Multivariable rational equations

At times we'd like to take an equation that has at least one fraction with a variable in its denominator and write the equation in a different way. We'll call an equation like this an **abstract fractional equation**. This lesson will look at how to do that.

There are a few things we want to remember about rational functions in general.

1. Multiplying a fraction by its reciprocal will always give us a value of 1.

   For example x/y has a reciprocal of y/x because

   $$\frac{x}{y} \cdot \frac{y}{x} = 1$$

   Remember that we can't divide by 0, so we should explicitly state that neither x nor y can be 0.

   $$\frac{x}{y} \cdot \frac{y}{x} = 1, \ x, y \neq 0$$

2. To clear a fraction from an equation, multiply both sides of the equation by the fraction's denominator.

   For example, to clear the b from the fraction in

   $$ax + \frac{m}{b} = c$$

   multiply both sides of the equation by b.

$$ax + \frac{m}{b} = c$$

$$b\left(ax + \frac{m}{b}\right) = b(c)$$

$$b(ax) + b\left(\frac{m}{b}\right) = b(c)$$

$$abx + m = bc$$

Remember that we can't divide by 0, so this new equation is true only if $b \neq 0$.

Let's solve some abstract fractional equations for specific variables.

Example

Solve the equation for n, if $n \neq 0$.

$$\frac{m}{n} + x + ab = c$$

In order to get rid of the fraction, we have to multiply both sides of the equation by the denominator of m/n.

$$\frac{m}{n} + x + ab = c$$

$$n\left(\frac{m}{n} + x + ab\right) = n(c)$$

$$n \cdot \frac{m}{n} + n(x) + n(ab) = n(c)$$

$$m + nx + nab = nc$$

To solve for n, we'll need to collect all terms containing n on one side of the equation, and then factor out the n.

Let's move m to the right side and nc to the left side.

$$nx + nab - nc = -m$$

Now factor out n.

$$n(x + ab - c) = -m$$

Divide both sides by $(x + ab - c)$.

$$n = \frac{-m}{x + ab - c}, \ n \neq 0$$

We could also write this with the negative sign in front.

$$n = -\frac{m}{x + ab - c}, \ n \neq 0$$

Let's try another one.

Example

Solve for x if $x \neq 0$ and $y \neq 0$.

$$\frac{1}{x} - \frac{m}{y} = p$$

In order to get rid of the fractions, we have to multiply both sides of the equation by the denominators of both fractions, x and y.

$$\frac{1}{x} - \frac{m}{y} = p$$

$$xy\left(\frac{1}{x} - \frac{m}{y}\right) = xy(p)$$

$$xy\left(\frac{1}{x}\right) - xy\left(\frac{m}{y}\right) = xy(p)$$

$$y - mx = xyp$$

To solve for x we'll need to collect all terms containing x on one side of the equation, and then factor out the x.

Let's move mx to the right side.

$$y = mx + xyp$$

It's nice to end up with the variable that we're solving for (in this case x) on the left side, so we'll switch the two sides of this equation.

$$mx + xyp = y$$

Now factor out the x.

$$x(m + yp) = y$$

Divide both sides by $m+yp$.

$$\frac{x(m+yp)}{m+yp}=\frac{y}{m+yp}$$

$$x=\frac{y}{m+yp},\ x,\ y\neq 0$$