Even, odd, or neither

We can classify functions as even, odd, or neither even nor odd. Each of these classifications corresponds to a particular type of symmetry of the graph of the function.

In fact, it's often easiest to tell whether a function is even, odd, or neither by looking at its graph. Sometimes it's difficult or impossible to graph a function, so there is an algebraic way to check as well.

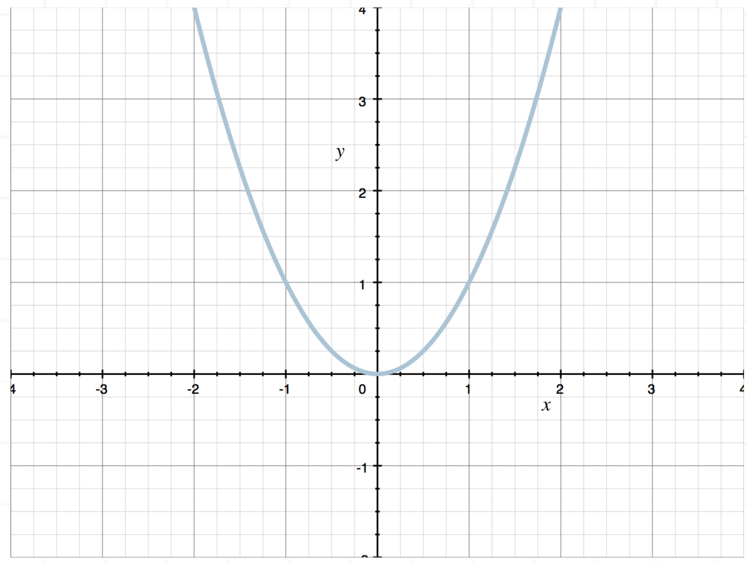
Even functions

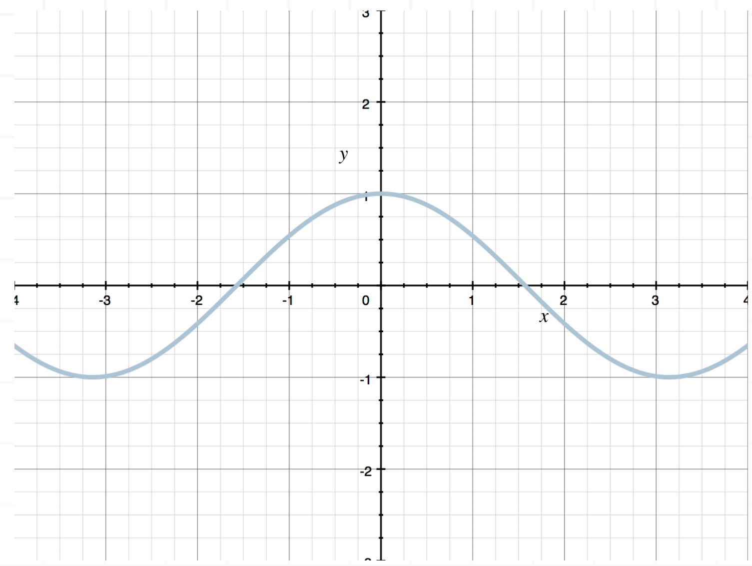
Functions that are even are symmetric with respect to the y-axis. When we plug $-x$ into the expression for an even function, it will simplify to the expression for the original function. This means that it doesn't matter whether we plug in x or $-x$, our output will be the same.

