

Exercise 2.1: Polynomials

Q.1: Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes and their coefficients:

(i) $f(x) = x^2 - 2x - 8$

(ii) $g(s) = 4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $h(t) = t^2 - 15$

(v) $p(x) = x^2 + 2\sqrt{2}x - 6$

(vi) $q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$

(vii) $f(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

(viii) $g(x) = a(x^2 + 1) - x(a^2 + 1)$

Solution:

(i) $f(x) = x^2 - 2x - 8$

We have,

$$\begin{aligned}
 f(x) &= x^2 - 2x - 8 \\
 &= x^2 - 4x + 2x - 8 \\
 &= x(x-4) + 2(x-4) \\
 &= (x+2)(x-4)
 \end{aligned}$$

Zeros of the polynomials are -2 and 4.

Now,

$$\text{Sum of the zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$-2 + 4 = -\frac{-2}{1}$$

$$2 = 2$$

$$\text{Product of the zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$-8 = -\frac{8}{1}$$

$$-8 = -8$$

Hence, the relationship is verified.

$$(ii) \quad g(s) = 4s^2 - 4s + 1$$

We have,

$$\begin{aligned}
 g(s) &= 4s^2 - 4s + 1 \\
 &= 4s^2 - 2s - 2s + 1 \\
 &= 2s(2s-1) - 1(2s-1) \\
 &= (2s-1)(2s-1)
 \end{aligned}$$

Zeroes of the polynomials are $12\frac{1}{2}$ and $12\frac{1}{2}$.

$$\text{Sum of zeroes} = \frac{-\text{coefficient of } s}{\text{coefficient of } s^2}$$

$$12 + 12 = -(-4)4\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$$

$$1 = 1$$

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{Coefficient of } x}$$

$$12 \times 12 = 14\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \quad 14 = 14\frac{1}{4} = \frac{1}{4}$$

Hence, the relationship is verified.

$$\text{(iii) } 6s^2 - 3 - 7x6s^2 - 3 - 7x$$

$$= 6s^2 - 7x - 36s^2 - 7x - 3 = (3x + 11)(2x - 3)$$

Zeros of the polynomials are 32 and $-13\frac{3}{2}$ and $\frac{-1}{3}$

$$\text{Sum of the zeros} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$-13 + 32 = -(-7)6\frac{-1}{3} + \frac{3}{2} = \frac{-(-7)}{6} \quad 76 = 76\frac{7}{6} = \frac{7}{6}$$

$$\text{Product of the zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$-13 \times 32 = -36\frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6} \quad -36 = -36\frac{-3}{6} = \frac{-3}{6}$$

Hence, the relationship is verified.

$$\text{(iv) } h(t) = t^2 - 15h(t) = t^2 - 15$$

We have,

$$h(t) = t^2 - 15h(t) = t^2 - 15$$

$$= t^2 - \sqrt{15}t^2 - \sqrt{15}$$

$$= (t + \sqrt{15})(t - \sqrt{15})(t + \sqrt{15})(t - \sqrt{15})$$

Zeros of the polynomials are $-\sqrt{15}$ and $\sqrt{15}$ and $\sqrt{15}$ and $-\sqrt{15}$

Sum of the zeroes = 0

$$-\sqrt{15} + \sqrt{15} - \sqrt{15} + \sqrt{15} = 0$$

$$0 = 0$$

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{Coefficient of } x} = -15 \frac{-15}{1}$$

$$-\sqrt{15} \times \sqrt{15} = -15 \times \sqrt{15} \times \sqrt{15} = -15$$

$$-15 = -15$$

Hence, the relationship verified.

$$(v) p(x) = x^2 + 2\sqrt{2}x - 6$$

We have,

$$p(x) = x^2 + 2\sqrt{2}x - 6$$

$$= x^2 + 3\sqrt{2}x + 3\sqrt{2}x - 6x^2 + 3\sqrt{2}x + 3\sqrt{2}x - 6$$

$$= x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2})x (x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2})$$

$$= (x + 3\sqrt{2})(x - \sqrt{2})(x + 3\sqrt{2})(x - \sqrt{2})$$

Zeros of the polynomials are $3\sqrt{2}$ and $-\sqrt{2}$ and $-\sqrt{2}$ and $3\sqrt{2}$

$$\text{Sum of the zeroes} = -2\sqrt{2} \frac{-2\sqrt{2}}{1}$$

$$\sqrt{2} - 3\sqrt{2} = -2\sqrt{2} \sqrt{2} - 3\sqrt{2} = -2\sqrt{2} - 2\sqrt{2} = -2\sqrt{2}$$

$$\text{Product of the zeroes} = \frac{\text{constant term}}{\text{Coefficient of } x}$$

$$\sqrt{2} \times -3\sqrt{2} = -6 \sqrt{2} \times -3\sqrt{2} = \frac{-6}{1}$$

$$-6 = -6$$

Hence, the relationship is verified.

$$(vi) \ q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$$

$$= \sqrt{3}x^2 + 7x + 3x + 7\sqrt{3}$$

$$= \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3})$$

$$= (x + \sqrt{3})(7 + \sqrt{3})$$

Zeros of the polynomials are $-\sqrt{3}$ and $-\frac{7}{\sqrt{3}}$

$$\text{Sum of zeros} = -\sqrt{3} - \frac{7}{\sqrt{3}}$$

$$-\sqrt{3} - \frac{7}{\sqrt{3}} = -\frac{3 + 7}{\sqrt{3}} = -\frac{10}{\sqrt{3}}$$

Product of the polynomials are $-\sqrt{3} \cdot -\frac{7}{\sqrt{3}} = 7$

$$7 = 7$$

Hence, the relationship is verified.

$$(vii) \ h(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$$

$$= x^2 - \sqrt{3}x - x + \sqrt{3}$$

$$= x(x - \sqrt{3}) - 1(x - \sqrt{3})$$

$$= (x - \sqrt{3})(x - 1)$$

Zeros of the polynomials are 1 and $\sqrt{3}$

$$\text{Sum of zeros} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{-(\sqrt{3} + 1)}{1} = \sqrt{3} + 1$$

$$1 + \sqrt{3} = \sqrt{3} + 1$$

$$\text{Product of zeros} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = \sqrt{3} \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = \sqrt{3}$$

$$\sqrt{3} = \sqrt{3} \sqrt{3} = \sqrt{3}$$

Hence, the relationship is verified

$$\text{(viii) } g(x) = a[(x^2+1)-x(a^2+1)]^2 a [(x^2+1)-x(a^2+1)]^2$$

$$= ax^2 + a - a^2x - xax^2 + a - a^2x - x$$

$$= ax^2 - [(a^2x+1)] + aax^2 - [(a^2x+1)] + a$$

$$= ax^2 - a^2x - x + aax^2 - a^2x - x + a$$

$$= ax(x-a) - 1(x-a)ax(x-a) - 1(x-a) = (x-a)(ax-1)(x-a)(ax-1)$$

Zeros of the polynomials are $\frac{1}{a}$ and 1 and 1

$$\text{Sum of the zeros} = a[-a^2-1]a \frac{a[-a^2-1]}{a}$$

$$\frac{1}{a} + a = \frac{a^2+1}{a} \quad a^2 + \frac{1}{a} = \frac{a^2+1}{a} \quad \frac{1}{a} + a = \frac{a^2+1}{a} = \frac{a^2+1}{a}$$

Product of zeros = a/a

$$\frac{1}{a} \times a = a \frac{1}{a} \times a = \frac{a}{a}$$

$$1 = 1$$

Hence, the relationship is verified.

Q.2: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 5x + 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution: We have,

α and β are the roots of the quadratic polynomial.

$$f(x) = x^2 - 5x + 4$$

$$\text{Sum of the roots} = \alpha + \beta = 5$$

$$\text{Product of the roots} = \alpha\beta = 4$$

So,

$$1\alpha + 1\beta - 2\alpha\beta \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \beta + \alpha - 2\alpha\beta \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta$$

$$= 5 - 2 \times 4 = 5 - 8 = -3 \quad \frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8 = \frac{-27}{4}$$

Q.3: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 5x + 4$, find the value of $1\alpha + 1\beta - 2\alpha\beta \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial.

$$p(y) = x^2 - 5x + 4$$

$$\text{Sum of the zeroes} = \alpha + \beta = 5$$

$$\text{Product of the roots} = \alpha\beta = 4$$

So,

$$1\alpha + 1\beta - 2\alpha\beta \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$$

$$= \beta + \alpha - 2\alpha\beta \frac{\beta + \alpha}{\alpha\beta}$$

$$= (\alpha + \beta) - 2(\alpha\beta) \frac{(\alpha + \beta) - 2(\alpha\beta)}{\alpha\beta}$$

$$= (5) - 2(4) \frac{(5) - 2(4)}{4}$$

$$= 5 - 2 \times 4 \frac{5 - 8}{4} = 5 - 2 \times 4 \frac{-3}{4} = 5 - 2 \times (-3) = 5 + 6 = 11$$

Q.4: If α and β are the zeroes of the quadratic polynomial $p(y)=5y^2-7y+1$, find the value of $\alpha + \beta \frac{1}{\alpha} + \frac{1}{\beta}$.

