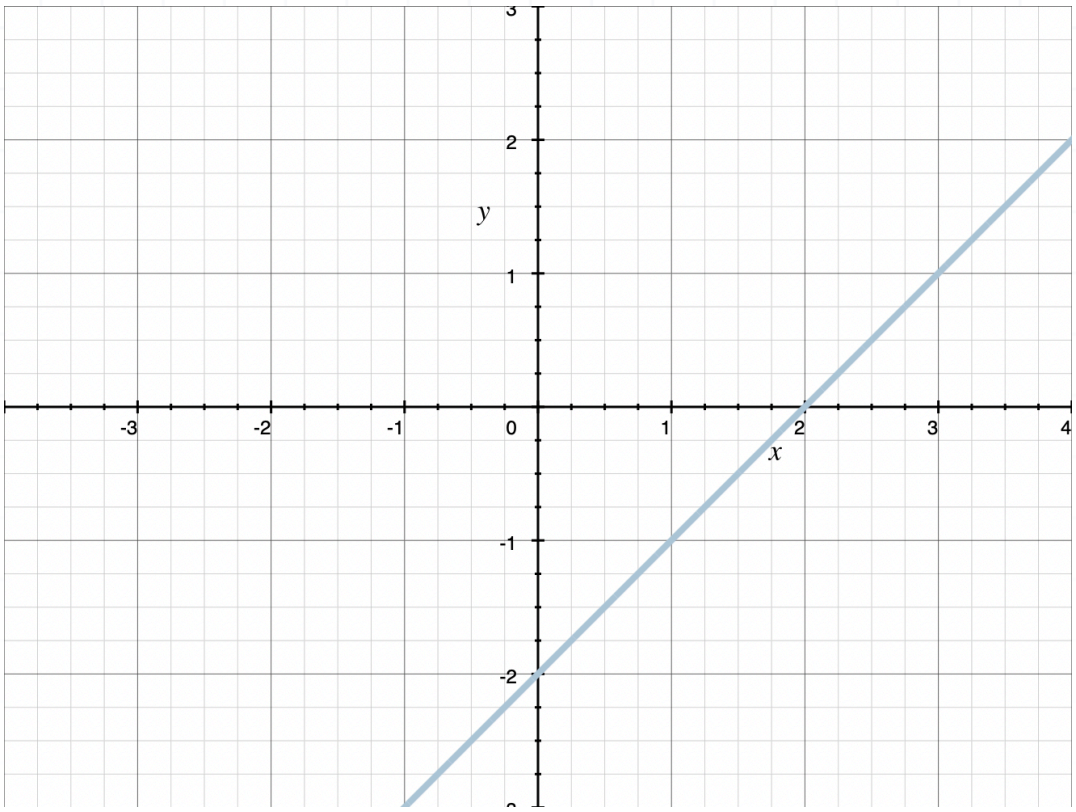
Slope

We've learned to plot points in the Cartesian coordinate system, and now we want to be able to plot lines in the same plane. For example, the line $y = x - 2$ is sketched in the plane as



How to find the slope

Every line has its own slope, where the **slope** is the "steepness" of the line, or the rate of change of the y-coordinates of the points on the graph as we move horizontally from left to right. If we think about the vertical change as "rise" and the horizontal change as "run," then we can write the slope formula as

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

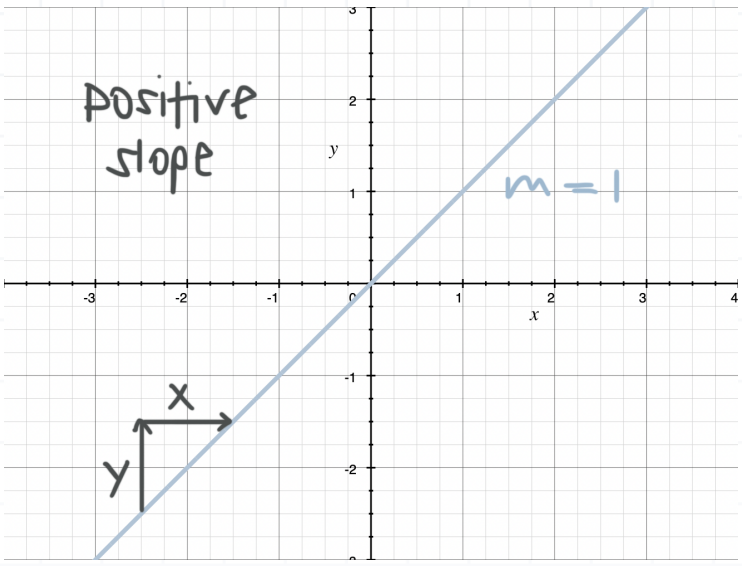
where (x_1, y_1) is one point on the line and (x_2, y_2) is another point on the line. This is the algebraic way of finding the slope, which we can use when we can't look at the graph, or when it's difficult to figure out rise/run by looking at the graph.

