Pre-Algebra Workbook Solutions

Mixed numbers

krista king
MATH

MIXED NUMBERS AND IMPROPER FRACTIONS

■ 1. Mixed numbers are a representation of what operation (addition, subtraction, multiplication, division)?

Solution:

Addition, because each mixed number is the sum of a whole number and a fraction.

■ 2. Convert $15/4$ into a mixed number.

Solution:

4 goes into 15 three times. $4 \cdot 3 = 12$, and $15 - 12 = 3$, so 4 goes into 15 three times, with a remainder of 3.

$$\frac{15}{4} = 3\frac{3}{4}$$

■ 3. Convert $34/6$ into a mixed number.

Solution:

6 goes into 34 five times. $6 \cdot 5 = 30$, and $34 - 30 = 4$, so 6 goes into 34 five times, with a remainder of 4.

$$\frac{34}{6} = 5\frac{4}{6} = 5\frac{2}{3}$$

■ **4. Write $-114/25$ as a mixed number.**

Solution:

25 goes into 114 four times. $25 \cdot 4 = 100$, and $114 - 100 = 14$, so 25 goes into 114 four times, with a remainder of 14.

$$-\left(\frac{100 + 14}{25}\right)$$

$$-\left(\frac{25 \cdot 4 + 14}{25}\right)$$

$$-\left(\frac{25 \cdot 4}{25} + \frac{14}{25}\right)$$

$$-\left(4 + \frac{14}{25}\right)$$

$$-4\frac{14}{25}$$

■ 5. Convert the mixed number into an improper fraction.

$$-2\frac{1}{6}$$

Solution:

The equivalent improper fraction will have the same denominator, 6. The numerator is the product of the denominator and the whole number, added to the numerator, $6 \cdot 2 + 1 = 12 + 1 = 13$.

$$-\left(2 + \frac{1}{6}\right)$$

$$-\left(\frac{6 \times 2}{7} + \frac{1}{6}\right)$$

$$-\left(\frac{6 \times 2 + 1}{6}\right)$$

$$-\frac{13}{6}$$

■ 6. Convert the mixed number into an improper fraction.

$$8\frac{4}{9}$$

Solution:

The equivalent improper fraction will have the same denominator, 9. The numerator is the product of the denominator and the whole number, added to the numerator, $9 \cdot 8 + 4 = 72 + 4 = 76$.

$$\frac{76}{9}$$

ADDING AND SUBTRACTING MIXED NUMBERS

■ 1. Simplify the expression.

$$5\frac{2}{3} + 1\frac{1}{12}$$

Solution:

To add the mixed numbers, add the whole numbers separately from the fractions.

$$5 + 1 + \frac{2}{3} + \frac{1}{12}$$

To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by 4/4.

$$6 + \frac{2}{3}\left(\frac{4}{4}\right) + \frac{1}{12}$$

$$6 + \frac{2 \cdot 4}{3 \cdot 4} + \frac{1}{12}$$

$$6 + \frac{8}{12} + \frac{1}{12}$$

Then add the numerators, while keeping the denominator the same.

$$6 + \frac{8 + 1}{12}$$

$$6\frac{9}{12}$$

$$6\frac{3}{4}$$

■ 2. Simplify the expression.

$$8\frac{7}{8} - 2\frac{1}{8}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$8 - 2 + \frac{7}{8} - \frac{1}{8}$$

The fractions already have the same denominator, so we can find the difference directly, which we'll do by finding the difference of the numerators while keeping the denominator the same.

$$6 + \frac{7-1}{8}$$

$$6\frac{6}{8}$$

$$6\frac{3}{4}$$

■ 3. Simplify the expression.

$$7\frac{4}{5} - 6\frac{1}{15}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$7 - 6 + \frac{4}{5} - \frac{1}{15}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by 3/3.

$$1 + \frac{4}{5}\left(\frac{3}{3}\right) - \frac{1}{15}$$

$$1 + \frac{4 \cdot 3}{5 \cdot 3} - \frac{1}{15}$$

$$1 + \frac{12}{15} - \frac{1}{15}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$1 + \frac{12 - 1}{15}$$

$$1\frac{11}{15}$$

■ 4. Simplify the expression.

$$15\frac{1}{2} - 11\frac{1}{4}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$15 - 11 + \frac{1}{2} - \frac{1}{4}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by 2/2.

$$4 + \frac{1}{2}\left(\frac{2}{2}\right) - \frac{1}{4}$$

$$4 + \frac{1 \cdot 2}{2 \cdot 2} - \frac{1}{4}$$

$$4 + \frac{2}{4} - \frac{1}{4}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$4 + \frac{2 - 1}{4}$$

$$4\frac{1}{4}$$

■ 5. Joey and Alex are both solving the following problem.

