Algebra 2 Workbook Solutions

Ratios and proportions

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RATIOS AND PROPORTIONS

■ 1. The class has 12 girls and 18 boys. What is the ratio of boys to the total number of students in the class?

Solution:

We know that there are 18 boys and we're looking for the ratio

$$\frac{\text{boys}}{\text{total}}$$

We need to find the total number of students in the class by adding the number of boys and girls together.

$$12 + 18 = 30$$

Then the ratio of boys to the total number of students is

$$\frac{\text{boys}}{\text{total}} = \frac{18}{30} = \frac{6(3)}{6(5)} = \frac{3}{5}$$

There are 3 boys for every 5 students.

■ 2. The ratio of boys to girls in the class is $4 : 3$. The total number of students in the class is 28. How many girls are in the class?

Solution:

We know that the ratio of boys to girls in the class is $4:3$. This means that there are 3 girls in each group of 7 students.

$$\frac{\text{girls in group}}{\text{total in group}} = \frac{3}{7}$$

This needs to be proportional to the total number of girls in the class.

$$\frac{\text{girls in group}}{\text{total in group}} = \frac{\text{total girls in class}}{\text{total in class}}$$

Let g be the total number of girls in the class. Then the number of girls in the class is

$$\frac{3}{7} = \frac{g}{28}$$

$$7g = 84$$

$$\frac{7g}{7} = \frac{84}{7}$$

$$g = 12$$

There are 12 girls in the class.

■ 3. The ratio of boys to girls in the class is $5:3$. The total number of students in the class is 32. How many boys are in the class?

Solution:

We know that the ratio of boys to girls in the class is $5 : 3$. This means that there are 5 boys in each group of 8 students.

$$\frac{\text{boys in group}}{\text{total in group}} = \frac{5}{8}$$

This needs to be proportional to the total number of boys in the class.

$$\frac{\text{boys in group}}{\text{total in group}} = \frac{\text{total boys in class}}{\text{total in class}}$$

Let b be the total number of boys in the class. Then the number of boys in the class is

$$\frac{5}{8} = \frac{b}{32}$$

$$8b = 160$$

$$\frac{8b}{8} = \frac{160}{8}$$

$$b = 20$$

There are 20 boys in the class.

■ 4. Two numbers have a ratio of 1 to 4 and a sum of 40. What are the two numbers?

Solution:

Let's call the two numbers x and y and set up a ratio.

$$\frac{x}{y} = \frac{1}{4}$$

Solve for a variable by cross multiplying.

$$4x = 1y$$

$$y = 4x$$

Set up an equation for the sum of the two numbers.

$$x + y = 40$$

Use substitution and plug $4x$ in for y.

$$x + y = 40$$

$$x + 4x = 40$$

$$5x = 40$$

$$\frac{5x}{5} = \frac{40}{5}$$

$$x = 8$$

Now use $y = 4x$ to solve for y by plugging 8 in for x.

$$y = 4x$$

$$y = 4(8)$$

$$y = 32$$

The two numbers are 8 and 32.

■ 5. There are 11 quarters, 9 dimes, and 13 nickels. What is the ratio of dimes to the total number of coins?

Solution:

We know that there are 9 dimes and we're looking for the ratio

$$\frac{\text{dimes}}{\text{total}}$$

We need to find the total number of coins by adding all the groups together.

$$11 + 9 + 13 = 33$$

Then the ratio of dimes to total coins is

$$\frac{\text{dimes}}{\text{total}} = \frac{9}{33}$$

Now we need to reduce the ratio.

$$\frac{3(3)}{3(11)}$$

$$\frac{3}{11}$$

There are 3 dimes for every 11 coins.

■ 6. The ratio of dimes to quarters is $3:2$. The total value of the coins is $\$2.40$. How many quarters are there?

Solution:

Let d be the number of dimes and q be the number of quarters. Set up the ratio of dimes to quarters.

$$\frac{d}{q}=\frac{3}{2}$$

Solve for a variable by cross multiplying.

$$3q=2d$$

$$q=\frac{2}{3}d$$

Set up an equation for the total value of the coins.

$$0.1d+0.25q=2.40$$

Clear out the decimals by multiplying through by 100.

$$(100)0.1d+(100)0.25q=(100)2.40$$

$$10d+25q=240$$

Use substitution and plug $(2/3)d$ in for q.

$$10d + 25q = 240$$

$$10d + 25\left(\frac{2}{3}d\right) = 240$$

Solve for d by simplifying and combining like terms.

$$10d + \frac{50}{3}d = 240$$

$$\frac{30}{3}d + \frac{50}{3}d = 240$$

$$\frac{80}{3}d = 240$$

$$\frac{3}{80} \cdot \frac{80}{3}d = 240 \cdot \frac{3}{80}$$

$$d = 3 \cdot 3$$

$$d = 9$$

Now use $q = (2/3)d$ to solve for q by plugging 9 in for d.

$$q = \frac{2}{3}d$$

$$q = \frac{2}{3}(9)$$

$$q = 2(3)$$

$$q = 6$$

There are 6 quarters.

CHEMICAL COMPOUNDS

■ 1. Find the molar mass for one molecule of table salt in grams per mole. Table salt has the molecular formula NaCl.

Sodium (Na) has a molar mass of 22.989770 g/mol

Chlorine (Cl) has a molar mass of 35.453 g/mol

Solution:

A molecule of table salt has one sodium atom and one chlorine atom. To find the molar mass of table salt, add the molar mass of sodium and chlorine together.

$$22.989770 + 35.453 = 58.44277$$

The molecular mass of one molecule of table salt is 58.44277 g/mol.