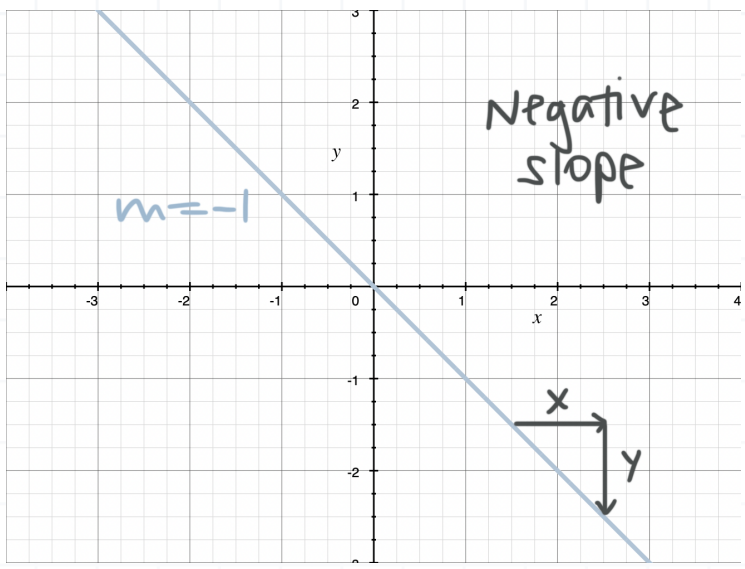
It never matters which point we call (x_1, y_1) and which one we call (x_2, y_2). The slope will be the same either way. Also, it doesn't matter which pair of points on the line we use to find the slope. The slope will always be the same.

Sign of the slope

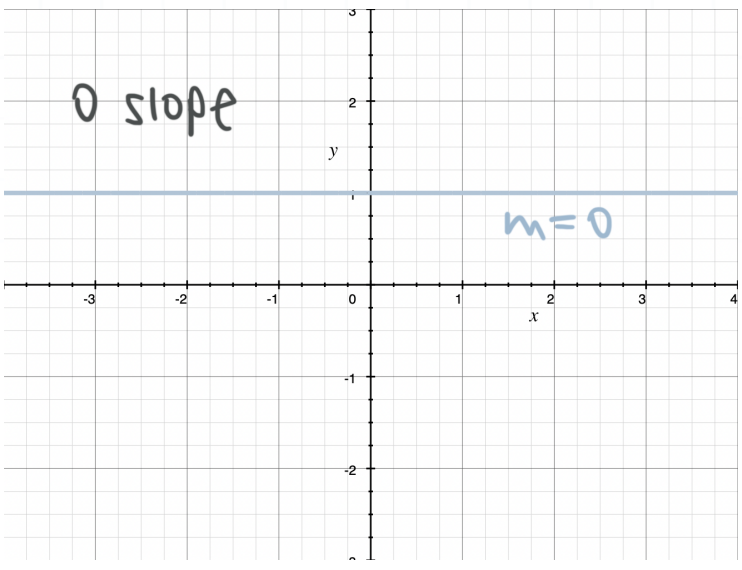
The slope is given by the change in y compared to the change in x, which means slopes can be positive, negative, zero (horizontal), and undefined (vertical).

