Algebra 1 Workbook Solutions

Functions and graphing

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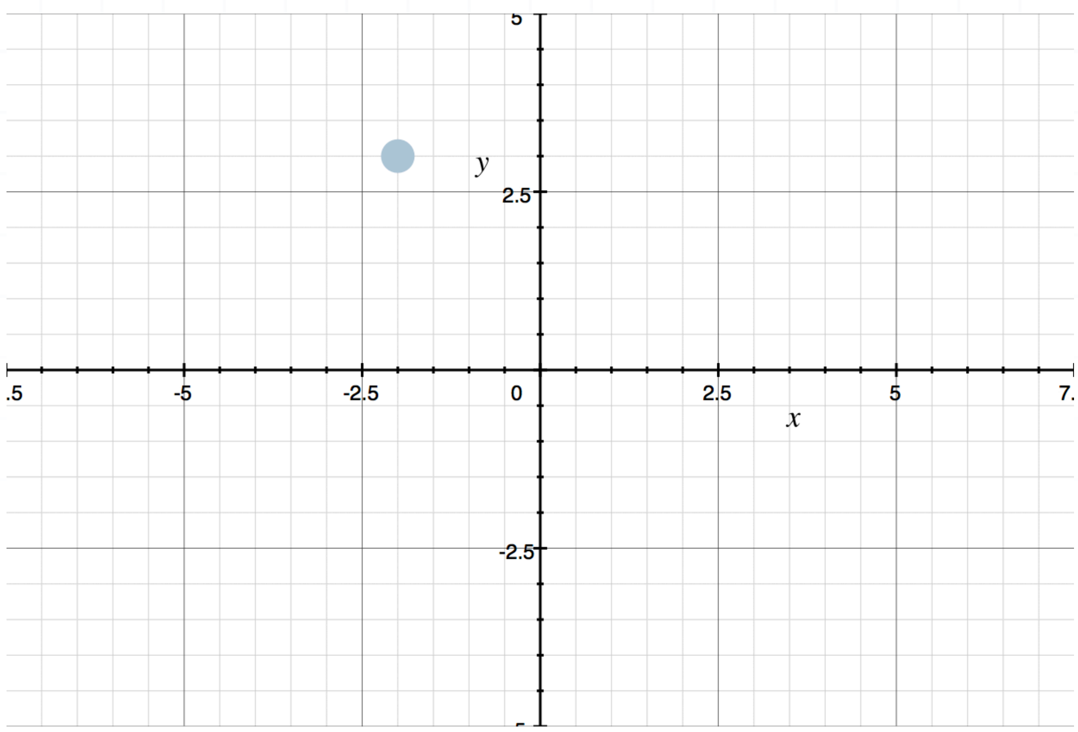
CARTESIAN COORDINATE SYSTEM

1. Graph the point $(-2,3)$ in the Cartesian plane.

Solution:

The graph of the point is



2. In which quadrant should we plot the point $(1,6)$?

Solution:

Since both the x- and the y-coordinates are positive, this point is graphed in Quadrant I.

3. What is the y-coordinate of any point that lies on the x-axis? Give an example of a coordinate point that lies on the x-axis.

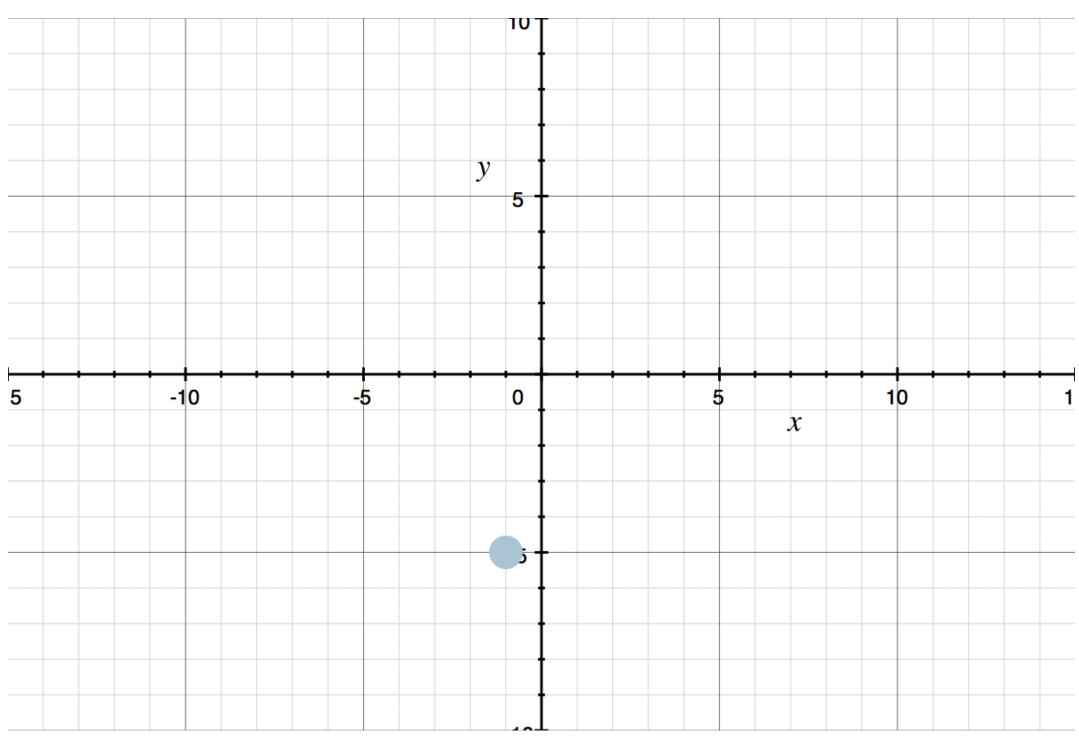
Solution:

The y-coordinate of any point on the x-axis is always $y = 0$. For example, $(3,0)$ is a point on the x-axis.

4. Graph the point $(-1, -5)$ in the Cartesian plane.

Solution:

The graph of the point is



5. In which quadrant should we plot $(3, -7)$?

Solution:

Since the x-coordinate is positive and the y-coordinate is negative, this point is graphed in Quadrant IV.

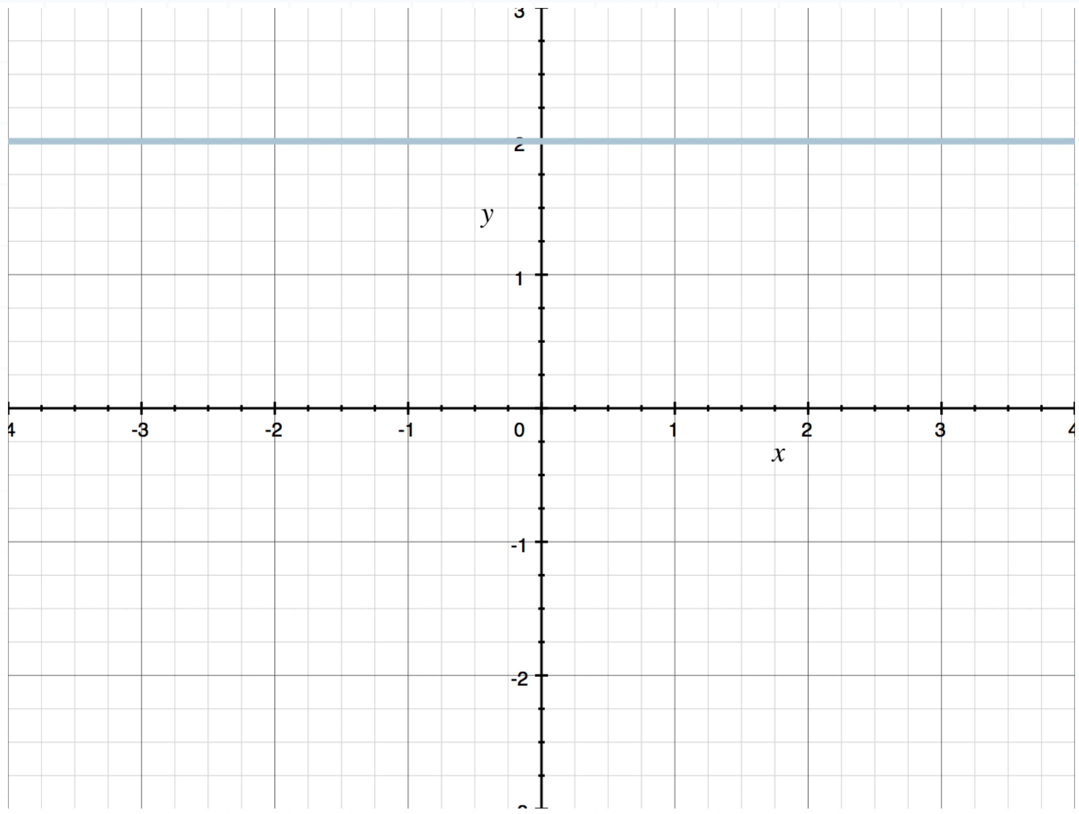
6. What is the x-coordinate of any point that lies on the y-axis? Give an example of a coordinate point that lies on the y-axis.

