Systems with quadratic inequalities

We know how to solve systems of equations, including systems with a quadratic equation.

All we want to do now is translate that into systems of inequalities, where at least one of the inequalities in the system is a quadratic inequality.

Solving by graphing

To solve a system of inequalities by graphing, we'll start by sketching the equation associated with each inequality.

Each curve that we sketch will be solid if the inequality is a $\leq$ or $\geq$ inequality, and dashed if the inequality is a $<$ or $>$ inequality.

We'll shade above the curve for $>$ or $\geq$ inequalities, and below the curve for $<$ or $\leq$ inequalities. And we can always use the origin as a test point to determine where to shade.

The solution to the system of inequalities will be any region where the shading from both inequalities overlaps.

Let's look at an example to see how this works.

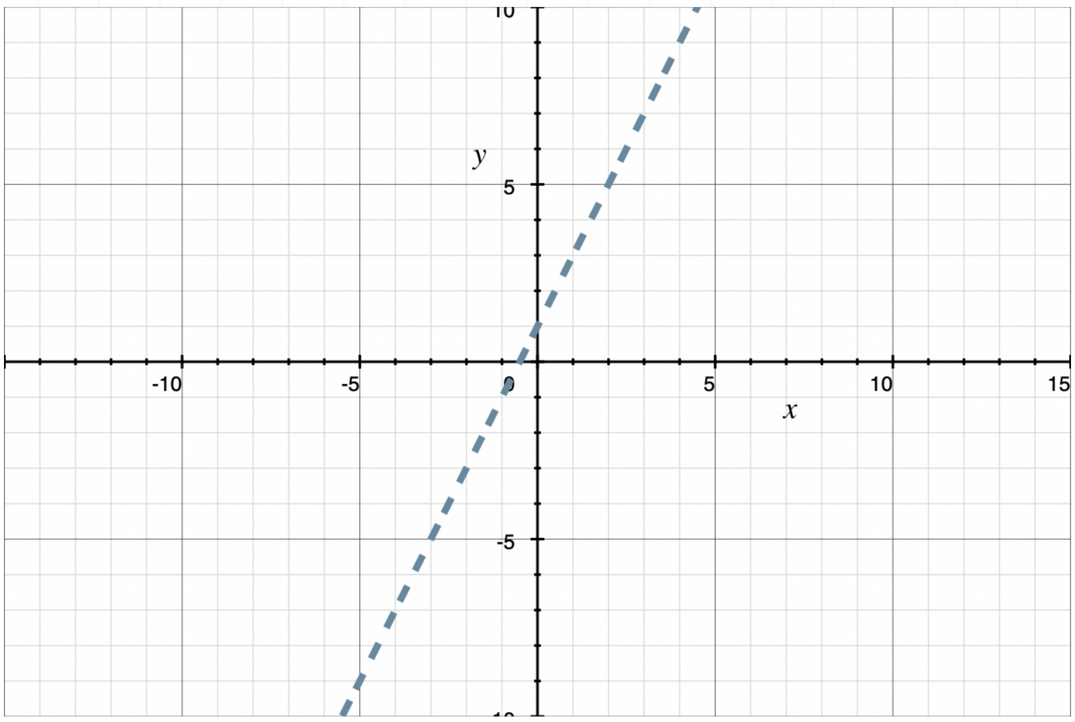
Example

Sketch the solution to the system of inequalities.

$$y > 2x + 1$$

$$y \le (2x + 1)(x - 3)$$

Begin by graphing the line $y = 2x + 1$ using the y-intercept of 1 and the slope of 2. The line will be dashed because of the $>$ sign.



