Algebra 2 Workbook Solutions

Systems of equations

krista king
MATH

SYSTEMS WITH SUBSCRIPTS

■ 1. Solve the system of equations using any method.

$$d_t = x_t - 3$$

$$d_t = \frac{1}{2}x_t + 2$$

Solution:

Substitute $d_t = x_t - 3$ into $d_t = (1/2)x_t + 2$ to solve for x_t.

$$d_t = \frac{1}{2}x_t + 2$$

$$x_t - 3 = \frac{1}{2}x_t + 2$$

$$\frac{1}{2}x_t - 3 = 2$$

$$\frac{1}{2}x_t = 5$$

$$x_t = 10$$

Now substitute $x_t = 10$ into either equation. We'll use $d_t = x_t - 3$ since it's a little simpler.

$$d_t = x_t - 3$$

$$d_t = 10 - 3$$

$$d_t = 7$$

Then the solution to the system is

$$(x_t, d_t) = (10,7)$$

2. Solve the system of equations using any method.

$$4A_x - B_y = 15$$

$$B_y = 12 - 5A_x$$

Solution:

Substitute $B_y = 12 - 5A_x$ into $4A_x - B_y = 15$ to solve for A_x.

$$4A_x - B_y = 15$$

$$4A_x - (12 - 5A_x) = 15$$

$$4A_x - 12 + 5A_x = 15$$

$$9A_x - 12 = 15$$

$$9A_x = 27$$

$$A_x = 3$$

Now substitute A_x into either equation. We'll use $B_y = 12 - 5A_x$ since it's a little simpler.

$$B_y = 12 - 5A_x$$

$$B_y = 12 - 5(3)$$

$$B_y = 12 - 15$$

$$B_y = -3$$

Then the solution to the system is

$$(A_x, B_y) = (3, -3)$$

■ 3. Solve the system of equations using any method.

$$2T_x + 5T_y = 120.8$$

$$T_y = 17T_x + 17.2$$

Solution:

Substitute $T_y = 17T_x + 17.2$ into $2T_x + 5T_y = 120.8$ to solve for T_x.

$$2T_x + 5T_y = 120.8$$

$$2T_x + 5(17T_x + 17.2) = 120.8$$

$$2T_x + 85T_x + 86 = 120.8$$

$$87T_x + 86 = 120.8$$

$$87T_x = 34.8$$

$$T_x = 0.4$$

Now substitute T_x into either equation. We'll use $T_y = 17T_x + 17.2$ since it's a little simpler.

$$T_y = 17T_x + 17.2$$

$$T_y = 17(0.4) + 17.2$$

$$T_y = 6.8 + 17.2$$

$$T_y = 24$$

Then the solution to the system is

$$(T_x, T_y) = (0.4, 24)$$

■ 4. Solve the system of equations using any method.

$$T_uR_u = 480$$

$$R_d = \frac{1}{2}R_u$$

$$T_dR_d = 320$$

$$T_d = 2 + T_u$$

Solution:

Substitute $R_d = (1/2)R_u$ and $T_d = 2 + T_u$ into $T_dR_d = 320$.

$$T_dR_d = 320$$

$$(2 + T_u)\left(\frac{1}{2}R_u\right) = 320$$

Distribute $(1/2)R_u$.

$$R_u + \frac{1}{2}T_uR_u = 320$$

Substitute $T_uR_u = 480$.

$$R_u + \frac{1}{2}T_uR_u = 320$$

$$R_u + \frac{1}{2}(480) = 320$$

$$R_u + 240 = 320$$

Solve for R_u.

$$R_u + 240 = 320$$

$$R_u = 80$$

Now substitute $R_u = 80$ into $T_uR_u = 480$ to solve for T_u.

$$T_u R_u = 480$$

$$T_u(80) = 480$$

$$T_u = 6$$

Substitute $R_u = 80$ into $R_d = (1/2)R_u$ to solve for R_d.

$$R_d = \frac{1}{2}R_u$$

$$R_d = \frac{1}{2}(80)$$

$$R_d = 40$$

Finally, substitute $R_d = 40$ into $T_d R_d = 320$ to solve for T_d.

$$T_d R_d = 320$$

$$T_d(40) = 320$$

$$T_d = 8$$

Then the solution to the system is

$$(R_d, T_d) = (40,8)$$

$$(R_u, T_u) = (80,6)$$

■ 5. Solve the system of equations using any method.

$$2X_1 + 3Y_1 = 1$$

$$X_2 = 4X_1$$

$$X_2 + Y_2 = 9$$

$$Y_2 = Y_1 + 2$$

Solution:

Substitute $X_2 = 4X_1$ and $Y_2 = Y_1 + 2$ into the equation $X_2 + Y_2 = 9$.

$$X_2 + Y_2 = 9$$

$$4X_1 + Y_1 + 2 = 9$$

$$4X_1 + Y_1 = 7$$

Use the method of elimination to solve for Y_1. Multiply $2X_1 + 3Y_1 = 1$ by -2 so that X_1 will be eliminated.

$$-2(2X_1 + 3Y_1 = 1)$$

$$-4X_1 - 6Y_1 = -2$$

and then adding the equations together gives

$$-4X_1 - 6Y_1 + (4X_1 + Y_1) = -2 + (7)$$

$$-4X_1 - 6Y_1 + 4X_1 + Y_1 = -2 + 7$$

$$-5Y_1 = 5$$

$$Y_1 = -1$$

Substitute $Y_1 = -1$ into $2X_1 + 3Y_1 = 1$ to solve for X_1.

$$2X_1 + 3Y_1 = 1$$

$$2X_1 + 3(-1) = 1$$

$$2X_1 - 3 = 1$$

$$2X_1 = 4$$

$$X_1 = 2$$

Now substitute $X_1 = 2$ into $X_2 = 4X_1$ to solve for X_2.

$$X_2 = 4X_1$$

$$X_2 = 4(2)$$

$$X_2 = 8$$

Finally, substitute $Y_1 = -1$ into $Y_2 = Y_1 + 2$ to solve for Y_2.

$$Y_2 = Y_1 + 2$$

$$Y_2 = -1 + 2$$

$$Y_2 = 1$$

Then the solution to the system is

$$(X_1, Y_1) = (2, -1)$$

$$(X_2, Y_2) = (8,1)$$

■ 6. Solve the system of equations using any method.

$$6X_1 + 12Y_1 = 78$$

$$X_2 = 2X_1$$

$$2X_2 - 3Y_2 = -24$$

$$Y_2 = 3Y_1 - 3$$

Solution:

Substitute $X_2 = 2X_1$ and $Y_2 = 3Y_1 - 3$ into $2X_2 - 3Y_2 = -24$.

$$2X_2 - 3Y_2 = -24$$

$$2(2X_1) - 3(3Y_1 - 3) = -24$$

$$4X_1 - 9Y_1 + 9 = -24$$

$$4X_1 - 9Y_1 = -33$$

Use the method of elimination to solve for Y_1. Multiply $4X_1 - 9Y_1 = -33$ by -3,

$$-3(4X_1 - 9Y_1 = -33)$$

$$-12X_1 + 27Y_1 = 99$$

and multiply $6X_1 + 12Y_1 = 78$ by 2,

$$2(6X_1 + 12Y_1 = 78)$$

$$12X_1 + 24Y_1 = 156$$

and then add the equations together so that X_1 will be eliminated.

$$-12X_1 + 27Y_1 + (12X_1 + 24Y_1) = 99 + (156)$$

$$-12X_1 + 27Y_1 + 12X_1 + 24Y_1 = 99 + 156$$

$$51Y_1 = 255$$

$$Y_1 = 5$$

Substitute $Y_1 = 5$ into $6X_1 + 12Y_1 = 78$ to solve for X_1.

$$6X_1 + 12Y_1 = 78$$

$$6X_1 + 12(5) = 78$$

$$6X_1 + 60 = 78$$

$$6X_1 = 18$$

$$X_1 = 3$$

Now substitute $X_1 = 3$ into $X_2 = 2X_1$ to solve for X_2.

$$X_2 = 2X_1$$

$$X_2 = 2(3)$$

$$X_2 = 6$$

Finally, substitute $Y_1 = 5$ into $Y_2 = 3Y_1 - 3$ to solve for Y_2.

$$Y_2 = 3Y_1 - 3$$

$$Y_2 = 3(5) - 3$$

$$Y_2 = 15 - 3$$

$$Y_2 = 12$$

Then the solution to the system is

$$(X_1, Y_1) = (3,5)$$

$$(X_2, Y_2) = (6,12)$$

UNIFORM MOTION PROBLEMS

■ 1. Kaitlyn is driving at a constant rate of 55 mph on the highway. 1 hour later, her friend Charlotte starts from the same point and drives at a constant rate of 65 mph. How many hours will each woman need to travel before Charlotte catches Kaitlyn. At that point, how far have each of them traveled?

