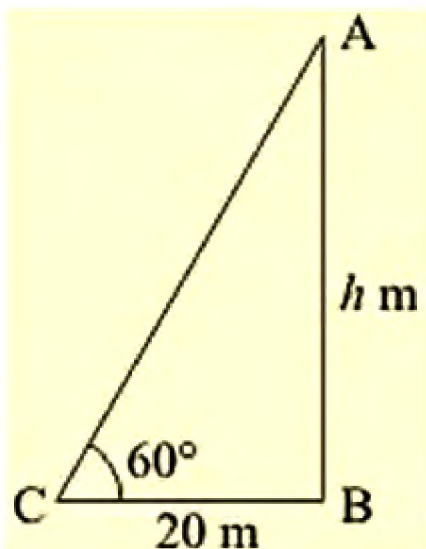


## Exercise 12.1: Some Applications of Trigonometry

**Q1.** A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top the tower is  $60^\circ$ . What is the height of the tower?

**Soln:**

Given:



Distance between point of observation and foot of tower = 20 m = BC

Angle of elevation of top of tower =  $60^\circ = \theta$

Height of tower  $H = AB$

Now from fig ABC

$\triangle ABC$  is a right angle triangle

$$\tan \theta = \frac{\text{opposite side}(AB)}{\text{Adjacent side}(BC)} = \frac{1}{\tan \theta} = \frac{\text{opposite side}(AB)}{\text{Adjacent side}(BC)}$$

$$\text{i.e. } \tan C = \frac{AB}{BC} \quad \tan C = \frac{AB}{BC}$$

$$AB = BC \tan 60^\circ$$

$$H = 20 \times \sqrt{3} \times \sqrt{3}$$

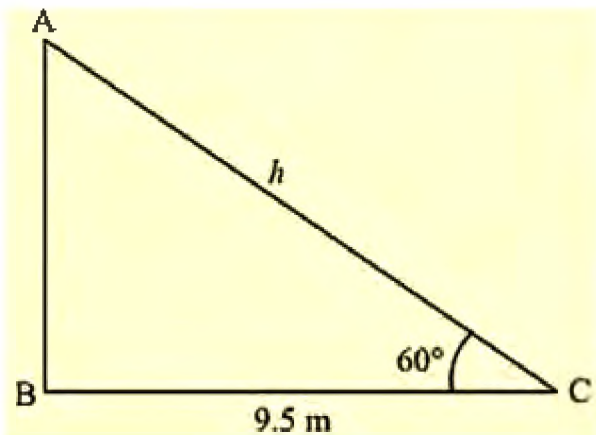
$$= 20 \times 3 = 60$$

$$\therefore \text{height of the tower} = 60 \text{ m} \therefore \text{height of the tower} = 60 \text{ m}$$

**Q2.** The angle of elevation of a ladder against a wall is  $60^\circ$  and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

**Soln:**

Given



Distance between foot of the ladder and wall = 9.5 m

Angle of elevation  $\theta = 60^\circ$

Length of the ladder = L = AC

Now figure forms a right angle triangle ABC

We know

$$\cos \Theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \cos \Theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \cos 60^\circ = \frac{BC}{AC} \quad \cos 60^\circ = \frac{BC}{AC}$$

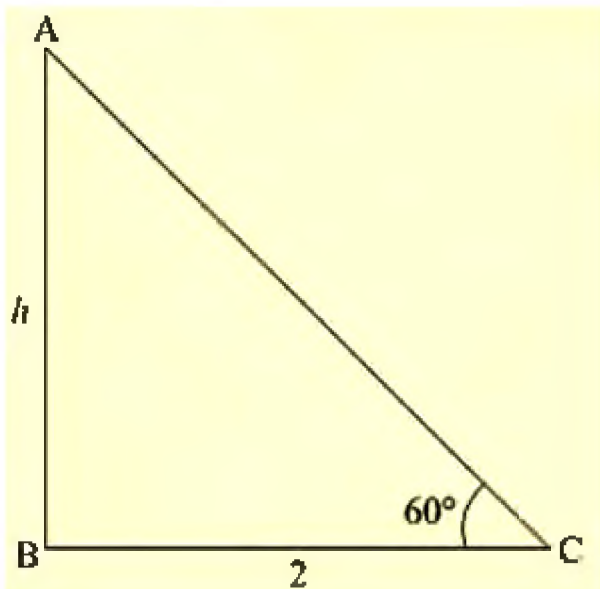
$$12 = 9.5 \times \frac{1}{2} = \frac{9.5}{AC}$$

$$AC = 2 \times 9.5 = 19 \text{ m} \quad 2 \times 9.5 = 19 \text{ m}$$

$\therefore$  length of the ladder (L) = 19 m

**Q3.** A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of  $60^\circ$  with the level of the ground. Determine the height of the wall.

**Soln:**



Distance between foot of the ladder and wall = 2 m = BC

Angle made by ladder with ground  $\Theta = 60^\circ$

Height of the wall H = AB

Now figure of ABC forms a right angle triangle

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan 60^\circ = \frac{AB}{BC} \quad \tan 60^\circ = \frac{AB}{2} \quad \sqrt{3} = \frac{AB}{2}$$

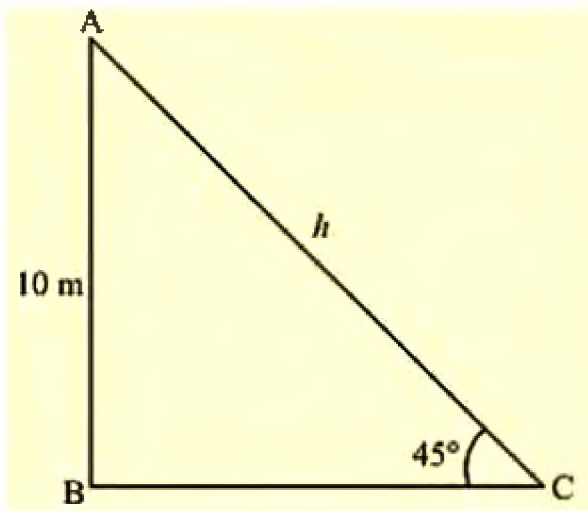
$$\sqrt{3} = \frac{AB}{2}$$

$$AB = 2\sqrt{3}$$

$\therefore$  height of the wall =  $2\sqrt{3}$  m  $\therefore$  height of the wall =  $2\sqrt{3}$  m

**Q4.**An electric pole is 10 m high. A steel wire to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of  $45^\circ$  with the horizontal through the foot of the pole, find the length of the wire.

**Soln:**



Height of the electric pole  $H = 10 \text{ m} = AB$

Angle made by steel wire with ground (horizontal)  $\Theta = 45^\circ$

Let length of wire  $= L = AC$

If we represent above data in form of a figure then it forms a right angle triangle ABC.

Here

$$\sin \Theta = \frac{\text{opposite side}}{\text{hypotenuse}} \quad \sin 45^\circ = \frac{AB}{AC} \quad \sin 45^\circ = \frac{10}{L} \quad \frac{1}{\sqrt{2}} = \frac{10}{L}$$

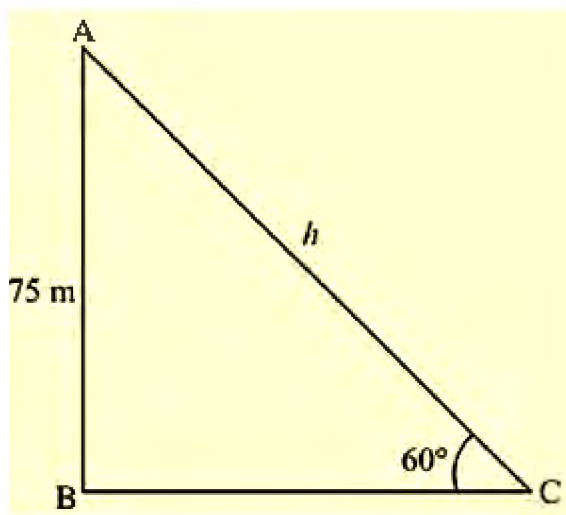
$$\frac{1}{\sqrt{2}} = \frac{10}{L}$$

$$L = 10\sqrt{2} \text{ m}$$

$$\therefore \text{length of the wire (L)} = 10\sqrt{2} \text{ m} \therefore \text{length of the wire (L)} = 10\sqrt{2} \text{ m}$$

**Q5.**A kite is flying at a height of 75 meters from the ground level, attached to a string inclined at  $60^\circ$  to the horizontal. Find the length of the string to the nearest meter.

**Soln:**



Given

Height of kite from ground = 75 m = AB

Inclination of string with ground  $\Theta = 60^\circ$

Length of string  $L = AC$

If we represent the above data in form of figure as shown the it forms a right angle triangle ABC

Here,

$$\sin \Theta = \frac{\text{opposite side}}{\text{hypotenuse}} \quad \sin 60^\circ = \frac{AB}{AC} \quad \sin 60^\circ = \frac{75}{L} \quad \frac{\sqrt{3}}{2} = \frac{75}{L}$$

$$\frac{\sqrt{3}}{2} = \frac{75}{L}$$

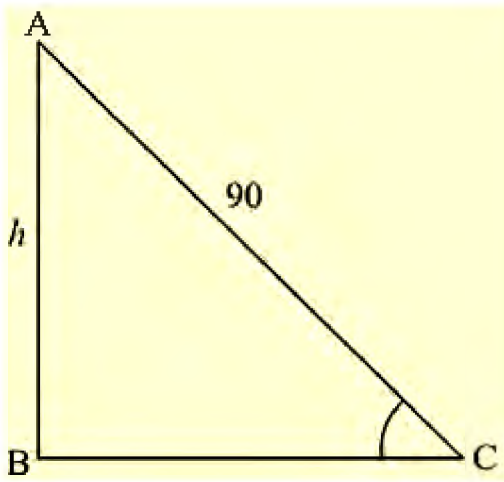
$$L = 50\sqrt{3} \text{ m}$$

$$\text{Length of string } L = 50\sqrt{3} \text{ m}$$

**Q6.** The length of a string between a kite and a point on the ground is 90 meters. If the string makes an angle  $\Theta$  with the ground level such that  $\tan \Theta = \frac{15}{8}$

$\tan \Theta = \frac{15}{8}$ . How high is the kite? Assume that there is no slack in the string

**Soln:**



length of the string between point on ground and kite = 90 m

Angle made by string with ground is  $\Theta$

$$\text{And } \tan \Theta = \frac{15}{8}$$

$$\Theta = \tan^{-1} \left( \frac{15}{8} \right)$$

Height of the kite be 'H' m

If we represent the above data in figure as shown the it forms right angle triangle ABC

We have

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \Theta = \frac{AB}{BC} \quad \tan \Theta = \frac{15}{8} = \frac{H}{BC}$$

$$\frac{15}{8} = \frac{H}{BC}$$

$$BC = \frac{8H}{15} \quad \text{--- (a)}$$

In  $\Delta ABC$ , by Pythagoras theorem we have

$$AC^2 = BC^2 + AB^2 \quad 90^2 = \left( \frac{8H}{15} \right)^2 + H^2$$

$$90^2 = \frac{(8H)^2 + (15H)^2}{15^2} \quad 90^2 \times 15^2 = (8^2 + 15^2) H^2 \quad 90^2 \times 15^2 = 17 H^2$$

$$H^2 = \frac{(90 \times 15)^2}{17} \quad H = \frac{90 \times 15}{\sqrt{17}}$$

$$H = \frac{90 \times 15}{\sqrt{17}}$$

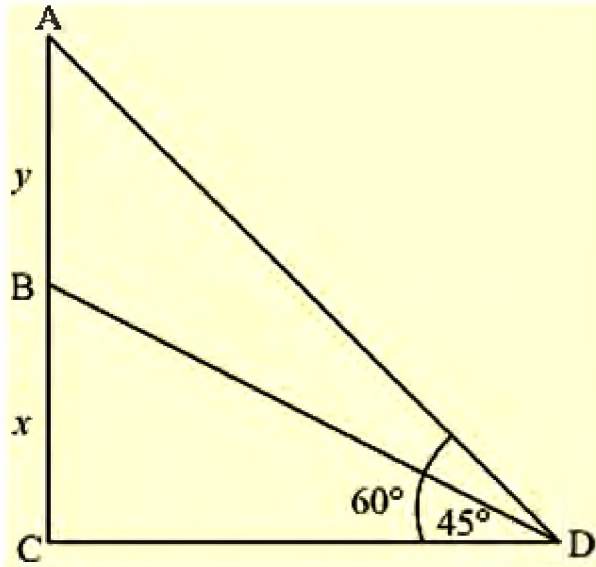
$$H = 79.41$$

$\therefore$  height of the kite from ground = 79.43m

$\therefore$  height of the kite from ground = 79.43m

**Q7. A vertical tower stands on a horizontal place and is surmounted by a vertical flag staff. At a point on the plane 70 meters away from the tower, an observer notices that the angles of elevation of the meters away from the flag staff are respectively  $60^\circ$  and  $45^\circ$ . Find the height of the flag staff and that of the tower**

**Soln:**



Given

Vertical tower is surmounted by flag staff.

Distance between tower and observer = 70 m = BC

Angle of elevation of top of tower  $\alpha = 45^\circ$

Angle of elevation of top of flag staff  $\beta = 60^\circ$

Height of flag staff =  $h = AD$

Height of tower =  $H = AB$

If we represent the above data in the figure then it forms right angle triangles  $\triangle ABC$  and  $\triangle BCD$

When  $\Theta$  is angle in right angle triangle we know that

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Now ,

$$\tan \alpha = \frac{AB}{BC} \tan \alpha = \frac{AB}{BC} \tan 45^{\text{circ}} = \frac{H}{80} \tan 45^{\text{circ}} = \frac{H}{80}$$

$$H = 70$$

$$\therefore H = 70$$

Again,

$$\tan \beta = \frac{DB}{BC} \tan \beta = \frac{DB}{BC} \tan 60^{\text{circ}} = \frac{AD+AB}{70} = \frac{h+H}{80} \tan 60^{\text{circ}} = \frac{AD+AB}{70} = \frac{h+H}{80}$$

$$h+70 = 80\sqrt{3}$$

$$h = 70(\sqrt{3}-1)$$

$$h = 70(1.732-1)$$

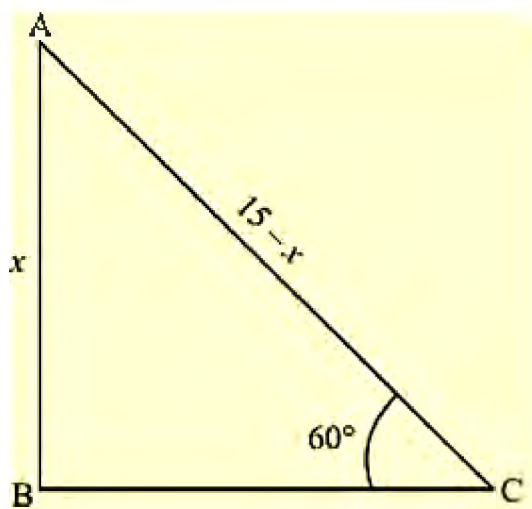
$$h = 51.24 \text{ m}$$

$$\therefore h = 51.24 \text{ m}$$

Height of tower = 80 m and height of flag staff = 51.24 m

**Q8. A vertically straight tree, 15 m high; is broken by the wind in such a way that its top just touches the ground and makes an angle of  $60^\circ$  with the ground. At what height from the ground did the tree break?**

**Soln:**



Initial height of tree  $H = 15 \text{ m} = AB$

Let us assume that it is broken at point C.



Then given that angle made by broken part with the ground  $\Theta = 60^\circ$

Height from ground to broken points =  $h = BC$

$$AB = AC + BC$$

$$H = AC + h \Rightarrow AC = (H - h) \text{ m}$$

If we represent the above data in the figure as shown then it forms right angle triangle ABC from figure

$$\sin \Theta = \frac{\text{opposite side}}{\text{hypotenuse}} \sin \Theta = \frac{\text{opposite side}}{\text{hypotenuse}} \sin 60^\circ = \frac{BC}{CA} \sin 60^\circ = \frac{BC}{CA} \frac{\sqrt{3}}{2} = \frac{h}{H-h}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{H-h} \sqrt{3}(15-h) = 2h \sqrt{3} (15-h) = 2h (2+\sqrt{3})h = 15\sqrt{3}(2+\sqrt{3}) h = 15\sqrt{3}$$

$$h = 15\sqrt{3} \times \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{15\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

Rationalizing denominator rationalizing factor of  $a + \sqrt{b}$  is  $a - \sqrt{b}$

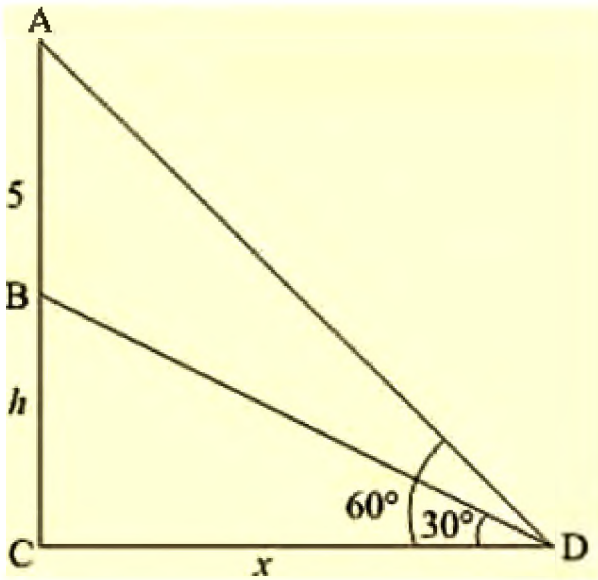
$$h = \frac{(15\sqrt{3})(2-\sqrt{3})}{2^2-3^2} = \frac{15(2\sqrt{3}-3)}{2^2-3^2} = 15(2\sqrt{3}-3)$$

$$h = 15(2\sqrt{3}-3) = 15(2\sqrt{3}-3)$$

$$\therefore \therefore \text{ height of broken points from ground } = 15(2\sqrt{3}-3) \text{ m} = 15(2\sqrt{3}-3) \text{ m}$$

**Q9. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 5 meters. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are respectively  $30^\circ$  and  $60^\circ$ . Find the height of the tower.**

**Soln:**



Height of the flag staff  $h = 5 \text{ m} = AB$

Angle of elevation of the top of flag staff  $= 60^\circ = \alpha$

Angle of elevation of the bottom of the flagstaff  $= 30^\circ = \beta$

Let height of tower be 'H' m = AB

If we represent the above data in forms of figure then it forms a right angle triangle CBD in which ABC is included

In right angle triangle, if

Angle is  $\theta$  then

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \alpha = \frac{BD}{BC} \quad \tan \alpha = \frac{BD}{BC}$$

$$\tan 60^\circ = \frac{AB + BD}{BC} \quad \tan 60^\circ = \frac{AB + BD}{BC}$$

$$\sqrt{3} = \frac{H + 5}{BC} \quad \sqrt{3} = \frac{H + 5}{BC} \quad \text{--- (a)}$$

$$\tan \beta = \frac{AB}{BC} \quad \tan \beta = \frac{AB}{BC} \quad \tan 30^\circ = \frac{H}{BC} \quad \tan 30^\circ = \frac{H}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{H}{BC} \quad \frac{1}{\sqrt{3}} = \frac{H}{BC} \quad \text{--- (b)}$$

Equating (1) and (2)

$$\sqrt{3} \cdot \frac{1}{\sqrt{3}} = \frac{H + 5}{BC} \cdot \frac{H}{BC} = \frac{H + 5}{BC} \cdot \frac{H}{BC}$$

$$3 = H + 5H \cdot \frac{H + 5}{H}$$

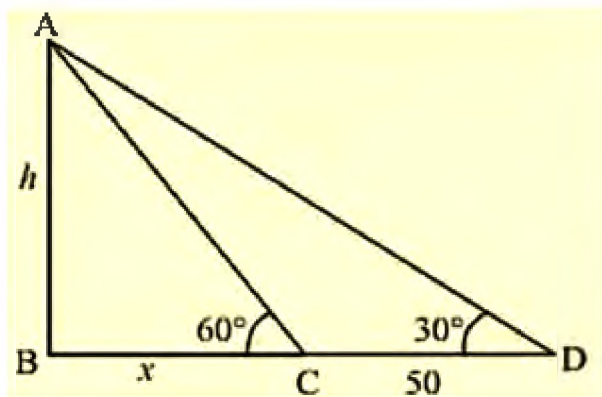
$$3H = H + 5$$

$$H = 2.5 \text{ m}$$

Height of the tower = 2.5 m

**Q10.** A person observed the angle of elevation of a tower as  $30^\circ$ . He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as  $60^\circ$ . Find the height of the tower.

