DOUBLEROOT

Cheat Sheet – Quadratic Equations

Roots of $ax^2 + bx + c = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of Roots of $ax^2 + bx + c = 0$

Real & Distinct

$b^2 - 4ac > 0$

Real & Equal

$b^2 - 4ac = 0$

Complex / Imaginary

$b^2 - 4ac < 0$

Rational

$a, b, c \in Q$; $b^2 - 4ac \longrightarrow$ perfect square

Integers

$a = 1$; $b, c \in Z$; $b^2 - 4ac \longrightarrow$ perfect square

Relation between roots and coefficients

Sum of the roots: $\alpha + \beta = -b/a$

Product of the roots: $\alpha\beta = c/a$

Common Roots

Equations: $ax^2 + bx + c = 0$ & $px^2 + qx + r = 0$

One root common

$$\frac{\alpha^2}{br - cq} = \frac{\alpha}{cp - ar} = \frac{1}{aq - bp}$$

where α is the common root

Both roots common

$a/p = b/q = c/r$

Range of a quadratic function: $ax^2 + bx + c$

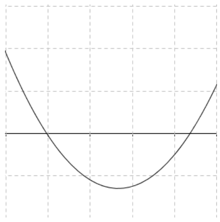
Condition	Range
$a > 0$	$[-D/4a, \infty)$
$a < 0$	$(-\infty, -D/4a]$

Sign of a quadratic function: $ax^2 + bx + c$

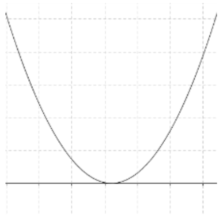
Condition	Sign
$a > 0, D > 0$	$> 0\ \forall\ x \in (-\infty, \alpha) \cup (\beta, \infty)$; $< 0\ \forall\ x \in (\alpha, \beta)$
$a > 0, D = 0$	$\geq 0\ \forall\ x \in R$
$a > 0, D < 0$	$> 0\ \forall\ x \in R$
$a < 0, D > 0$	$> 0\ \forall\ x \in (\alpha, \beta)$; $< 0\ \forall\ x \in (-\infty, \alpha) \cup (\beta, \infty)$
$a < 0, D = 0$	$\leq 0\ \forall\ x \in R$
$a < 0, D < 0$	$< 0\ \forall\ x \in R$

$D = b^2 - 4ac$

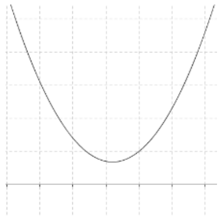
Graph of a quadratic function $f(x) = ax^2 + bx + c$



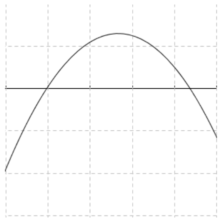
$a > 0; D > 0$



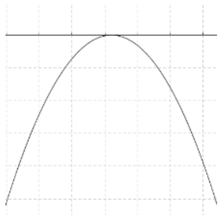
$a > 0; D = 0$



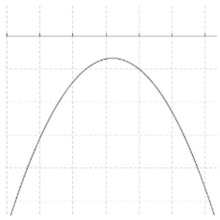
$a > 0; D < 0$



$a < 0; D > 0$



$a < 0; D = 0$



$a < 0; D < 0$

Location of Roots of $f(x) = ax^2 + bx + c$

Both roots positive

$D \geq 0$; $a.f(0) > 0$; $-b/2a > 0$

Both roots negative

$D \geq 0$; $a.f(0) > 0$; $-b/2a < 0$

Opposite signs

$a.f(0) < 0$

Equal and opposite signs

$D > 0$; $b = 0$

'k' lies between the roots

$a.f(k) < 0$

Both roots greater than 'k'

$D \geq 0$; $a.f(k) > 0$; $-b/2a > k$

Both roots less than 'k'

$D \geq 0$; $a.f(k) > 0$; $-b/2a < k$

Both roots lie inside the interval (k_1, k_2)

$D \geq 0$; $a.f(k_1) > 0$; $a.f(k_2) > 0$; $k_1 < -b/2a < k_2$

Exactly one root lies in the interval (k_1, k_2) if

$f(k_1).f(k_2) < 0$

One root smaller than 'k_1', other greater than 'k_2'

$a.f(k_1) < 0$; $a.f(k_2) < 0$