Decomposing composite functions

Now that we understand how to build a composite function from two other functions, we want to learn to go the other way and decompose a composite into its component functions.

To write a function as a composite of two other functions, we're essentially looking for a "function within a function."

And it's important to remember that there are almost always multiple ways to decompose the composite, and breaking the composite into one pair of functions isn't necessarily better than breaking it into a different pair of functions.

Let's work through an example.

Example

Write $f(x)=\sqrt{x^3-3}$ as the composite of two functions.

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x)=h(g(x))$. To do this, we look for a function inside a function within $f(x)$.

If we notice that the expression x^3-3 is inside the square root, it's fairly straightforward to decompose the function into

$$g(x)=x^3-3 \text{ and } h(x)=\sqrt{x}$$

Alternatively, we could also have decomposed the composite into

$$g(x) = x^3 \text{ and } h(x) = \sqrt{x - 3}$$

Let's look at another example.

Example

Write $f(x) = (x-1)^3 + 4(x-1)^2 - 3(x-1) + 5$ as the composite of two functions.

We're looking for two functions, $g(x)$ and $h(x)$, such that $f(x) = h(g(x))$. To do this, we look for a function inside a function within $f(x)$.

If we notice that the expression $x - 1$ is raised to power of 3, 2, 1, and 0, it's fairly straightforward to decompose the function into

$$g(x) = x - 1 \text{ and } h(x) = x^3 + 4x^2 - 3x + 5$$

Alternatively, we could have also decomposed the composite into

$$g(x) = (x-1)^3 + 4(x-1)^2 - 3(x-1) \text{ and } h(x) = x + 5$$