**Soln:**



Given

Angle of elevation of top of tower from first point of elevation (A)  $\alpha = 30^\circ$

Let the person walked 50 m from first point (A) to (B) then  $AB = 50 \text{ m}$

Angle of elevation from second point (B)  $\beta = 60^\circ$

Now let us represent the given data in form of figure

Then it forms  $\triangle ACD$  and  $\triangle BCD$  in which  $\angle C = 90^\circ$

Let the height of the tower be 'H' m = CD

$BC = x \text{ m}$ .

If in a right angle triangle  $\theta$  is the angle then

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \alpha = \frac{CD}{AC} \quad \tan \alpha = \frac{CD}{AC}$$

$$\tan 30^\circ = \frac{H}{AB + BC} = \frac{H}{50 + x}$$

$$1\sqrt{3} = H50+x \frac{1}{\sqrt{3}} = \frac{H}{50+x}$$

$$50+x = H\sqrt{3} \quad \text{--- (a)}$$

$$\tan\beta = \frac{CD}{BC} \tan \beta = \frac{CD}{BC} \tan 60^{\text{circ}} = Hx \tan 60^{\text{circ}} = \frac{H}{x} \sqrt{3} = Hx \sqrt{3} = \frac{H}{x}$$

$$x = H\sqrt{3} \frac{H}{\sqrt{3}} \quad \text{--- (b)}$$

Equating (a) and (b)

$$\Rightarrow 50 + H\sqrt{3} \frac{H}{\sqrt{3}} = H\sqrt{3} H\sqrt{3}$$

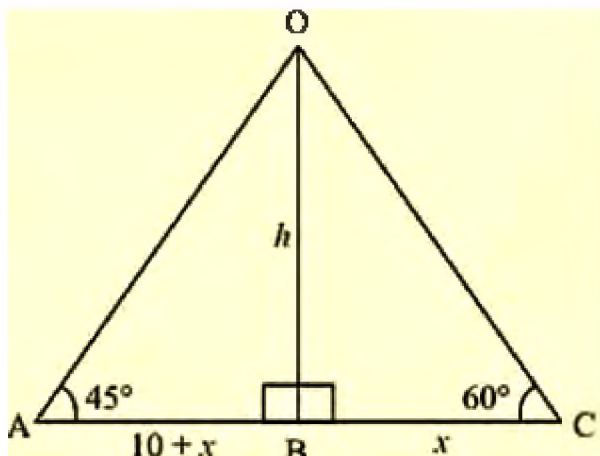
$$\Rightarrow H\sqrt{3} - H\sqrt{3} H\sqrt{3} - \frac{H}{\sqrt{3}} = 50$$

$$\Rightarrow H = 25\sqrt{3} 25\sqrt{3}$$

$$\therefore \therefore \text{ height of tower } H = 25\sqrt{3} 25\sqrt{3}$$

**Q11.** The shadow of a tower, when the angle of elevation of the sun is  $45^\circ$ , is found to be 10 m longer than when it was  $60^\circ$ . Find the height of the tower

**Soln:**



let the length of the shadow of the tower when angle of elevation is  $60^\circ$  be  $x$  m = BC.  
Then according to problem

Length of the shadow with angle of elevation  $45^\circ$  is  $(10+x)$  m = BD.

If we represent the above data in the form of figure then it forms a triangle ABD and triangle ABC where  $\angle B = 90^\circ$

Let height of the tower be 'H' m = AB

If in right angle triangle one of the angle is  $\Theta$  then

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \Theta = \frac{AB}{BC} \quad \tan \alpha = \frac{AB}{BC}$$

$$\tan 60^\circ = \frac{H}{x} \quad \tan 60^\circ = \frac{H}{x}$$

$$x = \frac{H}{\sqrt{3}} \quad \text{--- (a)}$$

$$\tan \beta = \frac{AB}{BD} \quad \tan \beta = \frac{AB}{BD} \quad \tan 45^\circ = \frac{H}{x+20} \quad \tan 45^\circ = \frac{H}{x+20}$$

$$x+10 = H$$

$$x = H - 10 \quad \text{--- (b)}$$

Substitute  $x = H - 10$  in (a)

$$H - 10 = \frac{H}{\sqrt{3}}$$

$$\sqrt{3}H - 10\sqrt{3} = H \quad \sqrt{3}H - 10\sqrt{3} = H \quad H = 10\sqrt{3} \quad \sqrt{3} - 1 \quad H = \frac{10\sqrt{3}}{\sqrt{3} - 1} \quad H = 10\sqrt{3} \times (\sqrt{3} + 1) \times (\sqrt{3} - 1) \times (\sqrt{3} + 1)$$

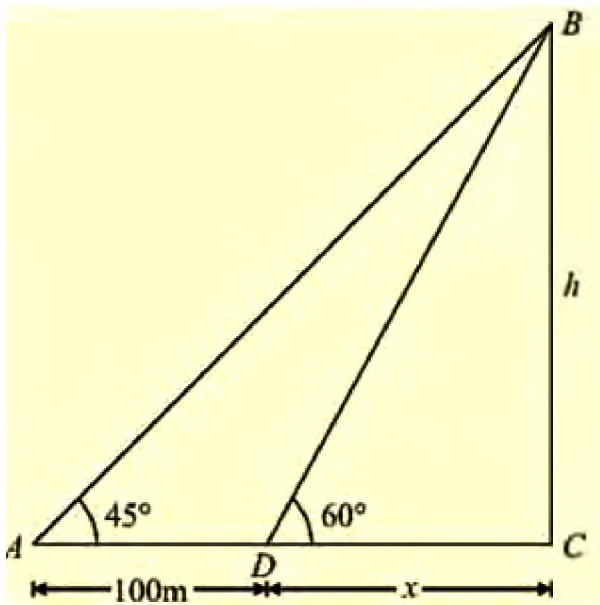
$$H = \frac{10\sqrt{3} \times (\sqrt{3} + 1)}{(\sqrt{3} - 1) \times (\sqrt{3} + 1)} \quad H = 10\sqrt{3}(\sqrt{3} + 1) \quad H = \frac{10\sqrt{3}(\sqrt{3} + 1)}{2}$$

$$H = 5(3 + \sqrt{3}) \quad 5(3 + \sqrt{3})$$

$$H = 23.66 \text{ m}$$

**Q12.** A parachute is descending vertically and makes angles of elevation of  $45^\circ$  and  $60^\circ$  at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of point where he falls on the ground from the just observation point.

**Soln:**



Let the parachute at highest point A. Let C and D be points which are 100 m apart on ground where from then  $CD = 100 \text{ m}$

Angle of elevation from point C =  $45^\circ = \alpha = \alpha$

Angle of elevation from point D =  $60^\circ = \beta = \beta$

Let B be the point just vertically down the parachute

Let us draw figure according to above data then it forms the figure as shown in which ABC and ABD are two triangles

Maximum height of the parachute from the ground

$AB = H \text{ m}$

Distance of point where parachute falls to just nearest observation point =  $x \text{ m}$

If in right angle triangle one of the included angle is  $\Theta$  then

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \Theta = \frac{AB}{BC} \quad \tan \alpha = \frac{AB}{BC} \quad \tan \alpha = \frac{AB}{BC}$$

$$\tan 45^\circ = \frac{H}{100} \quad \tan 45^\circ = \frac{H}{100}$$

$$100 + x = H \quad \text{--- (a)}$$

$$\tan \beta = \frac{AB}{BD} \quad \tan \beta = \frac{AB}{BD} \quad \tan 60^\circ = \frac{H}{x} \quad \tan 60^\circ = \frac{H}{x}$$

$$H = \sqrt{3}x \quad \text{--- (b)}$$

From (a) and (b)

$$\sqrt{3}x = 100 + x\sqrt{3} \Rightarrow x = 100 + x(\sqrt{3}-1) \Rightarrow x = 100(\sqrt{3}-1) \Rightarrow x = 100\sqrt{3}-100 \Rightarrow x = 100\sqrt{3}-100 \times \sqrt{3}+100 \Rightarrow x = 100$$

$$x = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \Rightarrow x = \frac{100(\sqrt{3}+1)}{2} \Rightarrow x = 50(\sqrt{3}+1) \Rightarrow x = 50(\sqrt{3}+1)$$

$x = 136.6$  m in (b)

$$H = \sqrt{3} \times 136.6 = 236.6 \text{ m} \quad H = \sqrt{3} \times 136.6 = 236.6 \text{ m}$$

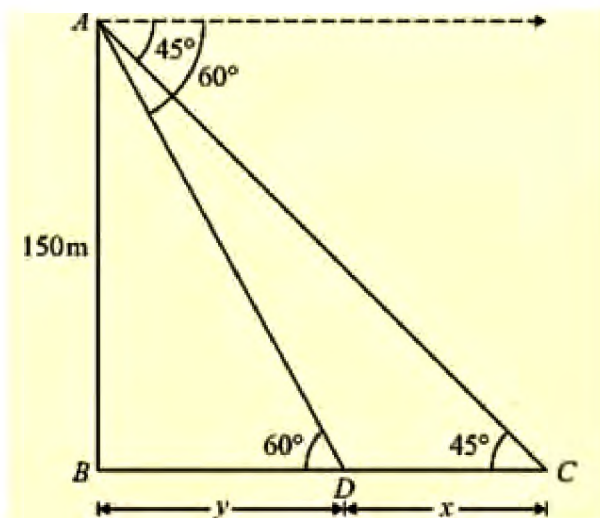
Maximum height of the parachute from the ground

$$H = 236.6 \text{ m} \quad H = 236.6 \text{ m}$$

Distance between the two points where parachute falls on the ground and just the observation is  $x = 136.6$  m

**Q13.** On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are  $45^\circ$  and  $60^\circ$ . If the height of the tower is 150 m, find the distance between the objects.

**Soln:**



Height of the tower,  $H = AB = 150$  m

Let A and B be two objects on the ground.

Angle of depression of object A'  $[\angle A'Ax] = \beta = 45^\circ = \angle AA'B$   $[Ax \parallel A'B]$   
 $[\angle A'Ax] = \beta = 45^\circ = \angle AA'B$   $[Ax \parallel A'B]$

Angle of depression of object B'  $[\angle xAB'] = \alpha = 60^\circ = \angle AB'B$   $[Ax \parallel A'B]$   
 $[\angle xAB'] = \alpha = 60^\circ = \angle AB'B$   $[Ax \parallel A'B]$

Let  $A'B' = x$  ;  $B'B = y$

If we represent the above data in form of a figure, Then it forms a right angle triangle with  $\angle B = 90^\circ$

In any right angle triangle if one of the include angle is  $\Theta$  then

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan 60^\circ = \frac{y}{150} \quad \tan 60^\circ = \frac{y}{150} \quad y = 150 \tan 60^\circ$$

$$y = 150\sqrt{3} \quad \text{--- (a)}$$

$$\tan \beta = \frac{AB}{A'B'} \quad \tan \beta = \frac{AB}{x+y} \quad \tan 45^\circ = \frac{150}{x+y} \quad x+y = 150 \quad \text{--- (b)}$$

$$x + y = 150 \quad \text{--- (b)}$$

Substituting (a) in (b)

$$x + 150\sqrt{3} = 150 \quad x = 150 - 150\sqrt{3}$$

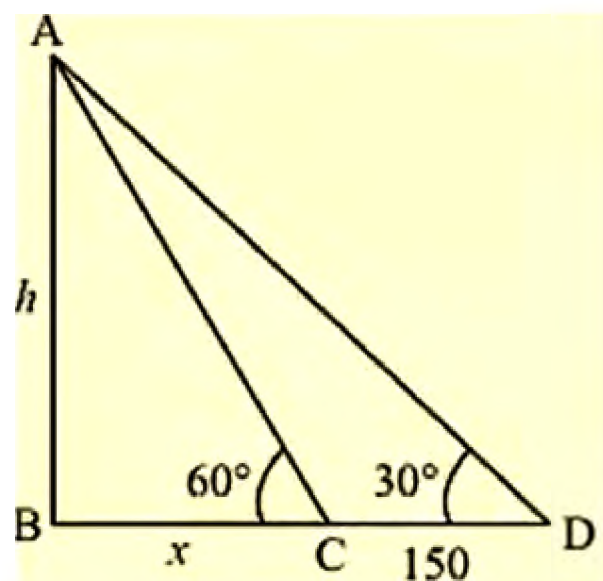
$$x = 150 - 150\sqrt{3}$$

$$x = 63.4 \text{ m}$$

Distance between object A'B' = 63.4 m

**Q14.** The angle of elevation of a tower from a point on the same level as the foot of the tower is  $30^\circ$ . On advancing 150 meters towards the foot of the tower, the angle of elevation of the tower becomes  $60^\circ$ . Show that the height of the tower is 129.9 m.

**Soln:**





Angle of elevation of top tower from first point A,  $\alpha = 30^\circ$

Let us advance through A to B by 150 m then AB = 150 m

Angle of elevation of top of tower from second point B,  $\beta = 60^\circ$

Let height of tower CD = H m

If we represent the above data in form of figure then it form a figure as shown with  $\angle D = 90^\circ$

In right angle triangle, one of included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan \alpha = \frac{CD}{AD} \tan \alpha = \frac{CD}{150+x} \tan 30^\circ = \frac{H}{150+x}$$

$$150 + x = H\sqrt{3} \quad \text{--- (a)}$$

$$\tan \beta = \frac{CD}{AD} \tan \beta = \frac{CD}{x} \tan 60^\circ = \frac{H}{x}$$

$$H = x\sqrt{3}$$

$$x = \frac{H}{\sqrt{3}} \quad \text{--- (b)}$$

Substituting (b) in (a)

$$150 + \frac{H}{\sqrt{3}} = H\sqrt{3}$$

$$H(\sqrt{3} - \frac{1}{\sqrt{3}}) = 150 \quad H\left(\frac{3-1}{\sqrt{3}}\right) = 150 \quad H\left(\frac{2}{\sqrt{3}}\right) = 150 \quad H = \frac{150 \times \sqrt{3}}{2}$$

$$H = \frac{150 \times \sqrt{3}}{2}$$

$$H = 75\sqrt{3}$$

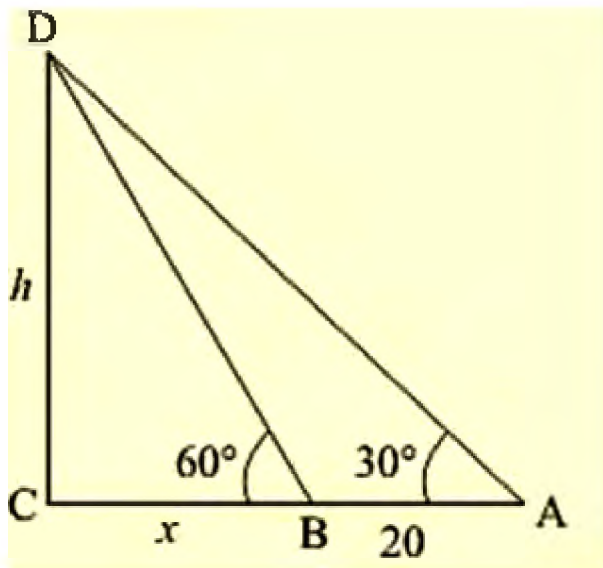
$$H = 129.9 \text{ m}$$

$\therefore$  height of the tower = 129.9 m

**Q16.** The angle of elevation of the top of a tower from a point A on the ground is  $30^\circ$ . Moving a distance of 20 meters towards the foot of the tower to a point B the

angle of elevation increases to  $60^\circ$ . Find the height of tower and the distance of the tower from the point A.

Soln:



Angle of elevation of top of the tower from point A,  $\alpha = 30^\circ$

Angle of elevation of top of tower from point B,  $\beta = 60^\circ$

Distance between A and B,  $AB = 20$  m

Let height of tower  $CD = 'h'$  m

Distance between second point B from foot of the tower be ' $x$ ' m

If we represent the above data in form of figure then it form a figure as shown with  $\angle C = 90^\circ$

In right angle triangle, one of included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan \alpha = \frac{CD}{AD} \quad \tan \alpha = \frac{CD}{20+x} \quad \tan 30^\circ = \frac{h}{20+x} = \frac{h}{20+x}$$

$$20 + x = h\sqrt{3} \quad \text{--- (a)}$$

$$\tan \beta = \frac{CD}{BD} \quad \tan \beta = \frac{CD}{x} \quad \tan 60^\circ = \frac{h}{x} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \quad \text{--- (b)}$$

Substituting (b) in (a)

$$20 + h\sqrt{3} = h\sqrt{3} \frac{h}{\sqrt{3}} = h\sqrt{3}$$

$$h(\sqrt{3} - 1\sqrt{3}) = 20h \left( \sqrt{3} - \frac{1}{\sqrt{3}} \right) = 20h(3 - 1\sqrt{3}) = 20h \left( \frac{3-1}{\sqrt{3}} \right) = 20h = 20 \times \sqrt{3} h = \frac{20 \times \sqrt{3}}{2}$$

$$h = 10\sqrt{3}$$

$$h = 17.32 \text{ m}$$

$$x = 10\sqrt{3} \frac{10\sqrt{3}}{\sqrt{3}}$$

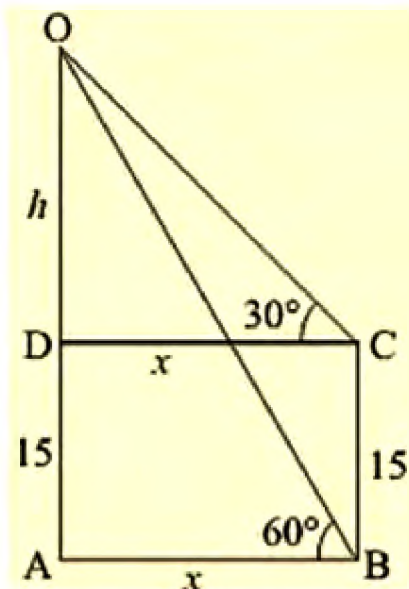
$$x = 10 \text{ m}$$

Height of the tower = 17.32 m

Distance of the tower from point A = (20 + 10) = 30 m

**Q17. From the top of a building 15 m high the angle of elevation of the top of tower is found to be  $30^\circ$ . From the bottom of the same building, the angle of elevation of the top of the tower is found to be  $60^\circ$ . Find the height of the tower and the distance between the building and the tower.**

**Soln:**



Let AB be the building and CD be the tower.

