Chapter 1

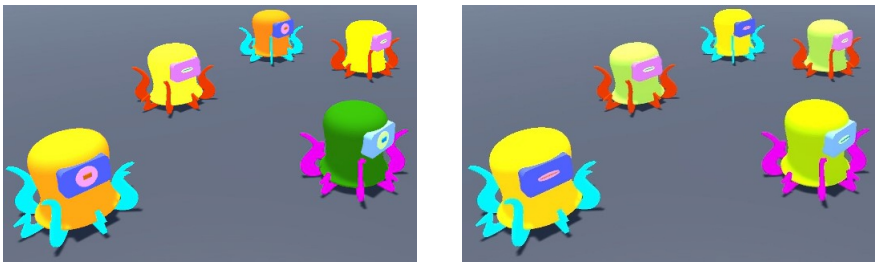
Implementation

This chapter gives an overview of the developed musical multi-robot system. The main goal of the implemented system is to allow for a multi-robot (musical) collective to interact with each other in order to achieve emergent and coordinating/co-operative behaviour—synchronization specifically in our case—with varying degrees of difficulty and certainty in the environment and communication. More specifically, the goal with the design is to enable the robot collective to achieve so-called *harmonic synchronization* within a relatively short time. What is meant by *harmonic synchronization* will be expounded in Subsection 1.1.3. These goals firstly require of the agents/nodes the modelling of oscillators with their properties, like phase and frequency, as explained further in Subsection 1.1.1. To allow for interaction and communication between the agents, mechanisms so that the agents can transmit "fire"-signals, as well as listen for other agents's "fire"-signals, is necessary as well, and is presented in Subsection 1.1.2.

1.1 The baseline: Achieving Decentralized Harmonic Synchronization in (Oscillators $\vee$ Musical Robot Collectives)

Envision that we have a multi-agent collective scenario consisting of musical robots modelled as oscillators, solely communicating through brief "fire"-like audio-signals—greatly inspired by K. Nymoen et al.'s approach for achieving *decentralized harmonic synchronization in mobile music systems* [1]. These agents are initially not synchronized in their firing of audio-signals, but as time goes, they are entraining to synchronize to each other by adjusting their phases and frequencies when or after hearing each other's audio-signals. An illustration of this is given in Figure 1.1.

These aforementioned audio-signals to be expounded further in Subsection 1.1.2, also referred to as "fire"-signals, are transmitted whenever an agent's oscillator *peaks* (i.e. after its cycle or period is finished, having phase $\phi(t) = 1$)—or actually every second *peak*, due to the target system goal of *harmonic synchrony*. All agents have the ability to listen for such transmitted "fire"-signals from their neighbours, which they then will use as a trigger to adjust



(a) The agents firing asynchronously at first. Here, only the two Dr. Squiggles with red tentacles are firing simultaneously, but the rest are not.

(b) Seconds later, after having listened to each other's fire-event signals and adapted themselves accordingly, the agents are here firing synchronously.

Figure 1.1: Decentralized Synchronization of phases achieved in a musical robot collective, consisting of M. J. Krzyzaniak and RITMO's Dr. Squiggles.

themselves according to some well-designed update-functions to be elaborated in Subsection 1.1.4.

1.1.1 The Node: the musical robot individual

BESKR.: [Om den enkelte noden/agenten med alle egenskaper den har osv. (som f.eks. en oscillator-komponent (jf. I.A., III.Intro., og 'Implementation' i Nymoens Firefly-paper))].

Notions like "agent", "node", "firefly", and "oscillator" will be used interchangeably throughout the thesis.

1.1.2 Robot communication: the "fire"-signal

No individual agent is able to adjust or modify the state of any other agent, and the only means of communication between the agents will be through these "fire"-signals.

- Short and impulsive audio sound/signal, representing a node's phase-climax and "fire"-/"flash"-event.
- Stigmergic co-ordination/communication.
- Facilitating PCO (pulse-coupled oscillators), different from phase-coupled oscillators with their differences.

GJØR: [Sjekk oppsummeringen av Nymoen-paperet i Essayet].

