Coefficients in quadratics

In this lesson we'll look at methods for factoring quadratic polynomials in which the coefficient of the x^2 term is neither 1 nor -1.

Factoring means we're taking an expression and rewriting it as parts that are being multiplied together (the factors).

Factoring a quadratic polynomial with coefficients means taking a quadratic polynomial $ax^2 + bx + c$ where a, b, and c are real numbers with $a \neq 1$ and $a \neq -1$, and writing it in the form $(px + r)(qx + s)$ where p, q, r, s are all real numbers.

Let's do a few examples.

Example

Factor the quadratic.

$$3x^2 + 5x - 2$$

Let's begin by looking at the factors of 3 (the coefficient of the x^2 term) and 2 (the absolute value of the constant term). The only factors of 3 are 3 and 1, so we know we'll have

$$(3x \qquad)(x \qquad)$$

The only factors of 2 are 2 and 1, which means we'll have one of the following:

$$(3x \quad 2)(x \quad 1)$$

$$(3x \quad 1)(x \quad 2)$$

We need to determine the signs of the constant terms in the individual factors such that when each constant term is multiplied by the x term in the opposite factor, and then those two products are added, we get the "middle term" (the x term) in the original quadratic polynomial.

Let's see what happens if we do the factoring the first way.

$$(3x \quad 2)(x \quad 1) = 3x^2 \quad 3x \quad 2x \quad 2$$

We need to combine $3x$ and $2x$ in such a way that we get the middle term, $5x$. But remember that in $3x^2 + 5x - 2$, the constant term (-2) is negative, which means that the sign of the constant term in exactly one of the factors has to be negative, so there are only two possibilities:

$$(3x + 2)(x - 1) = 3x^2 - 3x + 2x - 2 = 3x^2 - x - 2$$

$$(3x - 2)(x + 1) = 3x^2 + 3x - 2x - 2 = 3x^2 + x - 2$$

But neither of these is correct, because we don't get $5x$ for the middle term. Let's try doing the factoring the second way.

$$(3x \quad 1)(x \quad 2) = 3x^2 \quad 6x \quad x \quad 2$$

Can we get $5x$ by combining $6x$ and x? Yes, we can.

$$6x - x = 5x$$

Therefore, we have to use 2 as the constant term in the second factor (because $6x = 3x \cdot 2$), and -1 as the constant term in the first factor (because $-x = -1 \cdot x$), so we get

$$(3x - 1)(x + 2)$$

Let's try one more.

Example

Factor the quadratic.

$$15x^2 + 66x - 45$$

First, we'll factor out a 3, because 3 is the factor that's common to all three terms.

$$3(5x^2 + 22x - 15)$$

Now, let's factor $5x^2 + 22x - 15$.

The only factors of 5 are 5 and 1, so we know we'll have

$$(5x \quad)(x \quad)$$

The only pairs of factors of 15 are $(3,5)$ and $(15,1)$. From the pair $(3,5)$, we get two possibilities:

$$(5x \quad 3)(x \quad 5)$$

$$(5x \quad 5)(x \quad 3)$$

From the pair $(15,1)$, we get two possibilities:

$$(5x \quad 15)(x \quad 1)$$

$$(5x \quad 1)(x \quad 15)$$

For each possibility, let's look at the x terms we'll get when we multiply the constant term in each factor by the x term in the opposite factor, to see which possibility has a combination of x terms that will give us the middle term, $22x$.

Since there are so many possibilities, let's use a table to help keep them organized.

Possibility	Polynomial	x terms	Combine to 22x?
(5x 3)(x 5)	5x^2 25x 3x 15	25x and 3x	Yes: 25x-3x=22x
(5x 5)(x 3)	5x^2 15x 5x 15	15x and 5x	No
(5x 15)(x 1)	5x^2 5x 15x 15	5x and 15x	No
(5x 1)(x 15)	5x^2 75x x 15	75x and x	No

So we need to use $(5x \quad 3)(x \quad 5)$ and set it up to get $25x$ and $-3x$. Therefore, we have to use 5 as the constant term in the second factor (because $25x = 5x \cdot 5$), and -3 as the constant term in the first factor (because $-3x = -3 \cdot x$).

$$15x^2 + 66x - 45$$

$$3(5x^2 + 22x - 15)$$

$$3(5x - 3)(x + 5)$$