Probability & Statistics Final Exam Solutions

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Probability & Statistics Final Exam Answer Key

1. (5 pts)	A	B	C	D	■
2. (5 pts)	A	B	■	D	E
3. (5 pts)	A	B	C	■	E
4. (5 pts)	■	B	C	D	E
5. (5 pts)	A	B	■	D	E
6. (5 pts)	A	B	C	■	E
7. (5 pts)	A	■	C	D	E
8. (5 pts)	A	B	■	D	E

9. (15 pts) $y = 5{,}595.31x + 33{,}585.07$

10. (15 pts) Mean $\mu = 81.50$, Median 83.5, Mode 88

11. (15 pts) $72.42\,\%$

12. (15 pts) $21.12\,\%$ chance of rain

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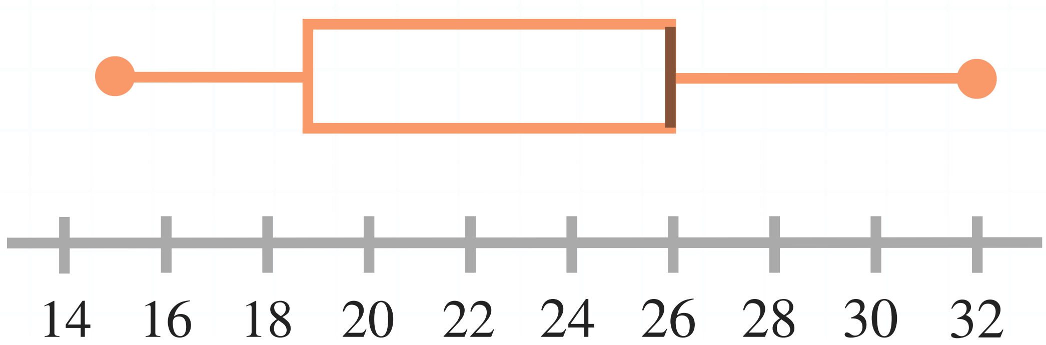
1. E. To determine the number of students surveyed, add all of the values listed in the Venn diagram.

$$8 + 3 + 7 + 10 = 28$$

2. C. First identify the five number summary from the box-and-whisker plot.



The minimum is 15, $Q_1 = 19$, the median is 26, $Q_3 = 26$, and the maximum is 32.

To determine which data set is represented by the graph, sort each set from smallest to largest, and then find the five number summary for each one. Since the median and third quartile have the same value, we're also looking for a data set that has three 26's, since there are 7 data points listed. The correct set is $15,\ 19,\ 19,\ 26,\ 26,\ 26,\ 32$.

3. D. Let X be the trial when we find our first person who plays a brass instrument. X follows a geometric distribution with one trial representing choosing a random student and recording whether or not they play a brass instrument. These trials will be independent, and the probability of success remains constant at $p = 0.27$. There are not a fixed number of trials.

To find the probability that a success S occurs on the nth trial when the probability of success is p, and therefore the probability of failure is $1 - p$, we have

$$P(S = n) = p(1 - p)^{n-1}$$

In this case, we'll set $n = 4$ and get

$$P(S = 4) = 0.27(1 - 0.27)^{4-1}$$

$$P(S = 4) = 0.1050$$

4. A. There are 6 possible scenarios where the sum of the dice is 7.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$P(7) = \frac{6}{36}$$

$$P(7) = \frac{1}{6}$$

5. C. This is a combination problem since the order in which we distribute the prizes doesn't matter. In this case, there are 100 raffle tickets and we choose 3 at a time.

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$_{100}C_3 = \frac{100!}{3!(100-3)!}$$

$$_{100}C_3 = \frac{100 \cdot 99 \cdot 98 \cdot 97!}{3!(97!)}$$

$$_{100}C_3 = \frac{100 \cdot 99 \cdot 98}{3 \cdot 2}$$

$$_{100}C_3 = \frac{970{,}200}{6}$$

$$_{100}C_3 = 161{,}700$$

6. D. We know that $\mu = 40.759$ and $\sigma = 0.55$. Since our competitor wants to finish in the top $3\,\%$, we'll look for the value in the body of the negative z-table that keeps us just under 0.0300. That value is 0.0294,

which gives us a z-score of $z = -1.89$. Using the z-score formula, we then get

$$z = \frac{x - \mu}{\sigma}$$

$$-1.89 = \frac{x - 40.759}{0.55}$$

$$-1.0395 = x - 40.759$$

$$40.759 - 1.0395 = x$$

$$39.720 = x$$

This means that the slowest possible time she can have is 39.720 seconds in order to finish among the top 3% of the kayakers.

7. B. Set up the inequality. Since our margin of error is $\pm 6\%$, we'll put 0.06 on the right side.

$$z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \le 0.06$$

If we want to find the smallest possible sample size n that keeps us within the margin of error, we need to optimize $\hat{p}(1 - \hat{p})$ by making it as large as possible. This happens when $\hat{p} = 0.5$.

$$z^* \sqrt{\frac{0.5(1 - 0.5)}{n}} \le 0.06$$

For a 90% confidence level, we're looking at the middle 90% probability in a normal distribution. This means we'll have 5% probability in each tail, and we're interested in the z-score that puts us at $100\% - 5\% = 95\%$ probability. Using a z-table, we find that 1.65 gets us a probability of $0.9505 \approx 0.9500$.

$$1.65\sqrt{\frac{0.5(1-0.5)}{n}} \le 0.06$$

$$1.65\sqrt{\frac{0.5(0.5)}{n}} \le 0.06$$

$$\frac{\sqrt{0.5^2}}{\sqrt{n}} \le \frac{0.06}{1.65}$$

$$\frac{\sqrt{n}}{0.5} \ge \frac{1.65}{0.06}$$

$$\sqrt{n} \ge \frac{1.65}{0.06}(0.5)$$

$$n \ge \left(\frac{1.65}{0.06}(0.5)\right)^2$$

$$n \ge 189.0625$$

Since we can't take part of a sheep, we round up to $n \ge 190$, and say that we'll need to sample at least 190 sheep in order to estimate the number of lambs in the herd within a 6% margin of error.

