Multiplying polynomials

We've learned to add and subtract polynomials (which was really just about adding and subtracting like terms), and now we want to learn to multiply and divide polynomials, starting with multiplication of polynomials.

In the same way that the sum of two polynomials is always itself a polynomial, the product of two polynomials will also always be a polynomial.

Binomial multiplication and FOIL

To multiply two polynomials, we'll always need to apply the Distributive Property. From what we already learned about the Distributive Property, we know how to distribute a single coefficient across parentheses, like this:

$$2(x+3)$$

$$2(x)+2(3)$$

$$2x+6$$

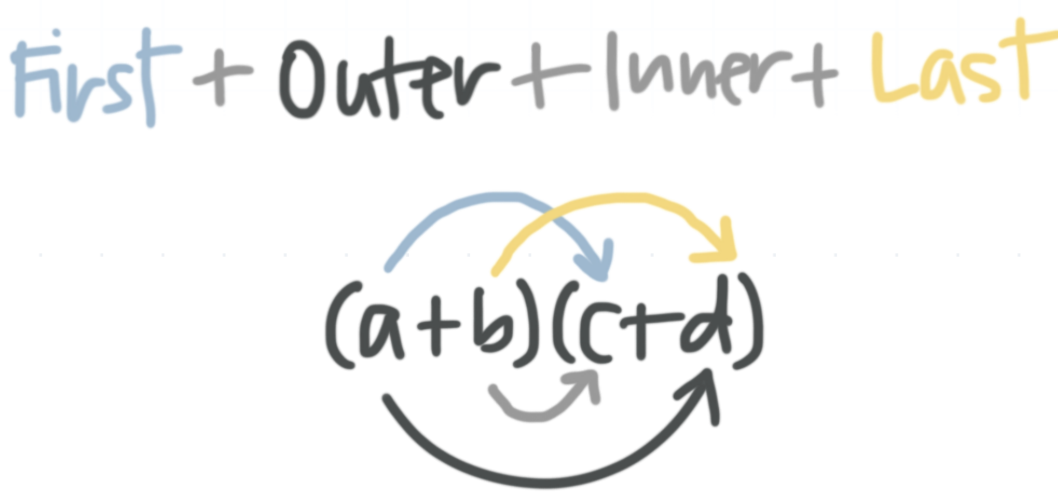
But now we want to replace the single coefficient with a polynomial coefficient. So in place of the single coefficient of 2 in the expression above, we could instead use a polynomial like $x+4$, in which case we'd have the product of two polynomials,

$$(x+4)(x+3)$$

When we're talking about polynomials, a single term is a **monomial**, two terms is a **binomial**, and three terms is a **trinomial**. We don't usually assign a special name to polynomials with four or more terms.

So the product above is the multiplication of two binomials, and whenever we multiply two binomials, we can use an acronym, FOIL, to help us with the multiplication.

FOIL is a way to help us remember to multiply each term in the first binomial by each term in the second binomial. FOIL stands for First, Outer, Inner, Last, which is the order of the four terms in the result of the multiplication. It also indicates which terms in the given binomials are multiplied to produce each term in the result.



The firsts are the first term in the first set of parentheses (which is a) and the first term in the second set of parentheses (which is c), so the first term in the result (first) is the product of a and c, or ac.

The outers are the outside term in the first set of parentheses (which is a) and the outside term in the second set of parentheses (which is d), so the second term in the result (outer) is the product of a and d, or ad.

Similarly, the inners are b and c, and the lasts are b and d, so the third term in the result (inner) is bc, and the fourth term in the result (last) is bd.

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(a+b)(c-d) = ac - ad + bc - bd$$

$$(a-b)(c+d) = ac + ad - bc - bd$$

$$(a-b)(c-d) = ac - ad - bc + bd$$

Really though, all we're doing with binomial multiplication is multiplying every term in the first polynomial by every term in the second polynomial. Looking at the example $(x+4)(x+3)$, we need to multiply both x and 4 from the first polynomial by the second polynomial.

$$(x+4)(x+3)$$

$$x(x+3) + 4(x+3)$$

$$x(x) + x(3) + 4(x) + 4(3)$$

$$x^2 + 3x + 4x + 12$$

$$x^2 + 7x + 12$$

We can also make a chart for the binomial multiplication $(a+b)(c+d)$ in which the terms a and b from the first set of parentheses go along the left side, and the terms c and d from the second set of parentheses go across the top. Then we multiply each term along the left side by both terms across the top, and write the individual results in the chart. The four results all get added together to make the expanded polynomial.

