Parallel and perpendicular lines

In this lesson we'll learn about the characteristics of parallel and perpendicular lines and how to identify them on a graph or in an equation.

The equation of a line in slope-intercept form is $y = mx + b$, where m is the slope of the line and b is the y-coordinate of the y-intercept. Remember, the y-intercept is the point at which the line crosses the y-axis. The x-coordinate of every point on the y-axis is 0, so the x-coordinate of the y-intercept is always 0. Therefore, we sometimes call b the y-intercept, even though (technically speaking) b is just the y-coordinate of the y-intercept.

Parallel lines

For two lines to be parallel, their slopes must be equal but their y-intercepts must be different (otherwise, they are the same line).

Algebraically, in slope-intercept form the equations of a pair of parallel lines would look like

$$\begin{cases} y = mx + b_1 \\ y = mx + b_2 \end{cases}$$

where $b_1 \neq b_2$.

Here are two examples of equations of a pair of parallel lines in slope-intercept form:

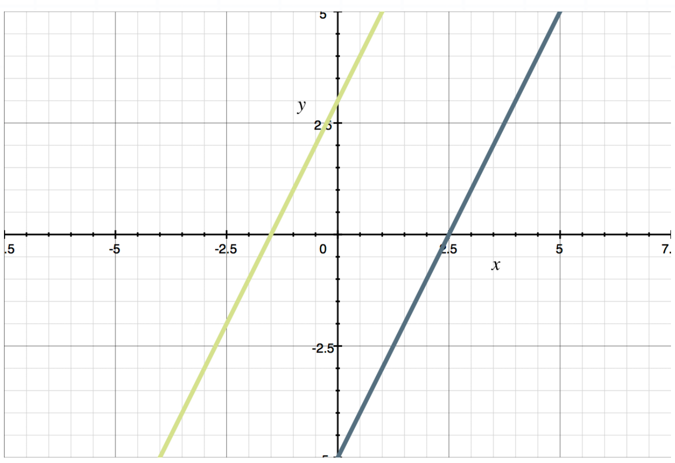
$$\begin{cases} y = 2x - 5 \\ y = 2x + 3 \end{cases}$$

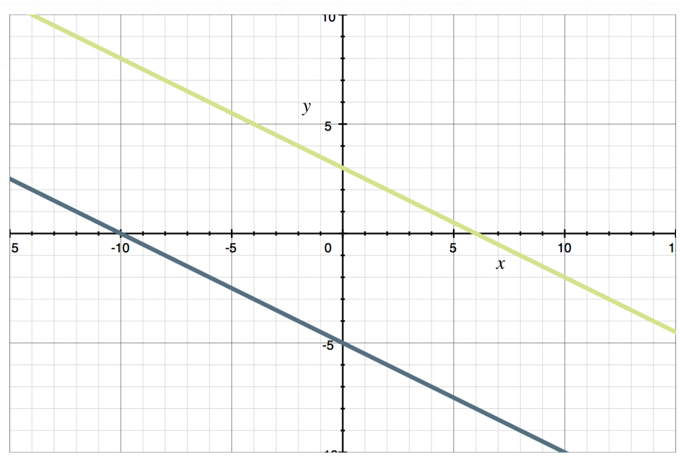
$$\begin{cases} y = -\frac{1}{2}x - 5 \\ y = -\frac{1}{2}x + 3 \end{cases}$$

Parallel lines go on forever in the same direction and never cross each other. Here are the graphs of those two examples of equations of a pair of parallel lines.

$$\begin{cases} y = 2x - 5 & \text{blue} \\ y = 2x + 3 & \text{green} \end{cases}$$

$$\begin{cases} y = -\frac{1}{2}x - 5 & \text{blue} \\ y = -\frac{1}{2}x + 3 & \text{green} \end{cases}$$





Perpendicular lines

Perpendicular lines have slopes that are negative reciprocals of each other. Remember, two numbers c, d are reciprocals of each other if $d = 1/c$. Therefore, if the slope of one of the lines in a pair of perpendicular lines is m, the slope of the other line is $-1/m$. Since the slopes are different, the y-intercepts could be the same.

