Greatest common factor

We've learned to apply the Distributive Property to distribute coefficients across polynomials, and even to multiply polynomials. But now we want to work that same process backwards.

In other words, if before we used the Distributive Property to change $3(x+2)$ to $3x+6$, now we want to learn how to change $3x+6$ into $3(x+2)$. This reverse process is called **factoring**, and we say that, from $3x+6$, we "factor out" a 3 to get $3(x+2)$. Factoring is like "un-distributing."

What to "factor out"

In order to factor some value out of a polynomial, all the terms in that polynomial need to contain the factor being taken out. Any factor that's shared by all the terms is a **common factor**, and the factor that consists of everything that's shared by all the terms is the **greatest common factor (GCF)**.

So 3 is a common factor in $6x^3+9x^2+12x$, because 3 can be factored out to get $3(2x^3+3x^2+4x)$. But $3x$ is the greatest common factor, because $3x$ is the most we can factor out of $6x^3+9x^2+12x$. When we do, we get $3x(2x^2+3x+4)$. We know $3x$ is the greatest common factor because there's no remaining common factor among $2x^2$, $3x$, and 4.

The first thing we want to do here is learn to factor the greatest common factor out of a polynomial. If we're having trouble identifying the greatest

common factor, we can always write out all the factors of each term. For example, we could write $2x^3y + 4x^2y^2 + 8xy$ as

$$2x^3y \quad + \quad 4x^2y^2 \quad + \quad 8xy$$

$$2 \cdot x \cdot x \cdot x \cdot y \quad + \quad 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \quad + \quad 2 \cdot 2 \cdot 2 \cdot x \cdot y$$

Now, to identify the greatest common factor, we just need to collect all the factors that are shared by each term. All three terms share one factor of 2,

$$2 \quad (x \cdot x \cdot x \cdot y \quad + \quad 2 \cdot x \cdot x \cdot y \cdot y \quad + \quad 2 \cdot 2 \cdot x \cdot y)$$

one factor of x,

$$2x \quad (x \cdot x \cdot y \quad + \quad 2 \cdot x \cdot y \cdot y \quad + \quad 2 \cdot 2 \cdot y)$$

and one factor of y.

$$2xy \quad (x \cdot x \quad + \quad 2 \cdot x \cdot y \quad + \quad 2 \cdot 2)$$

$$2xy(x^2 + 2xy + 4)$$

We can see that $2xy$ is the greatest common factor, because there are no remaining common factors between x^2, $2xy$, and 4. Remember, even though x^2 and $2xy$ still have a common factor of x, the 4 doesn't share that common factor, so it can't be factored out. Similarly, even though $2xy$ and 4 still have a common factor of 2, the x^2 doesn't share that common factor, so it can't be factored out either.

Let's do another example.

Example

Identify the greatest common factor and factor it out of the polynomial.

$$3x^2 - 15xy$$

Write out all the factors of each term.

$$3 \cdot x \cdot x - 3 \cdot 5 \cdot x \cdot y$$

Both terms share one factor of 3 and one factor of x. Factoring a $3x$ out of each term leaves just x in the first term, and just $5y$ in the second term.

$$3x(x - 5y)$$

Let's try another example with the greatest common factor.

Example

Factor out the greatest common factor.

$$4x^2y - 6x^4y^2 + 8x^3y^4$$

Write out all the factors of each term.

$$2 \cdot 2 \cdot x \cdot x \cdot y - 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y + 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$$

The only factors that are shared by all three terms are a 2, an $x \cdot x$, and a y, so the greatest common factor is $2x^2y$. When we factor out the $2x^2y$, we have to divide each term by $2x^2y$.

$$2x^2y(2 - 3x^2y + 4xy^3)$$

Let's do one more.

Example

Factor the trinomial by pulling out the greatest common factor.

$$4s^4t^3 + 16s^2t^2 - 24st^4$$

Write out all the factors of each term.

$$2 \cdot 2 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t + 2 \cdot 2 \cdot 2 \cdot 2 \cdot s \cdot s \cdot t \cdot t - 2 \cdot 2 \cdot 2 \cdot 3 \cdot s \cdot t \cdot t \cdot t \cdot t$$

The only factors that are shared by all three terms are a $2 \cdot 2$, an s, and a $t \cdot t$. Therefore, the greatest common factor is $4st^2$. Factoring out $4st^2$ gives

$$4st^2(s^3t + 4s - 6t^2)$$