Cheat Sheet – Ellipse

Equations

Focus: (±ae, 0), Directrix: x = ± a/e (Standard)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{where } b^2=a^2(1-e^2)$$

Focus: (0, ±ae), Directrix: y = ± a/e

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad \text{where } b^2=a^2(1-e^2)$$

Focus: (x_1, y_1), Directrix: ax + by + c = 0

$$(x-x_1)^2+(y-y_1)^2=e^2\frac{(ax+by+c)^2}{a^2+b^2}$$

where $e \to$ eccentricity, $0<e<1$

Parametric Equation

$$x=a\cos\phi \qquad \text{where } \phi \to \text{eccentric angle}$$
$$y=b\sin\phi$$

Notations (Standard)

$$S=\frac{x^2}{a^2}+\frac{y^2}{b^2}-1$$
$$T=\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$$
$$S_1=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$$

Position of a point (x_1, y_1) w.r.t. the ellipse

$$\text{Outside: } S_1>0, \text{On: } S_1=0, \text{Inside: } S_1<0$$

Tangent

Equation of the tangent having slope m

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

Equation of the tangent at the point (x_1, y_1)

$$T=0 \qquad (S_1=0)$$

Equation of the tangent at the point (ϕ)

$$\frac{x}{a}\cos\phi+\frac{y}{b}\sin\phi-1=0$$

Point of intersection of tangents at (ϕ_1) and (ϕ_2)

$$\left(a\frac{\cos\frac{\phi_1+\phi_2}{2}}{\cos\frac{\phi_1-\phi_2}{2}}, b\frac{\sin\frac{\phi_1+\phi_2}{2}}{\cos\frac{\phi_1-\phi_2}{2}}\right)$$

Pair of tangents from an external point (x_1, y_1)

$$SS_1=T^2 \qquad (S_1>0)$$

Normal

Equation of the normal at the point (x_1, y_1)

$$\frac{x-x_1}{\frac{x}{a^2}}=\frac{y-y_1}{\frac{y_1}{b^2}}$$

Equation of the normal at the point (ϕ)

$$ax\sec\phi-by\,\text{cosec}\,\phi=a^2-b^2$$

Equation of the normal having slope m

$$y=mx\pm\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$$

Chord

Chord with end points (ϕ_1) and (ϕ_2)

$$\frac{x}{a}\cos\frac{\phi_1+\phi_2}{2}+\frac{y}{b}\sin\frac{\phi_1+\phi_2}{2}=\cos\frac{\phi_1-\phi_2}{2}$$

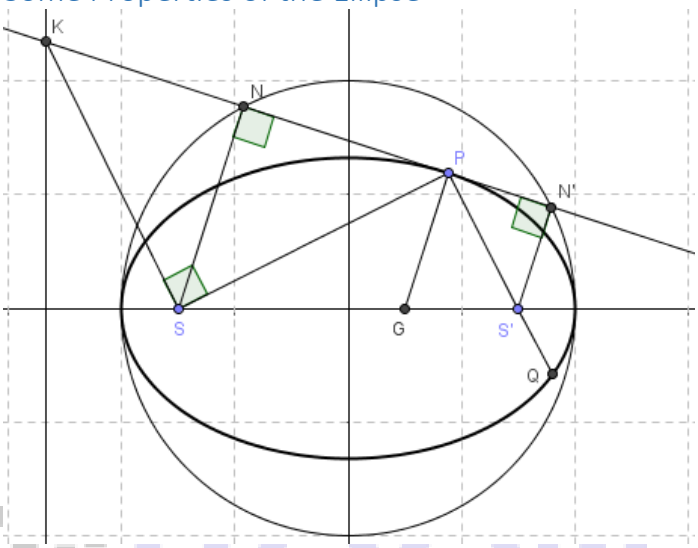
Chord of contact w.r.t the point (x_1, y_1)

$$T=0 \qquad (S_1>0)$$

Chord with mid-point (x_1, y_1)

$$T=S_1 \qquad (S_1<0)$$

Some Properties of the Ellipse



>> Sum of the focal distances of any point on the ellipse is equal to the major axis (PS + PS' = 2a)

>> Harmonic mean of the segments of a focal chord is equal to the semi latus rectum ($1/PS + 1/QS = 2a/b^2$)

>> Segment of tangent intercepted between point of contact and the directrix subtends right angle at focus (∠KSP=90°)

>> Feet of perpendicular from the foci upon any tangent lie on the auxiliary circle (SN⊥PN, S'N'⊥P'N')

>> Product of the lengths of perpendiculars from the foci upon any tangent is constant ($SN \times S'N' = b^2$)

>> Any passing through the focus passes through the other focus after reflection (∠SPG=∠S'PG)