Solution: Since, α and β are the zeroes of the quadratic polynomial.

$$p(y)=5y^2-7y+1$$

$$\text{Sum of the zeroes} = \alpha + \beta = 7$$

$$\text{Product of the roots} = \alpha\beta = 1$$

So,

$$\alpha + \beta \frac{1}{\alpha} + \frac{1}{\beta} = \alpha + \beta \frac{\alpha + \beta}{\alpha\beta} = 7 + \frac{7}{1} = 14$$

Q.5: If α and β are the zeroes of the quadratic polynomial $f(x)=x^2-x-4$, find the value of $\alpha + \beta - \alpha\beta \frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial.

We have,

$$f(x)=x^2-x-4$$

$$\text{Sum of zeroes} = \alpha + \beta = 1$$

$$\text{Product of the zeroes} = \alpha\beta = -4$$

So,

$$\alpha + \beta - \alpha\beta \frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \alpha + \beta - \alpha\beta \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta$$

$$= 1 - 4 - (-4) = -1 + 4 = 3$$

$$= -1 + 16 \frac{-1+16}{4} = 15 \frac{15}{4}$$

Q.6: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + x - 2$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial.

We have,

$$f(x) = x^2 + x - 2$$

$$\text{Sum of zeroes} = \alpha + \beta = 1$$

$$\text{Product of the zeroes} = \alpha\beta = -2$$

So,

$$\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta - \alpha}{\alpha\beta}$$

$$= \frac{\beta - \alpha}{\alpha\beta}$$

$$= \frac{\beta - \alpha}{\alpha\beta} \times \frac{(\alpha - \beta)}{(\alpha - \beta)} = \frac{\beta - \alpha}{\alpha\beta} \times \frac{-(\alpha - \beta)}{-(\alpha - \beta)} = \frac{\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}}{\alpha\beta} = \frac{\sqrt{1 + 8}}{\alpha\beta} = \frac{\sqrt{9}}{\alpha\beta} = \frac{3}{\alpha\beta}$$

$$= \frac{\sqrt{1+8}}{2} = \frac{\sqrt{9}}{2} = \frac{3}{2}$$

Q.7: If one of the zero of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is negative of the other, then find the value of k.

Solution:

Let, the two zeroes of the polynomial $f(x) = 4x^2 - 8kx - 9$ be α and $-\alpha$.

$$\text{Product of the zeroes} = \alpha \times -\alpha = -\alpha^2 = -9$$

$$\text{Sum of the zeroes} = \alpha + (-\alpha) = -8k = 0$$

$$\text{Since, } \alpha - \alpha = 0$$

$$\text{Since, } \alpha - \alpha = 0$$

$$\Rightarrow 8k=0 \Rightarrow 8k = 0 \Rightarrow k=0 \Rightarrow k = 0$$

Q.8: If the sum of the zeroes of the quadratic polynomial $f(t)=kt^2+2t+3k$ is equal to their product, then find the value of k .

Solution: Let the two zeroes of the polynomial $f(t)=kt^2+2t+3k$ be α and β .

$$\text{Sum of the zeroes} = \alpha + \beta = 2$$

$$\text{Product of the zeroes} = \alpha \times \beta = 3k$$

Now,

$$-2k = 3k \Rightarrow \frac{-2}{k} = \frac{3k}{k} \Rightarrow 3k = -2 \Rightarrow k = -\frac{2}{3}$$

$$\text{So, } k = 0 \text{ and } k = -\frac{2}{3}$$

Q.9: If α and β are the zeroes of the quadratic polynomial $p(x)=4x^2-5x-1$, find the value of $\alpha^2\beta + \alpha\beta^2$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial $p(x)=4x^2-5x-1$

$$\text{So, Sum of the zeroes} = \alpha + \beta = \frac{5}{4}$$

$$\text{Product of the zeroes} = \alpha \times \beta = -\frac{1}{4}$$

Now,

$$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$$

$$= \left(\frac{5}{4}\right) \left(-\frac{1}{4}\right)$$

$$= -\frac{5}{16}$$

Q.10: If α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - 4t + 3$

So, Sum of the zeroes = $\alpha + \beta = 4$

Product of the zeroes = $\alpha \times \beta = 3$

Now,

$$\begin{aligned}\alpha^4\beta^3 + \alpha^3\beta^4 &= \alpha^3\beta^3(\alpha + \beta) \\ &= (3)^3(4)(3) = 108\end{aligned}$$

Q.11: If α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$, find the value of $\alpha\beta + \beta\alpha \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$

Sum of the zeroes = $\alpha + \beta = -\frac{1}{6}$

Product of the zeroes = $\alpha \times \beta = -\frac{1}{3}$

Now,

$$\begin{aligned}\alpha\beta + \beta\alpha \frac{\alpha}{\beta} + \frac{\beta}{\alpha} &= (\alpha^2 + \beta^2) - 2\alpha\beta \frac{(\alpha^2 + \beta^2) - 2\alpha\beta}{\alpha\beta}\end{aligned}$$

By substitution the values of the sum of zeroes and product:

$$= -2512 \frac{-25}{12}$$

Q.12: If α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$, find the value of $\alpha\beta + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$.

$$\text{Sum of the zeroes} = \alpha + \beta = -\frac{1}{6}$$

$$\text{Product of the zeroes} = \alpha \times \beta = -\frac{2}{6} = -\frac{1}{3}$$

Now,

$$\alpha\beta + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$$

$$= \alpha^2\beta + \beta^2\alpha + 2\left[\frac{\alpha^2 + \beta^2}{\alpha\beta}\right] + 3\alpha\beta$$

$$= (\alpha + \beta)^2 - 2\alpha\beta + 2\left[\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right] + 3\alpha\beta$$

By substituting the values of sum and product of the zeroes, we will get

$$\alpha\beta + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta = 8$$

Q.13: If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Solution:

Let the two zeroes of the polynomial be α and β .

We have,

$$f(x) = x^2 + px + 45 \quad f(x) = x^2 + px + 45$$

Now,

$$\text{Sum of the zeroes} = \alpha + \beta = -p$$

$$\text{Product of the zeroes} = \alpha \times \beta = 45$$

So,

$$(\alpha + \beta)^2 - 4\alpha\beta = 144 \quad (\alpha + \beta)^2 - 4\alpha\beta = 144 \quad (p)^2 - 4 \times 45 = 144 \quad (p)^2 - 4 \times 45 = 144$$

$$(p)^2 = 144 + 180 \quad (p)^2 = 144 + 180 \quad (p)^2 = 324 \quad (p)^2 = 324 \quad p = \sqrt{324} \quad p = \sqrt{324} \quad p = \pm 18$$

Thus, in the given equation, p will be either 18 or -18.

Q.14: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$

$$f(x) = x^2 - px + q, \text{ prove that } \alpha^2\beta^2 + \beta^2\alpha^2 = p^4q^2 - 4p^2q + 2 \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2.$$

Solution:

Since, α and β are the roots of the quadratic polynomial given in the question.

$$f(x) = x^2 - px + q \quad f(x) = x^2 - px + q$$

Now,

$$\text{Sum of the zeroes} = p = \alpha + \beta$$

$$\text{Product of the zeroes} = q = \alpha \times \beta$$

$$\text{LHS} = \alpha^2\beta^2 + \beta^2\alpha^2 \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$$

$$= \alpha^4 + \beta^4 \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2}$$

$$= (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 \frac{(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 \frac{[(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$\begin{aligned}
&= [(p)^2 - 2q]^2 - 2(q)^2(q)^2 \frac{[(p)^2 - 2q]^2 - 2(q)^2}{(q)^2} \\
&= (p^4 + 4q^2 - 4p^2q) - 2q^2q^2 \frac{(p^4 + 4q^2 - 4p^2q) - 2q^2}{q^2} \\
&= p^4 + 2q^2 - 4p^2q \frac{p^4 + 2q^2 - 4p^2q}{q^2} \\
&= p^2q^2 + 2 - 4p^2q \frac{p^2}{q^2} + 2 - \frac{4p^2}{q} \\
&= p^2q^2 - 4p^2q + 2\frac{p^2}{q^2} - \frac{4p^2}{q} + 2
\end{aligned}$$

LHS = RHS

Hence, proved.

Q.15: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - p(x+1) - c$, show that $(\alpha+1)(\beta+1) = 1 - c$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial

$$f(x) = x^2 - p(x+1) - c$$

Now,

$$\text{Sum of the zeroes} = \alpha + \beta = p$$

$$\text{Product of the zeroes} = \alpha \times \beta = (-p - c)$$

So,

$$(\alpha+1)(\beta+1)$$

$$= \alpha\beta + \alpha + \beta + 1$$

$$= \alpha\beta + (\alpha + \beta) + 1$$

$$= (-p - c) + p + 1$$

$$= 1 - c = \text{RHS}$$

So, LHS = RHS

Hence, proved.

