DOUBLEROOT

Definition

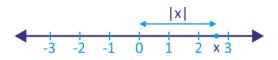
$$|x| = \sqrt{x^2} = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$$

Examples

$$|3| = 3$$
; $|0| = 0$; $|-4| = 4$

Geometrical Interpretation

|x|: Distance of x from the origin



Properties

 $|x| \ge 0 \ \forall \ x \in R$

$$||\mathbf{x}|| = |\mathbf{x}| = |-\mathbf{x}|$$

$$|xy| = |x||y|$$

$$|\mathbf{x}^{\mathbf{n}}| = |\mathbf{x}|^{\mathbf{n}}$$

$$\left|\frac{\mathbf{x}}{\mathbf{y}}\right| = \frac{|\mathbf{x}|}{|\mathbf{y}|} \ (\mathbf{y} \neq \mathbf{0})$$

$$|x + y| \le |x| + |y|$$

Equality holds when $xy \ge 0$

$$|x - y| \ge ||x| - |y||$$

Equality holds when $xy \ge 0$

$$|x - y| \le |x| + |y|$$

Equality holds when $xy \ge 0$

$$|x + y| \ge ||x| - |y||$$

Equality holds when $xy \le 0$

$$|x - y| \le |x - z| + |z - y|$$

Equality holds when $x \le z \le y$

$$\frac{\mathrm{d}}{\mathrm{d}x}|\mathbf{x}| = \frac{\mathbf{x}}{|\mathbf{x}|} \ (\mathbf{x} \neq \mathbf{0})$$

$$\int |x| dx = \frac{x|x|}{2} + c$$

Cheat Sheet – Modulus Function

Equations / Inequations

Equation	$ \mathbf{x} = \mathbf{a}$		
Condition	a > 0	a = 0	a < 0
Solution	$x = \pm a$	x = 0	x ∈ ф

Inequation	$ \mathbf{x} \geq a$		
Condition	a > 0	a = 0	a < 0
Solution	$x \in (-\infty, -a] \cup [a, \infty)$	$x \in R$	$x \in R$

Inequation	$ \mathbf{x} > \mathbf{a}$		
Condition	a > 0	a = 0	a < 0
Solution	$x \in (-\infty, -a) \cup (a, \infty)$	$x \in R - \{0\}$	$x \in R$

Inequation		$ \mathbf{x} \leq \mathbf{a}$	
Condition	a > 0	a = 0	a < 0
Solution	$x \in [-a, a]$	x = 0	$x \in \phi$

Inequation	$ \mathbf{x} < \mathbf{a}$		
Condition	a > 0	a = 0	a < 0
Solution	$x \in (-a, a)$	x ∈ φ	$x \in \varphi$

Graph