■ 2. Find the molar mass for one molecule of isopropyl chloride in grams per mole. Isopropyl chloride has the molecular formula C_3H_7Cl.

Carbon (C) has a molar mass of 12.0107 g/mol

Hydrogen (H) has a molar mass of 1.00794 g/mol

Chlorine (Cl) has a molar mass of 35.453 g/mol

Solution:

A molecule of isopropyl chloride has 3 carbon, 7 hydrogen, and 1 chlorine atom. To find the molar mass of isopropyl chloride, multiply each of the atoms by their molar mass.

Carbon: $12.0107(3) = 36.0321$

Hydrogen: $1.00794(7) = 7.05558$

Chlorine: $35.453(1) = 35.453$

Then add these together.

$$36.0321 + 7.05558 + 35.453 = 78.54068$$

The molecular mass of one molecule of isopropyl chloride is 78.54068 g/mol.

■ 3. Glucose, $C_6H_{12}O_6$, has a molar mass of 180.156 g/mol. What is the mass of oxygen in a 100 g glucose sample given that the molar mass of carbon is 12.01 g/mol and the molar mass of hydrogen is 1.00794 g/mol.

Solution:

A molecule of glucose has 6 carbon, 12 hydrogen, and 6 oxygen atoms. We first need to find the molar mass of oxygen in a glucose molecule. Since

we're only given the molar masses of carbon and hydrogen molecules, we can calculate the molar mass of the oxygen as

$$6(12.01) + 12(1.00794) + x = 180.156$$

$$x = 96.0007$$

The molar mass of oxygen in a glucose molecule is 96.0007 g/mol.

Now we can set up a proportion which consists of two ratios. The first is the ratio of the molar mass of oxygen in a glucose molecule to the molar mass of glucose, and the second is the ratio of the mass of oxygen, which we would denote as x, to the given mass of the glucose sample. Therefore, we have

$$\frac{96.0007}{180.156} = \frac{x}{100}$$

$$180.156(x) = 100(96.0007)$$

$$x \approx 53.2875 \text{ g}$$

■ 4. Find the total mass of a sample of silver phosphate, Ag_3PO_4, in grams per mole given that it contains 25.55 g of silver with the molar mass 107.8682 g/mol. The molar mass of Ag_3PO_4 is 418.58 g/mol.

Solution:

We can set up a proportion of two ratios. The first is the ratio of the molar mass of the silver in silver phosphate, to the molar mass of the silver phosphate itself, and the second is the ratio of the mass of silver to the total mass of the silver phosphate sample, which we would denote as x. Therefore, we have

$$\frac{107.8682(3)}{418.58} = \frac{25.55}{x}$$

$$107.8682(3)x = 25.55(418.58)$$

$$x \approx 33.0487 \text{ g}$$

■ 5. If vitamin C has the molecular formula $C_6H_8O_6$ and the molar mass of vitamin C is 176.12 g/mol, find the mass of each element in a 325 g vitamin C sample.

Carbon (C) has a molar mass of 12.01 g/mol

Hydrogen (H) has a molar mass of 1.00794 g/mol

Oxygen (O) has a molar mass of 15.9994 g/mol

Solution:

Before we can find the mass of each element in a 325 g vitamin C sample, we need to find the molar mass of each element in a vitamin C molecule. A molecule of vitamin C has 6 carbon, 8 hydrogen, and 6 oxygen atoms. Therefore, we have

Carbon: $12.01(6) = 72.06$ g/mol

Hydrogen: $1.00794(8) = 8.06352$ g/mol

Oxygen: $15.9994(6) = 95.9964$ g/mol

Now we can set up a proportion for each of three elements in vitamin C. Let's proceed one by one.

The proportion that we can use to find the mass of carbon consists of two ratios: the first is the ratio of the molar mass of carbon in the vitamin C molecule to the molar mass of vitamin C, and the second is the ratio of the mass of carbon, which we would denote as x, to the mass of the vitamin C sample. Therefore, we have

$$\frac{72.06}{176.12} = \frac{x}{325}$$

$$176.12x = 325(72.06)$$

$$x \approx 132.975 \text{ g}$$

The proportion that we can use to find the mass of hydrogen consists of two ratios: the first is the ratio of the molar mass of hydrogen in the vitamin C molecule to the molar mass of vitamin C, and the second is the ratio of the mass of hydrogen, which we would denote as x, to the mass of the vitamin C sample. Therefore, we have

$$\frac{8.06352}{176.12} = \frac{x}{325}$$

$$176.12x = 325(8.06352)$$

$$x \approx 14.8799 \text{ g}$$

The proportion that we can use to find the mass of oxygen consists of two ratios: the first is the ratio of the molar mass of oxygen in the vitamin C molecule to the molar mass of vitamin C, and the second is the ratio of the mass of oxygen, which we would denote as x, to the mass of the vitamin C sample. Therefore, we have

$$\frac{95.9964}{176.12} = \frac{x}{325}$$

$$176.12x = 325(95.9964)$$

$$x \approx 177.145 \text{ g}$$

■ 6. Find the molar mass of calcium (Ca) in one mole of calcium carbonate.

$CaCO_3$ has a molar mass of 100.0869 g/mol.

Carbon (C) has a mass of 12.0107 g/mol

Oxygen (O) has a mass of 15.9994 g/mol

Solution:

Let's set up what we know and what we don't know.

