Algebra 1
Workbook Solutions

Factoring

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MATH

GREATEST COMMON FACTOR

1. Factor out the greatest common factor.

$$3x^2y^3 + 12x^3y^2 - 9x^4y^4$$

Solution:

The greatest common factor is $3x^2y^2$, so the expression is factored as

$$3x^2y^2(y + 4x - 3x^2y^2)$$

2. Factor the polynomial in the numerator and simplify the resulting expression. Fill in the blank with the correct term.

$$\frac{3x^3 - 12x}{3x} = x^2 - \underline{\hspace{2em}}$$

Solution:

The blank should be filled in with 4.

$$\frac{3x^3 - 12x}{3x}$$

$$\frac{3x(x^2 - 4)}{3x}$$

$$x^2 - 4$$

3. Factor the expression.

$$9s^3t^2 + 15s^2t^5 - 24s^5t + 6s^4t^2$$

Solution:

The greatest common factor is $3s^2t$. When we factor out the $3s^2t$, we have to divide each term by $3s^2t$.

$$3s^2t(3st + 5t^4 - 8s^3 + 2s^2t)$$

4. What went wrong when the polynomial was factored?

$$10x^3y^4 - 5x^4y^2 - 20x^6y^3$$

$$x^3y^2(10y^2 - 5x - 20x^3y)$$

Solution:

There's a factor of 5 in each term that was not factored out. The factoring should have been

$$5x^3y^2(2y^2 - x - 4x^3y)$$

5. Factor the polynomial in the numerator and simplify the resulting expression.

$$\frac{4x^4 - 8x^3 - 32x^2}{4x^2}$$

Solution:

Factor the greatest common factor out of the numerator,

$$\frac{4x^4 - 8x^3 - 32x^2}{4x^2}$$

$$\frac{4x^2(x^2 - 2x - 8)}{4x^2}$$

then cancel like terms from the numerator and denominator.

$$x^2 - 2x - 8$$

6. Fill in the blank with the correct term.

$$4a^3b - 10ab^2 + \_\_\_\_ = 2ab(2a^2 - 5b + 3a^2b^2)$$

Solution:

The blank should be filled in with $6a^3b^3$. We can see this by distributing the $2ab$ across the parentheses.

$$2ab(2a^2 - 5b + 3a^2b^2)$$

$$4a^3b - 10ab^2 + 6a^3b^3$$

QUADRATIC POLYNOMIALS

1. Factor the quadratic expression.

$$2x^2 + 2x - 12$$

Solution:

The greatest common factor is 2, so we first factor out a 2.

$$2(x^2 + x - 6)$$

Since $(3)(-2) = -6$ and $(3) + (-2) = 1$, we see that $x^2 + x - 6$ factors as

$$(x + 3)(x - 2)$$

So the quadratic polynomial can be factored as

$$2(x + 3)(x - 2)$$

2. What went wrong when the polynomial was factored?

$$x^2 - 4x + 3$$

$$(x - 3)(x + 1)$$

Solution:

The second factor should have been $(x-1)$, instead of $(x+1)$. If we expand the expression $(x-3)(x+1)$, we get

$$(x-3)(x+1)$$

$$x^2+x-3x-3$$

$$x^2-2x-3$$

But if we instead factor x^2-4x+3 as $(x-3)(x-1)$, then we get back to the correct expression.

$$(x-3)(x-1)$$

$$x^2-x-3x+3$$

$$x^2-4x+3$$

3. Factor the quadratic expression.

$$x^2+3x-28$$

Solution:

Since $(-4)(7)=-28$ and $(-4)+(7)=3$, we see that $x^2+3x-28$ factors as

$$(x-4)(x+7)$$

4. Factor the quadratic expression.

$$x^2 - 9x + 18$$

Solution:

Since $(-3)(-6) = 18$ and $(-3) + (-6) = -9$, we see that $x^2 - 9x + 18$ factors as

$$(x - 3)(x - 6)$$

5. Fill in the blank with the correct term.

$$5x^2 - 40x + 60 = \_\_\_(x - 2)(x - \_\_\_)$$

Solution:

The greatest common factor of the polynomial on the left is 5, so we first factor out a 5.

$$5(x^2 - 8x + 12)$$

Since $(-6)(-2) = 12$ and $(-6) + (-2) = -8$, we see that $x^2 - 8x + 12$ factors as

$$(x - 6)(x - 2)$$

So the quadratic polynomial can be factored as

$$5(x - 6)(x - 2)$$

6. Factor the quadratic expression.

$$x^2 - x - 2$$

Solution:

