Topic: Sum of cubes

Question: Factor the expression.

$$64x^9y^{18} + 8r^3$$

Answer choices:

A $8(2x^3y^6 - r)(4x^6y^{12} - 2rx^3y^6)$

B $8(2x^3y^6 + r)(4x^6y^{12} - 2rx^3y^6 + r^2)$

C $(4x^3y^6 + 2r)(16x^6y^{12} + 8rx^3y^6 + 4r^2)$

D $(4x^3y^6 - 2r)(16x^6y^{12} + 8rx^3y^6 + 4r^2)$

Solution: B

We know we're dealing with the sum of cubes because we have two perfect cubes separated by a plus sign to indicate that the second perfect cube is to be added to the first perfect cube.

When that's the case, we take the cube root of each term.

The cube root of $64x^9y^{18}$ is $4x^3y^6$.

The cube root of $8r^3$ is $2r$.

The sum of cubes a^3+b^3 is always factored as

$$(a+b)(a^2-ab+b^2)$$

Since in this case $a=4x^3y^6$ and $b=2r$, we get

$$(4x^3y^6+2r)\left[(4x^3y^6)^2-(4x^3y^6)(2r)+(2r)^2\right]$$

$$(4x^3y^6+2r)(16x^6y^{12}-8rx^3y^6+4r^2)$$

$$2(2x^3y^6+r)(16x^6y^{12}-8rx^3y^6+4r^2)$$

$$2(2x^3y^6+r)(4)(4x^6y^{12}-2rx^3y^6+r^2)$$

$$8(2x^3y^6+r)(4x^6y^{12}-2rx^3y^6+r^2)$$

Topic: Sum of cubes

Question: Factor the expression.

$$27j^6k^9 + 125r^6t^3$$

Answer choices:

A $(3j^2k^3 + 5r^2t)(9j^4k^6 - 15j^2k^3r^2t + 25r^4t^2)$

B $(3j^2k^3 - 5r^2t)(9j^4k^6 + 15j^2k^3r^2t + 25r^4t^2)$

C $(3j^2k^3 + 5r^2t)(9j^4k^6 + 15j^2k^3r^2t + 25r^4t^2)$

D $(3j^2k^3 + 5r^2t)(9j^4k^6 - 15j^2k^2r^2t + 25r^4t^2)$

Solution: A

The formula for factoring the sum of two cubes is

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

a is the cube root of the first term, and b is the cube root of the second term.

In $27j^6k^9 + 125r^6t^3$,

a is the cube root of $27j^6k^9$, which is $3j^2k^3$.

b is the cube root of $125r^6t^3$, which is $5r^2t$.

Now we'll apply the formula given above.

$$(a + b)(a^2 - ab + b^2)$$

$$(3j^2k^3 + 5r^2t)(9j^4k^6 - 15j^2k^3r^2t + 25r^4t^2)$$

We can check our work by distributing each term in the binomial factor over all the terms in the trinomial factor.

$$(3j^2k^3)(9j^4k^6) + (3j^2k^3)(-15j^2k^3r^2t) + (3j^2k^3)(25r^4t^2)$$

$$+(5r^2t)(9j^4k^6) + (5r^2t)(-15j^2k^3r^2t) + (5r^2t)(25r^4t^2)$$

$$27j^6k^9 - 45j^4k^6r^2t + 75j^2k^3r^4t^2 + 45j^4k^6r^2t - 75j^2k^3r^4t^2 + 125r^6t^3$$

$$27j^6k^9 + 125r^6t^3$$

Topic: Sum of cubes

Question: Factor the expression.

$$343c^3p^3 + x^{12}y^9$$

Answer choices:

A $(7cp - x^4y^3)(49c^2p^2 + 7cpx^4y^3 + x^8y^6)$

B $(7cp + x^4y^3)(49c^2p^2 - 7cpx^4y^3 + x^8y^6)$

C $(7cp + x^4y^3)(49c^2p^2 + 7c^2p^2x^4y^3 + x^6y^3)$

D $(7cp + x^4y^3)(49c^2p^2 - 7cpx^4y^3 + x^6y^3)$

Solution: B

The formula for factoring the sum of two cubes is

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

a is the cube root of the first term, and b is the cube root of the second term.

In $343c^3p^3 + x^{12}y^9$,

a is the cube root of $343c^3p^3$, which is $7cp$.

b is the cube root of $x^{12}y^9$, which is x^4y^3.

Now we'll apply the formula for cube roots.

$$(a + b)(a^2 - ab + b^2)$$

$$(7cp + x^4y^3)(49c^2p^2 - 7cpx^4y^3 + x^8y^6)$$

We can check our work by distributing each term in the binomial factor over all the terms in the trinomial factor.

$$(7cp)(49c^2p^2) + (7cp)(-7cpx^4y^3) + (7cp)(x^8y^6)$$

$$+(x^4y^3)(49c^2p^2) + (x^4y^3)(-7cpx^4y^3) + (x^4y^3)(x^8y^6)$$

$$343c^3p^3 - 49c^2p^2x^4y^3 + 7cpx^8y^6 + 49c^2p^2x^4y^3 - 7cpx^8y^6 + x^{12}y^9$$

$$343c^3p^3 + x^{12}y^9$$