Exercise 5.1: Trigonometric Ratios

1.) Find the value of Trigonometric ratios in each of the following provided one of the six trigonometric ratios are given.

Sol.

(i) sinA=23 $\sin A = \frac{2}{3}$

Given:

sinA=23 $\sin A = \frac{2}{3}$ ….. (1)

By definition,

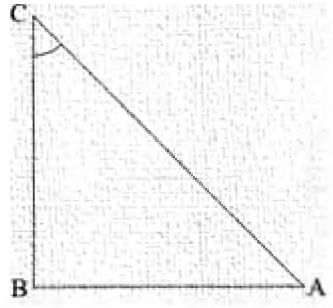
sinA=23 $\sin A = \frac{2}{3}$ = PerpendicularHypotenuse $\frac{Perpendicular}{Hypotenuse}$ ….(2)

By Comparing (1) and (2)

We get,

Perpendicular side = 2 and

Hypotenuse = 3



Therefore, by Pythagoras theorem,

$AC^2 = AB^{2\,+}\ BC^2$

Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side (AB)

Therefore,

$3^2 = AB^2 + 2^2$

$AB^2 = 3^2 – 2^2$

$AB^2 = 9 – 4$

$AB^{2\,=}\ 5$

AB= $\sqrt{5}\sqrt{5}$

Hence, Base = $\sqrt{5}\sqrt{5}$

Now, cosA= BaseHypotenuse $\cos A = \frac{Base}{Hypotenuse}$

cosA= √53 $\cos A = \frac{\sqrt{5}}{3}$

Now, cosec A = 1sinA $\frac{1}{sin A}$

Therefore,

cosec A = HypotenusePerependicular $\frac{Hypotenuse}{Perependicular}$

cosec A = 32 $\frac{3}{2}$

Now, sec A = HypotenuseBase $\frac{Hypotenuse}{Base}$

Therefore,

sec A = 3√5 $\frac{3}{\sqrt{5}}$

Now, tan A = PerependicularBase $\frac{Perependicular}{Base}$

tan A = 2√5 $\frac{2}{\sqrt{5}}$

Now, cot A = BasePerpendicular $\frac{Base}{Perpendicular}$

Therefore,

cot A = √52 $\frac{\sqrt{5}}{2}$

(ii) cosA=45 $\cos A = \frac{4}{5}$

Given: cosA=45 $\cos A = \frac{4}{5}$ .... (1)

By Definition,

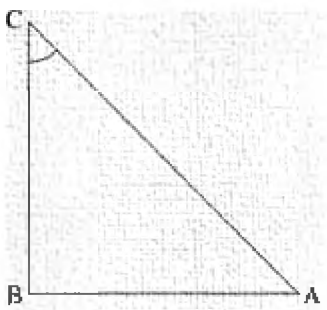
cosA=BaseHypotenuse $\cos A = \frac{Base}{Hypotenuse}$ .... (2)

By comparing (1) and (2)

We get,

Base =4 and

Hypotenuse = 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base (AB) and hypotenuse (AC) and get the perpendicular side

(BC)

$5^2 = 4^2 + BC^2$

$BC^2 = 5^2 - 4^2$

$BC^2 = 25 - 16$

$BC^2 = 9$

BC= 3

Hence, Perpendicular side = 3

Now,

sinA=23 $\sin A = \frac{2}{3}$ = PerpendicularHypotenuse $\frac{Perpendicular}{Hypotenuse}$

Therefore,

sinA=35 $\sin A = \frac{3}{5}$

Now, cosec A = 1sinA $\frac{1}{sinA}$

Therefore,

cosec A= 1sinA $\frac{1}{sinA}$

Therefore,

cosec A = HypotenusePerependicular $\frac{Hypotenuse}{Perependicular}$

cosec A = 53 $\frac{5}{3}$

Now, sec A = 1cosA $\frac{1}{cosA}$

Therefore,

sec A = HypotenuseBase $\frac{Hypotenuse}{Base}$

sec A = 54 $\frac{5}{4}$

Now, tan A = PerpendicularBase $\frac{Perpendicular}{Base}$

Therefore,

tan A =34 $\frac{3}{4}$

Now, cot A = 1tanA $\frac{1}{tanA}$

Therefore,

cot A = BasePerpendicular $\frac{Base}{Perpendicular}$

cot A = 43 $\frac{4}{3}$

(iii) tanΘ= 111 $\tan\Theta = \frac{11}{1}$

Given: tanΘ= 111 $\tan\Theta = \frac{11}{1}$ .... (1)

By definition,

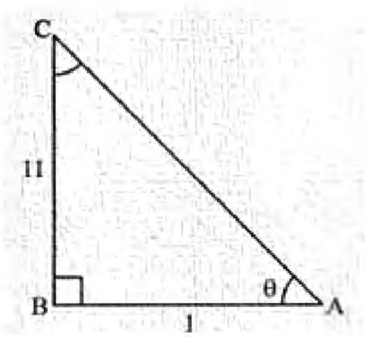
tanΘ= PerpendicularBase $\tan\Theta = \frac{Perpendicular}{Base}$ .... (2)

By Comparing (1) and (2)

We get,

Base= 1 and

Perpendicular side= 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and perpendicular side (BC) and get hypotenuse(AC)

$AC^2 = 1^2 + 11^2$

$AC^2 = 1 + 121$

$AC^2 = 122$

AC= √122 $\sqrt{122}$

Now, sinΘ= PerpendicularHypotenuse $\sin\Theta = \frac{Perpendicular}{Hypotenuse}$

Therefore,

sinΘ= 11√122 $\sin\Theta = \frac{11}{\sqrt{122}}$

Now, cosecΘ= 1sinΘ $\Theta = \frac{1}{sin\Theta}$

cosec Θ= √12211 $\Theta = \frac{\sqrt{122}}{11}$

Now, cosΘ= BaseHypotenuse $\cos\Theta = \frac{Base}{Hypotenuse}$

Therefore,

cosΘ= 1√122 $\cos\Theta = \frac{1}{\sqrt{122}}$

Now, secΘ= 1cosΘ $\sec\Theta = \frac{1}{cos\Theta}$

Therefore,

secΘ= HypotenuseBase $\sec\Theta = \frac{Hypotenuse}{Base}$ secΘ= √1221 $\sec\Theta = \frac{\sqrt{122}}{1}$ secΘ=√122 $\sec\Theta = \sqrt{122}$

Now, cotΘ= 1tanΘ $\cot\Theta = \frac{1}{tan\Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot\Theta = \frac{Base}{Perpendicular}$ cotΘ= 111 $\cot\Theta = \frac{1}{11}$

(iv) sinΘ=1115 $\sin\Theta = \frac{11}{15}$

Given: sinΘ=1115 $\sin\Theta = \frac{11}{15}$ .... (1)

By definition,

sinΘ=PerpendicularHypotenuse $\sin\Theta = \frac{Perpendicular}{Hypotenuse}$ .... (2)

By Comparing (1) and (2)

We get,

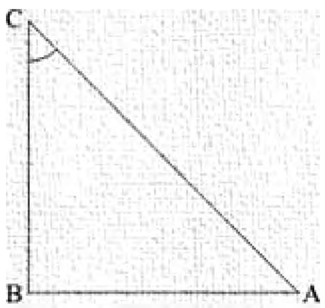
Perpendicular Side = 11 and

Hypotenuse= 15

Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$



Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side (AB)

$15^2 = AB^2 + 11^2$

$AB^2 = 15^2 - 11^2$

$AB^2 = 225 - 121$

$AB^2 = 104$

AB = √104 $\sqrt{104}$

AB= √2×2×2×13 $\sqrt{2 \times 2 \times 2 \times 13}$

AB= 2√2×13 $\sqrt{2 \times 13}$

AB= 2√26 $\sqrt{26}$

Hence, Base = 2√26 $\sqrt{26}$

Now, cosΘ=BaseHypotenuse $\cos\Theta = \frac{Base}{Hypotenuse}$

Therefore,

cosΘ=2√2615 $\cos\Theta = \frac{2\sqrt{26}}{15}$

Now, cosec[Θ=1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ=HypotenusePerpendicular $\Theta = \frac{Hypotenuse}{Perpendicular}$

cosecΘ=1511 $\Theta = \frac{15}{11}$

Now, secΘ=HypotenuseBase $\Theta = \frac{Hypotenuse}{Base}$

Therefore,

secΘ=152√26 $\Theta = \frac{15}{2\sqrt{26}}$

Now, tanΘ=PerpendicularBase $\tan\Theta = \frac{Perpendicular}{Base}$

Therefore,

tanΘ=112√26 $\tan\Theta = \frac{11}{2\sqrt{26}}$

Now, cotΘ=BasePerpendicular $\cot\Theta = \frac{Base}{Perpendicular}$

Therefore,

cotΘ=2√2611 $\cot\Theta = \frac{2\sqrt{26}}{11}$

(v) tanα=512 $\tan\alpha = \frac{5}{12}$

Given: tanα=512 $\tan\alpha = \frac{5}{12}$ .... (1)

By definition,

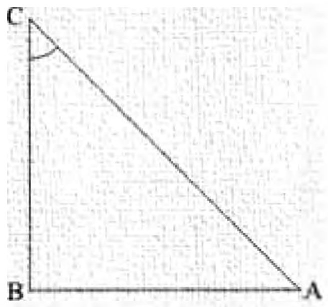
tanα=PerpendicularBase $\tan\alpha = \frac{Perpendicular}{Base}$ .... (2)

By comparing (1) and (2)

We get,

Base= 12 and

Perpendicular side = 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and the perpendicular side (BC) and gte hypotenuse (AC)

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

AC= 13

Hence Hypotenuse = 13

Now, sinα=PerpendicularHypotenuse $\sin\alpha = \frac{Perpendicular}{Hypotenuse}$

Therefore,

sinα=513 $\sin\alpha = \frac{5}{13}$

Now, cosecα=HypotenusePerpendicular $\alpha = \frac{Hypotenuse}{Perpendicular}$

cosecα=135 $\alpha = \frac{13}{5}$

Now, cosα=BaseHypotenuse $\cos \alpha = \frac{Base}{Hypotenuse}$

Therefore,

cosα=1213 $\cos \alpha = \frac{12}{13}$

Now, secα=1cosα $\sec \alpha = \frac{1}{cos\alpha}$

Therefore,

cotα=BasePerpendicular $\cot \alpha = \frac{Base}{Perpendicular}$ cotα=125 $\cot \alpha = \frac{12}{5}$

(vi) sinΘ=√32 $\sin \Theta = \frac{\sqrt{3}}{2}$

Given: sinΘ=√32 $\sin \Theta = \frac{\sqrt{3}}{2}$ .... (1)

By definition,

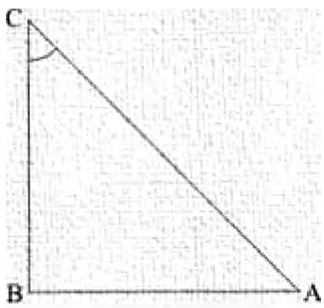
sinΘ=PerpendicularHypotenuse $\sin \Theta = \frac{Perpendicular}{Hypotenuse}$ ....(2)

By comparing (1) and (2)

We get,

Perpendicular Side = √3 $\sqrt{3}$

Hypotenuse = 2



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side (AB)

$2^2 = AB^2 + (\sqrt{3}\sqrt{3})^2$

$AB^2 = 2^2 - (\sqrt{3}\sqrt{3})^2$

$AB^2 = 4 - 3$

$AB^2 = 1$

AB= 1

Hence Base = 1

Now, cosΘ=BaseHypotenuse $\cos\Theta = \frac{Base}{Hypotenuse}$

Therefore,

cosΘ=12 $\cos\Theta = \frac{1}{2}$

Now, cosecΘ=1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ=HypotenusePerpendicualar $\Theta = \frac{Hypotenuse}{Perpendicualar}$

cosecΘ=2√3 $\Theta = \frac{2}{\sqrt{3}}$

Now, secΘ=HypotenuseBase $\sec\Theta = \frac{Hypotenuse}{Base}$

Therefore,

secΘ=21 $\sec\Theta = \frac{2}{1}$

Now, tanΘ=PerpendicularBase $\tan\Theta = \frac{Perpendicular}{Base}$

Therefore,

tanΘ=√31 $\tan\Theta = \frac{\sqrt{3}}{1}$

Now, cotΘ=BasePerpendicular $\cot\Theta = \frac{Base}{Perpendicular}$

Therefore,

cotΘ=1√3 $\cot\Theta = \frac{1}{\sqrt{3}}$

(vii) cosΘ=725 $\cos\Theta = \frac{7}{25}$

Given: cosΘ=725 $\cos\Theta = \frac{7}{25}$ …. (1)

By definition,

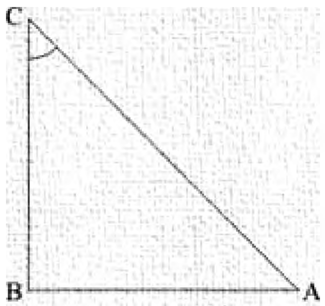
cosΘ=BaseHypotenuse $\cos\Theta = \frac{Base}{Hypotenuse}$

By comparing (1) and (2)

We get,

Base = 7 and

Hypotenuse = 25



Therefore

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$25^2 = 7^2 + BC^2$

$BC^2 = 25^2 – 7^2$

$BC^2 = 625 – 49$

BC = 576

BC= √576 $\sqrt{576}$

BC= 24

Hence, Perpendicular side = 24

Now, sinΘ=perpendicularHypotenuse $\sin \Theta = \frac{perpendicular}{Hypotenuse}$

Therefore,

sinΘ=2425 $\sin \Theta = \frac{24}{25}$

Now, cosecΘ=1sinΘ $\Theta = \frac{1}{\sin \Theta}$

Therefore,

cosecΘ=HypotenusePerpendicualar $\Theta = \frac{Hypotenuse}{Perpendicualar}$

cosecΘ=2524 $\Theta = \frac{25}{24}$

Now, secΘ=1cosΘ $\sec \Theta = \frac{1}{\cos \Theta}$

Therefore,

secΘ=HypotenuseBase $\sec \Theta = \frac{Hypotenuse}{Base}$ secΘ=257 $\sec \Theta = \frac{25}{7}$

Now, tanΘ=PerpendicularBase $\tan \Theta = \frac{Perpendicular}{Base}$

Therefore,

tanΘ=247 $\tan \Theta = \frac{24}{7}$

Now, cotΘ=1tanΘ $\cot \Theta = \frac{1}{\tan \Theta}$

Therefore,

cotΘ=BasePerpendicular $\cot \Theta = \frac{Base}{Perpendicular}$ cotΘ=724 $\cot \Theta = \frac{7}{24}$

(viii) tanΘ=815 $\tan \Theta = \frac{8}{15}$

Given: tanΘ=815 $\tan \Theta = \frac{8}{15}$ …. (1)

By definition,

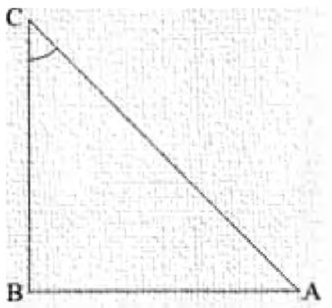
tanΘ=PerpendicularBase $\tan\Theta = \frac{Perpendicular}{Base}$ .... (2)

By comparing (1) and (2)

We get,

Base= 15 and

Perpendicular side = 8



Therefore,

By Pythagoras theorem,

AC^2= 15^2 + 8^2

AC^2= 225 + 64

AC^2 = 289

AC = √289 $\sqrt{289}$

AC= 17

Hence, Hypotenuse = 17

Now, sinΘ=PerpendicularHypotenuse $\sin\Theta = \frac{Perpendicular}{Hypotenuse}$

Therefore,

sinΘ=817 $\sin\Theta = \frac{8}{17}$

Now, cosecΘ=1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ=HypotenusePerpendicular $\Theta = \frac{Hypotenuse}{Perpendicular}$

Θ=178 $\Theta = \frac{17}{8}$

Now, cosΘ= BaseHypotenuse $\cos\Theta = \frac{Base}{Hypotenuse}$

Therefore,

cosΘ= 1517 $\cos\Theta = \frac{15}{17}$

Now, secΘ= 1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

secΘ= HypotenuseBase $\sec\Theta = \frac{Hypotenuse}{Base}$ secΘ= 1715 $\sec\Theta = \frac{17}{15}$

Now, cotΘ= 1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot\Theta = \frac{Base}{Perpendicular}$ cotΘ= 158 $\cot\Theta = \frac{15}{8}$

(ix) cotΘ= 125 $\cot\Theta = \frac{12}{5}$

Given: cotΘ= 125 $\cot\Theta = \frac{12}{5}$ .... (1)

By definition,

cotΘ= 1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

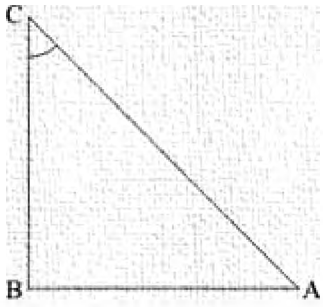
cotΘ= BasePerpendicular $\cot\Theta = \frac{Base}{Perpendicular}$ .... (2)

By comparing (1) and (2)

We get,

Base = 12 and

Perpendicular side = 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and perpendicular side(BC) and get the hypotenuse (AC)

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

AC = √169$\sqrt{169}$

AC = 13

Hence, Hypotenuse = 13

Now, sinΘ=PerpendicularHypotenuse $\sin\Theta = \frac{Perpendicular}{Hypotenuse}$

Therefore,

sinΘ=513 $\sin\Theta = \frac{5}{13}$

Now, cosecΘ=1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ=HypotenusePerpendicular $\Theta = \frac{Hypotenuse}{Perpendicular}$

cosecΘ=135 $\Theta = \frac{13}{5}$

Now, cosΘ=BaseHypotenuse $\cos\Theta = \frac{Base}{Hypotenuse}$

Therefore,

cosΘ=1213 $\cos\Theta = \frac{12}{13}$

Now, secΘ=1cosΘ $\sec\Theta = \frac{1}{cos\Theta}$

Therefore,

secΘ=HypotenuseBase $\sec\Theta = \frac{Hypotenuse}{Base}$ secΘ=1312 $\sec\Theta = \frac{13}{12}$

Now, tanΘ=1cotΘ $\tan\Theta = \frac{1}{\cot\Theta}$

Therefore,

tanΘ=PerpendicularBase $\tan\Theta = \frac{Perpendicular}{Base}$ tanΘ=512 $\tan\Theta = \frac{5}{12}$

(x) secΘ=135 $\sec\Theta = \frac{13}{5}$

Given: secΘ=135 $\sec\Theta = \frac{13}{5}$… (1)

By definition,

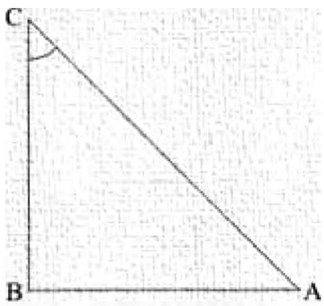
secΘ=1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$ …. (2)

By comparing (1) and (2)

We get,

Base=5

Hypotenuse = 13



Therefore,

By Pythagoras theorem,

Substituting the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$13^2 = 5^2 + BC^2$

$BC^2 = 13^2 – 5^2$

$BC^2=169 – 25$

$BC^2= 144$

BC= √144 $\sqrt{144}$

BC = 12

Hence, Perpendicular side = 12

Now, sinΘ=PerpendicularHypotenuse $\sin \Theta = \frac{Perpendicular}{Hypotenuse}$

Therefore,

sinΘ=1213 $\sin \Theta = \frac{12}{13}$

Now, cosec Θ=1sinΘ $\Theta = \frac{1}{\sin \Theta}$

Therefore,

cosecΘ=HypotenusePerpendicular $\Theta = \frac{Hypotenuse}{Perpendicular}$

cosecΘ=1312 $\Theta = \frac{13}{12}$

Now, cosΘ=1secΘ $\cos \Theta = \frac{1}{\sec \Theta}$

Therefore,

cosΘ=BaseHypotenuse $\cos \Theta = \frac{Base}{Hypotenuse}$ cosΘ=513 $\cos \Theta = \frac{5}{13}$

Now, tanΘ=PerpendicularBase $\tan \Theta = \frac{Perpendicular}{Base}$

Therefore,

tanΘ=125 $\tan \Theta = \frac{12}{5}$

Now, cotΘ=1tanΘ $\cot \Theta = \frac{1}{\tan \Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot \Theta = \frac{Base}{Perpendicular}$ cotΘ= 512 $\cot \Theta = \frac{5}{12}$

(xi) cosecΘ=√10$\Theta = \sqrt{10}$

Given: cosecΘ= √101 $\Theta = \frac{\sqrt{10}}{1}$... (1)

By definition

cosecΘ= 1sinΘ $\Theta = \frac{1}{\sin \Theta}$ ....(2)

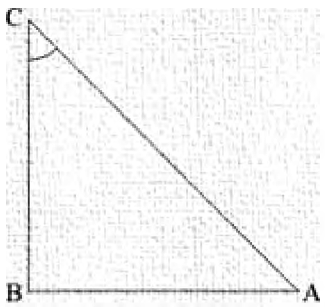
Θ= HypotenusePerpendicular $\Theta = \frac{Hypotenuse}{Perpendicular}$

By comparing (1) and(2)

We get,

Perpendicular side= 1 and

Hypotenuse = √10 $\sqrt{10}$



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side (AB)

$(\sqrt{10}\sqrt{10})^2 = AB^2 + 1^2$

$AB^2 = (\sqrt{10}\sqrt{10})^2 - 1^2$

$AB^2 = 10 - 1$

–

AB = √9$\sqrt{9}$

AB = 3

Hence, Base side = 3

Now, sinΘ=PerpendicularHypotenuse $\sin\Theta = \frac{Perpendicular}{Hypotenuse}$

Therefore,

sinΘ=1√10 $\sin\Theta = \frac{1}{\sqrt{10}}$

Now, cosΘ=BaseHypotenuse $\cos\Theta = \frac{Base}{Hypotenuse}$

Therefore,

cosΘ=3√10 $\cos\Theta = \frac{3}{\sqrt{10}}$

Now, secΘ=1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

secΘ=HypotenuseBase $\sec\Theta = \frac{Hypotenuse}{Base}$ secΘ=√103 $\sec\Theta = \frac{\sqrt{10}}{3}$

Now, tanΘ=PerpendicularBase $\tan\Theta = \frac{Perpendicular}{Base}$

Therefore,

tanΘ=13 $\tan\Theta = \frac{1}{3}$

Now, cotΘ=1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

cotΘ=31 $\cot\Theta = \frac{3}{1}$ cotΘ=3 $\cot\Theta = 3$

(xii) cosΘ 1215 $\cos\Theta\,\frac{12}{15}$

Given: cosΘ 1215 $\cos\Theta\,\frac{12}{15}$ .... (1)

By definition,

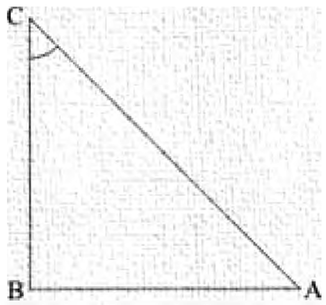
cosΘ=BaseHypotenuse $\cos \Theta = \frac{Base}{Hypotenuse}$ .... (2)

By comparing (1) and (2)

We get,

Base=12 and

Hypotenuse = 15



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$15^2 = 12^2 + BC^2$

$BC^2 = 15^2 - 12^2$

$BC^2 = 225 - 144$

$BC^2 = 81$

BC = √81 $\sqrt{81}$

BC = 9

Hence, Perpendicular side = 9

Now, sinΘ=PerpendicularHypotenuse $\sin \Theta = \frac{Perpendicular}{Hypotenuse}$

Therefore,

sinΘ=915 $\sin \Theta = \frac{9}{15}$

Now, cosecΘ= 1sinΘ $\Theta = \frac{1}{\sin \Theta}$

Therefore,

cosec Θ= HypotenusePerpendicular $\Theta = \frac{Hypotenuse}{Perpendicular}$

cosecΘ= 159 $\Theta = \frac{15}{9}$

Now, secΘ= 1cosΘ $\sec \Theta = \frac{1}{\cos \Theta}$

Therefore,

secΘ= HypotenuseBase $\sec \Theta = \frac{Hypotenuse}{Base}$ secΘ= 1512 $\sec \Theta = \frac{15}{12}$

Now, tanΘ= PerpendicularBase $\tan \Theta = \frac{Perpendicular}{Base}$

Therefore,

tanΘ= 912 $\tan \Theta = \frac{9}{12}$

Now, cotΘ= 1tanΘ $\cot \Theta = \frac{1}{\tan \Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot \Theta = \frac{Base}{Perpendicular}$ cotΘ= 129 $\cot \Theta = \frac{12}{9}$

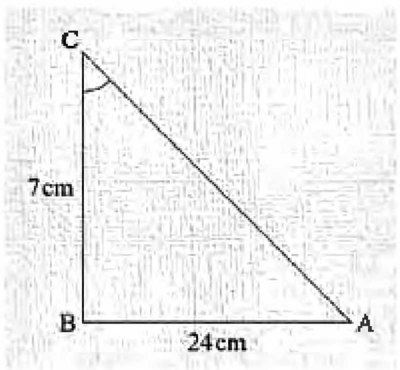
2.) In a ΔΔABC, right angled at B , AB – 24 cm , BC= 7 cm , Determine

(i) sin A , cos A

(ii) sin C, cos C

Sol.