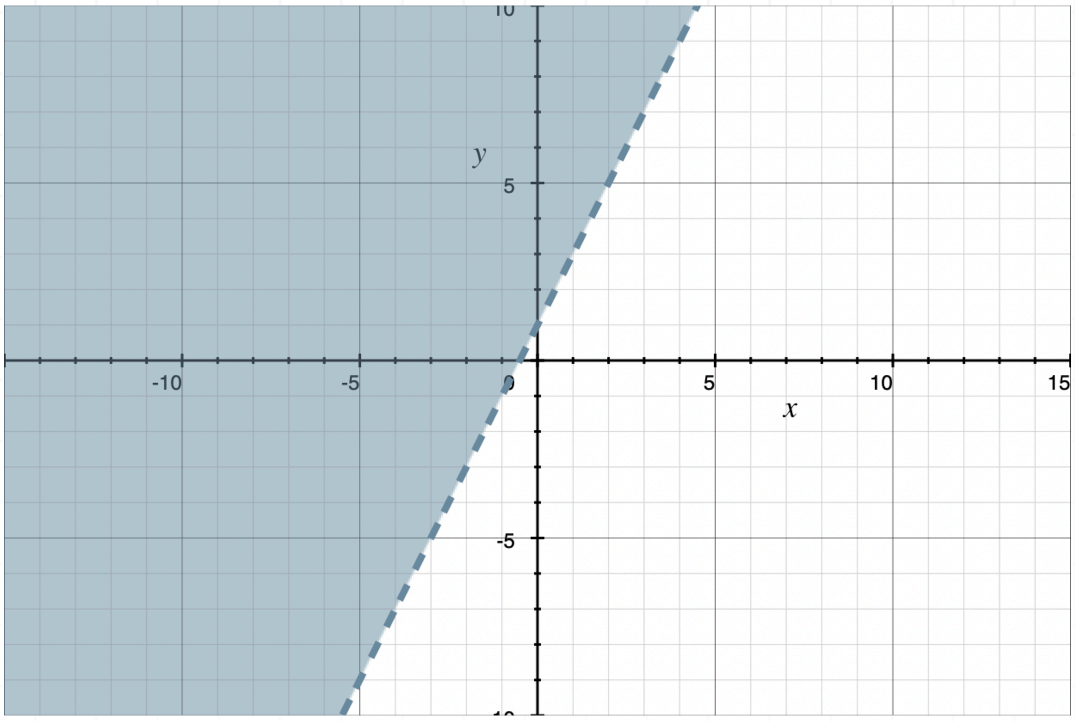
Let's test the origin to help us determine where to shade.

$$y > 2x + 1$$

$$0 > 2(0) + 1$$

$$0 > 1$$

Because this is a false statement, we shade away from the origin.



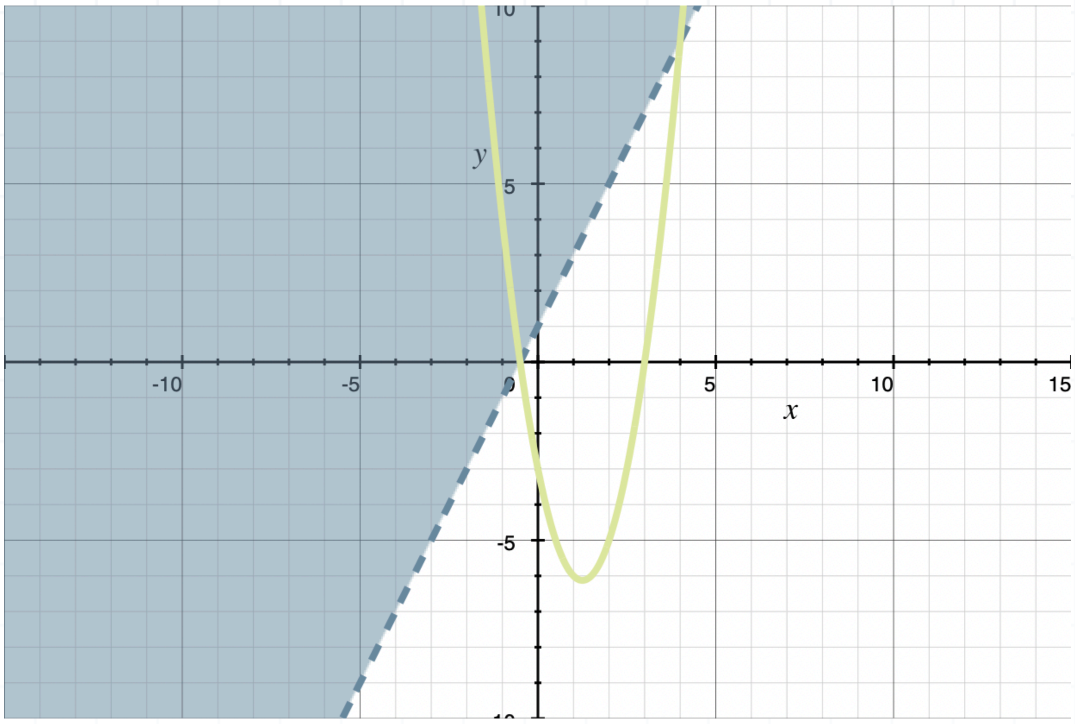
We can find the x-intercepts of the quadratic by solving the corresponding quadratic equation.

$$(2x+1)(x-3)=0$$

$2x+1=0$ and $x-3=0$

$x=-\dfrac{1}{2}$ and $x=3$

Now we can graph the parabola. The curve will be solid because of the $\leq$ sign.