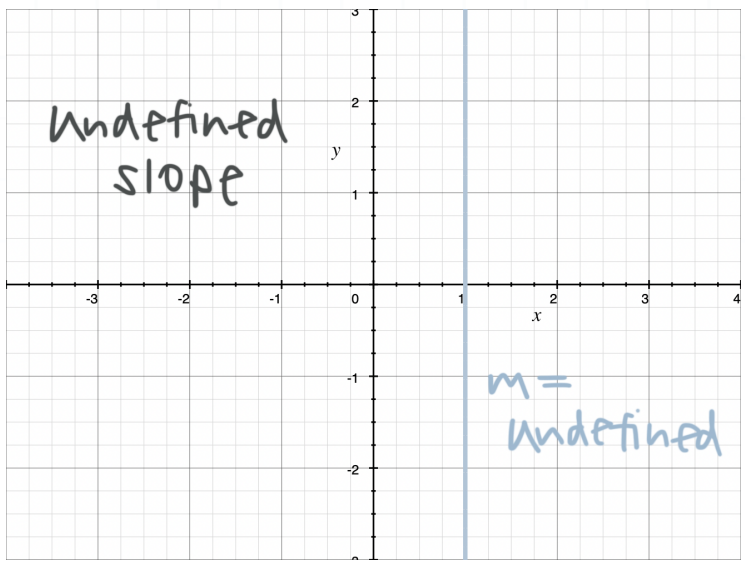
Lines with a positive slope move up as we move to the right, so the value of y increases as x increases. On the other hand, lines with a negative slope move down as we move to the right, so the value of y decreases as x increases.





Any perfectly horizontal line has a slope of 0, because there's no change in y for any change in x, so $m = 0/(x_2 - x_1) = 0$. On the other hand, any perfectly vertical line has a slope that's undefined, because there's no change in x for any change in y, so $m = (y_2 - y_1)/0$, but we can't divide by 0.