(i) The given triangle is below:



Given: In $\Delta\Delta$ ABC , AB= 24 cm

BC = 7cm

∠ABC$\angle ABC$ = 90°

To find: sin A, cos A

In this problem, Hypotenuse side is unknown

Hence we first find hypotenuse side by Pythagoras theorem

By Pythagoras theorem,

We get,

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

AC = √625$\sqrt{625}$

AC= 25

Hypotenuse = 25

By definition,

sinA=Perpendicularsideoppositeto∠AHypotenuse $\sin A = \frac{Perpendicularsideoppositeto\angle A}{Hypotenuse}$

sinA=BCAC $\sin A = \frac{BC}{AC}$ sinA=725 $\sin A = \frac{7}{25}$

By definition,

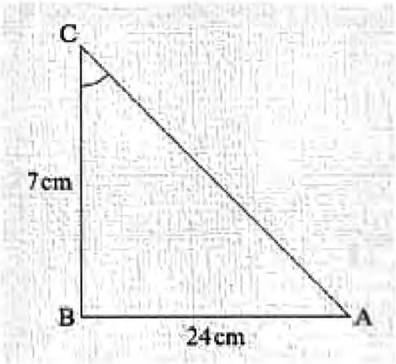
cosA= BasesideadjacenttoAHypotenuse $\cos A = \frac{Base side adjacent to \angle A}{Hypotenuse}$ cosA= ABAC

$\cos A = \frac{AB}{AC}$ cosA= 2425 $\cos A = \frac{24}{25}$

Answer:

sinA= 725 $\sin A = \frac{7}{25}$, **cosA= 2425** $\cos A = \frac{24}{25}$

(ii) The given triangle is below:



Given: In ΔΔ ABC , AB= 24 cm

BC = 7cm

∠ABC $\angle ABC$ = 90°

To find: sin C, cos C

In this problem, Hypotenuse side is unknown

Hence we first find hypotenuse side by Pythagoras theorem

By Pythagoras theorem,

We get,

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

AC = √625 $\sqrt{625}$

AC= 25

Hypotenuse = 25

By definition,

sinC=Perpendicularsideoppositeto∠CHypotenuse $\sin C = \frac{Perpendicular side opposite to \angle C}{Hypotenuse}$

sinC=ABAC $\sin C = \frac{AB}{AC}$ sinC=2425 $\sin C = \frac{24}{25}$

By definition,

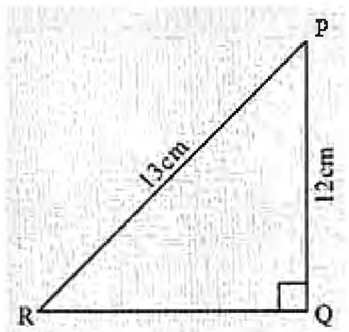
cosC=Basesideadjacentto∠CHypotenuse $\cos C = \frac{Base side adjacent to \angle C}{Hypotenuse}$ cosA=BCAC

$\cos A = \frac{BC}{AC}$ cosA=725 $\cos A = \frac{7}{25}$

Answer:

sinA=2425 $\sin A = \frac{24}{25}$**, cosA=725** $\cos A = \frac{7}{25}$

3.) In the below figure, find tan P and cot R. Is tan P = cot R?



To find, tan P, cot R

Sol.

In the given right angled ΔPQR ΔPQR, length of side OR is unknown

Therefore , by applying Pythagoras theorem in ΔPQR ΔPQR

We get,

$PR^2 = PQ^2 + QR^2$

Substituting the length of given side PR and PQ in the above equation

$13^2 = 12^2 + QR^2$

$QR^2 = 13^2 - 12^2$

$QR^2 = 169 - 144$

$QR^2 = 25$

QR = √25 $\sqrt{25}$

By definiton, we know that ,

tanP=Perpendicularsideoppositeto∠PBasesideadjacentto∠P $\tan P = \frac{Perpendicular side opposite to \angle P}{Base side adjacent to \angle P}$

tanP=QRPQ $\tan P = \frac{QR}{PQ}$

tanP=512 $\tan P = \frac{5}{12}$ .... (1)

Also, by definition, we know that

cotR=Basesideadjacentto∠RPerpendicularsideoppositeto∠R

$\cot R = \frac{Base\ side\ adjacent\ to\ \angle R}{Perpendicular\ \ side\ opposite\ to\ \angle R}$

cotR=QRPQ $\cot R = \frac{QR}{PQ}$

cotR=512 $\cot R = \frac{5}{12}$ .... (2)

Comparing equation (1) ad (2), we come to know that that R.H.S of both the equation are equal.

Therefore, L.H.S of both equations is also equal

tan P = cot R

Answer:

Yes , tan P =cot R = 512 $\frac{5}{12}$

4.) If sin A = 941 $\frac{9}{41}$, Compute cos A and tan A.

Sol.

Given: sinA=941 $\sin A = \frac{9}{41}$ .... (1)

To find: cos A, tan A

By definition,

sinA=Perpendicularsideoppositeto∠AHypotenuse $\sin A = \frac{Perpendicular\ side\ opposite\ to\ \angle A}{Hypotenuse}$ .... (2)

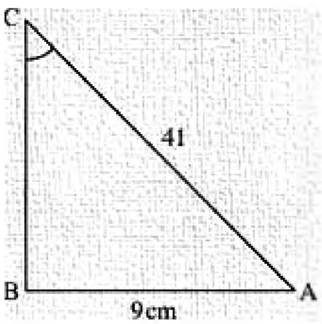
By comparing (1) and (2)

We get ,

Perpendicular side = 9 and

Hypotenuse = 41

Now using the perpendicular side and hypotenuse we can construct ΔABC ΔABC as shown below



Length of side AB is unknown is right angled ΔABC ΔABC ,

To find the length of side AB, we use Pythagoras theorem,

Therefore, by applying Pythagoras theorem in ΔABC ΔABC ,

We get,

$AC^2 = AB^2 + BC^2$

$41^2 = AB^2 + 9^2$

$AB^2 = 41^2 – 9^2$

$AB^2 = 168 – 81$

AB= 1600

AB = √1600 $\sqrt{1600}$

AB = 40

Hence, length of side AB= 40

Now

By definition,

cosA=Basesideadjacentto∠AHypotenuse $\cos A = \frac{Base\ side\ adjacent\ to\ \angle A}{Hypotenuse}$ cosA=ABAC

$\cos A = \frac{AB}{AC}$ cosA=4041 $\cos A = \frac{40}{41}$

Now,

By definition,

tanA=Perpendicularsideoppositeto∠ABasesideadjacentto∠A

$\tan A = \frac{Perpendicular\ side\ opposite\ to\ \angle A}{Base\ side\ adjacent\ to\ \angle A}$ tanA=BCAB $\tan A = \frac{BC}{AB}$ tanA=940 $\tan A = \frac{9}{40}$

Answer:

cosA=4041 $\cos A = \frac{40}{41}$ **, tanA=940** $\tan A = \frac{9}{40}$

5.) Given 15cot A=8, find sin A and sec A.

Answer:

Given: 15cot A = 8

To find: sin A, sec A

Since 15 cot A =8

By taking 15 on R.H.S

We get,

cotA=815 $\cot A = \frac{8}{15}$

By definition,

cotA=1tanA $\cot A = \frac{1}{\tan A}$

Hence,

cotA= 1 Perpendicularsideoppositeto∠ABasesideadjacentto∠A $\cot A = \frac{1}{\frac{\textit{Perpendicular side opposite to } \angle A}{\textit{Base side adjacent to } \angle A}}$

cotA= Basesideadjacentto∠APerpendicularsideoppositeto∠A

$\cot A = \frac{\textit{Base side adjacent to } \angle A}{\textit{Perpendicular side opposite to } \angle A}$ .... (2)

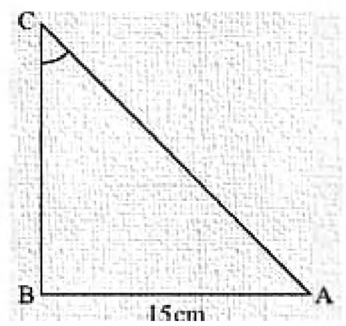
Comparing equation (1) and (2)

We get,

Base side adjacent to ∠A$\angle A$ = 8

Perpendicular side opposite to ∠A$\angle A$ = 15

ΔABCΔABC can be drawn below using above information



Hypotenuse side is unknown.

Therefore, we find side AC of ΔABCΔABC by Pythagoras theorem.

So, by applying Pythagoras theorem to ΔABCΔABC

We get,

$AC^2 = AB^2 + BC^2$

Substituting values of sides from the above figure

$AC^2 = 8^2 + 15^2$

$AC^2 = 64 + 225$

$AC^2 = 289$

AC = √289$\sqrt{289}$

AC = 17

Therefore, hypotenuse =17

Now by definition,

sinA=Perpendicularsideoppositeto∠AHypotenuse $\sin A = \frac{Perpendicular\ side\ opposite\ to\ \angle A}{Hypotenuse}$

Therefore, sinA=BCAC $\sin A = \frac{BC}{AC}$

Substituting values of sides from the above figure

sinA=1517 $\sin A = \frac{15}{17}$

By definition,

secA=1cosA $\sec A = \frac{1}{\cos A}$

Hence,

secA=1Basesideadjacentto∠AHypotenuse $\sec A = \frac{1}{\frac{Base\ side\ adjacent\ to\ \angle A}{Hypotenuse}}$

secA=HypotenuseBasesideadjacentto∠A $\sec A = \frac{Hypotenuse}{Base\ side\ adjacent\ to\ \angle A}$

Substituting values of sides from the above figure

secA=178 $\sec A = \frac{17}{8}$

Answer:

sinA=1517 $\sin A = \frac{15}{17}$**, secA=178** $\sec A = \frac{17}{8}$

6.) In ΔPQR ΔPQR , right angled at Q, PQ = 4cm and RQ= 3 cm .Find the value of sin P, sin R , sec P and sec R.

Sol.

Given:

ΔPQR ΔPQR is right angled at vertex Q.

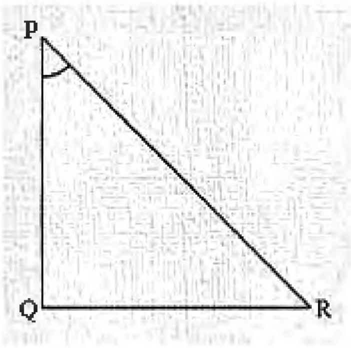
PQ= 4cm

RQ= 3cm

To find,

sin P, sin R , sec P , sec R

Given $\Delta PQR \Delta PQR$ is as shown below



Hypotenuse side PR is unknown.

Therefore, we find side PR of $\Delta PQR \Delta PQR$ by Pythagoras theorem

By applying Pythagoras theorem to $\Delta PQR \Delta PQR$

We get,

$PR^2 = PQ^2 + RQ^2$

Substituting values of sides from the above figure

$PR^2 = 4^2 + 3^2$

$PR^2 = 16 + 9$

$PR^2 = 25$

PR = $\sqrt{25}\sqrt{25}$

PR = 5

Hence, Hypotenuse =5

Now by definition,

sinP = Perpendicularsideoppositeto∠PHypotenuse $\sin P = \frac{Perpendicular side opposite to \angle P}{Hypotenuse}$

sinP = RQPR $\sin P = \frac{RQ}{PR}$

Substituting values of sides from the above figure

sinP = 35 $\sin P = \frac{3}{5}$

Now by definition,

sinR=Perpendicularsideoppositeto∠RHypotenuse $\sin R = \frac{\textit{Perpendicular side opposite to } \angle R}{\textit{Hypotenuse}}$

sinR=PQPR $\sin R = \frac{PQ}{PR}$

Substituting the values of sides from above figure

sinR=45 $\sin R = \frac{4}{5}$

By definition,

secP=1cosP $\sec P = \frac{1}{\cos P}$ secP=1Basesideadjacentto∠pHypotenuse $\sec P = \frac{1}{\frac{\textit{Base side adjacent to } \angle p}{\textit{Hypotenuse}}}$

secP=HypotenuseBasesideadjacentto∠P $\sec P = \frac{\textit{Hypotenuse}}{\textit{Base side adjacent to } \angle P}$

Substituting values of sides from the above figure

secP=PRPQ $\sec P = \frac{PR}{PQ}$ secP=54 $\sec P = \frac{5}{4}$

By definition,

secR=1cosR $\sec R = \frac{1}{\cos R}$ secR=1Basesideadjacentto∠RHypotenuse $\sec R = \frac{1}{\frac{\textit{Base side adjacent to } \angle R}{\textit{Hypotenuse}}}$

secR=HypotenuseBasesideadjacentto∠R $\sec R = \frac{\textit{Hypotenuse}}{\textit{Base side adjacent to } \angle R}$

Substituting values of sides from the above figure

secR=PRRQ $\sec R = \frac{PR}{RQ}$ secR=53 $\sec R = \frac{5}{3}$

Answer:

sinP=35 $\sin P = \frac{3}{5}$ **, sinR=45** $\sin R = \frac{4}{5}$**,**

secP=54 $\sec P = \frac{5}{4}$**, secR=53** $\sec R = \frac{5}{3}$

7.) If cotΘ=78 $\cot \Theta = \frac{7}{8}$**, evaluate**

(i) 1+sinΘ×1−sinΘ1+cosΘ×1−cosΘ $\frac{1+\sin \Theta \times 1-\sin \Theta}{1+\cos \Theta \times 1-\cos \Theta}$

(ii) cot²Θ $\cot^2 \Theta$

Sol.

Given: cotΘ=78 $\cot\Theta = \frac{7}{8}$

To evaluate: 1+sinΘ×1–sinΘ1+cosΘ×1–cosΘ $\frac{1+\sin\Theta\times1-\sin\Theta}{1+\cos\Theta\times1-\cos\Theta}$

1+sinΘ×1–sinΘ1+cosΘ×1–cosΘ $\frac{1+\sin\Theta\times1-\sin\Theta}{1+\cos\Theta\times1-\cos\Theta}$ ...(1)

We know the following formula

$(a + b)(a - b) = a^2 - b^2$

By applying the above formula in the numerator of equation (1)

We get,

(1+sinθ)×(1–sinθ)=1–sin2θ.....(2)(Where,a=1andb=sinθ)
$(1+sin\theta)\times(1-sin\theta) = 1-sin^2\theta.....(2) \qquad (Where,\ a = 1\ and\ b = sin\theta)$

Similarly,

By applying formula $(a + b)(a - b) = a^2 - b^2$ in the denominator of equation (1).

We get,

(1+cosΘ)(1–cosΘ)=12–cos2Θ $(1+\cos\Theta)(1-\cos\Theta) = 1^2-\cos^2\Theta$ ... (Where a=1 and b= cosΘ $\cos\Theta$

(1+cosΘ)(1–cosΘ)=1–cos2Θ $(1+\cos\Theta)(1-\cos\Theta) = 1-\cos^2\Theta$ ... (Where a=1 and b= cosΘ $\cos\Theta$

Substituting the value of numerator and denominator of equation (1) from equation (2), equation (3).

Therefore,

(1+sinΘ)(1–sinΘ)(1+cosΘ)(1–cosΘ) $\frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}$ = 1–sin2Θ1–cos2Θ $\frac{1-\sin^2\Theta}{1-\cos^2\Theta}$ ....(4)

Since,

cos2Θ+sin2Θ=1 $\cos^2\Theta+\sin^2\Theta = 1$

Therefore,

cos2Θ=1–sin2 Θ $\cos^2\Theta = 1-\sin^2\ \Theta$

Also, sin2Θ=1–cos2Θ $\sin^2\Theta = 1-\cos^2\Theta$

Putting the value of 1–sin^{2}Θ$1-\sin^2\Theta$ and 1–cos^{2}Θ$1-\cos^2\Theta$ in equation (4)

We get,

(1+sinΘ)(1–sinΘ)(1+cosΘ)(1–cosΘ) $\frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}$ = cos^{2}Θsin^{2}Θ $\frac{\cos^2\Theta}{\sin^2\Theta}$

We know that, cosΘsinΘ=cotΘ $\frac{\cos\Theta}{\sin\Theta}=\cot\Theta$

(1+sinΘ)(1–sinΘ)(1+cosΘ)(1–cosΘ) $\frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}$ = (cotΘ)^2$(\cot\Theta)^2$

Since, it is given that cotΘ=78 $\cot\Theta=\frac{7}{8}$

Therefore,

(1+sinΘ)(1–sinΘ)(1+cosΘ)(1–cosΘ) $\frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}$ = (78)^2$(\frac{7}{8})^2$

(1+sinΘ)(1–sinΘ)(1+cosΘ)(1–cosΘ) $\frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}$ = 7^{2}8^2 $\frac{7^2}{8^2}$

(1+sinΘ)(1–sinΘ)(1+cosΘ)(1–cosΘ) $\frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}$ = 4964 $\frac{49}{64}$

(ii) Given: cotΘ=78 $\cot\Theta=\frac{7}{8}$

To evaluate: cot^{2}Θ$\cot^2\Theta$

cotΘ=78 $\cot\Theta=\frac{7}{8}$

Squaring on both sides,

We get,

(cotΘ)^2=(78)^2$(\cot\Theta)^2=(\frac{7}{8})^2$

(cotΘ)^2$(\cot\Theta)^2$=4964 $\frac{49}{64}$

Answer:

4964 $\frac{49}{64}$

8.) If 3cotA=43 $\cot A = 4$, check whether 1–tan²A1+tan²A=cos²A–sin²A

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ **or not.**

Sol.

Given: 3cot A =4

To check whether 1–tan²A1+tan²A=cos²A–sin²A $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

3cot A =4

Dividing by 3 on both sides,

We get,

cot A = 43 $\frac{4}{3}$ .... (1)

By definition,

cotA=1tanA $\cot A = \frac{1}{\tan A}$

Therefore,

cotA=1PerpendicularsideoppositetoABasesideadjacenttoA $\cot A = \frac{1}{\frac{Perpendicular\ side\ opposite\ to\ \angle A}{Base\ side\ adjacent\ to\ \angle A}}$

cotA=BasesideadjacenttoAPerpendicularsideoppositetoA

$\cot A = \frac{Base\ side\ adjacent\ to\ \angle A}{Perpendicular\ side\ opposite\ to\ \angle A}$ .... (2)

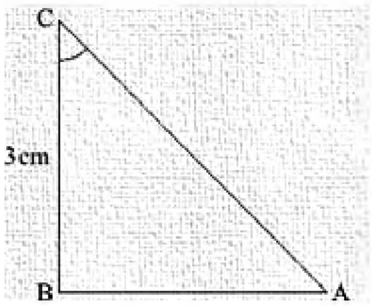
Comparing (1) and (2)

We get,

Base side adjacent to ∠A $\angle A$ = 4

Perpendicular side opposite to ∠A $\angle A$ = 3

Hence ΔABC ΔABC is as shown in figure below



In ΔABCΔABC , Hypotenuse is unknown

Hence, it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem inΔABCΔABC

We get

$AC^2 = AB^2 + BC^2$

Substituting the values of sides from the above figure

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

AC = √25$\sqrt{25}$

AC = 5

Hence, hypotenuse= 5

To check whether

To check whether 1–tan²A1+tan²A=cos²A–sin²A$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

We get thee values of tan A , cos A , sin A

By definition,

tan A = 1cotA$\frac{1}{\cot A}$

Substituting the value of cot A from equation (1)

We get,

tan A = 14$\frac{1}{4}$

tan A = 34 $\frac{3}{4}$ .... (3)

Now by definition,

cosA= Basesideadjacentto∠AHypotenuse $\cos A = \frac{Base\ side\ adjacent\ to\ \angle A}{Hypotenuse}$ cosA= ABAC

$\cos A = \frac{AB}{AC}$

Substituting the values of sides from the above figure

cosA= 45 $\cos A = \frac{4}{5}$ .... (5)

Now we first take L.H.S of equation 1–tan2A1+tan2A =cos2A–sin2A

$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$

L.H.S = 1–tan2A1+tan2A $\frac{1 - \tan^2 A}{1 + \tan^2 A}$

Substituting value of **tan A** from equation (3)

We get,

L.H.S= 1–(34)21+(34)2 $\frac{1 - (\frac{3}{4})^2}{1 + (\frac{3}{4})^2}$

1–tan2A1+tan2A $\frac{1 - \tan^2 A}{1 + \tan^2 A}$ = 1–(34)21+(34)2 $\frac{1 - (\frac{3}{4})^2}{1 + (\frac{3}{4})^2}$

1–tan2A1+tan2A $\frac{1 - \tan^2 A}{1 + \tan^2 A}$ = 1–9161+916 $\frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$

Taking L.C.M on both numerator and denominator

We get,

1–tan2A1+tan2A $\frac{1 - \tan^2 A}{1 + \tan^2 A}$ = 16–916 16+916 $\frac{\frac{16-9}{16}}{\frac{16+9}{16}}$

1–tan2A1+tan2A $\frac{1 - \tan^2 A}{1 + \tan^2 A}$ = 725 $\frac{7}{25}$ .... (6)

Now we take R.H.S of equation whether 1–tan2A1+tan2A =cos2A–sin2A

$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$

R.H.S = cos2A–sin2A $\cos^2 A - \sin^2 A$

Substituting value of sin A and cos A from equation (4) and (5)

We get,

R.H.S= $(45)^2 – (35^2)(\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2A–\sin^2A\cos^2 A - \sin^2 A = (45)^2 – (35^2)(\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2A–\sin^2A\cos^2 A - \sin^2 A$ = $1625 - 925 \frac{16}{25} - \frac{9}{25}$

$\cos^2A–\sin^2A\cos^2 A - \sin^2 A$ = $16–925 \frac{16–9}{25}$

$\cos^2A–\sin^2A\cos^2 A - \sin^2 A$ = $725 \frac{7}{25}$ ....(7)

Comparing (6) and (7)

We get.

$1–\tan^2A1+\tan^2A = \cos^2A–\sin^2A \frac{1–\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Answer:

Yes, $1–\tan^2A1+\tan^2A = \cos^2A–\sin^2A \frac{1–\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

9.) If $\tan\Theta = ab \tan\Theta = \frac{a}{b}$ **, find the value of** $\cos\Theta+\sin\Theta\cos\Theta–\sin\Theta \frac{\cos\Theta+\sin\Theta}{\cos\Theta–\sin\Theta}$.

