Algebra 1 Workbook Solutions

Polynomials

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ADDING AND SUBTRACTING POLYNOMIALS

1. Which part(s) of the terms stay the same when we add or subtract like terms?

Solution:

Both the base and the exponent stay the same when we add or subtract like terms. Only the coefficient changes.

2. Simplify the expression.

$$(2x^3 - 5x^2 + x - 3) - (x^2 - 2x + 7)$$

Solution:

Distribute the subtraction across the second set of parentheses.

$$(2x^3 - 5x^2 + x - 3) - (x^2 - 2x + 7)$$

$$2x^3 - 5x^2 + x - 3 - x^2 + 2x - 7$$

Combine like terms.

$$2x^3 - 6x^2 + 3x - 10$$

3. What went wrong in this set of steps?

$$6x^3 + 7 + x^2$$

$$7x^3 + 7$$

Solution:

The terms $6x^3$ and x^2 were added together but they aren't like terms. The exponents aren't the same, so they can't be added together.

4. Simplify the expression.

$$(10a^2b + 3ab^2 - ab) + (2ab^2 - a^2b + ab)$$

Solution:

Simplifying the expression by combining like terms.

$$(10a^2b + 3ab^2 - ab) + (2ab^2 - a^2b + ab)$$

$$10a^2b + 3ab^2 - ab + 2ab^2 - a^2b + ab$$

$$9a^2b + 5ab^2$$

5. Simplify the expression.

$$(x^4 - 5y^3 + z - xy) - (2y^4 + 6xy - z + x^4)$$

Solution:

Distribute the subtraction across the second set of parentheses.

$$(x^4 - 5y^3 + z - xy) - (2y^4 + 6xy - z + x^4)$$

$$x^4 - 5y^3 + z - xy - 2y^4 - 6xy + z - x^4$$

Combine like terms.

$$-5y^3 + 2z - 7xy - 2y^4$$

$$-2y^4 - 5y^3 - 7xy + 2z$$

6. What went wrong in this set of steps?

$$9 - x^3 + 3 + 4x^3$$

$$12 + 3x^6$$

Solution:

The terms $-x^3$ and $4x^3$ were added together. They're like terms, so we do want to add them, but when the terms were added, the exponents were added as well. The sum should be $3x^3$, not $3x^6$.

MULTIPLYING POLYNOMIALS

1. Use the Distributive Property to expand the expression.

$$\frac{1}{2}(6x+4)(x-1)$$

Solution:

Distribute the $1/2$ across the $(6x+4)$.

$$\frac{1}{2}(6x+4)(x-1)$$

$$\left(\frac{1}{2}(6x)+\frac{1}{2}(4)\right)(x-1)$$

$$(3x+2)(x-1)$$

Use FOIL to multiply the binomials.

$$(3x)(x)+(3x)(-1)+(2)(x)+(2)(-1)$$

$$3x^2-3x+2x-2$$

$$3x^2-x-2$$

2. What should we put in place of the “??” to make the expression true?

$$(2x+1)(5-x) = ?? + 10x - x + 5$$

Solution:

Use FOIL to expand the left side of the equation.

$$(2x+1)(5-x)$$

$$(2x)(5) + (2x)(-x) + (1)(5) + (1)(-x)$$

$$10x - 2x^2 + 5 - x$$

Matching this expanded left side to the form of the right side, $?? + 10x - x + 5$, we can see that the missing value is $-2x^2$.

3. What went wrong in this set of steps?

$$(a-2)^2$$

$$a^2 - 4$$

Solution:

The expression was not interpreted correctly, because the exponent was distributed to both terms directly, when it should have been expanded as

$$(a-2)^2$$

$$(a-2)(a-2)$$

Then FOIL should have been used to expand it.

$$a^2-2a-2a+4$$

$$a^2-4a+4$$

■ 4. Use the Distributive Property to expand the expression.

$$4(2-x)(3+2x)$$

Solution:

Use FOIL to multiply the binomials,

$$4(2-x)(3+2x)$$

$$4(6+4x-3x-2x^2)$$

$$4(6+x-2x^2)$$

then use the Distributive Property to distribute the 4 across the parentheses.

$$24+4x-8x^2$$

■ 5. Fill in the blank.

$$(3-a)(5+a) = 15 + \_\_\_ - a^2$$

Solution:

If we FOIL the product on the left, we get

$$(3-a)(5+a)$$

$$15 + 3a - 5a - a^2$$

$$15 - 2a - a^2$$

Comparing this to the right side of the original equation, the value that goes in the blank must be $-2a$.

