Product of functions

Similarly to the way we can add functions to get their sum, we can also multiply functions to get their product.

Multiplying functions

To find the product of functions, we multiply them, making sure to enclose both functions in parentheses and apply the Distributive Property appropriately.

$$(fg)(x) = f(x) \cdot g(x)$$

To find the value of the product at a specific value of x, we can either multiply the functions first and then evaluate the product at that value of x, or we can evaluate each function individually at that value, and then multiply those results.

Let's do an example so that we can see both methods.

Example

Find $(fg)(-4)$ if $f(x) = x + 2$ and $g(x) = x - 5$.

We need to find $(fg)(-4)$, which we could rewrite as $f(-4) \cdot g(-4)$. So we can substitute $x = -4$ into each function and then multiply the results.

First, let's find $f(-4)$.

$$f(x) = x + 2$$

$$f(-4) = -4 + 2$$

$$f(-4) = -2$$

Now let's find $g(-4)$.

$$g(x) = x - 5$$

$$g(-4) = -4 - 5$$

$$g(-4) = -9$$

Then the product $(fg)(-4)$ is

$$(fg)(-4) = f(-4) \cdot g(-4)$$

$$(fg)(-4) = -2 \cdot -9$$

$$(fg)(-4) = 18$$

We also could have multiplied the expressions for the functions, and then substituted $x = -4$ to get the answer.

$$(fg)(x) = (x + 2)(x - 5)$$

$$(fg)(x) = x^2 - 5x + 2x - 10$$

$$(fg)(x) = x^2 - 3x - 10$$

Evaluate the product at $x = -4$.

$$(fg)(-4) = (-4)^2 - 3(-4) - 10$$

$$(fg)(-4) = 16 + 12 - 10$$

$$(fg)(-4) = 28 - 10$$

$$(fg)(-4) = 18$$

Let's try another example where we find the product of functions, but don't evaluate at any specific value.

Example

Find $(gh)(x)$ if $g(x) = x + 6$ and $h(x) = x - 8$.

We need to find $(gh)(x)$ by multiplying the expressions for the functions. Our answer will be a new function instead of a single number, since there's no specific value of x at which we're evaluating.

$$(gh)(x) = (x + 6)(x - 8)$$

$$(gh)(x) = x^2 - 8x + 6x - 48$$

$$(gh)(x) = x^2 - 2x - 48$$