Height of the building is 15 m = h = AB

Angle of elevation of top of the tower from top of the building,  $\alpha = 30^\circ$

Angle of elevation of top of the tower from bottom of the building,  $\beta = 60^\circ$

Distance between the tower and the building  $BD = x$

Let height of the tower above building be 'a' m

If we represent the above data in form of figure then it form a figure as shown with  $\angle D = 90^\circ$  also  $AX \parallel BD, \angle AXC = 90^\circ$

Here ABDX is a rectangle

$\therefore, BD = DX = 'x'$  m  $AB = XD = h = 15$  m

In right triangle if one of the include angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan \alpha = \frac{CX}{AX} \tan \alpha = \frac{CX}{AX} \tan 30^\circ = \frac{a}{x}$$

$$x = a\sqrt{3} \quad \text{--- (1)}$$

$$\tan \beta = \frac{CD}{BD} \tan \beta = \frac{CD}{BD} \tan 60^\circ = \frac{a+15}{x}$$

$$x\sqrt{3} = a+15 \quad \text{--- (2)}$$

Substituting (1) in (2)

$$x\sqrt{3} = a\sqrt{3} + 15 \quad \frac{x}{\sqrt{3}} = \frac{a\sqrt{3} + 15}{\sqrt{3}}$$

$$a + 15 = a\sqrt{3}(\sqrt{3}) \quad a\sqrt{3}(\sqrt{3})$$

$$a + 15 = 3a$$

$$2a = 15$$

$$a = 15/2 = 7.5 \text{ m}$$

$$x = a\sqrt{3}$$

$$x = 7.5 \times \sqrt{3}$$

$$x = 12.99 \text{ m}$$

Height of the tower above ground =  $h + a$

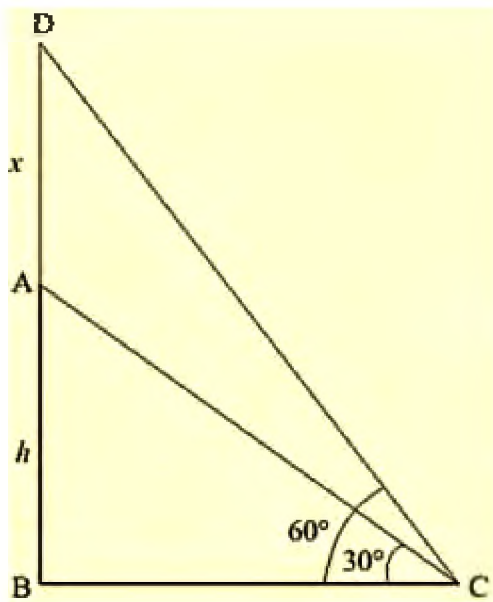
$$= 15 + 7.5$$

$$= 22.5 \text{ m}$$

Distance between tower and building = 12.99 m

**Q18.** On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 m away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are  $60^\circ$  and  $30^\circ$  respectively. Find the height of the tower and the flag pole mounted on it.

**Soln:**



let AB be the tower and BC be the flagstaff on the tower

Distance of the point of observation from foot of the tower BD = 9 m

Angle of elevation of top of flagstaff,  $\alpha = 60^\circ$

Angle of elevation of top of flagstaff,  $\beta = 30^\circ$

Let height of the tower = 'x' = AB

Height of the pole = 'y' = BC

If the above data is represented in form of figure a shown with  $\angle A = 90^\circ$

In right angle triangle, one of included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan \alpha = \frac{AC}{AD} \tan \alpha = \frac{AC}{AD} \tan 60^\circ = \frac{x+y}{9} \tan 60^\circ = \frac{x+y}{9}$$

$$x + y = 9\sqrt{3} \quad \text{--- (a)}$$

$$\tan \beta = \frac{AC}{AD} \quad \tan \beta = \frac{AC}{AD} \quad \tan 30^\circ = \frac{x}{9} \quad \tan 30^\circ = \frac{x}{9}$$

$$x = 9\sqrt{3} \cdot \frac{1}{\sqrt{3}}$$

$$x = 3\sqrt{3}$$

$$x = 5.196 \text{ m} \quad \text{--- (b)}$$

Substituting (b) in (a)

$$y = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}$$

$$y = 6\sqrt{3}$$

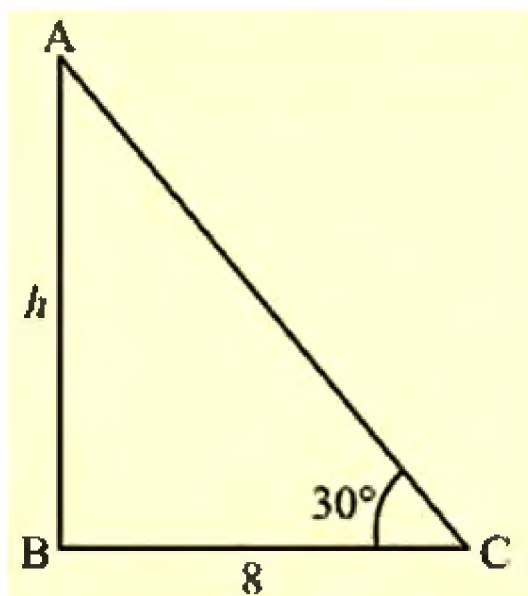
$$y = 10.392 \text{ m}$$

$\therefore$  Height of the tower  $x = 5.196 \text{ m}$

Height of the tower pole = 10.392 m

**Q19.** A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of  $30^\circ$  with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

**Soln:**



Let the initial height be AB

Let us assumed that the tree is broken at point C

Angle made by the broken part CB' with ground is  $30^\circ = \Theta 30^\circ = \Theta$

Distance between foot of tree to point where it touches the ground B'A = 8 m

Height of the tree = h = AC + CB' = AC + CB

The above information is represent in the form of figure as shown

$$\cos \Theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} \cos \Theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} \cos 30^\circ = \frac{AB'}{CB'} \cos 30^\circ = \frac{AB'}{CB'}$$

$$\sqrt{3}2 = 8CB' \frac{\sqrt{3}}{2} = \frac{8}{CB'}$$

$$CB' = 16\sqrt{3} \frac{16}{\sqrt{3}}$$

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \tan 30^\circ = \frac{CA}{AB'} \tan 30^\circ = \frac{CA}{AB'}$$

$$1\sqrt{3} = CA8 \frac{1}{\sqrt{3}} = \frac{CA}{8}$$

$$CA = 8\sqrt{3} \frac{8}{\sqrt{3}}$$

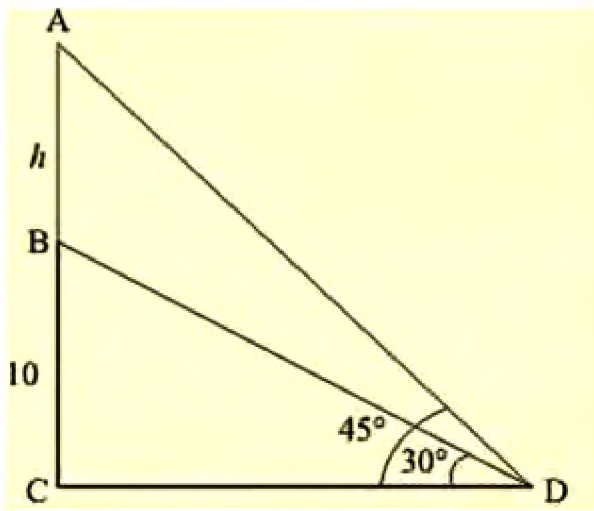
$$\text{Height of the tree} = CB' + CA = 16\sqrt{3} + 8\sqrt{3} \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}$$

$$h = 24\sqrt{3} \frac{24}{\sqrt{3}}$$

$$h = 8\sqrt{3}m8\sqrt{3}m$$

**Q20.** From a point P on the ground the angle of elevation of a 10 m tall building is  $30^\circ$ . A flag is hoisted at the top of the building and the angle of elevation of the flag staff from P is  $45^\circ$ . Find the length of the flag staff and the distance of the building from the point P.

**Soln:**



let AB be the tower and BD be the flag staff

Angle of elevation of top of the building from P  $\alpha = 30^\circ$

AB = height of the tower = 10 m

Angle of elevation of top of flag staff from P  $\beta = 45^\circ$

Let height of the flagstaff BD = 'a' m

The above information is represented in the form of figure as shown

with  $\angle A = 90^\circ$

In a right angle triangle if one of included angle is  $\Theta$

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \alpha = \frac{AB}{AP} \quad \tan \alpha = \frac{AB}{AP}$$

$$\tan 30^\circ = \frac{10}{AP} \quad \tan 30^\circ = \frac{10}{AP}$$

$$AP = 10\sqrt{3} \text{ m}$$

$$AP = 17.32 \text{ m}$$

$$\tan \beta = \frac{AD}{AP} \quad \tan \beta = \frac{AD}{AP} \quad \tan 45^\circ = \frac{10+a}{AP} \quad \tan 45^\circ = \frac{10+a}{AP}$$

$$10 + a = AP$$

$$a = 17.32 - 10 \text{ m}$$

$$a = 7.32 \text{ m}$$

Height of the flag staff 'a' = 7.32 m

Distance between P and foot of the tower = 17.32 m

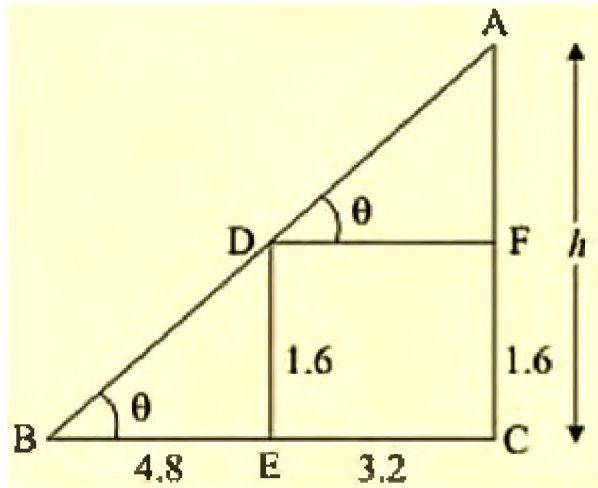


**Q21. A 1.6 m tall boy stands at a distance of 3.2 m from a lamp post and casts a shadow of 4.8 m on the ground. Find the height of the lamp post by using**

**(i) trigonometric ratio**

**(ii) properties of similar triangles**

**Soln:**



Let AC be the lamp post of height 'h'

We assume that ED = 1.6 m, BE = 4.8 m and EC = 3.2

We have to find the height of the lamp post

Now we have to find the height of the lamp post using similar triangles

Since triangle BDE and triangle ABC are similar

$$\frac{AC}{BC} = \frac{ED}{BE} \quad \frac{h}{4.8+3.2} = \frac{1.6}{4.8} \quad h = 1.6 \times \frac{4.8+3.2}{4.8} = \frac{1.6}{4.8} \times 8 = \frac{8}{3} m$$

Again we have to find the height of lamp post using trigonometric ratio

$$\text{In } \triangle ADE \quad \tan \theta = \frac{ED}{BE} = \frac{1.6}{4.8}$$

$$\tan \theta = \frac{1}{3}$$

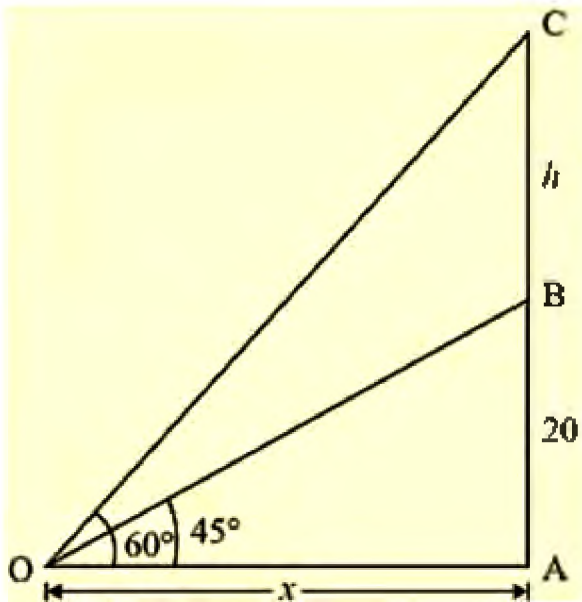
Again In  $\triangle ABC$

$$\tan \theta = \frac{h}{4.8+3.2} = \frac{1}{3} \quad h = \frac{1}{3} \times (4.8+3.2) = \frac{1}{3} \times 8 = \frac{8}{3} m$$

Hence the height of the lamp post is  $h = \frac{8}{3} m$

**Q24.** From a point on the ground the angle of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are  $45^\circ$  and  $60^\circ$  respectively. Find the height of the transmission tower.

**Soln:**



given height of building = 20 m = AB

Let height of tower above building = 'h' = BC

Height of tower + building = (h + 20) m [from ground] = CA

Angle of elevation of bottom of tower,  $\alpha = 45^\circ$

Angle of elevation of top of tower,  $\beta = 60^\circ$

Let distance between tower and observation point = 'x' m

The above data is represented in the form of figure as shown is

If one of the included angle in right angle triangle is  $\Theta$

Then  $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$   $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan \alpha = \frac{AB}{OA} \tan \alpha = \frac{AB}{OA} \tan 45^\circ = \frac{20}{x} \tan 45^\circ = \frac{20}{x}$$

$$x = 20 \text{ m}$$

$$\tan \beta = \frac{CA}{OA} \tan \beta = \frac{CA}{OA} \tan 60^\circ = \frac{h+20}{x} \tan 60^\circ = \frac{h+20}{x}$$

$$h + 20 = 20\sqrt{3} \text{ m}$$

$$h = 20(\sqrt{3}-1) \text{ m} \quad 20(\sqrt{3}-1) \text{ m}$$

$$\text{Height of the tower } h = 20(\sqrt{3}-1) \text{ m} \quad 20(\sqrt{3}-1) \text{ m}$$

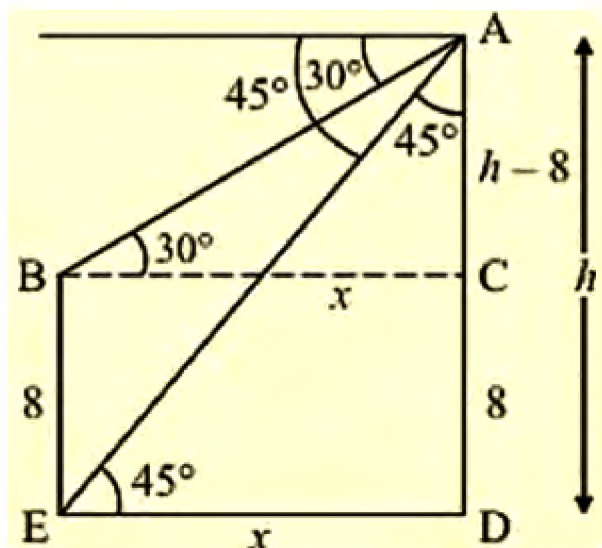
$$h = 20(1.73-1) \text{ m} \quad 20(1.73-1) \text{ m}$$

$$h = 20 \times .732$$

$$h = 14.64 \text{ m}$$

**Q25.** The angle of depression of the top and bottom of 8 m tall building from the top of a multistoried building are  $30^\circ$  and  $45^\circ$ . Find the height of the multistoried building and the distance between the two buildings.

**Soln:**



Let height of multistoried building 'h' m = AB

Height of the tall building = 8 m = CD

Angle of depression of top of the tall building,  $\alpha = 30^\circ$

Angle of depression of bottom of the tall building,  $\beta = 45^\circ$

Distance between the two buildings = 'x' m = BD

Let AO = s

AB = AO + BO but BO = CD  $[\because AOC \text{ is rectangle}]$

AB = (s + 8) m

The above information is represented in the form of figure as shown

If in right angle triangle one of the included angle is  $\theta$

Then  $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

In  $\triangle AOC$

$$\tan 30^\circ = \frac{AO}{CO} \tan 30^\circ = \frac{AO}{CO} \quad 1/\sqrt{3} = s/x \quad \frac{1}{\sqrt{3}} = \frac{s}{x}$$

$$x = s\sqrt{3} \quad x = s\sqrt{3} \quad \text{--- (1)}$$

In  $\triangle ABD$

$$\tan 45^\circ = \frac{AB}{BD} \tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{s+8}{x}$$

$$s + 8 = x \quad \text{--- (2)}$$

Substituting (a) in (2)

$$s + 8 = s\sqrt{3}$$

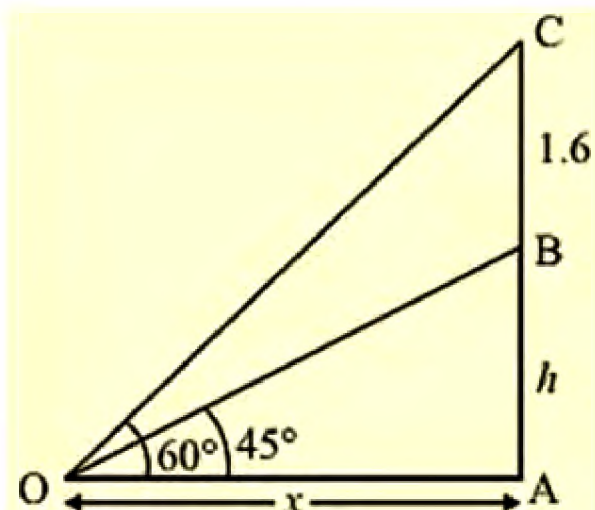
$$s(\sqrt{3}-1) = 8 \quad s(\sqrt{3}-1) = 8 \quad s = \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \quad s = 4(\sqrt{3}+1)$$

$$s = 4(\sqrt{3}+1) \quad s = 4(\sqrt{3}+1) \text{ m} \quad x = 4(3+\sqrt{3}) \text{ m} \quad x = 4(3+\sqrt{3}) \text{ m}$$

$$AB = s + 8 = 8 + 4(\sqrt{3}+1) \quad AB = s + 8 = 8 + 4(\sqrt{3}+1)$$

**Q26.** A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, angle of elevation of the top of the statue is  $60^\circ$  and from the same point the angle of elevation of the top of the pedestal is  $45^\circ$ . Find the height of the pedestal.

**Soln:**



let the height of the pedestal be 'h' m

Height of the statue = 1.6 m

Angle of elevation of the top of the statue  $\alpha = 30^\circ$

Angle of elevation of the top of the pedestal  $\beta = 60^\circ$

The above data is represented in the form of figure as shown.

If in right angle triangle one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan \alpha = \frac{BC}{DC} \quad \tan 45^\circ = \frac{h}{DC}$$

DC = 'h' m — (a)

$$\tan \beta = \frac{AC}{DC} \quad \tan 60^\circ = \frac{h+1.6}{DC}$$

$$DC = \frac{h+1.6}{\sqrt{3}} \quad \text{--- (b)}$$

From (a) and (b)

$$h = \frac{h+1.6}{\sqrt{3}}$$

$$h \times \sqrt{3} = h + 1.6 \quad h \times \sqrt{3} - h = 1.6 \quad h(\sqrt{3} - 1) = 1.6$$

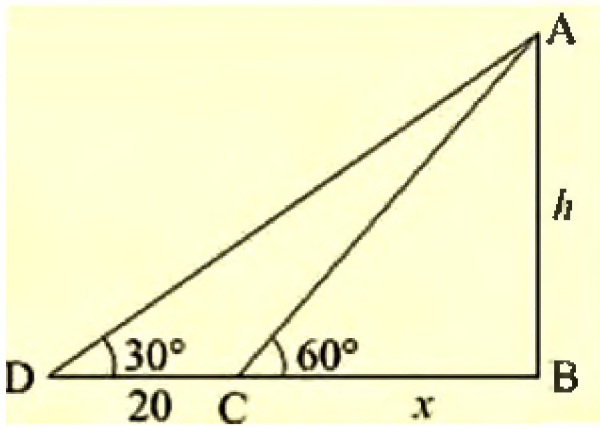
$$h = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$h = 0.5(\sqrt{3} + 1)$$

$$\text{Height of the pedestal} = 0.6(\sqrt{3} + 1)$$

**Q27. A T.V. tower stands vertically on a bank of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is  $60^\circ$ . From a point 20 m away this point on the same bank, the angle of elevation of the top of the tower is  $30^\circ$ . Find the height of the tower and the width of the river.**

**Soln:**



let AB be the T.V tower of height 'h' m on the bank of river and 'D' be the point on the opposite side of the river. An angle of elevation at the top of the tower is  $30^\circ$

Let AB = h and BC = x

Here we have to find height and width of the river.