Solution:

The x-coordinate of any point on the y-axis is always $x = 0$. For example, $(0, -7)$ is a point on the y-axis.

SLOPE

1. What is the slope of the line?



Solution:

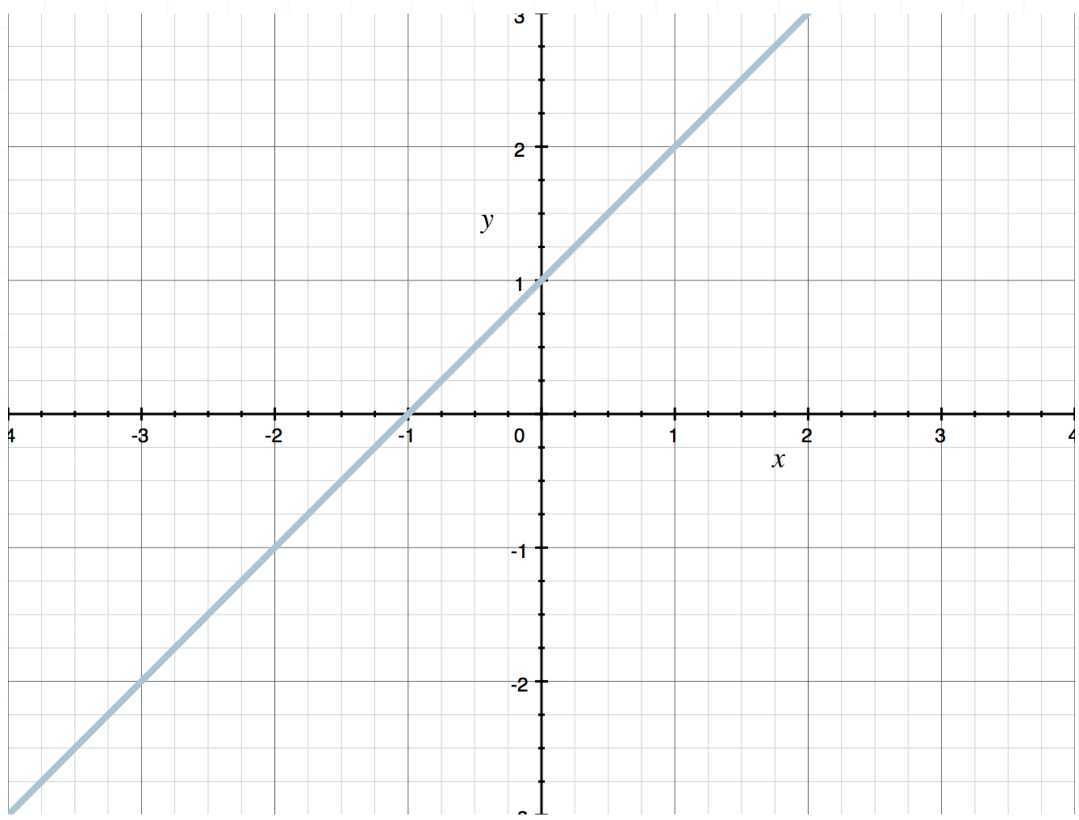
Since the line is a horizontal line, the slope is 0.

2. What direction is an undefined slope: horizontal or vertical? Use the formula for the slope to explain why.

Solution:

The direction of an undefined slope is vertical. It is because the change in x of a vertical line is 0, so the slope has a 0 in the denominator and is therefore undefined.

3. What is the slope of the line?



Solution:

Notice that the graph passes through the points $(-1,0)$ and $(0,1)$, which means the slope can be defined as

$$m = \frac{1-0}{0-(-1)}$$

$$m = \frac{1}{1}$$

$$m = 1$$

4. What is the slope of the line that passes through the points $(-1,3)$ and $(4,-7)$?

Solution:

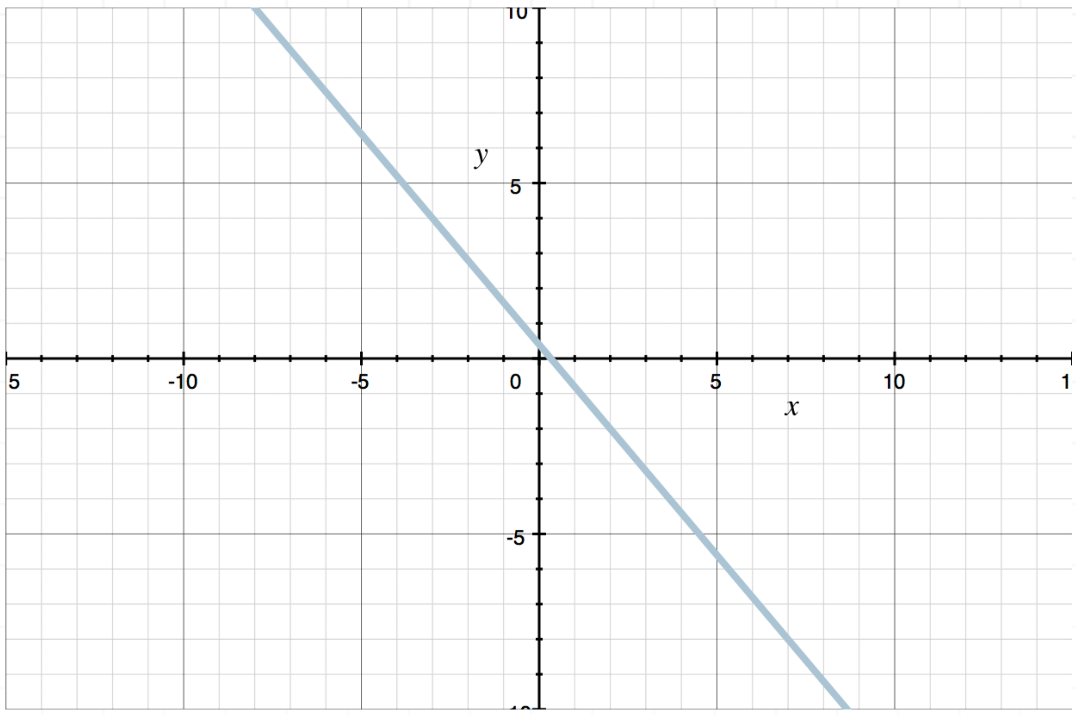
The graph passes through the points $(-1,3)$ and $(4,-7)$, so the slope is defined as

$$m = \frac{-7-3}{4-(-1)}$$

$$m = \frac{-10}{5}$$

$$m = -2$$

5. What is the slope of the line?



Solution:

Notice that the graph passes through the points $(-3,4)$ and $(2,-2)$, so the slope is

$$m = \frac{-2-4}{2-(-3)}$$

$$m = \frac{-6}{5}$$

$$m = -\frac{6}{5}$$

■ 6. Find the slope of the line that passes through $(3,5)$ and $(-1,5)$.

Solution:

The graph passes through the points $(3,5)$ and $(-1,5)$, so the slope is defined as

$$m = \frac{5-5}{-1-3}$$

$$m = \frac{0}{-4}$$

$$m = 0$$

POINT-SLOPE AND SLOPE-INTERCEPT FORMS OF A LINE

1. Find the equation of the line that passes through $(3,0)$ with slope -2.

Solution:

Using point-slope form, the equation of the line is

$$y - 0 = -2(x - 3)$$

2. Find the equation of the line that passes through the points $(-2,3)$ and $(2,-4)$.

Solution:

We first need to calculate the slope of the line as follows

$$m = \frac{-4 - 3}{2 - (-2)}$$

$$m = \frac{-7}{4}$$

$$m = -\frac{7}{4}$$

Using point-slope form, the equation of the line is either of the following:

$$y - 3 = -\frac{7}{4}(x + 2)$$

$$y + 4 = -\frac{7}{4}(x - 2)$$

■ 3. Find the equation of the line that passes through the points $(5, -4)$ and $(6,0)$.

