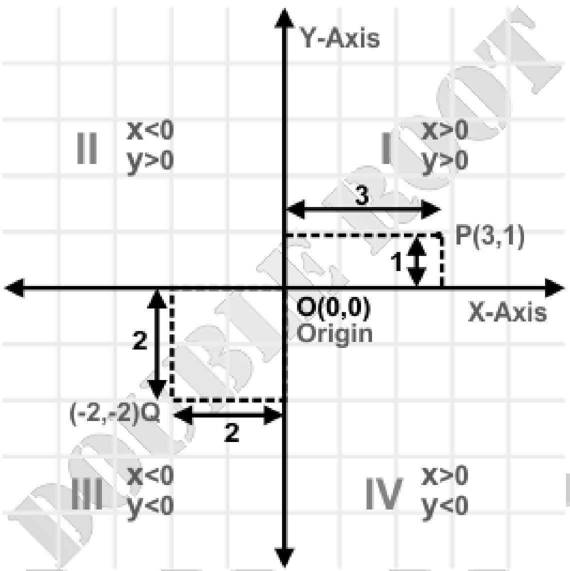
1. The Cartesian Coordinate System



2. Distance Formula

Distance between two given points P(x_1, y_1) and Q(x_2, y_2): $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. Section Formula

Coordinates of the point which divides the line joining two given points P(x_1, y_1) and Q(x_2, y_2) in a given ratio, m:n, i.e. PR:QR = m:n

(i) Internal Division: $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

(ii) External Division: $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

4. Mid-Point

Coordinates of the mid-point of a line segment joining two points (x_1, y_1), and (x_2, y_2)

$$P \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

5. Area of a triangle

Area of the triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3). (in anticlockwise order)

$A = 1/2[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

6. Area of a Polygon

Area of the polygon whose vertices are (x_1, y_1), (x_2, y_2), … , (x_n, y_n) (in anticlockwise order)

$A = 1/2[(x_1y_2-x_2y_1) + (x_2y_3-x_3y_2) + \ldots (x_ny_1-x_1y_n)]$

7. Collinearity of three given points (P, Q, R)

(i) Using Distance Formula

One of the following will hold

PQ + QR = PR

PR + RQ = PQ

QP + PR = QR

(ii) Using Section Formula

One of the points divides the line joining the other two in some ratio

(iii) Using Area of a Triangle

Area of the triangle PQR will be zero

8. Points related to a triangle

Vertices: A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3)

(i) Centroid

Point of concurrency of the medians

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

(ii) Incentre

Point of concurrency of the internal angle bisectors

$$I \equiv \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

(iii) Excentre

Point of concurrency of two external angle bisectors and one internal angle bisector

$$I_1 \equiv \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

$$I_2 \equiv \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right)$$

$$I_3 \equiv \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right)$$

Here, a, b and c are the lengths of the sides BC, CA and AB respectively and I_1, I_2 and I_3 lie opposite A, B and C respectively (relative to the sides)