1.1.3 System target state: Harmonic Synchrony

- The goal and target state of the system

- All agents/nodes having integer-relation frequencies (between each other), so that given a fundamental frequency $\omega_{i,0}$ for the agent i with the lowest frequency in the collective — all other agents have "legal" harmonic frequencies; meaning, all nodes have frequencies in the mengde $\omega_{i,0} \cdot 2^{\mathbb{N}_0}$.
- (FOR Å DEMONSTRERE *Harmonic Synchrony*) LEGG INN ETT WAVEFORM-SPEKTROGRAM PLOTT AV EN STEMME ELLER EN LYD MED HARMONICS I SEG (F.EKS. FRA PRARIEDOGS-FYREN ELLER-NOE), OG SAMMENLIKN MED DE MUSIKALSKE NODENES FREKVENS-SPEKTROGRAM VED SIDEN AV.

1.1.4 Update/Adjustment functions: Phase- & Frequency-Adjustment

When it comes to the temporality and timing of when the updating functions are used and applied:

Musical agents's phases get updated/adjusted immediately as "fire"-/"flash"-events are perceived, whereas agents's frequencies do not get updated until the end of their oscillator-cycle (i.e. when having a phase-climax $\phi(t) = 1$). This is also the reason why frequencies are updated discretely, not continuously. So-called H-values however, being "contributions" with which the frequencies are to be updated according to, are immediately calculated and accumulated when agents are perceiving a "fire"-/"flash"-event — and then finally used for frequency-adjustment/-updating at phase-climaxes.

1.1.4.1 Phase Adjustment

If we again first assume constant and equal oscillator-frequencies in our agents, we can take a look at how the agents adjust their—initially random—phases in order to synchronize to each other. This was then in contrast to the case in Subsection 1.1.4.2 where heterogenous and randomly initialized oscillator-frequencies in the musical agents are implemented and utilized.

The goal state of the agents is for now to fire/flash more or less simultaneously, after having started firing/flashing at random initially. Note that this is a different goal from the final and ultimate goal of *harmonic synchrony*.

One relatively new approach to achieve this, firstly introduced by K. Nymoen et al. [1], is now presented:

Bi-Directional Phase Shifts

This Phase-adjustment, as in Figure 1.2, works very similarly to the Phase-adjustment performed in the "standard" ***Mirollo-Strogatz*** approach presented in the Background-chapter; The only difference being that now, nodes update their phase with the slightly more complex **phase update function** (1.1) when hearing a "fire"-event from one of the other musical nodes — allowing for both larger, but also smaller, updated phases:

$$P(\phi) = \phi - \alpha \cdot sin(2\pi\phi) \cdot |sin(2\pi\phi)| \tag{1.1}$$

The fact that new and updated phases can both be larger, but also smaller, compared to the old phases, is exactly what's meant by the Phase-adjustment

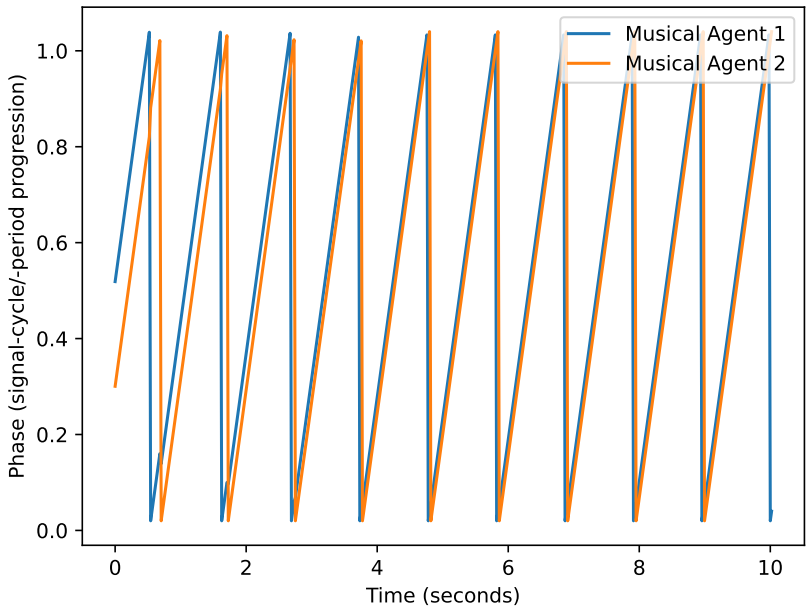


Figure 1.2: Bi-directional Phase-Adjustment with K. Nymoen et al.'s approach

being ***Bi-Directional***, or as the authors call it in the paper as using "*both excitatory and inhibitory phase couplings between oscillators*" [1].

