Combinations of functions

In this lesson we'll learn how to combine functions and use combination notation.

Let's say we have two functions, $f(x)$ and $g(x)$. We can find the sum, difference, product, or quotient of f and g, and each of these operations creates a **combination** of the functions. Let's look at each operation and how it's defined.

Sum	$(f+g)(x) = f(x) + g(x)$
Difference	$(f-g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$(f \div g)(x) = \dfrac{f(x)}{g(x)}$

Just as with these same operations on numbers, the order of the functions in addition or multiplication doesn't matter, but the order of the functions in subtraction or division does matter. With subtraction or division of two functions, if the f comes first, then we need to start with $f(x)$; if the g comes first, then we need to start with $g(x)$.

The domain of each of these combinations is given by the intersection of the domain of f and the domain of g. In other words, the combination won't be defined at $x = a$ unless $f(a)$ exists and $g(a)$ exists. And for the quotient in particular to be defined, the denominator can't be 0, which means $g(x)$ can't be 0.

We'll look at composite functions in the next lesson, and the notation for a composite function can look very similar to the notation for a product. Let's look closely here to be sure we see the difference.

Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Composite	$(f \circ g)(x) = f(g(x))$

There's a big difference in meaning between an open circle (for a composite function) and a closed circle (for a product of functions).

Let's do an example of each of the four combination operations.

Example

Find $(f - g)(x)$, if $f(x) = 3x^2 + 2x - 4$ and $g(x) = x^2 - 3x + 2$.

The combination $(f - g)(x)$ means the same thing as $f(x) - g(x)$, so we can find the difference.

$$(f - g)(x) = (3x^2 + 2x - 4) - (x^2 - 3x + 2)$$

$$(f - g)(x) = 3x^2 + 2x - 4 - x^2 + 3x - 2$$

$$(f - g)(x) = 3x^2 - x^2 + 2x + 3x - 4 - 2$$

$$(f - g)(x) = 2x^2 + 5x - 6$$

In the next example, we'll look at the sum of functions.

Example

Find $(f+g)(x)$, if $f(x)=3x-4$ and $g(x)=3x^2+5x+3$.

Remember that $(f+g)(x)$ is the same thing as $f(x)+g(x)$. Therefore,

$$(f+g)(x)=(3x-4)+(3x^2+5x+3)$$

$$(f+g)(x)=3x-4+3x^2+5x+3$$

$$(f+g)(x)=3x^2+3x+5x-4+3$$

$$(f+g)(x)=3x^2+8x-1$$

It's possible to use names other than f and g for the functions in a combination. Let's use some different function names for the product.

Example

Find $(h\cdot m)(x)$, if $h(x)=4x-3$ and $m(x)=-3x^2-1$.

The combination $(h\cdot m)(x)$ is the same as $h(x)\cdot m(x)$. Therefore,

$$(h\cdot m)(x)=(4x-3)(-3x^2-1)$$

We can find this product using the FOIL method.

$$(h \cdot m)(x) = -12x^3 - 4x + 9x^2 + 3$$

$$(h \cdot m)(x) = -12x^3 + 9x^2 - 4x + 3$$

Evaluating combinations

Since we already learned how to find the combination of functions, we can now easily evaluate combinations, and there are two ways to do that.

1. We can find the combination of the functions, and then substitute the value at which we want to evaluate the combination.

2. We can evaluate each function individually, and then find the combination of the results.

Let's try a problem where we find the quotient at a particular value.

Example

Find $(b \div w)(3)$, if $w(x) = 2x$ and $b(x) = 3$.

Remember that $(b \div w)(x)$ is the quotient $b(x)/w(x)$, so we could find the combination,

$$(b \div w)(x) = \frac{3}{2x}$$

and then evaluate it at $x = 3$.

$$(b \div w)(3) = \frac{3}{2(3)} = \frac{1}{2}$$

Alternatively, we could have evaluated each function individually at $x = 3$,

$$w(3) = 2(3) = 6$$

$$b(3) = 3$$

and then found the combination of those results.

$$(b \div w)(3) = \frac{3}{6} = \frac{1}{2}$$