Q.16: If α and β are the zeroes of the quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$, find a quadratic polynomial having α and β as its zeroes.

Solution:

We have,

$$\alpha + \beta = 24 \quad \text{.....E-1}$$

$$\alpha - \beta = 8 \quad \text{.....E-2}$$

By solving the above two equations accordingly, we will get

$$2\alpha = 32 \quad \alpha = 16$$

Substitute the value of α , in any of the equation. Let we substitute it in E-2, we will get

$$\beta = 16 - 8 \quad \beta = 8$$

Now,

$$\text{Sum of the zeroes of the new polynomial} = \alpha + \beta = 16 + 8 = 24$$

$$\text{Product of the zeroes} = \alpha\beta = 16 \times 8 = 128$$

Then, the quadratic polynomial is-

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes}) = x^2 - 24x + 128$$

$$\text{Hence, the required quadratic polynomial is } f(x) = x^2 + 24x + 128$$

Q.17: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 1$

find a quadratic polynomial whose zeroes are 2α and 2β .

Solution:

We have,

$$f(x) = x^2 - 1 \quad f(x) = x^2 - 1$$

$$\text{Sum of the zeroes} = \alpha + \beta \quad \alpha + \beta = 0$$

$$\text{Product of the zeroes} = \alpha\beta \quad \alpha\beta = -1$$

From the question,

$$\text{Sum of the zeroes of the new polynomial} = 2\alpha\beta \text{ and } 2\beta\alpha \frac{2\alpha}{\beta} \text{ and } \frac{2\beta}{\alpha}$$

$$= 2\alpha^2 + 2\beta^2 \alpha\beta \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$$

$$= 2(\alpha^2 + \beta^2) \alpha\beta \frac{2(\alpha^2 + \beta^2)}{\alpha\beta}$$

$$= 2((\alpha + \beta)^2 - 2\alpha\beta) \alpha\beta \frac{2((\alpha + \beta)^2 - 2\alpha\beta)}{\alpha\beta}$$

$$= 2(2)^2 - 1 \frac{2(2)^2 - 1}{-1} \quad \{ \text{By substituting the value of the sum and products of the zeroes} \}$$

As given in the question,

$$\text{Product of the zeroes} = (2\alpha)(2\beta) \alpha\beta \frac{(2\alpha)(2\beta)}{\alpha\beta} = 4\alpha\beta \alpha\beta \frac{4\alpha\beta}{\alpha\beta} = 4$$

Hence, the quadratic polynomial is

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$= kx^2 - (-4)x + 4x^2 - (-4)x + 4 = x^2 + 4x + 4x^2 + 4x + 4$$

$$\text{Hence, the required quadratic polynomial is } f(x) = x^2 + 4x + 4 \quad f(x) = x^2 + 4x + 4$$

Q.18: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 3x - 2$

$f(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are $12\alpha + \beta$ and $12\beta + \alpha$

$$\frac{1}{2\alpha + \beta} \text{ and } \frac{1}{2\beta + \alpha}.$$

Solution:

We have,

$$f(x) = x^2 - 3x - 2 \quad f(x) = x^2 - 3x - 2$$

$$\text{Sum of the zeroes} = \alpha + \beta \quad \alpha + \beta = 3$$

$$\text{Product of the zeroes} = \alpha\beta \quad \alpha\beta = -2$$

From the question,

$$\text{Sum of the zeroes of the new polynomial} = 12\alpha + \beta + 12\beta + \alpha \frac{1}{2\alpha + \beta} + \frac{1}{2\beta + \alpha}$$

$$= 2\beta + \alpha + 2\alpha + \beta(2\alpha + \beta)(2\beta + \alpha) \frac{2\beta + \alpha + 2\alpha + \beta}{(2\alpha + \beta)(2\beta + \alpha)}$$

$$= 3\alpha + 3\beta + 2(\alpha^2 + \beta^2) + 5\alpha\beta \frac{3\alpha + 3\beta}{2(\alpha^2 + \beta^2) + 5\alpha\beta}$$

$$= 3 \times 3 + 2[2(\alpha + \beta)^2 - 2\alpha\beta + 5 \times (-2)] \frac{3 \times 3}{2[2(\alpha + \beta)^2 - 2\alpha\beta + 5 \times (-2)]}$$

$$= 9 + 2[9 - (-4)] - 10 \frac{9}{2[9 - (-4)] - 10}$$

$$= 9 + 2[13] - 10 \frac{9}{2[13] - 10}$$

$$= 9 + 26 - 10 \frac{9}{26 - 10} = 9 + 16 \frac{9}{16}$$

$$\text{Product of the zeroes} = 12\alpha + \beta \times 12\beta + \alpha \frac{1}{2\alpha + \beta} \times \frac{1}{2\beta + \alpha}$$

$$= 1(2\alpha + \beta)(2\beta + \alpha) \frac{1}{(2\alpha + \beta)(2\beta + \alpha)}$$

$$= 14\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta \frac{1}{4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta}$$

$$= 15\alpha\beta + 2(\alpha^2 + \beta^2) \frac{1}{5\alpha\beta + 2(\alpha^2 + \beta^2)}$$

$$= 15\alpha\beta + 2((\alpha + \beta)^2 - 2\alpha\beta) \frac{1}{5\alpha\beta + 2((\alpha + \beta)^2 - 2\alpha\beta)}$$

$$= 15 \times (-2) + 2((3)^2 - 2 \times (-2)) \frac{1}{5 \times (-2) + 2((3)^2 - 2 \times (-2))}$$

$$= 1 - 10 + 26 \frac{1}{-10 + 26} = 116 \frac{1}{16}$$

So, the quadratic polynomial is,

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$= k(x^2 + 916x + 116)\left(x^2 + \frac{9}{16}x + \frac{1}{16}\right)$$

$$\text{Hence, the required quadratic polynomial is } k(x^2 + 916x + 116)\left(x^2 + \frac{9}{16}x + \frac{1}{16}\right).$$

Q.19: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + px + q$

form a polynomial whose zeroes are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

Solution:

We have,

$$f(x) = x^2 + px + q$$

$$\text{Sum of the zeroes} = \alpha + \beta = -p$$

$$\text{Product of the zeroes} = \alpha\beta = q$$

From the question,

$$\text{Sum of the zeroes of new polynomial} = (\alpha + \beta)^2 + (\alpha - \beta)^2$$

$$= (\alpha + \beta)^2 + \alpha^2 + \beta^2 - 2\alpha\beta$$

$$(\alpha + \beta)^2 + \alpha^2 + \beta^2 - 2\alpha\beta$$

$$= (\alpha + \beta)^2 + (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta(\alpha + \beta)^2 + (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$$

$$= (-p)^2 + (-p)^2 - 2 \times q - 2 \times q(-p)^2 + (-p)^2 - 2 \times q - 2 \times q$$

$$= p^2 + p^2 - 4qp^2 + p^2 - 4q$$

$$= 2p^2 - 4q - 4q$$

$$\text{Product of the zeroes of new polynomial} = (\alpha + \beta)^2(\alpha - \beta)^2$$

$$= (-p)^2((-p)^2-4q)(-p)^2 \left((-p)^2-4q \right)$$

$$= p^2(p^2-4q)p^2 (p^2-4q)$$

So, the quadratic polynomial is ,

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$= x^2 - (2p^2-4q)x + p^2(p^2-4q)$$

Hence, the required quadratic polynomial is $f(x) = k(x^2 - (2p^2-4q)x + p^2(p^2-4q))$

$$f(x) = k(x^2 - (2p^2-4q)x + p^2(p^2-4q)).$$

Q.20: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 2x + 3$

find a polynomial whose roots are:

(i) $\alpha+2, \beta+2$

(ii) $\alpha-1, \beta-1$

Solution:

We have,

$$f(x) = x^2 - 2x + 3$$

$$\text{Sum of the zeroes} = \alpha + \beta = 2$$

$$\text{Product of the zeroes} = \alpha\beta = 3$$

(i) Sum of the zeroes of new polynomial = $(\alpha+2) + (\beta+2)$

$$= \alpha + \beta + 4$$

$$= 2 + 4 = 6$$

Product of the zeroes of new polynomial = $(\alpha+1)(\beta+1)$

$$= \alpha\beta + 2\alpha + 2\beta + 4\alpha\beta + 2\alpha + 2\beta + 4$$

$$= \alpha\beta + 2(\alpha + \beta) + 4\alpha\beta + 2(\alpha + \beta) + 4 = 3 + 2(2) + 4 + 2(2) + 4 = 11$$

So, quadratic polynomial is:

