Adding and subtracting like terms

Now that we understand that $x = 1x^1$ (this is the concept of the "understood 1"), we can start talking about how to combine like terms. We'll begin here by looking at how to add and subtract like terms, and then later we'll talk about how to multiply and divide like terms.

What is a "like term"?

We define like terms differently depending on whether we're adding and subtracting, or multiplying and dividing. When we're adding and subtracting, **like terms** are terms with equivalent bases and equivalent exponents.

For example, x^2 and $3x^2$ are like terms because they both have base x and an exponent of 2. The coefficients are 1 and 3, but the coefficients don't have to match.

So when we add $x^2 + 3x^2$, we get $4x^2$. One trick for adding terms like these is to identify the like term, and then replace that like term with something easier to think about. It's silly, but let's pretend that, instead of x^2, we have "apples." It's can also be helpful to write in any "understood 1." So we can think about $x^2 + 3x^2$ as

$$1x^2 + 3x^2$$

$$1(\text{apples}) + 3(\text{apples})$$

$$4(\text{apples})$$

$$4x^2$$

Let's look at a much more complicated example, so that we can show that it's actually not that much more complicated at all.

Example

Simplify the expression.

$$2(x^2 + 1)^3 + (x^2 + 1)^3 - 3(x^2 + 1)^3 + 5(x^2 + 1)^3$$

This expression includes a lot more than we've learned how to deal with yet. But let's just focus in on the "like terms" concept.

All of these terms have the same base, $(x^2 + 1)$, and the same exponent, 3. The coefficients are 2, 1, 3, and 5. So let's rewrite the expression as

$$2 \text{ of these} + 1 \text{ of these} - 3 \text{ of these} + 5 \text{ of these}$$

If we think about just $2 + 1 - 3 + 5$, we get 5. So we can simplify the expression as

$$5 \text{ of these}$$

$$5(x^2 + 1)^3$$

Alternatively, if we don't like using text, we can always make a substitution with a different variable. For instance, if we substitute $A = (x^2 + 1)$, the expression is

$$2A + A - 3A + 5A$$

$$2A + 1A - 3A + 5A$$

$$(2 + 1 - 3 + 5)A$$

$$5A$$

$$5(x^2 + 1)^3$$

Let's try one more example of adding and subtracting like terms.

Example

Simplify the expression.

$$x - 3x^2 + 4x + 7x^2$$

We need to realize that the four terms in this expression aren't all like terms. The x and $4x$ terms are alike because the base is x and the exponent is the "understood 1." And the $-3x^2$ and $7x^2$ terms are alike because the base is x and the exponent is 2.

So we'll start by reordering terms in the expression to group like terms together.

$$x + 4x - 3x^2 + 7x^2$$

Combine the x terms.

$$(1 + 4)x - 3x^2 + 7x^2$$

$$5x - 3x^2 + 7x^2$$

Combine the x^2 terms.

$$5x + (-3 + 7)x^2$$

$$5x + 4x^2$$