4}}{x^{-9}}$$

Solution:

To make y^{-4} positive, move it from the numerator to the denominator.

$$\frac{2}{x^{-9}y^4}$$

To make the x^{-9} positive, move it from the denominator to the numerator.

$$\frac{2x^9}{y^4}$$

■ 6. Write the expression without any negative exponents.

$$\frac{1}{(3x^{-4}y^2)^{-3}}$$

Solution:

To make the exponents positive, move the term with the negative exponent from the denominator to the numerator.

$$(1)(3x^{-4}y^2)^3$$

$$3^3x^{-4\cdot 3}y^{2\cdot 3}$$

$$27x^{-12}y^6$$

To make the exponent positive, move the factor from the numerator to the denominator.

$$\frac{27y^6}{x^{12}}$$

FRACTIONAL EXPONENTS

■ 1. Simplify the expression.

$$b^2 \cdot b^{\frac{2}{3}}$$

Solution:

The base of both terms is b, so we need to add the exponents.

$$b^{2+\frac{2}{3}}$$

Find a common denominator inside the exponent, and then simplify.

$$b^{2\left(\frac{3}{3}\right)+\frac{2}{3}}$$

$$b^{\frac{6}{3}+\frac{2}{3}}$$

$$b^{\frac{8}{3}}$$

$$\sqrt[3]{b^8}$$

■ 2. Simplify the expression.

$$x^5 \cdot x^{\frac{1}{6}}$$

Solution:

The base of both terms is x, so we need to add the exponents.

$$x^{5+\frac{1}{6}}$$

Find a common denominator inside the exponent, and then simplify.

$$x^{5\left(\frac{6}{6}\right)+\frac{1}{6}}$$

$$x^{\frac{30}{6}+\frac{1}{6}}$$

$$x^{\frac{31}{6}}$$

$$\sqrt[6]{x^{31}}$$

■ 3. Simplify the expression.

$$\left(\frac{1}{16}\right)^{\frac{3}{2}}$$

Solution:

We know that $3/2$ can be rewritten as the product of $1/2$ and 3. Therefore, we can rewrite the expression as

$$\left(\left(\frac{1}{16}\right)^{\frac{1}{2}}\right)^{3}$$

Raising a value to the power $1/2$ is the same as taking the square root.

$$\left(\sqrt{\frac{1}{16}}\right)^3 = \left(\frac{1}{4}\right)^3 = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{64}$$

■ 4. Simplify the expression.

$$8^{\frac{2}{3}}$$

Solution:

We know that $2/3$ can be rewritten as the product of $1/3$ and 2. Therefore, we can rewrite the expression as

$$(8^{\frac{1}{3}})^2$$

Raising a value to the power $1/3$ is the same as taking the cube root.

$$(\sqrt[3]{8})^2 = 2^2 = 2 \cdot 2 = 4$$

■ 5. Simplify the expression.

$$3^{-\frac{3}{7}}$$

Solution:

First make the exponent positive.

$$\frac{1}{3^{\frac{3}{7}}}$$

In the fractional exponent, 3 is the power and 7 is the root, which means we can rewrite the expression as

$$\frac{1}{\sqrt[7]{3^3}}$$

$$\frac{1}{\sqrt[7]{27}}$$

■ 6. Simplify the expression.

$$(81a^4b^{\frac{1}{2}})^{-\frac{5}{4}}$$

Solution:

First make the exponent positive.

$$\frac{1}{(81a^4b^{\frac{1}{2}})^{\frac{5}{4}}}$$

Now we have an expression of the form $(x^c)^d$, so we can multiply the exponents.

$$\frac{1}{81^{\frac{5}{4}}(a^4)^{\frac{5}{4}}(b^{\frac{1}{2}})^{\frac{5}{4}}}$$

$$\frac{1}{(3^4)^{\frac{5}{4}}a^5b^{\frac{5}{8}}}$$

$$\frac{1}{3^5a^5b^{\frac{5}{8}}}$$

We can rewrite the expression as

$$\frac{1}{243a^5b^{\frac{5}{8}}}$$

$$\frac{1}{243a^5\sqrt[8]{b^5}}$$

RATIONALIZING THE DENOMINATOR

■ 1. Rationalize the denominator.

$$\frac{2}{\sqrt{5}}$$

Solution:

Multiply the numerator and denominator by the radical in the denominator.

$$\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\frac{2\sqrt{5}}{5}$$

■ 2. Rationalize the denominator.

$$\frac{1}{4\sqrt{3}}$$

Solution:

Multiply the numerator and denominator by the radical in the denominator.

