Cheat Sheet – Definite Integrals

Properties

$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = \int_a^b f(t)dt$$

$$\int_a^b f(x)dx = -\int_b^a f(t)dt$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$\int_{-a}^a f(x)dx = \int_0^a [f(x)+f(-x)]dx$$

$$\int_{-a}^a f(x)dx = 0, \text{if } f(x) \text{ is odd}$$

$$\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx\,, \text{if } f(x) \text{ is even}$$

$$\int_0^a f(x)dx = \int_0^{\frac{a}{2}} [f(x)+f(a-x)]dx$$

$$\int_0^a f(x)dx = 0, \quad \text{if } f(x) = -f(a-x)$$

$$\int_0^a f(x)dx = 2\int_0^{\frac{a}{2}} f(x)dx\,, \text{if } f(x) = f(a-x)$$

$$\int_0^a f(x)dx = (b-a)\int_0^1 f((b-a)x+a)dx$$

Periodic Functions

Let f(x) be a periodic function with period T

$$\int_a^{a+nT} f(x)dx = n\int_0^T f(x)dx$$

$$\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$$

$$\int_a^{a+T} f(x)dx = \int_0^T f(x)dx$$

$$\int_{mT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$$

$$\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$$

$m, n \in Z$

Inequalities

If $f(x) \geq g(x)\ \forall\ x \in [a, b]$ then

$$\int_a^b f(x)dx \geq \int_a^b g(x)dx$$

If $m \leq f(x) \leq M\ \forall\ x \in [a, b]$ then

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$$

$$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$$

Equality holds iff f(x) doesn't change sign in [a, b]

$$\left(\int_a^b f(x)g(x)dx\right)^2 \leq \left(\int_a^b [f(x)]^2 dx\right)\left(\int_a^b [g(x)]^2 dx\right)$$

Equality holds iff $f(x) = \alpha g(x)$, where α is constant

Differentiation under the Integral Sign

If $F(x) = \int_{g(x)}^{h(x)} f(t)dt$, then

$$\Rightarrow F'(x) = h'(x)f\big(h(x)\big) - g'(x)f\big(g(x)\big)$$

If $F(x) = \int_{g(x)}^{h(x)} f(x,t)dt$, then

$$F'(x) = \int_{g(x)}^{h(x)} \frac{\partial f(x,t)}{\partial x}dt + f\big(x,h(x)\big)h'(x) - f\big(x,g(x)\big)g'(x)$$

If $F(x) = \int_a^b f(x,t)dt$, then

$$F'(x) = \int_a^b \frac{\partial f(x,t)}{\partial x}dt$$

Integration as a Limit of Sum

$$\lim_{n\to\infty}\sum_{r=0}^{n-1}\left(\frac{b-a}{n}\right)f\left(a+\frac{(b-a)}{n}r\right) = \int_a^b f(x)dx$$

$$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$$