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Problem 1

$$T(n) = \begin{cases} T(n-1) + c & \text{if } n > 0 \\ b & \text{if } n = 0 \end{cases}$$

Cost when $N = 0$: $T(0) = b$

Cost for general case: $T(n) = T(n-1) + c$ (A)

Cost for $n-1$: $T(n-1) = T(n-2) + c$

Substitute $T(n-1)$ in (A)

$T(n) = (T(n-2) + c) + c = T(n-2) + 2c$ (B)

Cost for $n-2$: $T(n-2) = T(n-3) + c$

Substitute $T(n-2)$ in (B)

$T(n) = (T(n-3) + c) + 2c = T(n-3) + 3c$

$\vdots$

Hence, the function is $T(n) = T(n-k) + kc$

Problem 2

$$T(n) = \begin{cases} 3T(n-1) & \text{if } n > 0 \\ c & \text{if } n = 0 \end{cases}$$

$T(n) = \begin{cases} \\ c & \text{if } n=0 \end{cases}$

Cost when $n=0$: $T(0) = c$

Cost for general case: $T(n) = 3T(n-1)$ (A)

Cost for $n-1$: $T(n-1) = 3T(n-2)$

Substitute $T(n-1)$ in (A)

$T(n) = 3(3T(n-2)) = 9T(n-2)$ (B)

Cost for $n-2$: $T(n-2) = 3T(n-3)$

Substitute $T(n-2)$ in (B)

$T(n) = 9(3T(n-3) = 27T(n-3)$

$\vdots$

Hence the function is $T(n) = 3^K T(n-k)$