PEMDAS and order of operations

In any math expression, grouping symbols are used to group factors or terms together. When we evaluate a math expression, we have to perform the operations that are enclosed in grouping symbols before we perform other kinds of operations in that expression.

We're most familiar with parentheses as one kind of grouping symbol, but there are others.

Parentheses	( )
Brackets (square brackets)	[ ]
Braces (curly braces)	{ }
Absolute Value	| |

In some expressions, a division symbol (either the ÷ symbol or the "fraction line" that separates the numerator of a fraction from the denominator) is used as a grouping symbol. In the fraction

$$\frac{a}{b+c}$$

for example, the division symbol tells us that we have to first perform the addition in the denominator $(b+c)$, and then second divide the numerator a by the result.

Similarly, the absolute value in $a|b-c|$ tells us that we have to first perform the subtraction $b-c$, then second take the absolute value of the result, then third multiply by a.

PEMDAS and order of operations

When we're given an expression and we want to evaluate it, we have to perform the indicated operations in the correct order; that order has come to be known as the **order of operations**, or **PEMDAS**.

The first letter P tells us that the first thing we have to do is perform operations that are enclosed in grouping symbols; the reason why P is used for this is that **P**arentheses are the most commonly used grouping symbol. The order of operations for PEMDAS is

Parentheses	(all grouping symbols)
Exponents	(powers and roots)
Multiplication/**D**ivision	(From left to right and top to bottom, performing each multiplication/division as we come to it)
Addition/**S**ubtraction	(From left to right and top to bottom, performing each addition/subtraction as we come to it)

All grouping symbols other than the division symbol actually consist of a pair of symbols:

- an opening (left) parenthesis and a closing (right) parenthesis
- an opening (left) bracket and a closing (right) bracket

- an opening (left) brace and a closing (right) brace
- an opening absolute value line and a closing absolute value line

But keep in mind that when we refer in general to a "pair" of grouping symbols, in some cases we could be referring to just a division symbol.

Sometimes one pair of grouping symbols is inside another pair. When that happens, we have to start by performing the operation that's enclosed in the innermost pair of grouping symbols, then work our way outwards.

Let's do an example of how to apply PEMDAS.

Example

Simplify the expression.

$$[15 - (2 + 4)] \cdot 5 - 3$$

P Start by performing the operation that's enclosed in the innermost pair of grouping symbols (the addition in the parentheses).

$$[15 - (6)] \cdot 5 - 3$$

$$[15 - 6] \cdot 5 - 3$$

Now perform the operation that's enclosed in the remaining pair of grouping symbols (the subtraction in the brackets).

$$[9] \cdot 5 - 3$$

$$9 \cdot 5 - 3$$

E There are no exponents, **MD** so we'll perform all the multiplication and division together, working from left to right in the expression.

$$45 - 3$$

AS Finally, we'll do all the addition and subtraction together, working from left to right in the expression.

$$42$$

Let's try another example of applying PEMDAS.

Example

Simplify the expression.

$$3[(4 - 1) + 7] - (8 + 2)$$

P Start by performing the operation that's enclosed in the innermost pair of grouping symbols (the subtraction $4 - 1$ in the parentheses).

$$3[(3) + 7] - (8 + 2)$$

$$3[3 + 7] - (8 + 2)$$

Neither of the two remaining pairs of grouping symbols is inside the other pair, so perform the operations enclosed in those two pairs of grouping

symbols (the addition $3+7$ in the square brackets, and the addition $8+2$ in the parentheses) separately.

$$3[10]-(10)$$

$$3[10]-10$$

E There are no exponents, **MD** so we'll perform all the multiplication and division together, working from left to right in the expression.

$$30-10$$

AS Finally, we'll do all the addition and subtraction together, working from left to right in the expression.

$$20$$

We also want to make sure we know how to deal with grouping symbols in the denominator of a fraction.

Example

Simplify the expression.

$$\frac{3}{(4-1)+7}+\frac{(8+2)-4}{(12+2)-4}$$

P When we have grouping symbols within a fraction, we want to first simplify any grouping in the numerator,

$$\frac{3}{(4-1)+7}+\frac{10-4}{(12+2)-4}$$

then simplify any grouping in the denominator.

$$\frac{3}{3+7}+\frac{10-4}{14-4}$$

Before we can address the division represented by the fraction lines, we still need to simplify each numerator individually and each denominator individually. As we work inside each numerator and denominator, we still stick to PEMDAS. In the numerators and denominators we have left, we only have addition and subtraction, so we get

$$\frac{3}{10}+\frac{6}{10}$$

E There are no exponents, **MD** and there's no real value here to performing the division, since actually performing the division would just turn each fraction into a decimal number. **AS** So we'll perform all the addition and subtraction together, working from left to right in the expression.

$$\frac{3}{10}+\frac{6}{10}$$

$$\frac{9}{10}$$

Let's look at an example that includes exponents.

Example

Apply order of operations to simplify the expression.

$$3^3 + 9 \div (5 - 2) \cdot (4)^2$$

Parentheses

$$3^3 + 9 \div (5 - 2) \cdot (4)^2$$

$$3^3 + 9 \div (3) \cdot (4)^2$$

Exponents

$$27 + 9 \div (3) \cdot (16)$$

Multiplication and Division together, from left to right

$$27 + 3 \cdot 16$$

$$27 + 48$$

Addition and Subtraction together, from left to right

$$75$$

Let’s try one more example using the order of operations.

Example

Apply order of operations to simplify the expression.

$$6 + 2(3x + 1)$$

Parentheses

$3x$ and 1 are not like terms, so we can't simplify the expression inside the parentheses

Exponents

There are no exponents in the expression

Multiplication and Division together, from left to right

$$6 + 2(3x + 1)$$

$$6 + 2(3x) + 2(1)$$

$$6 + 6x + 2$$

Addition and Subtraction together, from left to right

$$6x + 8$$