8. C. To find standard deviation of a population, start by finding the mean.

$$\mu = \frac{22 + 17 + 29 + 12}{4} = \frac{80}{4} = 20$$

Find the difference between each data measure and the mean. Square the result.

$$22 - 20 = 2 \to 2^2 = 4$$

$$17 - 20 = -3 \to (-3)^2 = 9$$

$$29 - 20 = 9 \to 9^2 = 81$$

$$12 - 20 = -8 \to (-8)^2 = 64$$

Plug these results into the formula for standard deviation.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}}$$

$$\sigma = \sqrt{\frac{4 + 9 + 81 + 64}{4}} = \sqrt{\frac{158}{4}} = \sqrt{39.5}$$

$$\sigma \approx 6.2849$$

9. Find the slope and y-intercept using the correct formulas. It's helpful to create a chart of the values we'll need. We have 7 data points, so $n = 7$.

	x	y	xy	x^2
	0	$34,700	0	0
	1	$38,500	38,500	1
	2	$41,200	82,400	4
	4	$57,250	229,000	16
	6	$68,950	413,700	36
	8	$84,250	661,600	64
	10	$85,250	852,500	100
Sum	**31**	**$408,550**	**2,277,700**	**221**

Now we can plug everything into the formula for m,

$$m=\frac{n(\sum xy)-(\sum x)(\sum y)}{n(\sum x^2)-(\sum x)^2}$$

$$m=\frac{7(2{,}277{,}700)-31(408{,}550)}{7(221)-31^2}$$

$$m=\frac{15{,}943{,}900-12{,}665{,}050}{1{,}547-961}$$

$$m=\frac{3{,}278{,}850}{586}$$

$$m=5{,}595.31$$

and the formula for b.

$$b=\frac{(\sum y)-m(\sum x)}{n}$$

$$b = \frac{408{,}550 - 5{,}595.31(31)}{7}$$

$$b = \frac{235{,}095.48}{7}$$

$$b = 33{,}585.07$$

The line of best fit is therefore given by $y = 5{,}595.31x + 33{,}585.07$.

10. The measures of central tendency are mean, median, and mode. The mean is

$$\mu = \frac{\text{sum of all data points}}{\text{the number of data points}}$$

$$\mu = \frac{2{,}282}{28}$$

$$\mu = 81.50$$

If we line up all the data in ascending order, the two middle numbers are 83 and 84, so the median is their mean.

$$\mu = \frac{83 + 84}{2} = 83.5$$

The mode is the number that occurs the most. In this case, the score 88 occurs 4 times, which is more than any other score.

11. First we need to verify normality. Remember that our sample space should be no more than 10% of our population.

$$\frac{100}{1{,}250} = 0.08 = 8\%$$

This is less than 10%, so we've met the independence condition. We also satisfy the normal condition, because we have an expected number of successes and failures of at least 5.

$$100(0.7) = 70 \geq 5$$

$$100(0.3) = 30 \geq 5$$

The sample space was random, so we've met the conditions of normality. Now we'll find the mean and standard deviation for the sample.

$$\mu_{\hat{p}} = p = 0.7$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.7(0.3)}{100}} \approx 0.0458$$

We need to find the probability that our results are within 5% of the population proportion $p = 70\%$, which means we need to figure out how likely it is that the mean of the sample proportion falls between 65% and 75%. We can express 5% in terms of standard deviations.

$$\frac{0.05}{0.0458} \approx 1.09$$

This means we want to know the probability of $P(-1.09 < z < 1.09)$. Using a z-table, -1.09 gives us 0.1379 and 1.09 gives us 0.8621.

$$P(-1.09 < z < 1.09) = 0.8621 - 0.1379$$

$$P(-1.09 < z < 1.09) = 0.7242$$

There's a 72.42% chance that our sample proportion will fall within 5% of the ice cream shop's claim.

12. Use Bayes' Theorem to find the probability that it will rain during your day hiking the mountain.

$$P(A\,|\,B) = \frac{P(B\,|\,A) \cdot P(A)}{P(B)}$$

We're looking for the probability of rain given that it's already a cloudy morning, so $P(A\,|\,B) = P(\text{rain}\,|\,\text{cloudy})$, and Bayes' theorem is

$$P(\text{rain}\,|\,\text{cloudy}) = \frac{P(\text{cloudy}\,|\,\text{rain}) \cdot P(\text{rain})}{P(\text{cloudy})}$$

Each piece of Bayes' theorem is

$$P(\text{rain}) = 0.16$$

$$P(\text{cloudy}) = 0.50$$

$$P(\text{cloudy}\,|\,\text{rain}) = 0.66$$

So Bayes' theorem tells us that the probability that it will rain given that it's already a cloudy morning is

$$P(\text{rain}\,|\,\text{cloudy}) = \frac{(0.66)(0.16)}{0.50}$$

$$P(\text{rain}\,|\,\text{cloudy}) = \frac{0.1056}{0.50}$$

$$P(\text{rain}\,|\,\text{cloudy}) = 0.2112$$

$$P(\text{rain}\,|\,\text{cloudy}) = 21.12\,\%$$

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