$$\frac{1}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{1\sqrt{3}}{4 \cdot 3}$$

$$\frac{\sqrt{3}}{12}$$

■ 3. Simplify the expression, making sure to rationalize the denominator.

$$\sqrt{\frac{4}{12}} + \sqrt{\frac{9}{12}}$$

Solution:

Reduce the fractions if possible.

$$\sqrt{\frac{1}{3}} + \sqrt{\frac{3}{4}}$$

Apply the roots to the numerators and denominators separately.

$$\frac{\sqrt{1}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{4}}$$

$$\frac{1}{\sqrt{3}}+\frac{\sqrt{3}}{2}$$

Find a common denominator by multiplying each fraction by the denominator of the other fraction, then combine the fractions.

$$\frac{1}{\sqrt{3}}\cdot\frac{2}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{2}{2\sqrt{3}}+\frac{3}{2\sqrt{3}}$$

$$\frac{5}{2\sqrt{3}}$$

Rationalize the denominator by multiplying the numerator and denominator by the radical in the denominator.

$$\frac{5}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{5\sqrt{3}}{2\cdot 3}$$

$$\frac{5\sqrt{3}}{6}$$

■ 4. Simplify the expression, making sure to rationalize the denominator.

$$\sqrt{\frac{6}{25}}+\sqrt{\frac{20}{24}}$$

Solution:

Reduce the fractions if possible.

$$\sqrt{\frac{6}{25}}+\sqrt{\frac{5}{6}}$$

Apply the roots to the numerators and denominators separately.

$$\frac{\sqrt{6}}{\sqrt{25}}+\frac{\sqrt{5}}{\sqrt{6}}$$

$$\frac{\sqrt{6}}{5}+\frac{\sqrt{5}}{\sqrt{6}}$$

Find a common denominator by multiplying each fraction by the denominator of the other fraction, then combine the fractions.

$$\frac{\sqrt{6}}{5}\cdot\frac{\sqrt{6}}{\sqrt{6}}+\frac{\sqrt{5}}{\sqrt{6}}\cdot\frac{5}{5}$$

$$\frac{\sqrt{36}}{5\sqrt{6}}+\frac{5\sqrt{5}}{5\sqrt{6}}$$

$$\frac{\sqrt{36}+5\sqrt{5}}{5\sqrt{6}}$$

$$\frac{6+5\sqrt{5}}{5\sqrt{6}}$$

Rationalize the denominator by multiplying the numerator and denominator by the radical in the denominator.

$$\frac{6+5\sqrt{5}}{5\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{6\sqrt{6}+5\sqrt{30}}{5 \cdot 6}$$

$$\frac{6\sqrt{6}+5\sqrt{30}}{30}$$

■ 5. Simplify the expression, making sure to rationalize the denominator.

$$4\sqrt{\frac{2}{3}} - 7\sqrt{\frac{3}{2}} + \sqrt{96}$$

Solution:

Apply the roots to the numerators and denominators separately, and factor the 96.

$$4\frac{\sqrt{2}}{\sqrt{3}} - 7\frac{\sqrt{3}}{\sqrt{2}} + \sqrt{16 \cdot 6}$$

$$4\frac{\sqrt{2}}{\sqrt{3}}-7\frac{\sqrt{3}}{\sqrt{2}}+\sqrt{16}\sqrt{6}$$

$$\frac{4\sqrt{2}}{\sqrt{3}}-\frac{7\sqrt{3}}{\sqrt{2}}+4\sqrt{6}$$

Find a common denominator by multiplying each fraction by the denominator of the other fraction, then combine the fractions.

$$\frac{4\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{2}}{\sqrt{2}}-\frac{7\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+4\sqrt{6}\cdot\frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{4\cdot 2}{\sqrt{6}}-\frac{7\cdot 3}{\sqrt{6}}+\frac{4\cdot 6}{\sqrt{6}}$$

$$4\frac{2}{\sqrt{6}}-7\frac{3}{\sqrt{6}}+\frac{24}{\sqrt{6}}$$

$$\frac{8}{\sqrt{6}}-\frac{21}{\sqrt{6}}+\frac{24}{\sqrt{6}}$$

$$\frac{11}{\sqrt{6}}$$

Rationalize the denominator by multiplying the numerator and denominator by the radical in the denominator.

$$\frac{11}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{11\sqrt{6}}{6}$$

■ 6. Simplify the expression, making sure to rationalize the denominator.

$$5\sqrt{\frac{5}{7}}+\sqrt{\frac{7}{5}}-\sqrt{140}$$

Solution:

Apply the roots to the numerators and denominators separately, and factor the 140.