$$2\frac{1}{3} + 1\frac{3}{5}$$

Joey takes $2 + 1 = 3$ and then takes

$$\frac{1}{3} + \frac{3}{5} = \frac{14}{15}$$

Then he adds them together to get

$$3\frac{14}{15}$$

Alex decides to change both into improper fractions before adding. He gets

$$2\frac{1}{3} = \frac{7}{3} \text{ and } 1\frac{3}{5} = \frac{8}{5}$$

Then she finds common denominators and adds them together to get

$$\frac{59}{15}$$

Who solved this problem correctly?

Solution:

They both solved it correctly. We get Joey's answer when we convert Alex's answer to a mixed number, and we get Alex's answer when we convert Joey's answer to an improper fraction.

■ 6. Simplify the expression.

$$3\frac{2}{5} + \frac{3}{10} - 2\frac{3}{5}$$

Solution:

We'll start by adding the first two mixed numbers. To add the mixed numbers, add the whole numbers separately from the fractions.

$$3 + \frac{2}{5} + \frac{3}{10} - 2\frac{3}{5}$$

To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $2/2$.

$$3 + \frac{2}{5}\left(\frac{2}{2}\right) + \frac{3}{10} - 2\frac{3}{5}$$

$$3 + \frac{2 \cdot 2}{5 \cdot 2} + \frac{3}{10} - 2\frac{3}{5}$$

$$3 + \frac{4}{10} + \frac{3}{10} - 2\frac{3}{5}$$

Then add the numerators, while keeping the denominator the same.

$$3 + \frac{4 + 3}{10} - 2\frac{3}{5}$$

$$3\frac{7}{10} - 2\frac{3}{5}$$

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$3 - 2 + \frac{7}{10} - \frac{3}{5}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the second fraction by $2/2$.

$$1 + \frac{7}{10} - \frac{3}{5}\left(\frac{2}{2}\right)$$

$$1 + \frac{7}{10} - \frac{3 \cdot 2}{5 \cdot 2}$$

$$1 + \frac{7}{10} - \frac{6}{10}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$1 + \frac{7 - 6}{10}$$

$$1\frac{1}{10}$$

MULTIPLYING AND DIVIDING MIXED NUMBERS

■ 1. When we multiply and divide mixed numbers, we need to change the mixed numbers into _____ fractions before we do the multiplication or division.

Solution:

improper

■ 2. Simplify the expression.

$$3\frac{3}{7} \cdot 1\frac{1}{7}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{7 \cdot 3 + 3}{7} \cdot \frac{7 \cdot 1 + 1}{7}$$

$$\frac{24}{7} \cdot \frac{8}{7}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{24 \cdot 8}{7 \cdot 7}$$

$$\frac{192}{49}$$

$$3\frac{45}{49}$$

■ 3. Simplify the expression.

$$5\frac{1}{5} \cdot 2\frac{2}{3}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{5 \cdot 5 + 1}{5} \cdot \frac{3 \cdot 2 + 2}{3}$$

$$\frac{26}{5} \cdot \frac{8}{3}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{26 \cdot 8}{5 \cdot 3}$$

$$\frac{208}{15}$$

$$13\frac{13}{15}$$

■ 4. Simplify the expression.

$$2\frac{3}{4} \div 5\frac{1}{8}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{4 \cdot 2 + 3}{4} \div \frac{8 \cdot 5 + 1}{8}$$

$$\frac{11}{4} \div \frac{41}{8}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{11}{4} \cdot \frac{8}{41}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{11 \cdot 8}{4 \cdot 41}$$

$$\frac{88}{164}$$

$$\frac{22}{41}$$

■ 5. Simplify the expression.

$$4\frac{5}{9} \div 2\frac{1}{4}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{9 \cdot 4 + 5}{9} \div \frac{4 \cdot 2 + 1}{4}$$

$$\frac{41}{9} \div \frac{9}{4}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{41}{9} \cdot \frac{4}{9}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{41 \cdot 4}{9 \cdot 9}$$

$$\frac{164}{81}$$

$$2\frac{2}{81}$$

■ 6. Simplify the expression.

$$1\frac{4}{5} \div 3\frac{3}{8}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{5 \cdot 1 + 4}{5} \div \frac{8 \cdot 3 + 3}{8}$$

$$\frac{9}{5} \div \frac{27}{8}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{9}{5} \cdot \frac{8}{27}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{9 \cdot 8}{5 \cdot 27}$$

$$\frac{72}{135}$$

$$\frac{8}{15}$$

RELATIONSHIPS OF NUMBERS

■ 1. Which fraction is larger?

$$\frac{1}{8} \text{ or } \frac{1}{6}$$

Solution:

When the numerators are equivalent, the larger fraction is the one with the smaller denominator, so $1/6$ is the larger fraction.