Sol.

Given:

$\tan\Theta = ab \tan\Theta = \frac{a}{b}$ .... (1)

Now, we know that $\tan\Theta = \sin\Theta\cos\Theta \tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore equation (1) become as follows

$\sin\Theta\cos\Theta \frac{\sin\Theta}{\cos\Theta} = ab \frac{a}{b}$

Now, by applying invertendo

We get,

cosΘsinΘ = ba $\frac{\cos \Theta}{\sin \Theta} = \frac{b}{a}$

Now by applying Componendo – dividendo

We get,

cosΘ+sinΘcosΘ–sinΘ = b+ab–a $\frac{\cos \Theta + \sin \Theta}{\cos \Theta - \sin \Theta} = \frac{b+a}{b-a}$

Therefore,

cosΘ+sinΘcosΘ–sinΘ = b+ab–a $\frac{\cos \Theta + \sin \Theta}{\cos \Theta - \sin \Theta} = \frac{b+a}{b-a}$

10.) If 3tanΘ=4 $3 \tan \Theta = 4$ **, find the value of 4cosΘ–sinΘ2cosΘ+sinΘ** $\frac{4 \cos \Theta - \sin \Theta}{2 \cos \Theta + \sin \Theta}$

Sol.

Given: If 3tanΘ=4 $3 \tan \Theta = 4$

Therefore,

tanΘ=43 $\tan \Theta = \frac{4}{3}$ .... (1)

Now, we know that tanΘ=sinΘcosΘ $\tan \Theta = \frac{\sin \Theta}{\cos \Theta}$

Therefore equation (1) becomes

sinΘcosΘ = 43 $\frac{\sin \Theta}{\cos \Theta} = \frac{4}{3}$ ....(2)

Now, by applying Invertendo to equation (2)

We get,

cosΘsinΘ = 34 $\frac{\cos \Theta}{\sin \Theta} = \frac{3}{4}$ .... (3)

Now, multiplying by 4 on both sides

We get

4× cosΘsinΘ =4× 34 $4 \times \frac{\cos \Theta}{\sin \Theta} = 4 \times \frac{3}{4}$

Therefore

4cosΘ–sinΘsinΘ = 3–11 $\frac{4 \cos \Theta - \sin \Theta}{\sin \Theta} = \frac{3-1}{1}$

4cosΘ–sinΘsinΘ=21 $\frac{4\cos\Theta - \sin\Theta}{\sin\Theta} = \frac{2}{1}$ …. (4)

Now, multiplying by 2 on both sides of equation (3)

We get,

2cosΘsinΘ=32 $\frac{2\cos\Theta}{\sin\Theta} = \frac{3}{2}$

Now by applying componendo in above equation

2cosΘ+sinΘsinΘ=3+22 $\frac{2\cos\Theta + \sin\Theta}{\sin\Theta} = \frac{3+2}{2}$

2cosΘ+sinΘsinΘ=52 $\frac{2\cos\Theta + \sin\Theta}{\sin\Theta} = \frac{5}{2}$ ….(5)

We get,

4cosΘ–sinΘsinΘ 2cosΘ+sinΘsinΘ=21 52 $\frac{\frac{4\cos\Theta - \sin\Theta}{\sin\Theta}}{\frac{2\cos\Theta + \sin\Theta}{\sin\Theta}} = \frac{\frac{2}{1}}{\frac{5}{2}}$

Therefore,

4cosΘ–sinΘsinΘ × sinΘ2cosΘ+sinΘ=21 × 25 $\frac{4\cos\Theta - \sin\Theta}{\sin\Theta} \times \frac{\sin\Theta}{2\cos\Theta + \sin\Theta} = \frac{2}{1} \times \frac{2}{5}$

Therefore, on L.H.S sinΘ$\sin\Theta$ cancels and we get,

4cosΘ–sinΘ2cosΘ+sinΘ=21 × 25 $\frac{4\cos\Theta - \sin\Theta}{2\cos\Theta + \sin\Theta} = \frac{2}{1} \times \frac{2}{5}$

Therefore,

4cosΘ–sinΘ=4$4\cos\Theta - \sin\Theta = 4$

11.) If 3cotΘ=2$3\cot\Theta = 2$**, find the value of 4sinΘ–3cosΘ2sinΘ+6cosΘ** $\frac{4\sin\Theta - 3\cos\Theta}{2\sin\Theta + 6\cos\Theta}$

Sol.

Given:

3cotΘ=2$3\cot\Theta = 2$

Therefore,

cotΘ=23$\cot\Theta = \frac{2}{3}$ …. (1)

Now, we know that cotΘ=cosΘsinΘ $\cot\Theta = \frac{\cos\Theta}{\sin\Theta}$

Therefore equation (1) becomes

cosΘsinΘ=23 $\frac{\cos\Theta}{\sin\Theta} = \frac{2}{3}$ ….(2)

Now , by applying invertendo to equation (2)

sinΘcosΘ=32 $\frac{\sin\Theta}{\cos\Theta} = \frac{3}{2}$ ….(3)

Now, multiplying by 43 $\frac{4}{3}$ on both sides,

We get,

43×sinΘcosΘ=43×32 $\frac{4}{3} \times \frac{\sin\Theta}{\cos\Theta} = \frac{4}{3} \times \frac{3}{2}$

Therefore, 3 cancels out on R.H.S and

We get,

4sinΘ3cosΘ=21 $\frac{4\sin\Theta}{3\cos\Theta} = \frac{2}{1}$

Now by applying invertendo dividendo in above equation

We get,

4sinΘ–3cosΘ3cosΘ=2–11 $\frac{4\sin\Theta - 3\cos\Theta}{3\cos\Theta} = \frac{2-1}{1}$

4sinΘ–3cosΘ3cosΘ=11 $\frac{4\sin\Theta - 3\cos\Theta}{3\cos\Theta} = \frac{1}{1}$ ….(4)

Now, multiplying by 26 $\frac{2}{6}$ on both sides of equation (3)

We get,

26×sinΘcosΘ=26×32 $\frac{2}{6} \times \frac{\sin\Theta}{\cos\Theta} = \frac{2}{6} \times \frac{3}{2}$

Therefore, 2 cancels out on R.H.S and

We get,

2sinΘ6cosΘ=36 $\frac{2\sin\Theta}{6\cos\Theta} = \frac{3}{6}$ 2sinΘ6cosΘ=12 $\frac{2\sin\Theta}{6\cos\Theta} = \frac{1}{2}$

Now by applying componendo in above equation

We get,

2cosΘ+6sinΘ6sinΘ=1+22 $\frac{2\cos\Theta+6\sin\Theta}{6\sin\Theta}=\frac{1+2}{2}$

2cosΘ+6sinΘ6sinΘ=32 $\frac{2\cos\Theta+6\sin\Theta}{6\sin\Theta}=\frac{3}{2}$ ….(5)

Now, by dividing equation (4) by (5)

We get,

4sinΘ–3cosΘ3sinΘ 2cosΘ+6sinΘ6sinΘ = 11 32 $\frac{\frac{4\sin\Theta-3\cos\Theta}{3\sin\Theta}}{\frac{2\cos\Theta+6\sin\Theta}{6\sin\Theta}}=\frac{\frac{1}{1}}{\frac{3}{2}}$

Therefore,

4sinΘ–3cosΘ3sinΘ × 6sinΘ2cosΘ+6sinΘ = 11 × 23 $\frac{4\sin\Theta-3\cos\Theta}{3\sin\Theta}\times\frac{6\sin\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{1}\times\frac{2}{3}$

4sinΘ–3cosΘ3sinΘ × 2×3sinΘ2cosΘ+6sinΘ = 11 × 23 $\frac{4\sin\Theta-3\cos\Theta}{3\sin\Theta}\times\frac{2\times3\sin\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{1}\times\frac{2}{3}$

Therefore, on L.H.S (3 sinΘ$\sin\Theta$) cancels out and we get,

2×4sinΘ–3cosΘ2cosΘ+6sinΘ = 11 × 23 $\frac{2\times4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{1}\times\frac{2}{3}$

Now, by taking 2 in the numerator of L.H.S on the R.H.S

We get,

4sinΘ–3cosΘ2cosΘ+6sinΘ = 23×2 $\frac{4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta}=\frac{2}{3\times2}$

Therefore, 2 cancels out on R.H.S and

We get,

4sinΘ–3cosΘ2cosΘ+6sinΘ = 13 $\frac{4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{3}$

Hence answer,

4sinΘ–3cosΘ2cosΘ+6sinΘ = 13 $\frac{4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{3}$

12.) If tanΘ=ab $\tan\Theta=\frac{a}{b}$**, prove that asinΘ–bcosΘasinΘ+bcosΘ = a²–b²a²+b²**

$\frac{a\sin\Theta-b\cos\Theta}{a\sin\Theta+b\cos\Theta}=\frac{a^2-b^2}{a^2+b^2}$

Sol.

Given:

tanΘab $\tan\Theta \frac{a}{b}$ .... (1)

Now, we know that tanΘ=sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore equation (1) becomes

sinΘcosΘ=ab $\frac{\sin\Theta}{\cos\Theta} = \frac{a}{b}$ ....(2)

Now, by multiplying by ab $\frac{a}{b}$ on both sides of equation (2)

We get,

ab × sinΘcosΘ=ab × ab $\frac{a}{b} \times \frac{\sin\Theta}{\cos\Theta} = \frac{a}{b} \times \frac{a}{b}$

Therefore,

asinΘbcosΘ=a2b2 $\frac{a\sin\Theta}{b\cos\Theta} = \frac{a^2}{b^2}$ ....(3)

Now by applying dividendo in above equation (3)

We get,

asinΘ–bcosΘbcosΘ=a2–b2b2 $\frac{a\sin\Theta - b\cos\Theta}{b\cos\Theta} = \frac{a^2-b^2}{b^2}$ ....(4)

Now by applying componendo in equation (3)

We get,

asinΘ+bcosΘbcosΘ=a2+b2b2 $\frac{a\sin\Theta + b\cos\Theta}{b\cos\Theta} = \frac{a^2+b^2}{b^2}$ ....(5)

Now, by dividing equation (4) by equation (5)

We get,

asinΘ–bcosΘbcosΘ asinΘ+bcosΘbcosΘ = a2–b2b2 a2+b2b2 $\frac{\frac{a\sin\Theta - b\cos\Theta}{b\cos\Theta}}{\frac{a\sin\Theta + b\cos\Theta}{b\cos\Theta}} = \frac{\frac{a^2-b^2}{b^2}}{\frac{a^2+b^2}{b^2}}$

Therefore,

asinΘ–bcosΘbcosΘ × bcosΘasinΘ+bcosΘ = a2–b2b2 × b2a2+b2

$$\frac{a\sin\Theta - b\cos\Theta}{b\cos\Theta} \times \frac{b\cos\Theta}{a\sin\Theta + b\cos\Theta} = \frac{a^2-b^2}{b^2} \times \frac{b^2}{a^2+b^2}$$

Therefore, bcosΘ $b\cos\Theta$ and b^2 cancels on L.H.S and R.H.S respectively asinΘ–bcosΘasinΘ+bcosΘ = a^2–b^{2}a^2+b^2 $\frac{a \sin \Theta - b \cos \Theta}{a \sin \Theta + b \cos \Theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Hence, it is proved that

asinΘ–bcosΘasinΘ+bcosΘ = a^2–b^{2}a^2+b^2 $\frac{a \sin \Theta - b \cos \Theta}{a \sin \Theta + b \cos \Theta} = \frac{a^2 - b^2}{a^2 + b^2}$

13.) If secΘ=135 $\sec \Theta = \frac{13}{5}$**, show that 2sinΘ–3cosΘ4sinΘ–9cosΘ=3** $\frac{2 \sin \Theta - 3 \cos \Theta}{4 \sin \Theta - 9 \cos \Theta} = 3$

Sol.

Given:

secΘ=135 $\sec \Theta = \frac{13}{5}$

To show that 2sinΘ–3cosΘ4sinΘ–9cosΘ=3 $\frac{2 \sin \Theta - 3 \cos \Theta}{4 \sin \Theta - 9 \cos \Theta} = 3$

Now, we know that cosΘ=1secΘ $\cos \Theta = \frac{1}{\sec \Theta}$

Therefore,

cosΘ=1135 $\cos \Theta = \frac{1}{\frac{13}{5}}$

Therefore,

cosΘ=513 $\cos \Theta = \frac{5}{13}$ …. (1)

Now, we know that

cosΘ=BasesideadjacenttoΘ∠Hypotenuse $\cos \Theta = \frac{Base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$

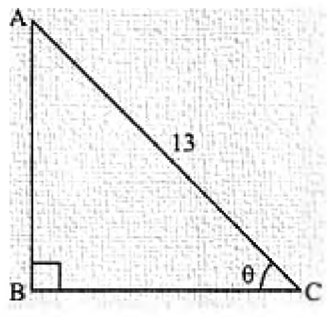
Now, by comparing equation (1) and(2)

We get,

Base side adjacent to ∠Θ$\angle\Theta$ = 5

And

Hypotenuse =13



Therefore from above figure

Base side BC = 5

Hypotenuse AC = 13

Side AB is unknown. It can be determined by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values of known sides

We get,

$13^2 = AB^2 + 5^2$

Therefore,

$AB^2 = 13^2 – 5^2$

$AB^2 = 169 – 25$

$AB^2 = 144$

AB = √144 $\sqrt{144}$

Therefore,

AB = 12 …. (3)

Now, we know that

sinΘ=ABAC $\sin\Theta = \frac{AB}{AC}$

sinΘ=1213 $\sin\Theta = \frac{12}{13}$ …. (4)

Now L.H.S of the equation to be proved is as follows

L.H.S = 2sinΘ–3tanΘ4sinΘ–3cosΘ $\frac{2\sin\Theta - 3\tan\Theta}{4\sin\Theta - 3\cos\Theta}$

Substituting the value cosΘ$\cos\Theta$ of sinΘ$\sin\Theta$and from equation (1) and (4) respectively

We get,

2×1213–3×5134×1213–9×513 $\frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}}$

Therefore,

L.H.S = 2×12–3×54×12–9×5 $\frac{2\times12-3\times5}{4\times12-9\times5}$

L.H.S = 24–1548–45 $\frac{24-15}{48-45}$

L.H.S= 93 $\frac{9}{3}$

L.H.S= 3

Hence proved that,

2sinΘ–3tanΘ4sinΘ–3cosΘ $\frac{2\sin\Theta - 3\tan\Theta}{4\sin\Theta - 3\cos\Theta}$ = 3

14.) If cosΘ=1213 $\cos\Theta = \frac{12}{13}$, show that sinΘ(1–tanΘ)=35156
$\sin\Theta(1-\tan\Theta) = \frac{35}{156}$

Sol.

Given: cosΘ=1213 $\cos\Theta = \frac{12}{13}$ …. (1)

To show that sinΘ(1–tanΘ)=35156 $\sin\Theta(1-\tan\Theta) = \frac{35}{156}$

Now we know that cosΘ=BasesideadjacenttoΘ∠Hypotenuse $\cos\Theta = \frac{Base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$

….(2)

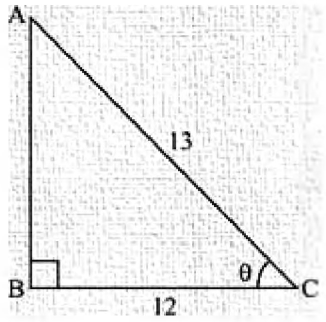
Therefore, by comparing equation (1) and (2)

We get,

Base side adjacent to ∠Θ$\angle\Theta$ = 12

And

Hypotenuse = 13



Therefore from above figure

Base side BC= 12

Hypotenuse AC= 13

Side AB is unknown and it can be determined by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values of known sides

We get,

$13^2 = AB^2 + 12^2$

Therefore,

$AB^2 = 13^2 - 12^2$

$AB^2 = 169 - 144$

AB =25

AB= $\sqrt{25}\sqrt{25}$

Therefore,

AB = 5 …. (3)

Now, we know that

sinΘ=Perpendicularsideoppositeto∠ΘHypotenuse $\sin\Theta = \frac{\textit{Perpendicular side opposite to } \angle\Theta}{\textit{Hypotenuse}}$

Now from figure (a)

We get,

sinΘ=ABAC $\sin\Theta = \frac{AB}{AC}$

Therefore,

sinΘ=512 $\sin\Theta = \frac{5}{12}$ …. (5)

Now L.H.S of the equation to be proved is as follows

L.H.S of the equation to be proved is as follows

L.H.S = sinΘ(1–tanΘ] $\sin\Theta(1-\tan\Theta]$ …. (6)

Substituting the value of sinΘ $\sin\Theta$ and tanΘ $\tan\Theta$ from equation (4) and (5)

We get,

L.H.S = 513(1−512) $\frac{5}{13}(1-\frac{5}{12})$

Taking L.C.M inside the bracket

We get,

L.H.S= 513(1×121×12−512) $\frac{5}{13}(\frac{1\times 12}{1\times 12}-\frac{5}{12})$

Therefore,

L.H.S= 513(12–512) $\frac{5}{13}(\frac{12-5}{12})$

L.H.S = 513(712) $\frac{5}{13}(\frac{7}{12})$

Now by opening the bracket and simplifying

We get,

L.H.S = 5×713×12 $\frac{5\times 7}{13\times 12}$

L.H.S= 35136 $\frac{35}{136}$

From equation (6) and (7) ,it can be shown that

that sinΘ(1–tanΘ) $\sin\Theta(1-\tan\Theta)$ **= 35136** $\frac{35}{136}$

15.) If cotΘ=1√3 $\cot\Theta = \frac{1}{\sqrt{3}}$ **, show that 1–cos²Θ2–sin²Θ=35** $\frac{1-\cos^2\Theta}{2-\sin^2\Theta} = \frac{3}{5}$

Sol.

Given: cotΘ=1√3 $\cot\Theta = \frac{1}{\sqrt{3}}$ …. (1)

To show that 1–cos²Θ2–sin²Θ=35 $\frac{1-\cos^2\Theta}{2-\sin^2\Theta} = \frac{3}{5}$

Now, we know that cotΘ=1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Since tanΘ=PerpendicularsideoppositetoΘBasesideadjacenttoΘ

$$\tan\Theta = \frac{\textit{Perpendicular side opposite to } \angle\Theta}{\textit{Base side adjacent to } \angle\Theta}$$

Therefore,

tanΘ=1PerpendicularsideoppositetoΘBasesideadjacenttoΘ $\tan\Theta = \frac{1}{\frac{\textit{Perpendicular side opposite to } \angle\Theta}{\textit{Base side adjacent to } \angle\Theta}}$

Therefore,

cotΘ=BasesideadjacenttoΘPerpendicularsideoppositetoΘ

$$\cot\Theta = \frac{\textit{Base side adjacent to } \angle\Theta}{\textit{Perpendicular side opposite to } \angle\Theta} \text{ …. (2)}$$

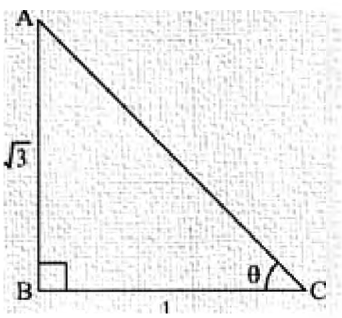
Comparing Equation (1) and (2)

We get.

Base side adjacent to ∠Θ$\angle\Theta$ = 1

Perpendicular side opposite to ∠Θ$\angle\Theta$ = √3$\sqrt{3}$

Therefore, triangle representing angle √3$\sqrt{3}$ is as shown below



Therefore, by substituting the values of known sides

We get,

AC^2 = (√3)^2$(\sqrt{3})^2$ + 1^2

Therefore,

$AC^2 = 3 + 1$

$AC^2 = 4$

AC= √4$\sqrt{4}$

Therefore,

AC = 2 …. (3)

Now, we know that

sinΘ=PerpendicularsideoppositetoΘ∠Hypotenuse $\sin\Theta = \frac{Perpendicular\ side\ opposite\ to\ \angle\Theta}{Hypotenuse}$

Now from figure (a)

sinΘ=ABAC $\sin\Theta = \frac{AB}{AC}$

Therefore from figure (a) and equation (3),

sinΘ=√32 $\sin\Theta = \frac{\sqrt{3}}{2}$

Now we know that

cosΘ Basesideadjacentto∠ΘHypotenuse $\cos\Theta\,\frac{Base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$

Now from figure (a)

We get,

BCAC $\frac{BC}{AC}$

Therefore from figure (a) and equation (3),

cosΘ=12 $\cos\Theta = \frac{1}{2}$ …. (5)

Now, L.H.S of the equation to be proved is as follows

L.H.S = 1–cos²Θ2–sin²Θ $\frac{1-\cos^2\Theta}{2-\sin^2\Theta}$

Substituting the value of from equation (4) and (5)

We get,

L.H.S = 1–(12)^{2}2–(√32)^2 $\frac{1-(\frac{1}{2})^2}{2-(\frac{\sqrt{3}}{2})^2}$

L.H.S = 1–142–34 $\frac{1-\frac{1}{4}}{2-\frac{3}{4}}$

Now by taking L.C.M in numerator as well as denominator

We get,

L.H.S= (4×1)–14 (4×2)–34 $\frac{\frac{(4\times1)-1}{4}}{\frac{(4\times2)-3}{4}}$

Therefore,

L.H.S = 4–14 8–34 $\frac{\frac{4-1}{4}}{\frac{8-3}{4}}$

L.H.S = 34 × 45 $\frac{3}{4} \times \frac{4}{5}$

L.H.S = 35 $\frac{3}{5}$ = R.H.S

Therefore,

1–cos^{2}Θ2–sin^{2}Θ = 35 $\frac{1-\cos^2\Theta}{2-\sin^2\Theta} = \frac{3}{5}$

16.) If tanΘ= 1√7 $\tan\Theta = \frac{1}{\sqrt{7}}$**, then show that** cosec^{2}Θ–sec^{2}Θcosec^{2}Θ+sec^{2}Θ

$\frac{cosec^2\Theta-\sec^2\Theta}{cosec^2\Theta+\sec^2\Theta}$ **= 34** $\frac{3}{4}$

Sol.

Given: tanΘ= 1√7 $\tan\Theta = \frac{1}{\sqrt{7}}$ …. (1)

To show that cosec^{2}Θ–sec^{2}Θcosec^{2}Θ+sec^{2}Θ $\frac{cosec^2\Theta-\sec^2\Theta}{cosec^2\Theta+\sec^2\Theta}$ = 34 $\frac{3}{4}$

Now, we know that

Since, tanΘ= Perpendicularsideopositeto∠ΘBasesideadjacentto∠Θ

$\tan\Theta = \frac{Perpendicular\ side\ oposite\ to\ \angle\Theta}{Base\ side\ adjacent\ to\ \angle\Theta}$ ….(2)

Therefore,

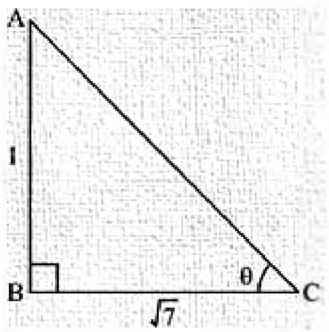
Comparing equation (1) and (2)

We get.