Calcium: x g/mol

Carbon: $12.0107(1) = 12.0107$ g/mol

Oxygen: $15.9994(3) = 47.9982$ g/mol

$CaCO_3$: 100.0869 g/mol

Set up an equation to solve for the molar mass of calcium in calcium carbonate.

$$x + 12.0107 + 47.9982 = 100.0869$$

$$x + 60.0089 = 100.0869$$

$$x = 40.078$$

The molecular mass of calcium in calcium carbonate is 40.078 g/mol.

FRACTIONS TO DECIMALS TO PERCENTS

■ 1. Convert $60\,\%$ to a fraction in lowest terms.

Solution:

A percent can be expressed as part of 100.

$$\frac{60}{100}$$

Reduce the fraction to lowest terms.

$$\frac{6(10)}{10(10)} = \frac{6}{10} = \frac{3(2)}{5(2)} = \frac{3}{5}$$

■ 2. Convert $33.5\,\%$ to a decimal.

Solution:

To change a percent to a decimal, divide it by 100.

$$\frac{33.5}{100}$$

$$0.335$$

■ 3. Convert $2/3$ to a percent.

Solution:

To change a fraction to a percent, first change the fraction to a decimal, then multiply the decimal by 100.

$$\frac{2}{3}$$

$$0.6667$$

$$0.6667(100)$$

$$66.67\,\%$$

■ 4. Find $15\,\%$ of 48.

Solution:

To find a percent of a number, first change the percent to a decimal by dividing it by 100.

$$\frac{15}{100}$$

0.15

Multiply the decimal by 48.

$$0.15(48)$$

$$7.2$$

■ 5. Find a mixed fraction that represents $8\,\%$ of 120.

Solution:

Divide 8 by 100 and multiply it by 120.

$$\frac{8}{100} \cdot 120$$

$$\frac{2}{25} \cdot 120$$

$$\frac{240}{25}$$

$$\frac{48}{5}$$

5 goes into 48 nine times with a remainder of 3, so the mixed number is

$$9\frac{3}{5}$$

■ 6. Convert $100/160$ to a percent.

Solution:

Reduce the fraction to lowest terms.

$$\frac{10(10)}{16(10)} = \frac{10}{16} = \frac{5(2)}{8(2)} = \frac{5}{8}$$

Set up a proportion to find the percent.

$$\frac{5}{8} = \frac{x}{100}$$

$$8x = 500$$

$$x = \frac{500}{8}$$

$$x = 62.5\,\%$$

PERCENT MARKUP

■ 1. A book store purchases a book for $\$6.00$ and sells it for $\$9.00$. What percentage of the original price is the markup amount?

Solution:

First find the markup amount by subtracting the original price from the selling price.

$$9.00 - 6.00$$

$$3.00$$

Let x represent the percentage of the original price.

$$6.00x = 3.00$$

$$x = \frac{3}{6}$$

$$x = \frac{1}{2}$$

$$x = 0.5$$

$$x = 50\,\%$$

The book store marks up the book by $50\,\%$.

■ 2. A bike shop buys a used bike for $\$130$ and marks up the price by $35\,\%$. What is the markup amount?

Solution:

Find the markup amount by multiplying the original price by the percentage, divided by 100.

$$130\left(\frac{35}{100}\right)$$

$$130(0.35)$$

$$45.50$$

The bike shop marks up the bike by $\$45.50$.

■ 3. It costs a car manufacturer $\$12{,}800$ to produce a car. The percent markup is $48\,\%$. What is the selling price of the car.

Solution:

The selling price of the car is

$$12{,}800\left(1+\frac{48}{100}\right)$$

$$12{,}800(1 + 0.48)$$

$$12{,}800(1.48)$$

$$18{,}944$$

The car manufacturer sells the car for $\$18{,}944$.

■ 4. A bakery purchases a dozen sugar cookies for $\$2.25$. The markup percent is $60\,\%$. What is the selling price of the dozen sugar cookies?

Solution:

The selling price of the cookies is

$$2.25\left(1 + \frac{60}{100}\right)$$

$$2.25(1 + 0.60)$$

$$2.25(1.60)$$

$$3.60$$

The bakery sells the dozen sugar cookies for $\$3.60$.

■ 5. A store purchases dresses from a manufacturer, marks them up by $75\,\%$, and sells each dress for $\$91$. How much did the store pay the manufacturer for each dress?

Solution:

If the store paid x dollars for each dress and marked it up by $75\,\%$, then the price they're selling it for is 1.75 times the price they paid for each dress.

$$1.75x = 91$$

$$\frac{1.75x}{1.75} = \frac{91}{1.75}$$

$$x = 52$$

The store buys each dress from the manufacturer for $\$52$.

■ 6. If a furniture store purchases a chair from a manufacturer, marks it up by $24\,\%$, and sells the chair for $\$84.94$. How much did the furniture store pay the manufacturer for the chair?

Solution:

If the furniture store paid x dollars for the chair and marked it up by $24\,\%$, then the price they're selling it for is 1.24 times the price they paid for it.

$$1.24x = 84.94$$

$$\frac{1.24x}{1.24} = \frac{84.94}{1.24}$$

$$x = 68.50$$

The store buys the chair from the manufacturer for $\$68.50$.

PERCENT MARKDOWN

■ 1. A computer has an original price of $\$375$ and is now on sale for $\$255$. What is the percent markdown?

