Cheat Sheet – Indefinite Integrals

I Basic

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$$

$$\int \frac{1}{x}dx = \log|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\log a} + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$$

$$\int \tan x\,dx = \log|\sec x| + C$$

$$\int \cot x\,dx = \log|\sin x| + C$$

$$\int \sec x\,dx = \log|\sec x + \tan x| + C$$

$$\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$$

II Special Integrals

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x + \sqrt{x^2+a^2}| + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x + \sqrt{x^2-a^2}| + C$$

$$\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + C$$

$$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + C$$

$$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

III Substitution Methods

$$\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

Express ax^2+bx+c as $a\left(x+\frac{b}{2a}\right)^2 - \frac{D}{4a}$

and put $x + \frac{b}{2a} = t \quad (D = b^2 - 4ac)$

$$\int \frac{(px+q)dx}{ax^2+bx+c}, \int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$$

Express $px+q$ as $k_1(2ax+b) + k_2$

$$\int \frac{dx}{a\sin x + b\cos x + c}$$

Convert to half angles and put $\tan\frac{x}{2} = t$

$$\int \frac{(p\sin x + q\cos x + r)dx}{a\sin x + b\cos x + c} = \left(\frac{N_r}{D_r}\right)$$

Express N_r as $k_1(D_r) + k_2(D_r') + k_3$

$$\int \frac{dx}{a\sin^2 x + b\cos^2 x + c\sin x\cos x + d}$$

Multiply and Divide by $\sec^2 x$ and put $\tan x = t$

$$\int \frac{x^2 \pm 1}{x^4 + kx^2 + 1}dx$$

Divide N_r and D_r by x^2 and put $\left(x \mp \frac{1}{x}\right) = t$

III Rational Functions (Partial Fractions)

$$\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}\ (a \neq b)$$

$$\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$$

$$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$$

$$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{(x^2+bx+c)}$$

IV Integration by Parts

$$\int f.g\,dx = f\int g\,dx - \int\left(f'\int g\,dx\right)dx$$

where f and g are functions of x.

Order of f and g will be determined by the ILATE rule

$$\int e^x\big(f(x) + f'(x)\big)\,dx = e^x.f(x) + C$$