Perpendicular side opposite to $\angle\Theta\angle\Theta$ = 1

Base side adjacent to $\angle\Theta\angle\Theta$ = $\sqrt{7}\sqrt{7}$

Therefore, Triangle representing $\angle\Theta\angle\Theta$ is shown below



Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values of known sides

We get,

$AC^2 = (1)^2 + (\sqrt{7})^2(\sqrt{7})^2$

Therefore,

$AC^{2} = 1 + 7$

$AC^2 = 8$

AC = $\sqrt{8}\sqrt{8}$

AC = $\sqrt{2\times2\times2}\sqrt{2 \times 2 \times 2}$

Therefore,

AC = $2\sqrt{2}2\sqrt{2}$ …. (3)

Now we know that

sinΘ=Perpendicularsideoppositeto∠ΘHypotenuse $\sin\Theta = \dfrac{Perpendicular\ side\ opposite\ to\ \angle\Theta}{Hypotenuse}$

sinΘ=ABAC $\sin\Theta = \dfrac{AB}{AC}$

sinΘ=12√2 $\sin\Theta = \dfrac{1}{2\sqrt{2}}$ .... (4)

Now, we know that cosec Θ=1sinΘ $\Theta = \dfrac{1}{\sin\Theta}$

Therefore, from equation (4)

We get,

cosec Θ=2√2Θ $= 2\sqrt{2}$ .... (5)

Now, we know that

cosΘ=Basesideadjacentto∠ΘHypotenuse $\cos\Theta = \dfrac{Base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$

Now from figure (a)

We get,

cosΘ=BCAC $\cos\Theta = \dfrac{BC}{AC}$

Therefore from figure (a) and equation (3)

cosΘ=√72√2 $\cos\Theta = \dfrac{\sqrt{7}}{2\sqrt{2}}$ .... (6)

Now we know that secΘ=1cosΘ $\sec\Theta = \dfrac{1}{\cos\Theta}$

Therefore, from equation (6)

We get,

secΘ=1√72√2 $\sec\Theta = \dfrac{1}{\frac{\sqrt{7}}{2\sqrt{2}}}$

secΘ=2√2√7 $\sec\Theta = \dfrac{2\sqrt{2}}{\sqrt{7}}$ .... (7)

Now, L.H.S of the equation to be proved is as follows

L.H.S = cosec²Θ–sec²Θcosec²Θ+sec²Θ $\dfrac{cosec^2\Theta - \sec^2\Theta}{cosec^2\Theta + \sec^2\Theta}$

Substituting the value of cosecΘΘ andsecΘsec Θ from equation (6) and (7)

We get,

L.H.S = $[(2\sqrt{2})]^2-(2\sqrt{2}\sqrt{7})^2[(2\sqrt{2})]^2+(2\sqrt{2}\sqrt{7})^2$ $\frac{[(2\sqrt{2})]^2-\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{[(2\sqrt{2})]^2+\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$

L.H.S= (8)−(87)(8)+(87) $\frac{(8)-\left(\frac{8}{7}\right)}{(8)+\left(\frac{8}{7}\right)}$

Therefore,

56–87 56+87 $\frac{\frac{56-8}{7}}{\frac{56+8}{7}}$

L.H.S = 487 647 $\frac{\frac{48}{7}}{\frac{64}{7}}$

Therefore,

L.H.S = 4864 $\frac{48}{64}$

L.H.S = 34 $\frac{3}{4}$ = R.H.S

Therefore,

cosec²Θ–sec²Θcosec²Θ+sec²Θ $\frac{cosec^2\,\Theta-\sec^2\Theta}{cosec^2\,\Theta+\sec^2\Theta}$ = 34 $\frac{3}{4}$

Hence proved that

cosec²Θ–sec²Θcosec²Θ+sec²Θ $\frac{cosec^2\,\Theta-\sec^2\Theta}{cosec^2\,\Theta+\sec^2\Theta}$ = 34 $\frac{3}{4}$

17.) If secΘ=54 $\sec\Theta=\frac{5}{4}$,find the value of sinΘ–2cosΘtanΘ–cotΘ $\frac{\sin\Theta-2\cos\Theta}{\tan\Theta-\cot\Theta}$

Sol.

Given: secΘ=54 $\sec\Theta=\frac{5}{4}$ …. (1)

To find the value of sinΘ–2cosΘtanΘ–cotΘ $\frac{\sin\Theta-2\cos\Theta}{\tan\Theta-\cot\Theta}$

Now we know that secΘ=1cosΘ $\sec\Theta=\frac{1}{\cos\Theta}$

Therefore,

cosΘ=1secΘ $\cos\Theta = \frac{1}{\sec\Theta}$

Therefore from equation (1)

cosΘ=15 $\cos\Theta = \frac{1}{5}$

cosΘ=45 $\cos\Theta = \frac{4}{5}$ .... (2)

Also, we know that cos2Θ+sin2Θ=1 $\cos^2\Theta + \sin^2\Theta = 1$

Therefore,

sin2Θ=1–cos2Θ $\sin^2\Theta = 1 - \cos^2\Theta$ sinΘ=√1–cos2Θ $\sin\Theta = \sqrt{1-\cos^2\Theta}$

Substituting the value of cosΘ $\cos\Theta$ from equation (2)

We get,

sinΘ=√1−(45)2 $\sin\Theta = \sqrt{1-\left(\frac{4}{5}\right)^2}$

= √1−1625 $\sqrt{1-\frac{16}{25}}$

= 925 $\frac{9}{25}$

= 35 $\frac{3}{5}$

Therefore,

sinΘ=35 $\sin\Theta = \frac{3}{5}$ .... (3)

Also, we know that

sec2Θ=1+tan2Θ $\sec^2\Theta = 1 + \tan^2\Theta$

Therefore,

tan2Θ=(54)2−1 $\tan^2\Theta = \left(\frac{5}{4}\right)^2 - 1$ tanΘ=(√916) $\tan\Theta = \left(\sqrt{\frac{9}{16}}\right)$

Therefore,

tanΘ=34 $\tan\Theta = \frac{3}{4}$ .... (4)

Also, cotΘ=1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore from equation (4)

We get,

cotΘ=43 $\cot\Theta = \frac{4}{3}$ .... (5)

Substituting the value of cosΘ $\cos\Theta$, cotΘ $\cot\Theta$ and tanΘ $\tan\Theta$ from the equation (2), (3),(4) and (5) respectively in the expression below

sinΘ–2cosΘtanΘ–cotΘ $\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta}$

We get,

sinΘ–2cosΘtanΘ–cotΘ $\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta}$ = 35−2(45)34−43 $\frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$

= 127 $\frac{12}{7}$

Therefore, sinΘ–2cosΘtanΘ–cotΘ $\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta}$ = 127 $\frac{12}{7}$

18.) If sinΘ=1213 $\sin\Theta = \frac{12}{13}$, find the value of 2sinΘcosΘcos²Θ–sin²Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta - \sin^2\Theta}$

Sol.

Given: sinΘ=1213 $\sin\Theta = \frac{12}{13}$ .... (1)

To, find the value of 2sinΘcosΘcos²Θ–sin²Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta - \sin^2\Theta}$

Now, we know the following trigonometric identity

cosec² Θ=1+tan²ΘΘ $= 1 + \tan^2\Theta$

Therefore, by substituting the value of tanΘ $\tan\Theta$ from equation (1)

We get,

cosec² Θ=1+(1213)² $\Theta = 1 + \left(\frac{12}{13}\right)^2$

= 1+12²13² $1 + \frac{12^2}{13^2}$

= 1+144169 $1+\frac{144}{169}$

By taking L.C.M on the R.H.S

We get,

cosec^{2}Θ= 169+144169 $\Theta=\frac{169+144}{169}$

= 313169 $\frac{313}{169}$

Therefore

cosecΘ=√313169 $\Theta=\sqrt{\frac{313}{169}}$

=Θ= √31313 $\Theta=\frac{\sqrt{313}}{13}$

Therefore

cosecΘΘ = Θ= √31313 $\Theta=\frac{\sqrt{313}}{13}$ …. (2)

Now, we know that

cosecΘ$cosec$Θ = 1sinΘ $\frac{1}{\sin\Theta}$

sinΘ= 1 √31313 $\sin\Theta=\frac{1}{\frac{\sqrt{313}}{13}}$

Therefore

sinΘ= 13√313 $\sin\Theta=\frac{13}{\sqrt{313}}$ …. (3)

Now, we know the following trigonometric identity

cos^{2}Θ+sin^{2}Θ=1 $\cos^2\Theta+\sin^2\Theta=1$

Therefore,

cos^{2}Θ=1−sin^{2}Θ $\cos^2\Theta=1-\sin^2\Theta$

Now by substituting the value of sinΘ $\sin\Theta$ from equation (3)

We get,

cos^{2}Θ=1−(13√313)^2 $\cos^2\Theta=1-\left(\frac{13}{\sqrt{313}}\right)^2$

=1−169313 $1-\frac{169}{313}$

Therefore, by taking L.C.M on R.H.S

We get,

$\cos^2\Theta$= 144313 $\cos^2\Theta = \frac{144}{313}$

Now, by taking square root on both sides

We get,

$\cos\Theta$= 12√313 $\cos\Theta = \frac{12}{\sqrt{313}}$

Therefore,

$\cos\Theta$= 12√313 $\cos\Theta = \frac{12}{\sqrt{313}}$ …. (4)

Substituting the value of $\sin\Theta \sin\Theta$ and $\cos\Theta \cos\Theta$ from equation (3) and (4) respectively in the equation below

2sinΘcosΘcos²Θ−sin²Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta - \sin^2\Theta}$

Therefore,

2sinΘcosΘcos²Θ−sin²Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta - \sin^2\Theta}$ = 2× 13√313 × 12√313 (13√313)²−(12√313)² $\frac{2\times\frac{13}{\sqrt{313}}\times\frac{12}{\sqrt{313}}}{(\frac{13}{\sqrt{313}})^2-(\frac{12}{\sqrt{313}})^2}$

= 312313 25313 $\frac{\frac{312}{313}}{\frac{25}{313}}$

31225 $\frac{312}{25}$

Therefore

2sinΘcosΘcos²Θ−sin²Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta - \sin^2\Theta}$ **=**

31225 $\frac{312}{25}$

19.) If $\cos\Theta$= 35 $\cos\Theta = \frac{3}{5}$ **, find the value of sinΘ−1tanΘ2tanΘ** $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$

Sol.

Given: cosΘ=35 $\cos\Theta = \frac{3}{5}$ .... (1)

To find the value of sinΘ−1tanΘ2tanΘ $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$

Now we know the following trigonometric identity

cos2Θ+sin2Θ=1 $\cos^2\Theta + \sin^2\Theta = 1$

Therefore by substituting the value of cosΘ $\cos\Theta$ from equation (1)

We get,

(35)2+sin2Θ=1 $(\frac{3}{5})^2 + \sin^2\Theta = 1$

Therefore,

sin2Θ=1−(35)2 $\sin^2\Theta = 1 - (\frac{3}{5})^2$ sin2Θ=1−(925) $\sin^2\Theta = 1 - (\frac{9}{25})$ sin2Θ=25−925 $\sin^2\Theta = \frac{25-9}{25}$ sin2Θ=1625 $\sin^2\Theta = \frac{16}{25}$

Therefore by taking square root on both sides

We get,

sinΘ=45 $\sin\Theta = \frac{4}{5}$ .... (2)

Now, we know that

tanΘ=sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore by substituting the value of sinΘ $\sin\Theta$ and cosΘ $\cos\Theta$ from equation (2) and (1) respectively

We get,

tanΘ=4535=43 $\tan\Theta = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$ .... (4)

Now, by substituting the value of sinΘ $\sin\Theta$ and of tanΘ $\tan\Theta$ from equation (2) and equation (4) respectively in the expression below

sinΘ−1tanΘ2tanΘ $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$

We get,

sinΘ−1tanΘ2tanΘ $\frac{\sin\Theta-\frac{1}{\tan\Theta}}{2\tan\Theta}$ = 45−142×43 $\frac{\frac{4}{5}-\frac{1}{4}}{2\times\frac{4}{3}}$

sinΘ−1tanΘ2tanΘ $\frac{\sin\Theta-\frac{1}{\tan\Theta}}{2\tan\Theta}$ = 1620−152083 $\frac{\frac{16}{20}-\frac{15}{20}}{\frac{8}{3}}$

sinΘ−1tanΘ2tanΘ $\frac{\sin\Theta-\frac{1}{\tan\Theta}}{2\tan\Theta}$ = 3160 $\frac{3}{160}$

Therefore,

sinΘ−1tanΘ2tanΘ $\frac{\sin\Theta-\frac{1}{\tan\Theta}}{2\tan\Theta}$ **= 3160** $\frac{3}{160}$

20.) If sinΘ=35 $\sin\Theta=\frac{3}{5}$ **, find the value of cosΘ−1tanΘ2cotΘ** $\frac{\cos\Theta-\frac{1}{\tan\Theta}}{2\cot\Theta}$

Sol.

Given:

sinΘ=35 $\sin\Theta=\frac{3}{5}$ …. (1)

To find the value of cosΘ−1tanΘ2cotΘ $\frac{\cos\Theta-\frac{1}{\tan\Theta}}{2\cot\Theta}$

Now, we know the following trigonometric identity

cos2Θ+sin2Θ=1 $\cos^2\Theta+\sin^2\Theta=1$

Therefore by substituting the value of cosΘ $\cos\Theta$ from equation (1)

We get,

cos2Θ+(35)2=1 $\cos^2\Theta+(\frac{3}{5})^2=1$

Therefore,

cos2Θ=1−(35)2 $\cos^2\Theta=1-(\frac{3}{5})^2$ cos2Θ=1−925 $\cos^2\Theta=1-\frac{9}{25}$

Now by taking L.C.M

We get,

cos2Θ=25−925 $\cos^2\Theta=\frac{25-9}{25}$ cos2Θ=25−925 $\cos^2\Theta=\frac{25-9}{25}$

Therefore, by taking square roots on both sides

We get,

cosΘ=45 $\cos\Theta = \frac{4}{5}$

Therefore,

cosΘ=45 $\cos\Theta = \frac{4}{5}$ …. (2)

Now we know that

tanΘ=sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore by substituting the value of sinΘ$\sin\Theta$ and cosΘ$\cos\Theta$ from equation (1) and (2) respectively

We get,

tanΘ=3545 $\tan\Theta = \frac{\frac{3}{5}}{\frac{4}{5}}$

tanΘ=34 $\tan\Theta = \frac{3}{4}$ …. (3)

Also, we know that

cotΘ=1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore from equation (3)

We get,

cotΘ=134 $\cot\Theta = \frac{1}{\frac{3}{4}}$

cotΘ=43 $\cot\Theta = \frac{4}{3}$ …. (4)

Now by substituting the value of cosΘ$\cos\Theta$, tanΘ$\tan\Theta$ and cotΘ$\cot\Theta$ from equation (2) ,(3) and (4) respectively from the expression below

cosΘ−1tanΘ2cotΘ $\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta}$

We get,

cosΘ−1tanΘ2cotΘ $\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta}$ = 45−132×43 $\frac{\frac{4}{5}-\frac{1}{3}}{2\times\frac{4}{3}}$

cosΘ−1tanΘ2cotΘ $\frac{\cos\Theta-\frac{1}{\tan\Theta}}{2\cot\Theta}$ = 1215−2015 83 $\frac{\frac{12}{15}-\frac{20}{15}}{\frac{8}{3}}$

= −815 83 $\frac{\frac{-8}{15}}{\frac{8}{3}}$

= −15 $\frac{-1}{5}$

Therefore, cosΘ−1tanΘ2cotΘ $\frac{\cos\Theta-\frac{1}{\tan\Theta}}{2\cot\Theta}$ = −15 $\frac{-1}{5}$

21.) If tanΘ=247 $\tan\Theta=\frac{24}{7}$, find that sinΘ+cosΘ $\sin\Theta+\cos\Theta$

Sol.

Given:

tanΘ=247 $\tan\Theta=\frac{24}{7}$ …. (1)

To find,

sinΘ+cosΘ $\sin\Theta+\cos\Theta$

Now we know that tanΘ $\tan\Theta$ is defined as follows

tanΘ= Perpendicularsideoppositeto∠ΘBasesideadjacentto∠Θ

$\tan\Theta=\frac{\textit{Perpendicular side opposite to } \angle\Theta}{\textit{Base side adjacent to } \angle\Theta}$ …. (2)

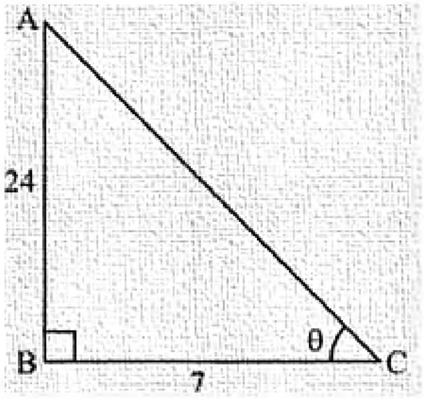
Now by comparing equation (1) and (2)

We get,

Perpendicular side opposite to ∠Θ $\angle\Theta$ = 24

Base side adjacent to ∠Θ $\angle\Theta$ = 7

Therefore triangle representing ∠Θ $\angle\Theta$ is as shown below



Side AC is unknown and can be found by using Pythagoras theorem

Therefore,

$AC^2 = AB^2 + BC^2$

Now by substituting the value of unknown sides from figure

We get,

$AC^2 = 24^2 + 7^2$

AC = 576 + 49

AC= 625

Now by taking square root on both sides,

We get,

AC = 25

Therefore H hy

Hypotenuse side AC = 25 …. (3)

Now we know sinΘ$\sin \Theta$ is defined as follows

sinΘ=Perpendicularsideoppositeto∠ΘHypotenuse $\sin \Theta = \frac{Perpendicular\ side\ opposite\ to\ \angle\Theta}{Hypotenuse}$

Therefore from figure (a) and equation (3)

We get,

sinΘ=ABAC $\sin \Theta = \frac{AB}{AC}$

sinΘ=2425 $\sin \Theta = \frac{24}{25}$ …. (4)

Now we know that $\cos\Theta$ is defined as follows

$$\cos\Theta = \frac{Base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$$

Therefore by substituting the value of $\sin\Theta$ and $\cos\Theta$ from equation (4) and (5) respectively, we get

$$\sin\Theta + \cos\Theta = \frac{24}{25} + \frac{7}{25}$$

$$\sin\Theta + \cos\Theta = \frac{31}{25}$$

Hence, $\sin\Theta + \cos\Theta = \frac{31}{25}$

22.) If $\sin\Theta = \frac{a}{b}$, find $\sec\Theta + \tan\Theta$ in terms of a and b.

Sol.

Given:

$\sin\Theta = \frac{a}{b}$ .... (1)

To find: $\sec\Theta + \tan\Theta$

Now we know, $\sin\Theta$ is defined as follows

$\sin\Theta = \frac{Perpendicular\ side\ opposite\ to\ \angle\Theta}{Hypotenuse}$ ....

(2)

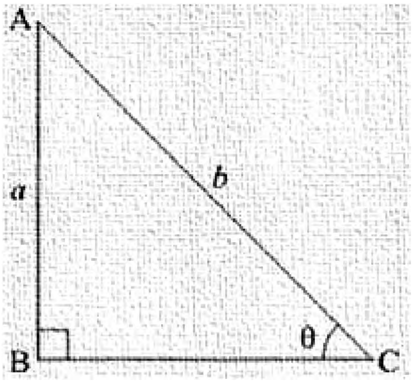
Now by comparing equation (1) and (2)

We get,

Perpendicular side opposite to $\angle\Theta$ = a

Hypotenuse = b

Therefore triangle representing $\angle\Theta$ is as shown below



Hence side BC is unknown

Now we find BC by applying Pythagoras theorem to right angled ΔABCΔABC

Therefore,

$AC^2 = AB^2 + BC^2$

Now by substituting the value of sides AB and AC from figure (a)

We get,

$b^2 = a^2 + BC^2$

Therefore,

$BC^2 = b^2 - a^2$

Now by taking square root on both sides

We get,

BC= √b2–a2$\sqrt{b^2 - a^2}$

Therefore,

Base side BC = √b2–a2$\sqrt{b^2 - a^2}$ …. (3)

Now we know cosΘ$\cos \Theta$ is defined as follows

cosΘ=BasesideadjacenttoΘHypotenuse $\cos \Theta = \frac{Base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$

Therefore from figure (a) and equation (3)

We get,

cosΘ=BCAC $\cos \Theta = \frac{BC}{AC}$

= √b2–a2b $\frac{\sqrt{b^2-a^2}}{b}$

cosΘ=BCAC $\cos\Theta = \frac{BC}{AC}$

= √b2–a2b $\frac{\sqrt{b^2-a^2}}{b}$ …. (4)

Now we know, secΘ=1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

secΘ=b√b2–a2 $\sec\Theta = \frac{b}{\sqrt{b^2-a^2}}$ …. (5)

Now we know, tanΘ=sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Now by substituting the values from equation (1) and (3)

We get,

tanΘ=ab√b2–a2b $\tan\Theta = \frac{\frac{a}{b}}{\frac{\sqrt{b^2-a^2}}{b}}$ tanΘ=a√b2–a2 $\tan\Theta = \frac{a}{\sqrt{b^2-a^2}}$

Therefore,

tanΘ=a√b2–a2 $\tan\Theta = \frac{a}{\sqrt{b^2-a^2}}$ …. (6)

Now we need to find secΘ+tanΘ $\sec\Theta + \tan\Theta$

Now by substituting the values of secΘ $\sec\Theta$ and tanΘ $\tan\Theta$ from equation (5) and (6) respectively

We get,

secΘ+tanΘ $\sec\Theta + \tan\Theta$ = b√b2–a2+a√b2–a2 $\frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}}$

secΘ+tanΘ $\sec\Theta + \tan\Theta$ = b+a√b2–a2 $\frac{b+a}{\sqrt{b^2-a^2}}$ …. (7)

We get,

secΘ+tanΘ $\sec\Theta + \tan\Theta$ = b+a√b+a–√b–a $\frac{b+a}{\sqrt{b+a}-\sqrt{b-a}}$

Now by substituting the value in above expression

We get,

secΘ+tanΘ$\sec\Theta+\tan\Theta$ = √b+a×√b+a√b+a−√b−a $\frac{\sqrt{b+a}\times\sqrt{b+a}}{\sqrt{b+a}-\sqrt{b-a}}$

Now, √b+a$\sqrt{b+a}$ present in the numerator as well as denominator of above denominator of above expression gets cancels we get,

secΘ+tanΘ=√b+a√b−a$\sec\Theta+\tan\Theta=\frac{\sqrt{b+a}}{\sqrt{b-a}}$

Square root is present in the numerator as well as denominator of above expression

Therefore we can place both numerator and denominator under a common square root sign

Therefore, secΘ+tanΘ=√b+a√b−a$\sec\Theta+\tan\Theta=\frac{\sqrt{b+a}}{\sqrt{b-a}}$

23.) If 8tanA=15$8\tan A=15$ **, find sinA−cosA**$\sin A-\cos A$

Sol.

