Topic: Graphing exponential functions

Question: Will the graph of the equation have a vertical asymptote or a horizontal asymptote?

$$x = -\left(\frac{1}{2}\right)^{y-6} + 1$$

Answer choices:

A It will have a vertical asymptote at $x = -1$

B It will have a vertical asymptote at $x = 1$

C It will have a horizontal asymptote at $y = -1$

D It will have a horizontal asymptote at $y = 1$

Solution: B

Because the equation expresses x in terms of y, its graph will have a vertical asymptote. To determine what the asymptote is, we can plug both $y=100$ and $y=-100$ into the equation.

For $y=100$:

$$x=-\left(\frac{1}{2}\right)^{100-6}+1$$

$$x=-\left(\frac{1}{2}\right)^{94}+1$$

$$x=-\frac{1^{94}}{2^{94}}+1$$

$$x=-\frac{1}{\text{a very large number}}+1$$

$$x=-0+1$$

$$x=1$$

For $y=-100$:

$$x=-\left(\frac{1}{2}\right)^{-106}+1$$

$$x=-\frac{1}{\left(\frac{1}{2}\right)^{106}}+1$$

$$x = -\frac{1}{\frac{1^{106}}{2^{106}}} + 1$$

$$x = -\frac{1}{\frac{1}{\text{a very large number}}} + 1$$

$$x = -1 \cdot \frac{\text{a very large number}}{1} + 1$$

$$x = -1 \cdot \text{a very large number} + 1$$

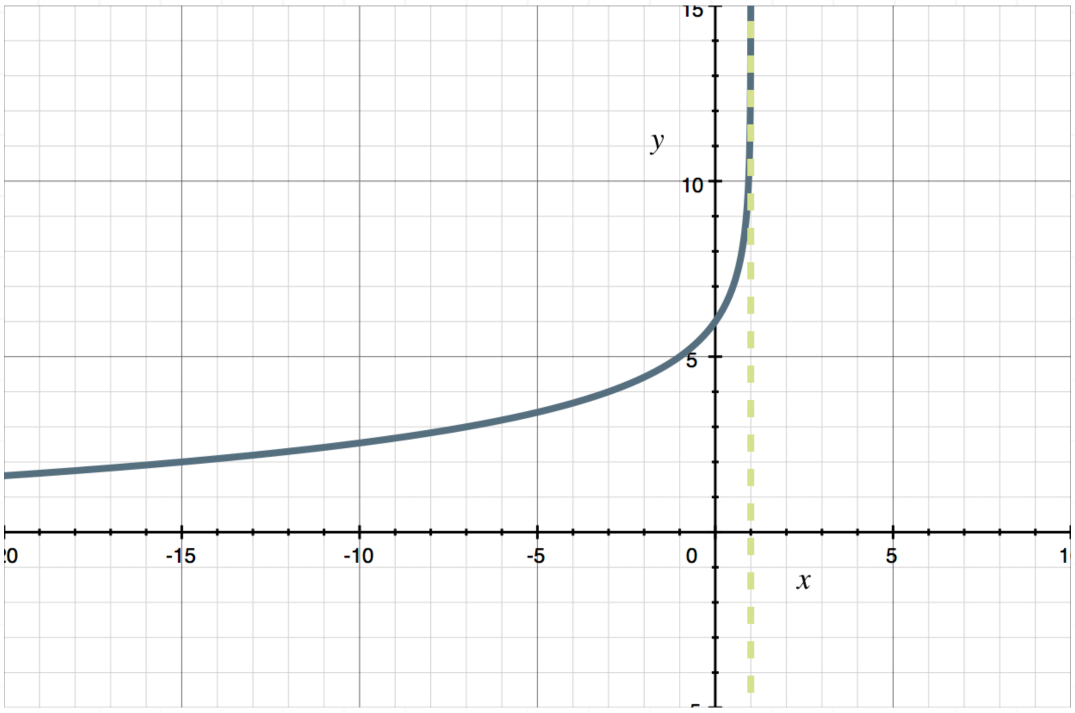
$$x = -\text{a very large number} + 1$$

$$x = -\infty + 1$$

$$x = -\infty$$

Plugging in $y = 100$ and $y = -100$ gives us a picture of the end behavior of the graph of the function. The results tell us that the function has a vertical asymptote at $x = 1$, and that the graph will tend toward $-\infty$ as $x \to -\infty$.

