Grouping

In this lesson we'll look at factoring a polynomial using a method called grouping.

When we have a polynomial, sometimes we can use grouping to help us find the factors. To do this, we need to look for a way to split the terms of the polynomial into two groups in such a way that each group can be factored separately and there's a factor that's common to the two groups.

Let's look at an example.

Example

Factor by grouping.

$$11z + 11qr + pyz + pyqr$$

Since we've been asked to use grouping to factor the polynomial, we need to look for a way to split the terms of the polynomial into two groups that can be factored separately. Since the first two terms have a factor of 11 in common, and the last two terms have a factor of py in common, we'll group the first two terms separately from the last two terms.

$$11z + 11qr + pyz + pyqr$$

$$(11z + 11qr) + (pyz + pyqr)$$

With our terms grouped, we need to look for the greatest common factor in each group. In this case, those are the factors we identified earlier (11 in the first group, and py in the second group). Factoring these out of the respective groups separately, we get

$$11(z+qr)+py(z+qr)$$

Notice that the two groups of terms do indeed have a factor in common (specifically, $z+qr$), so we can now factor that out of each group.

$$(11+py)(z+qr)$$

This is the correct solution, but it can also be written as $(z+qr)(11+py)$ or even $(qr+z)(11+py)$.

There are usually different ways to group our terms before we factor. We could have used grouping to factor our polynomial this way:

$$11z+11qr+pyz+pyqr$$

$$11qr+pyqr+11z+pyz$$

$$(11qr+pyqr)+(11z+pyz)$$

$$qr(11+py)+z(11+py)$$

$$(qr+z)(11+py)$$

Let’s do another example.

Example

Factor by grouping.

$$pqx^2 - psx + qrx - rs$$

Since we've been asked to use grouping to factor the polynomial, we need to look for a way to split the terms of the polynomial into two groups that can be factored separately. Since the first two terms have a factor of px in common, and the last two terms have a factor of r in common, we'll group the first two terms separately from the last two terms.

$$pqx^2 - psx + qrx - rs$$

$$(pqx^2 - psx) + (qrx - rs)$$

With our terms grouped, we need to look for the greatest common factor in each group. In this case, those are the factors we identified earlier (px in the first group, and r in the second group). Factoring these out of the respective groups separately, we get

$$px(qx - s) + r(qx - s)$$

Notice that the two groups of terms do indeed have a factor in common (specifically, $qx - s$), so we can now factor that out of each group.

$$(px + r)(qx - s)$$

As we mentioned before, there are multiple ways of writing this, such as $(-s + qx)(r + px)$, and it all depends on how we choose to group the factors.

We can also use grouping to factor quadratics. We already know how to factor quadratics of the form

$$ax^2 + bx + c$$

by looking at the factors of a and c and trying to figure out which combination of factors can be used to get the coefficient of the middle term, b. But that's not the only way to factor quadratics. We can also use grouping.

To use grouping to factor a quadratic, the first step is to find $a \cdot c$, then look for factors of $a \cdot c$ that sum to b. Then we can rewrite the quadratic, and factor by grouping.

Let's look at an example with numbers to see how this works.

Example

Factor the quadratic.

$$11x^2 + 13x + 2$$

In this case $a = 11$, $c = 2$, and $b = 13$, so we need to find the pair of factors of $a \cdot c = 11 \cdot 2 = 22$ whose sum is 13. The pairs of factors of 22 are $(1,22)$ and $(2,11)$. Which of these pairs of factors have a sum of 13?

$$1 + 22 = 23$$

and

$$2 + 11 = 13$$

So we need the pair $(2,11)$. Let's rewrite $11x^2 + 13x + 2$ as $11x^2 + 2x + 11x + 2$.

Now, since we want to use grouping to factor the polynomial, we need to look for a way to split the terms of the polynomial into two groups that can be factored separately. Since the first two terms have an x in common, we'll group the first two terms together separately from the last two terms.

$$(11x^2 + 2x) + (11x + 2)$$

Let's factor out the x from the terms in the first group.

$$x(11x + 2) + (11x + 2)$$

Remember we can write this as

$$x(11x + 2) + 1(11x + 2)$$

Notice that the two groups of terms do indeed have a factor in common (specifically, $11x + 2$), so we can now factor that out of each group.

$$(11x + 2)(x + 1)$$

If we're struggling with factoring a quadratic, factoring by grouping can give us a nice procedure to follow.