The effects then of adjusting phases, upon hearing "fire"-events, according to this newest update-function (1.1) are that the nodes's phases now get decreased if ϕ is lower than 0.5, increased if ϕ is higher than 0.5, and neither—at least almost—if the phases are close to $0.5 = \frac{1}{2}$. This is due to the negative and positive sign of the sinewave-component in Equation (1.1), as well as the last attenuating factor in it of $|sin(2\pi\phi)| \approx |sin(2\pi\frac{1}{2})| = |sin(\pi)| = |0| = 0$.

1.1.4.2 Frequency Adjustment

GJØR: [Presenter hva Frequency-adjustment er, bygg opp fremgangsmåten på hvordan man oppnår Frequency-adjustment (som i Worklog'en, og Mattermost-chatten med Kyrre evt.) — med passende figurer og illustrasjoner].

Now over to the slightly more complex issue of synchronizing frequencies in the agents/oscillators. We introduce randomly initialized, non-constant, and heterogenous oscillator-frequencies in our musical agents — allowing in the music collective the playing of rhythmic patterns like doubles, fourths, eights (FRA NYMOEN: subdivisions of a measure, i.e. quarter notes and 8ths. "Temporal components in music tend to appear in an integer-ratio relation to each other (e.g., beats, measures, phrases, or quarter notes, 8ths, 16ths)") etc. The agents are now then required to not only synchronize their phases to each other, but now also their initially different and random frequencies — so that frequencies are "legal" and *harmonically synchronized*.

This approach to Frequency Adjustment stands in contrast to previous approaches to synchronization in oscillators cite(fixed_freqs, fixed_range_freqs) where the oscillators's frequencies are either equal and fixed, or where frequencies are bound to initialize and stay within a fixed interval/range.

The goal state now will not be exactly equal frequencies and phases, but rather the goal state of *harmonic synchrony*, as expounded in Subsection 1.1.3.

In order to achieve this goal of *harmonic synchrony* in conjunction with—or rather through—frequency adjustment, we have to go through a few steps to build a sophisticated enough update-function able to help us achieve this.

Each agent i update their frequency, on their own phase-climax (i.e. when $\phi_i(t) = 1$), according to the frequency-update function $\omega_i(t^+)$:

$$\omega_i(t^+) = \omega_i(t) \cdot 2^{F(n)}, \tag{1.2}$$

where t^+ denotes the time-step immediately after phase-climax, $\omega_i(t)$ is the old frequency of the agent at time t, and $F(n) \in [-1, 1]$ is a quantity denoting how much and in which direction a node should update its frequency after having received its nth "fire"-signal.

This is how we obtain the aforementioned $F(n)$-quantity:

Step 1: The "in/out-of synch" Error-Measurement/-Score, $\epsilon(\phi(t))$

Describing the error measurements at the n-th "fire"-event, we introduce an Error Measurement function.

The Error Measurement function (1.3), plotted in Figure 1.3, is calculated immediately by each agent i, having phase $\phi_i(t)$, when a "fire"-event signal from another agent is detected by agent i at time t.

$$\epsilon(\phi_i(t)) = sin^2(\pi\phi_i(t)) \tag{1.3}$$

As we can see from this Error-Function, the error-score is close to 0 when the agent's phase $\phi_i(t)$ is itself close to 0 or 1 (i.e. the agent either just fired/flashed, or is about to fire/flash very soon). The error-score is the largest when an agent perceives a "fire"-signal while being half-way through its own phase (i.e. having phase $\phi(t) = 0.5$). We could also then ask ourselves, does not this go against the main/target goal of the system, being *harmonic synchrony* — if agents are allowed to be "half as fast" as each other?

