Variable Separable

Form

$$\frac{dy}{dx} = F(x).G(y)$$

Solution

Separate the variables: $\frac{dy}{G(y)} = F(x)dx$

Integrate both sides: $\int \frac{dy}{G(y)} = \int F(x)dx$

Reducible to Variable Separable

Form

$$\frac{dy}{dx} = f(ax + by + c)$$

Solution

Substitute $ax + by + c = t \Rightarrow a + b\frac{dy}{dx} = \frac{dt}{dx}$

Convert the equation: $\frac{1}{b}\left(\frac{dt}{dx} - a\right) = f(t)$

Separate the variables: $\frac{dt}{bf(t) + a} = dx$

Integrate both sides: $\int \frac{dt}{bf(t) + a} = \int dx$

Homogeneous

Form

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

Solution

Put $y = vx \Rightarrow v + x\frac{dv}{dx} = \frac{dy}{dx}$

Convert the equation: $v + x\frac{dv}{dx} = f(v)$

Separate the variables: $\frac{dv}{f(v) - v} = \frac{dx}{x}$

Integrate both sides: $\int \frac{dv}{f(v) - v} = \int \frac{dx}{x}$

Reducible to Homogeneous

Form

$$\frac{dy}{dx} = \frac{ax + by + c}{px + qy + r}; \qquad \frac{a}{p} \neq \frac{b}{q}$$

Solution

Put $x = X + h, y = Y + k$

where $ah + bk + c = 0, ph + qk + r = 0$

Convert the equation: $\frac{dY}{dX} = \frac{aX + bY}{pX + qY}$

Solve as a homogeneous differential equation

Linear

Form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Solution

Calculate Integrating Factor: $I(x) = e^{\int P(x)dx}$

Solution: $y.I(x) = \int Q(x).I(x)dx$

Reducible to Linear

Form

$$f'(y)\frac{dy}{dx} + P(x)f(y) = Q(x)$$

Solution

Substitute $f(y) = t \Rightarrow f'(y)\frac{dy}{dx} = \frac{dt}{dx}$

Convert the equation: $\frac{dt}{dx} + P(x)t = Q(x)$

Solve as a linear differential equation