Solution:

We first need to calculate the slope of the line as

$$m = \frac{0 - (-4)}{6 - 5}$$

$$m = \frac{4}{1}$$

$$m = 4$$

Using point-slope form, the equation of the line is then either of the following:

$$y + 4 = 4(x - 5)$$

$$y = 4(x - 6)$$

■ 4. Identify the y-intercept and slope m defining the line.

$$y = -\frac{1}{4}(x + 12)$$

Solution:

We need to rewrite the equation in slope-intercept form, $y = mx + b$.

$$y = -\frac{1}{4}(x + 12)$$

$$y = -\frac{1}{4}x - 3$$

Notice that the slope of the line given is $-1/4$ and the y-intercept (when $x = 0$) is $(0, -3)$.

5. Convert the point-slope equation into a slope-intercept equation.

$$y - 3 = \frac{1}{3}(x - 6)$$

Solution:

Converting to slope-intercept form means that we need to solve for y, and simplify as much as we can.

$$y - 3 = \frac{1}{3}(x - 6)$$

$$y - 3 = \frac{1}{3}x - 2$$

$$y = \frac{1}{3}x - 2 + 3$$

$$y = \frac{1}{3}x + 1$$

6. Find the equation of a line that passes through the points $(1, -1)$ and $(0,3)$. Write the solution in slope-intercept form.

Solution:

We first need to calculate the slope of the line as

$$m = \frac{3 - (-1)}{0 - 1}$$

$$m = \frac{4}{-1}$$

$$m = -4$$

Using slope-intercept form, noting that the y-intercept is 3, the equation of the line is

$$y = -4x + 3$$

GRAPHING LINEAR EQUATIONS

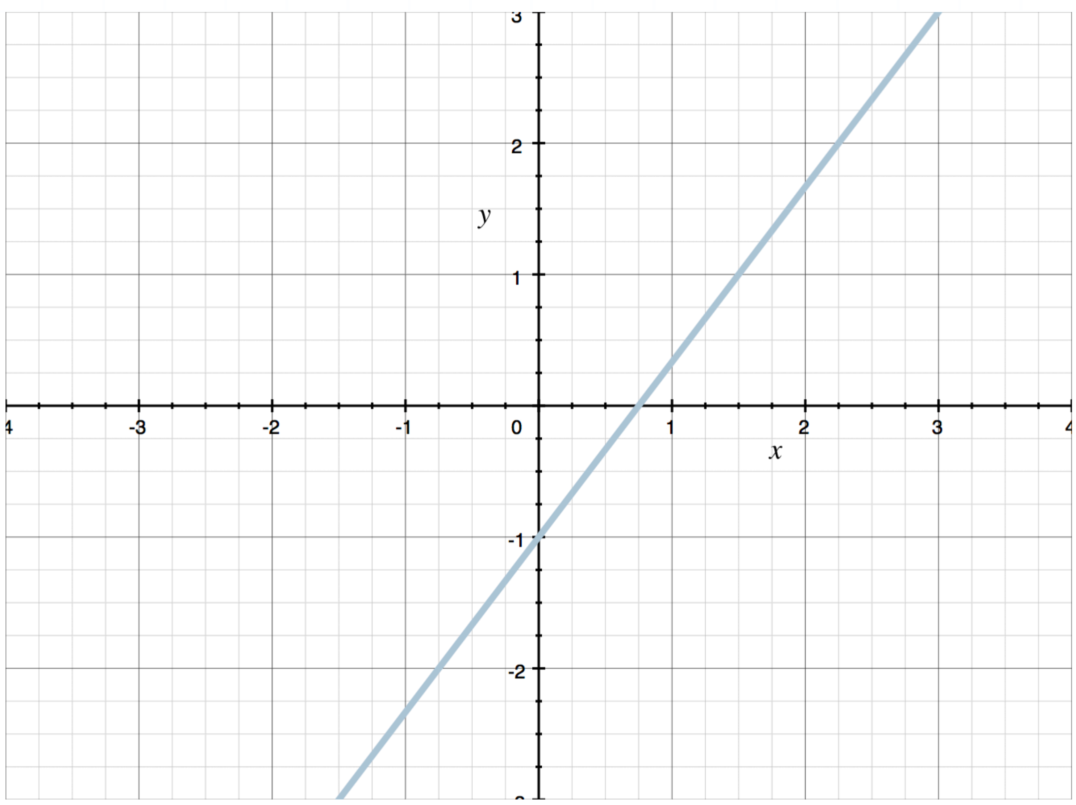
1. Graph the line.

$$y = \frac{4}{3}x - 1$$

Solution:

The linear equation is already in slope-intercept form, so we can see that the slope is $m = 4/3$ and the y-intercept is $b = -1$.

Since the slope is positive, we know that the line will lean to the right. The graph of the line is



2. Describe how we would use the slope to find another point on the line if the slope is $m = 2/3$ and the line passes through $(x_1, y_1) = (-1,2)$.

Solution:

Starting at the point $(-1,2)$, move up 2 and to the right 3 to get the point $(2,4)$, or move down 2 and to the left 3 to get the point $(-4,0)$.

3. Graph the line.

$$y + 2 = -3x + 1$$

Solution:

The linear equation isn't already in slope-intercept form, so we'll subtract 2 from both sides in order to solve for y.

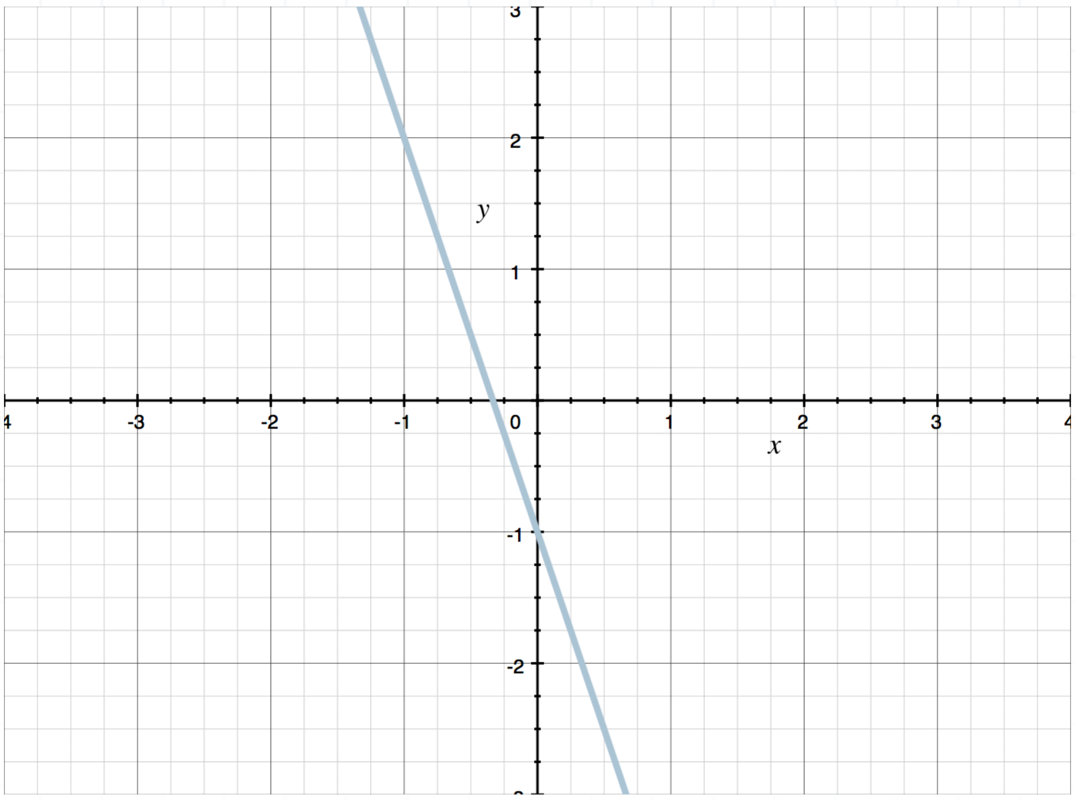
$$y + 2 = -3x + 1$$

$$y + 2 - 2 = -3x + 1 - 2$$

$$y = -3x - 1$$

With the equation in slope-intercept form, we can identify that the slope is $m = -3$ and the y-intercept is -1.

Since the slope is negative, we know that the line will lean to the left. The graph of the line is



4. Use the slope $m = 1/3$ to find two more points on the line passing through $(1,2)$. Move right to determine one point and left to determine another.

Solution:

Going right, we get the point $(4,3)$. Going left, we get the point $(-2,1)$.

