Algebra 2
Final Exam Solutions

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MATH

Algebra 2 Final Exam Answer Key

1. (5 pts)	A	B	■	D	E
2. (5 pts)	A	■	C	D	E
3. (5 pts)	■	B	C	D	E
4. (5 pts)	A	B	C	D	■
5. (5 pts)	■	B	C	D	E
6. (5 pts)	A	B	C	■	E
7. (5 pts)	A	B	■	D	E
8. (5 pts)	■	B	C	D	E

9. (15 pts) $(4,6)$ and $(-1,-4)$

10. (15 pts) $\dfrac{-21-i}{13}$

11. (15 pts) 30 miles

12. (15 pts) $f(g(x))=\dfrac{4x}{1+2x}$ with $x\neq -1/2,\ 0$

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1. C. If the woman spends x to make the soap and marked it up by $72\,\%$, then the price she's selling it to customers for is 1.72 times the cost to make it.

$$1.72x = \$18$$

$$\frac{1.72x}{1.72} = \frac{\$18}{1.72}$$

$$x = \$10.47$$

It costs the woman $\$10.47$ to make a 4-pack of soap. Her markup is $72\,\%$, or $\$10.47(0.72) = \$7.53$, and she sells the soap for $\$18$.

2. B. Multiply by the conjugate of the denominator.

$$\frac{4-\sqrt{2}}{\sqrt{2}+3}$$

$$\frac{4-\sqrt{2}}{\sqrt{2}+3} \cdot \frac{\sqrt{2}-3}{\sqrt{2}-3}$$

Use FOIL to multiply the numerators and denominators.

$$\frac{(4-\sqrt{2})(\sqrt{2}-3)}{(\sqrt{2}+3)(\sqrt{2}-3)}$$

$$\frac{4\sqrt{2} - 12 - 2 + 3\sqrt{2}}{2 - 3\sqrt{2} + 3\sqrt{2} - 9}$$

$$\frac{-14 + 7\sqrt{2}}{-7}$$

$$\frac{14 - 7\sqrt{2}}{7}$$

3. A. Square both sides of the equation.

$$\sqrt{x^2 - 10x - 16} = x - 4$$

$$(\sqrt{x^2 - 10x - 16})^2 = (x - 4)^2$$

The square and square root will cancel on the left. Use FOIL to expand the right side of the equation.

$$x^2 - 10x - 16 = x^2 - 4x - 4x + 16$$

$$x^2 - 10x - 16 = x^2 - 8x + 16$$

Solve for x.

$$x^2 - x^2 - 10x - 16 = x^2 - x^2 - 8x + 16$$

$$-10x - 16 = -8x + 16$$

$$-2x - 16 = 16$$

$$-2x = 32$$

$$x = -16$$

4. E. Direct variation is modeled by $x = ky$. If we let w be weight and d be distance, we can write $w = kd$. Plugging the pair $(d, w) = (4,120)$ into the direct variation equation gives

$$120 = k \cdot 4$$

$$k = 30$$

Plugging in the other value of w (135) and $k = 30$ gives

$$135 = 30d$$

$d = 4.5$ inches

5. A. Remember that the standard form of a quadratic expression is $ax^2 + bx + c$. For the equation $8x^2 - 10x + 3$, we identify $a = 8$, $b = -10$, and $c = 3$. Multiply $a \cdot c$ to get $8 \cdot 3 = 24$, then find factors of the result that combine to b. We'll make a table with all of the factors, and their sum.

Factors of 24	Sum
-1, -24	-25
-2, -12	-14
-3, -8	-11
-4, -6	-10

We know $-4+-6=-10$, so they're the factors we're looking for. Now we'll divide each factor by a and reduce the fractions, if possible.

$$\frac{-4}{8}=\frac{-1}{2}$$

One factor of the quadratic is $(2x-1)$ because the denominator of the reduced fraction becomes the coefficient on x. Then we add or subtract the numerator depending on the sign (in this case we'll subtract since -1 was the numerator).

$$\frac{-6}{8}=\frac{-3}{4}$$

The other factor of the quadratic is $(4x-3)$ because the denominator of the reduced fraction becomes the coefficient on x. Then we add or subtract the numerator depending on the sign (in this case we'll subtract since -3 was the numerator).

$$(2x-1)(4x-3)$$

Use FOIL to check your work.