Given:

8tanA=15$8\tan A=15$

Therefore,

tanA=158$\tan A=\frac{15}{8}$ .... (1)

To find:

sinA−cosA$\sin A-\cos A$

Now we know tan A is defined as follows

tanA=Perpendicularsideoppositeto∠ABasesideadjacentto∠A

$\tan A=\frac{\textit{Perpendicular side opposite to } \angle A}{\textit{Base side adjacent to } \angle A}$ .... (2)

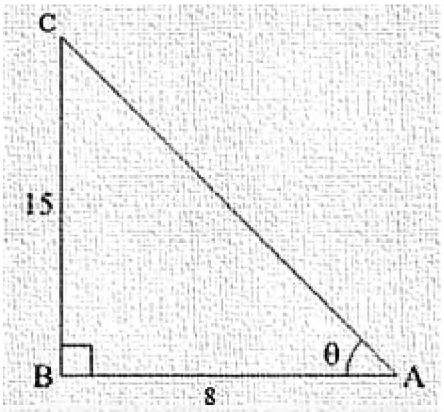
Now by comparing equation (1) and (2)

We get

Perpendicular side opposite to ∠A$\angle A$ = 15

Base side adjacent to ∠A$\angle A$ = 8

Therefore triangle representing angle A is as shown below



Side AC= is unknown and can be found by using Pythagoras theorem

Therefore,

$AC^2 = AB^2 + BC^2$

Now by substituting the value of known sides from figure (a)

We get,

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

AC = 289

Now by taking square root on both sides

We get,

AC = √289 $\sqrt{289}$

AC = 17

Therefore Hypotenuse side AC=17 …. (3)

Now we know, sin A is defined as follows

sinA=Perpendicularsideoppositeto∠AHypotenuse $\sin A = \frac{Perpendicular\ side\ opposite\ to\ \angle A}{Hypotenuse}$

Therefore from figure (a) and equation (3)

We get,

sinA=BCAC $\sin A = \frac{BC}{AC}$

sinA=1517 $\sin A = \frac{15}{17}$ …. (4)

Now we know, cos A is defined as follows

cosA=Basesideadjacentto∠AHypotenuse $\cos A = \frac{Base\ side\ adjacent\ to\ \angle A}{Hypotenuse}$

Therefore from figure (a) and equation (3)

We get,

cosA=ABAC $\cos A = \frac{AB}{AC}$

cosA=817 $\cos A = \frac{8}{17}$ …. (5)

Now we find the value of expression sinA–cosA$\sin A - \cos A$

Therefore by substituting the value the value of sinA$\sin A$ and cosA$\cos A$ from equation (4) and (5) respectively , we get,

sinA–cosA=1517−817 $\sin A - \cos A = \frac{15}{17} - \frac{8}{17}$ sinA–cosA=15–817
$\sin A - \cos A = \frac{15-8}{17}$ sinA–cosA=717 $\sin A - \cos A = \frac{7}{17}$

Hence, sinA–cosA=717 $\sin A - \cos A = \frac{7}{17}$

24.) If tanΘ=2021 $\tan \Theta = \frac{20}{21}$**, show that 1–sinΘ–cosΘ1+sinΘ+cosΘ=37**
$\frac{1-\sin \Theta - \cos \Theta}{1+\sin \Theta + \cos \Theta} = \frac{3}{7}$

Sol.
Given:

tanΘ=2021 $\tan \Theta = \frac{20}{21}$

To show that 1–sinΘ+cosΘ1+sinΘ+cosΘ=37 $\frac{1-\sin \Theta + \cos \Theta}{1+\sin \Theta + \cos \Theta} = \frac{3}{7}$

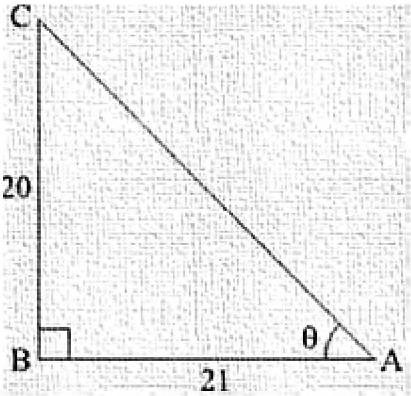
Now we know that

tanΘ=Perpendicularsideoppositeto∠ΘBasesideadjacentto∠Θ

$\tan \Theta = \frac{Perpendicular\ side\ opposite\ to\ \angle \Theta}{Base\ side\ adjacent\ to\ \angle \Theta}$

Therefore,

tanΘ=2021 $\tan\Theta = \frac{20}{21}$



Side AC be the hypotenuse and can be found by applying Pythagoras theorem

Therefore,

$AC^2 = AB^2 + BC^2$

$AC^2 = 21^2 + 20^2$

$AC^2 = 441 + 400$

$AC^2 = 841$

Now by taking square root on both sides

We get,

AC = √841 $\sqrt{841}$

AC= 29

Therefore Hypotenuse side AC= 29

Now we know, sinΘ $\sin\Theta$ is defined as follows,

sinAΘ= PerpendicularsideoppositetoㄥΘHypotenuse $\sin A\Theta = \frac{Perpendicular\ side\ opposite\ to\ \angle\Theta}{Hypotenuse}$

Therefore from figure and above equation

We get,

sinΘ=ABAC $\sin\Theta = \frac{AB}{AC}$ sinΘ=2029 $\sin\Theta = \frac{20}{29}$

Now we know cosΘ $\cos\Theta$ is defined as follows

cosΘ= BasesideadjacenttoㄥΘHypotenuse $\cos\Theta = \frac{Base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$

Therefore from figure and above equation

We get,

cosΘ=ABAC $\cos\Theta = \frac{AB}{AC}$ cosΘ=2129 $\cos\Theta = \frac{21}{29}$

Now we need to find the value of expression 1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$

Therefore by substituting the value of sinΘ$\sin\Theta$ and cosΘ$\cos\Theta$from above equations, we get

1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$ =

29–20+2129 7029 $\frac{\frac{29-20+21}{29}}{\frac{70}{29}}$

Therefore after evaluating we get,

1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$= 37 $\frac{3}{7}$

Hence,

1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$ **=**

37 $\frac{3}{7}$

25.) If cosecA=2$cosecA = 2$ **, find 1tanA+sinA1+cosA** $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$

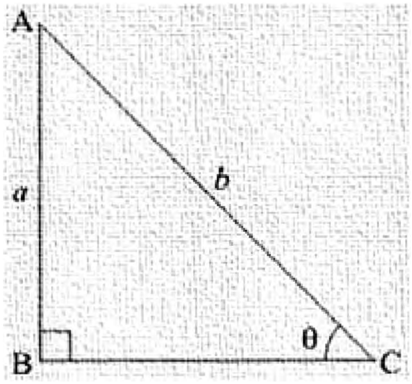
Sol.

Given:

cosecA=2$cosecA = 2$

To find 1tanA+sinA1+cosA $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$

Now cosec A = HypotenuseOppositeside $\frac{Hypotenuse}{Oppositeside}$ = 21 $\frac{2}{1}$



Here BC is the adjacent side,

By applying Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$4 = 1 + BC^2$

$BC^2 = 3$

BC = √3 $\sqrt{3}$

Now we know that

sinA=1cosecA $\sin A = \frac{1}{cosecA}$

sinA=12 $\sin A = \frac{1}{2}$ .... (1)

tanA=ABBC $\tan A = \frac{AB}{BC}$

tanA=1√3 $\tan A = \frac{1}{\sqrt{3}}$ .... (2)

cosA=BCAC $\cos A = \frac{BC}{AC}$

cosA=√32 $\cos A = \frac{\sqrt{3}}{2}$ .... (3)

Substitute all the values of sinA $\sin A$, cosA $\cos A$ and tanA $\tan A$ from the equations(1) , (2) and (3) respectively

We get.

1tanA + sinA1+cosA $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$ = 11√3 + 121+√32 $\frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$

−

= √3+ 12+√3 $\sqrt{3}+\frac{1}{2+\sqrt{3}}$

= 2(2+√3)2+√3 $\frac{2(2+\sqrt{3})}{2+\sqrt{3}}$

= 2

Hence,

1tanA + sinA1+cosA $\frac{1}{\tan A}+\frac{\sin A}{1+\cos A}$ **= 2**

26.) If ∠A $\angle A$ and ∠B $\angle B$ are acute angles such that cos A =cos B , then show that ∠A $\angle A$= ∠B $\angle B$

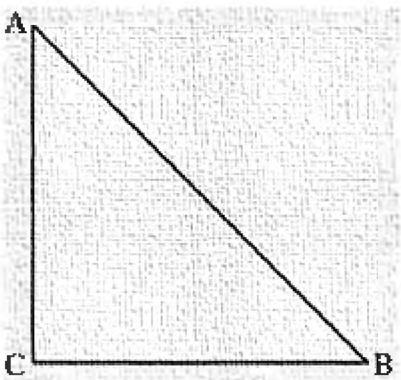
Sol.

Given:

∠A $\angle A$ and ∠B $\angle B$ are acute angles

cos A = cos B such that ∠A $\angle A$ = ∠B $\angle B$

Let us consider right angled triangle ACB



Now since cos A = cos B

Therefore

ACAB = BCAB $\frac{AC}{AB}=\frac{BC}{AB}$

Now observe that denominator of above equality is same that is AB

Hence ACAB = BCAB $\frac{AC}{AB}=\frac{BC}{AB}$ only when AC=BC

Therefore AC=BC

We know that when two sides of triangle are equal, then opposite of the sides are also

Equal.

Therefore

We can say that

Angle opposite to side AC = angle opposite to side BC

Therefore,

∠B$\angle B$ = ∠A$\angle A$

Hence, ∠A$\angle A$ = ∠B$\angle B$

27.) In a ΔABCΔABC , right angled triangle at A, if tan C = √3$\sqrt{3}$, find the value of sin B cos C + cos B sin C.

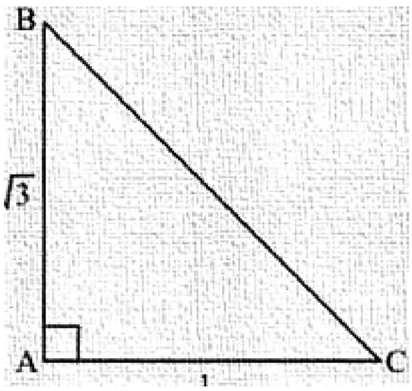
Sol.

Given:

ΔABCΔABC

To find : sin B cos C + cos B sin C

The given a ΔABCΔABC is as shown in figure



Side BC is unknown and can be found using Pythagoras theorem,

Therefore,

$BC^2 = AB^2 + AC^2$

$BC^2 = \sqrt{3}^2 \sqrt{3}^2 + 1^2$

$BC^2 = 3 + 1$

$BC^2 = 4$

Now by taking square root on both sides

We get,

$BC = \sqrt{4}\sqrt{4}$

BC= 2

Therefore Hypotenuse side BC= 2 …. (1)

Now, sin B = Perpendicularsideoppositeto∠BHypotenuse $\frac{\textit{Perpendicular side opposite to } \angle B}{\textit{Hypotenuse}}$

Therefore,

sinB= ACBC $\sin B = \frac{AC}{BC}$

Now by substituting the values from equation (1) and figure

We get,

sin B = 12 $\frac{1}{2}$ …. (2)

Now, cos B= basesideadjacentto∠BHypotenuse $\frac{\textit{base side adjacent to } \angle B}{\textit{Hypotenuse}}$

Therefore,

cos B = ABBC $\frac{AB}{BC}$

Now substituting the value from equation

cos B= √32 $\frac{\sqrt{3}}{2}$ …. (3)

Similarly

sin C = √32 $\frac{\sqrt{3}}{2}$ …. (4)

Now by definition,

tanC= sinCcosC $\tan C = \frac{sinC}{cosC}$

So by evaluating

cosC= 12 $\cos C = \frac{1}{2}$ .... (5)

Now, by substituting the value of sinB, cosB,sin C and cosC from equation (2) ,(3) ,(4) and (5) respectively in sinB cosC + cosB sin C

sinB cosC + cosB sin C= 12×12+√32×√32 $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

= 14+34 $\frac{1}{4} + \frac{3}{4}$

= 1

Hence,

sinB cosC + cosB sin C = 1

28.) State whether the following are true or false. Justify your answer.

(i) The value od tan A is always less than 1.

(ii) sec A = 125 $\frac{12}{5}$ for some value of ∠A$\angle A$.

(iii) cos A is the abbreviation used for the cosecant of ∠A$\angle A$.

(iv) sinΘ=43 $\sin \Theta = \frac{4}{3}$ for some angle ΘΘ.

Sol.

(i) tan A <$<$ 1

Value of tan A at 45^{o} i.e… tan 45 = 1

As value os A increases to 90^{o}

Tan A becomes infinite

So given statement is false.

(ii) sec A = 125 $\frac{12}{5}$ for some value of angle if

M-I

sec A = 2.4

sec A > 1

So given statements is true.

M- II

For sec A = 125 $\frac{12}{5}$ we get adjacent side = 13

Subtending 9i at B.

So, given statement is true.

(iii) Cos A is the abbreviation used for cosecant of angle A.

The given statement is false.

As such cos A is the abbreviation used for cos of angle A , not as cosecant of angle A.

(iv) Cot A is the product of cot A and A

Given statement is false

∵ cot A is a co-tangent of angle A and co-tangent of angle A = adjacentsideOpositeside

$\frac{\textit{adjacent side}}{\textit{Oposite side}}$.

(v) sinΘ=43 $\sin\Theta = \frac{4}{3}$ for some angle ΘΘ.

Given statement is false

Since value of sinΘ$\sin\Theta$ is less than(or) equal to one.

Here value of sinΘ$\sin\Theta$ exceeds one,

So given statement is false.

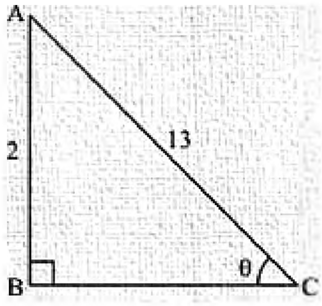
29.) If sinΘ=1213 $\sin\Theta = \frac{12}{13}$ find sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

Sol.

Given: sinΘ=1213 $\sin\Theta = \frac{12}{13}$

To Find: sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

As shown in figure



Here BC is the adjacent side,

By applying Pythagoras theorem,

$AC^2=AB^2+BC^2$

$169 = 144 + BC^2$

$BC^2 = 169 - 144$

$BC^2 = 25$

BC = 5

Now we know that,

cosΘ=basesideadjacentto∠ΘHypotenuse $cos\Theta = \frac{base\ side\ adjacent\ to\ \angle\Theta}{Hypotenuse}$ cosΘ=BCAC $\cos\Theta = \frac{BC}{AC}$ cosΘ=513 $\cos\Theta = \frac{5}{13}$

We also know that,

tanΘ=sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore, substituting the value of sinΘ $\sin\Theta$ and cosΘ $\cos\Theta$ from above equations

We get,

tanΘ=125 $\tan\Theta = \frac{12}{5}$

Now substitute all the values of sinΘ $\sin\Theta$, cosΘ $\cos\Theta$ and tanΘ $\tan\Theta$ from above equations in sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

We get,

$\sin^2\Theta - \cos^2\Theta 2\sin\Theta\cos\Theta \times 1\tan^2\Theta \ \frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = $(1213)^2 - (513)^2 2\times(1213)\times(513) \times 1(125)^2$

$$\frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\times\left(\frac{12}{13}\right)\times\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

Therefore by further simplifying we get,

$\sin^2\Theta - \cos^2\Theta 2\sin\Theta\cos\Theta \times 1\tan^2\Theta \ \frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = $119169 \times 169120 \times 25144$

$$\frac{119}{169} \times \frac{169}{120} \times \frac{25}{144}$$

Therefore,

$\sin^2\Theta - \cos^2\Theta 2\sin\Theta\cos\Theta \times 1\tan^2\Theta \ \frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = $5953456 \ \frac{595}{3456}$

Hence,

$\sin^2\Theta - \cos^2\Theta 2\sin\Theta\cos\Theta \times 1\tan^2\Theta \ \frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = $5953456 \ \frac{595}{3456}$

30.) If cosΘ= $513\cos\Theta = \frac{5}{13}$**, find the value of** $\sin^2\Theta - \cos^2\Theta 2\sin\Theta\cos\Theta \times 1\tan^2\Theta$

$$\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$$

Sol.

Given: If cosΘ= $513\cos\Theta = \frac{5}{13}$

To find:

The value of expression $\sin^2\Theta - \cos^2\Theta 2\sin\Theta\cos\Theta \times 1\tan^2\Theta \ \frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

Now we know that

cosΘ$cos\Theta$= basesideadjacentto∠ΘHypotenuse $\frac{\textit{base side adjacent to } \angle\Theta}{\textit{Hypotenuse}}$ …. (2)

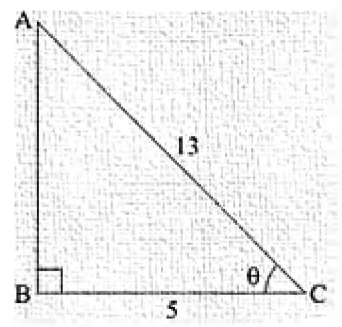
Now when we compare equation (1) and (2)

We get,

Base side adjacent to ∠Θ$\angle\Theta$ = 5

Hypotenuse = 13

Therefore, Triangle representing ∠Θ∠Θ is as shown below



Perpendicular side AB is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values ogf known sides,

$AB^2 = 13^2 – 5^2$

$AB^2 = 169 - 25$

$AB^2 = 144$

AB = 12 …. (3)

Now we know from figure and equation,

sinΘ=1213 $\sin\Theta = \frac{12}{13}$ …. (4)

Now we know that,

tanΘ=sinΘcosΘ $tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

tanΘ=125 $tan\Theta = \frac{12}{5}$ …. (5)

Now w substitute all the values from equation (1), (4) and (5) in the expression below,

sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

Therefore

We get,

sin^{2}Θ−cos^{2}Θ2sinΘcosΘ × 1tan^{2}Θ $\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = (1213)^2−(513)^{2}2×(1213)×(513) × 1(125)^2

$$\frac{\left(\frac{12}{13}\right)^2-\left(\frac{5}{13}\right)^2}{2\times\left(\frac{12}{13}\right)\times\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

Therefore by further simplifying we get,

sin^{2}Θ−cos^{2}Θ2sinΘcosΘ × 1tan^{2}Θ $\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = 119169 × 169120 × 25144

$$\frac{119}{169} \times \frac{169}{120} \times \frac{25}{144}$$

Therefore,

sin^{2}Θ−cos^{2}Θ2sinΘcosΘ × 1tan^{2}Θ $\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = 5953456 $\frac{595}{3456}$

Hence,

sin^{2}Θ−cos^{2}Θ2sinΘcosΘ × 1tan^{2}Θ $\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = 5953456 $\frac{595}{3456}$

31.) If sec A = 178 $\frac{17}{8}$, verify that 3–4sin^{2}A4cos^{2}A–3 = 3–tan^{2}A1–3tan^{2}A

$$\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$$

Sol.

Given: sec A = 178 $\frac{17}{8}$

To verify: 3–4sin^{2}A4cos^{2}A–3 = 3–tan^{2}A1–3tan^{2}A $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

Now we know that cosA= 1secA $\cos A = \frac{1}{\sec A}$

Now, by substituting the value of sec A

We get,

cosA= 817 $\cos A = \frac{8}{17}$

Now we also know that,

sin^{2}A+cos^{2}A=1 $\sin^2 A + \cos^2 A = 1$

Therefore

sin^{2}A=1–cos^{2}A $\sin^2 A = 1-\cos^2 A$

$= (817)^2 \left(\frac{8}{17}\right)^2$

$=225289 \frac{225}{289}$

Now by taking square root on both sides,

We get,

sinA= 1517 $\sin A = \frac{15}{17}$

We also know that , tanA= sinAcosA $\tan A = \frac{\sin A}{\cos A}$

Now by substituting the value of all the terms ,

We get,

tanA= 158 $\tan A = \frac{15}{8}$

Now from the expression of above equation which we want to prove:

L.H.S = 3–4sin^{2}A4cos^{2}A–3 $\frac{3–4 \sin^2 A}{4 \cos^2 A–3}$

Now by substituting the value of cos A ad sin A from equation (3) and (4)

We get,

L.H.S = 3–4 225289 4–64289–3 $\frac{3–4 \frac{225}{289}}{4–\frac{64}{289}–3}$

= 867–900256–867 $\frac{867–900}{256–867}$

= 33611 $\frac{33}{611}$

From expression

R.H.S = frac3–tan^{2}A1–3tan^{2}A $frac3–\tan^2 A1–3 \tan^2 A$

Now by substituting the value of tan A from above equation

We get,

R.H.S= 3–$(158)^2$1–3$(158)^2$ $\frac{3–\left(\frac{15}{8}\right)^2}{1–3\left(\frac{15}{8}\right)^2}$

= -3364 -61164 $\frac{\frac{-33}{64}}{\frac{-611}{64}}$

= 33611 $\frac{33}{611}$

Therefore,

We can see that,

3–4sin^{2}A4cos^{2}A–3 = 3–tan^{2}A1–3tan^{2}A $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

32.) If sinΘ=34 $\sin\Theta = \frac{3}{4}$**, prove that** $\sqrt{\text{cosec}^2\Theta-\cot^2\Theta\sec^2-1}$ **=** $\sqrt{7}3$

$$\sqrt{\frac{cosec^2\Theta-\cot^2\Theta}{\sec^2-1}} = \frac{\sqrt{7}}{3}$$

Sol.

Given: sinΘ=34 $\sin\Theta = \frac{3}{4}$ …. (1)

To prove:

$\sqrt{\text{cosec}^2\Theta-\cot^2\Theta\sec^2-1}$ = $\sqrt{7}3$ $\sqrt{\frac{cosec^2\Theta-\cot^2\Theta}{\sec^2-1}} = \frac{\sqrt{7}}{3}$ …. (2)

By definition,

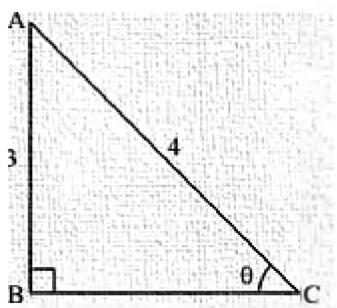
sin A = Perpendicularsideoppositeto∠AHypotenuse $\frac{Perpendicular\ side\ opposite\ to\ \angle A}{Hypotenuse}$ …. (3)

By comparing (1) and (3)

We get,

Perpendicular side = 3 and

Hypotenuse = 4



Side BC is unknown.

So we find BC by applying Pythagoras theorem to right angled $\Delta ABC \Delta ABC$

Hence,

$AC^2 = AB^2 + BC^2$

Now we substitute the value of perpendicular side (AB) and hypotenuse (AC) and get the base side (BC)

Therefore,

$4^2 = 3^2 + BC^2$

$BC^2 = 16 - 9$

$BC^2 = 7$

$BC = \sqrt{7}\sqrt{7}$

Hence, Base side BC $= \sqrt{7}\sqrt{7}$ .... (3)

Now cos A = BCAC $\frac{BC}{AC}$

$\sqrt{7}4 \; \frac{\sqrt{7}}{4}$ .... (4)

Now , cosecA= 1sinA $cosecA = \frac{1}{\sin A}$

Therefore, from fig and equation (1)

cosecA= HypotenusePerpendicular $cosecA = \frac{Hypotenuse}{Perpendicular}$

cosecA= 43 $cosecA = \frac{4}{3}$ .... (5)

Now, similarly

secA= $4\sqrt{7}$ $\sec A = \frac{4}{\sqrt{7}}$ .... (6)

Further we also know that

cotA= cosAsinA $\cot A = \frac{\cos A}{\sin A}$

Therefore by substituting th values from equation (1) and (4),

We get,

cotA=√73 $\cot A = \frac{\sqrt{7}}{3}$ .... (7)

Now by substituting the value of cosec A, sec A and cot A from the equations (5), (6), and (7) in the L.H.S of expression (2)

$\sqrt{\text{cosec}^2\Theta–\text{cot}^2\Theta\text{sec}^2–1}$ $\sqrt{\frac{cosec^2\Theta-\cot^2\Theta}{\sec^2-1}}$ = □□ □ $\sqrt{(43)^2-(\sqrt{73})^2(4\sqrt{7})^2-1}$ $\sqrt{\frac{\left(\frac{4}{3}\right)^2-\left(\frac{\sqrt{7}}{3}\right)^2}{\left(\frac{4}{\sqrt{7}}\right)^2-1}}$

= 169−79 167−1 $\frac{\frac{16}{9}-\frac{7}{9}}{\frac{16}{7}-1}$

= √73 $\frac{\sqrt{7}}{3}$

Hence it is proved that,

$\sqrt{\text{cosec}^2\Theta–\text{cot}^2\Theta\text{sec}^2–1}$ = √73 $\sqrt{\frac{cosec^2\Theta-\cot^2\Theta}{\sec^2-1}} = \frac{\sqrt{7}}{3}$

33.) If secA=178 $\sec A = \frac{17}{8}$ **, verify that 3–4sin²A4cos²A–3 = 3–tan²A1–3tan²A**

$\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

Sol.