5. Graph the line.

$$y = -2(3x + 1)$$

Solution:

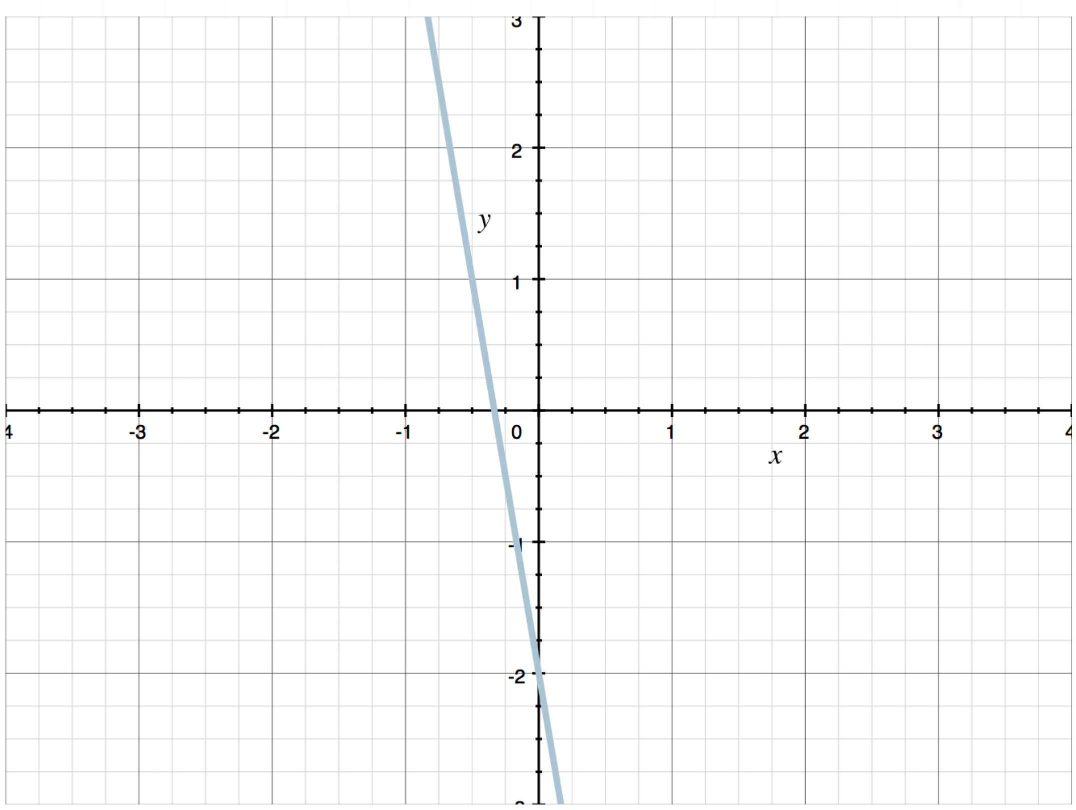
The linear equation isn't already in slope-intercept form, so we'll distribute the -2 across the parentheses.

$$y = -2(3x + 1)$$

$$y = -6x - 2$$

With the equation in slope-intercept form, we can identify that the slope is $m = -6$ and the y-intercept is -2.

Since the slope is negative, we know that the line will lean to the left. The graph of the line is



6. Give two points that lie on the line, find the slope, and graph the line.

$$y + 3 = -\frac{1}{2}(4x + 10)$$

Solution:

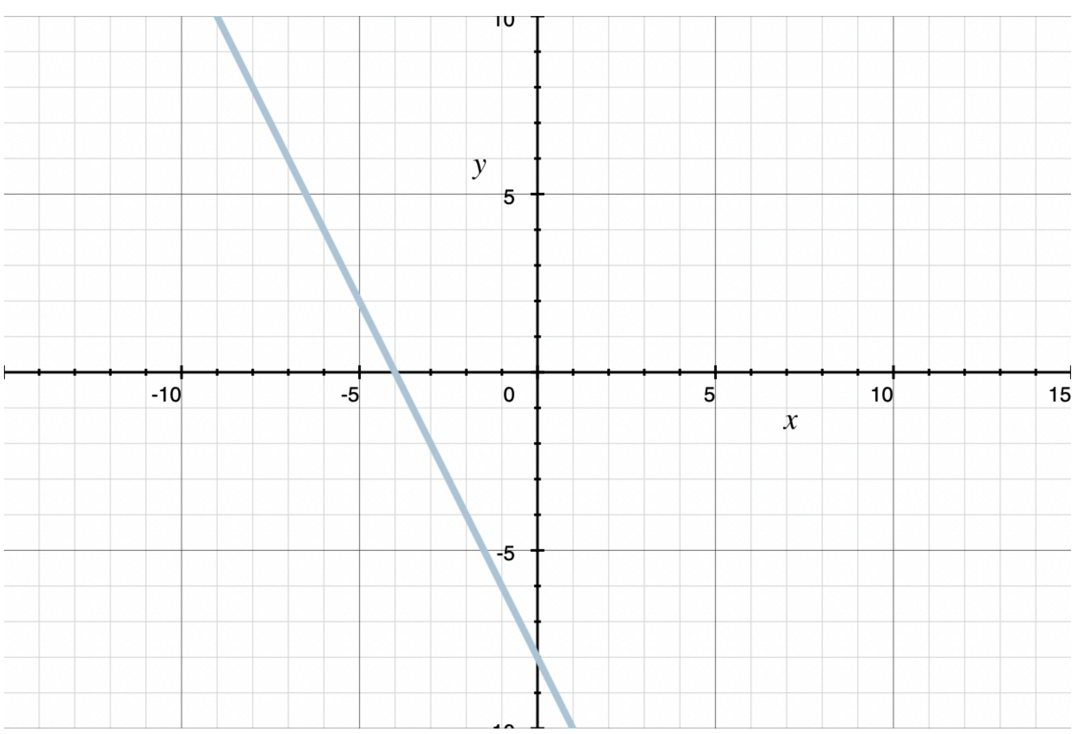
The linear equation isn't already in slope-intercept form, so we'll rewrite it.

$$y + 3 = -\frac{1}{2}(4x + 10)$$

$$y + 3 = -2x - 5$$

$$y = -2x - 8$$

With the equation in slope-intercept form, we can identify that the slope is $m = -2$ and the y-intercept is -8. There are an infinite number of points on the line, like $(0, -8)$ and $(-1, -6)$. The graph of the line is



FUNCTION NOTATION

1. Find and simplify $f(x+1)$ if $f(x)=4x-5$.

Solution:

To find $f(x+1)$, we plug $x+1$ into $f(x)$ in place of x.

$$f(x+1)=4(x+1)-5$$

$$f(x+1)=4x+4-5$$

$$f(x+1)=4x-1$$

2. What went wrong in this set of steps?

Evaluate $f(x)=x^2+1$ at $x=-2$.

$$f(-2)=-2^2+1$$

$$f(-2)=-4+1$$

$$f(-2)=-3$$

Solution:

To find $f(-2)$, we need to plug $x = -2$ into $f(x)$. But we needed to wrap the -2 in parentheses, to force the exponent to apply to the negative sign, not just to the 2. So the evaluation should have been

$$f(-2) = (-2)^2 + 1$$

$$f(-2) = 4 + 1$$

$$f(-2) = 5$$

■ 3. Find and simplify $h(s^2)$ if $h(s) = -s^2 + 3s - 1$.

Solution:

To find $h(s^2)$, we plug s^2 into $h(s)$ in place of s.

$$h(s^2) = -(s^2)^2 + 3(s^2) - 1$$

$$h(s^2) = -s^4 + 3s^2 - 1$$

■ 4. If $g(x) = x^3 - x + 1$, what do we need to plug into the function in order to get the following expression?

$$g(??) = (2x + 1)^3 - (2x + 1) + 1$$

Solution:

Notice that everywhere there's an x in $g(x)$, there's a $2x+1$ in the new function. Therefore, the value that got plugged in must have been $2x+1$.

5. Find the value of the expression if $f(x)=x^2+x-1$.

$$\frac{f(x+h)-f(x)}{h}$$

Solution:

To find $f(x+h)$, we plug $x+h$ into $f(x)$ in place of x.

$$f(x+h)=(x+h)^2+(x+h)-1$$

$$f(x+h)=x^2+2hx+h^2+x+h-1$$

Now substitute and simplify.

$$\frac{f(x+h)-f(x)}{h}=\frac{x^2+2hx+h^2+x+h-1-(x^2+x-1)}{h}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{x^2+2hx+h^2+x+h-1-x^2-x+1}{h}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{2hx+h^2+h}{h}$$

$$\frac{f(x+h)-f(x)}{h}=2x+h+1$$

6. What went wrong in this set of steps?

Find $f(1)$ if $f(x)=x^3+3x^2-5x+2$.

$$f(1)=1^3+3(1)^2-5(1)+2$$

$$f(1)=1+9-5+2$$

$$f(1)=7$$

Solution:

When evaluating $f(1)$, notice that instead of evaluating $3(1)^2$ as $3(1)^2=3(1)=3$, the steps show $3(1)^2=9$. However, the 3 is not being squared, so that's incorrect. The steps should have been

$$f(1)=1^3+3(1)^2-5(1)+2$$

$$f(1)=1+3-5+2$$

$$f(1)=1$$

DOMAIN AND RANGE

1. Find the domain of $f(x)$.

$$f(x)=\frac{3}{x(x+1)}+x^2$$

Solution:

In this function, the denominator can't be 0. The values of x that make the denominator 0 are $x=0$ and $x=-1$. So the domain of the function is all $x\neq 0,-1$.