6. D. Substitute $4x+3$ for x into $f(x)$.

$$f(g(x))=(4x+3)^2+4(4x+3)-7$$

$$f(g(x))=(16x^2+24x+9)+(16x+12)-7$$

$$f(g(x))=16x^2+40x+14$$

$$f(g(x)) = 2(8x^2 + 20x + 7)$$

7. C. Use these two rules to evaluate the expression.

$$\log_a x + \log_a y = \log_a xy$$

If $\log_a y = x$, then $a^x = y$.

Applying the first rule to the given expression gives

$$\log_2 \frac{1}{8} + \log_2 16$$

$$\log_2 \left(\frac{1}{8} \cdot 16\right)$$

$$\log_2 2$$

It's probably obvious from this that $\log_2 2 = 1$, but if not, use the second rule above. If we let $x = \log_2 2$, then

$$2^x = 2$$

$$x = 1$$

8. A. Switch x and y in the original equation.

$$y = \frac{x-2}{3}$$

$$x = \frac{y-2}{3}$$

Solve for y.

$$3x = y - 2$$

$$3x + 2 = y$$

9. Use the second equation to solve for y.

$$y - 2x = -2$$

$$y = 2x - 2$$

Plug $y = 2x - 2$ into the first equation and solve for x.

$$y = x^2 - x - 6$$

$$2x - 2 = x^2 - x - 6$$

$$0 = x^2 - 3x - 4$$

$$0 = (x - 4)(x + 1)$$

$x = 4$ and $x = -1$

Plug $x = 4$ into the equation where we've already solved for y.

$$y = 2(4) - 2$$

$$y = 6$$

Plug $x = -1$ into the equation where we've already solved for y.

$$y = 2(-1) - 2$$

$$y = -4$$

The solutions are

$(4,6)$ and $(-1, -4)$

10. Simplify the powers of i by remembering that $i^2 = -1$.

$$\frac{3+5i}{3i^3+2i^6}$$

$$\frac{3+5i}{3(-1)i+2(-1)(-1)(-1)}$$

$$\frac{3+5i}{-3i-2}$$

$$\frac{3+5i}{-2-3i}$$

Use the conjugate method to get the imaginary number out of the denominator.

$$\frac{3+5i}{-2-3i} \cdot \frac{-2+3i}{-2+3i}$$

$$\frac{(3+5i)(-2+3i)}{(-2-3i)(-2+3i)}$$

Use the FOIL method to multiply the binomials in the numerator and denominator.

$$\frac{-6+9i-10i+15i^2}{4-6i+6i-9i^2}$$

$$\frac{-6-i+15i^2}{4-9i^2}$$

Plug in $i^2 = -1$.

$$\frac{-6-i+15(-1)}{4-9(-1)}$$

$$\frac{-21-i}{13}$$

11. We'll use the formula for distance.

Distance $=$ Rate $\times$ Time

$$D = RT$$

Julie's rate is 20 mph, and her time is 1.5 hours. Therefore,

$$\text{Distance } = 20\ \frac{\text{miles}}{\text{hour}} \times 1.5 \text{ hours}$$

$$\text{Distance } = \frac{20 \cdot 1.5 \text{ miles} \cdot \text{hour}}{\text{hour}}$$

$$\text{Distance } = \frac{30 \text{ miles} \cdot \text{hour}}{\text{hour}}$$

Distance $= 30$ miles

12. First, find the domain of $g(x)$. The expression $1/x$ is undefined if the denominator is 0. That means $x = 0$ isn't in the domain of $g(x)$. Therefore, the domain of $g(x)$ is all real numbers x such that $x \neq 0$.

The algebraic expression for the composite function is

$$f(g(x)) = \frac{4}{\frac{1}{x} + 2}$$

$$f(g(x)) = \frac{4}{\frac{1}{x} + 2\left(\frac{x}{x}\right)}$$

$$f(g(x)) = \frac{4}{\frac{1+2x}{x}}$$

$$f(g(x)) = 4 \cdot \frac{x}{1 + 2x}$$

$$f(g(x)) = \frac{4x}{1 + 2x}$$

The domain of a rational functions is all real numbers such that the denominator is not equal to 0.

$$1 + 2x \neq 0$$

$$2x \neq -1$$

$$x \neq -\frac{1}{2}$$

Putting both exclusions together, the domain of the composite function is all real numbers except $-1/2$ and 0, so

$$f(g(x)) = \frac{4x}{1+2x} \text{ with } x \neq -1/2,\ 0$$