Given: secA=178 $\sec A = \frac{17}{8}$ .... (1)

To verify:

3–4sin²A4cos²A–3 = 3–tan²A1–3tan²A $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$ .... (2)

Now we know that sec A = 1cosA $\frac{1}{cosA}$

Therefore cosA=1secA $\cos A = \frac{1}{secA}$

We get,

cosA=817 $\cos A = \frac{8}{17}$ .... (3)

Similarly we can also get,

sin A= sinA=1517 $\sin A = \frac{15}{17}$ .... (4)

An also we know that tanA=sinAcosA $\tan A = \frac{sinA}{cosA}$

tanA=158 $\tan A = \frac{15}{8}$ …. (5)

Now from the expression of equation (2)

L.H.S: Missing close brace Missing close brace

Now by substituting the value of cos A and sin A from equation (3) and (4)

We get,

L.H.S = 3–4(1517)^{2}4(817)^2–3 $\frac{3–4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2–3}$

= 867–900289 256–867289 $\frac{\frac{867-900}{289}}{\frac{256–867}{289}}$

=33611 $\frac{33}{611}$ …. (6)

R.H.S = 3–tan^{2}A1–3tan^{2}A $\frac{3-\tan^2 A}{1–3\tan^2 A}$

Now by substituting the value of tan A from equation (5)

We get,

R.H.S=3–(1518)^2 1–3(158)^2 $\frac{3–\left(\frac{15}{18}\right)^2}{1–3\left(\frac{15}{8}\right)^2}$

-3364 -61164 $\frac{\frac{-33}{64}}{\frac{-611}{64}}$

=33611 $\frac{33}{611}$ …. (7)

Now by comparing equation (6) and (7)

We get,

3–4sin^{2}A4cos^{2}A–3 = 3–tan^{2}A1–3tan^{2}A $\frac{3–4\sin^2 A}{4\cos^2 A–3} = \frac{3-\tan^2 A}{1–3\tan^2 A}$

34.) If cotΘ=34 $\cot\Theta = \frac{3}{4}$, prove that secΘ–cosecΘsecΘ+cosecΘ=1√7

$$\frac{\sec\Theta - cosec\Theta}{\sec\Theta + cosec\Theta} = \frac{1}{\sqrt{7}}$$

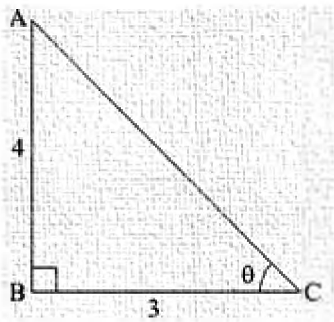
Sol.

Given: cotΘ=34 $\cot\Theta = \frac{3}{4}$

Prove that: secΘ–cosecΘsecΘ+cosecΘ=1√7 $\frac{\sec\Theta - cosec\Theta}{\sec\Theta + cosec\Theta} = \frac{1}{\sqrt{7}}$

Now we know that

secΘ–cosecΘsecΘ+cosecΘ=1√7 $\frac{\sec\Theta - cosec\Theta}{\sec\Theta + cosec\Theta} = \frac{1}{\sqrt{7}}$



Here AC is the hypotenuse and we can find that by applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

AC = 5

Similarly

secΘ=ACBC $\sec\Theta = \frac{AC}{BC}$ secΘ=53 $\sec\Theta = \frac{5}{3}$ cosec=ACAB $cosec = \frac{AC}{AB}$ cosec=54 $cosec = \frac{5}{4}$

Now on substituting the values in equations we get,

secΘ–cosecΘsecΘ+cosecΘ=1√7 $\frac{\sec\Theta - cosec\Theta}{\sec\Theta + cosec\Theta} = \frac{1}{\sqrt{7}}$

Therefore,

secΘ–cosecΘsecΘ+cosecΘ=1√7 $\frac{\sec\Theta - cosec\Theta}{\sec\Theta + cosec\Theta} = \frac{1}{\sqrt{7}}$

35.) If 3cosΘ–4sinΘ=2cosΘ+sinΘ$3\cos\Theta - 4\sin\Theta = 2\cos\Theta + \sin\Theta$ **,find tanΘ** $\tan\Theta$

Sol.

Given: 3cosΘ–4sinΘ=2cosΘ+sinΘ$3\cos\Theta - 4\sin\Theta = 2\cos\Theta + \sin\Theta$

To find: tanΘ$\tan\Theta$

We can write this as:

3cosΘ–4sinΘ=2cosΘ+sinΘ$3\cos\Theta - 4\sin\Theta = 2\cos\Theta + \sin\Theta$ cosΘ=5sinΘ $\cos\Theta = 5\sin\Theta$

Dividing both the sides by cosΘ$\cos\Theta$,

We get,

cosΘcosΘ=5sinΘcosΘ $\frac{\cos\Theta}{\cos\Theta} = \frac{5\sin\Theta}{\cos\Theta}$ 1=5tanΘ$1 = 5\tan\Theta$ tanΘ=1$\tan\Theta = 1$

Hence,

tanΘ=1$\tan\Theta = 1$

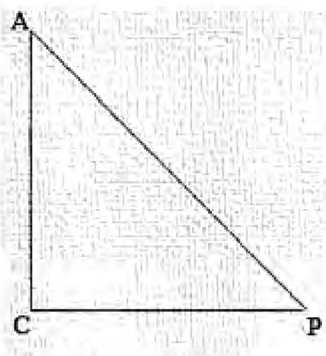
36.) If ∠A$\angle A$ **and ∠P** $\angle P$ **are acute angles such that tan A = tan P, then show ∠A=∠P**$\angle A = \angle P$

Sol.

Given: A and P are acute angles tan A =tan P

Prove that: ∠A=∠P$\angle A = \angle P$

Let us consider right angled triangle ACP



We know tanΘ=oppositesideadjacentside $\tan\Theta = \frac{opposite side}{adjacent side}$

tan A =PCAC $\frac{PC}{AC}$

tan P =ACPC $\frac{AC}{PC}$

∴∵ tan A =tan P

PCAC=ACPC $\frac{PC}{AC} = \frac{AC}{PC}$

PC =AC [∵∵Angle opposite to equal sides are equal]

∠ ∠ $\angle A = \angle P$

Exercise 5.2: Trigonometric Ratios

Evaluate each of the following:

Q 1 . sin 45°45° sin 30°30° + cos 45°45° cos 30°30°

Solution:

Sin 45°45°sin 30°30° + cos 45°45° cos 30°30°

[1]

We know that by trigonometric ratios we have ,

sin45°=1√2 $sin 45^\circ = \frac{1}{\sqrt{2}}$ sin30°=12 $sin 30^\circ = \frac{1}{2}$

cos45°=1√2 $cos 45^\circ = \frac{1}{\sqrt{2}}$ cos30°=√32 $cos 30^\circ = \frac{\sqrt{3}}{2}$

Substituting the values in equation 1 , we get

1√2 · 12 + 1√2 · √32 $\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

= 1√2 · √32√2 $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2\sqrt{2}}$

= $\sqrt{3}$+12$\sqrt{2}$ $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Q 2 . sin 60°60° cos 30°30° + cos 60°60° sin 30°30°

Solution:

sin 60°60° cos 30°30° + cos 60°60° sin 30°30° [1]

By trigonometric ratios we have ,

sin60°=$\sqrt{3}$2 $sin60^\circ = \frac{\sqrt{3}}{2}$ sin30°=12 $sin30^\circ = \frac{1}{2}$

cos30°=$\sqrt{3}$2 $cos30^\circ = \frac{\sqrt{3}}{2}$ cos60°=12 $cos60^\circ = \frac{1}{2}$

Substituting the values in equation 1 , we get

= $\sqrt{3}$2 · $\sqrt{3}$2 + 12 · 12 $\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

= 34 + 14 $\frac{3}{4} + \frac{1}{4}$ = 44 $\frac{4}{4}$ = 1

Q 3 . cos 60°60° cos 45°45° – sin 60°60° sin 45°45°

Solution:

cos 60°60° cos 45°45° – sin 60°60° sin 45°45° [1]

We know that by trigonometric ratios we have ,

cos60°=12 $cos60^\circ = \frac{1}{2}$ cos45°=1$\sqrt{2}$ $cos45^\circ = \frac{1}{\sqrt{2}}$

sin60°=$\sqrt{3}$2 $sin60^\circ = \frac{\sqrt{3}}{2}$ sin45°=1$\sqrt{2}$ $sin45^\circ = \frac{1}{\sqrt{2}}$

Substituting the values in equation 1 , we get

12 · 1√2 − √32 · 1√2 $\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

= 1−√32√2 $\frac{1-\sqrt{3}}{2\sqrt{2}}$

Q.4: sin²30°+sin²45°+sin²60°+sin²90°$sin^2 30° + sin^2 45° + sin^2 60° + sin^2 90°$

Solution:

sin²30°+sin²45°+sin²60°+sin²90°
$sin^2 30° + sin^2 45° + sin^2 60° + sin^2 90°$ [1]

We know that by trigonometric ratios we have ,

sin30°=12 $sin 30° = \frac{1}{2}$ sin45°=1√2 $sin 45° = \frac{1}{\sqrt{2}}$

sin60°=√32 $sin 60° = \frac{\sqrt{3}}{2}$ sin90° $sin 90°$ = 1

Substituting the values in equation 1 , we get

= [12]²+[1√2]²+[√32]²+1 $\left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + 1$

= 14+12+34+1 $\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

= 52 $\frac{5}{2}$

Q 5. cos²30°+cos²45°+cos²60°+cos²90°
$cos^2 30° + cos^2 45° + cos^2 60° + cos^2 90°$

Solution:

cos²30°+cos²45°+cos²60°+cos²90°
$cos^2 30° + cos^2 45° + cos^2 60° + cos^2 90°$ [1]

We know that by trigonometric ratios we have , cos30°=√32 $cos 30° = \frac{\sqrt{3}}{2}$ cos45°=1√2 $cos 45° = \frac{1}{\sqrt{2}}$

cos60°=12 $cos 60° = \frac{1}{2}$ cos90° $cos 90°$ = 0

Substituting the values in equation 1 , we get

[√32]^2+[1√2]^2+[12]^2+0 $\left[\frac{\sqrt{3}}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{1}{2}\right]^2 + 0$

= 34+12+14 $\frac{3}{4} + \frac{1}{2} + \frac{1}{4}$

= 32 $\frac{3}{2}$

Q 6 . tan²30°+tan²45°+tan²60° $tan^2 30° + tan^2 45° + tan^2 60°$

Solution:

tan²30°+tan²45°+tan²60° $tan^2 30° + tan^2 45° + tan^2 60°$
[1]

We know that by trigonometric ratios we have ,

tan30°=1√3 $tan 30° = \frac{1}{\sqrt{3}}$ tan60°=√3 $tan 60° = \sqrt{3}$

tan45°=1 $tan 45° = 1$

Substituting the values in equation 1 , we get

[1√3]^2+[√3]^2+1 $\left[\frac{1}{\sqrt{3}}\right]^2 + \left[\sqrt{3}\right]^2 + 1$

= 13+3+1 $\frac{1}{3} + 3 + 1$

= 133 $\frac{13}{3}$

Q 7 . 2sin²30°−3cos²45°+tan²60° $2sin^2 30° - 3cos^2 45° + tan^2 60°$

Solution:

2sin^{2}30°−3cos^{2}45°+tan^{2}60°$2sin^2 30^\circ - 3cos^2 45^\circ + tan^2 60^\circ$ [1]

We know that by trigonometric ratios we have ,

sin30°=12$sin30^\circ = \frac{1}{2}$ cos45°=1√2$cos45^\circ = \frac{1}{\sqrt{2}}$

tan60°=√3$tan60^\circ = \sqrt{3}$

Substituting the values in equation 1 , we get

= 2(12)^2−3(1√2)^2+(√3)^2$2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2$

= 2(14)−3(12)+32$\left(\frac{1}{4}\right) - 3\left(\frac{1}{2}\right) + 3$

= 1−3+62$\frac{1-3+6}{2}$

= 2

Q8:sin^{2}30°cos^{2}45°+4tan^{2}30°+12sin^{2}90°−2cos^{2}90°+124cos^{2}0°
$sin^2 30^\circ cos^2 45^\circ + 4tan^2 30^\circ + \frac{1}{2} sin^2 90^\circ - 2cos^2 90^\circ + \frac{1}{24} cos^2 0^\circ$

Solution:

sin^{2}30°cos^{2}45°+4tan^{2}30°+12sin^{2}90°−2cos^{2}90°+124cos^{2}0°
$sin^2 30^\circ cos^2 45^\circ + 4tan^2 30^\circ + \frac{1}{2} sin^2 90^\circ - 2cos^2 90^\circ + \frac{1}{24} cos^2 0^\circ$
[1]

We know that by trigonometric ratios we have ,

sin30°=12$sin30^\circ = \frac{1}{2}$ cos45°=1√2$cos45^\circ = \frac{1}{\sqrt{2}}$ tan30°=1√3$tan30^\circ = \frac{1}{\sqrt{3}}$

sin90°$sin90^\circ$ = 1

cos90°$cos90^\circ$ = 0

cos0°$cos0^\circ$ = 1

Substituting the values in equation 1 , we get

[12]^2·[1√2]^2+4[1√3]^2+12[1]^2−2[0]^2+124[1]^2

$$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$$

= 18+43+12+124 $\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

= 4824 $\frac{48}{24}$ = 2

Q 9 . 4(sin⁴60°+cos⁴30°)−3(tan²60°−tan²45°)+5cos²45°

$$4\left(sin^4 60° + cos^4 30°\right) - 3\left(tan^2 60° - tan^2 45°\right) + 5cos^2 45°$$

Solution:

4(sin⁴60°+cos⁴30°)−3(tan²60°−tan²45°)+5cos²45°

$$4\left(sin^4 60° + cos^4 30°\right) - 3\left(tan^2 60° - tan^2 45°\right) + 5cos^2 45°$$

[1]

We know that by trigonometric ratios we have ,

sin60°=√32 $sin 60° = \frac{\sqrt{3}}{2}$ cos45°=1√2 $cos 45° = \frac{1}{\sqrt{2}}$

tan60°=√3 $tan 60° = \sqrt{3}$ cos30°=√32 $cos 30° = \frac{\sqrt{3}}{2}$

Substituting the values in equation 1 , we get

4([√32]^4+[√32]^4)−3(3)^2−1^2+5[1√2]^2 $4\left(\left[\frac{\sqrt{3}}{2}\right]^4 + \left[\frac{\sqrt{3}}{2}\right]^4\right) - 3(3)^2 - 1^2 + 5\left[\frac{1}{\sqrt{2}}\right]^2$

= 4·1816−6+52 $4 \cdot \frac{18}{16} - 6 + \frac{5}{2}$

= 14−6+52 $\frac{1}{4} - 6 + \frac{5}{2}$

= 142−6 $\frac{14}{2} - 6$ = 7 – 6 = 1

Q 10 . (cosec²45°sec²30°)(sin²30°+4cot²45°−sec²60°)
$\left(cosec^2 45^\circ sec^2 30^\circ\right)\left(sin^2 30^\circ + 4cot^2 45^\circ - sec^2 60^\circ\right)$

Solution:

(cosec²45°sec²30°)(sin²30°+4cot²45°−sec²60°)
$\left(cosec^2 45^\circ sec^2 30^\circ\right)\left(sin^2 30^\circ + 4cot^2 45^\circ - sec^2 60^\circ\right)$
[1]

We know that by trigonometric ratios we have ,

cosec45°=√2 $cosec 45^\circ = \sqrt{2}$ sec30°=2√3 $sec 30^\circ = \frac{2}{\sqrt{3}}$

sin30°=12 $sin 30^\circ = \frac{1}{2}$ cot45° $cot 45^\circ$ = 1

sec60° $sec 60^\circ$ = 2

Substituting the values in equation 1 , we get

([√2]².[2√3]²)([12]²+4(1)(2)²) $\left(\left[\sqrt{2}\right]^2 . \left[\frac{2}{\sqrt{3}}\right]^2\right)\left(\left[\frac{1}{2}\right]^2 + 4\left(1\right)\left(2\right)^2\right)$

= 3·43·143 $\cdot \frac{4}{3} \cdot \frac{1}{4}$

= 23 $\frac{2}{3}$

Q11. cosec³30°cos60°tan³45°sin²90°sec²45°cot30°
$cosec^3 30^\circ cos 60^\circ tan^3 45^\circ sin^2 90^\circ sec^2 45^\circ cot 30^\circ$

Solution:

Given,

= cosec³30°cos60°tan³45°sin²90°sec²45°cot30°
$cosec^3 30^\circ cos 60^\circ tan^3 45^\circ sin^2 90^\circ sec^2 45^\circ cot 30^\circ$

= 2³(12)(1³)(1²)(√2²)(√3) $2^3\left(\frac{1}{2}\right)\left(1^3\right)\left(1^2\right)\left(\sqrt{2}^2\right)\left(\sqrt{3}\right)$

= $-133\,\frac{-13}{3}$

Q13. (cos0°+sin45°+sin30°)(sin90°−cos45°+cos60°)

$$(cos0^\circ + sin45^\circ + sin30^\circ)(sin90^\circ - cos45^\circ + cos60^\circ)$$

Solution:

Given,

= (2)^3×(12)×(1^3)×(1^2)×(√2^2)×(√3)$(2)^3 \times (\frac{1}{2}) \times (1^3) \times (1^2) \times (\sqrt{2}^2) \times (\sqrt{3})$

= 8×(12)×(1)×(1)×(2)×(√3)$8 \times (\frac{1}{2}) \times (1) \times (1) \times (2) \times (\sqrt{3})$

= 8√3$8\sqrt{3}$

Q12. cot²30°−2cos²60°− 34 sec²45°–4sec²30°

$cot^2 30° - 2cos^2 60° - \frac{3}{4} sec^2 45° - 4sec^2 30°$

Solution:

Given,

= cot²30°−2cos²60°− 34 sec²45°–4sec²30°

$cot^2 30° - 2cos^2 60° - \frac{3}{4} sec^2 45° - 4sec^2 30°$

= (√3^2)×2(12)^2×(34 ×√2^2)×(4×(2√3)^2)$(\sqrt{3}^2) \times 2(\frac{1}{2})^2 \times (\frac{3}{4} \times \sqrt{2}^2) \times (4 \times (\frac{2}{\sqrt{3}})^2)$

= 3− 12 − 32 − 163 $3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3}$

(cos0°+sin45°+sin30°)(sin90°−cos45°+cos60°)(1+1√2+1√2)(1−1√2+1√2)

(32+1√2)(32−1√2)((32)2−(1√2)2)94−12 74

$(cos0^\circ + sin45^\circ + sin30^\circ)(sin90^\circ - cos45^\circ + cos60^\circ)$

$(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})(1- \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})$

$(\frac{3}{2} + \frac{1}{\sqrt{2}})(\frac{3}{2} - \frac{1}{\sqrt{2}})$

$((\frac{3}{2})^2 - (\frac{1}{\sqrt{2}})^2)\frac{9}{4} - \frac{1}{2}\frac{7}{4}$

Q14. sin30°−sin90°+2cos0°tan30°tan60° $\frac{sin30^\circ - sin90^\circ + 2cos0^\circ}{tan30^\circ tan60^\circ}$

Solution:

Given,

$$\frac{sin30^\circ - sin90^\circ + 2cos0^\circ}{tan30^\circ tan60^\circ}$$

$$\frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

sin30°−sin90°+2cos0°tan30°tan60° 12−1+21√3×√3 32 $\frac{3}{2}$

Q15. 4cot²30° + 1sin²60° −cos²45° $\frac{4}{cot^2 30^\circ} + \frac{1}{sin^2 60^\circ} - cos^2 45^\circ$

Solution:

Given,

$4cot^2 30° + 1 sin^2 60° - cos^2 45° = 4(\sqrt{3})^2 + 1(\sqrt{3}2)^2 - (1\sqrt{2})^2 = 43 + 43 - 12 = 16–36 = 136$

$$\frac{4}{cot^2 30^c} + \frac{1}{sin^2 60°} - cos^2 45°$$
$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{(\frac{\sqrt{3}}{2})^2} - (\frac{1}{\sqrt{2}})^2$$
$$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2}$$
$$= \frac{16–3}{6}$$
$$= \frac{13}{6}$$

Q16. 4(sin⁴30°+cos²60°)−3(cos²45°−sin²90°)−sin²60°

$$4(sin^4 30° + cos^2 60°) - 3(cos^2 45° - sin^2 90°) - sin^2 60°$$

Solution:

Given,

$4(sin^4 30°+cos^2 60°)-3(cos^2 45°-sin^2 90°)-sin^2 60°=4((12)^4+(12)^2)-3((1\sqrt{2})^2-1)-$

$(\sqrt{3}2)^2=4(116+14)+32-34=84=2$

$$4(sin^4 30° + cos^2 60°) - 3(cos^2 45° - sin^2 90°) - sin^2 60°$$
$$= 4((\frac{1}{2})^4 + (\frac{1}{2})^2) - 3((\frac{1}{\sqrt{2}})^2 - 1) - (\frac{\sqrt{3}}{2})^2$$
$$= 4(\frac{1}{16} + \frac{1}{4}) + \frac{3}{2} - \frac{3}{4}$$
$$= \frac{8}{4} = 2$$

Q17. **tan²60°+4cos²45°+3sec²30°+5cos²90°cosec30°+sec60°−cot²30°**

$\frac{tan^2 60°+4cos^2 45°+3sec^2 30°+5cos^2 90°}{cosec30°+sec60°-cot^2 30°}$

Solution:

Given,

tan²60°+4cos²45°+3sec²30°+5cos²90°cosec30°+sec60°−cot²30° = (√3)²+4(1√2)²+3(2√3)²+5(0)2+2–

$$\frac{tan^2 60°+4cos^2 45°+3sec^2 30°+5cos^2 90°}{cosec30°+sec60°-cot^2 30°}$$

$$= \frac{(\sqrt{3})^2+4(\frac{1}{\sqrt{2}})^2+3(\frac{2}{\sqrt{3}})^2+5(0)}{2+2-(\sqrt{3})^2}$$

$$= 3 + 2 + 4$$

(√3)²=3+2+4=9$= 9$

Q18. **sin30°sin45° + tan45°sec60° − sin60°cot45° − cos30°sin90°** $\frac{sin30°}{sin45°} + \frac{tan45°}{sec60°} - \frac{sin60°}{cot45°} - \frac{cos30°}{sin90°}$

Solution:

Given,

sin30°sin45° + tan45°sec60° − sin60°cot45° − cos30°sin90° = 12 1√2 + 12 − √32 1 − √32 1 = √22 + 12 − √32 −

$$\frac{sin30^\circ}{sin45^\circ}+\frac{tan45^\circ}{sec60^\circ}-\frac{sin60^\circ}{cot45^\circ}-\frac{cos30^\circ}{sin90^\circ}$$
$$=\frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}+\frac{1}{2}-\frac{\frac{\sqrt{3}}{2}}{1}-\frac{\frac{\sqrt{3}}{2}}{1}$$
$$=\frac{\sqrt{2}}{2}+\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}$$

√32 = √2+1−2√32 $= \frac{\sqrt{2}+1-2\sqrt{3}}{2}$

Q19. **tan45°cosec30° + sec60°cot45° + ssin90°2cos0°** $\frac{tan45^\circ}{cosec30^\circ}+\frac{sec60^\circ}{cot45^\circ}+\frac{ssin90^\circ}{2cos0^\circ}$

Solution:

Given,

tan45°cosec30° + sec60°cot45° + ssin90°2cos0° = 12 + 21 − 5(1)2(1) = 52 − 52 = 0

$$\frac{tan45^\circ}{cosec30^\circ}+\frac{sec60^\circ}{cot45^\circ}+\frac{ssin90^\circ}{2cos0^\circ}$$
$$=\frac{1}{2}+\frac{2}{1}-\frac{5(1)}{2(1)}$$
$$=\frac{5}{2}-\frac{5}{2}$$
$$=0$$