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$= x^2 - 6x + 11$$

Hence, the required quadratic polynomial is $f(x) = k(x^2 - 6x + 11)$

$$(ii) \text{ Sum of the zeroes of new polynomial} = \alpha - 1 + \beta - 1 + \frac{\alpha - 1}{\alpha + 1} + \frac{\beta - 1}{\beta + 1}$$

$$= (\alpha - 1)(\beta + 1) + (\beta - 1)(\alpha + 1) + \frac{(\alpha - 1)(\beta + 1) + (\beta - 1)(\alpha + 1)}{(\alpha + 1)(\beta + 1)}$$

$$= \alpha\beta + \alpha - \beta - 1 + \alpha\beta + \beta - \alpha - 1 + \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta + \beta - \alpha - 1}{(\alpha + 1)(\beta + 1)}$$

$$= 3 - 1 + 3 - 1 + 2 \frac{3 - 1 + 3 - 1}{3 + 1 + 2} = 4 + 2 \frac{4}{6} = 4 + \frac{4}{3} = \frac{16}{3}$$

$$\text{Product of the zeroes of new polynomial} = \alpha - 1 + \beta - 1 + \frac{\alpha - 1}{\alpha + 1} + \frac{\beta - 1}{\beta + 1}$$

$$= 26 - 13 \frac{2}{6} = \frac{1}{3}$$

So, the quadratic polynomial is,

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$= x^2 - 23x + 13$$

Thus, the required quadratic polynomial is $f(x) = k(x^2 - 23x + 13)$

Q.21: If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate:

$$(i) \alpha - \beta \alpha - \beta$$

$$(ii) 1\alpha - 1\beta \frac{1}{\alpha} - \frac{1}{\beta}$$

$$(ii) 1\alpha + 1\beta - 2\alpha\beta \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$$

$$(iv) \alpha^2\beta + \alpha\beta^2 \alpha^2\beta + \alpha\beta^2$$

$$(v) \alpha^4 + \beta^4 \alpha^4 + \beta^4$$

$$(vi) 1a\alpha + b + 1a\beta + b \frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$$

$$(vii) \beta a\alpha + b + \alpha a\beta + b \frac{\beta}{a\alpha + b} + \frac{\alpha}{a\beta + b}$$

$$(viii) a[\alpha^2\beta + \beta^2\alpha] + b[\alpha a + \beta a] a \left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} \right] + b \left[\frac{\alpha}{a} + \frac{\beta}{a} \right]$$

Solution:

$$f(x) = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

Here,

$$\text{Sum of the zeroes of polynomial} = \alpha + \beta \alpha + \beta = -ba \frac{-b}{a}$$

$$\text{Product of zeroes of polynomial} = \alpha\beta \alpha\beta = ca \frac{c}{a}$$

Since, $\alpha + \beta \alpha + \beta$ are the roots (or) zeroes of the given polynomial, so

$$(i) \alpha - \beta \alpha - \beta$$

The two zeroes of the polynomials are-

$$-b + \sqrt{b^2 - 4ac} 2a - (-b - \sqrt{b^2 - 4ac} 2a) \frac{-b + \sqrt{b^2 - 4ac}}{2a} - \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right)$$

$$= -b + \sqrt{b^2 - 4ac} + b + \sqrt{b^2 - 4ac} 2a \frac{-b + \sqrt{b^2 - 4ac} + b + \sqrt{b^2 - 4ac}}{2a}$$

$$= 2\sqrt{b^2-4ac} \cdot 2a \frac{2\sqrt{b^2-4ac}}{2a} = \sqrt{b^2-4ac} \cdot \frac{\sqrt{b^2-4ac}}{a}$$

$$(ii) \quad 1\alpha - 1\beta \quad \frac{1}{\alpha} - \frac{1}{\beta}$$

$$= \beta - \alpha \quad \beta = -(\alpha - \beta) \quad \alpha\beta \frac{\beta - \alpha}{\alpha\beta} = \frac{-(\alpha - \beta)}{\alpha\beta} \quad \dots E.1$$

From previous question we know that,

$$\alpha - \beta \quad \alpha - \beta = \sqrt{b^2-4ac} \cdot \frac{\sqrt{b^2-4ac}}{a}$$

Also,

$$\alpha\beta\alpha\beta = ca \quad \frac{c}{a}$$

Putting the values in E.1, we will get

$$\begin{aligned} & -(\sqrt{b^2-4ac} \cdot ca) - \left(\frac{\sqrt{b^2-4ac}}{\frac{a}{c}} \right) \\ & = -(\sqrt{b^2-4ac} \cdot cc) - \left(\frac{\sqrt{b^2-4ac}}{c} \right) \end{aligned}$$

$$(iii) \quad 1\alpha + 1\beta - 2\alpha\beta \quad \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$$

$$= [1\alpha + 1\beta] - 2\alpha\beta \left[\frac{1}{\alpha} + \frac{1}{\beta} \right] - 2\alpha\beta$$

$$= [\alpha + \beta] - 2\alpha\beta \left[\frac{\alpha + \beta}{\alpha\beta} \right] - 2\alpha\beta \quad \dots E-1$$

Since,

$$\text{Sum of the zeroes of polynomial} = \alpha + \beta \quad \alpha + \beta = -ba \quad \frac{-b}{a}$$

$$\text{Product of zeroes of polynomial} = \alpha\beta \quad \alpha\beta = ca \quad \frac{c}{a}$$

After substituting it in E-1, we will get

$$-ba \times ac - 2ca \cdot \frac{-b}{a} \times \frac{a}{c} - 2\frac{c}{a}$$

$$= -bc - 2ca - \frac{b}{c} - 2\frac{c}{a}$$

$$= -ab - 2c^2ac \frac{-ab - 2c^2}{ac}$$

$$= -[bc + 2ca] - \left[\frac{b}{c} + \frac{2c}{a} \right]$$

$$(iv) \alpha^2\beta + \alpha\beta^2 \alpha^2\beta + \alpha\beta^2$$

$$= \alpha\beta(\alpha + \beta)\alpha\beta(\alpha + \beta) \dots\dots\dots E-1.$$

Since,

$$\text{Sum of the zeroes of polynomial} = \alpha + \beta \alpha + \beta = -ba \frac{-b}{a}$$

$$\text{Product of zeroes of polynomial} = \alpha\beta\alpha\beta = ca \frac{c}{a}$$

After substituting it in E-1, we will get

$$ca(-ba) \frac{c}{a} \left(\frac{-b}{a} \right)$$

$$= -bca^2 \frac{-bc}{a^2}$$

$$(v) \alpha^4 + \beta^4 \alpha^4 + \beta^4$$

$$= (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

$$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - (2\alpha\beta)^2 ((\alpha + \beta)^2 - 2\alpha\beta)^2 - (2\alpha\beta)^2 \dots\dots\dots E-1$$

Since,

$$\text{Sum of the zeroes of polynomial} = \alpha + \beta \alpha + \beta = -ba \frac{-b}{a}$$

$$\text{Product of zeroes of polynomial} = \alpha\beta\alpha\beta = ca \frac{c}{a}$$

After substituting it in E-1, we will get

$$[(-ba) - 2(ca)]^2 - [2(ca)]^2 \left[\left(-\frac{b}{a} \right) - 2\left(\frac{c}{a} \right) \right]^2 - \left[2\left(\frac{c}{a} \right)^2 \right]$$

$$= [b^2 - 2aca^2]^2 - 2c^2a^2 \left[\frac{b^2 - 2ac}{a^2} \right]^2 - \frac{2c^2}{a^2}$$

$$= (b^2 - 2ac)^2 - 2a^2c^2a^4 \frac{(b^2 - 2ac)^2 - 2a^2c^2}{a^4}$$

$$(vi) \quad 1a\alpha + b + 1a\beta + b \frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$$

$$= a\beta + b + a\alpha + b(a\alpha + b)(a\beta + b) \frac{a\beta + b + a\alpha + b}{(a\alpha + b)(a\beta + b)}$$

$$= a(\alpha + \beta) + 2ba^2\alpha\beta + ab\alpha + ab\beta + b^2 \frac{a(\alpha + \beta) + 2b}{a^2\alpha\beta + ab\alpha + ab\beta + b^2}$$

$$= a(\alpha + \beta) + 2ba^2\alpha\beta + ab(\alpha + \beta) + b^2 \frac{a(\alpha + \beta) + 2b}{a^2\alpha\beta + ab(\alpha + \beta) + b^2}$$

Since,

$$\text{Sum of the zeroes of polynomial} = \alpha + \beta \quad \alpha + \beta = -ba \frac{-b}{a}$$

$$\text{Product of zeroes of polynomial} = \alpha\beta \quad \alpha\beta = ca \frac{c}{a}$$