Let's test the origin to help us determine where to shade.

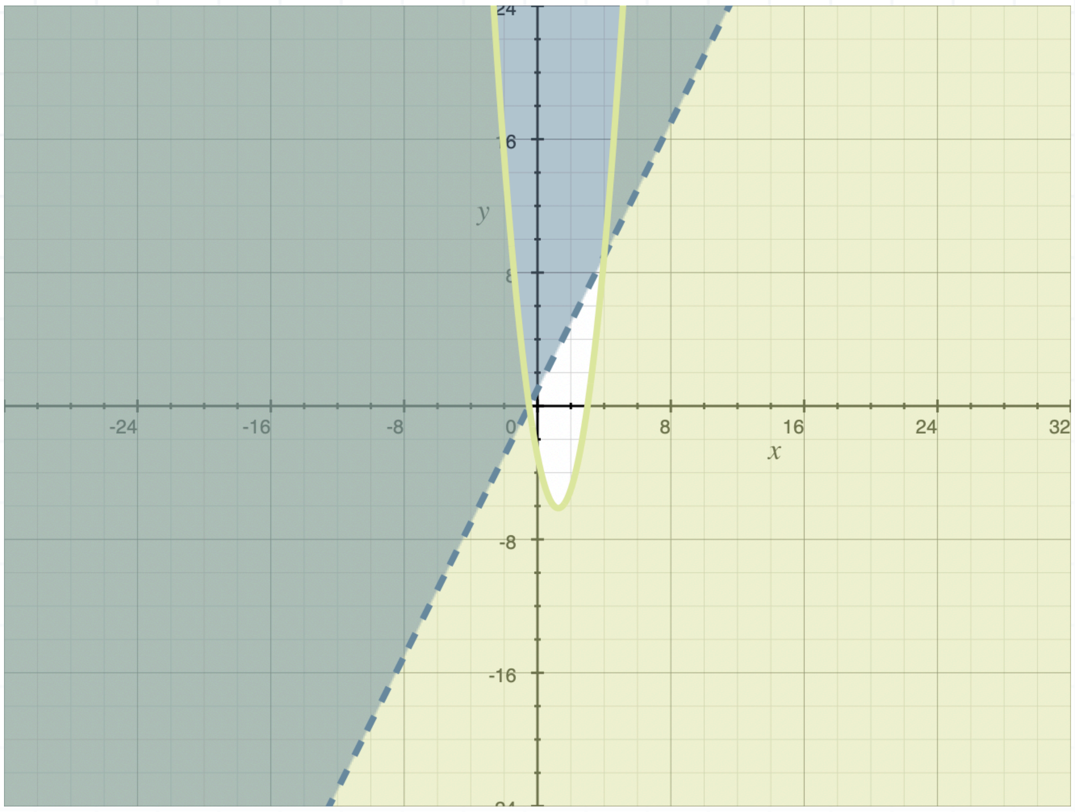
$$y \le (2x+1)(x-3)$$

$$0 \le (2(0)+1)(0-3)$$

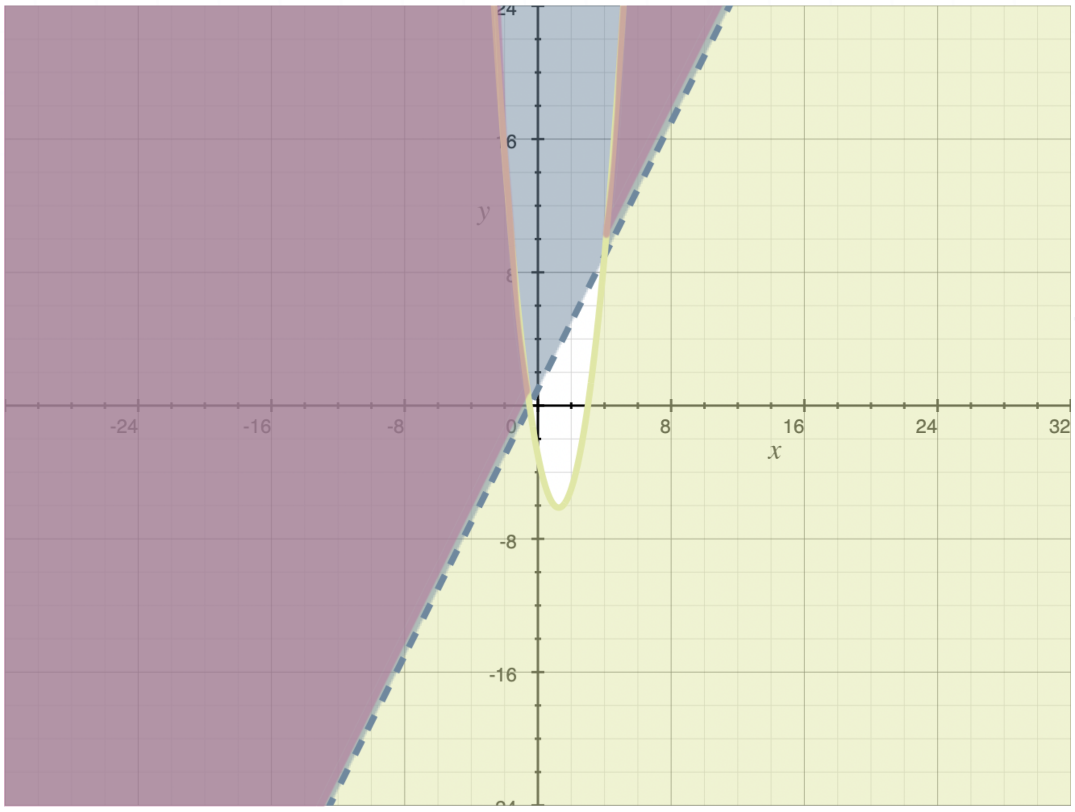
$$0 \le (1)(-3)$$

$$0 \le -3$$

Because this is a false statement, we shade away from the origin.



The regions where the shading overlaps is the solution to the system of inequalities.