If we continue on to sketch the function, we can see this end behavior.

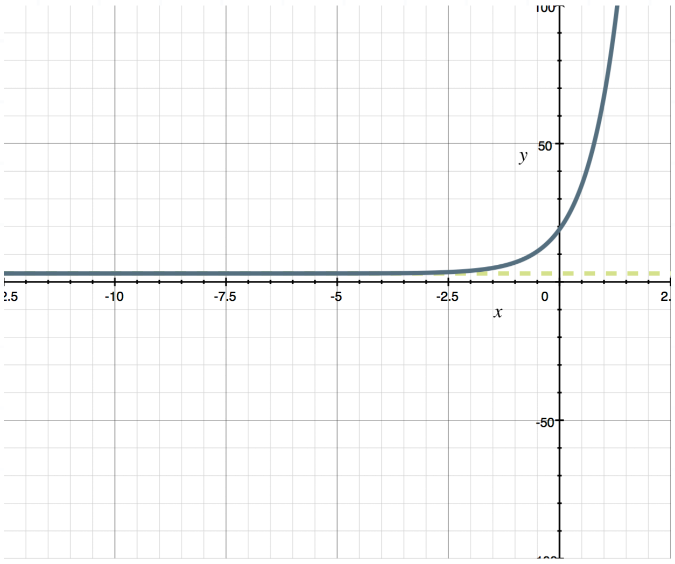

15
10
5
y
-15
-10
-5
0
5
x

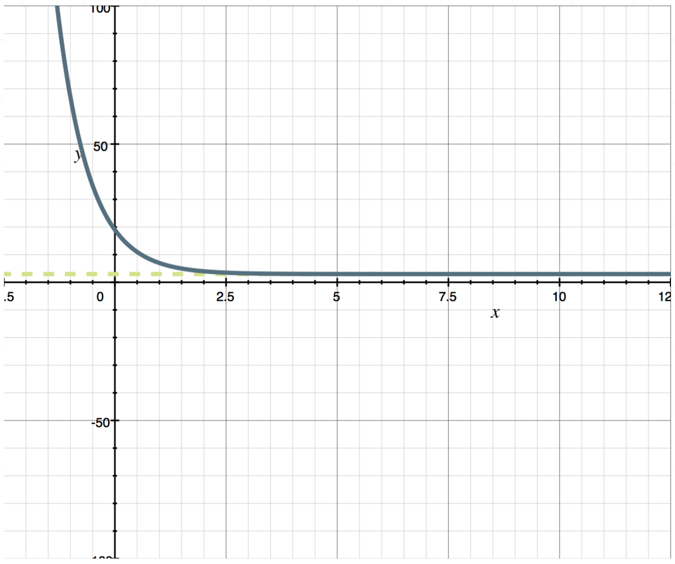
Topic: Graphing exponential functions

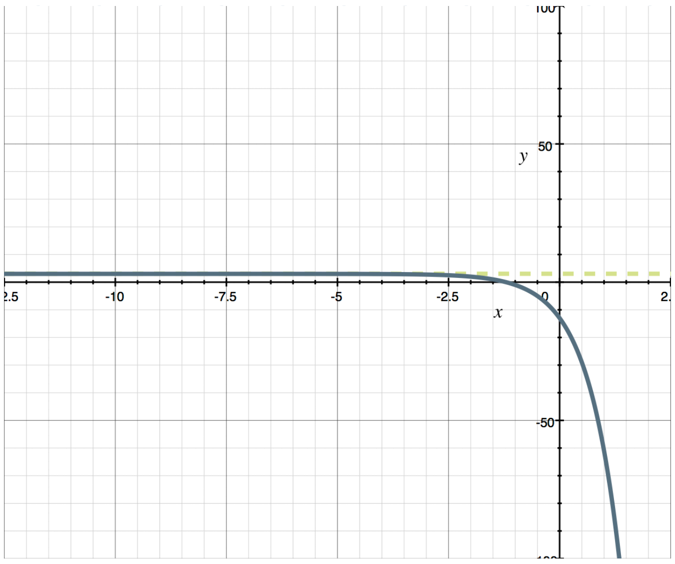
Question: Sketch the graph of the exponential function.