Solution:

Use the formula for distance.

Distance = Rate · Time

$$D = RT$$

Use subscripts to create a separate equation for each person.

Kaitlyn: $D_K = R_K T_K$

Charlotte: $D_C = R_C T_C$

When they're at the same spot on the highway, they'll have traveled the same distance. We also know the rate each person drove and that Charlotte left one hour later than Kaitlyn. Organize the information we know about each person.

$$D_K = D_C$$

$R_K = 55$ mi/hr

$T_K = \ ?$ hr

and

$D_C = \ ?$

$R_C = 65$ mi/hr

$T_C = (T_K - 1)$ hr

Plug all known values into both equations.

$D_K = R_K T_K$

$D_K = 55$ mi/hr T_K hr

$D_K = 55T_K$ mi

and

$D_C = R_C T_C$

$D_C = 65$ mi/hr $(T_K - 1)$ hr

$D_C = 65\ (T_K - 1)$ mi

Substitute $D_C = 55T_K$ mi into $D_C = 65\ (T_K - 1)$ mi.

$D_C = 65\ (T_K - 1)$ mi

$55T_K$ mi $= 65\ (T_K - 1)$ mi

$$55T_K \text{ mi} = 65T_K \text{ mi} - 65 \text{ mi}$$

Move all T_K terms to one side of the equation by subtracting $65T_K$ mi from both sides.

$$55T_K \text{ mi} - 65T_K \text{ mi} = 65T_K \text{ mi} - 65T_K \text{ mi} - 65 \text{ mi}$$

$$-10T_K \text{ mi} = -65 \text{ mi}$$

$$T_K = 6.5$$

$$T_K = 6 \text{ hr } 30 \text{ min}$$

Kaitlyn drove 6 hr 30 min.

Since Charlotte left 1 hour later than Kaitlyn, we can find Charlotte's time by subtracting 1 hr.

$$T_C = (T_K - 1) \text{ hr}$$

$$T_C = 6 \text{ hr } 30 \text{ min} - 1 \text{ hr}$$

$$T_C = 5 \text{ hrs } 30 \text{ min}$$

Charlotte drove 5 hrs 30 min. Find the distance using either Kaitlyn or Charlotte's distance formula.

$$D_K = 55 \text{ mi/hr}(6.5 \text{ hr})$$

$$D_K = 357.5 \text{ mi}$$

Kaitlyn and Charlotte each traveled 357.5 miles.

■ 2. Sam walked 4 mph from the bus stop to the school. John walked 5 mph from the same bus stop to the school and it took him 1 hour to get there. How long did it take Sam to get to the school?

Solution:

Use the formula for distance.

Distance = Rate · Time

$D = RT$

Use subscripts to create a separate equation for each person.

Sam: $D_S = R_S T_S$

John: $D_J = R_J T_J$

Since Sam and John both walk from the same bus stop to the school, they travel the same distance. We also know the rate each person walked and that John walked for 1 hour. Organize the information we know about each person.

$D_S = D_J$

$R_S = 4$ mi/hr

$T_S = \ ?$

and

$D_J = \ ?$

$R_J = 5$ mi/hr

$T_J = 1$ hr

Find the distance by plugging all known values into $D_J = R_J T_J$.

$D_J = R_J T_J$

$D_J = (5 \text{ mi/hr})(1 \text{ hr})$

$D_J = 5$ mi

Now find the time Sam walked by plugging all known values into $D_S = R_S T_S$.

$D_S = R_S T_S$

$5 \text{ mi} = 4 \text{ mi/hr } T_S$

$T_S = 1.25$ hr

Sam walked 1 hr 15 min to get from the bus stop to the school.

■ 3. McKenzie and Daisy plan a trip to the mountains together. McKenzie travels at 45 mph and Daisy travels at 60 mph. Daisy's trip took 3 hours less than McKenzie's. How far did each of them travel?

Solution:

Use the formula for distance.

Distance = Rate · Time

$D = RT$

Use subscripts to create a separate equation for each person.

McKenzie: $D_M = R_M T_M$

Daisy: $D_D = R_D T_D$

Since McKenzie and Daisy are going to the same place, they travel the same distance. We also know the rate at which each person drove, and we know that Daisy arrived 3 hours before McKenzie. Organize the information we know about each person.

$D_M = D_D$

$R_M = 45$ mi/hr

$T_M = \ ?$

and

$D_D = \ ?$

$R_D = 60$ mi/hr

$T_D = (T_M - 3)$ hr

Plug all known values into both equations.

$$D_D = R_D T_D$$

$$D_D = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$$

Substitute $D_D = 45 \text{ mi/hr } T_M$ into $D_D = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$.

$$D_D = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$$

$$45 \text{ mi/hr } T_M = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$$

$$45 \text{ mi/hr } T_M = 60 \text{ mi/hr } T_M - 180 \text{ mi}$$

Move all T_M terms to one side of the equation by subtracting $60 \text{ mi } T_M$ from both sides.

$$45 \text{ mi/hr } T_M - 60 \text{ mi/hr } T_M = 60 \text{ mi/hr } T_M - 60 \text{ mi/hr } T_M - 180 \text{ mi}$$

$$-15 \text{ mi/hr } T_M = -180 \text{ mi}$$

$$T_M = 12 \text{ hrs}$$

Find the distance using McKenzie's distance formula.

$$D_M = 45 \text{ mi/hr}(12 \text{ hrs})$$

$$D_M = 540 \text{ mi}$$

McKenzie and Daisy traveled 540 miles.

■ 4. Talan and Emily participate in a race together. Talan runs 10 mph and finishes 1 hour and 18 minutes before Emily does. If Emily runs 5 mph, how long does it take each person to finish the race and how far did they run?

Solution:

Use the formula for distance.

Distance = Rate · Time

$D = RT$

Use subscripts to create a separate equation for each person.

Talan: $D_T = R_T T_T$

Emily: $D_E = R_E T_E$

Since Talan and Emily are running in the same race, they travel the same distance. We also know the rate each person ran and that Talan arrived 1 hour and 18 minutes, or 1.3 hrs before Emily. Organize the information we know about each person.

$D_T = \ ?$

$R_T = 10$ mi/hr

$T_T = (T_E - 1.3)$ hr

and

$D_E = D_T$

$R_E = 5 \text{ mi/hr}$

$T_E = \ ?$

Plug all known values into both equations.

$D_T = R_T T_T$

$D_T = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$

and

$D_E = R_E T_E$

$D_T = 5 \text{ mi/hr } T_E$

Substitute $D_T = 5 \text{ mi/hr } T_E$ into $D_T = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$.

$D_T = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$

$5 \text{ mi/hr } T_E = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$

$5 \text{ mi/hr } T_E = 10 \text{ mi/hr } T_E - 13 \text{ mi}$

Move all T_E terms to one side of the equation by subtracting $10 \text{ mi/hr } T_E$ from both sides.

$5 \text{ mi/hr } T_E - 10 \text{ mi/hr } T_E = 10 \text{ mi/hr } T_E - 10 \text{ mi/hr } T_E - 13 \text{ mi}$

$-5 \text{ mi/hr } T_E = -13 \text{ mi}$

$T_E = 2.6 \text{ hr}$

Emily ran for 2 hr 36 min.