For the binomial multiplication, $(a+b)(c+d)$, we get $ac+ad+bc+bd$.

	c	d
a	ac	ad
b	bc	bd

For the binomial multiplication, $(a-b)(c-d)$, we get $ac-ad-bc+bd$.

	c	-d
a	ac	-ad
-b	-bc	bd

Let's do an example with binomial multiplication.

Example

Expand the expression.

$$(x+2)(x-7)$$

If we multiply the firsts x and x, the outers x and -7, the inners 2 and x, and the lasts 2 and -7, and add all the results together, we get

$$(x)(x)+(x)(-7)+(2)(x)+(2)(-7)$$

$$x^2-7x+2x-14$$

$$x^2-5x-14$$

Let’s try another example of multiplying binomials.

Example

Expand the expression.

$$(x+3)^2$$

This is “$(x+3)$ squared,” which means that the binomial $(x+3)$ is multiplied by itself.

$$(x+3)(x+3)$$

$$(x)(x)+(x)(3)+(3)(x)+(3)(3)$$

$$x^2+3x+3x+9$$

$$x^2+6x+9$$

Multiplying more than two binomials

We now know how to multiply two binomials together, but we’re not limited to just two binomials, nor are we limited to only binomials. We can multiply together as many binomials as we like, and we can multiply polynomials that have more than two terms (trinomials, etc.).

No matter the number and length of the polynomials, the key here is just to make sure we multiply every term in each polynomial by every term in every other polynomial.

For instance, let's say we want to multiply $(x+3)(x+3)(x+3)$. We take two polynomials at a time, so we'll multiply the first and second $(x+3)$ binomials together.

$$[(x+3)(x+3)](x+3)$$

$$[(x)(x)+(x)(3)+(3)(x)+(3)(3)](x+3)$$

$$[x^2+3x+3x+9](x+3)$$

$$(x^2+6x+9)(x+3)$$

Now we'll multiply each term in the trinomial by each term in the binomial.

$$x^2(x+3)+6x(x+3)+9(x+3)$$

$$x^2(x)+x^2(3)+6x(x)+6x(3)+9(x)+9(3)$$

$$x^3+3x^2+6x^2+18x+9x+27$$

$$x^3+9x^2+27x+27$$

Let's do an example, and this time we'll use tables to organize our results.

Example

Use the Distributive Property to expand the expression.

$$3x(x+4)(x+1)(x-2)$$

We can do the multiplication in any order, but let's start by distributing the $3x$ across the $x+4$.

$$[3x(x+4)](x+1)(x-2)$$

$$[3x(x)+3x(4)](x+1)(x-2)$$

$$(3x^2+12x)(x+1)(x-2)$$

Now let's use a chart to distribute the $3x^2+12x$ across the $x+1$.

	x	1
$3x^2$	$3x^3$	$3x^2$
$12x$	$12x^2$	$12x$

When we add all the results in the chart together and then combine like terms, we get

$$3x^3+3x^2+12x^2+12x$$

$$3x^3+15x^2+12x$$

So the remaining expression is now

$$(3x^3+15x^2+12x)(x-2)$$

Finally, we'll distribute the $3x^3+15x^2+12x$ across the $x-2$. We'll use another chart.

	x	-2
$3x^3$	$3x^4$	$-6x^3$
$15x^2$	$15x^3$	$-30x^2$
$12x$	$12x^2$	$-24x$

When we add all the results in the chart together and then combine like terms, we get

$$3x^4 - 6x^3 + 15x^3 - 30x^2 + 12x^2 - 24x$$

$$3x^4 + 9x^3 - 18x^2 - 24x$$