Algebra 2
Final Exam Solutions

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MATH

Algebra 2 Final Exam Answer Key

1. (5 pts)	A	B	C	D	■
2. (5 pts)	A	B	C	■	E
3. (5 pts)	■	B	C	D	E
4. (5 pts)	A	B	C	■	E
5. (5 pts)	A	■	C	D	E
6. (5 pts)	A	B	■	D	E
7. (5 pts)	A	■	C	D	E
8. (5 pts)	A	B	C	D	■

9. (15 pts) 38

10. (15 pts) $x = 5$

11. (15 pts) $x^2 + y^2 - 4y - 12 = 0$

12. (15 pts) $c = \dfrac{144{,}000 - T}{20}$

Algebra 2 Final Exam Solutions

1. E. Set up a proportion, by equation the ratio x/y to the ratio $7/13$, and solve for one of the variables in terms of the other.

$$\frac{a}{b} = \frac{6}{5}$$

Solve for a.

$$b \cdot \frac{a}{b} = \frac{6}{5} \cdot b$$

$$a = \frac{6}{5}b$$

Next, set up an equation for a and b using what we know about their difference.

$$a - b = 6$$

Substitute $a = \frac{6}{5}b$ and solve for b.

$$\frac{6}{5}b - b = 6$$

$$\frac{1}{5}b = 6$$

$$5 \cdot \frac{1}{5}b = 6 \cdot 5$$

$$b = 30$$

Now solve for a.

$$a = \frac{6}{5}b$$

$$a = \frac{6}{5} \cdot 30$$

$$a = 36$$

Since $a = 36$ and $b = 30$, $a = 36$ is the larger of the two numbers.

2. D. First simplify the fraction in the second radical to lowest terms.

$$\frac{13}{39} = \frac{1}{3}$$

Now we have

$$\sqrt{\frac{7}{36}} + \sqrt{\frac{1}{3}}$$

When we take the square root of a fraction, we can take the square roots of the numerator and denominator separately. Therefore, we can rewrite the expression as

$$\frac{\sqrt{7}}{\sqrt{36}} + \frac{\sqrt{1}}{\sqrt{3}}$$

Rewrite this by taking the square roots of any perfect squares.

$$\frac{\sqrt{7}}{6}+\frac{1}{\sqrt{3}}$$

Now we need to find a common denominator. Since we have only two terms, we can do this by multiplying the numerator and denominator of each fraction by the denominator of the other fraction.

$$\frac{\sqrt{7}}{6}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)+\frac{1}{\sqrt{3}}\left(\frac{6}{6}\right)$$

$$\frac{\sqrt{7}\sqrt{3}}{6\sqrt{3}}+\frac{1(6)}{6\sqrt{3}}$$

$$\frac{\sqrt{21}}{6\sqrt{3}}+\frac{6}{6\sqrt{3}}$$

Now that we have a common denominator, add the fractions.

$$\frac{6+\sqrt{21}}{6\sqrt{3}}$$

Rationalize the denominator.

$$\frac{6\sqrt{3}+\sqrt{21}\sqrt{3}}{6\sqrt{3}\sqrt{3}}$$

$$\frac{6\sqrt{3}+3\sqrt{7}}{18}$$

$$\frac{2\sqrt{3}+\sqrt{7}}{6}$$

3. A. Instead of dividing by the fractions in the denominators, we can multiply by their reciprocals.

$$\frac{8}{3}\cdot\frac{2}{5}=\frac{x}{9}\cdot\frac{3}{5}$$

$$\frac{16}{15}=\frac{3x}{45}$$

Multiply both sides by 45.

$$45\cdot\frac{16}{15}=3x$$

Divide both sides by 3 to solve for x. Then multiply fractions to simplify.

$$x=\frac{45}{3}\cdot\frac{16}{15}$$

$$x=16$$

4. D. A computer was originally $\$1499$, but the price is now reduced by $\$299(\$1499-\$1200=\$299)$. Use the proportion:

$$\frac{\text{Discount Amount}}{\text{Original Price}}=\frac{\text{Percent Markdown}}{100}$$

$$\frac{299}{1,499} = \frac{x}{100}$$

$$100 \cdot \frac{299}{1,499} = x$$

$$\frac{29,900}{1,499} = x$$

$$x \approx 19.95$$

The percent markdown is approximately $19.95\,\%$.

5. B. Factor each polynomial completely.

$$\frac{(3x^3 + x^2 - 10x)(x^2 + x - 12)}{(2x^2 + 3x - 2)(3x^2 + 7x - 20)}$$

$$\frac{x(3x - 5)(x + 2)(x + 4)(x - 3)}{(2x - 1)(x + 2)(3x - 5)(x + 4)}$$

Simplify.

$$\frac{x(x - 3)}{2x - 1}$$

6. C. Rearrange the equation to get the term with b by itself on one side.

$$\frac{c}{d} - \frac{a}{b} = \frac{e}{f}$$

$$\frac{c}{d} - \frac{a}{b} + \frac{a}{b} = \frac{e}{f} - \frac{e}{f} + \frac{a}{b}$$

$$\frac{c}{d} - \frac{e}{f} = \frac{a}{b}$$

Multiply both sides by b to get it out of the denominator.

