## Identities

If A, B, C are angles of a triangle, then

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$$

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

$$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$$

$$\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$$

$$\cos^2 A + \cos^2 B + \cos^2 C + 2\cos A \cos B \cos C = 1$$

$$\sin^2 A + \sin^2 B + \sin^2 C - 2\cos A \cos B \cos C = 2$$

$$\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\frac{B+C}{4}\sin\frac{C+A}{4}\sin\frac{A+B}{4}$$

$$\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\frac{B+C}{4}\cos\frac{C+A}{4}\cos\frac{A+B}{4}$$

## Inequalities

If A, B, C are angles of a triangle, then

$$0 < \sin A + \sin B + \sin C \le \frac{3\sqrt{3}}{2}$$

$$1 < \cos A + \cos B + \cos C \le \frac{3}{2}$$

$$\cot^2 A + \cot^2 B + \cot^2 C \ge 1$$

$$\tan^2 A + \tan^2 B + \tan^2 C \ge 9$$

$$\tan A + \tan B + \tan C \ge 3\sqrt{3}$$
(A, B, C acute)

$$\cot A + \cot B + \cot C \ge \sqrt{3}$$
(A, B, C acute)

$$\cos A \cos B \cos C \le \frac{1}{8}$$

$$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}$$

$$\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \ge 1$$

$$\csc\frac{A}{2} + \csc\frac{B}{2} + \csc\frac{C}{2} \ge 6$$