Point-slope and slope-intercept forms of a line

We have two options for writing the equation of a line: point-slope form and slope-intercept form.

Point-slope form	$y - y_1 = m(x - x_1)$
Slope-intercept form	$y = mx + b$

Both forms require that we know at least two of the following pieces of information about the line:

1. One point, (x_1, y_1)
2. A second point, (x_2, y_2)
3. The slope, m
4. The y-intercept, b (the y-value where the line crosses the y-axis)

If we know any two of these values, we can find the equation of the line in both forms.

Point-slope form

The equation of a line in point-slope form is

$$y - y_1 = m(x - x_1)$$

In this form, (x_1, y_1) is a point on the line, and m is the slope. To use this form when we know two points on the line but we don't know the slope, we'll find m as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Then we'll plug the slope m and the point (x_1, y_1) into point-slope form. Let's do an example where we know the slope and one point on the line.

Example

Write the equation of the line in point-slope form.

$$m = -\frac{1}{4}$$

$$(-6,1)$$

Since we've been given the slope of the line and a point on the line, we can use the point-slope form to find the equation of the line. We'll plug $m = -1/4$ and the point $(-6,1)$ into the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{4}(x - (-6))$$

$$y - 1 = -\frac{1}{4}(x + 6)$$

Let's try an example where we know two points on the line.

Example

Find the point-slope form of the equation of the line that passes through the points $(-2,-4)$ and $(3,5)$.

We'll start by finding the slope of the line. It never matters which point we use for (x_1, y_1) and which one we use for (x_2, y_2), as long as we stay consistent. Let's set

$$(x_1, y_1) = (-2,-4)$$

$$(x_2, y_2) = (3,5)$$

Plug these into the formula for slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5-(-4)}{3-(-2)}$$

$$m = \frac{9}{5}$$

Next, substitute $m = 9/5$ and the point $(-2,-4)$ into point-slope form.

$$y - (-4) = \frac{9}{5}(x - (-2))$$

$$y + 4 = \frac{9}{5}(x + 2)$$

Slope-intercept form

The equation of a line in slope-intercept form is

$$y = mx + b$$

where m is the slope and b is the y-intercept. Since the y-intercept, b, is the y-coordinate of the point at which the graph crosses the y-axis, and since the x-coordinate of every point on the y-axis is 0, the point where the graph crosses the y-axis is $(0,b)$.

Let's try an example where we know the slope and a point on the line.

Example

Find the equation of the line in slope-intercept form.

$$m = -3$$

$$(0,1)$$

If we recognize that the point $(0,1)$ lies on the y-axis (since its x-coordinate is 0), then we can recognize that the y-intercept is $y = b = 1$, and we can substitute $m = -3$ and $b = 1$ into slope-intercept form.

$$y = mx + b$$

$$y = -3x + 1$$

Even if we were unsure about whether $(0,1)$ gave us the y-intercept, we could still plug the slope $m = -3$ and the point $(0,1)$ into the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -3(x - 0)$$

$$y - 1 = -3x$$

$$y = -3x + 1$$

Let's try an example where we know two points on the line.

Example

Find the slope-intercept form of the equation of the line that passes through $(-1, -2)$ and $(3, -4)$.

First we need to find the slope, so let's set

$$(x_1, y_1) = (-1, -2)$$

$$(x_2, y_2) = (3, -4)$$

and then plug these points into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4 - (-2)}{3 - (-1)}$$

$$m = \frac{-2}{4}$$

$$m = -\frac{1}{2}$$

Next, substitute $m = -1/2$ and $(x_1, y_1) = (-1, -2)$ into the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{1}{2}(x - (-1))$$

$$y + 2 = -\frac{1}{2}(x + 1)$$

$$y + 2 = -\frac{1}{2}x - \frac{1}{2}$$

$$y = -\frac{1}{2}x - \frac{5}{2}$$

Converting between the forms

Regardless of which form we start with, point-slope or slope-intercept, we can easily convert back and forth between the two.

We've worked four examples so far in this lesson. In this first two, we found equations in point-slope form,

$$y - 1 = -\frac{1}{4}(x + 6) \qquad\qquad y + 4 = \frac{9}{5}(x + 2)$$

To convert equations in point-slope form to equations in slope-intercept form, we simplify the right side of the equation and then solve for y.

$$y - 1 = -\frac{1}{4}x - \frac{6}{4} \qquad\qquad y + 4 = \frac{9}{5}x + \frac{18}{5}$$

$$y = -\frac{1}{4}x - \frac{6}{4} + 1 \qquad\qquad y = \frac{9}{5}x + \frac{18}{5} - 4$$

$$y = -\frac{1}{4}x - \frac{6}{4} + \frac{4}{4} \qquad\qquad y = \frac{9}{5}x + \frac{18}{5} - \frac{20}{5}$$

$$y = -\frac{1}{4}x - \frac{1}{2} \qquad\qquad y = \frac{9}{5}x - \frac{2}{5}$$

Now the equations are in slope-intercept form, $y = mx + b$.

In the third and fourth examples, we found equations in slope-intercept form,

$$y = -3x + 1 \qquad\qquad y = -\frac{1}{2}x - \frac{5}{2}$$

To convert equations in slope-intercept form to equations in point-slope form, we factor out the coefficient on x,

$$y = -3\left(x - \frac{1}{3}\right) \qquad\qquad y = -\frac{1}{2}(x + 5)$$

Now the equations are in slope-intercept form, $y - y_1 = m(x - x_1)$. When we convert this way, we'll always have $y_1 = 0$.

When lines can't be written in either form

Perfectly vertical lines can't be written in either point-slope or slope-intercept forms, because the slope of a vertical line is undefined.

Given two points on a vertical line (x_1, y_1) and (x_2, y_2), we know that $x_1 = x_2$, which means $x_2 - x_1 = 0$ and the slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{0}$$

which is undefined because division by 0 is undefined. So instead of writing vertical lines in point-slope or slope-intercept form, we'll write them as $x = c$. For example, $x = 3$ is the equation of the vertical line that passes through the point $(3,0)$ on the horizontal axis.