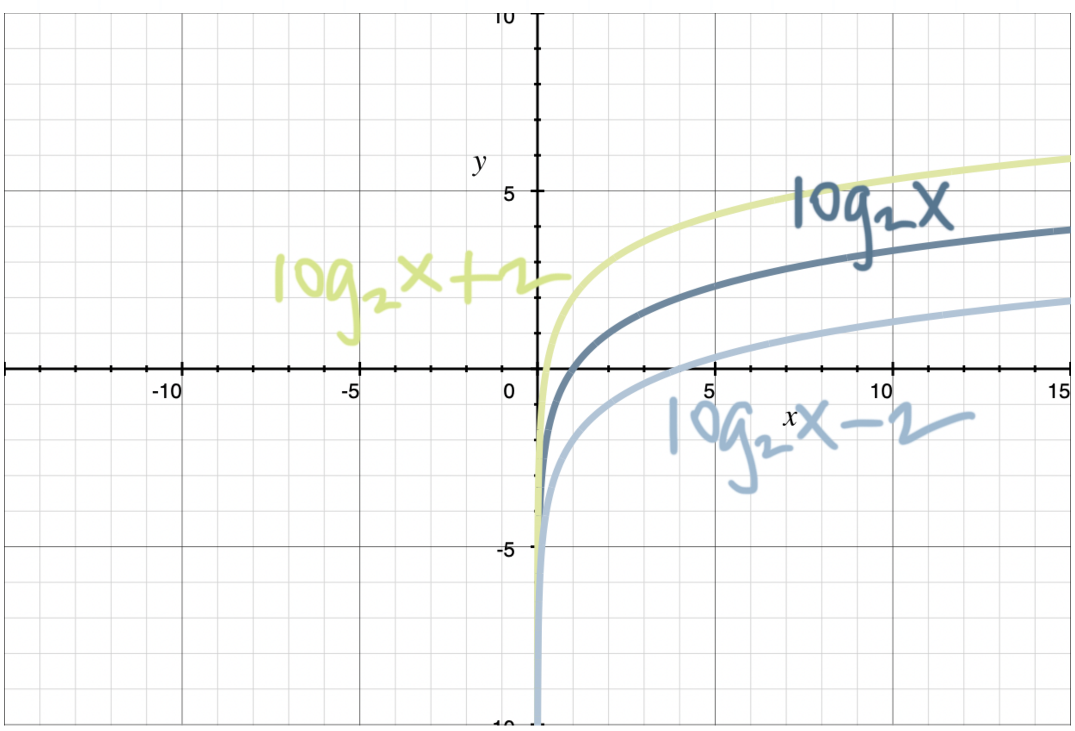
Graphing transformations of log functions

In the same way that we learned to transform exponential functions, we can also transform logarithmic functions. We'll again consider vertical and horizontal shifts, stretch and compressions, and reflections.

