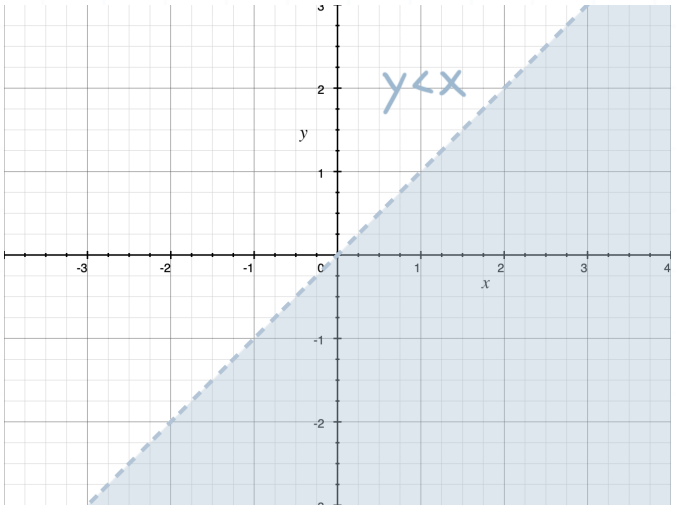
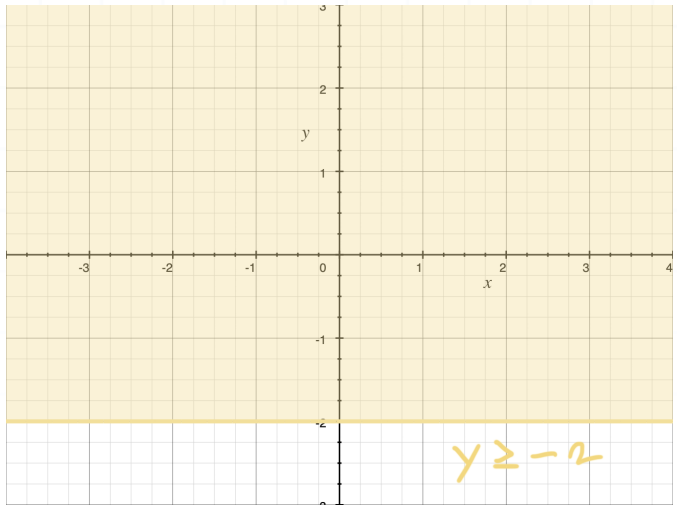
Systems of linear inequalities

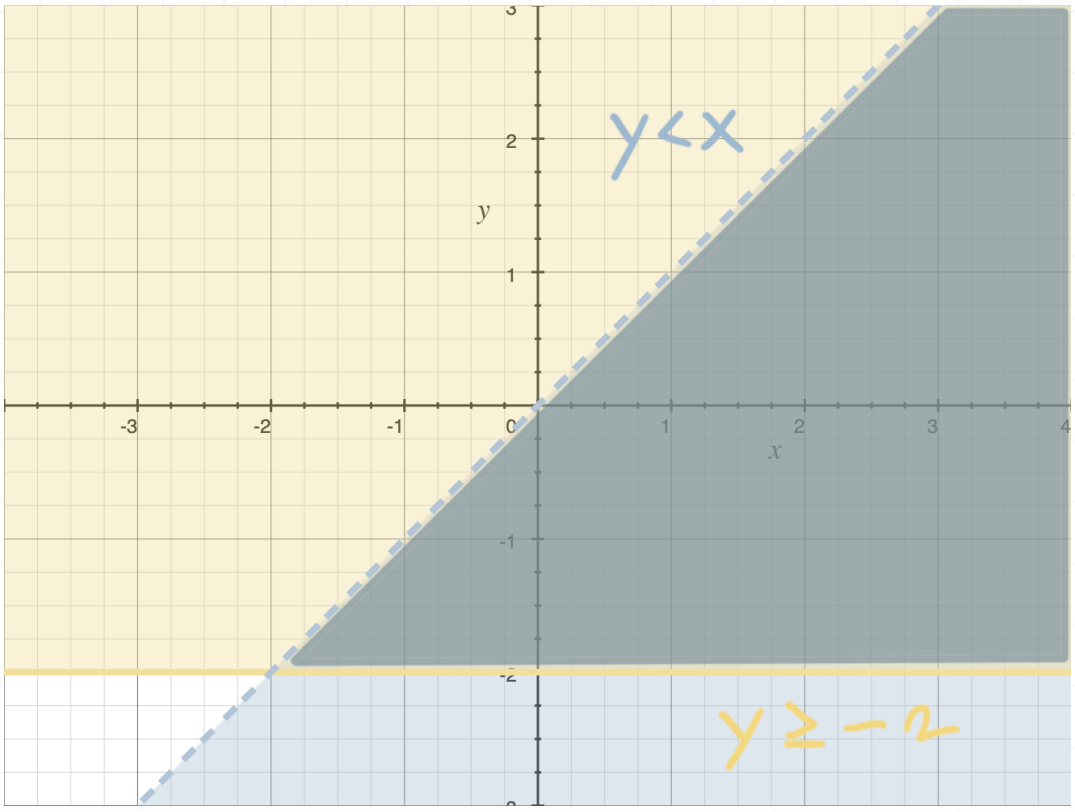
Now that we know how to solve systems of linear equations using substitution, elimination, and graphing, we want to learn to solve systems of inequalities.