Solution:

Find the difference between the sale price and the original price.

$$\$375 - \$255 = \$120$$

Set up a ratio and plug in the values we've found.

$$\frac{\text{Discount Amount}}{\text{Original Price}} = \frac{\text{Percent Markdown}}{100}$$

$$\frac{120}{375} = \frac{x}{100}$$

Solve for percent markdown.

$$\frac{8(15)}{25(15)} = \frac{x}{100}$$

$$\frac{8}{25} = \frac{x}{100}$$

$$25x = 800$$

$$\frac{25x}{25} = \frac{800}{25}$$

$$x = 32$$

The percent markdown is $32\,\%$.

■ 2. A sweater has an original price of $\$34$ and is now on sale for $\$25.50$. What is the percent markdown?

Solution:

Find the difference between the sale price and the original price.

$$\$34 - \$25.50 = \$8.5$$

Set up a ratio and plug in the values we've found.

$$\frac{\text{Discount Amount}}{\text{Original Price}} = \frac{\text{Percent Markdown}}{100}$$

$$\frac{8.5}{34} = \frac{x}{100}$$

Solve for percent markdown.

$$34x = 850$$

$$\frac{34x}{34} = \frac{850}{34}$$

$$x = 25$$

The percent markdown is $25\,\%$.

■ 3. The regular price of an item is $\$75$, but the item is now on clearance for $40\,\%$ off the regular price. What is the sale price of the item?

Solution:

Find $40\,\%$ of $\$75$ by multiplying 75 by 0.4.

$$75 \cdot 0.4 = 30$$

The price is marked down by $\$30$, so subtract $\$30$ from the regular price of the item, $\$75$.

$$\$75 - \$30 = \$45$$

The sale price of the item is $\$45$.

■ 4. The regular price of the latest smartphone is $\$749$. After two years, the smartphone is on sale for $25\,\%$ off. What is the sale price of the item?

Solution:

Find $25\,\%$ of $\$749$ by multiplying 749 by 0.25.

$$749 \cdot 0.25 = 187.25$$

The price is marked down $\$187.25$. Subtract $\$187.25$ from the regular price of the smartphone, $\$749$.

$$\$749 - \$187.25 = \$561.75$$

The sale price of the smartphone is $\$561.75$.

■ 5. The price of a house was marked down to $\$250{,}000$, and the sale price was $12\,\%$ off of the original price. What was the original price of the house?

Solution:

Let's let x represent the original price of the house. If we take $12\,\%$ off the original price, that means the sale price is $88\,\%$ of the original price ($100\% - 12\,\% = 88\,\%$). And we know that $88\,\%$ of the original price is $\$250{,}000$.

$$88\,\% \text{ of } x = \$250{,}000$$

$$\frac{88}{100} \cdot x = \$250{,}000$$

$$x = \frac{100}{88} \cdot \$250{,}000$$

$$x = \$284{,}091$$

So the original price of the house is $\$284{,}091$.

■ 6. The sale price of a shirt is $\$68.00$, and the shirt is on sale for $25\,\%$ off the original price. What was the original price of the shirt?

Solution:

Let's let x represent the original price of the shirt. If we take $25\,\%$ off the original price, that means the sale price is $75\,\%$ of the original price ($100\% - 25\,\% = 75\,\%$). And we know that $75\,\%$ of the original price is $\$68.00$.

$$75\,\% \text{ of } x = \$68.00$$

$$\frac{75}{100} \cdot x = \$68.00$$

$$x = \frac{100}{75} \cdot \$68.00$$

$$x \approx \$90.67$$

So the original price of the shirt is $x \approx \$90.67$.

CALCULATING COMMISSION

■ 1. A makeup company advertises that we can make $15\,\%$ commission on sales of their product. If we sell $\$3{,}252$ worth of product, how much money do we earn?

Solution:

Find the commission by multiplying the sales by the commission percentage.

$$\$3{,}252 \cdot 0.15 = \$487.80$$

We earn $\$487.80$ in commission by selling $\$3{,}252$ worth of product.

■ 2. An employee at a clothing store earned $\$1{,}450$ in hourly pay for the month. She also sold $\$4{,}250$ worth of merchandise and will earn a commission of $6\,\%$ on those sales. What is the employee's expected paycheck before tax deductions?

Solution:

Find the commission by multiplying the sales by the commission percentage.

$$\$4{,}250 \cdot 0.06 = \$255$$

Now add $\$255$ to the hourly pay to find the total amount.

$$\$1{,}450 + 255 = \$1{,}705$$

The employee should expect $\$1{,}705$ on her paycheck.

■ 3. A local bakery sells croissants for $\$5.00$ each. A sales clerk makes a $6\,\%$ commission on the selling price of each croissant he sells. How many croissants does he need to sell to earn $\$60$ in commission?

Solution:

First, let's determine how much money the sales clerk needs to make in sales to earn $\$60$ in commission.

$$\$60 = 6\,\% \text{ of } x$$

where x is the amount of money he needs to make in sales.

$$\$60 = 0.06x$$

$$\frac{60}{0.06} = x$$

$$\$1{,}000 = x$$

Since each croissant costs $\$5.00$, we can find the number of croissants the sales clerk needs to sell to earn $\$60$ in commission.

$$\frac{\$1{,}000}{\$5.00} = 200$$

■ 4. A car salesman earns $\$48{,}000$ per year plus a commission of $12\,\%$ on all the cars he sells. If he wants a yearly salary of $\$72{,}500$, how much money in car sales does he need to make?

Solution:

Subtract his salary from the amount he wants to earn for the year.

$$\$72{,}500 - \$48{,}000 = \$24{,}500$$

Let x be the amount of car sales.

$$0.12x = \$24{,}500$$

$$\frac{0.12x}{0.12} = \frac{\$24{,}500}{0.12}$$

$$x = \$204{,}166.67$$

To earn a salary of $\$72{,}500$, the car salesman needs to sell $\$204{,}166.67$ worth of cars.