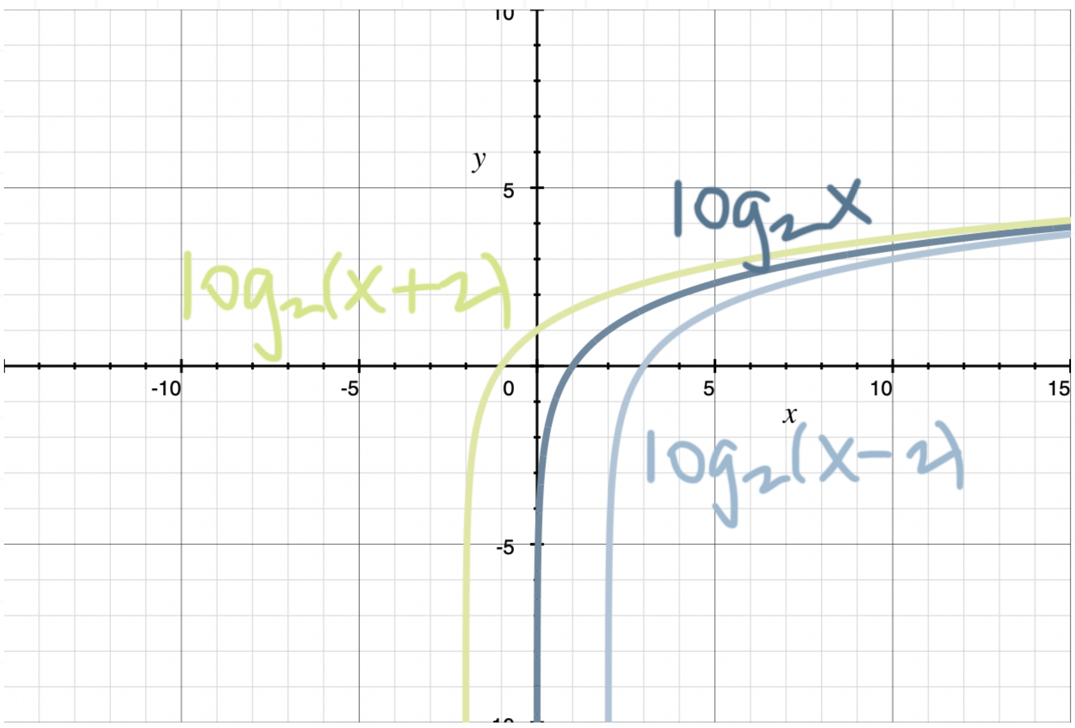
Vertical and horizontal shifts

Let's consider the parent function $f(x) = \log_b(x)$. Adding a constant d to the parent function gives us a vertical shift d units in the same direction as the sign of d.

For example, a sketch of the parent function $f(x) = \log_2(x)$ and this same function shifted vertically up 2 units and down 2 units gives



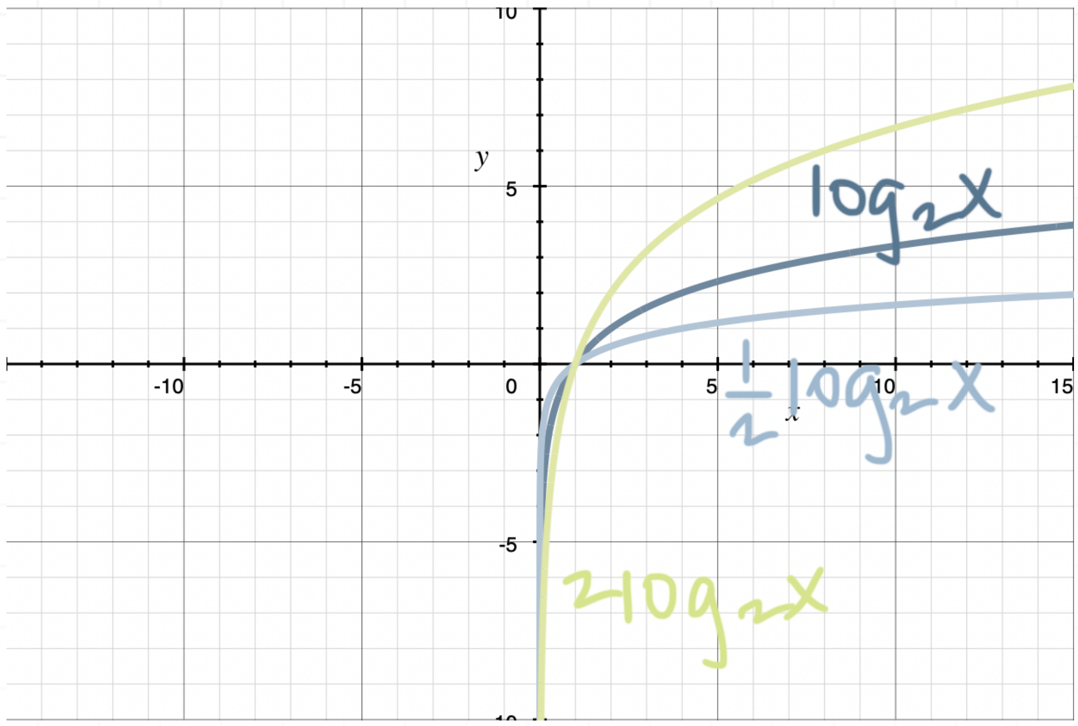
To shift a curve horizontally, we can add a constant c to the input of the parent function $f(x) = \log_b(x)$, but the direction of the shift is opposite the sign of c. So a sketch of the parent function $f(x) = \log_2(x)$ and this same function shifted horizontally left and right 2 units gives