Since Talan finished 1 hour and 18 minutes before Emily, we can find Talan's time by subtracting 1 hr 18 min.

$$T_T = T_E - 1 \text{ hr } 18 \text{ min}$$

$$T_T = 2 \text{ hrs } 36 \text{ min} - 1 \text{ hr } 18 \text{ min}$$

$$T_T = 1 \text{ hr } 18 \text{ min}$$

Talan ran for 1 hr 18 min.

Find the distance using either Talan or Emily's distance formula.

$$D_E = 5 \text{ mi/hr}(2.6 \text{ hrs})$$

$$D_E = 13 \text{ mi}$$

Talan and Emily ran 13 miles.

■ 5. Adeline and Ellie live 10 miles away from each other. Adeline started walking towards Ellie at $1:00$ p.m.. Ellie left 1 hour later and walked at a rate of 4 mph. If they met at $3:00$ p.m., how fast did Adeline walk?

Solution:

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts for two distance formulas, one for Adeline and one for Ellie.

$$D_A = R_A T_A$$

$$D_E = R_E T_E$$

We know that in order to meet, they covered the 10 miles between them.

$$D_A + D_E = 10 \text{ mi}$$

Substitute $D_A = R_A T_A$ and $D_E = R_E T_E$.

$$D_A + D_E = 10 \text{ mi}$$

$$R_A T_A + R_E T_E = 10 \text{ mi}$$

We know that Adeline walks from $1:00$ p.m. to $3:00$ p.m., which means she walks for 2 hours. We also know that Ellie walks from $2:00$ p.m. to $3:00$ p.m., which means Ellie walks for 1 hour, at a rate of 4 mph.

$$T_A = 2 \text{ hr}$$

$$T_E = 1 \text{ hr}$$

$$R_E = 4 \text{ mi/hr}$$

Then we can say

$$R_A T_A + R_E T_E = 10 \text{ mi}$$

$$R_A(2 \text{ hr}) + (4 \text{ mi/hr})(1 \text{ hr}) = 10 \text{ mi}$$

$$2R_A \text{ hr} + 4 \text{ mi} = 10 \text{ mi}$$

Subtract 4 mi from both sides and divide by 2 hr.

$$2R_A \text{ hr} = 6 \text{ mi}$$

$$R_A = 3 \text{ mi/hr}$$

Adeline walks at a rate of 3 mph.

■ 6. A train traveled 420 miles at 48 mph and arrived 1 hour and 45 minutes later than it was scheduled to arrive. How fast should the train have traveled in order to arrive on time?

Solution:

First we need to find how much time it took the train to travel 420 miles at 48 mph. Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Identify what we know and what we need to find.

$$D = 420 \text{ mi}$$

$$R = 48 \text{ mi/hr}$$

$$T = ?$$

Plug these values into the distance formula.

$$420 \text{ mi} = 48 \text{ mi/hr} \cdot T$$

Divide both sides by 48 mi/hr to solve for time.

$$\frac{420 \text{ mi}}{48 \text{ mi/hr}} = \frac{48 \text{ mi/hr} \cdot T}{48 \text{ mi/hr}}$$

$$T = 8.75 \text{ hr}$$

If the train was 1 hour 45 minutes late (or 1.75 hr), then the train needs to travel $8.75 - 1.75 = 7$ hrs to arrive on time. Use the distance formula again with the new time to solve for the new rate.

Identify what we know and what we need to find.

$$D = 420 \text{ mi}$$

$$R = ?$$

$$T = 7 \text{ hr}$$

Plug these values into the distance formula.

$$420 \text{ mi} = R \cdot 7 \text{ hr}$$

Divide both sides by 7 hr to solve for rate.

$$\frac{420 \text{ mi}}{7 \text{ hr}} = \frac{R \cdot 7 \text{ hr}}{7 \text{ hr}}$$

$$R = 60 \text{ mi/hr}$$

NUMBER WORD PROBLEMS

■ 1. The sum of two consecutive odd integers is -8. What are the two numbers?

Solution:

Let X represent the first number. Because the numbers are consecutive odd numbers, the next integer will be $X+2$. Write the equation that represents the sum of the two numbers.

$$X+X+2=-8$$

Solve for X.

$$2X+2=-8$$

$$2X=-10$$

$$X=-5$$

If $X=-5$, the next odd integer is $X+2$.

$$-5+2=-3$$

The integers are -5 and -3.

■ 2. The product of two consecutive integers is 182. What are the two numbers if they are both negative?

Solution:

Let X represent the first number. Because the numbers are consecutive numbers, the next integer will be $X+1$. Write the equation that represents the product of the two numbers.

$$X(X+1)=182$$

$$X^2+X=182$$

$$X^2+X-182=0$$

$$(X+14)(X-13)=0$$

$$X=-14,\ 13$$

We were asked to find two negative integers, so we'll use $X=-14$. If $X=-14$, the next integer is $X+1$.

$$-14+1=-13$$

The integers are -14 and -13.

■ 3. The sum of the digits of a two-digit counting number is 11. When the digits are reversed, the new number is 63 greater than the original number. What was the original number?

Solution:

Let T be the tens digit and U be the units digit of the original number. Write the equation for the sum of the digits.

$$T + U = 11$$

The value of the original number is $10T + U$, and reversing the digits gives us a number whose value is $10U + T$. The new number is 63 greater than the original number, so

$$10T + U + 63 = 10U + T$$

Move all the T and U terms to the left side of the equation and move the constant to the right side.

$$9T + U + 63 = 10U$$

$$9T - 9U + 63 = 0$$

$$9T - 9U = -63$$

$$T - U = -7$$

Add the sum of the digits equation to the equation we just found to solve this as a system of equations.

$$T - U + (T + U) = -7 + (11)$$

$$T + T - U + U = -7 + 11$$

$$2T = 4$$

$$T = 2$$

Plug $T = 2$ into $T + U = 11$ and solve for U.

$$2 + U = 11$$

$$U = 9$$

The value of the original number is $10T + U$.

$$10(2) + 9$$

$$20 + 9 = 29$$

The original number is 29.

■ 4. The sum of the digits of a two-digit counting number is 14. The ratio of the units digit to the tens digit is 3 to 4. What is the number?

Solution:

Let T be the tens digit and U be the units digit of the number. Write the equation for the sum of the digits.

$$T + U = 14$$

Write the equation for the ratio of the units digit and tens digit.

$$\frac{U}{T} = \frac{3}{4}$$

Multiply both sides by T to solve for U.

$$U = \frac{3}{4}T$$

Plug $U = (3/4)T$ into $T + U = 14$.

$$T + \frac{3}{4}T = 14$$

$$\frac{7}{4}T = 14$$

$$7T = 56$$

$$T = 8$$

Plug $T = 8$ into $T + U = 14$ and solve for U.

$$8 + U = 14$$

$$U = 6$$

The value of the number is $10T + U$.

$$10(8) + 6$$

$$80 + 6 = 86$$

The number is 86.

5. The sum of the digits of a two-digit counting number is 8. The ratio of the tens digit to the units digit is 3 to 1. What is the number?

Solution:

Let T be the tens digit and U be the units digit of the number. Write the equation for the sum of the digits.

$$T + U = 8$$

Write the equation for the ratio of the tens digit and units digit.

$$\frac{T}{U} = \frac{3}{1}$$

Multiply both sides by U to solve for T.

$$T = 3U$$

Plug $T = 3U$ into $T + U = 8$.

$$3U + U = 8$$

$$4U = 8$$

$$U = 2$$

Plug $U = 2$ into $T + U = 8$ and solve for T.

$$T + 2 = 8$$

$$T = 6$$

The value of the number is $10T+U$.

$$10(6)+2$$

$$60+2=62$$

The number is 62.

■ 6. Find three negative consecutive odd integers such that the sum of the first and third is 49 less than the product of the second and third.