After substituting it, we will get

$$bac - b^2 + b^2 \frac{b}{ac - b^2 + b^2}$$

$$= bac \frac{b}{ac}$$

$$(vii) \quad \beta a\alpha + b + \alpha a\beta + b \frac{\beta}{a\alpha + b} + \frac{\alpha}{a\beta + b}$$

$$= \beta(a\beta + b) + \alpha(a\alpha + b)(a\alpha + b)(a\beta + b) \frac{\beta(a\beta + b) + \alpha(a\alpha + b)}{(a\alpha + b)(a\beta + b)}$$

$$= a\beta^2 + b\beta + \alpha a^2 + ba^2\alpha\beta + ab\alpha + ab\beta + b^2 \frac{a\beta^2 + b\beta + \alpha a^2 + b\alpha}{a^2\alpha\beta + ab\alpha + ab\beta + b^2}$$

$$= a\alpha^2 + b\beta^2 + ba + b\beta a^2 \times ca + ab(\alpha + \beta) + b^2 \frac{a\alpha^2 + b\beta^2 + b\alpha + b\beta}{a^2 \times \frac{c}{a} + ab(\alpha + \beta) + b^2}$$

Since,

$$\text{Sum of the zeroes of polynomial} = \alpha + \beta \quad \alpha + \beta = -ba \frac{-b}{a}$$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta = ca \frac{c}{a}$

After substituting it , we will get

$$\begin{aligned}
 & a[(\alpha+\beta)^2+b(\alpha+\beta)]ac \frac{a[(\alpha+\beta)^2+b(\alpha+\beta)]}{ac} \\
 &= a[(\alpha+\beta)^2-2\alpha\beta]-b^2ac \frac{a[(\alpha+\beta)^2-2\alpha\beta]-\frac{b^2}{a}}{ac} \\
 &= a[b^2a-2ca]-b^2ac \frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac} \\
 &= a[b^2-2ca]-b^2ac \frac{a\left[\frac{b^2-2c}{a}\right]-\frac{b^2}{a}}{ac} \\
 &= a[b^2-2c-b^2a]ac \frac{a\left[\frac{b^2-2c-b^2}{a}\right]}{ac} \\
 &= b^2-2c-b^2ac \frac{b^2-2c-b^2}{ac} \\
 &= -2cac \frac{-2c}{ac} = -2a \frac{-2}{a}
 \end{aligned}$$

$$\begin{aligned}
 & \text{(viii) } a[\alpha^2\beta + \beta^2\alpha] + b[\alpha a + \beta a]a \left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} \right] + b \left[\frac{\alpha}{a} + \frac{\beta}{a} \right] \\
 &= a[\alpha^2+\beta^2\alpha\beta] + b(\alpha^2+\beta^2\alpha\beta)a \left[\frac{\alpha^2+\beta^2}{\alpha\beta} \right] + b \left(\frac{\alpha^2+\beta^2}{\alpha\beta} \right) \\
 &= a[(\alpha+\beta)^2-2\alpha\beta] + b((\alpha+\beta)^2-2\alpha\beta)\alpha\beta \frac{a[(\alpha+\beta)^2-2\alpha\beta] + b((\alpha+\beta)^2-2\alpha\beta)}{\alpha\beta}
 \end{aligned}$$

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha + \beta = -ba \frac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta = ca \frac{c}{a}$

After substituting it , we will get

$$a[(-ba)^2-3ca] + b((-ba)^2-2ca)ca \frac{a\left[\left(\frac{-b}{a}\right)^2-3\times\frac{c}{a}\right] + b\left(\left(\frac{-b}{a}\right)^2-2\frac{c}{a}\right)}{\frac{c}{a}}$$

$$= a^2c \left[-b^2a^2 + 3bca^2 + b^2a^2 - 2bca^2 \right] \frac{a^2}{c} \left[\frac{-b^2}{a^2} + \frac{3bc}{a^2} + \frac{b^2}{a^2} - \frac{2bc}{a^2} \right]$$

$$= \left[-b^2a^2a^2c + 3bca^2a^2c + b^2a^2a^2c - 2bca^2a^2c \right] \left[\frac{-b^2a^2}{a^2c} + \frac{3bca^2}{a^2c} + \frac{b^2a^2}{a^2c} - \frac{2bca^2}{a^2c} \right]$$

$$= -b^2ac + 3b + b^2ac - 2b \frac{-b^2}{ac} + 3b + \frac{b^2}{ac} - 2b = b$$

Exercise – 2.2: Polynomials

Q.1: Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also, verify the relationship between the zeroes and coefficients in each of the following cases:

(i) $f(x) = 2x^3 + x^2 - 5x + 2$; $12, 1, -2$ $f(x) = 2x^3 + x^2 - 5x + 2$; $\frac{1}{2}, 1, -2$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2$; $2, 1, 1$ $g(x) = x^3 - 4x^2 + 5x - 2$; $2, 1, 1$

Sol:

(i) $f(x) = 2x^3 + x^2 - 5x + 2$; $12, 1, -2$ $f(x) = 2x^3 + x^2 - 5x + 2$; $\frac{1}{2}, 1, -2$

(a) By putting $x = 12 \frac{1}{2}$ in the above equation, we will get

$$f(12) = 2(12)^3 + (12)^2 - 5(12) + 2 \quad f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= f(12) = 2(18) + 14 - 52 + 2 \quad f\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) + \frac{1}{4} - \frac{5}{2} + 2$$

$$= f\left(\frac{1}{2}\right) = 14 + 14 - 52 + 2f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = 0$$

(b) By putting $x = 1$ in the above equation, we will get

$$\begin{aligned} f(1) &= 2(1)^3 + (1)^2 - 5(1) + 2f(1) = 2(1)^3 + (1)^2 - 5(1) + 2 \\ &= 2 + 1 - 5 + 2 = 0 \end{aligned}$$

(c) By putting $x = -2$ in the above equation, we will get

$$\begin{aligned} f(-2) &= 2(-2)^3 + (-2)^2 - 5(-2) + 2f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 \\ &= -16 + 4 + 10 + 2 = -16 + 16 = 0 \end{aligned}$$

Now,

$$\text{Sum of zeroes} = \alpha + \beta + \gamma = -\frac{b}{a}$$

$$\Rightarrow 12 + 1 - 2 = -12 \Rightarrow \frac{1}{2} + 1 - 2 = \frac{-1}{2} \quad -12 = -12 \frac{-1}{2} = \frac{-1}{2}$$

$$\text{Product of the zeroes} = \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\begin{aligned} 12 \times 1 + 1 \times (-2) + (-2) \times 12 &= -52 \quad \frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2} = \frac{-5}{2} \quad 12 - 2 - 1 = -52 \\ \frac{1}{2} - 2 - 1 &= \frac{-5}{2} \quad -52 = -52 \frac{-5}{2} = \frac{-5}{2} \end{aligned}$$

Hence, verified.

$$\text{(ii) } g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1 \quad g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$$

(a) By putting $x = 2$ in the given equation, we will get

$$\begin{aligned} g(2) &= (2)^3 - 4(2)^2 + 5(2) - 2g(2) = (2)^3 - 4(2)^2 + 5(2) - 2 \\ &= 8 - 16 + 10 - 2 = 18 - 18 = 0 \end{aligned}$$

(b) By putting $x = 1$ in the given equation, we will get

$$g(1)=(1)^3-4(1)^2+5(1)-2 \quad g(1) = (1)^3-4(1)^2+5(1)-2$$

$$= 1 - 4 + 5 - 2 = 0$$

Now,

$$\text{Sum of zeroes} = \alpha + \beta + \gamma \quad \alpha + \beta + \gamma = -\frac{b}{a}$$

$$\Rightarrow 2+1+1=-(-4) \Rightarrow 2+1+1=-(-4)$$

$$4 = 4$$

$$\text{Product of the zeroes} = \alpha\beta + \beta\gamma + \alpha\gamma \quad \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$2 \times 1 + 1 \times 1 + 1 \times 2 = 5 \quad 2 \times 1 + 1 \times 1 + 1 \times 2 = 5$$

$$2 + 1 + 2 = 5$$

$$5 = 5$$

$$\alpha\beta\gamma = -(-2) \quad \alpha\beta\gamma = -(-2)$$

$$2 \times 1 \times 2 = 2 \quad 2 \times 1 \times 2 = 2$$

$$2 = 2$$

Hence, verified.

Q.2: Find a cubic polynomial with the sum, sum of the product of its zeroes is taken two at a time, and product of its zeroes as 3, -1 and -3 respectively.

Sol:

Any cubic polynomial is of the form ax^3+bx^2+cx+d :

$$=x^3 - (\text{sum of the zeroes})x^2 + (\text{sum of the products of its zeroes})x - (\text{product of the zeroes})$$

$$= x^3 - 3x^2 + (-1)x + (-3) \quad = x^3 - 3x^2 + (-1)x + (-3)$$

$$= k[x^3 - 3x^2 - x - 3] \quad = k[x^3 - 3x^2 - x - 3]$$

k is any non-zero real numbers.

Q.3: If the zeroes of the polynomial $f(x)=2x^3-15x^2+37x-30$

$f(x) = 2x^3 - 15x^2 + 37x - 30$, find them.