The above data is represented in the form of figure as shown.

In  $\triangle ACB$

$$\tan 60^\circ = \frac{AB}{BC} \tan 60^\circ = \frac{h}{x} \sqrt{3} = h \sqrt{3} \Rightarrow h = \frac{x\sqrt{3}}{\sqrt{3}} = x$$

Again in  $\triangle DBA$ ,

$$\tan 30^\circ = \frac{AB}{BD} \tan 30^\circ = \frac{h}{20+x} \frac{1}{\sqrt{3}} = \frac{h}{20+x} \Rightarrow h = \frac{(20+x)\sqrt{3}}{\sqrt{3}} = 20+x$$

$$\sqrt{3}h = 20 + h \Rightarrow \sqrt{3}h - h = 20 \Rightarrow h(\sqrt{3} - 1) = 20 \Rightarrow h = \frac{20}{\sqrt{3} - 1}$$

$$h = \frac{20(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{20(\sqrt{3} + 1)}{3 - 1} = 10(\sqrt{3} + 1)$$

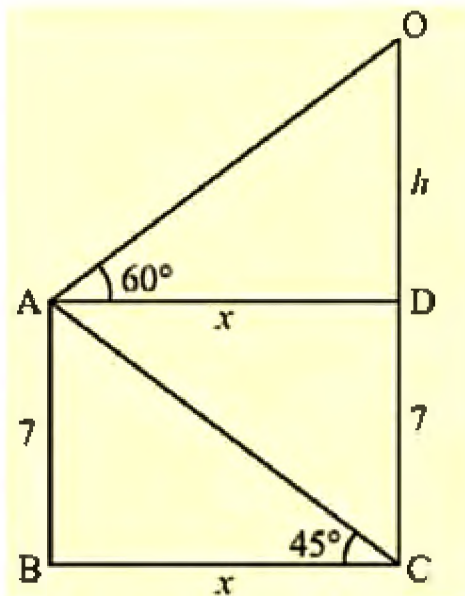
$$x = h = 10(\sqrt{3} + 1)$$

$$x = 10$$

Hence the height of the tower is  $10\sqrt{3}m$  and width of the river is 10 m.

**Q28.** From the top of a 7 m high building, the angle of elevation of the top of a cable is  $60^\circ$  and the angle of depression of its foot is  $45^\circ$ . Determine the height of the tower.

**Soln:**



Given

Height of the building = 7 m = AB

Height of the cable tower = 'H' m = CD

Angle of elevation of the top of the building  $\alpha = 60^\circ$

Angle of elevation of the bottom of the building  $\beta = 45^\circ$

The above data is represented in form of figure as shown

Let CX = 'x' m

CD = DX + XC = 7 m + 'x' m

= x + 7m

In  $\triangle ADC$

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan 45^\circ = \frac{7}{x} \quad \tan 45^\circ = \frac{7}{x}$$

x = 7m

$$\tan 60^\circ = \frac{h}{x} \quad \tan 60^\circ = \frac{h}{7} \quad \sqrt{3} = \frac{h}{7} \quad h = 7\sqrt{3}$$

$$x = 7\sqrt{3}$$

But CD = x + 7

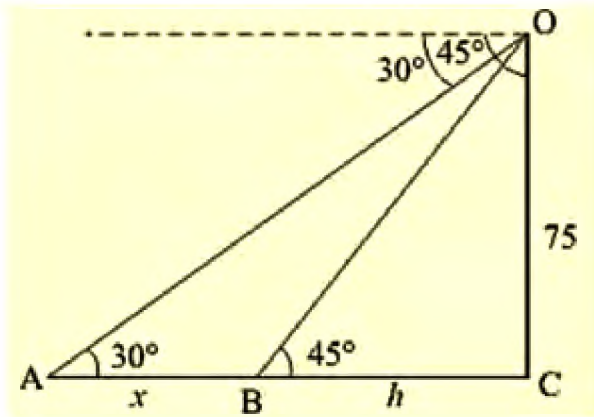
$$= 7\sqrt{3} + 7$$

$$= 7(\sqrt{3}+1)m \quad 7(\sqrt{3} + 1) m$$

$$\text{Height of the cable tower} = 7(\sqrt{3}+1)m \quad 7(\sqrt{3} + 1) m$$

**Q29.** As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are  $30^\circ$  and  $45^\circ$ . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

**Soln:**



Given;

Height of the lighthouse = 75m = 'h' m = OC

Angle of depression of ship 1,  $\alpha = 30^\circ$

Angle of depression of bottom of the tall building,  $\beta = 45^\circ$

The above data is represented in form of figure as shown

Let distance between ships be 'x' m = AB

If in right angle triangle one of the included angle is  $\theta$

$$\text{Then } \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan \alpha = \frac{OC}{AB + BC} \quad \tan 30^\circ = \frac{75}{x + BC} \quad \tan 30^\circ = \frac{75}{x + BC}$$

$$x + BC = 75\sqrt{3} \quad \text{--- (1)}$$

$$\tan \beta = \frac{OC}{BC} \quad \tan 45^\circ = \frac{75}{BC} \quad \tan 45^\circ = \frac{75}{BC}$$

$$BC = 75 \quad \text{--- (2)}$$



Substituting (2) in (1)

$$x + 75 = 75\sqrt{3}$$

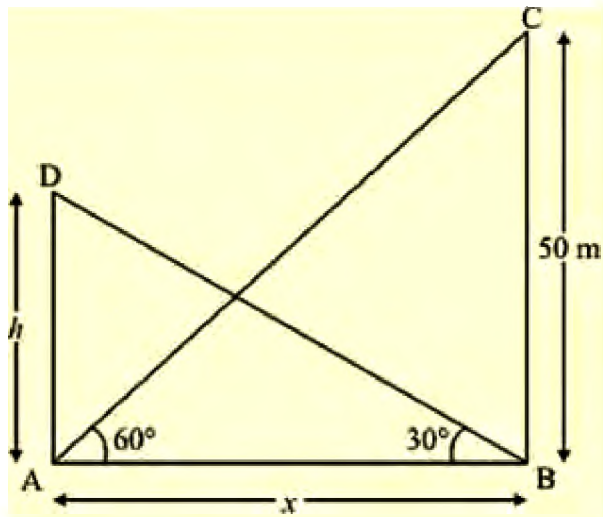
$$x = 75(\sqrt{3} - 1)$$

$\therefore$  distance between ships = 'x' m

$$x = 75(\sqrt{3} - 1)$$

**Q30.** The angle of elevation of the top of the building from the foot of the tower is  $30^\circ$  and the angle of the top of the tower from the foot of the building is  $60^\circ$ . If the tower is 50m high, find the height of the building.

**Soln:**



Angle of elevation of top of the building from foot of tower =  $30^\circ = \alpha = 30^\circ = \alpha$

Angle of elevation of top of the building from foot of tower =  $60^\circ = \beta = 60^\circ = \beta$

Height of the tower = 50m = AB

Height of building = 'h' m = CD

The above data is represented in the form of figure as shown

In right triangle if one of the included angle is  $\Theta$

$$\text{Then } \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

In  $\triangle ABD$

$$\tan \beta = \frac{AB}{BD} \quad \tan \beta = \frac{AB}{BD} \quad \tan 60^\circ = \frac{50}{BD} \quad \tan 60^\circ = \frac{50}{BD}$$

$$BD = 50\sqrt{3} \frac{50}{\sqrt{3}}$$

$$\tan \alpha = \frac{CD}{BD} \tan \alpha = \frac{CD}{BD}$$

$$\tan 30^\circ = \frac{h}{50\sqrt{3}} \frac{h}{\frac{50}{\sqrt{3}}}$$

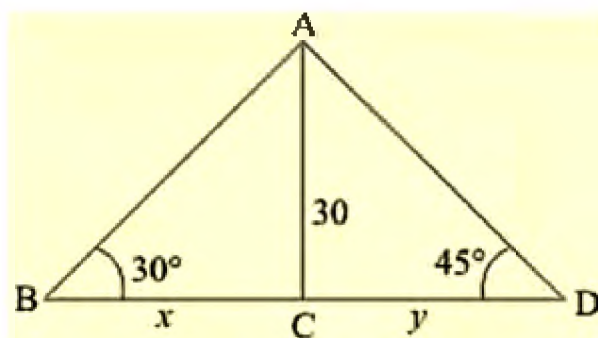
$$h = 50\sqrt{3} \times \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}}$$

$$h = 503 \frac{50}{3}$$

$$\therefore \therefore \text{height of the building} = 503 \text{ m} \frac{50}{3} \text{ m}$$

**Q31. From a point on a bridge across a river the angle of depression of the banks on opposite side of the river are  $30^\circ$  and  $45^\circ$  respectively. If the bridge is at the height of 30m from the banks, find the width of the river.**

**Soln:**



Height of the bridge = 30m [AB]

Angle of depression of bank 1 i.e,  $\alpha = 30^\circ$

Angle of depression of bank 2 i.e,  $\beta = 45^\circ$

Given banks are on opposite sides

Distance between banks  $B_1B_2 = BB_1 + BB_2$

The above data is represented in the form of figure as shown

In right angle triangle, if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

In  $\triangle ABB_1$

$$\tan \alpha = \frac{AB}{B_1B} \tan \alpha = \frac{AB}{B_1B} \tan 30^\circ = \frac{30}{B_1B} \tan 30^\circ = \frac{30}{B_1B} \quad B_1B = 30\sqrt{3}m$$

$$B_1B = 30\sqrt{3}m$$

In  $\triangle ABB_2$

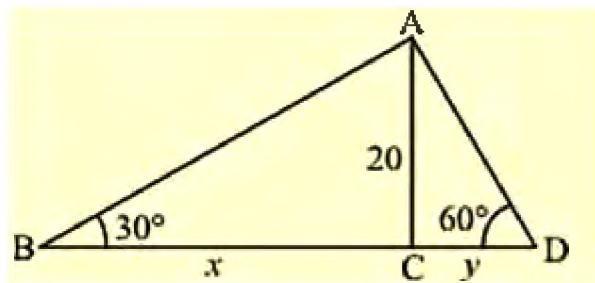
$$\tan \beta = \frac{AB}{B_2B} \tan \beta = \frac{AB}{B_2B} \tan 45^\circ = \frac{30}{B_2B} \tan 45^\circ = \frac{30}{B_2B} \quad B_2B = 30m$$

$$B_1B_2 = BB_1 + BB_2 = 30\sqrt{3} + 30 = 30(\sqrt{3} + 1)$$

$$\text{Distance between banks} = 30(\sqrt{3} + 1)m$$

**Q33.** A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are  $60^\circ$  and  $30^\circ$  respectively. Find the width of the river.

**Soln:**



Height of the tree  $AB = 20m$

Angle of depression of the pole 1 feet  $\alpha = 60^\circ$

Angle of depression of the pole 2 feet  $\beta = 30^\circ$

$B_1C_1$  be one pole and  $B_2C_2$  be the other pole

Given poles are on opposite sides.

Width of the river =  $B_1B_2$

$$= B_1B + BB_2$$

The above data is represented in the form of figure as shown

In right angle triangle, if one of the included angle is  $\Theta$

$$\text{Then } \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan \alpha = \frac{AB}{B_1B} \quad \tan 60^\circ = \frac{20}{B_1B} \quad \tan 60^\circ = \frac{20}{B_1B} \quad B_1B = \frac{20}{\sqrt{3}} \quad B_1B = \frac{20}{\sqrt{3}}$$

$$\tan \beta = \frac{AB}{BB_2} \quad \tan 30^\circ = \frac{20}{BB_2} \quad \tan 30^\circ = \frac{20}{BB_2} \quad BB_2 = \frac{20}{\frac{1}{\sqrt{3}}} = 20\sqrt{3} \quad BB_2 = 20\sqrt{3} \quad m$$

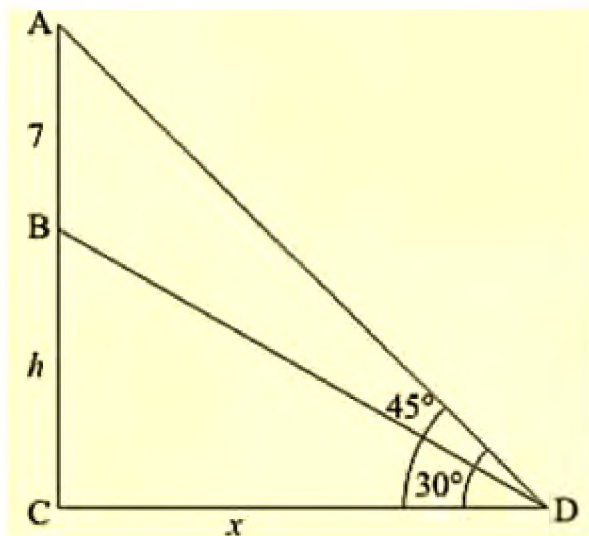
$$B_1B_2 = B_1B + BB_2 = \frac{20}{\sqrt{3}} + 20\sqrt{3} = \frac{20}{\sqrt{3}} + 20\sqrt{3}$$

$$20 + 3\sqrt{3} = 80\sqrt{3} \quad \frac{1+3}{\sqrt{3}} = \frac{80}{\sqrt{3}}$$

$$\text{Width of the river} = 80\sqrt{3} \quad \frac{80}{\sqrt{3}} \quad m$$

**Q34.** A vertical tower stands on a horizontal plane and is surmounted by a flag staff of height 7m. From a point on the plane, the angle of elevation of the bottom of flag staff is  $30^\circ$  and that of the top of the flag staff is  $45^\circ$ . Find the height of the tower.

**Soln:**



Given

Height of the flag staff = 7m = BC

Let height of the tower = 'h' m = AB

Angle of elevation of top of the bottom of the flagstaff  $\alpha = 30^\circ$

Angle of elevation of top of the top of the flagstaff  $\beta = 45^\circ$

Points of observation be 'p'

The above data is represented in the form of figure as shown

In right angle triangle, if one of the included angle is  $\Theta$

$$\text{Then } \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan \alpha = \frac{AB}{AP} \quad \tan \alpha = \frac{AB}{AP} \quad \tan 30^\circ = \frac{h}{AP} \quad \tan 30^\circ = \frac{h}{AP}$$

$$AP = h\sqrt{3} \quad AP = h\sqrt{3} \quad \text{--- (1)}$$

$$\tan \beta = \frac{AC}{AP} \quad \tan \beta = \frac{AC}{AP} \quad \tan 45^\circ = \frac{h+7}{AP} \quad \tan 45^\circ = \frac{h+7}{AP}$$

$$AP = h + 7 \quad \text{--- (2)}$$

From (1) and (2)

$$h\sqrt{3} = h + 7 \quad h\sqrt{3} - h = 7 \quad h(\sqrt{3} - 1) = 7 \quad h(\sqrt{3} - 1) = 7$$

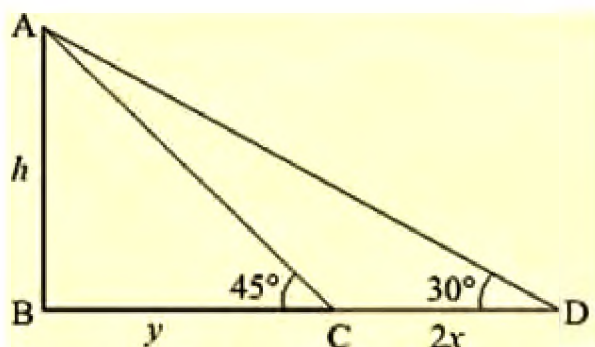
$$h = \frac{7}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad h = \frac{7(\sqrt{3} + 1)}{3 - 1} \quad h = \frac{7(\sqrt{3} + 1)}{2} \quad h = 3.5(\sqrt{3} + 1) \text{ m}$$

$$h = 3.5(\sqrt{3} + 1) \text{ m}$$

$$\text{Height of the tower} = 3.5(\sqrt{3} + 1) \text{ m}$$

**Q35.** The length of the shadow of a tower standing on level plane is found to be  $2x$  meters longer when the sun's altitude is  $30^\circ$  than when it was  $45^\circ$ . Prove that the height of tower is  $x(\sqrt{3} + 1)$  meters.

**Soln:**



Let

Length of shadow be 'a' m [BC] when sun altitude be  $\alpha = 45^\circ$

Length of shadow be  $(2x + a)$  m [BD] when sun altitude be  $\beta = 30^\circ$

Let height of tower be 'h' m = AB

The above data is represented in the form of figure as shown

In right angle triangle one of the included angle is  $\Theta$

$$\text{Then } \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

In  $\triangle ABC$

$$\tan \alpha = \frac{AB}{BC} \tan \alpha = \frac{AB}{BC} \tan 45^\circ = \frac{h}{a} \tan 45^\circ = \frac{h}{a}$$

$$h = a \text{ --- (1)}$$

In  $\triangle ADB$

$$\tan \beta = \frac{AB}{BC} \tan \beta = \frac{AB}{(2x+a)BC} \tan 30^\circ = \frac{h}{2x+a} \tan 30^\circ = \frac{h}{2x+a}$$

$$2x+a = h\sqrt{3} \text{ --- (2)}$$

Substituting (1) in (2)

$$2x+h = h\sqrt{3} \quad 2x + h = h\sqrt{3} \quad h(\sqrt{3}-1) = 2x \quad h(\sqrt{3}-1) = 2x$$

$$h = \frac{2x}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

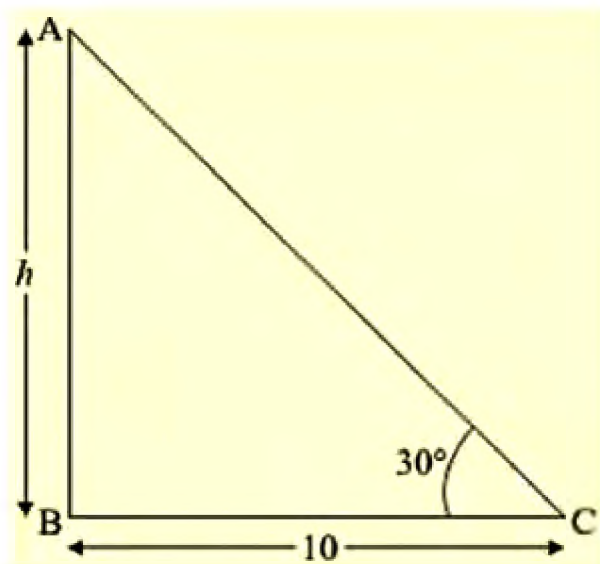
$$h = \frac{2x(\sqrt{3}+1)}{2}$$

$$h = x(\sqrt{3}+1)$$

$$\text{Height of the tower} = x(\sqrt{3}+1)m$$

**Q36.** A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of  $30^\circ$  with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 meters. Find the height of the tree.

