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A New Concentric Circle Detection Method Based on Hough Transform

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***Abstract*—A new approach of concentric circle detection is proposed in this paper. Firstly, the image is preprocessed by denosing, edge detection, and then the circle centers are allocated by the gradient Hough transform, at last, the radius are detected by the improved one-dimensional Hough transform. The detection efficiency is enhanced by image discretization and reduced resolution ratio in the process of circle center detection, and proves that the circle center is on the gradient line of circle edge points; meanwhile, the radius detection accuracy is improved by merging the similar radius in the process of radius detection. Experimental results show that the method combined with gradient Hough transform and one-dimensional Hough transform has good reliability and is high adaptive to noise, distortion, area of incomplete, edge discontinuous. Analysis shows that this new concentric circle detection algorithm reduces time complexity and improves anti-interference compared with the traditional concentric circle detection algorithm based on chord mid-point Hough transforms.**

***Index Terms*—gradient Hough transform, concentric circle, one-dimensional Hough transform, resolution ratio, similar radius**

I. INTRODUCTION

Different types of objects have different characteristics, such as form characteristics, texture characteristics, color characteristics and so on, we usually detect the object by detecting the object characteristics in machine vision field. The object including concentric circle characteristics exist widely in our daily lives and industrial production, such as iris of eye automatic detection in face recognition, the inhibition halos of antibacterial activity detection [1] in food industry, the camera calibration in optical study [2], concentric circles precise identify of printed circuit board (PCB) round hole photoelectric image in industrial production [3], concentric circles ring recognition of targeting system [4] and PET bottle of on-line inspection in blowing machines and beverage packaging industry and so on, are all need to use concentric circles detection technology, therefore the research of the concentric circles detection algorithm for machine vision development has a certain research significance.

At present, the point, line and circle detection algorithm research are widely studied, the most common algorithm is Hough transform algorithm [5], but as so far, the specific case of concentric circle detection has received little attention [6] [7]. Several scholars made some study of concentric circles detection. Referring from the collected documents, most algorithms apply the characteristic points on a circumference to detect concentric circles, such as in [7] and [8], firstly, three different points in the same circumference are found; then the parameters of circle general equation are obtained according to the three points' coordinates to get all the circles of the concentric circle; finally, the concentric circle can be located. As in [5], different circle area of the concentric circle are segmented according to the regional division method, then go to get the circle center and radius of these circles according to the points coordinates on a circle in different areas. The position of the circle center needs to be predicted in this algorithm, then get every circle by least square fitting method. As in [6], two characteristic points are fond in a circle, then link these two points to obtain the circle chord, and get the circle center according to the geometrical characteristic that every circle whose diameter is the chord above must intersect at the circle center. After that, accumulate every distance which is from each characteristic point to the circle center, the accumulation value which is larger than a set threshold is the right corresponding radius. The premise condition is to find out the two points in a same circle to get a diameter in this algorithm, but it's not described in this document. Therefore, appropriate constraint conditions need to be set for the algorithm based on feature points, and it has certain limitation. The algorithms based on feature points have bad anti-interference and high demands of preprocessing when there exists noise, distortion or edge discontinuous, which would increase the time complexity of the algorithms.

Circle center is got by accumulating the intersection of the gradient lines of every feature point in the algorithm of

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gradient Hough transform. Because every point' gradient line must go through the circle center, the circle center would get the maximum accumulation value. Even if there exists much noise points, gradient Hough transform is little influenced by the noise. What's more, the one-dimensional Hough transform can detect the entire radius at one time, so it has the advantage of high efficiency and precision.

As described above, a new concentric circle detection algorithm is proposed by combining with the gradient Hough transform and one-dimensional transform [9]. What's more, the radius detection method is improved to enhance the detection accuracy. The concentric circle detection algorithm based on chord mid-point is a typical concentric circle detection method based on feature points. Compare with this algorithm, the algorithm in this paper does not need to set constraint conditions, which reduces the detection limitation on the one hand, and improves the detection efficiency on the other hand. Especially when there exists much concentric circles in an image, the advantage of this algorithm is more obvious. The experimental result shows the high efficiency and accuracy of this new algorithm.

