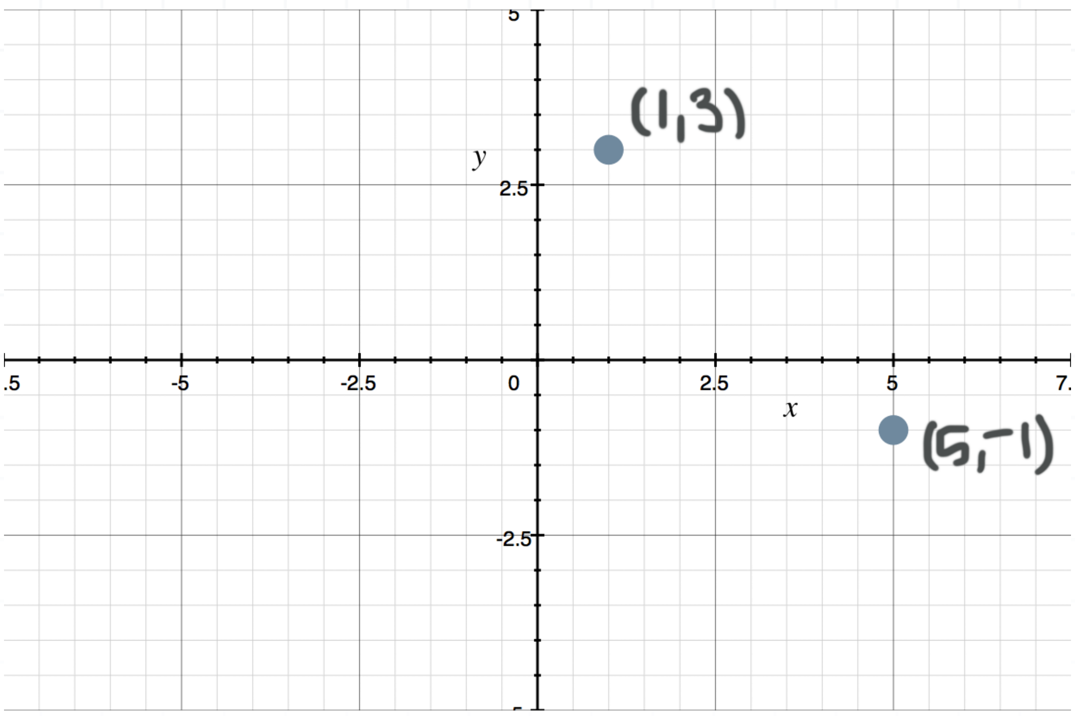
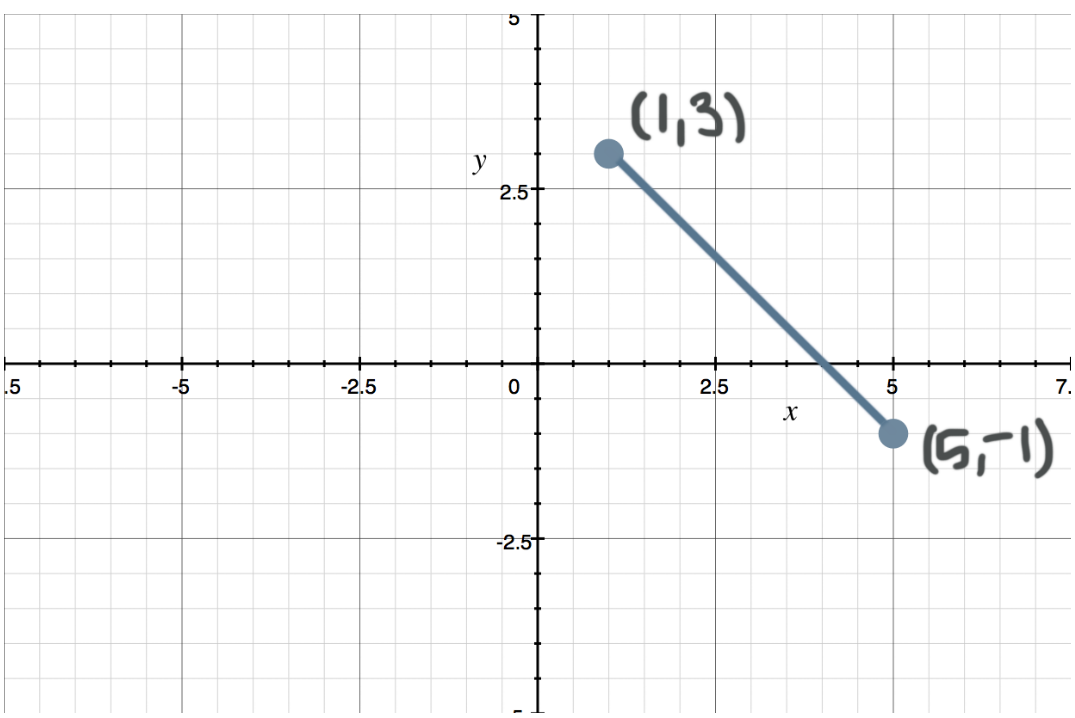
# Distance between two points