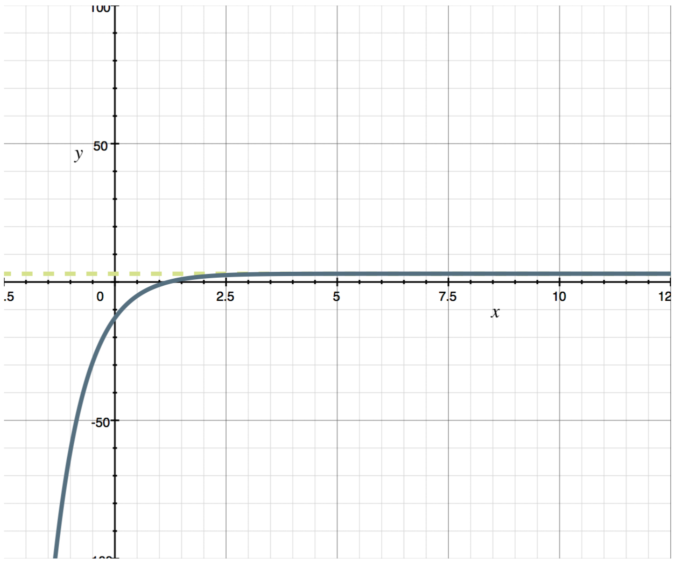
$$f(x) = -4^{2-x} + 3$$

Answer choices:

A


B


C


D


Solution: D

First, plug in $x = 100$ and $x = -100$ to see what the function is doing as x starts getting close to $-\infty$ or $+\infty$.

For $x = 100$:

$$f(100) = -4^{2-100} + 3$$

$$f(100) = -4^{-98} + 3$$

$$f(100) = -\frac{1}{4^{98}} + 3$$

$$f(100) = -\frac{1}{\text{a very large number}} + 3$$

$$f(100) = -0 + 3$$

$$f(100) = 3$$

For $x = -100$:

$$f(-100) = -4^{2+100} + 3$$

$$f(-100) = -4^{102} + 3$$

$$f(-100) = -\text{a very large number} + 3$$

$$f(-100) = -\text{a very large number}$$

$$f(-100) = -\infty$$

Therefore, $y = 3$ will be a horizontal asymptote, and as x tends toward $-\infty$, the function will curl down toward $-\infty$.

We'll plug in a few easy-to-calculate points, like $x = -1, 0, 1$ in order to get a couple of points that we can plot.

For $x = 0$:

$$f(0) = -4^{2-0} + 3$$

$$f(0) = -4^2 + 3$$

$$f(0) = -16 + 3$$

$$f(0) = -13$$

For $x = -1$:

$$f(-1) = -4^{2-(-1)} + 3$$

$$f(-1) = -4^{2+1} + 3$$

$$f(-1) = -4^3 + 3$$

$$f(-1) = -64 + 3$$

$$f(-1) = -61$$

For $x = 1$:

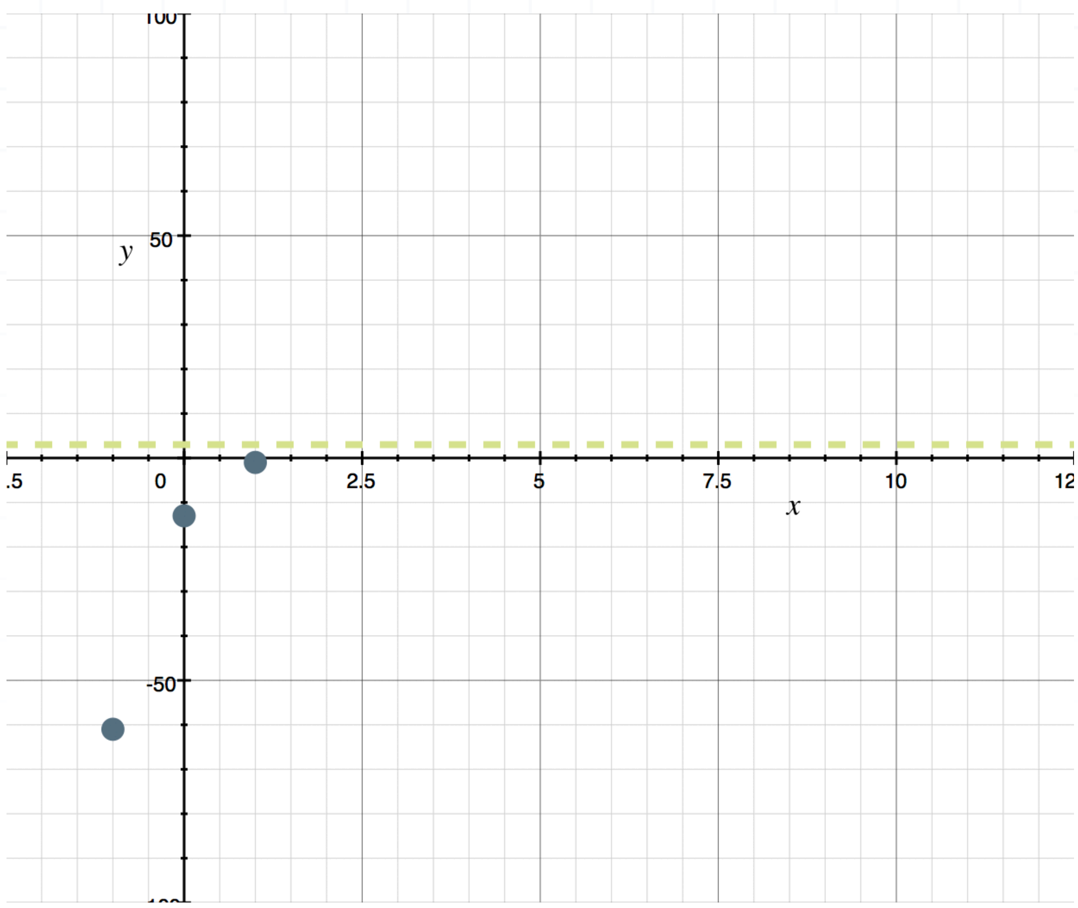
$$f(1) = -4^{2-1} + 3$$

$$f(1) = -4^1 + 3$$

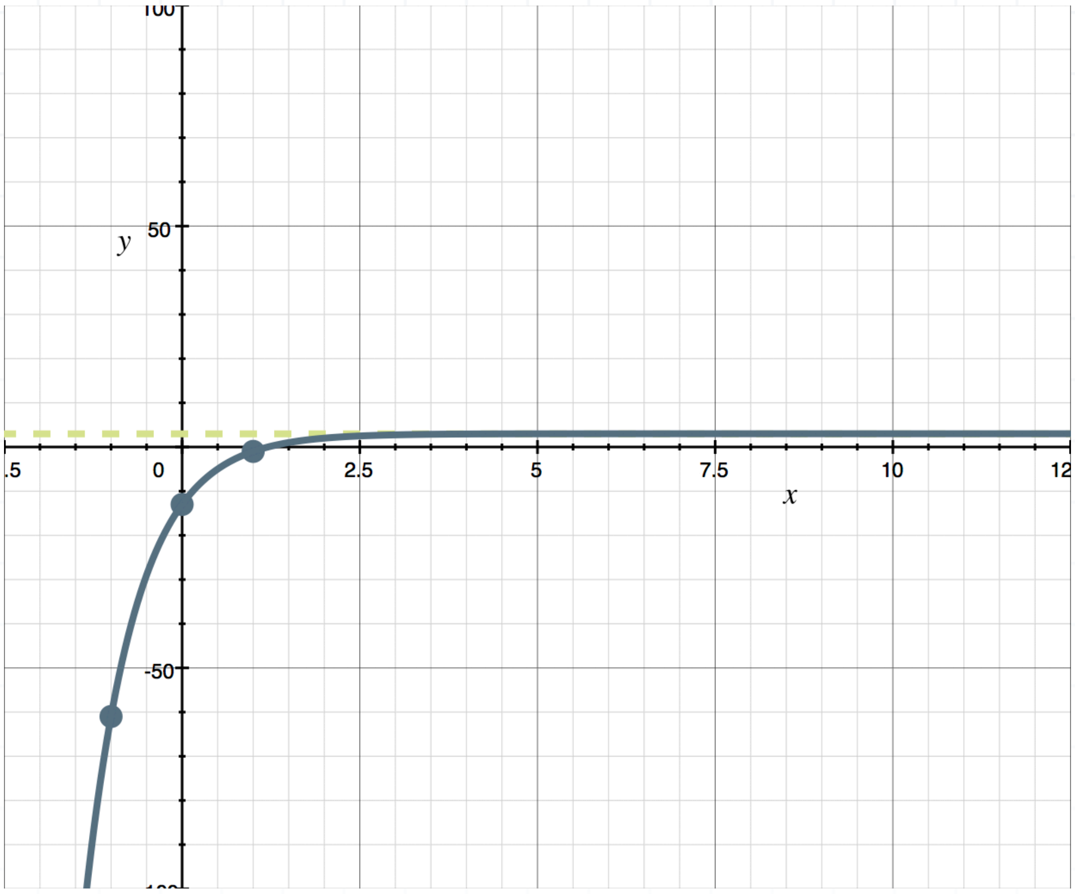
$$f(1) = -4 + 3$$

$$f(1) = -1$$

Now we have three points on the graph of f: $(0, -13)$, $(-1, -61)$, and $(1, -1)$. If we plot these three points and draw the horizontal asymptote $y = 3$, we get



We can see, as we expected, that the exponential function will skim along the horizontal asymptote $y = 3$, and then as $x \to -\infty$, the function's value also heads toward $-\infty$. Connecting the points on the function gives