II. Principle of Circle Detection Hough Transform

Hough transform is the earliest put forward by Paul Hough in 1962 [10], and the image is detected by mapping from image space to parameter space, the difficult global problem in image apace is transformed to a simple local peak detection problem with the method of voting and statistics [11]. Hough transform is the most widely used circle detection method at present. Because that traditional Hough transform need three-dimensional voting for the three parameters (circle center and radius) in the parameter space, a lot of storage space and time consumption are necessary [12]. The principle of the traditional Hough transform is as follow.

The general equation of circle is as follow:

$$(x-a)^2 + (y-b)^2 = r^2 \qquad (1)$$

where (a, b) is the circle center, and r is the radius.

In image space, the feature point (x, y) is treated as unknown quantity, (a, b) is treated as known quantity. The computer can only record the coordinates of the feature points in the image by scanning, but it couldn't get the value of a and b.

After switching the circle from *x-y* plane to *a-b-r* parameter space, the circle equation in parameter space is as in (2):

$$(a-x)^2 + (b-y)^2 = r^2 \qquad (2)$$

In the parameter space, the detected coordinates of (x, y) is known quantity, but the parameter (a, b) and r change into unknown quantity, thus every feature point (x, y) can constitute of a cone in *a-b-r* parameter space. Each cone which is corresponding to every feature point in a circle in the image space must intersect at one point. Different cones have much different intersections, but all the cones must intersect at one point, this point which has the most intersection time is the right circle center, we can detect the circle parameter by detecting this point, and conclude that the coordinates of (a, b, r) is the circle center and radius. In the process of the Hough transform algorithm, feature points are mapped to parameter space firstly, then discrete the parameters space, a three-dimensional accumulator (a, b, r) is used to storage all the points in the parameter space, when there exists repetitive intersections in a same position, the accumulator add one at this position. Set a threshold, when the accumulator value at a position surpasses the threshold, this point at this position is regarded as the circle center. The circle in image space map into the parameter space is as shown in Fig. 1.

The traditional Hough transform use three-dimensional accumulator to storage the circle parameters, it need large storage that it's impossible to realize in the practical application.

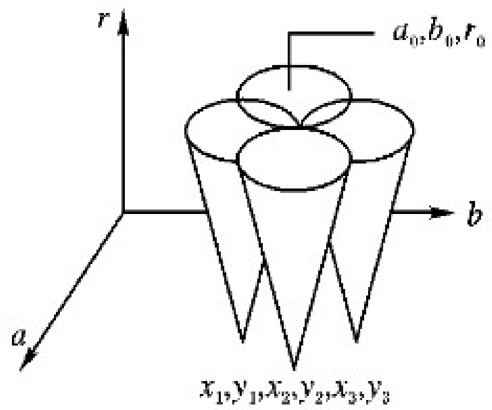


Figure 1. Circle in the parameter space.

III. Concentric Circle detection Method Based on Gradient Hough Transform

A. The Definitetion of Gradient

The definition of Gradient of *2-D* function $f(x, y)$ can be expressed as [14]:

$$\nabla f = [G_x, G_y]^T = [\partial f/\partial x, \partial f/\partial y] \qquad (3)$$

And the amplitude is expressed as:

$$\nabla f = mag(\nabla f) = [G_x + G_y]^{1/2} \qquad (4)$$

For simplicity, $\nabla f \approx |G_x| + |G_y|$ sometimes can be approximately calculated by omitting the square item as following:

$$\nabla f \approx |G_x| + |G_y| \qquad (5)$$

Gradient vector means that is the biggest change rate direction at the point (x, y), and then, the angle can be worked out as:

$$\partial(x, y) = \arctan(G_x/G_y) \qquad (6)$$

B. The Proof of the Direction of Gradient Passing through the Circle Center

The direction of gradient of the edge point of the circle is accordant with its normal direction; the gradient line is point to the center. The specific proving process was given below.