Vertical and horizontal stretches and compressions

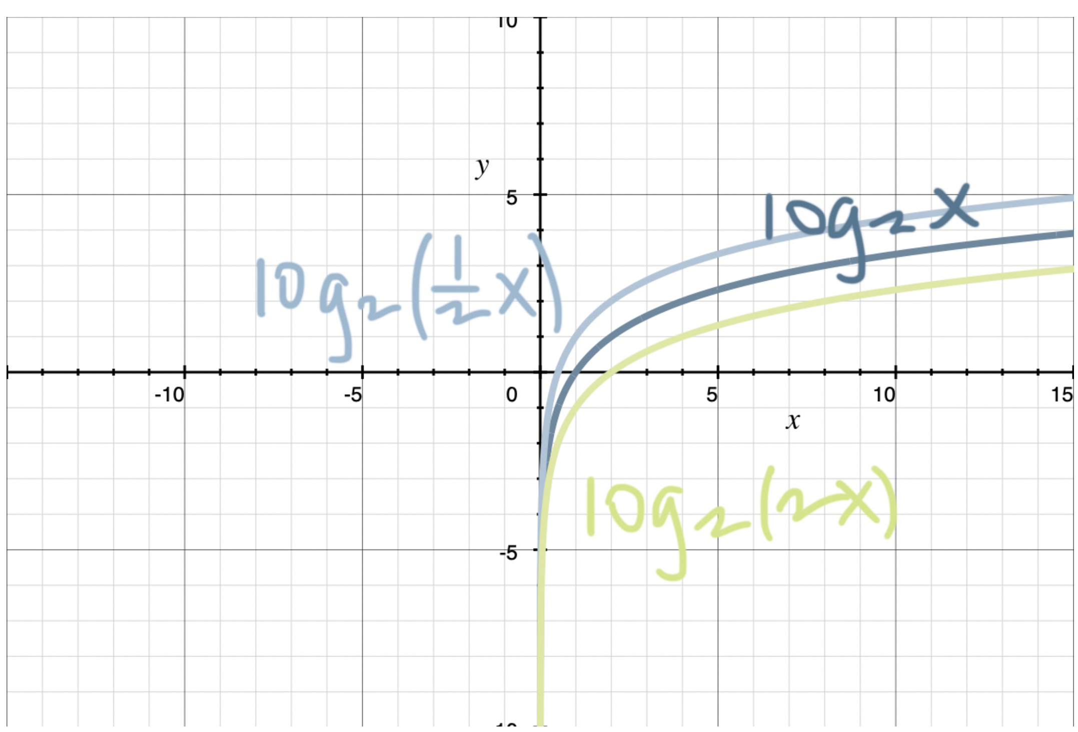
Let's consider the parent function $f(x) = \log_b(x)$. If we multiply the parent by some a where $0 < |a| < 1$ to get $f(x) = a\log_b(x)$, then $f(x) = \log_b(x)$ is being compressed vertically by a factor of a. But when $1 < |a|$, then $f(x) = \log_b(x)$ is being stretched.

For example, a sketch of the parent function $f(x) = \log_2(x)$ and this same function stretched and compressed vertically by a factor of 2 gives



If instead we multiply the input x by some k where $0 < |k| < 1$ to get $f(x) = \log_b(kx)$, then $f(x) = \log_b(x)$ is being stretched horizontally by a factor of k. But when $1 < |k|$, then $f(x) = \log_b(x)$ is being compressed.

For example, a sketch of the parent function $f(x) = \log_2(x)$ and this same function stretched and compressed horizontally by a factor of 2 gives