$$5\frac{\sqrt{5}}{\sqrt{7}}+\frac{\sqrt{7}}{\sqrt{5}}-\sqrt{14\cdot 10}$$

$$5\frac{\sqrt{5}}{\sqrt{7}}+\frac{\sqrt{7}}{\sqrt{5}}-\sqrt{2\cdot 7\cdot 2\cdot 5}$$

$$5\frac{\sqrt{5}}{\sqrt{7}}+\frac{\sqrt{7}}{\sqrt{5}}-\sqrt{4\cdot 35}$$

$$5\frac{\sqrt{5}}{\sqrt{7}}+\frac{\sqrt{7}}{\sqrt{5}}-\sqrt{4}\sqrt{35}$$

$$\frac{5\sqrt{5}}{\sqrt{7}}+\frac{\sqrt{7}}{\sqrt{5}}-2\sqrt{35}$$

Find a common denominator by multiplying each fraction by the denominator of the other fraction, then combine the fractions.

$$\frac{5\sqrt{5}}{\sqrt{7}}\cdot\frac{\sqrt{5}}{\sqrt{5}}+\frac{\sqrt{7}}{\sqrt{5}}\cdot\frac{\sqrt{7}}{\sqrt{7}}-2\sqrt{35}\cdot\frac{\sqrt{35}}{\sqrt{35}}$$

$$\frac{5\cdot 5}{\sqrt{35}}+\frac{7}{\sqrt{35}}-\frac{2\cdot 35}{\sqrt{35}}$$

$$5\frac{5}{\sqrt{35}}+\frac{7}{\sqrt{35}}-\frac{70}{\sqrt{35}}$$

$$\frac{25}{\sqrt{35}}+\frac{7}{\sqrt{35}}-\frac{70}{\sqrt{35}}$$

$$-\frac{38}{\sqrt{35}}$$

Rationalize the denominator by multiplying the numerator and denominator by the radical in the denominator.

$$-\frac{38}{\sqrt{35}}\cdot\frac{\sqrt{35}}{\sqrt{35}}$$

$$-\frac{38\sqrt{35}}{35}$$

RATIONALIZING WITH CONJUGATE METHOD

■ 1. Simplify the expression.

$$\frac{2-\sqrt{5}}{\sqrt{5}-7}$$

Solution:

Use the conjugate method to rationalize the denominator. The conjugate of $\sqrt{5}-7$ is $\sqrt{5}+7$. We want to multiply both the numerator and the denominator by $\sqrt{5}+7$.

$$\frac{2-\sqrt{5}}{\sqrt{5}-7}\cdot\frac{\sqrt{5}+7}{\sqrt{5}+7}$$

Use the FOIL method (First + Outside + Inside + Last) to multiply the binomial expressions. Using the conjugate method, the two middle terms in the denominator will always cancel, and the radicals in the denominator will be eliminated.

$$\frac{(2-\sqrt{5})(\sqrt{5}+7)}{(\sqrt{5}-7)(\sqrt{5}+7)}$$

$$\frac{2\sqrt{5}+14-5-7\sqrt{5}}{5+7\sqrt{5}-7\sqrt{5}-49}$$

$$\frac{9-5\sqrt{5}}{-44}$$

Multiply both the numerator and denominator by -1 to remove the negative sign from the denominator.

$$\frac{9-5\sqrt{5}}{-44}\cdot\frac{-1}{-1}$$

$$\frac{5\sqrt{5}-9}{44}$$

■ 2. Simplify the expression.

$$\frac{\sqrt{3}+\sqrt{6}}{\sqrt{6}-\sqrt{3}}$$

Solution:

Use the conjugate method to rationalize the denominator. The conjugate of $\sqrt{6}-\sqrt{3}$ is $\sqrt{6}+\sqrt{3}$. We want to multiply both the numerator and the denominator by $\sqrt{6}+\sqrt{3}$.

$$\frac{\sqrt{3}+\sqrt{6}}{\sqrt{6}-\sqrt{3}}\cdot\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$$

Use the FOIL method (First + Outside + Inside + Last) to multiply the binomial expressions. Using the conjugate method, the two middle terms in the denominator will always cancel, and the radicals in the denominator will be eliminated.

$$\frac{(\sqrt{3}+\sqrt{6})(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})}$$

$$\frac{\sqrt{18}+3+6+\sqrt{18}}{6+\sqrt{18}-\sqrt{18}-3}$$

$$\frac{9+2\sqrt{18}}{3}$$

Simplify $\sqrt{18}$.