2. Find the domain and range of the point set.

$$(-1,-3),\quad (0,5),\quad (-3,6),\quad (0,-3)$$

Solution:

The domain is all the x-values and the range is all the y-values. Therefore the domain and range are

Domain: $-3,\ -1,0$

Range: $-3,5,6$

3. Find the domain and range of $g(x)$.

$$g(x) = \frac{\sqrt{x-2}}{3}$$

Solution:

In this function, the radicand (the expression under the square root) must be 0 or positive. So $x - 2 \geq 0$ is the domain. Since the square root function can't be negative, the numerator is guaranteed to be positive or zero, while the denominator is guaranteed to be positive. Since a positive divided by a positive is a positive, and zero divided by a positive is zero, the range is $g(x) \geq 0$.

4. Find the domain and range of the function.

$$f(x) = \frac{2}{x} + 1$$

Solution:

In this function, the denominator can't be 0, which means $x \neq 0$. Therefore the domain of the function is $x \neq 0$.

Since $2/x$ will never be 0, $f(x)$ can never be 1. Therefore the range of the function is $f(x) \neq 1$.

5. Find the domain and range of $g(x)$.

$$g(x) = -x^2 + 5$$

Solution:

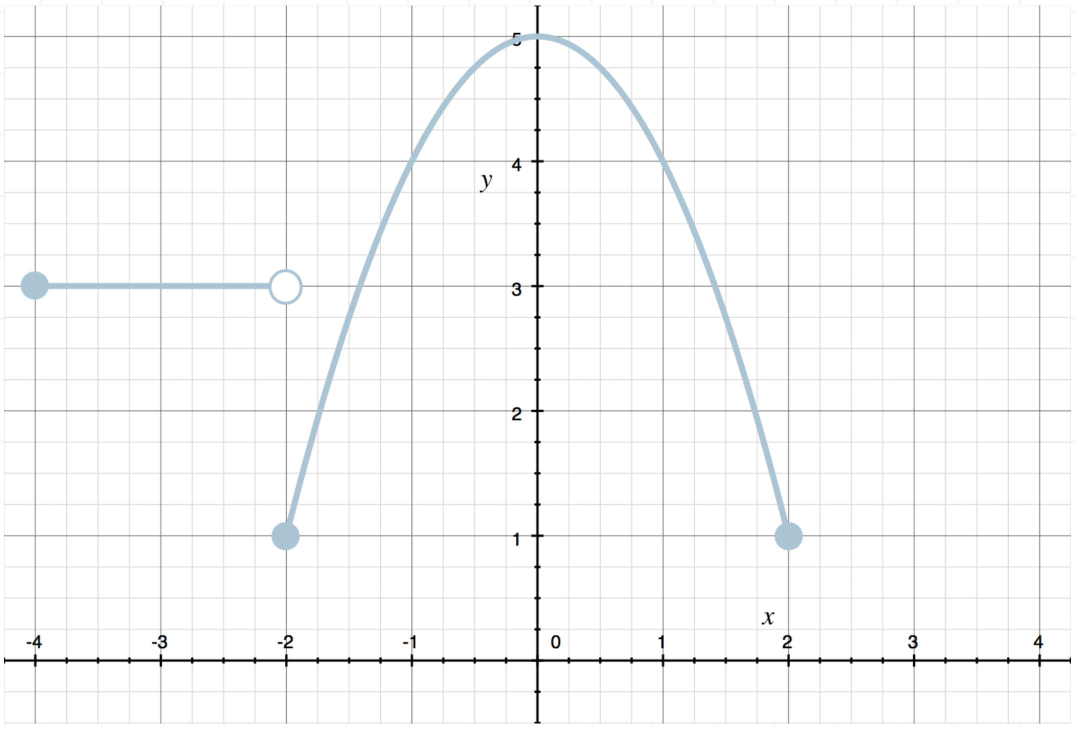
There is no real number that makes the expression undefined. So, the domain of the function is all real numbers.

The range is the set of all valid y values. If we switch the order of the terms to rewrite the function as

$$g(x) = 5 - x^2$$

we can see that we'll always be subtracting zero or a positive number from 5. Since x^2 will never be negative, it can only take on a zero value, in which case we'll get $f(x) = 5 - 0 = 5$, or a positive value, in which case we'll find some value less than 5. So the range is $g(x) \leq 5$.

6. What is the domain and range of the graph? Hint: An empty circle indicates that exact point *is not* included as part of the graph, while a solid circle indicates that exact point *is* included as part of the graph.



Solution:

The domain of the graph is determined by the x-values, which are defined between $x = -4$ and $x = 2$. The solid circles indicate that $x = -4$ and $x = 2$ are included in the graph. There's an empty circle at $x = -2$, but also a solid circle at the same value of x, which means $x = -2$ is also included in the domain. Therefore, the domain is $-4 \leq x \leq 2$.

The range is determined by the y-values, which are defined between $y = 1$ and $y = 5$. The solid circles indicate that $y = 1$ is included in the graph. Therefore, the range is $1 \leq y \leq 5$.

TESTING FOR FUNCTIONS

1. Determine whether or not the point set represents a function.

$$(2,-1), (-1,0), (0,-1), (3,2)$$

Solution:

For every x-value, there is only one y-value, so the set of points represents a function. The x-values are -1, 0, 2, 3 and the y-values are -1, 0, 2. Even though both $x=2$ and $x=0$ are mapping to $y=-1$, the point set is still a function.

The point set would not be a function if we found two different y-values mapping to the same x-value.

2. Fill in the blanks in the definition of a function.

For every __________, there is only one unique __________.

Solution:

x (or input), y (or output)

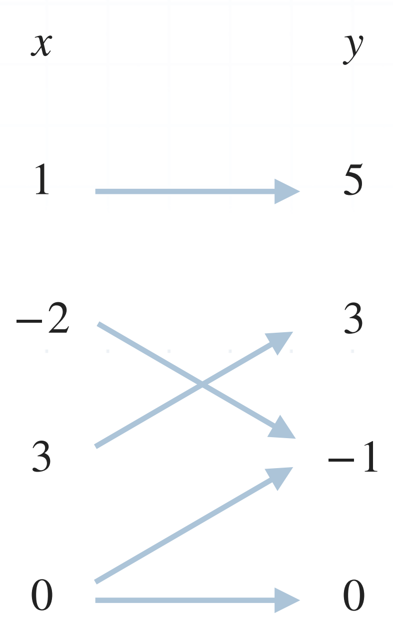
3. Determine whether or not the point set represents a function.

$$(1,2),\ (-1,5),\ (1,-3),\ (0,1)$$

Solution:

Notice that for $x = 1$, there are two y-values, $y = 2$ and $y = -3$. Since for one input there are two different outputs, the point set doesn't represent a function.

4. Determine whether the mapping represents a function.



Solution:

Notice that for $x = 0$, there are two y-values: $y = 0$ and $y = -1$. Since for one input there are two different outputs, this doesn't represent a function.

5. Determine algebraically whether or not the equation represents a function.

$$(x-1)^2+y=3$$

Solution:

Solve the equation for y.

$$(x-1)^2+y=3$$

$$y=3-(x-1)^2$$

Simplify the right side.

$$y=3-(x^2-2x+1)$$

$$y=3-x^2+2x-1$$

$$y=-x^2+2x+2$$

Each value we plug in for x will give a unique value for y, so the equation represents a function.

6. Determine algebraically whether or not the equation represents a function.

$$y^2=x+1$$

Solution:

Solve the equation for y.