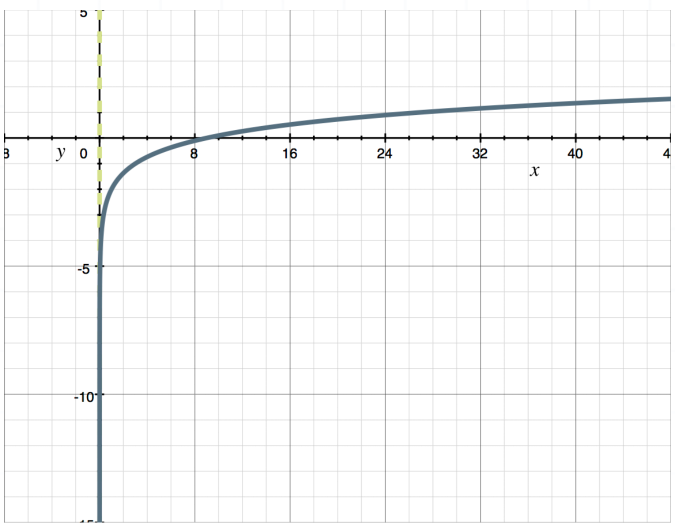
100
50
y
-50
.5
0
2.5
5
7.5
10
12
x

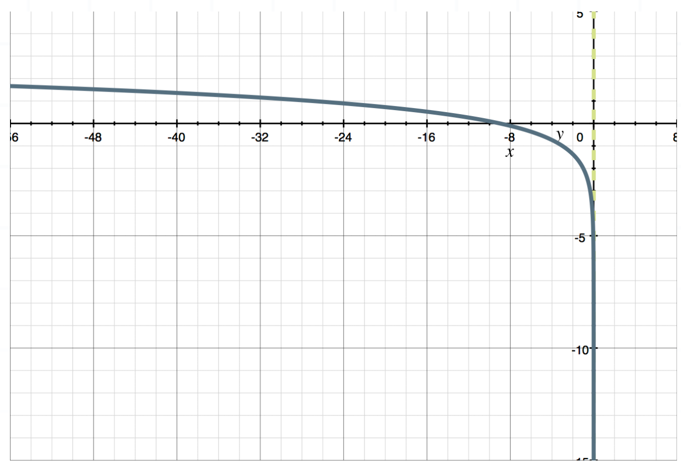
Topic: Graphing exponential functions

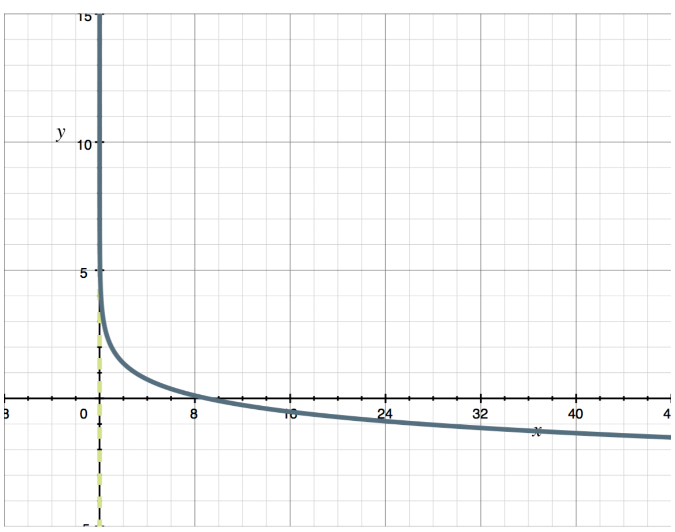
Question: Sketch the graph of the exponential equation.

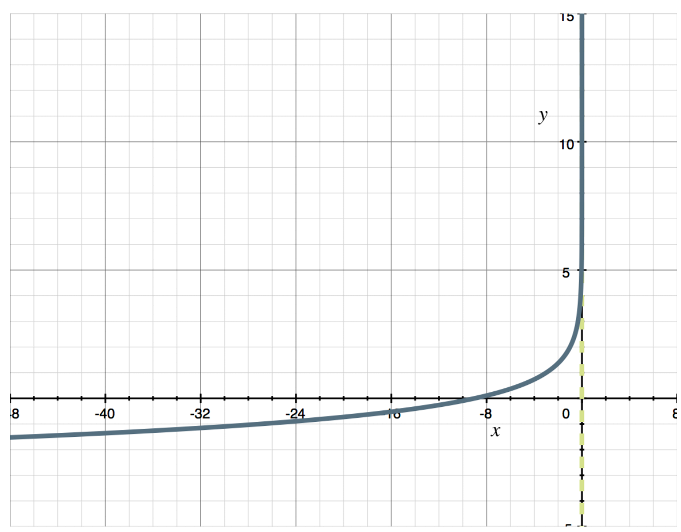
$$x = 3^{y+2}$$

Answer choices:

A


B


C


D


Solution: A

First, plug $y = 100$ and $y = -100$ into the equation to determine what happens to the value of x as $y \to \infty$ and $y \to -\infty$.

For $y = 100$:

$$x = 3^{100+2}$$

$$x = 3^{102}$$

$$x = \text{a very large positive number}$$

$$x = \infty$$

For $y = -100$:

$$x = 3^{-100+2}$$

$$x = 3^{-98}$$

$$x = \frac{1}{3^{98}}$$

$$x = \frac{1}{\text{a very large positive number}}$$

$$x = \text{a very small positive number}$$

$$x = 0$$

Therefore, $x = 0$ will be a vertical asymptote, and as x tends toward ∞, the function will curl up toward ∞.