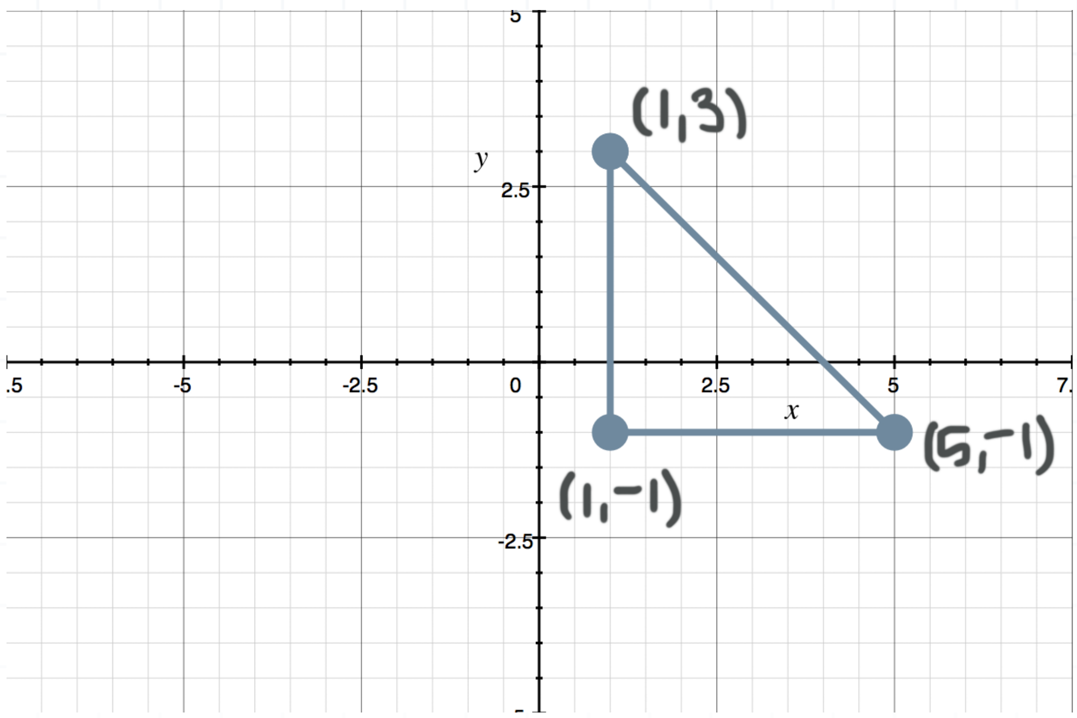
In this lesson we'll learn how to use the distance formula to calculate the distance between two points.