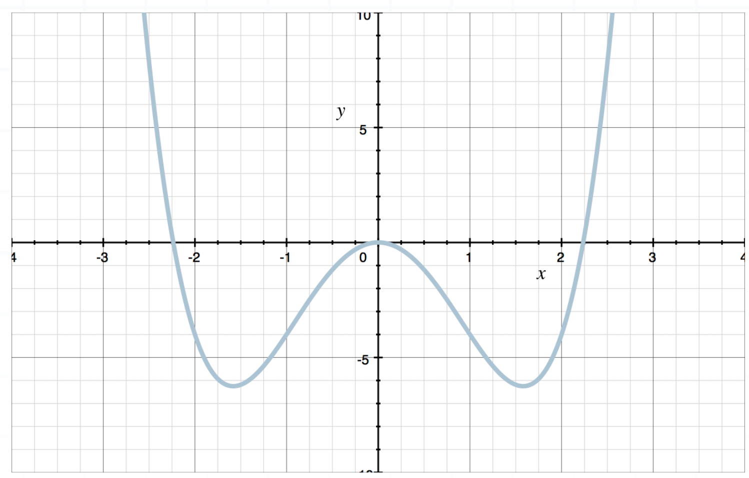
$$f(-x) = f(x)$$

What this means in terms of the graph of an even function is that the part that's to the left of the y-axis is a mirror image of the part that's to the right of the y-axis.

Below are graphs that are symmetric with respect to the y-axis and therefore represent even functions.







We can also identify even functions given points in a table. If a function is even, then opposite values of x will have equivalent values of y. For instance, $x=1$ and $x=-1$ will give the same value of y, $x=2$ and $x=-2$ will give the same value of y, $x=3$ and $x=-3$ will give the same value of y, etc.

As an example, this table of values could represent an even function.

