Definition

$$a^x = N \Leftrightarrow x = \log_a N$$

$$(a > 0, a \neq 1)$$

Properties

$$a^{\log_a N} = N$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_a M + \log_a N = \log_a MN$$

$$\log_a M - \log_a N = \log_a \frac{M}{N}$$

$$\log_a M^p = p \log_a M$$

$$\log_{a^p} M = \frac{1}{p} \log_a M$$

$$a > 0, a \neq 1; \ M, N > 0$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$a, b, c > 0; \ a, c \neq 1$$

$$\log_a b = \frac{1}{\log_b a}$$

$$a, b > 0; \ a, b \neq 1$$

$$a^{\log_b c} = c^{\log_b a}$$

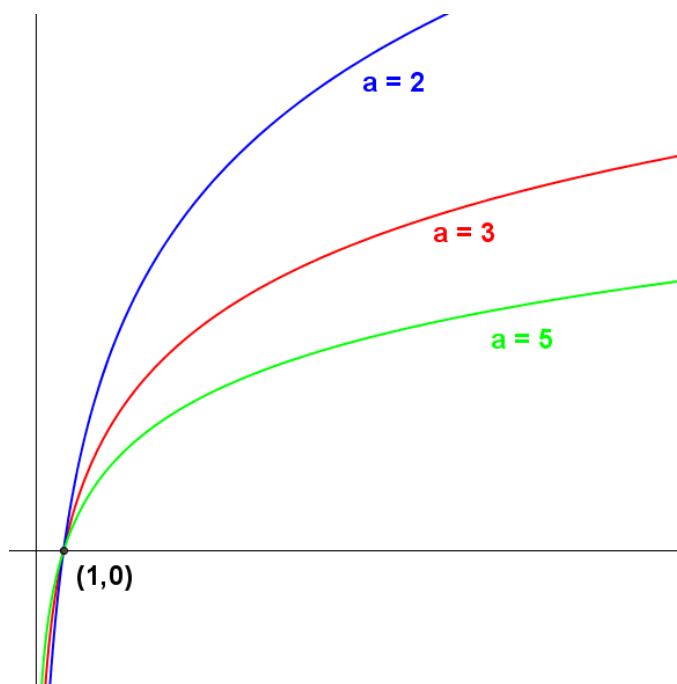$$a, b, c > 0; \ b \neq 1$$

Inequalities

If $a > 1$, then

$$0 < x < y \Leftrightarrow \log_a x < \log_a y$$

If $0 < a < 1$, then

$$0 < x < y \Leftrightarrow \log_a x > \log_a y$$

Graph

$$f(x) = \log_a x, \qquad a > 1$$

a = 2

a = 3

a = 5

(1,0)

$$f(x) = \log_a x, \qquad 0 < a < 1$$

(1,0)

a = 1/4

a = 1/3

a = 1/2