Algebraically, in slope-intercept form the equations of a pair of perpendicular lines would look like

$$\begin{cases} y = mx + b_1 \\ y = -\frac{1}{m}x + b_2 \end{cases}$$

where b_1 and b_2 could be the same.

Here are two examples of equations of a pair of perpendicular lines in slope-intercept form:

$$\begin{cases} y = 2x - 3 \\ y = -\frac{1}{2}x - 3 \end{cases}$$

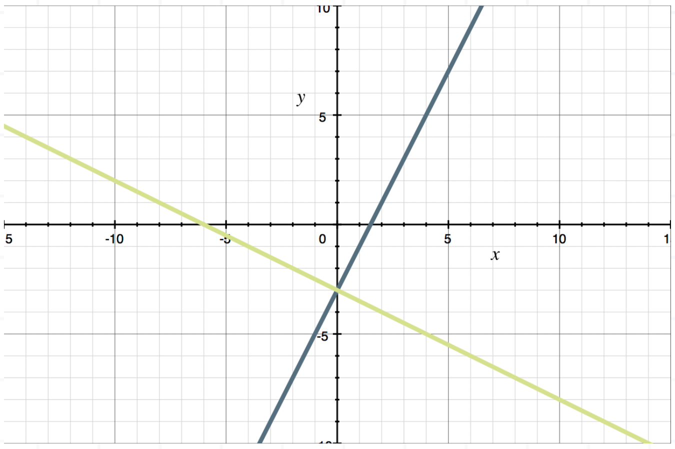
$$\begin{cases} y = -5x + 2 \\ y = \frac{1}{5}x - 4 \end{cases}$$

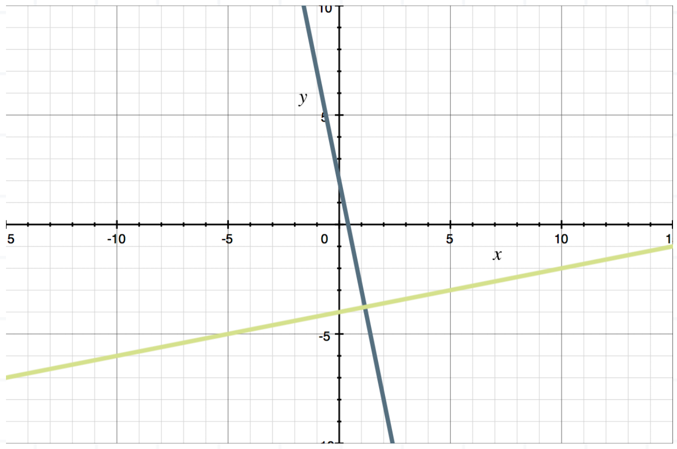
Perpendicular lines cross each other at a single point and form four right angles (four 90-degree angles) at their point of intersection. Be aware that checking this on a calculator might not always help, because the graphing window settings can disguise right angles.

Here are the graphs of those two examples of equations of a pair of perpendicular lines.

$$\begin{cases} y = 2x - 3 & \text{blue} \\ y = -\frac{1}{2}x - 3 & \text{green} \end{cases}$$

$$\begin{cases} y = -5x + 2 & \text{blue} \\ y = \frac{1}{5}x - 4 & \text{green} \end{cases}$$





Let's go ahead and look at a few of the types of problems involving parallel or perpendicular lines that we'll need to know how to solve.

Example

Write the equation of the line that's parallel to $5x + 2y = 10$ and has a y-intercept of 4.

For two lines to be parallel, their slopes must be equal.

