Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(x + 3y)(4x^2 + 2xy - 1)$$

Answer choices:

A $4x^3 + 14x^2y + 6xy^2 - x - 3y$

B $2x^3 + 24x^2y - 6xy + x - 3y$

C $4x^3 + 12x^2 + 6xy + x - 3y$

D $2x^4 - 6xy^2 - x + 3y$

Solution: A

When we multiply one polynomial by another, we need to make sure we multiply every term in the first polynomial by every term in the second polynomial.

$$(x + 3y)(4x^2 + 2xy - 1)$$

$$x(4x^2 + 2xy - 1) + 3y(4x^2 + 2xy - 1)$$

$$4x^3 + 2x^2y - x + 12x^2y + 6xy^2 - 3y$$

Rearrange the terms by descending power of x.

$$4x^3 + 2x^2y + 12x^2y + 6xy^2 - x - 3y$$

Group like terms, then combine them.

$$4x^3 + (2 + 12)x^2y + 6xy^2 - x - 3y$$

$$4x^3 + 14x^2y + 6xy^2 - x - 3y$$

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Question: Simplify the expression.

$$(3x + 3y)(x + y) + (x + y)(2x - 2y)$$

Answer choices:

A $5x^2 - 5y^2$

B $6x^2 - 6xy - 6y^2$

C $5x^2 - 5xy - 5y^2$

D $5x^2 + 6xy + y^2$

Solution: D

Use FOIL on each pair of binomials.

$$(3x + 3y)(x + y) + (x + y)(2x - 2y)$$

$$[3x(x) + 3x(y) + 3y(x) + 3y(y)] + [x(2x) + x(-2y) + y(2x) + y(-2y)]$$

$$3x^2 + 3xy + 3xy + 3y^2 + 2x^2 - 2xy + 2xy - 2y^2$$

Rearrange the terms by descending power of x.

$$3x^2 + 2x^2 + 3xy + 3xy - 2xy + 2xy + 3y^2 - 2y^2$$

Group like terms, then combine them.

$$(3 + 2)x^2 + (3 + 3 - 2 + 2)xy + (3 - 2)y^2$$

$$5x^2 + 6xy + y^2$$

Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(a+2b-c)(a-2b+c)$$

Answer choices:

A $a^2-4b^2+4bc-c^2$

B $a^2+4ab-4b^2+4bc-c^2$

C $a^2-4ab-4b^2+2ac-c^2$

D $a^2-4b^2+4ac-c^2$

Solution: A

Distribute each term in the first trinomial across each term in the second trinomial.

$$(a+2b-c)(a-2b+c)$$

$$a(a-2b+c)+2b(a-2b+c)-c(a-2b+c)$$

$$a(a)+a(-2b)+a(c)+2b(a)+2b(-2b)+2b(c)+(-c)(a)+(-c)(-2b)+(-c)(c)$$

$$a^2-2ab+ac+2ab-4b^2+2bc-ac+2bc-c^2$$

Rearrange the terms by descending power of a, then b, then c.

$$a^2-2ab+2ab-4b^2+ac-ac+2bc+2bc-c^2$$

$$a^2+(-2+2)ab-4b^2+(1-1)ac+(2+2)bc-c^2$$

$$a^2-4b^2+4bc-c^2$$