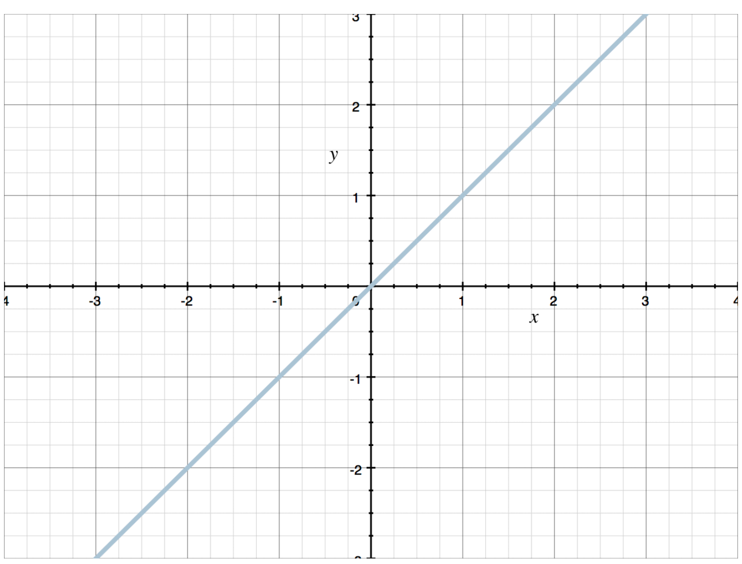
x	-3	-2	-1	0	1	2	3
f(x)	1	-1	2	4	2	-1	1

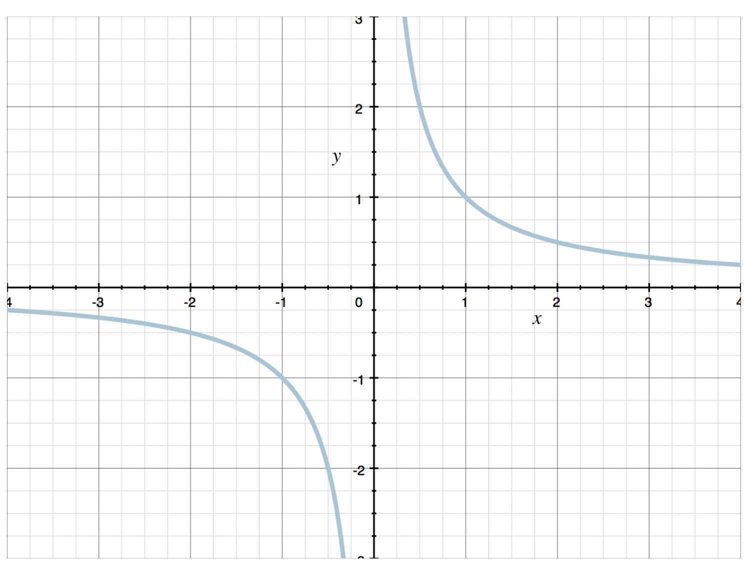
Odd functions

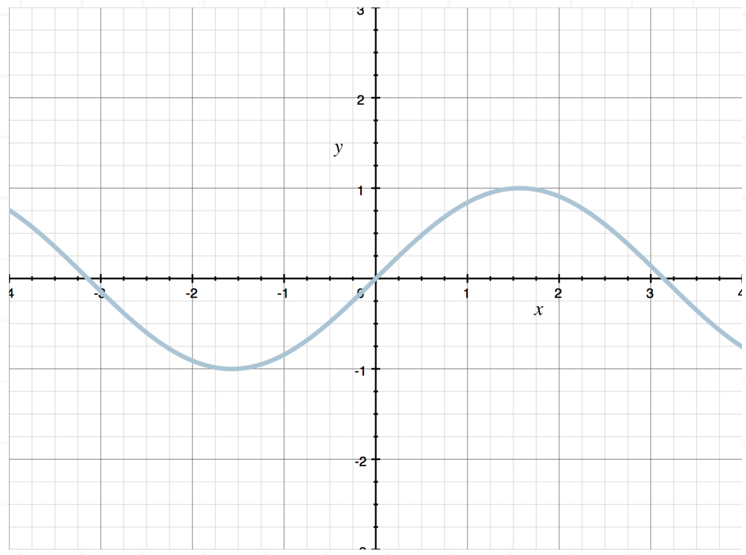
Functions that are odd are symmetric with respect to the origin. When we plug $-x$ into the expression for an odd function, it will simplify to the negative of the expression for the original function, or the expression for the original function multiplied by -1. This means that when we plug in $-x$, we'll get essentially the same output that we get when you plug in x, the only difference being that its sign will be opposite the sign of the original output.

$$f(-x) = -f(x)$$

Below are graphs that are symmetric with respect to the origin and therefore represent odd functions. Be sure to visually compare quadrants that are diagonal from each other (quadrants I and III, and quadrants II and IV). For every first-quadrant point (x, y) in the graph of an odd function, there's a third-quadrant point on the graph with coordinates $(-x, -y)$. Similarly, for every second-quadrant point (x, y) in the graph of an odd function, there's a fourth-quadrant point on the graph with coordinates $(-x, -y)$.







We can also identify odd functions given points in a table. If a function is odd, then opposite values of x will have opposite values of y. For instance, $x = 1$ and $x = -1$ might give $y = -2$ and $y = 2$, $x = 2$ and $x = -2$ might give $y = 5$ and $y = -5$, $x = 3$ and $x = -3$ might give $y = -1$ and $y = 1$, etc.

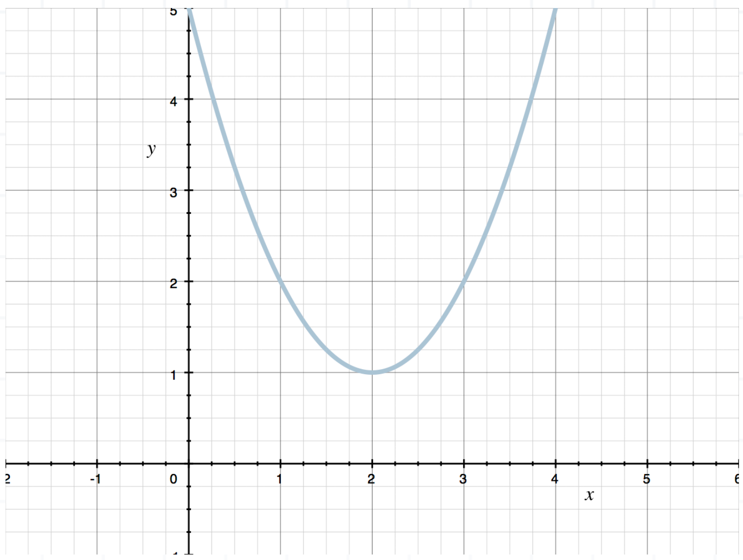
As an example, this table of values could represent an odd function.