We need to realize that the solution to a system of linear inequalities (if the system has a solution) will be a region in the plane. For instance, given two lines $y < x$ and $y \geq -2$, we can sketch each inequality separately in the plane,





and then identify the region of overlap as the solution to the system of inequalities.



Graphing the solution to the system

To find and graph the region of the solution to the system of inequalities, we'll

1. Graph the boundary lines
2. Determine whether the boundary lines are dashed ($<$ or $>$) or solid ($\leq$ or $\geq$)
3. Determine which side of each boundary line to shade
4. Identify the overlapping shaded region as the solution, keeping only the overlap shaded

Let's work through an example so that we can see these steps in practice.

Example

Graph the solution to the system of linear inequalities.

$$y < -2x + 3$$

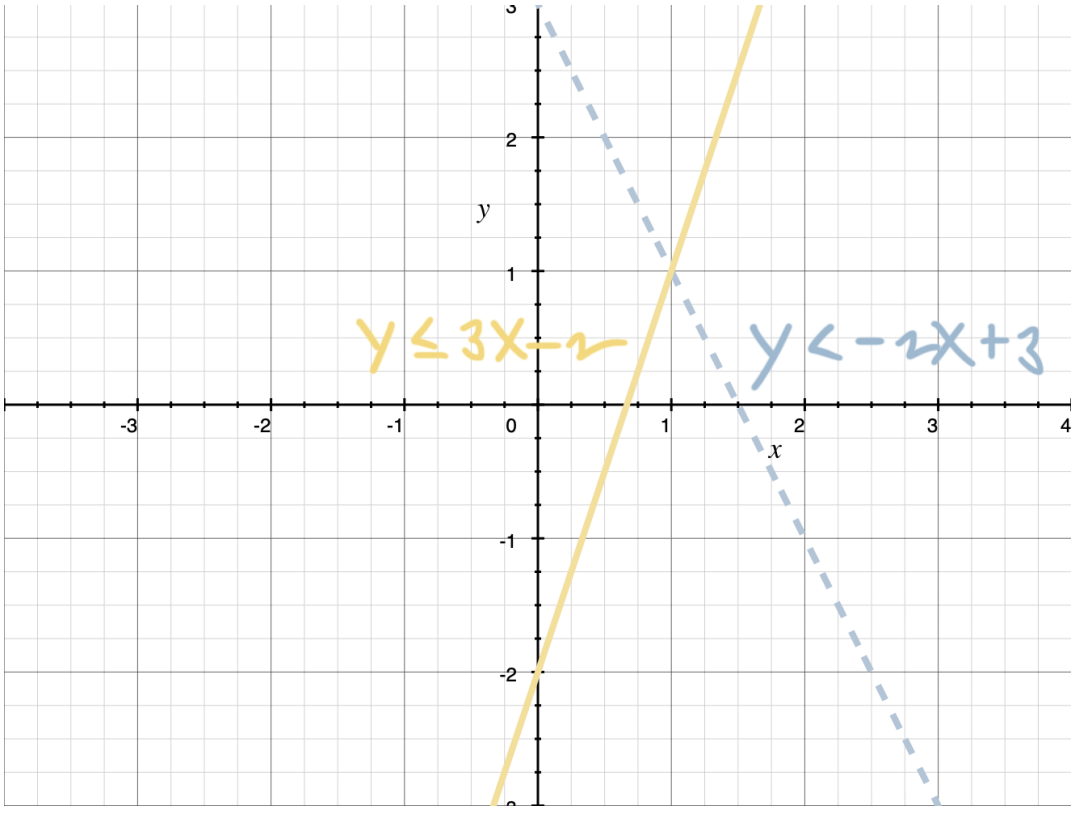
$$y \leq 3x - 2$$

First, we need to sketch the boundary lines of each inequality by graphing the corresponding equations,

$$y = -2x + 3$$

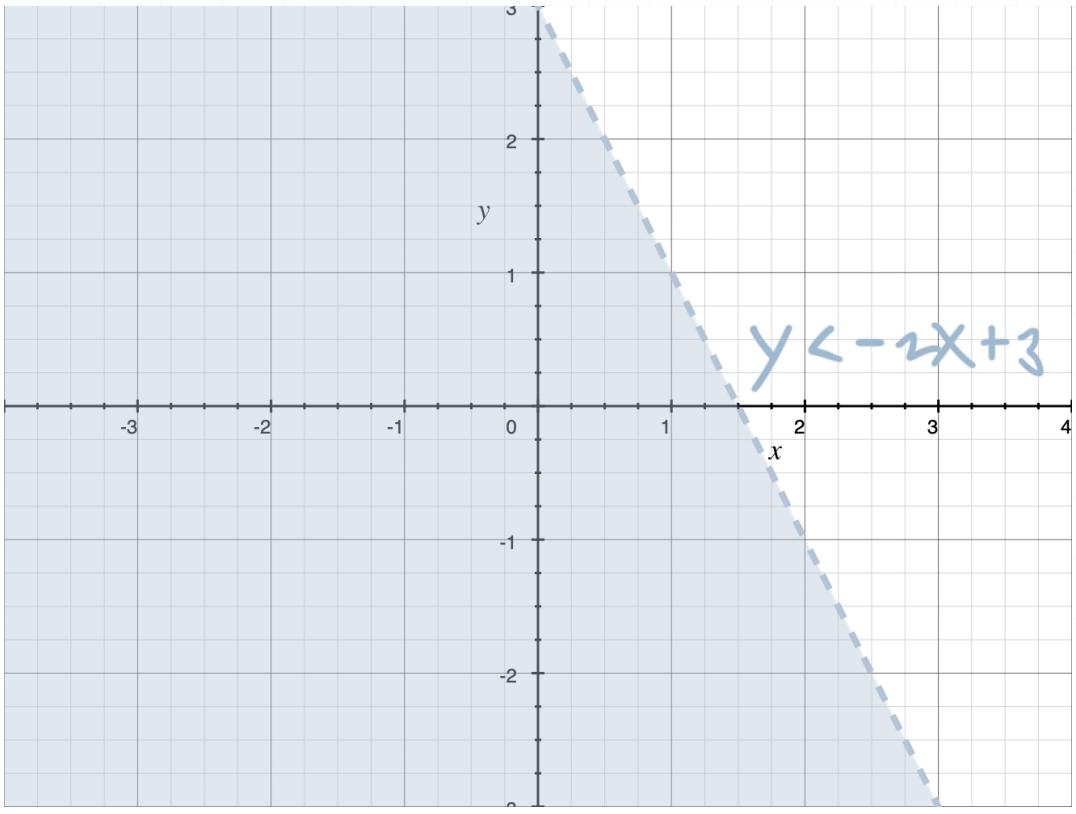
$$y = 3x - 2$$

Both lines are in slope-intercept form, so a sketch of the two of them together is