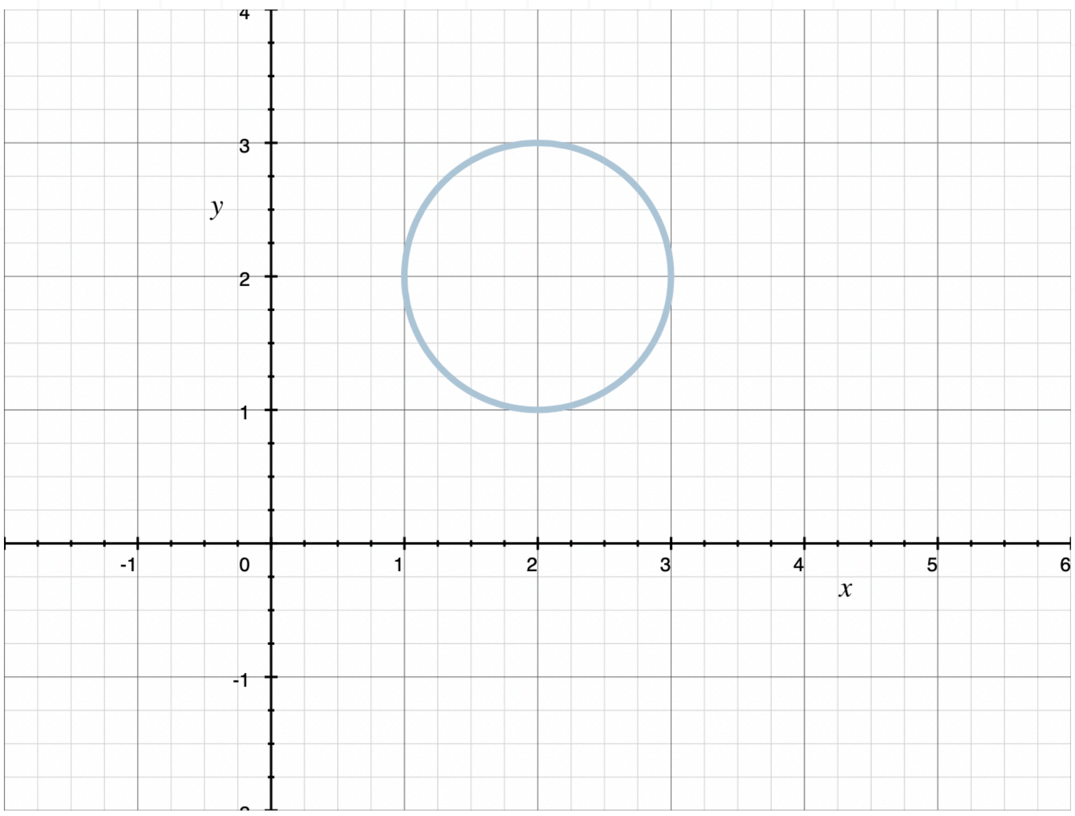
$$y^2 = x + 1$$

$$y = \pm \sqrt{x + 1}$$

Given this equation for y, there are values of x that will give multiple values for y. For instance, at $x = 0$ y takes on the values $y = -1$ and $y = 1$. So for one input there are two outputs and the equation doesn't represent a function.

VERTICAL LINE TEST

1. Use the Vertical Line Test to determine whether or not the graph is the graph of a function.

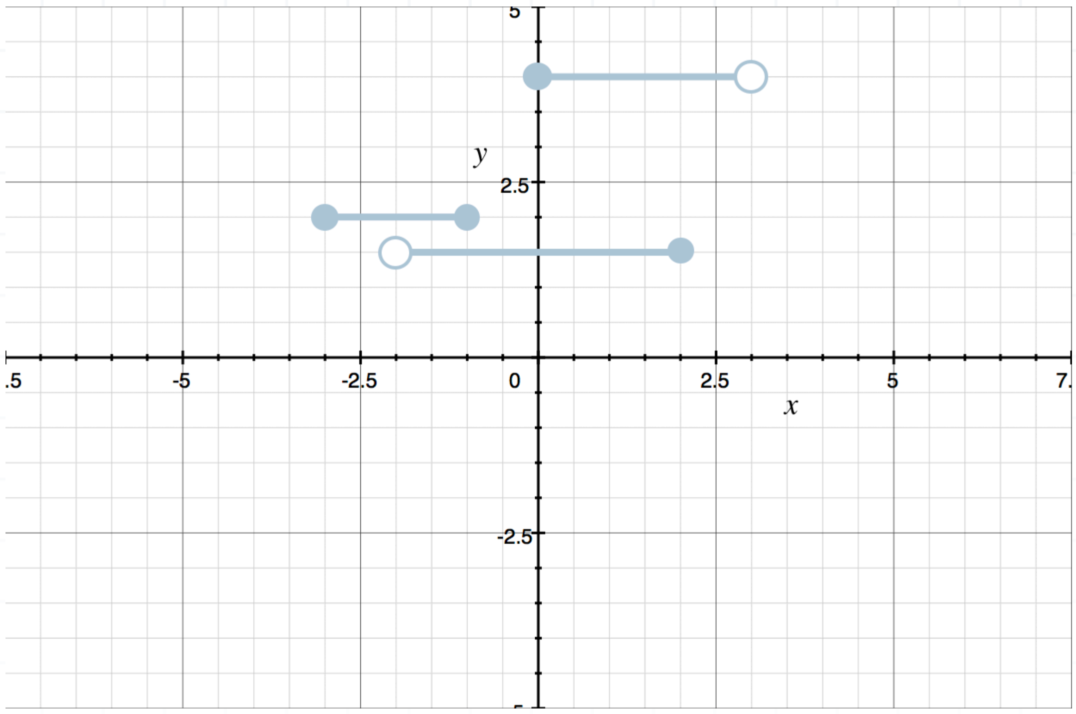


Solution:

The graph does not pass the Vertical Line Test, because any vertical line between the left edge of the circle and the right edge of the circle intersects the graph more than once. Therefore, the graph doesn't represent a function.

2. Use the Vertical Line Test to determine whether or not the graph represents a function. Hint: an empty circle indicates that exact point isn't

included in the graph, where a solid circle indicates that exact point is included in the graph.



Solution:

There are different vertical lines that intersect the graph more than once. An example would be $x = 0$, which intersects the graph at $y = 3/2$ and $y = 4$. So by the Vertical Line Test, the graph is not a graph of a function.

■ 3. Explain why the Vertical Line Test can determine whether or not a graph represents a function.

Solution:

There are many correct answers. But they should all more or less say something like:

> "The Vertical Line Test can show whether or not a graph represents a function, because if any perfectly vertical line crosses the graph more than once, it proves that there are two output values of y for the one input value of x."

4. Fill in the blanks using the words "equations" and "functions."

Not all __________ are __________.

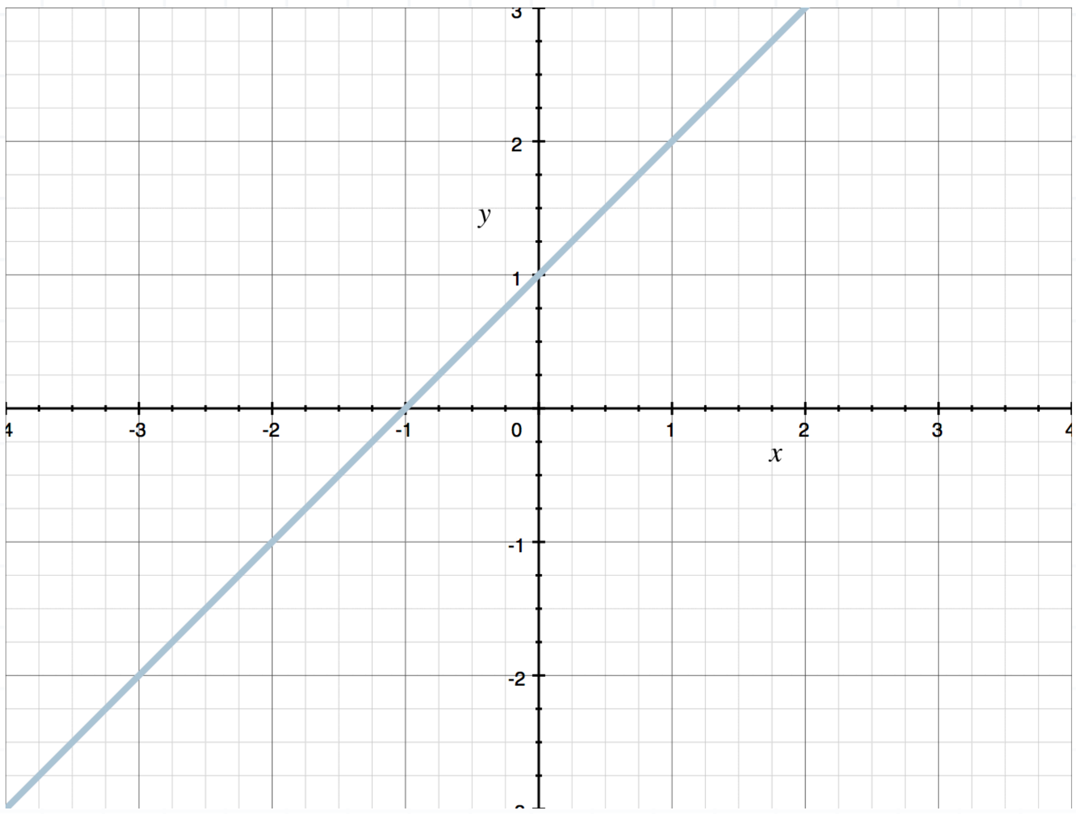
Solution:

The statement should read "Not all equations are functions."