Q20. **2sin3x=√32**$2sin3x=\sqrt{3}$

Solution:

Given,

$$2sin3x = \sqrt{3}$$
$$=> sin3x = \frac{\sqrt{3}}{2}$$
$$=> sin3x = sin60^\circ$$
$$=> 3x = 60^\circ$$

2sin3x=√3=>sin3x=√32=>sin3x=sin60°=>3x=60°=>x=20° $=> x = 20^\circ$

Q21) 2sinx2=1,x=? $2sin\frac{x}{2} = 1,\ x = ?$

Solution:

sinx2=12 $sin\frac{x}{2} = \frac{1}{2}$ sinx2=sin30 ^{0} $sin\frac{x}{2} = sin30^0$ x2=30 ^{0} $\frac{x}{2} = 30^0$

x = 60^0

Q22) √3sinx=cosx $\sqrt{3}sinx = cosx$

Solution:

√3tanx=1 $\sqrt{3}\ tanx = 1$ tanx=1√3 $tanx = \frac{1}{\sqrt{3}}$ ∴tanx=tan45 ^{0} $\therefore tanx = tan45^0$

x = 45^0

Q23) Tan x = sin 45^0 cos 45^0 + sin 30^0

Solution:

Tanx=1√2.1√2+12[∵sin45 ^{0}=1√2cos45 ^{0}=1√2sin30 ^{0}=12]

$Tanx = \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} + \frac{1}{2} \quad [\because sin45^0 = \frac{1}{\sqrt{2}}\ cos45^0 = \frac{1}{\sqrt{2}}\ sin30^0 = \frac{1}{2}]$ Tanx=12+12

$Tanx = \frac{1}{2} + \frac{1}{2}$

Tan x = 1

Tan x = 45^0

x = 45^0

Q24) **√3Tan2x=cos60^0+sin45^{0}cos45^0** $\sqrt{3}\ Tan2x = cos60^0 + sin45^0 cos45^0$

Solution:

√3Tan2x=12+1√2.1√2[∵cos60^0=12sin45^0=cos45^0=1√2]

$\sqrt{3}\ Tan2x = \frac{1}{2} + \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} \qquad [\because cos60^0 = \frac{1}{2}\ sin45^0 = cos45^0 = \frac{1}{\sqrt{2}}]$

√3Tan2x=1√3⇒tan2x=tan30^0 $\sqrt{3}\ Tan2x = \frac{1}{\sqrt{3}} \Rightarrow tan2x = tan30^0$

2x = 30^0

x = 15^0

Q25) **cos2x=cos60^{0}cos30^0+sin60^{0}sin30^0**
$cos2x = cos60^0\ cos30^0 + sin60^0 sin30^0$

Solution:

cos2x=12.√32+√32.12[∵cos60^0=sin30^0=12sin60^0=cos30^0=√32]

$cos2x = \frac{1}{2}.\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}.\frac{1}{2} \qquad [\because cos60^0 = sin30^0 = \frac{1}{2} sin60^0 = cos30^0 = \frac{\sqrt{3}}{2}]$

cos2x=2.√34 $cos2x = 2.\frac{\sqrt{3}}{4}$ cos2x=√32 $cos2x = \frac{\sqrt{3}}{2}$ cos2x=cos30^0 $cos2x = cos30^0$

2x=30^0 $2x = 30^0$ x=15^0 $x = 15^0$

Q26) $If\ \theta = 30^0,\ verify$

Ifθ=30^0,verify(i)Tan2θ=2Tanθ1−tan²θ $(i) Tan2\theta = \frac{2Tan\theta}{1-tan^2\theta}$

Solution:

$Tan2\theta = \frac{2Tan\theta}{1-tan^2\theta} \ldots\ldots (i)$

Substitute $\theta = 30^0$ in equation (i)

LHS = Tan 60^0 = $\sqrt{3}$

RHS = $\frac{2Tan30^0}{1+(Tan30^0)^2} = \frac{2-\frac{1}{\sqrt{2}}}{1-(\frac{1}{\sqrt{2}})^2} = \sqrt{3}$

Therefore, LHS = RHS

(ii) $sin\theta = \frac{2tan\theta}{1-tan^2\theta}$

Substitute $\theta = 30^0$

$sin60^0 = \frac{2tan30^0}{(1-tan30^0)^2}$

=> $\frac{\sqrt{3}}{2} = \frac{2.\frac{1}{\sqrt{2}}}{1+(\frac{1}{\sqrt{2}})^2}$

=> $\frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}}.\frac{3}{4}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Therefore, LHS = RHS.

(iii) $cos2\theta = \frac{1-tan^2\theta}{1+tan^2\theta}$

Substitute $\theta = 30^0$

LHS = $cosec\theta$

RHS = $\frac{1-tan^2\theta}{1+tan^2\theta}$

= cos 2(30^0)

$= \frac{1-tan^2 30^0}{1+tan^2 30^0}$

Cos 60^0 = $\frac{1}{2}$

$= \frac{1-(\frac{1}{\sqrt{2}})^2}{1+(\frac{1}{\sqrt{2}})^2} = \frac{\frac{2}{2}}{\frac{1}{2}} = \frac{1}{2}$

Therefore, LHS = RHS

(iv) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Solution:

LHS = $\cos 3\theta$

Substitute $\theta = 30^0$

= cos 3 (30^0) = cos 90^0

= 0

RHS = $4\cos^3\theta - 3\cos\theta$

= $4\cos^3 30^0 - 3\cos 30^0$

= $4(\frac{\sqrt{3}}{2})^3 - 3.\frac{\sqrt{3}}{2}$

= $3.\frac{\sqrt{3}}{2} - 3.\frac{\sqrt{3}}{2}$

= 0

Therefore, LHS = RHS.

Q27) If A = B = 60^0. Verify (i) Cos (A – B) = Cos A Cos B + Sin A Sin B

Solution:

Cos (A – B) = Cos A Cos B + Sin A Sin B…….(i)

Substitute A and B in (i)

=>cos (60^0 – 60^0) = cos 60^0 cos 60^0 + sin 60^0 sin 60^0

=>cos 0^0 = $(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2$

=>1 = $\frac{1}{4} + \frac{3}{4}$

=>1 = 1

Therefore, LHS = RHS

(ii) Substitute A and B in (i)

=>sin $(60^0 - 60^0)$ = sin 60^0 cos 60^0 – cos 60^0 sin 60^0

=> sin $0^0 = 0$

=>0 = 0

Therefore, LHS = RHS

(iii) Tan(A−B)=TanA−TanB1+TanATanB $Tan(A-B) = \frac{TanA - TanB}{1 + TanA\ TanB}$

A = 60^0, B = 60^0 we get,

Tan(60^0−60^0)=Tan60^0−Tan$60^0$1+Tan60^0Tan60^0 $Tan(60^0 - 60^0) = \frac{Tan60^0 - Tan60^0}{1 + Tan60^0\ Tan60^0}$

Tan $0^0 = 0$

0 = 0

Therefore, LHS = RHS

Q28) If A = 30^0, B = 60^0 verify:

(i) Sin (A + B) = Sin A Cos B + Cos A Sin B

Solution:

A = 30^0, B = 60^0 we get

Sin $(30^0 + 60^0)$ = Sin 30^0 Cos 60^0 + Cos 30^0 Sin 60^0

Sin (90^0) = 12.12+√32.√32 $\frac{1}{2} . \frac{1}{2} + \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2}$

Sin (90^0) = 1 => 1 = 1

Therefore, LHS = RHS

(ii) Cos (A + B) = Cos A Cos B – Sin A Sin B

$A = 30^0$, $B = 60^0$ we get

$\cos(30^0 + 60^0) = \cos 30^0 \cos 60^0 - \sin 30^0 \sin 60^0$

$\cos(90^0) =$ 12.√32−√32.12 $\frac{1}{2} . \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} . \frac{1}{2}$

0 = 0

Therefore, LHS = RHS

Q29. If sin(A+B) = 1 and cos(A-B) = 1, 0°<A+B≤90°$0^\circ < A + B \leq 90^\circ$, A≥B find A and B.

Sol:

Given,

sin(A+B) = 1 this can be written as sin(A+B) = sin(90°)$sin(90^\circ)$

cos(A-B) = 1 this can be written as cos(A-B) = cos(0°)$cos(0^\circ)$

=> A + B = 90°90°

A – B = 0°0°

2A = 90°90°

A = 90°2 $\frac{90^\circ}{2}$

A = 45°45°

Substitute A value in A – B = 0°0°

45°45° – B = 0°0°

B =45°45°

Hence, the value of A = 45°45° and B =45°45°

Q30. If tan(A-B) = 1√3 $\frac{1}{\sqrt{3}}$ and tan(A+B) = √3$\sqrt{3}$, 0°<A+B≤90°$0^\circ < A + B \leq 90^\circ$,

A>B find A and B

Solution:

Given,

tan(A-B) = $1\sqrt{3}\frac{1}{\sqrt{3}}$

A – B = $\tan^{-1}(1\sqrt{3}) tan^{-1}(\frac{1}{\sqrt{3}})$

A – B = $30°30°$ ——- 1

tan(A+B) = $\sqrt{3}\sqrt{3}$

A + B = $\tan^{-1}\sqrt{3} tan^{-1}\sqrt{3}$

A + B = $60°60°$ ——- 2

Solve equations 1 and 2

A + B = $30°30°$

A – B = $60°60°$

2A = $90°90°$

A = $90°2\frac{90°}{2}$

A = $45°45°$

Substitute the value of A in equation 1

$45°45°$ + B = $30°30°$

B = $30°30° – 45°45°$

B = $15°15°$

The value of A = $45°45°$ and B = $15°15°$

Q31. If sin(A-B) = $12\frac{1}{2}$ and cos(A+B) = $12\frac{1}{2}$, 0°<A+B≤90°$0° < A + B \leq 90°$, A<B find A and B.

Solution:

Given,

sin(A-B) = 12 $\frac{1}{2}$

A – B = $\sin^{-1}$(12)$sin^{-1}(\frac{1}{2})$

A – B = 30°30° —— 1

cos(A+B) = 12 $\frac{1}{2}$

A + B = $\cos^{-1}$(12)$cos^{-1}(\frac{1}{2})$

A + B = 60°60° —— 2

Solve equations 1 and 2

A + B = 60°60°

A – B = 30°30°

2A = 90°90°

A = 90°2 $\frac{90^\circ}{2}$

A = 45°45°

Substitute the value of A in equation 2

45°45° + B = 60°60°

B = 60°60° – 45°45°

B = 15°15°

The value of A = 45°45° and B = 15°15°

Q32. In a ΔΔ ABC right angled triangle at B, ∠A=∠C$\angle A = \angle C$.Find the values of:

1. sinAcosC + cosAsinC

Solution:

since, it is given as ∠A=∠C$\angle A = \angle C$

the value of A and C is 45°45°, the value of angle B is 90°90°

because the sum of angles of triangle is 180°180°

=> sin(45°45°)cos(45°45°) + cos(45°45°)sin(45°45°)

=> (1√2×1√2)($\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$) + (1√2×1√2)($\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$)

=> 12$\frac{1}{2}$ + 12$\frac{1}{2}$

=> 1

The value of sinAcosC + cosAsinC is 1

2. sinAsinB + cosAcosB

Solution:

since, it is given as ∠A=∠C$\angle A = \angle C$

the value of A and C is 45°45°, the value of angle B is 90°90°

because the sum of angles of triangle is 180°180°

=> sin(45°45°)sin(90°90°) + cos(45°45°)sin(90°90°)

=> 1√2$\frac{1}{\sqrt{2}}$(1) + 1√2$\frac{1}{\sqrt{2}}$(0)

=> 1√2$\frac{1}{\sqrt{2}}$ + 0

=> 1√2$\frac{1}{\sqrt{2}}$

The value of sinAsinB + cosAcosB is 1√2$\frac{1}{\sqrt{2}}$

Q33. Find the acute angle A and B, if sin(A+2B) = √32$\frac{\sqrt{3}}{2}$ and cos(A+4B) = 0, A>B.

Solution:

Given,

sin(A+2B) = √32 $\frac{\sqrt{3}}{2}$

A + 2B = $\sin^{-1}$ √32 $sin^{-1}\frac{\sqrt{3}}{2}$

A + 2B = 60°60° ——— 1

Cos(A+4B) = 0

A + 4B = $\sin^{-1}(90) sin^{-1}(90)$

A + 4B = 90°90° ——— 2

Solve equations 1 and 2

A + 2B = 60°60°

A + 4B = 90°90°

(-) (-) (-)

-2B = –30°30°

2B = 30°30°

B = 30°2 $\frac{30^\circ}{2}$

B = 15°15°

Substitute B value in eq 2

A + 4B = 90°90°

A + 4(15°15°) = 90°90°

A + 60°60° = 90°90°

A = 90°90° – 60°60°

A = 30°30°

The value of A = $30°30^\circ$ and B = $15°15^\circ$

Q 34. In ΔPQRΔPQR, right angled at Q, PQ = 3 cm and PR = 6 cm. Determine ∠$\angle$ P and ∠$\angle$ R.

Solution:

Given,

In ΔPQRΔPQR, right angled at Q, PQ = 3 cm and PR = 6 cm

By Pythagoras theorem,

PR²=PQ²+QR²=>6²=3²+QR²=>QR²=36–9=>QR=√27=>QR=3√3

$$PR^2 = PQ^2 + QR^2$$
$$=> 6^2 = 3^2 + QR^2$$
$$=> QR^2 = 36 - 9$$
$$=> QR = \sqrt{27}$$
$$=> QR = 3\sqrt{3}$$

sin R = 36=12=sin30° $\frac{3}{6} = \frac{1}{2} = sin 30^\circ$

∠R=30° $\angle R = 30^\circ$

As we know, Sum of angles in a triangle = 180

∠P+∠Q+∠R=180°=>∠P+90°+30°=180°=>∠P=180°–120°=>∠P=60°

$$\angle P + \angle Q + \angle R = 180^\circ$$
$$=> \angle P + 90^\circ + 30^\circ = 180^\circ$$
$$=> \angle P = 180^\circ - 120^\circ$$
$$=> \angle P = 60^\circ$$

Therefore, ∠R=30° $\angle R = 30^\circ$

And, ∠P=60° $\angle P = 60^\circ$

Q35. If sin(A – B) = sin A cos B – cos A sin B and cos (A – B) = cos A cos B + sin A sin B, find the values of sin 15 and cos 15.

Solution:

Given,

sin(A – B) = sin A cos B – cos A sin B

And, cos (A – B) = cos A cos B + sin A sin B

We need to find, sin 15 and cos 15.

Let A = 45 and B = 30

sin 15 = sin (45- 30) = sin 45 cos 30 – cos 45 sin 30

= $= (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2})-(\frac{1}{\sqrt{2}} \times \frac{1}{2})$

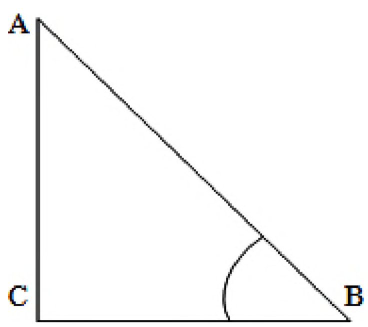
=(1√2×√32)–(1√2×12)=√3–12√2= $\frac{\sqrt{3}-1}{2\sqrt{2}}$

cos 15 = cos (45- 30) = cos 45 cos 30 – sin 45 sin 30

= $= (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}) + (\frac{1}{\sqrt{2}} \times \frac{1}{2})$

=(1√2×√32)+(1√2×12)=√3+12√2= $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Q36. In a right triangle ABC, right angled at C, if ∠B=60° $\angle B = 60^\circ$ and AB=15 units. Find the remaining angles and sides.



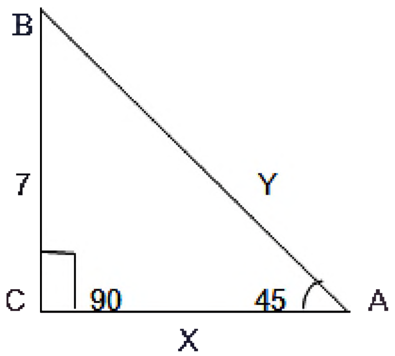
Solution:

sin60°=x15 √32=x15x=15√32unitscos60°=x15 12=x15x=152x=7.5units

$$sin60^{\circ} = \frac{x}{15}$$
$$\frac{\sqrt{3}}{2} = \frac{x}{15}$$
$$x = \frac{15\sqrt{3}}{2} units$$
$$cos60^{\circ} = \frac{x}{15}$$
$$\frac{1}{2} = \frac{x}{15}$$
$$x = \frac{15}{2}$$
$$x = 7.5 units$$

Q37. In ΔΔABC is a right triangle such that ∠C=90° $\angle C = 90^{\circ}$**, ∠A=45°** $\angle A = 45^{\circ}$ **and BC = 7 units. Find the remaining angles and sides.**

Solution:



Here, ∠C=90° $\angle C = 90^\circ$ and ∠A=45° $\angle A = 45^\circ$

We know that,

∠A+∠B+∠C $\angle A + \angle B + \angle C$ = 180° 180°

=> 45° 45° + 90° 90° + ∠C $\angle C$ = 180° 180°

=> 135° 135° + ∠C $\angle C$ = 180° 180°

=> ∠C $\angle C$ = 180° 180° – 135° 135°

=> ∠C $\angle C$ = 45° 45°

The value of the remaining angle C is 45° 45°

Now, we need to find the sides x and y

here,

cos(45) = BCAB $\frac{BC}{AB}$

1√2 $\frac{1}{\sqrt{2}}$ = 7y $\frac{7}{y}$

y = 7√2 $7\sqrt{2}$ units

sin(45) = ACAB $\frac{AC}{AB}$

1√2 $\frac{1}{\sqrt{2}}$ = xy $\frac{x}{y}$

1√2 $\frac{1}{\sqrt{2}}$ = x7√2 $\frac{x}{7\sqrt{2}}$

x = 7√2√2 $\frac{7\sqrt{2}}{\sqrt{2}}$

x = 7 units

the value of x = 7 units and y = 7√2 $7\sqrt{2}$ units

Q 38 . In a rectangle ABCD , AB = 20 cm , ∠$\angle$BAC = 60° 60° , calculate side BC and diagonals AC and BD .

Solution:

Let AC = x cm and CB = y cm

Since , cosθ$cos\theta$ = basehypotenuse $\frac{base}{hypotenuse}$

Therefore , cos60°=20x $cos60^{\circ} = \frac{20}{x}$

⇒12=20x⇒ $\frac{1}{2} = \frac{20}{x}$ [since,cos60°=12 $cos60^{\circ} = \frac{1}{2}$]

⇒⇒ x= 40 cm = AC

Similarly BD = 40 cm

Now ,

Since , sinθ$sin\theta$ = perpendicularhypotenuse $\frac{perpendicular}{hypotenuse}$

Therefore , sin60°=BCAC $sin60^{\circ} = \frac{BC}{AC}$

⇒√32=y40⇒ $\frac{\sqrt{3}}{2} = \frac{y}{40}$ ⇒y=40√32⇒ $y = \frac{40\sqrt{3}}{2}$

⇒y=20√3⇒ $y = 20\sqrt{3}$ cm .

Q39:If A & B are acute angles such that tanA=1/2 tanB=1/3 and tan(A+B)= tanA+tanB1−tanAtanB $\frac{tanA+tanB}{1-tanA\ tanB}$, find A+B.

Solution:

Tan(A+B)= 12+131−12.13 $Tan(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}$ = 3+2656 $= \frac{\frac{3+2}{6}}{\frac{5}{6}}$ = 5656 $= \frac{\frac{5}{6}}{\frac{5}{6}}$

Tan(A+B)=56×65 $Tan(A+B) = \frac{5}{6} \times \frac{6}{5}$ (A+B)=Tan⁻¹(1) $(A+B) = Tan^{-1}(1)$

(A+B) = 45^{0}

Q 40: Prove that : (√3−1)(3−cot30°)=tan³60−2sin60°
$(\sqrt{3}-1)(3-cot30^{\circ}) = tan^{3}60 - 2sin60^{\circ}$

Ans:

L.H.S => $(\sqrt{3}+1)(3-\cot 30^\circ)(\sqrt{3}+1)(3-cot30^\circ)$

$=(\sqrt{3}+1)(3-\sqrt{3}) \because \cot 30^\circ=\sqrt{3}(\sqrt{3}+1)(3-\sqrt{3}) \quad \because cot30^\circ=\sqrt{3}$

$=(\sqrt{3}+1)(\sqrt{3}-1)\sqrt{3}(\sqrt{3}+1)(\sqrt{3}-1)\sqrt{3}$

$=((\sqrt{3})^2-(1)^2)\sqrt{3}((\sqrt{3})^2-(1)^2)\sqrt{3}$

$=2\sqrt{3}2\sqrt{3}$

R.H.S => $\tan^3 60 - 2\sin 60° tan^3 60 - 2sin 60°$

= $(\sqrt{3})^3 - 2\times \sqrt{3}2 (\sqrt{3})^3 - 2 \times \frac{\sqrt{3}}{2}$

= $3\sqrt{3} - \sqrt{3}3 3\sqrt{3} - \sqrt{3}$

= $2\sqrt{3}2 2\sqrt{3}$

L.H.S = R.H.S

Hence proved.

