Topic: Rational equations

Question: Solve the equation.

$$\frac{5}{x+1} = \frac{2}{x}$$

Answer choices:

A $x = \frac{2}{7}$

B $x = \frac{3}{2}$

C $x = \frac{2}{3}$

D $x = -\frac{5}{3}$

Solution: C

If $x + 1 = 0$ or $x = 0$, then the equation

$$\frac{5}{x+1} = \frac{2}{x}$$

is undefined, so this equation is true only if $x \neq -1$ and $x \neq 0$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $x(x+1)$. Multiply both sides of the equation by the LCD, $x(x+1)$.

$$\left(\frac{5}{x+1} = \frac{2}{x}\right) x(x+1)$$

$$\frac{5}{x+1} \cdot x(x+1) = \frac{2}{x} \cdot x(x+1)$$

$$5x = 2(x+1)$$

$$5x = 2x + 2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

Topic: Rational equations

Question: Solve the equation.

$$\frac{3x}{x-2} = 5$$

Answer choices:

A $x = 5$

B $x = -\frac{5}{2}$

C $x = \frac{1}{5}$

D $x = \frac{1}{2}$

Solution: A

If $x - 2 = 0$, then the equation

$$\frac{3x}{x-2} = 5$$

is undefined, so this equation is true only if $x \neq 2$.

Multiply both sides of the equation by $x - 2$.

$$\frac{3x}{x-2} \cdot (x-2) = 5 \cdot (x-2)$$

$$3x = 5x - 10$$

$$-2x = -10$$

$$x = 5$$

Topic: Rational equations

Question: Solve the equation.

$$\frac{x}{4} - \frac{2}{x} = \frac{1}{2}$$

Answer choices:

A $x = 2$ and $x = -4$

B $x = -2$ and $x = 0$

C $x = -\sqrt{10}$ and $x = \sqrt{10}$

D $x = -2$ and $x = 4$

Solution: D

If $x = 0$, then the equation

$$\frac{x}{4} - \frac{2}{x} = \frac{1}{2}$$

is undefined, so this equation is true only if $x \neq 0$.

Now we need to find the least common denominator of all denominators in the equation. The LCD is $4x$. Multiply both sides of the equation by the LCD, $4x$.

$$\left(\frac{x}{4} - \frac{2}{x} = \frac{1}{2}\right) 4x$$

$$\frac{x}{4} \cdot 4x - \frac{2}{x} \cdot 4x = \frac{1}{2} \cdot 4x$$

$$x^2 - 8 = 2x$$

$$x^2 - 2x - 8 = 0$$

$$(x + 2)(x - 4) = 0$$

$x = -2$ and $x = 4$