■ 5. Brittany earns $\$1{,}772.10$ in commission of makeup products. If she earns $18\,\%$ commission, how much money in makeup sales did she make?

Solution:

Let x be the amount of makeup sales.

$$0.18x = \$1{,}772.10$$

$$\frac{0.18x}{0.18} = \frac{\$1{,}772.10}{0.18}$$

$$x = \$9{,}845$$

Brittany sold $\$9{,}845$ worth of makeup products to earn $\$1{,}772.10$ in commission.

■ 6. Anthony works at a clothing store and earned $\$1{,}644.75$ last month before tax deductions. If he earns $7.5\,\%$ in commission and his hourly pay was $\$975$ for the month, how much clothing did he sell?

Solution:

Subtract the hourly pay from the total paycheck for the month.

$$\$1644.75 - \$975 = \$669.75$$

Let x be the amount of clothing sales.

$$0.075x = \$669.75$$

$$\frac{0.075x}{0.075} = \frac{\$669.75}{0.075}$$

$$x = \$8{,}930$$

Anthony sold $\$8{,}930$ worth of clothes.

CALCULATING SIMPLE INTEREST

■ 1. If we deposit $\$300$ into a savings account and it earns $2\,\%$ in simple interest, how much interest will we earn on the account in 7 years?

Solution:

We know the values of P, r, and t.

$$P = 300$$

$$r = \frac{2}{100} = 0.02$$

$t = 7$ years

Plug into the formula for simple interest.

$$I = Prt$$

$$I = 300(0.02)(7)$$

$$I = 42$$

In 7 years, we'll earn $\$42$ in interest.

■ 2. If we invest $\$500$ that earns $13\,\%$ in simple interest, how much interest will we earn in 12 years?

Solution:

We know the values of P, r, and t.

$$P = 500$$

$$r = \frac{13}{100} = 0.13$$

$$t = 12 \text{ years}$$

Plug into the formula for simple interest.

$$I = Prt$$

$$I = 500(0.13)(12)$$

$$I = 780$$

In 12 years, we'll earn $\$780$ in interest.

■ 3. What is the simple interest rate if we invest $\$7{,}000$ and earn $\$3{,}250$ in interest in 15 years?

Solution:

We know the values of I, P, and t, and we need to find r.

$$P = \$7{,}000$$

$$I = \$3{,}250$$

$$t = 15 \text{ years}$$

Rewrite the original formula $I = Prt$ for r and substitute the values we know.

$$r = \frac{I}{Pt}$$

$$r = \frac{\$3{,}250}{\$7{,}000(15)}$$

$$r \approx 0.031$$

$$r \approx 3.1\,\%$$

■ 4. If we deposit $\$275$ into a savings account that earns $4\,\%$ simple interest, how much is in the account after 2 years?

Solution:

We know the values of P, r, and t.

$$P = 275$$

$$r = \frac{4}{100} = 0.04$$

$$t = 2 \text{ years}$$

Plug into the formula for the total amount.

$$A = P(1 + rt)$$

$$A = 275(1 + 0.04 \cdot 2)$$

$$A = 297$$

In 2 years, we'll have $\$297$ in the account.

■ 5. If we invest $\$450$ that earns $15\,\%$ simple interest, how many years will it take to have $\$1{,}800$ in the account?

Solution:

We know the values of A, P, and r.

$$P = 450$$

$$r = 0.15$$

$$A = \$1{,}800$$

Rewrite the original formula, $A = P(1 + rt)$, for t.

$$1 + rt = \frac{A}{P}$$

$$rt = \frac{A}{P} - 1$$

$$t = \frac{\frac{A}{P} - 1}{r}$$

Now substitute the values we know and calculate t.

$$t = \frac{\frac{\$1{,}800}{\$450} - 1}{0.15}$$

$$t = \frac{4 - 1}{0.15}$$

$$t = 20 \text{ years}$$

■ 6. If we invest $\$1{,}230$ that earns $14\,\%$ simple interest, how much is in the account after 10 years?

Solution:

We know the values of P, r, and t.

$$P = 1{,}230$$

$$r = \frac{14}{100} = 0.14$$

$$t = 10 \text{ years}$$

Plug into the formula for the total amount.

$$A = P(1 + rt)$$

$$A = 1{,}230(1 + 0.14 \cdot 10)$$

$$A = 2{,}952$$

In 6 years, we'll have $\$2{,}952$ in the account.

COMPLEX FRACTIONS

■ 1. Simplify the expression.

$$\frac{\frac{3}{5}}{\frac{2}{7}}$$

Solution:

Multiply the numerator by the reciprocal of the denominator.

$$\frac{3}{5} \div \frac{2}{7}$$

$$\frac{3}{5} \cdot \frac{7}{2}$$

$$\frac{3 \cdot 7}{5 \cdot 2}$$

$$\frac{21}{10}$$

■ 2. Simplify the expression.

$$\frac{y}{\frac{x}{z}}$$

Solution:

Rewrite the given fraction so that the numerator is also a fraction.

$$\frac{\frac{y}{1}}{\frac{x}{z}}$$

Multiply the numerator by the reciprocal of the denominator.

$$\frac{y}{1} \div \frac{x}{z}$$

$$\frac{y}{1} \cdot \frac{z}{x}$$

$$y \cdot z$$

$$\overline{1 \cdot x}$$

$$\frac{yz}{x}$$

■ 3. Simplify the expression.

