Absolute value inequalities

Solving absolute value inequalities is really similar to solving absolute value equations.

Solving absolute value inequalities

We'll always start by isolating the absolute value expression on one side of the inequality.

Once we have the absolute value isolated on one side, we need to consider the sign of the opposite side of the inequality.

If the value on the other side is negative, the inequality either has no solutions, or the solution is the set of all real numbers.

Inequality	**Solution**
$\vert$absolute value$\vert$ $<$ negative	No solution
$\vert$absolute value$\vert$ $>$ negative	Solution is all real numbers

If the value on the other side is positive, we need to rewrite the inequality as a compound inequality.

Inequality	**Solution**
$\vert$absolute value$\vert$ $<$ positive a	Conjunction
	$-a <$ absolute value $< a$

$\lvert \text{absolute value} \rvert >$ positive a	Disjunction
	absolute value $< -a$
	or absolute value $> a$

Once we have the correct inequality statement(s) set up, all we have left to do is solve them. Let's work through an example.

Example

Solve the absolute value inequality.

$$|4x+5|+3<1$$

First, we'll isolate the absolute value on one side of the inequality. In this case, we can do that by subtracting 3 from both sides.

$$|4x+5|<-2$$

This is the "$\lvert$absolute value$\rvert$ $<$ negative" case. We know the left side will be positive, and the right side is negative, and it can't be true that we'll have some positive value less than a negative value.

So there are no values that satisfy the inequality, and we can say that it has no solution.

Let's try another example.

Example

Solve the absolute value inequality.

$$|2x - 5| + 2 \geq 5$$

Isolate the absolute value on one side of the inequality.

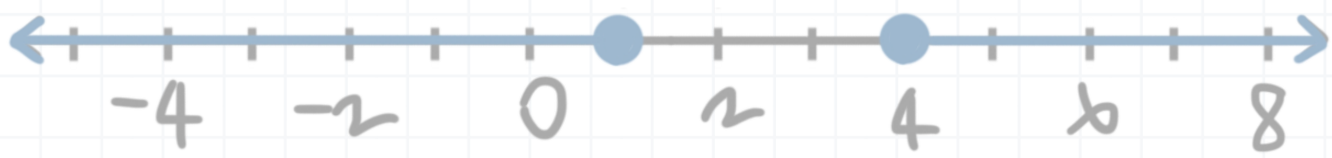
$$|2x - 5| + 2 - 2 \geq 5 - 2$$

$$|2x - 5| \geq 3$$

This is the "|absolute value| > positive a" case, which means the solution is a disjunction, and we can write the set of solutions as

absolute value $< -a$	or	absolute value $> a$
$2x - 5 \leq -3$	or	$2x - 5 \geq 3$
$2x - 5 + 5 \leq -3 + 5$	or	$2x - 5 + 5 \geq 3 + 5$
$2x \leq 2$	or	$2x \geq 8$
$\frac{2}{2}x \leq \frac{2}{2}$	or	$\frac{2}{2}x \geq \frac{8}{2}$
$x \leq 1$	or	$x \geq 4$

We can sketch the solution set as a disjunction on the number line.



Let's do one more example, this time where the solution is a conjunction.

Example

Solve the absolute value inequality.

$$|3x-1|+3<6$$

Isolate the absolute value on one side of the inequality.

$$|3x-1|+3-3<6-3$$

$$|3x-1|<3$$

This is the "|absolute value| < positive a" case, which means the solution is a conjunction, and we can write the set of solutions as

$$-3<3x-1<3$$

$$-3+1<3x-1+1<3+1$$

$$-2<3x<4$$

$$-\frac{2}{3}<\frac{3}{3}x<\frac{4}{3}$$

$$-\frac{2}{3} < x < \frac{4}{3}$$

We can sketch the solution set as a conjunction on the number line.