We could imagine a completely "legal" and harmonically synchronous scenario where two agents have half and double the frequency of each other. The agent with half the frequency of the faster agent would then have phase $\phi(t) = 0.5$ when it would hear the faster agent "fire"/"flash" — leading to its Error-score $\epsilon(0.5) = sin^2(\pi/2) = 1$, which then makes it seem like the slower agent is maximally out of synch, when it is actually perfectly and harmonically synchronized. ??? **INKL.:** [This calls out for an attenuating mechanism in our frequency update function, in order to "cancel out" this contribution so that perfectly harmonically synchronized agents will not be adjusted further despite their high Error-measurement.] **?**

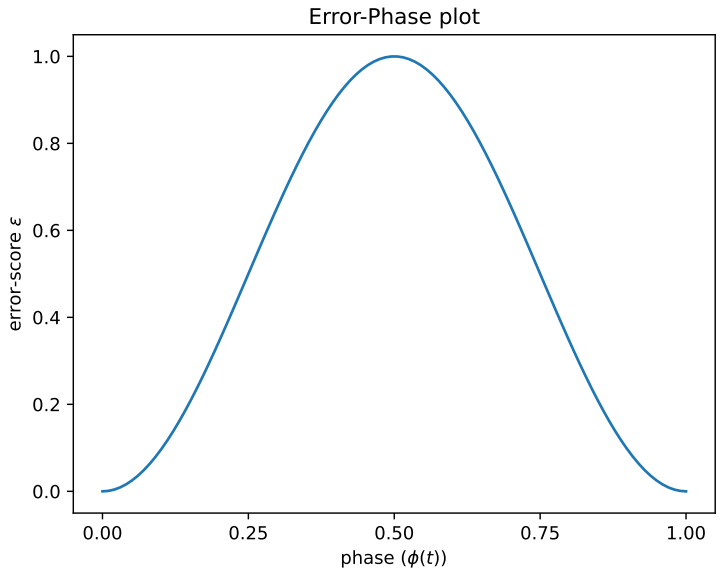


Figure 1.3: Error Measurement (1.3) plotted as a function of Phase

This Error-Measurement/-Score forms the basis and fundament for the first component of Self-awareness, being the *self-assessed synchrony-score* $s(n)$.

Step 2: The first Self-awareness component, s(n)
This aforementioned self-assessed synchrony-score, $s(n)$, is in fact simply the median of a list (was originally in K. Nymoen et al.'s approach a running median filter over the discrete error measurement function) containing m Error-scores ϵ. Such a list is easily implemented in C# by declaring a List<float> called *errorBuffer* e.g. (i.e. *errorBuffer* is a list containing floating point values):

$$errorBuffer = \{\epsilon(n), \epsilon(n-1), ..., \epsilon(n-m)\}, \tag{1.4}$$

then leading to:

$$\begin{aligned} s(n) &= median(errorBuffer) \\ &= median(\{\epsilon(n), \epsilon(n-1), ..., \epsilon(n-m)\}) \in [0,1], \end{aligned} \tag{1.5}$$

where n is the latest observed "fire-event", and m is the number of the last observed "fire"-events we would like to take into account when calculating the self-assessed synch-score.

If we then have a high $s(n)$-score, it tells us that the median of the k last error-scores is high, or in other words that we have mainly high error-scores — indicating that this agent is out of synch. Conversely, if we have a low $s(n)$-score, indicating mainly low error-scores for the agent — then we have an indication that the agent is in synch, hence leading to low error scores, and in turn low $s(n)$-scores.

In other words, we then have a way for each agent to assess themselves how much in or out of synch they believe they are compared to the rest of the agents. This is then the first degree/aspect of (public?) Self-awareness in the design.

Step 3: Frequency Update Amplitude- & Sign-factor, $\rho(n)$

Describing the amplitude and sign of the frequency-modification of the n-th "fire-event" received. It is used to say something about in which direction, and in how much, the frequency should be adjusted.

$$\rho(\phi) = -sin(2\pi\phi(t)) \in [-1, 1] \tag{1.6}$$

For example, if a node i has phase $\phi_i(t) = 1/4$, it gets a value $\rho(1/4) = -sin(\pi/2) = -1$; meaning, the node's frequency should be decreased (with the highest amplitude actually) in order to "slow down" to wait for the other nodes. Conversely, if a node j has phase $\phi_j(t) = 3/4$, it gets a value $\rho(3/4) = -sin(3/2\pi) = -(-1) = 1$; meaning, the node's frequency should be increased (with the highest amplitude) in order to getting "pushed forward" to catch up with the other nodes.