$$\frac{\frac{x}{b}}{n}$$

Solution:

Rewrite the given fraction so that the denominator is also a fraction.

$$\frac{\frac{x}{b}}{\frac{n}{1}}$$

Multiply the numerator by the reciprocal of the denominator.

$$\frac{x}{b} \div \frac{n}{1}$$

$$\frac{x}{b} \cdot \frac{1}{n}$$

$$\frac{x \cdot 1}{b \cdot n}$$

$$\frac{x}{bn}$$

4. Simplify the expression.

$$\frac{\frac{a}{m}}{n + \frac{1}{b}}$$

Solution:

Simplify just the denominator by finding a common denominator.

$$\frac{b}{b} \cdot n + \frac{1}{b}$$

$$\frac{bn}{b} + \frac{1}{b}$$

$$\frac{bn+1}{b}$$

Rewrite the given fraction with the simplified denominator.

$$\frac{\frac{a}{m}}{\frac{bn+1}{b}}$$

Multiply the numerator by the reciprocal of the denominator.

$$\frac{a}{m} \div \frac{bn+1}{b}$$

$$\frac{a}{m} \cdot \frac{b}{bn+1}$$

$$\frac{a \cdot b}{m(bn+1)}$$

$$\frac{ab}{bmn+m}$$

5. Simplify the expression.

$$\frac{\frac{1}{y} - \frac{1}{x}}{1 - \frac{1}{y}}$$

Solution:

Simplify just the numerator by finding a common denominator.

$$\frac{1}{y}-\frac{1}{x}$$

$$\frac{1}{y}\cdot\frac{x}{x}-\frac{1}{x}\cdot\frac{y}{y}$$

$$\frac{x}{xy}-\frac{y}{xy}$$

$$\frac{x-y}{xy}$$

Simplify just the denominator by finding a common denominator.

$$1-\frac{1}{y}$$

$$1\cdot\frac{y}{y}-\frac{1}{y}$$

$$\frac{y}{y}-\frac{1}{y}$$

$$\frac{y-1}{y}$$

Rewrite the original fraction with the simplified numerator and denominator.

$$\frac{\frac{x-y}{xy}}{\frac{y-1}{y}}$$

Multiply the numerator by the reciprocal of the denominator.

$$\frac{x-y}{xy} \cdot \frac{y}{y-1}$$

$$\frac{x-y}{x} \cdot \frac{1}{y-1}$$

$$\frac{x-y}{x(y-1)}$$

6. Simplify the expression.

$$\frac{\frac{1}{a-5} - \frac{1}{a+5}}{\frac{5}{a+5}}$$

Solution:

Simplify just the numerator by finding a common denominator.

$$\frac{1}{a-5} - \frac{1}{a+5}$$

$$\frac{1}{a-5} \cdot \frac{a+5}{a+5} - \frac{1}{a+5} \cdot \frac{a-5}{a-5}$$

$$\frac{a+5}{(a-5)(a+5)} - \frac{a-5}{(a-5)(a+5)}$$

$$\frac{a+5-a+5}{(a-5)(a+5)}$$

$$\frac{10}{(a-5)(a+5)}$$

Rewrite the given fraction with the simplified numerator.

$$\frac{\frac{10}{(a-5)(a+5)}}{\frac{5}{a+5}}$$

Multiply the numerator by the reciprocal of the denominator.

$$\frac{10}{(a-5)(a+5)} \cdot \frac{a+5}{5}$$

$$\frac{2}{a-5}$$

RATIOS AND PROPORTIONS WITH COMPLEX FRACTIONS

■ 1. Solve for the variable.

$$\frac{x}{\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{1}{4}}$$

Solution:

Cross multiply and then solve for x.

$$\frac{1}{4} \cdot x = \frac{1}{2} \cdot \frac{3}{4}$$

$$\frac{x}{4} = \frac{3}{8}$$

$$4 \cdot \frac{x}{4} = \frac{3}{8} \cdot 4$$

$$x = \frac{12}{8}$$

$$x = \frac{4 \cdot 3}{4 \cdot 2}$$

$$x = \frac{3}{2}$$

■ 2. Solve for the variable.

$$\frac{\frac{4}{7}}{\frac{1}{6}} = \frac{y}{\frac{7}{2}}$$

Solution:

Cross multiply and then solve for y.

$$\frac{1}{6} \cdot y = \frac{4}{7} \cdot \frac{7}{2}$$

$$\frac{y}{6} = \frac{4 \cdot 7}{2 \cdot 7}$$

$$\frac{y}{6} = \frac{4}{2}$$

$$\frac{y}{6} = 2$$

$$6 \cdot \frac{y}{6} = 2 \cdot 6$$

$$y = 12$$

■ 3. Solve for the variable.

$$\frac{\frac{x}{2}}{\frac{8}{3}} = \frac{\frac{3}{4}}{\frac{2}{5}}$$

Solution:

Cross multiply and then solve for x.

$$\frac{x}{2} \cdot \frac{2}{5} = \frac{8}{3} \cdot \frac{3}{4}$$

$$\frac{2x}{10} = \frac{24}{12}$$

$$\frac{2 \cdot x}{2 \cdot 5} = \frac{12 \cdot 2}{12}$$

$$\frac{x}{5} = 2$$

$$5 \cdot \frac{x}{5} = 2 \cdot 5$$

$$x = 10$$

4. Solve for the variable.

$$\frac{\frac{3}{8}}{\frac{x}{2}} = \frac{\frac{1}{4}}{\frac{4}{5}}$$

Solution:

Cross multiply and then solve for x.