Exercise 5.3: Trigonometric Ratios

Q.1) Evaluate the following : (i) sin20cos70 $\frac{sin20}{cos70}$

Sol (i) : Given that, sin20cos70 $\frac{sin20}{cos70}$

Since sin (90 – ΘΘ) = cosΘΘ

⇒⇒ sin20cos70 $\frac{sin20}{cos70}$ = sin(90−70)cos70 $\frac{sin(90-70)}{cos70}$

⇒⇒ sin20cos70 $\frac{sin20}{cos70}$ = cos70cos70 $\frac{cos70}{cos70}$

⇒⇒ sin20cos70 $\frac{sin20}{cos70}$ =1

Therefore sin20cos70 $\frac{sin20}{cos70}$ = 1

(ii) cos19sin71 $\frac{cos19}{sin71}$

Soln.(ii): Given that, cos19sin71 $\frac{cos19}{sin71}$

$\Rightarrow\Rightarrow$ cos19sin71 $\frac{cos19}{sin71}$ = cos(90−71)sin71 $\frac{cos(90-71)}{sin71}$

$\Rightarrow\Rightarrow$ cos19sin71 $\frac{cos19}{sin71}$ = sin71sin71 $\frac{sin71}{sin71}$

$\Rightarrow\Rightarrow$ cos19sin71 $\frac{cos19}{sin71}$ =1

Since cos(90-ΘΘ)= sin ΘΘ

Therefore cos19sin71 $\frac{cos19}{sin71}$ = 1

(iii) sin21cos69 $\frac{sin21}{cos69}$

Soln.(iii): Given that, sin21cos69 $\frac{sin21}{cos69}$

Since (90-ΘΘ) = cosΘΘ

$\Rightarrow\Rightarrow$ sin21cos69 $\frac{sin21}{cos69}$ = sin(90−69)cos69 $\frac{sin(90-69)}{cos69}$

$\Rightarrow\Rightarrow$ sin21cos69 $\frac{sin21}{cos69}$ = cos69cos69 $\frac{cos69}{cos69}$

$\Rightarrow\Rightarrow$ sin21cos69 $\frac{sin21}{cos69}$ =1

(iv) tan10cot80 $\frac{tan10}{cot80}$

Soln.(iv): We are given that, tan10cot80 $\frac{tan10}{cot80}$

Since tan(90-ΘΘ) = cotΘΘ

$\Rightarrow\Rightarrow$ tan10cot80 $\frac{tan10}{cot80}$ =tan(90−80)cot80 $\frac{tan(90-80)}{cot80}$

$\Rightarrow\Rightarrow$ tan10cot80 $\frac{tan10}{cot80}$ =

cot80cot80 $\frac{cot80}{cot80}$

⇒⇒ tan10cot80 $\frac{tan10}{cot80}$ = 1

Therefore tan10cot80 $\frac{tan10}{cot80}$ = 1

(v) sec11cosec79 $\frac{sec11}{cosec79}$

Soln.(v):

Given that, sec11cosec79 $\frac{sec11}{cosec79}$

Since sec(90-ΘΘ)=cosecΘΘ

⇒⇒ sec11cosec79 $\frac{sec11}{cosec79}$ = sec(90−79)cosec79 $\frac{sec(90-79)}{cosec79}$

⇒⇒ sec11cosec79 $\frac{sec11}{cosec79}$ = cosec79cosec79 $\frac{cosec79}{cosec79}$

⇒⇒ sec11cosec79 $\frac{sec11}{cosec79}$ = 1

Therefore sec11cosec79 $\frac{sec11}{cosec79}$ = 1

Q.2: EVALUATE THE FOLLOWING :

(i) $\left(\textbf{sin49°cos41°}\right)^{\mathbf{2}}\left(\frac{sin49^\circ}{cos41^\circ}\right)^2$ **+** $\left(\textbf{cos41°sin49°}\right)^{\mathbf{2}}\left(\frac{cos41^\circ}{sin49^\circ}\right)^2$

Soln.(i):

We have to find: $\left(\text{sin49°cos41°}\right)^2\left(\frac{sin49^\circ}{cos41^\circ}\right)^2$ + $\left(\text{cos41°sin49°}\right)^2\left(\frac{cos41^\circ}{sin49^\circ}\right)^2$

Since sec70°cosec20° $\frac{sec70^\circ}{cosec20^\circ}$ + sin59°cos31° $\frac{sin59^\circ}{cos31^\circ}$ sin(90°90° − ΘΘ) = cosΘΘ and cos(90° 90° − ΘΘ) = sinΘΘ

So

$\left(\sin(90^\circ-41^\circ)\cos41^\circ\right)^2 \left(\frac{sin(90^\circ-41^\circ)}{cos41^\circ}\right)^2 + \left(\cos(90^\circ-49^\circ)\sin49^\circ\right)^2 \left(\frac{cos(90^\circ-49^\circ)}{sin49^\circ}\right)^2 =$

$\left(\cos41^\circ\cos41^\circ\right)^2 \left(\frac{cos41^\circ}{cos41^\circ}\right)^2 + \left(\sin49^\circ\sin49^\circ\right)^2 \left(\frac{sin49^\circ}{sin49^\circ}\right)^2$

= 1+1 = 2

So value of $\left(\sin49^\circ\cos41^\circ\right)^2 \left(\frac{sin49^\circ}{cos41^\circ}\right)^2 + \left(\cos41^\circ\sin49^\circ\right)^2 \left(\frac{cos41^\circ}{sin49^\circ}\right)^2$ is 2

(ii) cos48°$cos48^\circ$ – sin42°$sin42^\circ$

Soln.(ii)

We have to find: cos48°$cos48^\circ$ – sin42°$sin42^\circ$

Since cos(90°−Θ$90^\circ - \Theta$) = sinΘΘ.So

cos48°$cos48^\circ$ – sin42°$sin42^\circ$ = cos(90°−42°)−sin42°$cos(90^\circ - 42^\circ) - sin42^\circ$

=sin42°$sin42^\circ$ – sin42°$sin42^\circ$ =0

So value of cos48°$cos48^\circ$ – sin42°$sin42^\circ$ is 0

(iii) cot40°tan50° − 12 (cos35°sin55°) $\frac{cot40^\circ}{tan50^\circ} - \frac{1}{2}\left(\frac{cos35^\circ}{sin55^\circ}\right)$

Soln.(iii)

We have to find:

cot40°tan50° − 12 (cos35°sin55°) $\frac{cot40^\circ}{tan50^\circ} - \frac{1}{2}\left(\frac{cos35^\circ}{sin55^\circ}\right)$

Since cot(90°−Θ$90^\circ - \Theta$) = tanΘΘ and cos(90°−Θ$90^\circ - \Theta$)=sinΘΘ

cot40°tan50° − 12 (cos35°sin55°) $\frac{cot40^\circ}{tan50^\circ} - \frac{1}{2}\left(\frac{cos35^\circ}{sin55^\circ}\right)$ = cot(90°−50°)tan50° − 12 (cos(90°−55°)sin55°) $\frac{cot(90^\circ-50^\circ)}{tan50^\circ} - \frac{1}{2}\left(\frac{cos(90^\circ-55^\circ)}{sin55^\circ}\right)$

$=\tan 50^\circ \tan 50^\circ - 12(\sin 55^\circ \sin 55^\circ)\frac{tan50^\circ}{tan50^\circ} - \frac{1}{2}\left(\frac{sin55^\circ}{sin55^\circ}\right)$

$= 1 - 12 1 - \frac{1}{2} = 12\frac{1}{2}$

So value of $\cot 40^\circ \tan 50^\circ - 12(\cos 35^\circ \sin 55^\circ)\frac{cot40^\circ}{tan50^\circ} - \frac{1}{2}\left(\frac{cos35^\circ}{sin55^\circ}\right)$ is $12\frac{1}{2}$

(iv) $\left(\sin 27^\circ \cos 63^\circ\right)^2\left(\frac{sin27^\circ}{cos63^\circ}\right)^2 - \left(\cos 63^\circ \sin 27^\circ\right)^2\left(\frac{cos63^\circ}{sin27^\circ}\right)^2$

Soln(iv)

We have to find: $\left(\sin 27^\circ \cos 63^\circ\right)^2\left(\frac{sin27^\circ}{cos63^\circ}\right)^2 - \left(\cos 63^\circ \sin 27^\circ\right)^2\left(\frac{cos63^\circ}{sin27^\circ}\right)^2$

Since sin $(90^\circ - \Theta 90^\circ - \Theta)$= $\cos\Theta\Theta$ and cos $(90^\circ - \Theta 90^\circ - \Theta)$=$\sin\Theta\Theta$

$\left(\sin 27^\circ \cos 63^\circ\right)^2\left(\frac{sin27^\circ}{cos63^\circ}\right)^2 - \left(\cos 63^\circ \sin 27^\circ\right)^2\left(\frac{cos63^\circ}{sin27^\circ}\right)^2 = \left(\sin(90^\circ - 63^\circ)\cos 63^\circ\right)^2\left(\frac{sin(90^\circ-63^\circ)}{cos63^\circ}\right)^2$

$- \left(\cos(90^\circ - 27^\circ)\sin 27^\circ\right)^2\left(\frac{cos(90^\circ-27^\circ)}{sin27^\circ}\right)^2$

$=\left(\cos 63^\circ \cos 63^\circ\right)^2\left(\frac{cos63^\circ}{cos63^\circ}\right)^2 - \left(\sin 27^\circ \sin 27^\circ\right)^2\left(\frac{sin27^\circ}{sin27^\circ}\right)^2$ =1-1 =0

So value of $\left(\sin 27^\circ \cos 63^\circ\right)^2\left(\frac{sin27^\circ}{cos63^\circ}\right)^2 - \left(\cos 63^\circ \sin 27^\circ\right)^2\left(\frac{cos63^\circ}{sin27^\circ}\right)^2$ is 0

(v) $\tan 35^\circ \cot 55^\circ + \cot 78^\circ \tan 12^\circ - 1\frac{tan35^\circ}{cot55^\circ} + \frac{cot78^\circ}{tan12^\circ} - 1$

Soln.(v)

We have to find:

$\tan 35^\circ \cot 55^\circ + \cot 78^\circ \tan 12^\circ - 1\frac{tan35^\circ}{cot55^\circ} + \frac{cot78^\circ}{tan12^\circ} - 1$

Since tan $(90^\circ - \Theta 90^\circ - \Theta)$= $\cot\Theta\Theta$ and cot $(90^\circ - \Theta 90^\circ - \Theta)$=$\tan\Theta\Theta$ =1

So value of tan35°cot55° + cot78°tan12° is 1 $\frac{tan35^\circ}{cot55^\circ}+\frac{cot78^\circ}{tan12^\circ} is 1$

(vi) sec70°cosec20° + sin59°cos31° $\frac{sec70^\circ}{cosec20^\circ}+\frac{sin59^\circ}{cos31^\circ}$

Soln.(vi)

We have to find: sec70°cosec20° + sin59°cos31° $\frac{sec70^\circ}{cosec20^\circ}+\frac{sin59^\circ}{cos31^\circ}$

Since sec70°cosec20° + sin59°cos31° $\frac{sec70^\circ}{cosec20^\circ}+\frac{sin59^\circ}{cos31^\circ}$ and sec(90°−Θ$90^\circ-\Theta$)=cosecΘΘ

So

sec70°cosec20° + sin59°cos31° $\frac{sec70^\circ}{cosec20^\circ}+\frac{sin59^\circ}{cos31^\circ}$ = (sec(90°−20°)cosec20°)^2 $\left(\frac{sec(90^\circ-20^\circ)}{cosec20^\circ}\right)^2$ − (sin(90°−31°)cos31°)^2 $\left(\frac{sin(90^\circ-31^\circ)}{cos31^\circ}\right)^2$

= cosec20°cosec20° + cos31°cos31° $\frac{cosec20^\circ}{cosec20^\circ}+\frac{cos31^\circ}{cos31^\circ}$ =1+1 =2

So value of sec70°cosec20° + sin59°cos31° $\frac{sec70^\circ}{cosec20^\circ}+\frac{sin59^\circ}{cos31^\circ}$ is 2

(vii) cosec31°−sec59°$cosec31^\circ-sec59^\circ$.

Soln(vii)

We have to find: cosec31°−sec59°$cosec31^\circ-sec59^\circ$

Since cosec(90°−Θ)$cosec(90^\circ-\Theta)$=secΘΘ.So

= cosec31°−sec59°$cosec31^\circ-sec59^\circ$

= cosec(90°−59°)−sec59$cosec\left(90^\circ-59^\circ\right)-sec59$ =sec59°−sec59° $sec59^\circ-sec59^\circ$ =0

So value of cosec31°−sec59°$cosec31^\circ-sec59^\circ$ is 0

(viii)(sin72°+cos18°)$(sin72^\circ + cos18^\circ)$ **(sin72°−cos18°)**$(sin72^\circ - cos18^\circ)$

Soln.(viii)

We have to find: (sin72°+cos18°)$(sin72^\circ + cos18^\circ)$ (sin72°−cos18°) $(sin72^\circ - cos18^\circ)$

Since sin(90°−Θ)$sin(90^\circ - \Theta)$=cosΘΘ,So

(sin72°+cos18°)$(sin72^\circ + cos18^\circ)$ (sin72°−cos18°)$(sin72^\circ - cos18^\circ)$= (sin72°)^2 $(sin72^\circ)^2$–(cos18°)^2$(cos18^\circ)^2$

=[sin(90°−18°)]^2−(cos18°)^2$[sin\,(90^\circ - 18^\circ)]^2 - (cos18^\circ)^2$

=(cos18°)^2$(cos18^\circ)^2$– (cos18°)^2$(cos18^\circ)^2$

=cos^{2}18°−cos^{2}18°$cos^2 18^\circ - cos^2 18^\circ$ =0

So value of (sin72°+cos18°)$(sin72^\circ + cos18^\circ)$ (sin72°−cos18°)$(sin72^\circ - cos18^\circ)$ is 0.

(ix)sin35°sin55°$sin35^\circ sin55^\circ$**–cos35°cos55°**$cos35^\circ cos55^\circ$

Soln(ix)

We find :

sin35°sin55°$sin35^\circ sin55^\circ$–cos35°cos55°$cos35^\circ cos55^\circ$

Since sin(90°−Θ)$sin(90^\circ - \Theta)$=cosΘΘ and cos(90°−Θ)$cos(90^\circ - \Theta)$=sinΘΘ

sin35°sin55°$sin35^\circ sin55^\circ$–cos35°cos55°$cos35^\circ cos55^\circ$= sin(90°−55°)sin55° $sin(90^\circ - 55^\circ) sin55^\circ$–cos(90°−55°)cos55°$cos(90^\circ - 55^\circ) cos55^\circ$ =1-1 =0

So value of sin35°sin55°$sin35^\circ sin55^\circ$–cos35°cos55°$cos35^\circ cos55^\circ$ is 0

(x) tan48°tan23°tan42°tan67°$tan48^\circ tan23^\circ tan42^\circ tan67^\circ$

Soln.(x)

We have to find $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

Since $\tan(90^\circ - \Theta) = \cot\Theta$. So

$\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ =$
$\tan(90^\circ - 42^\circ)\tan(90^\circ - 67^\circ)\tan 42^\circ \tan 67^\circ$

$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$

$= (\tan 67^\circ \cot 67^\circ)(\tan 42^\circ \cot 42^\circ) = 1 \times 1 = 1$

So value of $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$ is 1

(xi)$\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Soln.(xi)

We have to find $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Since $\cos(90^\circ - \Theta) = \sin\Theta$, $\sec(90^\circ - \Theta) = \operatorname{cosec}\Theta$ and $\sin\Theta . \operatorname{cosec}\Theta = 1$. So

$\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ =$
$\sec(90^\circ - 40^\circ)\sin 40^\circ + \cos(90^\circ - 50^\circ)\operatorname{cosec} 50^\circ = 1 + 1 = 2$

So value of $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$ is 2.

Q.3) Express $\cos 75^\circ + \cot 75^\circ$ in terms of angle between 0^0 and 30^0.

Soln. 3 :

Given that: $\cos 75^\circ + \cot 75^\circ$

=cos$75°75°$+cot$75°75°$

= cos(90°−15°)+cot(90°−15°)$cos\left(90° - 15°\right) + cot\left(90° - 15°\right)$

=sin $15°15°$+tan $15°15°$

Hence the correct answer is sin $15°15°$+tan $15°15°$

Q.4) If sin3A = cos(A – 26^0), where 3A is an acute angle, find the value of A.

Soln.4:

We are given 3A is an acute angle

We have: sin3A=cos(A-$26°26°$)

⇒⇒sin3A=sin($90°90°$-(A-$26°26°$))

⇒⇒ sin3A=sin($116°116°$-A)

⇒⇒ 3A=$116°116°$-A

⇒⇒ 4A=$116°116°$

⇒⇒A=$29°29°$

Hence thecorrect answer is $29°29°$

Q.5)If A, B, C are the interior angles of a triangle ABC, prove that,

(i) tan(C+A2)=cotB2$tan\left(\frac{C+A}{2}\right) = cot\frac{B}{2}$

(ii) sin(B+C2)=cosA2$sin\left(\frac{B+C}{2}\right) = cos\frac{A}{2}$

Soln.5:

(i)We have to prove: tan(C+A2)=cotB2$tan\left(\frac{C+A}{2}\right) = cot\frac{B}{2}$

Since we know that in triangle ABC

A+B+C=180

⇒⇒C+A=180°180°-B

⇒⇒ C+A2=90°−B2 $\frac{C+A}{2} = 90^\circ - \frac{B}{2}$

⇒⇒tan C+A2=tan(90° B2)$tan\frac{C+A}{2} = tan\left(90^\circ \frac{B}{2}\right)$

⇒⇒tan(C+A2)=cot B2 $tan\left(\frac{C+A}{2}\right) = cot\frac{B}{2}$

Hence proved

(ii)We have to prove : sin(B+C2)=cos A2 $sin\left(\frac{B+C}{2}\right) = cos\frac{A}{2}$

Since we know that in triangle ABC

A+B+C=180

⇒⇒B+C=180°180°-A

⇒⇒ B+C2=90°−A2 $\frac{B+C}{2} = 90^\circ - \frac{A}{2}$

⇒⇒sin B+C2=sin(90° A2)$sin\frac{B+C}{2} = sin\left(90^\circ \frac{A}{2}\right)$

sin(B+C2)=cos A2 $sin\left(\frac{B+C}{2}\right) = cos\frac{A}{2}$

Hence proved

Q.6)Prove that :

(i)tan20°20°tan35°35°tan45°45°tan55°55°tan70°70° = 1

(ii)sin48°48°.sec48°48°+cos48°48°.cosec42°42° = 2

(iii) sin70°cos20° + cosec20°sec70° −2cos70°.cosec20°=0

$$\frac{sin70^\circ}{cos20^\circ} + \frac{cosec20^\circ}{sec70^\circ} - 2cos70^\circ.cosec20^\circ = 0$$

(iv) $\frac{\cos 80^\circ}{\sin 10^\circ}$ **+cos59°.cosec31°=2** $\frac{cos80^\circ}{sin10^\circ} + cos59^\circ . cosec31^\circ = 2$

Soln.6:

(i)Therefore

tan20°20° tan35°35° tan45°45° tan55°55° tan70°70°

=tan(90°–70°)$tan\,(90^\circ - 70^\circ)$ tan(90°–55°)$tan\,(90^\circ - 55^\circ)$ tan45°45°tan55° 55°tan70°70°

=cot70°70° cot55°55° tan45°45° tan55°55° tan70°70°

= (tan70°cot70°)(tan55°cot55°)tan45°$(tan70^\circ cot70^\circ)\,(tan55^\circ cot55^\circ)\,tan45^\circ$
=1x1x1 =1

Hence proved

(ii) We will simplify the left hand side

sin48°48°.sec48°48°+cos48°48°.cosec42°42°=sin48°48°. sec(90°−48°) $sec\,(90^\circ - 48^\circ)$cos48°$48^\circ$.cosec(90°−48°)$cosec\,(90^\circ - 48^\circ)$

=sin48°48°.cos48°48°+cos48°48°.sin48°48° =1+1 =2

Hence proved

(iii) We have, $\frac{\sin 70^\circ}{\cos 20^\circ}$ + $\frac{\text{cosec}20^\circ}{\sec 70^\circ}$ −2cos70°.cosec20°=0

$$\frac{sin70^\circ}{cos20^\circ} + \frac{cosec20^\circ}{sec70^\circ} - 2cos70^\circ . cosec20^\circ = 0$$

So we will calculate left hand side

$\frac{\sin 70^\circ}{\cos 20^\circ}$ + $\frac{\text{cosec}20^\circ}{\sec 70^\circ}$ −2cos70°.cosec20°=0

$\frac{sin70^\circ}{cos20^\circ} + \frac{cosec20^\circ}{sec70^\circ} - 2cos70^\circ . cosec20^\circ = 0$=
$\frac{\sin 70^\circ}{\cos 20^\circ}$ + $\frac{\cos 70^\circ}{\sin 20^\circ}$ −2cos70°.cosec(90°−70°)

$$\frac{sin70^\circ}{cos20^\circ} + \frac{cos70^\circ}{sin20^\circ} - 2cos70^\circ . cosec(90^\circ - 70^\circ)$$

= sin(90°−20°)cos20° $\frac{sin(90°-20°)}{cos20°}$+ cos(90°−20°)sin20° $\frac{cos(90°-20°)}{sin20°}$–2cos70°.cosec(90°−70°) $2cos70°.cosec(90°-70°)$

= cos20°cos20° + sin20°sin20° −2×1 $\frac{cos20°}{cos20°}+\frac{sin20°}{sin20°}-2\times 1$ =1+1-2 =2-2 =0

Hence proved

(iv)We have cos80°sin10° +cos59°.cosec31°=2 $\frac{cos80°}{sin10°}+cos59°.cosec31°=2$

We will simplify the left hand side

cos80°sin10° +cos59°.cosec31° $\frac{cos80°}{sin10°}+cos59°.cosec31°$=

cos(90°−10°)sin10° +cos59°.cosec(90°−59°) $\frac{cos(90°-10°)}{sin10°}+cos59°.cosec\,(90°-59°)$

= sin10°sin10° +cos59°.sec59° $\frac{sin10°}{sin10°}+cos59°.sec59°$ =1+1 =2

Hence proved.

Question 7

If A,B,C are the interior of triangle ABC , show that

(i) sin(B+C2)=cos A2 $\sin(\frac{B+C}{2})=\cos\frac{A}{2}$

Solution

A+B+C=180^0

B + C = 180^0– A2 $\frac{A}{2}$

LHS=RHS

(ii)cos(90^0– A2)=sin A2 $\cos(90^0-\frac{A}{2})=\sin\frac{A}{2}$

LHS=RHS

Question 8

If $2\Theta+45^0$and$30-\Theta$ $2\Theta+45^0\, and\, 30-\Theta$ are acute angles , find the degree measure of

Θ Θ satisfying $\sin(20+45^0)=\cos(30^0+\Theta)$ $\sin(20+45^0)=\cos(30^0+\Theta)$

Solution

Here 20+45^0=$\sin(60^0+\Theta)$ $\sin(60^0+\Theta)$

We know that ,(($90^0-\Theta$)($90^0-\Theta$)= $\cos(\Theta)$ $\cos(\Theta)$

= $\sin(2\Theta+45^0)=\sin(90^0-(30^0-\Theta))$ $\sin(2\Theta+45^0)=\sin(90^0-(30^0-\Theta))$

= $\sin(2\Theta+45^0)=\sin(90^0-30^0-\Theta)$ $\sin(2\Theta+45^0)=\sin(90^0-30^0-\Theta)$

= $\sin(2\Theta+45^0)=\sin(60^0+\Theta)$ $\sin(2\Theta+45^0)=\sin(60^0+\Theta)$

On equating sin of angle of we get,

= $2\Theta+45^0=60^0+\Theta$ $2\Theta+45^0=60^0+\Theta$ = $\Theta=15^0$ $\Theta=15^0$

Question 9

If Θ Θ is appositive acute angle such that $\sec\Theta=\csc60^0$ $\sec\Theta=\csc60^0$, find

$2\cos^2\Theta-1$ $2\cos^2\Theta-1$

Solution

We know that, $\sec(90^0-\Theta)=\csc^2\Theta$ $\sec(90^0-\Theta)=\csc^2\Theta$

= $\sec(\Theta)=\sec(90^0-60^0)$ $\sec(\Theta)=\sec(90^0-60^0)$

= $\Theta=30^0$ $\Theta=30^0$ = $2\cos^2\Theta-1$ $2\cos^2\Theta-1$

= 2cos²30−12 $\cos^2 30 - 1$

= 2(√32)²−12($\frac{\sqrt{3}}{2}$)$^2 - 1$

= 2(34)−12($\frac{3}{4}$) $- 1$ =(32)−1($\frac{3}{2}$) $- 1$ = (12)($\frac{1}{2}$)

Q10.If sin3Θ=cos(Θ−6°)where3ΘandΘ−6^{circ}
$\sin 3\Theta = \cos(\Theta - 6°)\ where\ 3\Theta\ and\ \Theta - 6^{circ}$ **acute angles, find the value of ΘΘ.**

Soln:

We have, sin3Θ=cos(Θ−6°)$\sin 3\Theta = \cos(\Theta - 6°)$

cos(90°+3Θ)=cos(Θ−6°)$\cos(90° + 3\Theta) = \cos(\Theta - 6°)$ 90°–3Θ=Θ–6°
$90° - 3\Theta = \Theta - 6°$ −3Θ−Θ=6°−90°$-3\Theta - \Theta = 6° - 90°$ −4Θ=96°$-4\Theta = 96°$
Θ= −96°−4 =24°$\Theta = \frac{-96°}{-4} = 24°$

Q11.If sec2A=csc(A−42°)$\sec 2A = \csc(A - 42°)$ **where 2A is acute angle, find the value of A.**

Soln:we know that sec(90−3Theta)=cscΘ$\sec(90 - 3Theta) = \csc\Theta$

sec2A=sec(90–(A−42))$\sec 2A = \sec(90–(A - 42))$ sec2A=sec(90–A+42))
$\sec 2A = \sec(90– A + 42))$ sec2A=sec(132–A))$\sec 2A = \sec(132– A))$

Now equating both the angles we get

2A = 132 – A

A = = 1323$= \frac{132}{3}$

A = 44