Sol:

Let, $\alpha = a-d, \beta = a$ and $\gamma = a+d$ be the zeroes of the polynomial.

$$f(x) = 2x^3 - 15x^2 + 37x - 30 \quad f(x) = 2x^3 - 15x^2 + 37x - 30 \quad \alpha + \beta + \gamma = -(-15/2) = 15/2$$

$$\alpha + \beta + \gamma = -\left(\frac{-15}{2}\right) = \frac{15}{2} \quad \alpha\beta\gamma = -(-30/2) = 15 \quad \alpha\beta\gamma = -\left(\frac{-30}{2}\right) = 15$$

$$a - d + a + a + d = 15 \quad \frac{15}{2} \text{ and } a(a-d)(a+d) = 15 \quad a(a-d)(a+d) = 15$$

$$\text{So, } 3a = 15 \quad \frac{15}{2}$$

$$a = 5 \quad \frac{5}{2}$$

$$\text{And, } a(a^2 + d^2) = 15 \quad a(a^2 + d^2) = 15$$

$$d^2 = 14 \quad d^2 = \frac{1}{4} \quad d = \frac{1}{2} \quad d = \frac{1}{2}$$

$$\text{Therefore, } \alpha = 5 - \frac{1}{2} = \frac{9}{2} \quad \alpha = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$$

$$\beta = 5 \quad \beta = \frac{5}{2}$$

$$\gamma = 5 + \frac{1}{2} = \frac{11}{2} \quad \gamma = \frac{5}{2} + \frac{1}{2} = 3$$

Q.4: Find the condition that the zeroes of the polynomial $f(x)=x^3+3px^2+3qx+r$

$f(x) = x^3 + 3px^2 + 3qx + r$ may be in A.P.

Sol:

$$f(x) = x^3 + 3px^2 + 3qx + r \quad f(x) = x^3 + 3px^2 + 3qx + r$$

Let, $a - d, a, a + d$ be the zeroes of the polynomial.

Then,

$$\text{The sum of zeroes} = -ba \frac{-b}{a}$$

$$a + a - d + a + d = -3p$$

$$3a = -3p$$

$$a = -p$$

Since, a is the zero of the polynomial $f(x)$,

$$\text{Therefore, } f(a) = 0$$

$$f(a) = a^3 + 3pa^2 + 3qa + r = 0$$

$$\text{Therefore, } f(a) = 0 \Rightarrow f(a) = 0$$

$$\Rightarrow a^3 + 3pa^2 + 3qa + r = 0 \Rightarrow a^3 + 3pa^2 + 3qa + r = 0$$

$$\Rightarrow (-p)^3 + 3p(-p)^2 + 3q(-p) + r = 0 \Rightarrow (-p)^3 + 3p(-p)^2 + 3q(-p) + r = 0$$

$$= -p^3 + 3p^3 - pq + r = 0 \Rightarrow -p^3 + 3p^3 - pq + r = 0$$

$$= 2p^3 - pq + r = 0 \Rightarrow 2p^3 - pq + r = 0$$

Q.5: If zeroes of the polynomial $f(x) = ax^3 + 3bx^2 + 3cx + d$

$f(x) = ax^3 + 3bx^2 + 3cx + d$ are in A.P., prove that $2b^3 - 3abc + a^2d = 0$.

Sol:

$$f(x) = x^3 + 3px^2 + 3qx + r \Rightarrow f(x) = x^3 + 3px^2 + 3qx + r$$

Let, $a - d, a, a + d$ be the zeroes of the polynomial.

Then,

$$\text{The sum of zeroes} = -ba \frac{-b}{a}$$

$$a + a - d + a + d = -3ba \frac{-3b}{a}$$

$$\Rightarrow 3a = -3ba \Rightarrow 3a = -\frac{3b}{a} \Rightarrow a = -\frac{3b}{3a} = -\frac{b}{a} \text{ Since, } f(a)=0$$

$$\text{Since, } f(a) = 0 \Rightarrow a(a^2) + 3b(a)^2 + 3c(a) + d = 0 \Rightarrow a(a^2) + 3b(a)^2 + 3c(a) + d = 0$$

$$\Rightarrow a(-b)^3 + 3b^2a^2 - 3bca + d = 0 \Rightarrow a\left(\frac{-b}{a}\right)^3 + \frac{3b^2}{a^2} - \frac{3bc}{a} + d = 0 \quad 2b^3a^2 - 3bca + d = 0$$

$$\frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0 \quad 2b^3 - 3abc + a^2da^2 = 0 \quad \frac{2b^3 - 3abc + a^2d}{a^2} = 0 \quad 2b^3 - 3abc + a^2d = 0$$

$$2b^3 - 3abc + a^2d = 0$$

Q.6: If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + k$

$f(x) = x^3 - 12x^2 + 39x + k$ are in A.P., find the value of k .

Sol:

$$f(x) = x^3 - 12x^2 + 39x + k \quad f(x) = x^3 - 12x^2 + 39x + k$$

Let, $a-d, a, a+d$ be the zeroes of the polynomial $f(x)$.

The sum of the zeroes = 12

$$3a = 12$$

$$a = 4$$

Now,

$$f(a) = 0$$

$$f(a) = a^3 - 12a^2 + 39a + k \quad f(a) = a^3 - 12a^2 + 39a + k \quad f(4) = 4^3 - 12(4)^2 + 39(4) + k = 0$$

$$f(4) = 4^3 - 12(4)^2 + 39(4) + k = 0$$

$$64 - 192 + 156 + k = 0$$

$$k = -28$$

Exercise – 2.3: Polynomials

Points to note:

Division algorithms fall into two main categories: Slow division and fast division. Slow division algorithms produce one digit of the final quotient per iteration. Fast division methods start with an approximation to the final quotient and produce twice as many digits of the final quotient on each iteration.

Discussion will refer to the form $N \div D = (Q, R)$ where,

N = Numerator (dividend) & D = Denominator (divisor) is the input, and Q = Quotient & R = Remainder is the output.

Q.1: Apply division algorithm to find the quotient $q(x)$ and remainder $r(x)$ on dividing $f(x)$ by $g(x)$ in each of the following:

(i) $f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x^2 + x + 1$

$$f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x^2 + x + 1$$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105, g(x) = 2x^2$

$$10x^4 + 17x^3 - 62x^2 + 30x - 105, g(x) = 2x^2 + 7x + 1$$

$$(iii) f(x)=4x^3+8x^2+8x+7, g(x)=2x^2-x+1$$

$$f(x) = 4x^3 + 8x^2 + 8x + 7, g(x) = 2x^2 - x + 1$$

$$(iv) f(x)=15x^3-20x^2+13x-12, g(x)=x^2-2x+2$$

$$f(x) = 15x^3 - 20x^2 + 13x - 12, g(x) = x^2 - 2x + 2$$

Solution:

$$(i) f(x)=x^3-6x^2+11x-6, g(x)=x^2+x+1$$

$$f(x) = x^3 - 6x^2 + 11x - 6 \text{ and } g(x) = x^2 + x + 1$$

$x^2 + x + 1$	$x - 7$
	$x^3 - 6x^2 + 11x - 6$
	$x^3 + x^2 + x$
	<hr style="border: 0; border-top: 1px solid black; margin: 0;"/>
	$-7x^2 + 10x - 6$
	$-7x^2 - 7x - 7$
	<hr style="border: 0; border-top: 1px solid black; margin: 0;"/>
	$17x - 1$

$$(ii) f(x)=10x^4+17x^3-62x^2+30x-105, g(x)=2x^2+7x+1$$

$$f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105, g(x) = 2x^2 + 7x + 1$$

$$f(x)=10x^4+17x^3-62x^2+30x-105, g(x)=2x^2+7x+1$$

$$f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105, g(x) = 2x^2 + 7x + 1$$

	$5x^2 - 9x - 2$
$2x^2 + 7x + 1$	$10x^4 + 17x^3 - 62x^2 + 30x - 3$ $10x^4 + 35x^3 + 5x^2$
	$- 18x^3 - 67x^2 + 30x - 3$ $- 18x^3 - 63x^2 - 9x$
	$- 4x^2 + 39x - 3$ $4x^2 - 14x - 2$
	$53x - 1$

(iii) $f(x)=4x^3+8x^2+8x+7, g(x)=2x^2-x+1$

$f(x) = 4x^3 + 8x^2 + 8x + 7, g(x) = 2x^2 - x + 1$

	$2x + 5$
$2x^2 - x + 1$	$4x^3 + 8x^2 + 8x + 7$ $4x^3 - 2x^2 + 2x$
	$10x^2 + 6x + 7$ $10x^2 - 5x + 5$
	$11x + 2$

$f(x)=4x^3+8x^2+8x+7 f(x) = 4x^3 + 8x^2 + 8x + 7 g(x)=2x^2-x+1 g(x) = 2x^2 - x + 1$

