**DOUBLEROOT** Cheat Sheet – Properties of Triangles

Sides: a,b,c Angles: A,B,C Semi-perimeter: s Circum-radius: R In-radius: r Ex-radii: $r_1$, $r_2$, $r_3$

## Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

## Cosine Rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

## Tangent Rule

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$
$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$
$$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$$

## Projection Formula

$$a = b\cos C + c\cos B$$
$$b = c\cos A + a\cos C$$
$$c = a\cos B + b\cos A$$

## Area

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$$
$$\Delta = \frac{abc}{4R}$$
$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

## Half Angle Formula

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$
$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$
$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$
$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$
$$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$$
$$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$
$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$
$$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$
$$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

## In-Radius

$$r = \frac{\Delta}{s}$$
$$r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$$
$$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

## Ex-Radius

$$r_1 = \frac{\Delta}{s-a} = s\tan\frac{A}{2} = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$
$$r_2 = \frac{\Delta}{s-b} = s\tan\frac{B}{2} = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$$
$$r_3 = \frac{\Delta}{s-c} = s\tan\frac{C}{2} = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

## Distances between Centers

G – Centroid, O – Circumcenter, H – Orthocenter

I – Incenter, $I_1$, $I_2$, $I_3$ – Excenters

$$OI = \sqrt{R^2 - 2Rr}$$
$$OI_1 = \sqrt{R^2 + 2Rr_1}$$
$$OI_2 = \sqrt{R^2 + 2Rr_2}$$
$$OI_3 = \sqrt{R^2 + 2Rr_3}$$
$$OH = R\sqrt{1 - 8\cos A\cos B\cos C}$$
$$HG : OG = 2:1$$
$$IH = \sqrt{2r^2 - 4R^2\cos A\cos B\cos C}$$
$$II_1 = a\sec\frac{A}{2}, II_2 = b\sec\frac{B}{2}, II_3 = c\sec\frac{C}{2}$$
$$I_2I_3 = a\,\mathrm{cosec}\frac{A}{2}, I_3I_1 = b\,\mathrm{cosec}\frac{B}{2}, I_1I_2 = c\,\mathrm{cosec}\frac{C}{2}$$

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