Remember that the equation of a line in slope-intercept form is given by

$$y = mx + b$$

where m is the slope and b is the y-intercept. To get the slope of the original line (which will also be the slope of the new line), we'll first convert its equation to slope-intercept form.

$$5x + 2y = 10$$

$$5x - 5x + 2y = -5x + 10$$

$$2y = -5x + 10$$

$$\frac{1}{2} \cdot 2y = \frac{1}{2} \cdot -5x + \frac{1}{2} \cdot 10$$

$$y = -\frac{5}{2}x + 5$$

Therefore, the slope is $-5/2$.

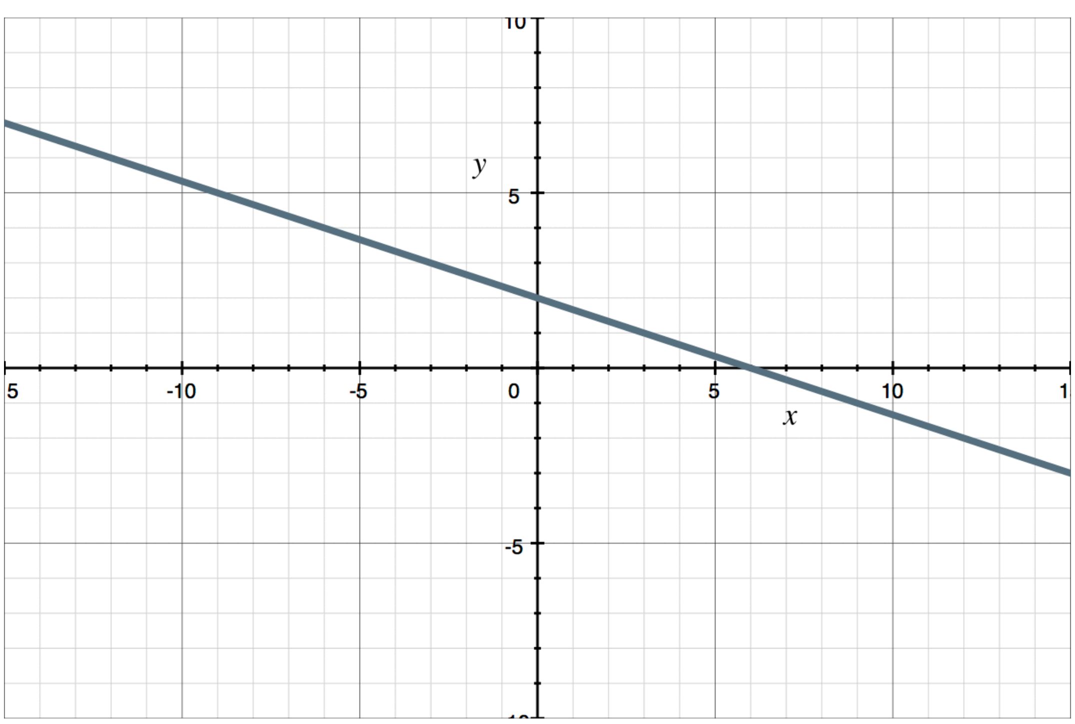
We want to write the equation of the line that has a slope of $-5/2$ and a y-intercept of 4. So $m = -5/2$ and $b = 4$. Therefore, the equation is

$$y = -\frac{5}{2}x + 4$$

Let's look at another example.

Example

Graph the line which is parallel to the line shown in the graph and has a y-intercept of -2.