Since $(-2)(1) = -2$ and $(-2) + 1 = -1$, we see that $x^2 - x - 2$ factors as

$$(x - 2)(x + 1)$$

DIFFERENCE OF SQUARES

1. Factor the expression.

$$4y^2 - 36$$

Solution:

The expression can be rewritten as

$$4y^2 - 36$$

$$(2y)^2 - (6)^2$$

and factored as

$$(2y - 6)(2y + 6)$$

2. What went wrong when the polynomial was factored?

$$9a^4 - 25b^2$$

$$(9a^2 - 25b)(9a^2 + 25b)$$

Solution:

The coefficients were not taken into consideration when factoring the expression. It should be first written as

$$9a^4 - 25b^2$$

$$(3a^2)^2 - (5b)^2$$

and then factored as the difference of squares.

$$(3a^2 - 5b)(3a^2 + 5b)$$

3. Factor the expression.

$$49x^6y^2 - 36z^4$$

Solution:

The expression can be rewritten as

$$49x^6y^2 - 36z^4$$

$$(7x^3y)^2 - (6z^2)^2$$

and factored as

$$(7x^3y - 6z^2)(7x^3y + 6z^2)$$

4. Fill in the blank with the correct term.

$\underline{\quad\quad} - 25y^2 = (2xz^2 - 5y)(2xz^2 + 5y)$

Solution:

The blank should be filled in with $4x^2z^4$.

5. Factor the expression.

$2x^2 - 288$

Solution:

The greatest common factor of this polynomial is 2, so we first factor out a 2.

$2(x^2 - 144)$

Since x^2 and 144 are both perfect squares (the squares of x and 12, respectively), $x^2 - 144$ is factored as $(x - 12)(x + 12)$, so the polynomial factors as

$2(x - 12)(x + 12)$

6. Factor the expression.

$5a^3 - 20ab^2$

Solution:

The greatest common factor of this polynomial is $5a$, so we first factor out a $5a$.

$$5a(a^2 - 4b^2)$$

Since a^2 and $4b^2$ are both perfect squares (the squares of a and $2b$, respectively), $a^2 - 4b^2$ is factored as $(a - 2b)(a + 2b)$, so the polynomial factors as

$$5a(a - 2b)(a + 2b)$$

ZERO THEOREM

1. Find the zeros of the function.

$$y = x^2 - 5x + 6$$

Solution:

The zeros are the x-values when $y = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 2 = 0$$

$$x = 2$$

and

$$x - 3 = 0$$

$$x = 3$$

The roots are $x = 2$ and $x = 3$.

2. Find the zeros of the function.

$$y = x^2 - 4x - 5$$

Solution:

The zeros are the x-values when $y = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 5 = 0$$

$$x = 5$$

and

$$x + 1 = 0$$

$$x = -1$$

The roots are $x = 5$ and $x = -1$.

3. Find the x-intercepts.

$$f(x) = x^2 + 10x + 24$$

Solution:

The x-intercepts are the x-values when $f(x) = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 + 10x + 24 = 0$$

$$(x + 6)(x + 4) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x + 6 = 0$$

$$x = -6$$

and

$$x + 4 = 0$$

$$x = -4$$

The solutions are $x = -6$ and $x = -4$.

4. Find the x-intercepts.

$$f(x) = x^2 - 7x + 6$$

Solution:

The x-intercepts are the x-values when $f(x) = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 7x + 6 = 0$$

$$(x - 6)(x - 1) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 6 = 0$$

$$x = 6$$

and

$$x - 1 = 0$$

$$x = 1$$

The solutions are $x = 6$ and $x = 1$.

5. Use the Zero Theorem to find the solutions to the quadratic equation.

$$4x^2 - 16 = 0$$

Solution:

Factor the left side as the difference of squares.

$$4x^2 - 16 = 0$$

$$(2x - 4)(2x + 4) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$2x - 4 = 0$$

$$2x = 4$$

$$x = 2$$

and

$$2x + 4 = 0$$

$$2x = -4$$

$$x = -2$$

The solutions are $x = 2$ and $x = -2$.

6. Use the Zero Theorem to find the solutions to the quadratic equation.

$$25 - 9x^2 = 0$$

Solution:

Factor the left side as the difference of squares.

$$25 - 9x^2 = 0$$

$$(5 - 3x)(5 + 3x) = 0$$

The Zero Theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$5 - 3x = 0$$

$$3x = 5$$

$$x = \frac{5}{3}$$

and

$$5 + 3x = 0$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

The solutions are $x = 5/3$ and $x = -5/3$.

