Multiplying rational functions

In this lesson we'll look at how to multiply rational functions, including factoring and cancellation to make the multiplication easier.

As we're canceling factors, we need to remember that numbers that are excluded from the domain of at least one of the rational functions in the product (because they result in the denominator of at least one of the functions being 0) won't be in the domain of the product.

After we cancel factors from the denominator of a product of rational functions, however, it may not be obvious (from the simplified form of the product) that the values which make those factors equal to 0 aren't in the domain of the product. Which is why we should explicitly state the "hidden" values of the variable that are excluded from the domain.

For example, in the product

$$\frac{x+2}{x-1} \cdot \frac{x-1}{x+4}$$

the factor $x-1$ will be canceled, so the simplified version should be written as

$$\frac{x+2}{x+4}, x \neq 1$$

Here we canceled a factor of $x-1$. Since there is no factor $x-1$ in the denominator of the simplified form of the product, it isn't obvious (from the simplified form) that the value of x which makes $x-1$ equal to 0 ($x=1$)

isn't in the domain of the product. This is why we should explicitly state that $x \neq 1$ when we give the simplified form of it.

It'll also help us with these kinds of problems to remember other factoring formulas, like the formulas for the difference and sum of squares:

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

Let's do a couple of examples.

Example

Simplify the expression by combining the two rational functions into a single rational function.

$$\frac{16a^2 - 1}{a^2 - 16} \cdot \frac{a - 4}{4a - 1}$$

To simplify the product

$$\frac{16a^2 - 1}{a^2 - 16} \cdot \frac{a - 4}{4a - 1}$$

we have to factor the top and bottom of the first fraction.

$$\frac{(4a - 1)(4a + 1)}{(a - 4)(a + 4)} \cdot \frac{a - 4}{4a - 1}$$

Cancel the factors that appear in both the numerator and the denominator. $4a - 1$ will cancel from the numerator of the first fraction and the denominator of the second fraction. $a - 4$ will cancel from the denominator of the first fraction and the numerator of the second fraction. So we're left with only

$$\frac{4a+1}{a+4}$$

We canceled a factor $4a - 1$ and a factor $a - 4$, and there's no factor of either of those two types in the denominator of the simplified form of the product. Therefore, it isn't obvious (from the simplified form) that the values of a which makes $4a - 1$ or $a - 4$ equal to 0 aren't in the domain of the product, so we should explicitly state the "hidden" values of a that are excluded from the domain.

To determine those values of a, we solve each of the equations $4a - 1 = 0$ and $a - 4 = 0$.

$$4a - 1 = 0$$

$$4a = 1$$

$$a = \frac{1}{4}$$

and

$$a - 4 = 0$$

$$a = 4$$

So we should write the product as

$$\frac{4a+1}{a+4}, a \neq \frac{1}{4}, 4$$

Let's do another example.

Example

Simplify the expression by combining the two rational functions into a single rational function.

$$\frac{a^2+3a-10}{a^2-7a+12} \cdot \frac{a^2-a-6}{a^2+8a+15}$$

To simplify the product, we have to factor the top and bottom of each fraction.

$$\frac{(a+5)(a-2)}{(a-3)(a-4)} \cdot \frac{(a-3)(a+2)}{(a+5)(a+3)}$$

Cancel the factors that appear in both the numerator and the denominator. We can cancel $a+5$ from the numerator of the first fraction and the denominator of the second fraction. We can cancel $a-3$ from the denominator of the first fraction and the numerator of the second fraction. This leaves us with only

$$\frac{(a-2)}{(a-4)} \cdot \frac{(a+2)}{(a+3)}$$

We canceled a factor $a+5$ and a factor $a-3$, and there's no factor of either of those two types in the denominator of the simplified form of the product. Therefore, it isn't obvious (from the simplified form) that the values of a which makes $a+5$ or $a-3$ equal to 0 aren't in the domain of the product, so we should explicitly state the "hidden" values of a that are excluded from the domain.

To determine those values of a, we solve each of the equations $a+5=0$ and $a-3=0$.

$$a+5=0$$

$$a=-5$$

and

$$a-3=0$$

$$a=3$$

So we should write the product as

$$\frac{(a-2)}{(a-4)}\cdot\frac{(a+2)}{(a+3)},\ a\neq-5,\ 3$$

We can further simplify the product by doing the multiplication in the numerator and the denominator separately.

$$\frac{(a-2)(a+2)}{(a-4)(a+3)},\ a\neq-5,\ 3$$

$$\frac{a^2+2a-2a-4}{a^2+3a-4a-12},\ a\neq-5,\ 3$$

$$\frac{a^2 - 4}{a^2 - a - 12}, a \neq -5, 3$$