Systems with subscripts

In math and science, we might encounter a variable with a subscript, but we shouldn't let the subscripts scare us. They're just a way to keep track of variables that could be related to each other in some way.

What does a variable with a subscript look like?

As an example, t_1, t_2, t_3 are all variables with subscripts. They could represent three different measurements of time for the same experiment. We read them as "time 1," "time 2," and "time 3," but it's shorter to write them with the subscripts instead of writing them out.

Even though the variables can be related in some way, that doesn't mean they have the same value. This means that if we're solving systems of equations that have variables with subscripts, we'll need to solve for each variable.

Let's do a few examples so we can get comfortable with the idea.

Example

Use any method to find the unique solution to the system of equations.

$$R_1T_1 = 500$$

$$R_2 = 10R_1$$

$$R_2T_2 = 800$$

$$T_2 = 4 - T_1$$

Let's come up with a plan. We know how to solve a pair of equations in two unknowns, so let's see if we can rewrite the third equation ($R_2T_2 = 800$) as an equation in terms of the variables R_1 and T_1, so we can solve the system that consists of that new equation and the first equation ($R_1T_1 = 500$).

Since the second equation ($R_2 = 10R_1$) is already solved for R_2, and the fourth equation ($T_2 = 4 - T_1$) is already solved for T_2, we can substitute the expressions $10R_1$ and $4 - T_1$ for R_2 and T_2, respectively, in the third equation.

$$R_2T_2 = 800$$

$$(10R_1)(4 - T_1) = 800$$

Use the distributive property.

$$10R_1(4) - 10R_1(T_1) = 800$$

$$40R_1 - 10R_1T_1 = 800$$

We can divide everything by 10 to make it a little easier.

$$4R_1 - R_1T_1 = 80$$

The first equation ($R_1T_1 = 500$) gives the value of R_1T_1 (500). If we substitute 500 for R_1T_1 in the equation we just found ($4R_1 - R_1T_1 = 80$), we have

$$4R_1 - 500 = 80$$

$$4R_1 - 500 + 500 = 80 + 500$$

$$4R_1 = 580$$

$$\frac{4R_1}{4} = \frac{580}{4}$$

$$R_1 = 145$$

We know that $R_2 = 10R_1$, so we get

$$R_2 = 10(145)$$

$$R_2 = 1{,}450$$

We can also use R_1 to find T_1, with the equation $R_1T_1 = 500$.

$$145(T_1) = 500$$

$$\frac{145(T_1)}{145} = \frac{500}{145}$$

$$T_1 = \frac{100}{29}$$

We can use R_2 to find T_2, with the equation $R_2T_2 = 800$.

$$1{,}450(T_2) = 800$$

$$\frac{1{,}450(T_2)}{1{,}450} = \frac{800}{1{,}450}$$

$$T_2 = \frac{16}{29}$$

Collecting all of our results, we get

$$(R_1, T_1) = \left(145, \frac{100}{29}\right)$$

$$(R_2, T_2) = \left(1{,}450, \frac{16}{29}\right)$$

Let's look at a system of two equations with subscripts.

Example

Solve the system of equations for h_t and x_t.

$$h_t = 2x_t - 4$$

$$h_t = \frac{1}{3}x_t + 3$$

Here the expression on the left-hand side of both equations is h_t, so we can equate the expressions on the right-hand side.

$$2x_t - 4 = \frac{1}{3}x_t + 3$$

Let's move the constant terms to the right.

$$2x_t - 4 + 4 = \frac{1}{3}x_t + 3 + 4$$

$$2x_t = \frac{1}{3}x_t + 7$$

Let's move the x_t terms to the left.

$$\frac{6}{3}x_t - \frac{1}{3}x_t = \frac{1}{3}x_t - \frac{1}{3}x_t + 7$$

$$\frac{5}{3}x_t = 7$$

Multiply both sides by $3/5$.

$$\frac{3}{5} \cdot \frac{5}{3}x_t = \frac{3}{5} \cdot 7$$

$$x_t = \frac{21}{5}$$

Now use either equation to solve for h_t. We'll use $h_t = 2x_t - 4$.

$$h_t = 2x_t - 4$$

$$h_t = 2\left(\frac{21}{5}\right) - 4$$

$$h_t = \frac{42}{5} - 4$$

$$h_t = \frac{42}{5} - \frac{20}{5}$$

$$h_t = \frac{22}{5}$$

So we get

$$(x_t, h_t) = \left(\frac{21}{5}, \frac{22}{5}\right)$$

As we can see, if we have subscripts in a system of equations, we can simply use the approach we normally would to solve it.