Algebra 1 Quizzes

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Topic: Variables

Question: Which of these is most likely to be used as a variable?

Answer choices:

A a

B b

C x

D m

Solution: C

We typically use letters at the end of the alphabet for variables, and letters at the beginning of the alphabet for constants, or constant coefficients.

Because x is the only letter from the end of the alphabet, it's more likely than all the other choices to be used as a variable.

Topic: Variables

Question: Identify the terms in the equation.

$$3\sqrt{x} + 6y^2 = ab^2 + 2$$

Answer choices:

A y^2

B $\sqrt{x}$ and y^2

C $6y^2$, ab^2, and 2

D $3\sqrt{x}$, $6y^2$, ab^2, and 2

Solution: D

A term is a single number or variable, or a number and variable multiplied together. Terms are separated by operators like addition and subtraction.

There are two terms on the left side of the equation, $3\sqrt{x}$ and $6y^2$, and two terms on the right side of the equation, ab^2 and 2.

Topic: Variables

Question: Identify the coefficient(s) in the expression.

$$ax^2 + bx + c$$

Answer choices:

A a, b, and c

B ax^2 and bx

C a and b

D c

Solution: C

A coefficient is a constant that multiplies a variable. The constant a is a coefficient on x^2, and b is a coefficient on x.

The value c is a constant, not a coefficient.

Topic: Identifying multiplication

Question: Which of these is multiplication?

Answer choices:

A $3 \times 2 = 6$

B $3 \cdot 2 = 6$

C $(3)(2) = 6$

D All of these

Solution: D

Answer choices A, B, and C all indicate multiplication.

Topic: Identifying multiplication

Question: Simplify the expression.

$$(3 \times 2 \cdot 2)(4) \times 2$$

Answer choices:

A $\quad 48$

B $\quad 96$

C $\quad 24$

D $\quad 12$

Solution: B

We need to identify every operation in this expression as multiplication, because the parentheses, the $\times$ symbol, and the $\cdot$ symbol all indicate multiplication.

So we'll perform one multiplication at a time. Remember that when it comes to multiplication, the order in which we perform the multiplication won't affect the result.

$$(3 \times 2 \cdot 2)(4) \times 2$$

$$(6 \cdot 2)(4) \times 2$$

$$(12)(4) \times 2$$

$$48 \times 2$$

$$96$$

Topic: Identifying multiplication

Question: Simplify the expression.

$$2(-3)(-5)\cdot 2\times 4$$

Answer choices:

A 60

B 240

C -120

D -240

Solution: B

We need to identify every operation in this expression as multiplication, because the parentheses, the $\times$ symbol, and the $\cdot$ symbol all indicate multiplication.

So we'll perform one multiplication at a time. Remember that when it comes to multiplication, the order in which we perform the multiplication won't affect the result.

$$2(-3)(-5)\cdot 2\times 4$$

$$(-6)(-5)\cdot 2\times 4$$

$$30\cdot 2\times 4$$

$$60\times 4$$

$$240$$

Topic: Associative Property

Question: Which of these equations best represents the Associative Property of Addition?

Answer choices:

A $(a+b)+c=a+(b+c)$

B $a+b+c=a+c+b$

C $a+b+c=b+a+c$

D $a(b+c)=ab+ac$

Solution: A

Answer choice A is the Associative Property of Addition, $(a+b)+c=a+(b+c)$. Order doesn't matter when adding three or more numbers. The other answer choices are properties we'll learn about later in this section.

Topic: Associative Property

Question: Which equation is true based on the Associative Property of Multiplication?

Answer choices:

A $(4 \cdot 3) \cdot 2 = 4 \cdot (3 \cdot 2)$

B $4 \cdot 3 \cdot 2 = 4 \cdot 2 \cdot 3$

C $4 \cdot 3 \cdot 2 = 3 \cdot 4 \cdot 2$

D $4(3 + 2) = (4)(3) + (4)(2)$

Solution: A

Answer choice A illustrates the Associative Property of Multiplication, which tells us that, when we're doing multiplication, we can group terms together in any order we'd like, and the result remains the same.

Topic: Associative Property

Question: Which equation shows the Associative Property of Addition?

Answer choices:

A $(x+y)+2z=x+y+2z$

B $x+(y+2z)=(x+(y+2z)$

C $x+y+2z=(x+2z+y)$

D $x+(y+2z)=(x+y)+2z$

Solution: D

The Associative Property has to do with different ways of grouping terms.

Answer choice A shows no grouping on the right, so rule out A.

Answer choice B shows a parenthesis error on the right side: two left parentheses, but only one right parenthesis. Rule out B.

Answer choice C shows no grouping on the left. Also, y and $2z$ are in a different order on the right. Rule out C.

Answer choice D correctly shows grouping one pair of terms, $(y+2z)$, on the left and a different pair of terms, $(x+y)$, on the right.

Topic: Commutative Property

Question: Which of the equations is true based on the Commutative Property?

Answer choices:

A $xm = mx$

B $a + c = c + a$

C $(x + 2)(x + 4) = (x + 4)(x + 2)$

D All of these

Solution: D

If the operation is addition or multiplication, the Commutative Property says that changing the order of the expressions to be added or multiplied doesn't change the result (the sum in the case of addition, or the product in the case of multiplication).

Topic: Commutative Property

Question: Which of the equations is true based on the Commutative Property?

Answer choices:

A $71 = 17$

B $3x + 2x = 2x + 3x$

C $(x + 4)(x - 6) = x^2 - 2x - 24$

D $(mx + b) + c = mx + (b + c)$

Solution: B

Answer choice A is not a true equation, and the Commutative Property doesn't have anything to do with flipping the digits in a number.

Answer choice C illustrates an algebraic property we'll learn later, and answer choice D illustrates the Associative Property.

Answer choice B is the only choice that illustrates the Commutative Property, which says that we can change the order of the expressions being added without changing the value of the sum.

Topic: Commutative Property

Question: Which of the equations is true based on the Commutative Property of Multiplication?

Answer choices:

A $a \cdot b = ab$

B $a \cdot b = b \cdot a$

C $(ab)c = a(bc)$

D $a \cdot b = a \cdot b$

Solution: B

The Commutative Property states that, in an operation, if we change the order of the terms, the new expression remains equal to the original expression.

Answer choice B is the only choice that shows a different order on each side of the equation.

Topic: Transitive Property

Question: What does the Transitive Property tell us about the set of equations?

$$y = 4x - 2$$

$$3z + 6 = y$$

Answer choices:

A $4x - 2 = 3z + 6$

B $4x - 2 = 3z$

C $y = 4x$

D $y = y$

Solution: A

We've been told that $y = 4x - 2$. If we turn this equation around (switch the left-hand side with the right-hand side), we get $4x - 2 = y$.

We've also been told that $3z + 6 = y$. If we turn this equation around, we get $y = 3z + 6$.

$$4x - 2 = y$$

$$y = 3x + 6$$

Since $4x - 2 = y$ and $y = 3x + 6$, the Transitive Property tells us that $4x - 2 = 3z + 6$.

Topic: Transitive Property

Question: What does the Transitive Property tell us about the set of equations?

$$3x = 2y - 5$$

$$7z + 4 = 2y - 5$$

Answer choices:

A $2y - 5 = 7z$

B $4 = 2y - 5$

C $3x = 7z + 4$

D $3x = 4$

Solution: C

We've been told that $7z + 4 = 2y - 5$. If we take this equation and turn it around (switch the left-hand side with the right-hand side), we get $2y - 5 = 7z + 4$. Now we can write the set of equations as

$$3x = 2y - 5$$

$$2y - 5 = 7z + 4$$

Since $3x = 2y - 5$ and $2y - 5 = 7z + 4$, the Transitive Property tells us that $3x = 7z + 4$.

Topic: Transitive Property

Question: Using the Transitive Property, choose the equation that would be the correct conclusion from the pair of equations.

$$w = r$$

$$r = b$$

Answer choices:

A $b = r$

B $b = b$

C $w = b$

D $r = w$

Solution: C

If we match the right side of $w = r$ to the left side of $r = b$, we see that we have matching values of r. If $w = r$ and $r = b$, then $w = b$.

Topic: Understood 1

Question: Choose an expression of equal value.

$$x$$

Answer choices:

A $\dfrac{x}{x}$

B $\dfrac{1x^1}{1}$

C $x \cdot x$

D 1

Solution: B

Answer choice A, x/x, simplifies to 1, which is not equal to x.

Answer choice C, $x \cdot x$, simplifies to x^2, which is not equal to x.

Answer choice D is 1, which is not equal to x.

The only answer choice that's equal to x is answer choice B, which we'll simplify one step at a time.

$$\frac{1x^1}{1}$$

$$\frac{1x}{1}$$

$$\frac{x}{1}$$

$$x$$

Topic: Understood 1

Question: Simplify the expression.

$$1(1x+1^1)-\frac{1}{1(-1x^1)}$$

Answer choices:

A $\quad x+1+\frac{1}{x}$

B $\quad x+1-\frac{1}{x}$

C $\quad 1(x+1)-\frac{1}{1x}$

D $\quad 1(x^1+1)-\frac{1}{1x^1}$

Solution: A

The idea of the "understood 1" is that we can multiply by 1, divide by 1, or raise something to the first power, and none of those operations changes the value.

It's unnecessary to write the 1 in these situations, so we'll simplify this expression one step at a time.

$$1(1x+1^1)-\frac{1}{1(-1x^1)}$$

$$(1x+1^1)-\frac{1}{1(-1x^1)}$$

$$(x+1^1)-\frac{1}{1(-1x^1)}$$

$$(x+1)-\frac{1}{1(-1x^1)}$$

$$(x+1)-\frac{1}{(-1x^1)}$$

$$(x+1)-\frac{1}{(-x^1)}$$

$$(x+1)-\frac{1}{(-x)}$$

$$x+1+\frac{1}{x}$$

Topic: Understood 1

Question: Which of these expressions, if any, would not be equal to xy?

Answer choices:

A $1x^1y^1$

B $\frac{x^1y}{1}$

C $\frac{1xy^1}{1}$

D $1+1x^1y^1$

Solution: D

Any number or variable can be thought of as having an understood 1 as a coefficient, an exponent, or a divisor. Writing the 1 in any of those positions won't change the value.

On the other hand, answer choice D shows adding 1, which would definitely change the value of the expression.

Topic: Adding and subtracting like terms

Question: Simplify the expression.

$$4x + 2x + 8x - 4x$$

Answer choices:

A $8x$

B $4x$

C $2x$

D $10x$

Solution: D

For addition and subtraction, like terms are terms whose bases (variables) are the same and and whose exponents are the same.

In other words, we can add $2x^2$ and $3x^2$, because they both have base x and an exponent of 2, but we can't add $2x^3$ and $3x^4$, because, while they have the same base x, they have different exponents.

Each term in the given expression has base x and an exponent of 1, so we can add all of the coefficients.

$$4x + 2x + 8x - 4x$$

$$(4 + 2 + 8 - 4)x$$

$$(6 + 8 - 4)x$$

$$(14 - 4)x$$

$$10x$$

Topic: Adding and subtracting like terms

Question: Simplify the expression.

$$x + x + 3x + 4x + x$$

Answer choices:

A $8x$

B $7x + x^3$

C $10x$

D $9x$

Solution: C

We can rewrite the expression as

$$x + x + 3x + 4x + x$$

$$1x + 1x + 3x + 4x + 1x$$

$$(1 + 1 + 3 + 4 + 1)x$$

$$(10)x$$

$$10x$$

Topic: Adding and subtracting like terms

Question: Simplify the expression.

$$2x + 7x + 4x^2 + 6x^2 + 10x$$

Answer choices:

A $10x + 9x^2$

B $19x + 10x^2$

C $-19x + 10x^2$

D $19x - 10x^2$

Solution: B

For addition and subtraction, like terms are terms whose bases (variables) are the same and and whose exponents are the same.

In other words, we can add $2x^2$ and $3x^2$, because they both have base x and an exponent of 2, but we can't add $2x^3$ and $3x^4$, because, while they have the same base x, they have different exponents.

So we'll group the terms together in this expression that have a matching base and matching exponent.

$$2x + 7x + 4x^2 + 6x^2 + 10x$$

$$(2x + 7x + 10x) + (4x^2 + 6x^2)$$

Then we'll do the addition.

$$(19x) + (10x^2)$$

$$19x + 10x^2$$

Topic: Multiplying and dividing like terms

Question: Simplify the expression.

$$\frac{x(x+x+m+b)}{x}$$

Answer choices:

A $2x+m+b$

B $x^2+mx+bx$

C $x^3+mx^2+bx^2$

D 1

Solution: A

We could expand the numerator, but before we do that we can cancel the x outside of the parentheses in the numerator with the x in the denominator.

$$\frac{x(x+x+m+b)}{x}$$

$$(x+x+m+b)$$

$$x+x+m+b$$

Now we just add like terms.

$$(1x+1x)+m+b$$

$$2x+m+b$$

Topic: Multiplying and dividing like terms

Question: Simplify the expression.

$$\frac{5x^2 \cdot 3y \cdot 2x}{2xy}$$

Answer choices:

A $30x^2y$

B $30x^4y^2$

C $15x^2y^2$

D $15x^2$

Solution: D

We'll group together the x terms in the numerator,

$$\frac{5x^2 \cdot 3y \cdot 2x}{2xy}$$

$$\frac{(5x^2 \cdot 2x) \cdot 3y}{2xy}$$

and then we'll apply product rule, keeping the base the same, and adding the exponents.

$$\frac{10x^{(2+1)} \cdot 3y}{2xy}$$

$$\frac{10x^3 \cdot 3y}{2xy}$$

$$\frac{30x^3y}{2xy}$$

We apply quotient rule, keeping the base the same, and subtracting the exponents.

$$15x^{3-1}y^{1-1}$$

$$15x^2y^0$$

$$15x^2$$

Topic: Multiplying and dividing like terms

Question: Simplify the expression.

$$\frac{t^3 \cdot t^4 \cdot d^5}{t^5 \cdot d^4}$$

Answer choices:

A $\frac{t^7}{d^4}$

B $t^{12}d^9$

C $\frac{t^2}{d}$

D t^2d

Solution: D

Add the exponents of the t terms in the numerator,

$$\frac{t^3 \cdot t^4 \cdot d^5}{t^5 \cdot d^4}$$

$$\frac{t^{3+4}d^5}{t^5d^4}$$

$$\frac{t^7d^5}{t^5d^4}$$

then subtract the exponents of t and d.

$$t^{7-5}d^{5-4}$$

$$t^2d$$

Topic: Distributive Property

Question: Which of these represents the Distributive Property?

Answer choices:

A $3(x + b) = 3x + b$

B $3(x + b) = 3x + 3b$

C $3(x + b) = 3 + x + b$

D $3(x + b) = x + 3b$

Solution: B

The Distributive Property tells us to multiply the factor outside the parentheses (the number 3) by each of the terms inside the parentheses.

$$3(x+b)$$

$$3(x)+3(b)$$

$$3x+3b$$

Topic: Distributive Property

Question: Use the Distributive Property to expand the expression.

$$\frac{1}{2}(4x+4)$$

Answer choices:

A $2x+2$

B $4x+4$

C $2x$

D $2+x$

Solution: A

The Distributive Property tells us to multiply the factor outside the parentheses (the number $1/2$) by each of the terms inside the parentheses.

$$\frac{1}{2}(4x+4)$$

$$\frac{1}{2}(4x)+\frac{1}{2}(4)$$

$$\frac{4x}{2}+\frac{4}{2}$$

$$2x+2$$

Topic: Distributive Property

Question: Use the Distributive Property to expand the expression.

$$2x(3 + x^2)$$

Answer choices:

A 8

B $8x$

C $6x + 2x$

D $6x + 2x^3$

Solution: D

The Distributive Property tells us to multiply the value outside the parentheses, $2x$, by each of the terms inside the parentheses.

$$2x(3 + x^2)$$

$$2x(3) + 2x(x^2)$$

$$6x + 2x^3$$

Topic: Distributive Property with fractions

Question: Use the Distributive Property to expand the expression.

$$\frac{2x}{yz}\left(\frac{5x}{3y}+\frac{x^2}{z}\right)$$

Answer choices:

A $\frac{10x^2}{3y^2z}+\frac{2x^3}{yz^2}$

B $\frac{10x}{3y^2}+\frac{2x^3}{yz^2}$

C $\frac{10x^2}{3yz}+\frac{2x^3}{yz^2}$

D $\frac{10x^2}{3y^2}+\frac{2x^2}{yz^2}$

Solution: A

Multiply each term in the parentheses by $2x/yz$.

$$\frac{2x}{yz}\left(\frac{5x}{3y}+\frac{x^2}{z}\right)$$

$$\left(\frac{2x}{yz}\cdot\frac{5x}{3y}\right)+\left(\frac{2x}{yz}\cdot\frac{x^2}{z}\right)$$

$$\frac{(2x)(5x)}{(yz)(3y)}+\frac{(2x)(x^2)}{(yz)(z)}$$

$$\frac{10x^2}{3y^2z}+\frac{2x^3}{yz^2}$$

Topic: Distributive Property with fractions

Question: Use the Distributive Property to expand the expression.

$$\left(abx-\frac{b^2x^2}{a^3}\right)\frac{bx^2}{a}$$

Answer choices:

A $\quad bx^2-\frac{b^3x^4}{a^3}$

B $\quad b^2x^2+\frac{b^3x^4}{a^4}$

C $\quad b^2x^3-\frac{b^3x^4}{a^4}$

D $\quad bx^3-\frac{b^2x^2}{a^4}$

Solution: C

Rewrite abx as $abx/1$.

$$\left(abx - \frac{b^2x^2}{a^3}\right)\frac{bx^2}{a}$$

$$\left(\frac{abx}{1} - \frac{b^2x^2}{a^3}\right)\frac{bx^2}{a}$$

Multiply each term in the parentheses by bx^2/a.

$$\left(\frac{bx^2}{a} \cdot \frac{abx}{1}\right) - \left(\frac{bx^2}{a} \cdot \frac{b^2x^2}{a^3}\right)$$

$$\frac{(bx^2)(abx)}{(a)(1)} - \frac{(bx^2)(b^2x^2)}{(a)(a^3)}$$

$$\frac{ab^2x^3}{a} - \frac{b^3x^4}{a^4}$$

Cancel the a from the numerator and denominator of the first term.

$$b^2x^3 - \frac{b^3x^4}{a^4}$$

Topic: Distributive Property with fractions

Question: Use the Distributive Property to expand the expression.

$$\frac{pb}{t^2}\left(\frac{2s}{t}-\frac{pj}{3t^2}+r\right)$$

Answer choices:

A $\frac{2pbs}{t^3}-\frac{p^2bj}{3t^4}-\frac{pbr}{t^2}$

B $\frac{2pbs}{t^3}-\frac{p^2bj}{3t^4}+\frac{pbr}{t^2}$

C $\frac{2pbs}{t^3}-\frac{2pbj}{3t^4}+\frac{pbr}{t^2}$

D $\frac{2pbs}{t^3}-\frac{2pbj}{3t^3}+\frac{pbr}{t^2}$

Solution: B

Rewrite r as $r/1$.

$$\frac{pb}{t^2}\left(\frac{2s}{t}-\frac{pj}{3t^2}+r\right)$$

$$\frac{pb}{t^2}\left(\frac{2s}{t}-\frac{pj}{3t^2}+\frac{r}{1}\right)$$

Multiply the three terms inside the parentheses by pb/t^2.

$$\frac{pb}{t^2}\left(\frac{2s}{t}\right)-\frac{pb}{t^2}\left(\frac{pj}{3t^2}\right)+\frac{pb}{t^2}\left(\frac{r}{1}\right)$$

$$\frac{(pb)(2s)}{(t^2)(t)}-\frac{(pb)(pj)}{(t^2)(3t^2)}+\frac{(pb)(r)}{(t^2)(1)}$$

$$\frac{2pbs}{t^3}-\frac{p^2bj}{3t^4}+\frac{pbr}{t^2}$$

Topic: PEMDAS and order of operations

Question: Simplify the expression.

$$[(5-4)3+2(6-4)]+2$$

Answer choices:

A 10

B 8

C 20

D 9

Solution: D

PEMDAS tells us that we have to perform the operations in the following order.

Parentheses

$$[(5 - 4)3 + 2(6 - 4)] + 2$$

$$[(1)3 + 2(2)] + 2$$

$$[3 + 4] + 2$$

$$[7] + 2$$

$$7 + 2$$

Exponents

Multiplication/Division

Addition/Subtraction

$$9$$

Topic: PEMDAS and order of operations

Question: Simplify the expression.

$$2[(5-2)+6]-(10-3)$$

Answer choices:

A $\quad 10$

B $\quad 11$

C $\quad 13$

D $\quad 2$

Solution: B

PEMDAS tells us that we have to perform the operations in the following order.

Parentheses

$$2[(5-2)+6]-(10-3)$$

$$2[3+6]-(7)$$

$$2[9]-(7)$$

Exponents

Multiplication/Division

$$18-7$$

Addition/Subtraction

$$11$$

Topic: PEMDAS and order of operations

Question: Use the order of operations to simplify the expression.

$$-5 + 3 \cdot 4 - 6 + (2 - 4) - 3^2$$

Answer choices:

A -10

B -11

C 10

D 5

Solution: A

PEMDAS tells us that we have to perform the operations in the following order.

Parentheses

$$-5 + 3 \cdot 4 - 6 + (2 - 4) - 3^2$$

$$-5 + 3 \cdot 4 - 6 + (-2) - 3^2$$

Exponents

$$-5 + 3 \cdot 4 - 6 + (-2) - 9$$

Multiplication/Division

$$-5 + 12 - 6 - 2 - 9$$

Addition/Subtraction

$$7 - 6 - 2 - 9$$

$$1 - 2 - 9$$

$$-1 - 9$$

$$-10$$

Topic: Evaluating expressions

Question: Evaluate the expression if $a = 3$ and $b = -6$.

$$a + b$$

Answer choices:

A -3

B -2

C -6

D -1

Solution: A

We'll plug the given values of a and b into the expression, and then simplify.

$$a+b$$

$$3+(-6)$$

$$3-6$$

$$-3$$

Topic: Evaluating expressions

Question: Evaluate the expression if $x = -1$, $y = 2$, and $z = -3$.

$$xy + y^2 + xyz$$

Answer choices:

A $\quad -2$

B $\quad 8$

C $\quad 3$

D $\quad 1$

Solution: B

We've been given the values of x, y and z, so we'll just plug them into the expression to find its value.

$$xy + y^2 + xyz$$

$$(-1)(2) + 2^2 + (-1)(2)(-3)$$

Simplify the exponent first.

$$(-1)(2) + 4 + (-1)(2)(-3)$$

Do the multiplication from left to right.

$$-2 + 4 + (-1)(2)(-3)$$

$$-2 + 4 + (-2)(-3)$$

$$-2 + 4 + 6$$

Do the addition and subtraction from left to right.

$$2 + 6$$

$$8$$

Topic: Evaluating expressions

Question: Use $a = -2$ and $b = 3$ to evaluate the expression.

$$ab^2 - b(a - b) + a$$

Answer choices:

A -4

B 5

C -3

D -5

Solution: D

We've been given the values of a and b, so we'll just plug them into the expression to find its value.

$$ab^2 - b(a - b) + a$$

$$(-2)(3)^2 - (3)[(-2) - (3)] + (-2)$$

Simplify parentheses first.

$$(-2)(3)^2 - (3)[-2 - 3] + (-2)$$

$$(-2)(3)^2 - (3)[-5] + (-2)$$

Simplify exponents.

$$(-2)(9) - (3)[-5] + (-2)$$

Perform multiplication from left to right.

$$-18 + 15 - 2$$

Perform addition and subtraction from left to right.

$$-5$$

Topic: Inverse operations

Question: Choose the pair of equations that represents inverse operations.

Answer choices:

A $3 \cdot 2 = 6$ and $2 \cdot 3 = 6$

B $3 + 2 = 5$ and $5 + 3 = 8$

C $1 + 3 = 4$ and $4 + 3 = 7$

D $1 + 3 = 4$ and $4 - 3 = 1$

Solution: D

Addition and subtraction are inverse operations of each other, and multiplication and division are inverse operations of each other. The only two equations that show addition and subtraction, or multiplication and division, are the equations in answer choice D.

Inverse operations always let us get back to our starting point. In this case, the first equation in answer choice D starts at 1, then we add 3 to it, and we get to 4. In the second equation, we start at the result of the first equation, 4, then we subtract 3 back out, and we get back to the starting point, 1. This proves that the addition in the first equation and the subtraction in the second equation are inverse operations.

Topic: Inverse operations

Question: Choose the pair of equations that represents inverse operations.

Answer choices:

A $4 \cdot 3 = 12$ and $3 \cdot 4 = 12$

B $4 + 3 = 7$ and $3 + 4 = 7$

C $1 + 3 = 4$ and $4 + 3 = 7$

D $4 + 3 = 7$ and $7 - 3 = 4$

Solution: D

Addition and subtraction are inverse operations of each other, and multiplication and division are inverse operations of each other. The only two equations that show addition and subtraction, or multiplication and division, are the equations in answer choice D.

Inverse operations always let us get back to our starting point. In this case, the first equation in answer choice D starts at 4, then we add 3 to it, and we get to 7. In the second equation, we start at the result of the first equation, 7, then we subtract 3 back out, and we get back to the starting point, 4. This proves that the addition in the first equation and the subtraction in the second equation are inverse operations.

Topic: Inverse operations

Question: Each choice below shows an equation before a change is made and then after the change is made. Three of them are wrong. Which one correctly shows the result of an inverse operation?

Answer choices:

A If $7+3=10$ then $7=10/3$

B If $-8+6=-2$ then $-8=-2+6$

C If $4\cdot5=20$ then $5=20/4$

D If $26/2=13$ then $26=13-2$

Solution: C

For answer choice A, the inverse of $+3$ is -3, so a correct answer would have been,

If $7+3=10$ then $7=10-3$

For answer choice B, the inverse of $+6$ is -6, so a correct answer would have been,

If $-8+6=-2$ then $-8=-2-6$

For answer choice C, the inverse of multiplying by 4 is dividing by 4, and that's exactly what's been done, so answer choice C is the correct answer.

If $4 \cdot 5=20$ then $5=20/4$

For answer choice D, the inverse of dividing by 2 is multiplying by 2, so a correct answer would have been,

If $26/2=13$ then $26=13 \cdot 2$

Topic: Simple equations

Question: Solve for the variable.

$$x + 5 = 10$$

Answer choices:

A $x = 5$

B $x = 10$

C $x = 15$

D $x = 20$

Solution: A

We need to get x by itself on the left side. In order to do that, we'll subtract 5 from both sides to move the 5 to the right side.

$$x + 5 - 5 = 10 - 5$$

$$x + 0 = 5$$

$$x = 5$$

Topic: Simple equations

Question: Solve for the variable.

$$4x + 2 = 10$$

Answer choices:

A $x = 4$

B $x = 2$

C $x = 8$

D $x = 10$

Solution: B

We need to get x by itself on the left side. In order to do that, we'll subtract 2 from both sides to move the 2 to the right side.

$$4x + 2 - 2 = 10 - 2$$

$$4x + 0 = 8$$

$$4x = 8$$

To finish getting x by itself, we have to divide both sides by 4.

$$\frac{4x}{4} = \frac{8}{4}$$

$$1 \cdot x = 2$$

$$x = 2$$

Topic: Simple equations

Question: Solve for the variable.

$$-3x + 4 = 16$$

Answer choices:

A $x = 4$

B $x = -4$

C $x = 12$

D $x = 3$

Solution: B

We need to get x by itself on the left side. In order to do that, we'll subtract 4 from both sides to move the 4 to the right side.

$$-3x + 4 - 4 = 16 - 4$$

$$-3x + 0 = 12$$

$$-3x = 12$$

To finish getting x by itself, we have to divide both sides by -3.

$$\frac{-3x}{-3} = \frac{12}{-3}$$

$$1 \cdot x = -4$$

$$x = -4$$

Topic: Balancing equations

Question: Solve for the variable.

$$3x + 2 = x - 10$$

Answer choices:

A $x = 6$

B $x = -4$

C $x = 4$

D $x = -6$

Solution: D

We need to get the x terms on the same side, and the other terms on the opposite side, making sure that anything we do to one side of the equation, we also do to the other side.

$$3x + 2 = x - 10$$

$$3x + 2 - 2 = x - 10 - 2$$

$$3x = x - 12$$

$$3x - x = x - x - 12$$

$$2x = -12$$

$$\frac{2x}{2} = \frac{-12}{2}$$

$$x = -6$$

Topic: Balancing equations

Question: Solve for the variable.

$$2x + 3x + 5 = x - 10$$

Answer choices:

A $x = \frac{15}{4}$

B $x = 4$

C $x = -3$

D $x = -\frac{15}{4}$

Solution: D

First, we'll collect like terms.

$$2x + 3x + 5 = x - 10$$

$$5x + 5 = x - 10$$

We need to get the x terms on the same side, and the other terms on the opposite side, making sure that anything we do to one side of the equation, we also do to the other side.

$$5x + 5 - 5 = x - 10 - 5$$

$$5x = x - 15$$

$$5x - x = x - x - 15$$

$$4x = -15$$

$$\frac{4x}{4} = -\frac{15}{4}$$

$$x = -\frac{15}{4}$$

Topic: Balancing equations

Question: Solve the equation for m.

$$4m - 2(3m + 2) + 4 = 3(4 - 2m) + 3m$$

Answer choices:

A 1

B 4

C 12

D 20

Solution: C

We'll use the distributive property to get

$$4m - 2(3m + 2) + 4 = 3(4 - 2m) + 3m$$

$$4m - 6m - 4 + 4 = 12 - 6m + 3m$$

Collect like terms.

$$-2m = 12 - 3m$$

$$-2m + 3m = 12 - 3m + 3m$$

$$m = 12$$

Topic: Equations with subscripts

Question: The pressure and volume of a gas are related by $P_1V_1 - P_2V_2 = 0$, where P_1 and V_1 are the original pressure and volume and P_2 and V_2 are the new pressure and volume. If the original pressure is 1.2, the original volume is 150, and the new pressure is 36, what is the new volume?