$$\frac{9+2\cdot 3\sqrt{2}}{3}$$

$$\frac{9+6\sqrt{2}}{3}$$

Reduce by dividing by 3.

$$3+2\sqrt{2}$$

■ 3. Simplify the expression.

$$\frac{8}{4+\sqrt{2}}$$

Solution:

Use the conjugate method to rationalize the denominator. The conjugate of $4+\sqrt{2}$ is $4-\sqrt{2}$. We want to multiply both the numerator and the denominator by $4-\sqrt{2}$.

$$\frac{8}{4+\sqrt{2}}\cdot\frac{4-\sqrt{2}}{4-\sqrt{2}}$$

Use the FOIL method (First + Outside + Inside + Last) to multiply the binomial expressions. Using the conjugate method, the two middle terms in the denominator will always cancel, and the radicals in the denominator will be eliminated.

$$\frac{8(4-\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})}$$

$$\frac{32-8\sqrt{2}}{16-4\sqrt{2}+4\sqrt{2}-2}$$

$$\frac{32-8\sqrt{2}}{14}$$

Reduce by dividing by 2.

$$\frac{16-4\sqrt{2}}{7}$$

■ 4. Simplify the expression.

$$\frac{x+\sqrt{5}}{-5\sqrt{x}+\sqrt{5}}$$

Solution:

Use the conjugate method to rationalize the denominator. The conjugate of $-5\sqrt{x}+\sqrt{5}$ is $-5\sqrt{x}-\sqrt{5}$. We want to multiply both the numerator and the denominator by $-5\sqrt{x}-\sqrt{5}$.

$$\frac{x+\sqrt{5}}{-5\sqrt{x}+\sqrt{5}} \cdot \frac{-5\sqrt{x}-\sqrt{5}}{-5\sqrt{x}-\sqrt{5}}$$

Use the FOIL method (First + Outside + Inside + Last) to multiply the binomial expressions. Using the conjugate method, the two middle terms in the denominator will always cancel, and the radicals in the denominator will be eliminated.

$$\frac{(x+\sqrt{5})(-5\sqrt{x}-\sqrt{5})}{(-5\sqrt{x}+\sqrt{5})(-5\sqrt{x}-\sqrt{5})}$$

$$\frac{-5x\sqrt{x}-x\sqrt{5}-5\sqrt{5x}-5}{25x+5\sqrt{5x}-5\sqrt{5x}-5}$$

$$\frac{-5x\sqrt{x}-x\sqrt{5}-5\sqrt{5x}-5}{25x-5}$$

5. Simplify the expression.

$$\frac{1+\sqrt{y}}{\sqrt{y}+\sqrt{3}}$$

Solution:

Use the conjugate method to rationalize the denominator. The conjugate of $\sqrt{y}+\sqrt{3}$ is $\sqrt{y}-\sqrt{3}$. We want to multiply both the numerator and the denominator by $\sqrt{y}-\sqrt{3}$.

$$\frac{1+\sqrt{y}}{\sqrt{y}+\sqrt{3}} \cdot \frac{\sqrt{y}-\sqrt{3}}{\sqrt{y}-\sqrt{3}}$$

Use the FOIL method (First + Outside + Inside + Last) to multiply the binomial expressions. Using the conjugate method, the two middle terms in the denominator will always cancel, and the radicals in the denominator will be eliminated.

$$\frac{(1+\sqrt{y})(\sqrt{y}-\sqrt{3})}{(\sqrt{y}+\sqrt{3})(\sqrt{y}-\sqrt{3})}$$

$$\frac{\sqrt{y}-\sqrt{3}+y-\sqrt{3y}}{y-\sqrt{3y}+\sqrt{3y}-3}$$

$$\frac{\sqrt{y}-\sqrt{3}+y-\sqrt{3y}}{y-3}$$

■ 6. Simplify the expression.

$$\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}$$

Solution:

Use the conjugate method to rationalize the denominator. The conjugate of $\sqrt{x}+\sqrt{y}$ is $\sqrt{x}-\sqrt{y}$. We want to multiply both the numerator and the denominator by $\sqrt{x}-\sqrt{y}$.

$$\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

Use the FOIL method (First + Outside + Inside + Last) to multiply the binomial expressions. Using the conjugate method, the two middle terms in the denominator will always cancel, and the radicals in the denominator will be eliminated.

$$\frac{\sqrt{x}(\sqrt{x}-\sqrt{y})}{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}$$

$$\frac{x-\sqrt{xy}}{x-\sqrt{xy}+\sqrt{xy}-y}$$

$$\frac{x - \sqrt{xy}}{x - y}$$

W W W . K R I S T A K I N G M A T H . C O M