Proving: the equation of the circle can be expressed as:

$$(x-a)^2+(y-b)^2=r^2 \qquad (7)$$

where (x, y) is a point on the circle and (a, b) is the center coordinates, and r repents the radius .

Thus the round curve equation can be expressed as:

$$f(x,y)=(x-a)^2+(y-b)^2-r^2 \qquad (8)$$

Then the derivative of x, y can be figured out as following:

$$f_x^{'}=2(x-a) \qquad (9)$$

$$f_x^{'}=2(x-a) \qquad (10)$$

Suppose that (x_0 , y_0) is the arbitrary point on the circle, then the vector of gradient can be calculated as [(x_0 - a)*,* (y_0 - b)], meanwhile, the vector of attachment line between (a, b) and (x_0, y_0) can be expressed as [(x_0 – a), (y_0 – b)] with the normal direction , thus we can conclude that direction of gradient must pass through the center of the circle.

C. Concentric Circle Center Detection

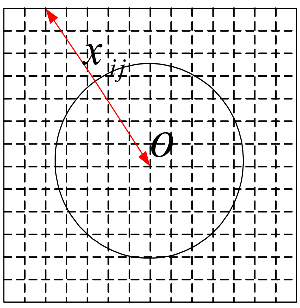


Figure 2. Schematic plot of solving the center by gradient hough transform method.

As it can be seen in Fig.2, x_{ij} is the characteristic point in the image, o represents the circle center, when x_{ij} was detected, calculate the gradient and angle of the gradient. At the same time, go on searching the characteristic point along the direction of the gradient until reach the boundary of the image, add the characteristics points into the accumulator register, because the gradient line of each edge point all go through the center, thus the maximum accumulated value in the accumulator register is the center.

IV. Radius Detection Method Besed On The Improved 1-D Hough Transform

A. Using the method based on 1-D Hough Transform to Detect the Radius

The *1-D* Hough transform method means to use a accumulated register to store the radius, the value added 1 until the radius value reaches to the given threshold, then the radius are regarded as the radius corresponding to the center, and all the corresponding radius can be seek out.

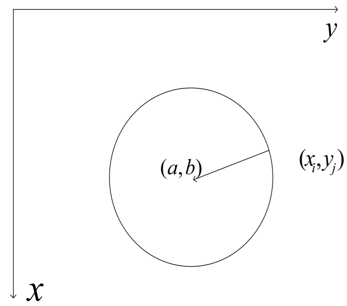


Figure 3. Schematic plot of the circle.

As is shown in Fig.3, denote the coordinate of the ith characteristic point as (x_{i}, x_{j}), (*a, b*) is the detected circle center, $D((x_{\text{i}}, x_{\text{j}}), (a, b))$ is the distance between the above two. A critical determines function is as follow:

$$F((a,b),r)=\begin{cases}1, & D((x_i,x_j),(a,b))=1\\ 0, & others\end{cases} \qquad (11)$$

Thus the sum function of the radius can be expressed as：

$$Sum((a,b),r)=\sum_{i=0}^{k}F((a_i,b_i),r) \qquad (12)$$

where k is the total number of the characteristic points whose distance to the center is r.

Set a threshold ε, and determine that at the point (a, b) there exists a circle with the radius r when the characteristic points in the binary image meets the condition that:

$$Sum((a,b),r)\geq\varepsilon \qquad (13)$$

B. Improved 1-D Hough Transform

Before circle detecting we'd like to deal with the image by reducing the image resolution ratio and discretize the image to be more effective and to reduce the time complexity. Considering without impacting the detection accuracy, the resolution ratio can be set to 2.

Use the one-dimension Hough transform to detect the radius after center detection by gradient transform. However, it is easily to result in wrong detection when there are a lot of noise points in the image, and there may appears much distance from the circle centre larger than the given threshold ε_1 that the number of require satisfied radius would be lager than the number of concentric circles. In order to improve the detection accuracy, in this paper, we merge such similar radius which is lager than ε_1, it means that when the D-value of these detected radius are smaller than a given threshold ε_2, these radius are similar radius and they are merged together. Then work out the average value of the similar radius, at the same time, add their accumulated value as the new radius accumulated value.