Solution:

Let X represent the first number. Because the numbers are consecutive odd integers, the next integer will be $X+2$ and the third will be $X+4$. The sum of the first and third integers will equal the product of the second and third, minus 49.

$$X+X+4=(X+2)(X+4)-49$$

FOIL out the binomials on the right side, then combine like terms.

$$2X+4=X^2+4X+2X+8-49$$

$$2X+4=X^2+6X-41$$

Move all the terms to the left side of the equation and then factor.

$$X^2+4X-45=0$$

$$(X+9)(X-5)=0$$

$$X=-9, 5$$

We were asked to find negative integers, so we'll use $X=-9$. If $X=-9$, the next two odd integers are $X+2$ and $X+4$.

$$-9+2=-7$$

$$-9+4=-5$$

The integers are -9, -7, and -5.

AGE WORD PROBLEMS

■ 1. In 16 years, Thorin will be nine times older than he is now. How old is he now?

Solution:

Let T represent Thorin's age now. If we add 16 years to T, it will equal 9 times T.

$$T + 16 = 9T$$

Solve for T.

$$16 = 8T$$

$$T = 2$$

Thorin is 2 years old now.

■ 2. In 12 years, Jake will be four times as old as he was nine years ago. How old will he be in 12 years?

Solution:

Let J represent Jake's age now. If we add 12 years to J, it will equal 4 times $(J - 9)$.

$$J + 12 = 4(J - 9)$$

Solve for J.

$$J + 12 = 4J - 36$$

$$48 = 3J$$

$$J = 16$$

Add 12 to 16 to find out how old Jake will be in 12 years.

$$16 + 12 = 28$$

Jake will be 28 years old in 12 years.

■ 3. In 16 years, Erica will be six times as old as she was 14 years ago. How old was she four years ago?

Solution:

Let E represent Erica's age now. If we add 16 years to E, it will equal 6 times $(E - 14)$.

$$E + 16 = 6(E - 14)$$

Solve for E.

$$E + 16 = 6E - 84$$

$$100 = 5E$$

$$E = 20$$

Subtract 4 from 20 to find out how old Erica was four years ago.

$$20 - 4 = 16$$

Erica was 16 years old four years ago.

■ 4. 10 years ago, Tim was three years younger than twice Sally's age. 17 years from now Sally will be seven years older than $5/8$ of Tim's age. How old are Sally and Tim now?

Solution:

Let T represent Tim's age now and let S represent Sally's age now. We need to multiply Sally's age 10 years ago by 2, and then subtract 3 in order to get Tim's age 10 years ago.

$$2(S - 10) - 3 = T - 10$$

$$2S - 20 - 3 = T - 10$$

$$2S - 23 = T - 10$$

$$2S - 13 = T$$

We need to subtract 7 from Sally's age in 17 years to get $5/8$ of Tim's age in 17 years.

$$S+17-7=\frac{5}{8}(T+17)$$

$$S+10=\frac{5}{8}(T+17)$$

Use substitution to plug $T=2S-13$ into $S+10=(5/8)(T+17)$.

$$S+10=\frac{5}{8}(T+17)$$

$$S+10=\frac{5}{8}(2S-13+17)$$

$$S+10=\frac{5}{8}(2S+4)$$

$$S+10=\frac{5}{4}S+\frac{5}{2}$$

$$\frac{15}{2}=\frac{1}{4}S$$

$$S=30$$

Sally is 30 years old.

Plug $S=30$ into $T=2S-13$ to find Tim's age.

$$T=2S-13$$

$$T=2(30)-13$$

$$T = 60 - 13$$

$$T = 47$$

Tim is 47 years old.

■ 5. Jessica is five years younger than Ryan. 12 years ago, Jessica was half the age Ryan will be in three years. How old is Jessica now?

Solution:

Let J represent Jessica's age now and let R represent Ryan's age now. We need to subtract 5 from Ryan's age to get Jessica's age.

$$J = R - 5$$

We need to multiply Jessica's age 12 years ago by 2 in order to get Ryan's age in three years.

$$2(J - 12) = R + 3$$

Use substitution to plug $J = R - 5$ into $2(J - 12) = R + 3$.

$$2(J - 12) = R + 3$$

$$2(R - 5 - 12) = R + 3$$

$$2(R - 17) = R + 3$$

$$2R - 34 = R + 3$$

$$R = 37$$

Ryan is 37 years old.

Plug $R = 37$ into $J = R - 5$ to find Jessica's age.

$$J = R - 5$$

$$J = 37 - 5$$

$$J = 32$$

Jessica is 32 years old.

■ 6. Kate is five times older than Sam. Five years ago, Kate was 15 times older than Sam. How old will Sam be in three years?

Solution:

Let K represent Kate's age now and let S represent Sam's age now. We need to multiply Sam's age by 5 to get Kate's age.

$$K = 5S$$

We need to multiply Sam's age five years ago by 15 in order to get Kate's age five years ago.

$$15(S - 5) = K - 5$$

Use substitution to plug $K = 5S$ into $15(S - 5) = K - 5$.

$$15(S - 5) = K - 5$$

$$15(S - 5) = 5S - 5$$

$$15S - 75 = 5S - 5$$

$$10S = 70$$

$$S = 7$$

Add 3 to 7 to get Sam's age in three years.

$$7 + 3 = 10$$

Sam will be 10 years old in three years.

SYSTEMS WITH NON-LINEAR EQUATIONS

■ 1. Solve the system of equations.

$$y = x - 1$$

$$x^2 + y^2 = 1$$

Solution:

Use substitution to plug $y = x - 1$ into $x^2 + y^2 = 1$.

$$x^2 + y^2 = 1$$

$$x^2 + (x - 1)^2 = 1$$

Use the FOIL method to expand $(x - 1)^2$, and then combine like terms.

$$x^2 + x^2 - x - x + 1 = 1$$

$$2x^2 - 2x + 1 = 1$$

$$2x^2 - 2x = 0$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0, 1$$

Plug $x = 0$ and $x = 1$ into $y = x - 1$ to find the corresponding y -values.

$y = 0 - 1$

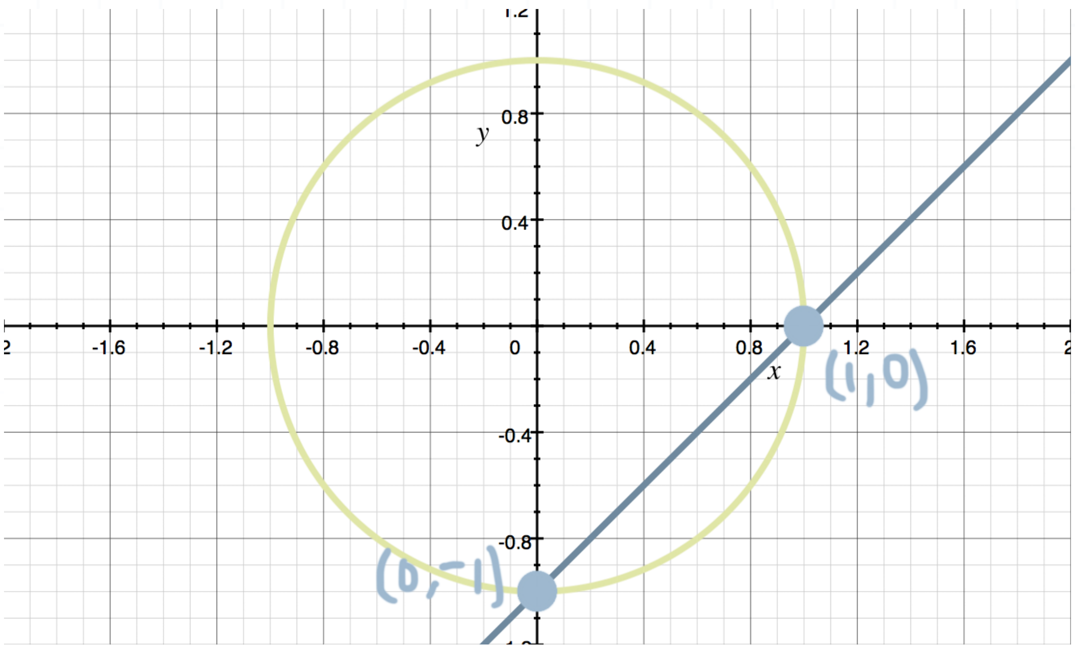
$y = -1$

and

$y = 1 - 1$

$y = 0$

The solutions are therefore $(0, -1)$ and $(1,0)$. If we plot both curves, we can see the points of intersection.