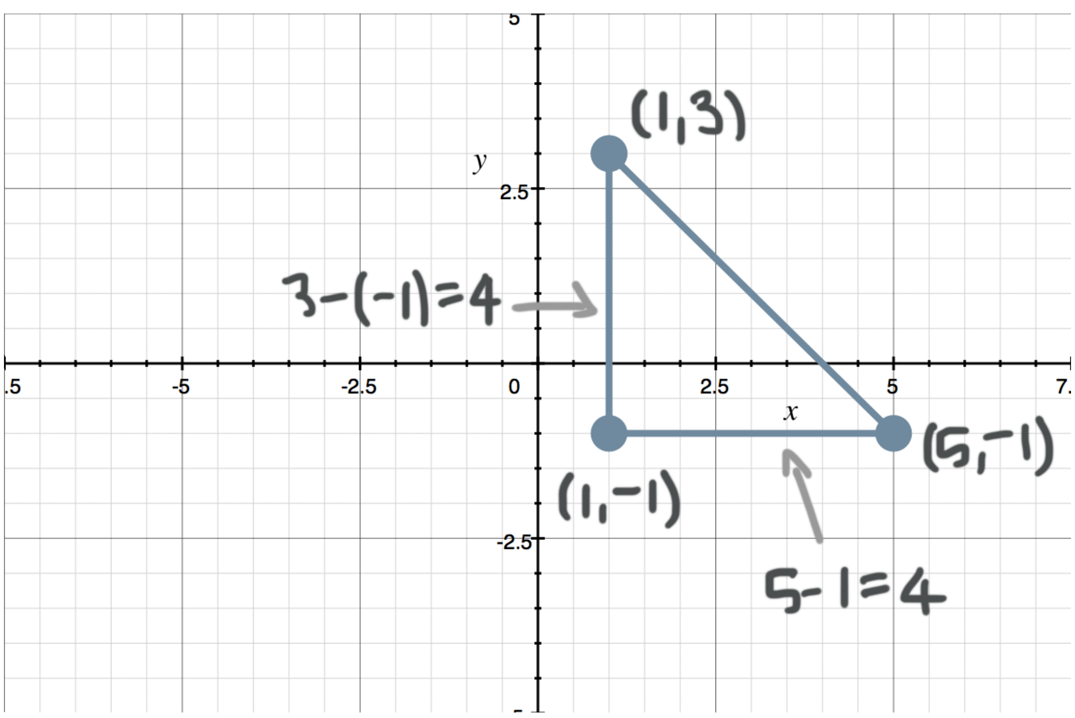

Let's begin by looking at two points on a graph. The distance formula will calculate the length of the straight line between the two points.



We can draw a right triangle that has the line between these two points as its hypotenuse.




We can find the lengths of the legs of the right triangle, $a$ and $b$ (where $a$ is the absolute value of the difference in the $x$-coordinates of the original two points and $b$ is the absolute value of the difference in their $y$-coordinates),




and use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find the length $c$ of the hypotenuse (the straight line between the original two points).

In this case, the original two points are $(1,3)$ and $(5, -1)$. The absolute value of the difference in their $x$-coordinates is

$$a = |5 - 1| = |4| = 4$$

and the absolute value of the difference in their $y$-coordinates is

$$b = |-1 - 3| = |-4| = 4$$

Substitute these values into the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$4^2 + 4^2 = c^2$$