Let's do another example.

Example

Graph the solution to the system of inequalities.

$$y + 6 > 5x^2$$

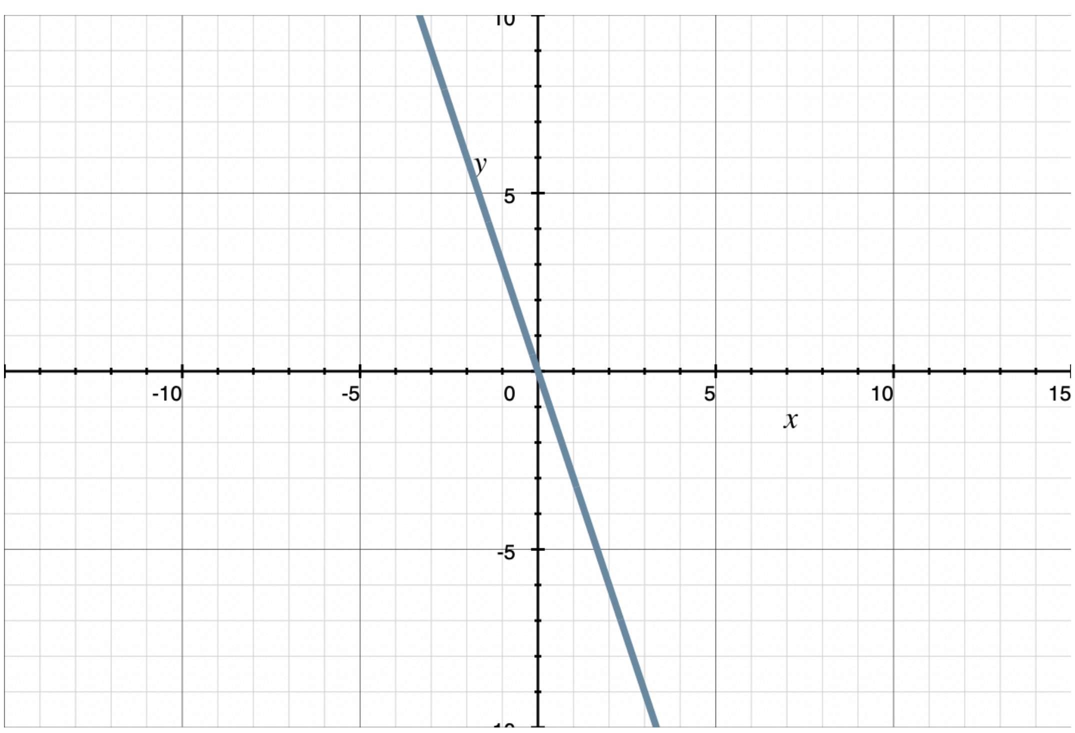
$$-\frac{1}{3}y \geq x$$

First we need to rewrite both inequalities by isolating y.

$$y > 5x^2 - 6$$

$$y \leq -3x$$

Begin by graphing the line $y = -3x$ using the y-intercept of 0 and the slope of -3. The line will be solid because of the $\leq$ sign.