■ 2. Solve the system of equations.

$y - 2x = 0$

$(x-2)^2 + (y-1)^2 = 9$

Solution:

Solve for y by adding $2x$ to both sides of $y - 2x = 0$.

$$y - 2x = 0$$

$$y = 2x$$

Use substitution to plug $y = 2x$ into $(x - 2)^2 + (y - 1)^2 = 9$.

$$(x - 2)^2 + (y - 1)^2 = 9$$

$$(x - 2)^2 + (2x - 1)^2 = 9$$

Use the FOIL method to expand and then combine like terms.

$$x^2 - 2x - 2x + 4 + 4x^2 - 2x - 2x + 1 = 9$$

$$x^2 - 4x + 4 + 4x^2 - 4x + 1 = 9$$

$$5x^2 - 8x + 5 = 9$$

$$5x^2 - 8x - 4 = 0$$

Next, factor and then set each factor equal to 0 to solve for x.

$$(5x + 2)(x - 2) = 0$$

$$5x + 2 = 0$$

$$5x = -2$$

$$x = -2/5$$

and

$$x - 2 = 0$$

$$x = 2$$

Plug $x = -2/5$ and $x = 2$ into $y = 2x$ to find the corresponding y-values.

$$y = 2\left(-\frac{2}{5}\right)$$

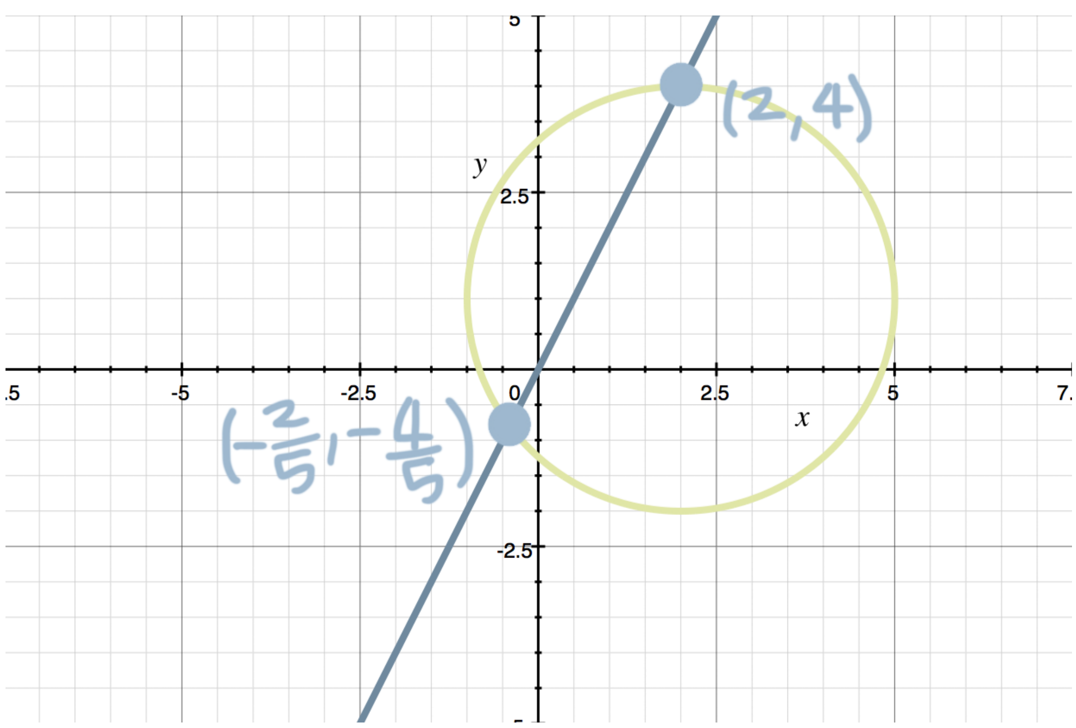
$$y = -\frac{4}{5}$$

and

$$y = 2(2)$$

$$y = 4$$

The solutions are therefore $(-2/5, -4/5)$ and $(2,4)$. If we plot both curves, we can see the points of intersection.



3. Solve the system of equations.

$$2y - x = -2$$

$$(x-6)^2 + 4(y-4)^2 = 16$$

Solution:

Solve $2y - x = -2$ for x

$$2y - x = -2$$

$$2y = x - 2$$

$$x = 2y + 2$$

Use substitution to plug $x = 2y + 2$ into $(x-6)^2 + 4(y-4)^2 = 16$.

$$(x-6)^2 + 4(y-4)^2 = 16$$

$$(2y + 2 - 6)^2 + 4(y-4)^2 = 16$$

$$(2y-4)^2 + 4(y-4)^2 = 16$$

Use the FOIL method to expand.

$$4y^2 - 8y - 8y + 16 + 4(y^2 - 4y - 4y + 16) = 16$$

$$4y^2 - 16y + 16 + 4(y^2 - 8y + 16) = 16$$

$$4y^2 - 16y + 16 + 4y^2 - 32y + 64 = 16$$

$$8y^2 - 48y + 80 = 16$$

$$8y^2 - 48y + 64 = 0$$

$$y^2 - 6y + 8 = 0$$

$$(y - 2)(y - 4) = 0$$

$$y = 2,\ 4$$

Plug $y = 2$ and $y = 4$ into $x = 2y + 2$ to find the corresponding x-values.

$$x = 2(2) + 2$$

$$x = 4 + 2$$

$$x = 6$$

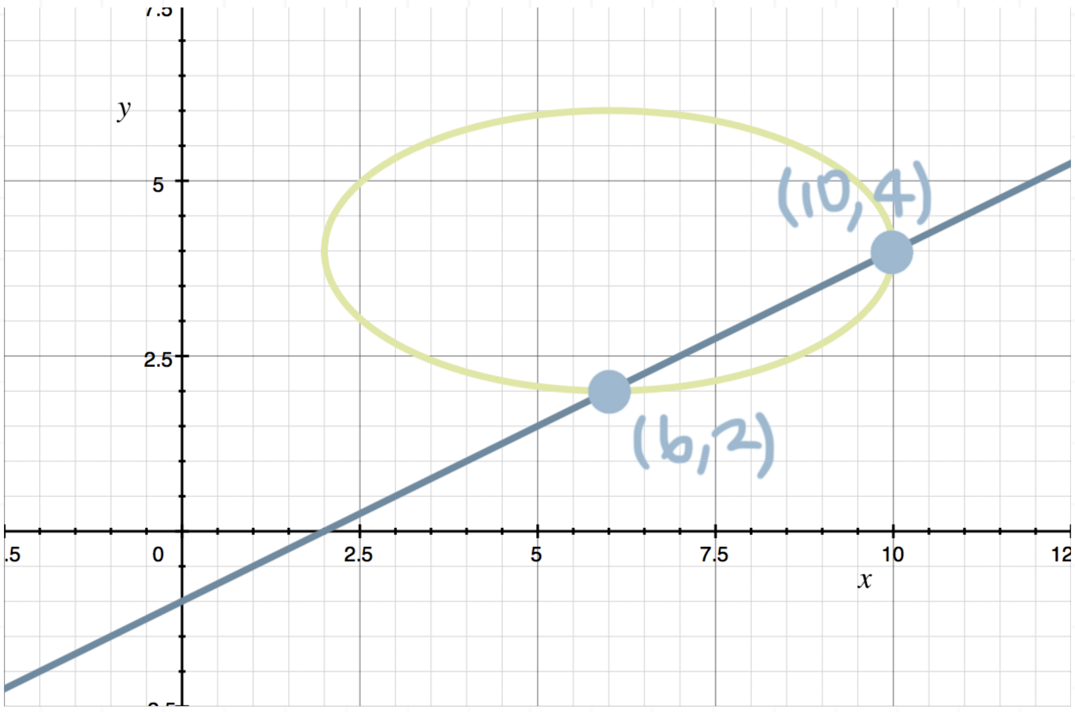
and

$$x = 2(4) + 2$$

$$x = 8 + 2$$

$$x = 10$$

The solutions are therefore $(6,2)$ and $(10,4)$. If we plot both curves, we can see the points of intersection.