**Soln:**



Let AB be the height of the tree and it is broken at point C and top touches ground at B'

Angle made by the top  $\alpha = 30^\circ$

Distance from foot of tree from point where it touches ground = 10 m

The above data is represented in form of figure as shown

Height of the tree = AB = AC + CB

= AC + CB'

In right angle triangle one of the included angle is  $30^\circ$

Then  $\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan 30^\circ = \frac{AC}{B'C} \quad \tan 30^\circ = \frac{AC}{10} \quad AC = 10\sqrt{3} \text{ m} \quad AC = \frac{10}{\sqrt{3}} \text{ m}$$

Again

$$\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} \quad \cos 30^\circ = \frac{B'C}{AB} \quad \cos 30^\circ = \frac{B'C}{10\sqrt{3} + \frac{10}{\sqrt{3}}}$$

$$\frac{\sqrt{3}}{2} = \frac{B'C}{10\sqrt{3} + \frac{10}{\sqrt{3}}} \quad B'C = \frac{10\sqrt{3} \cdot \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{10}{\sqrt{3}}$$

$$B'C = \frac{10\sqrt{3} \cdot \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{10}{\sqrt{3}}$$

AB = CA + CB'

$$10\sqrt{3} + \frac{10}{\sqrt{3}} + \frac{10}{\sqrt{3}}$$

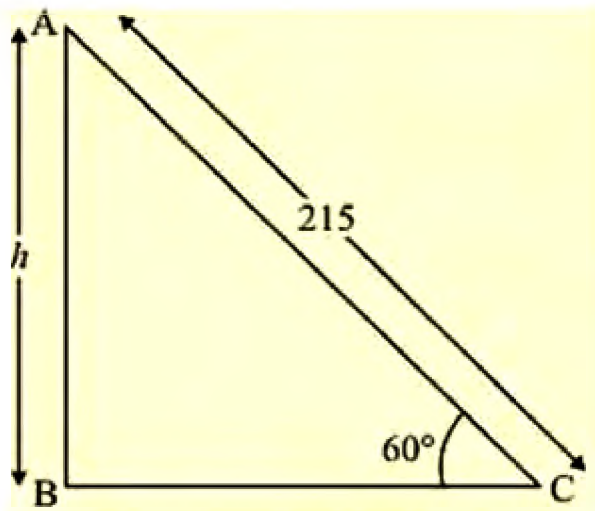
$$= 30\sqrt{3} \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3} \frac{10}{\sqrt{3}}$$

$$\text{Height of tree} = 10\sqrt{3} \text{ m} \frac{10}{\sqrt{3}} \text{ m}$$

**Q37.** A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at  $60^\circ$  to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

**Soln:**



Length of the cable connected to balloon = 215m [CB]

Angle of inclination of cable with ground  $\alpha = 60^\circ$

Height of the balloon from ground = 'h' m = AB

The above data is represented in form of figure as shown

In right angle triangle one of the included angle is  $\Theta$

$$\sin \Theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} \quad \sin 60^\circ = \frac{AB}{BC} \quad \sin 60^\circ = \frac{AB}{215} \quad \frac{\sqrt{3}}{2} = \frac{h}{215}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{215}$$

$$h = 215 \frac{\sqrt{3}}{2}$$

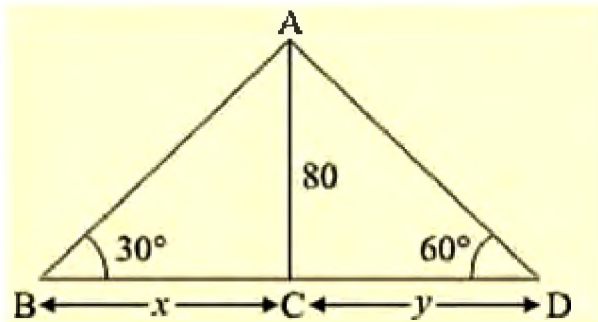
$$h = 107.5\sqrt{3} \text{ m}$$

$$\text{Height of the balloon from ground} = 107.5\sqrt{3} \text{ m}$$



**Q38. Two men on either side of the cliff 80m high observe the angles of elevation of the top of the cliff to be 30° and 60° respectively. Find the distance between the two men.**

**Soln:**



Height of cliff = 80m = AC

Angle of elevation from Man 1,  $\alpha = 30^\circ$  [M<sub>1</sub>]  $\alpha = 30^\circ$  [M<sub>1</sub>]

Angle of elevation from Man 2,  $\beta = 60^\circ$  [M<sub>2</sub>]  $\beta = 60^\circ$  [M<sub>2</sub>]

Distance between two men = M<sub>1</sub>M<sub>2</sub> = BM<sub>1</sub> + BM<sub>2</sub>  $M_1 M_2 = BM_1 + BM_2$

The above information is represented in form of figure as shown

In right angle triangle one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$   $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan \alpha = \frac{AC}{BM_1} \tan \alpha = \frac{80}{BM_1} \tan 30^\circ = \frac{80}{BM_1} \tan 30^\circ = \frac{80}{BM_1} \frac{1}{\sqrt{3}} \quad BM_1 = 80\sqrt{3} \quad BM_1 = 80\sqrt{3}$$

$$\tan \beta = \frac{AC}{BM_2} \tan \beta = \frac{80}{BM_2} \tan 60^\circ = \frac{80}{BM_2} \tan 60^\circ = \frac{80}{BM_2} \sqrt{3} \quad BM_2 = \frac{80}{\sqrt{3}}$$

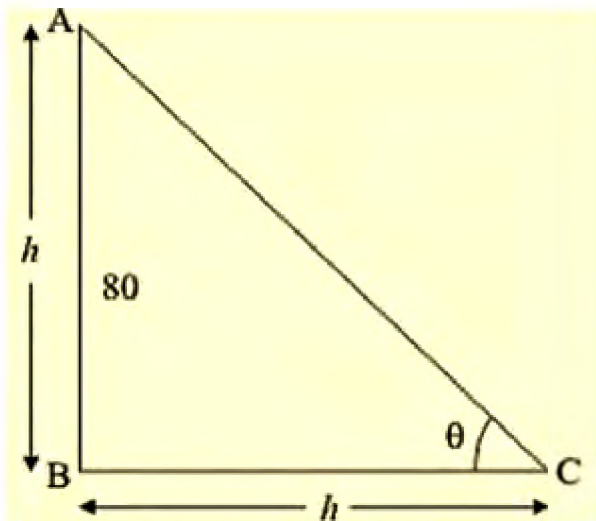
$$M_1 M_2 = BM_1 + BM_2 = 80\sqrt{3} + \frac{80}{\sqrt{3}}$$

$$80\sqrt{3} + \frac{80}{\sqrt{3}} = \frac{80 \times 3}{\sqrt{3}} + \frac{80}{\sqrt{3}} = \frac{240 + 80}{\sqrt{3}} = \frac{320}{\sqrt{3}}$$

Distance between men =  $\frac{320}{\sqrt{3}}$  meters

**Q39. Find the angle of elevation of the sun (sun's altitude) when the length of the shadow of a vertical pole is equal to its height.**

**Soln:**



Let

Height of the pole = 'h' m = sun's altitude from ground length of shadow be 'L'

Given that  $L = h$ .

Angle of elevation of sun's altitude be  $\Theta$

The above data is represented in form of figure as shown

In right angle triangle one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan \Theta = \frac{AB}{BC} \tan \Theta = \frac{AB}{BC} \tan \Theta = \frac{h}{L} \tan \Theta = \frac{h}{L}$$

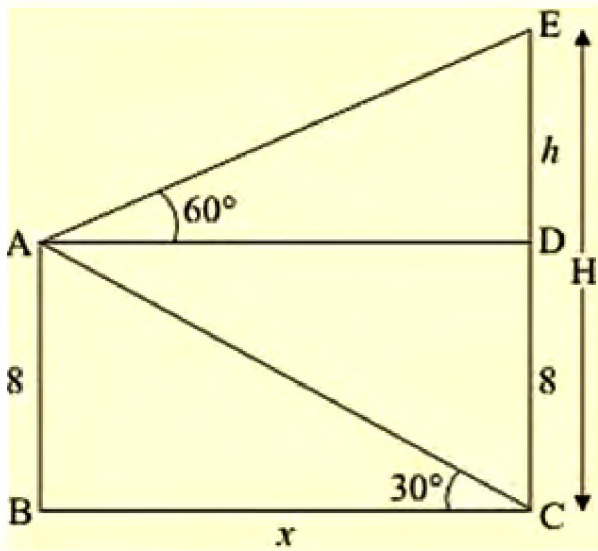
$$\tan \Theta = \frac{L}{L} \tan \Theta = \frac{L}{L} \text{ [since } h = L \text{]}$$

$$\Theta = \tan^{-1}(1) = 45^\circ \quad \Theta = \tan^{-1}(1) = 45^\circ$$

Angle of sun's altitude is  $45^\circ$

**Q42.** A man standing is on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as  $60^\circ$  and the angle of depression of the base of the hill as  $30^\circ$ . Calculate the distance of the hill from the ship and the height of the hill.

**Soln:**



Height of the ship above water level = 8m = AB

Angle of elevation of top of cliff (hill)  $\alpha = 60^\circ$

Angle of depression of the bottom of hill  $\beta = 30^\circ$

Height of the hill = CD

Distance between ship and hill = AX

Height of the hill above ship = CX = 'a' m

Height of hill = (a + 8) m

The above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan \alpha = \frac{CX}{AX} \tan \alpha = \frac{CX}{AX} \tan 60^\circ = \frac{a}{AX} \tan 60^\circ = \frac{a}{AX}$$

$$AX = a\sqrt{3} \frac{a}{\sqrt{3}}$$

$$\tan \beta = \frac{XD}{AX} \tan \beta = \frac{XD}{AX} \tan 30^\circ = \frac{B}{AX} \tan 30^\circ = \frac{B}{AX}$$

$$AX = 8\sqrt{3}$$

$$\therefore a\sqrt{3} = 8\sqrt{3} \therefore \frac{a}{\sqrt{3}} = 8\sqrt{3}$$

$$a = 24\text{m}$$

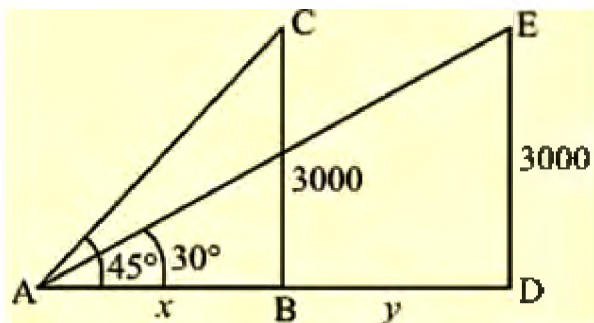
$$AX = 8\sqrt{3}m$$

$$\therefore \therefore \text{Height of the cliff hill} = (24+8) \text{ m} = 32\text{m}$$

$$\text{Distance between hill and ship } 8\sqrt{3}m$$

**Q44.** The angles of depression of two ships from the top of a light house and on the same side of it are found to be  $45^\circ$  and  $30^\circ$  respectively. If the ships are 200 m apart, find the height of the light house.

**Soln:**



Height of the light house AB = 'h' meters

Let  $S_1$  and  $S_2$  be ships

$$\text{Distance between ships } S_1S_2 = 200\text{m}$$

$$\text{Angle of depression of } S_1 (\alpha = 30^\circ)$$

$$\text{Angle of depression of } S_2 (\beta = 45^\circ)$$

The above data is represented in form of figure as shown

In  $\triangle ABS_2$

$$\tan \beta = \frac{AB}{BS_2} \tan \beta = \frac{AB}{BS_2} \tan 45^\circ = \frac{h}{BS_2}$$

$$BS_2 = h \quad \text{--- (1)}$$

In  $\triangle ABS_1$

$$\tan \alpha = \frac{AB}{BS_1} \tan \alpha = \frac{AB}{BS_1} \tan 30^\circ = \frac{h}{BS_1}$$

$$BS_1 = h\sqrt{3} \quad \text{--- (2)}$$

Subtracting (1) from (2)

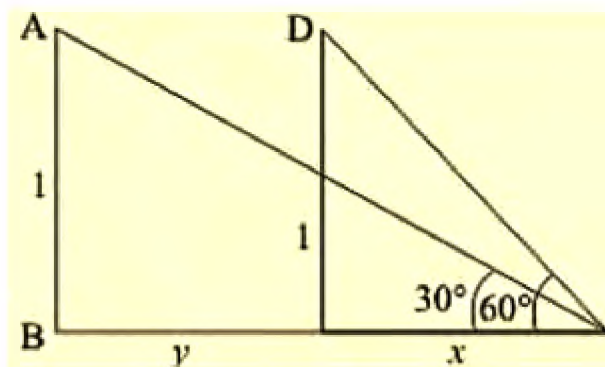
$$BS_1 - BS_2 = h(\sqrt{3} - 1) \quad BS_1 - BS_2 = h(\sqrt{3} - 1) \quad 200 = h(\sqrt{3} - 1)$$

$$200 = h(\sqrt{3} - 1) \quad h = 100(\sqrt{3} + 1) \text{ meters} \quad h = 100(\sqrt{3} + 1) \text{ meters}$$

Height of the light house = 273.2 meters

**Q45.** The angles of elevation from the top of a tower from two points at distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

**Soln:**



Height of the tower AB = 'h' meters

Let point C be 4 meters from B.

Angle of elevation is  $\alpha$  given point D is 9 meters from B, Angle of elevation is  $\beta$

Given  $\alpha, \beta$  are complementary,  $\alpha + \beta = 90^\circ \Rightarrow \beta = 90^\circ - \alpha$

Required to prove that h = 6 meters

The above data is represented in the form of figure as shown

In  $\triangle ABC$

$$\tan \alpha = \frac{AB}{BC} \quad \tan \alpha = \frac{h}{4} \quad \tan \alpha = \frac{h}{4}$$

$$h = 4 \tan \alpha \quad \text{--- (1)}$$

In  $\triangle ABD$

$$\tan \beta = \frac{AB}{BD} \quad \tan \beta = \frac{h}{9} \quad \tan(90^\circ - \alpha) = \frac{h}{9}$$

$$h = 9 \tan \alpha \quad \text{--- (2)}$$

Multiply (1) and (2)

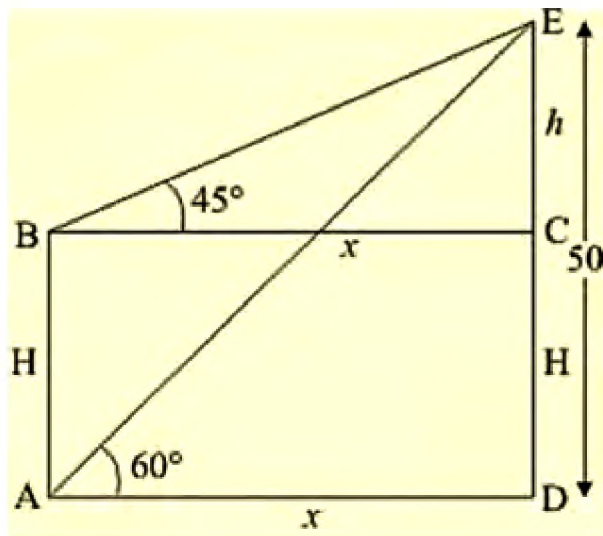
$$h \times h = 4 \tan \alpha \times 9 \cot \alpha \quad h \times h = 4 \tan \alpha \times 9 \cot \alpha \quad h^2 = 36(\tan \alpha \cot \alpha)$$

$$h^2 = 36(\tan \alpha \cot \alpha) \quad h = \sqrt{36} = 6m \quad h = \sqrt{36} = 6m$$

Height of the tower = 6 meters

**Q46.** From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be  $45^\circ$  and  $60^\circ$  respectively. Find the height of the pole.

**Soln:**



AB = height of the tower = 50m.

CD = height of the pole

Angle of depression of top of building  $\alpha = 45^\circ$

Angle of depression of bottom of building  $\beta = 60^\circ$

The above data is represented in the form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

$$\text{Then } \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan \alpha = \frac{AX}{CX} \quad \tan \alpha = \frac{AX}{CX} \quad \tan 45^\circ = \frac{AX}{CX} \quad \tan 45^\circ = \frac{AX}{CX}$$

$$AX = CX$$

$$\tan \beta = \frac{AB}{BD} \quad \tan \beta = \frac{AB}{BD} \quad \tan 60^\circ = \frac{50}{BD} \quad \tan 60^\circ = \frac{50}{BD}$$

$$CX = 50\sqrt{3} \frac{50}{\sqrt{3}}$$

$$AX = 50\sqrt{3} \text{ m} = BD \frac{50}{\sqrt{3}} \text{ m} = BD$$

$$CD + AB - AX = 50 - 50\sqrt{3} \frac{50}{\sqrt{3}}$$

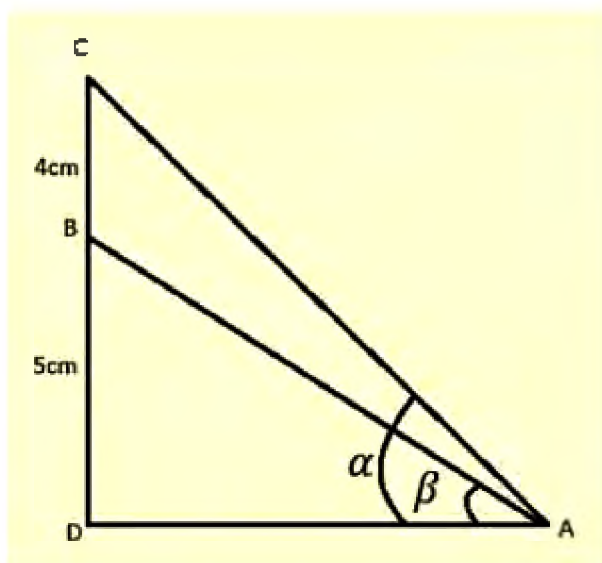
$$50(\sqrt{3}-1)\sqrt{3} = 50(3-\sqrt{3}) \frac{50(\sqrt{3}-1)}{\sqrt{3}} = \frac{50}{3}(3-\sqrt{3})$$

$$\text{Height of the building (pole)} = 50(3-\sqrt{3}) \frac{50}{3}(3-\sqrt{3})$$

$$\text{Distance between the pole and tower} = 50\sqrt{3} \text{ m} \frac{50}{\sqrt{3}} \text{ m}$$

**Q47.** The horizontal distance between two trees of different heights is 60 m. the angles of depression of the top of the first tree when seen from the top of the second tree is  $45^\circ$ . If the height of the second tree is 80 m, find the height of the first tree.

**Soln:**



Distance between the trees = 60 m [BD]

Height of second tree = 80 m [CD]

Let height of the first tree = 'h' m [AB]

Angle of depression from second tree top to first tree top  $\alpha = 45^\circ$

The above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

Draw  $CX \perp AB$

$$CX = BD = 60\text{m}$$

$$XB = CD = AB - AX$$

$$\tan \alpha = \frac{AX}{CX} \tan \alpha = \frac{AX}{60} \tan 45^\circ = \frac{AX}{60}$$

$$AX = 60 \text{ m}$$

$$XB = CD = AB - AX$$

$$= 80 - 60$$

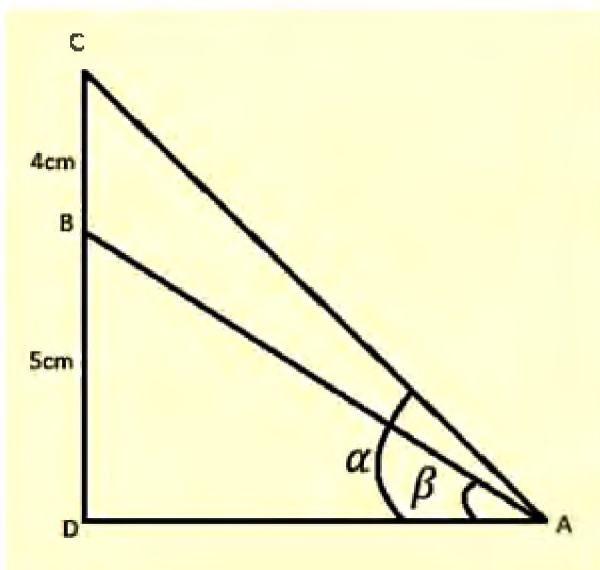
$$= 20 \text{ m}$$

Height of the second tree = 80 m

Height of the first tree = 20 m

**Q48.** A flag staff stands on the top of a 5 m high tower. From a point on the ground, angle of elevation of the top of the flag staff is  $60^\circ$  and from the same point, the angle of elevation of the top of the tower is  $45^\circ$ . Find the height of the flag staff.