(iv) $f(x)=15x^3-20x^2+13x-12, g(x)=x^2-2x+2$

$f(x) = 15x^3 - 20x^2 + 13x - 12, g(x) = x^2 - 2x + 2$



$f(x)=15x^3-20x^2+13x-12 f(x) = 15x^3 - 20x^2 + 13x - 12 g(x)=x^2-2x+2$

$g(x) = x^2 - 2x + 2$

Q.2: Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) $g(t)=t^2-3; f(t)=2t^4+3t^3-2t^2-9t-12$ $g(t) = t^2 - 3$; $f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $g(x)=x^2-3x+1; f(x)=x^5-4x^3+x^2+3x+1$

$g(x) = x^2 - 3x + 1$; $f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

(iii) $g(x)=2x^2-x+3; f(x)=6x^5-x^4+4x^3-5x^2-x-15$

$g(x) = 2x^2 - x + 3$; $f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Solution:

(i) $g(t)=t^2-3; f(t)=2t^4+3t^3-2t^2-9t-12$ $g(t) = t^2 - 3$; $f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

	$2t^2 + 3t + 4$
$t^2 - 3$	$2t^4 + 3t^3 - 2t^2 - 9t - 12$
	$2t^4 \quad - 6t^2$
	$3t^3 + 4t^2 - 9t$
	$3t^3 \quad - 9t$
	$4t^2 - 12$
	$4t^2 - 12$
	0

$g(t)=t^2-3$ $g(t) = t^2 - 3$ $f(t)=2t^4+3t^3-2t^2-9t-12$ $f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Therefore, $g(t)$ is the factor of $f(t)$.

(ii) $g(x)=x^2-3x+1; f(x)=x^5-4x^3+x^2+3x+1$

$g(x) = x^2 - 3x + 1$; $f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

	$x^2 - 1$
$x^3 - 3x + 1$	$x^5 - 4x^3 + x^2 + 3x + 1$
	$x^5 - 3x^3 + x^2$
	$-x^3 + 3x + 1$
	$-x^3 + 3x - 1$
	2

$$g(x) = x^2 - 3x + 1 \quad f(x) = x^5 - 4x^3 + x^2 + 3x + 1$$

$$f(x) = x^5 - 4x^3 + x^2 + 3x + 1$$

Therefore, $g(x)$ is not the factor of $f(x)$.

(iii) $g(x) = 2x^2 - x + 3$; $f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

$$g(x) = 2x^2 - x + 3 ; f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$$

	$3x^3 + x^2 - 2x - 5$
$2x^2 - x + 3$	$6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$
	$6x^5 - 3x^4 + 9x^3$
	$2x^4 - 5x^3 - 5x^2$
	$2x^4 - x^3 + 3x^2$
	$-4x^3 - 8x^2 - x$
	$-4x^3 + 2x^2 - 6x$
	$-10x^2 + 5x - 15$
	$-10x^2 + 5x - 15$
	0

$$g(x) = 2x^2 - x + 3 \quad f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$$

$$f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$$

Q.3: Obtain all zeroes of the polynomial $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ if two of its zeroes are -2 and -1.

Solution:

$$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6 \quad f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$$

If the two zeroes of the polynomial are -2 and -1, then its factors are $(x + 2)$ and $(x + 1)$

$$(x+2)(x+1) = x^2 + x + 2x + 2 = x^2 + 3x + 2$$

$$(x + 2)(x + 1) = x^2 + x + 2x + 2 = x^2 + 3x + 2$$

	$2x^2 - 5x - 3$
$x^2 + 3x + 2$	$2x^4 + x^3 - 14x^2 - 19x - 6$ $2x^4 + 6x^3 + 4x^2$ <hr/> $-5x^3 - 18x^2 - 19x$ $-5x^3 - 15x^2 - 10x$ <hr/> $-3x^2 - 9x - 6$ $-3x^2 - 9x - 6$ <hr/> 0

$$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6 \quad f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6 = (2x^2 - 5x - 3)(x^2 + 3x + 2)$$

$$= (2x + 1)(x - 3)(x + 2)(x + 1)$$

Therefore, zeroes of the polynomial = $-12 \frac{-1}{2}$, 3, -2, -1

Q-4: Obtain all zeroes of $f(x) = x^3 + 13x^2 + 32x + 20$

$f(x) = x^3 + 13x^2 + 32x + 20$, if one of its zeroes is -2.

Solution:

$$f(x) = x^3 + 13x^2 + 32x + 20 \quad f(x) = x^3 + 13x^2 + 32x + 20$$

Since, the zero of the polynomial is -2 so, it means its factor is $(x + 2)$.

	$x^2 + 11x + 10$
$x + 2$	$x^3 + 13x^2 + 32x + 20$
	$x^3 + 2x^2$
	$11x^2 + 32x + 20$
	$11x^2 + 22x$
	$10x + 20$
	$10x + 20$
	0

So, $f(x) = x^3 + 13x^2 + 32x + 20$
 $f(x) = x^3 + 13x^2 + 32x + 20 = (x^2 + 11x + 10)(x + 2)$
 $(x^2 + 11x + 10)(x + 2)$

$$= (x^2 + 10x + x + 10)(x + 2) = (x^2 + 10x + x + 10)(x + 2)$$

$$= (x + 10)(x + 1)(x + 2) = (x + 10)(x + 1)(x + 2)$$

Therefore, the zeroes of the polynomial are -1, -10, -2.

Q-5: Obtain all zeroes of the polynomial $f(x) = x^4 - 3x^3 - x^2 + 9x - 6$

$f(x) = x^4 - 3x^3 - x^2 + 9x - 6$, if the two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$ so, $-\sqrt{3}$ and $\sqrt{3}$.

Solution:

$$f(x) = x^4 - 3x^3 - x^2 + 9x - 6$$

Since, two of the zeroes of polynomial are $-\sqrt{3}$ and $\sqrt{3}$ so, $(x + \sqrt{3})(x - \sqrt{3})$
 $(x + \sqrt{3})(x - \sqrt{3}) = x^2 - 3$

	$x^2 - 3x + 2$
$x^2 - 3$	$x^4 - 3x^3 - x^2 + 9x - 6$
	$x^4 \quad - 3x^2$
	$- 3x^3 + 2x^2 + 9x$
	$- 3x^3 \quad + 9x$
	$2x^2 - 6$
	$2x^2 - 6$
	0

So, $f(x) = x^4 - 3x^2 - x^2 + 9x - 6$
 $f(x) = x^4 - 3x^2 - x^2 + 9x - 6 = (x^2 - 3)(x^2 - 3x + 2)$
 $(x^2 - 3)(x^2 - 3x + 2)$

$$= (x + \sqrt{3})(x - \sqrt{3})(x + \sqrt{3})(x - \sqrt{3})(x^2 - 2x - 2 + 2)(x^2 - 2x - 2 + 2)$$

$$= (x + \sqrt{3})(x - \sqrt{3})(x + \sqrt{3})(x - \sqrt{3})(x - 1)(x - 2)(x - 1)(x - 2)$$

Therefore, the zeroes of the polynomial are $-\sqrt{3}, \sqrt{3}, \sqrt{3}, 1, 2$.

Q-6: Obtain all zeroes of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1$

$f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1$, if the two of its zeroes are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$

$-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

Solution:

$$f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1$$

Since, $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are the zeroes of the polynomial, so the factors are

$$(x - \sqrt{\frac{3}{2}}) \text{ and } (x + \sqrt{\frac{3}{2}})$$

	$2x^2 - 2x - 4$
$x^2 - 3/2$	$2x^4 - 2x^3 - 7x^2 + 3x + 6$
	$2x^4 \quad - 3x^2$
	$- 2x^3 - 4x^2 + 3x + 6$
	$- 2x^3 \quad + 3x$
	$- 4x^2 + 6$
	$- 4x^2 + 6$
	0

So, $f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1$
 $f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1 = (x - \sqrt{32})(x + \sqrt{32})$

$$\left(x - \sqrt{\frac{3}{2}}\right) \left(x + \sqrt{\frac{3}{2}}\right) (2x^2 - 2x - 4)(2x^2 - 2x - 4)$$

$$= (x - \sqrt{32})(x + \sqrt{32}) \left(x - \sqrt{\frac{3}{2}}\right) \left(x + \sqrt{\frac{3}{2}}\right) (2x^2 - 4x + 2x - 4)(2x^2 - 4x + 2x - 4)$$

$$= (x - \sqrt{32})(x + \sqrt{32}) \left(x - \sqrt{\frac{3}{2}}\right) \left(x + \sqrt{\frac{3}{2}}\right) (x + 2)(x - 2)(x + 2)(x - 2)$$

Therefore, the zeroes of the polynomial = $x = -1, 2, -\sqrt{32}$ and $\sqrt{32}, -\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

Q.7: Find all the zeroes of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$

$x^4 + x^3 - 34x^2 - 4x + 120$, if the two of its zeroes are 2 and -2.