■ 4. Solve the system of equations.

$$y = \frac{1}{2}x + 1$$

$$x^2 - y^2 = 4$$

Solution:

Use substitution to plug $y = (1/2)x + 1$ into $x^2 - y^2 = 4$.

$$x^2 - y^2 = 4$$

$$x^2 - \left(\frac{1}{2}x + 1\right)^2 = 4$$

Use the FOIL method to expand the binomial.

$$x^2-\left(\frac{1}{4}x^2+\frac{1}{2}x+\frac{1}{2}x+1\right)=4$$

$$x^2-\left(\frac{1}{4}x^2+x+1\right)=4$$

$$x^2-\frac{1}{4}x^2-x-1=4$$

$$\frac{3}{4}x^2-x-1=4$$

$$3x^2-4x-4=16$$

$$3x^2-4x-20=0$$

$$(x+2)(3x-10)=0$$

$$x=-2,\ 10/3$$

Plug in $x=-2$ and $x=10/3$ into the equation $y=(1/2)x+1$ to find the corresponding y-values.

$$y=\frac{1}{2}(-2)+1$$

$$y=-1+1$$

$$y=0$$

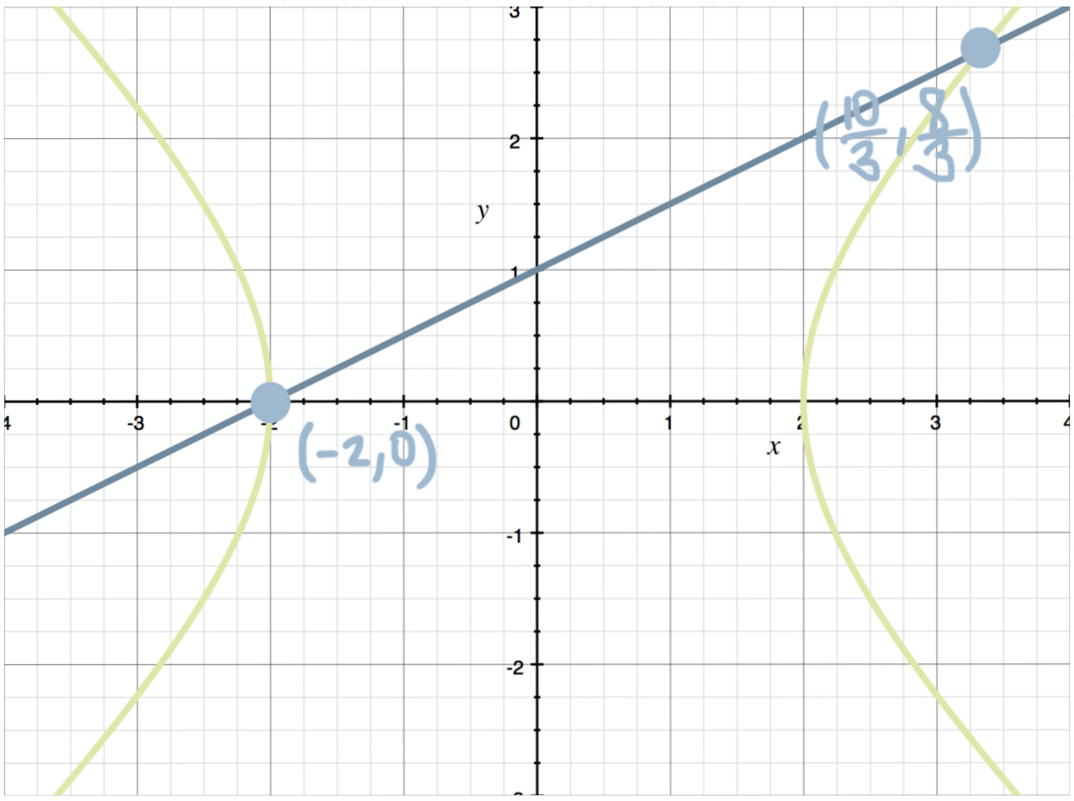
and

$$y=\frac{1}{2}\left(\frac{10}{3}\right)+1$$

$$y = \frac{5}{3} + \frac{3}{3}$$

$$y = \frac{8}{3}$$

The solutions are therefore $(-2,0)$ and $(10/3,8/3)$. If we plot both curves, we can see the points of intersection.



■ 5. Solve the system of equations.

$$y = -\frac{1}{2}x - 3$$

$$\frac{(x-3)^2}{9} - \frac{(y+3)^2}{9} = 1$$

Solution:

Use substitution to plug $y = -(1/2)x - 3$ into the second equation.

$$\frac{(x-3)^2}{9} - \frac{(y+3)^2}{9} = 1$$

$$\frac{(x-3)^2}{9} - \frac{\left(-\frac{1}{2}x - 3 + 3\right)^2}{9} = 1$$

$$\frac{(x-3)^2}{9} - \frac{\left(-\frac{1}{2}x\right)^2}{9} = 1$$

$$(x-3)^2 - \left(-\frac{1}{2}x\right)^2 = 9$$

Use the FOIL method to expand, combine like terms, and then multiply each term by 4 to simplify.

$$x^2 - 3x - 3x + 9 - \frac{1}{4}x^2 = 9$$

$$\frac{3}{4}x^2 - 6x + 9 = 9$$

$$3x^2 - 24x + 36 = 36$$

$$3x^2 - 24x = 0$$

$$x^2 - 8x = 0$$

$$x(x-8) = 0$$

$$x = 0, \ 8$$

Plug $x = 0$ and $x = 8$ into $y = -(1/2)x - 3$ to find the corresponding y-values.