**Soln:**



Height of tower =  $AB = 5$



Height of flag staff BC = 'h' m

Angle of elevation of top of flagstaff  $\alpha = 60^\circ$

Angle of elevation of bottom of flagstaff  $\beta = 45^\circ$

The above data is represented in form of figure as shown

In  $\triangle ABC$

$$\tan \beta = \frac{AB}{DA} \tan \beta = \frac{AB}{DA} \tan 45^\circ = \frac{AB}{DA} \tan 45^\circ = \frac{5}{DA}$$

$$DA = 5 \text{ cm}$$

In  $\triangle ADC$

$$\tan \alpha = \frac{AC}{DA} \tan \alpha = \frac{AC}{DA} \tan 60^\circ = \frac{AB+BC}{DA} \tan 60^\circ = \frac{h+5}{5} \tan 60^\circ = \frac{h+5}{5} \sqrt{3}$$

$$\sqrt{3} = \frac{h+5}{5}$$

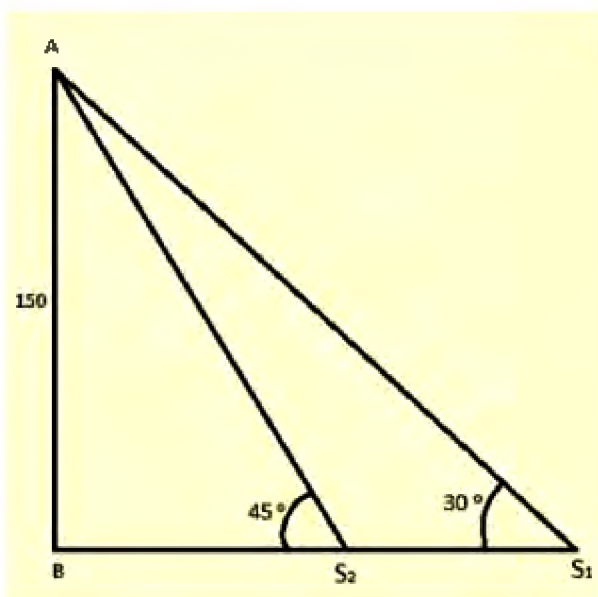
$$h+5 = 5\sqrt{3}$$

$$h = 5(\sqrt{3}-1) = 3.65 \text{ mts}$$

height of the flagstaff = 3.65 mts

**Q50.** As observed from the top of a 150m tall light house, the angles of depression of two ships approaching it are  $30^\circ$  and  $45^\circ$ . If one ship is directly behind the other, find the distance between the two ships.

**Soln:**



Height of the light house AB = 150 meters

Let  $S_1$  and  $S_2$  be two ships approaching each other

Angle of depression of  $S_1, \alpha = 30^\circ$

Angle of depression of  $S_2, \beta = 45^\circ$

Distance between ships =  $S_1S_2$

The above data is represented in the form of figure as shown

In  $\triangle ABS_2$

$$\tan \beta = \frac{AB}{BS_2} \tan \beta = \frac{AB}{BS_2} \tan 45^\circ = \frac{150}{BS_2} \tan 45^\circ = \frac{150}{BS_2} \quad BS_2 = 150 \text{ m}$$

In  $\triangle ABS_1$

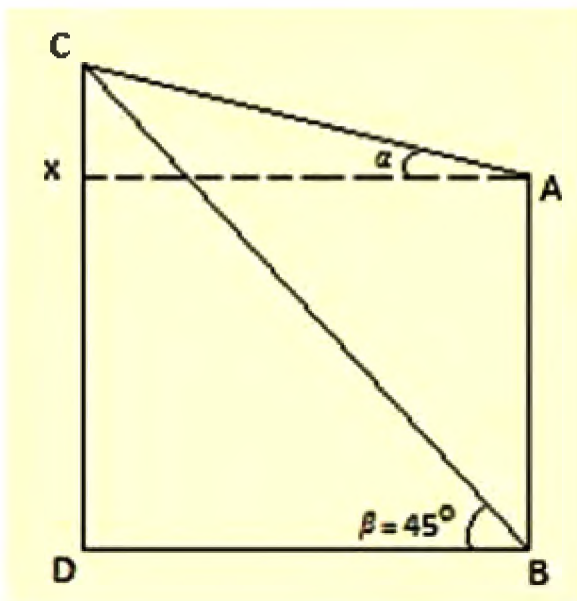
$$\tan \alpha = \frac{AB}{BS_1} \tan \alpha = \frac{AB}{BS_1} \tan 30^\circ = \frac{150}{BS_1} \tan 30^\circ = \frac{150}{BS_1} \quad BS_1 = 150\sqrt{3}$$

$$S_1S_2 = BS_1 - BS_2 = 150(\sqrt{3} - 1) \text{ meters}$$

$$\text{Distance between ships} = 150(\sqrt{3} - 1) \text{ meters}$$

**Q51.** The angle of elevation of the top of a rock from the top and foot of a 100 m high tower are respectively  $30^\circ$  and  $45^\circ$ . Find the height of the rock

**Soln:**



Height of the tower  $AB = 100 \text{ m}$

Height of rock CD = 'h' m

Angle of elevation of the top of rock from top of the tower  $\alpha = 30^{\text{circ}}$   $\alpha = 30^{\text{circ}}$

Angle of elevation of the top of rock from bottom of tower  $\beta = 60^{\text{circ}}$   $\beta = 60^{\text{circ}}$

The above data is represented in the form of figure as shown

Draw  $AX \perp CD$   $AX \perp CD$

$XD = AB = 100\text{m}$

$XA = DB$

In  $\triangle CXA$   $\triangle CXA$

$$\tan \alpha = \frac{CX}{AX} \tan \alpha = \frac{CX}{AX} \tan 30^\circ = \frac{CX}{DB} \tan 30^\circ = \frac{CX}{DB}$$

$$DB = CX \times \sqrt{3} \quad CX \times \sqrt{3} \quad \text{--- (1)}$$

In  $\triangle CBD$   $\triangle CBD$

$$\tan \beta = \frac{CD}{DB} = \frac{100 + CX}{DB} \tan \beta = \frac{CD}{DB} = \frac{100 + CX}{DB} \tan 45^\circ = \frac{100 + CX}{DB} \tan 45^\circ = \frac{100 + CX}{DB}$$

$$DB = 100 + CX \quad \text{--- (2)}$$

From (1) and (2)

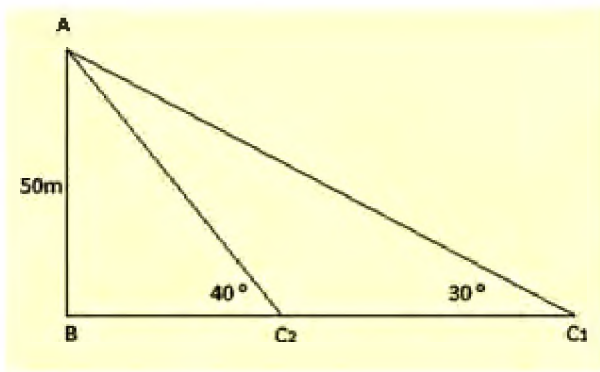
$$100 + CX = CX \sqrt{3} \quad 100 + CX = CX \sqrt{3} \quad 100 = CX(\sqrt{3} - 1) \quad 100 = CX(\sqrt{3} - 1) \\ CX = \frac{100}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad CX = 50(\sqrt{3} + 1) \quad CX = 50(\sqrt{3} + 1)$$

$$\text{Height of the hill} = 100 + 50(\sqrt{3} + 1) \quad 100 + 50(\sqrt{3} + 1)$$

$$= 150(3 + \sqrt{3}) \quad 150(3 + \sqrt{3}) \text{ meters}$$

**Q52.** A straight highway leads to the foot of a tower of height 50m. From the top of the tower, the angles of depression of two cars standing on the highway are  $30^\circ$  and  $60^\circ$  respectively. What is the distance between the two cars and how far is each car from the tower?

**Soln:**



Height of the tower  $AB = 50\text{m}$

$C_1$  and  $C_2$  be two cars

Angles of depression of  $C_1$  from top of the tower  $\alpha = 30^\circ$

Angles of depression of  $C_2$  from top of the tower  $\beta = 60^\circ$

Distance between cars  $C_1$  and  $C_2$

The above data is represented in form of figure as shown

In  $\triangle ABC_2$

$$\tan \beta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC_2} \tan 60^\circ = \frac{AB}{BC_2} \tan 60^\circ = 50 BC_2$$

$$\tan 60^\circ = \frac{50}{BC_2} \quad BC_2 = 50\sqrt{3} \quad BC_2 = \frac{50}{\sqrt{3}}$$

In  $\triangle ABC_1$

$$\tan \alpha = \frac{AB}{BC_1} \tan \alpha = \frac{AB}{BC_1} \tan 30^\circ = 50 BC_1 \tan 30^\circ = \frac{50}{BC_1} \quad BC_1 = 50\sqrt{3} \quad BC_1 = \frac{50}{\sqrt{3}}$$

$$C_1 \text{ and } C_2 = BC_1 - BC_2 \quad C_1 \text{ and } C_2 = BC_1 - BC_2 \quad 50\sqrt{3} - 50\sqrt{3} \quad 50\sqrt{3} - \frac{50}{\sqrt{3}} \quad 50(3 - \frac{1}{\sqrt{3}})$$

$$50 \left( \frac{3 - \frac{1}{\sqrt{3}}}{\sqrt{3}} \right)$$

$$100\sqrt{3} = 100\sqrt{3} \frac{100}{\sqrt{3}} = \frac{100}{3} \sqrt{3} \text{ meters.}$$

$$\text{Distance between cars } C_1 \text{ and } C_2 = 100\sqrt{3} \quad C_1 \text{ and } C_2 = \frac{100}{3} \sqrt{3}$$

$$\text{Distance of car1 from tower} = 50\sqrt{3} \quad 50\sqrt{3} \text{ meters}$$

$$\text{Distance of car2 from tower} = 50\sqrt{3} \quad \frac{50}{\sqrt{3}} \text{ meters}$$

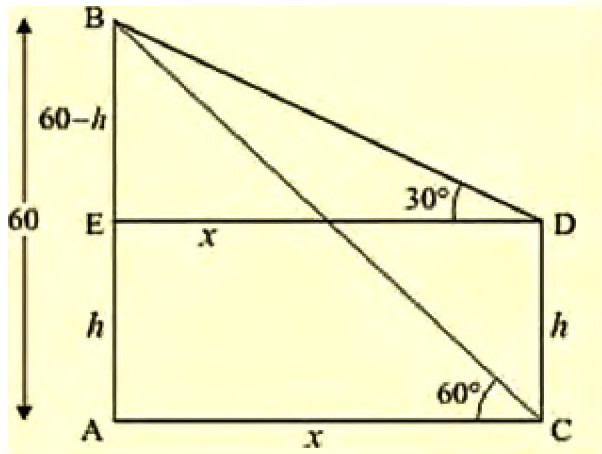
**Q53.** From the top of a building  $AB = 60\text{m}$ , the angles of depression of the top and bottom of a vertical lamp post  $CD$  are observed to be  $30^\circ$  and  $45^\circ$  respectively. Find

(i) The horizontal distance between  $AB$  and  $CD$

(ii) The height of the lamp post

(iii) The difference between the heights of the building and the lamp post.

**Soln:**



Height of building  $AB = 60\text{m}$

Height of lamp post  $CD = 'h' \text{ m}$

Angle of depression of top of lamp post from top of building  $\alpha = 30^\circ$

Angle of depression of bottom of lamp post from top of building  $\beta = 45^\circ$

The above data is represented in form of figure

Draw  $DX \perp AB$ ,  $DX \perp AB$ ,  $CX = AC$ ,  $CD = AX$

In  $\triangle BDX$

Then  $\tan \alpha = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BX}{DX}$

$$\tan 30^\circ = \frac{60 - CD}{AC} \quad \tan 30^\circ = \frac{60 - h}{AC} \quad \frac{1}{\sqrt{3}} = \frac{60 - h}{AC}$$

$$AC = (60 - h)\sqrt{3} \text{m} \quad (1)$$

In  $\triangle BCA$

$$\tan \beta = \frac{AB}{AC} \tan \beta = \frac{AB}{AC} \tan 60^\circ = \frac{60}{AC} \tan 60^\circ = \frac{60}{AC}$$

$$AC = 60\sqrt{3} = 20\sqrt{3}m \frac{60}{\sqrt{3}} = 20\sqrt{3}m \text{ --- (2)}$$

From (1) and (2)

$$(60-h)\sqrt{3} = 20\sqrt{3}(60-h) \sqrt{3} = 20\sqrt{3}$$

$$60 - h = 20$$

$$h = 40m$$

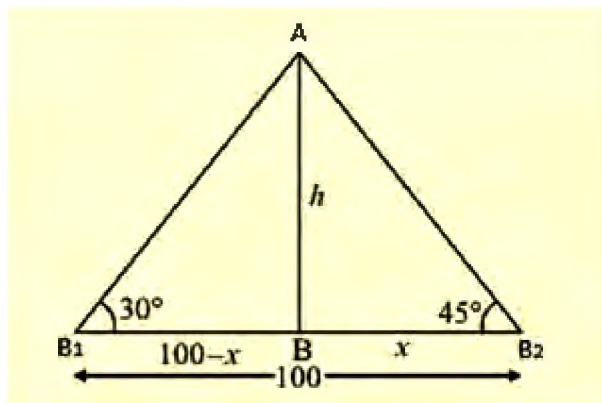
Height of the lamp post = 40m

$$\text{Distance between lamp post building AC} = 20\sqrt{3}m$$

Difference between heights of building and lamp post BX = 60 - h  $\Rightarrow$  60 - 40 = 20m.

**Q54. Two boats approach a light house in mid sea from opposite directions. The angles of elevation of the top of the light house from two boats are  $30^\circ$  and  $45^\circ$  respectively. If the distance between the two boats is 100m, find the height of the light house.**

**Soln:**



let B<sub>1</sub> be boat 1 and B<sub>2</sub> be boat 2

Height of light house = 'h' m = AB

Distance between B<sub>1</sub>B<sub>2</sub> = 100m

Angle of elevation of A from B<sub>1</sub>  $\alpha = 30^\circ$

Angle of elevation of B from B<sub>2</sub>  $\beta = 45^\circ$

The above data is represented in the form of figure as shown

Here In  $\triangle ABB_1 \triangle ABB_1$

$$\tan 30^\circ = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BB_1} \tan 30^\circ = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BB_1}$$

$$B_1B = AB\sqrt{3} = h\sqrt{3} \quad B_1B = AB\sqrt{3} = h\sqrt{3} \quad \text{--- (1)}$$

In  $\triangle ABB_2 \triangle ABB_2$

$$\tan 45^\circ = \frac{AB}{BB_2} \tan 45^\circ = \frac{AB}{BB_2}$$

$$h = BB_2 \quad \text{--- (2)}$$

Adding (1) and (2)

$$B_1B_2 = h(\sqrt{3} + 1) \quad B_1B_2 = h(\sqrt{3} + 1)$$

$$h = B_1B_2 \sqrt{3} + 1 \quad \frac{B_1B_2}{\sqrt{3} + 1}$$

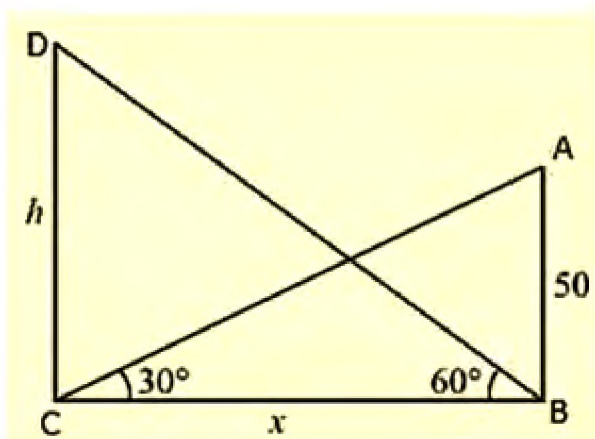
$$h = 100\sqrt{3} + 1 \times \sqrt{3} - 1 \sqrt{3} - 1 \quad \frac{100}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$h = 100(\sqrt{3} - 1) = 50(\sqrt{3} - 1) \frac{100(\sqrt{3} - 1)}{2} = 50(\sqrt{3} - 1)$$

$$\text{Height of the light house} = 50(\sqrt{3} - 1) \quad 50(\sqrt{3} - 1)$$

**Q55.** The angle of elevation of the top of a hill at the foot of a tower is  $60^\circ$  and the angle of elevation of the top of the tower from the foot of the hill is  $30^\circ$ . If tower is 50 m high, what is the height of the hill?

**Soln:**



Height of tower AB = 50m

Height of hill  $CD = 'h'$  m.

Angle of elevation of top of the hill from foot of the tower =  $\alpha = 60^\circ$

Angle of elevation of top of the tower from foot of the hill =  $\beta = 30^\circ$

The above data is represented in the form of figure as shown

From figure

In  $\triangle ABC$

$$\tan 30^\circ = \frac{AB}{BC} \quad \tan 30^\circ = \frac{AB}{BC} \cdot \frac{1}{\sqrt{3}} = \frac{50}{BC} \quad \frac{1}{\sqrt{3}} = \frac{50}{BC}$$

$$BC = 50\sqrt{3}$$

In  $\triangle BCD$

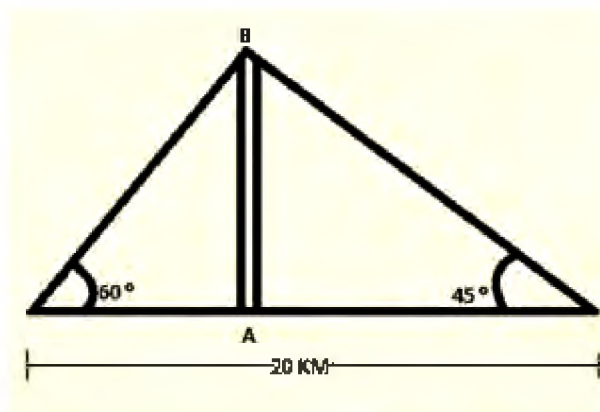
$$\tan 60^\circ = \frac{CD}{BC} \quad \tan 60^\circ = \frac{CD}{50\sqrt{3}} \quad \sqrt{3} = \frac{CD}{50\sqrt{3}} \quad CD = 50\sqrt{3} \cdot \sqrt{3} = \frac{CD}{50\sqrt{3}}$$

$$CD = 150\text{m}$$

Height of hill = 150m.

