Adding and subtracting rational functions

In this lesson we will look at how to add and subtract rational functions. In other words, how to add and subtract fractions that have variables in them as well as numbers.

Remember that when we add and subtract fractions, we need a common denominator. The lowest common denominator is the least common multiple of the denominators in the individual fractions.

Let's look at how to find the least common multiple of a group of terms. For instance, if we have the factors $5x$, xy^2, and $5y^4$, we can set up a table to help us organize the factors as individual terms. Put the terms down the far-left column, and a heading for each piece across the top row. Then fill in the table with the factors of each term.

	Coefficients	x's	y's
5x	5	x	
xy^2		x	y^2
$5y^4$	5		y^4

In order to generate the least common multiple, we have to take the least common multiple of the entries in each column, and then form the product of the results.

- The least common multiple of the entries in the coefficients column is 5. (The coefficient in xy^2 is 1, so we're taking the least common multiple of 1 and 5, which is 5.)

- The least common multiple of the entries in the base-x column is x. ($5x$ and xy^2 can be written as $5x^1$ and x^1y^2, respectively, and $5y^4$ can be written as $5x^0y^4$, so we're taking the least common multiple of x^0 and x^1, which is x^1, or x.

- The least common multiple of the entries in the base-y column is y^4. ($5x$ can be written as $5xy^0$, so we're taking the least common multiple of y^0, y^2, and y^4, which is y^4.)

Therefore, the least common multiple of our terms $5x$, xy^2, and $5y^4$ is

$$5xy^4$$

Now let's look at how to apply this idea to the addition and subtraction of rational functions.

Example

Simplify the expression by combining the three rational functions into a single rational function.

$$\frac{y}{5x} + \frac{a}{xy^2} - \frac{c}{5y^4}$$

We need to combine the three fractions in the expression into one fraction, which we'll do by finding a common denominator.

The lowest common denominator will be the least common multiple of the three denominators in

$$\frac{y}{5x}+\frac{a}{xy^2}-\frac{c}{5y^4}$$

The denominators are $5x$, xy^2, and $5y^4$. We found that their least common multiple is $5xy^4$.

Now we need to multiply the numerator and denominator of each fraction by whatever expression is needed to make its denominator equal to $5xy^4$.

$$\frac{y^4}{y^4}\cdot\frac{y}{5x}+\frac{5y^2}{5y^2}\cdot\frac{a}{xy^2}-\frac{x}{x}\cdot\frac{c}{5y^4}$$

$$\frac{y^5}{5xy^4}+\frac{5ay^2}{5xy^4}-\frac{cx}{5xy^4}$$

$$\frac{y^5+5ay^2-cx}{5xy^4}$$

Let’s do another example.

Example

Simplify the expression by combining the three rational functions into a single rational function.

$$\frac{a}{2bc^2}+\frac{m}{3c}+\frac{y}{4bc}$$

We need to combine the three fractions in the expression into one fraction, which we'll do by finding a common denominator.

The lowest common denominator will be the least common multiple of the three denominators in

$$\frac{a}{2bc^2}+\frac{m}{3c}+\frac{y}{4bc}$$

	Coefficients	b's	c's
$2bc^2$	2	b	c^2
3c	3		c
4bc	2*2	b	c

In order to generate the least common multiple, we have to take the least common multiple of the entries in each column, and then form the product of the results.

- The least common multiple of the entries in the coefficients column is 12. (We're taking the least common multiple of 2, 3, and $2 \cdot 2$, which is $2 \cdot 2 \cdot 3$, or 12.)

- The least common multiple of the entries in the base-b column is b. ($2bc^2$ and $4bc$ can be written as $2b^1c^2$ and $4b^1c$, respectively, and $3c$ can be written as $3b^0c$, so we're taking the least common multiple of b^0 and b^1, which is b^1, or b.)

- The least common multiple of the entries in the base-c column is c^2. ($3c$ and $4bc$ can be written as $3c^1$ and $4bc^1$, respectively, so we're taking the least common multiple of c^1 and c^2, which is c^2.)

Therefore, the least common multiple of $2bc^2$, $3c$, and $4bc$ is

$$12bc^2$$

So we need to multiply the numerator and denominator of each fraction by whatever expression is needed to make its denominator equal to

$$12bc^2$$

We get

$$\frac{6}{6}\cdot\frac{a}{2bc^2}+\frac{4bc}{4bc}\cdot\frac{m}{3c}+\frac{3c}{3c}\cdot\frac{y}{4bc}$$

$$\frac{6a}{12bc^2}+\frac{4bcm}{12bc^2}+\frac{3cy}{12bc^2}$$

$$\frac{6a+4bcm+3cy}{12bc^2}$$