$$\frac{x}{2} \cdot \frac{1}{4} = \frac{3}{8} \cdot \frac{4}{5}$$

$$\frac{x}{8} = \frac{12}{40}$$

$$\frac{x}{8} = \frac{4 \cdot 3}{4 \cdot 10}$$

$$\frac{x}{8} = \frac{3}{10}$$

$$8 \cdot \frac{x}{8} = \frac{3}{10} \cdot 8$$

$$x = \frac{24}{10}$$

$$x = \frac{2 \cdot 12}{2 \cdot 5}$$

$$x = \frac{12}{5}$$

5. Solve for the variable.

$$\frac{\frac{4}{5} - \frac{1}{2}}{\frac{3}{2}} = \frac{\frac{6}{7} + \frac{1}{7}}{\frac{b}{8}}$$

Solution:

Simplify just the numerators by finding a common denominator. We get

$$\frac{4}{5}-\frac{1}{2}=\frac{3}{10}$$

and

$$\frac{6}{7}+\frac{1}{7}=\frac{7}{7}=1$$

Replacing each numerator in the original equation, we get

$$\frac{\frac{3}{10}}{\frac{3}{2}}=\frac{1}{\frac{b}{8}}$$

Cross multiply and then solve for b.

$$\frac{3}{10}\cdot\frac{b}{8}=\frac{3}{2}\cdot 1$$

$$\frac{3b}{80}=\frac{3}{2}$$

$$\frac{3b}{80}\cdot 80=\frac{3}{2}\cdot 80$$

$$3b=120$$

$$b=40$$

6. Solve for the variable.

$$\frac{\frac{2}{3}}{\frac{1}{c}} = \frac{\frac{4}{5}}{\frac{7}{6}}$$

Solution:

Cross multiply and then solve for c.

$$\frac{1}{c} \cdot \frac{4}{5} = \frac{2}{3} \cdot \frac{7}{6}$$

$$\frac{4}{5c} = \frac{14}{18}$$

$$\frac{4}{5c} = \frac{2 \cdot 7}{2 \cdot 9}$$

$$\frac{4}{5c} = \frac{7}{9}$$

$$35c = 36$$

$$\frac{35c}{35} = \frac{36}{35}$$

$$c = \frac{36}{35}$$

IMAGINARY AND COMPLEX NUMBERS

■ 1. Simplify the imaginary expression.

$$2 - 6i - 4 + 9i$$

Solution:

Group, then combine like terms.

$$2 - 4 - 6i + 9i$$

$$-2 + 3i$$

■ 2. Simplify the imaginary expression.

$$-3 - 7i + 8 + 3i$$

Solution:

Group, then combine like terms.

$$-3 + 8 - 7i + 3i$$

$$5 - 4i$$

■ 3. Simplify the imaginary expression.

$$\sqrt{-4} + ii + 5i - 2i^3$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{4 \cdot -1} + i^2 + 5i - 2i^2 i$$

$$\sqrt{4}\sqrt{-1} + (-1) + 5i - 2(-1)i$$

$$2i - 1 + 5i + 2i$$

Group, then combine like terms.

$$-1 + 2i + 2i + 5i$$

$$-1 + 9i$$

■ 4. Simplify the imaginary expression.

$$\sqrt{27} - 3ii + 2i - 7i^3 + \sqrt{-36}$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{9 \cdot 3} - 3i^2 + 2i - 7i^2 i + \sqrt{36 \cdot -1}$$

$$\sqrt{9}\sqrt{3} - 3(-1) + 2i - 7(-1)i + \sqrt{36}\sqrt{-1}$$

$$3\sqrt{3} + 3 + 2i + 7i + 6i$$

Group, then combine like terms.

$$3 + 3\sqrt{3} + 15i$$

■ 5. Simplify the imaginary expression.

$$\sqrt{-9} + 2i^3 + 6i - \sqrt{25}\sqrt{-25} - 2\sqrt{-16}$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{9 \cdot -1} + 2i^2 i + 6i - 5\sqrt{25 \cdot -1} - 2\sqrt{16 \cdot -1}$$

$$\sqrt{9}\sqrt{-1} + 2(-1)i + 6i - 5\sqrt{25}\sqrt{-1} - 2\sqrt{16}\sqrt{-1}$$

$$3i - 2i + 6i - 5(5)i - 2(4)i$$

Simplify, then combine like terms.

$$i + 6i - 25i - 8i$$

$$7i - 33i$$

$-26i$

■ 6. Simplify the imaginary expression.

$$\sqrt{-4} + 2i^4 + 6i^5 - \sqrt{-49} - 2i^6$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{4 \cdot -1} + 2i^2i^2 + 6i^2i^2i - \sqrt{49 \cdot -1} - 2i^2i^2i^2$$

$$\sqrt{4}\sqrt{-1} + 2(-1)(-1) + 6(-1)(-1)i - \sqrt{49}\sqrt{-1} - 2(-1)(-1)(-1)$$

$$2i + 2 + 6i - 7i + 2$$

Group, then combine like terms.

$$2 + 2 + 2i + 6i - 7i$$

$$4 + 8i - 7i$$

$$4 + i$$

RATIONALIZING COMPLEX DENOMINATORS

■ 1. Use the conjugate method to simplify the imaginary expression.

$$\frac{2+6i}{3-i}$$

Solution:

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $3-i$ is $3+i$, so multiply the expression by $(3+i)/(3+i)$.

