Topic: Evaluating logs

Question: Find the exact value of the expression.

$$\log\sqrt{10}$$

Answer choices:

A $\frac{1}{2}$

B 1

C 2

D $-\frac{1}{2}$

Solution: A

The logarithm has an implied base 10, so if we rewrite the expression as $\log_{10}\sqrt{10} = x$, then we know we're looking for the value of x, and we can write the new equation

$$10^x = \sqrt{10}$$

$$10^x = 10^{\frac{1}{2}}$$

Because the bases are equal, in order for the equation to be true the exponents must be equal as well, so $x = 1/2$ and we can say that

$$\log\sqrt{10} = \frac{1}{2}$$

Topic: Evaluating logs

Question: Find the exact value of the expression.

$$\log_{32}(4)$$

Answer choices:

A 8

B $\frac{1}{8}$

C 2

D $\frac{2}{5}$

Solution: D

In this expression, the base is greater than the argument, which means the value of the expression will be less than 1.

We'll evaluate it by solving the exponential equation

$$32^x = 4$$

To do this, we'll express both 32 and 4 as powers of 2: $32 = 2^5$ and $4 = 2^2$. So we get

$$(2^5)^x = 2^2$$

$$2^{5x} = 2^2$$

Since the bases are equal, the exponents must also be equal.

$$5x = 2$$

$$x = \frac{2}{5}$$

This tells us that

$$\log_{32}(4) = \frac{2}{5}$$

Topic: Evaluating logs

Question: Find the exact value of the expression.

$$\log_9\left(\frac{1}{2{,}187}\right)$$

Answer choices:

A $x = \frac{7}{2}$

B $x = -\frac{7}{2}$

C $x = \frac{2}{3}$

D $x = -\frac{2}{3}$

Solution: B

In this expression, the argument is a fraction, but we'll be able to evaluate it by solving the exponential equation

$$9^x = \frac{1}{2{,}187}$$

Now $9 = 3^2$, and $2{,}187$ can also be expressed as a power of 3.

$$2{,}187 = 3 \cdot 729$$

$$2{,}187 = 3 \cdot 3 \cdot 243$$

$$2{,}187 = 3 \cdot 3 \cdot 3 \cdot 81$$

$$2{,}187 = 3^3 \cdot 81$$

$$2{,}187 = 3^3 \cdot 9 \cdot 9$$

$$2{,}187 = 3^3 \cdot 3^2 \cdot 3^2$$

$$2{,}187 = 3^7$$

Therefore, our equation becomes

$$(3^2)^x = \frac{1}{3^7}$$

$$3^{2x} = \frac{1}{3^7}$$

Using the negative exponent rule, we'll move 3^7 to the numerator by changing the sign of the exponent.

$$3^{2x} = 3^{-7}$$

Now that the bases are equal, the exponents must be equal as well.

$$2x = -7$$

$$x = -\frac{7}{2}$$

This tells us that

$$\log_9\left(\frac{1}{2{,}187}\right) = -\frac{7}{2}$$