Answer choices:

A $V_2 = 0.2$

B $V_2 = 0.288$

C $V_2 = 3.47$

D $V_2 = 5$

Solution: D

We know $P_1 = 1.2$, $V_1 = 150$, and $P_2 = 36$. Substituting these values into the equation that relates them gives

$$P_1V_1 - P_2V_2 = 0$$

$$1.2(150) - 36(V_2) = 0$$

$$180 - 36V_2 = 0$$

Use inverse operations to solve for V_2.

$$180 - 36V_2 + 36V_2 = 0 + 36V_2$$

$$180 = 36V_2$$

$$\frac{180}{36} = \frac{36V_2}{36}$$

$$5 = V_2$$

$$V_2 = 5$$

Topic: Equations with subscripts

Question: A car travels at 60 mph for 135 miles, then speeds up and travels at a new constant speed for another 216 miles. If the total time for the trip is 5.25 hours, how fast does the car travel during the second part of the trip? Use d_1 and d_2 as the first distance and the second distance, v_1 and v_2 as the first speed and the second speed, and t as the total time for the trip.

$$\frac{d_1}{v_1} + \frac{d_2}{v_2} = t$$

Answer choices:

A 70 mph

B 72 mph

C 74 mph

D 76 mph

Solution: B

We're starting with $d_1 = 135$, $d_2 = 216$, $v_1 = 60$, and $t = 5.25$, and we need to solve for v_2. Plug these values into the equation that relates them.

$$\frac{d_1}{v_1} + \frac{d_2}{v_2} = t$$

$$\frac{135}{60} + \frac{216}{v_2} = 5.25$$

Use inverse operations to isolate v_2.

$$\frac{135}{60} - \frac{135}{60} + \frac{216}{v_2} = 5.25 - \frac{135}{60}$$

$$\frac{216}{v_2} = 5.25 - 2.25$$

$$\frac{216}{v_2} = 3.00$$

$$\frac{216}{v_2}v_2 = 3.00v_2$$

$$216 = 3.00v_2$$

$$\frac{216}{3.00} = \frac{3.00v_2}{3.00}$$

$$v_2 = 72$$

Topic: Equations with subscripts

Question: A house has three grassy yards. The dimensions of the front yard are $l_f = 50$ ft by $w_f = 22$ ft. The side yard is $l_s = 40$ ft by $w_s = 12$ ft, and the back yard is $l_b = 50$ ft by an unknown w_b. If the total grassy area is $3{,}180$ ft^2, what is the width of the back yard?

$$A = l_f w_f + l_s w_s + l_b w_b$$

Answer choices:

A $\quad 32$ ft

B $\quad 51.2$ ft

C $\quad 76$ ft

D $\quad 95.2$ ft

Solution: A

Plugging everything we've been given into the formula for total area gives

$$A = l_f w_f + l_s w_s + l_b w_b$$

$$3{,}180 = 50(22) + 40(12) + 50(w_b)$$

$$3{,}180 = 1{,}100 + 480 + 50w_b$$

$$3{,}180 = 1{,}580 + 50w_b$$

Use inverse operations to isolate w_b.

$$3{,}180 - 1{,}580 = 1{,}580 - 1{,}580 + 50w_b$$

$$1{,}600 = 50w_b$$

$$\frac{1{,}600}{50} = \frac{50w_b}{50}$$

$$\frac{1{,}600}{50} = w_b$$

$$w_b = 32$$

Topic: Word problems into equations

Question: Write the phrase as an algebraic expression.

"three less than twice x"

Answer choices:

A $3-2x$

B $2x-3$

C $2x+x-3$

D $3-2x\cdot x$

Solution: B

The phrase we need to write with algebra is

"three less than twice x"

We know that "twice x" means "2 times x", and we can write it as $2x$. So now we can say that we're looking for

"three less than $2x$"

Therefore, the expression we're looking for is

$$2x - 3$$

Topic: Word problems into equations

Question: Find the value of the expression.

$$\frac{3}{4} \text{ of } 200$$

Answer choices:

A 170

B 200

C 150

D $\frac{4}{3}$

Solution: C

When we're turning a phrase into a mathematical expression or equation, the word "of" immediately after a fraction tells us to multiply. Therefore,

$$\frac{3}{4}(200)$$

$$\frac{3(200)}{4}$$

$$\frac{600}{4}$$

$$150$$

Topic: Word problems into equations

Question: Four times a number, decreased by 8, is 92. Find the number.

Answer choices:

A -25

B 20

C 100

D 25

Solution: D

When we're turning a phrase into a mathematical expression or equation, the word "times" tells us to multiply. The word "decreased" tells us to subtract, and the word "is" means equals. Therefore, the equation will be

$$4x - 8 = 92$$

Use inverse operations to solve for the number.

$$4x - 8 + 8 = 92 + 8$$

$$4x = 100$$

$$\frac{4x}{4} = \frac{100}{4}$$

$$x = 25$$

Topic: Consecutive integers

Question: Choose the group of consecutive integers.

Answer choices:

A $3, 5, 7$

B $-3, -2, -1$

C $2, 4, 6$

D $5, 10, 15$

Solution: B

Consecutive integers are positive or negative whole numbers that are one unit apart from each other.

Topic: Consecutive integers

Question: Find two consecutive integers that sum to 45.

Answer choices:

A $22,\ 23$

B $21,\ 24$

C $20,\ 25$

D $19,\ 26$

Solution: A

Consecutive integers are positive or negative whole numbers that are one unit apart from each other.

Which means two consecutive integers can be given by x and $x+1$. Therefore, we can set up the equation.

$$x+(x+1)=45$$

$$x+x+1=45$$

$$2x+1=45$$

Use inverse operations to solve for the smaller of the consecutive integers.

$$2x+1-1=45-1$$

$$2x=44$$

$$\frac{2x}{2}=\frac{44}{2}$$

$$x=22$$

With $x=22$, we know $x+1$ is $22+1=23$. So the two consecutive integers are 22 and 23. To double-check, $22+23=45$.

Topic: Consecutive integers

Question: In a string of three consecutive integers, the sum of the first two integers is 10 more than the third integer. What is the third integer?

Answer choices:

A 11

B 13

C 15

D 17

Solution: B

Because the integers are consecutive, it means they are three numbers like $3, 4, 5$ or $7, 8, 9$. Therefore, each integer is one more than the last which means we could represent the three integers as

First integer	x
Second integer	$x+1$
Third integer	$x+2$

The "sum of the first two integers" is,

$$x+(x+1)$$

$$2x+1$$

and "10 more than the third integer" is

$$(x+2)+10$$

$$x+12$$

We've been told that those two quantities are equivalent, so we'll set them equal to one another, and then use inverse operations to solve for x, the first integer.

$$2x+1=x+12$$

$$2x-x+1=x-x+12$$

$$x+1=12$$

$$x + 1 - 1 = 12 - 1$$

$$x = 11$$

The three integers are therefore

First integer	$x = 11$
Second integer	$x + 1 = 11 + 1 = 12$
Third integer	$x + 2 = 11 + 2 = 13$

We were asked for the third integer, and the third integer is 13.

Topic: Adding and subtracting polynomials

Question: Simplify the expression.

$$(9x^2 - 2x) - (5x^2 - 8x - 3)$$

Answer choices:

A $-4x^2 + 5$

B $4x^2 + 6x + 3$

C $12x^2 - 7$

D $14x^2 - 10x + 15$

Solution: B

We have to distribute the subtraction across each term in the second polynomial.

$$(9x^2 - 2x) - (5x^2 - 8x - 3)$$

$$9x^2 - 2x - 5x^2 - (-8x) - (-3)$$

$$9x^2 - 2x - 5x^2 + 8x + 3$$

Now we'll group like terms in in descending order of their exponents, and then combine them by adding their coefficients.

$$9x^2 - 5x^2 - 2x + 8x + 3$$

$$(9 - 5)x^2 + (-2 + 8)x + 3$$

$$4x^2 + 6x + 3$$

Topic: Adding and subtracting polynomials

Question: Simplify the expression.

$$(9x^3 + 4x^2 - 10x - 3) + (-2x^3 + 8x - 7x^2)$$

Answer choices:

A $11x^3 - 11x^2 + 3x - 3$

B $7x^3 - 3x^2 - 2x - 3$

C $7x^3 - 3x^2 + 2x - 3$

D $7x^3 - 11x^2 + 18x - 3$

Solution: B

We'll remove the parentheses,

$$(9x^3 + 4x^2 - 10x - 3) + (-2x^3 + 8x - 7x^2)$$

$$9x^3 + 4x^2 - 10x - 3 - 2x^3 + 8x - 7x^2$$

then group like terms together in descending order of their exponents, and then combine them by adding their coefficients.

$$9x^3 - 2x^3 + 4x^2 - 7x^2 - 10x + 8x - 3$$

$$(9 - 2)x^3 + (4 - 7)x^2 + (-10 + 8)x - 3$$

$$7x^3 - 3x^2 - 2x - 3$$

Topic: Adding and subtracting polynomials

Question: Simplify the expression.

$$(4t^5 + t^3 - 4 + 3t^3 - 7t^2) - (5t^4 + 2t^5 - 2t^3 - 3t - 5)$$

Answer choices:

A $6t^5 - 5t^4 + t^3 - 7t^2 + 3t - 1$

B $6t^5 - 5t^4 + 6t^3 - 7t^2 + 3t + 1$

C $2t^5 - 5t^4 + 6t^3 - 7t^2 - 3t - 9$

D $2t^5 - 5t^4 + 6t^3 - 7t^2 + 3t + 1$

Solution: D

We have to distribute the subtraction across each term in the second polynomial.

$$(4t^5 + t^3 - 4 + 3t^3 - 7t^2) - (5t^4 + 2t^5 - 2t^3 - 3t - 5)$$

$$4t^5 + t^3 - 4 + 3t^3 - 7t^2 - 5t^4 - 2t^5 - (-2t^3) - (-3t) - (-5)$$

$$4t^5 + t^3 - 4 + 3t^3 - 7t^2 - 5t^4 - 2t^5 + 2t^3 + 3t + 5$$

Now we'll group like terms in in descending order of their exponents, and then combine them by adding their coefficients.

$$4t^5 - 2t^5 - 5t^4 + t^3 + 3t^3 + 2t^3 - 7t^2 + 3t - 4 + 5$$

$$(4 - 2)t^5 - 5t^4 + (1 + 3 + 2)t^3 - 7t^2 + 3t + (-4 + 5)$$

$$2t^5 - 5t^4 + 6t^3 - 7t^2 + 3t + 1$$

Topic: Multiplying polynomials

Question: Expand the expression.

$$(x+3)(x+2)$$

Answer choices:

A $2x^2+5x+5$

B x^2+3x+2

C x^2+5x+6

D $x^2+x^2+3x+2x$

Solution: C

We'll use the FOIL method to expand this,

$$(x+3)(x+2)$$

$$(x)(x)+(x)(2)+(3)(x)+(3)(2)$$

and then we'll simplify.

$$x^2+2x+3x+6$$

$$x^2+5x+6$$

Topic: Multiplying polynomials

Question: Expand the expression.

$$2x(x-1)(x+3)(x-6)$$

Answer choices:

A $2x^4+16x^3-30x^2-12x$

B $2x^4-8x^3-30x^2-12x$

C $2x^4+16x^3-30x^2+36x$

D $2x^4-8x^3-30x^2+36x$

Solution: D

We'll start by distributing $2x$ across $(x-1)$.

$$2x(x-1)(x+3)(x-6)$$

$$(2x^2-2x)(x+3)(x-6)$$

Now we'll distribute $(2x^2-2x)$ across $(x+3)$.

$$(2x^3+6x^2-2x^2-6x)(x-6)$$

$$(2x^3+4x^2-6x)(x-6)$$

Finally, we'll distribute $(2x^3+4x^2-6x)$ across $(x-6)$.

$$2x^4-12x^3+4x^3-24x^2-6x^2+36x$$

$$2x^4-8x^3-30x^2+36x$$

Topic: Multiplying polynomials

Question: Simplify the expression.

$$(r-4)(r+3)(2r+5)$$

Answer choices:

A $\quad 2r^3 - 7r^2 - 19r - 60$

B $\quad 2r^3 + 3r^2 - 29r - 60$

C $\quad 2r^3 + 3r^2 - 19r - 60$

D $\quad 2r^3 - 7r^2 - 29r - 60$

Solution: B

We'll use the FOIL method on just the first two terms $(r-4)(r+3)$.

$$(r-4)(r+3)$$

$$r^2+3r-4r-12$$

$$r^2-r-12$$

Now we'll bring in the third binomial and multiply this result by $(2r+5)$.

$$(r^2-r-12)(2r+5)$$

$$r^2(2r)-r(2r)-12(2r)+r^2(5)-r(5)-12(5)$$

$$2r^3-2r^2-24r+5r^2-5r-60$$

$$2r^3+(-2+5)r^2+(-24-5)r-60$$

$$2r^3+3r^2-29r-60$$

Topic: Dividing polynomials

Question: Simplify the expression.

$$(x^2 + x + 8) \div (x - 1)$$

Answer choices:

A $\quad x + \dfrac{8}{x - 1}$

B $\quad x^2 + x + 4$

C $\quad x^2$

D $\quad x + 2 + \dfrac{10}{x - 1}$

Solution: D

We'll use polynomial long division.

$$\begin{array}{r l}
 & \quad x + 2 \quad R\ 10 \\
x-1 & \overline{\big)\, x^2 + x + 8} \\
 & -(x^2 - x) \\
\hline
 & \qquad\quad 2x + 8 \\
 & \qquad -(2x - 2) \\
\hline
 & \qquad\qquad\ \ 10
\end{array}$$

Topic: Dividing polynomials

Question: Simplify the expression.

$$(x^3 + 2x^2 + 12) \div (x - 1)$$

Answer choices:

A $2x^2 + 4x + 4$

B $x^2 + 3x + 3 + \dfrac{15}{x - 1}$

C $x^2 - 3x - 3 + \dfrac{15}{x - 1}$

D $x^2 + 3x - 3 + \dfrac{14}{x - 1}$

Solution: B

We'll use polynomial long division, making sure that we put in a placeholder of $0x$ for the missing term.

$$
\begin{array}{r|l}
 & x^2 \quad +3x \quad +3 \quad R\ 15 \\
\hline
x-1 & x^3+2x^2+0x+12 \\
 & \underline{-(x^3-x^2)} \\
 & \qquad 3x^2+0x \\
 & \qquad \underline{-(3x^2-3x)} \\
 & \qquad\qquad 3x+12 \\
 & \qquad\qquad \underline{-(3x-3)} \\
 & \qquad\qquad\qquad 15
\end{array}
$$

Topic: Dividing polynomials

Question: Find the quotient.

$$\frac{6x^4 - 17x^3 + 13x^2 - 24x + 10}{2x - 5}$$

Answer choices:

A $3x^3 - x^2 + 4x - 2$

B $3x^3 - 2x^2 + 4x - 10$

C $3x^3 - x^2 + 9x - 1$

D $3x^3 - x^2 + 4x - 5$

Solution: A

We'll use polynomial long division.

$$
\begin{array}{r|l}
 & \quad 3x^3 \quad - x^2 \quad + 4x \quad - 2 \\
\hline
2x-5 & 6x^4 - 17x^3 + 13x^2 - 24x + 10 \\
 & \underline{-(6x^4 - 15x^3)} \\
 & \qquad\quad -2x^3 + 13x^2 \\
 & \qquad\quad \underline{-(-2x^3 + 5x^2)} \\
 & \qquad\qquad\qquad 8x^2 - 24x \\
 & \qquad\qquad\quad \underline{-(8x^2 - 20x)} \\
 & \qquad\qquad\qquad\qquad -4x + 10 \\
 & \qquad\qquad\qquad\quad \underline{-(-4x + 10)} \\
 & \qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(x + 3y)(4x^2 + 2xy - 1)$$

Answer choices:

A $\quad 4x^3 + 14x^2y + 6xy^2 - x - 3y$

B $\quad 2x^3 + 24x^2y - 6xy + x - 3y$

C $\quad 4x^3 + 12x^2 + 6xy + x - 3y$

D $\quad 2x^4 - 6xy^2 - x + 3y$

Solution: A

When we multiply one polynomial by another, we need to make sure we multiply every term in the first polynomial by every term in the second polynomial.

$$(x+3y)(4x^2+2xy-1)$$

$$x(4x^2+2xy-1)+3y(4x^2+2xy-1)$$

$$4x^3+2x^2y-x+12x^2y+6xy^2-3y$$

Rearrange the terms by descending power of x.

$$4x^3+2x^2y+12x^2y+6xy^2-x-3y$$

Group like terms, then combine them.

$$4x^3+(2+12)x^2y+6xy^2-x-3y$$

$$4x^3+14x^2y+6xy^2-x-3y$$

Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(3x + 3y)(x + y) + (x + y)(2x - 2y)$$

Answer choices:

A $5x^2 - 5y^2$

B $6x^2 - 6xy - 6y^2$

C $5x^2 - 5xy - 5y^2$

D $5x^2 + 6xy + y^2$

Solution: D

Use FOIL on each pair of binomials.

$$(3x+3y)(x+y)+(x+y)(2x-2y)$$

$$[3x(x)+3x(y)+3y(x)+3y(y)]+[x(2x)+x(-2y)+y(2x)+y(-2y)]$$

$$3x^2+3xy+3xy+3y^2+2x^2-2xy+2xy-2y^2$$

Rearrange the terms by descending power of x.

$$3x^2+2x^2+3xy+3xy-2xy+2xy+3y^2-2y^2$$

Group like terms, then combine them.

$$(3+2)x^2+(3+3-2+2)xy+(3-2)y^2$$

$$5x^2+6xy+y^2$$

Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(a + 2b - c)(a - 2b + c)$$

Answer choices:

A $a^2 - 4b^2 + 4bc - c^2$

B $a^2 + 4ab - 4b^2 + 4bc - c^2$

C $a^2 - 4ab - 4b^2 + 2ac - c^2$

D $a^2 - 4b^2 + 4ac - c^2$

Solution: A

Distribute each term in the first trinomial across each term in the second trinomial.

$$(a+2b-c)(a-2b+c)$$

$$a(a-2b+c)+2b(a-2b+c)-c(a-2b+c)$$

$$a(a)+a(-2b)+a(c)+2b(a)+2b(-2b)+2b(c)+(-c)(a)+(-c)(-2b)+(-c)(c)$$

$$a^2-2ab+ac+2ab-4b^2+2bc-ac+2bc-c^2$$

Rearrange the terms by descending power of a, then b, then c.

$$a^2-2ab+2ab-4b^2+ac-ac+2bc+2bc-c^2$$

$$a^2+(-2+2)ab-4b^2+(1-1)ac+(2+2)bc-c^2$$

$$a^2-4b^2+4bc-c^2$$

Topic: Dividing multivariable polynomials

Question: Find the quotient.

$$\frac{x^3 + y^3}{x + y}$$

Answer choices:

A $x^2 + xy + y^2$

B $x^2 - xy - y^2$

C $x^2 - xy + y^2$

D $x^2 + 2xy + y^2$

Solution: C

If we use long division to find the quotient, we get

$$
\begin{array}{r|l}
 & \quad x^2 \quad - xy \quad + y^2 \\
\hline
x+y & x^3 + 0x^2y + 0xy^2 + y^3 \\
 & -(x^3 + x^2y) \\
\cline{2-2}
 & \qquad -x^2y + 0xy^2 \\
 & \quad -(-x^2y - xy^2) \\
\cline{2-2}
 & \qquad\qquad xy^2 + y^3 \\
 & \qquad -(xy^2 + y^3) \\
\cline{2-2}
 & \qquad\qquad\qquad 0
\end{array}
$$

Topic: Dividing multivariable polynomials

Question: Find the quotient.

$$\frac{2x^3 + 15yx^2 + 24y^2x - 16y^3}{x + 4y}$$

Answer choices:

A $2x^2 + 7xy - 4y^2$

B $2x^2 - 5xy - 4y^2$

C $2x^2 + 3xy - 4y^2$

D $2x^2 - 4xy - 4y^2$

Solution: A

If we use long division to find the quotient, we get

$$
\begin{array}{r|l}
 & \quad 2x^2 + 7xy - 4y^2 \\ \hline
x+4y & \quad 2x^3 + 15x^2y + 24xy^2 - 16y^3 \\
 & -(2x^3 + 8x^2y) \\ \hline
 & \qquad\qquad 7x^2y + 24xy^2 \\
 & \qquad\quad -(7x^2y + 28xy^2) \\ \hline
 & \qquad\qquad\qquad -4xy^2 - 16y^3 \\
 & \qquad\qquad\quad -(-4xy^2 - 16y^3) \\ \hline
 & \qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

Topic: Dividing multivariable polynomials

Question: Find the quotient.

$$\frac{3x^3 - 7x^2y - 7xy^2 + 3y^3}{x - 3y}$$

Answer choices:

A $3x^2 + 2xy - 3y^2$

B $3x^2 - 2xy - 3y^2$

C $3x^2 - 2xy + y^2$

D $3x^2 + 2xy - y^2$

Solution: D

If we use long division to find the quotient, we get

$$
\begin{array}{r|l}
 & 3x^2 + 2xy - y^2 \\
\hline
x-3y & 3x^3 - 7x^2y - 7xy^2 + 3y^3 \\
 & \underline{-(3x^3 - 9x^2y)} \\
 & \qquad\quad 2x^2y - 7xy^2 \\
 & \qquad \underline{-(2x^2y - 6xy^2)} \\
 & \qquad\qquad\qquad -xy^2 + 3y^3 \\
 & \qquad\qquad\quad \underline{-(-xy^2 + 3y^3)} \\
 & \qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

Topic: Greatest common factor

Question: Factor the polynomial.

$$10x^2y^2 - 5xy^3$$

Answer choices:

A $x(2xy - xy^2)$

B $5x^2(y^2 - xy)$

C $5xy^2(2x - y)$

D $y^2(2x^2 - x)$

Solution: C

We can see that each term in the numerator has a factor of 5, a factor of x, and a factor of y^2, so the greatest common factor is $5xy^2$.

$$10x^2y^2 - 5xy^3$$

$$5xy^2(2x - y)$$

Topic: Greatest common factor

Question: Identify the greatest common factor of the polynomial.

$$3s^4t^2v^2 - 6s^3tv + 15s^2t^3v^3$$

Answer choices:

A $3s^2tv$

B $3s^2t^2v$

C $3s^2tv^2$

D $3s^3tv$

Solution: A

We need to look for factors that are shared by the terms $3s^4t^2v^2$, $6s^3tv$, and $15s^2t^3v^3$. We can see that

- 3 is the greatest common factor of 3, 6, and 15
- s^2 is the greatest common factor of s^4, s^3, and s^2
- t is the greatest common factor of t^2, t, and t^3
- v is the greatest common factor of v^2, v, and v^3

Putting these common factors together gives a greatest common factor of $3s^2tv$.

Topic: Greatest common factor

Question: Factor the polynomial.

$$6t^4x - 3t^3x - 45t^2x$$

Answer choices:

A $(3t^2x)(2t^2 - t - 15)$

B $(3t^2x)(2t^2 + t - 15)$

C $(3tx)(2t^4 - 7t^2 - 15)$

D $(3tx)(2t^4 - t^2 - 15)$

Solution: A

We need to look for factors that are shared by the terms $6t^4x$, $3t^3x$, and $45t^2x$. We can see that

- 3 is the largest common integer factor
- t^2 is the largest power of t that's a common factor
- x is the largest power of x that's a common factor

Putting these common factors together gives a greatest common factor of $3t^2x$. Factoring out the $3t^2x$ gives

$$(3t^2x)(2t^2 - t - 15)$$

Topic: Quadratic polynomials

Question: Factor the quadratic.

$$x^2 - x - 42$$

Answer choices:

A $(x+6)(x-7)$

B $(x-6)(x+7)$

C $(x+6)(x+7)$

D $(x-6)(x-7)$

Solution: A

We're looking for a pair of factors of the constant term, -42, which sum to -1. The pairs of factors of -42 are

-1 and 42	1 and -42
-2 and 21	2 and -21
-3 and 14	3 and -14
-6 and 7	6 and -7

The only pair of factors that sum to -1 is the pair 6 and -7. So the quadratic factors as

$$(x+6)(x-7)$$

Topic: Quadratic polynomials

Question: Factor the quadratic.

$$x^2 - 4x - 21$$

Answer choices:

A $(x + 21)(x - 1)$

B $(x + 3)(x - 7)$

C $(x + 7)(x - 3)$

D $(x - 21)(x + 1)$

Solution: B

We're looking for a pair of factors of the constant term, -21, which sum to -4. The pairs of factors of -21 are

-1 and 21 $\qquad$ 1 and -21

-3 and 7 $\qquad$ 3 and -7

The only pair of factors that sum to -4 is the pair 3 and -7. So the quadratic factors as

$$(x+3)(x-7)$$

Topic: Quadratic polynomials

Question: Factor the quadratic.

$$t^2 + t - 20$$

Answer choices:

A $(t - 2)(t + 10)$

B $(t - 5)(t + 4)$

C $(t + 2)(t - 10)$

D $(t - 4)(t + 5)$

Solution: D

We're looking for a pair of factors of the constant term, -20, which sum to 1. The pairs of factors of -20 are

-1 and 20 1 and -20

-2 and 10 2 and -10

-4 and 5 4 and -5

The only pair of factors that sum to 1 is the pair -4 and 5. So the quadratic factors as

$$(t-4)(t+5)$$

Topic: Difference of squares

Question: Factor the binomial.

$$x^2 - 4$$

Answer choices:

A $x + 2$

B $(x + 2)(x - 2)$

C $(x + 2)(x + 2)$

D $(x - 2)(x - 2)$

Solution: B

The binomial is the difference of squares, because

$$x^2 = (x)^2$$

$$4 = (2)^2$$

So the difference of squares can be factored as

$$x^2 - 4$$

$$(x + 2)(x - 2)$$

Topic: Difference of squares

Question: Factor the binomial.

$$81x^2y^2 - 16t^6$$

Answer choices:

A $(9xy + 4t^3)(9xy + 4t^3)$

B $(9xy + 4t^2)(9xy - 4t^4)$

C $(9xy + 4t^3)(9xy - 4t^3)$

D $(9xy + 4t^2)(9xy - 4t^2)$

Solution: C

The binomial is the difference of squares, because

$$81x^2y^2 = (9xy)^2$$

$$16t^6 = (4t^3)^2$$

So the difference of squares can be factored as

$$81x^2y^2 - 16t^6$$

$$(9xy + 4t^3)(9xy - 4t^3)$$

Topic: Difference of squares

Question: Factor the binomial.

$$25r^4z^2 - 225a^4b^{10}$$

Answer choices:

A $(5r^2z + 15a^2b^5)(5r^2z - 15a^2b^5)$

B $(5r^2z - 25a^2b^5)(5r^2z - 25a^2b^5)$

C $(5r^2z + 15a^3b^2)(5r^2z - 15a^1b^5)$

D $(5r^2z + 25a^2b^5)(5r^2z - 25a^2b^5)$

Solution: A

The binomial is the difference of squares, because

$$25r^4z^2 = (5r^2z)^2$$

$$225a^4b^{10} = (15a^2b^5)^2$$

So the difference of squares can be factored as

$$25r^4z^2 - 225a^4b^{10}$$

$$(5r^2z + 15a^2b^5)(5r^2z - 15a^2b^5)$$

Topic: Zero Theorem

Question: Solve the quadratic equation.

$$x^2 + 3x - 4 = 0$$

Answer choices:

A $\quad x = -4, \ -1$

B $\quad x = 1, \ 4$

C $\quad x = -4, \ 1$

D $\quad x = -1, \ 4$

Solution: C

To factor the quadratic, we're looking for factors of -4 that sum to 3.

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

With the equation factored, the Zero Theorem tells us that we can set each factor equal to 0 individually.

$$x + 4 = 0$$

$$x = -4$$

and

$$x - 1 = 0$$

$$x = 1$$

The solutions are $x = -4$ and $x = 1$.

Topic: Zero Theorem

Question: Solve the quadratic equation.

$$x^2 - 5x - 6 = 0$$

Answer choices:

A $x = -2, 3$

B $x = -1, 6$

C $x = -6, 1$

D $x = -3, 2$

Solution: B

To factor the quadratic, we're looking for factors of -6 that sum to -5.

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

With the equation factored, the Zero Theorem tells us that we can set each factor equal to 0 individually.

$$x + 1 = 0$$

$$x = -1$$

and

$$x - 6 = 0$$

$$x = 6$$

The solutions are $x = -1$ and $x = 6$.