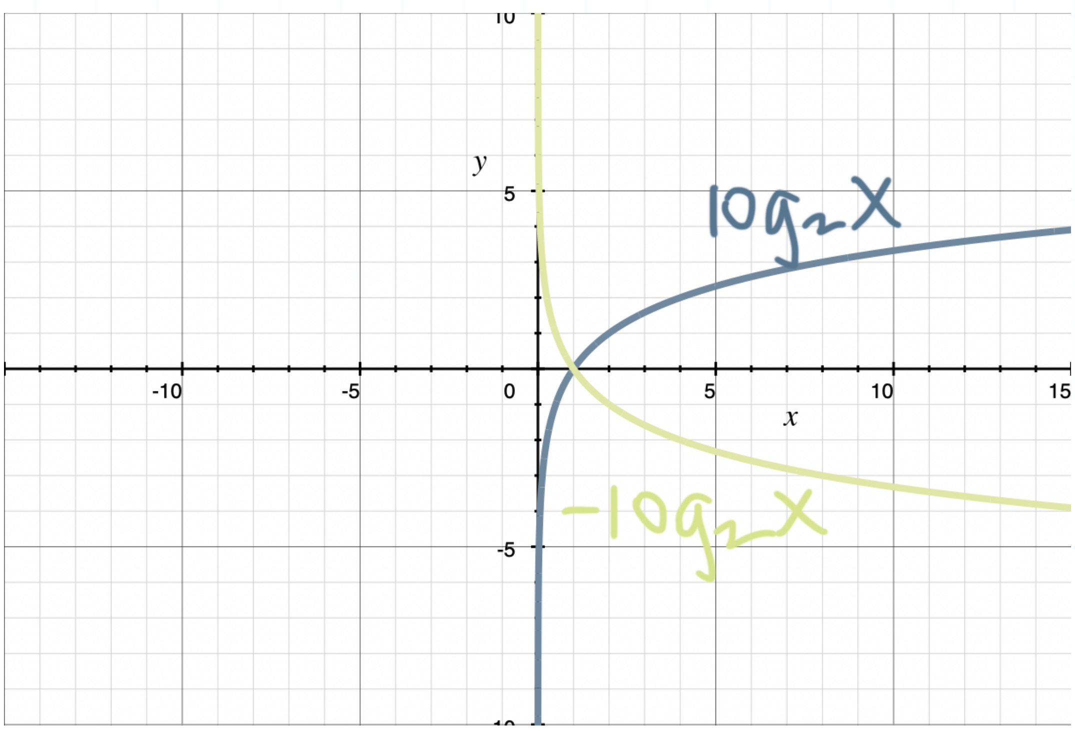


Let's do an example where we have to sketch a function with both a vertical and a horizontal stretch or compression.

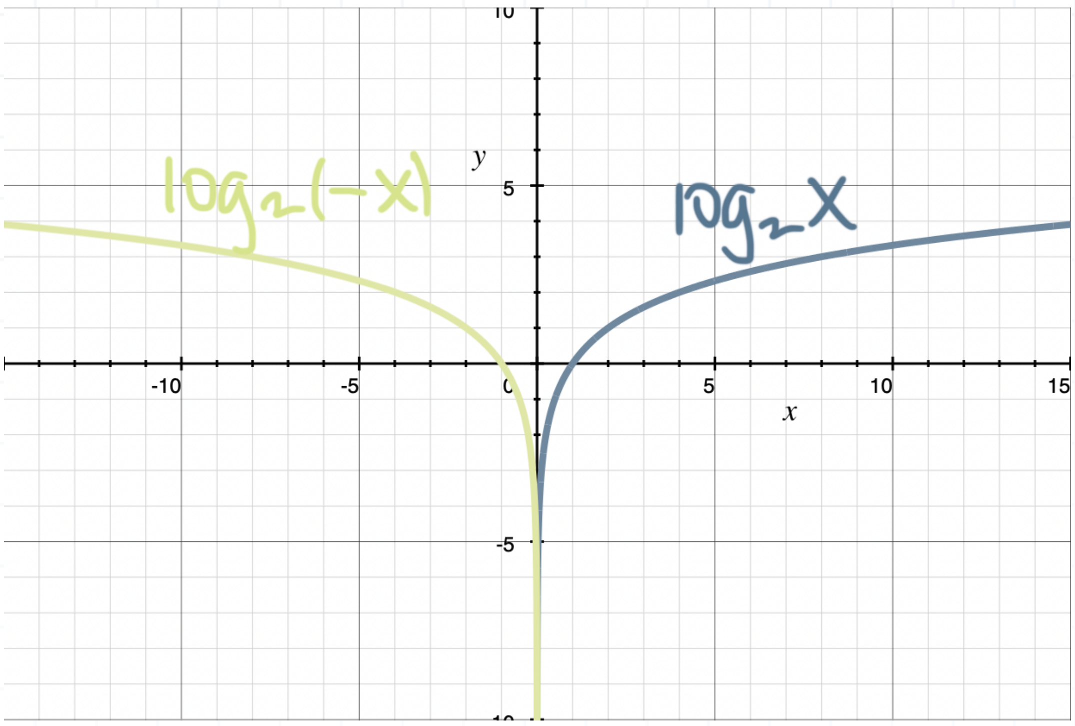
Vertical and horizontal reflections

It's also possible to reflect the graph across the x-axis and/or the y-axis. When we multiply the parent function $f(x) = \log_b(x)$ by -1, the graph gets reflected across the x-axis. But when we multiply the input of the parent by -1, the graph gets reflected across the y-axis.