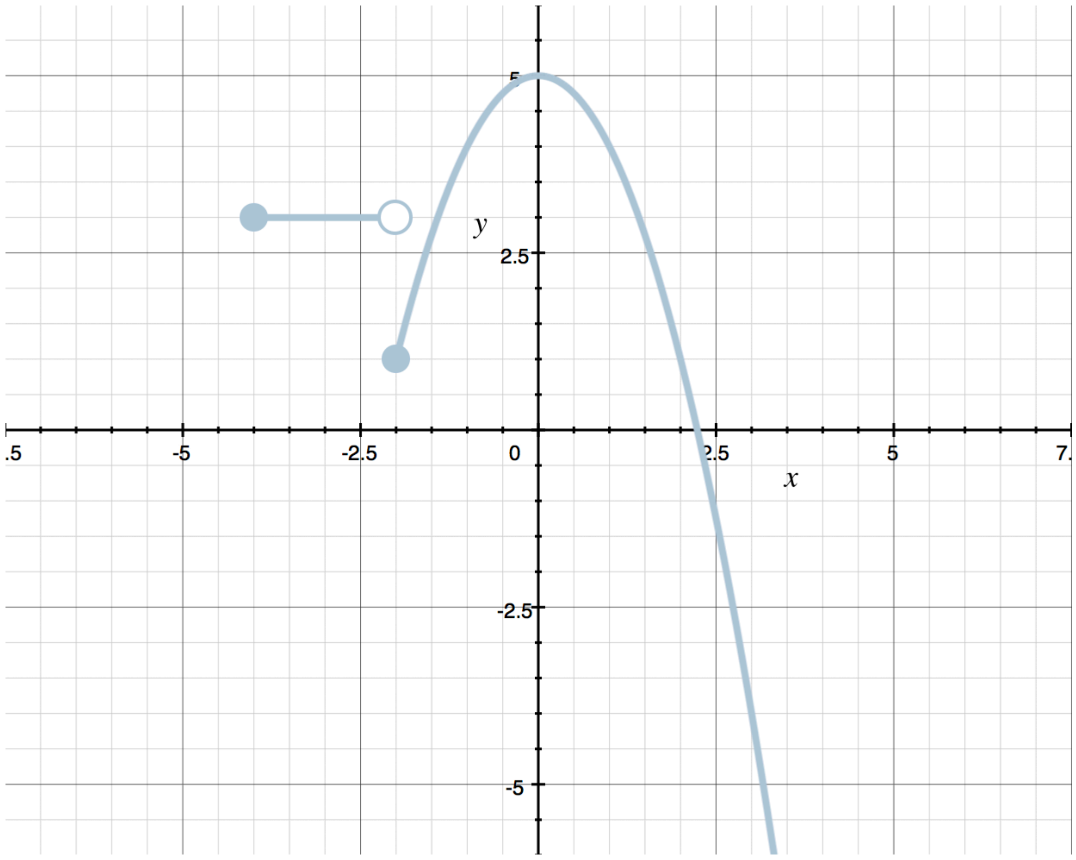
5. Draw a graph that represents a function, and explain why it's a function.

Solution:

There are many correct answers. Below is an example of a function because, for every input x, there is only one output y. In fact, every line that's not a perfectly vertical line will always be a function.



6. Determine whether or not the graph represents a function. Hint: an empty circle indicates that exact point isn't included in the graph, where a solid circle indicates that exact point is included in the graph.



Solution:

For every x-value, there's a single unique y-value. So this graph represents a function. At $x = -2$, it looks like the graph might include two different y-values. But the empty circle at $y = 3$ indicates that point is not included in the graph, which means the input $x = -2$ only produces the one output at $y = 1$.

SUM OF FUNCTIONS

1. Find $(f+h)(-1)$ if $f(x)=x^2+1$ and $h(x)=2x-2$.

Solution:

Find $(f+h)(x)$.

$$(f+h)(x)=(x^2+1)+(2x-2)$$

$$(f+h)(x)=x^2+2x-1$$

To find $(f+h)(-1)$, we plug $x=-1$ into $(f+h)(x)$.

$$(f+h)(-1)=(-1)^2+2(-1)-1$$

$$(f+h)(-1)=1-2-1$$

$$(f+h)(-1)=-2$$

2. Find and simplify $(h+g)(x)$ if $g(x)=x^2+3x-1$ and $h(x)=-2x^2+4x-5$.

Solution:

To find $(h+g)(x)$, we'll start by finding $h(x)+g(x)$.

$$(h+g)(x)=h(x)+g(x)=(-2x^2+4x-5)+(x^2+3x-1)$$

which simplifies as

$$(h+g)(x) = -2x^2 + 4x - 5 + x^2 + 3x - 1$$

$$(h+g)(x) = -x^2 + 7x - 6$$

■ 3. If $f(-2) = 6$, $g(-2) = -3$, and $h(-2) = 4$, find $(f+g+h)(-2)$.

Solution:

By the definition of the sum of functions, we get

$$(f+g+h)(-2) = f(-2) + g(-2) + h(-2)$$

$$(f+g+h)(-2) = 6 + (-3) + 4$$

$$(f+g+h)(-2) = 7$$

■ 4. Find $f(x)$ and $g(x)$.

$$(f+g)(x) = (-x^2 + 3x + 2) + (x - 7)$$

Solution:

By the definition of the sum of functions, we can see that

$f(x) = -x^2 + 3x + 2$ and $g(x) = x - 7$

It could also be correct to say that

$$g(x) = -x^2 + 3x + 2 \text{ and } f(x) = x - 7$$

■ 5. Let $a(x) = x^3 - x^2 + x - 1$ and $b(x) = -x^3 + x^2 + x - 1$. Determine the value of $(a + b)(-1)$.

Solution:

First, we can find the values for $a(-1)$ and $b(-1)$.

$$a(-1) = (-1)^3 - (-1)^2 + (-1) - 1$$

$$a(-1) = -1 - 1 - 1 - 1$$

$$a(-1) = -4$$

and

$$b(-1) = -(-1)^3 + (-1)^2 + (-1) - 1$$

$$b(-1) = 1 + 1 - 1 - 1$$

$$b(-1) = 0$$

Therefore,

$$(a + b)(-1) = a(-1) + b(-1)$$

$$(a + b)(-1) = -4 + 0$$

$$(a+b)(-1) = -4$$

6. If $f(0) = 3$ and $(f+g)(0) = 8$, find $g(0)$.

Solution:

By the definition of the sum of functions, we get

$$(f+g)(0) = f(0) + g(0)$$

$$(f+g)(0) = 3 + g(0)$$

Since $(f+g)(0) = 8$, we get

$$8 = 3 + g(0)$$

$$g(0) = 5$$

PRODUCT OF FUNCTIONS

1. Find and simplify $(ab)(x)$ if $a(x) = x + 3$ and $b(x) = 5x - 4$.

Solution:

By the definition of the product of two functions, we have

$$(ab)(x) = a(x)b(x)$$

$$(ab)(x) = (x + 3)(5x - 4)$$

$$(ab)(x) = 5x^2 + 15x - 4x - 12$$

$$(ab)(x) = 5x^2 + 11x - 12$$

2. Find $(fg)(-1)$ if $f(x) = x^2 + 3$ and $g(x) = x - 5$.

Solution:

We'll find $f(-1)$ and $g(-1)$.

$$f(-1) = (-1)^2 + 3 = 1 + 3 = 4$$

$$g(-1) = (-1) - 5 = -6$$

Then the product of these functions is

$$(fg)(-1) = f(-1)g(-1)$$

$$(fg)(-1) = (4)(-6)$$

$$(fg)(-1) = -24$$

3. If $g(0) = -2$ and $(gh)(0) = -14$, find $h(0)$.

Solution:

From the definition of the product of functions, we have

$$-14 = (gh)(0)$$

$$-14 = g(0)h(0)$$

$$-14 = (-2)h(0)$$

$$h(0) = 7$$

4. Given the expanded expression, determine $f(x)$ and $g(x)$.

$$(gf)(x) = x^2(x-7) - x(x-7) + 5(x-7)$$

Solution:

Factor the $(x-7)$ out of the expression.

$$(gf)(x) = (x-7)(x^2-x+5)$$

Then the two functions are $f(x) = (x-7)$ and $g(x) = x^2 - x + 5$. We could also define the functions as $g(x) = (x-7)$ and $f(x) = x^2 - x + 5$.