Topic: Zero Theorem

Question: Solve the quadratic equation.

$$x^2 - 18 = 3x$$

Answer choices:

A $x = -6, 3$

B $x = -3, 6$

C $x = -8, 4$

D $x = 4, 9$

Solution: B

Before we can factor the quadratic, we need to move $3x$ to the other side of the equation by subtracting $3x$ from both sides.

$$x^2 - 18 = 3x$$

$$x^2 - 3x - 18 = 3x - 3x$$

$$x^2 - 3x - 18 = 0$$

To factor the quadratic, we're looking for factors of -18 that sum to -3.

$$(x - 6)(x + 3) = 0$$

With the equation factored, the Zero Theorem tells us that we can set each factor equal to 0 individually.

$$x + 3 = 0$$

$$x = -3$$

and

$$x - 6 = 0$$

$$x = 6$$

The solutions are $x = -3$ and $x = 6$.

Topic: Completing the square

Question: Complete the square to solve the quadratic equation.

$$x^2 + 4x + 2 = 0$$

Answer choices:

A $\quad x = -2 \pm \sqrt{2}$

B $\quad x = 2 \pm \sqrt{2}$

C $\quad x = -2 \pm \sqrt{3}$

D $\quad x = 2 \pm \sqrt{3}$

Solution: A

The quadratic is given in standard form $ax^2 + bx + c$ with $a = 1$, so we can complete the square. We'll start by finding $(b/2)^2$,

$$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$$

then we'll add $(b/2)^2$ to both sides of the equation.

$$x^2 + 4x + 4 + 2 = 0 + 4$$

$$(x^2 + 4x + 4) + 2 = 4$$

$$x^2 + 4x + 4 = 2$$

The quadratic on the left factors as the perfect square $(x + 2)^2$.

$$(x + 2)(x + 2) = 2$$

$$(x + 2)^2 = 2$$

$$x + 2 = \pm\sqrt{2}$$

$$x = -2 \pm \sqrt{2}$$

The roots of the equation are therefore $x = -2 - \sqrt{2}$ and $x = -2 + \sqrt{2}$.

Topic: Completing the square

Question: Complete the square to find the roots of the quadratic.

$$u^2 - 4u + 3 = 0$$

Answer choices:

A $u = -1, \ -3$

B $u = 1, \ -3$

C $u = 1, \ 3$

D $u = -1, \ 3$

Solution: C

The quadratic is given in standard form $au^2 + bu + c$ with $a = 1$, so we can complete the square. We'll start by finding $(b/2)^2$,

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

then we'll add $(b/2)^2$ to both sides of the equation.

$$u^2 - 4u + 4 + 3 = 0 + 4$$

$$(u^2 - 4u + 4) + 3 = 4$$

$$u^2 - 4u + 4 = 1$$

The quadratic on the left factors as the perfect square $(u-2)^2$.

$$(u-2)(u-2) = 1$$

$$(u-2)^2 = 1$$

$$u - 2 = \pm\sqrt{1}$$

$$u = 2 \pm \sqrt{1}$$

$$u = 2 \pm 1$$

The roots of the equation are therefore $u = 2 - 1 = 1$ and $u = 2 + 1 = 3$.

Topic: Completing the square

Question: Solve the quadratic equation by completing the square.

$$x^2 - 2x + 9 = 0$$

Answer choices:

A $\quad x = 1 \pm \sqrt{2}i$

B $\quad x = 1 \pm 2\sqrt{2}i$

C $\quad x = 2 \pm \sqrt{2}i$

D $\quad x = 2 \pm 2\sqrt{2}i$

Solution: B

The quadratic is given in standard form ax^2+bx+c with $a=1$, so we can complete the square. We'll start by finding $(b/2)^2$,

$$\left(\frac{b}{2}\right)^2=\left(\frac{-2}{2}\right)^2=(-1)^2=1$$

then we'll add $(b/2)^2$ to both sides of the equation.

$$x^2-2x+1+9=0+1$$

$$(x^2-2x+1)+9=1$$

$$x^2-2x+1=-8$$

The quadratic on the left factors as the perfect square $(x-1)^2$.

$$(x-1)(x-1)=-8$$

$$(x-1)^2=-8$$

$$x-1=\pm\sqrt{-8}$$

$$x=1\pm\sqrt{-8}$$

Use the imaginary number to rewrite the solutions.

$$x=1\pm\sqrt{8(-1)}$$

$$x=1\pm\sqrt{8}\sqrt{-1}$$

$$x=1\pm2\sqrt{2}\sqrt{-1}$$

$$x = 1 \pm 2\sqrt{2}i$$

The roots of the equation are therefore $x = 1 - 2\sqrt{2}i$ and $x = 1 + 2\sqrt{2}i$.

Topic: Quadratic formula

Question: Use the quadratic formula to solve the quadratic.

$$3x^2 + 2x - 1 = 0$$

Answer choices:

A $x = \frac{1}{3}, 1$

B $x = -1, \frac{1}{3}$

C $x = -\frac{1}{3}, 1$

D $x = 2, 3$

Solution: B

If we compare the standard form of a quadratic ax^2+bx+c to the quadratic we've been given $3x^2+2x-1$, we can identify

$$a=3$$

$$b=2$$

$$c=-1$$

Plugging these values into the quadratic formula, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(3)(-1)}}{2(3)}$$

$$x=\frac{-2\pm\sqrt{4+12}}{6}$$

$$x=\frac{-2\pm\sqrt{16}}{6}$$

$$x=\frac{-2\pm4}{6}$$

So the solutions to the quadratic equation are

$$x=\frac{-2-4}{6}=\frac{-6}{6}=-1 \qquad x=\frac{-2+4}{6}=\frac{2}{6}=\frac{1}{3}$$

Topic: Quadratic formula

Question: Use the quadratic formula to find the solutions of the quadratic equation.

$$2x^2 - 7x - 3 = 0$$

Answer choices:

A $x = \dfrac{7 - \sqrt{73}}{4}$ and $x = \dfrac{7 + \sqrt{73}}{4}$

B $x = \dfrac{7 + \sqrt{73}}{2}$ and $x = \dfrac{-7 + \sqrt{73}}{2}$

C $x = \dfrac{-7 - \sqrt{73}}{4}$ and $x = \dfrac{-7 + \sqrt{73}}{4}$

D $x = \dfrac{7 - \sqrt{73}}{2}$ and $x = \dfrac{7 + \sqrt{73}}{2}$

Solution: A

If we compare the standard form of a quadratic $ax^2 + bx + c$ to the quadratic we've been given $2x^2 - 7x - 3$, we can identify

$$a = 2$$

$$b = -7$$

$$c = -3$$

Plugging these values into the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{49 + 24}}{4}$$

$$x = \frac{7 \pm \sqrt{73}}{4}$$

So the solutions to the quadratic equation are

$$x = \frac{7 - \sqrt{73}}{4} \qquad x = \frac{7 + \sqrt{73}}{4}$$

Topic: Quadratic formula

Question: Find the roots of the equation using the quadratic formula.

$$3x^2 + 10x + 5 = 0$$

Answer choices:

A $x = \dfrac{5 \pm \sqrt{10}}{3}$

B $x = \dfrac{-5 \pm \sqrt{10}}{3}$

C $x = \dfrac{-5 \pm \sqrt{10}}{6}$

D $x = \dfrac{5 \pm \sqrt{10}}{6}$

Solution: B

If we compare the standard form of a quadratic $ax^2 + bx + c$ to the quadratic we've been given $3x^2 + 10x + 5$, we can identify

$$a = 3$$

$$b = 10$$

$$c = 5$$

Plugging these values into the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{-10 \pm \sqrt{100 - 60}}{6}$$

$$x = \frac{-10 \pm \sqrt{40}}{6}$$

$$x = \frac{-10 \pm 2\sqrt{10}}{6}$$

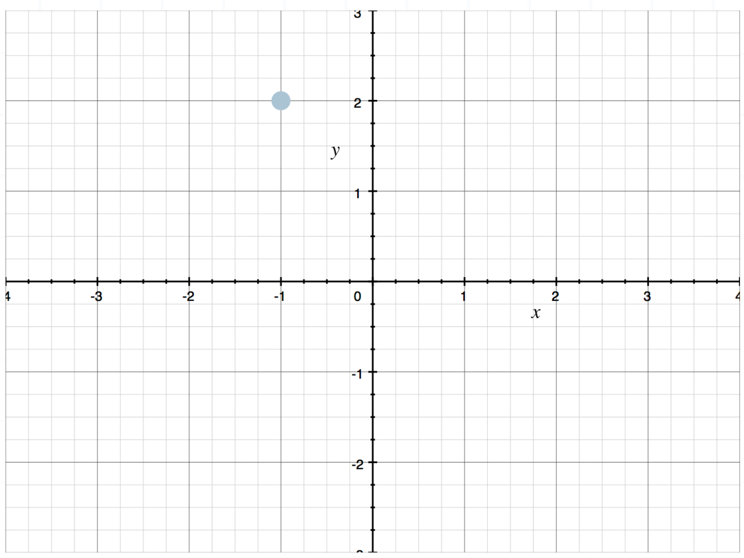
So the solutions to the quadratic equation are

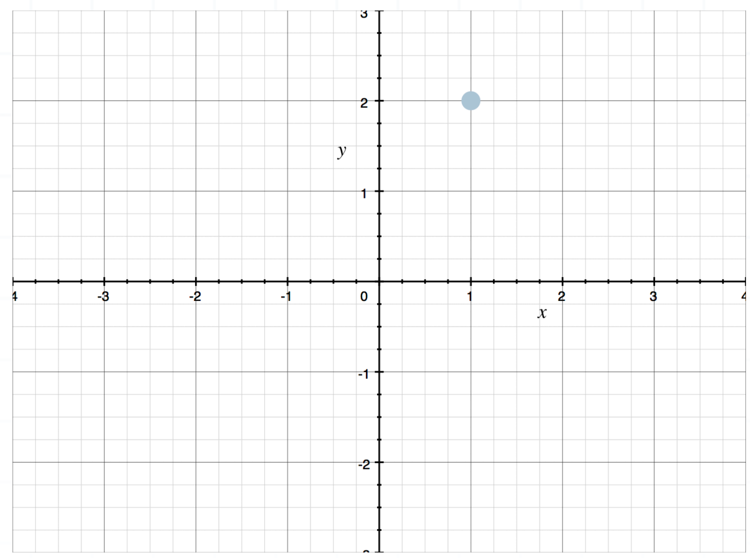
$$x = \frac{-5 \pm \sqrt{10}}{3}$$

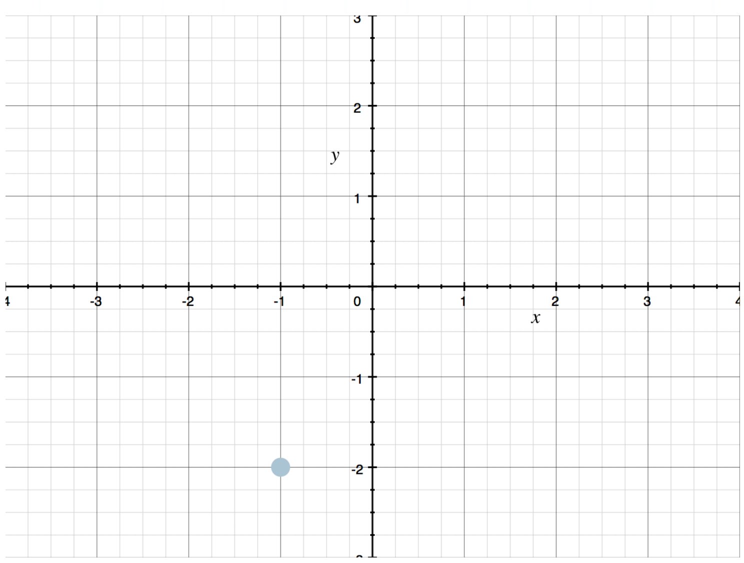
Topic: Cartesian coordinate system

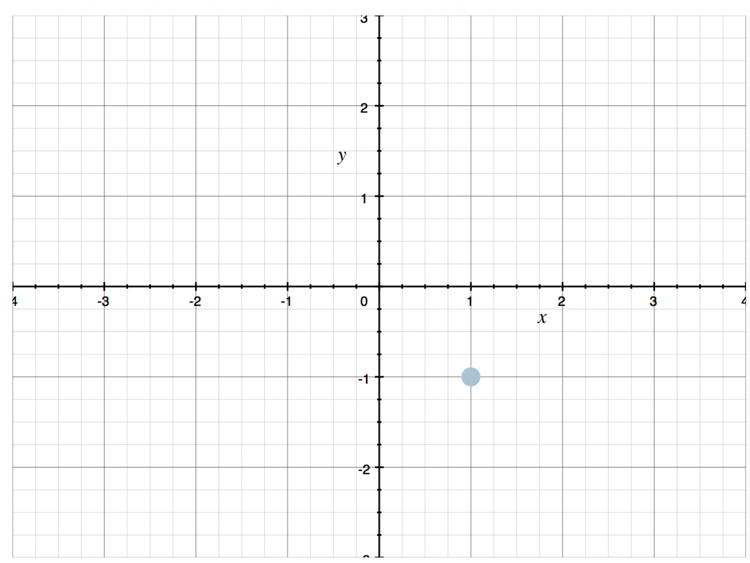
Question: Graph $(1,2)$ in the Cartesian coordinate system.

Answer choices:

A 

B 

C 

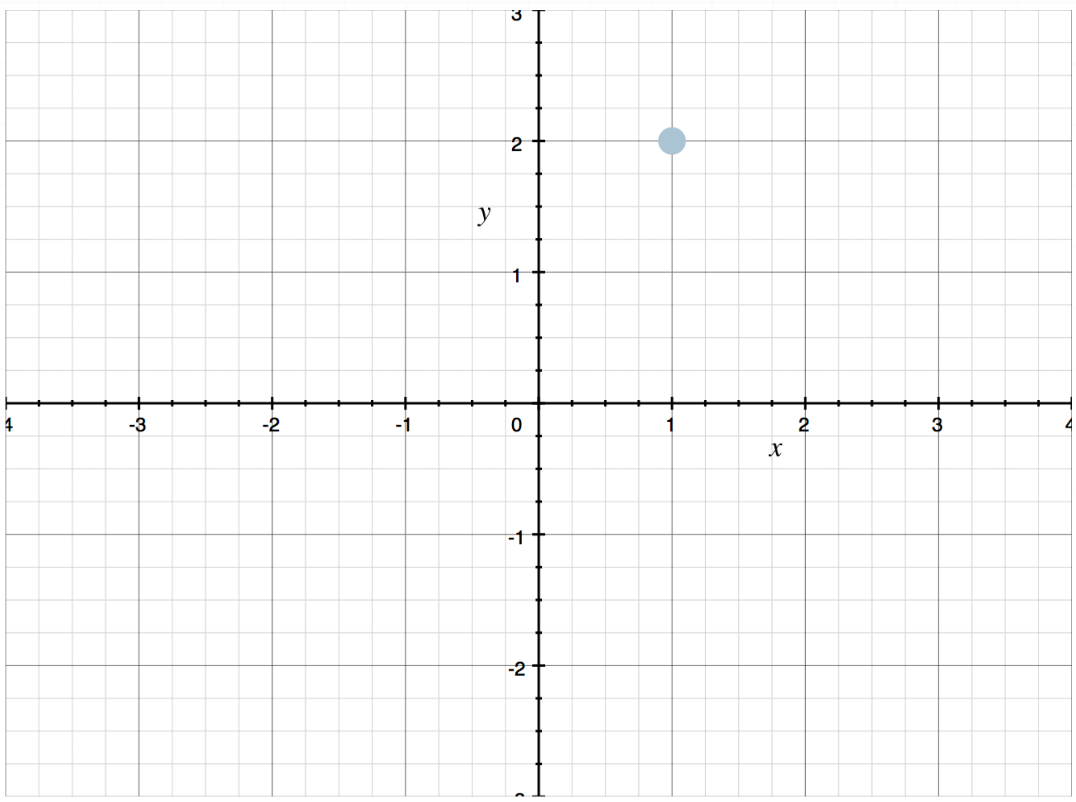
D 

Solution: B

Since Cartesian coordinates are always given in the form (x, y), we know that the x-coordinate is 1 and the y-coordinate is 2.

Since the x-coordinate is positive, we need to move 1 unit from the origin in the direction of the positive x-axis, which is to the right.

Then, since the y-coordinate is positive, we need to move 2 units from there in the direction of the positive y-axis, which is up.



Topic: Cartesian coordinate system

Question: Which point lies in Quadrant II of the Cartesian plane?

Answer choices:

A $(3,1)$

B $(3,-1)$

C $(-3,-1)$

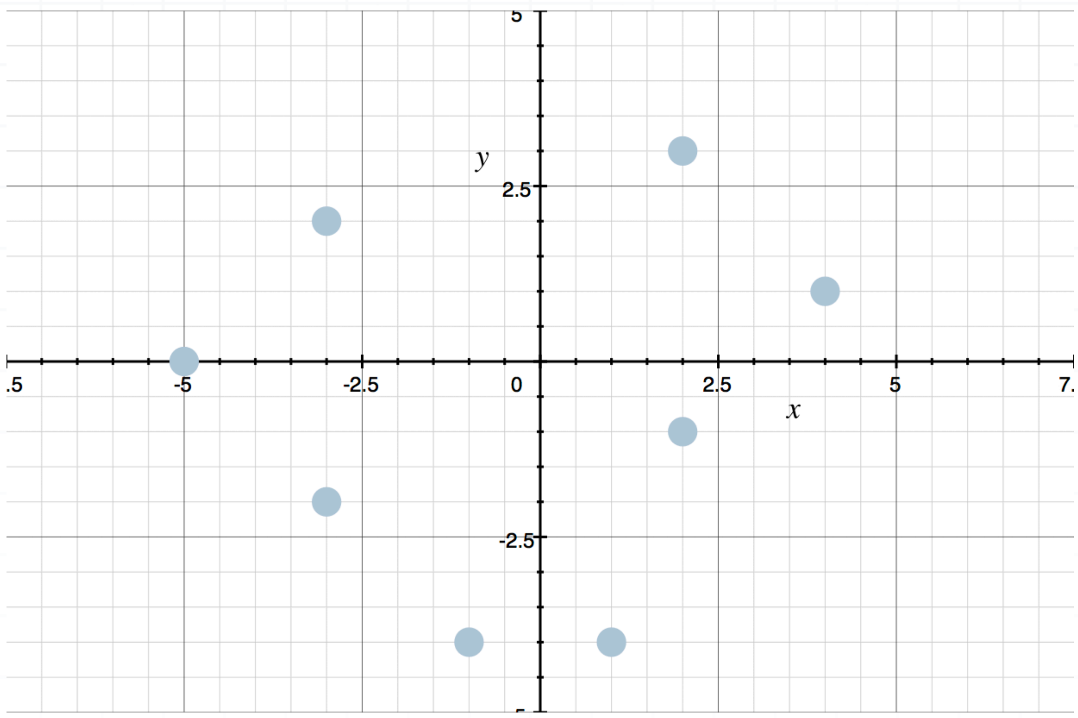
D $(-3,1)$

Solution: D

Quadrant II is the upper-left quadrant, where x is negative and y is positive. The only point among the answer choices where this is the case is answer choice D, $(-3,1)$.

Topic: Cartesian coordinate system

Question: Which group of points contains a point not plotted in the plane?



Answer choices:

A $(2,-1),\ (-1,-4),\ (-3,-2),\ (2,3)$

B $(2,3),\ (3,2),\ (2,-1),\ (-5,0)$

C $(1,-4),\ (-3,-2),\ (4,1),\ (-5,0)$

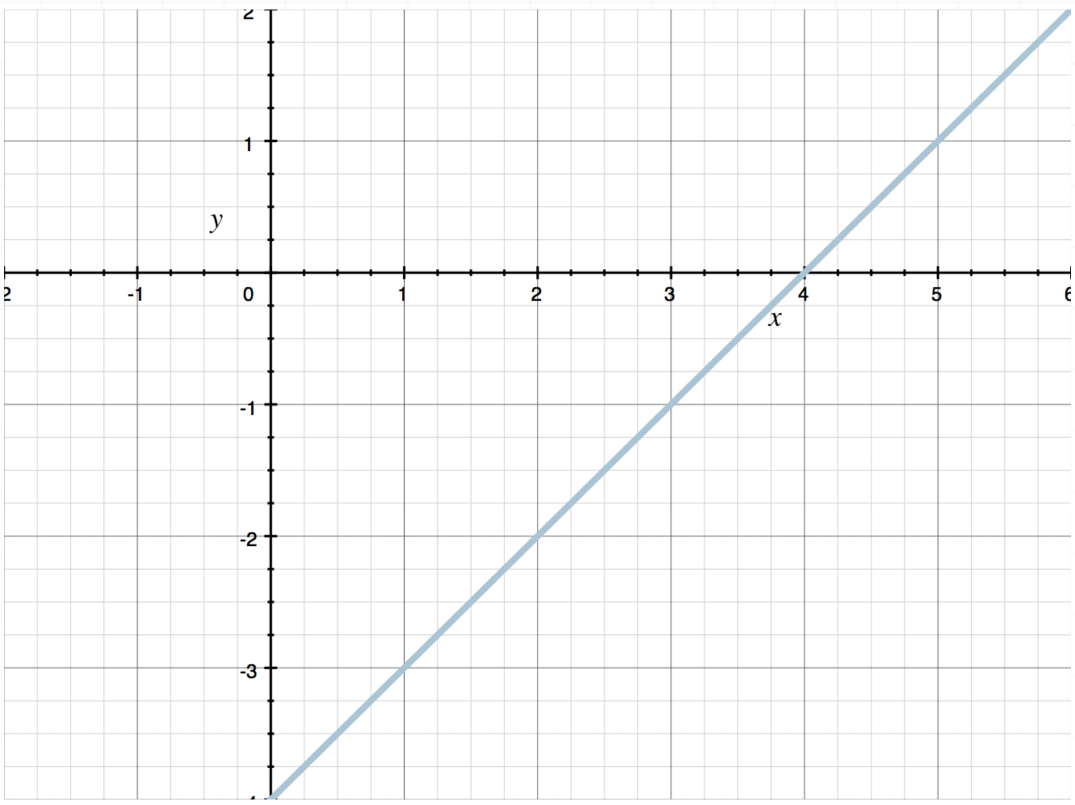
D $(4,1),\ (2,3),\ (1,-4),\ (2,-1)$

Solution: B

The point $(3,2)$ is listed in answer B, but is not plotted in the plane.

Topic: Slope

Question: What is the slope of the line?



Answer choices:

A 1

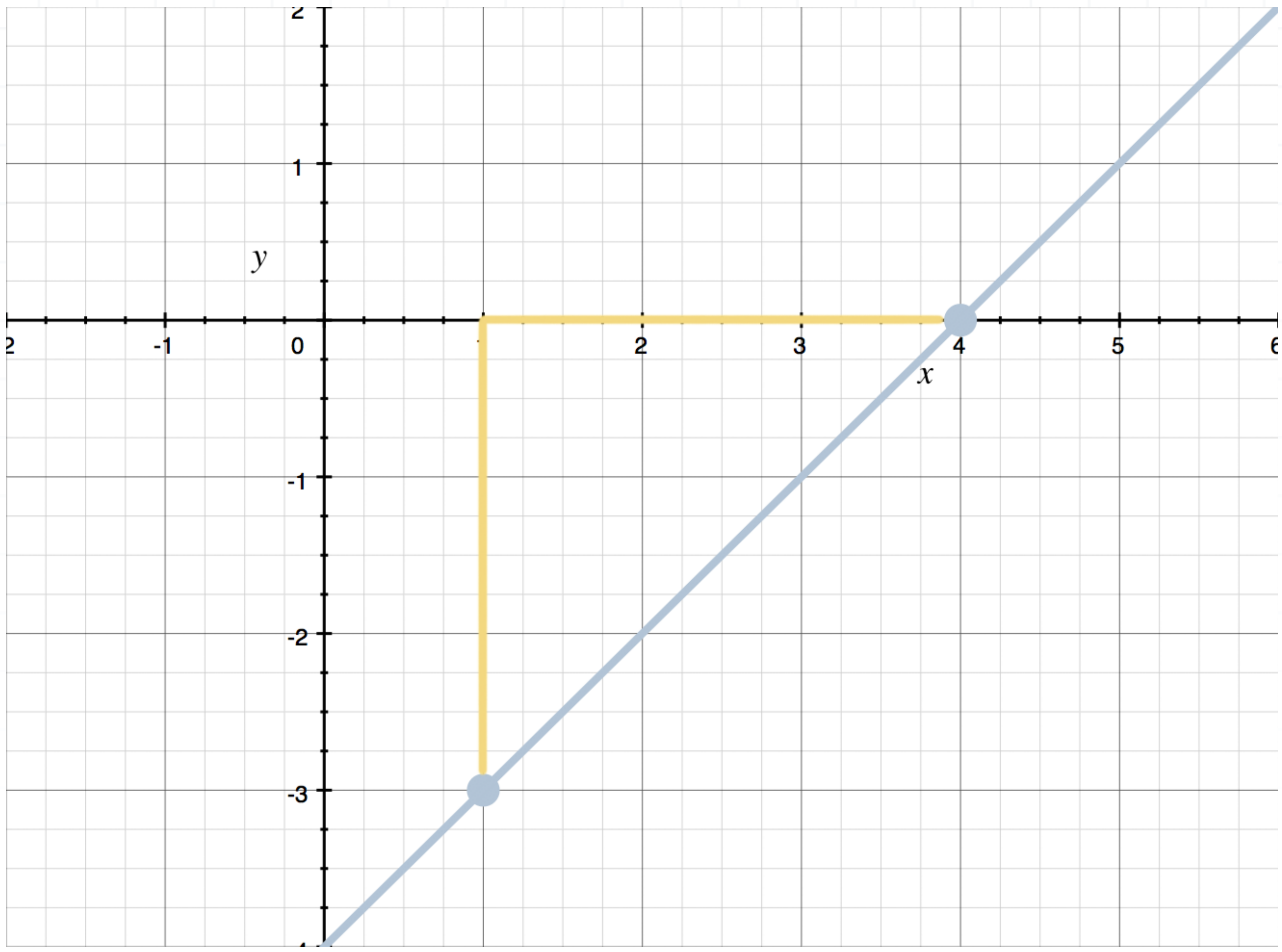
B 1.5

C 2

D 3

Solution: A

Any two points on the line can be used to find the slope. If we use the two points $(1,-3)$ and $(4,0)$, we'll get a rise of 3 and a run of 3, for a slope of $3/3=1$.



Topic: Slope

Question: What is the slope of the line that passes through the points $(3,-2)$ and $(-7,3)$?

Answer choices:

A 1

B -1

C $\frac{1}{2}$

D $-\frac{1}{2}$

Solution: D

Plug the given points into the slope equation,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2 - 3}{3 - (-7)}$$

then simplify.

$$m = \frac{-5}{10}$$

$$m = -\frac{1}{2}$$

Topic: Slope

Question: If each answer choice gives a pair of points on a line, which line has the steepest slope?

Answer choices:

A $(3,-4)$ and $(7,0)$

B $(5,1)$ and $(6,4)$

C $(4,5)$ and $(3,0)$

D $(6,2)$ and $(1,1)$

Solution: C

The slope between the points in answer choice A is

$$m = \frac{-4-0}{3-7} = \frac{-4}{-4} = 1$$

The slope between the points in answer choice B is

$$m = \frac{1-4}{5-6} = \frac{-3}{-1} = 3$$

The slope between the points in answer choice C is

$$m = \frac{5-0}{4-3} = \frac{5}{1} = 5$$

The slope between the points in answer choice D is

$$m = \frac{2-1}{6-1} = \frac{1}{5}$$

The slope is largest in answer choice C, which means the line that passes through those points is steeper than the lines given in any of the other answer choices.

Topic: Point-slope and slope-intercept forms of a line

Question: Find the equation of the line.

$$m = -\frac{2}{3}$$

$$(-7,2)$$

Answer choices:

A $\quad y + 2 = \frac{2}{3}(x - 7)$

B $\quad y - 2 = \frac{2}{3}(x + 7)$

C $\quad y + 2 = -\frac{2}{3}(x - 7)$

D $\quad y - 2 = -\frac{2}{3}(x + 7)$

Solution: D

When we're given a point and the slope, we can use the point-slope form of the equation of a line,

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is a point on the line.

We'll first plug in the slope and the coordinates of the point we've been given, and then simplify the equation by solving for y.

$$y - 2 = -\frac{2}{3}(x - (-7))$$

$$y - 2 = -\frac{2}{3}(x + 7)$$

Topic: Point-slope and slope-intercept forms of a line

Question: Find the equation, in point-slope form, of the line that passes through $(2,3)$ and $(4,11)$. Use $(2,3)$ for (x_1, y_1).

Answer choices:

A $y - 3 = 4(x - 2)$

B $y - 3 = 8(x - 2)$

C $y + 3 = 4(x + 2)$

D $y - 3 = 4(x + 2)$

Solution: A

First, find the slope of the line by plugging $(2,3)$ and $(4,11)$ into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 3}{4 - 2} = \frac{8}{2} = 4$$

Next, substitute $m = 4$ and the point $(2,3)$ into the point-slope formula for the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 4(x - 2)$$

Topic: Point-slope and slope-intercept forms of a line

Question: Find the slope-intercept form of the line that passes through $(0,-2)$ and has a slope of $1/2$.