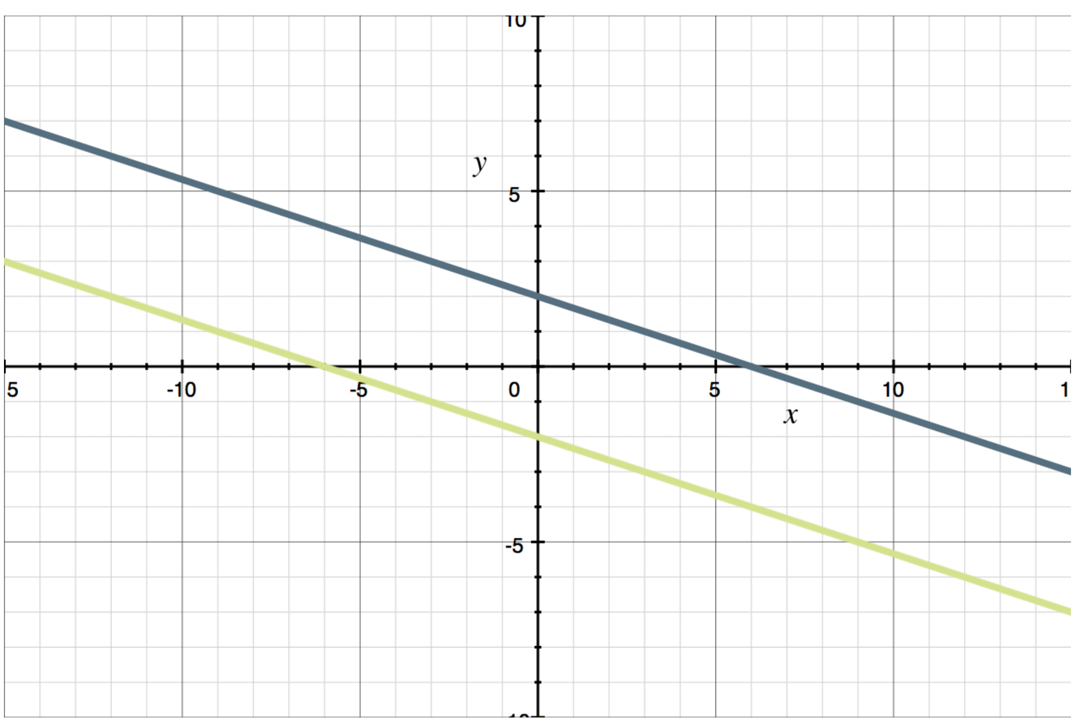


Parallel lines have the same slope. We want to draw the line that has the same slope as this one but goes through the point $(0,-2)$.

Start by graphing the point $(0,-2)$. To get the slope of the original line, we'll start from its y-intercept (the point $(0,2)$) and use the fact that the slope of a line is given by the ratio of the "rise" to the "run." Note that the point $(3,1)$ is on the original line, and that we get there from the point $(0,2)$ by going 1 unit down (from $y=2$ to $y=1$, which is a rise of -1) and 3 units to the right (from $x=0$ to $x=3$, which is a run of 3 units). Therefore, the slope of the given line (and also of the new line) is $-1/3$.

Then we'll start from the y-intercept of the new line (the point $(0,-2)$) and find the point that we get to by using that same rise and run (1 unit down and 3 units to the right). The coordinates of that point are $(0+3,-2-1)=(3,-3)$.

Connect the points $(0,-2)$ and $(3,-3)$, and draw the new line.



The equation of the new line is

$$y = -\frac{1}{3}x - 2$$

Let's look at an example of perpendicular lines.

Example

Write the equation of the line that passes through the point $(-2,5)$ and is perpendicular to the line

$$y = -\frac{4}{7}x - 2$$

Remember, perpendicular lines have slopes that are negative reciprocals of each other. In other words, the slope of the new line needs to be the negative reciprocal of $-4/7$, which means the slope of the new line is

$$\frac{7}{4}$$

The equation of a line in slope-intercept form is $y = mx + b$. For our new line, we know the slope, $7/4$, and one point on it, $(-2,5)$.

We can plug the slope and the coordinates of that point into the equation $y = mx + b$ and solve for b.

$$y = mx + b$$

$$5 = \frac{7}{4}(-2) + b$$

$$5 = -\frac{7}{2} + b$$

$$5 + \frac{7}{2} = -\frac{7}{2} + \frac{7}{2} + b$$

$$\frac{10}{2} + \frac{7}{2} = -\frac{7}{2} + \frac{7}{2} + b$$

$$\frac{17}{2} = b$$

The equation of the new line is

$$y = \frac{7}{4}x + \frac{17}{2}$$