COMPLETING THE SQUARE

1. Solve for x by completing the square.

$$x^2 - 6x + 5 = 0$$

Solution:

Completing the square gives

$$x^2 - 6x = -5$$

$$x^2 - 6x + 9 = -5 + 9$$

$$(x-3)^2 = 4$$

$$x - 3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 1, 5$$

2. Fill in the blank with the correct term.

$$x^2 - \_\_\_\_ + \frac{9}{4} = -2 + \frac{9}{4}$$

Solution:

We have the equation in the form

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

Therefore,

$$\left(\frac{b}{2}\right)^2 = \frac{9}{4}$$

$$\frac{b}{2} = \frac{3}{2}$$

$$b = 3$$

The blank should be the term $3x$.

3. Complete the square but don't solve for the roots.

$$y^2 - 4y + 1 = 0$$

Solution:

To complete the square, we first write the expression as

$$y^2 - 4y = -1$$

Now complete the square as

$$y^2 - 4y + 4 = -1 + 4$$

$$(y-2)^2 = 3$$

4. Solve for y by completing the square.

$$y^2 + 3y = 1$$

Solution:

Completing the square gives

$$y^2 + 3y + \frac{9}{4} = 1 + \frac{9}{4}$$

$$\left(y + \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$y = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

$$y = -\frac{3 \pm \sqrt{13}}{2}$$

5. Solve for x by completing the square.

$$x^2 + 6x + 11 = 0$$

Solution:

Completing the square gives

$$x^2 + 6x = -11$$

$$x^2 + 6x + 9 = -11 + 9$$

$$(x + 3)^2 = -2$$

$$x + 3 = \pm\sqrt{-2}$$

$$x = -3 \pm \sqrt{2}i$$

6. Solve for x by completing the square.

$$2x^2 + 8x + 35 = 0$$

Solution:

Completing the square gives

$$2x^2 + 8x = -35$$

$$x^2 + 4x = -\frac{35}{2}$$

$$x^2 + 4x + 4 = -\frac{35}{2} + 4$$

$$(x+2)^2 = -\frac{27}{2}$$

$$x + 2 = \pm\sqrt{-\frac{27}{2}}$$

$$x = -2 \pm \sqrt{\frac{27}{2}}i$$

$$x = -2 \pm 3\sqrt{\frac{3}{2}}i$$

QUADRATIC FORMULA

1. Write the quadratic formula for the following quadratic equation.

$$x^2 - 5x - 24 = 0$$

Solution:

In this problem $a = 1$, $b = -5$, and $c = -24$. The quadratic formula for the expression is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-24)}}{2(1)}$$

We could continue to simplify to solve for the roots.

$$x = \frac{5 \pm \sqrt{25 + 96}}{2}$$

$$x = \frac{5 \pm \sqrt{121}}{2}$$

$$x = \frac{5 \pm 11}{2}$$

$$x = -3, 8$$

2. What went wrong in the way the quadratic formula was applied?

$$3x^2 - 5x + 10 = 0$$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(10)}}{2(3)}$$

Solution:

The $-b$ at the beginning of the quadratic formula is written as -5, but $b = -5$. Which means it should be written as $-(-5)$.

3. Solve for z using the quadratic formula.

$$z^2 = z + 3$$

Solution:

Rewrite the expression as

$$z^2 = z + 3$$

$$z^2 - z - 3 = 0$$

In this problem $a = 1$, $b = -1$, and $c = -3$. Then the quadratic formula gives

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$$z = \frac{1 \pm \sqrt{13}}{2}$$

4. Fill in the blank with the correct term if the quadratic formula below was built from the quadratic equation.

$$\_\_\_\_ x^2 + 3x - 5 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(-2)(-5)}}{2(-2)}$$

Solution:

The blank should be filled in with -2.

5. What went wrong if the quadratic formula below was built from the quadratic equation?

$$x^2 + 2x = 7$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(7)}}{2(1)}$$

Solution:

The expression was not written in the correct form before using the quadratic formula. It should be written as $x^2 + 2x - 7 = 0$, for which the quadratic formula would then be

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-7)}}{2(1)}$$

■ 6. Solve for t using the quadratic formula.

$$4t^2 - 1 = -8t$$

Solution:

Rewrite the expression as

$$4t^2 - 1 = -8t$$

$$4t^2 + 8t - 1 = 0$$

In this problem $a = 4$, $b = 8$, and $c = -1$. Then the quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-8 \pm \sqrt{8^2 - 4(4)(-1)}}{2(4)}$$

$$t = \frac{-8 \pm \sqrt{64 + 16}}{8}$$

$$t = \frac{-8 \pm 4\sqrt{5}}{8}$$

$$t = \frac{-2 \pm \sqrt{5}}{2}$$

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