Set the radius whose accumulated value is larger than the given threshold ε_1 as r_1, r_2, …….. r_n, while the corresponding accumulated value was noted as t_1, t_2…….. t_n, such that:

$$|r_i - r_j| < \varepsilon_2, i \neq j \tag{14}$$

where ε_2 is a given threshold.

Then calculate the average value after remerging the two radius as:

$$r_a = (r_i + r_j)/2 \tag{15}$$

where r_a is the novel radius with its accumulated value:

$$t_a = t_i + t_j \tag{16}$$

Similar radius is merged together in this algorithm, and improves the radius detection efficiency.

V. STEP OF THE ALGORITHM

To sum up, the implementations of the algorithm can be described as follows:

Step1: Reduce the image resolution ratio to improve the detection efficiency

Step2: Gray the image and smooth with Gauss filter

Step3: Deal the preprocessed image with canny edge detection to change the image into binary edge image

Step4: Gradient Hough transform for the edge detected image and record the center of the circle detected.

Step5: Calculated the distance between the characteristic points and the corresponding center separately, record the radius whose accumulated value is larger than the given threshold ε_1.

Step6: Merge the similar radius corresponding to the different centers as the novel radius, and set the sum of accumulated value of the similar radius as the accumulated value of the novel radius.

Step7: Calculate the number of the concentric circles, the number of the concentric circle is equal with the number of the radius, it explains that there exists concentric circle when the number is larger than 1.

Step8: Draw all the concentric circles according to the center and its corresponding radius.

VI. ANALYSIS OF THE ALGORITHM AND THE SIMULATION RESULTS

A. Time Complexity Analysis of the Algorithm

Assuming that there are n characteristic points in the image after image preprocessing, thus:

Every gradient of the feature points need to be calculated when getting the circle center, so the center accumulated complexity function can be expressed as:

$$f_1(n) = n \tag{17}$$

Each feature point need to be scanned and the distance between the point to the circle center need to be calculated when we get the radius corresponding to each center, so the radius accumulated complexity function can be expressed as:

$$f_2(n) = n \tag{18}$$

The total time complexity of the algorithm can be calculated as:

$$f(n) = 2n \rightarrow o(n) \tag{19}$$

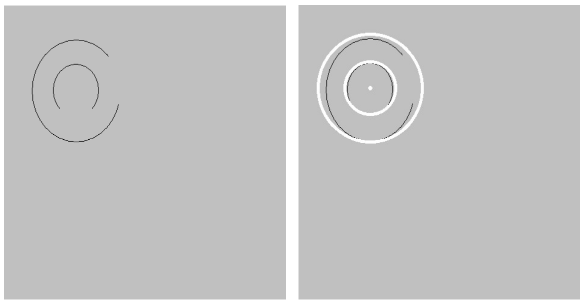
From the described above, the time complexity of the algorithm proposed in the paper is very low, which is superior to the time complexity of the algorithm in document [6] based on the chord midpoint Hough transform method whose time complexity is $o(n^2)$.

B. Anti-interference Performance Analysis of the Algorithm

In document [5], the circle detection algorithm based on chord mid-point Hough transform acquire to get the points on a same circle, thus how to get the points in a circle become key point of the problem, especially when the circles are intersected or discontinuous, the algorithm proposed in this paper is more effective.

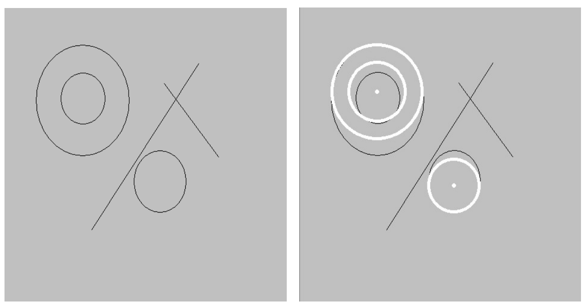
The simulation experiments perform in the 2.6 GHz PC with VS 2008 software platform to complete with C++ as the programming language and OpenCV-2.1 to deal with the images.