For example, let's choose $f(x) = \log_2(x)$ again as the parent function. Its reflection across the x-axis is $g(x) = -\log_2(x)$,



and its reflection across the y-axis is $h(x) = \log_2(-x)$.



Combining transformations

Now that we've seen this collection of transformations, let's summarize the order in which we should apply them, given multiple transformations in the same equation.

1. Horizontal stretch or compression

2. Horizontal shift

3. Horizontal reflection

4. Vertical stretch or compression

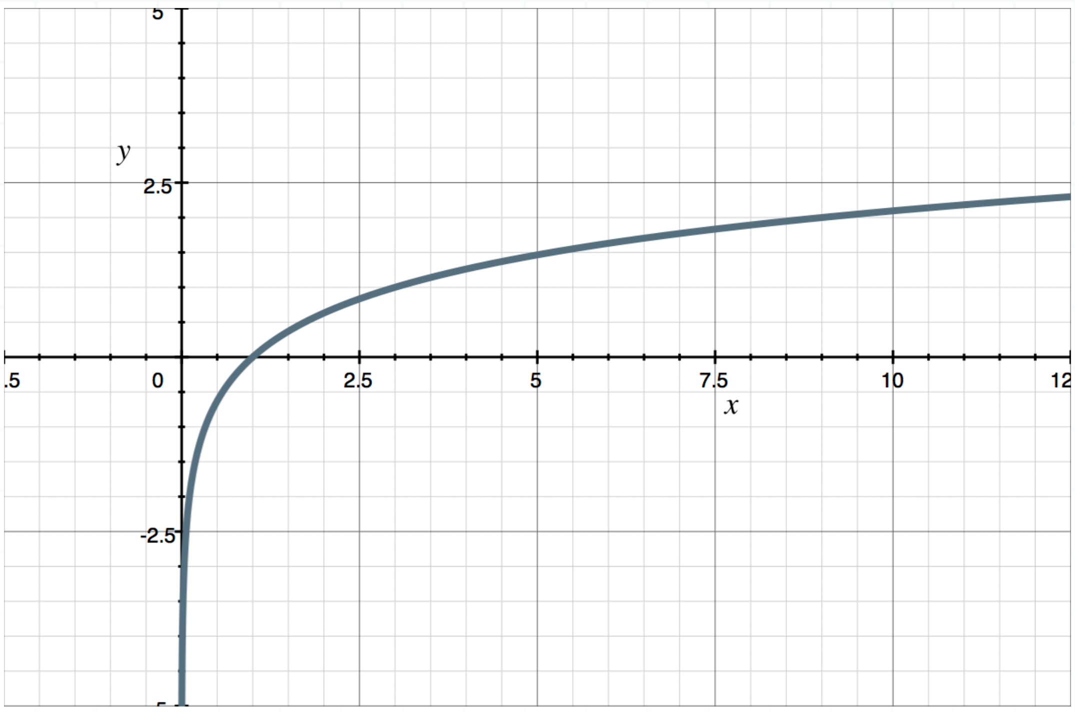
5. Vertical reflection

6. Vertical shift

Let's do an example where we apply these transformations in order.

Example

The graph of the logarithmic function $y = \log_3 x$ is given. Use that graph to sketch the graph of the function $y = -\log_3(x-1)$.