$$b \cdot \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} \cdot b$$

$$b\left(\frac{c}{d} - \frac{e}{f}\right) = a$$

$$b = \frac{a}{\frac{c}{d} - \frac{e}{f}}$$

7. B. Apply the rules of logarithms to simplify.

$$\log_a x + \log_a y = \log_a xy$$

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

$$b \log_a x = \log_a x^b$$

Apply the third rule to the middle term.

$$\log 8x + 3\log x - \log 2x^2$$

$$\log 8x + \log x^3 - \log 2x^2$$

Apply the first rule to the first two terms.

$$\log 8x \cdot x^3 - \log 2x^2$$

$$\log 8x^4 - \log 2x^2$$

Apply the second rule.

$$\log \frac{8x^4}{2x^2}$$

Simplify.

$$\log 4x^2$$

8. E. Start by rewriting the radicals, using the fact that $\sqrt{-1} = i$.

$$4i^5 - \sqrt{-9} + 4i^7 - 12i^4 + \sqrt{-16} - 7i^6 + 5i^2$$

$$4i^5 - \sqrt{9}\sqrt{-1} + 4i^7 - 12i^4 + \sqrt{16}\sqrt{-1} - 7i^6 + 5i^2$$

$$4i^5 - 3i + 4i^7 - 12i^4 + 4i - 7i^6 + 5i^2$$

Then we'll factor each expression of the form i^n with $n > 2$, using i and/or i^2 as factors.

$$4i^2i^2i - 3i + 4i^2i^2i^2i - 12i^2i^2 + 4i - 7i^2i^2i^2 + 5i^2$$

Replace each i^2 with -1.

$$4(-1)(-1)i - 3i + 4(-1)(-1)(-1)i - 12(-1)(-1)$$

$$+4i - 7(-1)(-1)(-1) + 5(-1)$$

$$4i - 3i - 4i - 12 + 4i + 7 - 5$$

$$i - 10$$

9. Let T and U be the tens digit and units digit, respectively, of the original number.

The value of the original number is

$$10T + U$$

Reversing the digits gives us a number whose value is

$$10U + T$$

The second number is 45 greater than the original number, so we can write

$$\text{original number} + 45 = \text{second number}$$

$$(10T + U) + 45 = (10U + T)$$

$$10T + U + 45 = 10U + T$$

$$9T - 9U + 45 = 0$$

Dividing through by 9 gives

$$T - U + 5 = 0$$

$$T = U - 5$$

We know that the product of the digits is 24, so we'll substitute the expression we just found for T into the equation $T \cdot U = 24$, and then solve for U.

$$T \cdot U = 24$$

$$(U - 5) \cdot U = 24$$

$$U^2 - 5U = 24$$

$$U^2 - 5U - 24 = 0$$

$$(U - 8)(U + 3) = 0$$

$U - 8 = 0$ or $U + 3 = 0$

$U = 8$ or $U = -3$

Since the product of the two digits is positive, then both digits are positive and $U = 8$ is the only solution.

Our next step is to plug this value of U into the equation $T \cdot U = 24$, and then solve for T.

$$T \cdot U = 24$$

$$T \cdot 8 = 24$$

$$T = 3$$

The original number is 38. When we reverse the digits, we get 83, which is indeed 45 greater than 38: $83 = 38 + 45$.

10. Add x to both sides.

$$\sqrt{5+4x}-x=0$$

$$\sqrt{5+4x}-x+x=0+x$$

$$\sqrt{5+4x}=x$$

Square both sides.

$$(\sqrt{5+4x})^2=x^2$$

The square and square root will cancel on the left.

$$5+4x=x^2$$

Get all of the terms to the right side and then factor to solve.

$$0=x^2-4x-5$$

$$0=(x-5)(x+1)$$

$x=5$ or $x=-1$

Check both possible solutions in the original equation.

Let $x=5$.

$$\sqrt{5+4(5)}-5=0$$

$$\sqrt{25}-5=0$$

$$5 - 5 = 0$$

$$0 = 0$$

Let $x = -1$.

$$\sqrt{5 + 4(-1)} - (-1) = 0$$

$$\sqrt{1} + 1 = 0$$

$$1 + 1 = 0$$

$$2 \neq 0$$

The only correct solution is $x = 5$.

11. The given information tells us that $h = 0$, $k = 2$, and $r = 4$. Substitute the values of h, k, and r into the equation $(x - k)^2 + (y - k)^2 = r^2$, then expand and simplify.

$$(x - 0)^2 + (y - 2)^2 = 4^2$$

$$x^2 + y^2 - 4y + 4 = 16$$

$$x^2 + y^2 - 4y - 12 = 0$$

12. Let c be the number of children and a the number of adults. The total money taken in is

$$T = 40c + 60a$$

We also know that the total number of people who came to the amusement park is $2{,}400$, that is, $c + a = 2{,}400$. So $a = 2{,}400 - c$. Substituting $2{,}400 - c$ for a into the equation $T = 40c + 60a$ gives

$$T = 40c + 60(2{,}400 - c)$$

$$T = 40c + 144{,}000 - 60c$$

$$T = -20c + 144{,}000$$

Now solve for c.

$$T + 20c = 144{,}000$$

$$20c = 144{,}000 - T$$

$$c = \frac{144{,}000 - T}{20}$$

FOR THE LOVE OF MATH

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