Deal with the 3 images of 468×473 and a image of 867×522 for detection simulation, and the results can be shown in Fig. 4 to Fig. 7.



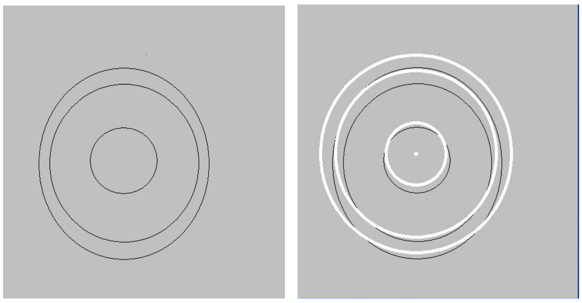
(a) original image (b) detection result

Figure 4. Detect the concentric circle with gap.



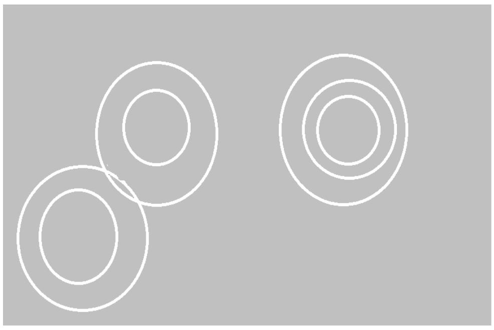
(a) original image (b) detection result

Figure 5. Detect the concentric circle with certain disturbance.

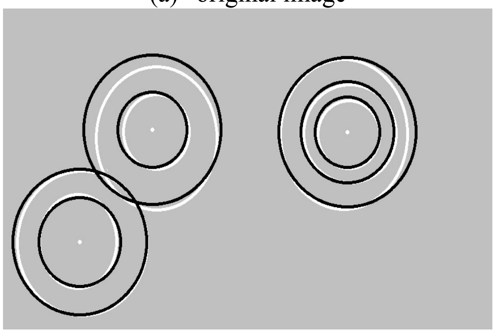


(a) original image (b) detection result

Figure 6. Detect a concentric circle with three circles.



(a) original image



（b）detection result

Figure 7. Detect the concentric circle with intersection.

Dealing results of the three images were shown in Table I for Fig. 4 to Fig. 7.

Table I. DETECTION RESULTS WITH THE ALGORITHM PROPOSED

Testing Pictures	**Concentric Circles Centers**	**Concentric Circles Radius/mm**	**Accumulated Valve of the Radius**	**Process Time/ms**
figure 4	(118,134)	77	500	10.311
		38	344	
figure 5	(128,136)	75	510	11.205
		47	427	
figure 6	(198,240)	159	1643	13.757
		134	1425	
		50	612	
figure 7	(264,196)	121	1517	20.112
		61	780	
figure 7	(104,374)	118	1581	20.112
		75	947	
	(604,204)	112	1530	
		76	1056	
		54	716	

The results show that the algorithm has good anti-interference and high efficiency. The analysis shows that the algorithm proposed in this paper outperforms the algorithm based on the chord mid-point though transform in time complexity and anti-interference, especially when there exists many concentric circles.

VII. CONCLUSION

Referring to the achievements in the former, put forward a new concentric circle detection algorithm based on gradient Hough transform and one-dimensional Hough transform, the anylysis shows that the altorithm has lower time complexity and smaller limitation than the concentric circle detection algorithm based on chord mid-point. In the process of circle centre detection by gradietn Hough transform, detection time is decreased by reducing the image resolution ratio, what's more, the theoem that circle center is on the gradient line of every circle edge point is proved. The one-dimensional Hough transform is improved in the process of radius detection, similar radius are merged to enhance the detection efficiency. Experimental result shows that the new proposed algorithm can detect the concentric circles which are intersected, discontinuous or have certain interference, and the algorithm has high detection efficiency so that it has certain practical value.

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