# Domains of composite functions

In this lesson we'll look at how to find the domain of a composite function.

Remember that the **domain** of a function is the set of $x$-values where the function is defined. To determine the domain of a composite function, we need to consider the domains of the original functions.

The domain of a composite $f(g(x))$ must exclude all values of $x$ that aren't in the domain of the "inside" function $g$, and all values of $x$ for which $g(x)$ isn't in the domain of the "outside" function $f$. In other words, given the composite $f(g(x))$, the domain will exclude all values of $x$ where $g(x)$ is undefined, and all values of $x$ where $f(g(x))$ is undefined.

Therefore, to find the domain of a composite function $f(g(x))$, we'll

1. Find the domain of $g$.

2. Find the domain of $f$.

3. Set $g$ equal to any values that are excluded from the domain of $f$ and solve that equation for $x$.

Any values excluded from the domain of $g$, as well as any values where $g$ is equivalent to values excluded from the domain of $f$, will be excluded from the domain of the composite $f(g(x))$.

Let's look at a few examples.

**Example**

What is the domain of $f \circ g$, if $f(x) = x^2 - 3$ and $g(x) = \sqrt{x+9}$?

First, find the domain of $g(x)$. The expression $\sqrt{x+9}$ is undefined where $x+9$ is negative. For example, if $x = -10$, then $x+9$ is $-1$. In general, if $x$ is any number less than $-9$, then $x+9$ is negative. However, $-9$ itself is okay, because $\sqrt{-9+9} = 0$. Therefore, the domain of $g(x)$ is all real numbers $x$ such that $x \geq -9$.

The composite function is

$$f(g(x)) = (\sqrt{x+9})^2 - 3$$

$$f(g(x)) = (x+9) - 3$$

$$f(g(x)) = x + 6$$

For this simple binomial $(x+6)$, no real numbers are excluded, so its domain is all real numbers. But because the domain of $g(x)$ excludes all $x < -9$, those values of $x$ also have to be excluded from the domain of the composite function $f(g(x))$.

That means the domain of $f(g(x))$ is $x \geq -9$.

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Let's try another example.

**Example**

What is the domain of $f \circ g$?

$$f(x) = \frac{2}{2x + 4}$$

$$g(x) = \frac{3}{x - 5}$$

First, find the domain of $g(x)$. The expression $3/(x - 5)$ is undefined if the denominator is $0$. That means $x = 5$ isn't in the domain of $g(x)$. Therefore, the domain of $g(x)$ is all real numbers $x$ such that $x \neq 5$.

The composite function is

$$f(g(x)) = \frac{2}{2\left(\frac{3}{x-5}\right) + 4}$$

$$f(g(x)) = \frac{2}{\left(\frac{6}{x-5}\right) + 4\left(\frac{x-5}{x-5}\right)}$$

$$f(g(x)) = \frac{2}{\left(\frac{6 + 4x - 20}{x-5}\right)}$$

$$f(g(x)) = \frac{2}{\frac{4x - 14}{x-5}}$$

$$f(g(x)) = 2\left(\frac{x-5}{4x-14}\right)$$

$$f(g(x)) = \frac{2(x-5)}{2(2x-7)}$$

$$f(g(x)) = \frac{x-5}{2x-7}$$

For this rational function, any numbers that make the denominator $0$ are excluded from the domain.

$$2x - 7 = 0 \quad \to \quad 2x = 7 \quad \to \quad x = \frac{7}{2}$$

Putting both exclusions together, the domain of the composite is all real numbers except $7/2$ and $5$, so

$$f(g(x)) = \frac{x-5}{2x-7}, \ x \neq \frac{7}{2}, \ 5$$

Alternatively, if we didn't need the composite function and only needed its domain, we could first find the domain of $g(x) = 3/(x-5)$ by recognizing that $g$ is undefined when $x - 5 = 0$, which means its domain is $x \neq 5$.

Then we would find the domain of $f(x) = 2/(2x+4)$ by recognizing that $f$ is undefined when $2x + 4 = 0$, which means its domain is $x \neq -2$.

Given the domain of $f(x)$, we have to exclude from the domain of the composite any values where $g(x) = -2$.

$$\frac{3}{x-5} = -2$$

$$-\frac{3}{2} = x - 5$$

$$-\frac{3}{2} + 5 = x$$

$$x = \frac{7}{2}$$

Merging both constraints, we know that the domain of the composite is all real numbers except $7/2$ and $5$.

$$x \neq \frac{7}{2}, 5$$