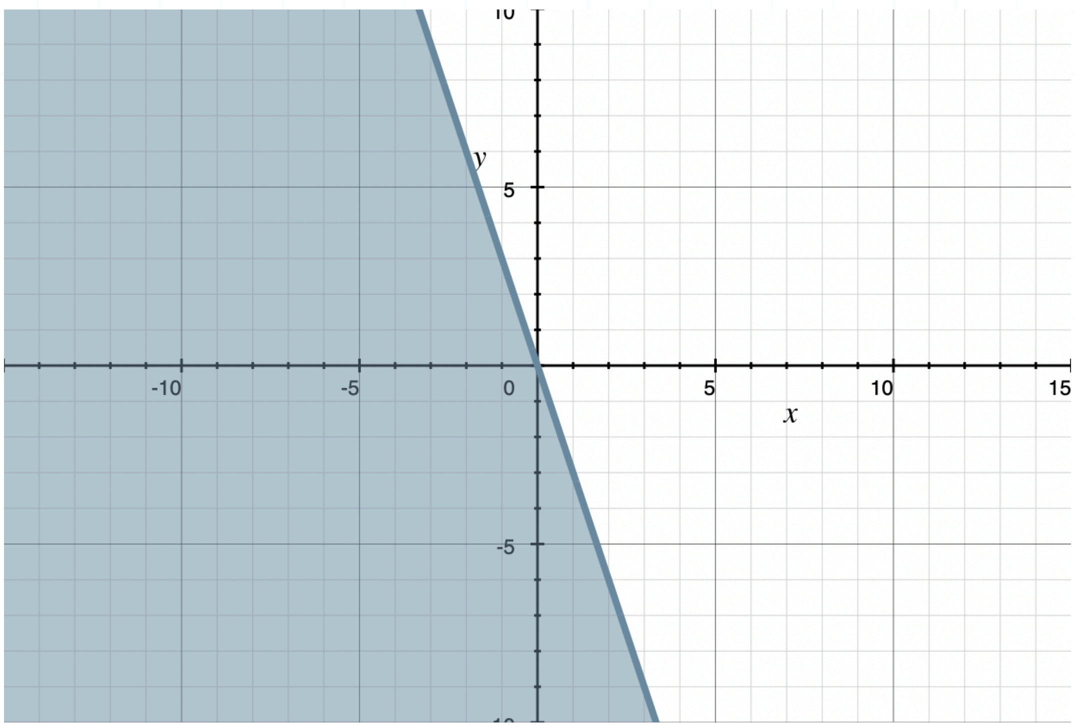
We can't use the origin as a test point because the line intersects the origin, so let's use $(0,1)$ instead.

$$y \leq -3x$$

$$1 \leq -3(0)$$

$$1 \leq 0$$

Because this is a false statement, we shade on the opposite side of the line from the test point $(0,1)$.



We can find the x-intercepts of the quadratic by solving the corresponding quadratic equation.