Answer choices:

A $\quad y=\frac{1}{2}(x-2)$

B $\quad y=\frac{1}{2}x+2$

C $\quad y=\frac{1}{2}x-2$

D $\quad y=\frac{1}{2}x-1$

Solution: C

First we'll plug $m = 1/2$ and $(x_1, y_1) = (0, -2)$ in the point-slope formula for the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{1}{2}(x - 0)$$

$$y + 2 = \frac{1}{2}x$$

Now we'll subtract 2 from both sides to convert the equation into slope-intercept form.

$$y = \frac{1}{2}x - 2$$

Topic: Graphing linear equations

Question: What is the y-intercept of the line?

$$y + 4 = -5(x - 2)$$

Answer choices:

A -5

B -2

C 10

D 6

Solution: D

The linear equation isn't already in slope-intercept form, so we need to first convert the equation.

$$y + 4 = -5(x - 2)$$

$$y + 4 = -5x + 10$$

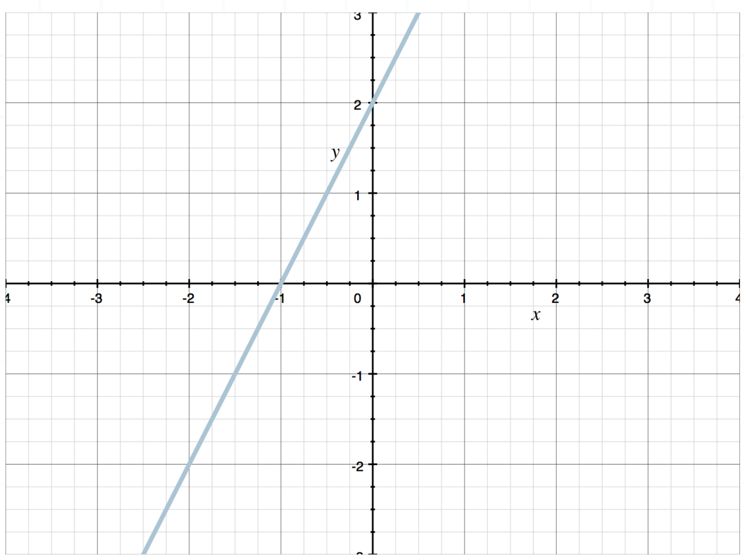
$$y = -5x + 6$$

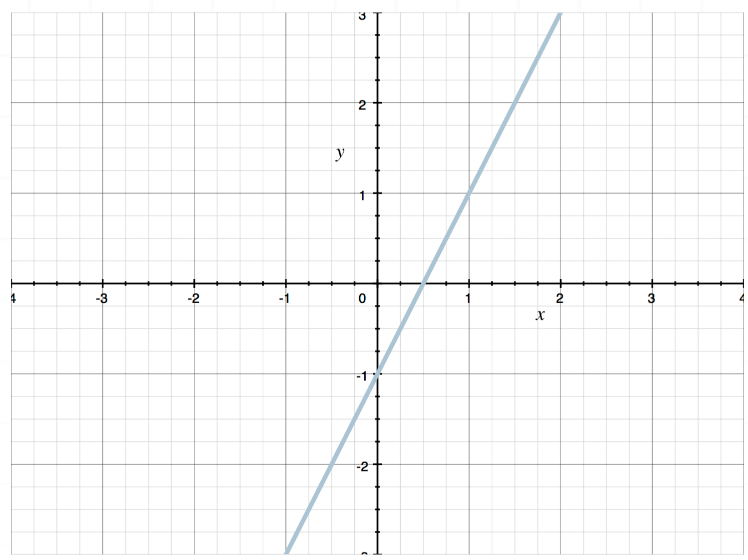
With the equation now in slope-intercept form, we can identify that the slope is $m = -5$ and the y-intercept is $b = 6$.

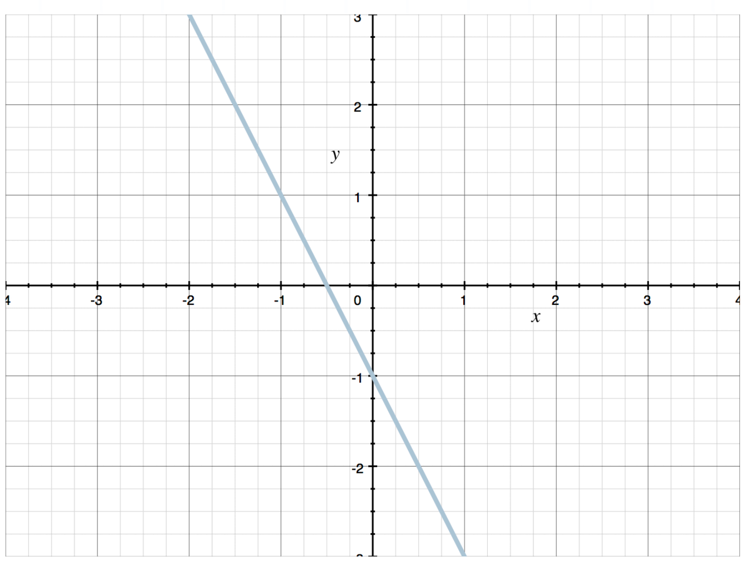
Topic: Graphing linear equations

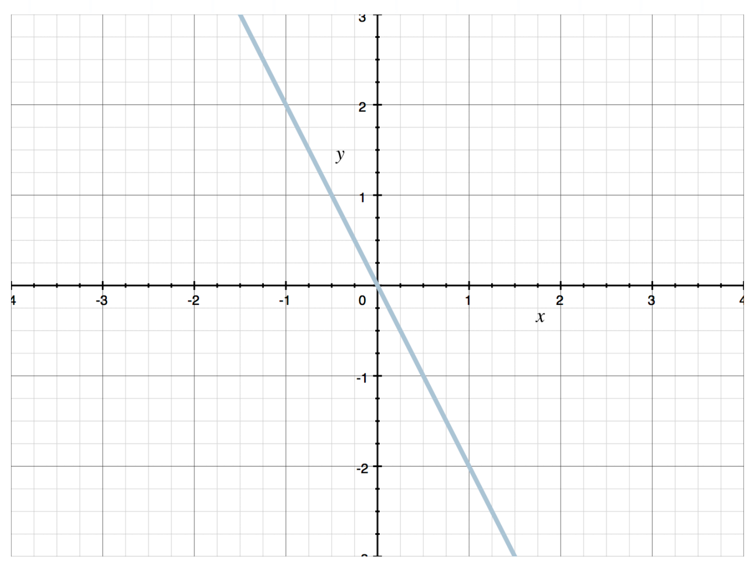
Question: Which graph is the sketch of $y = 2x - 1$?

Answer choices:

A 

B 

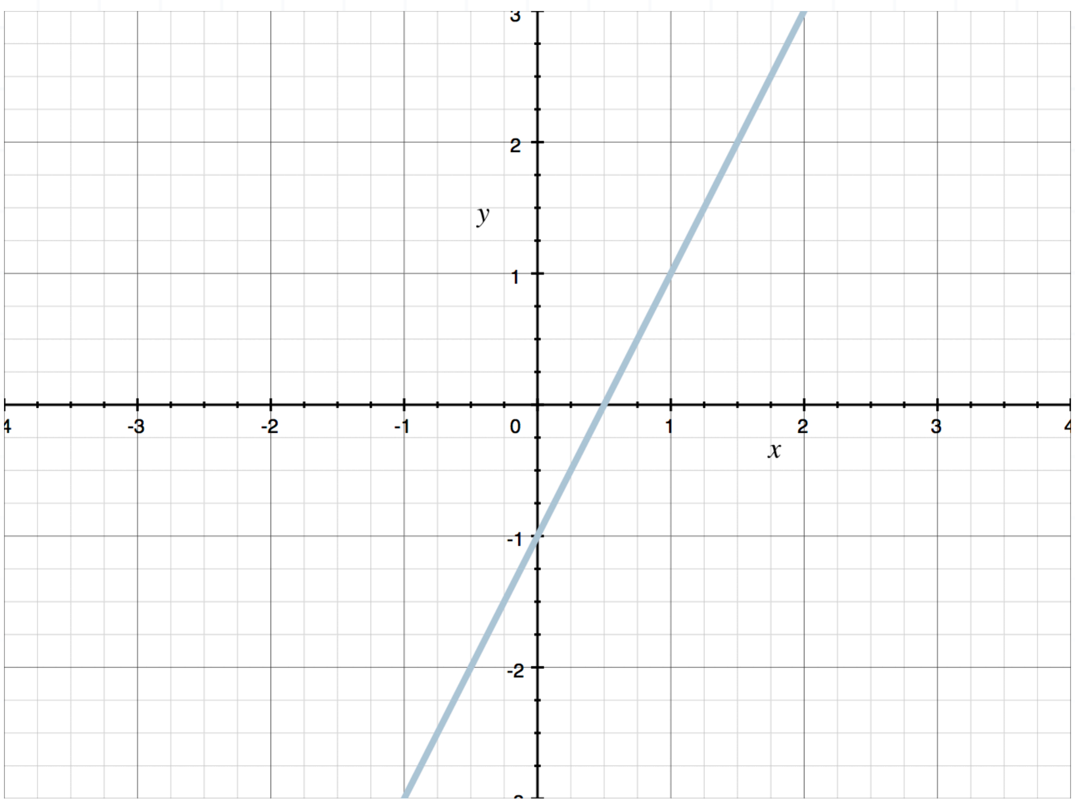
C 

D 

Solution: B

The linear equation is already in slope-intercept form, so we can see that the slope is $m = 2$ and the y-intercept is $b = -1$.

Since the slope is positive, we know that the line will lean to the right, with a rise of 2 and a run of 1, crossing the vertical axis at $y = -1$.

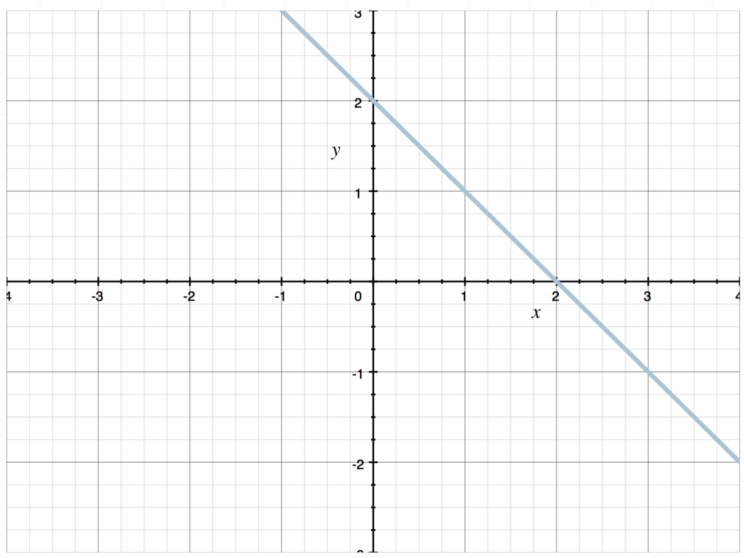


Topic: Graphing linear equations

Question: Which graph is the sketch of $y = -x + 2$?

Answer choices:

A



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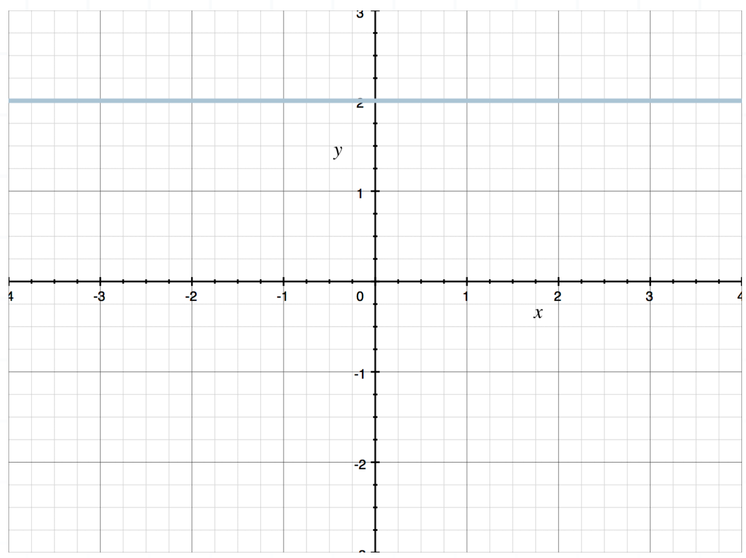

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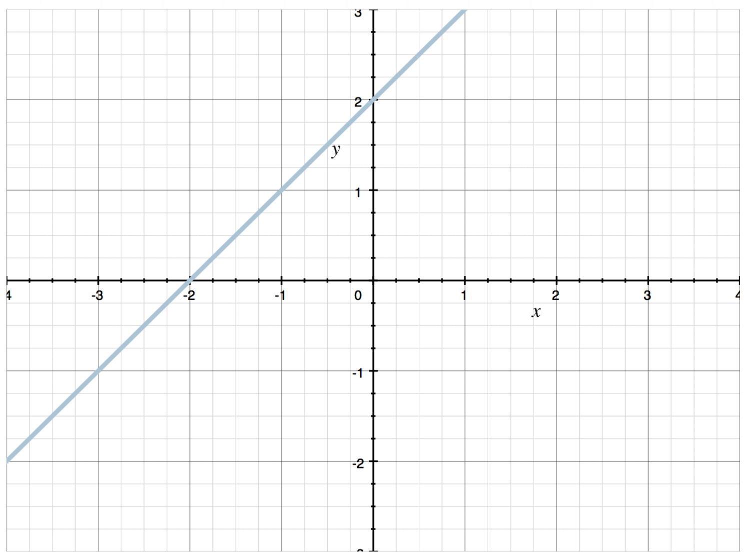
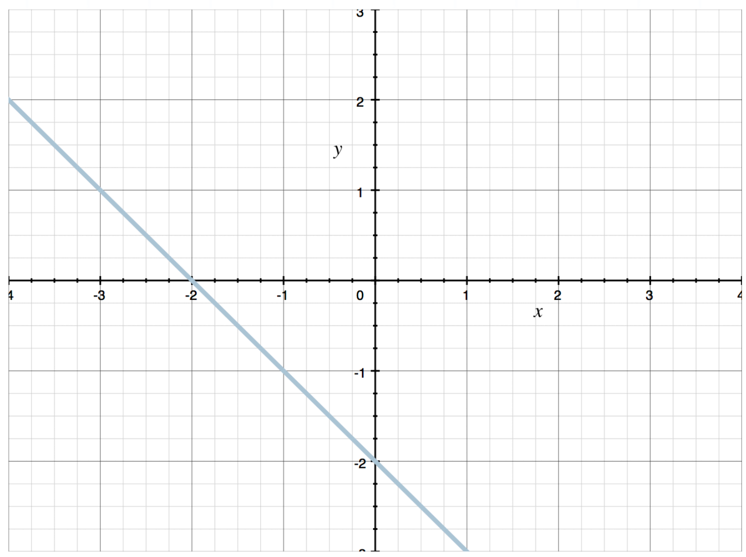


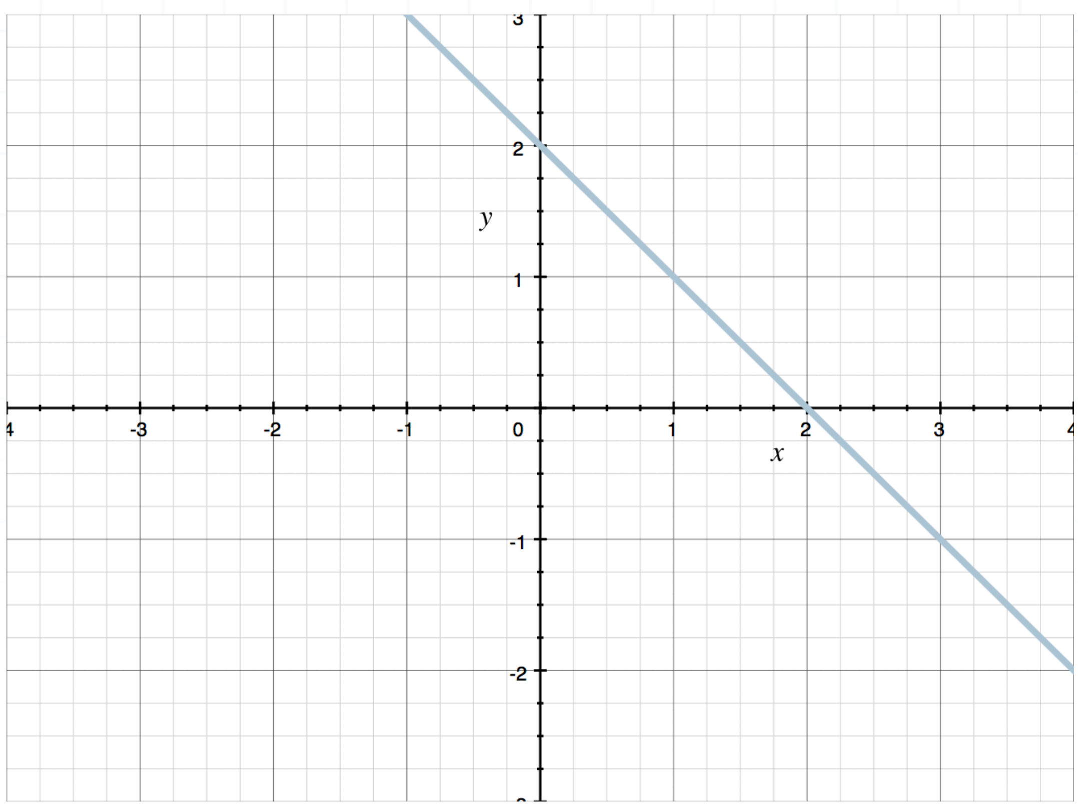
D



Solution: A

The linear equation is already in slope-intercept form, so we can see that the slope is $m = -1$ and the y-intercept is $b = 2$.

Since the slope is negative, we know that the line will lean to the left, with a rise of -1 and a run of 1, crossing the vertical axis at $y = 2$.



Topic: Function notation

Question: Find the value of $f(-11)$.

$$f(x) = 12 - 3x$$

Answer choices:

A -21

B 45

C -45

D 21

Solution: B

To find the value of $f(-11)$, substitute $x = -11$ into $f(x)$.

$$f(x) = 12 - 3x$$

$$f(-11) = 12 - 3(-11)$$

$$f(-11) = 12 + 33$$

$$f(-11) = 45$$

Topic: Function notation

Question: Find the value of $g(-5)$.

$$g(x) = 2x^2 + 3x - 32$$

Answer choices:

A 33

B -97

C -27

D 3

Solution: D

To find the value of $g(-5)$, substitute $x = -5$ into $g(x)$.

$$g(x) = 2x^2 + 3x - 32$$

$$g(-5) = 2(-5)^2 + 3(-5) - 32$$

$$g(-5) = 2(25) + 3(-5) - 32$$

$$g(-5) = 50 - 15 - 32$$

$$g(-5) = 35 - 32$$

$$g(-5) = 3$$

Topic: Function notation

Question: Find the value of $h(12)$.

$$h(x) = x^2 - 9x + 3$$

Answer choices:

A $\quad 6$

B $\quad 39$

C $\quad -27$

D $\quad -3$

Solution: B

To find the value of $h(12)$, substitute $x = 12$ into $h(x)$.

$$h(x) = x^2 - 9x + 3$$

$$h(12) = 12^2 - 9(12) + 3$$

$$h(12) = 144 - 108 + 3$$

$$h(12) = 36 + 3$$

$$h(12) = 39$$

Topic: Domain and range

Question: What are the domain and range of the function defined by the set of coordinate points?

$$(3,4), (4,1), (5,2), (7,1)$$

Answer choices:

A	The domain is $3, 4, 5, 7$.	The range is $1, 2, 4$.
B	The domain is $3, 7$.	The range is $1, 4$.
C	The domain is $3, 4, 5, 7$.	The range is $1, 2$.
D	None of these	

Solution: A

Remember that the coordinates of points in the Cartesian coordinate system are given in the form (x, y).

Since the domain of a function is all of the x-values, we can see that the domain of this function is

$$3, 4, 5, 7$$

The range of a function is all of the y-values, so we can see that the range of this function is

$$4, 1, 2, 1$$

We don't need to include the same value more than once, so we'll list 1 only once, and rearrange the numbers so that they're in ascending order. The range is

$$1, 2, 4$$

Topic: Domain and range

Question: What are the domain and range of the function?

$$y = \frac{2}{x}$$

Answer choices:

A Domain: all real numbers except 2 Range: all real numbers except 2

B Domain: all real numbers except 0 Range: all real numbers except 0

C Domain: all real numbers except 0 Range: all real numbers except 2

D Domain: all real numbers except 2 Range: all real numbers except 0

Solution: B

The domain of a function is all of the x-values for which the function is defined. The range of a function is all of the y-values that correspond to the x-values in the domain.

To solve for the domain of a function, we look for any values where the function is not defined. The function $y=2/x$ is undefined for $x=0$, because division by 0 is undefined. However, this function is defined for all other values of x, so its domain consists of all real numbers except 0.

To find the range, we need to look for the y-values that correspond to values in the domain and for those that don't.

For every nonzero real number y, there's some nonzero real number x such that

$$y=\frac{2}{x}$$

To see this, multiply both sides of this equation by x/y.

$$y\left(\frac{x}{y}\right)=\left(\frac{2}{x}\right)\left(\frac{x}{y}\right)$$

$$x=\frac{2}{y}$$

So for any nonzero real number y, we divide 2 by y to get a nonzero real number x for which $y=2/x$. However, there's no nonzero real number x such that

$$0 = \frac{2}{x}$$

To see this, multiply both sides of this equation by x.

$$0(x) = \left(\frac{2}{x}\right)(x)$$

$$0 = 2$$

This gives us the false equation $0 = 2$.

Combining these results, we find that the range of this function is all real numbers except 0.

Topic: Domain and range

Question: What is the domain of the function?

$$f(x) = \sqrt{4x^3}$$

Answer choices:

A The domain is all values of x that make $4x^3$ positive

B The domain is all values of x that make $4x^3$ negative

C The domain is all values of x that make $4x^3$ either 0 or positive

D The domain is all values of x that make $4x^3$ either 0 or negative

Solution: C

When we're dealing with real numbers, we can only take the square root of 0 or positive values.

In other words, we won't be able to find the square root of $4x^3$ unless the value of $4x^3$ is positive, or equal to 0.

Therefore, any values of x that make $4x^3$ equivalent to 0, or equivalent to any positive value, will be included in the domain of the function.

Topic: Testing for functions

Question: Does the equation represent a function?

$$2x^2 + 2y^2 = 18$$

Answer choices:

A Yes, because there are values of x that will give multiple values for y.

B No, because there are values of x that will give multiple values for y.

C No, because every value of x will give a unique value for y.

D Yes, because every value of x will give a unique value for y.

Solution: B

Solve the equation for y.

$$2x^2 + 2y^2 = 18$$

$$x^2 + y^2 = 9$$

$$y^2 = 9 - x^2$$

$$y = \pm\sqrt{9 - x^2}$$

Now that we have the equation in this form, we can find values of x that return multiple y-values. For instance, at $x = 1$,

$$y = \pm\sqrt{9 - 1^2}$$

$$y = \pm\sqrt{8}$$

$$y = \pm 2\sqrt{2}$$

Because the equation takes on the values $y = -2\sqrt{2}$ and $y = 2\sqrt{2}$ at the single value $x = 1$, we know the equation doesn't represent a function.

Topic: Testing for functions

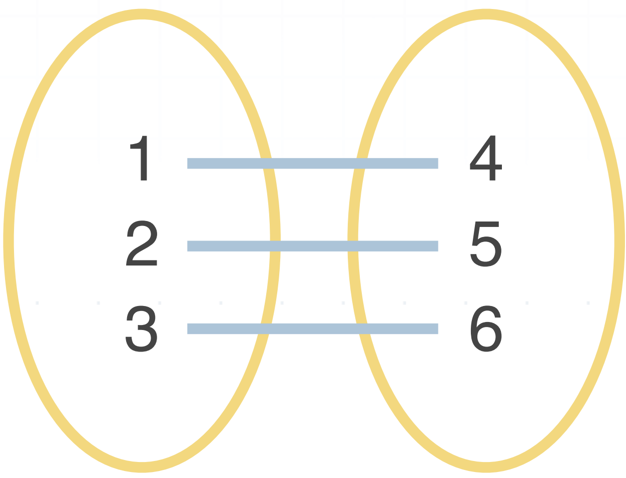
Question: Which of these could represent a function?

Answer choices:

A $(-1,-1)$, $(2,0)$, $(3,1)$, $(-1,2)$

B The relation whose graph consists of the points with coordinates $(1,2)$, $(1,3)$, and $(1,4)$.

C $(-2,-1)$, $(-3,0)$, $(-3,-3)$, $(3,2)$

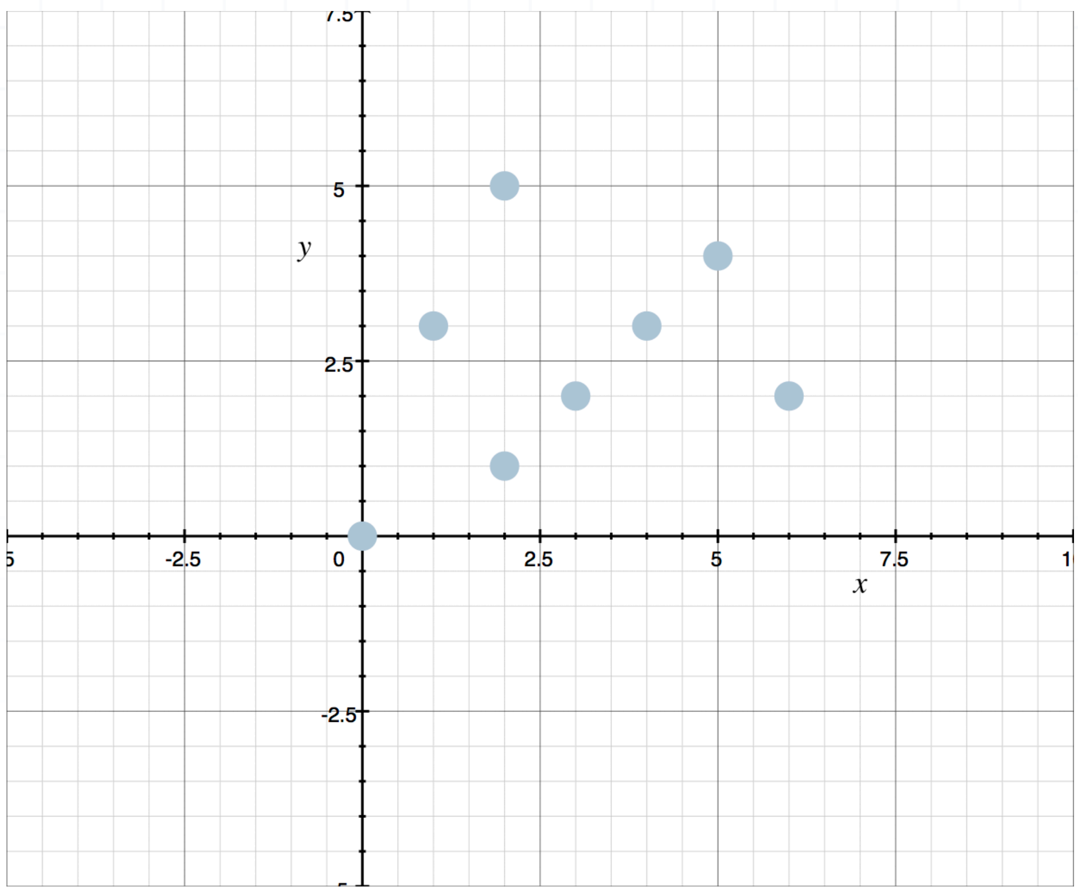
D 

Solution: D

Answer choice D is the only expression that could represent a function, because it's the only answer choice that shows just one y-value for every x-value.

Topic: Testing for functions

Question: The graph shows eight points that define a relation between x and y. Identify the pair of points which prove that the relation is not a function.



Answer choices:

A $(1,3)$ and $(4,3)$

B $(5,4)$ and $(2,1)$

C $(3,2)$ and $(6,2)$

D $(2,1)$ and $(2,5)$

Solution: D

If one x-value gives two different y-values, then the relation is not a function.

Answer choice D shows $x = 2$, $y = 1$ and $x = 2$, $y = 5$. In other words, the same x-value but two different y-values. Therefore, we know that the relation is not a function.

Topic: Vertical Line Test

Question: If a perfectly straight vertical line crosses a graph at more than one point, the graph fails the Vertical Line Test.

Answer choices:

A True

B False

Solution: A

A graph passes the Vertical Line Test if it’s impossible to draw a perfectly straight vertical line that crosses the graph more than once.

If we can draw a perfectly straight vertical line that crosses the graph more than once, then the graph fails the Vertical Line Test, and the graph does not represent a function.

Topic: Vertical Line Test

Question: Which figure will never pass the Vertical Line Test and therefore can never represent a function?

Answer choices:

A A horizontal line

B A set of six points, all of which have different x-coordinates

C A “slanted” line (neither vertical nor horizontal)

D A circle

Solution: D

A graph fails the Vertical Line Test when we can draw a vertical line that crosses the graph more than once.

