The general log rule

The general log rule that we introduced earlier was

Given the equation $a^x = y$, the associated log is $\log_a(y) = x$, and vice versa.

What this tells us is that

$\log_a(y) = x$ and $a^x = y$ are equivalent

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Remember that inverse functions have their x- and y-values swapped. This means that when we graph inverse functions on the same set of axes, the graphs are mirror images of one another, just reflected over the line $y = x$.

We can see that $\log_a(y) = x$ and $\log_a(x) = y$ have their x- and y-values swapped, and that $a^x = y$ and $a^y = x$ have their x- and y-values swapped. Which means that

Both $\log_a(x) = y$ and $a^y = x$ are inverses of $\log_a(y) = x$

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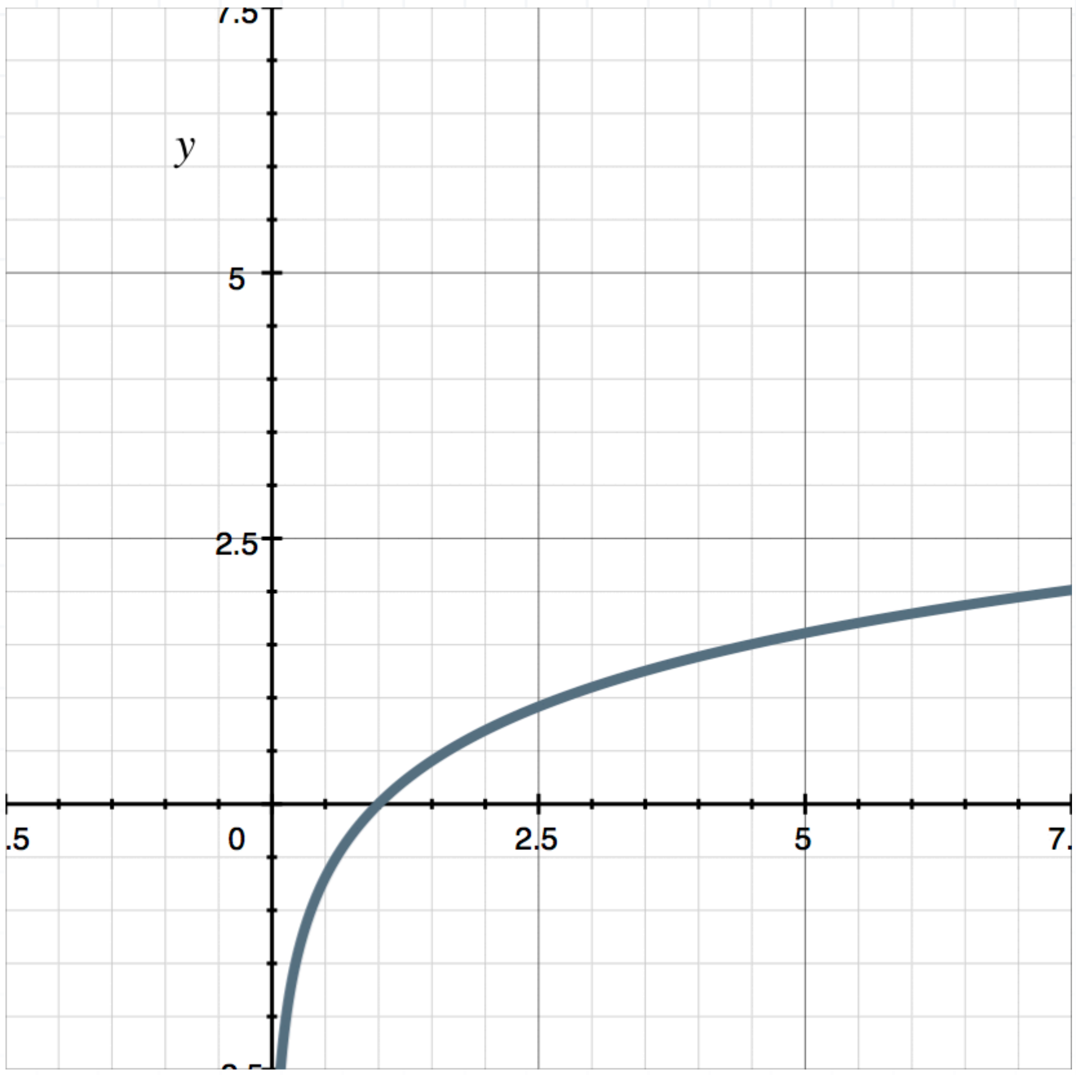
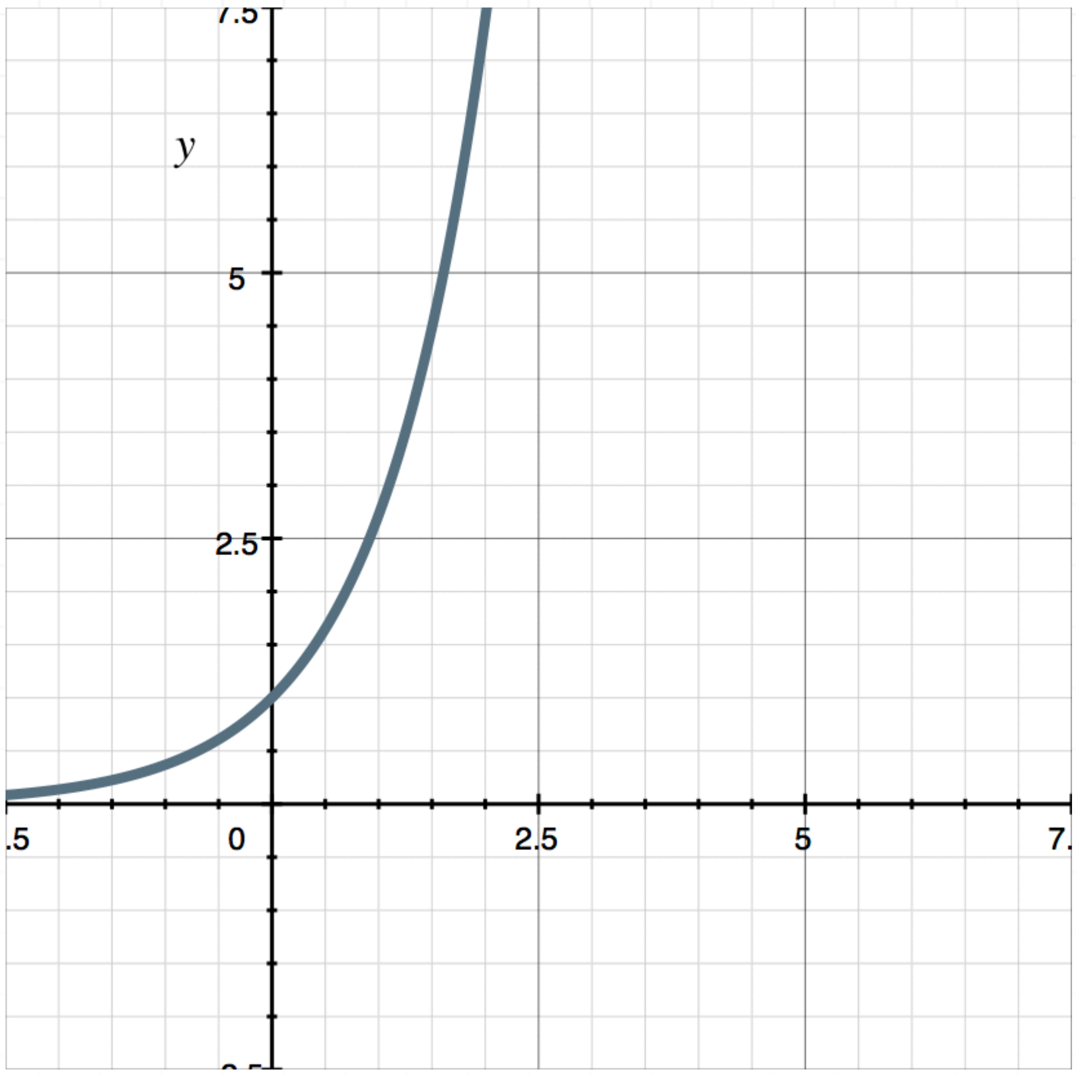
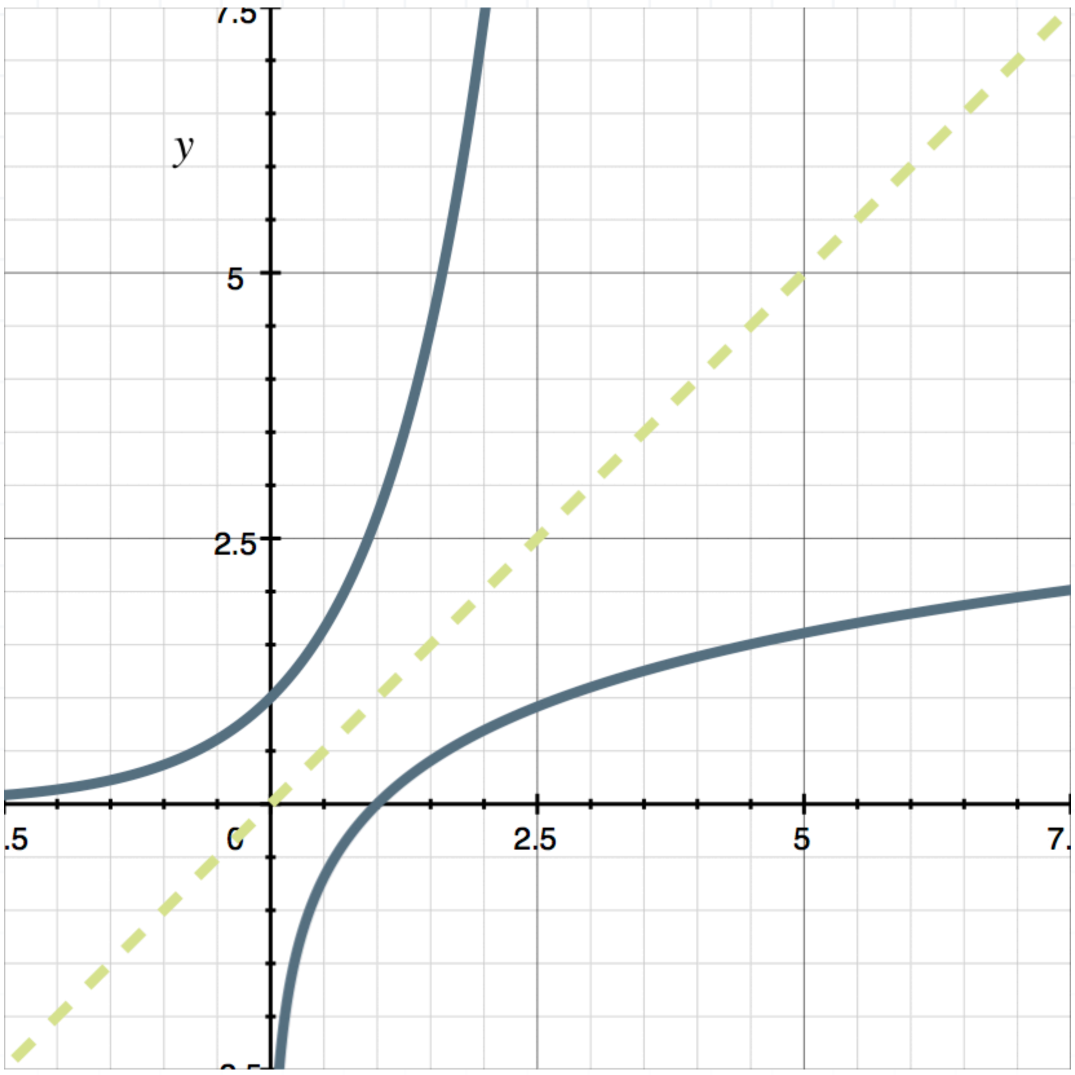
For example, the graph of $\log_a(x) = y$ (or equivalently $a^y = x$) is



And the graph of $\log_a(y) = x$ (or equivalently $a^x = y$) is



And we can see that these are inverses of one another, because they are a reflection of each other over the line $y = x$.



When functions are inverses of one another, we can also express their points in tables. For instance, given the equations $a^x = y$ and $\log_a(x) = y$, we can express points that satisfy each of these equations in tables.

If a point set that satisfies $a^x = y$ is

x	1	2	3	4
$y=2^x$	2	4	8	16

then the point set satisfying its inverse $\log_a(x) = y$ is

x	2	4	8	16
$y=\log_2 x$	1	2	3	4

And if we sketch these points on a graph, we can see again how they are mirror images of one another over the line $y = x$.