Cheat Sheet – Modulus Function

Definition

$$|x| = \sqrt{x^2} = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Examples

$|3| = 3;\ |0| = 0;\ |-4| = 4$

Geometrical Interpretation

$|x|$: Distance of x from the origin

Properties

$|x| \geq 0\ \forall\ x \in R$

$||x|| = |x| = |-x|$

$|xy| = |x||y|$

$|x^n| = |x|^n$

$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}\ (y \neq 0)$

$|x + y| \leq |x| + |y|$

Equality holds when $xy \geq 0$

$|x - y| \geq ||x| - |y||$

Equality holds when $xy \geq 0$

$|x - y| \leq |x| + |y|$

Equality holds when $xy \geq 0$

$|x + y| \geq ||x| - |y||$

Equality holds when $xy \leq 0$

$|x - y| \leq |x - z| + |z - y|$

Equality holds when $x \leq z \leq y$

$\frac{d}{dx}|x| = \frac{x}{|x|}\ (x \neq 0)$

$\int |x| dx = \frac{x|x|}{2} + c$

Equations / Inequations

Equation	$\|x\| = a$		
Condition	$a > 0$	$a = 0$	$a < 0$
Solution	$x = \pm a$	$x = 0$	$x \in \phi$

Inequation	$\|x\| \geq a$		
Condition	$a > 0$	$a = 0$	$a < 0$
Solution	$x \in (-\infty, -a] \cup [a, \infty)$	$x \in R$	$x \in R$

Inequation	$\|x\| > a$		
Condition	$a > 0$	$a = 0$	$a < 0$
Solution	$x \in (-\infty, -a) \cup (a, \infty)$	$x \in R - \{0\}$	$x \in R$

Inequation	$\|x\| \leq a$		
Condition	$a > 0$	$a = 0$	$a < 0$
Solution	$x \in [-a, a]$	$x = 0$	$x \in \phi$

Inequation	$\|x\| < a$		
Condition	$a > 0$	$a = 0$	$a < 0$
Solution	$x \in (-a, a)$	$x \in \phi$	$x \in \phi$

Graph