5. Find $(fh)(5)$ if $f(x) = -x^2 + 2x$ and $h(x) = 2x + 7$.

Solution:

By the definition of the product of functions, we have

$$(fh)(x) = f(x)h(x)$$

$$(fh)(x) = (-x^2+2x)(2x+7)$$

$$(fh)(x) = -2x^3 + 4x^2 - 7x^2 + 14x$$

$$(fh)(x) = -2x^3 - 3x^2 + 14x$$

Evaluating the product at $x = 5$ gives

$$(fh)(5) = -2(5)^3 - 3(5)^2 + 14(5)$$

$$(fh)(5) = -250 - 75 + 70$$

$$(fh)(5) = -255$$

6. Find and simplify $(gh)(x)$ if $g(x) = x^2 + 1$ and $h(x) = 2x^2 + 3$.

Solution:

By the definition of the product of functions, we have

$$(gh)(x) = g(x)h(x)$$

$$(gh)(x) = (x^2 + 1)(2x^2 + 3)$$

$$(gh)(x) = 2x^4 + 3x^2 + 2x^2 + 3$$

$$(gh)(x) = 2x^4 + 5x^2 + 3$$

EVEN, ODD, OR NEITHER

1. Is the function even, odd, or neither?

$$f(x) = -x^5 + 2x^2 - 1$$

Solution:

Substitute $-x$ for x.

$$f(-x) = -(-x)^5 + 2(-x)^2 - 1$$

$$f(-x) = x^5 + 2x^2 - 1$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(-x^5 + 2x^2 - 1) = x^5 - 2x^2 + 1$$

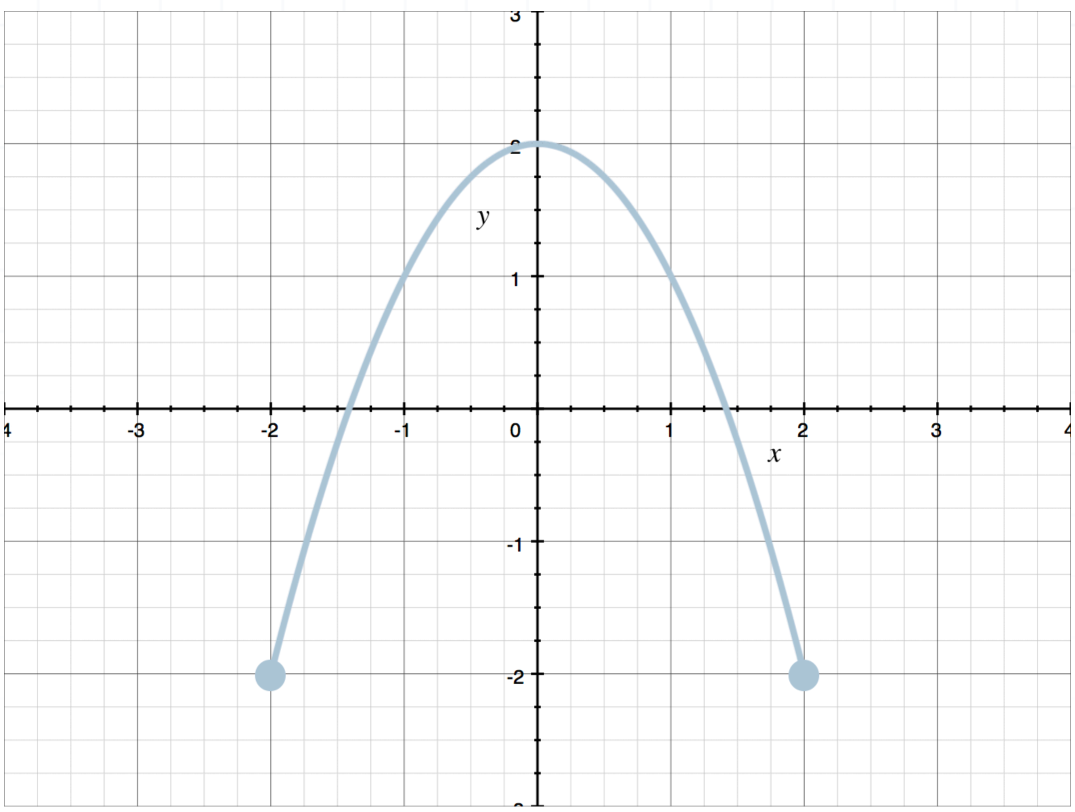
Because $f(-x) \neq -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

2. Describe the symmetry of an even function, and give an example of an even function.

Solution:

An even function is symmetric about the y-axis. There are many examples of even functions, one being $f(x) = x^2$.

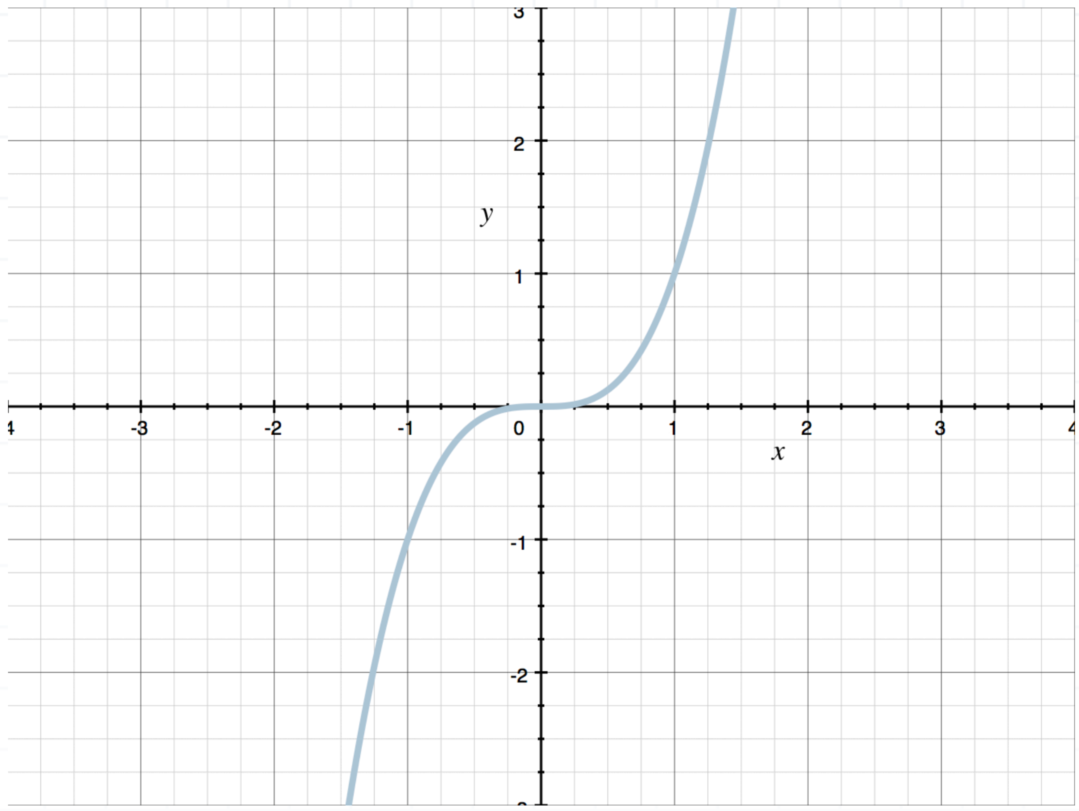
3. Determine whether the graph represents a function that's even, odd, or neither.



Solution:

Notice that the graph is symmetric about the y-axis and therefore the graph is the graph of an even function.

4. Determine whether the graph represents a function that's even, odd, or neither.



Solution:

Notice that the graph is symmetric about the origin, and therefore the graph is the graph of an odd function.

5. Is the function even, odd, or neither?

$$h(x) = x^3 - 3x$$

Solution:

Substitute $-x$ for x.

$$h(-x) = (-x)^3 - 3(-x)$$

$$h(-x) = -x^3 + 3x$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(x^3 - 3x)$$

$$-f(x) = -x^3 + 3x$$

Because $f(-x) = -f(x)$, the function is odd.

6. Is the function even, odd, or neither?

$$(-2,3),\ (-1,0),\ (0,-1),\ (1,0),\ (2,3)$$

Solution:

The y-values are symmetric across the vertical axis, so $f(-x) = f(x)$ and the function is even. For example, $f(-2) = f(2)$ and $f(-1) = f(1)$.

W W W . K R I S T A K I N G M A T H . C O M