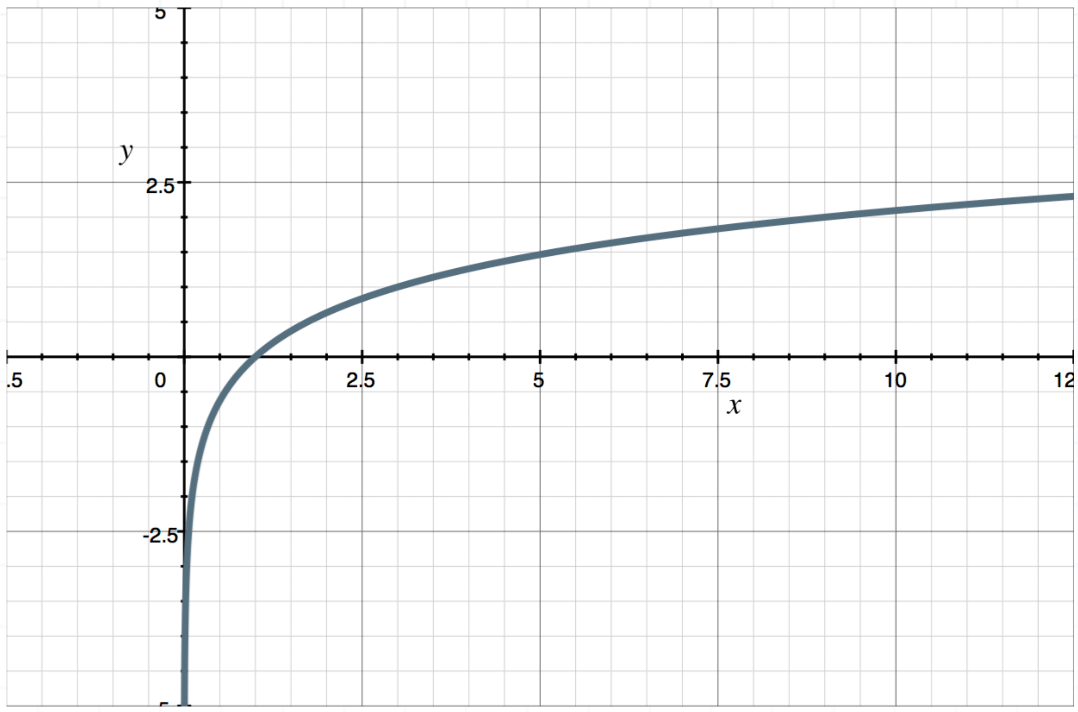
The function $-\log_3(x-1)$ is the result of applying a couple of transformations to the logarithmic function $\log_3 x$ in turn. We'll treat each transformation in a separate step, and we'll give different names to the function we obtain in different steps.