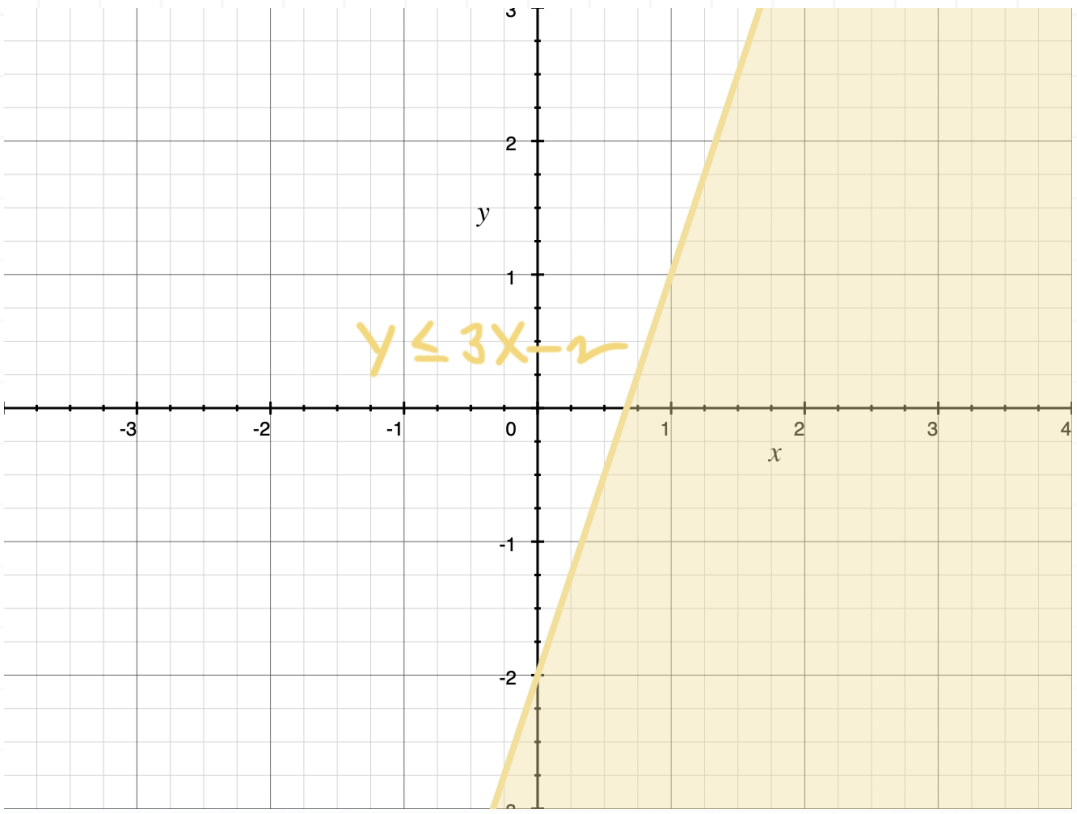


The line associated with $y < -2x + 3$ is dashed, given the "less than" sign, whereas the line associated with $y \leq 3x - 2$ is solid, given the "less than or equal to" sign.

Because $y < -2x + 3$ is a "less than" inequality, we shade below the dashed $y = -2x + 3$ line,



and because $y \leq 3x - 2$ is a "less than or equal to" inequality, we shade below the solid $y = 3x - 2$ line.



Putting these two regions together, we can identify the overlapping region as the solution to the system of inequalities.