Solution:

$$x^4 + x^3 - 34x^2 - 4x + 120$$

Since, the two zeroes of the polynomial given is 2 and -2

So, factors are $(x + 2)(x - 2) = x^2 + 2x - 2x - 4 = x^2 - 4$

	$x^2 + x - 30$
$x^2 - 4$	$x^4 + x^3 - 34x^2 - 4x + 120$
	$x^4 \quad - 4x^2$
	$x^3 - 30x^2 - 4x + 120$
	$x^3 \quad - 4x$
	$-30x^2 + 120$
	$-30x^2 + 120$
	0

So, $x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x^2 + x - 30)$
 $(x^2 - 4)(x^2 + x - 30)$

$= (x - 2)(x + 2)(x^2 + 6x - 5x - 30)(x - 2)(x + 2)(x^2 + 6x - 5x - 30)$

$= (x - 2)(x + 2)(x + 6)(x - 5)(x - 2)(x + 2)(x + 6)(x - 5)$

Therefore, the zeroes of the polynomial = $x = 2, -2, -6, 5$

Q-8: Find all the zeroes of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if the two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$ and $-\sqrt{2}$.

Solution:

$2x^4 + 7x^3 - 19x^2 - 14x + 30$

Since, $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of the polynomial given.

So, factors are $(x + \sqrt{2})$ and $(x - \sqrt{2})(x + \sqrt{2})$ and $(x - \sqrt{2}) = x^2 + \sqrt{2}x - \sqrt{2}x - 2$
 $x^2 + \sqrt{2}x - \sqrt{2}x - 2 = x^2 - 2$

	$2x^2 + 7x - 15$
$x^2 - 2$	$2x^4 + 7x^3 - 19x^2 - 14x + 30$
	$2x^4 \quad - 4x^2$
	$7x^3 - 15x^2 - 14x + 30$
	$7x^3 \quad - 14x$
	$-15x^2 + 30$
	$-15x^2 + 30$
	0

So, $2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 7x - 15)$
 $(x^2 - 2)(2x^2 + 7x - 15)$

$$= (2x^2 + 10x - 3x - 15)(x + \sqrt{2})(x - \sqrt{2})(2x^2 + 10x - 3x - 15)(x + \sqrt{2})(x - \sqrt{2})$$

$$= (2x - 3)(x + 5)(x + \sqrt{2})(x - \sqrt{2})(2x - 3)(x + 5)(x + \sqrt{2})(x - \sqrt{2})$$

Therefore, the zeroes of the polynomial is $\sqrt{2}, -\sqrt{2}, -5, 3, \sqrt{2}, -\sqrt{2}, -5, \frac{3}{2}$.

Q-9: Find all the zeroes of the polynomial $f(x) = 2x^3 + x^2 - 6x - 3$

$f(x) = 2x^3 + x^2 - 6x - 3$, if two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$

Solution:

$$f(x) = 2x^3 + x^2 - 6x - 3$$

Since, $-\sqrt{3}$ and $\sqrt{3}$ are the zeroes of the given polynomial

So, factors are $(x - \sqrt{3})$ and $(x + \sqrt{3})$ and $(x + \sqrt{3}) = (x^2 - \sqrt{3}x + \sqrt{3}x - 3)$
 $(x^2 - \sqrt{3}x + \sqrt{3}x - 3) = (x^2 - 3)(x^2 - 3)$

	$2x + 1$
$x^2 - 3$	$2x^3 + x^2 - 6x - 3$
	$2x^3 - 6x$
	$x^2 - 3$
	$x^2 - 3$
	0

So, $f(x) = 2x^3 + x^2 - 6x - 3$
 $f(x) = 2x^3 + x^2 - 6x - 3 = (x^2 - 3)(2x + 1)(x^2 - 3)(2x + 1)$
 $= (x - \sqrt{3})(x + \sqrt{3})(2x + 1)(x - \sqrt{3})(x + \sqrt{3})(2x + 1)$

Therefore, set of zeroes for the given polynomial = $\sqrt{3}, -\sqrt{3}, -12\sqrt{3}, -\sqrt{3}, \frac{-1}{2}$

Q-10: Find all the zeroes of the polynomial $f(x) = x^3 + 3x^2 - 2x - 6$

$f(x) = x^3 + 3x^2 - 2x - 6$, if the two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$ and $-\sqrt{2}$.

Solution:

$$f(x) = x^3 + 3x^2 - 2x - 6$$

Since, $\sqrt{2}$ and $-\sqrt{2}$ are the two zeroes of the given polynomial.

So, factors are $(x + \sqrt{2})$ and $(x - \sqrt{2})(x + \sqrt{2})$ and $(x - \sqrt{2}) = x^2 + \sqrt{2}x - \sqrt{2}x - 2$
 $x^2 + \sqrt{2}x - \sqrt{2}x - 2 = x^2 - 2$

	$x + 3$
$x^2 - 2$	$x^3 + 3x^2 - 2x - 6$
	$x^3 - 2x$
	$3x^2 - 6$
	$3x^2 - 6$
	0

By division algorithm, we have:

$$f(x) = x^3 + 3x^2 - 2x - 6 = (x^2 - 2)(x + 3)(x^2 - 2)(x + 3) \\ = (x - \sqrt{2})(x + \sqrt{2})(x + 3)(x - \sqrt{2})(x + \sqrt{2})(x + 3)$$

Therefore, the zeroes of the given polynomial is $-\sqrt{2}, \sqrt{2}$ and $-3 - \sqrt{2}, \sqrt{2}$ and -3 .

Q-11: What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$

$f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $g(x) = x^2 + 2x - 3$.

Sol:

$$f(x) = x^4 + 2x^3 - 2x^2 + x - 1$$

$x^2 + 2x - 3$	$x^2 - 1$
	$x^4 + 2x^3 - 2x^2 + x - 1$
	$x^4 + 2x^3 - 3x^2$
	<hr style="width: 100%;"/>
	$x^2 + x - 1$
	$x^2 + 2x - 3$
	<hr style="width: 100%;"/>
	$-x + 2$

We must add $(x - 2)$ in order to get the resulting polynomial exactly divisible by

$$g(x) = x^2 + 2x - 3$$

Q-12: What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$

$f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $g(x) = x^2 - 4x + 3$.

Sol:

$$f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21 \quad f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$$

$x^2 - 4x + 3$	$x^2 - 6x + 8$
	$x^4 + 2x^3 - 13x^2 - 12x + 21$
	$x^4 - 4x^3 + 3x^2$
	$+ 6x^3 - 16x^2 - 12x + 21$
	$2x^3 - 24x^2 - 18x$
	$8x^2 - 30x + 21$
	$8x^2 - 32x + 24$
	$2x - 3$

We must subtract $(2x - 3)$ in order to get the resulting polynomial exactly divisible by

$$g(x) = x^2 - 4x + 3 \quad g(x) = x^2 - 4x + 3.$$

Q-13: Given that $\sqrt{2}\sqrt{2}$ is a zero of the cubic polynomial $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ find its other two zeroes.

Solution:

$$f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \quad f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$$

Since, $\sqrt{2}\sqrt{2}$ is a zero of the cubic polynomial

So, factor is $(x - \sqrt{2})(x - \sqrt{2})$

	$6x^2 + 7\sqrt{2}x + 4$
$x - \sqrt{2}$	$6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$
	$6x^3 - 6\sqrt{2}x^2$
	$7\sqrt{2}x^2 - 10x - 4\sqrt{2}$
	$7\sqrt{2}x^2 - 14x$
	$4x - 4\sqrt{2}$
	$4x - 4\sqrt{2}$
	0

Since, $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

$$f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} = (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4)$$

$$= (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4)(x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4)$$

$$= (x - \sqrt{2})(3x + 2\sqrt{2})(2x + \sqrt{2})(x - \sqrt{2})(3x + 2\sqrt{2})(2x + \sqrt{2})$$

So, the zeroes of the polynomial is $-2\sqrt{2}3, -\sqrt{2}2, \sqrt{2} - \frac{2\sqrt{2}}{3}, -\frac{\sqrt{2}}{2}, \sqrt{2}$

Q-14: Given that $x - \sqrt{5}x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

Solution:

$$x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$$

In the question, it's given that $x - \sqrt{5}x - \sqrt{5}$ is a factor of the cubic polynomial.



Since, $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} = (x - \sqrt{5})(x^2 - 2\sqrt{5}x + 3)$

$$(x - \sqrt{5})(x^2 - 2\sqrt{5}x + 3)$$

$$= (x - \sqrt{5})(x - (\sqrt{5} + \sqrt{2}))(x - (\sqrt{5} - \sqrt{2}))(x - \sqrt{5})(x - (\sqrt{5} + \sqrt{2}))(x - (\sqrt{5} - \sqrt{2}))$$

So, the zeroes of the polynomial = $\sqrt{5}, (\sqrt{5} - \sqrt{2}), (\sqrt{5} + \sqrt{2}), \sqrt{5}, (\sqrt{5} - \sqrt{2}), (\sqrt{5} + \sqrt{2})$.