[1] $f(x) = \log_3 x$

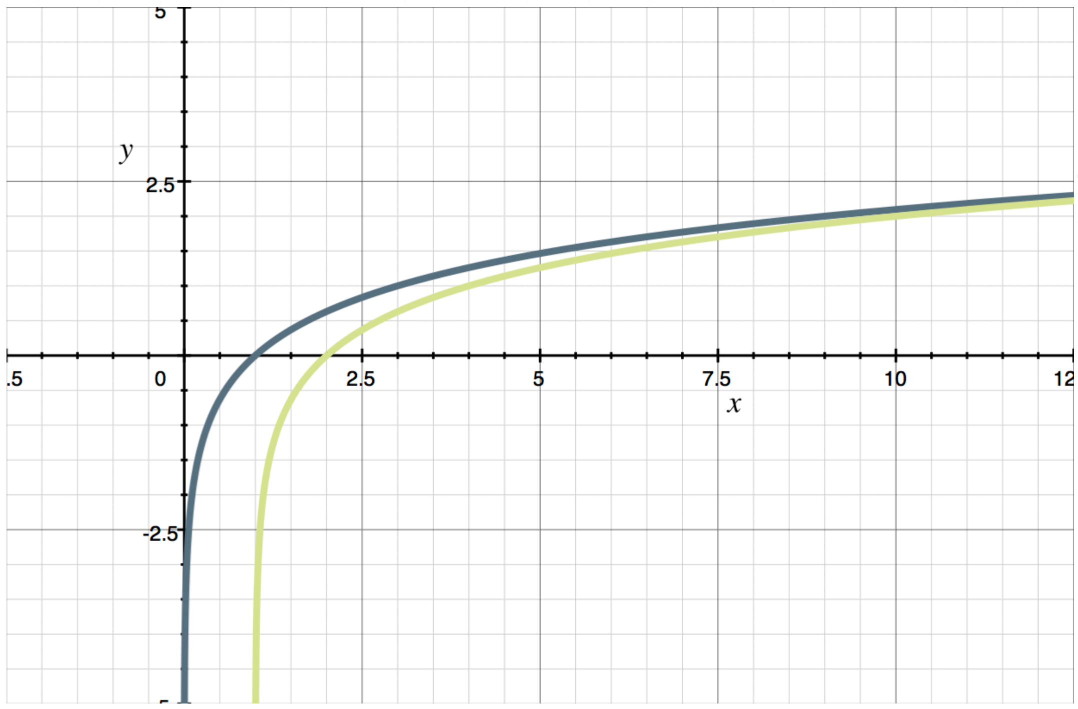
[2] $g(x) = \log_3(x-1)$

[3] $h(x) = -\log_3(x-1)$

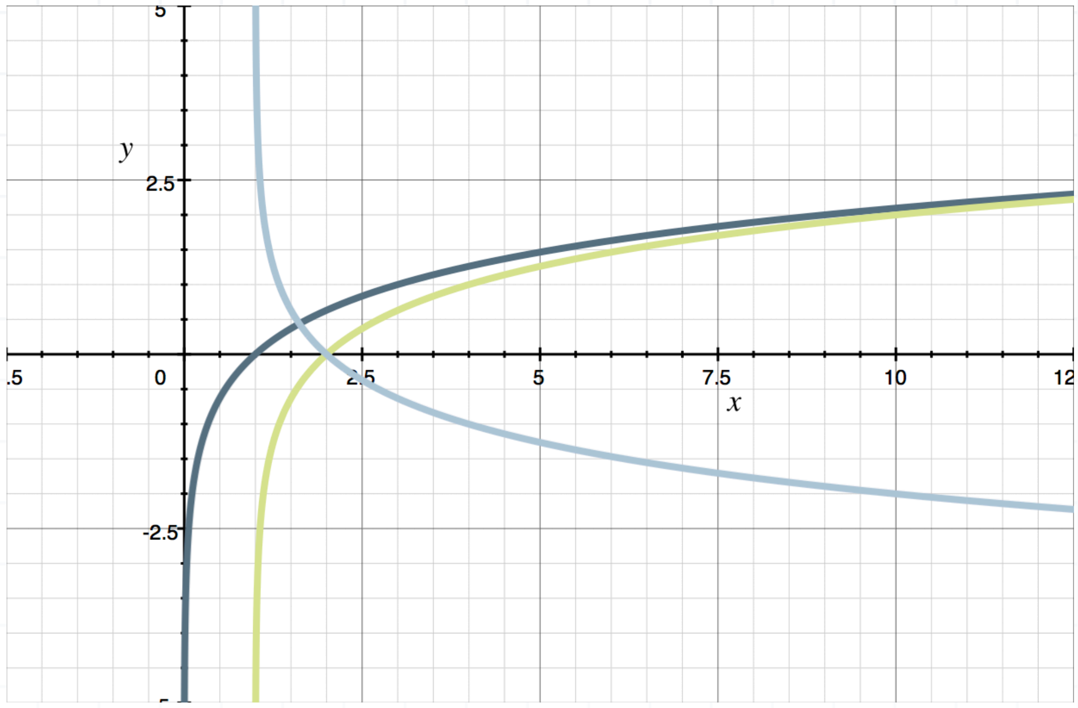
We were given the graph of $f(x)$.



If we substitute $x - 1$ for x in the expression for $f(x)$, we get the expression for $g(x)$: $\log_3(x - 1)$. This means that to get the graph of $g(x)$, we take the graph of $f(x)$ and shift it one unit to the right. If you're not sure about this, try plugging a few values of x into the function $g(x) = \log_3(x - 1)$. If we graph $g(x)$ on the same set of axes as $f(x)$, we get



To get the value of $h(x)$, we multiply the value of $g(x)$ by -1, which means that the graph of $h(x)$ is just the reflection of the graph of $g(x)$ with respect to the x-axis.



To summarize, we started with the function $\log_3 x$ and its graph. To get the graph of $-\log_3(x-1)$, we applied one transformation at a time.

[1] $f(x) = \log_3 x$

[2] $g(x) = f(x-1) = \log_3(x-1)$

[3] $h(x) = -1 \cdot g(x) = -\log_3(x-1)$

Finding the vertical asymptote

We know that the vertical asymptote of the log function $f(x) = \log_b(x)$ is always $x = 0$. But if that log function has undergone any kind of horizontal shift, the vertical asymptote will shift as well.

To find the shifted asymptote, we can set the argument of the shifted log function equal to 0. Let's do an example.

Example

What is the vertical asymptote of $f(x) = -2\ln(2x+6) - 4$?

To find the vertical asymptote of the logarithmic function, we'll set its argument equal to zero, then solve for x.

$$2x + 6 = 0$$

$$2x = -6$$

$$x = -3$$

The vertical asymptote is $x = -3$.