We'll plug in a few easy-to-calculate points, like $y = -1, 0, 1$ in order to get a couple of points that we can plot.

For $y = 0$:

$$x = 3^{0+2}$$

$$x = 3^2$$

$$x = 9$$

For $y = -1$:

$$x = 3^{-1+2}$$

$$x = 3^1$$

$$x = 3$$

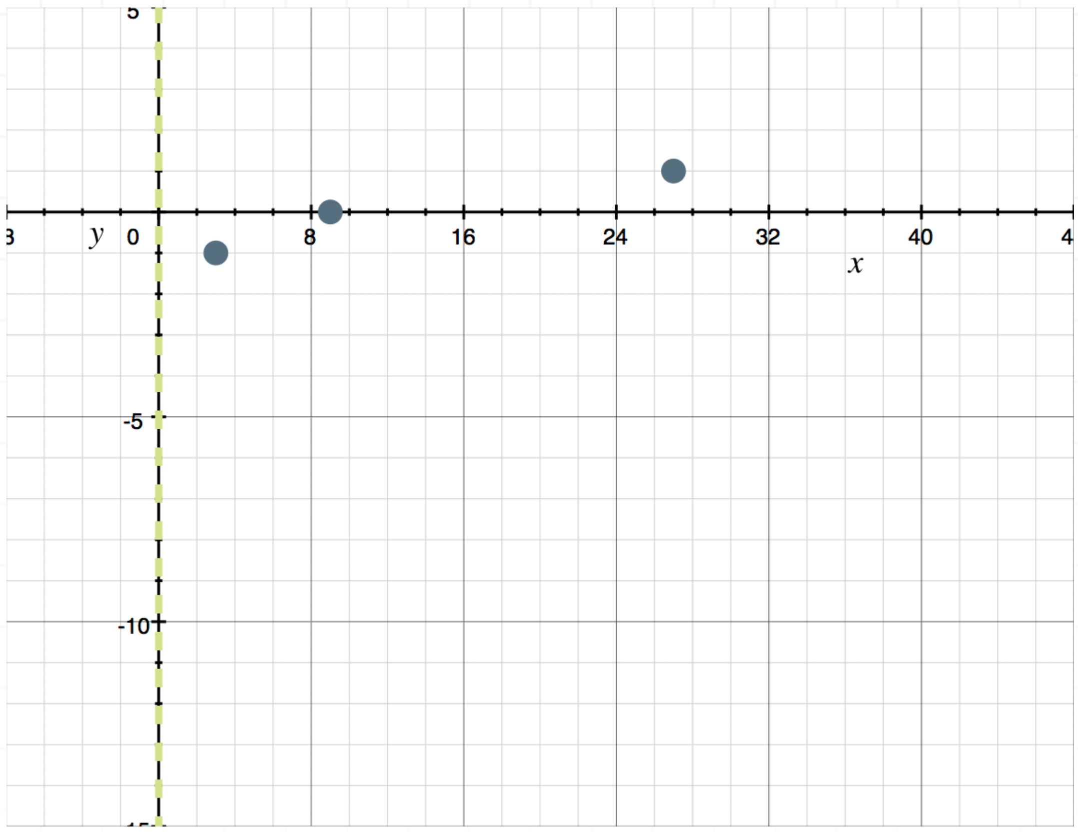
For $y = 1$:

$$x = 3^{1+2}$$

$$x = 3^3$$

$$x = 27$$

Now we have three points on the graph of the given exponential function, $(9,0)$, $(3, -1)$, and $(27,1)$. If we plot these three points and draw the vertical asymptote $x = 0$, we get



We can see, as we expected, that the exponential function will skim along the vertical asymptote $x=0$, and then as $x \to \infty$, the function's value also heads toward ∞. Connecting the points on the function gives