

Factoring to find a common denominator

A fraction in which the numerator and denominator are polynomials is known as a **rational expression**. In this lesson, we'll learn how to add rational expressions.

To do this, we need to find a common denominator, just like when we add fractions in which the numerator and denominator are just numbers. The difference is that finding the common denominator of rational expressions can be more complicated because their denominators can include variables.

Often we'll need to factor the denominators of rational expressions in order to find a common denominator.

Let's look at an example.

Example

Simplify the expression by combining the two fractions.

$$\frac{3}{x-3} + \frac{9}{x^2 + 2x - 15}$$

In order to add these fractions, we'll need a common denominator. Start by factoring the denominator of the second fraction.

$$\frac{3}{x-3} + \frac{9}{(x+5)(x-3)}$$



Now we can see that the common denominator is $(x + 5)(x - 3)$ and we need to multiply the first rational expression by

$$\frac{x + 5}{x + 5}$$

This is really just multiplying by a well-chosen expression for 1, and therefore doesn't break any rules of math.

$$\frac{3}{x - 3} \cdot \frac{x + 5}{x + 5} + \frac{9}{(x + 5)(x - 3)}$$

$$\frac{3(x + 5)}{(x + 5)(x - 3)} + \frac{9}{(x + 5)(x - 3)}$$

Distribute the 3 in the numerator of the first fraction.

$$\frac{3x + 15}{(x + 5)(x - 3)} + \frac{9}{(x + 5)(x - 3)}$$

Add the numerators, remembering that the denominator will stay the same.

$$\frac{3x + 15 + 9}{(x + 5)(x - 3)}$$

$$\frac{3x + 24}{(x + 5)(x - 3)}$$

In this case we could simplify the top a little by factoring out a 3.

$$\frac{3(x + 8)}{(x + 5)(x - 3)}$$



Let's try another example of factoring to find a common denominator.

Example

Simplify the expression by combining the two fractions.

$$\frac{x-5}{2x^2+x-10} + \frac{4}{2x+5}$$

In order to these fractions, we'll need a common denominator. Start by factoring the denominator of the first fraction.

$$\frac{x-5}{(2x+5)(x-2)} + \frac{4}{2x+5}$$

Now we can see that the common denominator is $(2x+5)(x-2)$ and we need to multiply the second rational expression by

$$\frac{x-2}{x-2}$$

Remember, this is just like multiplying by 1.

$$\frac{x-5}{(2x+5)(x-2)} + \frac{4}{2x+5} \cdot \frac{x-2}{x-2}$$

$$\frac{x-5}{(2x+5)(x-2)} + \frac{4(x-2)}{(2x+5)(x-2)}$$

Distribute the 4 in the numerator of the second fraction.



$$\frac{x - 5}{(2x + 5)(x - 2)} + \frac{4x - 8}{(2x + 5)(x - 2)}$$

Add the numerators, remembering that the denominator will stay the same.

$$\frac{x - 5 + 4x - 8}{(2x + 5)(x - 2)}$$

$$\frac{5x - 13}{(2x + 5)(x - 2)}$$

