Greatest Integer Function: [x]

Definition

$[x] = \max\{n \in Z \mid n \leq x\}$

$[x]$ = The greatest integer less than or equal to x

$[3] = 3;\ [3.9] = 3;\ [-3.2] = -4$

Properties

$[x] = n \Rightarrow x \in [n, n+1)$

$[[x]] = [x]$

$[x + n] = [x] + n$

$$[-x] = \begin{cases} -[x], & x \in Z \\ -1 - [x], & x \notin Z \end{cases}$$

$$[nx] = \sum_{k=0}^{n-1} \left[x + \frac{k}{n}\right]$$

Inequalities

$x - 1 < [x] \leq x < [x] + 1$

$[x] \geq n \Rightarrow x \in [n, \infty)$

$[x] > n \Rightarrow x \in [n+1, \infty)$

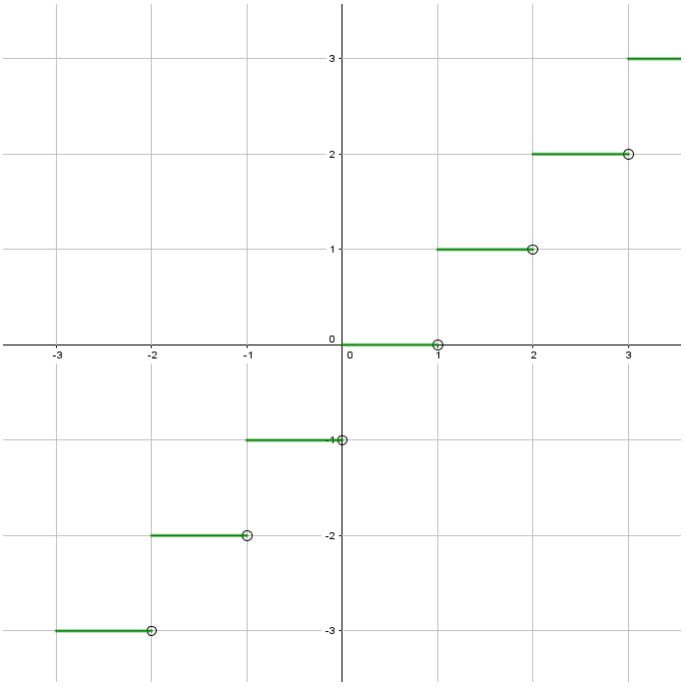
$[x] \leq n \Rightarrow x \in (-\infty, n+1)$

$[x] < n \Rightarrow x \in (-\infty, n)$

$[x] + [y] \leq [x + y] \leq [x] + [y] + 1$

$n \in Z; p \in [0,1)$

Graph



Fractional Part Function: {x}

Definition

$\{x\} = x - [x]$

$\{x\}$ = The fractional part of x

$\{3\} = 0;\ \{3.9\} = 0.9;\ \{-3.2\} = 0.8$

Properties

$\{x\} = p \Rightarrow x \in \bigcup \{n + p\}$

$\{\{x\}\} = \{x\}$

$\{x + n\} = \{x\}$

$\{[x]\} = 0 = [\{x\}]$

$$\{-x\} = \begin{cases} 0, & x \in Z \\ 1 - \{x\}, & x \notin Z \end{cases}$$

Inequalities

$0 \leq \{x\} < 1$

$\{x\} \geq p \Rightarrow x \in \bigcup [n + p, n + 1)$

$\{x\} > p \Rightarrow x \in \bigcup (n + p, n + 1)$

$\{x\} \leq p \Rightarrow x \in \bigcup [n, n + p]$

$\{x\} < p \Rightarrow x \in \bigcup [n, n + p)$

$n \in Z; p \in [0,1)$

Graph