$$\frac{2+6i}{3-i} \cdot \frac{3+i}{3+i}$$

$$\frac{(2+6i)(3+i)}{(3-i)(3+i)}$$

$$\frac{6+2i+18i+6i^2}{9+3i-3i-i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{6+20i+6(-1)}{9-(-1)}$$

$$\frac{6-6+20i}{9+1}$$

$$\frac{20i}{10}$$

$$2i$$

■ 2. Use the conjugate method to simplify the imaginary expression.

$$\frac{2-2i}{4i-1}$$

Solution:

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $4i-1$ is $-4i-1$, so multiply the expression by $(-4i-1)/(-4i-1)$.

$$\frac{2-2i}{4i-1} \cdot \frac{-4i-1}{-4i-1}$$

$$\frac{(2-2i)(-4i-1)}{(4i-1)(-4i-1)}$$

$$\frac{-8i-2+8i^2+2i}{-16i^2-4i+4i+1}$$

$$\frac{-6i-2+8i^2}{-16i^2+1}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-6i-2+8(-1)}{-16(-1)+1}$$

$$\frac{-6i-2-8}{16+1}$$

$$\frac{-6i-10}{17}$$

$$-\frac{10}{17}-\frac{6}{17}i$$

■ 3. Use the conjugate method to simplify the imaginary expression.

$$\frac{3i+2i^2}{5i^3+4i^4}$$

Solution:

Simplify the expression first.

$$\frac{3i+2i^2}{5i^3+4i^4}$$

$$\frac{3i+2i^2}{5i^2i+4i^2i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{3i+2(-1)}{5(-1)i+4(-1)(-1)}$$

$$\frac{-2+3i}{4-5i}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $4-5i$ is $4+5i$, so multiply the expression by $(4+5i)/(4+5i)$.

$$\frac{-2+3i}{4-5i}\cdot\frac{4+5i}{4+5i}$$

$$\frac{(-2+3i)(4+5i)}{(4-5i)(4+5i)}$$

$$\frac{-8-10i+12i+15i^2}{16+20i-20i-25i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-8+2i+15(-1)}{16-25(-1)}$$

$$\frac{-8-15+2i}{16+25}$$

$$\frac{-23+2i}{41}$$

$$-\frac{23}{41}+\frac{2}{41}i$$

■ 4. Use the conjugate method to simplify the imaginary expression.

$$\frac{2i+4i^2}{6-6i}$$

Solution:

Simplify the expression first by plugging in -1 for i^2.

$$\frac{2i+4i^2}{6-6i}$$

$$\frac{2i+4(-1)}{6-6i}$$

$$\frac{-4+2i}{6-6i}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $6-6i$ is $6+6i$, so multiply the expression by $(6+6i)/(6+6i)$.

$$\frac{-4+2i}{6-6i}\cdot\frac{6+6i}{6+6i}$$

$$\frac{(-4+2i)(6+6i)}{(6-6i)(6+6i)}$$

$$\frac{-24-24i+12i+12i^2}{36+36i-36i-36i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-24-12i+12(-1)}{36-36(-1)}$$

$$\frac{-24-12-12i}{36+36}$$

$$\frac{-36-12i}{72}$$

$$-\frac{1}{2}-\frac{1}{6}i$$

■ 5. Use the conjugate method to simplify the imaginary expression.

$$\frac{\sqrt{-5}\sqrt{-5}-7i^3}{3+i}$$

Solution:

Simplify the expression.

$$\frac{(\sqrt{5})^2 i^2-7i^2 i}{3+i}$$

$$\frac{5i^2-7i^2 i}{3+i}$$

Remember that $\sqrt{-1}=i$ and $i^2=-1$.

$$\frac{5(-1)-7(-1)i}{3+i}$$

$$\frac{-5+7i}{3+i}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $3+i$ is $3-i$, so multiply the expression by $(3-i)/(3-i)$.

$$\frac{-5+7i}{3+i}\cdot\frac{3-i}{3-i}$$

$$\frac{(-5+7i)(3-i)}{(3+i)(3-i)}$$

$$\frac{-15+5i+21i-7i^2}{9+3i-3i-i^2}$$

$$\frac{-15+26i-7i^2}{9-i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-15+26i-7(-1)}{9-(-1)}$$

$$\frac{-15+26i+7}{9+1}$$

$$\frac{-8+26i}{10}$$

$$-\frac{4}{5}+\frac{13}{5}i$$

■ 6. Use the conjugate method to simplify the imaginary expression.

$$\frac{\sqrt{-2}\sqrt{-2}+3i^3}{i-4}$$

Solution:

Simplify the expression.

$$\frac{(\sqrt{2})^2 i^2+3i^2 i}{i-4}$$

$$\frac{2i^2+3i^2 i}{i-4}$$

Remember that $\sqrt{-1}=i$ and $i^2=-1$.

$$\frac{2(-1)+3(-1)i}{i-4}$$

$$\frac{-2-3i}{i-4}$$

Use the conjugate method to get the imaginary number, i, out of the denominator. The conjugate of $i-4$ is $-i-4$, so multiply the expression by $(-i-4)/(-i-4)$.

$$\frac{-2-3i}{i-4}\cdot\frac{-i-4}{-i-4}$$

$$\frac{(-2-3i)(-i-4)}{(i-4)(-i-4)}$$

$$\frac{2i+8+3i^2+12i}{-i^2-4i+4i+16}$$

$$\frac{14i+8+3i^2}{-i^2+16}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{14i+8+3(-1)}{-(-1)+16}$$

$$\frac{14i+8-3}{1+16}$$

$$\frac{14i+5}{17}$$

$$\frac{5}{17}+\frac{14}{17}i$$

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