Acts as an attenuating factor, when $\phi(t) \approx 0.5$, in the making of the H-value — supporting the goal of *harmonic synchrony*.

Step 4: The H-value, and the H(n)-list

The following value, being "frequency-update-contributions", is then (as previously mentioned) calculated immediately when the agent perceives another agent's "flashing"-signal:

$$H(n) = \rho(n) \cdot s(n) \tag{1.7}$$

Here we then multiply a factor $\rho(n)$ representing how much, as well as in which direction, the node should adjust its frequency, together with a factor $s(n) \in [0, 1]$ of the adjusting node's self-assessed synch-score. To recall, the self-assessed synch-score $s(n)$ tells an adjusting node how in- or out-of-synch it was during the last m perceived "fire"-/"flash"-events — where $s(n) = 0$ signifies a mean of 0 in error-scores, and $s(n) = 1$ signifies a mean of 1 in error-scores. So then if this H-value is to be used to adjust the nodes's frequencies with, the frequency will then be adjusted in a certain direction and amount (specified by $\rho(n)$) — given that the node is *enough* "out of synch"/"unsynchronized" (in the case $s(n)$ is considerably larger than 0).

The H-value says something about how much "out of phase" the node was at the time the node's nth "flashing"-signal was perceived (and then followingly how much it should be adjusted, as well as in which direction after having been multiplied together with a sign-factor $\rho(n)$), given then that this H-value also consists of the *self-assessed synch score s(n)* — which again simply was the median of the list of error-scores, *errorBuffer* in our case.

We could look at this H-value as representing the direction and amplitude of the frequency adjustment weighted by the need to adjust (due to being out of synch) at the time of hearing "fire"-/"flash"-event n. Or in other words, this H-value is then the n-th contribution with which we want to adjust our frequency with.

Especially interesting cases are when we have $\phi(n) \approx 0.5 \implies \rho(n) \approx \pm 0$, as well as the last m Error-scores $\epsilon(n)$ being close to 0, also leading to $s(n) \approx 0$. In both of these two cases the entire frequency-adjustment contribution H would be cancelled out, due to harmonic synchronization (legally hearing a "fire"-/"flash"-event half-way through ones own phase) in the first case, and due to not being out of synch in the latter (having low Error-Measurements). Cancelling out the frequency adjustment contribution in these cases is then not something bad, but something wanted and something that makes sense. If these H-values then are cancelled out or very small, it is indicative of that nodes are already in *harmonic synchrony*, and hence should not be "adjusted away" from this goal state. On the other side, if these H-values then are different (e.g. closer to -1 and 1), it is indicative of that nodes are not yet in *harmonic synchrony*, and that they hence should be "adjusted closer" to the goal state.

All the calculated H-values are in my implementation accumulated and stored in an initially empty C#-list (of floats), referred to as $H(n)$, at once they are calculated. The $H(n)$-list is then consecutively "cleared out" or "flushed" when its H-values have been used for the current cycle/period's frequency adjustment (i.e. at the phase climax, when $\phi(t) = 1$), and is then ready to accumulate new H-values during the next cycle/period.

The final step: The Frequency Update function

Putting it all together.

When an agent i then has a phase-climax ($\phi_i(t) = 1$), it updates its frequency to the new $\omega_i(t^+)$ accordingly:

$$\omega_i(t^+) = \omega_i(t) \cdot 2^{F(n)}, \tag{1.8}$$

where t^+ denotes the time-step immediately after phase-climax, and $F(n)$ is found by:

$$F(n) = \beta \sum_{x=0}^{k-1} \frac{H(n-k)}{k}, \tag{1.9}$$

where β is the frequency coupling constant, k is the number of heard/received "fire-event"s from the start of the last cycle/period to the end (i.e. the phase-climax, or *now*) — and the rest of the values are as described above.

This $F(n)$-value then, as we see in Equation (1.9), is a weighted average of all the node's $H(n)$-values accumulated throughout the node's last cycle.

1.2 My new proposed algorithm: an additional Self-Awareness component