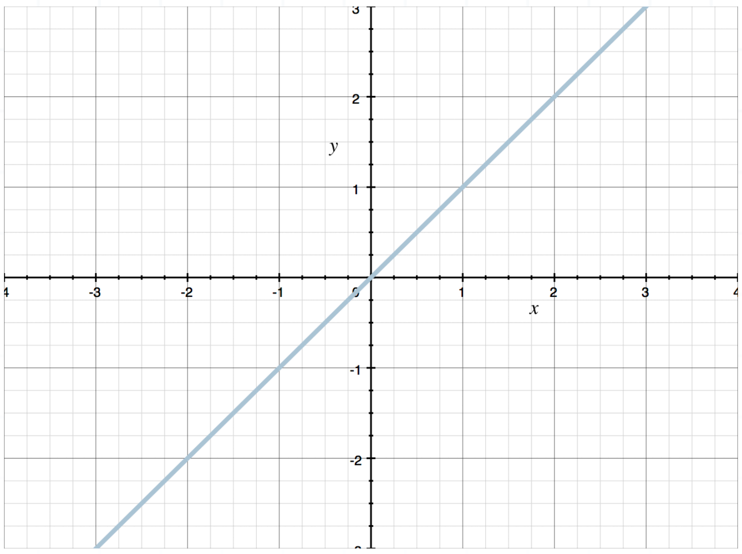
Since we'll always be able to draw a vertical line that crosses the graph of a circle more than once, a circle will always fail the Vertical Line Test, and therefore can never represent a function.

Topic: Vertical Line Test

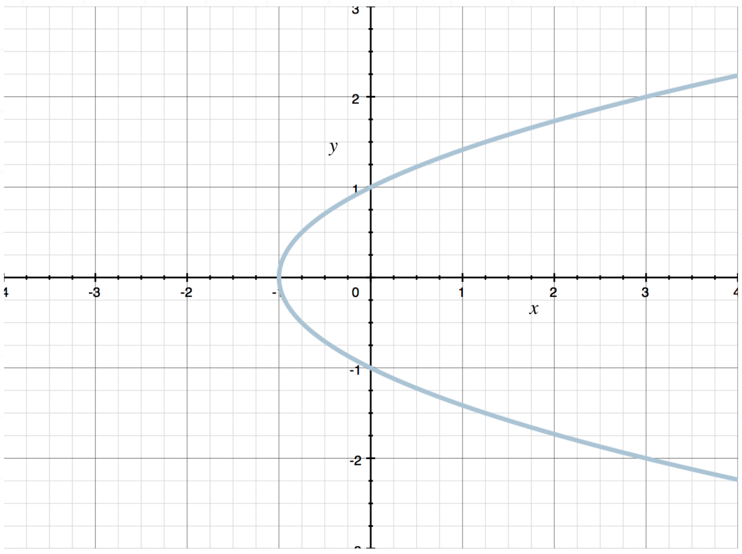
Question: Which graph represents a function?

Answer choices:

A



B



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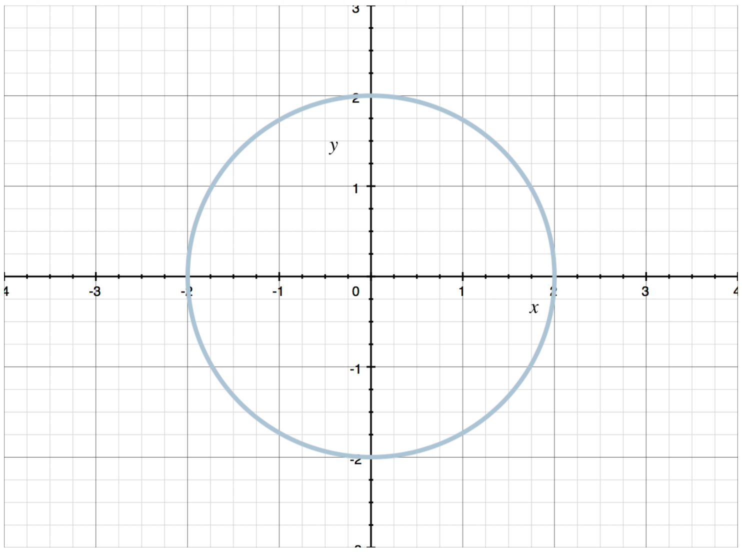
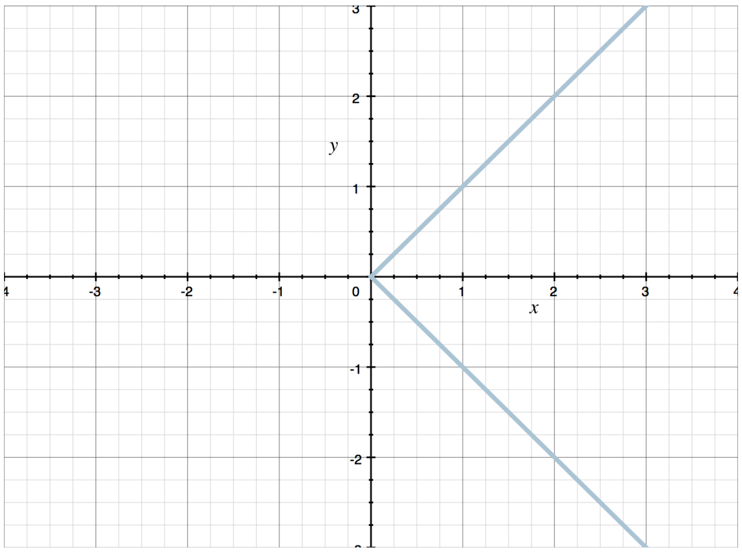


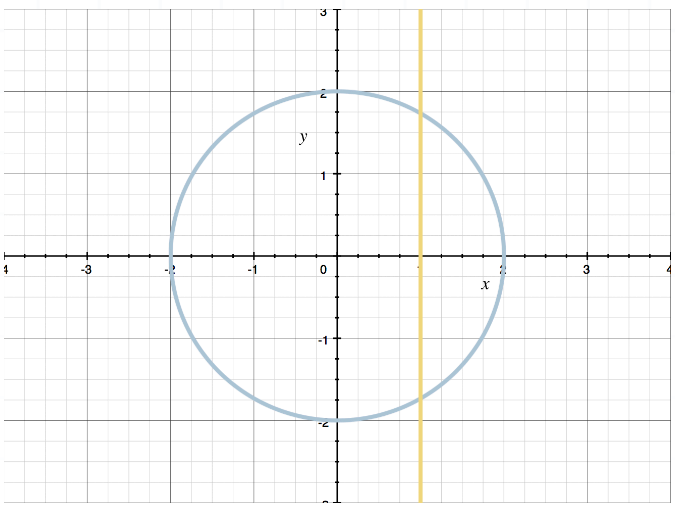
D

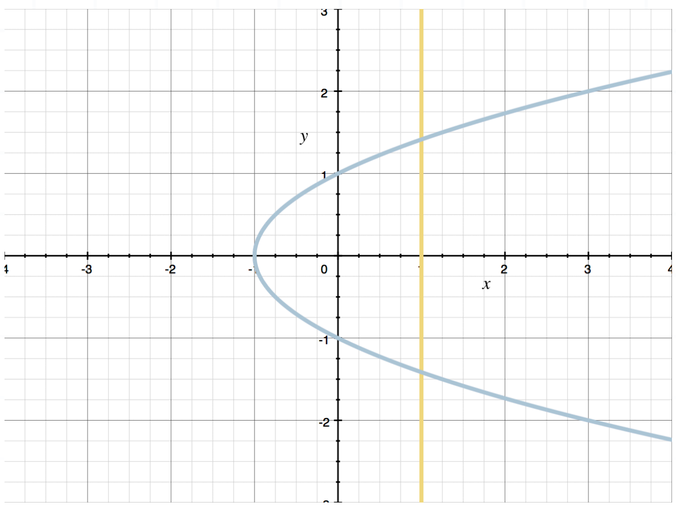


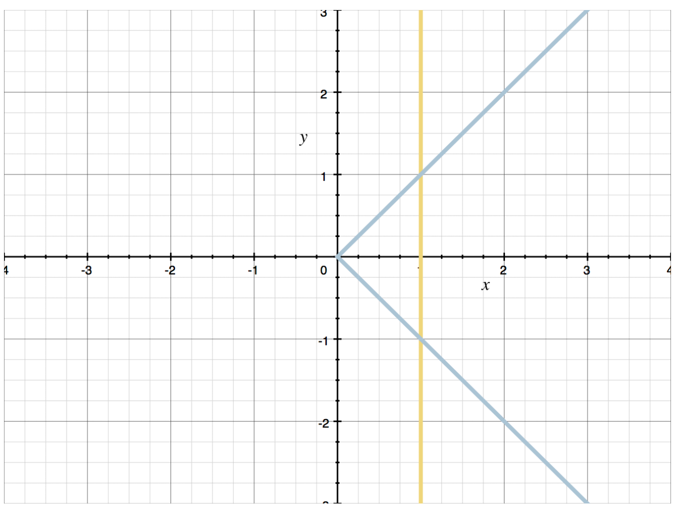
Solution: A

The graph in answer choice A represents a function because we can't draw any perfectly vertical line that crosses the graph in more than one place. Therefore, by the Vertical Line Test, the graph represents a function.

On the other hand, for each of the other three graphs, we can draw a perfectly vertical line that crosses the graph in more than one place. Therefore, those graphs don't represent functions.







Topic: Sum of functions

Question: Find $(g+h)(4)$.

$$g(x)=x^2-3x+1$$

$$h(x)=x+3$$

Answer choices:

A $(g+h)(4)=6$

B $(g+h)(4)=8$

C $(g+h)(4)=10$

D $(g+h)(4)=12$

Solution: D

We need to evaluate each of the functions at $x = 4$, and then add the results. For $g(4)$, we get

$$g(x) = x^2 - 3x + 1$$

$$g(4) = (4)^2 - 3(4) + 1$$

$$g(4) = 16 - 12 + 1$$

$$g(4) = 4 + 1$$

$$g(4) = 5$$

And for $h(4)$, we get

$$h(x) = x + 3$$

$$h(4) = 4 + 3$$

$$h(4) = 7$$

Now we can add the result to find $(g + h)(4)$.

$$(g + h)(4) = g(4) + h(4)$$

$$(g + h)(4) = 5 + 7$$

$$(g + h)(4) = 12$$

We could also have added the expressions for the functions, and then plugged in $x = 4$ to get the answer.

$$(g+h)(x) = (x^2 - 3x + 1) + (x + 3)$$

$$(g+h)(x) = x^2 - 3x + 1 + x + 3$$

$$(g+h)(x) = x^2 - 2x + 4$$

$$(g+h)(4) = 4^2 - 2(4) + 4$$

$$(g+h)(4) = 16 - 8 + 4$$

$$(g+h)(4) = 8 + 4$$

$$(g+h)(4) = 12$$

Topic: Sum of functions

Question: Find $(f+g)(4)$.

$$f(x) = x^2 + 4x$$

$$g(x) = -x + 2$$

Answer choices:

A $(f+g)(4) = 26$

B $(f+g)(4) = 30$

C $(f+g)(4) = 34$

D $(f+g)(4) = 38$

Solution: B

We know that

$$(f+g)(x)=f(x)+g(x)$$

Substituting the given expression for each function gives

$$(f+g)(x)=x^2+4x+(-x+2)$$

$$(f+g)(x)=x^2+4x-x+2$$

$$(f+g)(x)=x^2+3x+2$$

Substituting $x=4$ gives

$$(f+g)(4)=4^2+3(4)+2$$

$$(f+g)(4)=16+12+2$$

$$(f+g)(4)=28+2$$

$$(f+g)(4)=30$$

Topic: Sum of functions

Question: Find $(h+j)(3)$.

$$h(x) = (x-3)^2$$

$$j(x) = \sqrt{x^2+16}$$

Answer choices:

A $(h+j)(3) = 5$

B $(h+j)(3) = 18$

C $(h+j)(3) = 31$

D $(h+j)(3) = 36$

Solution: A

We know that

$$(h+j)(x) = h(x) + j(x)$$

Substituting the given expression for each function gives

$$(h+j)(x) = (x-3)^2 + \sqrt{x^2+16}$$

$$(h+j)(x) = x^2 - 3x - 3x + 9 + \sqrt{x^2+16}$$

$$(h+j)(x) = x^2 - 6x + 9 + \sqrt{x^2+16}$$

Substitute $x = 3$.

$$(h+j)(3) = 3^2 - 6(3) + 9 + \sqrt{3^2+16}$$

$$(h+j)(3) = 9 - 18 + 9 + \sqrt{9+16}$$

$$(h+j)(3) = 0 + \sqrt{25}$$

$$(h+j)(3) = 5$$

Topic: Product of functions

Question: Find $(gh)(-3)$.

$$g(x) = x - 4$$

$$h(x) = x + 1$$

Answer choices:

A $(gh)(-3) = 6$

B $(gh)(-3) = 8$

C $(gh)(-3) = 12$

D $(gh)(-3) = 14$

Solution: D

We need to find $(gh)(-3)$, which we could rewrite as

$$g(-3) \cdot h(-3)$$

This function notation tells us that we need to evaluate each of the functions at $x = -3$, and then multiply the results.

For $g(-3)$:

$$g(x) = x - 4$$

$$g(-3) = -3 - 4$$

$$g(-3) = -7$$

For $h(-3)$:

$$h(x) = x + 1$$

$$h(-3) = -3 + 1$$

$$h(-3) = -2$$

The product of the functions is

$$(gh)(-3) = g(-3) \cdot h(-3)$$

$$(gh)(-3) = -7 \cdot -2$$

$$(gh)(-3) = 14$$

We could have also multiplied the expressions for the functions, and then evaluated their product at $x = -3$.

$$(gh)(x) = (x-4)(x+1)$$

$$(gh)(x) = x^2 + x - 4x - 4$$

$$(gh)(x) = x^2 - 3x - 4$$

$$(gh)(-3) = (-3)^2 - 3(-3) - 4$$

$$(gh)(-3) = 9 + 9 - 4$$

$$(gh)(-3) = 18 - 4$$

$$(gh)(-3) = 14$$

Topic: Product of functions

Question: Find $(fg)(x)$.

$$f(x) = x + 7$$

$$g(x) = x - 5$$

Answer choices:

A $\quad (fg)(x) = x^2 + 2x - 35$

B $\quad (fg)(x) = x^2 - 2x - 35$

C $\quad (fg)(x) = x^2 + 2x + 35$

D $\quad (fg)(x) = x^2 - 2x + 35$

Solution: A

We need to find $(fg)(x)$, which we could rewrite as

$$f(x) \cdot g(x)$$

This function notation tells us that we need to multiply the expressions for the functions.

$$(fg)(x) = (x+7)(x-5)$$

$$(fg)(x) = x^2 - 5x + 7x - 35$$

$$(fg)(x) = x^2 + 2x - 35$$

Topic: Product of functions

Question: Find $(f \cdot g)(2)$.

$$f(x) = x^2 - 6$$

$$g(x) = 3x - 5$$

Answer choices:

A $(f \cdot g)(2) = -2$

B $(f \cdot g)(2) = 2$

C $(f \cdot g)(2) = 6$

D $(f \cdot g)(2) = 10$

Solution: A

We know that

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Substituting the given expression for each function gives

$$(f \cdot g)(x) = (x^2 - 6)(3x - 5)$$

Expanding the expression gives

$$(f \cdot g)(x) = 3x^3 - 5x^2 - 18x + 30$$

Substitute $x = 2$.

$$(f \cdot g)(x) = 3(2)^3 - 5(2)^2 - 18(2) + 30$$

$$(f \cdot g)(x) = 3(8) - 5(4) - 18(2) + 30$$

$$(f \cdot g)(x) = 24 - 20 - 36 + 30$$

$$(f \cdot g)(x) = 4 - 36 + 30$$

$$(f \cdot g)(x) = -32 + 30$$

$$(f \cdot g)(x) = -2$$

Topic: Even, odd, or neither

Question: Is the function even, odd, or neither?

$$f(x) = 2x^3 - x^7$$

Answer choices:

A Even

B Odd

C Neither

D The function can't be classified

Solution: B

A function is even if $f(x) = f(-x)$, odd if $f(-x) = -f(x)$, and neither if $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.

So to classify the given function, we'll substitute $-x$ into $f(x) = 2x^3 - x^7$ and then simplify the result.

$$f(-x) = 2(-x)^3 - (-x)^7$$

$$f(-x) = 2(-1x)^3 - (-1x)^7$$

$$f(-x) = 2(-1)^3(x^3) - (-1)^7(x^7)$$

$$f(-x) = 2(-1)x^3 - (-1)x^7$$

$$f(-x) = -2x^3 + x^7$$

$$f(-x) = -(2x^3 - x^7)$$

This function is odd, because $f(-x) = -f(x)$.

Topic: Even, odd, or neither

Question: Is the function even, odd, or neither?

$$f(x) = 5x^2 - 2x^3$$

Answer choices:

A Even

B Odd

C Neither

D The function can't be classified

Solution: C

A function is even if $f(x) = f(-x)$, odd if $f(-x) = -f(x)$, and neither if $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.

So to classify the given function, we'll substitute $-x$ into $f(x) = 5x^2 - 2x^3$ and then simplify the result.

$$f(-x) = 5(-x)^2 - 2(-x)^3$$

$$f(-x) = 5(-1x)^2 - 2(-1x)^3$$

$$f(-x) = 5(-1)^2(x^2) - 2(-1)^3(x^3)$$

$$f(-x) = 5(1)x^2 - 2(-1)x^3$$

$$f(-x) = 5x^2 + 2x^3$$

This function is neither even nor odd, because $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.

Topic: Even, odd, or neither

Question: Is the function even, odd, or neither?

$$f(x) = -x^4 - 6x^2$$

Answer choices:

A Even

B Odd

C Neither

D The function can't be classified

Solution: A

A function is even if $f(x) = f(-x)$, odd if $f(-x) = -f(x)$, and neither if $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.

So to classify the given function, we'll substitute $-x$ into $f(x) = -x^4 - 6x^2$ and then simplify the result.

$$f(-x) = -(-x)^4 - 6(-x)^2$$

$$f(-x) = -(-1x)^4 - 6(-1x)^2$$

$$f(-x) = -(-1)^4(x^4) - 6(-1)^2(x^2)$$

$$f(-x) = -(1)x^4 - 6(1)x^2$$

$$f(-x) = -x^4 - 6x^2$$

This function is even, because $f(x) = f(-x)$.

Topic: Trichotomy

Question: Which answer best illustrates the idea of trichotomy?

Answer choices:

A If $a = b$ and $b = c$, then $a = c$

B If $x \ngeq y$, then $x < y$

C If $x > y$, then $x > y$

D $x = 3$

Solution: B

Trichotomy is the idea that the relationship between two numbers a and b is always defined in one of three ways,

$$a > b$$

$$a = b$$

$$a < b$$

Answer choice B illustrates the trichotomy rule by saying that if $x \not> y$ and $x \neq y$, then it must be true that $x < y$, because we've removed two of the three options in the trichotomy, leaving only one option.

Topic: Trichotomy

Question: Solve the inequality.

$$4(1-x) \nleq 5(2-x)$$

Answer choices:

A $x < -6$

B $x \ngtr -6$

C $x > 6$

D $x < 6$

Solution: C

Expand both sides using the Distributive Property.

$$4(1-x) \nleq 5(2-x)$$

$$4-4x \nleq 10-5x$$

Combine like terms.

$$4-4x-4 \nleq 10-5x-4$$

$$-4x \nleq 6-5x$$

$$-4x+5x \nleq 6-5x+5x$$

$$x \nleq 6$$

If x is not less than 6 and also not equal to 6, the trichotomy law tells us that it must be greater than 6. Therefore, we can rewrite the solution as

$$x > 6$$

Topic: Trichotomy

Question: Solve the inequality.

$$-2(3-x) \nleq 3(5-x)+4x$$

Answer choices:

A $x > 21$

B $x > 9$

C $x > 3$

D $x < 21$

Solution: A

Expand both sides using the Distributive Property.

$$-2(3-x) \nleq 3(5-x)+4x$$

$$-6+2x \nleq 15-3x+4x$$

Combine like terms.

$$-6+2x \nleq 15+x$$

$$-6+2x-x \nleq 15+x-x$$

$$-6+x \nleq 15$$

$$-6+6+x \nleq 15+6$$

$$x \nleq 21$$

Since x isn't less than 21 and also isn't equal to 21, it can only be greater than 21, according to the trichotomy law.

$$x > 21$$

Topic: Inequalities and negative numbers

Question: Solve the inequality.

$$-x + 4 < 9$$

Answer choices:

A $x < -5$

B $x < 13$

C $x > -13$

D $x > -5$

Solution: D

Subtract 4 from both sides.

$$-x+4<9$$

$$-x+4-4<9-4$$

$$-x<5$$

Multiply both sides by -1. Because we're multiplying by a negative number, we also have to change the direction of the inequality sign.

$$(-1)(-x)>(-1)(5)$$

$$x>-5$$

Topic: Inequalities and negative numbers

Question: Solve the inequality.

$$-3x+7\leq 25$$

Answer choices:

A $x\leq -6$

B $x\geq -6$

C $x\geq 6$

D $x<6$

Solution: B

Subtract 7 from both sides.

$$-3x + 7 \leq 25$$

$$-3x + 7 - 7 \leq 25 - 7$$

$$-3x \leq 18$$

Divide both sides by -3. Because we're dividing by a negative number, we also have to change the direction of the inequality sign.

$$\frac{-3x}{-3} \geq \frac{18}{-3}$$

$$x \geq -6$$

Topic: Inequalities and negative numbers

Question: Solve the inequality.

$$2(x+6) \geq 4x + 10$$

Answer choices:

A $x \leq 1$

B $x \geq 1$

C $x \geq -2$

D $x \leq 2$

Solution: A

Use the Distributive Property to expand the left side.

$$2(x+6) \geq 4x+10$$

$$2x+12 \geq 4x+10$$

Subtract $4x$ from both sides.

$$2x+12-4x \geq 4x+10-4x$$

$$-2x+12 \geq 10$$

Subtract 12 from both sides.

$$-2x+12-12 \geq 10-12$$

$$-2x \geq -2$$

Divide both sides by -2. Because we're dividing by a negative number, remember to flip the direction of the inequality sign.

$$\frac{-2x}{-2} \leq \frac{-2}{-2}$$

$$x \leq 1$$

Topic: Graphing inequalities on a number line

Question: Graph $x > 1$ on a number line.

Answer choices:

A


B


C


D


Solution: A

Since the solution consists of all the numbers greater than 1, and "greater than" in the inequality $x > 1$ means "to the right of" on a number line, the ray we draw must start at 1 and extend out to the right. Since the solution does not include 1, we draw an open circle at 1.

Topic: Graphing inequalities on a number line

Question: Graph $x < -2$ on a number line.

Answer choices:

A

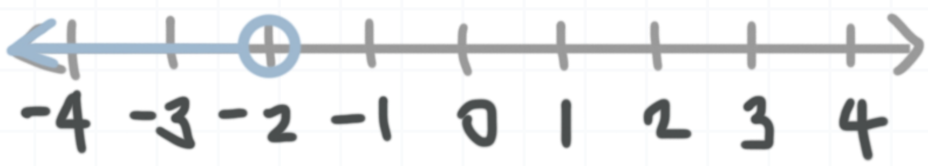
B

C

D

Solution: C

Since the solution consists of all the numbers less than -2, and "less than" in the inequality $x < -2$ means "to the left of" on a number line, the ray we draw must start at -2 and extend out to the left. Since the solution does not include -2, we draw an open circle at -2.

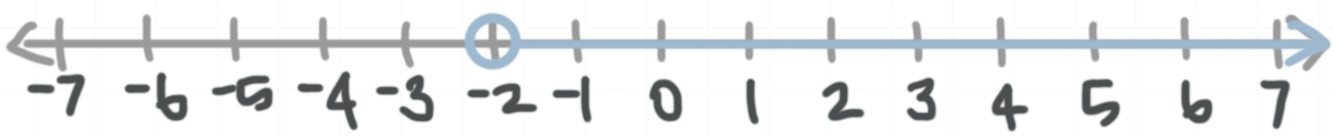


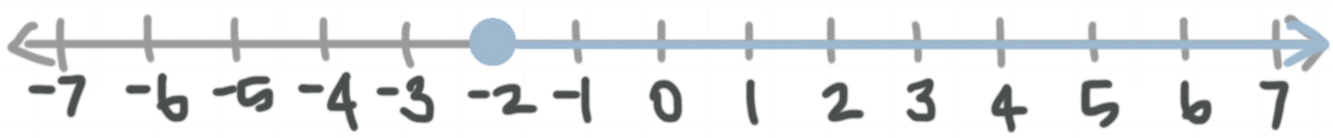
Topic: Graphing inequalities on a number line

Question: Graph $2x - 1 \ngtr x - 3$ on a number line.

Answer choices:

A 

B 

C 

D 

Solution: A

Start simplifying the inequality by adding 1 to both sides.

$$2x - 1 \ngtr x - 3$$

$$2x - 1 + 1 \ngtr x - 3 + 1$$

$$2x \ngtr x - 2$$

Subtract x from both sides.

$$2x - x \ngtr x - 2 - x$$

$$x \ngtr -2$$

If x is not greater than -2, the Trichotomy Law tells us that is must be less than or equal to -2. Therefore, we can rewrite the solution as

$$x \leq -2$$

and a sketch of the inequality on a number line is

Topic: Graphing disjunctions on a number line

Question: Graph the disjunction.

$x < -2$ or $x > 4$

Answer choices:

A 

B 

C 

D 

Solution: A

First, graph the two inequalities separately. The sketch of $x < -2$ is

and the sketch of $x > 4$ is

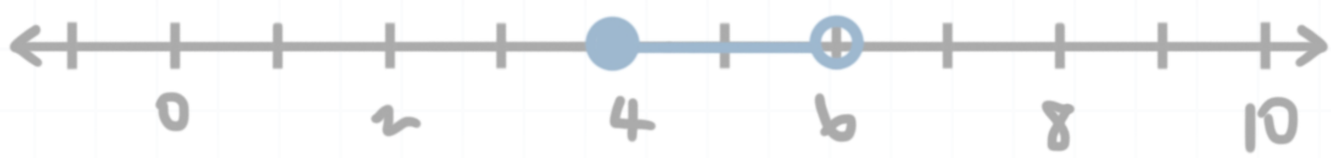
Sketching these two pieces together on the same number line, we can see the sketch of the complete disjunction.

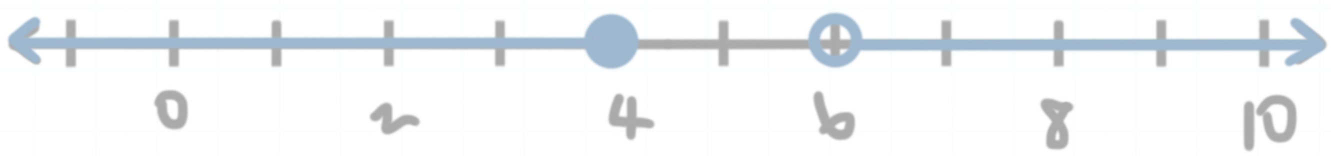
Topic: Graphing disjunctions on a number line

Question: Graph the disjunction.

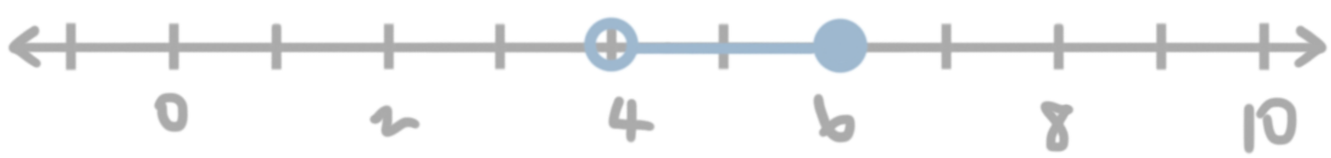
$$2x - 5 \geq 7 \text{ or } 3(x - 2) < 6$$

Answer choices:

A


B


C


D


Solution: C

Solve the first inequality,

$$2x - 5 \geq 7$$

$$2x \geq 12$$

$$x \geq 6$$

then sketch it on a number line.

Solve the second inequality,

$$3(x - 2) < 6$$

$$x - 2 < 2$$

$$x < 4$$

then sketch it on a number line.

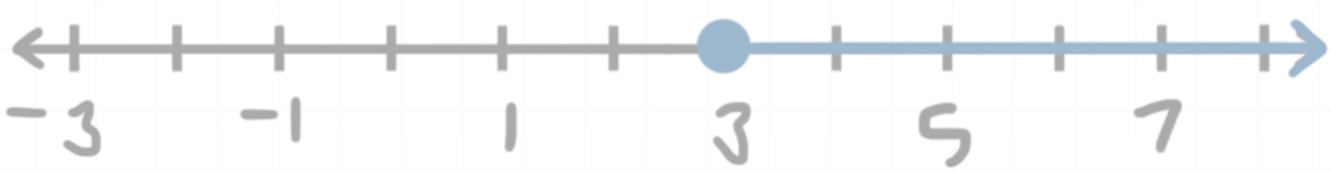
Sketching these two pieces together on the same number line, we can see the sketch of the complete disjunction.

Topic: Graphing disjunctions on a number line

Question: Graph the disjunction.