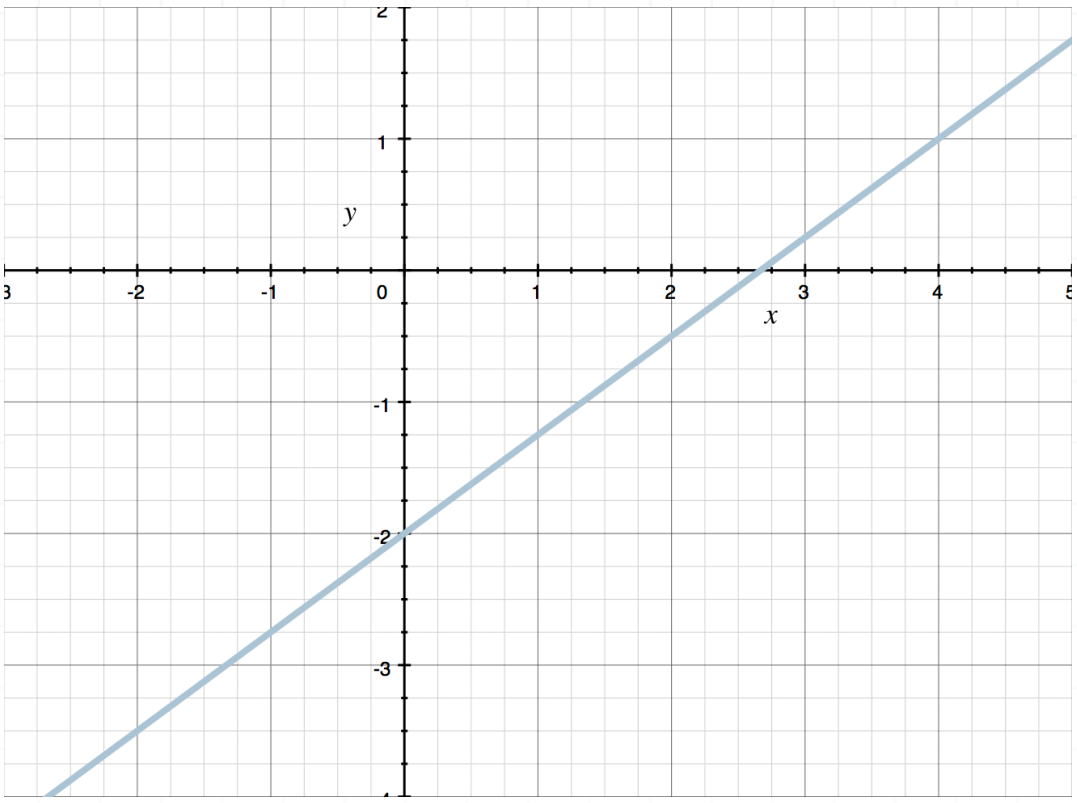




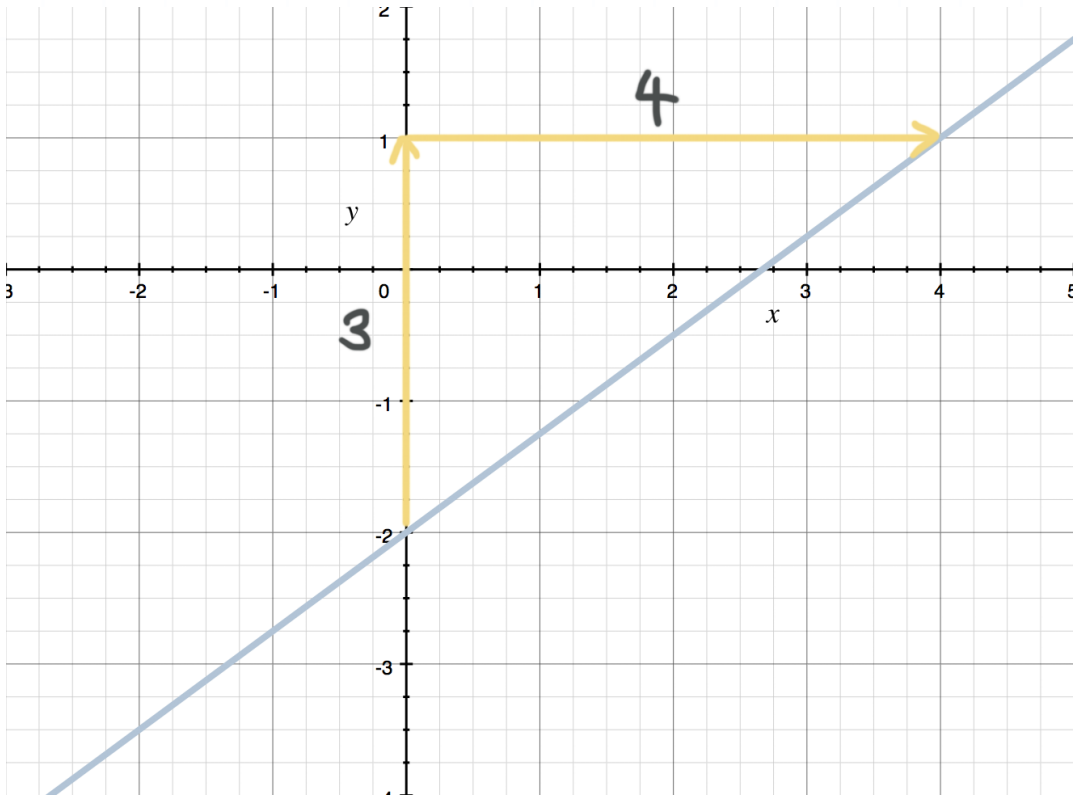
Let's do an example where we determine the slope of the line by looking at the graph.

Example

What is the slope of the line?



Graphically, we can find two points on the graph and count the rise (up and down) and run (to the right).



The rise is 3 and the run is 4, so the slope is $3/4$. We could have also used two points on the line, like $(0, -2)$ and $(4,1)$, to calculate the slope algebraically.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{4 - 0} = \frac{3}{4}$$

Let's try another example.

Example

What is the slope of the line that passes through $(-1,5)$ and $(3, -3)$?

It doesn't matter which point we label as (x_1, y_1) and which one we label as (x_2, y_2), we'll get the same slope either way. But let's say $(x_1, y_1) = (-1,5)$ and $(x_2, y_2) = (3, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-3 - 5}{3 - (-1)}$$

$$m = \frac{-8}{4}$$

$$m = -2$$