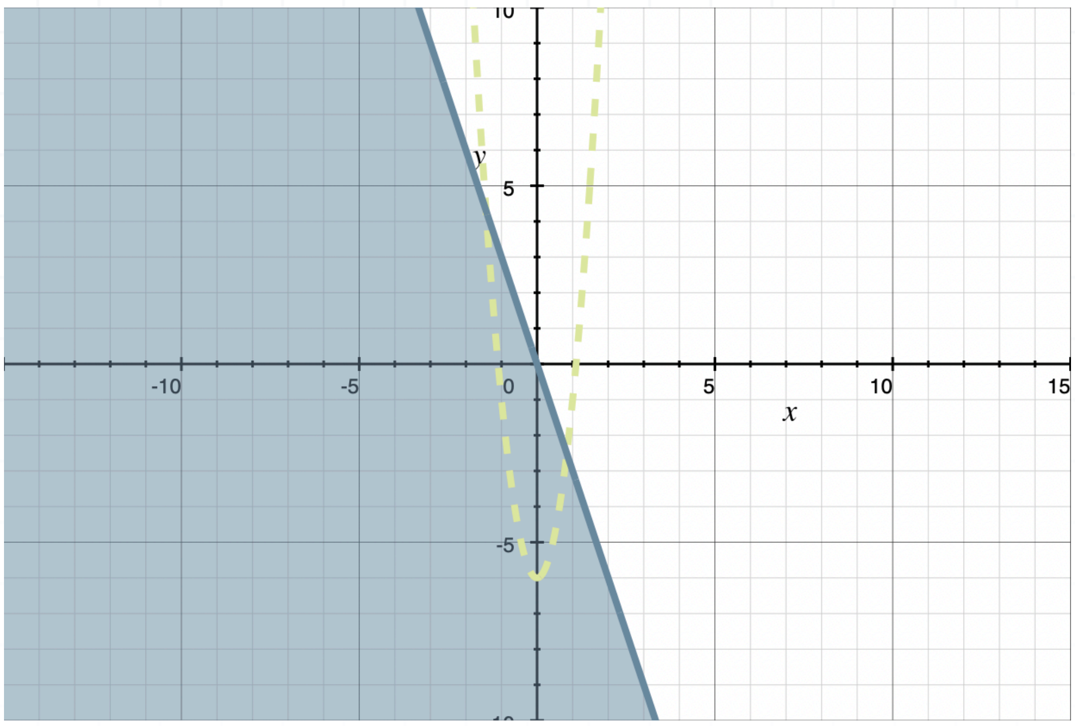
$$5x^2 - 6 = 0$$

$$5x^2 = 6$$

$$x^2 = \frac{6}{5}$$

$$x = \pm\sqrt{\frac{6}{5}}$$

Now we can graph the parabola. The curve will be dashed because of the $>$ sign.



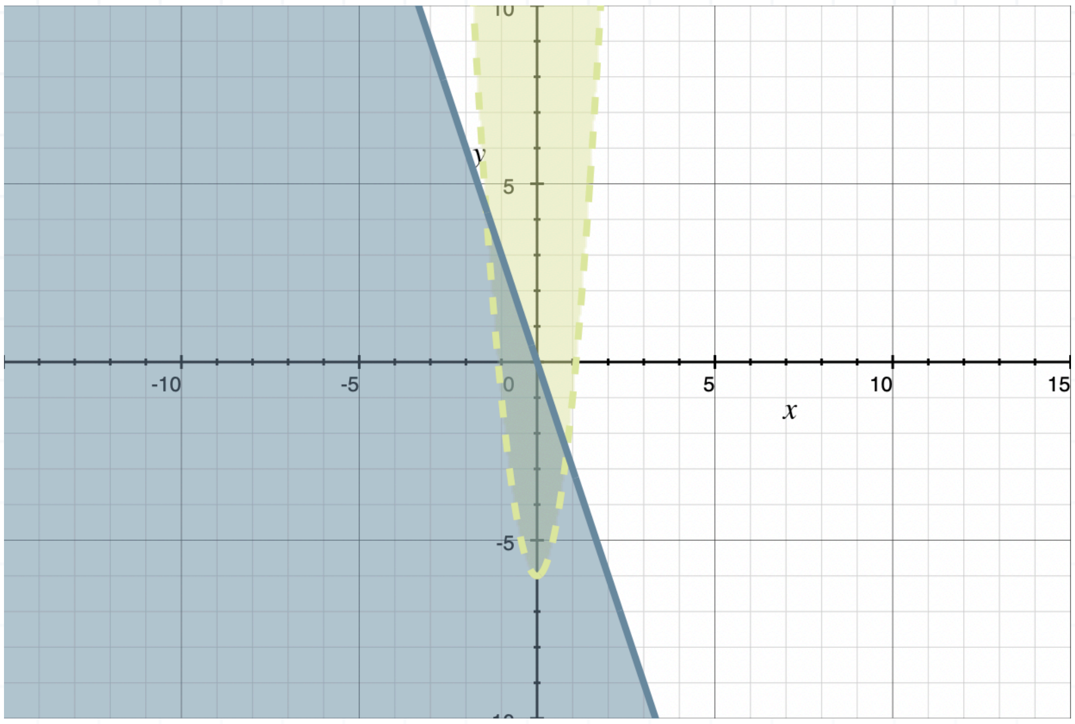
Let's test the origin to help us determine where to shade.

$$y > 5x^2 - 6$$

$$0 > 5(0^2) - 6$$

$$0 > -6$$

Because this is a true statement, we shade toward the origin.



The regions where the shading overlaps is the solution to the system of inequalities.