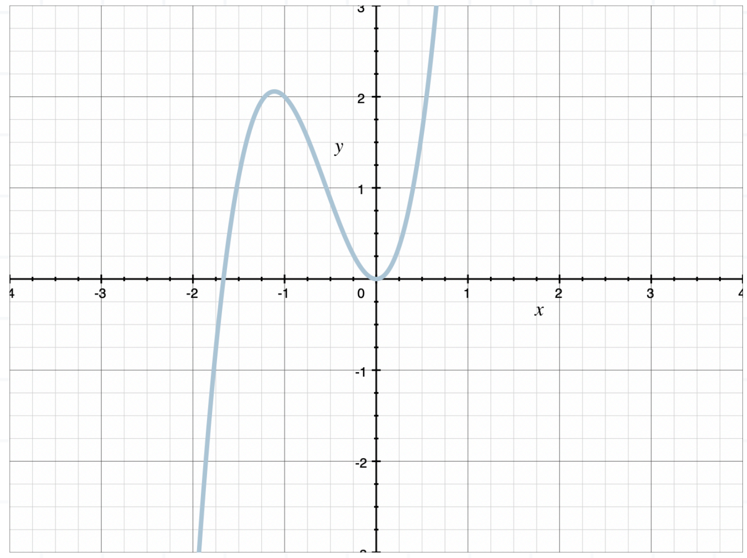
x	-3	-2	-1	0	1	2	3
f(x)	-1	-2	1	0	-1	2	1

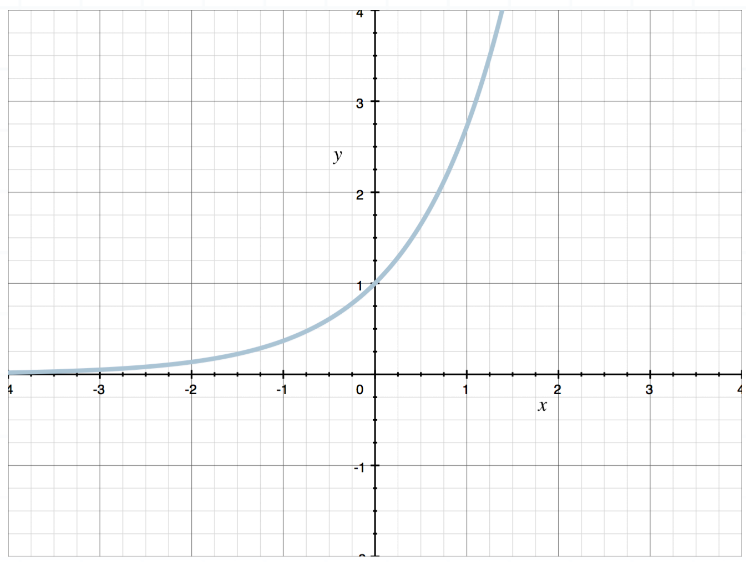
Neither even nor odd

Functions that aren't even and aren't odd are not symmetric with respect to the y-axis, and also not symmetric with respect to the origin

It's possible that a graph could be symmetric with respect to the x-axis, but then it wouldn't pass the Vertical Line Test and therefore wouldn't represent a function.







We can also identify functions that aren't even or odd given points in a table. If a function is neither even nor odd, then opposite values of x won't consistently correspond to equivalent values of y or opposite values of y. For instance, $x = 1$ and $x = -1$ might give $y = 2$ and $y = -1$, while $x = 2$ and $x = -2$ might give $y = 3$ and $y = 2$, etc.

As an example, this table of values could represent a function that's neither even nor odd.

x	-3	-2	-1	0	1	2	3
f(x)	1	-1	1	-3	2	0	5

Let's do an example where we determine whether a function is even, odd, or neither.