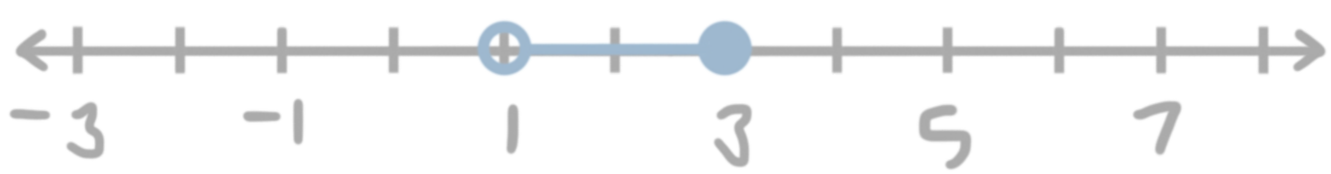
$$2(x-2)+3>1 \text{ or } 2(4-x)\leq x-1$$

Answer choices:

A 

B 

C 

D 

Solution: C

Solve the first inequality,

$$2(x-2)+3>1$$

$$2(x-2)>-2$$

$$x-2>-1$$

$$x>1$$

then sketch it on a number line.

Solve the second inequality,

$$2(4-x)\leq x-1$$

$$8-2x\leq x-1$$

$$8-3x\leq -1$$

$$-3x\leq -9$$

$$x\geq 3$$

then sketch it on a number line.

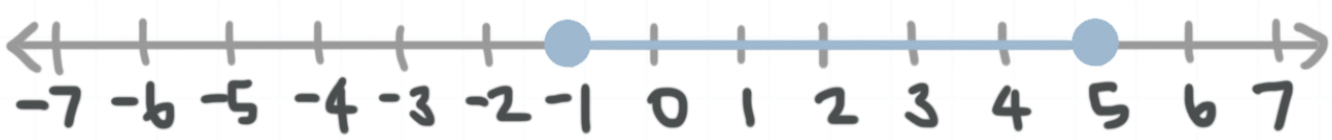
The solution of the disjunction is the set of values where both inequalities overlap. The set $x > 1$ includes the entire set $x \geq 3$, so a sketch of the complete disjunction is the sketch of $x > 1$.

Topic: Graphing conjunctions on a number line

Question: Graph the conjunction $-1 < x < 5$.

Answer choices:

A 

B 

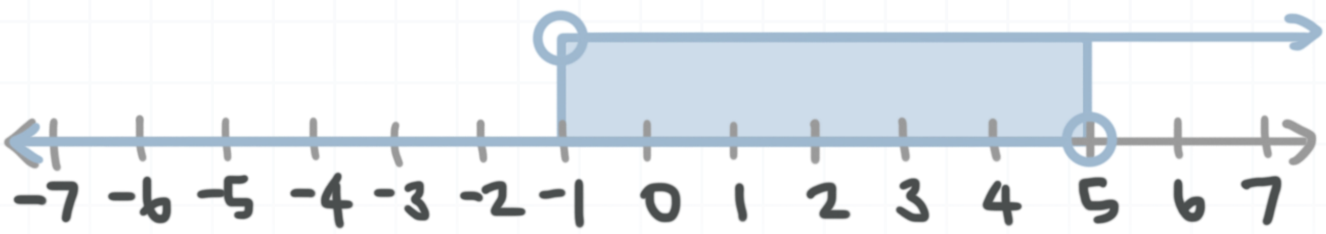
C 

D 

Solution: A

The solution of the conjunction $-1 < x < 5$ consists of all the numbers that are greater than -1 and less than 5.

Remember, $-1 < x$ is equivalent to $x > -1$, so the graph of the conjunction is the overlap of the graphs of the inequalities $x > -1$ and $x < 5$.



The shaded area shows the overlap, which is everything between -1 and 5. The overlap includes neither -1 nor 5, because there's an open circle at -1 on the graph of the inequality $x > -1$, and an open circle at 5 on the graph of the inequality $x < 5$.

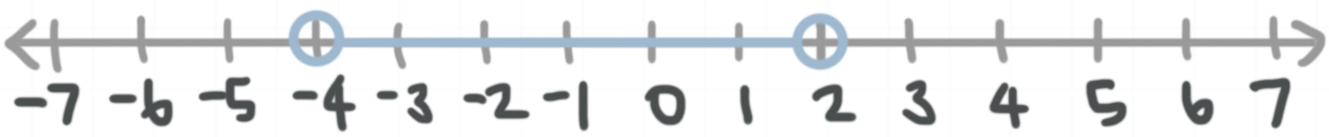
Topic: Graphing conjunctions on a number line

Question: Graph the conjunction $2 > x \geq -4$.

Answer choices:

A

B



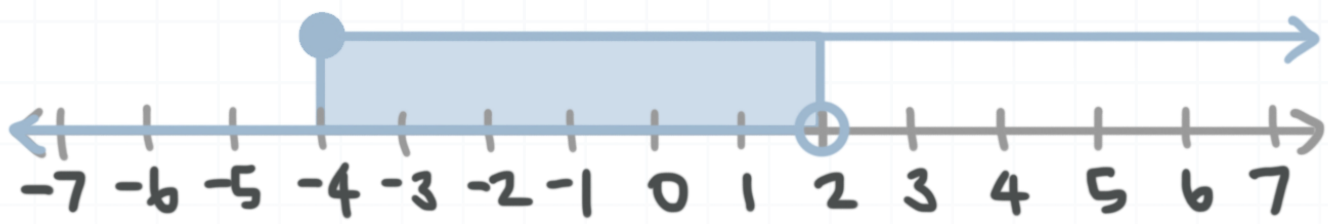
C

D

Solution: D

The conjunction $2 > x \geq -4$ can be thought of as the set of all x values that are greater than or equal to -4 and less than 2.

Remember, $2 > x$ is equivalent to $x < 2$, so in terms of graphing, that set would be the intersection (overlap) of $x \geq -4$ and $x < 2$.



The shaded area shows the overlap, which would be from -4 to 2, including -4, but not including 2.

Topic: Graphing conjunctions on a number line

Question: Graph the conjunction of the inequalities.

$$-6x + 11 > -7$$

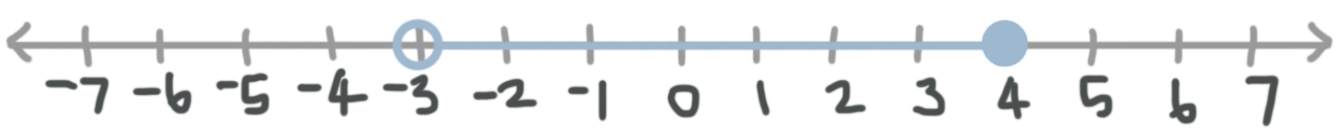
$$5x - 6 \geq -26$$

Answer choices:

A


B


C


D


Solution: B

Before graphing the conjunction of the inequalities $-6x + 11 > -7$ and $5x - 6 \geq -26$, we need to solve the inequalities separately. Begin solving $-6x + 11 > -7$.

$$-6x + 11 > -7$$

$$-6x > -18$$

$$x < 3$$

Solving $5x - 6 \geq -26$.

$$5x - 6 \geq -26$$

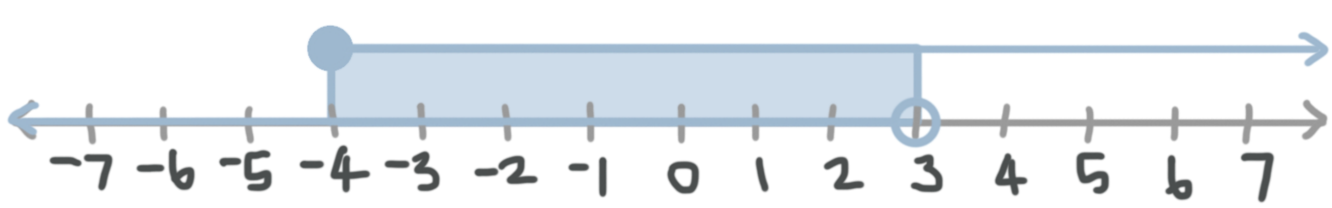
$$5x \geq -20$$

$$x \geq -4$$

Since the inequality $x \geq -4$ is equivalent to $-4 \leq x$, the conjunction of the inequalities $x \geq -4$ and $x < 3$ can be written as

$$-4 \leq x < 3$$

The solution of this conjunction consists of all the numbers that are greater than or equal to -4 and less than 3. The graph of the conjunction is the overlap of the graphs of the inequalities $x < 3$ and $x \geq -4$.



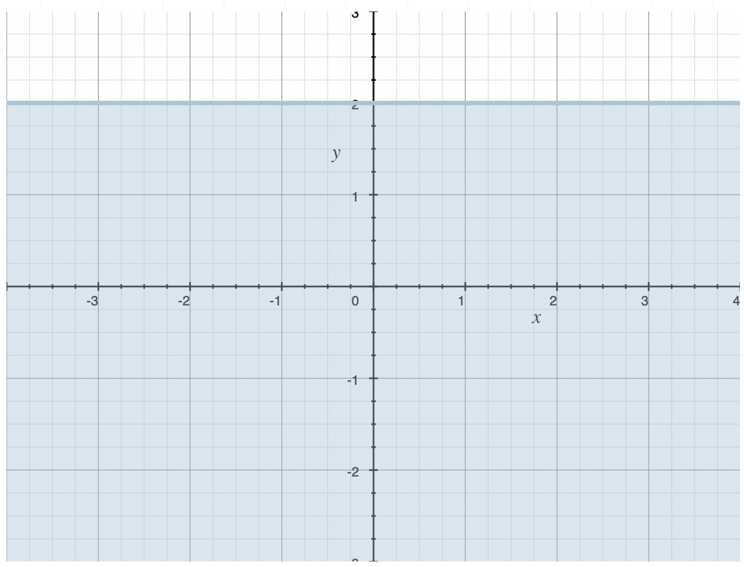
The overlap consists of an open circle at 3, a solid circle at -4, and everything between -4 and 3.

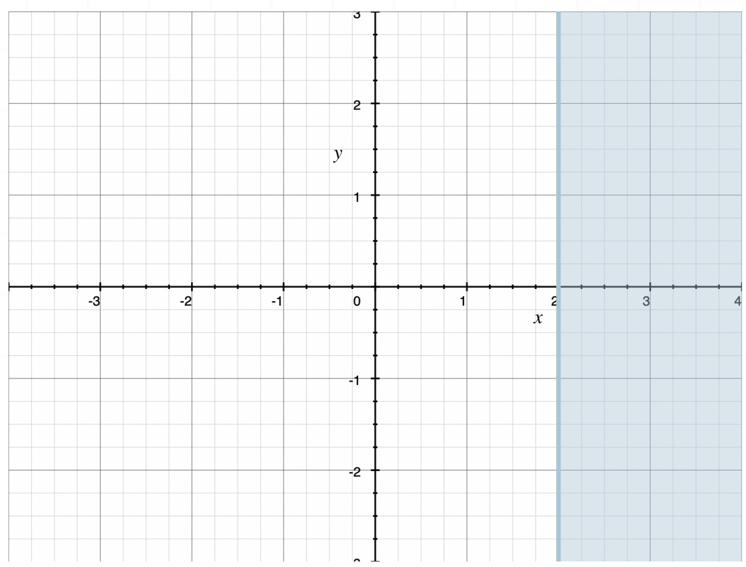
Topic: Graphing inequalities in the plane

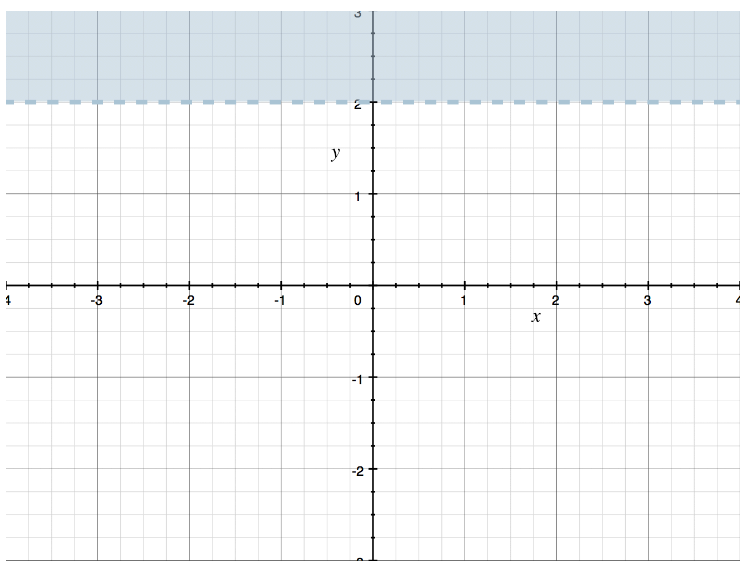
Question: Graph the linear inequality in the Cartesian coordinate system.

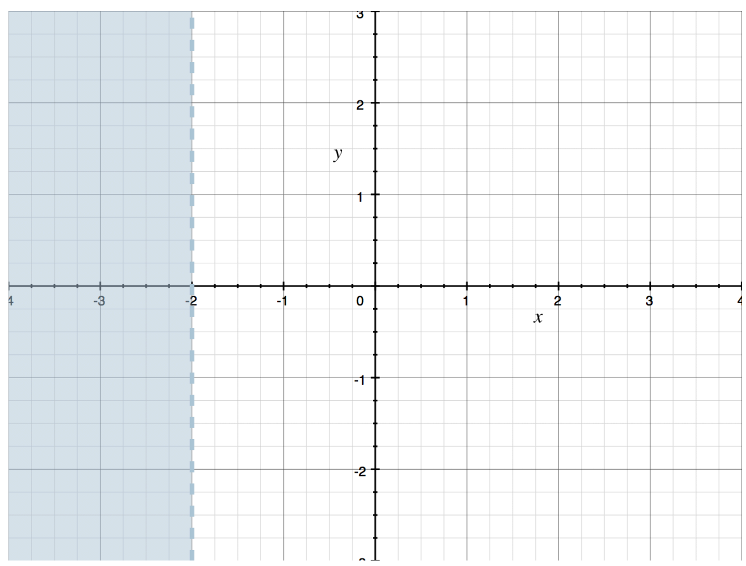
$$y > 2$$

Answer choices:

A 

B 

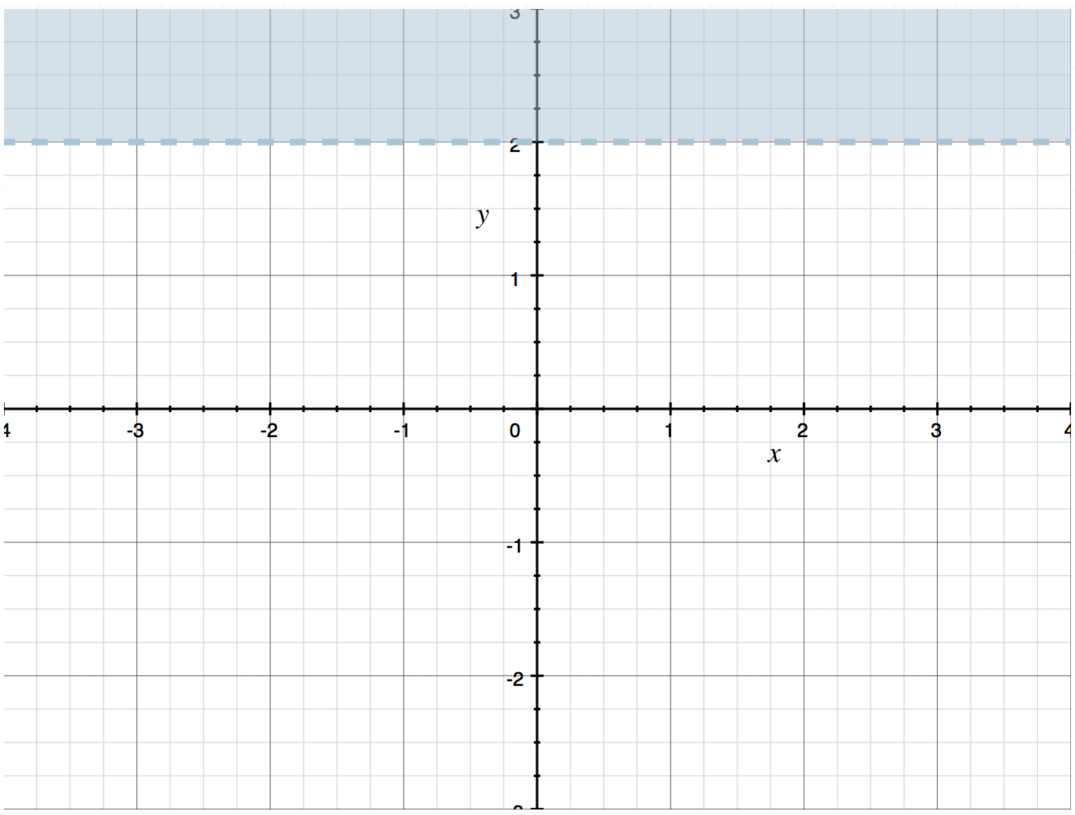
C 

D 

Solution: C

Since we're looking for all of the space where y is greater than 2, we need to draw the line $y=2$ and shade in everything above it.

Because the inequality is $y>2$ and not $y\geq 2$, we have to make sure we draw a dashed or dotted line at $y=2$ to indicate that the boundary line is not included in the graph of the inequality.

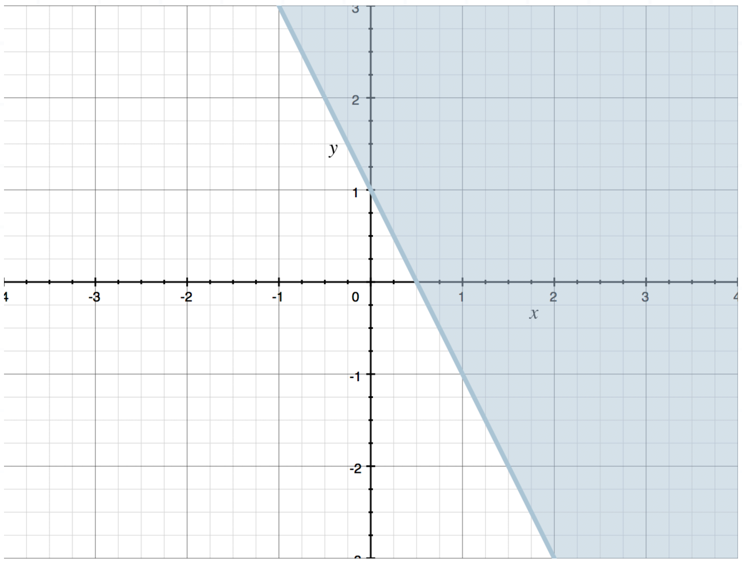


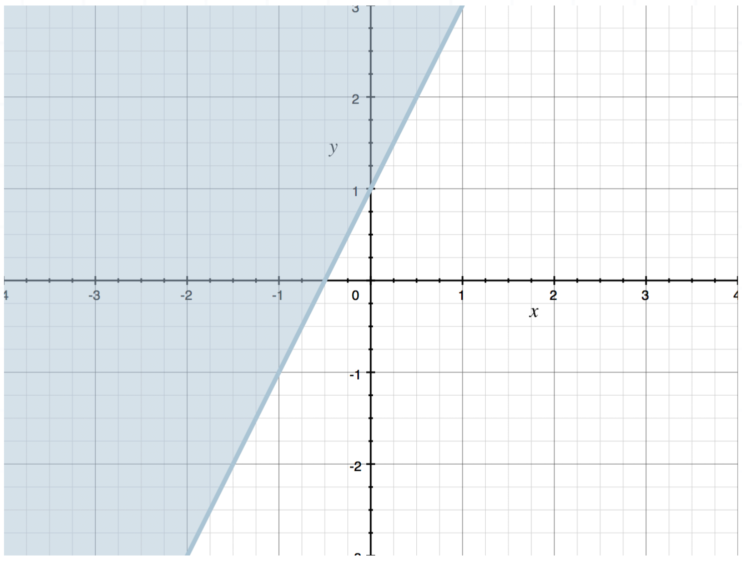
Topic: Graphing inequalities in the plane

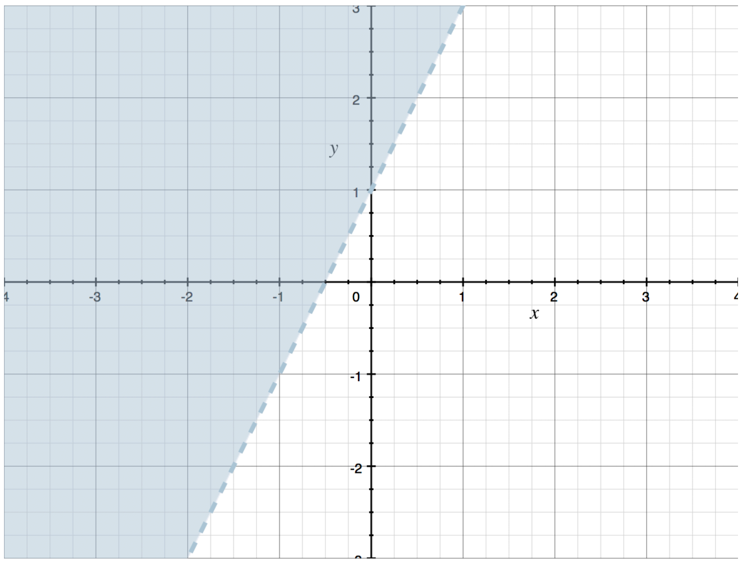
Question: Graph the linear inequality in the Cartesian coordinate system.

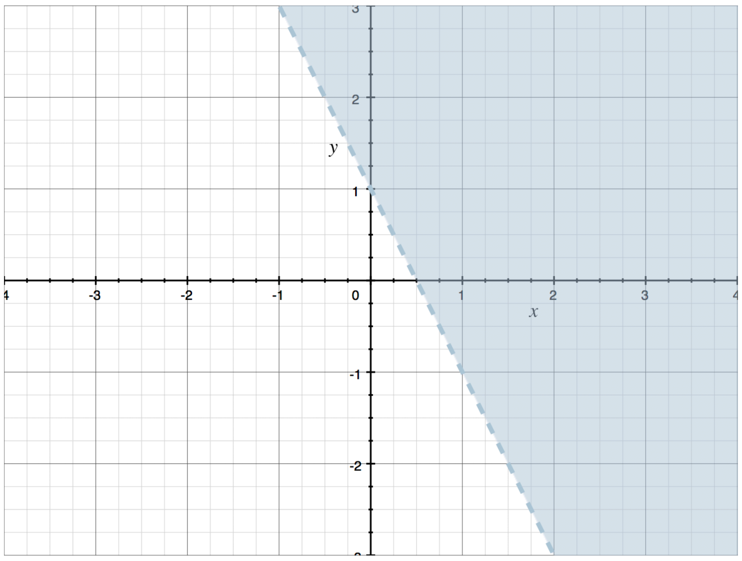
$$y \geq -2x + 1$$

Answer choices:

A 

B 

C 

D 

Solution: A

Since we're looking for all of the space where y is greater than or equal to $-2x+1$, we need to draw the line $y=-2x+1$ and shade in everything above it.

Because the inequality is $y \geq -2x+1$ and not $y > -2x+1$, we have to make sure we draw the line $y=-2x+1$ as a solid line, to indicate that the boundary line is included in the graph of the inequality.

To draw the boundary line, all we need to do is get the coordinates of any two points on it. If we let $x=0$, we get

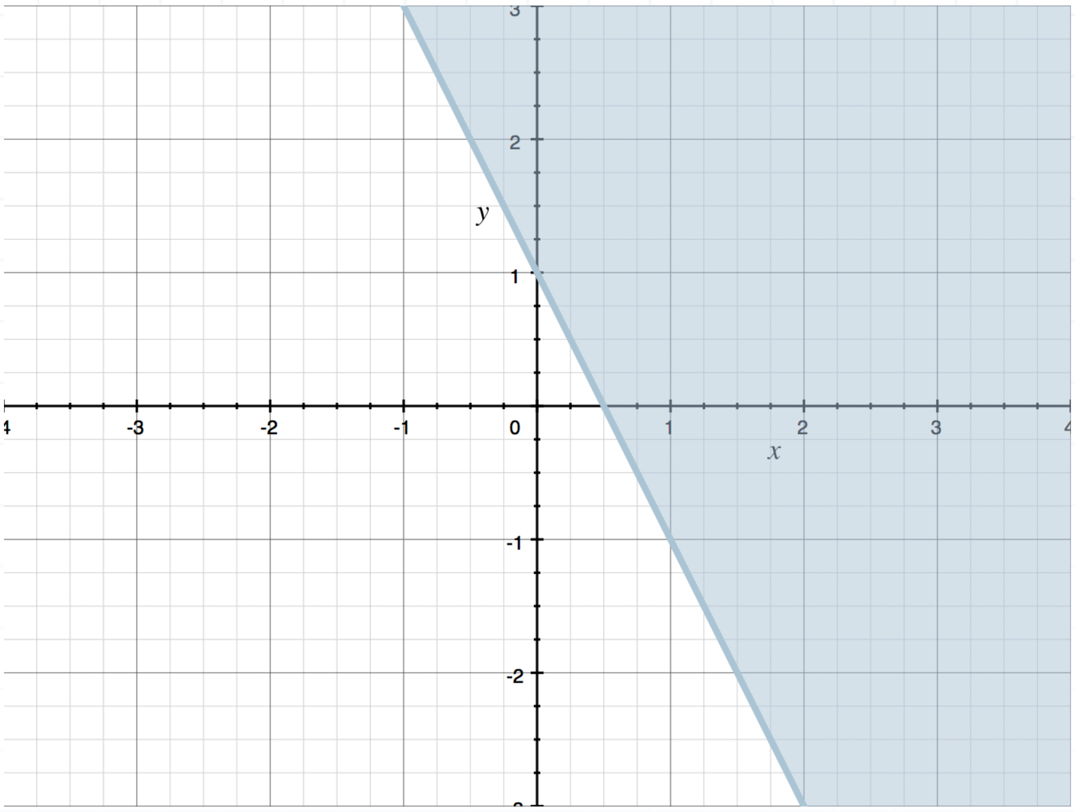
$$y=-2(0)+1=1$$

So the point $(0,1)$ is on the boundary line. If we let $x=1$, we get

$$y=-2(1)+1=-1$$

So the point $(1,-1)$ is also on the boundary line.

Now we'll draw the solid line that passes through those two points, and then shade in everything above that line.



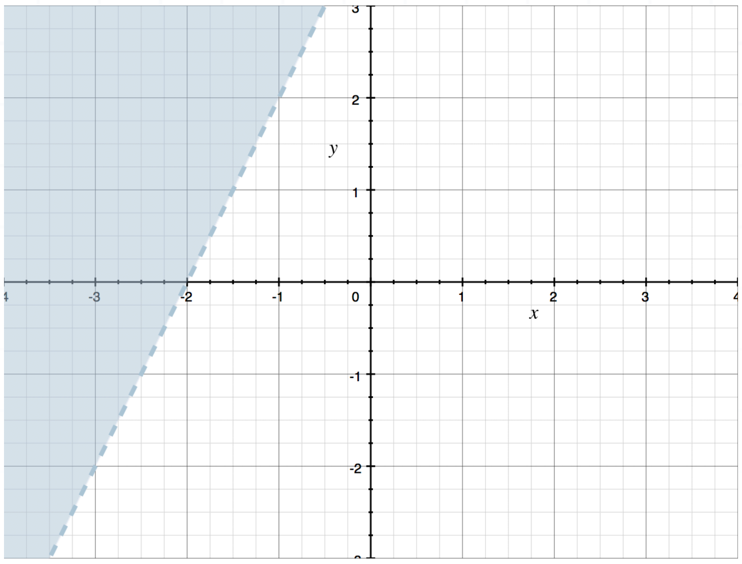
y
x
3
2
1
0
-1
-2
-3
-4
4

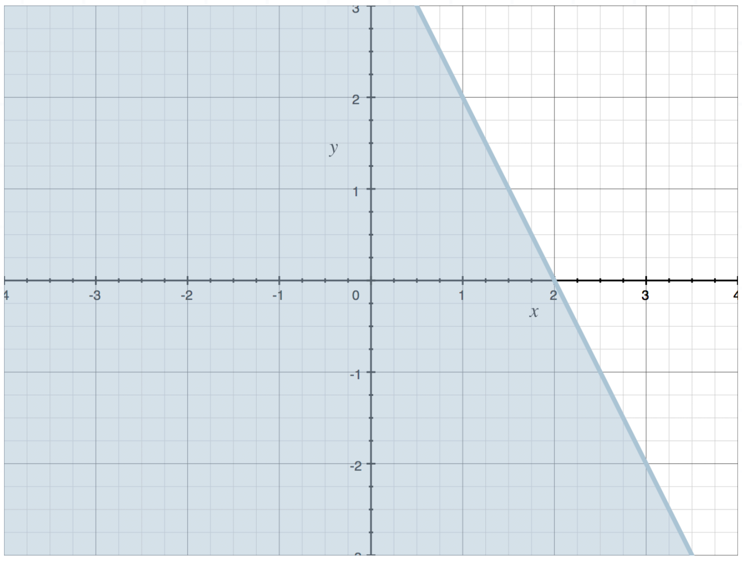
Topic: Graphing inequalities in the plane

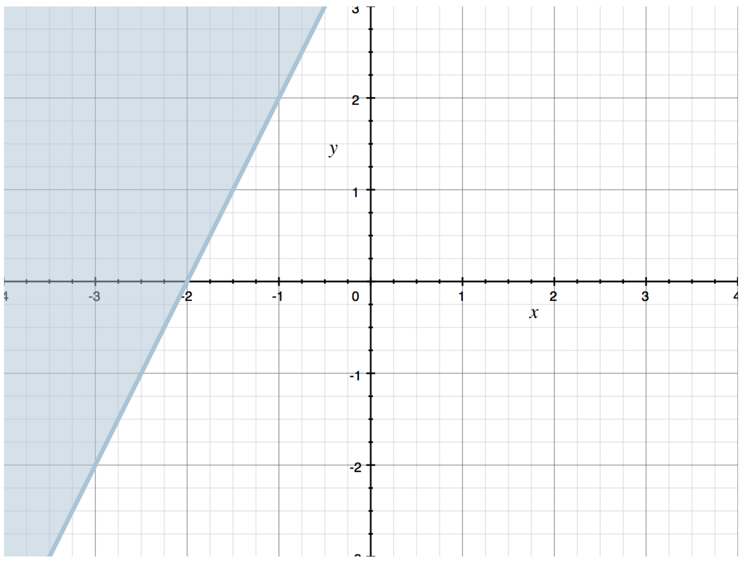
Question: Graph the linear inequality in the Cartesian coordinate system.

$$y \leq 2x + 4$$

Answer choices:

A


B


C


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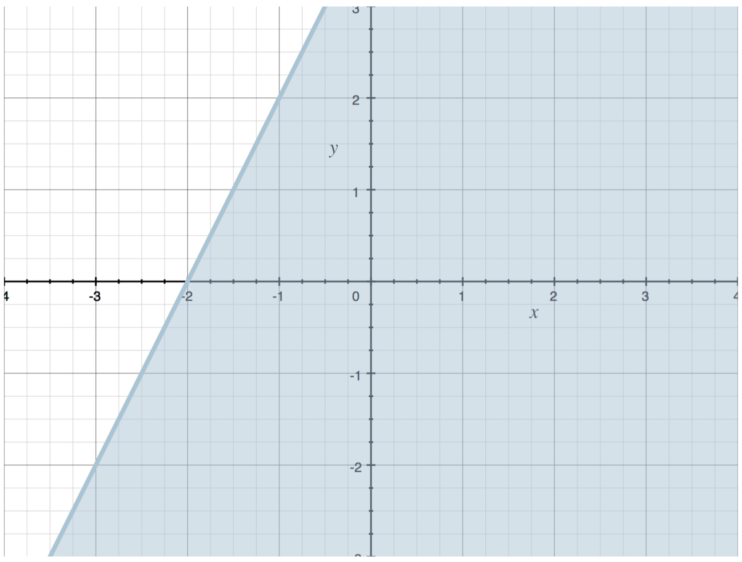


Solution: D

Since we're looking for all of the space where y is less than or equal to $2x+4$, we need to draw the line $y=2x+4$ and shade in everything below it.

