Equations

Focus: (a, 0), Directrix: x + a = 0

$$y^2 = 4ax \text{ (Standard)}$$

Shifted Parabola (Axis Parallel to X-axis)

$$x = ay^2 + by + c \quad \text{or}$$

$$\left(y + \frac{b}{2a}\right)^2 = 4\left(\frac{1}{4a}\right)\left(x + \frac{b^2 - 4ac}{4a}\right)$$

Focus: (0, a), Directrix: y + a = 0

$$x^2 = 4ay$$

Shifted Parabola (Axis Parallel to Y-axis)

$$y = ax^2 + bx + c \quad \text{or}$$

$$\left(x + \frac{b}{2a}\right)^2 = 4\left(\frac{1}{4a}\right)\left(y + \frac{b^2 - 4ac}{4a}\right)$$

Focus: (x_1, y_1), Directrix: ax + by + c = 0

$$(x - x_1)^2 + (y - y_1)^2 = \frac{(ax + by + c)^2}{a^2 + b^2}$$

Parametric Equation

$$x = at^2, y = 2at \quad \text{Point (t)}$$

Notations

$$S = y^2 - 4ax$$
$$T = yy_1 - 2a(x + x_1)$$
$$S_1 = y_1^2 - 4ax_1$$

Position of a point (x_1, y_1) w.r.t. a parabola

$$\text{Outside: } S_1 > 0, \text{On: } S_1 = 0, \text{Inside: } S_1 < 0$$

Tangent

Equation of the tangent having slope m

$$y = mx + a/m$$

Equation of the tangent at the point (x_1, y_1)

$$T = 0 \qquad (S_1 = 0)$$

Equation of the tangent at the point (t)

$$ty = x + at^2$$

Point of intersection of tangents at (t_1) and (t_2)

$$(at_1t_2, a(t_1 + t_2))$$

Pair of tangents from an external point (x_1, y_1)

$$SS_1 = T^2 \qquad (S_1 > 0)$$

Normal

Equation of the normal having slope m

$$y = mx - 2am - am^3$$

Equation of the normal at the point (x_1, y_1)

$$y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

Equation of the normal at the point (t)

$$y = -tx + 2at + at^3$$

Point of intersection of normals at (t_1) and (t_2)

$$\left(2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2)\right)$$

Three normals from a point P (h, k)

(t_1), (t_2), (t_3) are the feet of normals from P

$$t_1 + t_2 + t_3 = 0$$

$$t_1t_2 + t_2t_3 + t_3t_1 = \frac{2a - h}{a}$$

$$t_1t_2t_3 = \frac{k}{a}$$

Chord

Chord with end points (t_1) and (t_2)

$$(t_1 + t_2)y = 2(x + at_1t_2)$$

Chord of contact w.r.t the point (x_1, y_1)

$$T = 0 \qquad (S_1 > 0)$$

Chord with mid-point (x_1, y_1)

$$T = S_1 \qquad (S_1 < 0)$$

Some relations related to parametric form

(t_1) and (t_2) are the end points of a focal chord

$$t_1t_2 = -1$$

(t_1) and (t_2) subtend 90° at the vertex

$$t_1t_2 = -4$$

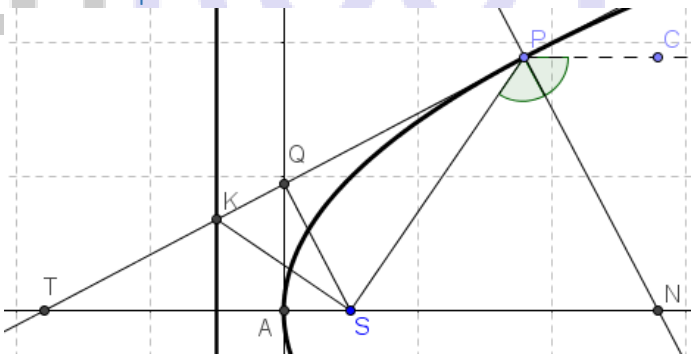
Normal at (t_1) meets the parabola at (t_2)

$$t_2 = -t_1 - \frac{2}{t_1}$$

Normals at (t_1) and (t_2) intersect on the parabola

$$t_1t_2 = 2$$

Some Properties of the Parabola



>> Foot of perpendicular from focus upon any tangent lies on the tangent at vertex (∠SQP=90°)

>> Segment of tangent intercepted between point of contact and the directrix subtends right angle at focus (∠KSP=90°)

>> Any line parallel to the axis passes through the focus after reflection (∠CPN=∠NPS)

>> Points of intersection of tangent and normal with the axis, and the point of contact are equidistant from the focus (SP = SN = ST)