Example

Is the function even, odd, or neither?

$$f(x) = x^5 - 3x^3$$

To use algebra to classify the function, we need to find the expression for $f(-x)$, so we'll replace every x (in the expression for $f(x)$) with $-x$.

$$f(-x) = (-x)^5 - 3(-x)^3$$

Remember that

$$(-x)^5 = (-1x)^5 = (-1)^5x^5$$

and

$$(-x)^3 = (-1x)^3 = (-1)^3x^3$$

Raising -1 to an odd power gives -1, so

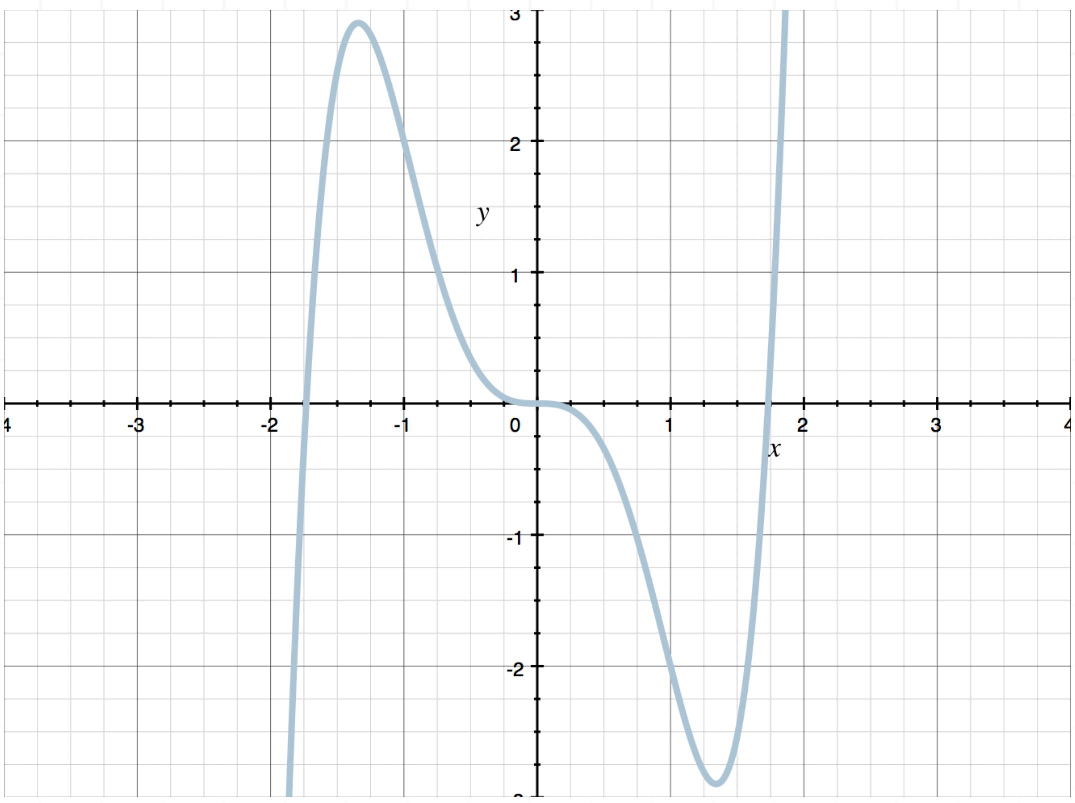
$$f(-x) = (-1)x^5 - 3(-1)x^3$$

Factor out a -1, and then simplify.

$$f(-x) = -1(x^5 - 3x^3)$$

$$f(-x) = -(x^5 - 3x^3)$$

Since $f(-x) = -f(x)$, the function is odd. We can see that the graph is symmetric with respect to the origin.



Let's try another example of even, odd, or neither.

Example

Is the function even, odd, or neither?

$$f(x) = 5x^2 - x^4$$

To use algebra to classify the function, we need to find the expression for $f(-x)$, so we'll replace every x (in the expression for $f(x)$) with $-x$.

$$f(-x) = 5(-x)^2 - (-x)^4$$

Remember that

$$(-x)^2 = (-1x)^2 = (-1)^2x^2$$

and

$$(-x)^4 = (-1x)^4 = (-1)^4x^4$$

Raising -1 to an even power gives 1, so

$$f(-x) = 5(1)x^2 - (1)x^4$$

$$f(-x) = 5x^2 - x^4$$

Since $f(-x) = f(x)$, the function is even. We can see that the graph is symmetric with respect to the y-axis.