An Investigation of the Effect of Deck Size on the Probability of Winning a Hand in the Game of War

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Abstract

The card Game of War is a simple game in which players flip cards, and the player with the higher card takes both flipped cards. This study aims to determine the correlation, if one exists, between the number of cards a player possesses and their chance of winning each duel. Python was used to code the game and data collection and to output the results in a graphical format. Graphs showed an initial and final negative correlation and a zero medial correlation. In summary, the results showed a correlation, but one dominated by the game's randomness when only a few games were played.

Background and Introduction

The game of War is a card game where players flip over the top card of their deck, called a duel in this study, and the player with the higher card takes both and places them at the bottom of their deck. In the case of a tie, both players place three cards face-down in the middle, then flip the fourth card over, and the player that flips the higher card wins all the cards in play. The variation I will use for this study is that instead of the winning player immediately placing the cards on the bottom of their deck, they put their winnings in a discard pile. They shuffle their discard pile, and it becomes their deck when their deck has run out of cards. There are very few studies on the card game of War and none that analyze the probability of winning a duel based on the number of cards in a player's possession (the combined size of the player's deck and discard pile); thus, this study aims to shed light on the correlation between the number of cards possessed and the probability of winning a duel. I hypothesized that as the number of cards in a player's possession decreases, their likelihood of winning the duel increases. The objective of this study is to measure the cards in a player's possession and the outcome of the succeeding duel to view any potential correlations that could exist.

Methods

This study follows a computer-simulated model since this version of the card game of War is entirely deterministic and random, i.e. there is no player strategy involved, making it a choice game for a computer to simulate. Additionally, an average card game of War can take between 10 and 20 minutes to play, and playing enough games to find a correlation would take a classical study a very long time, while a computer-based one would take very little time, comparatively. An original Python function that simulated a card game of War between two players was coded, along with parallel lists that tracked the size of a player's possessed cards and the outcome of the corresponding duel: 0 for a loss or tie, 1 for a win. A main class was coded to run the war function a set number of times for multiple trials (10000, 50000, and 100000), unpack the information into a usable format, separate the wins and non-wins, and calculate the experimental probability of a win and a non-win from equations **(1)** and **(2)**:

(1) $$Probability\ of\ Win\ with\ x\ Cards\ Possessed = \frac{Total\ Wins\ with\ x\ Cards\ Possessed}{Total\ Instances\ of\ x\ Cards\ Posseessed}$$

(2) $$Probability\ of\ Non-win\ with\ x\ Cards\ Possessed = \frac{Total\ Non\text{-}wins\ with\ x\ Cards\ Possessed}{Total\ Instances\ of\ x\ Cards\ Posseessed}$$

In this case, the variable x represents the player's number of cards. The final job of the main class is to graph the total occurrences, total wins, and total non-wins vs. the number of cards possessed, the relative probability of a win vs. the number of cards possessed, and the relative probability of a non-win vs. the number of cards possessed. The program will output the data in graphical format.

Results

10000 Game Trial:

Figures 1, 2, and 3 show the total occurrences of each outcome, the probability of a win, and the probability of a non-win, all plotted against the size of possession.

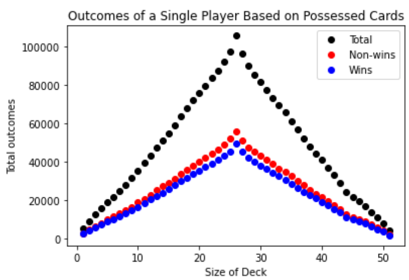

Figure 1: Total occurrences of each outcome plotted against the size of the player's possession

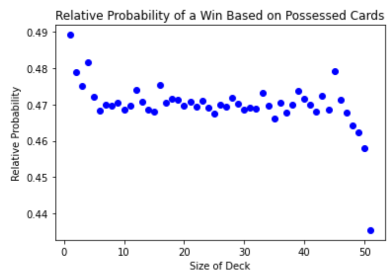

Figure 2: Relative probability of a win plotted against the size of the player's possession

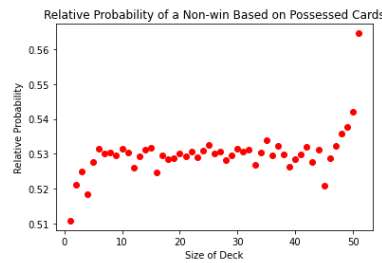

Figure 3: Relative probability of a non-win plotted against the size of the player's possession

50000 Game Trial:

Figures 4, 5, and 6 again show the total occurrences of each outcome, the probability of a win, and the probability of a non-win, all plotted against the size of possession.

