Cheat Sheet – Hyperbola

Equations

Focus: (±ae, 0), Directrix: x = ± a/e (Standard)

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{where } b^2=a^2(e^2-1)$$

Focus: (0, ±ae), Directrix: y = ± a/e

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1 \qquad \text{where } b^2=a^2(e^2-1)$$

Focus: (x_1, y_1), Directrix: ax + by + c = 0

$$(x-x_1)^2+(y-y_1)^2=e^2\frac{(ax+by+c)^2}{a^2+b^2}$$

$$\text{where } e \to \text{eccentricity}, e>1$$

Parametric Equation

$$x=a\sec\phi \qquad y=b\tan\phi \qquad \text{where } \phi \to \text{eccentric angle}$$

Notations (Standard)

$$S=\frac{x^2}{a^2}-\frac{y^2}{b^2}-1$$

$$T=\frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$$

$$S_1=\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$$

Position of a point (x_1, y_1) w.r.t. the hyperbola

$$\text{Outside: } S_1<0, \text{On: } S_1=0, \text{Inside: } S_1>0$$

Tangent

Equation of the tangent having slope m

$$y=mx\pm\sqrt{a^2m^2-b^2}$$

Equation of the tangent at the point (x_1, y_1)

$$T=0 \qquad (S_1=0)$$

Equation of the tangent at the point (ϕ)

$$\frac{x}{a}\sec\phi-\frac{y}{b}\tan\phi-1=0$$

Pair of tangents from an external point (x_1, y_1)

$$SS_1=T^2 \qquad (S_1<0)$$

Normal

Equation of the normal at the point (x_1, y_1)

$$\frac{x-x_1}{\frac{x}{a^2}}=\frac{y-y_1}{\frac{y_1}{-b^2}}$$

Equation of the normal at the point (ϕ)

$$ax\cos\phi+by\cot\phi=a^2+b^2$$

Equation of the normal having slope m

$$y=mx\pm\frac{(a^2+b^2)m}{\sqrt{a^2-b^2m^2}}$$

Chord

Chord with end points (ϕ_1) and (ϕ_2)

$$\frac{x}{a}\cos\frac{\phi_1-\phi_2}{2}-\frac{y}{b}\sin\frac{\phi_1+\phi_2}{2}=\cos\frac{\phi_1+\phi_2}{2}$$

Chord of contact w.r.t the point (x_1, y_1)

$$T=0 \qquad (S_1<0)$$

Chord with mid-point (x_1, y_1)

$$T=S_1 \qquad (S_1>0)$$

Asymptotes (Standard)

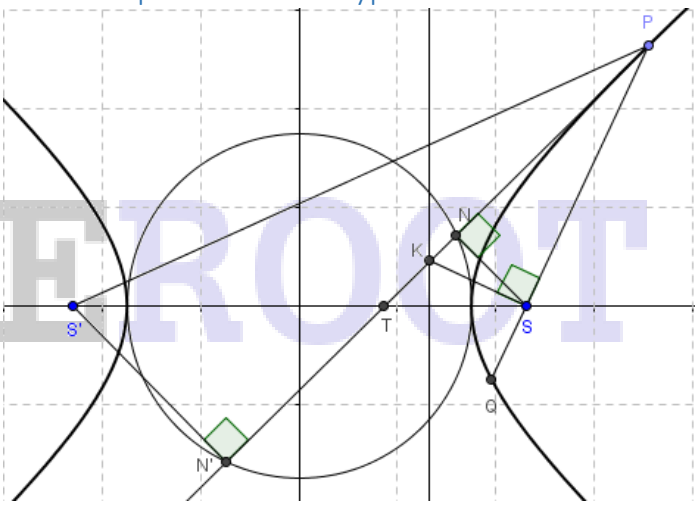
Equation:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$$

Angle between the asymptotes:

$$\theta=2\tan^{-1}\frac{b}{a}$$

Some Properties of the Hyperbola



>> Difference of the focal distances of any point on the hyperbola is constant (|PS - PS'| = 2a)

>> Harmonic mean of the segments of any focal chord is equal to the semi latus rectum ($1/PS + 1/QS = 2a/b^2$)

>> Segment of tangent intercepted between point of contact and the directrix subtends right angle at focus ($\angle KSP=90°$)

>> Feet of perpendicular from the foci upon any tangent lie on the auxiliary circle ($SN \perp PN$, $S'N' \perp P'N'$)

>> Product of the lengths of perpendiculars from the foci upon any tangent is constant ($SN \times S'N' = b^2$)

>> Tangent at any point P bisects $\angle SPS'$