**Q56. A fire in a building B is reported on telephone to two fire station P and Q 20 km apart from each other on a straight road. P observes that the fire is at an angle of  $60^\circ$  to the road and Q observes that it is at an angle of  $45^\circ$  to the road. Which station sends its team and how much will this team have to travel?**

**Soln:**



Let AB be the building

Angle of elevation from point P [fire station 1]  $\alpha = 60^\circ$



Angle of elevation from point Q [fire station 1]  $\beta = 45^\circ$

Distance between fire station PQ = 20 km

The above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan \alpha = \frac{AB}{AP} \quad \tan 60^\circ = \frac{AB}{AP}$$

$$AP = \frac{AB}{\sqrt{3}} \quad \text{--- (1)}$$

$$\tan \beta = \frac{AB}{AQ} \quad \tan 45^\circ = \frac{AB}{AQ}$$

$$AQ = AB \quad \text{--- (2)}$$

Adding (1) + (2)

$$AP + AQ = \frac{AB}{\sqrt{3}} + AB$$

$$AB \left( \frac{1}{\sqrt{3}} + 1 \right)$$

$$20 = AB \left( \frac{1}{\sqrt{3}} + 1 \right)$$

$$AB = \frac{20\sqrt{3}}{\sqrt{3}+1}$$

$$AB = \frac{20\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$AB = 10(3-\sqrt{3})$$

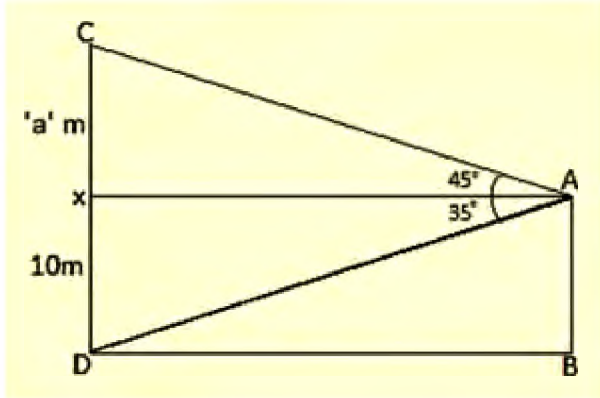
$$AQ = AB = 10(3-\sqrt{3}) = 12.64 \text{ km}$$

$$AP = \frac{AB}{\sqrt{3}} = \frac{10(3-\sqrt{3})}{\sqrt{3}} = 7.32 \text{ km}$$

Station 1 should send its team and they have to travel 7.32 km

**Q57.** A man on the deck of a ship is 10 m above the water level. He observes that the angle of elevation of the top of a cliff is  $45^\circ$  and the angle of depression of the base is  $30^\circ$ . Calculate the distance of the cliff from the ship and the height of the cliff.

**Soln:**



Height of the ship from water level = 10m = AB

Angle of elevation of top of the cliff  $\alpha = 45^\circ$

Angle of depression of bottom of the cliff  $\beta = 30^\circ$

Height of the cliff CD = 'h' m

Distance of the ship from foot of the tower cliff.

Height of cliff above ship be 'a' m

Then height of cliff = DX + XC = (10 + a) m

The above data is represented in form of figure as shown

In right angle triangle, if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$   $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan 45^\circ = \frac{CX}{AX} \quad \tan 45^\circ = \frac{a}{AX} \quad 1 = \frac{a}{AX}$$

$$AX = 'a' \text{ m}$$

$$\tan 30^\circ = \frac{XD}{AX} \quad \tan 30^\circ = \frac{10}{AX} \quad \frac{1}{\sqrt{3}} = \frac{10}{AX}$$

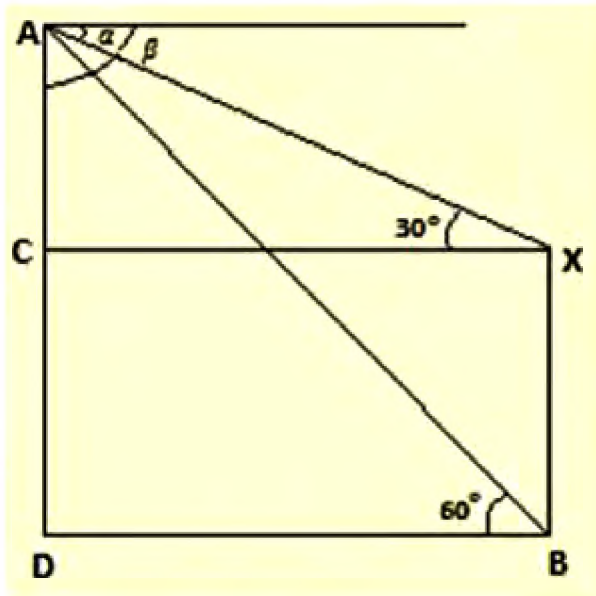
$$AX = 10\sqrt{3}$$

$$\therefore a = 10\sqrt{3} \text{ m} \therefore a = 10\sqrt{3} \text{ m}$$

$$\text{Height of the cliff} = 10 + 10\sqrt{3} \text{ m} = 10(\sqrt{3} + 1) \text{ m} = 10(\sqrt{3} + 1)$$

**Q59.** There are two temples one on each bank of a river opposite to one other. One temple is 50m high. From the top of this temple, the angles of depression of the top and foot of the other temple are  $30^\circ$  and  $60^\circ$  respectively. Find the width of the river and the height of the other temple.

**Soln:**



Height of the temple 1 (AB) = 50m

Angle of depression of top of temple 2,  $\alpha = 30^\circ$

Angle of depression of bottom of temple 2,  $\beta = 60^\circ$

Height of the temple 2 (CD) = 'h' m

Width of the river = BD = 'x' m

The above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

$$\text{Then } \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

Here BD = CX, CD = h

$$\tan \alpha = \frac{AX}{CX} \quad \tan 30^\circ = \frac{AX}{CX} \quad \tan 30^\circ = \frac{AX}{CX}$$

$$CX = \frac{AX}{\tan 30^\circ} = \frac{50}{\frac{1}{\sqrt{3}}} = 50\sqrt{3} \text{ m}$$

$$\tan \beta = \frac{AB}{BD} \tan \beta = \frac{AB}{BD} \tan 60^\circ = 50 \times \tan 60^\circ = \frac{50}{CX}$$

$$CX = 50\sqrt{3} \frac{50}{\sqrt{3}}$$

$$AX(\sqrt{3}) = 50\sqrt{3} \quad AX(\sqrt{3}) = \frac{50}{\sqrt{3}} \quad AX = 50\sqrt{3} \quad AX = \frac{50}{3}m$$

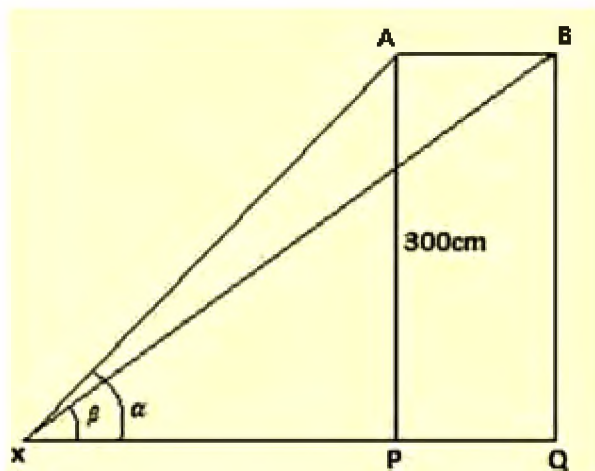
$$CD = XB = AB - AX = 50 - 50\sqrt{3} = 100\sqrt{3} \quad 50 - \frac{50}{3} = \frac{100}{3}m$$

$$\text{Width of river} = 50\sqrt{3} \frac{50}{\sqrt{3}}$$

$$\text{Height of temple 2} = 100\sqrt{3} \frac{100}{3}m$$

**Q60.** The angle of elevation of an airplane from a point on the ground is  $45^\circ$ . After a flight of 15 seconds, the elevation changes to  $30^\circ$ . If the airplane is flying at a height of 3000 meters, find the speed of the airplane.

**Soln:**



Let the airplane travelled from A to B in 15 sec

Angle of elevation of point A  $\alpha = 45^\circ$

Angle of elevation of point B  $\beta = 30^\circ$

Height of the airplane from ground = 3000 meters

= AP = BQ

Distance travelled in 15 seconds = AB = PQ

Velocity (or) speed = distance travelled time

If the above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

$$\text{Then } \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan \alpha = \frac{AP}{XP} \quad \tan 45^\circ = \frac{3000}{XP} \quad \tan 45^\circ = \frac{3000}{XP}$$

$$XP = 3000 \text{ m}$$

$$\tan \beta = \frac{BQ}{XQ} \quad \tan 30^\circ = \frac{3000}{XQ} \quad \tan 30^\circ = \frac{3000}{XQ}$$

$$XQ = 3000\sqrt{3}$$

$$PQ = XQ - XP = 3000(\sqrt{3} - 1) \text{ m}$$

$$\text{Speed} = \frac{PQ}{\text{time}} = \frac{3000(\sqrt{3} - 1)}{15} = 200(\sqrt{3} - 1) = 200(\sqrt{3} - 1)$$

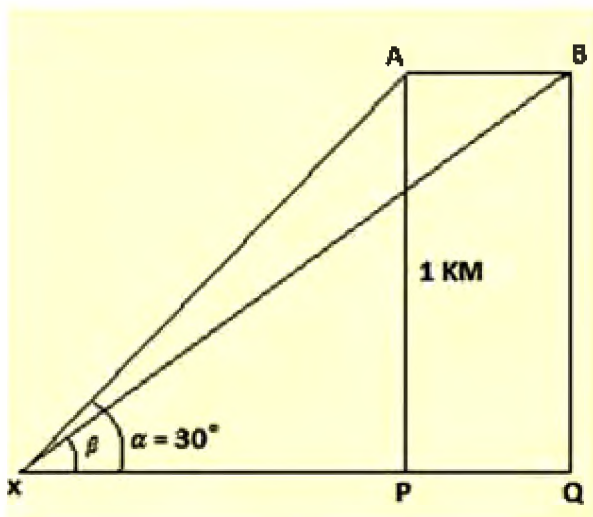
$$2000 \times 0.732 = 146.4$$

$$146.4 \text{ m/sec}$$

$$\text{Speed of the airplane} = 146.4 \text{ m/sec}$$

**Q61.** An airplane flying horizontally 1 km above the ground is observed at an elevation of  $60^\circ$ . After 10 seconds, its elevation is observed to be  $30^\circ$ . Find the speed of the airplane in km/hr.

**Soln:**



Let airplane travelled from A to B in 10 seconds

Angle of elevation of point A  $\alpha = 60^\circ$

Angle of elevation of point B  $\beta = 30^\circ$

Height of the airplane from ground = 1 km = AP = BQ

Distance travelled in 10 seconds = AB = PQ

The above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan \alpha = \frac{AP}{PX} \tan 60^\circ = \frac{1}{PX} \tan 60^\circ = \frac{1}{PX}$$

$$PX = \frac{1}{\sqrt{3}} \text{ km}$$

$$\tan \beta = \frac{BQ}{XQ} \tan 30^\circ = \frac{1}{XQ} \tan 30^\circ = \frac{1}{XQ}$$

$$XQ = \sqrt{3}$$

$$PQ = XQ - PX = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3 - 1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\text{Speed} = \frac{PQ}{\text{time}} = \frac{\frac{2}{\sqrt{3}}}{\frac{60}{60}} = \frac{2}{\sqrt{3}} \times 60 = 40\sqrt{3}$$

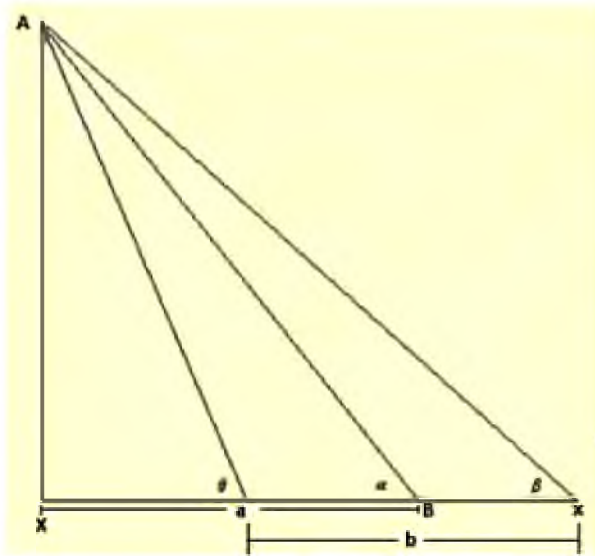
$$= 40\sqrt{3}$$

$$\text{Speed of the airplane} = 40\sqrt{3}$$

**Q62.** A tree standing on a horizontal plane is leaning towards east. At two points situated at distance  $a$  and  $b$  exactly due west on it, the angles of elevation of the top are respectively  $\alpha$  and  $\beta$ . Prove that the height of the top from the ground is

$$(b-a) \frac{\tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$$

**Soln:**



AB be the tree leaning east

From distance 'a' m from tree, angle of elevation be  $\alpha$  at point P.

From distance 'b' m from tree, angle of elevation be  $\beta$  at point Q.

The above data is represented in the form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$   $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

Draw  $AX \perp QB$   $AX \perp QB$  let  $BX = 'a'$  m

$$\tan \alpha = \frac{AX}{PX} \quad \tan \alpha = \frac{AX}{x+a} \quad \cot \alpha = \frac{x+a}{AX} \quad \cot \alpha = \frac{x+a}{AX}$$

$$x+a = AX \cot \alpha \quad AX \cot \alpha \quad \text{--- (1)}$$

$$\tan \beta = \frac{AX}{QX} \quad \tan \beta = \frac{AX}{x+b} \quad \cot \beta = \frac{x+b}{AX} \quad \cot \beta = \frac{x+b}{AX}$$

$$x+b = AX \cot \beta \quad AX \cot \beta \quad \text{--- (2)}$$

subtracting (1) from (2)

$$(x+b) - (x+a) = AX \cot \beta - AX \cot \alpha \quad AX \cot \beta - AX \cot \alpha$$

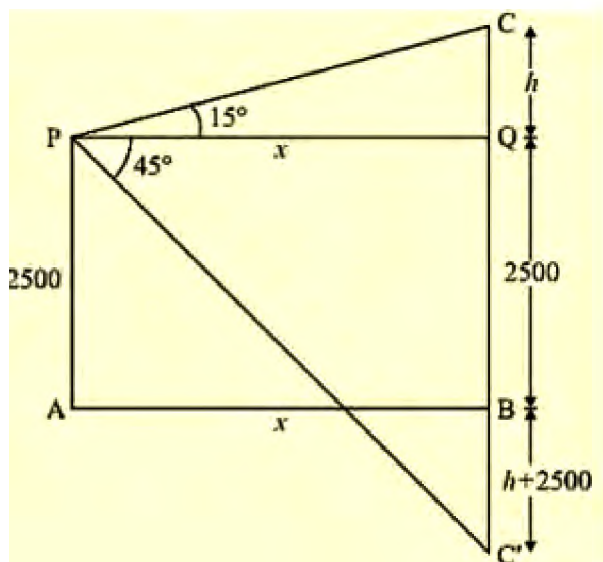
$$b - a = AX [\cot \beta - \cot \alpha] \quad AX \left[ \frac{\tan \alpha - \tan \beta}{\tan \alpha \tan \beta} \right]$$

$$AX = \frac{(b-a) \tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$$

$$\therefore \therefore \text{Height of top from ground} = (b-a)\tan\alpha.\tan\beta\tan\alpha-\tan\beta \frac{(b-a)\tan\alpha.\tan\beta}{\tan\alpha-\tan\beta}$$

**Q63.**The angle of elevation of a stationary cloud from a point 2500 m above a lake is  $15^\circ$  and the angle of depression of its reflection in the lake is  $45^\circ$ . What is the height of the cloud above the lake level?

**Soln:**



Let the cloud be at height PQ as represented from lake level

From point x, 2500 meters above the lake angle of elevation of top of cloud  $\alpha=15^\circ$

Angle of depression of shadow reflection in water  $\beta=45^\circ$

Here above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

$$\text{Then } \tan\Theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \tan\Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan 15^\circ = \frac{AQ}{AY} \tan 15^\circ = \frac{h}{AY} 0.268 = \frac{h}{AY} 0.268 = \frac{h}{AY}$$

$$AY = \frac{h}{0.268} \quad (1)$$

$$\tan 45^\circ = \frac{AB'}{AY} \tan 45^\circ = \frac{AP+PQ'}{AY} \tan 45^\circ = \frac{AP+PQ'}{AY}$$

$$AY = x + (h + x)$$

$$AY = h + 2x \quad (2)$$



From (1) and (2)

$$h \tan 0.268 = h + 2x \frac{h}{0.268} = h + 2x$$

$$3.131h - h = 2 \times 2500$$

$$h = 1830.8312$$

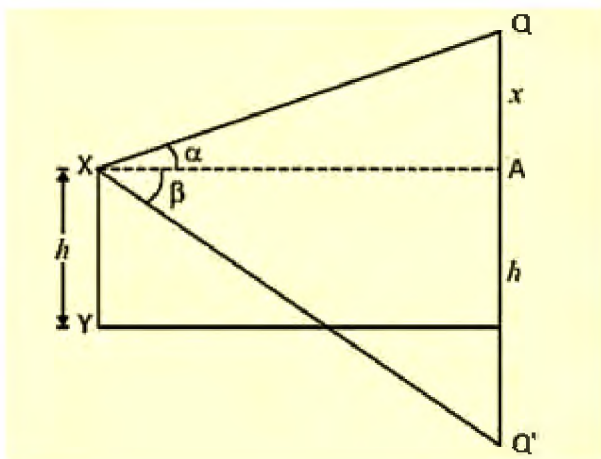
Height of the cloud above lake =  $h + x$

$$= 1830.8312 + 2500$$

$$= 4330.8312 \text{ m}$$

**Q64.** If the angle of elevation of a cloud from a point  $h$  meters above a lake is  $\alpha$  and the angle of depression of its reflection in the lake be  $\beta$ , prove that the distance of the cloud from the point of observation is  $\frac{2h \sec \alpha}{\tan \beta - \tan \alpha}$

**Soln:**



Let  $x$  be point ' $h$ ' meters above lake

Angle of elevation of cloud from  $x = \alpha$

Angle of depression of cloud reflection in lake =  $\beta$

Height of the cloud from lake =  $PQ$

$PQ'$  be the reflection then  $PQ' = PQ$

Draw  $XA + PQ$ ,  $AQ = 'x' \text{ m}$ ,  $AP = 'h' \text{ m}$

Distance of cloud from point of observation is  $XQ$ .

The above data is represented in form of figure as shown

In  $\triangle A Q X$

$$\tan \alpha = \frac{AQ}{AX}$$

$$\tan \alpha = \frac{x}{AX} \quad \text{--- (a)}$$

In  $\triangle A X Q'$

$$\tan \beta = \frac{AQ'}{AX}$$

$$\tan \beta = \frac{h+x+h}{AX} \quad \text{--- (2)}$$

Subtracting (1) from (2)

$$\tan \beta - \tan \alpha = \frac{2h}{AX} \quad AX = \frac{2h}{\tan \beta - \tan \alpha}$$

In  $\triangle A Q X$

$$\cos \alpha = \frac{AX}{XQ}$$

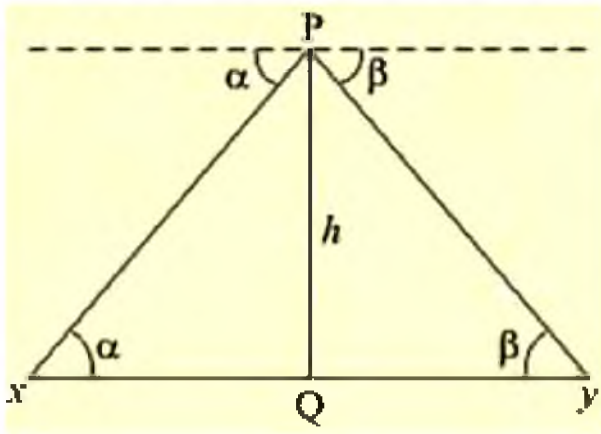
$$XQ = AX \sec \alpha$$

$$XQ = \frac{2h \sec \alpha}{\tan \beta - \tan \alpha}$$

$$\therefore \therefore \text{distance of cloud from point of observation} = \frac{2h \sec \alpha}{\tan \beta - \tan \alpha}$$

**Q65.** From an airplane vertically above a straight horizontal road, the angle of depression of two consecutive mile stones on opposite side of the airplane are observed to be  $\alpha$  and  $\beta$ . Show that the height in miles of airplane above the road is given by  $\frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$ .