$$y = -\frac{1}{2}(0) - 3$$

$$y = 0 - 3$$

$$y = -3$$

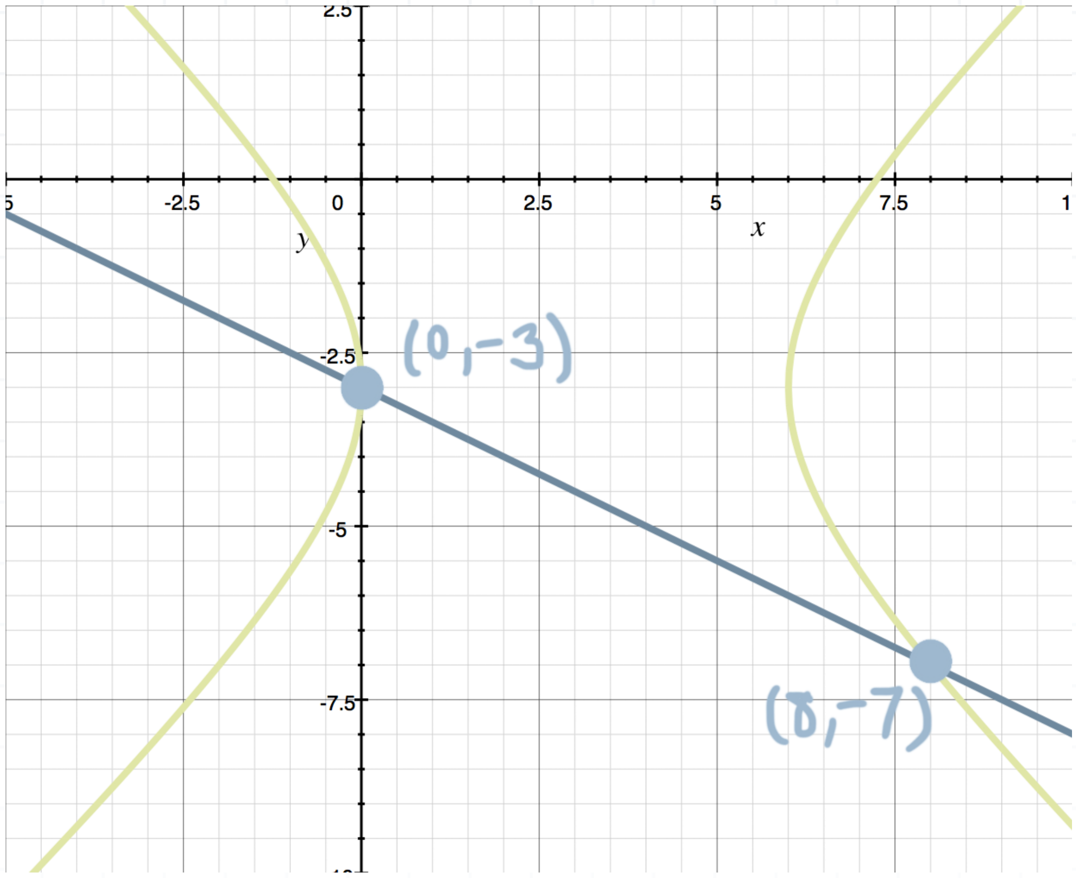
and

$$y = -\frac{1}{2}(8) - 3$$

$$y = -4 - 3$$

$$y = -7$$

The solutions are therefore $(0, -3)$ and $(8, -7)$. If we plot both curves, we can see the points of intersection.



■ 6. Solve the system of equations.

$$2y - 3x = 14$$

$$\frac{(y+2)^2}{9} + \frac{(x+4)^2}{4} = 1$$

Solution:

Solve $2y - 3x = 14$ for y.

$$2y - 3x = 14$$

$$2y = 3x + 14$$

$$y = \frac{3}{2}x + 7$$

Use substitution to plug $y = (3/2)x + 7$ into the second equation in the system.

$$\frac{(y+2)^2}{9} + \frac{(x+4)^2}{4} = 1$$

$$\frac{\left(\frac{3}{2}x + 7 + 2\right)^2}{9} + \frac{(x+4)^2}{4} = 1$$

$$\frac{\left(\frac{3}{2}x + 9\right)^2}{9} + \frac{(x+4)^2}{4} = 1$$

Multiply every term by 36 to simplify.

$$\frac{\left(\frac{3}{2}x + 9\right)^2}{9}(36) + \frac{(x+4)^2}{4}(36) = 1(36)$$

$$4\left(\frac{3}{2}x + 9\right)^2 + 9(x+4)^2 = 36$$

Use the FOIL method to expand and then combine like terms.

$$4\left(\frac{9}{4}x^2 + \frac{27}{2}x + \frac{27}{2}x + 81\right) + 9\left(x^2 + 4x + 4x + 16\right) = 36$$

$$4\left(\frac{9}{4}x^2 + \frac{54}{2}x + 81\right) + 9(x^2 + 8x + 16) = 36$$

$$9x^2 + 108x + 324 + 9x^2 + 72x + 144 = 36$$

$$18x^2 + 180x + 468 = 36$$

$$18x^2 + 180x + 432 = 0$$

$$x^2 + 10x + 24 = 0$$

$$(x + 6)(x + 4) = 0$$

$$x = -6, \ -4$$

Plug $x = -6$ and $x = -4$ into $y = (3/2)x + 7$ to find the corresponding y-values.

$$y = \frac{3}{2}(-6) + 7$$

$$y = -9 + 7$$

$$y = -2$$

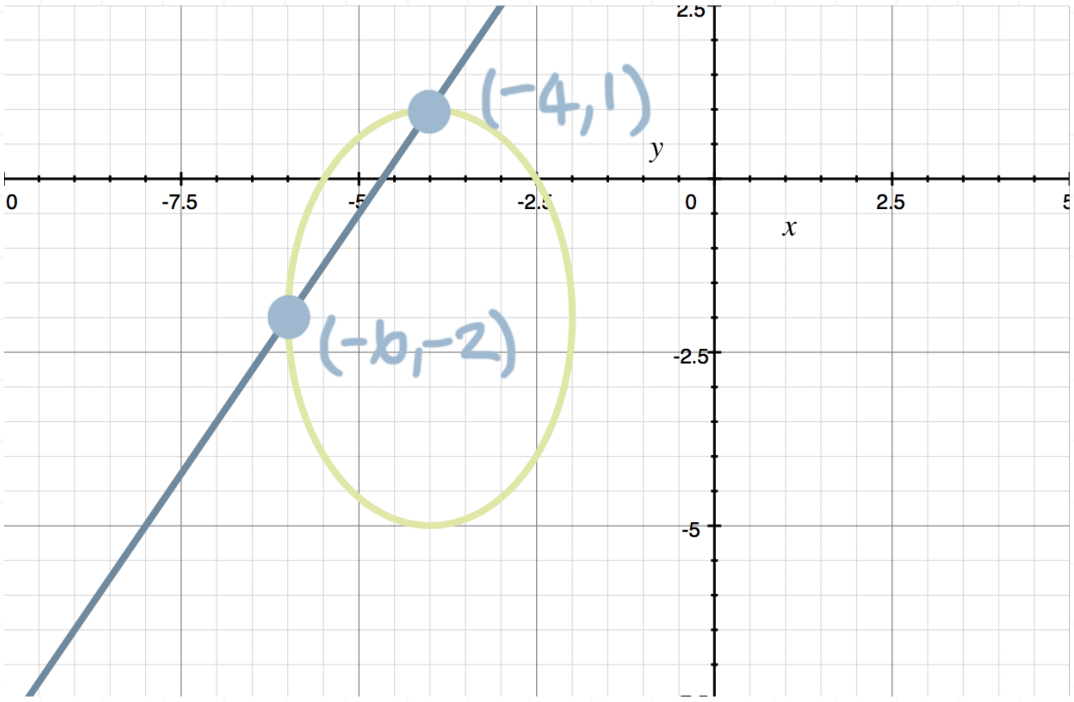
and

$$y = \frac{3}{2}(-4) + 7$$

$$y = -6 + 7$$

$$y = 1$$

The solutions are therefore $(-6, -2)$ and $(-4,1)$. If we plot both curves, we can see the points of intersection.


(−4, 1)
(−6, −2)
2.5
-2.5
-5
-7.5
-5
-2.5
0
2.5
5
x
y

SYSTEMS OF THREE EQUATIONS

■ 1. Find the unique solution to the system of equations.

$$2x + y - z = 3$$

$$x - y + z = 0$$

$$x - 2y - 3z = 4$$

Solution:

Let's number the equations to stay organized.

[1] $2x + y - z = 3$

[2] $x - y + z = 0$

[3] $x - 2y - 3z = 4$

Add equations [1] and [2] so that y and z will be eliminated.

$$(2x + y - z) + (x - y + z) = (3) + (0)$$

$$2x + y - z + x - y + z = 3$$

$$3x + y - y + z - z = 3$$

$$3x = 3$$

$$x = 1$$

Let's plug $x = 1$ into equations [2] and [3] to put them in terms of y and z.

$$x - y + z = 0$$

$$1 - y + z = 0$$

[4] $-y + z = -1$

and

$$x - 2y - 3z = 4$$

$$1 - 2y - 3z = 4$$

[5] $-2y - 3z = 3$

Multiply equation [4] by 3,

$$3(-y + z = -1)$$

$$-3y + 3z = -3$$

and then add this to equation [5] so that z can be eliminated.

$$(-3y + 3z) + (-2y - 3z) = (-3) + (3)$$

$$-3y + 3z - 2y - 3z = 0$$

$$-3y - 2y = 0$$

$$-5y = 0$$

$$y = 0$$

Plug $y = 0$ into equation [5] to solve for z.

$$-2y - 3z = 3$$

$$-2(0) - 3z = 3$$

$$-3z = 3$$

$$z = -1$$

Therefore, the solution to the system is $(x, y, z) = (1,0,-1)$.

■ 2. Find the unique solution to the system of equations.

$$3x + y - z = -2$$

$$x - 2y + 3z = 23$$

$$2x + 3y + 2z = 5$$

Solution:

Let's number the equations to stay organized.

