Topic: Multiplying rational functions

Question: Simplify the expression by combining the two rational functions into a single rational function.

$$\frac{4t^2 - 1}{t^2 - 4} \cdot \frac{t - 2}{2t - 1}$$

Answer choices:

A $\quad \dfrac{2t - 1}{t - 2}$

B $\quad \dfrac{2t - 1}{t + 2}$

C $\quad \dfrac{2t + 1}{t - 2}$

D $\quad \dfrac{2t + 1}{t + 2}$

Solution: D

To simplify the product

$$\frac{4t^2 - 1}{t^2 - 4} \cdot \frac{t - 2}{2t - 1}$$

we have to factor the top and bottom of the first fraction.

$$\frac{(2t + 1)(2t - 1)}{(t + 2)(t - 2)} \cdot \frac{t - 2}{2t - 1}$$

Cancel the factors that appear in both the numerator and the denominator.

$$\frac{(2t + 1)\cancel{(2t - 1)}}{(t + 2)\cancel{(t - 2)}} \cdot \frac{\cancel{t - 2}}{\cancel{2t - 1}}$$

$$\frac{2t + 1}{t + 2}$$

We canceled a factor $2t - 1$ and a factor $t - 2$, and there's no factor of either of those two types in the denominator of the simplified form of the product. Therefore, it isn't obvious (from the simplified form) that the values of t which makes $2t - 1$ or $t - 2$ equal to 0 aren't in the domain of the product. To determine those values of t, we solve each of the equations $2t - 1 = 0$ and $t - 2 = 0$.

$$2t - 1 = 0$$

$$2t = 1$$

$$t = \frac{1}{2}$$

and

$$t - 2 = 0$$

$$t = 2$$

So we would usually write the product as

$$\frac{2t + 1}{t + 2}, y \neq \frac{1}{2}, 2$$

Topic: Multiplying rational functions

Question: Simplify the expression by combining the two rational functions into a single rational function.

$$\frac{y^2 + 2y - 15}{y^2 - 6y + 8} \cdot \frac{y^2 - 3y - 4}{y^2 + 6y + 5}$$

Answer choices:

A $\frac{y - 3}{y - 2}$

B $\frac{y + 5}{y + 1}$

C $\frac{y - 2}{y + 1}$

D $\frac{y - 4}{y - 2}$

Solution: A

To simplify the product

$$\frac{y^2+2y-15}{y^2-6y+8}\cdot\frac{y^2-3y-4}{y^2+6y+5}$$

we have to factor the top and bottom of each fraction.

$$\frac{(y-3)(y+5)}{(y-2)(y-4)}\cdot\frac{(y-4)(y+1)}{(y+5)(y+1)}$$

Cancel the factors that appear in both the numerator and the denominator.

$$\frac{(y-3)\cancel{(y+5)}}{(y-2)\cancel{(y-4)}}\cdot\frac{\cancel{(y-4)}\cancel{(y+1)}}{\cancel{(y+5)}\cancel{(y+1)}}$$

$$\frac{y-3}{y-2}$$

We canceled a factor $y+5$, a factor $y-4$, and a factor $y+1$, and there's no factor of any of those three types in the denominator of the simplified form of the product. Therefore, it isn't obvious (from the simplified form) that the values of y which make $y+5$, $y-4$, or $y+1$ equal to 0 aren't in the domain of the product. To determine those values of y, we solve each of the equations $y+5=0$, $y-4=0$, and $y+1=0$.

$$y+5=0$$

$$y=-5$$

and

$$y - 4 = 0$$

$$y = 4$$

and

$$y + 1 = 0$$

$$y = -1$$

So we would usually write the product as

$$\frac{y-3}{y-2}, \ y \neq -5, 4, \ -1$$

Topic: Multiplying rational functions

Question: Simplify the expression by combining the two rational functions into a single rational function.

$$\frac{m+3}{7m+35} \cdot \frac{7m^2-21m-280}{m-8}$$

Answer choices:

A $\frac{m+3}{7}$

B $m+3$

C $\frac{7}{m+5}$

D $m-8$

Solution: B

To simplify the product

$$\frac{m+3}{7m+35}\cdot\frac{7m^2-21m-280}{m-8}$$

we have to factor the bottom of the first fraction and the top of the second fraction.

$$\frac{m+3}{7\cdot(m+5)}\cdot\frac{7\cdot(m^2-3m-40)}{(m-8)}$$

$$\frac{m+3}{7\cdot(m+5)}\cdot\frac{7\cdot(m+5)(m-8)}{(m-8)}$$

Cancel the factors that appear in both the numerator and the denominator. Then we are left with just

$$m+3$$

We canceled a factor $m+5$ and a factor $m-8$, and there's no factor of either of those two types in the denominator of the simplified form of the product. Therefore, it isn't obvious (from the simplified form) that the values of m which make $m+5$ or $m-8$ equal to 0 aren't in the domain of the product. To determine those values of m, we solve each of the equations $m+5=0$ and $m-8=0$.

$$m+5=0$$

$$m=-5$$

and

$$m - 8 = 0$$

$$m = 8$$

So we would usually write the product as

$$m + 3, \ m \neq -5, \ 8$$