Because the inequality is $y \le 2x+4$ and not $y < 2x+4$, we have to make sure we draw the line $y=2x+4$ as a solid line, to indicate that the boundary line is included in the graph of the inequality.

We'll get the coordinates of two points on the boundary line, and then draw the line that passes through those two points. If we let $x=0$, we get

$$y=2(0)+4=4$$

So the point $(0,4)$ is on the boundary line. If we let $x=1$, we get

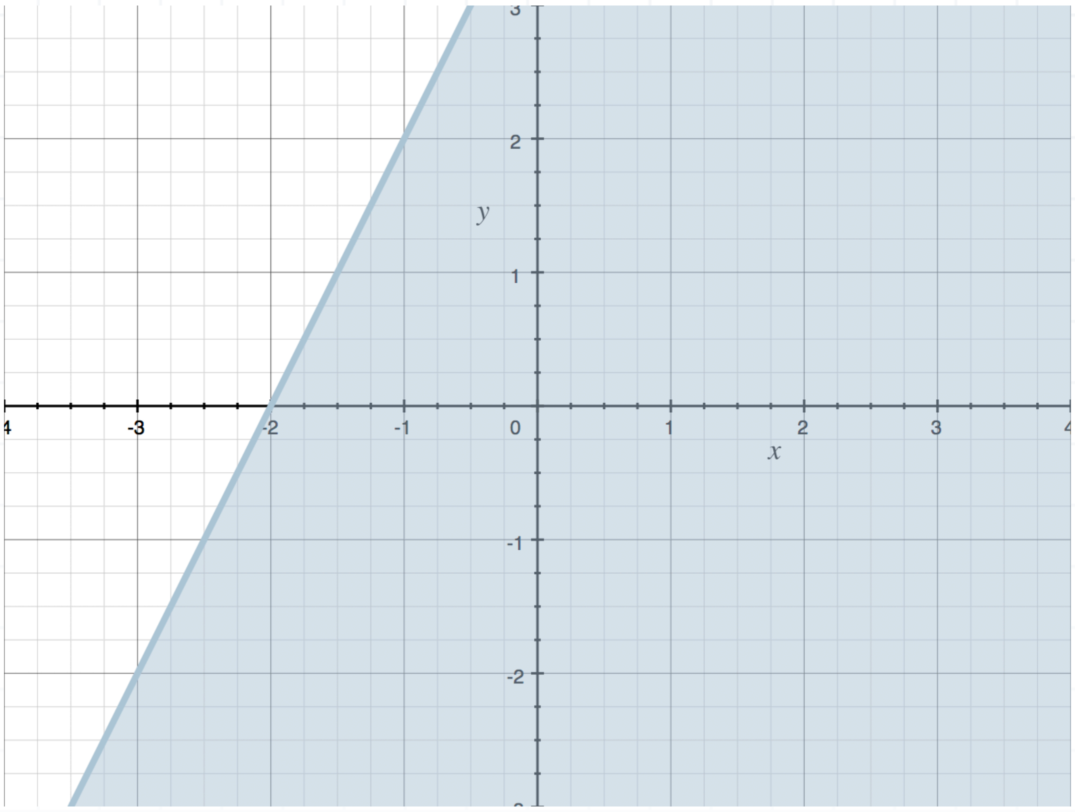
$$y=2(1)+4=6$$

So the point $(0,4)$ is on the boundary line. If we let $x=1$, we get

$$y=2(1)+4=6$$

So the point $(1,6)$ is also on the boundary line.

Now we'll draw the solid line that passes through those two points, and then shade in everything below that line.



y
x
3
2
1
-1
-2
-4
-3
-2
-1
0
1
2
3
4

Topic: Absolute value equations

Question: Solve the equation.

$$|3x - 2| = 5$$

Answer choices:

A $x = -\frac{7}{3}$ or $x = 1$

B $x = \frac{7}{3}$

C $x = \frac{7}{3}$ or $x = -1$

D $x = 1$

Solution: C

We can split the absolute value equation $|3x-2|=5$ into two related equations.

$$3x-2=5 \qquad\qquad 3x-2=-5$$

$$3x=7 \qquad\qquad 3x=-3$$

$$x=\frac{7}{3} \qquad\qquad x=-1$$

Let's now check each solution by substituting them into the original absolute value equation.

$$|3x-2|=5 \qquad\qquad |3x-2|=5$$

$$\left|3\cdot\frac{7}{3}-2\right|=5 \qquad\qquad |3(-1)-2|=5$$

$$|7-2|=5 \qquad\qquad |-3-2|=5$$

$$|5|=5 \qquad\qquad |-5|=5$$

Both equations are true, so $x=7/3$ and $x=-1$ are the solutions to the given absolute value equation.

Topic: Absolute value equations

Question: Solve the equation.

$$|2x| = x + 6$$

Answer choices:

A $x = -2$

B $x = 6$

C $x = 2$ or $x = 6$

D $x = -2$ or $x = 6$

Solution: D

We can split the absolute value equation $|2x| = x + 6$ into two related equations.

$$2x = x + 6$$

$$x = 6$$

$$2x = -(x + 6)$$

$$2x = -x - 6$$

$$3x = -6$$

$$x = -2$$

Let's now check each solution by substituting them into the original absolute value equation.

$$|2x| = x + 6$$

$$|2(6)| = 6 + 6$$

$$|12| = 12$$

$$|2x| = x + 6$$

$$|2(-2)| = -2 + 6$$

$$|-4| = 4$$

Both equations are true, so $x = 6$ and $x = -2$ are the solutions to the given absolute value equation.

Topic: Absolute value equations

Question: How many solutions does the equation have?

$$|2x - 12| = 2x$$

Answer choices:

A $\quad 0$

B $\quad 1$

C $\quad 2$

D $\quad 3$

Solution: B

We can split the absolute value equation $|2x-12|=2x$ into two related equations.

$$2x-12=2x$$

$$-12=0$$

No solution

$$2x-12=-2x$$

$$4x=12$$

$$x=3$$

Let's now check whether $x=3$ is a solution by substituting it into the original absolute value equation.

$$|2x-12|=2x$$

$$|2(3)-12|=2(3)$$

$$|6-12|=6$$

$$|-6|=6$$

The equation is true, so $x=3$ is the only solution to the given absolute value equation.

Topic: Absolute value inequalities

Question: Graph the inequality.

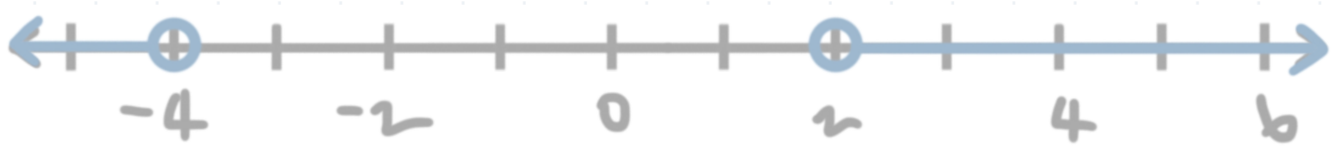
$$2|x-1|+3>9$$

Answer choices:

A


B


C


D


Solution: B

Isolate the absolute value expression on the left side of the inequality.

$$2|x-1|+3>9$$

$$2|x-1|>6$$

$$|x-1|>3$$

Since $3>0$, taking away the absolute value sign, we get

$x-1>3$ or $x-1<-3$

$x>4$ or $x<-2$

Then we can graph the disjunction $x>4$ or $x<-2$ as

Topic: Absolute value inequalities

Question: Graph the inequality.

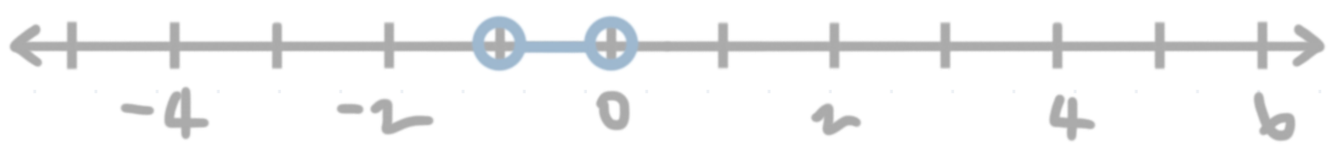
$$3(|2x+1|-2)+2<-1$$

Answer choices:

A No solution

B 

C 

D 

Solution: D

Isolate the absolute value expression on the left side of the inequality.

$$3(|2x+1|-2)+2<-1$$

$$3(|2x+1|-2)<-3$$

$$|2x+1|-2<-1$$

$$|2x+1|<1$$

Since $1>0$, taking away the absolute value sign, we get

$$-1<2x+1<1$$

$$-2<2x<0$$

$$-1<x<0$$

Then we can graph the conjunction as

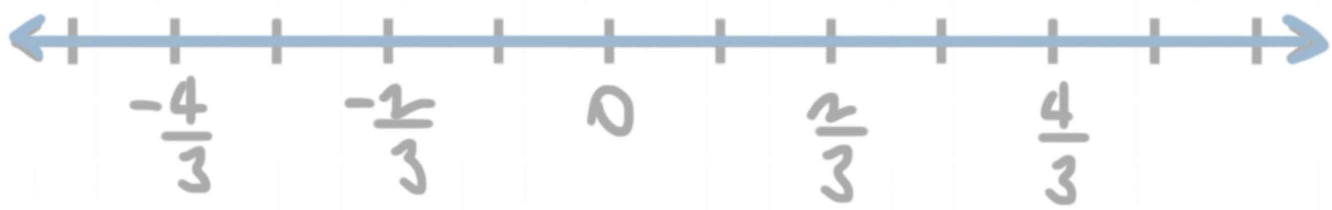
Topic: Absolute value inequalities

Question: Graph the inequality.

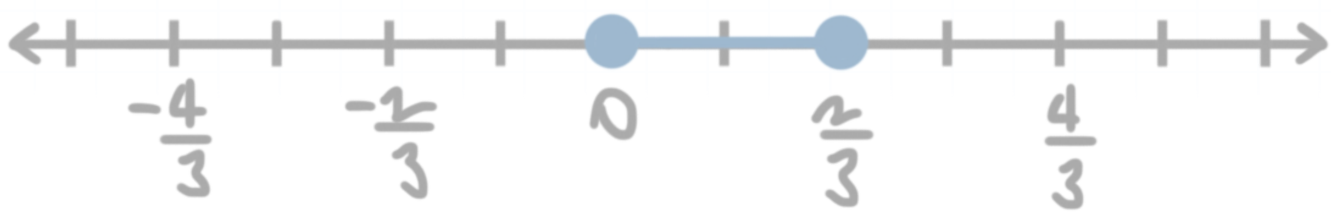
$$2 - |3x - 1| \leq 3$$

Answer choices:

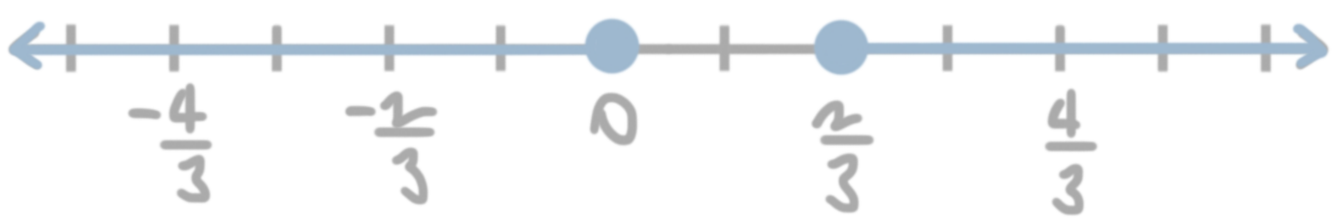
A No solution



B



C



D

Solution: B

Isolate the absolute value expression on the left side of the inequality.

$$2 - |3x - 1| \le 3$$

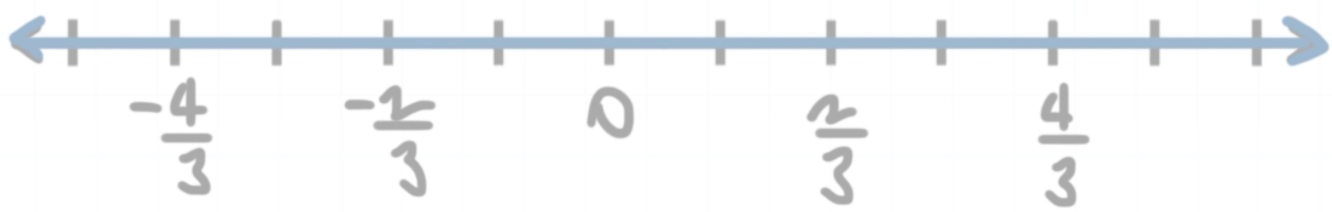
$$-|3x - 1| \le 1$$

$$|3x - 1| \ge -1$$

Because $|3x - 1|$ is always positive, we have an inequality that tells us

positive $\ge$ negative

so the solution is the set of all real numbers, and a sketch of the graph is



Topic: Two-step problems

Question: If $x + 5 = 10$, what is $x + 3$?

Answer choices:

A 8

B 10

C 5

D 13

Solution: A

First, we'll solve the equation $x+5=10$ to find the value of x.

$$x+5=10$$

$$x+5-5=10-5$$

$$x=5$$

Now we'll take the value we found for x and plug it into the expression $x+3$ to answer the question we've been asked.

$$x+3$$

$$5+3$$

$$8$$

Topic: Two-step problems

Question: If $6(2x - 5) = 54$, what is $3x - 4$?

Answer choices:

A 25

B 17

C 24

D 19

Solution: B

First, we'll solve the equation $6(2x-5)=54$ to find the value of x.

$$6(2x-5)=54$$

$$12x-30=54$$

$$12x=84$$

$$x=7$$

Now we'll take the value we found for x and plug it into the expression $3x-4$ to answer the question we've been asked.

$$3x-4$$

$$3(7)-4$$

$$21-4$$

$$17$$

Topic: Two-step problems

Question: If $9t - 4 = 3 - 5t$, then what is the value of $12t^2 + 2$?

Answer choices:

A 5

B 6

C 8

D 10

Solution: A

First, we'll solve the equation $9t - 4 = 3 - 5t$ to find the value of t.

$$9t - 4 = 3 - 5t$$

$$14t - 4 = 3$$

$$14t = 7$$

$$t = \frac{1}{2}$$

Now we'll take the value we found for t and plug it into the expression $12t^2 + 2$ to answer the question we've been asked.

$$12\left(\frac{1}{2}\right)^2 + 2$$

$$12\left(\frac{1}{4}\right) + 2$$

$$3 + 2$$

$$5$$

Topic: Solving systems with substitution

Question: Use substitution to find the unique solution to the system of equations.

$$y = x + 7$$

$$x + 2y = -16$$

Answer choices:

A $(10,3)$

B $(-10,3)$

C $(10,-3)$

D $(-10,-3)$

Solution: D

Since the first equation is already solved for y, we'll make a substitution for y in the second equation, so that we can get the second equation in terms of only x and then solve for x.

$$x + 2y = -16$$

$$x + 2(x + 7) = -16$$

$$x + 2x + 14 = -16$$

$$3x + 14 = -16$$

$$3x = -30$$

$$x = -10$$

Now we'll take the value we found for x and plug it into the first equation to find the value of y.

$$y = x + 7$$

$$y = -10 + 7$$

$$y = -3$$

Topic: Solving systems with substitution

Question: Use substitution to find the unique solution to the system of equations.

$$y = x + 3$$

$$2x + y = 10$$

Answer choices:

A $\left(-\frac{16}{3}, -\frac{7}{3}\right)$

B $\left(-\frac{7}{3}, -\frac{16}{3}\right)$

C $\left(\frac{16}{3}, \frac{7}{3}\right)$

D $\left(\frac{7}{3}, \frac{16}{3}\right)$

Solution: D

Since the first equation is already solved for y, we'll make a substitution for y in the second equation, so that we can get the second equation in terms of only x and then solve for x.

$$2x + y = 10$$

$$2x + (x + 3) = 10$$

$$2x + x + 3 = 10$$

$$3x + 3 = 10$$

$$3x = 7$$

$$x = \frac{7}{3}$$

Now we'll take the value we found for x and plug it into the first equation to find the value of y.

$$y = x + 3$$

$$y = \frac{7}{3} + 3$$

$$y = \frac{7}{3} + \frac{9}{3}$$

$$y = \frac{16}{3}$$

Topic: Solving systems with substitution

Question: Use substitution to find the unique solution to the system of equations.

$$3x - y = -5$$

$$y = -2x - 5$$

Answer choices:

A $(2,1)$

B $(-2,1)$

C $(-2,-1)$

D $(2,-1)$

Solution: C

Since the second equation is already solved for y, we'll make a substitution for y in the first equation, so that we can get the first equation in terms of only x and then solve for x.

$$3x - y = -5$$

$$3x - (-2x - 5) = -5$$

$$3x + 2x + 5 = -5$$

$$5x + 5 = -5$$

$$5x = -10$$

$$x = -2$$

Now we'll take the value we found for x and plug it into the second equation to find the value of y.

$$y = -2x - 5$$

$$y = -2(-2) - 5$$

$$y = 4 - 5$$

$$y = -1$$

Topic: Solving systems with elimination

Question: Use elimination to find the unique solution to the system of equations.

$$x - 3y = -7$$

$$2x - 3y = 4$$

Answer choices:

A $(12,7)$

B $(11,6)$

C $(9,3)$

D $(-11,-6)$

Solution: B

Since the y-term in each equation is $-3y$, we'll subtract the second equation from the first equation.

$$x - 3y - (2x - 3y) = -7 - (4)$$

$$x - 3y - 2x + 3y = -7 - 4$$

$$-x = -11$$

$$x = 11$$

Now that we have the value of x, we'll plug it into the original first equation and solve for y.

$$x - 3y = -7$$

$$11 - 3y = -7$$

$$-3y = -18$$

$$y = 6$$

Topic: Solving systems with elimination

Question: Use elimination to find the unique solution to the system of equations.

$$x - 2y = -1$$

$$2x - 3y = 4$$

Answer choices:

A $(11,6)$

B $(-11,-6)$

C $(-11,6)$

D $(11,-6)$

Solution: A

We'll multiply through the first equation by 2 so that the x-term in each equation will be $2x$.

$$x - 2y = -1$$

$$2(x - 2y) = 2(-1)$$

$$2x - 4y = -2$$

Now that the x-term in each equation is $2x$, we'll subtract the original second equation from the new first equation.

$$2x - 4y - (2x - 3y) = -2 - (4)$$

$$2x - 4y - 2x + 3y = -2 - 4$$

$$-y = -6$$

$$y = 6$$

Now that we have the value of y, we'll plug it into the original first equation and solve for x.

$$x - 2y = -1$$

$$x - 2(6) = -1$$

$$x - 12 = -1$$

$$x = 11$$

Topic: Solving with elimination

Question: Use elimination to find the unique solution to the system of equations.

$$3x - 4y = 7$$

$$2x - 7y = -4$$

Answer choices:

A $(-5,2)$

B $(5,2)$

C $(-5,-2)$

D $(5,-2)$

Solution: B

Multiply through the first equation by 2 and the second equation by 3 so that both equations will contain a $6x$.

$$2(3x-4y=7)$$

$$2(3x)-2(4y)=2(7)$$

$$6x-8y=14$$

and

$$3(2x-7y=-4)$$

$$3(2x)-3(7y)=3(-4)$$

$$6x-21y=-12$$

Then the new system is

$$6x-8y=14$$

$$6x-21y=-12$$

Now that both equations include a $6x$, we should be able to subtract one from the other in order to eliminate it.

$$6x-8y-(6x-21y)=14-(-12)$$

$$6x-8y-6x+21y=14+12$$

$$13y=26$$

$$y = 2$$

Now that we have a value for y, we can plug it back into one of the original equations to solve for the corresponding value of x.

$$3x - 4y = 7$$

$$3x - 4(2) = 7$$

$$3x - 8 = 7$$

$$3x = 15$$

$$x = 5$$

Topic: Solving systems three ways

Question: Which solution is not being done with substitution? The steps are not explained, so we'll need to figure out what was done in each step.

Answer choices:

A

$$\begin{aligned} y &= x-3 \\ y &= 4x-9 \\ \hline x-3 &= 4x-9 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

B

$$\begin{aligned} y &= x-3 \\ y &= 4x-9 \\ \hline 0 &= -3x+6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

C

$$\begin{aligned} y &= x-3 \\ y &= 4x-9 \\ \hline x &= y+3 \\ y &= 4(y+3)-9 \\ y &= 4y+12-9 \\ y &= 4y+3 \\ -3y &= 3 \\ y &= -1 \end{aligned}$$

D

$$\begin{aligned} y &= x-3 \\ y &= 4x-9 \\ \hline x &= \frac{y+9}{4} \\ y &= \frac{y+9}{4}-3 \\ 4y &= y+9-12 \\ 3y &= -3 \\ y &= -1 \end{aligned}$$

Solution: B

Looking at answer choice B,

$$\begin{array}{l} y = x - 3 \\ \underline{y = 4x - 9} \\ 0 = -3x + 6 \end{array}$$

this first step consisted of subtracting the second equation from the first equation, so this is solving by elimination, not substitution.

Topic: Solving systems three ways

Question: To solve the system by elimination, which of these would not be a useful first step?

$$5x + y = 13$$

$$x - 2y = 7$$

Answer choices:

A Multiply the second equation by -5.

B Multiply the first equation by 2.

C Subtract the second equation from the first.

D Divide the second equation by 2.

Solution: C

If we did the step in answer choice A, we'd have the following system of equations:

$$5x + y = 13$$

$$-5x + 10y = -35$$

We could then add the two equations and eliminate x. So this would be a useful first step.

If we did the step in answer choice B, we'd have the following system of equations:

$$10x + 2y = 26$$

$$x - 2y = 7$$

We could then add the two equations and eliminate y. So this would be a useful first step.

If we did the step in answer choice C, we'd have the following equation:

$$5x + y - x + 2y = 13 - 7$$

$$4x + 3y = 6$$

At this point there's no single step that could be done to eliminate x or y, so this wouldn't be a useful first step.

If we did the step in answer choice D, we'd have the following system of equations:

$$5x + y = 13$$

$$\frac{1}{2}x - y = \frac{7}{2}$$

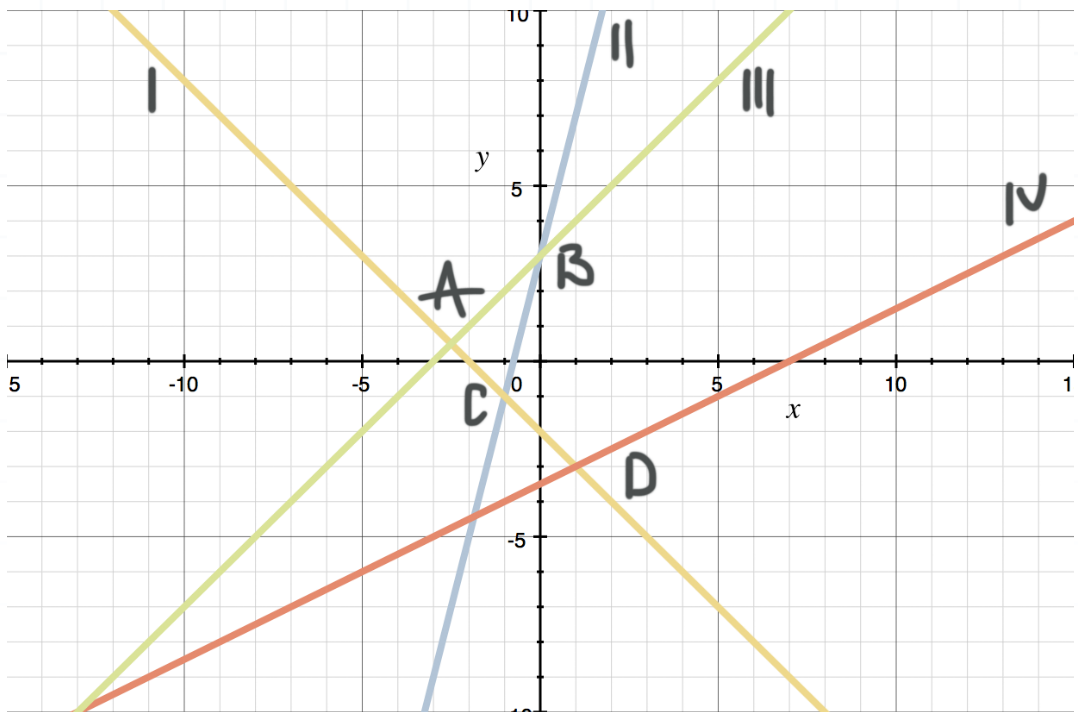
We could then add the two equations and eliminate y. So this would be a useful first step.

Topic: Solving systems three ways

Question: Which point in the graph is the solution to the system?

$$4x - y = -3$$

$$x + y = -2$$



Answer choices:

A Point A

B Point B

C Point C

D Point D

Solution: C

We can rewrite the two equations in slope-intercept form, and then see which two intersecting graphs belong to those equations. Rewriting $4x - y = -3$ gives

$$y = 4x + 3$$

This line has a slope of 4 and a y-intercept of 3, which means it corresponds to line II. Rewriting $x + y = -2$ gives

$$y = -x - 2$$

This line has a slope of -1 and a y-intercept of -2, which means it corresponds to line I. Lines I and II meet at point C.