[1] $3x + y - z = -2$

[2] $x - 2y + 3z = 23$

[3] $2x + 3y + 2z = 5$

Multiply equation [1] by 2,

$$2(3x + y - z = -2)$$

$$6x + 2y - 2z = -4$$

and add this to equation [3] so that z can be eliminated.

$$(6x + 2y - 2z) + (2x + 3y + 2z) = (-4) + (5)$$

$$6x + 2y - 2z + 2x + 3y + 2z = 1$$

$$6x + 2y + 2x + 3y = 1$$

[4] $8x + 5y = 1$

Multiply equation [1] by 3,

$$3(3x + y - z = -2)$$

$$9x + 3y - 3z = -6$$

and add this to equation [2] so that z can be eliminated.

$$(9x + 3y - 3z) + (x - 2y + 3z) = (-6) + (23)$$

$$9x + 3y - 3z + x - 2y + 3z = 17$$

$$9x + 3y + x - 2y = 17$$

[5] $10x + y = 17$

Multiply equation [5] by -5,

$$-5(10x + y = 17)$$

$$-50x - 5y = -85$$

and then add this to equation [4] so that y will be eliminated.

$$(8x + 5y) + (-50x - 5y) = (1) + (-85)$$

$$8x + 5y - 50x - 5y = -84$$

$$8x - 50x = -84$$

$$-42x = -84$$

$$x = 2$$

Let's plug $x = 2$ into equation [5] to solve for y.

$$10x + y = 17$$

$$10(2) + y = 17$$

$$20 + y = 17$$

$$y = -3$$

Plug $x = 2$ and $y = -3$ into any of the original equations to solve for z. We'll use equation [1].

$$3x + y - z = -2$$

$$3(2) + (-3) - z = -2$$

$$6 - 3 - z = -2$$

$$3 - z = -2$$

$$-z = -5$$

$$z = 5$$

Therefore, the solution to the system is $(x, y, z) = (2, -3, 5)$.

■ 3. Find the unique solution to the system of equations.

$$-2x + 3y - 4z = 10$$

$$4x + 3y + 2z = 4$$

$$x - 6y + 4z = -19$$

Solution:

Let's number the equations to stay organized.

[1] $-2x + 3y - 4z = 10$

[2] $4x + 3y + 2z = 4$

[3] $x - 6y + 4z = -19$

Multiply equation [2] by 2,

$$2(4x + 3y + 2z = 4)$$

$$8x + 6y + 4z = 8$$

and then add this to equation [1] so that z will be eliminated.

$$(8x + 6y + 4z) + (-2x + 3y - 4z) = (8) + (10)$$

$$8x + 6y + 4z - 2x + 3y - 4z = 18$$

$$8x + 6y - 2x + 3y = 18$$

$$6x + 9y = 18$$

[4] $2x + 3y = 6$

Add equation [1] to equation [3] so that z will be eliminated.

$$(-2x + 3y - 4z) + (x - 6y + 4z) = (10) + (-19)$$

$$-2x + 3y - 4z + x - 6y + 4z = -9$$

$$-2x + 3y + x - 6y = -9$$

[5] $-x - 3y = -9$

Solve for x in equation [5].

$$-x - 3y = -9$$

$$-x = -9 + 3y$$

[6] $x = 9 - 3y$

Plug $x = 9 - 3y$ into equation [4] to solve for y.

$$2x + 3y = 6$$

$$2(9 - 3y) + 3y = 6$$

$$18 - 6y + 3y = 6$$

$$18 - 3y = 6$$

$$-3y = -12$$

$$y = 4$$

Let's plug $y = 4$ into equation **[6]** to solve for x.

$$x = 9 - 3y$$

$$x = 9 - 3(4)$$

$$x = 9 - 12$$

$$x = -3$$

Plug $x = -3$ and $y = 4$ into any of the original equations to solve for z. We'll use equation **[2]**.

$$4x + 3y + 2z = 4$$

$$4(-3) + 3(4) + 2z = 4$$

$$-12 + 12 + 2z = 4$$

$$2z = 4$$

$$z = 2$$

Therefore, the solution to the system is $(x, y, z) = (-3,4,2)$.

■ 4. Find the unique solution to the system of equations.

$$2x - y + z = 9$$

$$4x - 2y + 2z = 18$$

$$-2x + y - z = -9$$

Solution:

Let's number the equations to stay organized.

[1] $2x - y + z = 9$

[2] $4x - 2y + 2z = 18$

[3] $-2x + y - z = -9$

Add equation **[1]** to equation **[3]** so that z will be eliminated.

$$(2x - y + z) + (-2x + y - z) = (9) + (-9)$$

$$2x - y + z - 2x + y - z = 0$$

$$-y + z + y - z = 0$$

$$z - z = 0$$

$$0 = 0$$

When all the variables eliminate and we get a true statement, it means all points (x, y, z) are a solution to the system. So far, this is the case with equations **[1]** and **[3]**. If this also happens with equation **[2]**, then the whole system is an "identity" and there are infinite solutions.

Let's check the second equation to see if this is the case. Multiply equation **[3]** by 2,

$$2(-2x + y - z = -9)$$

$$-4x + 2y - 2z = -18$$

and then add it to equation **[2]**.

$$(-4x + 2y - 2z) + (4x - 2y + 2z) = (-18) + (18)$$

$$-4x + 2y - 2z + 4x - 2y + 2z = 0$$

$$2y - 2z - 2y + 2z = 0$$

$$-2z + 2z = 0$$

$$0 = 0$$

Since all the variables eliminate and we get a true statement, the system is an identity and there are infinite solutions.

■ 5. Find the unique solution to the system of equations.

$$x + 2y - z = 9$$

$$3x + y - z = 5$$

$$-x - 4y + z = 2$$

Solution:

Let's number the equations to stay organized.

[1] $x + 2y - z = 9$

[2] $3x + y - z = 5$

[3] $-x - 4y + z = 2$

Add equations [1] and [3] together so that x will be eliminated.

$$(x + 2y - z) + (-x - 4y + z) = (9) + (2)$$

$$x + 2y - z - x - 4y + z = 11$$

$$2y - 4y = 11$$

$$-2y = 11$$

[4] $y = -\dfrac{11}{2}$

Add equation [2] to equation [3] so that z will be eliminated.

$$(3x + y - z) + (-x - 4y + z) = (5) + (2)$$

$$3x + y - z - x - 4y + z = 7$$

$$3x + y - x - 4y = 7$$

[5] $2x - 3y = 7$

Plug equation [4] into equation [5] to solve for x.

$$2x - 3y = 7$$

$$2x - 3\left(-\frac{11}{2}\right) = 7$$

$$2x + \frac{33}{2} = 7$$

$$4x + 33 = 14$$

$$4x = -19$$

$$x = -\frac{19}{4}$$

Plug in $x = -19/4$ and $y = -11/2$ into any of the original equations to solve for z. We'll use equation [3].

$$-x - 4y + z = 2$$

$$-\left(-\frac{19}{4}\right) - 4\left(-\frac{11}{2}\right) + z = 2$$

$$\frac{19}{4} + \frac{44}{2} + z = 2$$

$$19 + 88 + 4z = 8$$

$$4z = -99$$

$$z = -\frac{99}{4}$$

Therefore, the solution to the system is $(x, y, z) = (-19/4, -11/2, -99/4)$.

■ 6. Find the unique solution to the system of equations.

$$-x + y - z = 12$$

$x - y + z = 2$

$2x - 2y + 2z = 9$

Solution:

Let's number the equations to stay organized.

[1] $-x + y - z = 12$

[2] $x - y + z = 2$

[3] $2x - 2y + 2z = 9$

Add equations **[1]** and **[2]** together so that x will be eliminated.

$(-x + y - z) + (x - y + z) = (12) + (2)$

$-x + y - z + x - y + z = 14$

$y - z - y + z = 14$

$-z + z = 14$

[4] $0 = 14$

Since all the variables were eliminated and we're left with a false statement (0 can't equal 14), it means that there's no solution to this system.

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