6. Expand the expression.

$$(x^2-3)(2-x)$$

Solution:

The expression is expanded and simplified as

$$(x^2-3)(2-x)$$

$$2x^2 - x^3 - 6 + 3x$$

$$-x^3 + 2x^2 + 3x - 6$$

DIVIDING POLYNOMIALS

1. Simplify the expression using polynomial long division.

$$(3x^3 - x^2 + 5) \div (x + 2)$$

Solution:

Using polynomial long division, we get

$$
\begin{array}{r l}
 & 3x^2 - 7x + 14 \\
x+2 & \overline{)\,3x^3 - x^2 + 0x + 5} \\
 & \underline{-(3x^3 + 6x^2)} \\
 & -7x^2 + 0x \\
 & \underline{-(-7x^2 - 14x)} \\
 & 14x + 5 \\
 & \underline{-(14x + 28)} \\
 & -23
\end{array}
$$

Therefore, the solution is

$$3x^2 - 7x + 14 - \frac{23}{x+2}$$

2. What went wrong in setting up the long division problem?

$(5x^4 - 3x^2 + x - 2) \div (x^2 + 1)$

$$5x^4 - 3x^2 + x - 2 \,\big)\overline{\; x^2 + 1 \;}$$

Solution:

The dividend and divisor were placed incorrectly. We should have set up the division problem as

$$x^2 + 1 \,\big)\overline{\; 5x^4 - 3x^2 + x - 2 \;}$$

3. Express the full solution of the polynomial long division.

$$
\begin{array}{r l}
 & \quad 3x \qquad -1 \\
x^2 - 3 & \big)\overline{\; 3x^3 - x^2 \;\; + x \;\; - 5 \;} \\
 & -(3x^3 + 0x^2 - 9x) \\
\hline
 & \qquad -x^2 + 10x - 5 \\
 & \quad -(-x^2 + 0x + 3) \\
\hline
 & \qquad\qquad 10x - 8
\end{array}
$$

Solution:

The solution should be written as

$$\text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

$$3x - 1 + \frac{10x - 8}{x^2 - 3}$$

4. Simplify the expression using polynomial long division.

$$(2x^5 - 3x^3 + x^2 + 4x - 1) \div (x^2 + 2)$$

Solution:

Using polynomial long division, we get

$$
\begin{array}{r}
2x^3 \quad -7x \quad +1 \\
x^2+2 \,\big)\, \overline{2x^5 - 3x^3 + x^2 + 4x - 1} \\
-(2x^5 + 4x^3) \qquad\qquad\qquad \\
\hline
-7x^3 + x^2 + 4x \quad\; \\
-(-7x^3 + 0x^2 - 14x) \quad\; \\
\hline
x^2 + 18x - 1 \\
-(x^2 + 0x + 2) \\
\hline
18x - 3
\end{array}
$$

Therefore, the solution is

$$2x^3 - 7x + 1 + \frac{18x - 3}{x^2 + 2}$$

5. Simplify the expression using polynomial long division.

$$\frac{x^5 - x^3 + 4x^2 - x + 6}{2x^3 - 5}$$

Solution:

Using polynomial long division, we get

$$\begin{array}{r} \frac{1}{2}x^2 - \frac{1}{2} \\ 2x^3-5 \,\overline{\big)\, x^5 - x^3 + 4x^2 - x + 6} \\ -(x^5 + 0x^3 - \frac{5}{2}x^2) \\ \hline -x^3 + \frac{13}{2}x^2 - x + 6 \\ -(-x^3 + 0x^2 + 0x + \frac{5}{2}) \\ \hline \frac{13}{2}x^2 - x + \frac{7}{2} \end{array}$$

Therefore, the solution is

$$\frac{1}{2}x^2 - \frac{1}{2} + \frac{\frac{13}{2}x^2 - x + \frac{7}{2}}{2x^3 - 5}$$

6. Simplify the expression using polynomial long division.

$$(3x^2 + 2x + 5) \div (3x + 5)$$

Solution:

Using polynomial long division, we get

$$
\begin{array}{r|l}
 & \quad x \qquad -1 \\ \cline{2-2}
3x+5 & 3x^2+2x+5 \\
 & -(3x^2+5x) \\ \cline{2-2}
 & \qquad\quad -3x+5 \\
 & \qquad -(-3x-5) \\ \cline{2-2}
 & \qquad\qquad\quad 10
\end{array}
$$

Therefore, the solution is

$$x - 1 + \frac{10}{3x + 5}$$

MULTIPLYING MULTIVARIABLE POLYNOMIALS

1. Simplify the expression.

$$(a-3y)(2a+y)$$

Solution:

Use FOIL to expand the product of the binomials.

$$(a-3y)(2a+y)$$

$$(a)(2a)+(a)(y)+(-3y)(2a)+(-3y)(y)$$

$$2a^2+ay-6ay-3y^2$$

$$2a^2-5ay-3y^2$$

2. Simplify the expression.

$$(x-2y)(x+y)+(3x-y)(4x+4y)$$

Solution:

Use FOIL to expand each pair of binomials.

$$(x-2y)(x+y)+(3x-y)(4x+4y)$$

$$(x)(x) + (x)(y) + (-2y)(x) + (-2y)(y)$$

$$+(3x)(4x) + (3x)(4y) + (-y)(4x) + (-y)(4y)$$

$$x^2 + xy - 2xy - 2y^2 + 12x^2 + 12xy - 4xy - 4y^2$$

$$x^2 - xy - 2y^2 + 12x^2 + 8xy - 4y^2$$

$$13x^2 + 7xy - 6y^2$$

3. Fill in the blanks with the correct terms.

$$(5a - b)(7b - 3a)$$

$$35ab - 15a^2 + \_\_\_ + 3ab$$

$$\_\_\_ - 15a^2 + \_\_\_$$

Solution:

Expanding and simplifying the binomial expression gives

$$(5a - b)(7b - 3a)$$

$$35ab - 15a^2 - 7b^2 + 3ab$$

$$38ab - 15a^2 - 7b^2$$

Therefore, the first blank should be filled with $-7b^2$, the second blank with $38ab$, and the last blank with $-7b^2$.