**Soln:**



let PQ be the height of airplane from ground x and y be two mile stones on opposite side of the airplane  $xy = 1$  mile

Angle of depression of x from p =  $\alpha$

Angle of depression of y from p =  $\beta$

The above data is represented in the form of figure as shown

In right angle triangle if one of the included angle is  $\Theta$

Then  $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$   $\tan \Theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

In  $\triangle PXQ$

$$\tan \alpha = \frac{PQ}{XQ} \quad \tan \alpha = \frac{PQ}{XQ}$$

$$XQ = \frac{PQ}{\tan \alpha} \quad XQ = \frac{PQ}{\tan \alpha} \quad (1)$$

In  $\triangle XQY$

$$\tan \beta = \frac{PQ}{QY} \quad \tan \beta = \frac{PQ}{QY}$$

$$QY = \frac{PQ}{\tan \beta} \quad QY = \frac{PQ}{\tan \beta} \quad (2)$$

Adding (1) and (2)

$$XQ + QY = \frac{PQ}{\tan \alpha} + \frac{PQ}{\tan \beta}$$

$$1 = PQ \left( \frac{1}{\tan \alpha} + \frac{1}{\tan \beta} \right)$$

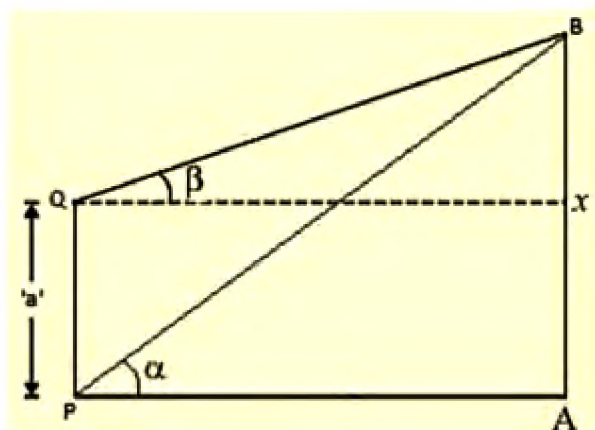
$$1 = PQ \left( \frac{\tan \beta + \tan \alpha}{\tan \alpha \tan \beta} \right)$$

$$PQ = \tan \alpha \cdot \tan \beta \tan \beta + \tan \alpha \frac{\tan \alpha \cdot \tan \beta}{\tan \beta + \tan \alpha}$$

$$\text{Height of airplane} = \tan \alpha \cdot \tan \beta \tan \beta + \tan \alpha \frac{\tan \alpha \cdot \tan \beta}{\tan \beta + \tan \alpha} \text{ miles.}$$

**Q66.** PQ is a post given height 'a' m and AB is a tower at same distance. If  $\alpha$  and  $\beta$  are the angles of elevation of B, the top of the tower, at P and Q respectively. Find the height of the tower and its distance from the post.

**Soln:**



PQ is part height = 'a' m AB is tower height

Angle of elevation of B from P =  $\alpha$

Angle of elevation of B from Q =  $\beta$

The above data is represented in form of figure as shown

In right angle triangle if one of the included angle is  $\theta$

$$\text{Then } \tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$QX \perp AB, PQ = a, QX \perp AB, PQ = a$$

In  $\triangle BQX$

$$\tan \beta = \frac{BX}{QX} \quad \tan \beta = \frac{AB - AX}{QX} \quad \tan \beta = \frac{AB - a}{QX}$$

$$\tan \beta = \frac{AB - a}{QX} \quad \tan \beta = \frac{AB - a}{QX} \quad \text{--- (1)}$$

In  $\triangle BPA$

$$\tan \alpha = \frac{AB}{AP} \quad \tan \alpha = \frac{AB}{AP}$$

$$\tan \alpha = \frac{AB}{QX} \quad \tan \alpha = \frac{AB}{QX} \quad \text{--- (2)}$$

Dividing (1) by (2)

$$\frac{\tan \beta}{\tan \alpha} = \frac{AB - a}{AB} = 1 - \frac{a}{AB} \quad \frac{\tan \beta}{\tan \alpha} = \frac{AB - a}{AB} = 1 - \frac{a}{AB} \quad a = AB(1 - \frac{\tan \beta}{\tan \alpha}) = AB(1 - \frac{\tan \beta}{\tan \alpha})$$

$$\frac{a}{AB} = 1 - \frac{\tan \beta}{\tan \alpha} = \frac{\tan \alpha - \tan \beta}{\tan \alpha}$$

$$AB = \frac{a \tan \alpha}{\tan \alpha - \tan \beta}$$

$$QX = AB \tan \alpha = \frac{a \tan \alpha}{\tan \alpha - \tan \beta} \tan \alpha = \frac{a \tan^2 \alpha}{\tan \alpha - \tan \beta}$$

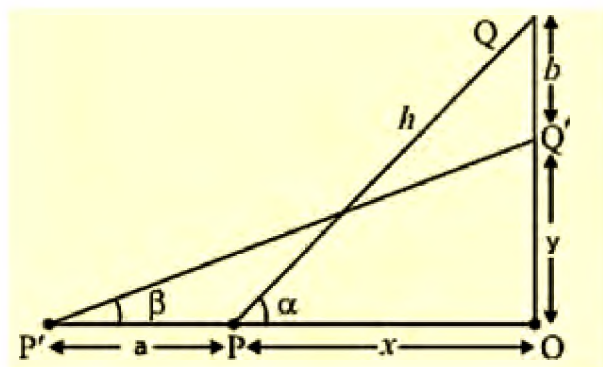
$$\text{Height of tower} = \frac{a \tan \alpha}{\tan \alpha - \tan \beta}$$

$$\text{Distance between post and tower} = \frac{a \tan \alpha}{\tan \alpha - \tan \beta}$$

**Q67.** A ladder rests against a wall at an angle  $\alpha$  to the horizontal. Its foot is pulled away from the wall through a distance  $a$ , so that it slides a distance  $b$  down the wall making an angle  $\beta$  with the horizontal. Show that  $\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$

$$\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$$

**Soln:**



Let  $AB$  be the ladder initially at an inclination  $\alpha$  to ground

When its foot is pulled through distance ' $a$ ' let  $BB' = 'a'$  m and  $AA' = 'b'$  m

New angle of elevation from  $B' = B$  the above data is represented in form of figure as shown

Let  $AP \perp \text{ground}$ ,  $B'P, AB = A'B', AP \perp \text{ground}$ ,  $B'P, AB = A'B'$

$$A'P = x \quad BP = y$$

In  $\triangle ABP$   $\triangle ABP$

$$\sin \alpha = \frac{AP}{AB} \sin \alpha = \frac{AP}{AB}$$

$$\sin \alpha = \frac{x+b}{AB} \sin \alpha = \frac{x+b}{AB} \quad \text{--- (1)}$$

$$\cos \alpha = \frac{BP}{AB} \cos \alpha = \frac{BP}{AB}$$

$$\cos \alpha = \frac{y}{AB} \cos \alpha = \frac{y}{AB} \quad \text{--- (2)}$$

In  $\triangle A'B'P$   $\triangle A'B'P$

$$\sin \beta = \frac{A'P}{A'B'} \sin \beta = \frac{A'P}{A'B'}$$

$$\sin \beta = \frac{x}{AB} \sin \beta = \frac{x}{AB} \quad \text{--- (3)}$$

$$\cos \beta = \frac{B'P}{A'B'} \cos \beta = \frac{B'P}{A'B'}$$

$$\cos \beta = \frac{y+a}{AB} \cos \beta = \frac{y+a}{AB} \quad \text{--- (4)}$$

Subtracting (3) from (1)

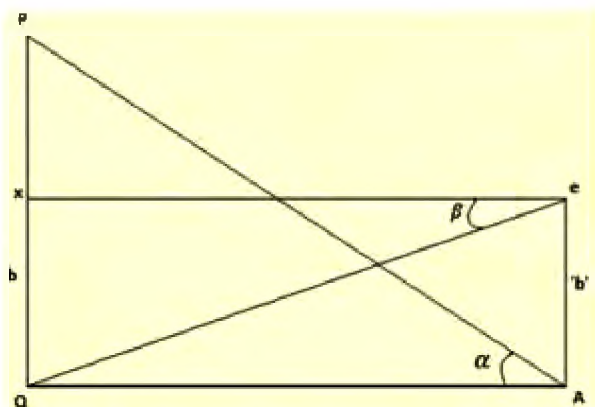
$$\sin \alpha - \sin \beta = \frac{b}{AB} \sin \alpha - \sin \beta = \frac{b}{AB}$$

Subtracting (4) from (2)

$$\cos \beta - \cos \alpha = \frac{a}{AB} \cos \beta - \cos \alpha = \frac{a}{AB} \quad \text{--- (5)}$$

**Q68.** A tower subtends an angle  $\alpha$  at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b meters just above A is  $\beta$ . Prove that the height of the tower is  $b \tan \alpha \cot \beta$ .

**Soln:**



let height of the tower be 'h' m = PQ

Angle of elevation at point A on ground =  $\alpha$

Let B be the point 'b' m above the A

Angle of depression of foot of the tower from B =  $\beta$

The above data is represented in form of figure as shown

Let  $BX \perp P$ ,  $AB = A'B'$ ,  $BX \perp P$ ,  $AB = A'B'$

In  $\triangle PBX$  and  $\triangle PBX$

$$\tan \alpha = \frac{PQ}{BX} \quad \tan \alpha = \frac{PQ}{BX} \quad \text{--- (1)}$$

In  $\triangle QBX$  and  $\triangle QBX$

$$\tan \beta = \frac{QX}{BX} \quad \tan \beta = \frac{QX}{BX} \quad \text{--- (2)}$$

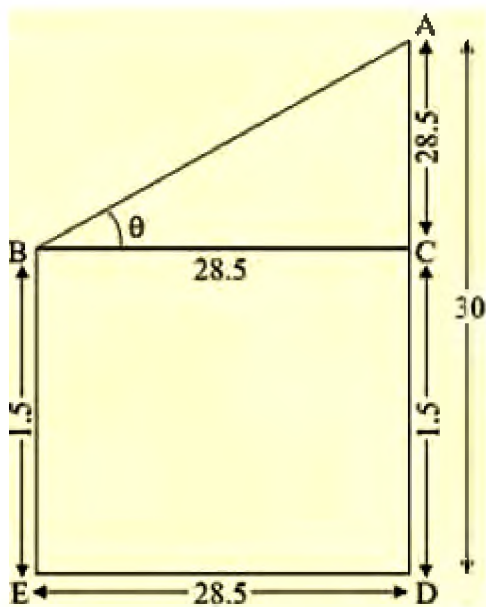
Dividing (1) by (2)

$$\tan \alpha \tan \beta = \frac{PQ}{QX} \quad \frac{\tan \alpha}{\tan \beta} = \frac{PQ}{QX}$$

$$PQ = QX \cdot \tan \alpha \tan \beta = b \tan \alpha \cdot \cot \beta = b \tan \alpha \cdot \cot \beta$$

**Q69.** An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Determine the angle of elevation of the top of the tower from his eye.

**Soln:**



Height of observer = AB = 1.5 m

Height of tower = PQ = 30 m

Height of tower above the observer eye = 30 – 1.5

QX = 28.5m

Distance between tower and observer XB = 28.5m

$\theta$  be angle of elevation of tower top from eye

The above data is represented in the form of figure

$$\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \tan \theta = \frac{QX}{BX} = \frac{28.5}{28.5} = 1$$

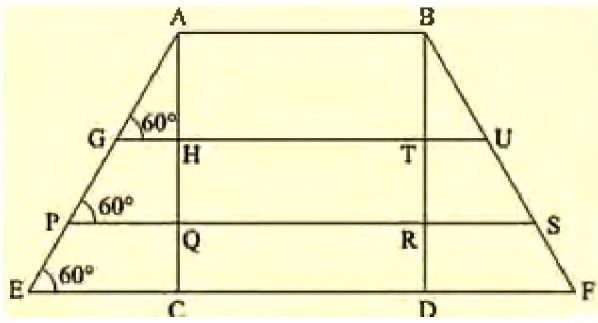
$$\tan \theta = \frac{QX}{BX} = \frac{28.5}{28.5} = 1 \quad \theta = \tan^{-1}(1) = 45^\circ \quad \theta = \tan^{-1}(1) = 45^\circ$$

Angle of elevation =  $45^\circ$

**Q70. A carpenter makes stools for electricians with a square top of side 0.5 m and at a height of 1.5 m above the ground. Also each leg is inclined at an angle of  $60^\circ$  to the ground. Find the length of each leg and also the lengths of two steps to be put at equal distance.**

**Soln:**





let AB be the height of stool = 1.5 m

Let P and Q be equal distance then AP = 0.5. AQ = 1 m

The above data is represented in form of figure as shown

BC = length of leg

$$\sin 60^\circ = \frac{AB}{BC} \sin 60^\circ = \frac{1.5}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1.5}{BC}$$

$$BC = 1.5 \times \frac{2}{\sqrt{3}} = \frac{1.5 \times 2}{\sqrt{3}}$$

Draw  $PX \perp AB$ ,  $QZ \perp AB$ ,  $XY \perp CA$ ,  $ZW \perp CA$

$PX \perp AB$ ,  $QZ \perp AB$ ,  $XY \perp CA$ ,  $ZW \perp CA$

$$\sin 60^\circ = \frac{XY}{XC} \sin 60^\circ = \frac{XY}{XC}$$

$$XC = 0.5 \times \frac{2}{\sqrt{3}} \times \sqrt{3} = 1.1077 \text{ m}$$

$$\frac{\sqrt{3}}{2} \times \frac{1.5}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{1.5}{\sqrt{3}}$$

$$XC = 1.1077 \text{ m}$$

$$\sin 60^\circ = \frac{ZW}{CZ} \sin 60^\circ = \frac{ZW}{CZ}$$

$$CZ = \frac{1.5}{\frac{\sqrt{3}}{2}} = \frac{1.5 \times 2}{\sqrt{3}}$$

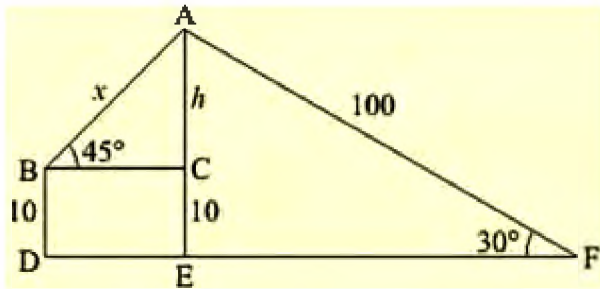
$$CZ = \frac{1.5 \times 2}{\sqrt{3}} = 1.654 \text{ m}$$

$$CZ = 1.654 \text{ m}$$

**Q71.** A boy is standing on the ground and flying a kite with 100m of string at an elevation of  $30^\circ 30'$ . Another boy is standing on the roof of a 10m high building and is

flying his kite at an elevation of  $45^\circ$ . Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the kites meet.

**Soln:**



For boy 1

Length of the string AB = 100 m

Angle made by string with ground =  $\alpha = 30^\circ$

For boy 2

Height of the building CD = 10m

Angle made by string with building top =  $\beta = 45^\circ$

Length of the kite thread of boy 2 if both the kites meet must be 'DB'

The above data is represented in form of figure as shown

Draw  $BX \perp AC, YD \perp BC$

In  $\triangle ABX$

$$\tan 30^\circ = \frac{BX}{AX} \quad \sin 30^\circ = \frac{BX}{AB} \quad \sin 30^\circ = \frac{BX}{100} \quad \frac{1}{2} = \frac{BX}{100}$$

$$BX = 50\text{m}$$

$$BY = BX - XY = 50 - 10 = 40\text{m}$$

In  $\triangle BYD$

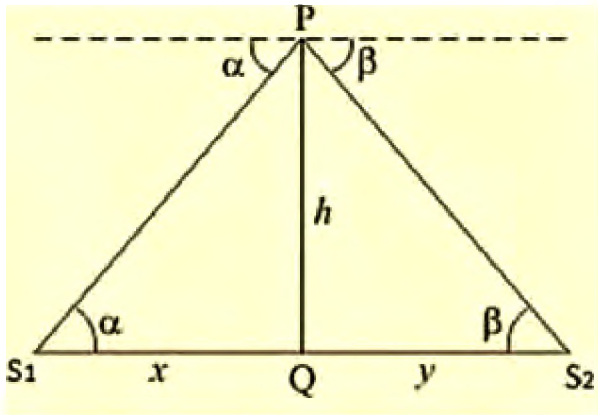
$$\sin 45^\circ = \frac{BY}{BD} \quad \sin 45^\circ = \frac{40}{BD} \quad \frac{1}{\sqrt{2}} = \frac{40}{BD}$$

$$BD = 40\sqrt{2}$$

Length of thread or string of boy 2 =  $40\sqrt{2}$

**Q72. From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be  $\alpha$  and  $\beta$ . If the height of the light house be  $h$  meters and the line joining the ships passes through the foot of the light house, show that the distance  $h(\tan \alpha + \tan \beta)$**

**Soln:**



Height of light house = 'h' meters = AB

$S_1$  and  $S_2$  be two ships on opposite sides of the light house

Angle of depression of  $S_1$  from top of light house =  $\alpha$

Angle of depression of  $S_2$  from top of light house =  $\beta$

Required to prove that

Distance between ships =  $h(\tan \alpha + \tan \beta)$  meters

The above data is represented in the form of figure as shown

In  $\triangle ABS_1$

$$\tan \alpha = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{S_1B} \quad \tan \alpha = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{S_1B}$$

$$S_1B = h \tan \alpha \quad S_1B = \frac{h}{\tan \alpha} \quad \text{--- (1)}$$

In  $\triangle ABS_2$

$$\tan \beta = \frac{AB}{S_2B} \quad \tan \beta = \frac{AB}{S_2B}$$

$$S_2B = h \tan \beta \quad S_2B = \frac{h}{\tan \beta} \quad \text{--- (2)}$$

Adding (1) and (2)

$$\Rightarrow S_1 B + S_2 = h \tan \alpha + h \tan \beta \quad S_1 B + S_2 = \frac{h}{\tan \alpha} + \frac{h}{\tan \beta}$$

$$\Rightarrow S_1 S_2 = h \{ \tan \alpha + \tan \beta \} S_1 S_2 = h \left\{ \frac{1}{\tan \alpha} + \frac{1}{\tan \beta} \right\}$$

$$\Rightarrow h(\tan \alpha + \tan \beta) \tan \alpha \tan \beta \frac{h(\tan \alpha + \tan \beta)}{\tan \alpha \tan \beta}$$

$$\text{Distance between ships} = h(\tan \alpha + \tan \beta) \tan \alpha \tan \beta \frac{h(\tan \alpha + \tan \beta)}{\tan \alpha \tan \beta} \text{ meters}$$