Topic: Systems of linear inequalities

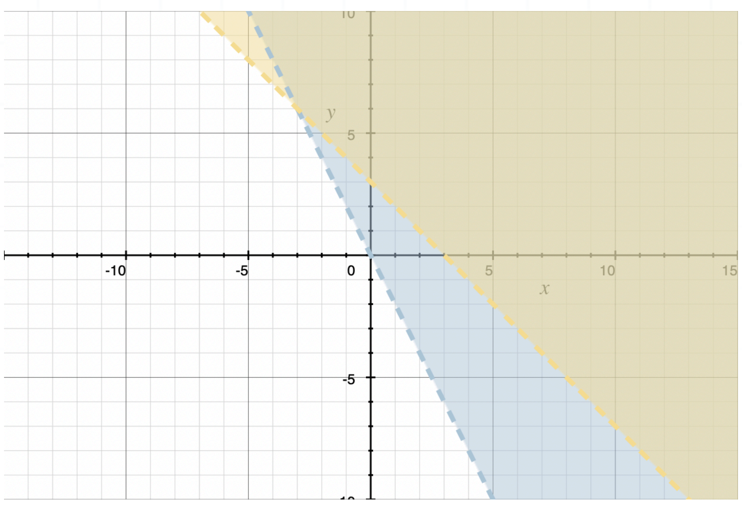
Question: Graph the solution of the system of linear inequalities.

$$2x + y > 0$$

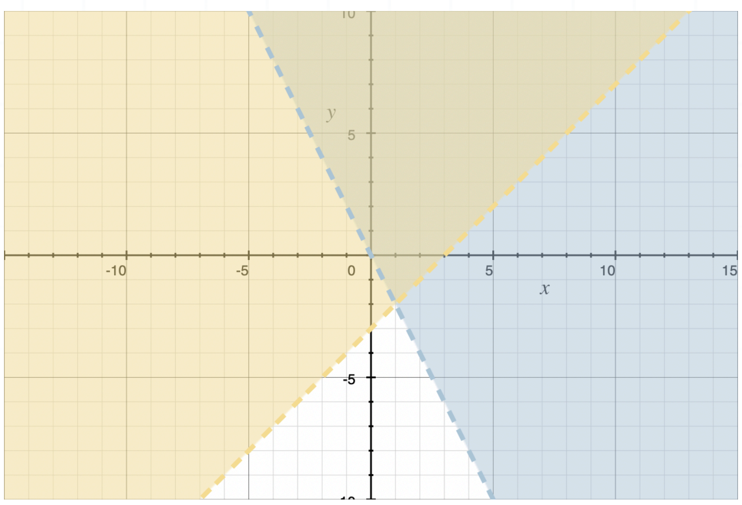
$$x - y < 3$$

Answer choices:

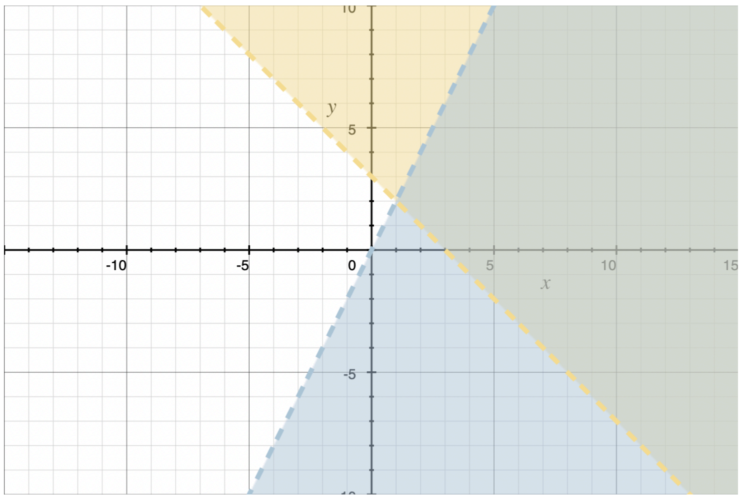
A



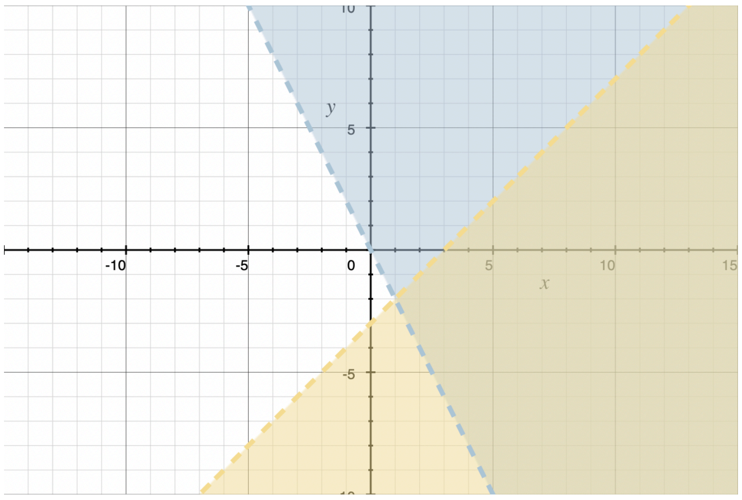
B



C



D



Solution: B

We need to find the boundary lines of each inequality by graphing their corresponding equations. We'll rewrite the corresponding equations in slope-intercept form.

$$2x + y = 0$$

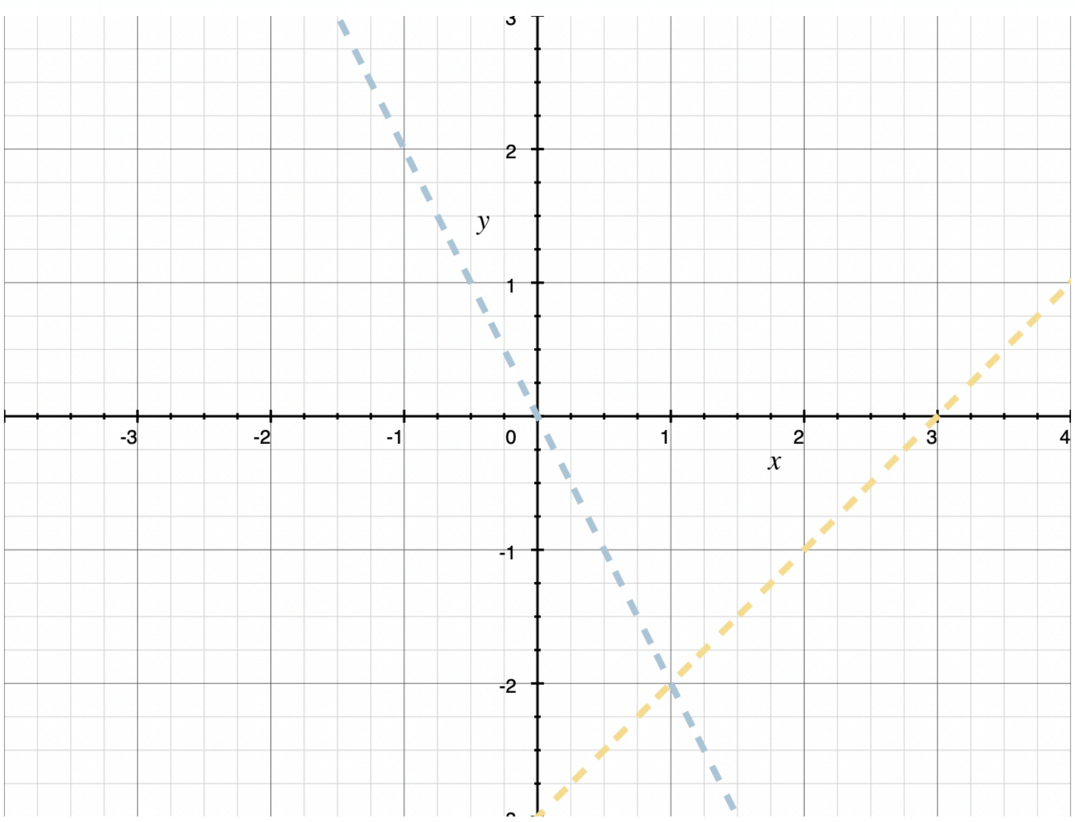
$$y = -2x$$

and

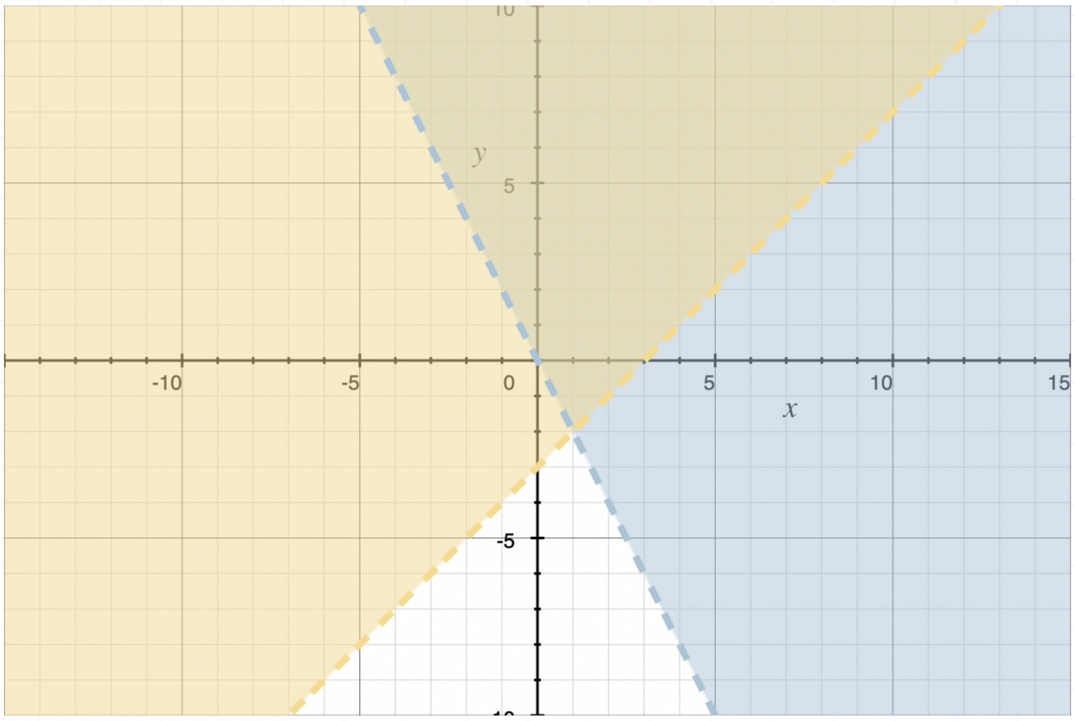
$$x - y = 3$$

$$y = x - 3$$

Because the inequalities are $<$ and $>$, the lines should be dashed.



We need to shade above $y = -2x$ and below $y = x - 3$, and the solution to the system of inequalities is



10
y
5
-10
-5
0
5
10
15
x
-5

Topic: Systems of linear inequalities

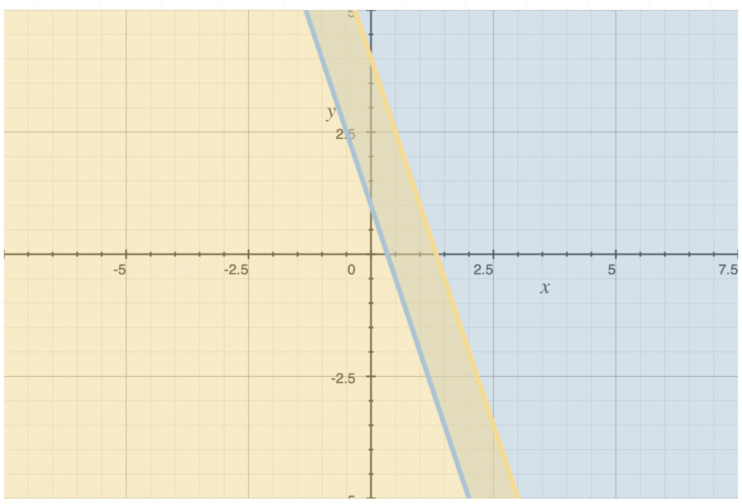
Question: Graph the solution of the system of linear inequalities.

$$3x + y \geq 1$$

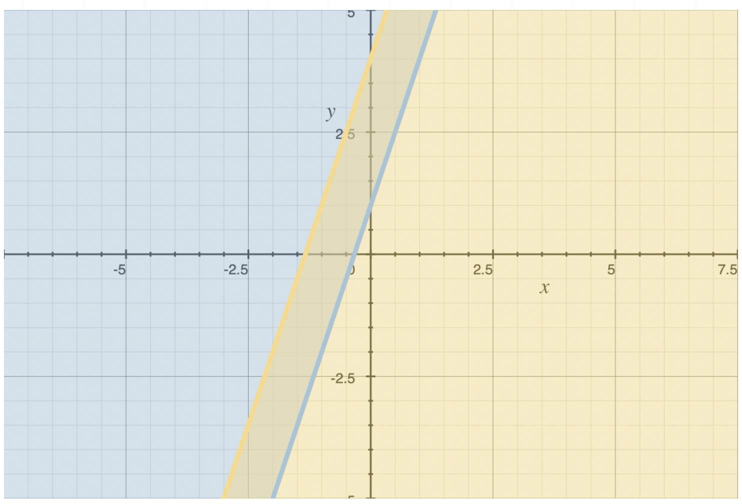
$$3x + y \leq 4$$

Answer choices:

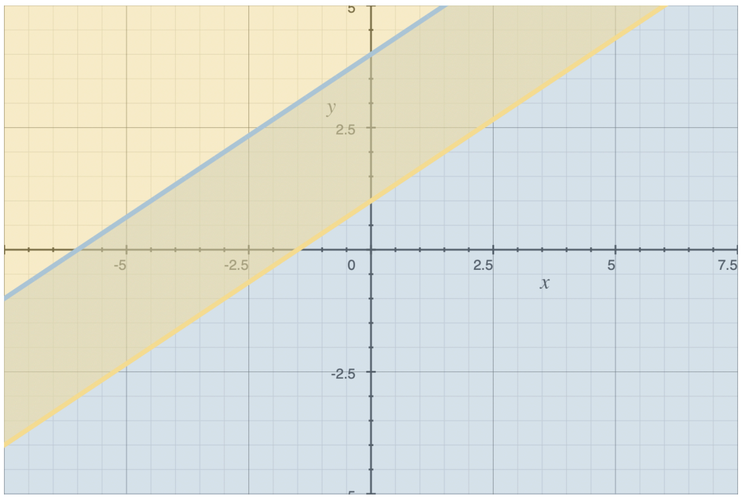
A



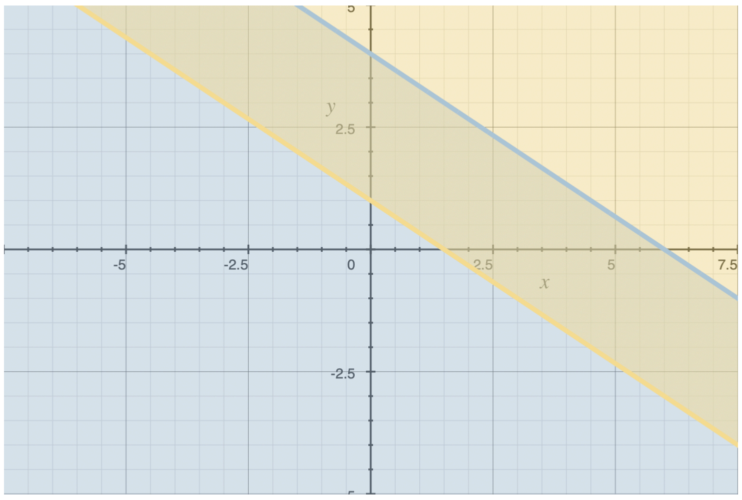
B



C



D



Solution: A

We need to find the boundary lines of each inequality by graphing their corresponding equations. We'll rewrite the corresponding equations in slope-intercept form.

$$3x + y = 1$$

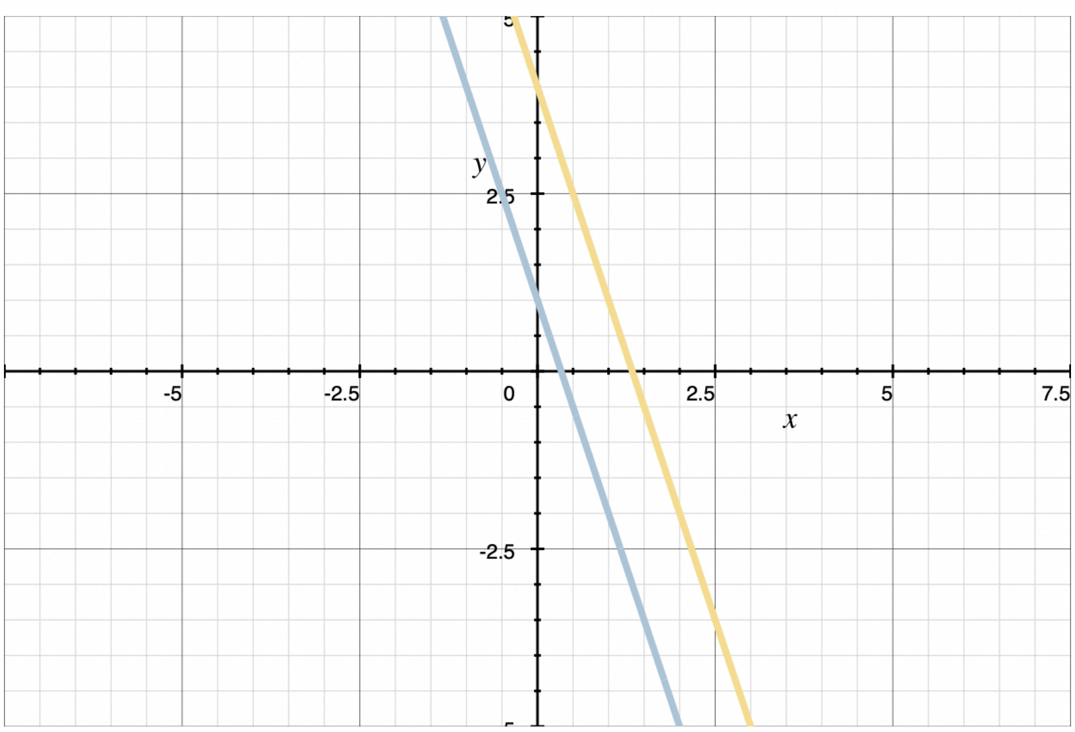
$$y = -3x + 1$$

and

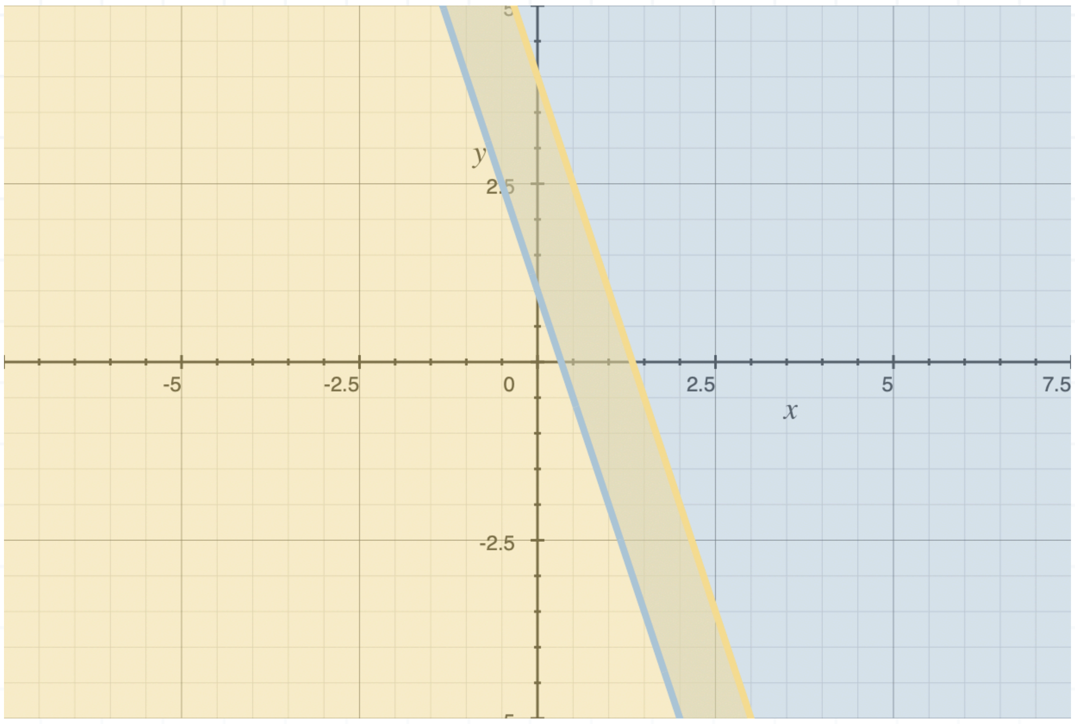
$$3x + y = 4$$

$$y = -3x + 4$$

Because the inequalities are $\leq$ and $\geq$, the lines should be solid.



We need to shade above $y = -3x + 1$ and below $y = -3x + 4$, and the solution to the system of inequalities is



y
2.5
-2.5
-5
-2.5
0
2.5
5
7.5
x

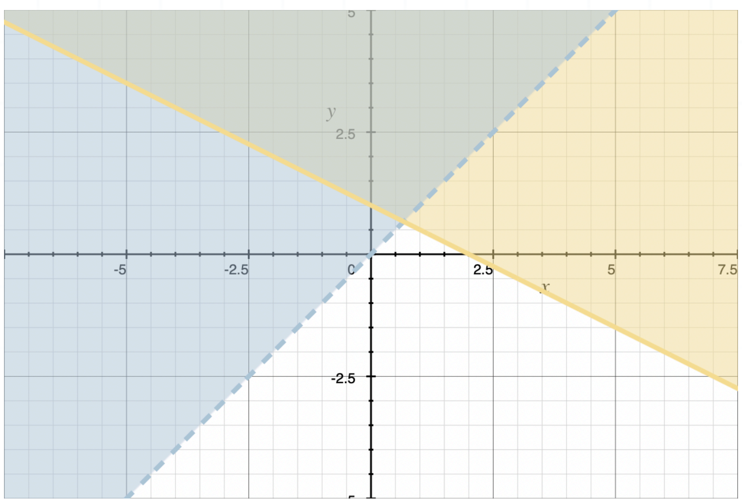
Topic: Systems of linear inequalities

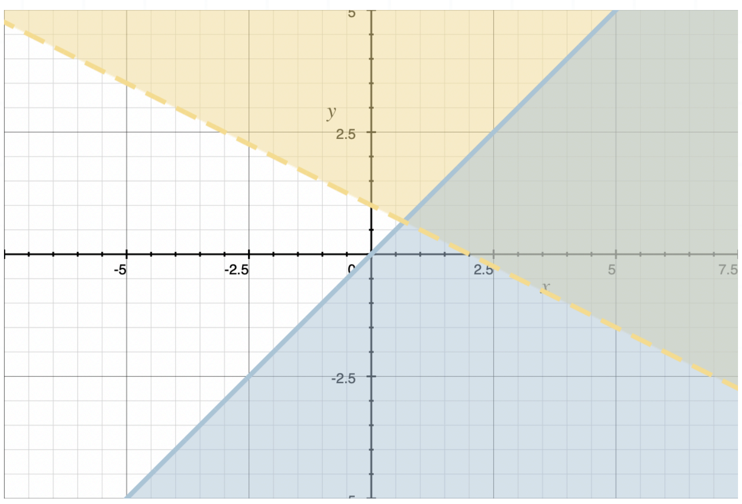
Question: Graph the solution of the system of linear inequalities.

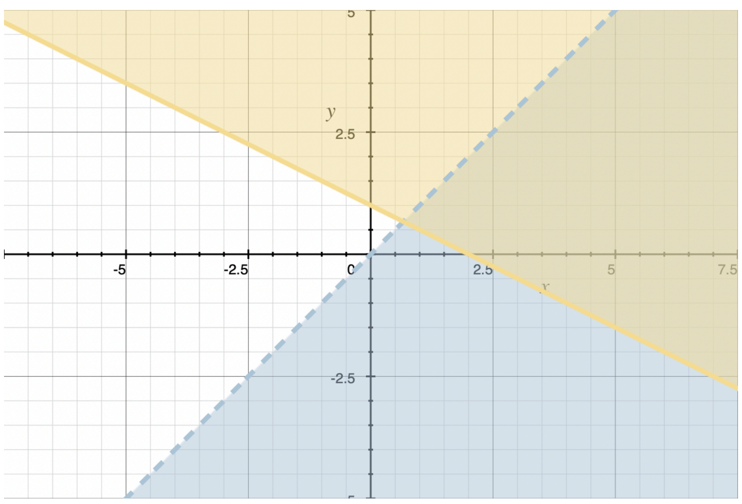
$$x > y$$

$$x + 2y \geq 2$$

Answer choices:

A 

B 

C 

D 

Solution: C

We need to find the boundary lines of each inequality by graphing their corresponding equations. We'll rewrite the corresponding equations in slope-intercept form.

$$x = y$$

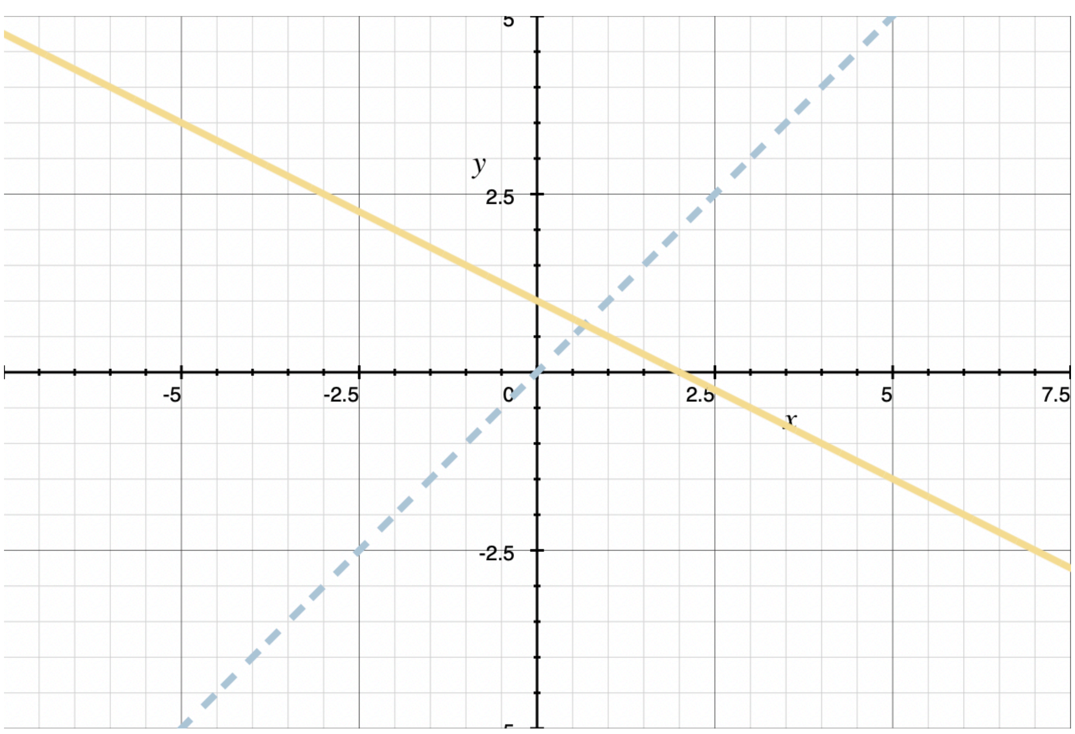
$$y = x$$

and

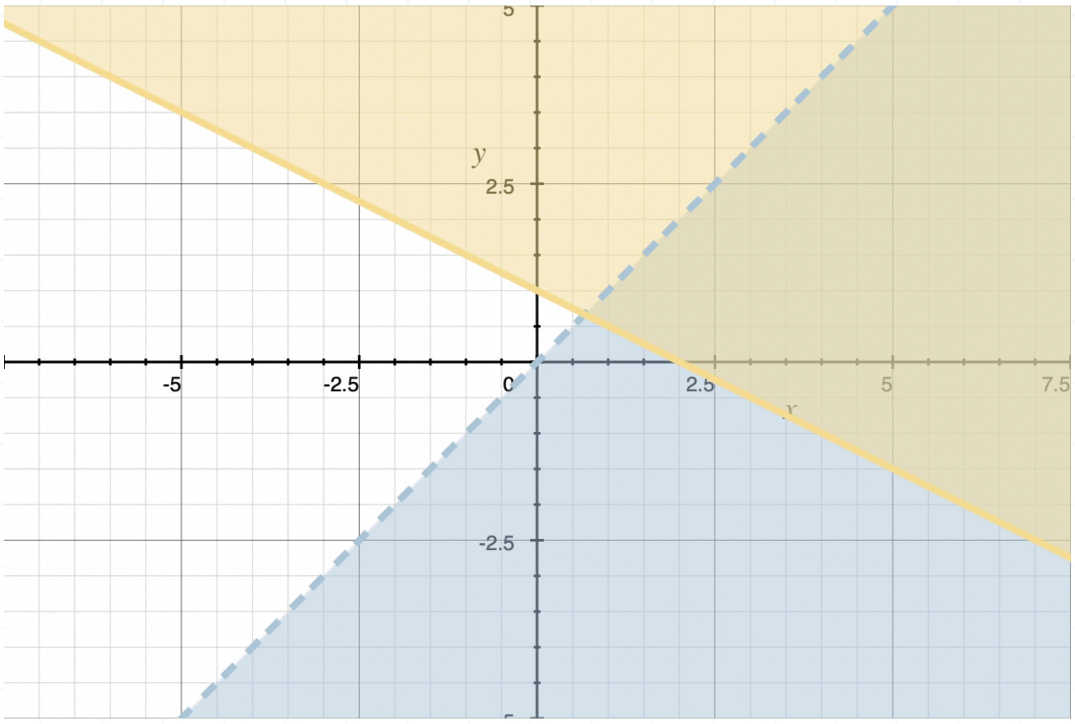
$$x + 2y = 2$$

$$y = -\frac{1}{2}x + 1$$

Because the inequalities are $\geq$ and $>$, one line will be dashed and the other will be solid.



We need to shade below $y = x$ and above $y = -(1/2)x + 1$, and the solution to the system of inequalities is


5
y
2.5
-5
-2.5
0
2.5
5
7.5
x
-2.5

W W W . K R I S T A K I N G M A T H . C O M