Cheat Sheet – Differentiation

First Principle

$$y = f(x)$$

$$\Rightarrow \frac{dy}{dx} \text{ or } f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Rules

Multiplication by constant

$$(cf(x))' = cf'(x)$$

Sum Rule

$$(f(x) + g(x))' = f'(x) + g'(x)$$

Difference Rule

$$(f(x) - g(x))' = f'(x) - g'(x)$$

Product Rule

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

Chain Rule

$$f(g(x))' = f'(g(x))g'(x)$$

Parametric Functions

$$x = f(t), y = g(t)$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{g'(t)}{f'(t)}$$

Powers of Functions

$$y = f(x)^{g(x)}$$

$$\Rightarrow \frac{dy}{dx} = f(x)^{g(x)} \left(g(x) \frac{f'(x)}{f(x)} + g'(x) \ln f(x) \right)$$

Higher Order Derivatives

$$(f'(x))' \text{ or } \frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x) \text{ or } \frac{d^2y}{dx^2}$$

$$(f''(x))' \text{ or } \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = f'''(x) \text{ or } \frac{d^3y}{dx^3}$$

$$f''' \text{ (n times) } '(x) = f^{(n)}(x) \text{ or } \frac{d^ny}{dx^n}$$

Formulas

f(x) or y	f'(x) or dy/dx
c (constant)	0
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
e^x	e^x
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \ln a}$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cot^{-1} x$	$\frac{-1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{\lvert x \rvert \sqrt{x^2-1}}$
$\csc^{-1} x$	$\frac{-1}{\lvert x \rvert \sqrt{x^2-1}}$