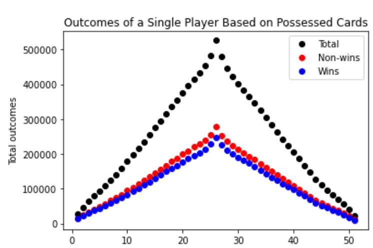

Figure 4: Total occurrences of each outcome plotted against the size of the player's possession

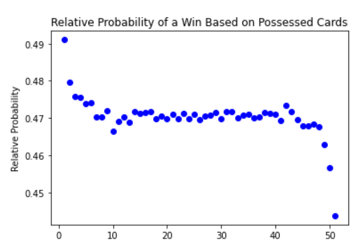

Figure 5: Relative probability of a win plotted against the size of the player's possession

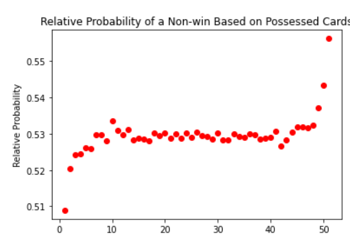

Figure 6: Relative probability of a non-win plotted against the size of the player's possession

100000 Game Trial:

Figures 7, 8, and 9 also show the total occurrences of each outcome, the probability of a win, and the probability of a non-win, all plotted against the size of possession.

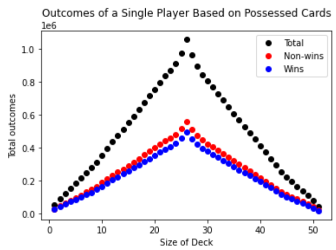

Figure 7: Total occurrences of each outcome plotted against the size of the player's possession

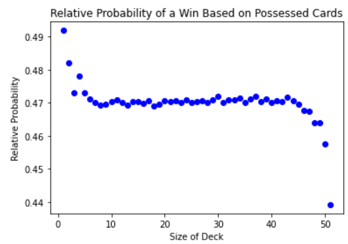

Figure 8: Relative probability of a win plotted against the size of the player's possession

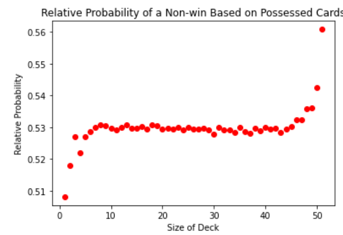

Figure 9: Relative probability of a non-win plotted against the size of the player's possession

Discussion

In the 10000 Game Trial, an initial negative correlation exists that evens out closer to the middle of the graph and then comes back near the end. Still, the nature of the correlation is unclear due to the disorganized nature of the data points. As the number of games per trial increased, the graph became tighter, as is expected if a correlation exists. In the 50000 and 100000 Game Trials, we see that the correlation is non-linear. The shape of this graph shows an elevated chance of winning at a smaller size of possession, along with a corresponding lower chance of winning at a larger size of possession, as hypothesized; however, the effect of the probability boosting is insignificant, as it only has a tangible impact at a possession size of 5 cards or fewer, and the increased likelihood of a win is between 0.02 and 0.03. This could be because, as possession size decreases, a player's weaker cards will lose more of the time, and the stronger ones will win more. This effect selects for the stronger cards to remain. In contrast, the weaker ones go to the opponent, which means that the probability that the player with a few strong cards remaining will increase while the probability of the player with many cards and their strong ones diluted by all their weak cards will decrease. Another thing to note is that, in the trials with fewer games, the volatility of the graph goes up. Considering how volatile the 10,000 Game graph is, we can guess that graphs of any values smaller than that would be too random to show a coherent trend.

Conclusion

This study proves that you will likely win more at a vast sample size when you have very few cards. While interesting, this point has very little practical use, as people are unlikely to play very many Games of War, and the correlation between cards in a player's possession gets dominated significantly by the randomness of the game when the number of games decreases to within the realm of possibility that human players would play. In conclusion, this study determined that a correlation between a player's size of possession and their likelihood of winning exists, but that it has little to no practical use.