4. What went wrong in this set of steps?

$$(a^2 + 6b)(-a - b^2)$$

$$-a^3 - a^2b^2 - 6ab - b^3$$

$$-a^3 - 7ab - b^3$$

Solution:

In the first step, the terms $6b$ and $-b^2$ were multiplied incorrectly. Their product was shown as $-b^3$, but it should have been $-6b^3$. In the second step, the terms $-a^2b^2$ and $-6ab$ were added, but they shouldn't have been added because they're not like terms.

5. Fill in the the multiplication chart with the correct terms, given the following product of binomials.

$$(4a + 3b)(-a + 2b^2)$$

		3b
-a		-3ab

Solution:

The chart should be filled in as

	4a	3b
-a	$-4a^2$	-3ab
$2b^2$	$8ab^2$	$6b^3$

6. Simplify the expression.

$$(5ax - 3by)(a + y) - (a - y)(2ax + 4by)$$

Solution:

We'll use FOIL to expand both pairs of binomials.

$$(5ax - 3by)(a + y) - (a - y)(2ax + 4by)$$

$$(5a^2x + 5axy - 3aby - 3by^2) - (2a^2x + 4aby - 2axy - 4by^2)$$

Distribute the subtraction across the second set of parentheses,

$$5a^2x + 5axy - 3aby - 3by^2 - 2a^2x - 4aby + 2axy + 4by^2$$

then combine like terms.

$$3a^2x + 7axy - 7aby + by^2$$

DIVIDING MULTIVARIABLE POLYNOMIALS

1. Find the quotient.

$$\frac{3x^2 + 6xy - 2y^2}{x - 2y}$$

Solution:

Using polynomial long division,

$$\begin{array}{r|l}
 & 3x \quad +12y \\
\hline
x-2y & 3x^2+6xy-2y^2 \\
 & -(3x^2-6xy) \\
\hline
 & \qquad\quad 12xy-2y^2 \\
 & \quad -(12xy-24y^2) \\
\hline
 & \qquad\qquad\quad 22y^2
\end{array}$$

we can see that the solution is

$$3x + 12y + \frac{22y^2}{x - 2y}$$

2. Identify the quotient, remainder, and divisor.

$$
\begin{array}{r|l}
 & x^2 \quad -xy \qquad +y^2 \\
\hline
x+y & x^3+0x^2y+0xy^2+y^3 \\
 & \underline{-(x^3+x^2y)} \\
 & \qquad -x^2y+0xy^2 \\
 & \qquad \underline{-(-x^2y-xy^2)} \\
 & \qquad\qquad\quad xy^2+y^3 \\
 & \qquad\qquad\quad \underline{-(xy^2+y^3)} \\
 & \qquad\qquad\qquad\qquad 0
\end{array}
$$

Solution:

The quotient is $x^2 - xy + y^2$, the remainder is 0, and the divisor is $x + y$.

3. How should we rewrite the expression before starting the long division?

$$\frac{2y^3 - xy^2 + x^3}{x - y}$$

Solution:

Because the leading term int the divisor is x, we want to reorder the terms in the dividend by descending power of x, which means we should rewrite the quotient as

$$\frac{x^3 - xy^2 + 2y^3}{x - y}$$

4. Find the quotient.

$$\frac{6x^2 - xy + 2y^2}{2x - y}$$

Solution:

Using polynomial long division,

$$\begin{array}{r|l}
 & 3x \quad + y \\
\hline
2x - y & 6x^2 - xy + 2y^2 \\
 & -(6x^2 - 3xy) \\
\hline
 & \qquad\quad 2xy + 2y^2 \\
 & \qquad -(2xy - y^2) \\
\hline
 & \qquad\qquad\quad 3y^2
\end{array}$$

we can see that the solution is

$$3x + y + \frac{3y^2}{2x - y}$$

5. In words, what's the first question we should ask when solving this long division problem?

$$2x+3y\,\overline{\big)\,6x^4-x^2y+xy^2+4y^4}$$

Solution:

To begin the long division, the first question we need to ask is "What do we need to multiply by $2x$ in order to get $6x^4$?" The answer to that question will be the first term in the quotient.

6. Find the quotient.

$$(y^2+xy-3x^2)\div(y+x)$$

Solution:

Using polynomial long division,

$$
\begin{array}{r}
-3x \quad +4y \\
x+y \;\overline{\big)\, -3x^2 + xy + y^2} \\
\underline{-(-3x^2 - 3xy)} \\
4xy + y^2 \\
\underline{-(4xy + 4y^2)} \\
-3y^2
\end{array}
$$

we can see that the solution is

$$-3x + 4y - \frac{3y^2}{x+y}$$

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