■ 2. Which fraction is smaller?

$$\frac{3}{7} \text{ or } \frac{3}{8}$$

Solution:

When the numerators are equivalent, the smaller fraction is the one with the larger denominator, so $3/8$ is the smaller fraction.

■ 3. Find a number that's $1/5$ of the way from $2/5$ to $3/10$.

Solution:

First, we'll find the distance between 3/10 and 2/5.

$$\frac{2}{5} - \frac{3}{10}$$

In order to do the subtraction, we have to find a common denominator.

$$\frac{2}{5}\left(\frac{2}{2}\right) - \frac{3}{10}$$

$$\frac{4}{10} - \frac{3}{10}$$

$$\frac{1}{10}$$

Now we want to find 1/5 of this distance, which means we need to multiply it by 1/5.

$$\frac{1}{10} \times \frac{1}{5}$$

$$\frac{1}{50}$$

This is 1/5 the distance from 3/10 to 2/5, and since we want to end up exactly 1/5 of the way from 3/10 to 2/5, we add 1/50 to the smaller fraction, 3/10.

$$\frac{3}{10} + \frac{1}{50}$$

In order to do the addition, we have to find a common denominator.

$$\frac{3}{10}\left(\frac{5}{5}\right) + \frac{1}{50}$$

$$\frac{15}{50} + \frac{1}{50}$$

$$\frac{16}{50}$$

$$\frac{8}{25}$$

So $8/25$ is the number that's $1/5$ of the way from $3/10$ to $2/5$.

■ 4. Find a number that's $2/3$ of the way from $-2/3$ to $1/4$.

Solution:

First, we'll find the distance between $-2/3$ and $1/4$.

$$\frac{1}{4} - \left(-\frac{2}{3}\right)$$

In order to do the subtraction, we have to find a common denominator.

$$\frac{1}{4}\left(\frac{3}{3}\right) - \left(-\frac{2}{3}\right)\left(\frac{4}{4}\right)$$

$$\frac{3}{12}+\frac{8}{12}$$

$$\frac{11}{12}$$

Now we want to find $2/3$ of this distance, which means we need to multiply it by $2/3$.

$$\frac{11}{12}\times\frac{2}{3}$$

$$\frac{22}{36}$$

$$\frac{11}{18}$$

This is $2/3$ of the distance from $-2/3$ to $1/4$, and since we want to end up exactly $2/3$ of the way from $-2/3$ to $1/4$, we add $11/18$ to the smaller fraction, $-2/3$.

$$-\frac{2}{3}+\frac{11}{18}$$

In order to do the addition, we have to find a common denominator.

$$-\frac{2}{3}\left(\frac{6}{6}\right)+\frac{11}{18}$$

$$-\frac{12}{18}+\frac{11}{18}$$

$$-\frac{1}{18}$$

So $-1/18$ is the number that's $2/3$ of the way from $-2/3$ to $1/4$.

■ 5. Find the fraction halfway between $1/2$ and $2/5$.

Solution:

If we find a common denominator among the fractions, $1/2$ converts to $5/10$, and $2/5$ converts to $4/10$. Now that the denominators are equivalent, we know the smaller fraction is the one with the smaller numerator, so $4/10$ is smaller than $5/10$.

We can now find the distance between these fractions by subtracting the smaller fraction from the larger fraction.

$$\frac{5}{10} - \frac{4}{10}$$

$$\frac{5-4}{10}$$

$$\frac{1}{10}$$

We need half of this total distance, so we'll divide $1/10$ by 2 to find that half of the distance is $1/20$. Now to find the fraction that's halfway between $4/10$ and $5/10$, we'll add this half distance to $4/10$.

$$\frac{4}{10} + \frac{1}{20}$$

$$\frac{4}{10}\left(\frac{2}{2}\right) + \frac{1}{20}$$

$$\frac{4 \cdot 2}{10 \cdot 2} + \frac{1}{20}$$

$$\frac{8}{20} + \frac{1}{20}$$

$$\frac{8 + 1}{20}$$

$$\frac{9}{20}$$

■ 6. Find the fraction halfway between $1/10$ and $8/13$.