$$32 = c^2$$

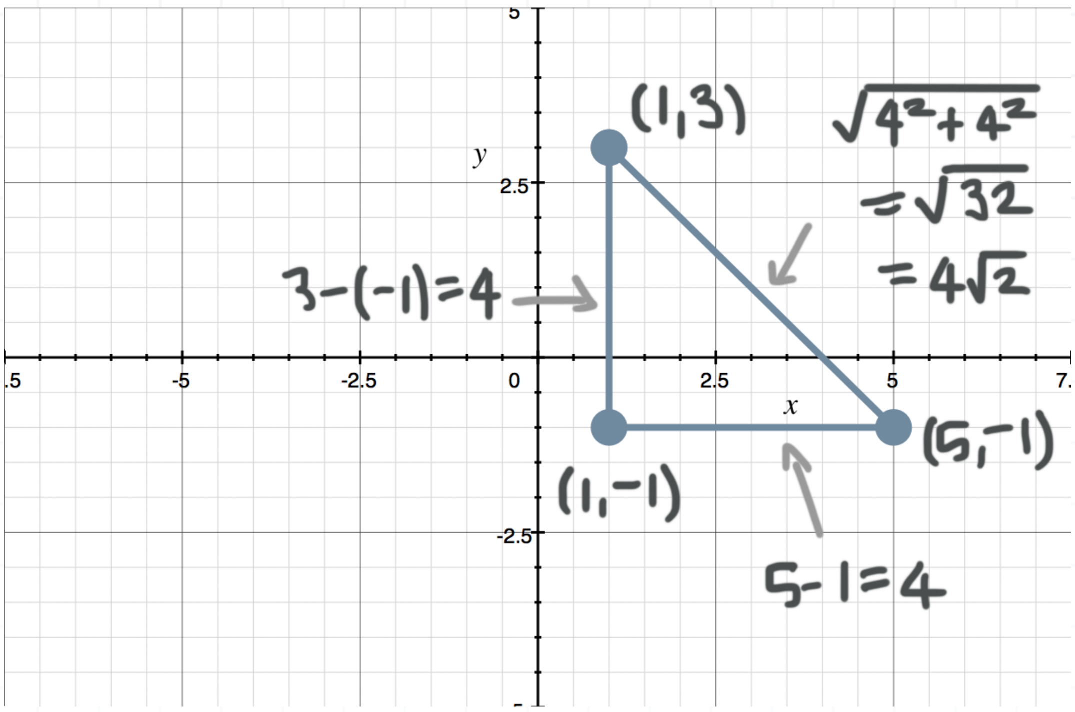
$$c = \sqrt{32}$$

$$c = \sqrt{16 \cdot 2}$$

$$c = \sqrt{16} \cdot \sqrt{2}$$

$$c = 4\sqrt{2}$$

Now we have all three side lengths of the triangle.




So what did we do to get the length between the two points? When we used the Pythagorean Theorem, we got

$$c = \sqrt{(5-1)^2 + (-1-3)^2}$$

or in other words,

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

And that's the distance formula! However, the distance formula uses $d$ instead of $c$. It says that the distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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Let's do a few examples where we use the distance formula directly.

**Example**

What is the distance between the two points?

$(5,7)$

$(-3,5)$

We'll plug the given points $(x_1, y_1) = (5,7)$ and $(x_2, y_2) = (-3,5)$ into the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-3 - 5)^2 + (5 - 7)^2}$$

$$d = \sqrt{(-8)^2 + (-2)^2}$$

$$d = \sqrt{64 + 4}$$

$$d = \sqrt{68}$$

$$d = \sqrt{4 \cdot 17}$$

$$d = \sqrt{4} \cdot \sqrt{17}$$

$$d = 2\sqrt{17}$$

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The distance formula works with irrational numbers as well.

**Example**

Find the distance between the points.

$$(3,\sqrt{2})$$

$$(2,-\sqrt{2})$$

We'll plug the given points $(x_1, y_1) = (3,\sqrt{2})$ and $(x_2, y_2) = (2,-\sqrt{2})$ into the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 3)^2 + (-\sqrt{2} - \sqrt{2})^2}$$

$$d = \sqrt{(-1)^2 + (-2\sqrt{2})^2}$$

$$d = \sqrt{1 + [(-2)^2(\sqrt{2})^2]}$$

$$d = \sqrt{1 + (4 \cdot 2)}$$

$$d = \sqrt{1 + 8}$$

$$d = \sqrt{9}$$

$$d = 3$$

## Midpoint between two points

We can also find the midpoint between two points, and doing so is just an extension of what we've already done to find the distance between two points.

The midpoint between points will always fall exactly half way between the $x$-values of their coordinate points, and half way between the $y$-values of their coordinate points, which means the midpoint $M$ is given by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

For instance, let's continue on with the last example, and find the midpoint between the points

$(3,\sqrt{2})$ and $(2,-\sqrt{2})$

Plugging both points into the midpoint formula will give the midpoint between them.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{3 + 2}{2}, \frac{\sqrt{2} - \sqrt{2}}{2}\right)$$

$$M = \left(\frac{5}{2}, \frac{0}{2}\right)$$

$$M=\left(\frac{5}{2},0\right)$$