Complex fractions

A complex fraction is an algebraic expression with fraction(s) in either the numerator, the denominator, or both. The main steps of solving complex fractions are

1. Simplify both the numerator and denominator so we can have a simple fraction.
2. Simplify the numerator and the denominator as much as possible.

What do we need to remember?

A reciprocal of a fraction is just that fraction "flipped upside down."

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

The reciprocal of $\frac{x}{1}$ is $\frac{1}{x}$

A fraction bar can be thought of as a division sign.

$$\frac{x}{y} = x \div y$$

To divide by a fraction, we can multiply by its reciprocal.

$$\frac{x}{\left(\frac{a}{b}\right)} = x \div \frac{a}{b} = x \cdot \frac{b}{a}$$

Any number or variable can be written as itself divided by 1.

$$x = \frac{x}{1}$$

Let's look at a few examples.

Example

Simplify the expression.

$$\frac{\left(\frac{2}{3}\right)}{\left(\frac{3}{4}\right)}$$

Here, we're dividing the fraction in the numerator ($2/3$) by the fraction in the denominator ($3/4$).

$$\frac{2}{3} \div \frac{3}{4}$$

Now that we have a fraction divided by another fraction, instead of dividing by the fraction that was originally in the denominator, we can multiply by its reciprocal.

$$\frac{2}{3} \cdot \frac{4}{3}$$

For fraction multiplication, multiply the numerators and denominators separately.

$$\frac{2 \cdot 4}{3 \cdot 3}$$

$$\frac{8}{9}$$

We can do the same thing with variables.

Example

Simplify the expression.

$$\frac{x}{\left(\frac{a}{b}\right)}$$

We have to rewrite the given fraction.

$$\frac{\left(\frac{x}{1}\right)}{\left(\frac{a}{b}\right)}$$

$$\frac{x}{1} \div \frac{a}{b}$$

Now that we have a fraction divided by another fraction, instead of dividing by the fraction that was originally in the denominator, we can multiply by its reciprocal.

$$\frac{x}{1} \cdot \frac{b}{a}$$

For fraction multiplication, multiply the numerators and denominators separately.

$$\frac{xb}{1a}$$

$$\frac{xb}{a}$$

Let's now solve a more complex example.

Example

Simplify the expression.

$$\frac{\frac{1}{a} - 1}{\frac{1}{b} - \frac{1}{a}}$$

Simplify just the numerator by finding a common denominator.

$$\frac{1}{a} - 1$$

$$\frac{1}{a} - \frac{a}{a}$$

$$\frac{1-a}{a}$$

Simplify just the denominator by finding a common denominator.

$$\frac{1}{b}-\frac{1}{a}$$

$$\frac{1}{b}\cdot\frac{a}{a}-\frac{1}{a}\cdot\frac{b}{b}$$

$$\frac{a}{ab}-\frac{b}{ab}$$

$$\frac{a-b}{ab}$$

Rewrite the given fraction with the simplified numerator and denominator.

$$\frac{\frac{1-a}{a}}{\frac{a-b}{ab}}$$

Multiply the numerator by the reciprocal of the denominator.

$$\frac{1-a}{a}\div\frac{a-b}{ab}$$

$$\frac{1-a}{a}\cdot\frac{ab}{a-b}$$

$$\frac{1-a}{1}\cdot\frac{b}{a-b}$$

$$\frac{b-ab}{a-b}$$