Solution:

If we find a common denominator among the fractions, $1/10$ converts to $13/130$, and $8/13$ converts to $80/130$. Now that the denominators are equivalent, we know the smaller fraction is the one with the smaller numerator, so $13/130$ is smaller than $80/130$.

We can now find the distance between these fractions by subtracting the smaller fraction from the larger fraction.

$$\frac{80}{130} - \frac{13}{130}$$

$$\frac{80 - 13}{130}$$

$$\frac{67}{130}$$

We need half of this total distance, so we'll divide $67/130$ by 2 to find that half of the distance is $67/260$. Now to find the fraction that's halfway between $13/130$ and $80/130$, we'll add this half distance to $13/130$.

$$\frac{13}{130} + \frac{67}{260}$$

$$\frac{13}{130} \left(\frac{2}{2} \right) + \frac{67}{260}$$

$$\frac{13 \cdot 2}{130 \cdot 2} + \frac{67}{260}$$

$$\frac{26}{260} + \frac{67}{260}$$

$$\frac{93}{260}$$

ADDING MIXED MEASURES

■ 1. Add the mixed measures.

4 seconds, 11 minutes, 3 hours, 35 minutes, 56 minutes, 35 seconds

Solution:

Organize and combine the measures we've been given.

3 hours, $11 + 35 + 56$ minutes, $4 + 35$ seconds

3 hours, 102 minutes, 39 seconds

Since we have more than 60 minutes, we want to convert some of those minutes into hours.

3 hours, $60 + 42$ minutes, 39 seconds

3 hours, 1 hour, 42 minutes, 39 seconds

$3 + 1$ hours, 42 minutes, 39 seconds

4 hours, 42 minutes, 39 seconds

■ 2. Add the mixed measures.

34 inches, 2 yards, 5 feet, 8 inches, 13 feet, 1 yard

Solution:

Organize and combine the measures we've been given.

$2 + 1$ yards, $5 + 13$ feet, $34 + 8$ inches

3 yards, 18 feet, 42 inches

Since we have more than 12 inches, we want to convert some of those inches into feet.

3 yards, 18 feet, $36 + 6$ inches

3 yards, 18 feet, 3 feet, 6 inches

3 yards, $18 + 3$ feet, 6 inches

3 yards, 21 feet, 6 inches

Since we have more than 3 feet, we want to convert some of those feet into yards.

3 yards, 7 yards, 6 inches

10 yards, 6 inches

■ 3. Add the mixed measures.

25 seconds, 1 hour, 15 minutes, 45 seconds, 22 minutes

Solution:

Organize and combine the measures we've been given.

$\qquad$ 1 hour, $15 + 22$ minutes, $25 + 45$ seconds

$\qquad$ 1 hour, 37 minutes, 70 seconds

Since we have more than 60 seconds, we want to convert some of those seconds into minutes.

$\qquad$ 1 hour, 37 minutes, $60 + 10$ seconds

$\qquad$ 1 hour, 37 minutes, 1 minute, 10 seconds

$\qquad$ 1 hour, $37 + 1$ minutes, 10 seconds

$\qquad$ 1 hour, 38 minutes, 10 seconds

■ 4. Add the mixed measures.

$\qquad$ 13 inches, 45 feet, 35 inches, 27 feet, 9 yards

Solution:

Organize and combine the measures we've been given.

$\qquad$ 9 yards, $45 + 27$ feet, $13 + 35$ inches

9 yards, 72 feet, 48 inches

Since we have more than 12 inches, we want to convert some of those inches into feet.

9 yards, 72 feet, 4 feet

9 yards, 72 + 4 feet

9 yards, 76 feet

Since we have more than 3 feet, we want to convert some of those feet into yards.

9 yards, 75 + 1 feet

9 yards, 25 yards, 1 foot

34 yards, 1 foot

■ 5. Add the mixed measures.

1 foot, 38 inches

Solution:

Since we have more than 12 inches, we want to convert some of those inches into feet.

1 foot, 38 inches

1 foot, $36 + 2$ inches

1 foot, 3 feet $+ 2$ inches

$1 + 3$ feet $+ 2$ inches

4 feet, 2 inches

■ 6. Add the mixed measures.

1 hour, 85 minutes, 55 seconds, 20 minutes

Solution:

Organize and combine the measures we've been given.

1 hour, $85 + 20$ minutes, 55 seconds

1 hour, 105 minutes, 55 seconds

Since we have more than 60 minutes, we want to convert some of those minutes into hours.

1 hour, $60 + 45$ minutes, 55 seconds

1 hour, 1 hour, 45 minutes, 55 seconds

$1 + 1$ hour, 45 minutes, 55 seconds

2 hour, 45 minutes, 55 seconds