Cheat Sheet – Limits

Indeterminate Forms

$$\frac{0}{0};0.\infty;\frac{\infty}{\infty};\infty-\infty;0^0;\infty^0;1^\infty$$

Properties of Limits

If $\lim_{x\to a} f(x) = l_1$ and $\lim_{x\to a} f(x) = l_2$ then

$$\lim_{x\to a} c.f(x) = c.l_1 \ (c \to \text{constant})$$

$$\lim_{x\to a} f(x) \pm g(x) = l_1 \pm l_2$$

$$\lim_{x\to a} f(x).g(x) = l_1.l_2$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{l_1}{l_2} \ (l_2 \neq 0)$$

$$\lim_{x\to a} \left(f(x)\right)^n = (l_1)^n, n \in N$$

$$\lim_{x\to a} \left(f(x)\right)^{1/n} = (l_1)^{1/n}, n \in N; (l_1 > 0 \text{ if } n = 2k)$$

Limits of the form $f(x)^{g(x)}$

If $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = \infty$ then,

$$\lim_{x\to a} f(x)^{g(x)} = e^{\lim_{x\to a} g(x)(f(x)-1)}$$

otherwise

$$\lim_{x\to a} f(x)^{g(x)} = e^{\lim_{x\to a} g(x)\log(f(x))}$$

Sandwich Theorem

If $g(x) \le f(x) \le h(x)$ and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then

$$\lim_{x\to a} f(x) = L$$

L'Hôpital's Rule

If both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x) = 0$ or $\pm\infty$ then,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Common Limits

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to a} \frac{x^n - a^n}{x - a} = n\, a^{n-1}$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$$

$$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$$

$$\lim_{x\to \infty} \left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{x\to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x\to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x\to \infty} a^x = \infty \ (a > 1)$$

$$\lim_{x\to \infty} a^x = 0 \ (0 < a < 1)$$

$$\lim_{x\to -\infty} a^x = 0 \ (a > 1)$$

$$\lim_{x\to -\infty} a^x = \infty \ (0 < a < 1)$$

If $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = a$ then

$$\lim_{x\to c} \frac{\sin f(x)}{f(x)} = 1$$

$$\lim_{x\to c} \frac{g(x)^n - a^n}{g(x) - a} = n\, a^{n-1}$$

$$\lim_{x\to c} \frac{\ln(1+f(x))}{f(x)} = 1$$

$$\lim_{x\to c} \frac{e^{f(x)} - 1}{f(x)} = 1$$

$$\lim_{x\to c} \frac{a^{f(x)} - 1}{f(x)} = \ln a$$

$$\lim_{x\to c} (1+f(x))^{\frac{1}{f(x)}} = e$$